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Participation, Recruitment Selection, and the Minimum Wage: Dynamics and Directed Search

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Abstract

This note proves that results extend to a dynamic setting. It also shows that *laissez-faire* is efficient when wages are derived from directed search in lieu of Nash bargaining.

Key words : Search and matching, heterogeneous workers, applicant ranking, public policies, efficiency.

JEL Classification numbers: D8, J6.

1 A dynamic model

This appendix provides an extension of the main points of the analysis to a dynamic setting¹. We first show that participation remains (constrained) insufficient. Next we prove that introducing a minimum wage still raises participation and job creation. Consequently, a small minimum wage improves market efficiency. For the sake of simplicity, time is discrete². It is also assumed that the destruction probability of jobs, s , does not depend on workers' productivities. For the sake of expositional

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¹Unless otherwise stated, the notation used is the same as in text.

²Following Moen (1999) and Gavrel (2012), switching to continuous time is straightforward.

simplicity, the welfare analysis is restricted to the case in which the time preference rate, r , is very close to zero. See Hosios (1990).

From the usual Bellman relations, we deduce the equilibrium equation for job creation

$$-c + \int_{y^*}^1 q(y) \frac{y - z + \gamma}{r + s + \beta p(y)} dy = 0 \quad (1)$$

with c being the cost at keeping a vacancy open.

Notice that in this equation, the term $S(y) = \frac{y - z + \gamma}{r + s + \beta p(y)}$, which represents the surplus of a match with a worker of productivity y , should grow with y . Ranking workers by ability would not otherwise make sense. This consistency requirement imposes that

$$1 - \beta q(y) S(y) > 0$$

This existence condition is assumed to be satisfied henceforth. It comes from the fact that the ranking of applicants by ability is ordinal. See Moscarini (1997).

With respect to participation, the corresponding equation now is

$$\gamma = \beta p(y^*) S(y^*) \quad (2)$$

In this dynamic setting, the distribution of productivities, $F(y)$, among the total population remains exogenous, but the distribution among unemployed workers becomes endogenous. It is denoted by $G(y)$ and defined on $[y^*, 1]$. Its density is $g(y)$. Below, we study how this endogenous distribution is affected by an increase in job creation or in participation.

1.1 Preliminary results

1.1.1 Flow equilibrium and hiring probabilities

Let u denote the number of unemployed workers (searching for a job). The market tightness, θ , is then determined by the ratio v/u . For almost all $z > y^*$, flow equilibrium imposes the condition that

$$s[f(z)n - g(z)u] = p(z)g(z)u$$

Integrating the previous equation on $[y^*, 1]$ gives

$$s[(1 - F(y^*))n - u] = P(\theta)u$$

with $P(\theta) = \int_{y^*}^1 p(y)g(y)dy = \theta(1 - e^{(-1/\theta)})$ being the average probability of an unemployed worker finding a job.

Combining the two latter equations yields

$$sg(z) + \theta q(z) = \frac{s + P(\theta)}{1 - F(y^*)}f(z)$$

By integrating this equation on $[y, 1]$, we obtain

$$s[1 - G(y)] + \theta[1 - p(y)] = \frac{s + P(\theta)}{1 - F(y^*)}[1 - F(y)] \quad (3)$$

As

$$\ln p(y) = -\frac{1 - G(y)}{\theta}$$

equation (3) can be rewritten as

$$p(y) + s \ln p(y) = 1 - \frac{s + P(\theta)}{\theta} \frac{1 - F(y)}{1 - F(y^*)} \quad (4)$$

As $\theta = v/u$ and

$$u = \frac{s(1 - F(y^*))n}{s + P(\theta)}$$

we also have

$$p(y) + s \ln p(y) = 1 - \frac{s(1 - F(y))n}{v} \quad (5)$$

According to the above equation, the probabilities $p(y)$ still grow with a job creation increase when taking into account the incidence of market tightness on the repartition

function $G(y)$. Likewise, they remain independent of the cutoff y^* . Consequently their derivatives (with respect to y), $q(y) = p(y)\frac{g(y)}{\theta}$, are also independent of y^* .

