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## To cite this version:

Olivier Cardi, Romain Restout. Imperfect Mobility of Labor Across Sectors A Reappraisal of the Balassa-Samuelson Effect. 2015. halshs-01252478

HAL Id: halshs-01252478
https://shs.hal.science/halshs-01252478
Preprint submitted on 7 Jan 2016

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## Document de Recherche du Laboratoire d'Économie d'Orléans

DR LEO 2015-19

Imperfect Mobility of Labor Across Sectors
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Balassa-Samuelson Effect

Olivier CARDI<br>Romain RESTOUT

# IMPERFECT MOBILITY OF LABOR ACROSS SECTORS: A REAPPRAISAL OF THE BALASSA-SAMUELSON EFFECT* 

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#### Abstract

This paper investigates the relative price and relative wage effects of a higher productivity in the traded sector compared with the non traded sector in a two-sector open economy model with imperfect substitutability in hours worked across sectors. The BalassaSamuelson [1964] model predicts that a rise in the sectoral productivity ratio by $1 \%$ raises the relative price of non tradables by $1 \%$ while leaving the non traded wage-traded wage ratio unchanged. Applying cointegration methods to a panel of fourteen OECD countries over the period 1970-2007, our estimates show that the relative price rises by only $0.78 \%$ and the relative wage falls by $0.27 \%$. While our first set of empirical findings cast doubt on the quantitative predictions of the Balassa-Samuelson model, our second set of evidence highlights the role of imperfect labor mobility: the relative price responds more to a productivity differential between tradables and non tradables while the reaction of the relative wage is more muted in countries with higher intersectoral reallocation of labor. We show that the ability of the two-sector model to account for our evidence quantitatively relies upon two ingredients: i) imperfect mobility of labor across sectors, and ii) physical capital accumulation. Finally, our numerical results reveal that the model predicts the relative price response fairly well, and to a lesser extent the relative wage response.


Keywords: Relative price of non tradables; Sectoral wages; Productivity growth; Sectoral labor reallocation; Investment;
JEL Classification: E22; F11; F41; F43;

[^0]
## 1 Introduction

One of the strongest relationships established in the empirical macroeconomic literature is the positive correlation between the price of non traded goods in terms of traded goods and relative productivity in the traded and non traded sectors, see e.g., De Gregorio et al. [1994], Canzoneri et al. [1999], Kakkar [2003], Lane and Milesi-Ferretti [2002]. Balassa [1964] and Samuelson [1964] have provided the benchmark setup to explain the movements in the long run of the relative price of non tradables in terms of the productivity differential between tradables and non tradables. Quantitatively, the Balassa-Samuelson (BS hereafter) model predicts that a rise by $1 \%$ in productivity in the traded sector relative to productivity in the non traded sector raises the relative price of non tradables by the same amount while leaving the ratio of the non traded wage to the traded wage (relative wage hereafter) unchanged, due to the assumption of perfect mobility of labor across sectors. However, using a panel of 14 OECD countries over the period 1970-2007, our empirical estimates cast doubt on these predictions as the relative price increases by less than $1 \%$ while the relative wage falls. We find that theory can be reconciled with evidence once we consider imperfect mobility of labor across sectors and allow for physical capital accumulation.

While the analysis of the consequences of a productivity differential between tradables and non tradables has recently received growing attention in the literature, all studies assume that wages equalize across sectors, see e.g., Bergin et al. [2006], Ghironi and Melitz [2005], Mejean [2008]. More precisely, these studies focus on the real exchange rate by analyzing the implications of entry and exit of firms and/or heterogeneous productivity, allowing for wages to vary across countries instead of across sectors. ${ }^{1}$ Applying panel unit root tests, we find that the ratio of sectoral wages is integrated of order one, thus revealing that the relative wage is non stationary and invalidating the hypothesis of (long-run) wage equalization between the traded and non traded sectors imposed in the literature. ${ }^{2}$ Our analysis complements the papers mentioned above by focusing mainly on the movements in the relative price of non tradables, departing from the wage equalization hypothesis. To our knowledge, our paper is the first attempt to address quantitatively the long-run relative price of non-tradables and relative wage responses to higher productivity growth in tradables relative to non tradables. ${ }^{3}$

[^1]Because we aim to assess the ability of the two-sector model with tradables and non tradables to account for the evidence, we first estimate the relative price and relative wage effects of technological change biased toward the traded sector. Applying cointegration methods to a panel of 14 OECD countries over the period 1970-2007, we find that a 1 percentage point increase in the productivity differential between tradables and non tradables appreciates the relative price of non tradables by $0.78 \%$ and lowers the relative wage by $0.27 \%$. Furthermore, using an intersectoral labor reallocation index, estimates reveal that countries with larger movements of labor across sectors experience a more pronounced increase in the relative price and a smaller decline in the relative wage following a productivity differential, thus confirming the role of labor mobility in the determination of the relative price and relative wage responses.

In order to account for the long-run movements in the relative price and the relative wage, we put forward a variant of the two-sector model with tradables and non tradables. ${ }^{4}$ We assume that the economy is small in world goods (and capital) markets so that real exchange rate movements are exclusively driven by the long-run adjustment in the relative price of non tradables. One major feature of our setup is that we consider imperfect mobility of labor across sectors by assuming limited substitutability in hours worked, along the lines of Horvath [2000]..$^{5}$ When hours worked are perfect substitutes, as in the standard BS model, the allocation of labor supply across sectors is perfectly elastic to the ratio of sectoral wages; hence, the demand side of the labor market determines the sectoral allocation of the labor force while the relative wage remains fixed. With limited substitutability of labor, the agent is willing to devote more hours worked to one sector only if firms pay higher wages. The more reluctant workers are to shift hours worked across sectors, the larger the required increase in the sectoral wage. This difficulty in reallocating labor across sectors can be interpreted as a preference for the status quo or psychological costs when switching sectors (Dix-Carneiro [2014]), or it may capture other barriers to mobility that are not included in the model such as sector-specific human capital (Lee and Wolpin [2006]), geographic mobility costs (Kennan and Walker [2011]), and firing costs (Kambourov [2009]). The assumption of limited substitutability of hours worked is a convenient shortcut to produce a difficulty in reallocating hours worked across sectors which allows us i) to provide analytical results, ii) to estimate precisely a deep parameter of the model capturing the degree of labor mobility across sectors

[^2]for each country in our sample, and iii) to compare our results with those obtained in the standard BS model since the situation of perfect mobility of labor emerges as a special case.

To shed light on the transmission mechanism of technical change biased toward the traded sector in a model with imperfect mobility of labor, we analytically break down the relative price and relative wage effects into three components: i) a productivity channel when keeping sectoral capital-labor ratios and the capital stock unchanged, ii) a capital reallocation channel stemming from the shift of capital across sectors, iii) a capital accumulation channel caused by the investment boom along the transitional path. These three channels impinge on both the relative price and the relative wage as long as there is a difficulty in reallocating labor across sectors.

When it abstracts first from physical capital accumulation while assuming imperfect mobility of labor, the model can account for the evidence through the productivity channel only if traded and non traded goods are substitutes in consumption. Intuitively, higher productivity growth biased toward the traded sector has an expansionary effect on traded output so that the relative price of non tradables must appreciate to clear the goods market. With an elasticity of substitution in consumption greater than one, the relative price of non tradables increases less than proportionately. In this case, the relative wage falls because the consecutive increased share of tradables in total expenditure has an expansionary effect on labor demand in the traded sector. Conversely, with an elasticity of substitution smaller than one, the relative price growth exceeds the productivity differential while the relative wage rises instead of decreasing, in contradiction with our evidence, due to the expansionary effect on labor demand in the non traded sector.

Introducing physical capital along with imperfect labor mobility improves the predictive power of the model. More precisely, when the elasticity of substitution between traded and non traded goods is smaller than one, we find analytically that both the reallocation of capital across sectors and physical capital accumulation counteract the productivity channel. First, keeping the aggregate stock of physical capital fixed while assuming perfect mobility of capital, a productivity differential between tradables and non tradables produces a shift of capital towards the non traded sector, thereby raising non traded output and exerting a negative impact on the relative price and thus on the relative wage. Second, by giving rise to an investment boom, a productivity shock leads to a current account deficit which requires a long-run improvement in the trade balance for the intertemporal solvency condition to hold. When labor is imperfectly mobile, a steady-state increase in net exports depreciates the relative price of non tradables and lowers the relative wage by raising the demand for
tradables.
The role of trade balance surplus in determining the adjustment in the relative price of non tradables concurs closely with the empirical findings documented by Lane and MilesiFerretti [2002], [2004]. More precisely, their estimates reveal that countries with net external liabilities tend to run a trade balance surplus and to have a more depreciated relative price of non tradables. Intuitively, a net external debt produces a negative wealth effect that lowers consumption and raises labor supply. The fall in consumption of tradables raises net exports. The combined effect of a decline in consumption of non tradables and a higher labor supply to the non traded sector leads to a depreciation in the relative price of non tradables. ${ }^{6}$

In the assessment of the ability of our model to replicate our empirical findings, our numerical results show that regardless of the value of the elasticity of substitution between traded and non traded goods, the model can produce the less than proportional increase in the relative price of non tradables and the decline in the relative wage, as long as we allow for imperfect mobility of labor and physical capital. The final exercise we perform is to compare the responses of the relative price and relative wage for each OECD economy in our sample to our empirical estimates. To do so, we estimate the parameter capturing the degree of labor mobility and the elasticity of substitution in consumption between tradables and non tradables for each country. Allowing the two pivotal parameters to vary across countries, the model is found to predict the relative price growth fairly well but tends to overstate the decline in the relative wage.

The remainder of the paper is organized as follows. In section 2, we provide evidence on the relative price and relative wage effects of relative productivities in the long run. In section 3 , we develop an open economy version of the two-sector model with imperfect mobility of labor across sectors. We abstract from physical capital, allowing us to explore analytically the role of imperfect mobility of labor for the relative price and relative wage effects of higher productivity of tradables relative to non tradables. In section 4, we introduce physical capital and analytically break down the relative price and relative wage responses to a productivity differential between tradables and non tradables. In section 5 , we discuss numerical results and investigate the ability of the model to replicate our empirical findings for each OECD economy. Section 6 summarizes our main results and concludes.

[^3]
## 2 Empirical evidence

In this section, we confront the predictions of the BS model with data and thereby revisit the evidence regarding the relationships between relative price, relative wage and relative productivity. ${ }^{7}$ Throughout the paper, we denote the level of the variable in upper case, the logarithm in lower case, and the percentage deviation from its initial steady-state by a hat.

### 2.1 Revisiting the Balassa-Samuelson Effect

To set the stage for the empirical analysis, we find it useful to revisit the theory that Balassa [1964] and Samuelson [1964] constructed to explain the appreciation of the relative price of non tradables following technological change biased toward the traded sector. In contrast with the original framework, we relax the assumption of perfect labor mobility across sectors so that sectoral wages no longer equalize.

As is commonly assumed, the country is small in terms of both world goods and capital markets, and thus faces an exogenous international price for the traded good $P^{T, \star}$ and a given world interest rate, $r^{\star}$. Each sector produces $Y^{j}$ by using physical capital, $K^{j}$, and labor, $L^{j}$, according to Cobb-Douglas production functions:

$$
\begin{equation*}
Y^{j}=Z^{j}\left(L^{j}\right)^{\theta^{j}}\left(K^{j}\right)^{1-\theta^{j}}, \tag{1}
\end{equation*}
$$

where $Z^{j}$ represents the total factor productivity (TFP) index and $\theta^{j}$ the labor income share in the value added of sector $j=T, N$. In perfect competition, prices $P^{j}$ equalize with unit costs for producing $\frac{\Psi^{j}\left(W^{j}\right)^{\theta^{j}}\left(R^{j}\right)^{1-\theta^{j}}}{Z^{j}}$ where $\Psi^{j}=\left(\theta^{j}\right)^{\theta^{j}}\left(1-\theta^{j}\right)^{1-\theta^{j}}, W^{j}$ and $R^{j}$ are the wage rate and the rental rate of capital paid in sector $j$, respectively. Assuming that the law of one price holds so that $P^{T}=P^{T, \star}$, normalizing the price of the traded good on world good markets to unity, and taking the traded good as the numeraire, the price of non tradables in terms of tradables can be written as follows:

$$
\begin{equation*}
\frac{P^{N}}{P^{T}} \equiv P=\frac{\Psi^{T}}{\Psi^{N}} \frac{Z^{T}}{Z^{N}} \frac{\left(W^{N}\right)^{\theta^{N}}\left(R^{N}\right)^{1-\theta^{N}}}{\left(W^{T}\right)^{\theta^{T}}\left(R^{T}\right)^{1-\theta^{T}}} \tag{2}
\end{equation*}
$$

Using the fact that in the traded sector, the unit cost for producing is equal to one, we have $W^{T}=\left(Z^{T}\right)^{\frac{1}{\theta^{T}}}\left(\Psi^{T}\right)^{\frac{1}{\theta^{T}}}\left(R^{T}\right)^{-\left(\frac{1-\theta^{T}}{\theta^{T}}\right)}$. Denoting by $\delta_{K}$ the fixed depreciation rate of physical capital and assuming perfect capital mobility across sectors so that $R^{T}=R^{N}=R$ where

[^4]$R=P\left(r^{\star}+\delta_{K}\right)$ when investment is non-traded, eq. (2) reduces to: ${ }^{8}$
\[

$$
\begin{equation*}
P=\frac{\Psi^{T}}{\left(\Psi^{N}\right)^{\theta^{T} / \theta^{N}}} \frac{Z^{T}}{\left(Z^{N}\right)^{\theta^{T} / \theta^{N}}}\left(\frac{W^{N}}{W^{T}}\right)^{\theta^{T}}\left(r^{\star}+\delta_{K}\right)^{\frac{\theta^{T}-\theta^{N}}{\theta^{N}}} . \tag{3}
\end{equation*}
$$

\]

Taking logarithm, and denoting by $\omega=\ln \left(W^{N} / W^{T}\right)$ the (logged) relative wage yields:

$$
\begin{equation*}
p=c+\left(z^{T}-\frac{\theta^{T}}{\theta^{N}} z^{N}\right)+\theta^{T} \omega, \tag{4}
\end{equation*}
$$

where $c=\ln \Psi^{T}-\frac{\theta^{T}}{\theta^{N}} \ln \Psi^{N}+\left(\frac{\theta^{T}-\theta^{N}}{\theta^{N}}\right) \ln \left(r^{\star}+\delta_{K}\right)$ is a constant. Imposing perfect labor mobility across sectors, as in the standard BS model, both sectors pay the same wage so that the wage differential across sectors vanishes, i.e., $\omega=0$. As a result, the relative price of non tradables appreciates by the same amount as the productivity differential. The explanation is intuitive. The wage in the non traded sector rises at the same speed as in the traded sector while productivity gains are smaller. To compensate for the rise in the non-tradable unit labor cost, prices must increase by $\left(z^{T}-\frac{\theta^{T}}{\theta^{N}} z^{N}\right)$ in that sector. Conversely, if labor is not perfectly mobile, there is no longer wage equalization across sectors and therefore $\omega$ may change. When labor demand expands in the traded sector due to higher productivity in tradables relative to non tradables, the relative wage $\omega$ falls as the traded sector must pay higher wages to induce workers to shift hours worked toward this sector. As a result, eq. (4) implies that the relative price appreciates less than the productivity differential.

Eq. (4) establishes a relationship between the relative price, the productivity gap and the relative wage. While this equation allows us to explain how imperfect labor mobility modifies the long-run response of the relative price to a productivity differential between tradables and non tradables, eq. (4) is determined by abstracting from the goods market equilibrium, which matters as long as labor is not perfectly mobile across sectors. In section 4, we show that the steady-state can be solved for the relative price and the relative wage, i.e., $p=p\left(z^{T}, z^{N}\right)$ and $\omega=\omega\left(z^{T}, z^{N}\right)$. Because all variables display trends, our empirical strategy consists in estimating the cointegrating relationships with the productivity discrepancy between tradables and non tradables.

### 2.2 Data Construction

Before empirically exploring the relative price and relative wage effects of a productivity differential, we briefly describe the dataset we use and provide details about data construction below and in Appendix A. Our sample consists of a panel of fourteen OECD countries: Belgium,

[^5]Denmark, Spain, Finland, France, Germany, Ireland, Italy, Japan, Korea, the Netherlands, Sweden, the UK and the US. ${ }^{9}$ Our sample covers the period 1970-2007 (except for Japan: 1974-2007), for eleven 1-digit ISIC-rev. 3 industries.

To split these eleven industries into traded and non traded sectors, we follow the classification suggested by De Gregorio et al. [1994]. Agriculture, hunting, forestry and fishing; Mining and quarrying; Total manufacturing; Transport, storage and communication are classified as traded industries. Following Jensen and Kletzer [2006], we updated the classification of De Gregorio et al. [1994] by treating Financial intermediation as a traded industry. Electricity, gas and water supply; Construction; Wholesale and retail trade; Hotels and restaurants; Real estate, renting and business services; Community, social and personal services are classified as non traded industries. ${ }^{10}$ Once industries have been classified as tradable or non tradable, series for sectoral value added in current (constant) prices are constructed by adding value added in current (constant) prices of all sub-industries in sector $j=T, N$.

We use the EU KLEMS [2011] database which provides domestic currency series of value added in current and constant prices, labor compensation and employment (number of hours worked) for the eleven industries that we classify either as tradable or non tradable, permitting the construction of price indices $p^{j}$ (in $\log$ ) which correspond to sectoral value added deflators, sectoral wage rates $w^{j}$ (in log), and sectoral measures of productivities $z^{j}$ (in log). The relative price of non tradables $p$ is the log of the ratio of the non traded value added deflator to the traded value added deflator (i.e., $p=p^{N}-p^{T}$ ). The relative wage $\omega$ is the $\log$ of the ratio of the non traded wage to the traded wage (i.e., $\omega=w^{N}-w^{T}$ ). We use sectoral total factor productivities $Z^{j}$ (TFPs) to approximate technical change. Sectoral TFPs (in log) $z_{t}^{j}$ at time $t$ are constructed as Solow residuals from constant-price (domestic currency) series of value added $y_{t}^{j}$ and capital stock $k_{t}^{j}$, and employment $l_{t}^{j}$ :

$$
\begin{equation*}
z_{t}^{j}=y_{t}^{j}-\theta^{j} l_{t}^{j}-\left(1-\theta^{j}\right) k_{t}^{j} \tag{5}
\end{equation*}
$$

where $\theta^{j}$ is labor's share in value added in sector $j=T, N$ defined as the ratio of the compensation of employees to value added in the $j$ th sector, averaged over the period 1970-2007. To obtain series for sectoral capital stock, we first compute the overall capital stock by adopt-

[^6]ing the perpetual inventory approach, using constant-price investment series taken from the OECD's Annual National Accounts. Following Garofalo and Yamarik [2002], we split the gross capital stock into traded and non traded industries by using sectoral valued added shares. ${ }^{11}$ Assuming that investment expenditures are non traded, we compute the labor share-adjusted
TFP differential as follows $z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N} .{ }^{12}$

### 2.3 Tests of BS predictions

We begin by examining the behavior of the series for 14 OECD economies over the period 1970-2007. Figures 1(a) and 1(b) plot the average relative price growth and average relative wage growth against the average productivity growth differential between tradables and non tradables, respectively. Quantitatively, the BS model predicts that a 1 percentage point increase in the productivity differential i) raises the relative price of non tradables by $1 \%$, ii) while leaving the relative wage unchanged. The first prediction implies that graphically, all countries should be positioned on the $45^{\circ}$ line in Figure 1(a). However, we find that all countries are positioned below the $45^{\circ}$ line which suggests that higher productivity in tradables relative to non tradables is not fully reflected in the relative price. According to the second prediction, all countries should be positioned on the X -axis in the bottom right panel. However, as shown in Figure 1(b), all countries are below the X-axis which suggests that a productivity differential between tradables and non tradables lowers the relative wage.

While the data seem to challenge the conclusions of the standard BS model, in the following we use unit root tests and cointegration methods to confirm these findings and to estimate precisely the effects of the higher productivity in tradables relative to non tradables on both the relative price of non tradables and the relative wage.
$<$ Please insert Figure 1 about here >
We test for the presence of unit roots in the logged relative wage, $\omega$, and in the difference between the ( $\log$ ) relative price $p$ and the (log) relative TFPs, i.e., $p-\left[z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}\right]$. If the predictions of the BS model were right, the relative wage should be stationary due to the assumption of perfect labor mobility, which implies wage equalization across sectors. Because

[^7]sectoral wages increase at the same speed, the difference between the logged relative price and the logged relative sectoral productivity should also be integrated of order zero.
< Please insert Table 1 about here >
We consider five panel unit root tests developed by Levin, Lin and Chu [2002], Breitung [2000], Im, Pesaran and Shin [2003], Maddala and Wu [1999], and Hadri [2000]. Results are summarized in Table 1. To begin with, as shown in the first and the third column of Table 1, all unit root tests applied to the relative price and the relative productivity of tradables confirm that these two variables are non-stationary. On the basis of all tests shown in the second column, except for Levin et al.'s [2002] unit root test, the relative wage variable is found to be non-stationary. Hence, the data reject the wage equalization hypothesis. On the contrary, the sectoral wage differential persists in the long run, casting doubt on the assumption of perfect mobility of labor. The p-values shown in the last column of Table 1 reveal that the relative price of non tradables and the ratio of sectoral labor share-adjusted TFPs are not cointegrated with a unit cointegrating vector. Put differently, the change in the ratio of sectoral TFPs is not fully reflected in the relative price $p$.

To get some sense of the magnitude of the long-run effects that a productivity differential might generate, we now estimate the cointegrating relationships. Using $i$ to index countries and $t$ to index years, we regress the $(\log )$ relative wage $\omega$ and the ( $\log$ ) relative price $p$ on the (log) relative productivity, respectively:

$$
\begin{align*}
\omega_{i, t} & =\delta_{i}+\beta\left[z_{i, t}^{T}-\left(\theta_{i}^{T} / \theta_{i}^{N}\right) z_{i, t}^{N}\right]+v_{i, t},  \tag{6a}\\
p_{i, t} & =\alpha_{i}+\gamma\left[z_{i, t}^{T}-\left(\theta_{i}^{T} / \theta_{i}^{N}\right) z_{i, t}^{N}\right]+u_{i, t}, \tag{6b}
\end{align*}
$$

where $\delta_{i}$ and $\alpha_{i}$ are country fixed effects and $v_{i, t}$ and $u_{i, t}$ are i.i.d. error terms. While the standard BS model predicts $\beta=0$ and $\gamma=1$, we expect $\beta<0$ and $0<\gamma<1$, in line with our empirical findings above.

Having verified that the assumption of cointegration is empirically supported, we estimate the cointegrating relationships by using fully modified OLS (FMOLS) and dynamic OLS (DOLS) procedures for the cointegrated panel proposed by Pedroni [2000], [2001]. ${ }^{13}$ Both estimators give the same results and estimated coefficients of the cointegrating relationships are significant at $1 \%$. Estimates reported in column 1 of panel A (for $\beta$ ) and panel B (for $\gamma$ ) of Table 2 reveal that a 1 percentage point increase in the productivity differential lowers the relative wage by $0.27 \%$ and appreciates the relative price by $0.78 \%$. As shown in the last row of panel A (panel B) of Table 2, imposing the restriction that the slope of the cointegrating

[^8]vector $\beta$ is equal to zero ( $\gamma$ is equal to one) is strongly rejected at a $1 \%$ significance level.
< Please insert Table 2 about here >
< Please insert Table 3 about here >
To get a sense of the interval of estimates across countries, we again run regressions of (6a)-(6b) by letting $\beta$ and $\gamma$ vary across countries. Table 3 shows results for the fourteen countries in our sample, using both DOLS and FMOLS cointegration procedures. Despite large cross-country variations, higher productivity growth in tradables relative to non tradables significantly lowers the relative wage in all countries (except Sweden and the United States) while the estimated coefficient for the relative price is always significantly smaller than one. When considering statistically significant estimates, the response of the relative wage to a 1 percentage point increase in the productivity differential ranges from a low of -0.59 for Germany to a high of -0.14 for the United Kingdom and Belgium while the reaction of the relative price of non tradables varies between 0.47 for Denmark to 0.97 for Japan.

### 2.4 Interpreting the Puzzle: Imperfect Mobility of Labor across Sectors

How to explain the less than proportional increase in the relative price and the fall in the relative wage? We conjecture that imperfect mobility of labor could rationalize the evidence as unit root tests applied to the (logged) relative wage $\omega$ indicate that the sectoral wage differential persists in the long run, a finding uncovered recently by the literature using a structural empirical approach, see e.g., Artuç et al. [2010], Lee and Wolpin [2006]. According to the relative price equation (4) which has been derived without imposing wage equalization, imperfect mobility moderates the relative price response to a productivity differential by lowering the non traded wage relative to the traded wage.

## $<$ Please insert Figure 2 about here >

Intuitively, technological change biased toward the traded sector has an expansionary effect on labor demand in the traded sector. To hire more workers, firms in the traded sector have to pay higher wages to compensate for the mobility cost. The higher the mobility cost, the more the ratio of non traded wage to traded wage should fall when traded firms experience higher productivity gains than non traded producers. Because the non tradable unit labor cost increases less than if labor were perfectly mobile, the relative price must be increased by a smaller amount.

We thus empirically expect countries with lower interindustry gross flows of workers to experience a smaller appreciation in the relative price and a larger decline in the relative wage in response to productivity growth biased toward the traded sector. Using FMOLS
estimates, Figure 2 illustrates this by depicting the relationship between the relative price response and the relative wage reaction (in absolute value) to a 1 percentage point increase in the productivity differential. The trend line in Figure 2 shows that the estimated responses of these two variables are inversely related across countries.

In the next subsection, we thus test our conjecture whereby the relative price of non tradables is more responsive to higher productivity growth in tradables relative to non tradables while the reaction of the relative wage becomes more muted, as labor becomes more mobile across sectors.

### 2.5 The Role of Imperfect Labor Mobility

To evaluate the role of imperfect mobility of labor across sectors in explaining the relationship between $p$ and $\omega$ and productivity, we proceed as follows. First, we construct an index capturing the extent of labor shifts across sectors. Then we empirically explore our conjecture by interacting the measure of intersectoral labor reallocation and the productivity differential.

### 2.5.1 Measures of Sectoral Labor Movements

For our empirical analysis, we construct an indicator capturing the extent of labor mobility across sectors. The data are taken from EU KLEMS. Following Wacziarg and Wallack [2004], we compute the labor reallocation index in year $t$ for country $i$ denoted by $L R_{i, t}$ by calculating the ratio of the absolute change in sectoral employment resulting from labor reallocation to average employment over $\tau$ years:

$$
\begin{equation*}
L R_{i, t}(\tau)=\frac{\sum_{j=T}^{N}\left|L_{i, t}^{j}-L_{i, t-\tau}^{j}\right|-\left|\sum_{j=T}^{N} L_{i, t}^{j}-\sum_{j=T}^{N} L_{i, t-\tau}^{j}\right|}{0.5 \sum_{j=T}^{N}\left(L_{i, t-\tau}^{j}+L_{i, t}^{j}\right)} . \tag{7}
\end{equation*}
$$

where $L_{i, t}^{j}$ denotes employment in sector $j=T, N$; the changes are computed over $\tau=2$ and $\tau=3$ years. ${ }^{14}$ The first term in the numerator of (7) captures the change in employment over two (three) years in sector $j$ while the second term "filters" the change in labor arising from total employment growth. The term in the denominator of (7) is a measure of total employment in the economy (i.e., the average employment computed over $t$ and $t-\tau$ ). Dividing one by the other gives the rate of workers that have shifted from one sector to another over two (three) years.

[^9]
### 2.5.2 Empirical results

To test our conjecture, we add interaction terms in (6) and explore the following relationships empirically:

$$
\begin{align*}
\omega_{i, t} & =\delta_{i}+\beta\left[z_{i, t}^{T}-\left(\theta_{i}^{T} / \theta_{i}^{N}\right) z_{i, t}^{N}\right]+\beta_{L}\left[z_{i, t}^{T}-\left(\theta_{i}^{T} / \theta_{i}^{N}\right) z_{i, t}^{N}\right] \times L R_{i, t}+v_{i, t}  \tag{8a}\\
p_{i, t} & =\alpha_{i}+\gamma\left[z_{i, t}^{T}-\left(\theta_{i}^{T} / \theta_{i}^{N}\right) z_{i, t}^{N}\right]+\gamma_{L}\left[z_{i, t}^{T}-\left(\theta_{i}^{T} / \theta_{i}^{N}\right) z_{i, t}^{N}\right] \times L R_{i, t}+u_{i, t} \tag{8b}
\end{align*}
$$

In light of our conjecture, we expect coefficients of interaction terms $\beta_{L}$ and $\gamma_{L}$ to be positive in both regressions (8a) and (8b). Such a result would imply that higher productivity in tradables relative to non tradables lowers the relative wage less and raises the relative price more in countries where workers are more mobile across sectors.

We estimate cointegrating vectors by using DOLS and FMOLS estimators. The estimates are reported in columns 2 and 3 of Table 2. Both DOLS and FMOLS cointegration procedures yield similar results. The first line of panel A and B of Table 2 confirms that higher productivity growth in tradables relative to non tradables lowers the relative wage and raises the relative price less than proportionately. Importantly, as shown in the second and third lines of panels A and B of Table 2, the coefficients $\beta_{L}$ and $\gamma_{L}$ of interaction terms are positive (and statistically significant at conventional level), regardless of whether the labor reallocation index is computed over 2 or 3 years. Hence, as labor mobility across sectors increases, the relative price becomes more responsive to a productivity differential while the reaction of the relative wage is more muted.

In the following, we construct a dynamic general equilibrium model with a traded and a non traded sector by allowing for imperfect mobility of labor across sectors. In particular, our aim is to assess its ability to account for the following set of empirical findings. A 1 percentage point increase in the productivity differential between tradables and non tradables: i) raises the relative price of non tradables $p$ by $0.78 \%$, ii) lowers the relative wage $\omega$ by $0.27 \%$, iii) appreciates $p$ more while lowering $\omega$ less as labor becomes more mobile across sectors. In order to shed light on the role of imperfect mobility of labor, we first solve analytically the model in the next section by abstracting from physical capital.

## 3 A Simple Two-Sector Model with Limited Substitutability of Labor

We consider a small open economy that is populated by a constant number of identical households and firms that have perfect foresight and live forever. The country is small in terms of both world goods and capital markets, and faces a given world interest rate, $r^{\star}$. One
sector produces a traded good denoted by the superscript $T$ that can be exported at no cost and consumed domestically. A second sector produces a non traded good denoted by the superscript $N$ which is consumed domestically. The traded good is chosen as the numeraire. We denote by $P$ the relative price of nontradables $\left(P \equiv P^{N} / P^{T}\right)$. To derive a number of analytical results which enables us to build intuition about the transmission mechanism, we abstract first from physical capital and assume that both traded and non-traded goods are produced by using labor only. Time is continuous and indexed by $t$.

### 3.1 Households

At each instant the representative household consumes traded and non traded goods denoted by $C^{T}(t)$ and $C^{N}(t)$, respectively, which are aggregated by means of a CES function:

$$
\begin{equation*}
C(t)=\left[\varphi^{\frac{1}{\phi}}\left(C^{T}(t)\right)^{\frac{\phi-1}{\phi}}+(1-\varphi)^{\frac{1}{\phi}}\left(C^{N}(t)\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{9}
\end{equation*}
$$

where $0<\varphi<1$ is the weight attached to the traded good in the CES sub-utility function $C($.$) and \phi$ is the elasticity of substitution between traded goods and non traded goods.

The representative household supplies labor $L^{T}(t)$ and $L^{N}(t)$ in the traded and non traded sectors, respectively. The standard BS model assumes that hours worked are perfect substitutes. Because workers are willing to devote their whole time to the sector that pays the highest wages, sectors pay the same wage. However, our unit root tests applied to the ratio of sectoral wages reject the wage equalization between sectors in the long-run. A shortcut to produce a persistent wage differential across sectors is to assume limited substitutability in hours worked. Along the lines of Horvath [2000], we assume that hours worked in the traded and the non traded sectors are aggregated by means of a CES function:

$$
\begin{equation*}
L(t)=\left[\vartheta^{-1 / \epsilon}\left(L^{T}(t)\right)^{\frac{\epsilon+1}{\epsilon}}+(1-\vartheta)^{-1 / \epsilon}\left(L^{N}(t)\right)^{\frac{\epsilon+1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon+1}}, \tag{10}
\end{equation*}
$$

and $0<\vartheta<1$ is the weight of labor supply to the traded sector in the labor index $L($.$) and$ $\epsilon$ measures the ease with which worked hours can be substituted for each other and thereby captures the degree of labor mobility. The case of perfect labor mobility is nested under the assumption that $\epsilon$ tends towards infinity; in this case, (10) reduces to $L(t)=L^{T}(t)+L^{N}(t)$ which implies that hours worked are perfectly substitutable across sectors. When $\epsilon<\infty$, hours worked are no longer perfect substitutes. More specifically, as $\epsilon$ becomes smaller, labor mobility across sectors becomes lower as workers perceive a higher cost (in utility terms) of shifting and therefore become more reluctant to reallocate labor across sectors.

Producing imperfect labor mobility across sectors by means of (10) is a convenient shortcut
which has several advantages over the alternatives. ${ }^{15}$ First, the CES form (10) for the aggregate labor index allows us to consider the range of all degrees of labor mobility across sectors. Specifically, if we let $\epsilon$ be zero or tend towards infinity, the situations of total immobility $(\epsilon=0)$ and perfect mobility $(\epsilon \rightarrow \infty)$ of labor emerge as special cases. Second, by combining first-order conditions for labor supply and labor demand, the formulation (10) allows us to estimate precisely the parameter $\epsilon$ for each country in our sample. ${ }^{16}$ Hence, the formulation (10) serves our purpose which is to assess quantitatively the ability of the two-sector model to account for our evidence. Third, as emphasized by Horvath [2000], this formulation introduces partial labor mobility across sectors without deviating from the tractable representative agent framework. Fourth, several papers introduce intersectoral adjustment costs to produce imperfect mobility of labor across sectors (see e.g., Shi [2011]). Such formulation implies that labor frictions are absent in the steady-state while our evidence reveals that sectoral wages do not equalize in the long run. Since we focus on the long-run relative price and relative wage effects of a productivity differential, we need to set up a model that can produce a persistent sectoral wage differential. The aggregator function (10) is consistent with our objective.

The representative agent is endowed with one unit of time, supplies a fraction $L(t)$ as labor, and consumes the remainder $1-L(t)$ as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Assuming that the felicity function is additively separable in consumption and labor, the representative household maximizes the following objective function: ${ }^{17}$

$$
\begin{equation*}
U=\int_{0}^{\infty}\left\{\frac{1}{1-\frac{1}{\sigma_{C}}} C(t)^{1-\frac{1}{\sigma_{C}}}-\frac{1}{1+\frac{1}{\sigma_{L}}} L(t)^{1+\frac{1}{\sigma_{L}}}\right\} e^{-\beta t} \mathrm{~d} t \tag{11}
\end{equation*}
$$

where $\beta$ is the discount rate, $\sigma_{C}>0$ corresponds to the intertemporal elasticity of substitution

[^10]for consumption, and $\sigma_{L}>0$ is the Frisch elasticity of labor supply or intertemporal elasticity of substitution for (aggregate) labor supply.

Labor income is derived by supplying labor at a wage rate $W(t)$. In addition, households accumulate internationally traded bonds, $B(t)$, that yield net interest rate earnings of $r^{\star} B(t)$. The flow budget constraint is equal to households' income less consumption expenditure:

$$
\begin{equation*}
\dot{B}(t)=r^{\star} B(t)+W\left(W^{T}(t), W^{N}(t)\right) L(t)-P_{C}(P(t)) C(t), \tag{12}
\end{equation*}
$$

where the consumption-based price index $P_{C}($.$) is increasing with the relative price of non$ tradables $P$. The aggregate wage index $W($.$) associated with the labor index (10) is:$

$$
\begin{equation*}
W(t)=\left[\vartheta\left(W^{T}(t)\right)^{\epsilon+1}+(1-\vartheta)\left(W^{N}(t)\right)^{\epsilon+1}\right]^{\frac{1}{\epsilon+1}} \tag{13}
\end{equation*}
$$

where $W^{T}(t)$ and $W^{N}(t)$ are wages paid in the traded and the non traded sectors, respectively.
Denoting the co-state variable associated with eq. (12) by $\lambda$ the first-order conditions characterizing the representative household's optimal plans are:

$$
\begin{align*}
C(t) & =\left[P_{C}(t) \lambda(t)\right]^{-\sigma_{C}},  \tag{14a}\\
L(t) & =[\lambda(t) W(t)]^{\sigma_{L}},  \tag{14b}\\
\dot{\lambda}(t) & =\lambda(t)\left(\beta-r^{\star}\right), \tag{14c}
\end{align*}
$$

and the transversality condition $\lim _{t \rightarrow \infty} \lambda B(t) e^{-\beta t}=0$. For the sake of clarity, we drop the time argument below when this causes no confusion.

Applying Shephard's lemma gives $C^{N}=P_{C}^{\prime} C$ where $P_{C}^{\prime}=\partial P_{C} / \partial P$; denoting by $\alpha_{C}$ the share of non-traded goods in the consumption expenditure, we have $C^{N}=\alpha_{C} P_{C} C / P$ and $C^{T}=\left(1-\alpha_{C}\right) P_{C} C .{ }^{18}$ Intra-temporal allocation of consumption follows from the following optimal rule:

$$
\begin{equation*}
\left(\frac{1-\varphi}{\varphi}\right) \frac{C^{T}}{C^{N}}=P^{\phi} . \tag{15}
\end{equation*}
$$

An appreciation in the relative price of non tradables $P$ increases expenditure on tradables relative to expenditure on non tradables (i.e., $C^{T} / P C^{N}$ ), only when $\phi>1$.

As for consumption, intra-temporal allocation of hours worked across sectors follows from Shephard's Lemma. We therefore obtain labor income by supplying hours worked in the non traded and the traded sectors, i.e., $W^{N} L^{N}=\alpha_{L} W L$ and $W^{T} L^{T}=\left(1-\alpha_{L}\right) W L$, with $\alpha_{L}$ being the share of non-tradable labor revenue in the labor income. ${ }^{19}$ Denoting by $\Omega \equiv$

[^11]$W^{N} / W^{T}$ the relative wage, workers allocate hours worked in the traded and the non traded sectors according to the following optimal rule:
\[

$$
\begin{equation*}
\left(\frac{1-\vartheta}{\vartheta}\right) \frac{L^{T}}{L^{N}}=\Omega^{-\epsilon} \tag{16}
\end{equation*}
$$

\]

If the traded sector pays higher wages (i.e., if $\Omega$ falls) workers are induced to shift hours worked towards the traded sector, but less so as $\epsilon$ is lower. Put differently, the worker is reluctant to shift hours worked from the non traded to the traded sector, unless the wage differential across sectors is large enough to compensate for the cost (in utility terms) of moving hours worked across sectors.

### 3.2 Firms

A large number of identical and perfectly competitive firms produces a traded and a non traded good using labor $L^{j}$ as the sole input in a linear (constant returns to scale) technology:

$$
\begin{equation*}
Y^{j}=A^{j} L^{j} \tag{17}
\end{equation*}
$$

where $Y^{j}$ denotes output and $A^{j}$ is the labor productivity index in sector $j$. Since the labor market is assumed to be competitive, the ratio of marginal revenues of labor must equalize the ratio of sectoral wages:

$$
\begin{equation*}
P \frac{A^{N}}{A^{T}}=\Omega . \tag{18}
\end{equation*}
$$

According to (18), the non traded wage falls relative to the traded wage if the relative price of non tradables appreciates less than the productivity differential between tradables and non tradables.

### 3.3 Model Closure and Equilibrium

To fully describe the equilibrium, we impose goods market clearing conditions. The non traded good market clearing condition requires that output is equalized with consumption:

$$
\begin{equation*}
Y^{N}=C^{N} \tag{19}
\end{equation*}
$$

Plugging this condition into the flow budget constraint (12) and using firms' optimal conditions yields the market clearing condition for tradables or the current account equation:

$$
\begin{equation*}
\dot{B}=r^{\star} B+Y^{T}-C^{T}=r^{\star} B+N X \tag{20}
\end{equation*}
$$

where the second term on the RHS, i.e., $Y^{T}-C^{T} \equiv N X$, corresponds to net exports.
In an open economy model with a representative agent having perfect foresight, a constant rate of time preference and perfect access to world capital markets, we impose $\beta=r^{\star}$ in order
to generate an interior solution. Setting $\beta=r^{\star}$ into (14c) yields $\lambda=\bar{\lambda}$. As the shadow value of wealth must remain constant over time, invoking the transversality condition and denoting the long-term value with a tilde implies that the intertemporal solvency condition reduces to:

$$
\begin{equation*}
\tilde{B}=B_{0} \tag{21}
\end{equation*}
$$

where $B_{0}$ is the initial stock of traded bonds. Because $B(t)=B_{0}$ must hold at each point of time, the dynamics degenerate so that the economy adjusts instantaneously to its steady-state.

The equilibrium which comprises (15)-(16), (18), and (19)-(21), can be reduced to two equations. Combining the optimal rule for intra-temporal allocation of consumption (15) with market clearing conditions for the non traded and the traded good, i.e., (19)-(20), together with (21), yields the goods market equilibrium (henceforth GME):

$$
\begin{equation*}
\frac{\tilde{Y}^{T}}{\tilde{Y}^{N}}=\left(\frac{\varphi}{1-\varphi}\right) \frac{\tilde{P}^{\phi}}{1+\omega_{B}} \tag{22}
\end{equation*}
$$

where we denote by $\omega_{B} \equiv r^{\star} B_{0} / Y^{T}$ the ratio of interest receipts to traded output.
Using the production functions, i.e., $L^{j}=Y^{j} / A^{j}$, and combining optimal rules for labor supply (16) with labor demand (18) to eliminate $\Omega$, yields the labor market equilibrium (henceforth LME):

$$
\begin{equation*}
\frac{\tilde{Y}^{T}}{\tilde{Y}^{N}}=\left(\frac{\vartheta}{1-\vartheta}\right)\left(\frac{A^{T}}{A^{N}}\right)^{\epsilon+1} \tilde{P}^{-\epsilon} . \tag{23}
\end{equation*}
$$

### 3.4 Graphical Apparatus

Before turning to the derivation of steady-state effects of a productivity differential, we characterize the long-run equilibrium graphically. Because the dynamics degenerate and thus the open economy is always at steady-state, the tilde is suppressed for the purposes of clarity. The long-run equilibrium can be described by considering alternatively the labor market or the goods market. The initial steady-state is represented by $E_{0}$ in Figure 3.

When focusing on the labor market, the model can be summarized graphically by two schedules in the $\left(l^{T}-l^{N}, \omega\right)$-space, as shown in Figure 3(a). Applying logarithm to eq. (16) yields the (relative) labor supply-schedule ( $L S$ henceforth):

$$
\begin{equation*}
\left.\left(l^{T}-l^{N}\right)\right|^{L S}=-\epsilon \omega+d, \tag{24}
\end{equation*}
$$

where $d=\ln \left(\frac{\vartheta}{1-\vartheta}\right)$. When the traded sector pays higher wages, the consecutive decline in $\omega$ provides an incentive to shift labor supply from the non-traded sector towards the traded sector. Hence the $L S$-schedule is downward-sloping in the $\left(l^{T}-l^{N}, \omega\right)$-space where the slope is equal to $-1 / \epsilon$. In the polar case of perfect labor mobility, $\epsilon$ tends towards infinity so
that the $L S$-schedule becomes horizontal. Inserting the first-order conditions for the firm's maximization problem given by eq. (18) into (22), using production functions (17) to eliminate sectoral outputs, yields the labor demand-schedule ( $L D$ henceforth). Formally, the (relative) $L D$-schedule is given by:

$$
\begin{equation*}
\left.\left(l^{T}-l^{N}\right)\right|^{L D}=\phi \omega+(\phi-1)\left(a^{T}-a^{N}\right)+x, \tag{25}
\end{equation*}
$$

where $x=\ln \left(\frac{\varphi}{1-\varphi} \frac{1}{1+\omega_{B}}\right)$. The $L D$-schedule is upward-sloping in the $\left(l^{T}-l^{N}, \omega\right)$-space where the slope is equal to $1 / \phi$. If the non-traded sector pays higher wages, that sector raises its prices to compensate for the increased unit labor cost. As a result, consumers substitute traded for non-traded goods which in turn produces an expansionary effect on labor demand in the traded sector relative to the non-traded sector (i.e., $L^{T} / L^{N}$ rises), and all the more so the larger the elasticity of substitution $\phi$ between traded and non-traded goods.

We turn now to the goods market which can be summarized graphically by two schedules in the $\left(y^{T}-y^{N}, p\right)$-space, as shown in Figure 3(b). The $G M E$-equilibrium (see (22)) is upward-sloping in the $\left(y^{T}-y^{N}, p\right)$-space with a slope equal to $1 / \phi$. Intuitively, a rise in traded output relative to non traded output produces an appreciation in the relative price of non tradables which induces agents to substitute the traded good for the non traded. The $L M E$-schedule (see (23)) is downward-sloping in the $\left(y^{T}-y^{N}, p\right)$-space with a slope equal to $-1 / \epsilon$. A rise in the relative price of non-tradables $p$ raises the non traded wage which in turn induces workers to shift hours worked to the non traded sector. As a consequence, the ratio of sectoral outputs $Y^{T} / Y^{N}$ declines. Assuming that hours worked are perfect substitutes (i.e., $\epsilon \rightarrow \infty)$, sectors pay the same wage. Graphically, the LME-schedule becomes an horizontal line.
< Please insert Figure 3 about here >

### 3.5 Relative Price and Relative Wage Effects

This section analyzes graphically and analytically the consequences on the relative price and the relative wage of an increase in relative sectoral productivity $A^{T} / A^{N}$. It compares the steady-state of the model before and after the productivity shock biased towards the traded sector.

To begin with, an inspection of eq. (25) shows that higher productivity in tradables relative to non-tradables has an expansionary effect on labor demand in the traded sector relative to the non traded sector, if and only if the elasticity of substitution $\phi$ between traded and nontraded goods is larger than one. The reason is as follows. Higher productivity in tradables and the labor inflow in that sector increases output of tradables relative to non-tradables.

For the market clearing condition to hold (see eq. (22)), the relative price of non-tradables must rise. With an elasticity of substitution $\phi$ greater than one, the demand for tradables rises more than proportionally. The increased share of tradables in total expenditure has an expansionary effect on labor demand in tradables relative to non tradables and therefore lowers the relative wage $\omega$ (see eq. (18)). Graphically, as shown in Figure 3(a), the $L D$ schedule shifts to the right along the $L S$-supply schedule which produces a fall in relative wage from $\omega_{0}$ to $\omega_{1}$. Because the traded sector pays higher wages, workers shift hours worked towards that sector (see eq. (16)).

To determine the change in the relative wage formally, we equate labor demand described by (25) and labor supply described by (24). Differentiating and denoting by a hat the deviation from initial steady-state in percentage terms gives the relative wage growth:

$$
\begin{equation*}
\hat{\omega}=-(\phi-1) \Theta^{L}\left(\hat{a}^{T}-\hat{a}^{N}\right), \quad \Theta^{L}=\left(\frac{1}{\epsilon+\phi}\right) \tag{26}
\end{equation*}
$$

According to (26), higher productivity growth in tradables relative to non tradables produces a fall in the ratio of the non-traded wage to the traded wage if and only if $\phi>1$. In terms of Figure $3(\mathrm{a})$, the new steady-state $\left(E_{1}\right)$ lies to the south east of the old equilibrium $\left(E_{0}\right)$ along the initial $L S$-schedule. By contrast, when the elasticity of substitution $\phi$ is smaller than one, demand for tradables increases less than proportionately, and therefore the share of tradables in total expenditure falls. In this case, graphically, higher productivity growth in tradables relative to non tradables shifts the $L D$-schedule to the left which results in an increase in the relative wage $\omega$. The second parameter which plays a major role in the determination of changes in $\omega$ is $\epsilon$ which captures the degree of labor mobility across sectors. As workers are less reluctant to shift hours worked from the non-traded to the traded sector, as reflected by a higher $\epsilon$, the response of the relative wage to a productivity differential is moderated. Graphically, the $L D$-schedule shifts along a flatter $L S$-schedule. When labor is perfectly mobile, as in the standard BS model, $\epsilon$ tends toward infinity so that (26) implies that $\hat{\omega}=0$. In terms of Figure 3(a), the $L D$-schedule shifts along a horizontal $L S$-schedule, thus leaving unchanged the relative wage at $\omega_{0}$ while the intersect of the two schedules is at $B S_{1}$.

Having explored the change in the relative wage, let us now examine the response of the relative price of non-tradables. Equating (22) to (23), taking logarithm, and differentiating leads to the long-run adjustment of the relative price in percentage terms:

$$
\begin{equation*}
\hat{p}=(\epsilon+1) \Theta^{L}\left(\hat{a}^{T}-\hat{a}^{N}\right) \tag{27}
\end{equation*}
$$

According to (27), the relative price increases by less than $1 \%$ following a 1 percentage point increase in the productivity differential only when $\phi>1$. Intuitively, because consumers are
relatively more prone to substitute the traded for the non traded good, the relative price of non tradables must appreciate less than proportionately to clear the goods market as traded output relative to non traded output. Graphically, the $L M E$-schedule shifts to the right as shown in Figure $3(\mathrm{~b})$. When $\phi>1$, the $G M E$-schedule is flatter than the $45^{\circ}$ line, so that the new steady-state $E_{1}$ is below the long-run equilibrium $B S_{1}$ reached by assuming perfect labor mobility. Moreover, as workers are more willing to switch from one sector to another (i.e., as $\epsilon$ takes higher values), the relative price of non tradables appreciates by a larger amount because traded output increases more. Graphically, the $L M E$-schedule becomes flatter. When labor is perfectly mobile across sectors, the $L M E$-schedule become a horizontal line (see Figure $3(\mathrm{~b})$ ). Consequently, higher productivity growth of tradables relative to non tradables by $1 \%$ shifts the $L M E$-schedule higher, thus leading to an appreciation in the relative price by $1 \%$.

In conclusion, the two-sector model can account for the fall in the relative wage and the less than proportional increase in the relative price of non-tradables after a 1 percentage point increase in the productivity differential between tradables and non tradables as long as labor is imperfectly mobile across sectors (i.e., $\epsilon<\infty$ ) and the elasticity of substitution $\phi$ is larger than one. Because previous empirical studies and our estimates reveal that the elasticity substitution may take values smaller than one, in the following section we add a new ingredient to improve the predictive power of our model. ${ }^{20}$

## 4 Introducing Physical Capital

We introduce physical capital in the framework which is assumed to be perfectly mobile across sectors. As will become clear later, this ingredient makes the two-sector model able to account for the evidence related to the effects of sectoral productivity shocks, as long as imperfect mobility of labor across sectors is assumed. For the purpose of clarity, we assume that investment expenditures are non traded. ${ }^{21}$ For reasons of space and since the BalassaSamuelson effect is a long-run phenomenon, we restrict ourselves to the discussion of the steady-state. We drop the time argument to refer to long-run values.

Before discussing the steady-state, we emphasize very briefly how introducing capital modifies the framework. First, households' factor income is derived by supplying labor $L(t)$

[^12]at a wage rate $W(t)$, and capital $K(t)$ at a rental rate $R(t)$. To rent capital, agents must invest an amount $I(t)$, thus giving rise to capital accumulation $\dot{K}(t)=I(t)-\delta_{K} K(t)$, where $0 \leq$ $\delta_{K}<1$ is a fixed depreciation rate. First-order conditions characterizing the representative household's optimal plans include (14) and the equality $R(t) / P(t)-\delta_{K}+\dot{P}(t) / P(t)=r^{\star}$ which states that the return on domestic capital must equalize the return on traded bonds. Second, firms produce according to constant returns to scale technology (1) while aggregate capital $(K(t))$ is allocated to the traded $\left(K^{T}(t)\right)$ and the non traded sector $\left(K^{N}(t)\right)$, i.e., $K(t)=K^{T}(t)+K^{N}(t)$.

