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Participation, Recruitment Selection, and the Minimum Wage

Frédéric Gavrel* †

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Abstract

This paper reexamines the efficiency of participation with heterogeneous workers in a search-matching model with bargained wages and free entry. Assuming that firms hire their best applicants, we show that participation is always too low. The reason for this is a hold-up phenomenon: To be active, a worker must pay the entire search cost whereas part of the gain from this investment goes to the firm. As a consequence, introducing a (small) minimum wage raises participation, job creation, and employment. Therefore, net aggregate income of the economy is increased.

Key words : Search and matching, heterogeneous workers, applicant ranking, public policies, efficiency.

JEL Classification numbers: D8, J6.

*CREM and TEPP, CNRS and Normandy University, Caen, 19 rue Claude Bloch, BP 5186, 14032 Caen Cedex, France; frederic.gavrel@unicaen.fr.

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I. Introduction

In this paper we study the efficiency of participation in a search-matching model with heterogeneous workers and recruitment selection. We also describe the effects of the minimum wage in this environment.

Generally speaking, participation is governed by the return to market activity relative to non-market activity. The standard approach is to model heterogeneity in the returns to non-market activity as in Pissarides (2000). An alternative approach is to model heterogeneity in the returns to market activity, as in Albrecht *et al.* (2010). In the latter case individuals are inactive because the income gain from activity is lower than the (search) cost of entering the market.

Building on Albrecht *et al.*, Gavrel (2011) finds that too many individuals participate in the labor market according to the social surplus criterion. This result is obtained under a very strong assumption: recruitment is random. Hence all workers face the same probability of finding a job, independently of their productivity. We instead assume that firms rank their applicants and hire the best. Under this assumption highly-skilled job-seekers have a greater chance of success than low-skilled ones. This assumption also implies that too few individuals participate in the labor market. The reason is that the probability of a firm hiring a high-skilled worker is not affected by the entry of lower-skilled ones. Consequently, any individual whose expected (market) output compensates for her search investment should enter the labor market. However, part of the gain from the search investment goes to the firm. Hence the worker pays the entire cost and does not receive the entire benefit from entering. This hold-up phenomenon explains why entry is socially insufficient. Furthermore, participation remains insufficient when job creation is determined by free-entry. The entry of lower-skilled workers gives firms the opportunity of filling their vacancies in a less productive but profitable manner. This stimulates job creation, and hence

creates a positive effect on all participating workers.

What could be done to improve the efficiency of the labor market? We show that the introduction of a mandatory minimum wage is an effective instrument in correcting this market failure. Intuitively, a minimum wage (provided it is not too high) increases participation. More surprisingly, labor demand as well as the employment rate increase. Again, the entry of lower-skilled workers tends to raise firms' expected profits leading to the creation of additional vacancies. Therefore, despite the unemployment rate increase, (net) aggregate income necessarily rises. Furthermore, introducing a minimum wage appears to raise the expected income of all participants. Subsidies to participants (like unemployment compensation or a negative income tax) also improve efficiency.

In the relevant literature we can distinguish between two subsets. First, some papers build on Pissarides (2000)[1]. In Pissarides' model, job creation and individuals' participation decisions are efficient if and only if the elasticity of the matching function with respect to unemployment (usually denoted by η) coincides with workers' bargaining strengths (usually denoted by β): the famous Hosios condition. Conversely, if the bargaining power of workers is not high enough (if β is lower than η), participation will be insufficient. In this case, the minimum wage could raise aggregate income by acting as a substitute for an increase in workers' bargaining strengths. Empirically, however, it seems that the bargaining power of workers is too high. For instance, Flinn (2011) finds $\beta = 0.4$ and $\eta < 0.4$. By contrast, assuming vertically differentiated workers and recruitment selection, we show that minimum wage efficiency no longer depends on workers' bargaining strengths.

The other subset includes papers which attempt to derive the matching function from first principles. As Petrongolo and Pissarides (2001) put it, this literature seeks to

[1]See for instance Lehmann *et al.* (2011) for a study of optimal income taxation with endogenous participation.

throw some light into the black box. Many of these articles are, like the present one, based on the urn-ball model. Some authors assume recruitment selection.[2]

To summarize, our contributions are twofold. First, relative to Gavrel (2011)[3] we show that participation is unavoidably insufficient when firms rank their applicants. Second, relative to the usual search-matching model with endogenous participation (Flinn 2006, 2011), we find that a (small) minimum wage necessarily increases employment and improves market efficiency.

The paper is organized as follows. Section II presents our setting. Next we define a *laissez-faire* equilibrium and study its welfare properties (section III). The results concerning the introduction of a minimum wage are exposed in section IV while section V provides an analysis of the impact of subsidies to participants.[4] Section VI concludes.

II. Endogenous participation with applicant ranking

Market structure

Apart from the matching process, the market structure used here is very close to Albrecht *et al.* (2010) and Gavrel (2011). We study the efficiency of participation in the following simplified static environment with risk-neutral agents[5].

The population (participants and non-participants) has a positive measure n. When holding a job, workers' productivities y are distributed according to the (strictly) increasing c.d.f. $F(y)$ of support $[0, 1]$. The density $F'(y)$ is denoted by $f(y)$ ($f(y) >$

[2]See for instance Blanchard and Diamond (1994), Moscarini (1997), Moen (1999), Gavrel (2009, 2012).

[3]Relative to Gavrel (2012), who assumes recruitment selection, we extend the analysis to vertically differentiated workers and endogenous participation.

[4]An online appendix proves that results extend to a dynamic setting. It is also shown that directed search restores market efficiency.

[5]This static setting helps us present our results with clarity. See the online appendix for the extension to a dynamic model.

0). When inactive all individuals earn the same positive income z. For expositional simplicity, it is assumed that z does not depend on y, but the analysis extends to the case in which z grows with y as long as the increase in domestic output is lower than the increase in market output. Expressed in words, individuals need to hold a job to fully take advantage of their skills.