1.1.2 Employment shares and market tightness

In the following, we will need to know how the employment shares $\rho(y)$ are affected by θ . To that end we make use of (4). We have

$$\rho(y) = \frac{q(y)}{Q(\theta)} = \frac{p(y)g(y)}{P(\theta)}$$

From

$$g(y) = \frac{1}{1 - F(y^*)} \frac{s + P(\theta)}{s + p(y)} f(y)$$

we deduce

$$\rho(y) = \frac{1}{1 - F(y^*)} \frac{p(y)(s + P(\theta))}{(s + p(y))P(\theta)} f(y)$$

From paragraph 1.1.1, we obtain that the derivative of $\rho(y)$ with respect to θ has the same sign as

$$\Gamma = \frac{1}{p(y)(s + p(y))} \frac{\partial p(y)}{\partial \theta} - \frac{P'(\theta)}{P(s + P)}$$

Differentiating (4) gives

$$\frac{\partial p(y)}{\partial \theta} = \frac{p(y)}{s + p(y)} \left[\frac{s}{\theta^2} - Q'(\theta) \right] \frac{1 - F(y)}{1 - F(y^*)}$$

Substitution into the expression Γ yields

$$\Gamma = \frac{1 - F(y)}{(s + p(y))^2} \left[\frac{s}{\theta^2} - Q'(\theta) \right] \frac{1}{1 - F(y^*)} - \frac{P'(\theta)}{P(s + P)}$$

This shows that $\frac{\partial \rho(y)}{\partial \theta}$ is a strictly decreasing function in y .

As

$$\int_{y^*}^1 \rho(y) dy = 1$$

it results by continuity that there exists some median output \tilde{y} ($1 > \tilde{y} > y^*$) such that

$$\frac{\partial \rho(\tilde{y})}{\partial \theta} = 0$$

and

$$y < (>) \tilde{y} \iff \frac{\partial \rho(y)}{\partial \theta} > (<) 0.$$

This preliminary result is used to show that, for any (strictly) increasing function $\varphi(y)$ defined on $[y^, 1]$,*

$$\int_{y^*}^1 \frac{\partial \rho(y)}{\partial \theta} \varphi(y) dy < 0$$

The proof is a straightforward extension of Appendix A. Notice that setting $\varphi(y) = y$ shows that an increase in market tightness still reduces the average output, meaning then that the externality which was highlighted in the static model (Proposition 1b) extends to the dynamic setting.

On the other hand, because a market tightness increase lowers $S(y)$, setting $\varphi(y) = S(y)$ shows that

$$\int_{y^*}^1 \rho(y) S(y) dy$$

is a decreasing function of θ .

1.1.3 Job creation

Here we can study the derivatives of

$$H \equiv \int_{y^*}^1 q(y)S(y)dy = Q(\theta) \int_{y^*}^1 \rho(y)S(y)dy$$

with respect to θ (hence to v) as well as to y^* .

(i) We first show that the derivative $\frac{\partial H}{\partial \theta}$, which is negative, is bounded above.

Integrating H by parts gives

$$H = S(1) - p(y^*)S(y^*) - \int_{y^*}^1 \frac{p(y)}{r+s+\beta p(y)} [1 - \beta q(y)S(y)] dy$$

$-p(y^*)S(y^*)$ is a decreasing function of θ . On the other hand, developing the derivative of the integral yields

$$- \int_{y^*}^1 \frac{\partial \frac{p(y)}{r+s+\beta p(y)}}{\partial \theta} [1 - \beta q(y)S(y)] dy + \beta \int_{y^*}^1 \frac{p(y)}{r+s+\beta p(y)} \frac{\partial(q(y)S(y))}{\partial \theta} dy$$

Due to the existence condition, we have

$$- \int_{y^*}^1 \frac{\partial \frac{p(y)}{r+s+\beta p(y)}}{\partial \theta} [1 - \beta q(y)S(y)] dy < 0$$