### 4.1 The Steady-State

Considering the technology of production described by (1), since capital can move freely between the two sectors, marginal revenues of capital in the traded and the non-traded sector equalize while costly labor mobility implies a persistent wage differential across sectors:

$$
\begin{gather*}
Z^{T}\left(1-\theta^{T}\right)\left(k^{T}\right)^{-\theta^{T}}=P Z^{N}\left(1-\theta^{N}\right)\left(k^{N}\right)^{-\theta^{N}} \equiv R,  \tag{28a}\\
Z^{T} \theta^{T}\left(k^{T}\right)^{1-\theta^{T}} \equiv W^{T}, \quad P Z^{N} \theta^{N}\left(k^{N}\right)^{1-\theta^{N}} \equiv W^{N}, \tag{28b}
\end{gather*}
$$

where we denote by $k^{j} \equiv K^{j} / L^{j}$ the capital-labor ratio for sector $j=T, N$, and $\theta^{j}$ represents the labor income share in output of sector $j$. Using the fact that $K^{j}=k^{j} L^{j}$, the resource constraint for capital can be written as follows:

$$
\begin{equation*}
k^{T} L^{T}+k^{N} L^{N}=K \tag{29}
\end{equation*}
$$

When the relative price of non-tradables adjusts to its steady-state value (i.e., $\dot{P}=0$ ), we obtain the equality between the return on domestic capital and the world interest rate:

$$
\begin{equation*}
R / P-\delta_{K}=r^{\star}, \tag{30}
\end{equation*}
$$

where $R$ is given by (28a). Additionally, both non traded and traded goods markets must clear, i.e., $Y^{N}=C^{N}+I$ with $I=\delta_{K} K$ and $Y^{T}+r^{\star} B=C^{T}$. Denoting by $v_{I} \equiv \delta_{K} K / Y^{N}$ and $v_{B} \equiv r^{\star} B / Y^{T}$ the ratio of investment to non traded output and the ratio of interest receipts to traded output, respectively, the market-clearing condition can be written as follows:

$$
\begin{equation*}
\frac{Y^{T}\left(1+v_{B}\right)}{Y^{N}\left(1-v_{I}\right)}=\frac{C^{T}}{C^{N}}, \tag{31}
\end{equation*}
$$

where the allocation of aggregate consumption expenditure between traded and non traded goods follows from (15).

Finally, the open economy must satisfy the intertemporal solvency condition:

$$
\begin{equation*}
B-B_{0}=\Phi\left(K-K_{0}\right), \tag{32}
\end{equation*}
$$

where $\Phi \equiv\left[\partial Y^{T} / \partial K+\left(\partial Y^{T} / \partial P-\partial C^{T} / \partial P\right) \omega_{2}^{1}\right] /\left(\nu_{1}-r^{\star}\right)<0$, with all partial derivatives evaluated at the steady-state and $\omega_{2}^{1}$ the element of the eigenvector associated with the stable eigenvalue $\nu_{1}<0$, and $B_{0}, K_{0}$ are the initial stocks of traded bonds and physical capital, respectively. ${ }^{22}$

Using production functions (1), the system consisting of (15)-(16), (30)-(31), and (28a)(28b) can be solved for $C^{T} / C^{N}, L^{T} / L^{N}, k^{T}, k^{N}, W^{T}, W^{N}$ and $P$ as functions of $Z^{T}, Z^{N},\left(\frac{1-v_{I}}{1+v_{B}}\right)$ which is taken as exogenous for pedagogical purposes. ${ }^{23}$ Hence, when solving the steady-state in this way, we thus assume that the capital stock and traded bonds holding are exogenous. This procedure to solve for the steady-state enables us to break down analytically the relative price and relative wage effects of a productivity differential between tradables and non tradables in three components: i) a productivity channel when keeping sectoral capital-labor ratios $k^{j}$ and the capital stock $K$ (and thus $B$ ) unchanged, ii) a capital reallocation channel stemming from the shift of capital across sectors, iii) a capital accumulation channel caused by the investment boom along the transitional path. We build intuition about these three channels below. ${ }^{24}$

Besides the productivity channel discussed in section 3, introducing physical capital produces two additional channels through which higher productivity of tradables relative to non tradables impinges on the relative price and the relative wage.

First, changes in sectoral TFPs shift capital across sectors (i.e., modify $k^{j}$ ) and thus influence the relative price by modifying sectoral outputs. ${ }^{25}$ Further, as shown by (28b), a change in the relative price influences labor demand in the non traded sector and thereby the relative wage. Therefore, keeping unchanged the overall capital stock $K$ (and the stock of foreign bonds $B$ ), the capital reallocation channel impinges on the relative price and the relative wage by shifting capital across sectors.

Second, households hold financial wealth which consists of physical capital and foreign bonds. A productivity shock increases the marginal product of capital above the rate of return on traded bonds which triggers capital accumulation. Because the economy has perfect access

[^13]to external borrowing, capital accumulation can be financed by running a current account deficit along the transitional path. For the intertemporal solvency condition to hold, the country must run a trade balance surplus in the long run. Increased net exports raise the demand for tradables which in turn impinges on the relative price and the relative wage. Hence, compared with a model abstracting from physical capital, a productivity differential affects $P$ and $\Omega \equiv W^{N} / W^{T}$ through a capital accumulation channel stemming from changes in $K$ and $B$.

### 4.2 Relative Price and Relative Wage Effects

Before turning to the numerical analysis, we analytically break down the relative price and relative wage effects of a productivity differential between tradables and non tradables in three components.

We first explore the relative wage effect of a productivity differential by equating relative labor supply (24) and relative labor demand to eliminate $l^{T}-l^{N} .{ }^{26}$ Differentiating, and noting that $v_{B}=-v_{N X}$ where we denote by $v_{N X} \equiv\left(Y^{T}-C^{T}\right) / Y^{T}$ the ratio of net exports to traded output, yields the deviation in percentage of the relative wage from its initial steadystate: ${ }^{27}$

$$
\begin{equation*}
\hat{\omega}=-(\phi-1)\left[\Theta^{L}+\left(\Theta^{K}-\Theta^{L}\right)\right]\left[\hat{z}^{T}-\left(\theta^{T} / \theta^{N}\right) \hat{z}^{N}\right]-\Theta^{K}\left(\mathrm{~d} v_{N X}-\mathrm{d} v_{I}\right), \tag{33}
\end{equation*}
$$

where $\Theta^{K} \equiv \frac{1}{\left[(\epsilon+1)+\theta^{T}(\phi-1)\right]}>0, \Theta^{L}=\left(\frac{1}{\epsilon+\phi}\right)>0($ see $(26))$, and $\hat{z}^{T}-\left(\theta^{T} / \theta^{N}\right) \hat{z}^{N}$ is the labor share-adjusted TFP differential.

Eq. (33) breaks down $\hat{\omega}$ into three components. Setting the labor income share $\theta^{T}$ to 1 in (33) implies $\Theta^{K}=\Theta^{L}$ and $\mathrm{d} v_{N X}=\mathrm{d} v_{I}=0$. Hence, when abstracting from physical capital accumulation, (33) reduces to $\hat{\omega}=-(\phi-1) \Theta^{L}\left(\hat{z}^{T}-\hat{z}^{N}\right)$ (see eq. (26)). In this case, the relative wage is only affected through the productivity channel, as captured by $-(\phi-1) \Theta^{L} \lessgtr 0$. In a model abstracting from physical capital, the relative wage falls only when the elasticity of substitution in consumption, $\phi$, between traded and non traded goods is larger than one since only in this case does the share of tradables rise.

[^14]Eq. (33) reveals that introducing physical capital (i.e., $\theta^{j}<1$ ) produces two additional effects on the relative wage. First, the effect of a productivity differential on $\omega$ stemming from the shift of capital across sectors is captured by the term $-(\phi-1)\left(\Theta^{K}-\Theta^{L}\right)<0$, as shown in the RHS of (33). Hence the capital reallocation channel exerts a negative impact on $\omega$ irrespective of whether $\phi>1$ or $\phi<1$. If $\phi<1$, the shift of capital towards the non traded sector lowers $p$ by raising non traded output; the fall in the relative price exerts a negative impact on the marginal product of labor in the non traded sector and therefore on the relative wage. When $\phi>1$, the productivity differential shifts capital towards the traded sector which raises the marginal product of labor in this sector and thus reduces the relative wage further. In terms of Figure $3(\mathrm{a})$, when considering capital, the $L D$-schedule becomes steeper (if $\phi>1$ ) so that it intercepts the $L S$-schedule for a relative wage below $\omega_{1}$.

Second, when introducing physical capital, the productivity differential impinges on $\hat{\omega}$ through a capital accumulation channel, as captured by $-\Theta^{K}\left(\mathrm{~d} v_{N X}-\mathrm{d} v_{I}\right)<0$. Because higher productivity raises the rate of return on domestic capital, it is optimal for the economy to accumulate physical capital by running a current account deficit which must be matched in the long run by a trade balance surplus. Further, the improvement in the trade balance must exceed the investment boom because along the transitional path, the current account deficit is induced by the combined effect of capital accumulation and reduced savings. ${ }^{28}$ Formally, we have $\mathrm{d} v_{N X}-\mathrm{d} v_{I}>0$. Higher steady-state net exports raise demand for tradables, with an expansionary effect on labor demand in the traded sector, thereby lowering $\omega$. Graphically, in terms of Figure 3(a), the capital accumulation channel shifts the $L D$-schedule to the right, regardless of the value of the elasticity of substitution between traded and non traded goods.

We now explore the long-run response of the relative price of non tradables to a productivity differential by equating demand (15) and supply of tradables in terms of non tradables to eliminate $y^{T}-y^{N} .{ }^{29}$ Differentiating yields the deviation in percentage of the relative price from its initial steady-state:

$$
\begin{equation*}
\hat{p}=(1+\epsilon)\left[\Theta^{L}+\left(\Theta^{K}-\Theta^{L}\right)\right]\left[\hat{z}^{T}-\left(\theta^{T} / \theta^{N}\right) \hat{z}^{N}\right]-\theta^{T} \Theta^{K}\left(\mathrm{~d} v_{N X}-\mathrm{d} v_{I}\right) \tag{34}
\end{equation*}
$$

When assuming perfect mobility of labor across sectors as in the standard BS model, (34) reduces to $\hat{p}=\hat{z}^{T}-\left(\theta^{T} / \theta^{N}\right) \hat{z}^{N}$.

[^15]Conversely, assuming imperfect mobility of labor across sectors while abstracting from physical capital accumulation (by setting $\theta^{j}=1$ ), (34) reduces to $\hat{p}=(1+\epsilon) \Theta^{L}\left[\hat{z}^{T}-\hat{z}^{N}\right]$. In this case, only the productivity channel, reflected by $(1+\epsilon) \Theta^{L}>0$, is in effect (see eq. (27)). According to the productivity channel, a 1 percentage point increase in the productivity differential raises the relative price of non tradables less (more) than proportionately if the elasticity of substitution $\phi$ is larger (smaller) than one.

Introducing physical capital produces two additional channels through which a productivity differential may impinge on the relative price of non tradables. First, the effect of a productivity differential on $p$ stemming from the shift of capital across sectors is captured by the term $(1+\epsilon)\left(\Theta^{K}-\Theta^{L}\right) \gtrless 0$, as shown in the RHS of (34), depending on whether $\phi \gtrless 1$. Hence, the capital reallocation channel may reinforce the increase in $p$ triggered by the productivity channel if $\phi>1$ or may moderate it if $\phi<1$. In the latter case, capital shifts towards the non traded sector, thereby raising output in that sector, which lowers $p$. When $\phi>1$, the shift of capital towards the traded sector amplifies the increase in $p$ by raising the marginal product of labor (and hence wages in that sector) so that the consecutive labor inflow raises traded output.

When introducing physical capital, a productivity differential also impinges on $p$ through a capital accumulation channel captured by $-\theta^{T} \Theta^{K}\left(\mathrm{~d} v_{N X}-\mathrm{d} v_{I}\right)<0$ (see the second term on the RHS of (34)). The capital accumulation channel always exerts a negative impact on $p$. As mentioned above, the long-run improvement in the trade balance raises the demand for tradables which produces a fall in $p$. Graphically, in terms of Figure 3(b), the capital accumulation channel shifts the $G M E$-schedule to the right, regardless of the value of $\phi$.

To conclude, we have to consider two cases depending on whether the elasticity of substitution between traded and non traded goods is larger or smaller than one:

- If $\phi>1$, when abstracting from physical capital, higher productivity growth in tradables relative to non tradables lowers the relative wage and increases the relative price of non tradables less than proportionately, in line with our evidence. Introducing physical capital exerts two opposite effects on the relative price while both channels reduce the relative wage. First, a productivity differential induces a shift of capital towards the traded sector, which pushes up the relative price and lowers the relative wage. Second, increased demand for tradables due to the long-run trade balance surplus drives down both the relative price of non tradables and the relative wage.
- When $\phi<1$, a model without physical capital predicts that higher productivity growth in tradables relative to non tradables raises the relative wage and more than proportion-
ately increases the relative price, in contradiction to our evidence. Introducing physical capital produces two novel channels which lower the relative price and the relative wage. First, by shifting capital towards the non traded sector, a productivity differential exerts a negative impact on $p$ and $\omega$. Second, the trade balance surplus further reduces $p$ and $\omega$.

While in the latter case (i.e., $\phi<1$ ), the capital reallocation and accumulation channels counteract the productivity channel, we have to determine numerically if they are large enough to produce a decline in the relative wage and a less than proportional increase in the relative price following a productivity differential between tradables and non tradables.

## 5 Quantitative Analysis

In this section, we analyze the effects of a 1 percentage point increase in the labor shareadjusted TFP differential quantitatively. For this purpose we solve the model numerically. ${ }^{30}$ Therefore, first we discuss parameter values before turning to the long-term consequences of higher productivity growth in tradables relative to non tradables.

### 5.1 Calibration

To calibrate our model, we estimated a set of parameters so that the initial steady-state is consistent with the key empirical properties of a representative OECD economy. While at the end of the section we move a step further and calibrate the model for each economy, we first have to evaluate the ability of the two-sector open economy model with physical capital and imperfect mobility of labor to accommodate the less than proportional increase in the relative price and the decline in the relative wage. Our sample covers the fourteen OECD economies in our dataset. Our reference period for the calibration corresponds to the period 1990-2007. Since we calibrate a two-sector model with tradables and non tradables, we pay particular attention to the adequacy of the non-tradable content of the model to the data. Table 7 in Appendix B summarizes our estimates of the non-tradable content of GDP, employment, consumption, and gives the share of government spending on the traded and non traded goods in the sectoral output, the shares of labor income in output in both sectors, for all countries in our sample. ${ }^{31}$ Targeted ratios when calibrating to the representative OECD economy are the fourteen OECD countries' unweighted average shown in the last line of Table 7.

[^16]We start by describing the calibration of consumption-side parameters that we use as a baseline. The world interest rate which is equal to the subjective time discount rate $\beta$ is set to $4 \%$. One period of time corresponds to a year. In light of our discussion above, both $\epsilon$ and $\phi$ play a key role in the determination of the relative price and the relative wage responses to a productivity differential. ${ }^{32}$ Building on our panel data estimations, we set the elasticity of substitution to 1 in the baseline calibration but conduct a sensitivity analysis by considering alternatively a value of $\phi$ smaller or larger than one (i.e., $\phi$ is set to 0.5 and 1.5 , respectively). ${ }^{33}$

The degree of labor mobility captured by $\epsilon$ is set to 0.8 which corresponds roughly to the average of our estimates shown in column 1 of Table 6 in Appendix B. Our estimates display a wide dispersion across countries and we therefore conduct a sensitivity analysis with respect to this parameter. Excluding the estimate of $\epsilon$ for Denmark which is not statistically significant at $10 \%$, estimates of $\epsilon$ range from a low of 0.22 for the Netherlands to a high of 1.80 for the United States. Hence, we allow for $\epsilon$ to vary between 0.2 and 1.8 in the sensitivity analysis.

The weight of consumption in non tradables $1-\varphi$ is set to 0.43 to target a non-tradable content in total consumption expenditure (i.e., $\alpha_{C}$ ) of $43 \%$, in line with the average of our estimates shown in the last line of Table 7. The intertemporal elasticity of substitution for consumption $\sigma_{C}$ is set to 1 . In our baseline parametrization, we set the intertemporal elasticity of substitution for labor supply $\sigma_{L}$ to 0.5 , in line with evidence reported by Domeij and Flodén [2006], but conduct a sensitivity analysis with respect to this parameter. The weight of labor supply to the non traded sector, $1-\vartheta$, is set to 0.6 to target a non-tradable content of labor compensation of $65 \%$, in line with the average of our estimates shown in the last line of Table 7.

We now describe the calibration of production-side parameters. We assume that physical capital depreciates at a rate $\delta_{K}=6 \%$ to target an investment-GDP ratio of $20 \%$. The shares of sectoral labor income in output take two different values depending on whether the traded sector is more or less capital intensive than the non traded sector. If $k^{T}>k^{N}$, labor shares in the traded $\left(\theta^{T}\right)$ and the non traded sector $\left(\theta^{N}\right)$ are set to 0.6 and 0.7 , respectively, which correspond roughly to the averages for countries with $k^{T}>k^{N}$. When $k^{N}>k^{T}$, we use

[^17]reverse but symmetric values, i.e., $\theta^{T}=0.7$ and $\theta^{N}=0.6$. As in Ghironi and Melitz [2005], we assume that traded firms are 50 percent more productive than non traded firms; hence we set $Z^{T}$ and $Z^{N}$ to 1.5 and 1 , respectively.

For calibration purposes, we introduce government spending on traded and non traded goods in the setup. We set $G^{N}$ and $G^{T}$ so as to yield a non-tradable share of government spending of $90 \%$, and government spending as a share of GDP of $20 \% .^{34}$ In line with the averages of the values reported in the last line of Table 7 , the ratios $G^{T} / Y^{T}$ and $G^{N} / Y^{N}$ are $5 \%$ and $28 \%$ in the baseline calibration.

We consider a permanent increase in the TFP index $Z^{j}$ of both sectors biased towards the traded sector so that the labor share-adjusted productivity differential between tradables and non tradables, i.e., $\hat{z}^{T}-\left(\theta^{T} / \theta^{N}\right) \hat{z}^{N}$, is $1 \%$. While in our baseline calibration we set $\phi=1, \epsilon=0.8, \sigma_{L}=0.5, \theta^{T}=0.6$, we conduct a sensitivity analysis with respect to these four parameters by setting alternatively: $\phi$ to 0.5 and $1.5, \epsilon$ to 0.2 and $1.8, \sigma_{L}$ to 0.2 and 1 , and the sectoral labor income share $\theta^{T}$ to 0.7 .

### 5.2 Discussion

The relative price and relative wage responses are summarized in Table 4. For comparison purposes, column 1 summarizes our empirical evidence for the whole sample. Since a twosector model (with imperfect mobility of labor) abstracting from physical capital accumulation fails to account for the evidence when the elasticity of substitution between traded and non traded goods is smaller than one, we first discuss the numerical results in this configuration. Panels C and D of Table 4 report the long-run changes (in percentage) for the (log) relative price of non traded goods $p$ and the (log) relative wage $\omega \equiv w^{N}-w^{T}$. The numbers reported in the first line of each panel give the (overall) responses of these variables to a 1 percentage point increase in the productivity differential between tradables and non tradables.

Column 2 of Table 4 shows that the standard two-sector model assuming perfect mobility of labor across sectors predicts an unchanged relative wage and an increase in the relative price of $1 \%$. While the standard BS model fails to account for the evidence, the predictive power of the two-sector model improves when we introduce two ingredients: imperfect mobility of labor across sectors and physical capital. More precisely, the results summarized in column 3 for the benchmark scenario reveal that $\omega$ falls by $0.24 \%$ while $p$ increases by $0.85 \%$.

To emphasize the key role of physical capital in improving the predictive power of the model, it is useful to break down the responses of the relative wage and relative price into three

[^18]components: a productivity effect, a capital reallocation effect, and a capital accumulation effect stemming from changes in the overall capital stock and therefore in net exports. When breaking down the effects, the second line of panel D shows that a model abstracting from physical capital predicts an increase in $p$ by $1.38 \%$. The reason is that when the elasticity of substitution $\phi$ is smaller than one, the relative price must increase more than proportionately to clear the goods market. Moreover, as shown in the second line of panel C, the relative wage increases instead of decreasing as expenditure on non tradables rises relative to expenditure on tradables, therefore producing an expansionary effect on labor demand in the non traded sector.

The third line and the fourth line of panels C and D show that both the capital reallocation and capital accumulation channels counteract the productivity channel. More precisely, the third line of panel D reveals that the capital reallocation channel produces a fall in the relative price of non tradables (by $0.18 \%$ ) by raising non traded output as capital shifts towards the non traded sector. The decline in $p$ lowers the marginal product of labor in the non traded sector, which reduces $\omega$ (by $0.05 \%$ ), as shown in the third line of panel C. Productivity growth in tradables relative to non tradables also lowers $p$ and $\omega$ through the capital accumulation channel. More precisely, the long-run improvement in the trade balance raises the demand for tradables, which substantially reduces the relative price by $0.34 \%$, as shown in the fourth line of panel D. Additionally, to hire more workers, the traded sector must pay higher wages which significantly drives down the relative wage by $0.57 \%$. Importantly, numerical results show that both the capital reallocation and accumulation channels are large enough to produce a less than proportional increase in $p$ and a decline in $\omega$, in line with the evidence established in section 2.

Columns 6 and 7 of Table 4 reveal that the degree of labor mobility substantially modifies the results. As labor mobility increases (i.e., $\epsilon$ is raised from 0.2 to 1.8), the first line of panel $C$ and panel $D$ indicates that $p$ increases more while $\omega$ falls less, in line with the evidence documented in section 2. Introducing physical capital plays a key role in accommodating the data. As shown in the fourth line of panel $C$ and $D$, raising labor mobility across sectors significantly moderates the capital accumulation channel. As workers are more willing to shift hours worked across sectors, traded wages increase by a smaller amount, dampening the decline in $\omega$ from $-0.87 \%$ to $-0.35 \%$. Because traded output increases by a larger amount as $\epsilon$ is raised from 0.2 to 1.8 , the relative price must fall less to clear the goods market.

As shown in columns 4 and 5, the elasticity of labor supply merely affects the results by modifying the capital accumulation channel. ${ }^{35}$ Finally, columns 8 and 9 of Table 4 show

[^19]results when it is assumed that the non traded sector is more capital intensive than the traded sector. If the assumption of perfect mobility across sectors is imposed, the responses of $\omega$ and $p$ shown in column 8 are unchanged compared with those displayed in column 2 where we impose $k^{T}>k^{N}$. When assuming imperfect labor mobility, a comparison of the responses of $\omega$ and $p$ in column 9 with those shown in column 3 indicates that our results are robust to sectoral capital intensities. ${ }^{36}$

Let us briefly discuss the scenario of an elasticity of substitution between $C^{T}$ and $C^{N}$ larger than one. As shown in column 3 of Table 4, the second line of panels E and F reveals that a model abstracting from physical capital predicts the responses of $\omega$ and $p$ estimated empirically fairly well. However, as shown in the first line of panel E , the model tends to overstate the decline in the relative wage because both the capital reallocation and the capital accumulation channels reinforce the productivity channel.

Finally, we explore the relative wage and relative price effects when the elasticity of substitution between tradables and non tradables is set to one. This case is shown in panel A and panel B of Table 4. The second line reveals that a model with imperfect labor mobility across sectors abstracting from physical capital, yields identical results to those obtained in the standard BS framework assuming $\epsilon \rightarrow \infty$, because the share of tradables in total expenditure remains unchanged. However, by producing a trade balance surplus in the long run, the model with physical capital and imperfect mobility of labor lowers $\omega$ and exerts a negative impact on $p$ (thus producing a less than proportional increase in $p$ ), as shown in the fourth line of panel A and panel B of Table 4. Alternative scenarios yield similar results to those discussed above and therefore do not merit further comment.

$$
<\text { Please insert Table } 4 \text { about here }>
$$

### 5.3 Taking the Model to the Data

We now compare the predicted values for $\hat{p}$ and $\hat{\omega}$ with empirical estimates of $\beta$ and $\gamma$ for each country and the whole sample. To do so, we keep the baseline calibration unchanged, except for the parameter capturing the degree of labor mobility across sectors (i.e., $\epsilon$ ) and the elasticity of substitution between traded and non traded goods (i.e., $\phi$ ) which play a major

[^20]role in the determination of responses of $p$ and $\omega .{ }^{37}$ When numerically computing $\hat{\omega}_{i}$ and $\hat{p}_{i}$ for each country $i$, we set $\epsilon_{i}$ and $\phi_{i}$ in accordance with their empirical estimates shown in columns 1 and 2 of Table 5 . When contrasting predicted with empirically estimated values for $\hat{p}$ and $\hat{\omega}$ for the whole sample, we set $\epsilon$ to 0.61 and $\phi$ to 0.66 which correspond to their estimates for the whole sample, i.e., when assuming $\epsilon_{i}=\epsilon$ and $\phi_{i}=\phi$.

Results are shown in Table 5. Columns 3 and 6 of Table 5 give the predicted responses of $\hat{p}$ and $\hat{\omega}$ to a 1 percentage point increase in the productivity differential between tradables and non tradables. Columns 4 and 7 report fully modified OLS estimates of $\hat{p}$ and $\hat{\omega}$ for each country and the whole sample. ${ }^{38}$ Column 5 gives the ratio between the actual and the predicted value for the relative price response; when the ratio is smaller (larger) than one, the model tends to overstate (understate) the actual values. As shown in the last line of Table 5 , for the whole sample, our two-sector model with imperfect mobility of labor across sectors predicts the actual response (i.e., $0.78 \%$ ) of the relative price remarkably well. Column 5 also reveals that our model's predictions for $\hat{p}$ are close to the evidence, i.e., the error does not exceed $10 \%$, for nine of the countries in our sample, and to a lesser extent for Belgium, Finland and Korea.
< Please insert Table 5 about here >
When we turn to the relative wage response, we find that the model tends to substantially overstate the response of the relative wage. For the whole sample, the model predicts a decline in $\omega$ by $0.35 \%$ while the relative wage is found to fall by $0.27 \%$ in the data. The difference between actual and predicted values reported in the last column reveals that the two-sector model is able to predict the relative wage response for six countries pretty well, including Germany, Italy, Korea, Spain, Sweden, and the United Kingdom and to a lesser extent for France, Japan and the United States. By and large, the figures shown in the last column of Table 5 indicate that the model tends to overstate the decline in the relative wage as the difference between actual and predicted values shown in the last column is positive for ten countries.

[^21]
## 6 Conclusion

In this paper we have analyzed the relative price and the relative wage effects of higher productivity growth in tradables relative to non tradables in a two-sector small open economy model with imperfect mobility of labor across sectors. To guide our theoretical analysis, we have estimated the responses of the relative price of non tradables and the ratio of the non traded wage to the traded wage to a productivity differential. For a sample covering fourteen OECD countries over the period 1970-2007, three major results emerge. Following a 1 percentage point increase in the productivity differential, i) the relative price increases by $0.78 \%, \mathrm{ii}$ ) the relative wage declines by $0.27 \%$, and iii) the relative price rises more while the relative wage falls less as the degree of labor mobility across sectors increases.

We find analytically that two parameters play a major role in the determination of the relative price and relative wage responses to technological change biased toward the traded sector: the elasticity of substitution in consumption between traded and non traded goods and the extent of the difficulty in reallocating labor across sectors. After estimating these two parameters and calibrating the two-sector model, the numerical results reveal that two ingredients are necessary to account for the set of evidence: imperfect mobility of labor across sectors and physical capital accumulation. Quantitatively, we find that our calibrated and simulated two-sector model predicts the response of the relative price fairly well but tends to overstate the decline in the relative wage.

Our quantitative exercise suggests that further work has to be done to improve the predictive power of the two-sector model regarding the response of the relative wage. We believe that our assumption of perfectly competitive labor markets is too strong; in this regard, extending the setup to labor market frictions in the tradition of Diamond-Mortensen-Pissarides should improve the model's performance. Such a framework also has the advantage of clarifying the nature of switching costs across sectors, in particular induced by labor market regulation, such as firing costs, generous unemployment benefit schemes, or high worker bargaining power. Moreover, while we assume that labor is homogenous, the change in the skill premium could potentially impinge on the relative price and the relative wage of non tradables as long as the low-skilled (or high-skilled) labor income shares are heterogenous across sectors. We leave a further analysis of these issues for future research.


Figure 1: Relative Price and Relative Wage Growth against Productivity Growth Differential. Notes: Figure 1(a) plots the average relative price growth ( $Y$-axis) against the average productivity growth differential between tradables and non tradables ( $X$-axis) while Figure 1 (b) plots the average relative wage growth ( $Y$-axis) against the average productivity growth differential ( $X$-axis) over the period 1970-2007 (1974-2007 for Japan).

Table 1: Panel Unit Root Tests (p-values)

| Test | Stat | Variables |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: |
|  |  | $p$ | $\omega$ | $z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}$ | $p-\left(z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}\right)$ |
| Levin et al. [2002] | t-stat | 0.840 | 0.001 | 0.991 | 0.295 |
| Breitung [2000] | t-stat | 0.730 | 0.520 | 0.592 | 0.138 |
| Im et al. [2003] | $W_{\text {tbar }}$ | 1.000 | 0.398 | 1.000 | 0.930 |
| Maddala and $\mathrm{Wu}[1999]$ | $\mathrm{P}($ ADF-stat $)$ | 1.000 | 0.174 | 0.997 | 0.811 |
|  | $\mathrm{P}($ PP-stat $)$ | 0.982 | 0.250 | 1.000 | 0.911 |
| Hadri [2000] | $Z_{\mu}$ | 0.000 | 0.000 | 0.000 | 0.000 |

Notes: For all tests, except for Hadri [2000], the null of a unit root is not rejected if p-value $\geq 0.05$ at a $5 \%$ significance level. For Hadri [2000], the null of stationarity is rejected if p-value $\leq 0.05$ at a $5 \%$ significance level.


Figure 2: Relative Price against Relative Wage Responses. Notes: Figure plots fully modified OLS estimates of relative price responses to a labor-share adjusted TFPs differential against relative wage responses. FMOLS estimates for each country are taken from Table 3.

Table 2: Panel Cointegration Estimates of $\beta$ and $\gamma$ for the Whole Sample

| Panel A | Dependent variable: Relative wage ( $\omega$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | eq. (6a) |  | eq. (8a) |  |  |  |
|  | (1) |  | (2) |  | (3) |  |
|  | DOLS | FMOLS | DOLS | FMOLS | DOLS | FMOLS |
| $\left(z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}\right)$ | $\begin{gathered} -0.270^{a} \\ (-22.27) \end{gathered}$ | $\begin{gathered} -0.270^{a} \\ (-27.83) \end{gathered}$ | $\begin{gathered} -0.262^{a} \\ (-28.18) \end{gathered}$ | $\begin{gathered} -0.269^{a} \\ (-30.85) \end{gathered}$ | $\underset{(-25.55)}{-0.253^{a}}$ | $\begin{gathered} -0.258^{a} \\ (-29.43) \end{gathered}$ |
| $\left(z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}\right) \times L R(2)$ |  |  | $\underset{(1.78)}{0.212^{c}}$ | $\underset{(1.80)}{0.164^{c}}$ |  |  |
| $\left(z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}\right) \times L R(3)$ |  |  |  |  | ${\underset{(2.80)}{0.162^{a}}}^{2}$ | $\underset{(2.38)}{0.153^{b}}$ |
| $t(\beta)=0$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Number of countries | 14 | 14 | 14 | 14 | 14 | 14 |
| Number of observations | 529 | 529 | 501 | 501 | 487 | 487 |


| Panel B | Dependent variable: Relative price ( $p$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | eq. (6b) |  | eq. (8b) |  |  |  |
|  | (1) |  | (2) |  | (3) |  |
|  | DOLS | FMOLS | DOLS | FMOLS | DOLS | FMOLS |
| $\left(z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}\right)$ | $\underset{(91.83)}{0.782^{a}}$ | $\begin{aligned} & 0.779^{a} \\ & (108.91) \end{aligned}$ | $\begin{aligned} & 0.796^{a} \\ & (107.34) \end{aligned}$ | $\begin{aligned} & 0.780^{a} \\ & (118.24) \end{aligned}$ | $\begin{aligned} & 0.802^{a} \\ & (105.21) \end{aligned}$ | $\begin{aligned} & 0.788^{a} \\ & (118.74) \end{aligned}$ |
| $\left(z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}\right) \times L R(2)$ |  |  | ${\underset{(2.39)}{0.125^{b}}}^{\text {b }}$ | $\underbrace{0.126^{b}}_{(2.38)}$ |  |  |
| $\left(z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}\right) \times L R(3)$ |  |  |  |  | ${\underset{(2.43)}{0.084^{b}}}^{b}$ | $\underbrace{0.089^{b}}_{(2.46)}$ |
| $t(\gamma)=1$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Number of countries | 14 | 14 | 14 | 14 | 14 | 14 |
| Number of observations | 529 | 529 | 501 | 501 | 487 | 487 |

Notes: All regressions include country fixed effects. Heteroskedasticity and autocorrelation consistent tstatistics are reported in parentheses. ${ }^{a},^{b}$ and ${ }^{c}$ denote significance at $1 \%, 5 \%$ and $10 \%$ levels. The rows $t(\beta)=0$ and $t(\gamma)=1$ report the p-value of the test of $H_{0}: \beta=0$ and $H_{0}: \gamma=1$ respectively, where in this context, $\beta$ and $\gamma$ refer to the estimated coefficients. $L R(2)$ and $L R(3)$ are labor reallocation indices, developed by Wacziarg and Wallack [2004], measuring the rate of workers who shift from one sector to another over 2 and 3 years, respectively.

Table 3: Panel Cointegration Estimates of $\beta_{i}$ and $\gamma_{i}$ for Each Country (eq. (6))

| Country | Relative wage equation |  | Relative price equation |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\hat{\beta}_{i}^{\text {DOLS }}$ | $\hat{\beta}_{i}^{F M O L S}$ | $\hat{\gamma}_{i}^{\text {DOLS }}$ | $\hat{\gamma}_{i}^{F M O L S}$ |
| BEL | $\begin{gathered} l \\ \hline-0.160^{a} \\ (-5.44) \end{gathered}$ | $\underset{(-5.10)}{-0.142^{a}}$ | $\begin{aligned} & 1.815^{a} \\ & (23.26) \end{aligned}$ | $\begin{aligned} & \frac{1 \imath}{0.825^{a}} \\ & (24.75) \end{aligned}$ |
| DEU | $\underset{\left(-0.590^{a}\right.}{\substack{a \\ \hline}}$ | $\underset{\left(-0.582^{a}\right.}{-18.35)}$ | $\begin{aligned} & 0.624^{a} \\ & (10.14) \end{aligned}$ | $\begin{aligned} & 0.606^{a} \\ & (11.58) \end{aligned}$ |
| DNK | $\underset{(-3.90)}{-0.452^{a}}$ | $\underset{(-5.53)}{-0.453^{a}}$ | $\begin{aligned} & 0.469^{a} \\ & (10.13) \end{aligned}$ | $\underset{(14.68)}{0.470^{a}}$ |
| ESP | $\underset{(-6.97)}{-0.277^{a}}$ | $\begin{gathered} -0.280^{a} \\ (-10.70) \end{gathered}$ | ${ }_{(21.99)}^{0.821^{a}}$ | $\underset{(29.26)}{0.836^{a}}$ |
| FIN | $\underset{(-9.07)}{-0.224^{a}}$ | $\underset{(-11.66)}{-0.221^{a}}$ | ${ }_{(25.24)}^{0.762^{a}}$ | $\underset{(28.38)}{0.733^{a}}$ |
| FRA | $\underset{(-5.15)}{-0.413^{a}}$ | $\underset{(-6.47)}{-0.412^{a}}$ | ${ }_{(33.66)}^{0.840^{a}}$ | $\underset{(40.31)}{0.841 a}$ |
| GBR | $\underset{(-1.52)}{-0.122}$ | $\underset{(-2.29)}{-0.141^{b}}$ | $\underbrace{0.941^{a}}_{(8.82)}$ | ${ }_{(10.63)}^{0.922^{a}}$ |
| IRL | $\underset{(-2.05)}{-0.171^{b}}$ | $\underset{(-3.34)}{-0.210^{a}}$ | ${ }_{(16.37)}^{0.733^{a}}$ | $\underset{(22.50)}{0.737^{a}}$ |
| ITA | $\underset{(-9.64)}{-0.274^{a}}$ | $\begin{gathered} -0.290^{a} \\ (-10.50) \end{gathered}$ | ${ }_{(34.72)}^{0.786^{a}}$ | $\begin{aligned} & 0.767^{a} \\ & (32.49) \end{aligned}$ |
| JPN | $\underset{(-9.63)}{-0.192^{a}}$ | $\begin{gathered} -0.178^{a} \\ (-10.25) \end{gathered}$ | $\begin{aligned} & 0.950^{a} \\ & (54.10) \end{aligned}$ | $\begin{aligned} & 0.970^{a} \\ & (56.74) \end{aligned}$ |
| KOR | $\begin{gathered} -0.499^{a} \\ (-9.28) \end{gathered}$ | $\underset{(-12.19)}{-0.482^{a}}$ | $\begin{aligned} & 0.648^{a} \\ & (40.34) \end{aligned}$ | $\underset{(52.48)}{0.651 a}$ |
| NLD | $\underset{(-5.59)}{-0.375^{a}}$ | $\underset{(-6.12)}{-0.345^{a}}$ | $\underset{(20.44)}{0.796^{a}}$ | $\underset{(24.85)}{0.800^{a}}$ |
| SWE | $\frac{-0.012}{(-0.35)}$ | $\frac{-0.004}{(-0.23)}$ | ${ }_{(11.33)}^{0.925^{a}}$ | ${ }_{(23.80)}^{0.918^{a}}$ |
| USA | $\begin{array}{r} -0.018 \\ (-0.68) \\ \hline \end{array}$ | $\begin{gathered} -0.034 \\ (-1.40) \\ \hline \end{gathered}$ | $\begin{array}{r} 0.837^{a} \\ (33.06) \\ \hline \end{array}$ | $\begin{aligned} & 0.822^{a} \\ & (35.05) \\ & \hline \end{aligned}$ |
| Whole sample | $\begin{gathered} -0.270^{a} \\ (-22.27) \\ \hline \end{gathered}$ | $\begin{gathered} -0.270^{a} \\ (-27.83) \\ \hline \end{gathered}$ | $\begin{aligned} & 0.782^{a} \\ & (91.83) \\ & \hline \end{aligned}$ | $\begin{array}{r} 0.779^{a} \\ (108.91) \\ \hline \end{array}$ |

Notes: Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses. ${ }^{a},{ }^{b}$ and ${ }^{c}$ denote significance at $1 \%, 5 \%$ and $10 \%$ levels.


Figure 3: Relative Price and Relative Wage Effects of an Increase in $a^{T}-a^{N}$

Table 4: Long-Term Relative Price and Relative Wage Responses (in \%) to Higher Productivity Growth in Tradables relative to Non Tradables


Notes: Effects of a 1 percentage point increase in the labor share-adjusted TFPs differential between tradables and non tradables. Column (1) gives estimates for the whole sample taken from the first column of Table 2 while columns (2)-(9) show numerical results. Panels A and B show the deviation in percentage relative to the steady-state for the ( $\log$ ) relative price of non tradables, $p \equiv p^{N}-p^{T}$, and the ( $\log$ ) ratio of the non traded to the traded wage, $\omega \equiv w^{N}-w^{T}$, respectively, and break down changes in a productivity effect (keeping unchanged sectoral capitallabor ratios $k^{j}$, the overall capital stock $K$ and the stock of foreign bonds $B$ ), a capital reallocation effect (induced by changes in $k^{j}$ keeping unchanged $K$ and $B$ ), a capital accumulation effect (stemming from the investment boom causing a current account deficit in the short-run and therefore requiring a steady-state improvement in the balance of trade). While panels A and B show the results when setting $\phi$ to one, panels C and D show results for $\phi<1$ and panels E and F show results for $\phi>1 ; \phi$ is the elasticity of substitution between tradables and non tradables; $\epsilon$ captures the degree of labor mobility across sectors; $\sigma_{L}$ is the elasticity of labor supply.

Table 5: Comparison of Predicted Values with Empirical Estimates

| Country | Parameters |  | Relative price response |  | Relative wage response |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ | $(6)$ | $(7)$ | $(8)$ |
|  | Mobility $\epsilon$ | Substitutability $\phi$ | $\hat{p}^{\text {predict }}$ | $\hat{p}^{\text {FMOLS }}$ | $(4) /(3)$ | $\hat{\omega}^{\text {predict }}$ | $\hat{\omega}^{F M O L S}$ | $(7)-(6)$ |
| BEL | 0.32 | 0.77 | 0.691 | 0.825 | 1.19 | -0.498 | -0.142 | 0.36 |
| DEU | 0.73 | 1.24 | 0.668 | 0.606 | 0.91 | -0.535 | -0.582 | -0.05 |
| DNK | 0.12 | 1.42 | 0.445 | 0.470 | 1.06 | -0.905 | -0.453 | 0.45 |
| ESP | 1.65 | 0.78 | 0.844 | 0.836 | 0.99 | -0.245 | -0.280 | -0.04 |
| FIN | 0.53 | 1.04 | 0.662 | 0.733 | 1.11 | -0.545 | -0.221 | 0.32 |
| FRA | 1.26 | 0.75 | 0.823 | 0.841 | 1.02 | -0.279 | -0.412 | -0.13 |
| GBR | 0.99 | 0.48 | 0.867 | 0.922 | 1.06 | -0.207 | -0.141 | 0.07 |
| IRL | 0.25 | - | 0.720 | 0.737 | 1.02 | -0.450 | -0.210 | 0.24 |
| ITA | 0.77 | - | 0.798 | 0.767 | 0.96 | -0.321 | -0.290 | 0.03 |
| JPN | 0.99 | 0.81 | 0.786 | 0.970 | 1.23 | -0.340 | -0.178 | 0.16 |
| KOR | 1.80 | 1.79 | 0.744 | 0.651 | 0.88 | -0.410 | -0.482 | -0.07 |
| NLD | 0.22 | 0.93 | 0.608 | 0.800 | 1.32 | -0.635 | -0.345 | 0.29 |
| SWE | 0.42 | 0.23 | 0.976 | 0.918 | 0.94 | -0.026 | -0.004 | 0.02 |
| USA | 1.80 | 0.58 | 0.884 | 0.822 | 0.93 | -0.178 | -0.034 | 0.14 |
| Whole sample | 0.61 | 0.66 | 0.779 | 0.779 | 1.00 | -0.353 | -0.270 | 0.08 |

Notes: $\epsilon$ captures the degree of labor mobility across sectors; $\phi$ is the intratemporal elasticity of substitution between traded goods and non traded goods. We denote by superscripts "predict" and "FMOLS" the numerically computed values and fully modified OLS estimates taken from Table 3, respectively; column (5) gives the ratio of estimates to predicted values for the percentage change in the relative price of non tradables while column (8) shows the difference between estimates and predicted values for the percentage change in the relative wage of non tradables.

## A Data for Empirical Analysis

Coverage: Our sample consists of a panel of 14 countries: Belgium (BEL), Denmark (DNK), Finland (FIN), France (FRA), Germany (DEU) Ireland (IRL), Italy (ITA), Japan (JPN), Korea (KOR), the Netherlands (NLD), Spain (ESP), Sweden (SWE), the United Kingdom (GBR), and the United States (USA). The period is running from 1970 to 2007, except for Japan (1974-2007).

Source: We use the EU KLEMS [2011] database (the March 2011 data release, available at http://www.euklems.net) which provides annual data for eleven 1-digit ISIC-rev. 3 industries.

Traded Sector comprises the following industries: Agriculture, Hunting, Forestry and Fishing; Mining and Quarrying; Total Manufacturing; Transport, Storage and Communication; and Financial Intermediation.

Non Traded Sector comprises the following industries: Electricity, Gas and Water Supply; Construction; Wholesale and Retail Trade; Hotels and Restaurants; Real Estate, Renting and Business Services; and Community Social and Personal Services.

In the following, we provide details on data construction (mnemonics are in parentheses):

- The relative price of non tradables, $P_{t}$, corresponds to the ratio of the value added deflator of non traded goods $P_{t}^{N}$ to the value added deflator of traded goods $P_{t}^{T}$, i.e., $P_{t}=P_{t}^{N} / P_{t}^{T}$. The sectoral value-added deflator $P_{t}^{j}$ for sector $j=T, N$ is calculated by dividing value added at current prices (VA) by value added at constant prices (VA_QI) in sector $j$.
- The relative wage of non tradables, $\Omega_{t}$, is calculated as the ratio of the nominal wage in the non traded sector $W_{t}^{N}$ to the nominal wage in the traded sector $W_{t}^{T}$, i.e., $\Omega_{t}=W_{t}^{N} / W_{t}^{T}$. The sectoral nominal wage $W_{t}^{j}$ for sector $j=T, N$ is calculated by dividing labor compensation in sector $j$ (LAB) by total hours worked by persons engaged (H_EMP) in that sector.
- The relative productivity of tradables, $Z_{t}^{T} /\left(Z_{t}^{N}\right)^{\theta^{T} / \theta^{N}}$, is calculated as the ratio of traded TFP $Z_{t}^{T}$ to the labor share-adjusted non traded TFP $\left(Z_{t}^{N}\right)^{\theta^{T} / \theta^{N}}$ where sectoral TFPs are constructed as Solow residuals from constant-price domestic currency series of value added (VA_QI), capital, labor shares $\theta^{j}$, and employment (H_EMP) in sector $j ; \theta^{j}$ is the ratio of the compensation of employees (LAB) to value added (VA) in the $j$ th sector, averaged over the period 1970-2007 (except Japan: 1974-2007). Aggregate capital stocks are estimated from the perpetual inventory approach by using real gross capital formation from the OECD Economic Outlook Database (data in millions of national currency, constant prices) and assuming a depreciation rate of $5 \%$. Following Garofalo and Yamarik [2002], the capital stock is then allocated to traded and non traded industries by using sectoral output shares $K_{t}^{j}=\varpi_{t}^{j} K_{t}$ where $\varpi_{t}^{j}$ is the share of sector $j$ 's value added in overall output.
- The labor reallocation index (see eq. (7)), $L R_{t}(\tau)$, is computed by using data for labor (H_EMP). Series for the rate of workers that have shifted from one sector to another over two $(L R(2))$ or three ( $L R(3)$ ) years cover the period 1972-2007 (1976-2007 for Japan) and 1973-2007 (1977-2007 for Japan), respectively.


## B Data for Calibration

Table 7 shows the non-tradable content of GDP, consumption, government spending, labor, labor compensation, and gives the share of government spending on traded and non traded goods in the sectoral value added, the shares of labor income in value added in both sectors. Our sample covers the 14 OECD countries mentioned in section A. Our reference period for the calibration corresponds to the period 1990-2007. The choice of this period has been dictated by data availability.

To calculate the non-tradable share of output, employment and labor compensation, we split the eleven industries into traded and non-traded sectors by adopting the classification proposed by De Gregorio et al. [1994] and updated by Jensen and Kletzer [2006] (Source: EU KLEMS [2011]). The non-tradable shares of output and labor, shown in columns 1 and 4 of Table 7, average to $63 \%$ and $65 \%$, respectively. We calculate the non-tradable share of labor compensation as $\alpha_{L}=W^{N} L^{N} / W L$ where $W L$ corresponds to overall labor compensation (Source: EU KLEMS [2011]). The non-tradable share of compensation of employees, shown in column 5 of Table 7, averages to $65 \%$.

To split consumption expenditure (at current prices) into consumption in traded and non traded goods, we made use of the Classification of Individual Consumption by Purpose (COICOP) published by the United Nations (Source: United Nations [2011]). Among the twelve items, the following ones
are treated as consumption in traded goods: Food and Non-Alcoholic Beverages; Alcoholic Beverages Tobacco and Narcotics; Clothing and Footwear; Furnishings, Household Equipment; Transport; Miscellaneous Goods and Services. The remaining items are treated as consumption in non traded goods: Housing, Water, Electricity, Gas and Fuels; Health; Communication; Education; Restaurants and Hotels. Because the item 'Recreation and Culture' is somewhat problematic, we decided to consider it as both tradable $(50 \%)$ and non tradable ( $50 \%$ ) with equal shares. Note that the non-tradable share of consumption shown in column 2 of Table 7 averages to $43 \%$, in line with the share reported by Stockman and Tesar [1995].

Sectoral government expenditure data were obtained from the Government Finance Statistics Yearbook (Source: IMF [2011]) and the OECD General Government Accounts database (Source: OECD [2011]). Adopting Morshed and Turnovsky's [2004] methodology, the following four items were treated as traded: Fuel and Energy; Agriculture, Forestry, Fishing, and Hunting; Mining, Manufacturing, and Construction; Transport and Communications. Items treated as non traded are: Government Public Services; Defense; Public Order and Safety; Education; Health; Social Security and Welfare; Environment Protection; Housing and Community Amenities; Recreation Cultural and Community Affairs. The non-tradable component of government spending shown in column 3 of Table 7 averages to $90 \%$. The proportion of government spending on the traded and non traded good (i.e., $G^{T} / Y^{T}$ and $G^{N} / Y^{N}$ ) are shown in columns 6 and 7 of Table 7 . They average $5 \%$ and $28 \%$, respectively.

The labor income share for sector $j$ denoted by $\theta^{j}$ is calculated as the ratio of labor compensation to value added in current prices (Source: EU KLEMS [2011]). The labor income shares for the traded and the non traded sector (i.e., $\theta^{T}$ and $\theta^{N}$ ) shown in columns 8 and 9 of Table 7 average 0.63 and 0.68 , respectively. When $k^{T}>k^{N}$, the shares of labor income average 0.61 and 0.69 for the traded and the non traded sector, respectively, while if $k^{N}>k^{T}, \theta^{T}$ and $\theta^{N}$ average 0.72 and 0.64 , respectively.

Columns 1 and 2 of Table 5 show our estimates of $\epsilon$ which captures the degree of labor mobility across sectors and of the elasticity of substitution $\phi$ between traded and non traded goods. We detail our empirical strategy to estimate these two parameters for each country and the whole sample as well below.