Participating workers must pay some positive search cost, γ, and find a job with probability $p(.,y)$. They are then unemployed with probability $[1-p(.,y)]$. In this case, they all enjoy the same domestic income z as non-participants . All firms are identical and freely enter the market. Each active firm creates a single vacancy incurring some positive cost c to that end.

When matched with a worker of productivity y (an event which occurs with probability density $q(.,y)$), a job generates the (private) surplus $(y-z)$. This surplus is divided between the two parties according to their bargaining strengths. The wage, $w(y)$, of an employed worker is then given by

$$w(y) = z + \beta(y-z) \tag{1}$$

with β being workers' bargaining strengths $(0 < \beta < 1)$.

Other arguments of functions $p(.,y)$ and $q(.,y)$ will be made explicit when we turn to the matching process.

Participation choices

Ignoring corner points, one can surmise that high-skilled workers (workers whose productivities are higher that some cutoff y^*) will choose to join the labor market, whereas low-skilled individuals (individuals whose productivities are lower than the cutoff y^*) will prefer to be inactive. This cutoff is thus determined by the following indifference condition

$$z = -\gamma + p(., y^*)w(y^*) + [1 - p(., y^*)]z$$

Or

$$-\gamma + p(., y^*)\beta(y^* - z) = 0 \tag{2}$$

In the next paragraph, we shall show that in conformity with intuition, the probability $p(., y)$ increases with the output y. It follows that

$$-\gamma + p(., y)w(y) + [1 - p(., y)]z > (<)z \Longleftrightarrow y > (<)y^*$$

Consequently, labor supply has a measure equal to $[1 - F(y^*)]n$ while the set of non-participants has a measure equal to $F(y^*)n$. Active firms offer a set of jobs of measure v. Hence the tightness of the labor market, θ, is given by the ratio

$$\theta = \frac{v}{[1 - F(y^*)]n}$$

As already noted, the main innovative feature of our paper involves the matching process. Contrary to Albrecht *et al.* (2010) and Gavrel (2011), (active) firms' recruitment is not random. As in Moen (1999) and Gavrel (2009, 2012), they rank their applicants and select the best. As a consequence, workers do not face the same probability of finding a job. In accordance with empirical evidence, we find that high-skilled job seekers have a greater chance of success.

Matching process

We now provide a brief account of the modeling of meetings in the labor market.[6] Applying the usual urn-ball model, we assume that each job seeker draws one firm at random. In general, firms will have several applicants of differing productivity. We

[6]See Gavrel (2012) for a more detailed exposition.

assume that they have full knowledge of the sample of their applicants. They then recruit the best one.

Let us consider the probability density, $q(.,y)$, for a firm to hire a worker of productivity y. One can prove the following preliminary result

Lemma 1. *With recruitment selection, the probability density* $q(.,y)$ $(y^* \leq y \leq 1)$ *is*

$$q(.,y) = q((v/n),y) = \exp\left(-\frac{[1-F(y)]n}{v}\right)\frac{f(y)n}{v} \tag{3}$$

Proof. The proof follows the same line as Gavrel (2012).

The above expression for the density $q((v/n),y)$ is quite intuitive. As the second term, $f(y)n/v$, is the probability density for a firm to meet at least one applicant of productivity y, the exponential term, $\exp(-[1-F(y)]n/v)$, represents the probability of (a firm) not meeting any applicant of productivity higher than y. Because a firm will not hire a worker of productivity y if another applicant is more skilled the expression for $q((v/n),y)$ follows.

Integrating $q((v/n),y)$ on the range $[y^*,1]$ gives the probability of filling a vacancy, $Q((v/n),y^*)$. We get

$$Q((v/n),y^*) = 1 - \exp\left(-\frac{[1-F(y^*)]n}{v}\right) \tag{4}$$

From the density $q(.,y)$, we also deduce the probability of finding a job, $p(.,y)$. It follows that

$$p(.,y) = p((v/n),y) = \exp\left(-\frac{[1-F(y)]n}{v}\right) \tag{5}$$

Integrating $[p(.,y)f(y)/(1-F(y^*))]$ on the range $[y^*,1]$ gives the average probability of finding a job, $P((v/n),y^*)$. We obtain

$$P((v/n), y^*) = \theta Q((v/n), y^*)$$

It is worth noting that neither $q((v/n), y)$ nor $p((v/n), y)$ depend on the cutoff y^*. This point will have important implications in what follows.

$\rho((v/n), y, y^*)$ will denote the density of the output y among employed workers (i.e. the set of occupied jobs). This density is defined as follows

$$\rho((v/n), y, y^*) \equiv \frac{q((v/n), y)}{Q((v/n), y^*)} \tag{6}$$

Next, from the density $\rho((v/n), y, y^*)$, we deduce the expected output, $\bar{y}$, of a job when matched with a worker

$$\bar{y} = \int_{y^*}^{1} \rho((v/n), y, y^*) y dy \tag{7}$$

III. Equilibrium and efficiency

We now define a labor market equilibrium, and will then study its welfare properties.

Equilibrium and social optimum

As indicated above, firms freely enter the labor market. This means that the profit expected from a vacancy should be equal to the cost of its creation (c). Job creation is then formally deduced from the following equation

$$-c + (1 - \beta) \int_{y^*}^{1} q((v/n), y)(y - z) dy = 0 \tag{8}$$

Consequently, a labor market equilibrium can be defined as follows

Definition 1. *A laissez-faire equilibrium is a pair* (y^*, v) *which jointly satisfies equations*(2) *and* (8).

From the equilibrium pair (y^*, v), one can deduce the equilibrium values of all other endogenous variables.

Our welfare criterion is the social surplus (also referred to as the aggregate income). Denoted by Σ, the social surplus per head is defined as

$$\Sigma = (v/n)Q((v/n), y^*)\bar{y} + [1 - Q((v/n), y^*)(v/n)]z - [1 - F(y^*)]\gamma - (v/n)c \quad (9)$$

In the previous expression, the quantity $(v/n)Q((v/n), y^*)$ measures the employment rate. It is also equal to $P((v/n), y^*)[1 - F(y^*)]$.

A social optimum is then determined as follows

Definition 2. *A social optimum is a pair* (y^*, v) *which maximizes the social surplus* Σ.