The last term of the derivative can be rewritten as

$$\begin{aligned} \beta \int_{y^*}^1 \frac{p(y)}{r+s+\beta p(y)} \frac{\partial(q(y)S(y))}{\partial \theta} dy &= \beta \int_{y^*}^1 \frac{p(y)}{r+s+\beta p(y)} q(y) \frac{\partial S(y)}{\partial \theta} dy \\ + \beta \int_{y^*}^1 \frac{p(y)}{r+s+\beta p(y)} S(y) Q'(\theta) \rho(y) dy &+ \beta \int_{y^*}^1 Q \frac{p(y)}{r+s+\beta p(y)} S(y) \frac{\partial \rho(y)}{\partial \theta} dy \end{aligned}$$

As $\frac{\partial S(y)}{\partial \theta} < 0$, the first term of the right hand side (of the previous equation) is strictly negative. From $Q'(\theta) < 0$, we deduce that the same holds for the second

term. Consider the expression $\frac{p(y)}{r+s+\beta p(y)}S(y)$. This is a strictly increasing function of y . So, by setting $\varphi(y) = \frac{p(y)}{r+s+\beta p(y)}S(y)$, we obtain that the third term is also (strictly) negative. See paragraph 1.1.2.

This shows that

$$\frac{\partial H(\theta, y^*)}{\partial \theta} < -\frac{\partial p(y^*)S(y^*)}{\partial \theta} < 0 \quad (6)$$

Differentiating θ with respect to v gives

$$\frac{\partial \theta}{\partial v} = \frac{1}{u} - \frac{v}{u^2} \frac{\partial u}{\partial v}$$

From³

$$\frac{\partial u}{\partial v} = -\frac{u}{s+P} Q(1-\eta) \frac{\partial \theta}{\partial v}$$

we deduce

$$\frac{\partial \theta}{\partial v} = \frac{s+P}{(s+\eta P)u} > 0$$

Thus, we also obtain

$$\frac{\partial H(v, y^*)}{\partial v} < -\frac{\partial p(y^*)S(y^*)}{\partial v} < 0 \quad (7)$$

(ii) We now turn to the derivative of H with respect to y^* . As $q(y)S(y)$ does not depend on y^* for all y in $]y^*, 1]$ (see equation (5)), *this derivative reduces to*

$$\frac{\partial H(v, y^*)}{\partial y^*} = -q(y^*)S(y^*) < 0 \quad (8)$$

So, according to the job creation equation (1), an increase in participation (a decrease in y^) stimulates job creation: $v'(y^*) < 0$.*

³Remember that $\eta(\theta)$ is the elasticity of $Q(\theta)$ with respect to θ in absolute value.

1.2 Constrained (in)efficiency of participation

Let $l(y)$ and $u(y)$ denote the employment and unemployment levels of y -workers respectively. The social surplus per period is defined as follows⁴

$$\Sigma = \int_{y^*}^1 l(y)ydy + \int_{y^*}^1 u(y)(z - \gamma)dy + F(y^*)nz - vc$$

Let l be the total employment level. As $l(y) = \frac{q(y)l}{Q}$ and $sl = Qv$, the job creation equation (1) can be written as

$$-vc + \int_{y^*}^1 l(y)[y - w(y)]dy = 0$$

with $w(y)$ being the wage of y -workers.

Σ is also equal to

$$\Sigma = -vc + \int_{y^*}^1 l(y)[y - w(y)]dy + \int_{y^*}^1 l(y)w(y)dy + \int_{y^*}^1 u(y)(z - \gamma)dy + F(y^*)nz$$

As equation (1) is treated as a constraint, the social surplus is reduced to

$$\Sigma_{cp} = \int_{y^*}^1 l(y)w(y)dy + \int_{y^*}^1 [f(y)n - l(y)](z - \gamma)dy + F(y^*)nz$$

Or

$$\Sigma_{cp} = \int_{y^*}^1 l(y)[w(y) - z + \gamma]dy - \gamma[1 - F(y^*)]n + nz$$

As usual, wages satisfy

$$w(y) = rU(y) + \beta(y - rU(y))$$

with $U(y)$ being the asset value of unemployment:

⁴In this efficiency study, the interest rate is close to zero.

$$rU(y) = z - \gamma + p(y)\beta S(y)$$

Σ_{cp} is a function of the form

$$\Sigma_{cp} = \Sigma_{cp}(v(y^*), y^*)$$

with $v(y^*)$ being the implicit (decreasing) function derived from equation (1).