Along the lines of Horvath [2000], we derive a testable equation by combining optimal rules for labor supply and labor demand and estimate $\epsilon$ by running the regression of the worker inflow in sector $j=T, N$ of country $i$ at time $t$ arising from labor reallocation across sectors computed as $\hat{l}_{i, t}^{j}-\hat{l}_{i, t}$ on the relative labor's share percentage changes in sector $j \hat{\beta}_{i, t}^{j}:{ }^{39}$

$$
\begin{equation*}
\hat{l}_{i, t}^{j}-\hat{l}_{i, t}=f_{i}+f_{t}+\gamma_{i} \hat{\beta}_{i, t}^{j}+\nu_{i, t}^{j}, \tag{35}
\end{equation*}
$$

where we denote logarithm in lower case and the deviation from initial steady-state by a hat; $\nu_{i, t}^{j}$ is an i.i.d. error term; country fixed effects are captured by country dummies, $f_{i}$, and common macroeconomic shocks by year dummies, $f_{t}$. The LHS term of (35) is calculated as the difference between changes (in percentage) in hours worked in sector $j, \hat{l}_{i, t}^{j}$, and in total hours worked $\hat{l}_{i, t}$. The RHS term $\beta^{j}$ corresponds to the fraction of labor's share of output accumulating to labor in sector $j$. Denoting by $P_{t}^{j} Q_{t}^{j}$ the output at current prices in sector $j=T, N$ at time $t, \beta_{t}^{j}$ is computed as $\frac{\xi^{j} P_{t}^{j} Q_{t}^{j}}{\sum_{j=N}^{T} \xi^{j} P_{t}^{j} Q_{t}^{j}}$ where $\xi^{j}$ is labor's share in output in sector $j=T, N$ defined as the ratio of the compensation of employees to output in the $j$ th sector, averaged over the period 1971-2007. ${ }^{40}$ Because hours worked are aggregated by means of a CES function, total hours percentage change $\hat{l}_{i, t}$ is calculated as a weighted average of sectoral employment percentage changes, i.e., $\hat{l}_{t}=\sum_{j=N}^{T} \beta_{t-1}^{j} \hat{l}_{t}^{j}$. Data are taken from EU KLEMS [2011] and the sample includes the 14 OECD countries mentioned above over the period 1971-2007 (except for Japan: 1975-2007). When exploring empirically (35), the coefficient $\gamma$ is alternatively assumed to be identical, i.e., $\gamma_{i}=\gamma$, or to vary across countries. Building on our panel data estimations, we calculate $\epsilon_{i}$ by computing $\frac{\gamma_{i}}{1-\gamma_{i}}$ where in this context, $\gamma_{i}$ refers to the estimated coefficient. Table 6 reports empirical estimates over the period 1971-2007 with t-stats. All values are statistically significant at $10 \%$, except for Denmark. Estimates of $\epsilon$ are also reported in column 1 of Table 5 when calibrating the model for each country. ${ }^{41}$

[^22]To estimate the elasticity of substitution in consumption $\phi$ between traded and non traded goods, we first derive a testable equation by inserting the optimal rule for intra-temporal allocation of consumption (15) into the goods market equilibrium which gives $\frac{C^{T}}{C^{N}}=\frac{Y^{T}-N X-G^{T}-I^{T}}{Y^{N}-G^{N}-I^{N}}=\left(\frac{\varphi}{1-\varphi}\right) P^{\phi}$ where $N X \equiv \dot{B}-r^{\star} B$ is net exports, $I^{j}$ and $G^{j}$ are investment in physical capital and government spending in sector $j$, respectively. Isolating $\left(Y^{T}-N X\right) / Y^{N}$ and taking logarithm yields $\ln \left(\frac{Y^{T}-N X}{Y^{N}}\right)=\alpha+\phi \ln P$. Adding an error term $\mu$, we estimate $\phi$ by running the regression of the (logged) output of tradables adjusted with net exports at constant prices in terms of output of non tradables on the (logged) relative price of non tradables:

$$
\begin{equation*}
\ln \left(\frac{Y^{T}-N X}{Y^{N}}\right)_{i, t}=f_{i}+f_{t}+\alpha_{i} t+\phi_{i} \ln P_{i, t}+\mu_{i, t} \tag{36}
\end{equation*}
$$

where $f_{i}$ and $f_{t}$ are the country fixed effects and time dummies, respectively. Because the term $\alpha \equiv \ln \frac{\left(1-v_{G^{N}}-v_{I^{N}}\right)}{\left(1-v_{G^{T}}-v_{I^{T}}\right)}+\ln \left(\frac{\varphi}{1-\varphi}\right)$ is composed of ratios, denoted by $v_{G^{j}}$ and $v_{I^{j}}$, of $G^{T}\left(G^{N}\right)$ and $I^{T}$ $\left(I^{N}\right)$ to $Y^{T}-N X\left(Y^{N}\right)$ and hence may display a trend over time, we add country-specific linear trends, as captured by $\alpha_{i} t .{ }^{42}$

Instead of using time series for sectoral value added, we can alternatively make use of series for sectoral labor compensation by inserting the first-order condition equating the marginal revenue of labor and the sectoral wage, i.e., $\frac{\theta^{j} P^{j} Y^{j}}{L^{j}}=W^{j}$, into the goods market clearing condition. Eliminating $Y^{j}$, denoting by $\gamma^{T}=\left(W^{T} L^{T}-\theta^{T} P^{T} N X\right)$ and $\gamma^{N}=W^{N} L^{N}$, and taking logarithm yields $\ln \left(\frac{\gamma^{T}}{\gamma^{N}}\right)=$ $\eta+\phi \ln P$ where $\eta$ is a term composed of both preference (i.e., $\varphi$ ) and production (i.e., $\theta^{j}$ ) parameters, and (logged) ratios of $G^{T}\left(G^{N}\right)$ and $I^{T}\left(I^{N}\right)$ to $W^{T} L^{T}-\theta^{T} P^{T} N X\left(W^{N} L^{N}\right)$. We estimate $\phi$ by exploring alternatively the following empirical relationship:

$$
\begin{equation*}
\ln \left(\gamma^{T} / \gamma^{N}\right)_{i, t}=g_{i}+g_{t}+\sigma_{i} t+\phi_{i} \ln P_{i, t}+\zeta_{i, t} \tag{37}
\end{equation*}
$$

where $g_{i}$ and $g_{t}$ are the country fixed effects and time dummies, respectively, and we add countryspecific trends, as captured by $\sigma_{i} t$, because $\eta$ is composed of ratios that may display a trend over time.

Time series for sectoral value added at constant prices, labor compensation, and the relative price of non tradables are taken from EU KLEMS [2011] (see section A). Net exports correspond to the external balance of goods and services at current prices taken from the OECD Economic Outlook Database. To construct time series for net exports at constant prices $N X$, data are deflated by the value added deflator of traded goods $P^{T}$.

Since LHS terms of (36) and (37) and relative price of non tradables display trends, we ran unit root and then cointegration tests. Having verified that these two assumptions are empirically supported, we estimate the cointegrating relationships by using the fully modified OLS (FMOLS) procedure for cointegrated panels proposed by Pedroni [2000], [2001]. FMOLS estimates are reported in the second and the third column of Table 6, considering alternatively eq. (36) or eq. (37). Estimates of $\phi$ are reported in column 2 of Table 5 when calibrating the model for each country. As a reference model, we consider eq. (36); exploring the empirical relationship (36) gives an estimate for the whole sample of 0.66 which is roughly halfway between estimates documented by cross-section studies, notably Stockman and Tesar [1995] who find a value for $\phi$ of 0.44 and Mendoza [1995] who reports an estimate of 0.74 . Yet, as shown in column 2 of Table 6, estimates for Belgium, Denmark, and Korea are not statistically significant and thus we take consistent estimates obtained when exploring the empirical relationship (37) for these three economies. Because estimates for Italy and Ireland are either negative
may potentially show up. Our empirical results reveal that for the majority of the countries in our sample, the dependent variable does not Granger-cause the explanatory variable. A second econometric issue is related to the correlation of dependent variables across sectors within each country. We thus relax the homoscedasticity assumption and allow standard errors of the residuals to be clustered at the country level. By and large, our results suggest that allowing errors to be correlated within each country merely influences the statistical significance of our estimates of the degree of labor mobility across sectors for the whole sample and the majority of the countries.
${ }^{42}$ Because an endogeneity problem of relative prices may potentially affect our econometric results, we ran Granger causality tests. Our empirical results reveal that for the majority of the countries in our sample, the dependent variable does not Granger-cause the explanatory variable. Our results show that one can consider the regressor in eq. (36) as exogenous with respect to the dependent variable.

Table 6: Panel Data Estimates of the Degree of Labor Mobility ( $\epsilon$ ) and the Elasticity of Substitution in Consumption between Tradables and Non Tradables ( $\phi$ )

| Country | Labor Mobility $\epsilon$ | Elasticity of substitution $\phi$ |  |
| :---: | :---: | :---: | :---: |
|  | eq. (35) (1971-2007) | eq. (36) (1970-2007) | eq. (37) (1970-2007) |
|  | $\epsilon_{i}$ | $\phi_{i}^{\text {FMOLS }}$ | $\phi_{i}^{\text {FMOLS }}$ |
| BEL | ${\underset{(2.14)}{0.320^{b}}}^{6}$ | $\begin{aligned} & 0.205 \\ & (0.90) \end{aligned}$ | $\underbrace{0.770^{a}}_{(4.78)}$ |
| DEU | ${\underset{(2.47)}{0.733^{b}}}^{b}$ | ${\underset{(3.98)}{1.236^{a}}}^{a}$ | ${\underset{(4.96)}{1.581}}^{a}$ |
| DNK | $\underset{(1.05)}{0.119}$ | $\underset{(0.63)}{0.493}$ | ${\underset{(2.69)}{1.416^{a}}}^{a}$ |
| ESP | ${\underset{(2.61)}{1.649^{a}}}^{a}$ | $\underbrace{0.779^{a}}_{(5.29)}$ | $\underbrace{0.284^{c}}_{(1.90)}$ |
| FIN | ${ }_{(3.09)}^{0.525^{a}}$ | ${ }_{(8.99)}^{1.041^{a}}$ | $1_{(7.71)}^{1.354^{a}}$ |
| FRA | ${\underset{(2.11)}{1.262^{b}}}^{2}$ | $\underbrace{0.749^{a}}_{(4.95)}$ | $\underset{(4.34)}{0.824^{a}}$ |
| GBR | $0_{(3.29)}^{0.994^{a}}$ | ${ }_{(8.50)}^{0.482^{a}}$ | ${ }_{(16.87)}$ |
| IRL | ${ }_{(2.66)}^{0.249^{a}}$ | $\underset{(0.52)}{0.133}$ | $\underset{(-0.27)}{-0.147}$ |
| ITA | $\underset{(2.48)}{0.768^{b}}$ | $\underset{(-0.05)}{-0.006}$ | $\underset{(0.88)}{0.091}$ |
| JPN | $\underbrace{0.994^{b}}_{(2.53)}$ | ${\underset{(4.16)}{0.811^{a}}}^{a}$ | $\underbrace{0.785^{a}}_{(6.60)}$ |
| KOR | ${ }_{(3.06)}^{1.795^{a}}$ | $\begin{gathered} 1.580 \\ (1.58) \end{gathered}$ | $\underset{(2.99)}{1.786^{a}}$ |
| NLD | $\underset{(1.66)}{0.216^{c}}$ | $\underset{(2.56)}{0.927^{b}}$ | $\underset{(1.87)}{0.786^{c}}$ |
| SWE | $\underbrace{0.419^{a}}_{(3.06)}$ | ${\underset{(2.15)}{0.231^{b}}}^{b}$ | $\underbrace{0.864^{a}}_{(5.97)}$ |
| USA | $\begin{gathered} 1.800^{c} \\ (1.84) \\ \hline \end{gathered}$ | $\begin{gathered} 0.577^{a} \\ (3.02) \\ \hline \end{gathered}$ | $\begin{gathered} 0.593^{c} \\ \hline(1.73) \\ \hline \end{gathered}$ |
| Whole sample | $\begin{aligned} & \hline 0.607^{a} \\ & (10.22) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0.660^{a} \\ & (12.61) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0.871^{a} \\ & (16.84) \\ & \hline \end{aligned}$ |

Notes: Fixed effects (country) regressions for $\epsilon$ and panel cointegration estimates for $\phi$. ${ }^{a},{ }^{b}$ and ${ }^{c}$ denote significance at $1 \%, 5 \%$ and $10 \%$ levels; t-statistics are reported in parentheses.
or not statistically significant by using alternatively eq. (36) or eq. (37), estimates for $\phi$ for these two countries are left blank and $\phi$ is set to our panel data estimation for the whole sample, i.e., 0.66 , when calibrating the model for each country.

Table 7: Data to Calibrate the Two-Sector Model (1990-2007)

| Country | Non tradable Share |  |  |  |  | $G^{j} / Y^{j}$ |  | Labor Share |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|  | Output | Consumption | Gov. Spending | Labor | Lab. comp. | $G^{T} / Y^{T}$ | $G^{N} / Y^{N}$ | $\theta^{T}$ | $\theta^{N}$ |
| BEL | 0.65 | 0.42 | 0.91 | 0.68 | 0.66 | 0.06 | 0.30 | 0.61 | 0.63 |
| DEU | 0.65 | 0.40 | 0.91 | 0.65 | 0.60 | 0.05 | 0.27 | 0.75 | 0.63 |
| DNK | 0.66 | 0.42 | 0.94 | 0.68 | 0.68 | 0.05 | 0.36 | 0.63 | 0.70 |
| ESP | 0.64 | 0.46 | 0.88 | 0.66 | 0.67 | 0.06 | 0.24 | 0.60 | 0.66 |
| FIN | 0.58 | 0.43 | 0.89 | 0.63 | 0.63 | 0.06 | 0.34 | 0.59 | 0.73 |
| FRA | 0.70 | 0.40 | 0.94 | 0.69 | 0.68 | 0.05 | 0.31 | 0.70 | 0.64 |
| GBR | 0.64 | 0.40 | 0.93 | 0.70 | 0.65 | 0.04 | 0.29 | 0.70 | 0.73 |
| IRL | 0.52 | 0.43 | 0.89 | 0.62 | 0.62 | 0.04 | 0.26 | 0.46 | 0.69 |
| ITA | 0.64 | 0.38 | 0.91 | 0.63 | 0.62 | 0.05 | 0.28 | 0.71 | 0.64 |
| JPN | 0.63 | 0.43 | 0.86 | 0.64 | 0.65 | 0.06 | 0.22 | 0.55 | 0.60 |
| KOR | 0.52 | 0.44 | 0.76 | 0.58 | 0.56 | 0.06 | 0.18 | 0.73 | 0.83 |
| NLD | 0.65 | 0.40 | 0.90 | 0.70 | 0.69 | 0.07 | 0.32 | 0.60 | 0.70 |
| SWE | 0.64 | 0.45 | 0.92 | 0.68 | 0.67 | 0.06 | 0.39 | 0.63 | 0.71 |
| USA | 0.68 | 0.51 | 0.90 | 0.73 | 0.69 | 0.05 | 0.21 | 0.61 | 0.66 |
| Mean | 0.63 | 0.43 | 0.90 | 0.65 | 0.65 | 0.05 | 0.28 | 0.63 | 0.68 |

Notes: $G^{j} / Y^{j}$ is the share of government spending in good $j$ in output of sector $j . \theta^{j}$ is the share of labor income in output of sector $j=T, N$.

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# IMPERFECT MOBILITY OF LABOR ACROSS SECTORS: A REAPPRAISAL OF THE BALASSA-SAMUELSON EFFECT 

TECHNICAL APPENDIX<br>NOT MEANT FOR PUBLICATION

Olivier CARDI and Romain RESTOUT

## A Data Description

## A. 1 Data: Source and Construction

Coverage: Our sample consists of a panel of 14 countries: Belgium (BEL), Denmark (DNK), Finland (FIN), France (FRA), Germany (DEU) Ireland (IRL), Italy (ITA), Japan (JPN), Korea (KOR), the Netherlands (NLD), Spain (ESP), Sweden (SWE), the United Kingdom (GBR), and the United States (USA). These countries have the most extensive coverage of variables of our interest.

Source: We use the EU KLEMS [2011] database (the March 2011 data release, available at http://www.euklems.net) which provides annual data for eleven 1-digit ISIC-rev. 3 industries. The period is running from 1970 to 2007, except for Japan (1974-2007).

The eleven 1-digit ISIC-rev. 3 industries are split into tradables and non tradables sectors. To do so, we adopt the classification proposed by De Gregorio et al. [1994] who treat an industry as traded when it exports at least $10 \%$ of its output. Following Jensen and Kletzer [2006], we have updated the classification suggested by De Gregorio et al. [1994] by treating "Financial Intermediation" as a traded industry. Jensen and Kletzer [2006] use the geographic concentration of service activities within the United States to identify which service activities are traded domestically. The authors classify activities that are traded domestically as potentially traded internationally. The idea is that when a good or a service is traded, the production of the activity is concentrated in a particular region to take advantage of economies of scale in production.

Jensen and Kletzer [2006] use the two-digit NAICS (North American Industrial Classification System) to identify tradable and non tradable sectors. We map their classification into the NACE-ISIC-rev. 3 used by the EU KLEMS database. The mapping was clear for all sectors except for "Real Estate, Renting and Business Services". According to the EU KLEMS classification, the industry labelled "Real Estate, Renting and Business Services" is an aggregate of five sub-industries: "Real estate activities" (NACE code: 70), "Renting of Machinery and Equipment" (71), "Computer and Related Activities" (72), "Research and Development" (73) and "Other Business Activities" (74). While Jensen and Kletzer [2006] find that industries 70 and 71 can be classified as tradable, they do not provide information for industries 72,73 and 74 . We decided to classify "Real Estate, Renting and Business Services" as non tradable but conduct a robustness check by contrasting our empirical findings when "Real Estate, Renting and Business Services" is non traded with those when "Real Estate, Renting and Business Services" is traded in section G.2. As shown in panel E of Table 15, our conclusions hold and remain unsensitive to the classification.

Traded Sector comprises the following industries: Agriculture, Hunting, Forestry and Fishing; Mining and Quarrying; Total Manufacturing; Transport, Storage and Communication; and Financial Intermediation.

Non Traded Sector comprises the following industries: Electricity, Gas and Water Supply; Construction; Wholesale and Retail Trade; Hotels and Restaurants; Real Estate, Renting and Business Services; and Community Social and Personal Services.

Relevant to our work, the EU KLEMS database provides data, for each industry and year, on value added at current and constant prices, permitting the derivation of sectoral deflators of value added, as well as details on labor compensation and employment data, allowing the construction of sectoral wage rates. We describe below the construction for the data employed in Section 2 (mnemonics are given in parentheses):

- Sectoral value-added deflator $P_{t}^{j}$ for $j=T, N$ : value added at current prices (VA) over value added at constant prices (VA_QI) in sector $j$. Source: EU KLEMS database. The relative price of non tradables $P_{t}$ corresponds to the ratio of the value added deflator of non traded goods to the value added deflator of traded goods: $P_{t}=P_{t}^{N} / P_{t}^{T}$.
- Sectoral labor $L_{t}^{j}$ for $j=T, N$ : total hours worked by persons engaged (H_EMP) in sector $j$. Source: EU KLEMS database.
- Sectoral nominal wage $W_{t}^{j}$ for $j=T, N$ : labor compensation in sector $j$ (LAB) over total hours worked by persons engaged (H_EMP) in that sector. Source: EU KLEMS database. The relative wage, $\Omega_{t}$ is calculated as the ratio of the nominal wage in the non traded sector $W^{N}$ to the nominal wage in the traded sector: $\Omega_{t}=W_{t}^{N} / W_{t}^{T}$.
Summary statistics of the data used in the empirical analysis are displayed in Table 9. As shown in the first three columns, all countries of our sample experience technological change biased toward the traded sector, an appreciation in the relative price of non tradables and a decline in the ratio of the non traded wage relative to the traded wage.

Because data source and construction are heterogenous across variables as a result of different nomenclatures, Table 8 provides a summary of the classification adopted to split value added and its demand components as well intro traded and non traded goods.

## A. 2 Construction of Sectoral TFPs

To calculate the total factor productivity for sector $j=T, N$, we assume that traded and non traded goods are produced with capital $K^{j}$ and labor $L^{j}$ according a constant returns to scale technology:

$$
\begin{equation*}
Y_{t}^{j}=Z_{t}^{j}\left(K_{t}^{j}\right)^{1-\theta^{j}}\left(L_{t}^{j}\right)^{\theta^{j}} \tag{38}
\end{equation*}
$$

where $Y_{t}^{j}$ is value added, $K_{t}^{j}$ is capital input, $L_{t}^{j}$ is labor input, and $\theta^{j}$ corresponds to the labor share in value added in sector $j$. Data for the series of constant-price value-added (VA_QI) and labor (H_EMP) are taken from EU KLEMS database. The sectoral labor share in output corresponds to the labor compensation in sector $j$ (LAB) over value added at current prices (VA) averaged over the period 1970-2007 (1974-2007 for Japan). Source: EU KLEMS database.

To construct the series for the sectoral capital stock, we proceed as follows. Capital stocks are estimated from the perpetual inventory approach. In order to apply this method, we need ( $i$ ) real gross capital formation series, (ii) the initial capital stock in the base year, which is set to be 1970 and (iii) the rate of depreciation of the existing capital stock. Real gross capital formation is obtained from OECD Economic Outlook Database (data in millions of national currency, constant prices).

Consistent with the neoclassical growth model, the initial capital stock, $K_{1970}$, is computed using the following formula:

$$
K_{1970}=\frac{I_{1970}}{g_{I}+\delta_{K}},
$$

where $I_{1970}$ corresponds to the real gross capital formation in the base year 1970, $g_{I}$ is the average growth rate from 1970 to 2007 of the real gross capital formation series and $\delta$ is the depreciation rate which is assumed to be $5 \%$ (see Hall and Jones [1999]). The capital stock is obtained by using the standard capital accumulation equation: $K_{t+1}=(1-\delta) K_{t}+I_{t}$ for $t=1970, \ldots, 2007$ and where $K_{t}$ is the capital stock at the beginning of period $t$. Following Garofalo and Yamarik [2002], the gross capital stock is then allocated to traded and non traded industries by using sectoral output shares:

$$
K_{t}^{j}=\varpi_{t}^{j} K_{t}
$$

where $\varpi_{t}^{j}$ is the share of sector $j$ 's value added in overall output.
Sectoral TFPs (in log) $z_{t}^{j}$ at time $t$ are constructed as Solow residuals from constant-price (domestic currency) series of value added $y_{t}^{j}$ and capital stock $k_{t}^{j}$, and employment $l_{t}^{j}$ :

$$
\begin{equation*}
z_{t}^{j}=y_{t}^{j}-\theta^{j} l_{t}^{j}-\left(1-\theta^{j}\right) k_{t}^{j} . \tag{39}
\end{equation*}
$$

Finally, the (log) relative productivity variable is computed as the labor share-adjusted ratio of traded TFP to non traded TFP in the form $z_{t}^{T}-\left(\frac{\theta^{T}}{\theta^{N}}\right) z_{t}^{N}$. In cointegrating regressions, all variables of our interest, namely $P_{t}, \Omega_{t}$ and $Z_{t}^{T} /\left(Z_{t}^{N}\right)^{\theta^{T} / \theta^{N}}$ are converted into index $1995=100$ and are expressed in log levels. Summary statistics of the relative productivity of tradables used in the empirical analysis are displayed in Table 9. As shown in the third column, all countries of our sample experience a positive productivity differential between tradables and non tradables.

## A. 3 Construction of the Labor Reallocation Index

The labor reallocation index denoted by $L R_{t}(\tau)$ is the ratio of the absolute change in sectoral employment resulting from labor reallocation to average employment over $\tau$ years, with $\tau=2$, 3:

$$
L R_{t}(\tau)=\frac{\sum_{j=T}^{N}\left|L_{t}^{j}-L_{t-\tau}^{j}\right|-\left|\sum_{j=T}^{N} L_{t}^{j}-\sum_{j=T}^{N} L_{t-\tau}^{j}\right|}{0.5 \sum_{j=T}^{N}\left(L_{t-\tau}^{j}+L_{t}^{j}\right)}
$$

Data for labor (H_EMP), used to compute $L R_{t}(\tau)$, are taken from EU KLEMS database.
Summary statistics of the the labor reallocation index used in the empirical analysis are displayed in Table 9. Columns 4 and 5 show the rate of workers who shift from one sector to another over alternatively two or three years. On average, $1.33 \%$ of workers have shifted from one sector to another
Table 8: Construction of Variables and Data Sources

| Variable | Countries covered | Period | Construction and aggregation | Database |
| :---: | :---: | :---: | :---: | :---: |
| Value added $Y^{T} \& Y^{N}$ (constant prices) | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA | 1970-2007 | T: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation <br> $N$ : Electricity, Construction, Trade, Hotels, Real Estate, Personal Services | EU KLEMS |
| Value added $P^{T} Y^{T} \& P^{N} Y^{N}$ (current prices) | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA | 1970-2007 | T: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation <br> $N$ : Electricity, Construction, Trade, Hotels, Real Estate, Personal Services | EU KLEMS |
| Labor $L^{T} \& L^{N}$ (total hours worked by persons engaged) | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA | 1970-2007 | T: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation <br> $N$ : Electricity, Construction, Trade, Hotels, Real Estate, Personal Services | EU KLEMS |
| Labor compensation $L A B^{T}$ $\& L A B^{N}$ (current prices) | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA | 1970-2007 | T: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation <br> $N$ : Electricity, Construction, Trade, Hotels, Real Estate, Personal Services | EU KLEMS |
| Investment $P^{T} I^{T} \& P^{N} I^{N}$ (current prices) | DNK, DEU, ESP, FIN, FRA, GBR, IRL (90-07), ITA, JPN, NLD, SWE (93-07), USA | 1970-2007 | T: Products of Agriculture, Machinery, Transport, Other (50\%) <br> $N$ : Housing, Other Constructions, Other (50\%) | OECD Input-Output |
| $\begin{aligned} & \text { Consumption } C^{T} \& C^{N} \\ & \text { (constant prices) } \end{aligned}$ | BEL (95-07), DEU (91-07), DNK, ESP (95-07), FIN (75-07), FRA, ITA, GBR (90-07), IRL (96-07) JPN (80-07), KOR, NLD (80-07), SWE (93-07), USA | 1970-2007 | T: Food, Beverages, Clothing, Furnishings, Transport, Recreation, Other <br> $N$ : Housing, Health, Communication, Education, Restaurants, Recreation (Recreation is defined as $50 \%$ tradable and $50 \%$ non tradable) | COICOP |
| Government spending $P^{T} G^{T} \& P^{N} G^{N}$ (current prices) | BEL, DEU (91-07), DNK, ESP (95-07), FIN, FRA (95-07), GBR, IRL, ITA, JPN, KOR (00-07), NLD (95-07), SWE (95-07), USA | 1990-2007 | T: Energy, Agriculture, Manufacturing, Transport <br> $N$ : Public Services, Defense, Safety, Education, Health, Welfare, Housing, Environment, Recreation | OECD-FMI |
| Aggregate Investment $I$ (constant prices) | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, KOR, NLD, SWE, USA | 1970-2007 | Real gross capital formation | OECD |
| Trade balance $N X$ (constant prices) | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, KOR, NLD, SWE, USA | 1970-2007 | External balance of goods and services at current prices (source: OCDE) over price of traded goods $\left(P^{T}\right)$ | authors' calculations |
| Price $P^{T} \& P^{N}$ (value added deflator) | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA | 1970-2007 | Value added at current prices $\left(P^{j} Y^{j}\right)$ over value added at constant prices $\left(Y^{j}\right)$ | authors' calculations |
| Relative Price $P$ (index $1995=100$ ) | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA | 1970-2007 | Value added deflator of non traded goods $\left(P^{N}\right)$ over value added deflator of traded goods $\left(P^{T}\right)$ | authors' calculations |
| Wage $W^{T} \& W^{N}$ (nominal and per hour) | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA | 1970-2007 | Labor compensation $\left(L A B^{j}\right)$ over total hours worked by persons engaged $\left(L^{j}\right)$ | authors' calculations |
| Relative Wage $\Omega$ (index $1995=100$ ) | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA | 1970-2007 | Nominal wage in non tradables ( $W^{N}$ ) over nominal wage in tradables ( $W^{T}$ ) | authors' calculations |
| $\begin{aligned} & \text { Capital stock } K \\ & \text { (constant prices) } \end{aligned}$ | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, KOR, NLD, SWE, USA | 1970-2007 | Computed from the perpetual inventory approach using data of aggregate investment $I$ (depreciation rate: 5\%) | authors' calculations |
| Sectoral capital stocks $K^{T} \& K^{N}$ (constant prices) | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, KOR, NLD, SWE, USA | 1970-2007 | Computed from Garofalo and Yamarik [2002] using sectoral output shares | authors' calculations |
| Sectoral TPFs $Z^{T} \& Z^{N}$ (index 1995=100) | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA | 1970-2007 | Computed as Solow residuals from $Z^{j}=Y^{j} /\left[\left(K^{j}\right)^{1-\theta^{j}}\left(L^{j}\right)^{\theta^{j}}\right]$ $\theta^{j}$ : labor share in value added averaged over the period 1970-2007 | authors' calculations |
| Relative TPF (index 1995 =100) $Z^{T} /\left(Z^{N}\right)^{\theta^{T}} / \theta^{N}$ | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, JPN (74-07), KOR, NLD, ESP, SWE, USA | 1970-2007 | Labor share-adjusted ratio of traded TFP to non traded TFP | authors' calculations |
| Labor reallocation index $L R(\tau)$ | BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN $(74+\tau-07)$, KOR, NLD, SWE, USA | $\begin{gathered} 1970+\tau \\ -2007 \end{gathered}$ | Rate of workers who shift one sector to another over $\tau$ years. Computed from Wacziarg and Wallack [2004] | authors' calculations | Notes: times series for $I^{T} \& I^{N}$ are not available for BEL and KOR.

Table 9: Summary Statistics per Country (1970-2007)

| Countries | Variables |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ |
|  | $\hat{p}$ | $\hat{\omega}$ | $\hat{z}^{T}-\left(\theta^{T} / \theta^{N}\right) \hat{z}^{N}$ | $L R(2)$ | $L R(3)$ |
| BEL | 2.18 | -0.21 | 2.33 | 1.04 | 1.53 |
| DEU | 0.81 | -0.61 | 1.28 | 0.99 | 1.50 |
| DNK | 0.73 | -0.90 | 1.60 | 0.47 | 0.73 |
| ESP | 2.53 | -0.97 | 3.00 | 0.81 | 1.05 |
| FIN | 2.43 | -0.78 | 3.50 | 1.12 | 1.84 |
| FRA | 2.08 | -0.97 | 2.45 | 1.13 | 1.76 |
| GBR | 1.46 | -0.50 | 1.95 | 1.03 | 1.45 |
| IRL | 2.43 | -0.98 | 3.11 | 0.49 | 0.88 |
| ITA | 1.94 | -0.92 | 2.73 | 1.23 | 1.81 |
| JPN | 2.12 | -0.55 | 2.25 | 0.71 | 0.97 |
| KOR | 3.21 | -2.29 | 5.05 | 1.69 | 2.68 |
| NLD | 1.78 | -0.39 | 2.16 | 0.34 | 0.48 |
| SWE | 2.22 | -0.12 | 2.53 | 0.61 | 1.02 |
| USA | 1.66 | -0.25 | 2.27 | 0.49 | 0.90 |
| Average | 1.97 | -0.75 | 2.59 | 0.87 | 1.33 |

Notes: $\hat{p}$ is the average growth rate for the relative price of non tradables, $\hat{\omega}$ is the average growth rate for the relative wage of non tradables and $\hat{z}^{T}-\left(\theta^{T} / \theta^{N}\right) \hat{z}^{N}$ is the labor-share adjusted productivity differential between tradables and non tradables (average growth rate). For $\hat{p}, \hat{\omega}$ and $\hat{z}^{T}-\left(\theta^{T} / \theta^{N}\right) \hat{z}^{N}$ the data coverage is 1970-2007 (except Japan: 1974-2007). Columns $L R(2)$ and $L R(3)$ display the mean of the labor reallocation index $L R$ resulting from sectoral changes in labor over 2 and 3 years respectively (data coverage : 1972-2007 for $L R(2)$ (1976-2007 for Japan) and 1973-2007 for $L R(3)$ (1977-2007 for Japan)).
over any given 3 -year period. This result is in line with the evidence documented by Davis and Haltiwanger [1999] who find that most job reallocations are within sectors. There is considerable heterogeneity in this indicator, which varies from a low of 0.48 for the Netherlands to a high of 2.68 for Korea. It is worth noting that the labor reallocation indicator $L R$ is not a measure of the intersectoral labor mobility cost: some countries can exhibit a high $L R$ index due to a large productivity differential. We have to interact the $L R$ index with the productivity differential to investigate whether countries with larger labor shifts across sectors experience a more pronounced appreciation in the relative price of non tradables and a smaller decline in the ratio of sectoral wages.

The low $L R(2)$ index for the U.S. shown in Table 9 could be rationalized in the light of the estimates documented by Kambourov and Manovskii [2009] who find that educated workers exhibit lower occupational mobility than their less educated counterparts. Since the U.S. is endowed with a high human capital per worker and because human capital is not perfectly transferable across all occupations, it is expected to reduce workers' mobility.

## B Robustness Check for Panel Unit Tests and Cointegration Tests

## B. 1 Robustness Check for Panel Unit Root Tests

In the paper, we consider five panel unit root tests among the most commonly used in the literature: Levin, Lin and Chu ([2002], hereafter LLC), Breitung [2000], Im, Pesaran and Shin ([2003], hereafter IPS), Maddala and Wu ([1999], hereafter MW) and Hadri [2000]. All tests, with the exception of Hadri [2000], consider the null hypothesis of a unit root against the alternative that some members of the panel are stationary. Additionally, they are designed for cross sectionally independent panels. LLC and IPS are based on the use of the Augmented Dickey-Fuller test (ADF hereafter) to each individual series of the form $\Delta x_{i, t}=\alpha_{i}+\rho_{i} x_{i, t-1}+\sum_{j=1}^{q_{i}} \theta_{i, j} \Delta x_{i, t-j}+\varepsilon_{i, t}$, where $\varepsilon_{i, t}$ are assumed to be iid (the lag length $q_{i}$ is permitted to vary across individual members of the panel). Under the homogenous

Table 10: Panel Unit Root Tests (second generation)

| Test | Stat | Variables |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: |
|  |  | $p$ | $\omega$ | $z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}$ | $p-\left(z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}\right)$ |
| Bai and $\mathrm{Ng}[2002]$ | $Z_{\hat{e}}^{c}$ | 0.260 | 0.062 | 0.122 | 0.778 |
|  | $P_{\hat{e}}^{c}$ | 0.242 | 0.073 | 0.125 | 0.769 |
| Choi [2001] | $P_{m}$ | 0.996 | 0.000 | 0.993 | 0.240 |
|  | $Z$ | 1.000 | 0.002 | 1.000 | 0.322 |
| Pesaran [2007] | $L^{\star}$ | 1.000 | 0.001 | 1.000 | 0.343 |
|  | CIPS | 0.160 | 0.100 | 0.520 | 0.155 |
| Chang [2002] | CIPS | 0.160 | 0.100 | 0.520 | 0.155 |
|  | $S_{N}$ | 1.000 | 1.000 | 1.000 | 1.000 |

Notes: For all tests, the null of a unit root is not rejected if p-value $\geq 0.05$ at a $5 \%$ significance level.
alternative the coefficient $\rho_{i}$ in LLC is required to be identical across all units ( $\rho_{i}=\rho, \forall i$ ). IPS relax this assumption and allow for $\rho_{i}$ to be individual specific under the alternative hypothesis. MW propose a Fisher type test based on the p-values from individual unit root statistics (ADF or PhillipsPerron [1988] for instance). Like IPS, MW allow for heterogeneity of the autoregressive root $\rho_{i}$ under the alternative. We also apply the pooled panel unit root test developed by Breitung [2000] which does not require bias correction factors when individual specific trends are included in the ADF type regression. This is achieved by an appropriate variable transformation. As a sensitivity analysis, we also employ the test developed by Hadri [2000] which proposes a panel extension of the Kwiatkowski et al. [1992] test of the null that the time series for each cross section is stationary against the alternative of a unit root in the panel data. Breitung' and Hadri's tests, like LLC's test, are pooled tests against the homogenous alternative.

The common feature of these first generation tests is the restriction that all cross-sections are independent. In order to check that the cross-unit independence assumption does not affect the main conclusions we draw, we apply second generation unit root tests that allow for cross-unit dependencies as well. We consider the tests developed by: i) Bai and Ng [2002] based on a dynamic factor model, ii) Choi [2001] based on an error-component model, iii) Pesaran [2007] based on a dynamic factor model and iv) Chang [2002] who proposes the instrumental variable nonlinear test. The results of second generation unit root tests are shown in Table 10.

In all cases, except for the Choi's [2001] test applied to the relative wage variable ( $\omega$ ), we fail to reject the presence of a unit root in the relative price, the relative wage, the productivity differential, and the difference $z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}$, when cross-unit dependencies are taken into account.

## B. 2 Cointegration Tests and Robustness Check

To begin with, we report the results of parametric and non parametric cointegration tests developed by Pedroni ([1999]), ([2004]). Cointegration tests are based on the estimated residuals of equations (6a) and (6b). Table 11 reports the tests of the null hypothesis of no cointegration. All Panel tests reject the null hypothesis of no cointegration between $p$ and $z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}$ at the $1 \%$ significance level while three Panel tests reject the null hypothesis of no cointegration between $\omega$ and $z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}$ at the $5 \%$ significance level. Group-mean t-test confirms cointegration between $p$ and labor shareadjusted productivity differential and between $\omega$ and $z^{T}-\frac{\theta^{T}}{\theta^{N}} z^{N}$ at the $5 \%$ and $10 \%$ significance level, respectively. This is strong evidence in favor of cointegration between the relative price and relative productivity. Pedroni [2004] explores finite sample performances of the seven statistics. The results reveal that group-mean parametric t-test is more powerful than other tests in finite samples. While the results are somewhat less pervasive for the relative wage equation, group-mean parametric t-test indicatess that we cannot reject the null hypothesis of no cointegration for both the relative price and the relative wage equations.

As robustness checks, we contrast our group-mean FMOLS estimates and group-mean DOLS estimates with one lag $(q=1)$ shown in Table 2, with alternative estimators. Table 12 summarizes our estimates for cointegrating vectors by adopting alternative procedure. First, we consider the groupmean DOLS estimator with 2 lags $(q=2)$ and 3 lags $(q=3)$. Second, we estimate cointegration relationships (6a) and (6b) by using the panel DOLS estimator (Mark and Sul [2003]). We also use

Table 11: Panel cointegration tests results (p-values)

| Panel tests | wage equation | price equation |
| :--- | :---: | :---: |
|  | eq. (6a) | eq. (6b) |
|  | 0.020 |  |
| Non-parametric $\rho$ | 0.088 | 0.000 |
| Non-parametric $t$ | 0.030 | 0.003 |
| Parametric $t$ | 0.070 | 0.003 |
| Group-mean tests |  | 0.021 |
| Non-parametric $\rho$ | 0.399 |  |
| Non-parametric $t$ | 0.063 | 0.149 |
| Parametric $t$ | 0.014 | 0.017 |

Notes: The null hypothesis of no cointegration is rejected if the p-value is below 0.05 ( 0.10 resp.) at $5 \%$ ( $10 \%$ resp.) significance level.

Table 12: Alternative Cointegration Estimates of $\beta$ and $\gamma$

|  | Relative wage eq. (6a) |  | Relative price eq. (6b) |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\hat{\beta}$ | $t(\hat{\beta}=0)$ | $\hat{\gamma}$ | $t(\hat{\gamma}=1)$ |
| DOLS $(q=1)$ | $\begin{aligned} & -0.270^{a} \\ & (-22.27) \end{aligned}$ | 0.000 | $\begin{aligned} & 0.782^{a} \\ & (91.83) \end{aligned}$ | 0.000 |
| DOLS $(q=2)$ | $\frac{-0.268^{a}}{(-21.56)}$ | 0.000 | $\begin{aligned} & 0.788^{a} \\ & (97.24) \end{aligned}$ | 0.000 |
| DOLS $(q=3)$ | $\frac{-0.261^{a}}{(-20.83)}$ | 0.000 | ${ }_{(97.94)}^{0.792^{a}}$ | 0.000 |
| DFE | $\underset{(-3.46)}{-0.178^{a}}$ | 0.001 | $\begin{aligned} & 0.797^{a} \\ & (26.81)^{a} \end{aligned}$ | 0.000 |
| MG | $\underset{(-7.98)}{-0.208^{a}}$ | 0.000 | $\begin{aligned} & 0.760^{a} \\ & (34.39)^{a} \end{aligned}$ | 0.000 |
| PMG | $\frac{-0.190^{a}}{(-8.97)}$ | 0.000 | $\underset{(50.71)}{0.843^{a}} \underset{( }{a}$ | 0.000 |
| Panel DOLS $(q=1)$ | $\frac{-0.298^{a}}{(-6.82)}$ | 0.000 | $\begin{aligned} & 0.762^{a} \\ & (30.35) \end{aligned}$ | 0.000 |
| Panel DOLS $(q=2)$ | $\begin{aligned} & -0.298^{a} \\ & (-7.28) \end{aligned}$ | 0.000 | $\begin{aligned} & 0.762^{a} \\ & (32.35) \end{aligned}$ | 0.000 |
| Panel DOLS $(q=3)$ | ${ }_{(-7.57)}^{-0.294^{a}}$ | 0.000 | $\begin{aligned} & 0.763^{a} \\ & (34.44) \end{aligned}$ | 0.000 |

Notes: All regressions include country fixed effects. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses. ${ }^{a}$ denotes significance at $1 \%$ level. The columns $t(\hat{\beta})=0$ and $t(\hat{\gamma})=1$ report the p-value of the test of $H_{0}: \hat{\beta}=0$ and $H_{0}: \hat{\gamma}=1$ respectively.
alternative econometric techniques to estimate cointegrating relationships (6): the dynamic fixed effects estimator (DFE), the mean group estimator (MG, Pesaran and Smith [1995]), the pooled mean group estimator (PMG, Pesaran et al. [1999]). As shown in Table 12, our estimates of $\hat{\beta}$ and $\hat{\gamma}$ are close to those reported in Table 2 and discussed in the main text, except for the pooled mean group (PMG) estimator.

## C A Two-Sector Model with Imperfect Mobility of Labor across Sectors

This Appendix presents the formal analysis underlying the results when abstracting from physical capital. Time is continuous and indexed by $t$.

## C. 1 Households

At each instant the representative agent consumes traded goods and non-traded goods denoted by $C^{T}(t)$ and $C^{N}(t)$, respectively, which are aggregated by a constant elasticity of substitution function:

$$
\begin{equation*}
C\left(C^{T}(t), C^{N}(t)\right)=\left[\varphi^{\frac{1}{\phi}}\left(C^{T}(t)\right)^{\frac{\phi-1}{\phi}}+(1-\varphi)^{\frac{1}{\phi}}\left(C^{N}(t)\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{40}
\end{equation*}
$$

The representative agent must also decide on worked hours in the traded and the non traded sector denoted by $L^{T}(t)$ and $L^{N}(t)$ at each instant of time which are aggregated by a constant elasticity of substitution function:

$$
\begin{equation*}
L\left(L^{T}(t), L^{N}(t)\right)=\left[\vartheta^{-\frac{1}{\epsilon}}\left(L^{T}(t)\right)^{\frac{\epsilon+1}{\epsilon}}+(1-\vartheta)^{-\frac{1}{\epsilon}}\left(L^{N}(t)\right)^{\frac{\epsilon+1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon+1}} \tag{41}
\end{equation*}
$$

The agent is endowed with a unit of time and supplies a fraction $L(t)$ of this unit as labor, while the remainder, $1-L(t)$, is consumed as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Households decide on consumption and worked hours by maximizing lifetime utility:

$$
\begin{equation*}
U=\int_{0}^{\infty}\left\{\frac{1}{1-\frac{1}{\sigma_{C}}} C(t)^{1-\frac{1}{\sigma_{C}}}-\gamma \frac{1}{1+\frac{1}{\sigma_{L}}} L(t)^{1+\frac{1}{\sigma_{L}}}\right\} e^{-\beta t} \mathrm{~d} t \tag{42}
\end{equation*}
$$

where $\beta$ is the consumer's discount rate, $\sigma_{C}>0$ is the intertemporal elasticity of substitution for consumption, and $\sigma_{L}>0$ is the Frisch elasticity of labor supply.

Labor income is derived by supplying labor in the traded sector $L^{T}(t)$ and non traded sector $L^{N}(t)$ at a wage rate $W^{T}(t)$ and $W^{N}(t)$, respectively. In addition, households accumulate internationally traded bonds, $B(t)$, that yield net interest rate earnings of $r^{\star} B(t)$. The flow budget constraint is equal to households' income less consumption expenditure:

$$
\begin{equation*}
\dot{B}(t)=r^{\star} B(t)+W^{T}(t) L^{T}(t)+W^{N}(t) L^{N}(t)-C^{T}(t)-P(t) C^{N}(t) \tag{43}
\end{equation*}
$$

For the sake of clarity, we drop the time argument below when this causes no confusion.
The current-value for the household's optimization problem is (dropping the time index for the purposes of clarity):

$$
\mathcal{H}=U\left[C\left(C^{T}, C^{N}\right)\right]+V\left[L\left(L_{T}, L_{N}\right)\right]+\lambda\left(r^{\star} B+W^{T} L^{T}+W^{N} L^{N}-C^{T}-P C^{N}\right)
$$

where $B$ is the state variable, $\lambda$ is the corresponding co-state variable, and $C^{T}, C^{N}, L^{T}$ and $L^{N}$ are control variables. The first-order conditions are:

$$
\begin{gather*}
C^{-\frac{1}{\sigma_{C}}} \varphi^{\frac{1}{\phi}}\left(C^{T}\right)^{-\frac{1}{\phi}} C^{\phi}=\lambda,  \tag{44a}\\
C^{-\frac{1}{\sigma_{C}}}(1-\varphi)^{\frac{1}{\phi}}\left(C^{N}\right)^{-\frac{1}{\phi}} C^{\phi}=\lambda P,  \tag{44b}\\
\gamma L^{\frac{1}{\sigma_{L}}} \vartheta^{-\frac{1}{\epsilon}}\left(L^{T}\right)^{\frac{1}{\epsilon}} L^{-\frac{1}{\epsilon}}=\lambda W^{T},  \tag{44c}\\
\left.\gamma L^{\frac{1}{\sigma_{L}}}(1-\vartheta)\right)^{-\frac{1}{\epsilon}}\left(L^{N}\right)^{\frac{1}{\epsilon}} L^{-\frac{1}{\epsilon}}=\lambda W^{N},  \tag{44~d}\\
\dot{\lambda}=\lambda\left(\beta-r^{\star}\right), \tag{44e}
\end{gather*}
$$

and the transversality condition $\lim _{t \rightarrow \infty} \lambda B(t) e^{-\beta t}=0$.
Combining (44a) and (44b) yields:

$$
\begin{equation*}
\left(\frac{\varphi}{1-\varphi}\right) \frac{C^{N}}{C^{T}}=P^{-\phi} \tag{45}
\end{equation*}
$$

Combining (44c) and (44d) yields:

$$
\begin{equation*}
\left(\frac{\vartheta}{1-\vartheta}\right) \frac{L^{N}}{L^{T}}=\Omega^{\epsilon} \tag{46}
\end{equation*}
$$

where $\Omega \equiv W^{N} / W^{T}$.

## Consumption Price Index

The traded and the non traded goods are aggregated by means of a CES function given by (40) with $\phi>0$ the intratemporal elasticity of substitution between consumption of traded and non traded goods. At the first stage, the household minimizes the cost or total expenditure measured in terms of traded goods:

$$
\begin{equation*}
E_{C} \equiv C^{T}+P C^{N} \tag{47}
\end{equation*}
$$

for a given level of subutility, $C$, where $P$ is the price of non traded goods in terms of traded goods. For any chosen $C$, the optimal basket $\left(C^{T}, C^{N}\right)$ is a solution to:

$$
\begin{equation*}
P_{C}(P) C=\min _{\left\{C^{T}, C^{N}\right\}}\left\{C^{T}+P C^{N}: C\left(C^{T}, C^{N}\right) \geq C\right\} \tag{48}
\end{equation*}
$$

The subutility function $C$ (.) is linear homogeneous which implies that total expenditure in consumption goods can be expressed as $E_{C}(t)=P_{C}(P) C$, with $P_{C}(P)$ is the unit cost function dual (or consumption-based price index) to $C$. The unit cost dual function, $P_{C}($.$) , is defined as the minimum$ total expense in consumption goods, $E_{C}$, such that $C=C\left(C^{T}, C^{N}\right)=1$, for a given level of the relative price of non tradables, $P$. Its expression is given by

$$
\begin{equation*}
P_{C}=\left[\varphi+(1-\varphi) P^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{49}
\end{equation*}
$$

The minimized unit cost function depends on relative price of non tradables with the following properties:

$$
\begin{align*}
P_{C}^{\prime} & =(1-\varphi) P^{-\phi}\left(P_{C}\right)^{\phi}>0  \tag{50a}\\
-\frac{P_{C}^{\prime \prime} P}{P_{C}^{\prime}} & =\phi\left[1-\frac{(1-\varphi) P^{1-\phi}}{P_{C}^{1-\phi}}\right]=\phi\left(1-\alpha_{C}\right) . \tag{50b}
\end{align*}
$$

Intra-temporal allocations between non tradable goods and tradable goods follow from Shephard's Lemma (or the envelope theorem) applied to (48):

$$
\begin{align*}
C^{N} & =P_{C}^{\prime} C=(1-\varphi)\left(\frac{P}{P_{C}}\right)^{-\phi} C, \quad \text { and } \quad \frac{P C^{N}}{P_{C} C}=\alpha_{C}  \tag{51a}\\
C^{T} & =\left[P_{C}-P P_{C}^{\prime}\right] C=\varphi\left(\frac{1}{P_{C}}\right)^{-\phi} C, \quad \text { and } \quad \frac{C^{T}}{P_{C} C}=\left(1-\alpha_{C}\right), \tag{51b}
\end{align*}
$$

where the non tradable and tradable shares in total consumption expenditure are:

$$
\begin{align*}
\alpha_{C} & =\frac{(1-\varphi) P^{1-\phi}}{\varphi+(1-\varphi) P^{1-\phi}},  \tag{52a}\\
1-\alpha_{C} & =\frac{\varphi}{\varphi+(1-\varphi) P^{1-\phi}} \tag{52b}
\end{align*}
$$

Note that the share of non traded goods in consumption expenditure $\alpha_{C}$ coincides with the weight of the non traded good in the overall consumption bundle $1-\varphi$ when the elasticity of substitution $\phi$ between traded and non traded goods is equal to one. When calibrating the model, we consider an elasticity of substitution equal to one in the baseline scenario so that $\alpha_{C}=1-\varphi$. When $\phi \neq 1$, we have to set $\varphi$ so as to target a share of non traded goods in consumption expenditure $\alpha_{C}$ in line with our empirical estimates (i.e., 0.43 , see Table 7 ).

## Aggregate Wage Index

The representative household maximizes $1-L($.$) where L($.$) is a CES function given by (41) with$ $\epsilon>0$ the intratemporal elasticity of substitution between labor in the traded and non traded sector, given total labor income denoted by $R$ measured in terms of the traded good:

$$
\begin{equation*}
R \equiv W^{T} L^{T}+W^{N} L^{N} \tag{53}
\end{equation*}
$$

where $W^{T}$ is the wage rate in the traded sector and $W^{N}$ is the wage rate in the non traded sector. The linear homogeneity of the subutility function $L$ (.) implies that total labor income can be expressed as $R=W\left(W^{T}, W^{N}\right) L$, with $W\left(W^{T}, W^{N}\right)$ is the unit cost function dual (or aggregate wage index) to $L$. The unit cost dual function, $W($.$) , is defined as the minimum total labor income, R$, such that $L=L\left(L^{T}, L^{N}\right)=1$, for a given level of the wage rates $W^{T}$ and $W^{N}$. We derive below its expression.

Combining (46) together with total labor income denoted by $R$ measured in terms of the traded good, i.e. $R \equiv W^{T} L^{T}+W^{N} L^{N}$, we are able to express labor supply in the traded and non traded sector, respectively, as functions of total labor income:

$$
\begin{aligned}
L^{T} & =(1-\vartheta)\left(W^{T}\right)^{-1}\left[(1-\vartheta)+\vartheta\left(\frac{W^{N}}{W^{T}}\right)^{\epsilon+1}\right]^{-1} R \\
L^{N} & =\vartheta\left(W^{T}\right)^{-1}\left(\frac{W^{N}}{W^{T}}\right)^{\epsilon}\left[(1-\vartheta)+\vartheta\left(\frac{W^{N}}{W^{T}}\right)^{\epsilon+1}\right]^{-1} R .
\end{aligned}
$$

Plugging these equations into (41), setting $L=1$ and $R=W$, yields the aggregate wage index:

$$
\begin{equation*}
W=\left[\vartheta\left(W^{T}\right)^{\epsilon+1}+(1-\vartheta)\left(W^{N}\right)^{\epsilon+1}\right]^{\frac{1}{\epsilon+1}} . \tag{54}
\end{equation*}
$$

Intratemporal allocation of hours worked between the traded and the non traded sector follows from Shephard's Lemma (or the envelope theorem):

$$
\begin{align*}
L^{T} & =\frac{\partial W}{\partial W^{T}} L=W_{T} L, \quad \text { and } \quad \frac{W^{T} L^{T}}{W L}=1-\alpha_{L}  \tag{55a}\\
L^{N} & =\frac{\partial W}{\partial W^{N}} L=W_{N} L, \quad \text { and } \quad \frac{W^{N} L^{N}}{W L}=\alpha_{L} \tag{55b}
\end{align*}
$$

where the non tradable and tradable content of total labor income are:

$$
\begin{align*}
\alpha_{L} & =\frac{(1-\vartheta)\left(W^{N}\right)^{\epsilon+1}}{\left[\vartheta\left(W^{T}\right)^{\epsilon+1}+(1-\vartheta)\left(W^{N}\right)^{\epsilon+1}\right]},  \tag{56a}\\
1-\alpha_{L} & =\frac{\vartheta\left(W^{T}\right)^{\epsilon+1}}{\left[\vartheta\left(W^{T}\right)^{\epsilon+1}+(1-\vartheta)\left(W^{N}\right)^{\epsilon+1}\right]} \tag{56b}
\end{align*}
$$

## Alternative Way to Solve the Household's Maximization Problem

The representative household maximizes lifetime utility (42) subject to the budget constraint:

$$
\begin{equation*}
\dot{B}=r^{\star} B+W\left(W^{T}, W^{N}\right) L-P_{C}(P) C . \tag{57}
\end{equation*}
$$

Denoting the co-state variable associated with (57) by $\lambda$, the first-order conditions characterizing the representative household's optimal plans are:

$$
\begin{align*}
C & =\left(P_{C} \lambda\right)^{-\sigma_{C}}  \tag{58a}\\
L & =\left(\frac{W \lambda}{\gamma}\right)^{\sigma_{L}}  \tag{58b}\\
\dot{\lambda} & =\lambda\left(\beta-r^{\star}\right) \tag{58c}
\end{align*}
$$

and the transversality condition $\lim _{t \rightarrow \infty} \bar{\lambda} B(t) e^{-\beta t}=0$. In an open economy model with a representative agent having perfect foresight, a constant rate of time preference and perfect access to world capital markets, we impose $\beta=r^{\star}$ in order to generate an interior solution. This standard assumption made in the literature implies that the marginal utility of wealth, $\lambda$, will undergo a discrete jump when individuals receive new information and must remain constant over time from thereon, i.e., $\lambda=\bar{\lambda}$.