In what follows, we will restrict the welfare analysis to the neighborhood of a *laissez-faire* equilibrium. We first study the impact of participation (job creation) on the social surplus for a given value of job creation (participation). We will refer to our results as partial efficiency properties. Partial efficiency should be clearly distinguished from constrained efficiency, which is the second step of our investigation.

Partial efficiency

Let us first study the partial efficiency of the output cutoff y^*. Differentiating Σ with respect to y^* gives

$$\frac{\partial \Sigma}{\partial y^*} = -\frac{v}{n} q((v/n), y^*)(y^* - z) + f(y^*)\gamma$$

From

$$\frac{v}{n}q((v/n), y^*) = p((v/n), y^*)f(y^*)$$

we deduce that this derivative has the same sign as

$$Y \equiv \gamma - p((v/n), y^*)(y^* - z)$$

We obtain the following proposition

Proposition 1a. *In a laissez-faire equilibrium, the cutoff y^* is partially inefficient. Participation $[1 - F(y^*)]n$ is necessarily too low.*

Proof. The expression Y can be rewritten as follows

$$Y = \gamma - \beta p((v/n), y^*)(y^* - z) - (1 - \beta)p((v/n), y^*)(y^* - z)$$

Using equation (2), we obtain

$$Y = -(1 - \beta)p((v/n), y^*)(y^* - z) < 0$$

This proves that in the neighborhood of a *laissez-faire* equilibrium the cutoff y^* is too high whatever the value of job creation v is.

Q.E.D.

Proposition 1a contradicts the results of Gavrel (2011). In this paper, where firms' recruitment is random, participation is excessive. This contradiction requires interpretation. The intuition behind it is that, with random recruitment, a decrease in the cutoff lowers the average output per filled job. As Albrecht *et al.* (2010) put it, *the marginal participant reduces the average productivity of matches in the market.* On the contrary, when firms rank their applicants, the probability density, $q(., y)$, for a firm to hire a worker whose productivity y lies on the range $]y^*, 1]$ is not affected

by a decrease in the cutoff[7]. Consequently, any participant of productivity y raises the aggregate income by the amount: $-\gamma + p((v/n), y)(y - z)$.

In a competitive environment, all individuals whose productivities, y, are higher than the value of inactivity, z, should participate in the market. In this context, the productivity of the marginal participant should be equal to z. With matching frictions and recruitment selection, all individuals whose expected output $(p(., y)y + (1 - p(., y))z - \gamma)$ is higher than z should join the labor market. In this case, the expected output of the marginal participant should be equal to the value of inactivity.

Because productivities are higher than wages ($\beta < 1$), the private return to participation $(-\gamma + p((v/n), y)\beta(y - z))$ is lower than the social return. This clearly shows why the productivity of the marginal participant is too high with recruitment selection: the core of this inefficiency is a hold-up problem. To enter the market, a worker must pay the entire search cost whereas part of the gain from this investment goes to the firm.

This interpretation also explains why there is no value for the bargaining strengths of workers which could make the *laissez-faire* equilibrium coincide with a social optimum. When β tends to 1, the entire output goes to workers. The cutoff y^* becomes efficient, but job creation is reduced to zero.

It is worth noting that this insufficiency of participation does not stem from urn-ball matching by itself, but from recruitment selection. With random recruitment, the urn-ball matching function is $H = v(1 - e^{-1/\theta})$, which is no more than a particular specification of the usual CRTS function. Thus, the results would be the same as in Gavrel (2011).

Regarding the partial efficiency of job creation we build on Gavrel (2012) to prove the following proposition

[7] With random recruitment, this density is: $(1 - e^{-1/\theta})f(y)/(1 - F(y^*))$.

Proposition 1b. (i) *An increase in job creation* (v) *lowers the expected output* ($\bar{y}$).(ii) *Consequently, in the neighborhood of laissez-faire, job creation is partially inefficient under the Hosios rule. Firms create too many jobs.*

Proof. Statement (i) is proved in Appendix A. Let us prove statement (ii). Differentiating Σ with respect to v gives

$$n\frac{\partial \Sigma}{\partial v} = Q(.)(1-\eta(.))(\bar{y}-z) - c + vQ(.)\frac{\partial \bar{y}(.)}{\partial v}$$

with $\eta(.)$ being the elasticity of the probability $Q(.)$ with respect to v (in absolute value).

Let $\alpha(.)$ denote the elasticity of $(\bar{y}-z)$ with respect to v (in absolute value).

Since $\partial\bar{y}/\partial v < 0$, the derivative of Σ can be rewritten as

$$\frac{\partial \Sigma}{\partial v} = \frac{1}{n}[Q(.)(1-\eta(.)-\alpha(.))(\bar{y}-z) - c]$$

On the other hand, equation (8) is equivalent to

$$Q(.)(1-\beta)(\bar{y}-z) - c = 0$$

Applying the Hosios rule ($\beta = \eta$), we then obtain

$$\frac{\partial \Sigma}{\partial v} = -\frac{1}{n}Q(.)\alpha(.)(\bar{y}-z) < 0$$

This proves that job creation is too high in the neighborhood of an equilibrium, whatever the value of the cut off y^* is.

Q.E.D.

The intuition behind this result is that, relative to the basic matching model, job creation acquires a new congestion effect in the presence of recruitment selection.

As market tightness increases, the number of applications that a firm can expect to receive falls and accordingly the expected maximum output (*i.e.* the expected value of the nth order statistic) among its applicants is reduced. In other words, when the market becomes tighter, firms are compelled to hire lower-skilled workers. This lowers the average output $\bar{y}$. See Appendix A. Since firms do not not internalize this negative externality, job creation is excessive.

It is worth noting that, with recruitment selection, an increase in the number of vacancies exerts the same influence as a decrease in the share of skilled workers with random recruitment.

Constrained efficiency

Proposition 1a suggests that any policy measure that tends to raise participation is likely to improve the efficiency of the labor market. On the other hand, a participation increase will affect firms' entry. Hence to assess the value of this conjecture it is convenient to study the constrained efficiency of the cutoff y^*. See Moen and Rosén (2004). This amounts to examining the effect of an increase in the cutoff on aggregate income when job creation is set by free-entry (equation (8)).