We first show that the direct derivative $\frac{\partial \Sigma_{cp}(v, y^*)}{\partial y^*}$ is reduced to zero in a *laissez-faire* equilibrium. Noting that

$$l(y) = \frac{p(y)}{s + p(y)} f(y)n$$

one can see that this derivative is proportional to

$$\gamma - \frac{p(y^*)}{s + p(y^*)} [w(y^*) - z + \gamma]$$

Because

$$w(y^*) - z + \gamma = rU(y^*) - z + \gamma + \beta(y^* - rU(y^*)) = \beta(s + p(y^*))S(y^*)$$

we find that this direct derivative is zero in a *laissez-faire* equilibrium (equation (2)).

Let us now consider the derivative of Σ_{cp} with respect to v . As $p(y)$ grows with v , it results that $l(y)$, $rU(y)$ and, $w(y)$ are increasing functions of v .

From paragraph 1.1.3, we know that $v'(y^*) < 0$. Consequently, we obtain

$$\frac{\partial \Sigma_{cp}(v(y^*), y^*)}{\partial y^*} = \frac{\partial \Sigma_{cp}(v(y^*), y^*)}{\partial v(y^*)} v'(y^*) < 0$$

It results that participation is constrained suboptimal.

1.3 Introducing a minimum wage

In the presence of a binding minimum, the job creation equation is rewritten as⁵

$$-c + \int_{y^*}^{\hat{y}} q(y) \frac{y - \hat{w}}{r + s} dy + (1 - \beta) \int_{\hat{y}}^1 q(y) S(y) dy \quad (9)$$

while the indifference condition becomes

$$\gamma = p(y^*) \frac{\hat{w} - z + \gamma}{r + s + p(y^*)} \quad (10)$$

1.3.1 Impacts on participation and job creation

We claim that a small minimum wage stimulates job creation and participation⁶. To that end, let J denote the following quantity

$$J = \int_{y^*}^{\hat{y}} q(y) \frac{y - \hat{w}}{r + s} dy + (1 - \beta) \int_{\hat{y}}^1 q(y) S(y) dy$$

As in the static study, we examine the implications of introducing a binding minimum in the neighborhood of an equilibrium where all wages are bargained. As a consequence, we obtain (see paragraph 1.1.3)

$$\frac{\partial J}{\partial \hat{w}} = 0$$

$$\frac{\partial J}{\partial v} < -\frac{y^* - \hat{w}}{r + s} \frac{\partial p(y^*)}{\partial v} < 0$$

$$\frac{\partial J}{\partial y^*} = -\frac{y^* - \hat{w}}{r + s} q(y^*) < 0$$

We also have

⁵Remember that \hat{y} is the productivity level such that the bargained wage coincides with the minimum wage, \hat{w} .

⁶This result holds for any positive interest rate.

$$\frac{\partial J}{\partial \hat{y}} = 0$$

Differentiating (10) gives

$$\frac{(r+s)(\hat{w}-z+\gamma)}{[r+s+p(y^*)]^2} \left[\frac{\partial p(y^*)}{\partial v} \frac{\partial v}{\partial \hat{w}} + q(y^*) \frac{\partial y^*}{\partial \hat{w}} \right] + \frac{p(y^*)}{r+s+p(y^*)} = 0$$

This shows that

$$\frac{\partial p(y^*)}{\partial v} \frac{\partial v}{\partial \hat{w}} + q(y^*) \frac{\partial y^*}{\partial \hat{w}} < 0 \quad (11)$$

On the other hand, differentiating (9) yields

$$-\frac{\partial J}{\partial v} \frac{\partial v}{\partial \hat{w}} = \frac{\partial J}{\partial y^*} \frac{\partial y^*}{\partial \hat{w}}$$

Suppose by contradiction that $\frac{\partial y^*}{\partial \hat{w}} > 0$. As $\frac{\partial J}{\partial v} < 0$ and $\frac{\partial J}{\partial y^*} < 0$, we should have

$\frac{\partial v}{\partial \hat{w}} < 0$. This implies

$$-\frac{\partial J}{\partial v} \frac{\partial v}{\partial \hat{w}} < \frac{y^* - \hat{w}}{r+s} \frac{\partial p(y^*)}{\partial v} \frac{\partial v}{\partial \hat{w}}$$

We then obtain

$$\frac{y^* - \hat{w}}{r+s} \left[\frac{\partial p(y^*)}{\partial v} \frac{\partial v}{\partial \hat{w}} + q(y^*) \frac{\partial y^*}{\partial \hat{w}} \right] > 0$$

This inequality contradicts (11).