The homogeneity of $C($.$) allows a two-stage consumption decision: in the first stage, consumption$ is determined, and the intratemporal allocation between traded and non-traded goods is decided at the second stage. Applying Shephard's lemma gives $C^{N}=P_{C}^{\prime} C$ where $P_{C}^{\prime}=\partial P_{C} / \partial P$; denoting by $\alpha_{C}$ the share of non-traded goods in the consumption expenditure, we have $C^{N}=\alpha_{C} P_{C} C / P$ and $C^{T}=P_{C} C-P C^{N}=\left(1-\alpha_{C}\right) P_{C} C$. Dividing expenditure on traded goods by expenditure on non traded goods leads to:

$$
\begin{equation*}
\frac{C^{T}}{P C^{N}}=\frac{\left(1-\alpha_{C}\right) P_{C} C}{\alpha_{C} P_{C} C}, \quad \text { or } \quad\left(\frac{1-\varphi}{\varphi}\right) \frac{C^{T}}{C^{N}}=P^{\phi} \tag{59}
\end{equation*}
$$

Applying the same logic to the labor supply decision gives the tradable content of labor income $W^{T} L^{T}=\left(1-\alpha_{L}\right) W L$ and the non tradable content of total labor income $W^{N} L^{N}=\alpha_{L} W L$. Dividing
labor income in the traded sector by labor income in the non traded sector and using the definition of $\alpha_{L}$ given by (56), we find:

$$
\begin{equation*}
\frac{W^{T} L^{T}}{W^{N} L^{N}}=\frac{\left(1-\alpha_{L}\right) W L}{\alpha_{L} W L}, \quad \text { or } \quad\left(\frac{1-\vartheta}{\vartheta}\right) \frac{L^{T}}{L^{N}}=\left(\frac{W^{T}}{W^{N}}\right)^{\epsilon} \tag{60}
\end{equation*}
$$

We write out some useful properties:

$$
\begin{align*}
\frac{\partial W}{\partial W^{T}} \frac{W^{T}}{W} & =\left(1-\alpha_{L}\right), \quad \frac{\partial W}{\partial W^{N}} \frac{W^{N}}{W}=\alpha_{L}  \tag{61a}\\
\frac{\partial W_{T}}{\partial W^{T}} & =\frac{\partial^{2} W}{\partial\left(W^{T}\right)^{2}}=\vartheta \epsilon\left(W^{T}\right)^{\epsilon-1} W^{-\epsilon} \alpha_{L}  \tag{61b}\\
\frac{\partial W_{T}}{\partial W^{T}} \frac{W^{T}}{W_{T}} & =\epsilon \alpha_{L}>0  \tag{61c}\\
\frac{\partial W_{T}}{\partial W^{N}} \frac{W^{N}}{W_{T}} & =-\epsilon \alpha_{L}<0  \tag{61d}\\
\frac{\partial W_{N}}{\partial W^{N}} \frac{W^{N}}{W} & =\epsilon\left(1-\alpha_{L}\right)>0  \tag{61e}\\
\frac{\partial W_{N}}{\partial W^{T}} \frac{W^{T}}{W} & =-\epsilon\left(1-\alpha_{L}\right)<0 \tag{61f}
\end{align*}
$$

where $W_{j}=\frac{\partial W}{\partial W^{j}}($ with $j=T, N)$.

## C. 2 Firms

Both the traded and non-traded sectors use labor, $L^{T}$ and $L^{N}$, according to linearly homogenous production functions, $Y^{T}=A^{T} L^{T}$ and $Y^{N}=A^{N} L^{N}$. Both sectors face a labor cost equal to the wage rate, i.e., $W^{T}$ and $W^{N}$, respectively. The traded and non traded sectors are assumed to be perfectly competitive. The first order conditions derived from profit-maximization state that factors are paid to their respective marginal products:

$$
\begin{equation*}
A^{T}=W^{T}, \quad \text { and } \quad P A^{N}=W^{N} \tag{62}
\end{equation*}
$$

Dividing the second equality by the first yields

$$
\begin{equation*}
P=\Omega \frac{A^{T}}{A^{N}} \tag{63}
\end{equation*}
$$

## C. 3 Solving the Model

## Model Closure

Abstracting from capital accumulation and government spending, the market-clearing condition in the non traded good market requires that non traded output $Y^{N}$ is equalized with consumption $C^{N}$ :

$$
\begin{equation*}
C^{N}=Y^{N} \tag{64}
\end{equation*}
$$

Inserting (62) and plugging (64) into the accumulation equation of foreign bonds (57) yields the market clearing condition for the traded good or the current account dynamic equation:

$$
\begin{equation*}
\dot{B}=r^{\star} B+Y^{T}-C^{T} \tag{65}
\end{equation*}
$$

## Short-Run Static Solutions for Consumption and Labor

In this subsection, we compute short-run static solutions for consumption and labor supply. Static efficiency conditions (58a) and (58b) can be solved for consumption and labor which of course must hold at any point of time: ${ }^{43}$

$$
\begin{equation*}
C=C(\bar{\lambda}, P), \quad L=L\left(\bar{\lambda}, W^{T}, W^{N}\right) \tag{66}
\end{equation*}
$$

[^23]with
\[

$$
\begin{align*}
C_{\bar{\lambda}} & =\frac{\partial C}{\partial \bar{\lambda}}=-\sigma_{C} \frac{C}{\bar{\lambda}}<0,  \tag{67a}\\
C_{P} & =\frac{\partial C}{\partial P}=-\alpha_{C} \sigma_{C} \frac{C}{P}<0,  \tag{67b}\\
L_{\bar{\lambda}} & =\frac{\partial L}{\partial \bar{\lambda}}=\sigma_{L} \frac{L}{\bar{\lambda}}>0,  \tag{67c}\\
L_{W^{T}} & =\frac{\partial L}{\partial W^{T}}=\sigma_{L} L \frac{\left(1-\alpha_{L}\right)}{W^{T}}>0,  \tag{67d}\\
L_{W^{N}} & =\frac{\partial L}{\partial W^{N}}=\sigma_{L} L \frac{\alpha_{L}}{W^{N}}>0, \tag{67e}
\end{align*}
$$
\]

where we used the fact that $\frac{\partial W}{\partial W^{T}} \frac{W^{T}}{W}=\left(1-\alpha_{L}\right)$ and $\frac{\partial W}{\partial W^{N}} \frac{W^{N}}{W}=\alpha_{L}$ (see (61)); $\sigma_{C}$ and $\sigma_{L}$ correspond to the intertemporal elasticity of substitution for consumption and labor, respectively.

Inserting first the short-run solution for consumption (66), (51) can be solved for $C^{T}$ and $C^{N}$ :

$$
\begin{equation*}
C^{T}=C^{T}(\bar{\lambda}, P), \quad C^{N}=C^{N}(\bar{\lambda}, P), \tag{68}
\end{equation*}
$$

where partial derivatives are

$$
\begin{align*}
C_{\bar{\lambda}}^{T} & =-\sigma_{C} \frac{C^{T}}{\bar{\lambda}}<0  \tag{69a}\\
C_{P}^{T} & =\alpha_{C} \frac{C^{T}}{P}\left(\phi-\sigma_{C}\right) \lessgtr 0  \tag{69b}\\
C_{\bar{\lambda}}^{N} & =-\sigma_{C} \frac{C^{N}}{\bar{\lambda}}<0  \tag{69c}\\
C_{P}^{N} & =-\frac{C^{N}}{P}\left[\left(1-\alpha_{C}\right) \phi+\alpha_{C} \sigma_{C}\right]<0 \tag{69d}
\end{align*}
$$

where we used the fact that $-\frac{P_{C}^{\prime \prime} P}{P_{C}^{\prime}}=\phi\left(1-\alpha_{C}\right)>0$ and $P_{C}^{\prime} C=C^{N}$.
Inserting first the short-run solution for labor (66), into $L^{T}=\frac{\partial W\left(W^{T}, W^{N}\right)}{\partial W^{T}} L$ and $L^{N}=\frac{\partial W\left(W^{T}, W^{N}\right)}{\partial W^{N}} L$, we are able to solve for $L^{T}$ and $L^{N}$ :

$$
\begin{equation*}
L^{T}=L^{T}\left(\bar{\lambda}, W^{T}, W^{N}\right), \quad L^{N}=L^{N}\left(\bar{\lambda}, W^{T}, W^{N}\right), \tag{70}
\end{equation*}
$$

where partial derivatives are

$$
\begin{align*}
L_{\bar{\lambda}}^{T} & =\frac{\partial L^{T}}{\partial \bar{\lambda}}=\sigma_{L} \frac{L^{T}}{\bar{\lambda}}>0,  \tag{71a}\\
L_{W^{T}}^{T} & =\frac{\partial L^{T}}{\partial W^{T}}=\frac{L^{T}}{W^{T}}\left[\epsilon \alpha_{L}+\sigma_{L}\left(1-\alpha_{L}\right)\right]>0,  \tag{71b}\\
L_{W^{N}}^{T} & =\frac{\partial L^{T}}{\partial W^{N}}=\frac{L^{T}}{W^{N}} \alpha_{L}\left(\sigma_{L}-\epsilon\right) \gtrless 0,  \tag{71c}\\
L_{\bar{\lambda}}^{N} & =\frac{\partial L^{N}}{\partial \bar{\lambda}}=\sigma_{L} \frac{L^{N}}{\bar{\lambda}}>0,  \tag{71d}\\
L_{W^{N}}^{N} & =\frac{\partial L^{N}}{\partial W^{N}}=\frac{L^{N}}{W^{N}}\left[\epsilon\left(1-\alpha_{L}\right)+\sigma_{L} \alpha_{L}\right]>0,  \tag{71e}\\
L_{W^{T}}^{N} & =\frac{\partial L^{N}}{\partial W^{T}}=\frac{L^{N}}{W^{T}}\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right) \gtrless 0, \tag{71f}
\end{align*}
$$

where we used the fact that $\frac{W_{T T} W^{T}}{W_{T}}=\epsilon \alpha_{L}, \frac{W_{T N} W^{N}}{W_{T}}=-\epsilon \alpha_{L}, \frac{W_{N N} W^{N}}{W_{N}}=\epsilon\left(1-\alpha_{L}\right), \frac{W_{N T} W^{T}}{W_{N}}=$ $-\epsilon\left(1-\alpha_{L}\right)$.

Short-run Static Solutions for Sectoral Wages
First order conditions (62) can be solved for the sectoral wages:

$$
\begin{equation*}
W^{T}=W^{T}\left(A^{T}\right), \quad W^{N}=W^{N}\left(A^{N}, P\right), \tag{72}
\end{equation*}
$$

where partial derivatives are:

$$
\begin{align*}
W_{A^{T}}^{T} & =\frac{\partial W^{T}}{\partial A^{T}}=1  \tag{73a}\\
W_{A^{N}}^{N} & =\frac{\partial W^{N}}{\partial A^{N}}=P  \tag{73b}\\
W_{P}^{N} & =\frac{\partial W^{N}}{\partial P}=A^{N} \tag{73c}
\end{align*}
$$

Inserting (72) into (70) yields:

$$
\begin{equation*}
L^{T}=L^{T}\left(\bar{\lambda}, A^{T}, A^{N}, P\right), \quad L^{N}=L^{N}\left(\bar{\lambda}, A^{T}, A^{N}, P\right), \tag{74}
\end{equation*}
$$

where partial derivatives are

$$
\begin{align*}
L_{A^{T}}^{T} & =\frac{\partial L^{T}}{\partial A^{T}}=L_{W^{T}}^{T}=\frac{L^{T}}{W^{T}}\left[\epsilon \alpha_{L}+\sigma_{L}\left(1-\alpha_{L}\right)\right]>0  \tag{75a}\\
L_{A^{N}}^{T} & =\frac{\partial L^{T}}{\partial A^{N}}=L_{W^{N}}^{T} P=P \frac{L^{T}}{W^{N}} \alpha_{L}\left(\sigma_{L}-\epsilon\right) \gtrless 0  \tag{75b}\\
L_{P}^{T} & =\frac{\partial L^{T}}{\partial P}=L_{W^{N}}^{T} A^{N}=A^{N} \frac{L^{T}}{W^{N}} \alpha_{L}\left(\sigma_{L}-\epsilon\right) \gtrless 0  \tag{75c}\\
L_{A^{T}}^{N} & =\frac{\partial L^{N}}{\partial A^{T}}=L_{W^{T}}^{N}=\frac{L^{N}}{W^{T}}\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right) \gtrless 0  \tag{75d}\\
L_{A^{N}}^{N} & =\frac{\partial L^{N}}{\partial A^{N}}=L_{W^{N}}^{N} P=P \frac{L^{N}}{W^{N}}\left[\epsilon\left(1-\alpha_{L}\right)+\sigma_{L} \alpha_{L}\right] \gtrless 0  \tag{75e}\\
L_{P}^{N} & =\frac{\partial L^{N}}{\partial P}=L_{W^{N}}^{N} A^{N}=A^{N} \frac{L^{N}}{W^{N}}\left[\epsilon\left(1-\alpha_{L}\right)+\sigma_{L} \alpha_{L}\right]>0 \tag{75f}
\end{align*}
$$

and $L_{\bar{\lambda}}^{T}$ and $L_{\bar{\lambda}}^{N}$ are given by (71a) and (71d), respectively.

## C. 4 Equilibrium Dynamics

Inserting the short-run static solutions for labor in the non-traded sector and consumption in nontradables given by (74) and (68), respectively, into the non traded good market clearing condition (64), and linearizing around the steady-state implies that the dynamics for the relative price of non tradables degenerate, i.e., $P(t)=\tilde{P}$.

Inserting the short-run static solutions for labor in the traded sector and consumption in tradables given by (74) and (68) into the accumulation equation of foreign bonds (65), respectively, and linearizing around the steady-state yields:

$$
\begin{equation*}
\dot{B}(t)=r^{\star}(B(t)-\tilde{B}) \tag{76}
\end{equation*}
$$

Solving and invoking the transversality condition $\lim _{t \rightarrow \infty} \lambda B(t) e^{-r^{\star} t}=0$ leads to:

$$
\begin{equation*}
B(t)=B_{0} \tag{77}
\end{equation*}
$$

Hence, for the transversality condition to hold, the stock of traded bonds $B(t)$ must be equal to its initial predetermined level. Combining (77) with (65) gives:

$$
\begin{equation*}
r^{\star} B_{0}+Y^{T}=C^{T} \tag{78}
\end{equation*}
$$

Because the stock of foreign bonds must stick to its initial value, for the sake of simplicity and without loss of generality, we set $B_{0}=0$.

## C. 5 The Equilibrium

The equilibrium is defined by the following set of equations:

$$
\begin{align*}
\left(\frac{1-\varphi}{\varphi}\right) \frac{C^{T}}{C^{N}} & =P^{\phi}  \tag{79a}\\
\left(\frac{1-\vartheta}{\vartheta}\right) \frac{L^{T}}{L^{N}} & =\Omega^{-\epsilon}  \tag{79b}\\
P & =\Omega \frac{A^{T}}{A^{N}}  \tag{79c}\\
\frac{A^{T} L^{T}}{A^{N} L^{N}} & =\frac{C^{T}}{C^{N}} \tag{79d}
\end{align*}
$$

where $\Omega \equiv W^{N} / W^{T}$ is the non traded wage-traded wage ratio or the relative wage. Combining the market clearing conditions in the traded and the non traded sectors given by (78) and (64), respectively, yields ( 79 d ) which states that relative supply of tradables must be equal to relative demand of tradables.

Substituting the relative supply of labor in the traded sector given by (79b) and demand of tradables in terms of non traded goods given by (79a) yields:

$$
\frac{A^{T}}{A^{N}}\left(\frac{\vartheta}{1-\vartheta}\right) \tilde{\Omega}^{-\epsilon}=\left(\frac{\varphi}{1-\varphi}\right) P^{\phi} .
$$

Using (63) to eliminate the relative wage $\Omega$, the equation can be solved for the relative price of non tradables:

$$
\begin{equation*}
P=\Gamma\left(\frac{A^{T}}{A^{N}}\right)^{\frac{\epsilon+1}{\epsilon+\phi}}, \quad \Gamma \equiv\left[\left(\frac{\vartheta}{1-\vartheta}\right)\left(\frac{\varphi}{1-\varphi}\right)\right]^{\frac{1}{\epsilon+\phi}}>0 \tag{80}
\end{equation*}
$$

Denoting by a hat the percentage deviation relative to initial steady-state, (80) can be rewritten as:

$$
\begin{equation*}
\hat{p}=(\epsilon+1) \Theta^{L}\left(\hat{a}^{T}-\hat{a}^{N}\right), \quad \Theta^{L}=\frac{1}{\epsilon+\phi} . \tag{81}
\end{equation*}
$$

Plugging (63) into (80) allows us to solve for the relative wage:

$$
\begin{equation*}
\Omega=\Gamma\left(\frac{A^{T}}{A^{N}}\right)^{-\frac{\phi-1}{\epsilon+\phi}}, \quad \Gamma \equiv\left[\left(\frac{\vartheta}{1-\vartheta}\right)\left(\frac{\varphi}{1-\varphi}\right)\right]^{\frac{1}{\epsilon+\phi}}>0 \tag{82}
\end{equation*}
$$

Taking logarithm and differentiating (82) yields:

$$
\begin{equation*}
\hat{\omega}=-(\phi-1) \Theta^{L}\left(\hat{a}^{T}-\hat{a}^{N}\right), \quad \Theta^{L}=\left(\frac{1}{\epsilon+\phi}\right) \tag{83}
\end{equation*}
$$

## C. 6 Graphical Apparatus

To build intuition, we characterize the equilibrium graphically. We denote the logarithm of variables with lower-case letters. The steady state can be described by considering alternatively the goods market or the labor market.

## Goods Market Equilibrium- and Labor Market Equilibrium- Schedules

The model can be summarized graphically by Figure 3(b) that traces out two schedules in the $\left(y^{T}-y^{N}, p\right)$-space. System (79a)-(79d) described above can be reduced to two equations. Substituting (79a) into eq. (79d) yields the goods market equilibrium (henceforth labelled GME) schedule:

$$
\begin{equation*}
\left.\ln \left(\frac{Y^{T}}{Y^{N}}\right)\right|^{G M E}=\left.\left(y^{T}-y^{N}\right)\right|^{G M E}=\phi p+x \tag{84}
\end{equation*}
$$

where $x=\ln \left(\frac{\varphi}{1-\varphi}\right)$. Since a rise in the relative price $p$ raises consumption in tradables, the goods market equilibrium requires a rise in the traded output-non traded output ratio. Hence the goods market equilibrium is upward-sloping in the $\left(y^{T}-y^{N}, p\right)$-space where the slope is equal to $1 / \phi$.

Substituting (79b) into (79c) to eliminate $\omega$ yields the labor market equilibrium (LME) schedule:

$$
\begin{equation*}
\left.\left(y^{T}-y^{N}\right)\right|^{L M E}=-\epsilon p+(1+\epsilon)\left(a^{T}-a^{N}\right)+z \tag{85}
\end{equation*}
$$

where $z=\ln \left(\frac{\vartheta}{1-\vartheta}\right)$. A rise in the relative price $p$ raises the relative wage $\omega$ which induces agents to supply more labor in the non traded sector, and more so if $\epsilon$ is larger (i.e., agents are more willing to move across sectors). Hence the labor market equilibrium is downward-sloping in the $\left(y^{T}-y^{N}, p\right)$ space where the slope is equal to $-1 / \epsilon$. Assuming that the shift of labor across sectors is costless, i.e. $\epsilon$ tends to infinity, wages between traded and non traded sectors are equalized. Graphically, the $L M E$-schedule becomes an horizontal line. Conversely, as labor mobility becomes more costly, i.e. $\epsilon$ is smaller, the $L M E$-schedule becomes steeper in the $\left(y^{T}-y^{N}, p\right)$-space.

For a given relative price of non tradables, a rise in relative productivity $A^{T} / A^{N}$, shifts to the right the $L M E$-schedule by raising traded output relative to non traded output. Since the supply of traded goods is increased, the price of non traded goods in terms of traded goods $p$ must rise, and less so as the elasticity of substitution $\phi$ between $C^{T}$ and $C^{N}$ is larger.

Further, the more costly (in utility terms) labor mobility is, i.e., the smaller $\epsilon$, the less agents are willing to move from the traded towards the non traded good. Graphically, the $L M E$-schedule shifts to the right by a smaller amount.

## Labor Demand- and Labor Supply- Schedules

The labor market is summarized graphically in Figure 3(a). Eq. (79b) describes the labor supplyschedule ( $L S$ henceforth) in the $\left(l^{T}-l^{N}, \omega\right)$-space. Taking logarithm yields:

$$
\begin{equation*}
\left.\ln \left(\frac{L^{T}}{L^{N}}\right)\right|^{L S}=\left.\left(l^{T}-l^{N}\right)\right|^{L S}=-\epsilon \omega+d \tag{86}
\end{equation*}
$$

where $d=\ln \left(\frac{\vartheta}{1-\vartheta}\right)$. A rise in the non traded wage-traded wage ratio $\omega$ provides an incentive to shift labor supply from the traded sector towards the non traded sector. Hence the $L S$-schedule is downward-sloping in the $\left(l^{T}-l^{N}, \omega\right)$-space where the slope is equal to $-1 / \epsilon$.

Substituting demand for traded goods in terms of non traded goods (79a) into the market clearing condition given by (79d) yields:

$$
\begin{equation*}
\frac{\tilde{Y}^{T}}{\tilde{Y}^{N}}=\left(\frac{\varphi}{1-\varphi}\right) P^{\phi} \tag{87}
\end{equation*}
$$

Substituting first-order conditions from the firms' maximization problem given by (63) and using production functions, i.e. $L^{T}=Y^{T} / A^{T}$ and $L^{N}=Y^{N} / A^{N}$, we get:

$$
\frac{L^{T}}{L^{N}}=\left(\frac{\varphi}{1-\varphi}\right)\left(\frac{A^{T}}{A^{N}}\right)^{\phi-1} \Omega^{\phi}
$$

Taking logarithm yields the labor demand-schedule ( $L D$ henceforth) in the $\left(l^{T}-l^{N}, \omega\right.$ )-space is given by

$$
\begin{equation*}
\left.\left(l^{T}-l^{N}\right)\right|^{L D}=\phi \omega+(\phi-1)\left(a^{T}-a^{N}\right)+x \tag{88}
\end{equation*}
$$

where $x=\ln \left(\frac{\varphi}{1-\varphi}\right)$. A rise in the relative wage $\omega$ raises the cost of labor in the non traded sector relative to the traded sector. To compensate for the increased labor cost, the non traded sector sets higher prices which induces agents to substitute traded for non traded goods and therefore produces an expansionary effect on labor demand in the traded sector. Hence the $L D$-schedule is upward-sloping in the $\left(l^{T}-l^{N}, \omega\right)$-space where the slope is equal to $1 / \phi$.

Using (86) to eliminate $\omega$ and differentiating (88), the percentage change in the ratio $L^{T} / L^{N}$ is given by:

$$
\begin{equation*}
\left(\hat{l}^{T}-\hat{l}^{N}\right)=\frac{\epsilon(\phi-1)}{(\epsilon+\phi)}\left(\hat{a}^{T}-\hat{a}^{N}\right) \tag{89}
\end{equation*}
$$

Hence, depending on whether $\phi \gtrless 1$, a rise in the sectoral labor productivities ratio $a^{T}-a^{N}$ raises or lowers $l^{T}-l^{N}$.

## C. 7 Relative Price and Relative Wage Effects

We now analyze graphically and analytically the consequences on the relative price and the relative wage of an increase in the relative productivity $a^{T} / a^{N}$. Because our estimates capture the longterm effects of an increase in $a^{T} / a^{N}$, we compare the steady state of the model before and after the productivity shock biased towards the traded sector.

To begin with, an inspection of (88) shows that higher productivity in tradables relative to non tradables has an expansionary effect on labor demand in the traded sector relative to the non traded
sector, if and only if the elasticity of substitution $\phi$ between traded and non traded goods is larger than one. The reason is as follows. Higher productivity in tradables increases output of tradables relative to non tradables. For the market clearing condition to hold, the relative price of non tradables must rise. With an elasticity of substitution $\phi$ greater than one, the demand for tradables rises more than proportionately. The increased share of tradables in total expenditure has an expansionary effect on labor demand in tradables relative to non tradables and therefore lowers the relative wage $\omega$. Graphically, as shown in Figure 3(a), the $L D$-schedule shifts to the right along the $L S$-schedule, producing a fall in the relative wage from $\omega_{0}$ to $\omega_{1}$. Because the traded sector pays higher wages, workers shift hours worked towards that sector. The new steady state is $E_{1}$ and the ratio $L^{T} / L^{N}$ is higher.

Equating labor demand given by (88) and labor supply described by (87), differentiating and denoting by a hat the deviation from initial steady state in percentage terms, we find that the (log) relative wage $w^{N} / w^{T}$ declines in the long run as a result of a productivity differential between tradables and non tradables only if $\phi>1$ :

$$
\begin{equation*}
\hat{\omega}=-(\phi-1) \Theta^{L}\left(\hat{a}^{T}-\hat{a}^{N}\right), \quad \Theta^{L}=\left(\frac{1}{\epsilon+\phi}\right) \tag{90}
\end{equation*}
$$

As workers are more reluctant to shift hours worked from the non traded to the traded sector, as reflected by a lower $\epsilon$, the response of the relative wage to a productivity differential is amplified because the traded sector must pay higher wages to attract workers. Graphically, the $L D$-schedule shifts along a steeper $L S$-schedule. When $\epsilon \rightarrow \infty$, the new steady state is $B S_{1}$ and $\hat{\omega}=0$.

Having explored the change in the relative wage, let us now examine the response of the relative price of non tradables to a productivity differential between tradables and non tradables. Graphically, irrespective of whether $\phi \gtrless 1$, an increase in $a^{T}-a^{N}$ shifts the $L M E$-schedule to the right, as shown in Figure 3(b). As long as $\phi>1$, the $L M E$-schedule shifts along a flatter $G M E$-schedule than the $45^{\circ}$ line which implies that $p$ increases less than $a^{T} / a^{N}$, in line with our evidence. To show it formally, we equate (84) to (85) and differentiate:

$$
\begin{equation*}
\hat{p}=(\epsilon+1) \Theta^{L}\left(\hat{a}^{T}-\hat{a}^{N}\right), \tag{91}
\end{equation*}
$$

where $\Theta^{L}$ is given by (90). According to (91), following a productivity differential of $1 \%, p$ must increase by less than $1 \%$ only if $\phi>1$. In this case, the lower $\epsilon$, the smaller $\hat{p}$. Intuitively, because workers are more reluctant to shift hours worked across sectors, the ratio $L^{T} / L^{N}$ increases less, requiring a lower $\hat{p}$ to clear the market. Graphically, the $L M E$-schedule shifts to the right by a smaller amount. When $\epsilon \rightarrow \infty$, we have $\hat{p}=\hat{a}^{T}-\hat{a}^{N}$, i.e., a strict proportional relationship between $p$ and and the productivity differential $a^{T}-a^{N}$.

In conclusion, when assuming imperfect labor mobility, the two-sector model can account for our set of empirical findings but only if the elasticity of substitution is larger than one.

## D Introducing Physical Capital Accumulation

This Appendix presents the formal analysis underlying the results discussed in section 4 of the main text. We extend the small open economy model with tradables and non tradables and imperfect mobility of labor across sectors to physical capital accumulation. We assume that investment expenditure are non traded but relaxes this assumption in section F. We further impose perfect mobility of capital across sectors. This assumption is relaxed in section H. 2 and section I.

## D. 1 Consumer's Maximization Problem

The representative household chooses consumption, decides on labor supply, and investment that maximizes his/her lifetime utility:

$$
\begin{equation*}
U=\int_{0}^{\infty}\left\{\frac{1}{1-\frac{1}{\sigma_{C}}} C(t)^{1-\frac{1}{\sigma_{C}}}-\gamma \frac{1}{1+\frac{1}{\sigma_{L}}} L(t)^{1+\frac{1}{\sigma_{L}}}\right\} e^{-\beta t} \mathrm{~d} t \tag{92}
\end{equation*}
$$

where $\beta$ is the consumer's discount rate, $\sigma_{C}>0$ is the intertemporal elasticity of substitution for consumption, and $\sigma_{L}>0$ is the Frisch elasticity of labor supply.

Denoting the capital rental cost by $R(t)$, the stock of physical capital by $K(t)$ and investment by $I(t)$ which is assumed to be non-traded, the flow budget constraint is:

$$
\begin{equation*}
\dot{B}(t)=r^{\star} B(t)+R(t) K(t)+W\left(W^{T}(t), W^{N}(t)\right) L(t)-P_{C}(P(t)) C(t)-P(t) I(t) \tag{93}
\end{equation*}
$$

and capital accumulation which evolves as follows:

$$
\begin{equation*}
\dot{K}(t)=I(t)-\delta_{K} K(t) \tag{94}
\end{equation*}
$$

where $I$ corresponds to investment expenditure and $0 \leq \delta_{K}<1$ is a fixed depreciation rate.
Denoting the co-state variables associated with (93) and (94) by $\lambda(t)$ and $\psi(t)$, respectively, the first-order conditions characterizing the representative household's optimal plans are:

$$
\begin{gather*}
C(t)=\left(P_{C}(P(t)) \lambda(t)\right)^{-\sigma_{C}}  \tag{95a}\\
L(t)=\left(\frac{W(t) \lambda(t)}{\gamma}\right)^{\sigma_{L}}  \tag{95b}\\
\dot{\lambda}(t)=\lambda(t)\left(\beta-r^{\star}\right)  \tag{95c}\\
\frac{R(t)}{P(t)}-\delta_{K}+\frac{P(t)}{P(t)}=r^{\star} \tag{95d}
\end{gather*}
$$

and the transversality conditions $\lim _{t \rightarrow \infty} \bar{\lambda} B(t) e^{-\beta t}=0$ and $\lim _{t \rightarrow \infty} P(t) K(t) e^{-\beta t}=0$; to derive (95d), we used the fact that $\psi(t)=\lambda P(t)$. For the sake of clarity, we drop the time argument below when this causes no confusion.

Eqs. (95a) and (95b) can be solved for consumption and labor:

$$
\begin{equation*}
C=C(\bar{\lambda}, P), \quad L=L\left(\bar{\lambda}, W^{T}, W^{N}\right) \tag{96}
\end{equation*}
$$

where partial derivatives are given by (67).

## D. 2 Firm's Maximization Problem

Both the traded and non-traded sectors use physical capital, $K^{T}$ and $K^{N}$, and labor, $L^{T}$ and $L^{N}$, according to constant returns to scale production functions: $Y^{T}=Z^{T} F\left(K^{T}, L^{T}\right)$ and $Y^{N}=Z^{N} H\left(K^{N}, L^{N}\right)$, which are assumed to have the usual neoclassical properties of positive and diminishing marginal products:

$$
\begin{equation*}
Y^{j}=Z^{j}\left(L^{j}\right)^{\theta^{j}}\left(K^{j}\right)^{1-\theta^{j}}, \quad j=T, N . \tag{97}
\end{equation*}
$$

Both sectors face two cost components: a capital rental cost equal to $R$, and a labor cost equal to the wage rate, i.e., $W^{T}$ in the traded sector and $W^{N}$ in the non traded sector. Both sectors are assumed to be perfectly competitive.

Denoting by $k^{i} \equiv K^{i} / L^{i}$ the capital-labor ratio for sector $i=T, N$, enables us to express the production functions in intensive form, i.e., $f\left(k^{T}\right) \equiv F\left(K^{T}, L^{T}\right) / L^{T}$ and $h\left(k^{N}\right) \equiv H\left(K^{N}, L^{N}\right) / L^{N}$. Production functions are supposed to take a Cobb-Douglas form: $f\left(k^{T}\right)=\left(k^{T}\right)^{1-\theta^{T}}$ and $h\left(k^{N}\right)=$ $\left(k^{N}\right)^{1-\theta^{N}}$, where $\theta^{T}$ and $\theta^{N}$ represent the labor income share in output in the traded and the non traded sector, respectively. Since capital can move freely between the two sectors while the shift of labor across sectors is costly, only marginal revenues of capital in the traded and the non-traded sector equalize:

$$
\begin{gather*}
Z^{T}\left(1-\theta^{T}\right)\left(k^{T}\right)^{-\theta^{T}}=P Z^{N}\left(1-\theta^{N}\right)\left(k^{N}\right)^{-\theta^{N}} \equiv R  \tag{98a}\\
Z^{T} \theta^{T}\left(k^{T}\right)^{1-\theta^{T}} \equiv W^{T}  \tag{98b}\\
P Z^{N} \theta^{N}\left(k^{N}\right)^{1-\theta^{N}} \equiv W^{N} \tag{98c}
\end{gather*}
$$

These static efficiency conditions state that the sectoral marginal products must equal the labor cost $W^{j}$ and capital rental rate $R$.

The resource constraint for capital is:

$$
\begin{equation*}
k^{T} L^{T}+k^{N} L^{N}=K \tag{99}
\end{equation*}
$$

## D. 3 Solving the Model

Before providing details of derivation, we find convenient to describe the procedure to solve the model.

## Short-Run Static Solutions

Eqs. (95a)-(95b) can be solved for consumption $C=C(\bar{\lambda}, P)$ with $C_{\bar{\lambda}}<0, C_{P}<0$, and for labor $L=L\left(\bar{\lambda}, W^{T}, W^{N}\right)$ with $L_{\bar{\lambda}}>0, L_{W^{T}}>0$ and $L_{W^{N}}>0$. A rise in the shadow value of wealth induces agents to cut their real expenditure and to supply more labor. By raising the consumption price index, an appreciation in the relative price of non tradables drives down consumption. A rise in sectoral wage rates increases the aggregate wage index which provides an incentive to raise hours worked.

Using the fact that consumption in non tradables and tradables are given by $C^{N}=\frac{\partial P_{C}(P)}{\partial P_{N}} C$ and $C^{T}=\left(P_{C}-P P_{C}^{\prime}\right) C$ and inserting the short-run static solution for consumption yields: $C^{N}=$ $C^{N}(\bar{\lambda}, P)$ with $C_{\bar{\lambda}}^{N}<0$ and $C_{P}^{N}<0$, and $C^{T}=C^{T}(\bar{\lambda}, P)$ with $C_{\bar{\lambda}}^{T}<0$ and $C_{P}^{T} \gtrless 0$ (depending on whether $\left.\phi \gtrless \sigma_{C}\right)$.

Using the fact that hours worked in the traded and the non traded sector are given by $L^{T}=$ $\frac{\partial W\left(W^{T}, W^{N}\right)}{\partial W^{T}} L$ and $L^{N}=\frac{\partial W\left(W^{T}, W^{N}\right)}{\partial W^{N}} L$, respectively, and inserting the short-run static solution for labor yields: $L^{T}=L^{T}\left(\bar{\lambda}, W^{T}, W^{N}\right)$ with $L_{\bar{\lambda}}^{T}>0, L_{W^{T}}^{T}>0, L_{W^{N}}^{T} \lessgtr 0$, and $L^{N}=L^{N}\left(\bar{\lambda}, W^{T}, W^{N}\right)$ with $L_{\bar{\lambda}}^{N}>0, L_{W^{N}}^{N}>0, L_{W^{T}}^{N} \lessgtr 0$. The interpretation of these results deserves attention. A rise in the shadow value of wealth induces agents to supply more labor in both sectors. When the traded sector pays higher wages, i.e., $W^{T}$ rises, workers supply more labor in that sector. Higher wages in the traded sector exerts opposite effects on $L^{N}$. On the one hand, because increased $W^{T}$ raises the aggregate wage index in proportion of $\left(1-\alpha_{L}\right)$, workers are induced to supply more labor in the non traded sector. On the other hand, if the cost of shifting is not too high, i.e. if $\epsilon$ is not too small, workers are induced to reallocate hours worked towards the traded sector. If $\epsilon<\sigma_{L}$, a rise in $W^{T}$ lowers $L^{N}$. The same logic applies when analyzing the effect of a rise in $W^{N}$.

Plugging the short-run static solutions for $L^{T}$ and $L^{N}$, into the resource constraint for capital (99), (98a)-(98c) and (99) can be solved for the sectoral capital-labor ratio $k^{j}=k^{j}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right)$ and the sectoral wage $W^{j}=W^{j}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right)$ (with $\left.j=T, N\right)$. Inserting short-run static solutions for sectoral capital-labor ratios and sectoral labor into production functions (97) allows us to solve for sectoral output: $Y^{j}=Y^{j}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right)$. As in a model assuming perfect labor mobility, an increase in $Z^{j}$ stimulates output of sector $j$. A rise in the relative price of non tradables $P$ exerts opposite effects on sectoral outputs by shifting resources away from the traded sector towards the non traded output. Unlike the standard BS model, an increase in $\bar{\lambda}$ (in $K$ ) raises $Y^{T}$ and $Y^{N}$ as well, regardless of sectoral capital intensities, by raising labor $\left(k^{j}\right)$ in both sectors as long as $\epsilon$ is not too large.

## Solving for Sectoral Wage Rates and Sectoral Capital-Labor Ratios

Plugging the short-run static solutions for $L^{T}$ and $L^{N}$ given by (70) into the resource constraint for capital (99), the system of four equations comprising (98a)-(98c) and (99) can be solved for the sectoral wage rates $W^{j}$ and sectoral capital-labor ratios $k^{j}$. Log-differentiating (98a)-(98c) and (99) yields in matrix form:

$$
\left(\begin{array}{cccc}
-\theta^{T} & \theta^{N} & 0 & 0  \tag{100}\\
\left(1-\theta^{T}\right) & 0 & -1 & 0 \\
0 & \left(1-\theta^{N}\right) & 0 & -1 \\
(1-\xi) & \xi & \Psi_{W^{T}} & \Psi_{W^{N}}
\end{array}\right)\left(\begin{array}{c}
\hat{k}^{T} \\
\hat{k}^{N} \\
\hat{W}^{T} \\
\hat{W}^{N}
\end{array}\right)=\left(\begin{array}{c}
\hat{P}+\hat{Z}^{N}-\hat{Z}^{T} \\
-\hat{Z}^{T} \\
-\hat{P}-\hat{Z}^{N} \\
\hat{K}-\Psi_{\bar{\lambda}} \hat{\bar{\lambda}}
\end{array}\right)
$$

where we set:

$$
\begin{align*}
\Psi_{W^{T}} & =(1-\xi) \frac{L_{W^{T}}^{T} W^{T}}{L^{T}}+\xi \frac{L_{W^{T}}^{N} W^{T}}{L^{N}}  \tag{101a}\\
\Psi_{W^{N}} & =(1-\xi) \frac{L_{W^{N}}^{T} W^{N}}{L^{T}}+\xi \frac{L_{W^{N}}^{N} W^{N}}{L^{N}}  \tag{101b}\\
\xi & \equiv \frac{k^{N} L^{N}}{K}  \tag{101c}\\
\Psi_{\bar{\lambda}} & =(1-\xi) \sigma_{L}+\xi \sigma_{L} \tag{101d}
\end{align*}
$$

The determinant is:

$$
\begin{equation*}
G \equiv-\left\{\theta^{T}\left[\left(1-\theta^{N}\right) \Psi_{W^{N}}+\xi\right]+\theta^{N}\left[\left(1-\theta^{T}\right) \Psi_{W^{T}}+(1-\xi)\right]\right\} \lessgtr 0, \tag{102}
\end{equation*}
$$

where

$$
\begin{align*}
\Psi_{W^{T}} & =(1-\xi) \epsilon+\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right),  \tag{103a}\\
\Psi_{W^{N}} & =\xi \epsilon+\alpha_{L}\left(\sigma_{L}-\epsilon\right)  \tag{103b}\\
\Psi_{W^{T}}+\Psi_{W^{N}} & =\sigma_{L} . \tag{103c}
\end{align*}
$$

Because the sign of $\sigma_{L}-\epsilon$ is ambiguous, we cannot sign $G$; while for the baseline calibration, we have $\sigma_{L}<\epsilon$, because the discrepancy is small, we find convenient to assume $\sigma_{L}=\epsilon$ so that a rise in $W^{T}$ $\left(W^{N}\right)$ does not affect $L^{N}\left(L^{T}\right)$. Hence, we have $G<0$. In the following, for clarity purpose, when discussing the results, we assume that $\sigma_{L} \simeq \epsilon$ so that determinant $G$ is negative.

Sectoral wages can be solved as follows:

$$
\begin{equation*}
W^{T}=W^{T}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right), \quad W^{N}=W^{N}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right) \tag{104}
\end{equation*}
$$

with

$$
\begin{align*}
\frac{\hat{W}^{T}}{\hat{K}^{K}} & =-\frac{\left(1-\theta^{T}\right) \theta^{N}}{G}>0,  \tag{105a}\\
\frac{\hat{W}^{N}}{\hat{K}} & =-\frac{\left(1-\theta^{N}\right) \theta^{T}}{G}>0,  \tag{105b}\\
\frac{\hat{W}^{T}}{\hat{P}} & =\frac{\left(1-\theta^{T}\right)\left(\Psi_{W^{N}}+\xi\right)}{G}<0,  \tag{105c}\\
\frac{\hat{W}^{N}}{\hat{P}} & =-\frac{\left\{\theta^{T} \xi+\theta^{N}(1-\xi)++\theta^{N}\left(1-\theta^{T}\right) \Psi_{W^{T}}\right\}}{G}>0, \tag{105d}
\end{align*}
$$

and sectoral capital labor ratios:

$$
\begin{equation*}
k^{T}=k^{T}\left(\lambda, K, P, Z^{T}, Z^{N}\right), \quad k^{N}=k^{N}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right), \tag{106}
\end{equation*}
$$

with

$$
\begin{align*}
\frac{\hat{k}^{T}}{\hat{K}^{N}} & =-\frac{\theta^{N}}{G}>0  \tag{107a}\\
\frac{\hat{k}^{N}}{\hat{K}} & =-\frac{\theta^{T}}{G}>0  \tag{107b}\\
\frac{\hat{k}^{T}}{\hat{P}} & =\frac{\Psi_{W^{N}}+\xi}{G}<0  \tag{107c}\\
\frac{\hat{k}^{N}}{\hat{P}} & =\frac{\left\{\theta^{T} \Psi_{W^{N}}-\left[\left(1-\theta^{T}\right) \Psi_{W^{T}}+(1-\xi)\right]\right\}}{G}>0 \tag{107d}
\end{align*}
$$

Partial derivatives of short-run static solutions for sectoral capital-labor ratios are: $k_{\bar{\lambda}}^{j}<0$ and $k_{K}^{j}>0$ (with $j=T, N$ ), $k_{P}^{N} \gtrless 0$ and $k_{P}^{T}<0, k_{Z^{T}}^{N}<0$ and $k_{Z^{T}}^{T}>0, k_{Z^{N}}^{N}>0$ and $k_{Z^{N}}^{T}<0$. Partial derivatives of short-run static solutions for sectoral wage rates are: $W_{\bar{\lambda}}^{j}<0$ and $W_{K}^{j}>0$ (with $j=T, N), W_{P}^{N}>0$ and $W_{P}^{T}>0, W_{Z^{T}}^{N}<0$ and $W_{Z^{T}}^{T}>0, W_{Z^{N}}^{N}>0$ and $W_{Z^{N}}^{T}<0 .{ }^{44}$ An increase in the capital stock $K$ raises capital-labor ratios and thereby wage rates in both sectors. A rise in $\lambda$ induces agents to supply more labor which reduces capital-labor ratios and thereby wage rates in both sectors. In the standard model assuming perfect mobility of labor across sectors, an appreciation in the relative price of non tradables shifts resources in the non-traded sector and increases (lowers) $k^{N}$ and $k^{T}$ if the traded sector is more (less) capital intensive than the non-traded sector. As in the standard model, $k^{N}$ increases or decrease as $P$ appreciates depending on whether $k^{T} \gtrless k^{T}$. But the difficulty of reallocating labor across sectors, reduces the possibility to shift labor across sectors which moderates changes in sectoral capital-labor ratios. When $\epsilon$ is small, an appreciation in $P$ may result in a decline in $k^{T}$. ${ }^{45}$

[^24]
## Solving for Sectoral Labor and Output

Inserting sectoral wages (104) into (70) allows us to solve for sectoral labor:

$$
\begin{equation*}
L^{T}=L^{T}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right), \quad L^{N}=L^{N}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right) \tag{108}
\end{equation*}
$$

where

$$
\begin{align*}
\frac{\hat{L}^{T}}{\hat{K}} & =-\left\{\frac{\sigma_{L}\left(1-\theta^{T}\right) \theta^{N}+\alpha_{L}\left(\sigma_{L}-\epsilon\right)\left(\theta^{T}-\theta^{N}\right)}{G}\right\} \gtrless 0,  \tag{109a}\\
\frac{\hat{L}^{N}}{\hat{K}} & =-\left\{\frac{\sigma_{L}\left(1-\theta^{N}\right) \theta^{T}+\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)\left(\theta^{N}-\theta^{T}\right)}{G}\right\} \gtrless 0,  \tag{109b}\\
\frac{\hat{L}^{T}}{\hat{P}} & =\frac{\sigma_{L}\left(1-\theta^{T}\right)\left(\Psi_{W^{N}}+\xi\right)-\alpha_{L}\left(\sigma_{L}-\epsilon\right)\left[\xi+\theta^{N}(1-\xi)+\left(1-\theta^{T}\right)\left(\Psi_{W^{N}}+\theta^{N} \Psi_{W^{T}}\right)\right]}{G}(10400 \mathrm{c}) \\
\frac{\hat{L}^{N}}{\hat{P}} & =\sigma_{L} \frac{\hat{W}^{N}}{\hat{P}}+\frac{\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)\left[\xi+\theta^{N}(1-\xi)+\left(1-\theta^{T}\right)\left(\Psi_{W^{N}}+\theta^{N} \Psi_{W^{T}}\right)\right]}{G} \gtrless 0 . \tag{109d}
\end{align*}
$$

Substituting short-run static solutions for sectoral capital-labor ratios (106) and sectoral labor (108) into the production function of the traded and non traded sectors (97) yields:

$$
\begin{equation*}
Y^{T}=Y^{T}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right), \quad Y^{N}=Y^{N}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right) \tag{110}
\end{equation*}
$$

where

$$
\begin{align*}
\frac{\hat{Y}^{T}}{\hat{K}} & =\frac{\hat{L}^{T}}{\hat{K}}+\left(1-\theta^{T}\right) \frac{\hat{k}^{T}}{\hat{K}}=-\left\{\frac{\left(\sigma_{L}+1\right)\left(1-\theta^{T}\right) \theta^{N}+\alpha_{L}\left(\sigma_{L}-\epsilon\right)\left(\theta^{T}-\theta^{N}\right)}{G}\right\} \gtrless 0 \\
\frac{\hat{Y}^{N}}{\hat{K}^{N}} & =\frac{\hat{L}^{N}}{\hat{K}}+\left(1-\theta^{N}\right) \frac{\hat{k}^{N}}{\hat{K}}=-\left\{\frac{\left(\sigma_{L}+1\right)\left(1-\theta^{N}\right) \theta^{T}+\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)\left(\theta^{N}-\theta^{T}\right)}{G}\right\} \gtrless 0, \\
\frac{\hat{Y}^{T}}{\hat{P}} & =\frac{\hat{L}^{T}}{\hat{P}}+\left(1-\theta^{T}\right) \frac{\hat{k}^{T}}{\hat{P}} \\
& =\frac{\left(\sigma_{L}+1\right)\left(1-\theta^{T}\right)\left(\Psi_{W^{N}}+\xi\right)-\alpha_{L}\left(\sigma_{L}-\epsilon\right)\left[\xi+\theta^{N}(1-\xi)+\left(1-\theta^{T}\right)\left(\Psi_{W^{N}}+\theta^{N} \Psi_{W^{T}}\right)\right]_{(1)}}{G} \\
\frac{\hat{Y}^{N}}{\hat{P}} & =\frac{\hat{L}^{N}}{\hat{P}}+\left(1-\theta^{N}\right) \frac{\hat{k}^{N}}{\hat{P}} \gtrless 0 .
\end{align*}
$$

## D. 4 Model Closure

The non traded goods market must clear:

$$
\begin{equation*}
Y^{N}=C^{N}+I=C^{N}+\dot{K}-\delta_{K} K . \tag{112}
\end{equation*}
$$

Using the fact that $Y^{T}=R^{T} K^{T}+W^{T} L^{T}$ and $Y^{N}=R^{N} K^{N}+W^{N} L^{N}$ and inserting (112) into the flow budget constraint (93) gives the current account equation:

$$
\begin{equation*}
\dot{B}=r^{\star} B+Y^{T}-C^{T} . \tag{113}
\end{equation*}
$$

## D. 5 Equilibrium Dynamics

Inserting the short-run static solutions (110), (106) and (68) into the physical capital accumulation equation (112) and the dynamic equation for the relative price of non tradables (95d), the dynamic system is:

$$
\begin{align*}
\dot{K} & =Y^{N}(K, P, \bar{\lambda})-C^{N}(\bar{\lambda}, P)-\delta_{K} K  \tag{114a}\\
\dot{P} & =P\left[r^{\star}+\delta_{K}-Z^{N} h_{k}(K, P, \bar{\lambda})\right] \tag{114b}
\end{align*}
$$

where for the purposes of clarity, we abstract from time-constant arguments of short-run static solutions, i.e., $\bar{\lambda}, Z^{T}$, and $Z^{N}$.

Denoting with a tilde long-run values, linearizing these two equations around the steady-state yields in matrix form:

$$
\binom{\dot{K}(t)}{\dot{P}(t)}=\left(\begin{array}{cc}
\left(Y_{K}^{N}-\delta_{K}\right) & \left(Y_{P}^{N}-C_{P}^{N}\right)  \tag{115}\\
-\tilde{P} Z^{N} h_{k k} k_{K}^{N} & -\tilde{P} Z^{N} h_{k k} k_{P}^{N}
\end{array}\right)\binom{K(t)-\tilde{K}}{P(t)-\tilde{P}} .
$$

After some manipulations, we find that the trace of the Jacobian matrix denoted by $\operatorname{Tr} \mathrm{J}$ is:

$$
\begin{equation*}
\left(Y_{K}^{N}-\delta_{K}\right)-\tilde{P} Z^{N} h_{k k} k_{P}^{N}=r^{\star}>0 \tag{116}
\end{equation*}
$$

where we used the fact that $\frac{Y^{N}}{K}=\frac{Z^{N} \xi h_{k}}{1-\theta^{N}}$ and $P Z^{N} h_{k k} k_{P}^{N}=Z^{N} h_{k} \theta^{N} \frac{k_{P}^{N} \tilde{P}}{k^{N}}$.
The determinant of the Jacobian matrix denoted by Det J is:

$$
\begin{equation*}
\left(Y_{P}^{N}-C_{P}^{N}\right) \tilde{P} Z^{N} h_{k k} k_{K}^{N}-\left(Y_{K}^{N}-\delta_{K}\right) \tilde{P} Z^{N} h_{k k} k_{P}^{N} \tag{117}
\end{equation*}
$$

Saddle-path stability requires that (117) is negative. For all parametrization and irrespective of the relative capital intensities, this inequality holds. The stable solutions are:

$$
\begin{align*}
K(t) & =\tilde{K}+\left(K_{0}-\tilde{K}\right) e^{\mu_{1} t}  \tag{118a}\\
P(t) & =\tilde{P}+\omega_{2}^{1}\left(K_{0}-\tilde{K}\right) e^{\mu_{1} t} \tag{118b}
\end{align*}
$$

where $K_{0}$ is the initial capital stock and $\left(1, \omega_{2}^{1}\right)^{\prime}$ is the eigenvector associated with the stable negative eigenvalue $\mu_{1}<0$ :

$$
\begin{equation*}
\omega_{2}^{1}=\frac{\mu_{1}-\left(Y_{K}^{N}-\delta_{K}\right)}{\left(Y_{P}^{N}-C_{P}^{N}\right)} \tag{119}
\end{equation*}
$$

For all plausible sets of parameter values, we find numerically $\omega_{2}^{1}<0$, regardless of sectoral capital intensities, which implies that the relative price of nontradables is negatively correlated with investment along the stable transitional path.