As equation (8) is treated as a constraint, the social surplus, Σ_{cp}, can be rewritten as follows[8]

$$\Sigma_{cp} = \int_{y^*}^{1} [p((v/n), y)\beta(y - z) - \gamma] f(y) dy \tag{10}$$

On the other hand, according to equation (8) job creation (v) is an implicit function, $v(.)$, of the participation cutoff y^*. One can see that the derivative $v'(y^*)$ is negative. The reason for this is that with recruitment selection a participation increase raises the expected profits of vacancies, prompting firms to enter the market. Differentiating

[8]See Appendix B for detailed calculus.

Σ_{cp} with respect to y^* gives

$$\frac{\partial \Sigma_{cp}}{\partial y^*} = -f(y^*)[p((v/n), y^*)\beta(y^* - z) - \gamma] + \int_{y^*}^{1} \frac{\partial p((v/n), y)}{\partial v} \beta(y - z) f(y) dy v'(y^*)$$

Using equation (2) and noting that

$$\frac{\partial p((v/n), y)}{\partial v} > 0$$

the following results

$$\frac{\partial \Sigma_{cp}}{\partial y^*} = \int_{y^*}^{1} \frac{\partial p((v/n), y)}{\partial v} \beta(y - z) f(y) dy v'(y^*) < 0$$

So the participation cutoff appears to be too high. We have then proved the result below.

Proposition 2. *In a laissez-faire equilibrium, participation is constrained suboptimal.*

The intuition behind Proposition 2 is as follows. A participation increase tends to raise the value of vacancies. As free entry is imposed, this stimulates job creation, and hence creates a positive effect (an indirect externality) on all participating workers.

Symmetrically, as in Albrecht *et al.* (2010), we can study the constrained efficiency of job creation (v). This amounts to appraising the effect of an increase in v on the social surplus when the participation cutoff is set by individuals' choices, equation (2). From (2) we deduce the following derivative of y^*

$$\frac{\partial y^*}{\partial v} = -\frac{(1 - F(y^*))(y^* - z)}{v f(y^*)} < 0$$

As an increase in job creation raises the probability of getting a job for all workers, lower-skilled workers find it profitable to enter the market. In other words, the productivity of the marginal participant decreases. This indirect effect of job creation is similar to Albrecht *et al.*

We then obtain the following expression for the impact on the social surplus

$$\frac{\partial \Sigma_{cv}}{\partial v} = \frac{\partial \Sigma}{\partial v} + \frac{\partial \Sigma}{\partial y^*}\frac{\partial y^*}{\partial v}$$

Under the Hosios rule, in the neighborhood of *laissez-faire*, this derivative can be rewritten as follows (see Proposition 1b)

$$\frac{\partial \Sigma_{cv}}{\partial v} = \frac{\partial \Sigma}{\partial y^*}\frac{\partial y^*}{\partial v} - \frac{1}{n}Q(.)\alpha(.)(\bar{y} - z)$$

with $\alpha(.)$ being the elasticity of $(\bar{y} - z)$ with respect to v (in absolute value).

As $\partial\Sigma/\partial y^* < 0$ (Proposition 1a), it follows that the first term of the derivative is positive whereas the second term is negative. The constrained efficiency of job creation appears to be indeterminate. This contrasts with Albrecht *et al.* (2010) who obtain the result that job creation is excessive under the Hosios rule. The reason for this is twofold. First, holding the participation cutoff constant, the expected output no longer depends on job creation when recruitment is random (as in Albrecht *et al.*). Second , attracting lower-skilled workers improves market efficiency under recruitment selection, whereas this tends to depress efficiency under random recruitment. See our comment on Proposition 1a.

IV. Introducing a minimum wage

Participation is insufficient. What should be done to deal with this market failure?

In this section we provide a rationale for adopting a mandatory minimum wage. We show that introducing a minimum wage improves the efficiency of the economy (measured by the aggregate income). This improvement of efficiency does not require workers' bargaining strengths to be low. It holds for any value of the surplus shares. Moreover a small minimum also appears to raise the expected incomes of all participants.

We first study the effects of introducing a minimum wage on participation, labor demand and unemployment. This study is interesting by itself as the vertical differentiation of workers may be a better assumption than match specific productivities in assessing the merits of the minimum wage[9]. Surprisingly, we find that job creation is stimulated. We explain how recruitment selection causes this unexpected result.

The labor market in the presence of a binding minimum wage

In the presence of a binding minimum wage, $\hat{w}$, low-ability workers (workers whose productivities are lower that some trigger $\hat{y}$) now earn that minimum, whereas the wages of high-ability workers (workers whose productivities are greater that $\hat{y}$) are still subject to bargaining. Formally, we have

$$w(\hat{y}) = z + \beta(\hat{y} - z) = \hat{w}$$

and

$$y* \leq y \leq \hat{y} \Longleftrightarrow w(y) = \hat{w}$$

$$\hat{y} \leq y \leq 1 \Longleftrightarrow w(y) = z + \beta(y - z).$$

Notice that the first equation determines the trigger $\hat{y}$.

[9]Indeed, practical relevance requires that lower-skilled workers are on the minimum. Such a prediction is not compatible with horizontally differentiated workers.

The presence of a mandatory wage has two consequences. First, the equation which determines the cutoff y^* is now written as follows

$$z = -\gamma + p((v/n), y^*)\hat{w} + (1 - p((v/n), y^*))z$$

Or

$$\gamma = p((v/n), y^*)(\hat{w} - z) \tag{11}$$

For a given value of job creation v, an increase in the minimum wage $\hat{w}$ lowers the cutoff y^*, consequently increasing the participation level. We will show that this intuitive result holds true when job creation is endogenous.