It results that $\frac{\partial y^}{\partial \hat{w}} < 0$ and $\frac{\partial v}{\partial \hat{w}} > 0$. A small minimum wage stimulates job creation and participation.*

1.3.2 Impact on welfare

We claim that introducing a small minimum wage improves market efficiency⁷. This result stems from the constrained inefficiency of participation.

As equation (9) is satisfied, the social surplus per period reduces to

$$\Sigma = \int_{y^*}^{\hat{y}} l(y)\hat{w}dy + \int_{\hat{y}}^1 l(y)w(y)dy + \int_{y^*}^1 u(y)(-\gamma + z)dy + F(y^*)nz$$

Notice that in the previous equation, $w(y)$ is the bargained wage (defined as in the constrained efficiency study, subsection 1.2) for $y \geq \hat{y}$.

Σ can be rewritten as follows

$$\Sigma = \int_{y^*}^{\hat{y}} l(y)(\hat{w} - z + \gamma)dy + \int_{\hat{y}}^1 l(y)(w(y) - z + \gamma)dy + (1 - F(y^*))n(-\gamma + z) + F(y^*)nz$$

Let us consider the derivatives of $\Sigma(\hat{w}, \hat{y}, v, y^*)$. As \hat{y} initially coincides with y^* , we have

$$\frac{\partial \Sigma(\hat{w}, \hat{y}, v, y^*)}{\partial \hat{w}} = 0$$

Besides, we always have

$$\frac{\partial \Sigma(\hat{w}, \hat{y}, v, y^*)}{\partial \hat{y}} = 0$$

Because $p(y)$ grows with v , $l(y)$ and $w(y)$ are increasing functions of v . Consequently

$$\frac{\partial \Sigma(\hat{w}, \hat{y}, v, y^*)}{\partial v} > 0$$

On the other hand the derivative of $\Sigma(\hat{w}, \hat{y}, v, y^*)$ with respect to y^* is

$$\frac{\partial \Sigma(\hat{w}, \hat{y}, v, y^*)}{\partial y^*} = -l(y^*)(\hat{w} - z + \gamma) + f(y^*)n\gamma$$

⁷In this welfare study, the interest rate is very close to zero.

From equation (10), we deduce that this derivative is close to zero when r tends to zero. Consequently, we obtain

$$\Sigma'(\hat{w}) = \frac{\partial \Sigma(\hat{w}, \hat{y}, v, y^*)}{\partial v} \frac{\partial v}{\partial \hat{w}} > 0$$

Introducing a (small) minimum wage improves market efficiency.

2 Directed search

Building on Albrecht *et al.* (2006), we here assume that the determination of wages results from firms' competition for workers. As multiple applications (hence *ex post* competition) are ruled out, our model can be seen as an extension of Moen (1997) to heterogeneous workers with recruitment selection and endogenous participation. We find that with competitive search the wage of the marginal participant does coincide with her productivity. Participation then is efficient. In addition firms internalize both (negative) externalities of job creation. This implies that net aggregate income is maximized with wage posting.⁸ This result can be seen as an illustration of a *generalized* first welfare theorem. In general competitive analysis, agents are price takers. With directed search, firms treat workers' expected incomes as exogenous. We first define a *laissez-faire* equilibrium. Next we study its welfare properties and we provide an interpretation.

2.1 Model and Equilibrium

The population (participants and non-participants) has a positive measure n . When holding a job, workers' productivities y are distributed according to the (strictly) increasing c.d.f. $F(y)$ of support $[0, 1]$. The density $F'(y)$ is denoted by $f(y)$ ($f(y) > 0$). Participating workers must pay some positive search cost, γ , and find a job with probability $p(y)$. They are then unemployed with probability $[1 - p(y)]$. With no loss

⁸Relative to the literature on wage posting with heterogeneous workers, the present study considers continuously differentiated workers and makes participation endogenous.

in generality the inactivity income as well as the income of the unemployed are set to zero. All firms are identical and freely enter the market. Each active firm creates a single vacancy incurring some positive cost c to that end.