Inserting short-run static solutions for traded output (110) and consumption in tradables (68) into the market clearing condition for the traded good (113) gives:

$$
\begin{equation*}
\dot{B}=r^{\star} B+Y^{T}(K, P, \bar{\lambda})-C^{T}(P, \bar{\lambda}) \tag{120}
\end{equation*}
$$

Linearizing (120) around the steady state, substituting the solutions for $K(t)$ and $P(t)$, and invoking the transversality condition, yields the stable solution for the stock of foreign bonds:

$$
\begin{equation*}
B(t)=\tilde{B}+\Phi\left(K_{0}-\tilde{K}\right) e^{\mu_{1} t} \tag{121}
\end{equation*}
$$

where $\Phi=\left[Y_{K}^{T}+\left(Y_{P}^{T}-C_{P}^{T}\right) \omega_{2}^{1}\right] /\left(\mu_{1}-r^{\star}\right)$ is found to be negative numerically.
Finally, the intertemporal solvency condition of the economy is:

$$
\begin{equation*}
\tilde{B}-B_{0}=\Phi\left(\tilde{K}-K_{0}\right) \tag{122}
\end{equation*}
$$

where $B_{0}$ is the initial stock of traded bonds.

## D. 6 The Steady-State

We now characterize the steady-state and use tilde to denote long-run values. Setting $\dot{P}=\dot{K}=\dot{B}=0$ into (114b), (114a) and (120) yields the following set of equations:

$$
\begin{gather*}
Z^{N}\left(1-\theta^{N}\right)\left[k^{N}\left(\tilde{K}, \tilde{P}, \bar{\lambda}, Z^{T}, Z^{N}\right)\right]^{-\theta^{N}}=r^{\star}+\delta_{K}  \tag{123a}\\
Y^{N}\left(\tilde{K}, \tilde{P}, \bar{\lambda}, Z^{T}, Z^{N}\right)-C^{N}(\tilde{P}, \bar{\lambda})-\delta_{K} \tilde{K}=0  \tag{123b}\\
r^{\star} \tilde{B}+Y^{T}\left(\tilde{K}, \tilde{P}, \bar{\lambda}, Z^{T}, Z^{N}\right)-C^{T}(\tilde{P}, \bar{\lambda})=0  \tag{123c}\\
\tilde{B}-B_{0}=\Phi\left(\tilde{K}-K_{0}\right) \tag{123d}
\end{gather*}
$$

These four equations jointly determine $\tilde{P}, \tilde{K}, \tilde{B}$ and $\bar{\lambda}$.

## D. 7 Graphical Apparatus

To build intuition regarding steady-state changes, we investigate graphically the long-run effects of higher productivity of tradables relatives to non tradables. To do so, it is convenient to rewrite the steady-state as follows:

$$
\begin{gather*}
\frac{\tilde{C}^{T}}{\tilde{C^{N}}}=\frac{\varphi}{1-\varphi} \tilde{P}^{\phi},  \tag{124a}\\
\frac{\tilde{L}^{T}}{\tilde{L}^{N}}=\frac{\vartheta}{1-\vartheta} \tilde{\omega}^{-\epsilon},  \tag{124b}\\
\frac{\tilde{Y}^{T}\left(1+v_{B}\right)}{\tilde{Y}^{N}\left(1-v_{I}\right)}=\frac{\tilde{C}^{T}}{\tilde{C}^{N}},  \tag{124c}\\
Z^{N}\left(1-\theta^{N}\right)\left(\tilde{k}^{N}\right)^{-\theta^{N}} \equiv r^{\star}+\delta_{K},  \tag{124d}\\
Z^{T}\left(1-\theta^{T}\right)\left(\tilde{k}^{T}\right)^{-\theta^{T}}=\tilde{P} Z^{N}\left(1-\theta^{N}\right)\left(\tilde{k}^{N}\right)^{-\theta^{N}} \equiv \tilde{R},  \tag{124e}\\
Z^{T} \theta^{T}\left(\tilde{k}^{T}\right)^{1-\theta^{T}} \equiv \tilde{W}^{T},  \tag{124f}\\
P Z^{N} \theta^{N}\left(\tilde{k}^{N}\right)^{1-\theta^{N}} \equiv \tilde{W}^{N}, \tag{124~g}
\end{gather*}
$$

where $\tilde{\omega}=\tilde{W}^{N} / \tilde{W}^{T}$ is the steady-state relative wage and $\tilde{R} / \tilde{P}=r^{\star}+\delta_{K}$. We denote by $v_{I} \equiv \frac{\delta_{K} \tilde{K}}{\tilde{Y}^{N}}$ the ratio of investment to non traded output and by $v_{B} \equiv \frac{r^{\star} \tilde{B}}{\tilde{Y}^{T}}$ the ratio of interest receipts to traded output. Remembering that $\tilde{Y}^{T}=Z^{T} \tilde{L}^{T}\left(\tilde{k}^{T}\right)^{1-\theta^{T}}$ and $\tilde{Y}^{N}=Z^{N} \tilde{L}^{N}\left(\tilde{k}^{N}\right)^{1-\theta^{N}}$, the system (124) can be solved for $\tilde{C}^{T} / \tilde{C}^{N}, \tilde{L}^{T} / \tilde{L}^{N}, \tilde{k}^{T}, \tilde{k}^{N}, \tilde{W}^{T}, \tilde{W}^{N}$ and $\tilde{P}$ as functions of $Z^{T}, Z^{N},\left(\frac{1-v_{I}}{1+v_{B}}\right)$. Then substituting these functions into $\tilde{Y}^{N}=\tilde{C}^{N}+\tilde{I}, \tilde{K}=\tilde{k}^{T} \tilde{L}^{T}+\tilde{k}^{N} \tilde{L}^{N}$ and $\tilde{B}-B_{0}=\Phi\left(\tilde{K}-K_{0}\right)$ and substituting short-run static solutions for $L^{T}$ and $L^{N}$ (see eq. (70)) which obviously hold at the steady-state, the system can be solved for $\tilde{K}, \tilde{B}$ and $\bar{\lambda}$ as functions of $Z^{T}$ and $Z^{N}$. Hence, when solving the system (124), we assume that the aggregate capital stock, foreign bonds and the marginal utility of wealth are exogenous which allows us to separate the static reallocations (or intratemporal) effects from the dynamic (or intertemporal) effects.

Before breaking down the three channels analytically, we characterize the steady state graphically. We denote the logarithm of variables with lower-case letters. Because we restrict ourselves to the analysis of the long-run effects, the tilde is suppressed for the purposes of clarity. The steady state can be described by considering alternatively the labor market or the goods market.

## D. 8 The Goods Market

To begin with, we characterize the goods market equilibrium. The steady state can be summarized graphically in Figure 4(b) if $\phi>1$ and Figure 5(b) if $\phi<1$. Each figure traces out two schedules in the $\left(y^{T}-y^{N}, p\right)$-space which are derived below. System (124) which is described below can be reduced to two equations.

Combining (124a) and the market clearing condition (124c) yields:

$$
\begin{equation*}
\frac{C^{T}}{C^{N}}=\frac{\varphi}{1-\varphi} P^{\phi}=\frac{Y^{T}+r^{\star} B}{Y^{N}-\delta_{K} K} \tag{125}
\end{equation*}
$$

The ratio of traded output to non traded output is:

$$
\begin{equation*}
\frac{Y^{T}}{Y^{N}}=\frac{\left(1-v_{I}\right)}{\left(1+v_{B}\right)} \frac{\varphi}{1-\varphi} P^{\phi} \tag{126}
\end{equation*}
$$

Taking logarithm yields:

$$
\begin{equation*}
\left.\left(y^{T}-y^{N}\right)\right|^{G M E}=\phi p+x^{\prime} \tag{127}
\end{equation*}
$$

where $x^{\prime}=\ln \left(\frac{\varphi}{1-\varphi}\right)+\ln \left(\frac{1-v_{I}}{1+v_{B}}\right)$.

According to (127), as in the model without capital, the goods market equilibrium is upwardsloping in the $\left(y^{T}-y^{N}, p\right)$-space and the slope of the $G M E$-schedule is equal to $1 / \phi$.

Combining (124b) with the steady-state relative wage given by (124f)-(124g), and using the production functions for the traded sector and non traded sectors which imply $L^{T}=\frac{Y^{T}}{Z^{T}\left(k^{T}\right)^{1-\theta^{T}}}$ and $L^{N}=\frac{Y^{N}}{Z^{N}\left(k^{N}\right)^{1-\theta^{N}}}$, yields:

$$
\frac{Y^{T}}{Y^{N}}=\frac{\vartheta}{1-\vartheta}\left(\frac{Z^{T}}{Z^{N}}\right)^{\epsilon+1} P^{-\epsilon}\left(\frac{\theta^{T}}{\theta^{N}}\right)^{\epsilon}\left[\frac{\left(k^{T}\right)^{1-\theta^{T}}}{\left(k^{N}\right)^{1-\theta^{N}}}\right]^{1+\epsilon}
$$

To eliminate the sectoral capital-labor ratios, we use (124d)-(124e):

$$
\frac{\left(k^{T}\right)^{1-\theta^{T}}}{\left(k^{N}\right)^{1-\theta^{N}}}=P^{-\frac{1-\theta^{T}}{\theta^{T}}}\left(r^{\star}+\delta_{K}\right)^{\frac{1-\theta^{N}}{\theta^{N}}}-\frac{1-\theta^{T}}{\theta^{T}} \quad \frac{\left(Z^{T}\right)^{\frac{1-\theta^{T}}{\theta^{T}}}}{\left(Z^{N}\right)^{\frac{1-\theta^{N}}{\theta^{N}}}} \frac{\left(1-\theta^{T}\right)^{\frac{1-\theta^{T}}{\theta^{T}}}}{\left(1-\theta^{N}\right)^{\frac{1-\theta^{N}}{\theta^{N}}}} .
$$

Using the equation above, we have:

$$
\begin{equation*}
\frac{Y^{T}}{Y^{N}}=P^{-\left[\epsilon+\left(\frac{1-\theta^{T}}{\theta^{T}}\right)(1+\epsilon)\right]} \frac{\left(Z^{T}\right)^{\frac{1+\epsilon}{\theta^{T}}}}{\left(Z^{N}\right)^{\frac{1+\epsilon}{\theta N}}} \Pi^{\prime} \tag{128}
\end{equation*}
$$

where we set

$$
\begin{equation*}
\Pi^{\prime} \equiv \frac{\vartheta}{1-\vartheta}\left(r^{\star}+\delta_{K}\right)\left(\frac{\theta^{T}-\theta^{N}}{\theta^{T} \theta^{N}}\right)(1+\epsilon) \frac{\left[\left(\theta^{T}\right)^{\epsilon \theta^{T}}\left(1-\theta^{T}\right)^{\left(1-\theta^{T}\right)(1+\epsilon)}\right]^{1 / \theta^{T}}}{\left[\left(\theta^{N}\right)^{\epsilon \theta^{N}}\left(1-\theta^{N}\right)^{\left(1-\theta^{N}\right)(1+\epsilon)}\right]^{1 / \theta^{N}}>0 . . . . ~ . ~ . ~} \tag{129}
\end{equation*}
$$

Taking logarithm, (128) can be rewritten as follows:

$$
\begin{equation*}
\left.\left(y^{T}-y^{N}\right)\right|^{L M E}=-\left[\epsilon+\left(\frac{1-\theta^{T}}{\theta^{T}}\right)(1+\epsilon)\right] p+\left(\frac{1+\epsilon}{\theta^{T}}\right) z_{T}-\left(\frac{1+\epsilon}{\theta^{N}}\right) z_{N}+\pi^{\prime} \tag{130}
\end{equation*}
$$

where $\pi^{\prime}=\ln \Pi^{\prime}$.
If $\theta^{T}=1$, (130) reduces to (85). If $\theta^{T}<1$, the $L M E$-schedule (labelled $L M E^{K}$ in Figures $4(\mathrm{~b})$ and $5(\mathrm{~b}))$ becomes flatter than that in a model abstracting from physical capital in the $\left(y^{T}-\right.$ $\left.y^{N}, p\right)$-space. The $L M E$-schedule is downward-sloping in the $\left(y^{T}-y^{N}, p\right)$-space with a slope equal to $-1 /\left[\epsilon+\left(\frac{1-\theta^{T}}{\theta^{T}}\right)(1+\epsilon)\right]$. A rise in the relative price of non tradables $p$ allows the non traded sector to pay higher wages. Because the relative wage $\omega$ rises, workers are induced to shift hours worked from the traded sector to the non traded sector. As a consequence, the ratio of sectoral outputs $Y^{T} / Y^{N}$ declines. Introducing capital rotates to the left the $L M E$-schedule due to the shift of capital across sectors triggered by a change in $p$. Following an appreciation in $p$, the non traded sector experiences a capital inflow which amplifies the expansionary effect on non traded output triggered by the reallocation of labor, which results in a flatter $L M E$-schedule.

To sum up, the slope of the GME-schedule remains unchanged while the $L M E$-schedule is flatter than that in a model without capital. Higher productivity in tradables relative to non tradables produces a shift to the right the $L M E$-schedule. The relative price of non tradables rises more or increases less than in a model abstracting from physical capital depending on whether $\phi>1$ or $\phi<1$.

## D. 9 The Labor Market

When focusing on the labor market, the model can be summarized graphically by two schedules in the $\left(l^{T}-l^{N}, \omega\right)$-space, Applying logarithm to (124b) yields the labor supply-schedule (henceforth $L S$-schedule):

$$
\begin{equation*}
\left.\left(l^{T}-l^{N}\right)\right|^{L S}=-\epsilon \omega+d, \tag{131}
\end{equation*}
$$

where $d=\ln \left(\frac{\vartheta}{1-\vartheta}\right)$.
According to (131), as in the model without capital, a rise in the non traded wage-traded wage ratio $\omega$ provides an incentive to shift labor supply from the traded sector towards the non traded
sector. Hence the $L S$-schedule is downward-sloping in the $\left(l^{T}-l^{N}, \omega\right)$-space where the slope is equal to $-1 / \epsilon$.

We turn to the derivation of the labor demand-schedule. Dividing (124g) by (124f) yields:

$$
\begin{equation*}
\frac{P Z^{N} \theta^{N}\left(\tilde{k}^{N}\right)^{1-\theta^{N}}}{Z^{T} \theta^{T}\left(\tilde{k}^{T}\right)^{1-\theta^{T}}} \equiv \Omega \tag{132}
\end{equation*}
$$

To eliminate the sectoral capital-labor ratios, we use eqs. (124d)-(124e):

$$
\begin{equation*}
\frac{\left(k^{N}\right)^{1-\theta^{N}}}{\left(k^{T}\right)^{1-\theta^{T}}}=P^{\frac{1-\theta^{T}}{\theta^{T}}}\left(r^{\star}+\delta_{K}\right)^{\frac{1-\theta^{T}}{\theta^{T}}-\frac{1-\theta^{N}}{\theta^{N}}} \frac{\left[Z^{N}\left(1-\theta^{N}\right)\right]^{\frac{1-\theta^{N}}{\theta^{N}}}}{\left[Z^{T}\left(1-\theta^{T}\right)\right]^{\frac{1-\theta^{T}}{\theta^{T}}}} \tag{133}
\end{equation*}
$$

To eliminate the relative price of non tradables, we combine the market-clearing condition (124c) and the demand for tradables in terms of non traded goods (124a) together with production functions (97):

$$
\begin{equation*}
P=\left[\frac{1-\varphi}{\varphi} \frac{1+v_{B}}{1-v_{I}} \frac{Z^{T} L^{T}\left(k^{T}\right)^{1-\theta^{T}}}{Z^{N} L^{N}\left(k^{N}\right)^{1-\theta^{N}}}\right]^{\frac{1}{\phi}} \tag{134}
\end{equation*}
$$

Substituting (134) into (133) yields:

$$
\begin{align*}
\frac{\left(k^{N}\right)^{1-\theta^{N}}}{\left(k^{T}\right)^{1-\theta^{T}}}= & \left(r^{\star}+\delta_{K}\right)^{\frac{\phi\left(\theta^{N}-\theta^{T}\right)}{\theta^{N}\left[1+\theta^{T}(\phi-1)\right]}}\left[\frac{1-\varphi}{\varphi} \frac{1+v_{B}}{1-v_{I}} \frac{L^{T}}{L^{N}}\right]^{\frac{\left(1-\theta^{T}\right)}{\left[1+\theta^{T}(\phi-1)\right]}} \\
& \times\left[\frac{\left(1-\theta^{N}\right)^{\frac{\left(1-\theta^{N}\right) \theta^{T}}{\theta^{N}}}}{\left(1-\theta^{T}\right)^{\left(1-\theta^{T}\right)}}\right]^{\frac{\phi}{\left[1+\theta^{T}(\phi-1)\right]}} \frac{\left(Z^{N}\right)^{\frac{\left[\left(1-\theta^{N}\right) \theta^{T} \phi-\left(1-\theta^{T}\right) \theta^{N}\right]}{\theta^{N}\left[1+\theta^{T}(\phi-1)\right]}}}{\left(Z^{T}\right)^{\frac{\left(1-\theta^{T}\right)(\phi-1)}{\left[1+\theta^{T}(\phi-1)\right]}}} . \tag{135}
\end{align*}
$$

Substituting first (134) into (132) and then plugging (135) allows us to relate relative labor demand to the relative wage:

$$
\begin{equation*}
\frac{L^{T}}{L^{N}} \frac{\left(Z^{N}\right)^{\frac{(\phi-1) \theta^{T}}{\theta^{N}}}}{\left(Z^{T}\right)^{(\phi-1)}} \Theta=\Omega^{\left[1+\theta^{T}(\phi-1)\right]} \tag{136}
\end{equation*}
$$

where we set

$$
\begin{equation*}
\Theta \equiv\left(r^{\star}+\delta_{K}\right)^{\frac{\left(\theta^{N}-\theta^{T}\right)(\phi-1)}{\theta^{N}}}\left(\frac{1-\varphi}{\varphi}\right)\left(\frac{1+v_{B}}{1-v_{I}}\right)\left(\frac{\theta^{N}}{\theta^{T}}\right)^{\left[1+\theta^{T}(\phi-1)\right]}\left[\frac{\left(1-\theta^{N}\right)^{\left(1-\theta^{N}\right) \frac{\theta^{T}}{\theta^{N}}}}{\left(1-\theta^{T}\right)^{\left(1-\theta^{T}\right)}}\right]^{(\phi-1)} . \tag{137}
\end{equation*}
$$

Applying logarithm to (136) yields the labor demand-schedule (henceforth $L D$-schedule):

$$
\begin{equation*}
\left.\left(l^{T}-l^{N}\right)\right|^{L D}=\left[1+\theta^{T}(\phi-1)\right] \omega+(\phi-1)\left(z^{T}-\frac{\theta^{T}}{\theta^{N}} z^{N}\right)-\ln \Theta . \tag{138}
\end{equation*}
$$

Eq. (138) states that, as in a model abstracting from physical capital, the $L D$-schedule is upwardsloping in the $\left(l^{T}-l^{N}, \omega\right)$-space since an increase in $\omega$ induces non traded producers to set higher prices, increasing the demand for traded goods and therefore labor demand in that sector relative to the non traded sector.

When $\theta^{T}<1$, the $L D$-schedule (labelled $L D^{K}$ in Figures $4(\mathrm{a})$ and $5(\mathrm{a})$ ) is steeper or flatter than that in a model abstracting from physical capital (i.e., when $\theta^{T}=1$ ) depending on whether $\phi$ is larger or smaller than one. In both cases, following an increased non tradable labor cost, the non traded sector is induced to use more capital which raises non traded output and thereby produces a decline in $p$. Depending on whether $\phi$ is larger or smaller than one, the share of non tradables in total expenditure increases or decreases, as a result of the shift of capital towards the non traded sector. Hence, a given rise in $\omega$ produces a smaller or a larger expansionary effect on labor demand in the traded sector depending on whether $\phi$ exceeds or falls below unity.


Figure 4: Effects of an Increase in $Z^{T} /\left(Z^{N}\right)^{\theta^{T} / \theta^{N}}$ when $\phi>1$


Figure 5: Effects of an Increase in $Z^{T} /\left(Z^{N}\right)^{\theta^{T} / \theta^{N}}$ when $\phi<1$

## D. 10 The Relative Price and Relative Wage Effects: Imperfect Substitutability of Hours Worked across Sectors

In this subsection, we derive the long-run responses of the relative price of non tradables to a productivity differential between tradables and non tradables by assuming limited substitutability of hours worked across sectors.

Plugging (124b) into (136) to eliminate $L^{T} / L^{N}$ yields:

$$
\begin{equation*}
\Omega^{\left[(\epsilon+1)+\theta^{T}(\phi-1)\right]}=\Lambda\left(\frac{1+v_{B}}{1-v_{I}}\right)\left[\frac{Z^{T}}{\left(Z^{N}\right)^{\frac{\theta^{T}}{\theta}}}\right]^{-(\phi-1)} \tag{139}
\end{equation*}
$$

where

$$
\begin{equation*}
\Lambda \equiv\left(r^{\star}+\delta_{K}\right)^{\frac{\left(\theta^{N}-\theta^{T}\right)(\phi-1)}{\theta^{N}}}\left(\frac{1-\varphi}{\varphi}\right)\left(\frac{\vartheta}{1-\vartheta}\right)\left(\frac{\theta^{N}}{\theta^{T}}\right)^{\left[1+\theta^{T}(\phi-1)\right]}\left[\frac{\left(1-\theta^{N}\right)^{\left(1-\theta^{N}\right) \frac{\theta^{T}}{\theta^{N}}}}{\left(1-\theta^{T}\right)^{\left(1-\theta^{T}\right)}}\right]^{(\phi-1)} . \tag{140}
\end{equation*}
$$

Taking logarithm and differentiating (139) yields the percentage deviation of the relative wage from its initial steady-state following a productivity differential between tradables and non tradables:

$$
\begin{equation*}
\hat{\omega}=\frac{1}{\left[(\epsilon+1)+\theta^{T}(\phi-1)\right]}\left[\left(\mathrm{d} v_{B}+\mathrm{d} v_{I}\right)-(\phi-1)\left(\hat{z}^{T}-\frac{\theta^{T}}{\theta^{N}} \hat{z}^{N}\right)\right] . \tag{141}
\end{equation*}
$$

To derive the first term in brackets in the RHS of (141), take logarithm to $\left(\frac{1+v_{B}}{1-v_{I}}\right)$ which gives $\ln \left(1+v_{B}\right)-\ln \left(1-v_{I}\right)$, use a Taylor approximation at a first order which implies $\ln \left(1+v_{B}\right)-$ $\ln \left(1-v_{I}\right) \simeq v_{B}+v_{I}$, and differentiate which yields the first term in brackets in the RHS of (141). Since $(\epsilon+1)+\theta^{T}(\phi-1)>\epsilon+\phi, \omega$ falls by a larger amount in a model with capital than that in a model abstracting from physical capital (for given $K$ and $B$ ).

Setting $\Theta^{K} \equiv \frac{1}{\left[(\epsilon+1)+\theta^{T}(\phi-1)\right]}$, the long-run response of the relative wage (141) can be rewritten as follows:

$$
\hat{\omega}=-(\phi-1) \Theta^{K}\left(\hat{z}^{T}-\frac{\theta^{T}}{\theta^{N}} \hat{z}^{N}\right)+\Theta^{K}\left(\mathrm{~d} v_{B}+\mathrm{d} v_{I}\right)
$$

Adding and subtracting $\Theta^{L}=\left(\frac{1}{\epsilon+\phi}\right)$ (see (81)), and noting that $v_{B}=-v_{N X}$ where we denote by $v_{N X} \equiv\left(\tilde{Y}^{T}-\tilde{C}^{T}\right) / \tilde{Y}^{T}$ the ratio of net exports to traded output, allows us to break down the relative wage growth into three components: ${ }^{46}$

$$
\begin{equation*}
\hat{\omega}=-(\phi-1)\left[\Theta^{L}+\left(\Theta^{K}-\Theta^{L}\right)\right]\left[\hat{z}^{T}-\left(\theta^{T} / \theta^{N}\right) \hat{z}^{N}\right]-\Theta^{K}\left(\mathrm{~d} v_{N X}-\mathrm{d} v_{I}\right) \tag{142}
\end{equation*}
$$

Eq. (142) corresponds to eq. (33) in the text.
Equating (127) and (130) to eliminate $y^{T}-y^{N}$, taking logarithm and differentiating yields the percentage deviation of the relative price of non tradables from its initial steady-state following a productivity differential between tradables and non tradables:

$$
\begin{equation*}
\hat{p}=\frac{(1+\epsilon)}{\theta^{T}\left[(\epsilon+\phi)+\left(\frac{1-\theta^{T}}{\theta^{T}}\right)(1+\epsilon)\right]}\left[\hat{z}^{T}-\frac{\theta^{T}}{\theta^{N}} \hat{z}^{N}\right]+\frac{1}{\left[(\epsilon+\phi)+\left(\frac{1-\theta^{T}}{\theta^{T}}\right)(1+\epsilon)\right]}\left(\mathrm{d} v_{B}+\mathrm{d} v_{I}\right) \tag{143}
\end{equation*}
$$

According to (143), keeping unchanged the overall capital stock and the stock of foreign bonds (i.e., keeping fixed $v_{I}$ and $v_{B}$ ), following a productivity differential between tradables and non tradables of 1 percentage point, $p$ increases more or rises less than in a model abstracting from physical capital depending on whether $\phi$ is larger or smaller than one. The reason is that when $\phi<1$, the non traded sector experiences a capital inflow which exerts a negative impact on $p$; conversely, if $\phi>1$, the traded sector experiences a capital inflow which increases traded output and thereby raises more the relative price of non tradables.

[^25]Adopting the same procedure as for the relative wage, i.e., adding and subtracting $\Theta^{L}$, yields the deviation in percentage of the relative price from its initial steady state:

$$
\begin{equation*}
\hat{p}=(1+\epsilon)\left[\Theta^{L}+\left(\Theta^{K}-\Theta^{L}\right)\right]\left(\hat{z}^{T}-\frac{\theta^{T}}{\theta^{N}} \hat{z}^{N}\right)-\theta^{T} \Theta^{K}\left(\mathrm{~d} v_{N X}-\mathrm{d} v_{I}\right) \tag{144}
\end{equation*}
$$

where $\Theta^{K} \equiv \frac{1}{\left[(\epsilon+1)+\theta^{T}(\phi-1)\right]}>0$ and $\Theta^{L} \equiv \frac{1}{\epsilon+\phi}>0$. Eq. (144) corresponds to eq. (34) in the text.

## D. 11 Derivation of the Accumulation Equation of Financial Wealth

Remembering that the stock of financial wealth $A(t)$ is equal to $B(t)+P(t) K(t)$, differentiating w.r.t. time, plugging the dynamic equation (95d) for the relative price, inserting the accumulation equations for physical capital (94) and traded bonds (93), yields the accumulation equation for the stock of financial wealth or private savings dynamic equation:

$$
\begin{equation*}
\dot{A}(t)=r^{\star} A(t)+W(t) L(t)-P_{C}(P(t)) C(t) . \tag{145}
\end{equation*}
$$

We first determine short-run static solutions for aggregate labor supply and aggregate wage index. Inserting short-run static solutions for sectoral wages (104) into the short-run static solution for aggregate labor supply (96), we can solve for total hours worked:

$$
\begin{equation*}
L=L\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right) \tag{146}
\end{equation*}
$$

where partial derivatives are given by

$$
\begin{align*}
L_{K} & \equiv \frac{\partial L}{\partial K}=L_{W^{T}} W_{K}^{T}+L_{W^{N}} W_{K}^{N}  \tag{147a}\\
L_{P} & \equiv \frac{\partial L}{\partial P}=L_{W^{T}} W_{P}^{T}+L_{W^{N}} W_{P}^{N} \tag{147b}
\end{align*}
$$

Substituting (104) into $W \equiv W\left(W^{T}, W^{N}\right)$, we can solve for the aggregate wage index:

$$
\begin{equation*}
W=W\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right) \tag{148}
\end{equation*}
$$

where partial derivatives are given by

$$
\begin{align*}
W_{K} & \equiv \frac{\partial W}{\partial K}=W_{W^{T}} W_{K}^{T}+W_{W^{N}} W_{K}^{N}  \tag{149a}\\
W_{P} & \equiv \frac{\partial W}{\partial P}=W_{W^{T}} W_{P}^{T}+W_{W^{N}} W_{P}^{N} \tag{149b}
\end{align*}
$$

where $W_{W^{T}}=\left(W / W^{T}\right)\left(1-\alpha_{L}\right)$ and $W_{W^{N}}=\left(W / W^{N}\right) \alpha_{L}$.
Inserting short-run static solutions (146) and (148) into (145), and linearizing around the steadystate yields:

$$
\dot{A}(t)=r^{\star}(A(t)-\tilde{A})+M_{1}(P(t)-\tilde{P}),
$$

with $M_{1}$ given by

$$
M_{1}=\left\{\left(W_{K} \tilde{L}+\tilde{W} L_{K}\right)+\left[\left(W_{P} \tilde{L}+\tilde{W} L_{P}\right)-\tilde{C}^{N}-P_{C} C_{P}\right] \omega_{2}^{1}\right\}
$$

## D. 12 Solving the Full Model

Plugging the short-run static solutions for consumption in tradables and non tradables given by (68) and for hours worked in the traded and non traded sector given by (70), the steady-state is defined by
the following set of equations:

$$
\begin{gather*}
Z^{N}\left(1-\theta^{N}\right)\left(\tilde{k}^{N}\right)^{-\theta^{N}} \equiv r^{\star}+\delta_{K},  \tag{150a}\\
Z^{T}\left(1-\theta^{T}\right)\left(\tilde{k}^{T}\right)^{-\theta^{T}}=\tilde{P} Z^{N}\left(1-\theta^{N}\right)\left(\tilde{k}^{N}\right)^{-\theta^{N}},  \tag{150b}\\
Z^{T} \theta^{T}\left(\tilde{k}^{T}\right)^{1-\theta^{T}} \equiv \tilde{W}^{T},  \tag{150c}\\
P Z^{N} \theta^{N}\left(\tilde{k}^{N}\right)^{1-\theta^{N}} \equiv \tilde{W}^{N}  \tag{150d}\\
\tilde{k}^{T} L^{T}\left(\bar{\lambda}, \tilde{W}^{T}, \tilde{W}^{N}\right)+\tilde{k}^{N} L^{N}\left(\bar{\lambda}, \tilde{W}^{T}, \tilde{W}^{N}\right)=\tilde{K}  \tag{150e}\\
\tilde{Y}^{N}=C^{N}(\bar{\lambda}, \tilde{P})+\delta_{K} \tilde{K}  \tag{150f}\\
\tilde{Y}^{T}=C^{T}(\bar{\lambda}, \tilde{P})-r^{\star} \tilde{B}  \tag{150g}\\
\tilde{B}-B_{0}=\Phi\left(\tilde{K}-K_{0}\right), \tag{150h}
\end{gather*}
$$

where $\tilde{Y}^{T}=Z^{T} L^{T}\left(\bar{\lambda}, \tilde{W}^{T}, \tilde{W}^{N}\right)\left(\tilde{k}^{T}\right)^{1-\theta^{T}}$ and $\tilde{Y}^{T}=Z^{N} L^{N}\left(\bar{\lambda}, \tilde{W}^{T}, \tilde{W}^{N}\right)\left(\tilde{k}^{N}\right)^{1-\theta^{N}}$. This system of eight equations jointly solve for sectoral capital-labor ratios, $\tilde{k}^{T}$ and $\tilde{k}^{N}$, sectoral wage rates, $\tilde{W}^{T}$ and $\tilde{W}^{N}$, the relative price of non tradables, $\tilde{P}$, the capital stock, $\tilde{K}$, the stock of foreign assets, $\tilde{B}$, and the shadow value of wealth $\bar{\lambda}$.

Because we restrict ourselves to the analysis of the long-run effects, the tilde is suppressed for the purposes of clarity. Denoting by a hat the percentage deviation relative to initial steady-state, (150a) can be rewritten as:

$$
\begin{equation*}
\hat{Z}^{N}-\theta^{N} \hat{k}^{N}=0, \quad \hat{k}^{N}=\frac{\hat{Z}^{N}}{\theta^{N}}>0 . \tag{151}
\end{equation*}
$$

Hence a rise in $Z^{N}$ raises $k^{N}$. Taking logarithm and differentiating (150b) yields:

$$
\begin{equation*}
\hat{Z}^{T}-\theta^{T} \hat{k}^{T}=\hat{P}+\hat{Z}^{N}-\theta^{N} \hat{k}^{N}=\hat{P}>0, \quad \hat{k}^{T}=\frac{\hat{Z}^{T}-\hat{P}}{\theta^{T}} \tag{152}
\end{equation*}
$$

where we used (151) to get the last equality on the LHS. Taking logarithm and differentiating (150d) yields:

$$
\begin{equation*}
\hat{W}^{N}=\hat{P}+\hat{Z}^{N}+\left(1-\theta^{N}\right) \hat{k}^{N}=\hat{P}+\frac{\hat{Z}^{N}}{\theta^{N}}>0 \tag{153}
\end{equation*}
$$

where use has been made of (151). Hence a rise in $Z^{N}$ increases $W^{N}$ directly and indirectly by raising $P$ and $k^{N}$. Taking logarithm and differentiating (150c) yields:

$$
\begin{equation*}
\hat{W}^{T}=\hat{Z}^{T}+\left(1-\theta^{T}\right) \hat{k}^{T}=\hat{P}+\hat{k}^{T}=\frac{\hat{Z}^{T}}{\theta^{T}}-\left(\frac{1-\theta^{T}}{\theta^{T}}\right) \hat{P}>0 \tag{154}
\end{equation*}
$$

where use has been made of (152).
Before taking logarithm and differentiating the market-clearing condition, we express production functions for the traded and non traded sector as percentage deviations relative to initial steady-state. For traded output, we have:

$$
\begin{equation*}
\hat{Y}^{T}=\hat{Z}^{T}+\hat{L}^{T}+\left(1-\theta^{T}\right) \hat{k}^{T}=\hat{P}+\hat{k}^{T}+\hat{L}^{T}=\frac{\hat{Z}^{T}}{\theta^{T}}+\hat{L}^{T}-\left(\frac{1-\theta^{T}}{\theta^{T}}\right) \hat{P} \tag{155}
\end{equation*}
$$

where use has been made of (152). For non traded output, we have:

$$
\begin{equation*}
\hat{Y}^{N}=\hat{Z}^{N}+\hat{L}^{N}+\left(1-\theta^{N}\right) \hat{k}^{N}=\hat{L}^{N}+\hat{k}^{N}=\frac{\hat{Z}^{N}}{\theta^{N}}+\hat{L}^{N} \tag{156}
\end{equation*}
$$

where use has been made of (151).
To determine the steady-state changes of sectoral labor, we take logarithm and differentiate shortrun static solutions (70). For hours worked in the traded sector, we have:

$$
\hat{L}^{T}=\sigma_{L} \hat{\bar{\lambda}}+\left[\epsilon \alpha_{L}+\sigma_{L}\left(1-\alpha_{L}\right)\right] \hat{W}^{T}+\alpha_{L}\left(\sigma_{L}-\epsilon\right) \hat{W}^{N}
$$

For hours worked in the non-traded sector, we have:

$$
\hat{L}^{N}=\sigma_{L} \hat{\bar{\lambda}}+\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right) \hat{W}^{T}+\left[\epsilon\left(1-\alpha_{L}\right)+\sigma_{L} \alpha_{L}\right] \hat{W}^{N}
$$

Plugging the steady-state changes of sectoral wage rates given by (153) and (154), the percentage deviation relative to steady-state for hours worked in the traded sector is:

$$
\begin{align*}
\hat{L}^{T} & =\sigma_{L} \hat{\bar{\lambda}}+\left\{\alpha_{L}\left(\sigma_{L}-\epsilon\right)-\left[\epsilon \alpha_{L}+\sigma_{L}\left(1-\alpha_{L}\right)\right]\left(\frac{1-\theta^{T}}{\theta^{T}}\right)\right\} \hat{P} \\
& +\frac{\left[\epsilon \alpha_{L}+\sigma_{L}\left(1-\alpha_{L}\right)\right]}{\theta^{T}} \hat{Z}^{T}+\frac{\alpha_{L}\left(\sigma_{L}-\epsilon\right)}{\theta^{N}} \hat{Z}^{N}  \tag{157}\\
& =\sigma_{L} \hat{\bar{\lambda}}+\frac{\left[\epsilon \alpha_{L}+\sigma_{L}\left(1-\alpha_{L}\right)\right]}{\theta^{T}} \hat{Z}^{T}+\frac{\alpha_{L}\left(\sigma_{L}-\epsilon\right)}{\theta^{N}} \hat{Z}^{N} \\
& +\left[\frac{\alpha_{L}\left(\sigma_{L}-\epsilon\right)-\left(1-\theta^{T}\right) \sigma_{L}}{\theta^{T}}\right] \hat{P} .
\end{align*}
$$

Applying a similar procedure for hours worked in the non-traded sector, we have:

$$
\begin{align*}
\hat{L}^{N} & =\sigma_{L} \hat{\bar{\lambda}}+\left\{\left[\epsilon\left(1-\alpha_{L}\right)+\sigma_{L} \alpha_{L}\right]-\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)\left(\frac{1-\theta^{T}}{\theta^{T}}\right)\right\} \hat{P} \\
& +\frac{\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)}{\theta^{T}} \hat{Z}^{T}+\frac{\left[\epsilon\left(1-\alpha_{L}\right)+\sigma_{L} \alpha_{L}\right]}{\theta^{N}} \hat{Z}^{N}  \tag{158}\\
& =\sigma_{L} \hat{\bar{\lambda}}+\frac{\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)}{\theta^{T}} \hat{Z}^{T}+\frac{\left[\epsilon\left(1-\alpha_{L}\right)+\sigma_{L} \alpha_{L}\right]}{\theta^{N}} \hat{Z}^{N} \\
& +\left[\frac{\sigma_{L} \theta^{T}-\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)}{\theta^{T}}\right] \hat{P} .
\end{align*}
$$

Denoting by $\omega_{N} \equiv P Y^{N} / Y$ the non-tradable share of output, $\omega_{C} \equiv P_{C} C / Y$ the consumption-to-GDP ratio, $v_{I} \equiv P I / Y$ the investment-to-GDP ratio, taking logarithm and differentiating the market-clearing condition for the non traded goods $Y^{N}=C^{N}+I^{N}$ with $I^{N}=I=\delta_{K} K$ yields:

$$
\omega_{N} \hat{Y}^{N}=-\omega_{C} \alpha_{C} \sigma_{C} \hat{\bar{\lambda}}-\omega_{C} \alpha_{C}\left[\left(1-\alpha_{C}\right) \phi+\alpha_{C} \sigma_{C}\right] \hat{P}+v_{I} \hat{K}
$$

Substituting (158) into (156) and collecting terms allows us to rewrite the market-clearing condition for non-tradables as follows:

$$
\begin{align*}
& \hat{\bar{\lambda}}\left[\omega_{N} \sigma_{L}+\sigma_{C} \omega_{C} \alpha_{C}\right]+\hat{P}\left\{\omega_{N} \Psi_{N}+\omega_{C} \alpha_{C}\left[\left(1-\alpha_{C}\right) \phi+\alpha_{C} \sigma_{C}\right]\right\}-v_{I} \hat{K} \\
= & -\omega_{N}\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right) \frac{\hat{Z}^{T}}{\theta^{T}}-\omega_{N}\left[(\epsilon+1)+\alpha_{L}\left(\sigma_{L}-\epsilon\right)\right] \frac{\hat{Z}^{N}}{\theta^{N}}, \tag{159}
\end{align*}
$$

where

$$
\begin{align*}
\Psi_{N} & =\left\{\left[\epsilon\left(1-\alpha_{L}\right)+\sigma_{L} \alpha_{L}\right]-\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)\left(\frac{1-\theta^{T}}{\theta^{T}}\right)\right\} \\
& =\sigma_{L}-\frac{\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)}{\theta^{T}} \tag{160}
\end{align*}
$$

Denoting by $1-\omega_{N} \equiv Y^{T} / Y$ the tradable share of output, $v_{B} \equiv r^{\star} B / Y$ the interest receipts-toGDP ratio, inserting the intertemporal solvency condition (122), taking logarithm and differentiating the market-clearing condition for traded goods $r^{\star} B+Y^{T}=C^{T}$ with $B=B_{0}+\Phi\left(K-K_{0}\right)$ yields:

$$
\left(1-\omega_{N}\right) \hat{Y}^{T}=-\sigma_{C} \omega_{C}\left(1-\alpha_{C}\right) \hat{\bar{\lambda}}+\omega_{C}\left(1-\alpha_{C}\right) \alpha_{C}\left(\phi-\sigma_{C}\right) \hat{P}-v_{B} \Phi \frac{K}{B} \hat{K}
$$

Plugging (155) into (157) and collecting terms allows us to rewrite the market-clearing condition for tradables as follows:

$$
\begin{align*}
& \hat{\bar{\lambda}}\left[\left(1-\omega_{N}\right) \sigma_{L}+\sigma_{C} \omega_{C}\left(1-\alpha_{C}\right)\right]+\hat{P}\left\{\left(1-\omega_{N}\right) \Psi_{T}-\omega_{C}\left(1-\alpha_{C}\right) \alpha_{C}\left(\phi-\sigma_{C}\right)\right\}+v_{B} \Omega \frac{\tilde{K}}{\tilde{B}} \hat{K} \\
= & -\left(1-\omega_{N}\right)\left[(\epsilon+1)+\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)\right] \frac{\hat{Z}^{T}}{\theta^{T}}-\left(1-\omega_{N}\right) \alpha_{L}\left(\sigma_{L}-\epsilon\right) \frac{\hat{Z}^{N}}{\theta^{N}}, \tag{161}
\end{align*}
$$

where we set

$$
\begin{align*}
\Psi_{T} & =\left\{\alpha_{L}\left(\sigma_{L}-\epsilon\right)-\left[(\epsilon+1)+\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)\right]\left(\frac{1-\theta^{T}}{\theta^{T}}\right)\right\} \\
& =\left(\sigma_{L}+1\right)-\frac{\left[(\epsilon+1)+\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)\right]}{\theta^{T}} \tag{162}
\end{align*}
$$

Finally, denoting by $\xi_{N} \equiv K^{N} / K$ the non-tradable share of capital stock, taking logarithm and differentiating the resource constraint for capital given by (99) yields:

$$
\left(1-\xi_{N}\right) \hat{k}^{T}+\left(1-\xi_{N}\right) \hat{L}^{T}+\xi_{N} \hat{k}^{N}+\xi_{N} \hat{L}^{N}=\hat{K} .
$$

Plugging the steady-state changes of sectoral labor given by (157) and (158) into the equation above yields:

$$
\begin{align*}
& \sigma_{L} \hat{\bar{\lambda}}-\hat{K}+\hat{P}\left\{\sigma_{L}+\left(1-\xi_{N}\right)-\frac{\left[\left(1-\xi_{N}\right)(\epsilon+1)+\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)\right]}{\theta^{T}}\right\} \\
= & -\frac{\hat{Z}^{T}}{\theta^{T}}\left[\left(1-\xi_{N}\right)(\epsilon+1)+\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)\right]-\frac{\hat{Z}^{N}}{\theta^{N}}\left[\xi_{N}(\epsilon+1)+\alpha_{L}\left(\sigma_{L}-\epsilon\right)\right], \tag{163}
\end{align*}
$$

where $\sigma_{L}+\left(1-\xi_{N}\right)-\frac{\left[\left(1-\xi_{N}\right)(\epsilon+1)+\left(1-\alpha_{L}\right)\left(\sigma_{L}-\epsilon\right)\right]}{\theta^{T}}=\left(1-\xi_{N}\right) \Psi_{T}+\xi_{N} \Psi_{N}$.
The system (150) expressed in steady-state deviation relative to the steady-state can be reduced to three equations: i) the market-clearing condition for the non-traded good given by (159), ii) the market-clearing condition for the traded good given by (161), and iii) the resource constraint for physical capital given by (163). This system comprising three equations jointly determines $\tilde{P}, \tilde{K}, \bar{\lambda}$ in terms of exogenous productivity parameters $Z^{T}$ and $Z^{N}$. Denoting with a tilde the long-term values to avoid confusion, the system (150) can be solved for the relative price of non tradables, physical capital and the marginal utility of wealth:

$$
\begin{equation*}
\tilde{P}=P\left(Z^{T}, Z^{N}\right), \quad \tilde{K}=K\left(Z^{T}, Z^{N}\right), \quad \bar{\lambda}=\lambda\left(Z^{T}, Z^{N}\right) . \tag{164}
\end{equation*}
$$

Inserting $\hat{P}$, eqs. (153) and (154) can be solved for $\hat{W}^{N}$ and $\hat{W}^{T}$. Hence, we have:

$$
\begin{equation*}
\tilde{W}^{N}=W^{N}\left(Z^{T}, Z^{N}\right), \quad \tilde{W}^{T}=W^{T}\left(Z^{T}, Z^{N}\right), \quad \tilde{\Omega}=\Omega\left(Z^{T}, Z^{N}\right) \tag{165}
\end{equation*}
$$

## D. 13 Introducing Physical Capital: The Steady-State in a Compact Form

In section 4 in the main text and in section D.8-D. 9 in the Technical Appendix, we use a specific procedure to solve for the steady-state which allows us to break down analytically the relative wage and relative price responses to a productivity differential in three components. Below, we characterize the whole steady-state and use tilde to denote long-run values. Setting $\dot{P}=\dot{K}=\dot{B}=0$ into (114b), (114a) and (120), and inserting short-run static solutions for $k^{N}, Y^{N}$ and $Y^{T}, C^{N}$ and $C^{T}$ yields the following set of equations:

$$
\begin{gather*}
Z^{N}\left(1-\theta^{N}\right)\left[k^{N}\left(\tilde{K}, \tilde{P}, \bar{\lambda}, Z^{T}, Z^{N}\right)\right]^{-\theta^{N}}=r^{\star}+\delta_{K}  \tag{166a}\\
Y^{N}\left(\tilde{K}, \tilde{P}, \bar{\lambda}, Z^{T}, Z^{N}\right)-C^{N}(\tilde{P}, \bar{\lambda})-\delta_{K} \tilde{K}=0  \tag{166b}\\
r^{\star} \tilde{B}+Y^{T}\left(\tilde{K}, \tilde{P}, \bar{\lambda}, Z^{T}, Z^{N}\right)-C^{T}(\tilde{P}, \bar{\lambda})=0  \tag{166c}\\
\tilde{B}-B_{0}=\Phi\left(\tilde{K}-K_{0}\right) . \tag{166d}
\end{gather*}
$$

As shown in section D.12, these four equations jointly solve for the steady-state values for $\tilde{P}, \tilde{K}, \tilde{B}$ and $\bar{\lambda}$, in terms of exogenous productivity parameters $Z^{T}$ and $Z^{N}$.

## E Introducing Non-Separability between Consumption and Labor

In this section, we consider a more general form for preferences taken from Shimer [2011]. Since such preferences do not affect the first-order conditions from profit maximization, we do not repeat them and indicate major changes when solving the model.

## E. 1 Households

Previously, we assumed that preferences are separable in consumption and leisure. We relax this assumption which implies that consumption and leisure are substitutes. In particular, this more general specification implies that consumption can be affected by the wage rate while labor supply can be influenced by the change in the relative price of non tradables. As previously, the household's period utility function is increasing in its consumption $C$ and decreasing in its labor supply $L$, with functional form:

$$
\begin{equation*}
\frac{C^{1-\sigma} V(L)^{\sigma}-1}{1-\sigma}, \quad \text { if } \quad \sigma \neq 1, \quad V(L) \equiv\left(1+(\sigma-1) \gamma \frac{\sigma_{L}}{1+\sigma_{L}} L^{\frac{1+\sigma_{L}}{\sigma_{L}}}\right) \tag{167}
\end{equation*}
$$

and

$$
\begin{equation*}
\log C-\gamma \frac{\sigma_{L}}{1+\sigma_{L}} L^{\frac{1+\sigma_{L}}{\sigma_{L}}}, \quad \text { if } \quad \sigma=1 \tag{168}
\end{equation*}
$$

These preferences are characterized by two crucial parameters: $\sigma_{L}$ is the Frisch elasticity of labor supply, and $\sigma>0$ determines the substitutability between consumption and leisure; it is worthwhile noticing that if $\sigma>1$, the marginal utility of consumption is increasing in hours worked. Importantly, such preferences imply that the Frisch elasticity of labor supply is constant.

The representative household maximizes lifetime utility subject to the flow budget constraint (93) and the accumulation of physical capital (94).

Denoting the co-state variables associated with (93) and (94) by $\lambda$ and $\psi$, respectively, the firstorder conditions characterizing the representative household's optimal plans are:

$$
\begin{gather*}
C^{-\sigma} V(L)^{\sigma}=P_{C} \lambda,  \tag{169a}\\
C^{1-\sigma} \sigma \gamma L^{1 / \sigma_{L}} V(L)^{\sigma-1}=W \lambda,  \tag{169b}\\
\dot{\lambda}=\lambda\left(\beta-r^{\star}\right),  \tag{169c}\\
\frac{R}{P}-\delta+\frac{\dot{P}}{P}=r^{\star}, \tag{169d}
\end{gather*}
$$

and the transversality conditions $\lim _{t \rightarrow \infty} \bar{\lambda} B(t) e^{-\beta t}=0$ and $\lim _{t \rightarrow \infty} \psi(t) K(t) e^{-\beta t}=0$; to derive (169d), we used the fact that $\psi(t)=\lambda P(t)$.