Next, when matched with a worker whose productivity lies on the range $[y^*, \hat{y}]$, a job generates the profit $(y - \hat{w})$. Otherwise the profit remains equal to $[(1 - \beta)(y - z)]$. As a consequence, the job creation equation must be rewritten as

$$-c + \int_{y^*}^{\hat{y}} q((v/n), y)(y - \hat{w})dy + (1 - \beta) \int_{\hat{y}}^{1} q((v/n), y)(y - z)dy = 0 \tag{12}$$

This leads to the following definition

Definition 3. *In the presence of a binding minimum wage, a labor market equilibrium is a pair* (y^*, v) *which jointly satisfies equations* (11) *and* (12).

In what follows we examine the implications of introducing a binding minimum in the neighborhood of an equilibrium where all wages are bargained. In other words, this amounts to studying the effects of a (small) increase in the minimum wage in a situation where the legal minimum $\hat{w}$ is initially equal to the bargained wage: $z + \beta(y^* - z)$. This also means that in our benchmark, the cutoff y^* coincides with the trigger $\hat{y}$.

Effects of a minimum wage increase

With respect to the effects of introducing a small but binding minimum wage ($\hat{w}$ becomes higher than $[z + \beta(y^* - z)]$), we first show the following proposition

Proposition 3. *Introducing a minimum wage lowers the cutoff* (y^*) *and raises job creation* (v). *Despite the increase in labor demand, the unemployment rate* $(1 - P)$ *rises but the employment rate* $(P(1 - F(y^*)))$ *is enhanced.*

Proof. See Appendix C.

At first glance this increase in labor demand might look surprising. The intuition behind it is very simple. Introducing a (small) minimum wage makes lower-skilled workers (infra-marginal workers) join the market. Because their productivities remain higher than the wages they earn when holding a job, this tends to raise the expected profits of an offered job. On the other hand the wage increase lowers the profits of matches with incumbent workers on the minimum. In the neighborhood of *laissez-faire* this negative effect on expected profits is very small. Job creation is then stimulated. Conversely, implementing a minimum wage lowers market tightness, consequently increasing the unemployment rate. However, employment rises, so that fewer workers are jobless, whether inactive or unemployed.

It is worth noting that with vertically differentiated workers and ranking of candidates, the minimum wage effects are quite different from the usual search and matching setting (Flinn 2006, 2011). When all workers have the same (expected[10]) productivities, introducing a minimum wage tends to lower the (expected) profit of all matches[11], leading then to a reduction of job creation. On the contrary, in our model, a minimum wage only affects the profit of matches with low-skilled workers.

[10]Flinn assumes match specific productivities.

[11]Independently of the increase in the value of being unemployed that occurs in a dynamic setting.

Minimum wage and market efficiency

We can now study the effect of implementing a minimum wage on the efficiency of the labor market. Differentiating the social surplus Σ with respect to the minimum wage $\hat{w}$ yields

$$\frac{\partial \Sigma}{\partial \hat{w}} = \frac{\partial \Sigma}{\partial v}\frac{\partial v}{\partial \hat{w}} + \frac{\partial \Sigma}{\partial y^*}\frac{\partial y^*}{\partial \hat{w}}$$

We have seen that the cutoff y^* decreases with a minimum wage increase (Proposition 3). This increase in participation tends to improve market efficiency (Proposition 1a). On the other hand job creation is enhanced (Proposition 3). With applicant ranking this increase in labor demand tends to depress market efficiency under the Hosios rule (Proposition 1b). So, a priori, the impact on the social surplus would be indeterminate. However, when resulting from the introduction of a minimum wage, the stimulus to job creation follows from the decrease in the participation cutoff, which improves the expected income of vacancies. This is the reason why we can prove the result below.

Proposition 4. *Introducing a small but binding minimum wage improves the efficiency of the labor market.*

Proof. Proposition 4 results from the constrained inefficiency of participation (Proposition 2). See Appendix D.

Proposition 4 contradicts Gavrel (2011) where the efficiency of the labor market is improved by subsidizing non-participants (*i.e.* making inactivity more attractive to workers). The reason for this is that in this paper (as well as in Albrecht *et al.* 2010) random recruitment creates a mixing effect which tends to lower the social surplus when low-ability workers join the market (see the comment on Proposition 1b). On the contrary, with applicant ranking, the presence of bad workers in the market does

not affect the probability for a firm to hire a good one. That only enhances the probability of filling the vacancies in a less productive but efficient manner. Proposition 4 also contrasts the usual search-matching model in which minimum wage efficiency critically depends on workers' bargaining strengths.

Different authors prefer to focus on the value of being unemployed in lieu of the social surplus. In our static setting with heterogeneous workers this amounts to examining how the expected incomes of all individuals are affected by the minimum wage. Let us first consider the individuals whom the minimum wage prompts to enter the market. The expected incomes of these new participants obviously rise; otherwise they would remain inactive. Let us now turn to the workers who already participated in the market. As labor demand rises, all incumbent workers have a higher probability of finding a job. See equation (5). Because workers earn the minimum wage only if it is binding, we obtain the following corollary to Proposition 3:

Result. *A small minimum wage raises the expected incomes of all participating workers.*

As introducing a minimum wage lowers market tightness (Proposition 3), this result might look counterintuitive. The intuition behind this point is that the entry of lower-skilled workers does not directly affect the probabilities with which incumbent workers get a job. New entrants find it more difficult to obtain a job (relative to incumbent) but this is compensated by the minimum wage.

V. Other public policies

Basic income

Introducing a minimum wage is a simple policy measure. Nevertheless, as French (2012) points out in his review of Flinn (2011), other policy measures could be con-

sidered. We here study the implications of a lump-sum subsidy to participants paid for by a neutral tax[12]. See Holmlund (1998). This income, denoted by a, is received by all participants whether they find a job or not. Although non-participants do not profit from it, this subsidy resembles a basic income.[13]

In the presence of such a basic income, the surplus of a match remains equal to $(y-z)$. Consequently, the equilibrium equation for job creation is left unchanged.

On the other hand, the equilibrium equation for participation becomes

$$z = -\gamma + p(., y^*)[z + a + \beta(y^* - z)] + [1 - p(., y^*)](z + a)$$

Or

$$-(\gamma - a) + p(., y^*)\beta(y^* - z) = 0 \tag{13}$$

This basic income clearly acts as a subsidy to job search, tending then to make participation more attractive. In Appendix E, we show that an increase in the subsidy a necessarily lowers the cutoff y^*. So, as a consequence of Proposition 2, implementing a (small) basic income improves market efficiency.