According to the basic urn-ball process, participating workers send a unique application to one of the (active) firms. Firms have full knowledge of the distribution of productivity among the queue of their applicants (if any). They therefore rank them by ability and hire the best.

Following Albrecht *et al.* (2006), wage setting results from the competition of firms for workers. Each firm sets wages subject to the constraint that workers are indifferent between all firms. As a consequence, firms can attract additional applicants by posting higher wages. More precisely, an increase in the offered wage to y -workers will raise the probability (density) of this firm filling its vacancy with a y -worker.

2.1.1 Participation behavior

An individual of type y will choose to be active if and only if his/her expected income is higher than the entry cost: $p(y)w(y) \geq \gamma$. Below it is proved that the expected income $p(y)w(y)$ is an increasing function of y . Let y^* be the productivity of the marginal participant in the market. Participation equilibrium imposes the condition that

$$\gamma = p(y^*)w(y^*) \tag{12}$$

From the cutoff y^* we deduce the number of participants: $[1 - F(y^*)]n$.

2.1.2 Determination of wages

Consider a symmetric pooling equilibrium in which all (active) firms post the same wages $w(y)$ ($y^* \leq y \leq 1$). Let v denote the (very large) number of vacancies. As a consequence, (participating) workers are indifferent and send their (unique) application to all firms with the same probability intensity $1/v$. Market tightness, θ , is the ratio of vacancies over participants: $\theta = v/[1 - F(y^*)]n$.

As firms rank their applicants and hire the best, the probability density, $q(y)$, for a firm to hire a y -worker is

$$q(y) = e^{(1-F(y))(n/v)} \frac{f(y)n}{v}$$

while the probability of an y -worker finding a job is

$$p(y) = e^{(1-F(y))(n/v)}$$

The probability of a vacancy to be filled therefore is $Q = Q(\theta) = 1 - e^{-1/\theta}$ while the average probability of a worker finding a job is $P = P(\theta) = \theta Q(\theta)$.

$\rho(y) = q(y)/Q(\cdot)$ denotes the density of y among employed workers (i.e. the set of occupied jobs). With this notation the average output, \bar{y} , is defined as

$$\bar{y} = \int_{y^*}^1 \rho(y)y dy$$

In order to determine the wages, we study the behavior of a deviant firm D . To that end, we denote by $i(y)$ the probability density with which y -workers send their applications to firm D . The expected number of y -applications for firm D 's job is $a(y) = f(y)i(y)n$ with $y^* \leq y \leq 1$. Therefore the probability density for firm D to hire a y -worker, $\hat{q}(y)$, is

$$\hat{q}(y) = e^{-\int_y^1 a(z)dz} a(y)$$

This implies that D 's vacancy will be filled with probability $\hat{Q} = 1 - e^{-a}$, with a ($= \int_{y^*}^1 a(y)dy$) being the total expected number of applicants.

On the other hand, the probability of a y -applicant getting D 's job, $\hat{p}(y)$, is

$$\hat{p}(y) = e^{-\int_y^1 a(z)dz}$$

Firm D posts the wages $\hat{w}(y)$. Its expected profit (gross of job opening costs) then is

$$\hat{\pi} = \int_{y^*}^1 \hat{q}(y)(y - \hat{w}(y))dy$$

An application to D should have the same return as to other firms whatever the worker's type might be. This imposes the condition that

$$\hat{p}(y)\hat{w}(y) = p(y)w(y) \quad \iff \quad \hat{q}(y)\hat{w}(y) = a(y)p(y)w(y) \quad (13)$$

for (almost) all y in the interval $[y^*, 1]$.