First-order conditions (169a) and (169b) can be solved for consumption and labor as follows:

$$
\begin{equation*}
C=C(\bar{\lambda}, P, W), \quad L=L(\bar{\lambda}, P, W) . \tag{170}
\end{equation*}
$$

To derive the partial derivatives, we take logarithm and totally differentiate the system which yields in matrix form:

$$
\left(\begin{array}{cc}
-\sigma & \sigma\left(\frac{1+\sigma_{L}}{\sigma_{L}}\right)\left[\frac{V(L)-1}{V(L)}\right]  \tag{171}\\
(1-\sigma) & \left\{\frac{1}{\sigma_{L}}+(\sigma-1)\left(\frac{1+\sigma_{L}}{\sigma_{L}}\right)\left[\frac{V(L)-1}{V(L)}\right]\right\}
\end{array}\right)\binom{\hat{C}}{\hat{L}}\binom{\hat{\bar{\lambda}}+\alpha_{C} \hat{P}}{\hat{\bar{\lambda}}+\hat{W}}
$$

where we denote by a hat the deviation in percentage.
Partial derivatives are:

$$
\begin{align*}
\frac{\hat{C}}{\hat{\bar{\lambda}}} & =\frac{\left(1+\sigma_{L}\right)}{\sigma}\left[\frac{V(L)-1}{V(L)}\right]-\frac{1}{\sigma}<0  \tag{172a}\\
\frac{\hat{L}}{\hat{\bar{\lambda}}} & =\frac{\sigma_{L}}{\sigma}>0  \tag{172b}\\
\frac{\hat{C}}{\hat{W}} & =\left(1+\sigma_{L}\right)\left[\frac{V(L)-1}{V(L)}\right]>0  \tag{172c}\\
\frac{\hat{L}}{\hat{W}} & =\sigma_{L}>0,  \tag{172d}\\
\frac{\hat{C}}{\hat{P}} & =-\frac{\alpha_{C}}{\sigma}\left\{1+(\sigma-1)\left(1+\sigma_{L}\right)\left[\frac{V(L)-1}{V(L)}\right]\right\}<0,  \tag{172e}\\
\frac{\hat{L}}{\hat{P}} & =-\alpha_{C} \frac{(\sigma-1) \sigma_{L}}{\sigma}<0 . \tag{172f}
\end{align*}
$$

Using the fact that $W=W\left(W^{T}, W^{N}\right)$ with $\frac{\partial W}{\partial W^{T}} \frac{W^{T}}{W}=\left(1-\alpha_{L}\right)$ and $\frac{\partial W}{\partial W^{N}} \frac{W^{N}}{W}=\alpha_{L}$, we get:

$$
\begin{equation*}
L=L\left(\bar{\lambda}, P, W^{T}, W^{N}\right) \tag{173}
\end{equation*}
$$

where

$$
\begin{align*}
\frac{\hat{L}}{\hat{W}^{T}} & =\left(1-\alpha_{L}\right) \sigma_{L}>0  \tag{174a}\\
\frac{\hat{L}}{\hat{W}^{N}} & =\sigma_{L} \alpha_{L}>0 \tag{174b}
\end{align*}
$$

Inserting first the short-run static solution for consumption given by (170), consumption in nontradables, i.e., $C^{N}=P_{C}^{\prime} C$ and tradables, i.e., $C^{T}=\left[P_{C}-P P_{C}^{\prime}\right] C$, can be solved for $C^{N}$ and $C^{T}$ as follows:

$$
\begin{equation*}
C^{T}=C^{T}\left(\bar{\lambda}, P, W^{T}, W^{N}\right), \quad C^{N}=C^{N}\left(\bar{\lambda}, P, W^{T}, W^{N}\right) \tag{175}
\end{equation*}
$$

where partial derivatives are given by:

$$
\begin{align*}
C_{P}^{T} & =\frac{C^{T}}{P}\left(\alpha_{C} \phi+\frac{C_{P} P}{C}\right) \lessgtr 0,  \tag{176a}\\
C_{P}^{N} & =-\frac{C^{N}}{P}\left[\left(1-\alpha_{C}\right) \phi-\frac{C_{P} P}{C}\right]<0,  \tag{176b}\\
C_{W^{T}}^{T} & =\frac{C^{T}}{W^{T}}\left(1-\alpha_{L}\right) \frac{C_{W} W}{C}>0,  \tag{176c}\\
C_{W^{T}}^{N} & =\frac{C^{N}}{W^{T}}\left(1-\alpha_{L}\right) \frac{C_{W} W}{C}>0,  \tag{176d}\\
C_{W^{N}}^{T} & =\frac{C^{T}}{W^{N}} \alpha_{L} \frac{C_{W} W}{C}>0,  \tag{176e}\\
C_{W^{N}}^{N} & =\frac{C^{N}}{W^{N}} \alpha_{L} \frac{C_{W} W}{C}>0 . \tag{176f}
\end{align*}
$$

Inserting first the short-run solution for labor (173), into $L^{T}=\frac{\partial W\left(W^{T}, W^{N}\right)}{\partial W^{T}} L$ and $L^{N}=\frac{\partial W\left(W^{T}, W^{N}\right)}{\partial W^{N}} L$, we are able to solve for $L^{T}$ and $L^{N}$ :

$$
\begin{equation*}
L^{T}=L^{T}\left(\bar{\lambda}, W^{T}, W^{N}, P\right), \quad L^{N}=L^{N}\left(\bar{\lambda}, W^{T}, W^{N}, P\right) \tag{177}
\end{equation*}
$$

where partial derivatives w.r.t. $W^{T}$ and $W^{N}$ are given by (71) and partial derivatives w.r.t. $P$ are:

$$
\begin{align*}
\frac{\hat{L}^{T}}{\hat{P}} & =\frac{L^{T}}{P} \alpha_{C}(1-\sigma) \frac{\sigma_{L}}{\sigma}>0  \tag{178a}\\
\frac{\hat{L}^{N}}{\hat{P}} & =\frac{L^{N}}{P} \alpha_{C}(1-\sigma) \frac{\sigma_{L}}{\sigma}>0 \tag{178b}
\end{align*}
$$

## E. 2 Solving the Model

Plugging the short-run static solutions for $L^{T}$ and $L^{N}$ given by (177) into the resource constraint for capital (99), the system of four equations comprising (98a)-(98c) and (99) can be solved for sectoral wages and sectoral capital-labor ratios. Taking logarithm and differentiating (98a)-(98c) and (99) yields in matrix form:

$$
\left(\begin{array}{cccc}
-\theta^{T} & \theta^{N} & 0 & 0  \tag{179}\\
\left(1-\theta^{T}\right) & 0 & -1 & 0 \\
0 & \left(1-\theta^{N}\right) & 0 & -1 \\
(1-\xi) & \xi & \Psi_{W^{T}} & \Psi_{W^{N}}
\end{array}\right)\left(\begin{array}{c}
\hat{k}^{T} \\
\hat{k}^{N} \\
\hat{W}^{T} \\
\hat{W}^{N}
\end{array}\right)=\left(\begin{array}{c}
\hat{P}+\hat{Z}^{N}-\hat{Z}^{T} \\
-\hat{Z}^{T} \\
-\hat{P}-\hat{Z}^{N} \\
\hat{K}-\Psi_{\bar{\lambda}} \hat{\bar{\lambda}}-\Psi_{P} \hat{P}
\end{array}\right)
$$

where $\Psi_{W^{T}}$ and $\Psi_{W^{N}}$ are given by (103a) (103b), respectively, $\xi \equiv \frac{k^{N} L^{N}}{K}$ and we set:

$$
\begin{equation*}
\Psi_{P}=(1-\xi) \frac{L_{P}^{T} P}{L^{T}}+\xi \frac{L_{P}^{N} P}{L^{N}}=-\alpha_{C} \frac{(\sigma-1) \sigma_{L}}{\sigma}<0 \tag{180}
\end{equation*}
$$

Only the partial derivatives w.r.t. $P$ are modified when preferences are non separable in consumption and leisure. Hence, we limit ourselves to these partial derivatives. Short-run static solutions for sectoral wages are:

$$
\begin{equation*}
W^{T}=W^{T}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right), \quad W^{N}=W^{N}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right) \tag{181}
\end{equation*}
$$

with

$$
\begin{align*}
\frac{\hat{W}^{T}}{\hat{P}} & =-\frac{\left(1-\theta^{T}\right)\left(\Psi_{W^{N}}+\theta^{N} \Psi_{P}+\xi+\right)}{G}<0  \tag{182a}\\
\frac{\hat{W}^{N}}{\hat{P}} & =-\frac{\left\{1+\left(1-\theta^{T}\right) \Psi_{W^{T}}-\left(1-\theta^{T}\right) \xi-\theta^{T}\left(1-\theta^{N}\right) \Psi_{P}\right\}}{G}>0 \tag{182b}
\end{align*}
$$

and sectorial capital-labor ratios:

$$
\begin{equation*}
k^{T}=k^{T}\left(\lambda, K, P, Z^{T}, Z^{N}\right), \quad k^{N}=k^{N}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right), \tag{183}
\end{equation*}
$$

with

$$
\begin{align*}
\frac{\hat{k}^{T}}{\hat{P}} & =\frac{\Psi_{W^{N}}+\xi+\theta^{N} \Psi_{P}}{G}<0  \tag{184a}\\
\frac{\hat{k}^{N}}{\hat{P}} & =\frac{\left\{\theta^{T}\left(\Psi^{W^{N}}+\Psi_{P}\right)-\left[\left(1-\theta^{T}\right) \Psi_{W^{T}}+(1-\xi)\right]\right\}}{G}>0 \tag{184b}
\end{align*}
$$

To solve the model, insert first short-run static solutions for sectoral wages (181) into sectoral labor (177), then substitute the resulting solutions for sectoral labor and capital-labor ratios (184), production functions can be solved for sectoral outputs.

As mentioned in the text, we break down the long-run relative price and relative wage responses to a productivity differential into three channels: i) a productivity channel when keeping fixed sectoral capital-labor ratios and the overall capital stock, ii) a capital reallocation effect induced by the shift of capital across sectors, iii) a capital accumulation effect stemming from the investment boom causing a current account deficit in the short-run and therefore requiring a trade balance surplus in the longrun. As expected, non separable preferences in consumption and leisure modifies only the capital accumulation channel by influencing private savings and thereby the current account adjustment in the short-run.

## E. 3 Numerical Results: Discussion

We now briefly assess numerically to what extent our results depend on the assumptions regarding the form of preferences. We consider a more general specification for preferences which are assumed to be non-separable in consumption and leisure. Considering non separability in preferences implies a positive relationship between consumption and the aggregate wage index which modifies only the capital accumulation channel. More precisely, by raising the aggregate wage index, the productivity differential now induces agents to consume more and to reduce private savings further, thus resulting in a larger current account deficit. Because the economy must run a larger surplus in the balance of trade, the demand for tradables rises more, producing a larger negative impact on $p$ and $\omega$.

When numerically exploring the implications of non-separability in preferences between consumption and leisure, we set the substitutability between consumption and leisure captured by $\sigma$ to 2 , as in Shimer [2011], keeping unchanged the baseline calibration discussed in section 5.1. The results for the case of non-separability in preferences are shown in column 3 of Table 13, for our three alternative scenarios, i.e., $\phi=1, \phi<1$ and $\phi>1$. Neither the productivity channel nor the capital reallocation channel are modified when considering non-separability in preferences. As shown in the fourth line of panel A and B of Table 13, non-separability in preferences substantially amplifies the capital accumulation channel, in line with the theoretical predictions. When setting $\phi$ to $1, \omega$ falls by $0.54 \%$ instead of $0.45 \%$ for the benchmark scenario while $p$ rises by $0.66 \%$ instead of $0.72 \%$.

## F Introducing Traded Investment

The section examines implications of a two-sector model that differentiate between tradable and non-tradable goods in investment. The small open economy produces a traded and a

Table 13: Long-Term Relative Price and Relative Wage Responses to a Productivity Differential between Tradables and Non Tradables (in \%)

|  | BS | Bench | Non sep. | Traded inv. |
| :---: | :---: | :---: | :---: | :---: |
|  | $(\epsilon=\infty)$ | ( $\epsilon=0.8$ ) | ( $\sigma=2$ ) | ( $\varphi_{I}=0.42$ ) |
| $\phi=1$ |  |  |  |  |
| A.Relative Wage |  |  |  |  |
| Relative wage, $\hat{\omega}$ | 0.00 | -0.45 | -0.54 | -0.42 |
| Baseline effect | 0.00 | 0.00 | 0.00 | 0.00 |
| Capital reallocation effect | 0.00 | 0.00 | 0.00 | 0.00 |
| Capital accumulation effect | 0.00 | -0.45 | -0.54 | -0.42 |
| B.Relative Price |  |  |  |  |
| Relative price, $\hat{p}$ | 1.00 | 0.72 | 0.66 | 0.72 |
| Baseline effect | 1.00 | 1.00 | 1.00 | 1.00 |
| Capital reallocation effect | 0.00 | 0.00 | 0.00 | 0.00 |
| Capital accumulation effect | 0.00 | -0.27 | -0.33 | -0.27 |
| ( $\quad \underline{\text { c }}$ |  |  |  |  |
| C.Relative $\overline{\text { Wage }}$ |  |  |  |  |
| Relative wage, $\hat{\omega}$ | 0.00 | -0.24 | -0.39 | -0.22 |
| Baseline effect | 0.00 | 0.38 | 0.38 | 0.38 |
| Capital reallocation effect | 0.00 | -0.05 | -0.05 | -0.05 |
| Capital accumulation effect | 0.00 | -0.57 | -0.72 | -0.56 |
| D.Relative Price |  |  |  |  |
| Relative price, $\hat{p}$ | 1.00 | 0.85 | 0.76 | 0.85 |
| Baseline effect | 1.00 | 1.38 | 1.38 | 1.38 |
| Capital reallocation effect | 0.00 | -0.18 | -0.18 | -0.17 |
|  | 0.00 | -0.34 | -0.43 | -0.36 |
| $\begin{array}{r} \text { effect } \\ \phi>1 \\ \hline \end{array}$ |  |  |  |  |
| E.Relative Wage |  |  |  |  |
| Relative wage, $\hat{\omega}$ | 0.00 | -0.58 | -0.64 | -0.56 |
| Baseline effect | 0.00 | -0.22 | -0.22 | -0.22 |
| Capital reallocation effect | 0.00 | -0.02 | -0.02 | -0.02 |
| Capital accumulation effect | 0.00 | -0.34 | -0.41 | -0.33 |
| F.Relative Price |  |  |  |  |
| Relative price, $\hat{p}$ | 1.00 | 0.64 | 0.60 | 0.63 |
| Baseline effect | 1.00 | 0.78 | 0.78 | 0.78 |
| Capital reallocation effect | 0.00 | 0.07 | 0.07 | 0.07 |
| Capital accumulation effect | 0.00 | -0.21 | -0.25 | -0.21 |

Notes: Effects of a labor share-adjusted TFPs diffential between tradables and non tradables of $1 \%$. Panels A and B show the deviation in percentage relative to steady-state for the (log) relative price of non tradables $p \equiv p^{N}-p^{T}$ and the (log) relative wage of non traded workers $\omega \equiv w^{N}-w^{T}$, respectively, and break down changes in a productivity effect (keeping unchanged sectoral capital-labor ratios $k^{j}$, the overall capital stock $K$ and the stock of foreign bonds $B$ ), a capital reallocation effect (induced by changes in $k^{j}$ keeping unchanged $K$ and $B$ ), a capital accumulation effect (stemming from the investment boom causing a current account deficit in the short-run and therefore requiring a steady-state improvement in the balance of trade). While panels A and B show the results when setting $\phi$ to one, panels C and D show results for $\phi<1$ and panels E and F show results for $\phi>1 ; \phi$ is the elasticity of substitution between tradables and non tradables; $\epsilon$ captures the degree of labor mobility across sectors.
non traded good by means of a production technology described by Cobb-Douglas production functions that uses capital and labor. As previously, the output of the non traded good $\left(Y^{N}\right)$ can be used for private $\left(C^{N}\right)$ and public consumption $\left(G^{N}\right)$, and for investment $\left(I^{N}\right)$. The output of the traded good $\left(Y^{T}\right)$ can be consumed by households and the government ( $C^{T}$ and $G^{T}$ ), invested ( $I^{T}$ ), or exported ( $Y^{T}-C^{T}-G^{T}-I^{T}$ ).

As in De Cordoba and Kehoe [2000], the investment good is produced using inputs of the traded good and the non-traded good according to a constant-returns-to-scale function which is assumed to take a CES form:

$$
\begin{equation*}
I \equiv I\left(I^{T}, I^{N}\right)=\left[\varphi_{I}^{\frac{1}{\phi_{I}}}\left(I^{T}\right)^{\frac{\phi_{I}-1}{\phi_{I}}}+\left(1-\varphi_{I}\right)^{\frac{1}{\phi_{I}}}\left(I^{N}\right)^{\frac{\phi_{I}-1}{\phi_{I}}}\right]^{\frac{\phi_{I}}{\phi_{I}-1}}, \tag{185}
\end{equation*}
$$

where $\varphi_{I}$ is the weight of the investment traded input $\left(0<\varphi_{I}<1\right)$ and $\phi_{I}$ corresponds to the intratemporal elasticity of substitution between investment traded goods and investment non traded goods. At each instant, the investment sector minimizes the cost or total expenditure measured in terms of traded goods:

$$
\begin{equation*}
E_{I} \equiv P I^{N}+I^{T}, \tag{186}
\end{equation*}
$$

for a given level of output, $I$, where $P$ is the relative price of the non traded good. For any chosen $I$, the optimal basket $\left(I^{T}, I^{N}\right)$ is a solution to:

$$
\begin{equation*}
P_{I}(P) I=\min _{\left\{I^{T}, I^{N}\right\}}\left\{I^{T}+P I^{N}: I\left(I^{T}, I^{N}\right) \geq I\right\} . \tag{187}
\end{equation*}
$$

The aggregator function (185) is linear homogeneous implies that total expenditure in consumption goods can be expressed as $E_{I}=P_{I}(P) I$, with $P_{I}(P)$ is the unit cost function dual (or consumption-based price index) to $I$. The unit cost dual function, $P_{I}($.$) , is defined as the$ minimum total expense in investment goods, $E_{I}$, such that $I=I\left(I^{T}, I^{N}\right)=1$, for a given level of the relative price of non tradables, $P$. Its expression is given by:

$$
\begin{equation*}
P_{I}=\left[\varphi_{I}+\left(1-\varphi_{I}\right) P^{1-\phi_{I}}\right]^{\frac{1}{1-\phi_{I}}} . \tag{188}
\end{equation*}
$$

Intra-temporal allocations between non tradable goods and tradable goods follow from Shephard's Lemma (or the envelope theorem) applied to (187):

$$
\begin{align*}
& I^{N}=P_{I}^{\prime} I=\left(1-\varphi_{I}\right)\left(\frac{P}{P_{I}}\right)^{-\phi_{I}} I, \quad \text { and } \quad \frac{P I^{N}}{P_{I} I}=\alpha_{I},  \tag{189a}\\
& I^{T}=\left[P_{I}-P P_{I}^{\prime}\right] I=\varphi_{I}\left(\frac{1}{P_{I}}\right)^{-\phi_{I}} I, \quad \text { and } \quad \frac{I^{T}}{P_{I} I}=\left(1-\alpha_{I}\right), \tag{189b}
\end{align*}
$$

where the non tradable and tradable shares in total investment expenditure are:

$$
\begin{align*}
\alpha_{I} & =\frac{\left(1-\varphi_{I}\right) P^{1-\phi_{I}}}{\varphi_{I}+\left(1-\varphi_{I}\right) P^{1-\phi_{I}}}  \tag{190a}\\
1-\alpha_{I} & =\frac{\varphi_{I}}{\varphi_{I}+\left(1-\varphi_{I}\right) P^{1-\phi_{I}}} \tag{190b}
\end{align*}
$$

## F. 1 Households

The representative household chooses consumption $C$, decides on labor supply $L$, and investment $I$ that maximizes his/her lifetime utility (92) subject to the flow budget constraint:

$$
\begin{equation*}
\dot{B}(t)=r^{\star} B(t)+R(t) K(t)+W\left(W^{T}(t), W^{N}(t)\right) L(t)-P_{C}(P(t)) C(t)-P_{I}(P(t)) I(t), \tag{191}
\end{equation*}
$$

and capital accumulation which evolves as follows:

$$
\begin{equation*}
\dot{K}(t)=I(t)-\delta_{K} K(t), \tag{192}
\end{equation*}
$$

where $I$ corresponds to investment expenditure and $0 \leq \delta_{K}<1$ is a fixed depreciation rate.
Denoting the co-state variables associated with (191) and (192) by $\lambda(t)$ and $\psi(t)$, respectively, the first-order conditions characterizing the representative household's optimal plans are:

$$
\begin{gather*}
C(t)=\left(P_{C}(P(t)) \lambda\right)^{-\sigma_{C}},  \tag{193a}\\
L(t)=\left(\frac{W(t) \lambda(t)}{\gamma}\right)^{\sigma_{L}},  \tag{193b}\\
\dot{\lambda}(t)=\lambda(t)\left(\beta-r^{\star}\right),  \tag{193c}\\
\frac{R(t)}{P_{I}(P(t))}-\delta_{K}+\alpha_{I} \frac{\dot{P}(t)}{P(t)}=r^{\star}, \tag{193d}
\end{gather*}
$$

and the transversality conditions $\lim _{t \rightarrow \infty} \bar{\lambda} B(t) e^{-\beta t}=0$ and $\lim _{t \rightarrow \infty} \psi(t) K(t) e^{-\beta t}=0$; to derive (193d), we used the fact that $\psi(t)=\bar{\lambda} P_{I}$. Eqs. (193a) and (193b) can be solved for consumption and labor (see eq. (96)). For the sake of clarity, we drop the time argument below when this causes no confusion.

## F. 2 Equilibrium Dynamics

First-order conditions from profit maximization remains unchanged and therefore we do not repeat them (see section D.5). To solve the model, we adopt the same reasoning as in section D.5.

Remembering that the non traded input $I^{N}$ used to produce the capital good is equal to $P_{I}^{\prime} I$, using the fact that $I^{N}=Y^{N}-C^{N}-G^{N}$ and inserting $I=\dot{K}+\delta_{K}$, the capital accumulation equation becomes:

$$
\begin{equation*}
\dot{K}=\frac{Y^{N}-C^{N}-G^{N}}{P_{I}^{\prime}}-\delta_{K} K . \tag{194}
\end{equation*}
$$

Inserting short-run static solutions for non traded output (110), consumption in non tradables (68), and the capital-labor ratio in the non traded sector (106) into the physical capital accumulation equation (194) and the dynamic equation for the relative price of non tradables (193d), the dynamic system is:

$$
\begin{align*}
\dot{K} & =\frac{Y^{N}(K, P, \bar{\lambda})-C^{N}(\bar{\lambda}, P)-G^{N}}{P_{I}^{\prime}}-\delta_{K} K  \tag{195a}\\
\dot{P} & =\frac{P}{\alpha_{I}}\left[\left(r^{\star}+\delta_{K}\right)-\frac{P}{P_{I}(P)} Z^{N} h_{k}(K, P, \bar{\lambda})\right], \tag{195b}
\end{align*}
$$

where for the purposes of clarity, we abstract from time-constant arguments of short-run static solutions, i.e., $Z^{T}$, and $Z^{N}$.

Denoting with a tilde long-run values, linearizing these two equations around the steadystate yields in matrix form:

$$
\binom{\dot{K}(t)}{\dot{P}(t)}=\left(\begin{array}{ll}
a_{11} & a_{12}  \tag{196}\\
a_{21} & a_{22}
\end{array}\right)\binom{K(t)-\tilde{K}}{P(t)-\tilde{P}},
$$

where

$$
\begin{align*}
a_{11} & =\left(\frac{Y_{K}^{N}}{P_{I}^{\prime}}-\delta_{K}\right)>0,  \tag{197a}\\
a_{12} & =\frac{\left(Y_{P}^{N}-C_{P}^{N}\right)}{P_{I}^{\prime}}+\frac{\tilde{I}^{N} \phi_{I}\left(1-\alpha_{I}\right)}{\tilde{P} P_{I}^{\prime}},  \tag{197b}\\
a_{21} & =-\frac{\tilde{P}^{2} Z^{N} h_{k k} k_{K}^{N}}{\alpha_{I} P_{I}}>0,  \tag{197c}\\
a_{22} & =\frac{\tilde{P} Z^{N} h_{k}}{\alpha_{I} P_{I}}\left[\theta^{N} \frac{k_{P}^{N} \tilde{P}}{\tilde{k}^{N}}-\left(1-\alpha_{I}\right)\right] . \tag{197d}
\end{align*}
$$

Saddle path stability requires the determinant of the Jacobian matrix $\operatorname{Det} J$ given by $a_{11} a_{22}-a_{21} a_{12}$ to be negative. The term $a_{21} a_{12}$ is always negative, regardless of sectoral capital intensities while the term. If $k^{T}>k^{N}$, we have $Y_{K}^{N}<0$ and $k_{P}^{N}>0 ; a_{11}$ is negative while $a_{22}$ is positive as long as $\left(1-\alpha_{I}\right)$ which is the tradable content of investment expenditure is not too large. In this case, we have $a_{11} a_{22}<0$. If $k^{N}>k^{T}$, we have $Y_{K}^{N}>0$ and $k_{P}^{N}<0$. Hence, $a_{11}$ becomes positive while $a_{22}$ becomes unambiguously negative. As a result, we have $a_{11} a_{22}<0$. To conclude, the saddle-path stability condition is fulfilled regardless of sectoral capital intensities as long as $\left(1-\alpha_{I}\right)$ does not exceed the elasticity of $k^{N}$ with respect to $P$.

Assuming that the saddle-path stability condition is fulfilled, the stable solutions for $K$ and $P$ are:

$$
\begin{align*}
K(t) & =\tilde{K}+\left(K_{0}-\tilde{K}\right) e^{\mu_{1} t}  \tag{198a}\\
P(t) & =\tilde{P}+\omega_{2}^{1}\left(K_{0}-\tilde{K}\right) e^{\mu_{1} t} \tag{198b}
\end{align*}
$$

where $K_{0}$ is the initial capital stock and $\left(1, \omega_{2}^{1}\right)^{\prime}$ is the eigenvector associated with the stable negative eigenvalue $\mu_{1}$ :

$$
\begin{equation*}
\omega_{2}^{1}=\frac{\mu_{1}-a_{11}}{a_{12}} \tag{199}
\end{equation*}
$$

For all plausible sets of parameter values, we find numerically $\omega_{2}^{1}<0$, regardless of sectoral capital intensities, which implies that the relative price of non tradables and the stock physical capital move in opposite direction.

Remembering that $I^{T}=\left(1-\alpha_{I}\right) P_{I} I$ with $I=\dot{K}+\delta_{K} K$, the current account equation is given by:

$$
\begin{equation*}
\dot{B}=Y^{T}-C^{T}-G^{T}-\left(1-\alpha_{I}\right) P_{I}\left(\dot{K}+\delta_{K} K\right) \tag{200}
\end{equation*}
$$

Substituting the short-run static solutions for traded output (110) and consumption in tradables (68) into the accumulation equation of foreign bonds (200), linearizing, solving and invoking the transversality condition yields:

$$
\begin{equation*}
B(t)=\tilde{B}+\Phi\left(K_{0}-\tilde{K}\right) e^{\mu_{1} t} \tag{201}
\end{equation*}
$$

where $\Phi \equiv \frac{N_{1}}{\mu_{1}-r^{\star}}$ and

$$
\begin{align*}
N_{1}= & {\left[Y_{K}^{T}-\left(\frac{1-\alpha_{I}}{\alpha_{I}}\right) \tilde{P} Y_{K}^{N}\right]+\left\{\left(Y_{P}^{T}-C_{P}^{T}\right)\right.} \\
& \left.-\left(\frac{1-\alpha_{I}}{\alpha_{I}}\right) \tilde{P}\left(Y_{K}^{N}-C_{P}^{N}\right)-\phi_{I}\left(\frac{1-\alpha_{I}}{\alpha_{I}}\right) \tilde{I}^{N}\right\} \omega_{2}^{1} . \tag{202}
\end{align*}
$$

The intertemporal solvency condition of the economy is:

$$
\begin{equation*}
\tilde{B}-B_{0}=\Phi\left(\tilde{K}-K_{0}\right) \tag{203}
\end{equation*}
$$

where $B_{0}$ is the initial stock of traded bonds.

## F. 3 The Steady-State

We now describe the steady-state by abstracting from government spending for clarity purpose. Plugging the short-run static solutions for consumption in tradables and non tradables given by (68) and for hours worked in the traded and non traded sector given by (70), the steady-state is defined by the following set of equations:

$$
\begin{gather*}
\tilde{P} Z^{N}\left(1-\theta_{N}\right)\left(\tilde{k}^{N}\right)^{-\theta_{N}} \equiv P_{I}(\tilde{P})\left(r^{\star}+\delta_{K}\right),  \tag{204a}\\
Z^{T}\left(1-\theta_{T}\right)\left(\tilde{k}^{T}\right)^{-\theta_{T}}=\tilde{P} Z^{N}\left(1-\theta_{N}\right)\left(\tilde{k}^{N}\right)^{-\theta_{N}} \equiv \tilde{R},  \tag{204b}\\
Z^{T} \theta_{T}\left(\tilde{k}^{T}\right)^{1-\theta_{T}} \equiv \tilde{W}^{T},  \tag{204c}\\
P Z^{N} \theta_{N}\left(\tilde{k}^{N}\right)^{1-\theta_{N}} \equiv \tilde{W}^{N}  \tag{204d}\\
\tilde{k}^{T} L^{T}\left(\bar{\lambda}, \tilde{W}^{T}, \tilde{W}^{N}\right)+\tilde{k}^{N} L^{N}\left(\bar{\lambda}, \tilde{W}^{T}, \tilde{W}^{N}\right)=\tilde{K}  \tag{204e}\\
\tilde{Y}^{N}=C^{N}(\bar{\lambda}, \tilde{P})+P_{I}^{\prime}(\tilde{P}) \delta_{K} \tilde{K}  \tag{204f}\\
\tilde{Y}^{T}=C^{T}(\bar{\lambda}, \tilde{P})+\left(1-\alpha_{I}\right) P_{I}(\tilde{P}) \delta_{K} \tilde{K}-r^{\star} \tilde{B}  \tag{204~g}\\
\tilde{B}-B_{0}=\Phi\left(\tilde{K}-K_{0}\right) \tag{204h}
\end{gather*}
$$

where $\tilde{Y}^{T}=Z^{T} L^{T}\left(\bar{\lambda}, \tilde{W}^{T}, \tilde{W}^{N}\right)\left(\tilde{k}^{T}\right)^{1-\theta_{T}}$ and $\tilde{Y}^{T}=Z^{N} L^{N}\left(\bar{\lambda}, \tilde{W}^{T}, \tilde{W}^{N}\right)\left(\tilde{k}^{N}\right)^{1-\theta_{N}}$. This system of equations jointly solve for sectoral capital-labor ratios, $\tilde{k}^{T}$ and $\tilde{k}^{N}$, for sectoral wages, $\tilde{W}^{T}$ and $\tilde{W}^{N}$, the relative price of non tradables, $\tilde{P}$, the capital stock, $\tilde{K}$, the stock of foreign assets, $\tilde{B}$, and the shadow value of wealth $\bar{\lambda}$, in terms of exogenous productivity parameters $Z^{T}$ and $Z^{N}$.

## F. 4 Graphical Apparatus: Rewriting the Steady-State

Before breaking down the three channels analytically, we characterize the steady state graphically, which allows us to emphasize how introducing traded investment modifies the results. We assume that $I\left(I^{T}, I^{N}\right)$ takes a Cobb-Douglas form as evidence that $\phi_{I}=1$ (see Bems [2008]). The steady-state can be rewritten as follows:

$$
\begin{gather*}
\frac{\tilde{C}^{T}}{\tilde{C}^{N}}=\frac{\varphi}{1-\varphi} \tilde{P}^{\phi},  \tag{205a}\\
\frac{\tilde{L}^{T}}{\tilde{L}^{N}}=\frac{\vartheta}{1-\vartheta} \tilde{\Omega}^{-\epsilon},  \tag{205b}\\
\frac{\tilde{Y}^{T}\left(1+v_{B}-v_{I^{T}}\right)}{\tilde{Y}^{N}\left(1-v_{I^{N}}\right)}=\frac{\tilde{C}^{T}}{\tilde{C}^{N}},  \tag{205c}\\
\tilde{P} Z^{N}\left(1-\theta_{N}\right)\left(\tilde{k}^{N}\right)^{-\theta_{N}} \equiv P_{I}(\tilde{P})\left(r^{\star}+\delta_{K}\right),  \tag{205d}\\
Z^{T}\left(1-\theta_{T}\right)\left(\tilde{k}^{T}\right)^{-\theta_{T}}=\tilde{P} Z^{N}\left(1-\theta_{N}\right)\left(\tilde{k}^{N}\right)^{-\theta_{N}} \equiv \tilde{R},  \tag{205e}\\
Z^{T} \theta_{T}\left(\tilde{k}^{T}\right)^{1-\theta_{T}} \equiv \tilde{W}^{T},  \tag{205f}\\
P Z^{N} \theta_{N}\left(\tilde{k}^{N}\right)^{1-\theta_{N}} \equiv \tilde{W}^{N}, \tag{205~g}
\end{gather*}
$$

where $\tilde{\omega}=\tilde{W}^{N} / \tilde{W}^{T}$ is the steady-state relative wage and $\tilde{R} / \tilde{P}=r^{\star}+\delta_{K}$. We denoted by $v_{I^{N}} \equiv \frac{\tilde{I}^{N}}{\tilde{Y}^{N}}\left(v_{I^{T}} \equiv \frac{\tilde{I}^{T}}{\tilde{Y}^{T}}\right)$ the ratio of non traded (traded) investment to non traded (traded) output and by $v_{B} \equiv \frac{r^{\star} \tilde{B}}{\tilde{Y}^{T}}$ the ratio of interest receipts to traded output.

Because we restrict ourselves to the analysis of the long-run effects, the tilde is suppressed for the purposes of clarity.

## F. 5 Goods Market Equilibrium

Applying the same procedure as in section D.8, combining (205a) with (205c) yields the $G M E$-equilibrium schedule described by:

$$
\begin{equation*}
\left.\left(y^{T}-y^{N}\right)\right|^{G M E}=\phi p+x^{\prime} \tag{206}
\end{equation*}
$$

where $x^{\prime}=\ln \left(\frac{\varphi}{1-\varphi}\right)+\ln \left(\frac{1-v_{I N}}{1+v_{B}-v_{I} T}\right)$. The goods market equilibrium is upward-sloping in the $\left(y^{T}-y^{N}, p\right)$-space and its slope is equal to $1 / \phi$.

Combining (205b) with (205f)-(205g) and production functions, we get:

$$
\frac{Y^{T}}{Y^{N}}=\frac{\vartheta}{1-\vartheta}\left(\frac{Z^{T}}{Z^{N}}\right)^{\epsilon+1} P^{-\epsilon}\left(\frac{\theta_{T}}{\theta_{N}}\right)^{\epsilon}\left[\frac{\left(k^{T}\right)^{1-\theta_{T}}}{\left(k^{N}\right)^{1-\theta_{N}}}\right]^{1+\epsilon}
$$

Combining (205d) and (205e) yields:

$$
\begin{equation*}
\frac{\left(k^{N}\right)^{1-\theta_{N}}}{\left(k^{T}\right)^{1-\theta_{T}}}=P^{\frac{1-\theta_{N}}{\theta_{N}}}\left[P_{I}\left(r^{\star}+\delta_{K}\right)\right]^{\frac{1-\theta_{T}}{\theta_{T}}-\frac{1-\theta_{N}}{\theta_{N}}} \frac{\left[Z^{N}\left(1-\theta_{N}\right)\right]^{\frac{1-\theta_{N}}{\theta_{N}}}}{\left[Z^{T}\left(1-\theta_{T}\right)\right]^{\frac{1-\theta_{T}}{\theta_{T}}}} . \tag{207}
\end{equation*}
$$

Inserting (207) to eliminate sectorial capital-labor ratios yields the $L M E$-schedule:

$$
\begin{equation*}
\frac{Y^{T}}{Y^{N}}=P^{-\left[\epsilon+\left(\frac{1-\theta_{N}}{\theta_{N}}\right)(1+\epsilon)\right]} P_{I}^{\left(\frac{\theta_{T}-\theta_{N}}{\theta_{T} \theta_{N}}\right)(1+\epsilon)} \frac{\left(Z^{T}\right)^{\frac{1+\epsilon}{\theta_{T}}}}{\left(Z^{N}\right)^{\frac{1+\epsilon}{\theta_{N}}}} \Pi^{\prime}, \tag{208}
\end{equation*}
$$

where we set

$$
\begin{equation*}
\Pi^{\prime} \equiv \frac{\vartheta}{1-\vartheta}\left(r^{\star}+\delta_{K}\right)^{\left(\frac{\theta_{T}-\theta_{N}}{\theta_{T} \theta_{N}}\right)(1+\epsilon)} \frac{\left[\left(\theta_{T}\right)^{\epsilon \theta_{T}}\left(1-\theta_{T}\right)^{\left(1-\theta_{T}\right)(1+\epsilon)}\right]^{1 / \theta_{T}}}{\left[\left(\theta_{N}\right)^{\epsilon \theta_{N}}\left(1-\theta_{N}\right)^{\left(1-\theta_{N}\right)(1+\epsilon)}\right]^{1 / \theta_{N}}}>0 . \tag{209}
\end{equation*}
$$

As mentioned above, we assume that the aggregator function for inputs of the investment good is Cobb-Douglas since data suggest that $\phi_{I}=1$. Taking logarithm, (208) can be rewritten as follows:

$$
\begin{equation*}
\left.\left(y^{T}-y^{N}\right)\right|^{L M E}=-\left\{\epsilon+(1+\epsilon)\left[\left(\frac{1-\theta_{N}}{\theta_{N}}\right)-\left(1-\varphi_{I}\right)\left(\frac{\theta_{T}-\theta_{N}}{\theta_{T} \theta_{N}}\right)\right]\right\} p+\left(\frac{1+\epsilon}{\theta_{T}}\right)\left(z_{T}-\frac{\theta^{T}}{\theta_{N}} z_{N}\right)+\pi^{\prime}, \tag{210}
\end{equation*}
$$

where $\pi^{\prime}=\ln \Pi^{\prime}$.
Setting $\varphi_{I}=0$ into (210) implies that the $L M E$-schedule is unambiguously negative in the ( $y^{T}-y^{N}, p$ )-space. This result holds when $\varphi_{I}>0$ as long as $\theta^{T}>\theta^{N}$ or if $\theta^{T}$ is close to $\theta^{N}$ as data suggest. The slope of the $L M E$-schedule in the $\left(y^{T}-y^{N}, p\right)$-space is

$$
\begin{equation*}
\left.\frac{\mathrm{d} p}{\mathrm{~d}\left(y^{T}-y^{N}\right)}\right|_{\vartheta_{I}>0} ^{L M E}=-\frac{1}{\left\{\epsilon+(1+\epsilon)\left[\left(\frac{1-\theta_{N}}{\theta_{N}}\right)-\left(1-\varphi_{I}\right)\left(\frac{\theta_{T}-\theta_{N}}{\theta_{T} \theta_{N}}\right)\right]\right\}} \tag{211}
\end{equation*}
$$

The slope of the $L M E$-schedule in the $\left(y^{T}-y^{N}, p\right)$-space is unambiguously negative and varies between $\frac{1}{\epsilon+(1+\epsilon)\left(\frac{1-\theta^{N}}{\theta^{N}}\right)}$ if investment expenditure are traded only (i.e., $\varphi_{I}$ is set to one) and $\frac{1}{\epsilon+(1+\epsilon)\left(\frac{1-\theta^{T}}{\theta^{T}}\right)}$ if investment expenditure are non-traded only (i.e., $\varphi_{I}$ is set to zero).

First, we compare the slope of the $L M E$-schedule when investment expenditure are both traded and non traded with the slope of the $L M E$-schedule in a model abstracting from physical capital. We find that that the $L M E$-schedule in a model abstracting from physical capital is steeper in the $\left(y^{T}-y^{N}, p\right)$-space if the following condition $\theta^{N}\left(1-\theta^{T}\right)>\varphi_{I}\left(\theta^{N}-\theta^{T}\right)$ holds.

Second, we compare the slope of the $L M E$-schedule when investment expenditure are both traded and non traded with the slope of the $L M E$-schedule when investment expenditure are non-traded only (i.e., $\varphi_{I}$ is set to 0 ). Formally, we find that the former is steeper than the latter in the $\left(y^{T}-y^{N}, p\right)$-space if the following condition holds:

$$
\left(\theta_{N}-\theta_{T}\right) \varphi_{I}>0,
$$

where $\theta_{T}$ and $\theta_{N}$ correspond to the labor share in the traded and the non traded sectors, respectively. The $L M E$-schedule when $\varphi_{I}>0$ is steeper than the $L M E$-schedule when $\varphi_{I}=0$ in the $\left(y^{T}-y^{N}, p\right)$-space as long as $\theta^{N}>\theta^{T}$, i.e. if the traded sector is more capital intensive than the non traded sector.

At this stage, it is useful to summarize our results when focusing on the goods market equilibrium in the $\left(y^{T}-y^{N}, \omega\right)$-space. We have to consider two cases, depending on whether the traded sector is more or less capital intensive then the non traded sector:

- If $\theta^{N}>\theta^{T}$, the following inequalities hold:

$$
\left.\frac{\mathrm{d} p}{\mathrm{~d}\left(y^{T}-y^{N}\right)}\right|^{L M E}<\left.\frac{\mathrm{d} p}{\mathrm{~d}\left(y^{T}-y^{N}\right)}\right|_{\vartheta_{I}>0} ^{L M E}<\left.\frac{\mathrm{d} p}{\mathrm{~d}\left(y^{T}-y^{N}\right)}\right|_{\vartheta_{I}=0} ^{L M E}<0 .
$$

- If $\theta^{T}>\theta^{N}$, the following inequalities hold:

$$
\left.\frac{\mathrm{d} p}{\mathrm{~d}\left(y^{T}-y^{N}\right)}\right|^{L M E}<\left.\frac{\mathrm{d} p}{\mathrm{~d}\left(y^{T}-y^{N}\right)}\right|_{\vartheta_{I}=0} ^{L M M E}<\left.\frac{\mathrm{d} p}{\mathrm{~d}\left(y^{T}-y^{N}\right)}\right|_{\vartheta_{I}>0} ^{L M E}<0 .
$$

## F. 6 Labor Market Equilibrium

Taking logarithm, (205b) can be rewritten to give the labor supply-schedule (henceforth $L S$ schedule):

$$
\begin{equation*}
\left.\left(l^{T}-l^{N}\right)\right|^{L S}=-\epsilon \omega+d, \tag{212}
\end{equation*}
$$

where $d=\ln \left(\frac{\vartheta}{1-\vartheta}\right)$. The $L S$-schedule is downward-sloping in the $\left(l^{T}-l^{N}, \omega\right)$-space where the slope is equal to $-1 / \epsilon$.

We turn to the derivation of the labor demand-schedule. Dividing (205g) by (205f) yields:

$$
\begin{equation*}
\frac{P Z^{N} \theta_{N}\left(k^{N}\right)^{1-\theta_{N}}}{Z^{T} \theta_{T}\left(k^{T}\right)^{1-\theta_{T}}}=\Omega \tag{213}
\end{equation*}
$$

To eliminate the sectoral capital-labor ratios, we use (205d)-(205e), i.e.

$$
\begin{equation*}
\frac{\left(k^{N}\right)^{1-\theta_{N}}}{\left(k^{T}\right)^{1-\theta_{T}}}=P^{\frac{1-\theta_{N}}{\theta_{N}}}\left[P_{I}\left(r^{\star}+\delta_{K}\right)\right]^{\frac{\theta^{N}-\theta^{T}}{\theta^{T} \theta^{N}}} \frac{\left[Z^{N}\left(1-\theta_{N}\right)\right]^{\frac{1-\theta_{N}}{\theta_{N}}}}{\left[Z^{T}\left(1-\theta_{T}\right)\right]^{\frac{1-\theta_{T}}{\theta_{T}}}} . \tag{214}
\end{equation*}
$$

To eliminate the relative price of non tradables, combine the market-clearing condition (205c) and the demand for traded goods in terms of non traded goods (205a) together with production functions (97):

$$
\begin{equation*}
P=\left[\frac{1-\varphi}{\varphi} \frac{1+v_{B}-v_{I^{T}}}{1-v_{I^{N}}} \frac{Z^{T} L^{T}\left(k^{T}\right)^{1-\theta_{T}}}{Z^{N} L^{N}\left(k^{N}\right)^{1-\theta_{N}}}\right]^{\frac{1}{\phi}} . \tag{215}
\end{equation*}
$$

Substituting (215) into (214) yields:

$$
\begin{align*}
\frac{\left(k^{N}\right)^{1-\theta_{N}}}{\left(k^{T}\right)^{1-\theta_{T}}}= & \left(r^{\star}+\delta_{K}\right)^{\frac{\phi\left(\theta_{N}-\theta_{T}\right)}{\psi}}\left[\frac{1-\varphi}{\varphi} \frac{1+v_{B}-v_{I^{T}}}{1-v_{I^{N}}^{T}} \frac{L^{T}}{L^{N}}\right] \frac{\left[\left(1-\theta^{N}\right) \theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]}{\psi} \\
& \times\left[\frac{\left(1-\theta_{N}\right)\left(\frac{1-\theta_{N}}{\theta^{N}}\right)}{\left(1-\theta_{T}\right)\left(\frac{1-\theta_{T}}{\theta^{T}}\right)}\right]^{\frac{\phi \theta^{T} \theta^{N}}{\psi}} \frac{\left(Z^{N}\right)^{\frac{\left[\theta^{T}\left(1-\theta^{N}\right)(\phi-1)-\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]}{\psi}}}{\left(Z^{T}\right)^{\frac{\left(1-\theta^{T}\right) \phi^{N}-\left[\left(1-\theta^{N}\right) \theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]}{\psi}}}, \tag{216}
\end{align*}
$$

where we set

$$
\begin{equation*}
\psi \equiv \theta^{T}\left[1+\theta^{N}(\phi-1)\right]+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right) \tag{217}
\end{equation*}
$$

Substituting first (215) into (213), we get:

$$
\Omega=\frac{\theta^{N}}{\theta^{T}}\left[\left(\frac{1+v_{B}-v_{I^{T}}}{1-v_{I^{N}}}\right)\left(\frac{1-\varphi}{\varphi}\right) \frac{L^{T}}{L^{N}}\right]^{\frac{1}{\phi}}\left(\frac{Z^{N}}{Z^{T}}\right)^{\frac{\phi-1}{\phi}}\left[\frac{\left(k^{N}\right)^{1-\theta_{N}}}{\left(k^{T}\right)^{1-\theta_{T}}}\right]^{\frac{\phi-1}{\phi}}
$$

Then plugging (216) enables us to find a relationship between labor in tradables relative to non tradables and the relative wage along the $L D$-schedule:

$$
\begin{equation*}
\frac{L^{T}}{L^{N}}=\Omega^{\frac{\psi}{\left.\theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]}}+\left(\frac{Z^{T}}{\left(Z^{N}\right)^{\frac{\theta_{T}}{\theta_{N}}}}\right)^{\frac{(\phi-1) \theta^{N}}{\left[\theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]}} \Theta^{\prime} \tag{218}
\end{equation*}
$$

where we set
$\Theta^{\prime} \equiv\left(\frac{1-v_{I^{N}}}{1+v_{B}-v_{I^{T}}}\right)\left(\frac{\varphi}{1-\varphi}\right)\left[\left(r^{\star}+\delta_{K}\right)^{\left(\theta_{T}-\theta_{N}\right)}\left(\frac{\theta_{T}}{\theta_{N}}\right)^{\frac{\psi}{\phi-1}} \frac{\left(1-\theta_{T}\right)^{\left(1-\theta_{T}\right) \theta^{N}}}{\left(1-\theta_{N}\right)^{\left(1-\theta_{N}\right) \theta^{T}}}\right]^{\frac{(\phi-1)}{\left[\theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]}}$.
Taking logarithm, (218) can be rewritten to yield the labor demand-schedule (henceforth $L D$-schedule):

$$
\begin{equation*}
\left.\left(l^{T}-l^{N}\right)\right|^{L D}=\frac{\psi}{\left[\theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]} \omega+\frac{(\phi-1) \theta^{N}}{\left[\theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]}\left(z^{T}-\frac{\theta_{T}}{\theta_{N}} z^{N}\right)+\ln \Theta^{\prime} . \tag{220}
\end{equation*}
$$

The slope of the $L D$-schedule in the $\left(l^{T}-l^{N}, p\right)$-space is:

$$
\begin{equation*}
\left.\frac{\mathrm{d} \omega}{\mathrm{~d}\left(l^{T}-l^{N}\right)}\right|_{\vartheta_{I}>0} ^{L D}=\frac{\theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)}{\theta^{T}\left[1+\theta^{N}(\phi-1)\right]+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)}>0 . \tag{221}
\end{equation*}
$$

First, we compare the slope of the $L D$-schedule when investment expenditure are both traded and non traded with the slope of the $L D$-schedule in a model abstracting from physical capital. We find that the $L D$-schedule in a model abstracting from physical capital is steeper in the $\left(l^{T}-l^{N}, \omega\right)$-space if the following condition holds:

$$
\begin{equation*}
(1-\phi)\left[\theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]>0 \tag{222}
\end{equation*}
$$

The $L D$-schedule in a model abstracting from physical capital is steeper in the $\left(l^{T}-l^{N}, \omega\right)$ space than the $L D$-schedule in a model where $\varphi_{I}>0$ as long as $\phi<1$.

Second, we compare the slope of the $L D$-schedule when investment expenditure are both traded and non traded with the slope of the $L D$-schedule when investment expenditure are non traded only (i.e., $\varphi_{I}$ is set to 0 ). Formally, we find that the former is flatter than the latter in the $\left(l^{T}-l^{N}, \omega\right)$-space if the following condition holds:

$$
\begin{equation*}
(\phi-1)\left(\theta^{N}-\theta^{T}\right) \varphi_{I}>0 . \tag{223}
\end{equation*}
$$

According to (223), when $\phi<1$ and the traded sector is more capital intensive (i.e., $\theta^{N}>\theta^{T}$ ), the $L D$-schedule when investment expenditure are both traded and non traded is flatter than the $L D$-schedule when investment expenditure are non traded.

At this stage, it is useful to summarize our results when focusing on the labor market equilibrium in the $\left(l^{T}-l^{N}, \omega\right)$-space. We have to consider two cases, depending on whether $\phi$ is larger or smaller than one. For clarity purpose, we assume that the traded sector is more capital intensive than the non-traded sector (i.e., we impose $\theta^{N}>\theta^{T}$ ):

- If $\phi>1$ and $\theta^{N}>\theta^{T}$, these inequalities hold:

$$
\left.\frac{\mathrm{d} \omega}{\mathrm{~d}\left(l^{T}-l^{N}\right)}\right|_{\vartheta_{I}=0} ^{L D}>\left.\frac{\mathrm{d} \omega}{\mathrm{~d}\left(l^{T}-l^{N}\right)}\right|_{\vartheta_{I}>0} ^{L D}>\left.\frac{\mathrm{d} \omega}{\mathrm{~d}\left(l^{T}-l^{N}\right)}\right|^{L D}>0 .
$$

- If $\phi<1$ and $\theta^{N}>\theta^{T}$, these inequalities hold:

$$
\left.\frac{\mathrm{d} \omega}{\mathrm{~d}\left(l^{T}-l^{N}\right)}\right|^{L D}>\left.\frac{\mathrm{d} \omega}{\mathrm{~d}\left(l^{T}-l^{N}\right)}\right|_{\vartheta_{I}>0} ^{L D}>\left.\frac{\mathrm{d} \omega}{\mathrm{~d}\left(l^{T}-l^{N}\right)}\right|_{\vartheta_{I}=0} ^{L D}>0 .
$$

## F. 7 The Relative Price and Relative Wage Effects of a Productivity Differential

## The Relative Price Effect

Equating (206) and (210) to eliminate $y^{T}-y^{N}$ and differentiating yields the percentage deviation of the relative price of non tradables from its initial steady-state following a productivity differential between tradables and non tradables:

$$
\begin{equation*}
\hat{p}=\frac{(1+\epsilon)\left[\frac{\theta_{N}}{\theta_{T}} \hat{z}^{T}-\hat{z}^{N}\right]+\theta^{N} \mathrm{~d} \ln \left(\frac{1+v_{B}-v_{v^{T}}}{1-v_{I N}}\right)}{\theta^{N}\left\{(\epsilon+\phi)+(1+\epsilon)\left[\left(\frac{1-\theta_{N}}{\theta_{N}}\right)-\left(1-\varphi_{I}\right)\left(\frac{\theta_{T}-\theta_{N}}{\theta_{T} \theta_{N}}\right)\right]\right\}} . \tag{224}
\end{equation*}
$$

To ease the interpretation of the equation, we rewrite the term $\ln \left(\frac{1+v_{B}-v_{I T}}{1-v_{I} N}\right)$ as $\ln \left(1+v_{B}-v_{I^{T}}\right)-$ $\ln \left(1-v_{I^{N}}\right)$, by using a Taylor approximation at a first order which implies $\ln \left(1+v_{B}-v_{I^{T}}\right)-$ $\ln \left(1-v_{I^{N}}\right) \simeq v_{B}-v_{I^{T}}+v_{I^{N}}$. Then using the fact that $v_{B}=-v_{N X},(224)$ reads:

$$
\hat{p}=\frac{(1+\epsilon)\left[\frac{\theta_{N}}{\theta_{T}} \hat{z}^{T}-\hat{z}^{N}\right]+\theta^{N}\left(\mathrm{~d} v_{N X}+\mathrm{d} v_{I^{T}}-\mathrm{d} v_{I^{N}}\right)}{\theta^{N}\left\{(\epsilon+\phi)+(1+\epsilon)\left[\left(\frac{1-\theta_{N}}{\theta_{N}}\right)-\left(1-\varphi_{I}\right)\left(\frac{\theta_{T}-\theta_{N}}{\theta_{T} \theta_{N}}\right)\right]\right\}}
$$

When considering that investment is both non traded and traded investment, the labor share-adjusted TFP differential becomes:

$$
\begin{equation*}
\frac{\left[\frac{\theta_{N}}{\theta_{T}} \hat{z}^{T}-\hat{z}^{N}\right]}{\vartheta_{I}+\frac{\theta^{N}}{\theta^{T}}\left(1-\vartheta_{I}\right)} . \tag{225}
\end{equation*}
$$

Using (225) and rearranging terms, the long-run response of the relative price given by (224) becomes:

$$
\begin{equation*}
\hat{p}=(1+\epsilon) \Theta^{K T} \frac{\left[\frac{\theta_{N}}{\theta_{T}} \hat{z}^{T}-\hat{z}^{N}\right]}{\vartheta_{I}+\frac{\theta^{N}}{\theta^{T}}\left(1-\vartheta_{I}\right)}-\frac{\theta^{T} \theta^{N}}{\theta^{T} \vartheta_{I}+\theta^{N}\left(1-\vartheta_{I}\right)} \Theta^{K T}\left(\mathrm{~d} v_{N X}+\mathrm{d} v_{I^{T}}-\mathrm{d} v_{I^{N}}\right) \tag{226}
\end{equation*}
$$

where we set

$$
\begin{align*}
\Theta^{K T} & =\frac{\theta^{T} \vartheta_{I}+\theta^{N}\left(1-\vartheta_{I}\right)}{\theta^{N}\left\{(\epsilon+\phi)+(1+\epsilon)\left[\left(\frac{1-\theta_{N}}{\theta_{N}}\right)-\left(1-\varphi_{I}\right)\left(\frac{\theta_{T}-\theta_{N}}{\theta_{T} \theta_{N}}\right)\right]\right\}} \\
& =\frac{\theta^{T} \vartheta_{I}+\theta^{N}\left(1-\vartheta_{I}\right)}{(1+\epsilon)\left[\theta^{T} \vartheta_{I}+\theta^{N}\left(1-\vartheta_{I}\right)\right]+(\phi-1) \theta^{T} \theta^{N}} . \tag{227}
\end{align*}
$$

We now break down the long-run relative price response to a productivity differential into three components by adding and subtracting the following terms $\Theta^{K}$ and $\Theta^{L}$ in the RHS of (226):

$$
\begin{align*}
\hat{p}= & (1+\epsilon)\left[\Theta^{L}+\left(\theta^{K}-\theta^{L}\right)+\left(\Theta^{K T}-\Theta^{K}\right)\right] \frac{\left[\frac{\theta_{N}}{\theta_{T}} \hat{z}^{T}-\hat{z}^{N}\right]}{\vartheta_{I}+\frac{\theta^{N}}{\theta^{T}}\left(1-\vartheta_{I}\right)} \\
& -\frac{\theta^{T} \theta^{N}}{\theta^{T} \vartheta_{I}+\theta^{N}\left(1-\vartheta_{I}\right)} \Theta^{K T}\left(\mathrm{~d} v_{N X}+\mathrm{d} v_{I^{T}}-\mathrm{d} v_{I^{N}}\right), \tag{228}
\end{align*}
$$

where

$$
\begin{equation*}
\Theta^{K T}-\Theta^{K}=-\frac{\theta^{T}(\phi-1) \vartheta_{I}\left(\theta^{N}-\theta^{T}\right)}{\left\{(1+\epsilon)\left[\theta^{T} \vartheta_{I}+\theta^{N}\left(1-\vartheta_{I}\right)\right]+(\phi-1) \theta^{T} \theta^{N}\right\}\left\{(1+\epsilon)+(\phi-1) \theta^{T}\right\}} \lessgtr 0 . \tag{229}
\end{equation*}
$$

While the sign of the numerator is ambiguous as it depends on $\phi \gtrless 1$ and $\theta^{N} \gtrless \theta^{T}$, the sign of the denominator is unambiguously positive.