Unemployment compensation

As workers are assumed to be risk-neutral, unemployment insurance does not really make sense in our setup. See Pissarides (2001). Nevertheless unemployment compensation can be used as a means of attracting more individuals in the labor market. Contrary to a basic income, introducing unemployment benefits (denoted by b) changes the expression for the (private) surplus of a match. As a consequence, the job creation equation must be rewritten as

[12]One can also consider the case in which subsidies are financed by a tax on job creation. Not surprisingly, results are ambiguous.

[13]This lump-sum subsidy also resembles certain elements of a tax credit conditional on employment, like EITC in US or WTC in UK.

$$-c + (1-\beta)\int_{y^*}^{1} q((v/n), y)(y - z - b)dy = 0 \tag{14}$$

On the other hand, the participation cutoff is now derived from the following equation

$$z = -\gamma + p(., y^*)[z + b + \beta(y^* - z - b)] + [1 - p(., y^*)](z + b)$$

Or

$$-(\gamma - b) + p(., y^*)\beta(y^* - z - b) = 0 \tag{15}$$

Holding job creation (v) constant, unemployment compensation also tends to lower the participation threshold y^*. More workers enter the labor market because they receive the income b if they are not successful in finding a job. In Appendix F, we show that unemployment compensation actually increases participation when job creation is endogenous. But, due to the increase in all wages, the impact on job creation is indeterminate, even in the neighborhood of *laissez-faire*. Despite of this, we can prove that introducing (small) unemployment benefits improves market efficiency. To that end we make use of previous (partial) inefficiency results (Proposition 1a,b) when job creation is reduced. In the reverse case the proof is similar to that of Proposition 2. See Appendix F.

Which policy is better for welfare? To answer this question, numerical simulations would indicate the extent to which these different measures can raise the aggregate income (outside the neighborhood of *laissez-faire*). We can however mention two arguments which can be made in favor of minimum wages. First, unlike subsidies to workers, introducing a minimum wage does not require the collection of taxes, consequently creating administrative costs. Second, and more importantly, to be efficient, subsidies to participants imply that their job search is monitored. This will

generate a dead weight loss.

VI. Conclusion

Using a matching model with vertically differentiated workers, we show that in the presence of recruitment selection, participation in the labor market is insufficient. Insofar it is not too large, a minimum wage improves the efficiency of the economy by rendering activity more attractive to low-skilled workers. This result holds despite an increase in unemployment, implying then that the effects of minimum wages on unemployment are not what really matters. Subsidies to participants also enhance net aggregate income. An online appendix extends the analysis to a dynamic model. Although the distribution of productivities among unemployed workers becomes endogenous, results hold true in this setting. Participation remains (constrained) insufficient and introducing a minimum wage still improves market efficiency. This appendix also shows that directed search restores market efficiency.

In order to derive analytical insights, the study was restricted to the neighborhood of *laissez-faire*. A calibration of the dynamic model would allow one to go beyond and to compare the efficiency of different policy measures. Numerical analysis would also provide an assessment of the optimum minimum wage. This a topic for further work.

From an empirical perspective these analytical insights also make possible an alternative interpretation of the results from Card and Krueger's (1994) case study. According to these authors, the observed employment increase in the fast-food industry is likely to result either from a higher inflow of workers into this industry, or from a lower outflow. See Portugal and Cardoso (2006) for empirical findings that coincide with this explanation. In an earlier paper, we show that a minimum wage can induce an employment increase in the low-skill sector by prompting employees to reduce the intensity of their (on-the-job) search for better-paid jobs. A larger

proportion of vacancies is filled. However, due to the reduction of job creation, the (possible) employment growth in low-skill industries is accompanied by a macroeconomic fall in employment. This would imply that one cannot infer macroeconomic predictions from sectoral results. See Gavrel *et al.* (2012). The present paper clearly weakens this pessimistic view of minimum wages. By giving firms the opportunity of filling their vacancies with new participants, a mandatory wage instead stimulates job creation when employers are selective.

To conclude, we would like to emphasize that both (perfect) applicant ranking and random recruitment are strong assumptions. In the real world firms carry out a cost-benefit analysis when deciding on their recruitment process. In fact their selectiveness should be modeled as an endogenous variable that is likely to depend on market tightness as well as on the distribution of workers' productivities. We leave a more realistic modeling of alternative recruitment strategies for future research.

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Appendix A: Proof of Proposition 1b, Statement (i)

We here prove that an increase in v reduces the expected output $\bar{y}$

One can show that the derivative of $\rho((v/n), y, y^*)$ with respect to v has the same sign as the following expression

$$[\frac{n}{v}(1 - F(y) - 1]Q - (Q - 1)(1 - F(y^*)\frac{n}{v}$$

The previous expression is a decreasing function of y.

As

$$\int_{y^*}^{1} \frac{\partial \rho((v/n), y, y^*)}{\partial v} dy = 0$$

there exists some positive output $\tilde{y}$ ($\tilde{y} > y^*$) such that

$$\frac{\partial \rho((v/n), \tilde{y}, y^*)}{\partial v} = 0$$

and

$$y < (>) \tilde{y} \Longleftrightarrow \frac{\partial \rho((v/n), y, y^*)}{\partial v} > (<) 0$$

This implies that, $\forall y \in [y^*, \tilde{y}[\ \cup \ ]\tilde{y}, 1]$

$$\frac{\partial \rho((v/n), y, y^*)}{\partial v} y < \frac{\partial \rho((v/n), y, y^*)}{\partial v} \tilde{y}$$

Consequently we obtain

$$\frac{\partial \bar{y}}{\partial v} < \int_{y^*}^{1} \frac{\partial \rho((v/n), y, y^*)}{\partial v} \tilde{y} dy = 0$$

This proves that job creation lowers the expected output.