According to this indifference condition, an increase in $a(y)$ imposes an increase in $\hat{w}(y)$. Firm D must post a higher wage to attract more workers. It is worth noting that an increase in $a(y)$ also imposes an increase in $\hat{w}(z)$ for all z lower than y . The reason is that, as D ranks its applicants, the probability of a low-skilled worker being hired decreases when the deviant firm attracts additional high-skilled workers. Substituting into $\hat{\pi}$ and rearranging the terms give

$$\hat{\pi} = \int_{y^*}^1 \hat{q}(y)ydy - \int_{y^*}^1 a(y)p(y)w(y)dy \quad (14)$$

Firm D maximizes expected profit $\hat{\pi}$ with respect to function $a(y)$. Let $A(y)$ denote $\int_y^1 a(z)dz$. It follows that $A'(y) = -a(y)$ and the Hamiltonian of this problem is $H = e^{-A(y)}a(y)y - a(y)p(y)w(y) - \lambda(y)a(y)$ with $\lambda(y)$ being the costate variable. We obtain the two following optimality conditions:

$$\frac{\partial H}{\partial a(y)} = 0 \quad \iff \quad e^{-A(y)}y - p(y)w(y) - \lambda(y) = 0$$

$$\frac{\partial H}{\partial A(y)} + \lambda'(y) = 0 \quad \iff \quad -e^{-A(y)}a(y)y + \lambda'(y) = 0$$

From the second condition, we deduce⁹ $\lambda(y) = \int_{y^*}^y e^{-A(z)}a(z)zdz$. Substitution into the first optimality condition yields

⁹Since $A(y^*) = a > 0, \lambda(y^*) = 0$.

$$\hat{p}(y)y - p(y)w(y) - \int_{y^*}^y \hat{p}(z)a(z)zdz = 0 \quad (15)$$

2.1.3 *Laissez-faire* equilibrium

In symmetric equilibrium $i(y) = 1/v$, and $a(y) = f(y)n/v$. Therefore equation (15) determines workers' expected incomes as

$$p(y)w(y) = p(y)y - \int_{y^*}^y q(z)zdz \quad (16)$$

Notice that the derivative of expected income $p(y)w(y)$ with respect to y , equal to $p(y)$, is (strictly) positive. In other words, as assumed above, high-skilled workers have higher expected incomes. In addition, one can see that firms' profits, $y - w(y)$, increase with y . This ensures the consistency of recruitment selection. Ranking applicants by ability would otherwise not make sense.

From equation (16), we deduce

$$q(y)(y - w(y)) = \frac{n}{v}f(y) \int_{y^*}^y q(z)zdz$$

Free entry determines the equilibrium equation for job creation:

$$-c + \int_{y^*}^1 \int_{y^*}^y q(z)zdz f(y) \frac{n}{v} dy = 0 \quad (17)$$

A *laissez-faire* equilibrium can then be defined as

A laissez-faire equilibrium is a pair (v, y^) which jointly satisfies equations (12) and (17).*

2.2 Efficiency

2.2.1 Results

Our welfare criterion is (net) aggregate income. It can be defined as follows:

$$\Sigma = -vc + v \int_{y^*}^1 q(y)ydy \quad (18)$$

Differentiating Σ with respect to v gives

$$\frac{\partial \Sigma(v, y^*)}{\partial v} = -c + \int_{y^*}^1 q(y)y \frac{[1 - F(y)]n}{v} dy$$

On the other hand, consider the *laissez-faire* equation for job creation, (17). As

$$\int_{y^*}^1 \int_{y^*}^y q(z)z dz \frac{f(y)n}{v} dy = \left[\int_{y^*}^y q(z)z dz \frac{F(y)n}{v} \right]_{y^*}^1 - \int_{y^*}^1 q(y)y \frac{F(y)n}{v} dy$$

we deduce that the derivative of aggregate income with respect to v falls to zero in a *laissez-faire* equilibrium. This implies that job creation is efficient for any given value of the cutoff y^* .

Differentiating Σ with respect to y^* yields

$$\frac{\partial \Sigma(v, y^*)}{\partial y^*} = [-p(y^*)y^* + \gamma]f(y^*)n$$

With applicant ranking, all workers whose expected output ($p(y)y$) is higher than their search investment (γ) should enter the market. This is what occurs with *laissez-faire*: from equation (16), we deduce that the wage of the marginal participant, $w(y^*)$, coincides with his/her productivity, y^* . The equilibrium equation for participation, equation (12), then implies that the cutoff y^* is efficient for any given value of job creation. We obtain the following result:

With directed search, laissez-faire is efficient in terms of job creation and participation. Net aggregate income is maximized.