## The Relative Wage Effect

Equating (212) and (220) to eliminate $l^{T}-l^{N}$, taking logarithm and differentiating yields the percentage deviation of the relative wage $\omega$ from its initial steady-state following a productivity differential:

$$
\begin{aligned}
\hat{\omega}= & -\frac{(\phi-1) \theta^{N}\left(\hat{z}^{T}-\frac{\theta_{T}}{\theta_{N}} \hat{z}^{N}\right)+\left[\theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]-\mathrm{d} \ln \left(\frac{1-v_{I^{N}}}{1+v_{B}-v_{I^{T}}}\right)}{(1+\epsilon)\left[\theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]+\theta^{T} \theta^{N}(\phi-1)}, \\
& -\frac{(\phi-1) \theta^{N}\left(\hat{z}^{T}-\frac{\theta_{T}}{\theta_{N}} \hat{z}^{N}\right)+\left[\theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]\left(\mathrm{d} v_{N X}+\mathrm{d} v_{I^{T}}-\mathrm{d} v_{I^{N}}\right)}{(1+\epsilon)\left[\theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]+\theta^{T} \theta^{N}(\phi-1)}
\end{aligned}
$$

where the second line has been obtained by using a Taylor approximation at the first order to rewrite $\ln \left(\frac{1-v_{I^{N}}}{1+v_{B}-v_{I^{T}}}\right)$ as $\ln \left(1-v_{I^{N}}\right)-\left(1+v_{B}-v_{I^{T}}\right) \simeq-v_{I^{N}}-v_{B}+v_{I^{T}}=\left(v_{N X}+v_{I^{T}}-v_{I^{N}}\right)$.

Inserting $\Theta^{K T}$ given by (227) and using the labor share-adjusted TFPs differential (225), the long-run response of the relative wage (230) can be rewritten as follows:
$\hat{\omega}=-(\phi-1) \Theta^{K T} \frac{\left[\frac{\theta_{N}}{\theta_{T}} \hat{z}^{T}-\hat{z}^{N}\right]}{\vartheta_{I}+\frac{\theta^{N}}{\theta^{T}}\left(1-\vartheta_{I}\right)}-\Theta^{K T} \frac{\left[\theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]}{\vartheta_{I}+\frac{\theta^{N}}{\theta^{T}}\left(1-\vartheta_{I}\right)}\left(\mathrm{d} v_{N X}+\mathrm{d} v_{I^{T}}-\mathrm{d} v_{I^{N}}\right)$.
We now break down the long-run relative wage response to a productivity differential into three components by adding and subtracting $\Theta^{K}$ and $\Theta^{L}$ in the RHS of (231). We get:

$$
\begin{align*}
\hat{\omega}= & -(\phi-1)\left[\Theta^{L}+\left(\theta^{K}-\theta^{L}\right)+\left(\Theta^{K T}-\Theta^{K}\right)\right] \frac{\left[\frac{\theta_{N}}{\theta_{T}} \hat{z}^{T}-\hat{z}^{N}\right]}{\vartheta_{I}+\frac{\theta^{N}}{\theta^{T}}\left(1-\vartheta_{I}\right)} \\
& -\Theta^{K T}\left[\theta^{T}+\left(1-\varphi_{I}\right)\left(\theta^{N}-\theta^{T}\right)\right]\left(\mathrm{d} v_{N X}+\mathrm{d} v_{I^{T}}-\mathrm{d} v_{I^{N}}\right), \tag{232}
\end{align*}
$$

where $-(\phi-1)\left(\Theta^{K T}-\Theta^{K}\right)$ is positive as long as the non-traded sector is more labor intensive than the traded sector:
$-(\phi-1)\left(\Theta^{K T}-\Theta^{K}\right)=\frac{\theta^{T}(\phi-1)^{2} \vartheta_{I}\left(\theta^{N}-\theta^{T}\right)}{\left\{(1+\epsilon)\left[\theta^{T} \vartheta_{I}+\theta^{N}\left(1-\vartheta_{I}\right)\right]+(\phi-1) \theta^{T} \theta^{N}\right\}\left\{(1+\epsilon)+(\phi-1) \theta^{T}\right\}} \gtrless 0$.
In order to shed light analytically on the implications of considering that investment expenditure are both traded and non-traded, it is useful to break down the reallocation channel as follows $-(\phi-1)\left[\left(\theta^{K T}-\theta^{K}\right)+\left(\theta^{K}-\theta^{L}\right)\right]$. While $-(\phi-1)\left(\theta^{K T}-\theta^{K}\right)$ reflects the reallocation channel when investment is non-tradable, the novel term - $(\phi-1)\left(\theta^{K T}-\theta^{K}\right)$ captures
the user capital cost channel arising when investment expenditure are both traded and nontraded. To keep things simple, let us assume that the traded sector is more capital intensive than the non-traded sector (i.e., we set $\left.\theta^{N}>\theta^{T}\right)$. In this case, $-(\phi-1)\left(\Theta^{K T}-\Theta^{K}\right)>0 .{ }^{47}$ Hence, irrespective of whether $\phi$ is larger or smaller than one, introducing traded investment raises the relative wage compared with a model assuming $\varphi_{I}=0$. Intuitively, the user capital $\operatorname{cost} P_{I}\left(r^{\star}+\delta_{K}\right)$ increases less when $0<\varphi_{I}<1$ since the investment price index increases in proportion of the non tradable content of investment expenditure, following an appreciation in the relative price, which mitigates the decline in the traded capital-labor ratio $k^{T}$. As long as the traded sector is more capital intensive (i.e., $\theta^{N}>\theta^{T}$ ), the non-traded sector experiences a smaller capital inflow which moderates the rise in non traded output compared with that in a model abstracting from traded investment. Graphically, the $L D^{K}$-schedule shown in Figure 4(a) would become flatter if $\phi>1$ while the $L D^{K}$-schedule in Figure 5(a) would become steeper if $\phi<1$. Hence, in either cases, introducing traded investment moderates the decline in the relative wage induced by the capital reallocation channel.

## F. 8 Calibration and Discussion of Numerical Results

To split investment expenditure into traded and non traded goods, we follow the methodology proposed by Burstein et al. [2004] who treat Housing and Other Constructions as nontradable investment and Products of agriculture, forestry, fisheries and aquaculture, Metal products and machinery, Transport Equipment as tradable investment expenditure (Source: OECD Input-Output database [?]). Due to the lack of information, we consider the item 'Other products' as both tradable ( $50 \%$ ) and non tradable ( $50 \%$ ) with equal shares. For each country, the period is running from 1990 to 2007, except for Sweden (1993-2007). Time series are not available for Belgium and Korea, for which we rely on Bems's [2008] estimates and set $\alpha_{I}=0.59$ for both countries. Non tradable share of investment shown in column 3 of Table 14 averages to $58 \%$, in line with estimates provided by Burstein et al. [2004] and Bems [2008].

When assuming that investment expenditures are both traded and non traded, we set the elasticity of substitution $\phi_{I}$ between $I^{T}$ and $I^{N}$ to 1 , in line with the empirical findings documented by Bems [2008] for OECD countries. Further, the weight of non traded investment $\left(1-\varphi_{I}\right)$ is set to 0.58 to target a non-tradable content of investment expenditure of $58 \%$, in line with our estimates shown in the last line of column 3 of Table 14.

It is worth noting that introducing traded investment modifies the labor share-adjusted TFPs differential; more precisely, assuming perfect mobility of labor across sectors and considering both tradable and non-tradable investments, the long-run response of the relative price becomes $\hat{p}=\left[\frac{\theta^{N}}{\theta^{T}} \hat{z}^{T}-\hat{z}^{N}\right] /\left[\alpha_{I}+\frac{\theta^{N}}{\theta^{T}}\left(1-\alpha_{I}\right)\right]$. Hence, when running the simulations we now consider an increase by $1 \%$ in the modified labor share-adjusted TFPs differential.

When introducing traded investment (i.e., $\varphi_{I}$ is set to 0.42 ), the relative wage and relative price responses (in \%) following a productivity differential between tradables and non tradables of $1 \%$ are shown in the last column of Table 13 for three alternative scenarios. Considering traded investment merely affects the capital reallocation channel by influencing the user cost of capital $P_{I}\left(r^{\star}+\delta_{K}\right) .{ }^{48}$ Because the relative price and relative wage responses are almost unchanged if not identical, numerical results when considering traded investment

$$
\begin{aligned}
& { }^{47} \text { The sign of } \Theta^{K T}-\Theta^{K} \text { depends on sectoral capital intensities. Formally, we have: } \\
& \qquad-(\phi-1)\left(\Theta^{K T}-\Theta^{K}\right)=\frac{\theta^{T}(\phi-1)^{2} \vartheta_{I}\left(\theta^{N}-\theta^{T}\right)}{\left\{(1+\epsilon)\left[\theta^{T} \vartheta_{I}+\theta^{N}\left(1-\vartheta_{I}\right)\right]+(\phi-1) \theta^{T} \theta^{N}\right\}\left\{(1+\epsilon)+(\phi-1) \theta^{T}\right\}} \gtrless 0 .
\end{aligned}
$$

While the denominator is unambiguously positive, the sign of the numerator depends on $\left(\theta^{N}-\theta^{T}\right)$. If $\theta^{N}>\theta^{T}$, we have $-(\phi-1)\left(\Theta^{K T}-\Theta^{K}\right)>0$.
${ }^{48}$ Intuitively, following an appreciation in the relative price, the user cost of capital $P_{I}\left(r^{\star}+\delta_{K}\right)$ increases by a smaller amount when $0<\varphi_{I}<1$ since $P_{I}$ rises in proportion to the non-tradable content of investment expenditure. Thus, an appreciation in the relative price of non tradables raises the ratio $k^{N} / k^{T}$ but less than if $\varphi_{I}=0$ as long as $\theta^{N}>\theta^{T}$.
do not merit further comment.

## F. 9 The Relative Price and Relative Wage Effects: Imperfect vs. Perfect Substitutability of Hours Worked across Sectors

In this subsection, we derive long-run adjustments of the relative price and relative wage following a productivity differential by emphasizing the role of the degree of labor mobility.

Assuming that capital is perfectly mobile and labor is imperfectly mobile across sectors, first-order conditions from the firm's profit maximization are:

$$
\begin{array}{r}
Z^{T}\left(1-\theta^{T}\right)\left(k^{T}\right)^{-\theta^{T}}=P Z^{N}\left(1-\theta^{N}\right)\left(k^{N}\right)^{-\theta^{N}} \equiv R, \\
Z^{T} \theta^{T}\left(k^{T}\right)^{1-\theta^{T}} \equiv W^{T}, \\
P Z^{N} \theta^{N}\left(k^{N}\right)^{1-\theta^{N}} \equiv W^{N}, \tag{234c}
\end{array}
$$

where $R$ is the capital rental cost and $W^{j}$ the labor cost in sector $j=T, N$.
Setting $\dot{P}=0$ into (193d), the capital rental cost can be written as follows:

$$
\begin{equation*}
R=P_{I}\left(r^{\star}+\delta_{K}\right) . \tag{235}
\end{equation*}
$$

Dividing the marginal product of labor by the marginal product of capital in each sector yields the sectoral capital-labor ratios:

$$
\begin{equation*}
k^{T}=\frac{1-\theta^{T}}{\theta^{T}} \frac{W^{T}}{R}, \quad k^{N}=\frac{1-\theta^{N}}{\theta^{N}} \frac{W^{N}}{R} . \tag{236}
\end{equation*}
$$

Substituting sectoral capital-labor ratios (236) into (234b) and (234c) yields an expression of the steady-state relative price of non tradables :

$$
\begin{equation*}
P=\frac{\Psi^{T}}{\Psi^{N}} \frac{Z^{T}}{Z^{N}} \frac{\left(W^{N}\right)^{\theta^{N}}(R)^{1-\theta^{N}}}{\left(W^{T}\right)^{\theta^{T}}(R)^{1-\theta^{T}}} \tag{237}
\end{equation*}
$$

where

$$
\begin{equation*}
\Psi^{j}=\left(\theta^{j}\right)^{\theta^{j}}\left(1-\theta^{j}\right)^{1-\theta^{j}}, \quad j=T, N . \tag{238}
\end{equation*}
$$

According to (237), the relative price of non tradables is equal to unit cost of producing in the non traded sector relative to the unit cost for producing in the traded sector. The unit cost function is a weighted average of the wage rate $W^{j}$ and the rental rate of capital $R$, divided by the sectoral TFP $Z^{j}$.

Substituting the sectoral capital-labor ratio given by (236) into (234b), the wage rate in the traded sector can be rewritten as follows:

$$
\begin{equation*}
W^{T}=\left(Z^{T}\right)^{\frac{1}{\theta^{T}}}\left(\Psi^{T}\right)^{\frac{1}{\theta^{T}}} R^{-\left(\frac{1-\theta^{T}}{\theta^{T}}\right)} . \tag{239}
\end{equation*}
$$

Multiplying and dividing the RHS of (237) by $\left(W^{T}\right)^{\theta^{N}}$ and substituting (239) yields:

$$
\begin{align*}
P & =\frac{\Psi^{T}}{\Psi^{N}} \frac{Z_{T}}{Z_{N}}\left(\frac{W^{N}}{W^{T}}\right)^{\theta^{N}}\left(W^{T}\right)^{\left(\theta^{N}-\theta^{T}\right)} R^{\theta^{T}-\theta^{N}}, \\
& =\frac{\left(\Psi^{T}\right)^{\theta^{N}}}{\Psi^{N}} \frac{\left(Z_{T} \frac{\theta^{N}}{\theta^{T}}\right.}{Z_{N}}\left(\frac{W^{N}}{W^{T}}\right)^{\theta^{N}} R^{\frac{\theta^{T}-\theta^{N}}{\theta^{T}}} . \tag{240}
\end{align*}
$$

Totally differentiating (240) and (235), collecting terms, and denoting the percentage deviation from its initial steady-state by a hat gives:

$$
\begin{equation*}
\hat{p}=\frac{\hat{z}^{T}-\frac{\theta^{T}}{\theta^{N}} \hat{z}^{N}}{\alpha_{I}+\left(1-\alpha_{I}\right) \frac{\theta^{T}}{\theta^{N}}}+\frac{\theta^{T}}{\alpha_{I}+\left(1-\alpha_{I}\right) \frac{\theta^{T}}{\theta^{N}}} \hat{\omega}, \tag{241}
\end{equation*}
$$

Table 14: Data to Calibrate the Two-Sector Model (1990-2007)

| Countries | Non tradable Share |  |  |  |  |  | $G^{j} / Y^{j}$ |  | Labor Share |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
|  | Output | Consumption | Investment | Gov. Spending | Labor | Lab. comp. | $G^{T} / Y^{T}$ | $G^{N} / Y^{N}$ | $\theta^{T}$ | $\theta^{N}$ |
| BEL | 0.65 | 0.42 | 0.59 | 0.91 | 0.68 | 0.66 | 0.06 | 0.30 | 0.61 | 0.63 |
| DEU | 0.65 | 0.40 | 0.59 | 0.91 | 0.65 | 0.60 | 0.05 | 0.27 | 0.75 | 0.63 |
| DNK | 0.66 | 0.42 | 0.53 | 0.94 | 0.68 | 0.68 | 0.05 | 0.36 | 0.63 | 0.70 |
| ESP | 0.64 | 0.46 | 0.64 | 0.88 | 0.66 | 0.67 | 0.06 | 0.24 | 0.60 | 0.66 |
| FIN | 0.58 | 0.43 | 0.65 | 0.89 | 0.63 | 0.63 | 0.06 | 0.34 | 0.59 | 0.73 |
| FRA | 0.70 | 0.40 | 0.61 | 0.94 | 0.69 | 0.68 | 0.05 | 0.31 | 0.70 | 0.64 |
| GBR | 0.64 | 0.40 | 0.55 | 0.93 | 0.70 | 0.65 | 0.04 | 0.29 | 0.70 | 0.73 |
| IRL | 0.52 | 0.43 | 0.67 | 0.89 | 0.62 | 0.62 | 0.04 | 0.26 | 0.46 | 0.69 |
| ITA | 0.64 | 0.38 | 0.52 | 0.91 | 0.63 | 0.62 | 0.05 | 0.28 | 0.71 | 0.64 |
| JPN | 0.63 | 0.43 | 0.56 | 0.86 | 0.64 | 0.65 | 0.06 | 0.22 | 0.55 | 0.60 |
| KOR | 0.52 | 0.44 | 0.59 | 0.76 | 0.58 | 0.56 | 0.06 | 0.18 | 0.73 | 0.83 |
| NLD | 0.65 | 0.40 | 0.59 | 0.90 | 0.70 | 0.69 | 0.07 | 0.32 | 0.60 | 0.70 |
| SWE | 0.64 | 0.45 | 0.48 | 0.92 | 0.68 | 0.67 | 0.06 | 0.39 | 0.63 | 0.71 |
| USA | 0.68 | 0.51 | 0.57 | 0.90 | 0.73 | 0.69 | 0.05 | 0.21 | 0.61 | 0.66 |
| Mean | 0.63 | 0.43 | 0.58 | 0.90 | 0.65 | 0.65 | 0.05 | 0.28 | 0.63 | 0.68 |

[^26]where $\hat{\omega}=\hat{W}^{N}-\hat{W}^{T}$ and we used the fact that $\hat{R}=\alpha_{I} \hat{P}=\frac{\theta^{N}}{\theta^{T}}\left[\alpha_{I}+\left(1-\alpha_{I}\right) \frac{\theta^{T}}{\theta^{N}}\right]$.
A closed-form solution for the steady-state level of the relative price of non-tradables can be found when assuming that investment is non traded which implies that (235) reduces to:
\[

$$
\begin{equation*}
R=P\left(r^{\star}+\delta_{K}\right) . \tag{242}
\end{equation*}
$$

\]

Inserting (242) into (240) and collecting termes yields:

$$
\begin{equation*}
P=\frac{\Psi^{T}}{\left(\Psi^{N}\right)^{\theta^{T} / \theta^{N}}} \frac{Z^{T}}{\left(Z^{N}\right)^{\theta^{T} / \theta^{N}}}\left(\frac{W^{N}}{W^{T}}\right)^{\theta^{T}}\left(r^{\star}+\delta_{K}\right)^{\frac{\theta^{T}-\theta^{N}}{\theta^{N}}} . \tag{243}
\end{equation*}
$$

Eq. (243) corresponds to eq. (3) in the text.
Depending on whether investment is either traded or non trade, or both, three cases emerge:

- When investment is traded, i.e., setting $\alpha_{I}=0$ into (241), the productivity differential reduces to:

$$
\begin{equation*}
\frac{\theta^{N}}{\theta^{T}} \hat{z}^{T}-\hat{z}^{N} \tag{244}
\end{equation*}
$$

- When investment is non-traded, i.e., setting $\alpha_{I}=1$ into (241), the productivity differential reduces to:

$$
\begin{equation*}
\hat{z}^{T}-\frac{\theta^{T}}{\theta^{N}} \hat{z}^{N} . \tag{245}
\end{equation*}
$$

- When investment is non-traded, i.e., assuming that $\alpha_{I}>0$, the productivity differential reduces to:

$$
\begin{equation*}
\frac{\hat{z}^{T}-\frac{\theta^{T}}{\theta^{N}} \hat{z}^{N}}{\alpha_{I}+\left(1-\alpha_{I}\right) \frac{\theta^{T}}{\theta^{N}}} . \tag{246}
\end{equation*}
$$

The standard Balassa-Samuelson model imposes perfect mobility of labor across sectors so that sectoral wages increase at the same speed $\hat{W}^{N}=\hat{W}^{T}$. Since $\hat{\omega}=0$, eq. (240) implies that following a productivity differential $\frac{\hat{z}^{T}-\frac{\theta^{T}}{\theta^{N}} \hat{z}^{N}}{\alpha_{I}+\left(1-\alpha_{I} \frac{\theta^{T}}{\theta^{N}}\right.}$ by one percentage point, the relative price of non tradables appreciates by one percentage point. To avoid unnecessary complications, we assume that investment is non traded by setting $\alpha_{I}=1$ so that the productivity differential reduces to $\hat{z}^{T}-\frac{\theta^{T}}{\theta^{N}} \hat{z}^{N}$. As shown above, our numerical results are unsensitive to this assumption. As discussed in the next section, our empirical results are unsensitive to the assumption related to the non tradables content of investment expenditure as well.

## G Robustness Analysis

In this section we explore the extent to which our estimates of the relative price (i.e., $\hat{\gamma}$ ) and relative wage (i.e., $\hat{\beta}$ ) responses to a productivity differential between tradables and non tradables are robust to a variety of specification checks. More precisely, we run the regression of the relative wage and the relative price on the relative productivity of tradables (see eqs. (6)):

$$
\begin{align*}
& \omega_{i, t}=\delta_{i}+\beta \text {.relative productivity of } \text { tradables }_{i, t}+v_{i, t}  \tag{247a}\\
& p_{i, t}=\alpha_{i}+\gamma \text {.relative productivity of tradables }  \tag{247b}\\
& i, t
\end{align*}+u_{i, t} .
$$

To assess the robustness of the estimation results, we re-run the regressions and compare our empirical findings with those shown in column 1 of Table 2. Table 15 gives results when carrying out different types of robustness tests including alternatives measures of the
productivity of tradables in terms of non tradables, excluding Korea, calculating sectoral capital stocks using disaggregated capital stock data at the sectoral level obtained form the EU KLEMS database instead of breaking down physical capital into traded and non traded capital by using sectoral value added shares in the lines of Garofalo and Yamarik [2002]. Because DOLS and FMOLS estimates are very similar, for clarity purposes, Table 15 shows FMOLS estimates only.

## G. 1 Alternative Measures of Technological Change Biased toward the Traded Sector

We first conduct a robustness check by considering alternative measures of the productivity differential between tradables and non tradables. In section F.9, we show that, when investment is both traded and non traded, technological change biased toward the traded sector is given by:

$$
\begin{equation*}
\left[\frac{Z^{T}}{\left(Z^{N}\right)^{\frac{\theta^{T}}{\theta^{N}}}}\right]\left[\frac{1}{\alpha_{I}+\left(1-\alpha_{I}\right) \frac{\theta^{T}}{\theta^{N}}}\right] \tag{248}
\end{equation*}
$$

where $\alpha_{I}$ is the non tradable share in total investment expenditure.
Panel A of Table 15 summarizes our estimates for baseline regressions. In the main text, productivity of tradables in termes of non tradables is calculated as the ratio of the traded TFP to the labor share-adjusted non traded TFP when assuming that investment is non traded, i.e., setting $\alpha_{I}=1$ into (248).

Panel B of Table 15 shows estimates for the relative price and relative wage responses when considering alternative measures of productivity of tradables in terms of non tradables:

- Because earlier studies analyzing the effects of biased technological change toward the traded sector on the relative price of non tradables, see e.g., Canzoneri, Cumby and Diba [1999], or the real exchange rate, see e.g., Bergin, Glick and Rogoff [2006], use sectoral labor productivity, in order to make our results comparable, the first line of panel B of Table 15 gives results when technological change is measured by using the labor productivity index denoted by $A^{j}$. In this case, relative productivity of tradables (248) reduces to $A^{T} / A^{N}$. To measure labor productivity, we divide value-added at constant prices (VA_QI in KLEMS) by the total hours worked (H_EMP in KLEMS) for each sector in each country from 1970 to 2007 (from 1974 to 2007 for Japan). Source: EU KLEMS database.
- Kakkar [2003] analyzes the effects of a productivity differential between tradables and non tradables by assuming that investment is traded only, i.e., $\alpha_{I}=0$. In this case, relative productivity of tradables (248) reduces to:

$$
\begin{equation*}
\frac{\left(Z^{T}\right)^{\frac{\theta^{N}}{\theta^{T}}}}{Z^{N}} \tag{249}
\end{equation*}
$$

The second line of panel B of Table 15 gives the relative wage and relative prices responses when the relative productivity of tradables is given by (249)

- The third line of panel B of Table 15 shows the relative wage and relative price effects of technological change biased toward the traded sector when assuming that investment is both traded and non traded. In this case, the relative productivity of tradables is given by (248). Because the formula includes the non tradable share of investment expenditure, $\alpha_{I}$, we have to calculate it. To do so, we follow the methodology proposed by Burstein et al. [2004] who treat "Housing" and "Other Constructions" as non-tradable investment and "Products of Agriculture, Forestry, Fisheries and Aquaculture", "Metal

Products and Machinery", "Transport Equipment" as tradable investment expenditure (Source: OECD Input-Output database [?]). Due to the lack of information, we consider the item "Other Products"' as both tradable (50\%) and non tradable (50\%) with equal shares. For each country, the period is running from 1970 to 2007, except for Ireland (1990-2007) and Sweden (1993-2007). Time series are not available for Belgium and Korea, for which we rely on Bems's [2008] estimates (see Tables 7 and 8) and set $\alpha_{I}=0.59$ for both countries.

## G. 2 Robustness Check

Because Figures 1(a) and 1(b) could suggest that Korea is an outlier, we exclude that country from our sample when running the regression of the relative wage and the relative price on the relative productivity of tradables. Results are shown in panel C of Table 15. Additionally, in order to check that the trend line in Figures 1(a) and 1(b) is not driven by Korea, as a second attempt to circumvent this potential source of bias, we re-estimate the slope of the slope coefficient for the trend line by the method of quantile regressions (Koenker and Bassett [1978]). Because this approach uses the absolute value rather than the square of the residuals, it is less sensitive to extreme values and will be more efficient that OLS with respect to outliers.

Another concern is the way the tradable and non-tradable sectors are constructed. In particular, the classification of the items "Wholesale and Retail Trade", "Hotels and Restaurants", "Transport, Storage and Communication", "Financial Intermediation" and "Real Estate, Renting and Business Services" is somewhat problematic. In order to address this issue, we replicate regressions (6a)-(6b) and (8a)-(8a) by adopting alternative classifications in which one of the five mentioned above industries initially marked as tradable (non-tradable resp.) is treated as non tradable (tradable resp.), keeping the classification unchanged for the rest of industries. In doing so, the classification of only one industry is altered, allowing us to see if results are sensitive to the inclusion of a particular industry in the tradable or non-tradable sector. Results are shown in panel D of Table 15.

Finally, we examine whether our approach to construct series for sectoral capital stock, based on Garofalo and Yamarik [2002], could affect the main results. To do so, we rely on the EU KLEMS database which provides disaggregated capital stock data at the 1-digit level for up to 11 industries (K_GFCF), but only for eight countries of our sample for the period 1970-2007 (Denmark, Finland, Italy, Japan, the Netherlands, Spain, the United Kingdom, and the United States). ${ }^{49}$ Results are shown in panel E of Table 15. For comparison purposes, the bottom part of panel $E$ shows our estimates when exploring empirical relationships (6) for the same panel of countries (i.e., eight instead of fourteen countries) when adopting the methodology of Garofalo and Yamarik [2002] to split the national gross capital stock into traded and non traded industries by using sectoral output shares (see eq. (A.2)).

Remarkably, the cointegrating vectors estimates are robust across all these runs. For the response of the relative wage to a productivity differential of $1 \%$, the estimated coefficient of $\beta$ ranges from a low of -0.379 when the industry "Transport, Storage and Communication" moves to sector $N$ (see panel D of Table 15) to a high of -0.213 when "Financial Intermediation" is classified as non tradable (see panel D of Table 15). Across all specifications, estimates of $\beta$ are significantly different from zero. Additionally, estimates of $\gamma$, which captures the reaction of the relative price of non tradables to a productivity differential between tradables and non tradables of $1 \%$, vary from a low of 0.689 when technological change is measured with labor productivity (see Panel B of Table 15) to a high of 0.830 for the sample of eight countries providing data on capital stock data at the sectoral level (see panel E of Table 15).

[^27]Table 15: Robustness Tests: Panel FMOLS Estimates of Cointegrating Vectors

|  | Relative wage ( $\omega$ ) |  |  | Relative price ( $p$ ) |  |  | Number of countries |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | eq. (6a) | eq. (8a) |  | eq. (6b) | eq. (8b) |  |  |
|  | $\beta$ | $\beta$ | $\beta_{L}$ | $\gamma$ | $\gamma$ | $\gamma_{L}$ |  |
| A. Benchmark | $\begin{gathered} -0.270^{a} \\ (-27.83) \\ \hline \end{gathered}$ | $\begin{gathered} -0.269^{a} \\ (-30.85) \\ \hline \end{gathered}$ | $\underset{(1.80)}{0.164^{c}}$ | $\begin{aligned} & 0.779^{a} \\ & (108.91) \end{aligned}$ | $\begin{aligned} & 0.780^{a} \\ & (118.24) \end{aligned}$ | $\underbrace{0.126^{b}}_{(2.38)}$ | 14 |
| B. Technological change Labor productivity | $\frac{-0.239^{a}}{(-30.50)}$ | $\underset{(-33.61)}{-0.237^{a}}$ | $\underset{(1.62)}{0.165}$ | $\underset{(83.90)}{0.689^{a}}$ | ${ }_{(92.56)}^{0.691 a}$ | $\underset{(2.57)}{0.177^{b}}$ | 14 |
| TFP with $I^{T}$ | $\frac{-0.262^{a}}{(-27.83)}$ | $\frac{-0.262^{a}}{(-30.88)}$ | $\underset{(1.78)}{0.164^{c}}$ | $\begin{aligned} & 0.738^{a} \\ & (108.91) \end{aligned}$ | $\begin{aligned} & 0.739^{a} \\ & (118.34) \end{aligned}$ | $\underset{(2.37)}{0.127^{b}}$ | 14 |
| TFP with $I^{T} \& I^{N}$ | $\begin{gathered} -0.267^{a} \\ (-27.83) \\ \hline \end{gathered}$ | $\begin{array}{r} -0.267^{a} \\ (-30.86) \\ \hline \end{array}$ | ${ }_{(1.80)}^{0.164^{c}}$ | $\begin{aligned} & 0.763^{a} \\ & (108.91) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.764^{a} \\ & (118.27) \\ & \hline \end{aligned}$ | $\underset{(2.38)}{0.127^{b}}$ | 14 |
| C. Without Korea | $\begin{aligned} & -0.253^{a} \\ & (-25.50) \\ & \hline \end{aligned}$ | $\begin{gathered} -0.253^{a} \\ (-28.52) \\ \hline \end{gathered}$ | $0_{(2.41)}^{0.212^{a}}$ | $\begin{aligned} & 0.788^{a} \\ & (98.47) \end{aligned}$ | $\begin{aligned} & 0.790^{a} \\ & (108.86) \\ & \hline \end{aligned}$ | $\underset{(2.43)}{0.135^{b}}$ | 13 |
| D. Classification Wholesale/Retail Trade in $T$ | $\underset{(-21.05)}{-0.237^{a}}$ | ${ }_{(-23.14)}^{-0.235^{a}}$ | $0_{(3.37)}^{0.214^{a}}$ | ${ }_{(117.15)}^{0.820^{a}}$ | $\begin{aligned} & 0.818^{a} \\ & (128.91) \end{aligned}$ | $\underset{(2.79)}{0.158^{a}}$ | 14 |
| Hotels/Restaurants in $T$ | $\underset{(-17.91)}{-0.246^{a}}$ | $\underset{(-20.73)}{-0.247^{a}}$ | $\underset{(2.07)}{0.115^{b}}$ | $\underset{(87.71)}{0.788^{a}}$ | ${ }_{(97.41)}^{0.792^{a}}$ | $\underset{(1.89)}{0.129^{c}}$ | 14 |
| Transport/Communication in $N$ | $\frac{-0.379^{a}}{(-33.96)}$ | $\underset{(-39.49)}{-0.382^{a}}$ | $\underset{(2.32)}{0.155^{b}}$ | $\underset{(89.49)}{0.743^{a}}$ | $\underset{(99.43)}{0.746^{a}}$ | $\underbrace{0.099^{c}}_{(1.93)}$ | 14 |
| Financial Intermediation in $N$ | $\underset{(-24.34)}{-0.213^{a}}$ | $\underset{(-27.35)}{-0.216^{a}}$ | $\underset{(1.83)}{0.1633^{c}}$ | $\underset{(117.71)}{0.797^{a}}$ | $\begin{aligned} & 0.798^{a} \\ & (132.26) \end{aligned}$ | $\underset{(3.88)}{0.179^{a}}$ | 14 |
| Real Estate in $T$ | $\begin{array}{r} -0.370^{a} \\ (-34.65) \\ \hline \end{array}$ | $\begin{array}{r} -0.365^{a} \\ (-37.91) \\ \hline \end{array}$ | $\begin{gathered} 0.128^{c} \\ \hline \end{gathered}$ | $\begin{aligned} & 0.724^{a} \\ & (77.58) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.731^{a} \\ & (79.90) \\ & \hline \end{aligned}$ | $\underset{(3.35)}{0.227^{a}}$ | 14 |
| E. Capital stock |  |  |  |  |  |  |  |
| KLEMS data | $\underset{(-20.87)}{-0.304^{a}}$ | $\underset{(-24.49)}{-0.310^{a}}$ | $\underset{(1.33)}{0.104}$ | ${\underset{(17.13)}{0.794^{a}}}^{a}$ | $\underset{(18.12)}{0.822^{a}}$ | $\underset{(1.24)}{0.093}$ | 8 |
| Garofalo and Yamarik [2002] | $\begin{gathered} -0.249^{a} \\ (-20.25) \\ \hline \end{gathered}$ | $\begin{array}{r} -0.251^{a} \\ (-23.99) \end{array}$ | $\begin{gathered} 0.089 \\ (1.36) \end{gathered}$ | $\begin{gathered} 0.830^{a} \\ (80.86) \end{gathered}$ | $\underset{(87.27)}{0.823^{a}}$ | $\underset{(0.38)}{0.001}$ | 8 |

Notes: ${ }^{a},{ }^{b}$ and ${ }^{c}$ denote significance at $1 \%, 5 \%$ and $10 \%$ levels respectively. In benchmark specifications, we use the labor share-adjusted TFP differential $z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}$ and the labor reallocation index measuring changes in sectoral employment over two years $L R(2)$.

Table 16: Quantile regressions

| Dependent variable | Relative wage $(\omega)$ |  |  | Relative price $(p)$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left(z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}\right)$ | $\alpha=0.25$ | $\alpha=0.50$ | $\alpha=0.75$ | $\alpha=0.25$ | $\alpha=0.50$ | $\alpha=0.75$ |
|  | $-0.507^{a}$ | $-0.408^{a}$ | $-0.434^{b}$ | $0.594^{a}$ | $0.494^{a}$ | $0.392^{b}$ |
|  | $(-3.39)$ | $(-3.59)$ | $(-2.65)$ | $(5.75)$ | $(4.81)$ | 14 |
| Observations | 14 | 14 | 14 | 14 | 14 |  |
| Notes: variables are expressed in the |  |  |  |  |  |  |

Notes: variables are expressed in terms of average growth rates. All regressions include a constant. t-statistics are reported in parentheses. ${ }^{a}$ and ${ }^{b}$ denotes significance at $1 \%$ and $5 \%$ levels respectively.

Reassuringly, across all specifications, panel data estimations of $\gamma$ are significantly smaller than one. All these findings lend strong support to our results reported in Table 2. We provide more details below.

First, our estimates are robust to the measure of technological change biased toward the traded sector. Whether we measure technological change biased toward the traded sector with labor productivity, or sectoral TFPs with alternative assumptions regarding the non tradable content of investment expenditure, our main conclusions hold.

Next, when estimating cointegrating vectors by excluding Korea, our results reveal it merely moderates the decline of the relative wage from $0.27 \%$ to about $0.25 \%$ while the relative price response (captured by $\hat{\gamma}$ ) is virtually unchanged.

Another concern is related to the trend line in Figures 1(a) and 1(b). Because Korea could be viewed as an outlier in Figures 1(a) and 1(b), as a robustness check, following Koenker and Bassett [1978], we estimate the quantile regressions for different values of the quantiles, say $\alpha=0.25,0.5,0.75$ where $\alpha$ is the quantile requested. When including Korea, Figure 1(a) (Figure 1(b) resp.) reveals that there exists a clear positive (negative resp.) relation
between the average relative price (relative wage resp.) of non tradables growth and the average productivity differential between tradables and non tradables. Also reported in both Figures is a regression line. For the relative price, the slope coefficient and t-statistic are 0.64 and 6.92 , respectively, implying that the estimated coefficient is significant at the $1 \%$ level (the R-squared is 0.78 ). For the relative wage, the slope coefficient is -0.44 (with a t-statistic of -3.86 ) and a R-squared of 0.52 . Table 16 shows estimates when adopting the quantile regression procedure that allows us to check whether Korea is an outlier or not; each column gives the estimate of the slope coefficient when running the regression of the average relative price (relative wage) growth on the average productivity differential, each of which is a quantile regression using one of the above weights $\alpha$. In all regressions, our estimates of the slope coefficients are reassuringly robust (varying from -0.41 to -0.51 for the relative wage equation and from 0.39 to 0.59 for the relative price). While there are some differences across estimates, the estimated coefficients are all statistically significant and of similar magnitude to OLS estimates.

Furthermore, classification issues do not seem to drive the results. Specifically, when contrasting our estimates in panel A of Table 15 for the baseline scenario with those shown in panel D of Table 15 for alternative classifications, our main conclusions hold: the relative price of non tradables increases less than proportionately, the relative wage falls, the relative price response is larger while the relative wage reaction becomes more muted when labor mobility increases (i.e., coefficients of interaction terms $\beta_{L}$ and $\gamma_{L}$ are both positive and statistically significant, see eqs. (8)).

Finally, empirical findings related to the use of an alternative measure of sectoral capital stock series, which are shown in panel E of Table 15, suggest that our estimates obtained in the baseline regression (see panel A of Table 15) stay valid. Note that that across all specifications reported in panel E of 15 , estimates of coefficients of interaction terms, $\hat{\beta}_{L}$ and $\hat{\gamma}_{L}$, do not appear to be significant at conventional level. A possible explanation is that our sample covers only 8 countries instead of 14 . Given the shortened sample size, one should not be too demanding in terms of statistical significance for interaction term estimates.

## H Labor and Capital Adjustment Costs

In this subsection, we analyze an alternative modelling strategy to explain cross-sector differences in wages by introducing sectoral labor adjustment cost along the lines of Shi [2011]. We show that the optimal allocation rule of total hours worked to the traded and non traded sector is similar whether we produce imperfect mobility of labor by introducing a sectoral labor adjustment cost or assuming limited substitutability of hours worked across sectors along the lines of Horvath [2000], except that in a model with intersectoral adjustment cost, wages do no longer equalize in the long-run as the switching cost is in effect only along the transitional path.

We also analyze the implications of introducing imperfect capital mobility across sectors by considering sectoral capital adjustment cost along the lines of Morshed and Turnovsky [2004]. The framework with intersectoral capital adjustment costs developed by Morshed and Turnovsky [2004] is an interesting but analytically untractable model. While the authors solve numerically the model by assuming that capital does not depreciate, setting $\delta_{K}=0$ implies that the relative price of non tradables appreciates by the same amount as the rise in productivity of tradables relative to productivity of non tradables. When considering that physical capital depreciates and abstracting from the goods market clearing condition, we show that a productivity differential between tradables and non tradables of $1 \%$ produces an appreciation in the relative price of non tradables by less than $1 \%$, as in a model with imperfect labor mobility.

## H. 1 A Small Open Economy Model with Labor Adjustment Costs

Shi [2011] develops a two-sector small open economy model which can be viewed as an extension of the framework constructed by Devereux, Lane and Xu [2006]. Because we aim at comparing our two-sector model with limited substitutability of hours worked across sectors along the lines of Horvath [2000] with a two-sector setup with sectoral labor adjustment cost, we present a version of Shi's [2011] model in continuous time by abstracting from price stickiness and portfolio adjustment costs.

At each instant of time $t$, the representative agent consumes traded goods and non-traded goods denoted by $C^{T}(t)$ and $C^{N}(t)$, respectively, which are aggregated by a constant elasticity of substitution function:

$$
\begin{equation*}
C\left(C^{T}(t), C^{N}(t)\right)=\left[\varphi^{\frac{1}{\phi}}\left(C^{T}(t)\right)^{\frac{\phi-1}{\phi}}+(1-\varphi)^{\frac{1}{\phi}}\left(C^{N}(t)\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{250}
\end{equation*}
$$

The subutility function (250) is linear homogeneous which implies that total expenditure in consumption goods can be expressed as $E_{C}(t)=P_{C}(P(t)) C(t)$, with $P_{C}(P(t))$ is the unit cost function dual (or consumption-based price index) to $C(t)$. The unit cost dual function $P_{C}(P(t))$ is given by

$$
\begin{equation*}
P_{C}(P(t))=\left[\varphi+(1-\varphi) P(t)^{1-\phi}\right]^{\frac{1}{1-\phi}} . \tag{251}
\end{equation*}
$$

The agent is endowed with a unit of time and supplies a fraction $L(t)$ of this unit as labor, while the remainder, $1-L(t)$, is consumed as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Households decide on consumption and worked hours by maximizing lifetime utility:

$$
\begin{equation*}
U=\int_{0}^{\infty}\left\{\frac{1}{1-\frac{1}{\sigma_{C}}} C(t)^{1-\frac{1}{\sigma_{C}}}-\frac{1}{1+\frac{1}{\sigma_{L}}} L(t)^{1+\frac{1}{\sigma_{L}}}\right\} e^{-\beta t} \mathrm{~d} t \tag{252}
\end{equation*}
$$

where $\beta$ is the consumer's discount rate, $\sigma_{C}>0$ is the intertemporal elasticity of substitution for consumption, and $\sigma_{L}>0$ is the Frisch elasticity of labor supply.

Following Shi [2011], when the household wishes to supply more labor in sector $j$, the reallocation of employment toward sector $j$ produces an adjustment cost given by $\frac{\chi^{j}}{2}\left(L^{j}(t)-\tilde{L}^{j}\right)^{2}$ where $\tilde{L}^{j}$ corresponds to the labor supply to sector $j=T, N$ in steady-state and $\chi^{j}$ is a parameter that measures the size of the switching cost. Factor income is derived by supplying labor $L^{j}(t)$ at a wage rate $W^{j}(t)$, and capital $K(t)$ at a rental rate $R(t)$. For simplicity purpose, we assume that labor adjustment costs are supported by the traded sector. In addition, households accumulate internationally traded bonds, $B(t)$, that yield net interest rate earnings of $r^{\star} B(t)$. The households' flow budget constraint can be written as:

$$
\begin{align*}
\dot{B}(t)= & r^{\star} B(t)+R(t) K(t)+W^{T}(t) L^{T}(t)+W^{N}(t) L^{N}(t)-P_{C}(P(t)) C(t)-P(t) I(t) \\
& -\frac{\chi^{T}}{2}\left(L^{T}(t)-\tilde{L}^{T}\right)^{2}-\frac{\chi^{N}}{2}\left(L^{N}(t)-\tilde{L}^{N}\right)^{2} \tag{253}
\end{align*}
$$

where $P(t) I(t)$ corresponds to investment expenditure. Aggregate investment gives rise to overall capital accumulation according to the dynamic equation:

$$
\begin{equation*}
\dot{K}(t)=I(t)-\delta_{K} K(t), \tag{254}
\end{equation*}
$$

where $0 \leq \delta_{K}<1$ is a fixed depreciation rate. Aggregating labor and capital over the two sectors gives us the resource constraints:

$$
\begin{equation*}
L^{T}(t)+L^{N}(t)=L(t), \quad k^{T}(t) L^{T}(t)+k^{N}(t) L^{N}(t)=K(t) . \tag{255}
\end{equation*}
$$

For the sake of clarity, we drop the time argument below when this causes no confusion.
Households choose consumption, worked hours and investment in physical capital by maximizing lifetime utility (252) subject to (253) and (254). Denoting by $\lambda$ and $\psi$ the co-state variables associated with (253) and (254), the first-order conditions characterizing the representative household's optimal plans are:

$$
\begin{gather*}
C=\left(P_{C} \lambda\right)^{-\sigma_{C}},  \tag{256a}\\
L^{1 / \sigma_{L}}=\lambda\left[W^{T}-\chi^{T}\left(L^{T}-\tilde{L}^{T}\right)\right],  \tag{256b}\\
L^{1 / \sigma_{L}}=\lambda\left[W^{N}-\chi^{N}\left(L^{N}-\tilde{L}^{N}\right)\right],  \tag{256c}\\
\dot{\lambda}=\lambda\left(\beta-r^{\star}\right),  \tag{256d}\\
R / P-\delta_{K}+\dot{P} / P=r^{\star}, \tag{256e}
\end{gather*}
$$

and the transversality conditions $\lim _{t \rightarrow \infty} \lambda B(t) e^{-\beta t}=0, \lim _{t \rightarrow \infty} \psi(t) K(t) e^{-\beta t}=0 .{ }^{50}$
Combining (256b) and (256c), we find that the sectoral wage differential depends on the relative size of mobility cost:

$$
\begin{equation*}
W^{N}-W^{T}=\chi^{N}\left(L^{N}-\tilde{L}^{N}\right)-\chi^{T}\left(L^{T}-\tilde{L}^{T}\right) . \tag{257}
\end{equation*}
$$

In order to compare the optimal rule for the intra-temporal allocation of hours worked across sectors when workers experience a utility loss when switching from one sector to another with that when considering labor adjustment costs, it is useful to log-linearize (16) shown in the main text:

$$
\left(\frac{1-\vartheta}{\vartheta}\right) \frac{L^{T}}{L^{N}}=\Omega^{-\epsilon} .
$$

Denoting the percentage deviation from its initial steady-state by a hat, eq. (16) can be written as follows:

$$
\begin{equation*}
\left[\hat{L}^{N}-\hat{L}^{T}\right]=\epsilon\left[\hat{W}^{N}-\hat{W}^{T}\right], \tag{258}
\end{equation*}
$$

where $\epsilon$ captures the extent to which workers wish to shift hours worked between sectors. In order to derive a similar equation to (258), we totally differentiate (257), divide the LHS and the RHS by the initial steady-state value of aggregate wage $\tilde{L}$ :

$$
\frac{\mathrm{d} L^{N}}{\tilde{L}}-\frac{\mathrm{d} L^{T}}{\tilde{L}}=\Gamma\left[\frac{\mathrm{d} W^{N}}{\tilde{W}}-\frac{\mathrm{d} W^{T}}{\tilde{W}}\right],
$$

where we denote the share of the aggregate wage in switching costs $\frac{\tilde{W}}{\chi \tilde{L}}$ by $\Gamma$ and assume that the intersectoral adjustment cost parameter $\chi^{j}$ is identical across sectors, i.e., $\chi^{j}=\chi$. Denoting by the percentage deviation from initial steady-state by a hat, the above equation can be rewritten ad follows:

$$
\begin{equation*}
\left[\hat{L}^{N}-\hat{L}^{T}\right]=\Gamma\left[\hat{W}^{N}-\hat{W}^{T}\right] . \tag{259}
\end{equation*}
$$

When $\Gamma$ tends toward infinity, the mobility cost are so small that the sectoral wage differential is eliminated, as in the Horvath's[2000] model. In the calibration of the model, Shi [2011] sets $\chi=\frac{\tilde{W}}{\tilde{L}} \frac{1}{\Gamma}$ to match estimates of the elasticity $\epsilon$.

The major difference between a model with imperfect mobility of labor across sectors along the lines of Horvath's [2000] and a model with sectoral labor adjustment costs is that in the former case, the wage differential is persistent in the long-run while the discrepancy between sectoral wages is eliminated in the long-run in the latter case. Because unit root tests suggest a persistent wage differential across sectors in the long-run, it is appropriate to consider the model with limited substitutability of hours worked across sectors along the lines of Horvath [2000].

[^28]
## H. 2 A Small Open Economy Model with Capital Adjustment Costs

Following Morshed and Turnovsky [2004], we develop a two-sector model in which intersectoral capital movements involve adjustment costs, expressed as capital lost in the transformation process. For clarity purposes, we abstract from labor and rather assume that traded and non traded goods are produced by firms using capital as the sole input in a linear (constant returns to scale) technology:

$$
\begin{equation*}
Y^{T}(t)=A^{T} K^{T}(t), \quad Y^{N}(t)=A^{N} K^{N}(t) \tag{260}
\end{equation*}
$$

The output of the non traded good $\left(Y^{N}\right)$ can be used for consumption $\left(C^{N}\right)$ and for investment $\left(I^{N}\right)$. The output of the traded good $\left(Y^{T}\right)$ can be consumed $\left(C^{T}\right)$ or exported $\left(Y^{T}-C^{T}\right)$.

Factor income is derived by supplying capital $K^{j}(t)$ in sector $j=T, N$ at a rental rate $R^{j}(t)$. In addition, households accumulate internationally traded bonds, $B(t)$, that yield net interest rate earnings of $r^{\star} B(t)$. The households' flow budget constraint can be written as:

$$
\begin{equation*}
\dot{B}(t)=r^{\star} B(t)+R^{T}(t) K^{T}(t)+R^{N}(t) K^{N}(t)-P_{C}(P(t)) C(t)-P(P(t)) I(t) \tag{261}
\end{equation*}
$$

where $P_{C}(P(t))$ is the consumption-based price index and $P(t) I(t)$ corresponds to investment expenditure.