Appendix B: Constrained efficiency of participation

The aggregate income, Σ, can be rewritten as

$$\Sigma = -(v/n)c + (v/n)Q((v/n), y^*)(1-\beta)(\bar{y} - z) + (v/n)Q((v/n), y^*)[z + \beta(\bar{y} - z)] +$$

$$[1 - Q((v/n), y^*)(v/n)]z - [1 - F(y^*)]\gamma$$

As equation (8) is treated as a constraint, we obtain the following expression for the constrained (social) surplus

$$\Sigma_{cp} = (v/n)Q((v/n), y^*)\beta(\bar{y} - z) - [1 - F(y^*)]\gamma + z$$

Or

$$\Sigma_{cp} = \int_{y^*}^{1} [(v/n)q((v/n), y)\beta(y - z) - f(y)\gamma]dy + z$$

From $(v/n)q((v/n), y) = p((v/n), y)f(y)$, we deduce

$$\frac{\partial \Sigma_{cp}}{\partial y^*} = \int_{y^*}^{1} \frac{\partial p((v/n), y)}{\partial y^*} \beta(y - z)f(y)dy - [p((v/n), y^*)\beta(y^* - z) - \gamma]f(y^*)$$

Using equation (2), we obtain

$$\frac{\partial \Sigma_{cp}}{\partial y^*} = \int_{y^*}^{1} \frac{\partial p((v/n), y)}{\partial y^*} \beta(y - z)f(y)dy$$

On the other hand, equation (8) implies that $\forall y \in [y^*, 1]$

$$\frac{\partial p((v/n), y)}{\partial y^*} = \frac{\partial p((v/n), y)}{\partial v} v'(y^*) < 0$$

As $v'(y^*) < 0$ (see the text), this shows that $\partial \Sigma_{cp}/\partial y^* < 0$ and proves Proposition 2.

Appendix C: Proof of Proposition 3

Let J denote the expected profit of an offered vacancy (gross of job opening costs)

$$J \equiv J((v/n), \hat{y}, y^*, \hat{w}) = \int_{y^*}^{\hat{y}} q((v/n), y)(y - \hat{w})dy + (1 - \beta) \int_{\hat{y}}^{1} q((v/n), y)(y - z)dy$$

In the neighborhood of the *laissez-faire* equilibrium (Definition 2), the result is obtained

$$\frac{\partial J(.)}{\partial y^*} = -(y^* - \hat{w})q((v/n), y^*) = -(1-\beta)(y^* - z)q((v/n), y^*) < 0$$

$$\frac{\partial J(.)}{\partial \hat{y}} = 0$$

$$\frac{\partial J(.)}{\partial \hat{w}} = -\int_{y^*}^{\hat{y}} q((v/n), y)dy = 0$$

Let us now turn to the effect of job creation v on $J(.)$. Let H denote

$$H \equiv \int_{y^*}^{1} q((v/n), y)(y - z)dy$$

Integrating H by parts gives

$$H = 1 - z - p((v/n), y^*)(y^* - z) - \int_{y^*}^{1} p((v/n), y)dy$$

In the neighborhood of *laissez-faire*, the derivative of J with respect to v is (very) close to

$$(1-\beta)\frac{\partial H}{\partial v}$$

From

$$\frac{\partial p((v/n), y)}{\partial v} > 0$$

we deduce

$$\frac{\partial H}{\partial v} = -\frac{\partial p((v/n), y^*)}{\partial v}(y^* - z) - \int_{y^*}^{1} \frac{\partial p((v/n), y)}{\partial v}dy < -\frac{\partial p((v/n), y^*)}{\partial v}(y^* - z)$$

It results that

$$\frac{\partial J(.)}{\partial v} < -(1-\beta)\frac{\partial p((v/n), y^*)}{\partial v}(y^* - z) < 0$$

Now, differentiating equation (12) yields

$$-(1-\beta)(y^* - z)q((v/n), y^*)dy^* + \frac{\partial J(.)}{\partial v}dv = 0$$

From the two previous relations, we deduce

$$0 > \frac{dv}{dy^*} > -\frac{q((v/n), y^*)}{\frac{\partial p((v/n), y^*)}{\partial v}}.$$

As

$$\frac{\partial p((v/n), y^*)}{\partial v} = p((v/n), y^*)[1 - F(y^*)]\frac{n}{v^2}$$

and

$$q((v/n), y^*) = p((v/n), y^*)f(y^*)\frac{n}{v}$$

we obtain

$$\frac{dv}{dy^*} > -\frac{vf(y^*)}{1 - F(y^*)}$$

Next, differentiating equation (11) gives

$$1 + \frac{n}{v}[f(y^*) + \frac{1 - F(y^*)}{v}\frac{dv}{dy^*}](\hat{w} - z)\frac{\partial y^*}{\partial \hat{w}} = 0$$

From the previous inequality, we get

$$[f(y^*) + \frac{1-F(y^*)}{v}\frac{dv}{dy^*}](\hat{w}-z) = \frac{1-F(y^*)}{v}[\frac{vf(y^*)}{1-F(y^*)} + \frac{dv}{dy^*}](\hat{w}-z) > 0$$

This proves that introducing a minimum wage lowers the cutoff y^*. We already know that a decrease in y^* is associated with an increase in v ($dv/dy^* < 0$). Therefore, introducing a minimum wage raises job creation.

From the same equation (11), we deduce

$$\frac{\partial\theta}{\partial\hat{w}} = -\frac{\theta^2}{\hat{w}-z} < 0$$

This shows that introducing a minimum raises the (average) unemployment rate of participating workers.

The employment rate, $[1-F(y^*)]P(.)$, can be written as

$$[1-F(y^*)]P(.) = Q(.)\frac{v}{n}$$

We know that introducing a minimum wage increases v and lowers θ. Consequently, $Q(.)$ rises. This shows that the employment rate is enhanced, completing then the proof of Proposition 3.