Although they are not too difficult to prove, these results are not intuitive *a priori*. They need to be interpreted.

2.2.2 Interpretation

Efficiency of participation clearly requires that all individuals whose expected market output is higher than the search investment do participate in the market. So the question is: why do marginal participants receive the entire output of a match ($w(y^*) = y^*$) in *laissez-faire* equilibrium? To understand this point we can consider a discrete distribution of productivities with f^* denoting the share (in total population) of marginal participants. The specificity of lower-skilled participants is that a change in their wage does not affect other workers. Therefore, as in the basic wage-posting model, marginal participants will earn the wage $w^* = \eta^* y^*$ with η^* being the elasticity of $q^* = 1 - e^{-f^*(n/v)}$ with respect to v in absolute value. When their number f^*n tends to zero, this elasticity tends to one. This explains why participation is socially optimal.¹⁰

Let us now turn to the efficiency of job creation. To reach a better understanding of this result, notice that (net) aggregate income can also be defined as

$$\Sigma = -vc + vQ(\theta)\bar{y} - [1 - F(y^*)]n\gamma$$

Consequently, its derivative with respect to v is also equal to

$$\frac{\partial \Sigma(v, y^*)}{\partial v} = -c + Q(1 - \eta - \mu)\bar{y} \quad (19)$$

with η (resp. μ) being the elasticity of $Q(\cdot)$ (resp. $\bar{y}(\cdot)$) with respect to v in absolute value.

Like that of Q , the elasticity of \bar{y} is negative.¹¹ It follows that the Hosios rule no longer ensures the efficiency of job creation with bargained wages.¹² This is because job creation acquires another congestion effect with applicant ranking. As market tightness increases, the number of applications that a firm can expect to receive falls,

¹⁰As the set of marginal participants is infinitely small, almost all wages are lower than corresponding productivities.

¹¹See Appendix A.

¹²With bargained wages the equation for job creation is $-c + Q(1 - \eta)\bar{y} = 0$, under the Hosios rule.

and accordingly the expected maximum output (*i.e.* the expected value of the n^{th} order statistic) among its applicants is reduced. In other words, more vacancies lower the average output, \bar{y} . See the text. So the question is: how do firms internalize this negative externality with wage-posting? To answer this question, let us consider a deviant firm D who maximizes its profit with respect to the total expected number of applicants (a) while leaving their productivity distribution unchanged, *i.e.* $\frac{a(y)}{a} = \frac{f(y)}{1-F(y^*)}$. This density is denoted by $g(y)$ and $G(y)$ is the corresponding cumulative function. The probability of a y -applicant getting D 's job is $\hat{p}(y) = e^{-(1-G(y))a}$, while the probability density of firm D filling its job with a y -worker is $\hat{q}(y) = \hat{p}(y)ag(y) = e^{-a(1-G(y))}ag(y)$. With $\hat{\rho}(y)$ being the ratio $\frac{\hat{q}(y)}{\hat{Q}}$, the expected output of D 's job is $\hat{y} = \int_{y^*}^1 \hat{\rho}(y)ydy$. Thus the expected profit can be rewritten as

$$\hat{\pi} = \hat{Q}\hat{y} - a \int_{y^*}^1 g(y)p(y)w(y)dy \quad (20)$$

From this writing of $\hat{\pi}$, we deduce the following optimality condition for a

$$\frac{d\hat{\pi}}{da} = 0 \Leftrightarrow \int_{y^*}^1 g(y)p(y)w(y)dy = \frac{d\hat{Q}}{da}\hat{y} + \hat{Q}\frac{\partial\hat{y}}{\partial a} \quad (21)$$

In symmetric equilibrium, we have $\frac{d\hat{Q}}{da}a/\hat{Q} = \eta$ and $\frac{\partial\hat{y}}{\partial a}a/\hat{y} = \mu$. From the substitution of (21) into (20) we deduce $-c + \pi = \frac{\partial\Sigma(v,y^*)}{\partial v} = 0$. See equation (19). This clearly shows how firms are led to behave like a social planner in creating vacancies.

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