Appendix D: Proof of Proposition 4

In the presence of a binding minimum wage, the aggregate income can be rewritten as

$$\Sigma = (v/n)[-c + \int_{y^*}^{\hat{y}} q((v/n), y)(y-\hat{w})dy + \int_{\hat{y}}^{1}(1-\beta)q((v/n), y)(y-z)dy]$$

$$+(v/n)[\int_{y^*}^{\hat{y}} q((v/n),y)\hat{w}dy+\int_{\hat{y}}^{1} q((v/n),y)(z+\beta(y-z))]dy+[1-Q((v/n),y^*)(v/n)]z-[1-F(y^*)]\gamma$$

From the job creation equation (12), we deduce

$$\Sigma = (v/n)[\int_{y^*}^{\hat{y}} q((v/n),y)\hat{w}dy + \int_{\hat{y}}^{1} q((v/n),y)(z+\beta(y-z))dy]$$

$$+[1-Q((v/n),y^*)(v/n)]z - [1-F(y^*)]\gamma$$

Differentiating Σ with respect to $\hat{w}$ gives

$$\frac{\partial \Sigma}{\partial \hat{w}} = [\int_{y^*}^{\hat{y}} \frac{\partial p((v/n),y)}{\partial v} f(y)(\hat{w}-z)dy + \int_{\hat{y}}^{1} \frac{\partial p((v/n),y)}{\partial v} f(y)\beta(y-z)dy]\frac{\partial v}{\partial \hat{w}}$$

$$-[p((v/n),y^*)(\hat{w}-z)-\gamma]f(y^*)\frac{\partial y^*}{\partial \hat{w}}$$

From Proposition 3, we know that $\partial v/\partial \hat{w} > 0$. On the other hand we have $\partial p((v/n),y)/\partial v > 0$. From the indifference condition (11), we deduce that introducing a minimum wage improves market efficiency.

Appendix E: Basic income

Following the same reasoning as in the proof of Proposition 3 (Appendix C), we obtain

$$\frac{dv}{dy^*} > -\frac{vf(y^*)}{1-F(y^*)}$$

Next, differentiating equation (13) gives

$$1+\beta p((v/n),y^*)\{1+(y^*-z)\frac{(1-F(y^*))n}{v^2}[\frac{dv}{dy^*}+\frac{vf(y^*)}{1-F(y^*)}]\}\frac{\partial y^*}{\partial a}=0$$

From the previous inequality, we deduce

$$1+(y^*-z)\frac{(1-F(y^*))n}{v^2}[\frac{dv}{dy^*}+\frac{vf(y^*)}{1-F(y^*)}]>0$$

This proves that introducing a basic income (a) lowers the cutoff y^*.

Appendix F: Unemployment compensation

Participation

Let us consider the equilibrium equation for job creation (14). From

$$\int_{y^*}^{1} q((v/n),y)(y-z-b)dy = 1-z-b-p((v/n),y^*)(y^*-z-b)-\int_{y^*}^{1} p((v/n),y)dy$$

we deduce that (14) can be rewritten as

$$-c+(1-\beta)[1-z-b-p((v/n),y^*)(y^*-z-b)-\int_{y^*}^{1} p((v/n),y)dy]=0$$

On the other hand, the participation equation (15) gives

$$p((v/n),y^*)(y^*-z-b)=\frac{\gamma-b}{\beta}$$

Substitution into the job creation equation yields

$$-c+(1-\beta)[1-z-b-\frac{\gamma-b}{\beta}-\int_{y^*}^{1} p((v/n),y)dy]=0$$

Or

$$-c + (1-\beta)[\frac{\beta(1-z)-\gamma}{\beta} + \frac{1-\beta}{\beta}b - \int_{y^*}^{1} p((v/n), y)dy] = 0$$

Let us now prove that unemployment compensation (b) lowers the participation cutoff (y^*). Suppose that the derivative $\partial y^*/\partial b$ is positive. From the job creation equation (as previously rewritten) we deduce $\partial v/\partial b > 0$. Conversely the participation equation (15) imposes that $\partial v/\partial b < 0$. This contradiction proves that unemployment compensation enhances participation.

Efficiency

As the impact on job creation is indeterminate we are led to distinguish between the two cases.

Case 1. $\partial v/\partial b \leq 0$

In this case we use Propositions 1a,b. The derivative of Σ can be written as

$$\frac{\partial \Sigma}{\partial b} = \frac{\partial \Sigma}{\partial v}\frac{\partial v}{\partial b} + \frac{\partial \Sigma}{\partial y^*}\frac{\partial y^*}{\partial b}$$

From Propositions 1a-b, we know that, in the neighborhood of *laissez-faire*, $\partial\Sigma/\partial v$ is negative under the Hosios condition, while $\partial\Sigma/\partial y^*$ is always (strictly) negative. As $\partial y^*/\partial b < 0$, we obtain $\partial\Sigma/\partial b > 0$. This proves that, in this first case, introducing (small) unemployment benefits improves market efficiency under the Hosios condition.

Case 2. $\partial v/\partial b > 0$

In this case, we note that Σ can be written as

$$\Sigma = \frac{v}{n}[-c+(1-\beta)Q(.)(\bar{y}-z-b)]+(1-\beta)\frac{v}{n}Q(.)(b+z)+\beta\frac{v}{n}Q(.)\bar{y}+(1-\frac{v}{n}Q(.))z-(1-F(y^*)\gamma$$

As the pair (v, y^*) satisfies (14), Σ is reduced to

$$\Sigma = (1-\beta)\frac{v}{n}Q(.)(b+z) + \beta\frac{v}{n}Q(.)\bar{y} + (1-\frac{v}{n}Q(.))z - (1-F(y^*)\gamma$$

Or

$$\Sigma = (1-\beta)\frac{v}{n}Q(.)b + \beta\int_{y^*}^{1} p((v/n),y)f(y)(y-z)dy + z - (1-F(y^*)\gamma$$

One can check that Σ is an increasing function in b as in v. Differentiating Σ with respect to y^* gives

$$\frac{\partial\Sigma}{\partial y^*} = [\gamma - b - \beta p((v/n),y^*)(y^* - z - b) + (1-p((v/n),y^*))b]f(y^*)$$

From equation (15), we deduce that the previous derivative is close to zero in the neighborhood of *laissez-faire*. This proves that (small) benefits increase the aggregate income.