Following Morshed and Turnovsky [2004], only non-traded new output can be converted into capital, and once it becomes capital good in the nontraded sector, it takes extra resources to transform it into capital suitable for use in the traded sector. Accordingly, capital accumulation in this economy is described by:

$$
\begin{align*}
\dot{K}^{T}(t) & =X(t)-\delta_{K} K^{T}(t)  \tag{262a}\\
\dot{K}^{N}(t) & =I(t)-X(t)\left(1+\frac{h X(t)}{2 K^{N}(t)}\right)-\delta_{K} K^{N}(t) \tag{262b}
\end{align*}
$$

where $0 \leq \delta_{K}<1$ is a fixed depreciation rate. Summing (262a) and (262b), we find that aggregate investment gives rise to overall capital accumulation according to the following dynamic equation:

$$
\begin{equation*}
\dot{K}(t)=I(t)-\frac{h(X(t))^{2}}{2 K^{N}(t)}-\delta_{K}\left(K^{T}(t)+K^{N}(t)\right) \tag{263}
\end{equation*}
$$

where $\frac{h(X(t))^{2}}{2 K^{N}(t)}>0$ represents the intersectoral adjustment costs.
Abstracting from labor supply, the representative household maximizes the following objective function:

$$
\begin{equation*}
U=\int_{0}^{\infty}\left\{\frac{1}{1-\frac{1}{\sigma_{C}}} C(t)^{1-\frac{1}{\sigma_{C}}}\right\} e^{-\beta t} \mathrm{~d} t \tag{264}
\end{equation*}
$$

where $\beta$ is the discount rate, $\sigma_{C}>0$ corresponds to the intertemporal elasticity of substitution for consumption.

Households choose consumption, investment in physical capital, the capital allocation decisions, $K^{T}(t)$ and $K^{N}(t)$, and the rate of accumulation of traded bonds by maximizing lifetime utility (264) subject to (261), (262a), (263). Denoting by $\lambda(t), \psi^{T, \prime}(t)$, and $\psi^{N, \prime}(t)$, the co-state variables associated with (261), (262a), (263), the first-order conditions characterizing
the representative household's optimal plans are:

$$
\begin{gather*}
C(t)=\left(P_{C}(t) \lambda(t)\right)^{-\sigma_{C}}  \tag{265a}\\
\psi^{N}(t)=P(t),  \tag{265b}\\
\psi^{T}(t)=\psi^{N}(t)\left(1+\frac{h X(t)}{K^{N}(t)}\right),  \tag{265c}\\
\dot{\lambda}(t)=\lambda(t)\left(\beta-r^{\star}\right),  \tag{265d}\\
\dot{\psi}^{T}(t)=-R^{T}(t)+\left(r^{\star}+\delta_{K}\right) \psi^{T}(t),  \tag{265e}\\
\dot{\psi}^{N}(t)=-R^{N}(t)-\psi^{N}(t) \frac{h}{2}\left(\frac{X(t)}{K^{N}(t)}\right)^{2}+\left(r^{\star}+\delta_{K}\right) \psi^{N}(t), \tag{265f}
\end{gather*}
$$

and the transversality conditions $\lim _{t \rightarrow \infty} \lambda B(t) e^{-\beta t}=0, \lim _{t \rightarrow \infty} \lambda \psi^{T}(t) K^{T}(t) e^{-\beta t}=0$, $\lim _{t \rightarrow \infty} \lambda \psi^{N}(t) K^{N}(t) e^{-\beta t}=0$ where we denote by $\psi^{j} \equiv \psi^{j, \prime} / \lambda$ with $j=T, N$, and $\lambda(t) \equiv \bar{\lambda}$. Using (265b), eq. (265f) states that that that the relative price of non tradables equalize the rate of return on domestic capital to the rate of return on traded bonds:

$$
\frac{R^{N}}{P(t)}+\frac{h}{2}\left(\frac{X(t)}{K^{N}(t)}\right)^{2}+\frac{\dot{P}(t)}{P(t)}-\delta_{K}=r^{\star}
$$

where $R^{N}=P \frac{\partial Y^{N}}{\partial K^{N}}=P A^{N}$. We suppress the time index below.
Denoting the long-run values with a tilde, the steady-state is given by:

$$
\begin{gather*}
A^{N} \tilde{K}^{N}=P_{C}^{\prime} \tilde{C}+\tilde{I},  \tag{266a}\\
\tilde{X}=\delta_{K} \tilde{K}^{T},  \tag{266b}\\
\tilde{I}=\frac{h \tilde{X}^{2}}{2 \tilde{K}^{N}}-\delta_{K}\left(\tilde{K}^{T}+\tilde{K}^{N}\right),  \tag{266c}\\
r^{\star} \tilde{B}+A^{T} \tilde{K}^{T}=\left(1-\alpha_{C}\right) P_{C} \tilde{C}  \tag{266d}\\
\tilde{R}^{T}=\left(r^{\star}+\delta_{K}\right) \tilde{\psi}^{T},  \tag{266e}\\
\tilde{R}^{N}=\left[\left(r^{\star}+\delta_{K}\right)-\frac{h}{2}\left(\frac{\tilde{X}}{\tilde{K}^{N}}\right)^{2}\right] \tilde{\psi}^{N},  \tag{266f}\\
\tilde{\psi}^{T}=\tilde{\psi}^{N}\left(1+\frac{h \tilde{X}}{\tilde{K}^{N}}\right),  \tag{266~g}\\
\left.\tilde{B}-B_{0}\right)=\Phi^{T}\left(\tilde{K}^{T}-K_{0}^{T}\right)+\Phi^{N}\left(\tilde{K}^{N}-K_{0}^{N}\right) . \tag{266h}
\end{gather*}
$$

where $\tilde{C}=\left(P_{C} \bar{\lambda}\right)^{-\sigma_{C}}$ and (266i) is the (linearized version of the) intertemporal solvency condition with $K_{0}^{j}$ the initial (predetermined) initial capital stock in sector $j=T, N$.

Substituting first $\tilde{X}=\delta_{K} \tilde{K}^{T}$ into (266f) and (266g), denoting by $\tilde{k} \equiv \tilde{K}^{T} / \tilde{K}^{N}$ the steadystate ratio of the traded capital stock to the non traded capital stock, dividing (266f) by (266e) and eliminating $\tilde{\psi}^{N} / \tilde{\psi}^{T}$ by using $(266 \mathrm{~g})$, we get:

$$
\begin{equation*}
\tilde{P}=\frac{A^{T}}{A^{N}} \frac{\left[\left(r^{\star}+\delta_{K}\right)-\frac{h}{2}\left(\delta_{K}\right)^{2}(\tilde{k})^{2}\right]}{\left(r^{\star}+\delta_{K}\right)\left(1+h \delta_{K} \tilde{k}\right)} \tag{267}
\end{equation*}
$$

where we used the fact that $\tilde{R}^{T}=A^{T}$ and $\tilde{R}^{N}=\tilde{P} A^{N}$. If the capital stock does not depreciate as in Morshed and Turnovsky [2004], i.e., setting $\delta_{K}=0$, we have $\tilde{P}=\frac{A^{T}}{A^{N}}$; the reason is that
capital reallocation ceases in the long-run so that intersectoral adjustment costs do no longer influence the relative price in the long-run. When assuming that capital depreciates, eq. (267) reveals that the rise in $\tilde{k}$ moderates the appreciation in the relative price of non-tradables, and more so as the parameter $h$ in the adjustment cost function increases. Hence, intersectoral adjustment costs moderate the relative price appreciation following a productivity differential between tradables and non tradables by increasing $\tilde{R}^{T}$ relative to $\tilde{R}^{N}$. Intuitively, investors ask for higher return when capital shifts toward the traded sector in order to cover intersectoral adjustment costs.

## I Imperfect Capital Mobility across Sectors

The framework with intersectoral capital adjustment costs developed by Morshed and Turnovsky [2004] is an interesting but analytically untractable model. While the authors solve numerically the model by assuming that capital does not depreciate, setting $\delta_{K}=0$ into (267) implies that the relative price of non tradables appreciates by the same amount as the rise in productivity of tradables relative to productivity of non tradables. When introducing capital depreciation, as highlighted by eq. (267), the relative price appreciates by less than the productivity differential. Yet, eq. (267) has been determined by abstracting from the goods market equilibrium which implies that sectoral output changes trigger an adjustment in the relative price of non tradables in order to clear the goods market. Instead of considering intersectoral capital adjustment costs, we show below that introducing imperfect substitutability of capital across sectors along the lines of Horvath [2000], can produce similar results to those obtained in a model assuming intersectoral capital adjustment costs and has the advantage to be analytically tractable. This section presents the formal analysis when introducing imperfect mobility of capital across sectors while abstracting from labor to avoid unnecessary complications.

## I. 1 Households

The representative agent consumes traded goods and non-traded goods denoted by $C^{T}(t)$ and $C^{N}(t)$, respectively, which are aggregated by a constant elasticity of substitution function:

$$
\begin{equation*}
C\left(C^{T}(t), C^{N}(t)\right)=\left[\varphi^{\frac{1}{\phi}}\left(C^{T}(t)\right)^{\frac{\phi-1}{\phi}}+(1-\varphi)^{\frac{1}{\phi}}\left(C^{N}(t)\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{268}
\end{equation*}
$$

At any instant of time $t$, households derive utility from their consumption. The representative household maximizes the following objective function:

$$
\begin{equation*}
U=\int_{0}^{\infty}\left\{\frac{1}{1-\frac{1}{\sigma_{C}}} C(t)^{1-\frac{1}{\sigma_{C}}}\right\} e^{-\beta t} \mathrm{~d} t \tag{269}
\end{equation*}
$$

where $\beta$ is the discount rate, $\sigma_{C}>0$ corresponds to the intertemporal elasticity of substitution for consumption.

Factor income is derived by supplying capital $K(t)$ at a rental rate $R(t)$. In addition, households accumulate internationally traded bonds, $B(t)$, that yield net interest rate earnings of $r^{\star} B(t)$. The households' flow budget constraint can be written as:

$$
\begin{equation*}
\dot{B}(t)=r^{\star} B(t)+R\left(R^{T}(t), R^{N}(t)\right) K(t)-P_{C}(P(t)) C(t)-P(t) I(t), \tag{270}
\end{equation*}
$$

where $P(t) I(t)$ corresponds to investment expenditure and the consumption-based price index $P_{C}($.$) is increasing with the relative price of non tradables P(t)$. Aggregate investment gives rise to overall capital accumulation according to the dynamic equation:

$$
\begin{equation*}
\dot{K}(t)=I(t)-\delta_{K} K(t), \tag{271}
\end{equation*}
$$

where $0 \leq \delta_{K}<1$ is a fixed depreciation rate. For the sake of clarity, we drop the time argument below when this causes no confusion.

The representative household supplies capital $K^{T}$ and $K^{N}$ in the traded and non traded sectors, respectively. The standard BS model assumes that sectoral capital goods are perfect substitutes. Because agents are willing to devote their whole investment expenditure to the sector that pays the highest return, sectors pay the same return, i.e., $R^{T}=R^{N}=R$. A shortcut to produce imperfect capital mobility is to introduce limited substitutability in capital across sectors; along the lines of Horvath [2000] who introduce limited substitutability of hours worked, we assume that capital in the traded and the non traded sectors are aggregated by means of a CES function:

$$
\begin{equation*}
K\left(K^{T}, K^{N}\right)=\left[\vartheta^{-\frac{1}{\epsilon}}\left(K^{T}\right)^{\frac{\epsilon+1}{\epsilon}}+(1-\vartheta)^{-\frac{1}{\epsilon}}\left(K^{N}\right)^{\frac{\epsilon+1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon+1}} \tag{272}
\end{equation*}
$$

where $0<\vartheta<1$ is the fraction of aggregate capital supplied in the traded sector and $\epsilon$ measures the ease with which capital can be substituted for each other and thereby captures the degree of capital mobility. The case of perfect capital mobility is nested under the assumption that $\epsilon$ tends towards infinity; in this case, (272) reduces to $K=K^{T}+K^{N}$ which implies that capital is perfectly substitutable across sectors. When $\epsilon<\infty$, sectoral capital goods are no longer perfect substitutes. More specifically, as $\epsilon$ becomes smaller, capital mobility across sectors becomes lower as investors perceive a higher cost of shifting capital and therefore become more reluctant to reallocate capital across sectors. The aggregate capital rental rate index $R($.$) associated with the above defined capital index (272) is:$

$$
\begin{equation*}
R=\left[\vartheta\left(R^{T}\right)^{\epsilon+1}+(1-\vartheta)\left(R^{N}\right)^{\epsilon+1}\right]^{\frac{1}{\epsilon+1}}, \tag{273}
\end{equation*}
$$

where $R^{T}$ and $R^{N}$ are capital rental rates paid in the traded and the non traded sectors, respectively.

Households choose consumption, worked hours and investment in physical capital by maximizing lifetime utility (269) subject to (270) and (271). Denoting by $\lambda$ and $\psi$ the co-state variables associated with (270) and (271), the first-order conditions characterizing the representative household's optimal plans are:

$$
\begin{gather*}
C=\left(P_{C} \lambda\right)^{-\sigma_{C}},  \tag{274a}\\
\dot{\lambda}=\lambda\left(\beta-r^{\star}\right),  \tag{274b}\\
R / P-\delta_{K}+\dot{P} / P=r^{\star}, \tag{274c}
\end{gather*}
$$

and the transversality conditions $\lim _{t \rightarrow \infty} \lambda B(t) e^{-\beta t}=0, \lim _{t \rightarrow \infty} \psi(t) K(t) e^{-\beta t}=0 .{ }^{51}$
Applying Shephard's lemma (or the envelope theorem) yields expenditure in tradables and non tradables, i.e., $C^{N}=\frac{\partial P_{C}}{\partial P} C$, and $C^{T}=P_{C} C-P P_{C}^{\prime} C$. Intra-temporal allocation of consumption follows from the following optimal rule:

$$
\begin{equation*}
\left(\frac{1-\varphi}{\varphi}\right) \frac{C^{T}}{C^{N}}=P^{\phi}, \tag{275}
\end{equation*}
$$

where $\alpha_{C}$ is the share of non traded goods in consumption expenditure. ${ }^{52}$ An appreciation in the relative price of non tradables $P$ increases expenditure on tradables relative to expenditure on non tradables (i.e. $C^{T} / P C^{N}$ ), only when $\phi>1$.

As for consumption, intra-temporal allocation of capital across sectors follows from Shephard's Lemma. We therefore obtain capital income from supplying capital in the non traded

[^29]and the traded sectors, i.e. $R^{N} K^{N}=\alpha_{K} R K$ and $R^{T} K^{T}=\left(1-\alpha_{K}\right) R K$, with $\alpha_{K}$ being the share of non-tradable capital revenue in total capital income. ${ }^{53}$ Denoting by $\Gamma \equiv R^{N} / R^{T}$ the relative capital income, investors allocate capital in the traded and the non traded sectors according to the following optimal rule:
\[

$$
\begin{equation*}
\left(\frac{1-\vartheta}{\vartheta}\right) \frac{K^{T}}{K^{N}}=\Gamma^{-\epsilon} . \tag{276}
\end{equation*}
$$

\]

If the traded sector pays higher capital income (i.e., if $\Gamma$ falls) investors are induced to shift capital towards the traded sector, but less so as $\epsilon$ is lower.

## I. 2 Firms

A large number of identical and perfectly competitive firms produces a traded and a non traded good using capital as the sole input in a linear (constant returns to scale) technology:

$$
\begin{equation*}
Y^{T}=A^{T} K^{T}, \quad Y^{N}=A^{N} K^{N}, \tag{277}
\end{equation*}
$$

where $A^{T}$ and $A^{N}$ are capital productivity index in the traded and non traded sector, respectively. Since the capital market is assumed to be competitive, capital is paid its marginal products:

$$
\begin{equation*}
A^{T}=R^{T}, \quad \text { and } \quad P A^{N}=R^{N} \tag{278}
\end{equation*}
$$

Hence the relative price of non tradables must equalize with the capital unit cost in the non traded sector relative to the traded sector:

$$
\begin{equation*}
P=\Gamma \frac{A^{T}}{A^{N}}, \quad \Omega \equiv \frac{R^{N}}{R^{T}} . \tag{279}
\end{equation*}
$$

Relaxing the perfect capital mobility assumption implies that a sectoral capital income discrepancy captured by $\Gamma$ also influences the price of non tradables in terms of tradables.

## I. 3 Model Closure and Equilibrium

To fully describe the equilibrium, we impose two good market clearing conditions. The non traded good market clearing condition requires that non traded output is equalized with consumption in non-tradables and investment:

$$
\begin{equation*}
Y^{N}=C^{N}+I, \quad I=\dot{K}+\delta_{K} K \tag{280}
\end{equation*}
$$

Plugging this condition into the flow budget constraint (270) and substituting (278) yields the market clearing condition for tradables or the current account dynamic equation:

$$
\begin{equation*}
\dot{B}=r^{\star} B+Y^{T}-C^{T}, \tag{281}
\end{equation*}
$$

where the second term on the RHS, i.e., $Y^{T}-C^{T}$, corresponds to net exports .

## I. 4 Solving the Model

## Short-Run Static Solutions

Intratemporal allocation of capital between the traded and the non traded sector follows from Shephard's Lemma (or the envelope theorem):

$$
\begin{align*}
K^{T} & =\frac{\partial R}{\partial R^{T}} K=R_{T} L, \quad \text { and } \quad \frac{R^{T} K^{T}}{R K}=1-\alpha_{K},  \tag{282a}\\
K^{N} & =\frac{\partial R}{\partial R^{N}} K=R_{N} K, \quad \text { and } \quad \frac{R^{N} K^{N}}{R K}=\alpha_{K},  \tag{282b}\\
{ }^{53} \text { Specifically, we have } \alpha_{K} & =\frac{(1-\vartheta)\left(K^{N}\right)^{\epsilon+1}}{\left[\vartheta\left(K^{T}\right)^{c+1}+(1-\vartheta)\left(K^{N}\right)^{c+1}\right]} .
\end{align*}
$$

where

$$
\begin{align*}
\frac{\partial R}{\partial R^{T}} \equiv R_{T} & =\vartheta\left(R^{T}\right)^{\epsilon} R^{-\epsilon}  \tag{283a}\\
\frac{\partial R}{\partial R^{N}} \equiv R_{N} & =(1-\vartheta)\left(R^{N}\right)^{\epsilon} R^{-\epsilon} \tag{283b}
\end{align*}
$$

with the aggregate capital rental rate index $R$ is given by (273).
We write out some useful properties:

$$
\begin{align*}
\frac{\partial R}{\partial R^{T}} \frac{R^{T}}{R} & =\left(1-\alpha_{K}\right), \quad \frac{\partial R}{\partial R^{N}} \frac{R^{N}}{R}=\alpha_{L},  \tag{284a}\\
\frac{\partial R_{T}}{\partial R^{T}} & =\frac{\partial^{2} R}{\partial\left(R^{T}\right)^{2}}=\vartheta \epsilon\left(R^{T}\right)^{\epsilon-1} R^{-\epsilon} \alpha_{L},  \tag{284b}\\
\frac{\partial R_{T}}{\partial R^{T}} \frac{R^{T}}{R_{T}} & =\epsilon \alpha_{K}>0,  \tag{284c}\\
\frac{\partial R_{T}}{\partial R^{N}} \frac{R^{N}}{R_{T}} & =-\epsilon \alpha_{K}<0,  \tag{284d}\\
\frac{\partial R_{N}}{\partial R^{N}} \frac{R^{N}}{R} & =\epsilon\left(1-\alpha_{K}\right)>0,  \tag{284e}\\
\frac{\partial R_{N}}{\partial R^{T}} \frac{R^{T}}{R} & =-\epsilon\left(1-\alpha_{K}\right)<0, \tag{284f}
\end{align*}
$$

where $R_{j}=\frac{\partial R}{\partial R^{j}}($ with $j=T, N)$.
We compute short-run static solution for consumption supply. Static efficiency conditions (274a) can be solved for consumption which of course must hold at any point of time:

$$
\begin{equation*}
C=C(\bar{\lambda}, P), \tag{285}
\end{equation*}
$$

with

$$
\begin{align*}
C_{\bar{\lambda}} & =\frac{\partial C}{\partial \bar{\lambda}}=-\sigma_{C} \frac{C}{\bar{\lambda}}<0  \tag{286a}\\
C_{P} & =\frac{\partial C}{\partial P}=-\alpha_{C} \sigma_{C} \frac{C}{P}<0 \tag{286b}
\end{align*}
$$

where $\sigma_{C}$ corresponds to the intertemporal elasticity of substitution for consumption.
Intratemporal allocation between non tradable goods and tradable goods follows from Shephard's Lemma (or the envelope theorem):

$$
\begin{align*}
C^{N} & =P_{C}^{\prime} C=(1-\varphi)\left(\frac{P}{P_{C}}\right)^{-\phi} C, \quad \text { and } \quad \frac{P C^{N}}{P_{C} C}=\alpha_{C}  \tag{287a}\\
C^{T} & =\left[P_{C}-P P_{C}^{\prime}\right] C=\varphi\left(\frac{1}{P_{C}}\right)^{-\phi} C, \quad \text { and } \quad \frac{C^{T}}{P_{C} C}=\left(1-\alpha_{C}\right), \tag{287b}
\end{align*}
$$

where the non tradable and tradable shares in total consumption expenditure are:

$$
\begin{align*}
\alpha_{C} & =\frac{(1-\varphi) P^{1-\phi}}{\varphi+(1-\varphi) P^{1-\phi}},  \tag{288a}\\
1-\alpha_{C} & =\frac{\varphi}{\varphi+(1-\varphi) P^{1-\phi}} \tag{288b}
\end{align*}
$$

Inserting first the short-run solution for consumption (286), (287) can be solved for $C^{T}$ and $C^{N}$ :

$$
\begin{equation*}
C^{T}=C^{T}(\bar{\lambda}, P), \quad C^{N}=C^{N}(\bar{\lambda}, P), \tag{289}
\end{equation*}
$$

where partial derivatives are

$$
\begin{align*}
C_{\bar{\lambda}}^{T} & =-\sigma_{C} \frac{C^{T}}{\bar{\lambda}}<0  \tag{290a}\\
C_{P}^{T} & =\alpha_{C} \frac{C^{T}}{P}\left(\phi-\sigma_{C}\right) \lessgtr 0,  \tag{290b}\\
C_{\bar{\lambda}}^{N} & =-\sigma_{C} \frac{C^{N}}{\bar{\lambda}}<0,  \tag{290c}\\
C_{P}^{N} & =-\frac{C^{N}}{P}\left[\left(1-\alpha_{C}\right) \phi+\alpha_{C} \sigma_{C}\right]<0, \tag{290d}
\end{align*}
$$

where we used the fact that $-\frac{P_{C}^{\prime \prime} P}{P_{C}^{\prime}}=\phi\left(1-\alpha_{C}\right)>0$ and $P_{C}^{\prime} C=C^{N}$.
Inserting first the short-run solution $R=R\left(R^{T}, R^{N}\right)$ into $K^{T}=\frac{\partial R\left(R^{T}, R^{N}\right)}{\partial R^{T}} K$ and $K^{N}=$ $\frac{\partial R\left(R^{T}, R^{N}\right)}{\partial R^{N}} K$, we are able to solve for $K^{T}$ and $K^{N}$ :

$$
\begin{equation*}
K^{T}=K^{T}\left(R^{T}, R^{N}, K\right), \quad K^{N}=K^{N}\left(R^{T}, R^{N}, K\right) \tag{291}
\end{equation*}
$$

where partial derivatives are

$$
\begin{align*}
K_{K}^{T} & =\frac{\partial K^{T}}{\partial K}=\frac{\partial R}{\partial R^{T}}>0  \tag{292a}\\
K_{R^{T}}^{T} & =\frac{\partial K^{T}}{\partial R^{T}}=\frac{K^{T}}{R^{T}} \epsilon \alpha_{K}>0  \tag{292b}\\
K_{R^{N}}^{T} & =\frac{\partial K^{T}}{\partial R^{N}}=-\frac{K^{T}}{R^{N}} \alpha_{K} \epsilon<0  \tag{292c}\\
K_{K}^{N} & =\frac{\partial K^{N}}{\partial K}=\frac{\partial R}{\partial R^{N}}>0  \tag{292d}\\
K_{R^{N}}^{N} & =\frac{\partial K^{N}}{\partial R^{N}}=\frac{K^{N}}{R^{N}} \epsilon\left(1-\alpha_{K}\right)>0  \tag{292e}\\
K_{R^{T}}^{N} & =\frac{\partial K^{N}}{\partial R^{T}}=-\frac{K^{N}}{R^{T}}\left(1-\alpha_{K}\right) \epsilon<0 \tag{292f}
\end{align*}
$$

where we used the fact that $\frac{R_{T T} R^{T}}{R_{T}}=\epsilon \alpha_{K}, \frac{R_{T N} R^{N}}{R_{T}}=-\epsilon \alpha_{K}, \frac{R_{N N} R^{N}}{R_{N}}=\epsilon\left(1-\alpha_{K}\right), \frac{R_{N T} R^{T}}{R_{N}}=$ $-\epsilon\left(1-\alpha_{K}\right)$.

Inserting first-order conditions (278), we can solve for the aggregate capital rental rate index $R=R\left(R^{T}, R^{N}\right)$ as follows:

$$
\begin{equation*}
R=R\left(A^{T}, A^{N}, P\right), \tag{293}
\end{equation*}
$$

where

$$
\begin{equation*}
\frac{\partial R}{\partial P} \frac{P}{R^{N}}=A^{N} \frac{P}{R^{N}}=1 \tag{294}
\end{equation*}
$$

## Equilibrium Dynamics

Inserting the short-run static solutions for consumption in non tradables (289) and non traded capital (291) into the physical capital accumulation equation (280) and the short-run static solution for the aggregate capital rental rate index (293) into the dynamic equation for the relative price of non tradables (274c), the dynamic system is:

$$
\begin{align*}
\dot{K} & =A^{N} K^{N}\left[R^{T}\left(A^{T}\right), R^{N}\left(A^{N}, P\right), K\right]-C^{N}(\bar{\lambda}, P)-\delta_{K} K,  \tag{295a}\\
\dot{P} & =P\left[r^{\star}+\delta-\frac{R\left(A^{T}, A^{N}, P\right)}{P}\right] . \tag{295b}
\end{align*}
$$

Denoting with a tilde long-run values, linearizing (295a) around the steady-state yields:

$$
\begin{aligned}
\dot{K}= & {\left[\frac{A^{N} \tilde{K}^{N}}{\tilde{P}} \frac{\partial K^{N}}{\partial R^{N}} \frac{\tilde{R}^{N}}{\tilde{K}^{N}} \frac{\partial R^{N}}{\partial P} \frac{\tilde{P}}{\tilde{R}^{N}}-C_{P}^{N}\right](P(t)-\tilde{P}) } \\
& +\left(A^{N} \frac{K^{N}}{\partial K}-\delta_{K}\right)(K(t)-\tilde{K}) .
\end{aligned}
$$

and $\frac{\partial K^{N}}{\partial R^{N}} R^{N}=\epsilon\left(1-\alpha_{K}\right), C_{P}^{N}$ is given by (290d), $A^{N} \frac{K^{N}}{\partial K}=\frac{A^{N}}{\tilde{K}} \frac{K^{N}}{\partial K} \tilde{K}=\frac{A^{N} \tilde{K}^{N}}{\tilde{K}}=\frac{\tilde{R}^{N} \tilde{K}^{N}}{\tilde{R} \tilde{K}} \frac{\tilde{R}}{\tilde{P}}=$ $\alpha_{K}\left(\delta_{K}+r^{\star}\right)$.

Linearizing (295a) around the steady-state yields:

$$
\dot{P}=-\frac{\tilde{R}}{\tilde{P}}\left(1-\alpha_{K}\right)(P(t)-\tilde{P}),
$$

where $\frac{\tilde{R}}{\tilde{P}}=\left(\delta_{K}+r^{\star}\right)$.
In matrix form, we get:

$$
\binom{\dot{K}(t)}{\dot{P}(t)}=\left(\begin{array}{cc}
\alpha_{K}\left(\delta_{K}+r^{\star}\right)-\delta_{K} & a_{12}  \tag{296}\\
0 & \left(\delta_{K}+r^{\star}\right)\left(1-\alpha_{K}\right)
\end{array}\right)\binom{K(t)-\tilde{K}}{P(t)-\tilde{P}},
$$

where

$$
\begin{equation*}
a_{12}=\frac{\tilde{Y}^{N}}{\tilde{P}}\left\{\epsilon\left(1-\alpha_{K}\right)+\frac{\alpha_{C} \omega_{C}}{\omega_{N}}\left[\left(1-\alpha_{C}\right) \phi+\alpha_{C} \sigma_{C}\right]\right\}>0, \tag{297}
\end{equation*}
$$

with $\frac{P C^{N}}{P Y^{N}}=\frac{P C^{N}}{P_{C} C} \frac{P_{C} C}{Y} \frac{Y}{P Y^{N}}=\frac{\alpha_{C} \omega_{C}}{\omega_{N}}$ and $\omega_{N}=\frac{P Y^{N}}{Y}, \frac{P_{C} C}{Y}=\omega_{C}$.
Using the fact that the Trace is the sum of the diagonal elements, we find that the trace of the Jacobian matrix denoted by $\operatorname{Tr} \mathrm{J}$ is:

$$
\begin{equation*}
\operatorname{Tr} \mathrm{J}=r^{\star}>0 . \tag{298}
\end{equation*}
$$

The determinant of the Jacobian matrix denoted by Det J is:

$$
\begin{equation*}
\text { Det } \mathrm{J}=\left[\alpha_{K}\left(\delta_{K}+r^{\star}\right)-\delta_{K}\right]\left(\delta_{K}+r^{\star}\right)\left(1-\alpha_{K}\right) \tag{299}
\end{equation*}
$$

Saddle-path stability requires that $\left(1-\alpha_{K}\right) \delta_{K}>\alpha_{K} r^{\star}$ which can be rewritten as $A^{N} \tilde{K}^{N}=$ $\tilde{Y}^{N}<\delta_{K} \tilde{K}=\tilde{I}$. Since such a condition is not consistent with the market clearing condition for the non traded good (280), in order to eliminate unstable paths, both the relative price and the capital stock are jump variables which must adjust instantaneously to their steady-state:

$$
\begin{equation*}
P(t)=\tilde{P}, \quad K(t)=\tilde{K} . \tag{300}
\end{equation*}
$$

Inserting the short-run static solutions for capital in the traded sector (291) and consumption in tradables (289) into the accumulation equation of foreign bonds (281), linearizing around the steady-state and inserting (300) yields:

$$
\begin{equation*}
\dot{B}(t)=r^{\star}(B(t)-\tilde{B}) . \tag{301}
\end{equation*}
$$

Solving and invoking the transversality condition $\lim _{t \rightarrow \infty} \lambda B(t) e^{-r^{\star} t}=0$ yields:

$$
\begin{equation*}
B(t)=B_{0} . \tag{302}
\end{equation*}
$$

Hence, for the transversality condition to hold, the stock of traded bonds $B(t)$ must be equal to its initial predetermined level. Combining (302) with (300) yields:

$$
\begin{equation*}
r^{\star} B_{0}+Y^{T}=C^{T} . \tag{303}
\end{equation*}
$$

Because the stock of foreign bonds must stick to its initial value, for the sake of simplicity and without loss of generality, we set $B_{0}=0$.

## I. 5 The equilibrium

Because the dynamics degenerate, we drop the superscript tilde for the sake of simplicity. The equilibrium is defined by the following set of equations:

$$
\begin{align*}
\left(\frac{1-\varphi}{\varphi}\right) \frac{C^{T}}{C^{N}} & =P^{\phi},  \tag{304a}\\
\left(\frac{1-\vartheta}{\vartheta}\right) \frac{K^{T}}{K^{N}} & =\Gamma^{-\epsilon}  \tag{304b}\\
P & =\Gamma \frac{A^{T}}{A^{N}},  \tag{304c}\\
\frac{A^{T} K^{T}}{A^{N} K^{N}} & =\frac{C^{T}}{C^{N}}, \tag{304d}
\end{align*}
$$

where $\Gamma \equiv R^{N} / R^{T}$ is the ratio of the non traded rental rate to the traded rental rate or the relative capital rental rate.

## I. 6 Long-Run Change of the Relative Capital Rental Rate

We denote the logarithm in lower case and the percentage deviation from its initial steadystate by a hat. When focusing on the capital market, the model can be summarized graphically by two schedules in the ( $k^{T} / k^{N}, \gamma$ )-space, Applying logarithm to eq. (304b) yields the capital supply-schedule ( $C S$ henceforth):

$$
\begin{equation*}
\left.\frac{k^{T}}{k^{N}}\right|^{C S}=-\epsilon \ln \gamma+d \tag{305}
\end{equation*}
$$

where $d=\ln \left(\frac{\vartheta}{1-\vartheta}\right)$ and $\gamma=\ln \Gamma$. A rise in $\gamma$ provides an incentive to shift capital from the traded sector towards the non traded sector. Hence the $C S$-schedule is downward-sloping in the $\left(k^{T} / k^{N}, \gamma\right)$-space where the slope is equal to $-1 / \epsilon$.

Substituting demand for traded goods in terms of non traded goods (304a) into the market clearing condition given by (304d) yields:

$$
\begin{equation*}
\frac{Y^{T}}{Y^{N}}=\left(\frac{\varphi}{1-\varphi}\right) P^{\phi} \tag{306}
\end{equation*}
$$

Substituting first-order conditions from the firms' maximization problem given by (304c) and using production functions, i.e. $K^{T}=Y^{T} / A^{T}$ and $K^{N}=Y^{N} / A^{N}$, we get:

$$
\frac{K^{T}}{K^{N}}=\left(\frac{\varphi}{1-\varphi}\right)\left(\frac{A^{T}}{A^{N}}\right)^{\phi-1} \gamma^{\phi} .
$$

Taking logarithm yields the labor demand-schedule ( $L D$ henceforth) in the $\left(k^{T} / k^{N}, \gamma\right)$-space is given by

$$
\begin{equation*}
\left.\frac{k^{T}}{k^{N}}\right|^{C D}=\phi \omega+(\phi-1) \frac{a^{T}}{a^{N}}+x, \tag{307}
\end{equation*}
$$

where $x=\ln \left(\frac{\varphi}{1-\varphi}\right)$. A rise in the relative rental rate $\gamma$ raises the cost of capital in the non traded sector relative to the traded sector. To compensate for the increased capital cost, the non traded sector sets higher prices which induces agents to substitute traded for non traded goods and therefore produces an expansionary effect on capital demand in the traded sector. Hence the $L D$-schedule is upward-sloping in the $\left(k^{T} / k^{N}, \gamma\right)$-space where the slope is equal to $1 / \phi$.

Equating capital demand given by (307) and capital supply described by (305), differentiating and denoting by a hat the deviation from initial steady state in percentage terms,
we find that the relative capital rental rate $r^{N} / r^{T}$ declines in the long run as a result of a productivity differential between tradables and non tradables only if $\phi>1$ :

$$
\begin{equation*}
\hat{\gamma}=-(\phi-1) \Theta^{C}\left(\hat{a}^{T}-\hat{a}^{N}\right), \quad \Theta^{C}=\left(\frac{1}{\epsilon+\phi}\right) . \tag{308}
\end{equation*}
$$

As investors are more reluctant to shift capital from the non traded to the traded sector, as reflected by a lower $\epsilon$, the response of the relative rental rate to a productivity differential is amplified because the traded sector must pay higher return to attract investors. Graphically, the $C D$-schedule shifts along a steeper $C S$-schedule. When $\epsilon \rightarrow \infty$, the relative capital rental rate is unaffected, as in the standard BS model.

## I. 7 Long-Run Change of the Relative Price of Non Tradables

The model can be summarized graphically in the $\left(y^{T} / y^{N}, p\right)$-space. System (304a)-(304d) described above can be reduced to two equations. Substituting (304a) into eq. (304d) yields the goods market equilibrium (henceforth labelled GME) schedule:

$$
\begin{equation*}
\left.\frac{y^{T}}{y^{N}}\right|^{G M E}=\phi p+x \tag{309}
\end{equation*}
$$

where $x=\ln \left(\frac{\varphi}{1-\varphi}\right)$. Since a rise in the relative price $p$ raises consumption in tradables, the goods market equilibrium requires a rise in the traded output-non traded output ratio. Hence the goods market equilibrium is upward-sloping in the $\left(y^{T} / y^{N}, p\right)$-space where the slope is equal to $1 / \phi$.

Substituting (304c) into (304b) to eliminate $\gamma$ yields the capital market equilibrium (CME) schedule:

$$
\begin{equation*}
\left.\frac{y^{T}}{y^{N}}\right|^{L M E}=-\epsilon p+(1+\epsilon) \frac{a^{T}}{a^{N}}+z, \tag{310}
\end{equation*}
$$

where $z=\ln \left(\frac{\vartheta}{1-\vartheta}\right)$. A rise in the relative price $p$ raises the relative capital rental rate $\gamma$ which induces agents to supply more capital in the non traded sector, and more so if $\epsilon$ is larger (i.e., agents are more willing to shift capital across sectors). Hence the capital market equilibrium is downward-sloping in the $\left(y^{T} / y^{N}, p\right)$-space where the slope is equal to $-1 / \epsilon$. Assuming that the shift of capital across sectors is costless, i.e. $\epsilon$ tends to infinity, capital rental rates between traded and non traded sectors are equalized. Graphically, the CMEschedule becomes an horizontal line. Conversely, as capital mobility becomes more costly, i.e. $\epsilon$ is smaller, the $C M E$-schedule becomes steeper in the ( $y^{T} / y^{N}, p$ )-space.

Equating (309) and (310) and differentiating, we find that the relative price $p$ increases less than the productivity differential between tradables and non tradables only if $\phi>1$ :

$$
\begin{equation*}
\hat{p}=(\epsilon+1) \Theta^{C}\left(\hat{a}^{T}-\hat{a}^{N}\right), \tag{311}
\end{equation*}
$$

where $\Theta^{L}$ is given by (308). According to (311), following a productivity differential of $1 \%, p$ must increase by less than $1 \%$ only if $\phi>1$. In this case, the lower $\epsilon$, the smaller $\hat{p}$. Intuitively, because investors are more reluctant to shift capital across sectors, the ratio $k^{T} / k^{N}$ increases less, requiring a lower $\hat{p}$ to clear the market. Graphically, the $C M E$-schedule shifts to the right by a smaller amount. When $\epsilon \rightarrow \infty$, we have $\hat{p}=\hat{a}^{T}-\hat{a}^{N}$, i.e., a strict proportional relationship between $p$ and $a^{T} / a^{N}$.

## J A Two-Sector Model With Labor Market Frictions

In this section, we explore the link between the relative price (and relative wage) and technological change biased toward the traded sector by developing a two-sector framework with
unemployment in the labor market. We see our setup as an extension of the framework by Heijdra and Ligthart [2009] who solve analytically a dynamic open economy model with search unemployment and endogenous labor force participation. ${ }^{54}$ In the tradition of Diamond-Mortensen-Pissarides, unemployment arises because it takes time for firms to hire workers and for unemployed workers to find a job. Because firms face a cost by maintaining job vacancies, they receive a surplus equal to the marginal product of labor less the product wage. Symmetrically, so as to compensate for the cost of searching for a job, unemployed workers receive a surplus equal to the product wage less the reservation wage. Nash bargaining between firms and workers yields a product wage defined as the weighted sum of the marginal product of labor and a reservation wage. As Heijdra and Ligthart, we depart from the usual practice by assuming endogenous labor force participation which implies that the reservation wage varies over time.

The country is small in terms of both world goods and capital markets, and faces a given world interest rate, $r^{\star}$. ${ }^{55}$ The small open economy is populated by a constant number of identical households and firms that have perfect foresight and live forever. Households decide on labor market participation and consumption while firms decide on hours worked. The economy consists of two sectors. A sector produces a traded good denoted by the superscript $T$ that can be exported while the other sector produces a non-traded good denoted by the superscript $N$. The setup allows for traded and non-traded goods to be used for consumption. The traded good is chosen as the numeraire. The labor market, in the tradition of Diamond-Mortensen-Pissarides, consists of a matching process within each sector between the firms who post job vacancies and unemployed workers who search for a job.

## J. 1 Households

The economy that we consider consists of a representative household with a measure one continuum of identical infinitely lived members. At any instant, members in the household derive utility from consumption goods $C$ and experience disutility from working and searching efforts. More precisely, the representative household comprises members who engage in only one of the following activities: working and searching a job in each sector, or enjoying leisure. Assuming that the representative individual is endowed with one unit of time, leisure is defined as $l \equiv 1-L^{T}-L^{N}-U^{T}-U^{N}$, where $L^{j}$ denotes units of labor time and $U^{j}$ corresponds to time spent on searching for a job in sector $j$ (with $j=T, N$ ). Hence, the labor force is not constant which enables us to focus on both the transition between employment and unemployment on the one hand, and the transition between leisure and labor force on the other. Unemployed agents are randomly matched with job vacancies according to a matching function described later. Since the timing of a match is random, agents face idiosyncratic risks. To simplify the analysis, we assume that members in the household perfectly insure each other against variations in labor income.

We consider that the utility function is additively separable in the disutility received by working and searching in the two sectors. As will become clear later, such specification makes it impossible to switch immediately from one sector to the other. This can be justified on the grounds of sector specific skills. Technically, in order to work more in sector $j$ the agent must raise the time spent on searching for a job in sector $j$. The representative household chooses the time path of consumption and labor force to maximize the following objective function:

$$
\begin{equation*}
U=\int_{0}^{\infty}\left\{\frac{1}{1-\frac{1}{\sigma_{C}}} C(t)^{1-\frac{1}{\sigma_{C}}}-\frac{1}{1+\frac{1}{\sigma_{L}^{T}}} F^{T}(t)^{1+\frac{1}{\sigma_{L}^{T}}}-\frac{1}{1+\frac{1}{\sigma_{L}^{N}}} F^{N}(t)^{1+\frac{1}{\sigma_{L}^{N}}}\right\} e^{-\rho t} \mathrm{~d} t \tag{312}
\end{equation*}
$$

[^30]where $\rho$ is the consumer's subjective time discount rate, $\sigma_{C}>0$ is the intertemporal elasticity of substitution for consumption, and $\sigma_{L}^{j}>0$ is the elasticity of labor supply at the extensive margin in sector $j=T, N$. For later use, we denote by $u^{j}$ the sectoral unemployment rate defined as $u^{j}=\frac{U^{j}}{U^{j}+L^{j}}=\frac{U^{j}}{F^{j}}$ with $F^{j}=L^{j}+U^{j}$ the labor force in sector $j$.

At each instant of time, $m^{j} U^{j}$ unemployed agents find a job in sector $j=T, N$ and $s^{j} L^{j}$ employed individuals lose their job. Employment in sector $j$ evolves gradually according to:

$$
\begin{equation*}
\dot{L}^{j}(t)=m^{j} U^{j}(t)-s^{j} L^{j}(t), \quad j=T, N \tag{313}
\end{equation*}
$$

where $m^{j}$ denotes the rate at which unemployed agents find jobs and $s^{j}$ is the constant rate of job separation; $1 / m^{j}$ can be interpreted as the average unemployment duration; $m^{j}$ is a function of labor market tightness $\theta^{j}$ which is defined as the ratio of the number of job vacancies over unemployed agents in sector $j$.

Households supply $L^{j}(t)$ units of labor services for which they receive the product wage $w^{j}(t)$ in sector $j=T, N$. They accumulate internationally traded bonds, $B(t)$, that yield net interest rate earnings $r^{\star} B(t)$. We denote by $A(t)$ the stock of financial wealth held by households which comprises the shadow value of employment defined later. Denoting by $T$ the lump-sum taxes, the flow budget constraint is equal to households' real disposable income less consumption expenditure $P_{C} C$ :

$$
\begin{equation*}
\dot{A}(t)=r^{\star} A(t)+w^{T}(t) L^{T}(t)+w^{N}(t) L^{N}(t)+R^{T} U^{T}(t)+R^{N} U^{N}(t)-T(t)-P_{C}(P(t)) C(t) \tag{314}
\end{equation*}
$$

where $P_{C}$ is the consumption price index which is a function of the relative price of non-traded goods $P$ and $R^{j}$ represents unemployment benefits received by job seekers in sector $j$.

The representative household selects consumption, time dedicated for searching a job in sector $j$, and financial wealth: ${ }^{56}$

$$
\begin{gather*}
C=\left(P_{C} \lambda\right)^{-\sigma_{C}}  \tag{315a}\\
F^{j}=\left\{\lambda\left[m^{j}\left(\theta^{j}\right) \xi^{j}+R^{j}\right]\right\}^{\sigma_{L}^{j}}  \tag{315b}\\
\dot{\lambda}=\lambda\left(\rho-r^{\star}\right)  \tag{315c}\\
\dot{\xi}^{j}=\left(s^{j}+r^{\star}\right) \xi^{j}-\left(w^{j}-\left(F^{j}\right)^{1 / \sigma_{L}^{j}}\right), \tag{315~d}
\end{gather*}
$$

and the appropriate transversality conditions; $\lambda$ and $\xi^{j}$ denote the shadow prices of wealth and finding a job in sector $j$, respectively. Eq. (315b) shows that labor market participation is a positive function of the reservation wage $w_{R}^{j}$, which is defined as the sum of the expected value of a job $m^{j} \xi^{j}$ and the unemployment benefit $R^{j}$. Solving eq. (315d) forward and invoking the transversality condition yields:

$$
\begin{equation*}
\xi^{j}(t)=\int_{t}^{\infty}\left[w^{j}(\tau)-w_{R}^{j}(\tau)\right] e^{\left(s^{j}+r^{\star}\right)(t-\tau)} \mathrm{d} \tau \tag{316}
\end{equation*}
$$

Eq. (316) states that $\xi$ is equal to the present discounted value of the surplus from an additional job consisting of the excess of labor income over the household's outside option. Note that as described above, we consider a representative household who splits available time between leisure and market activities (i.e., time devoted to job search and work). While labor supply is elastic at the extensive margin, search effort and worked hours are supplied

[^31]inelastically. ${ }^{57}$ For the sake of clarity, we drop the time argument below when this causes no confusion.

Applying Shephard's lemma (or the envelope theorem) yields expenditure in tradables and non tradables, i.e., $P C^{N}=\alpha_{C} P_{C} C,\left(1-\alpha_{C}\right) P_{C} C$, with $\alpha_{C}$ being the share of non traded goods in consumption expenditure. ${ }^{58}$ Intra-temporal allocation of consumption follows from the following optimal rule:

$$
\begin{equation*}
\left(\frac{1-\varphi}{\varphi}\right) \frac{C^{T}}{C^{N}}=P^{\phi} . \tag{317}
\end{equation*}
$$

An appreciation in the relative price of non tradables $P$ increases expenditure on tradables relative to expenditure on non tradables (i.e. $C^{T} / P C^{N}$ ), only when $\phi>1$.

## J. 2 Firms

Each sector consists of a large number of identical firms. Both the traded and non-traded sectors use labor, $L^{T}$ and $L^{N}$, according to constant returns to scale production functions, $Y^{T}=A^{T} L^{T}$ and $Y^{N}=A^{N} L^{N}$. Firms post job vacancies $V^{j}$ to hire workers and face a cost per job vacancy $\kappa^{j}$ which is assumed to be constant and measured in terms of the traded good. Firms pay the wage $w^{j}$ decided by the generalized Nash bargaining solution. As producers face a labor cost $w^{j}$ per employee and a cost per hiring of $\kappa^{j}$, the profit function of the representative firm in the traded sector is:

$$
\begin{equation*}
\pi^{T}=A^{T} L^{T}-w^{T} L^{T}-\kappa^{T} V^{T}+x^{T} \dot{L}^{T} \tag{318}
\end{equation*}
$$

where $x^{T}$ is a firing tax when layoffs are higher than hirings, i.e., if $\dot{L}^{T}<0$ is negative (see e.g., Heijdra and Ligthart [2002]). This variable captures the extent of employment protection legislation. Symmetrically, denoting by $P$ the price of non traded goods in terms of traded goods, the profit function of the representative firm in the non traded sector is:

$$
\begin{equation*}
\pi^{T}=P A^{N} L^{N}-w^{N} L^{N}-\kappa^{N} V^{N}+x^{N} \dot{L}^{N}, \tag{319}
\end{equation*}
$$

where $x^{N}$ is a firing tax in the non traded sector when $\dot{L}^{N}<0$.
Denoting by $f^{j}$ the rate at which a vacancy is matched with unemployed agents, the law of motion for labor is given by:

$$
\begin{equation*}
\dot{L}^{j}=f^{j} V^{j}-s^{j} L^{j}, \tag{320}
\end{equation*}
$$

where $f^{j} V^{j}$ represents the flow of job vacancies which are fulfilled; note that $f^{j}$ decreases with labor tightness $\theta^{j}$.

Denoting by $\gamma^{j}$ the shadow price of employment to the firm, and keeping in mind that $f^{j}$ is taken as given, the maximization problem yields the following first-order conditions:

$$
\begin{align*}
\gamma^{j}+x^{j} & =\frac{\kappa^{j}}{f^{j}\left(\theta^{j}\right)},  \tag{321a}\\
\dot{\gamma}^{j} & =\gamma^{j}\left(r^{\star}+s^{j}\right)-\left(\Xi^{j}+r^{\star} x^{j}-w^{j}\right) \tag{321b}
\end{align*}
$$

where $\Xi^{T}=A^{T}$ and $\Xi^{N}=P A^{N}$. Eq. (321a) requires the marginal cost of vacancy, $\kappa^{j}$, to be equal to the marginal benefit of vacancy, $f^{j}().\left(\gamma^{j}+x^{j}\right)$. Solving equation (321b) forward and invoking the transversality condition yields:

$$
\begin{equation*}
\gamma^{j}(t)=\int_{t}^{\infty}\left[\Xi^{j}(\tau)-x^{j} s^{j}-w^{j}(\tau)\right] e^{\left(s^{j}+r^{\star}\right)(t-\tau)} \mathrm{d} \tau . \tag{322}
\end{equation*}
$$

[^32]Eq. (322) states that $\gamma^{j}$ is equal to the present discounted value of the cash flow earned on an additional worker, consisting of the excess of labor productivity $\Xi^{j}$ over the wage $w^{j}$ minus the firing cost $x^{j} s^{j}$.

Differentiating $\gamma^{j}(t) L^{j}(t)$ w. r. t. time and inserting the law of motion for employment $\dot{L}^{j}(t)$ together with the dynamic optimality condition (321b), using (321a), i.e., $\gamma^{j}=\kappa^{j} / f^{j}-$ $x^{j}$, and $\dot{L}^{j}=f^{j} \theta^{j}-s^{j} L^{j}$, solving forward, and making use of the transversality condition, we get: ${ }^{59}$

$$
\begin{equation*}
\gamma^{j}(t) L^{j}(t)=\int_{t}^{\infty} \pi^{j} e^{-r^{\star}(\tau-t)} \mathrm{d} \tau, \quad j=T, N \tag{323}
\end{equation*}
$$

Eq. (323) states that the value of human assets $\gamma^{j} L^{j}$ is equal to the value of the representative firm in sector $j$ which corresponds to the present discounted value of profits $\pi^{j}$.

## J. 3 Matching and Wage Determination

We now set the matching function and the wage determination scheme. As it is common in the literature, the matching function is assumed to take a Cobb-Douglas form:

$$
\begin{equation*}
M^{j}\left(V^{j}, U^{j}\right)=X^{j}\left(V^{j}\right)^{\alpha_{V}^{j}}\left(U^{j}\right)^{1-\alpha_{V}^{j}}, \quad \alpha_{V}^{j} \in(0,1) \tag{324}
\end{equation*}
$$

where $M^{j}$ describes the number of job matches, $\alpha_{V}^{j}$ represents the elasticity of vacancies in job matches and $X^{j}$ corresponds to the matching efficiency. We express the number of labor contracts per unemployment units: ${ }^{60}$

$$
\begin{equation*}
m^{j}=m^{j}\left(\theta^{j}\right)=X^{j}\left(\theta^{j}\right)^{\alpha_{V}^{j}}, \quad f^{j}=f^{j}\left(\theta^{j}\right)=\frac{m^{j}\left(\theta^{j}\right)}{\theta^{j}}=X^{j}\left(\theta^{j}\right)^{\alpha_{V}^{j}-1} \tag{325}
\end{equation*}
$$

with

$$
\begin{equation*}
\frac{\left(f^{j}\right)^{\prime} \theta^{j}}{f^{j}}=-\left(1-\alpha_{V}^{j}\right), \quad \frac{\left(m^{\prime}\right)^{j} \theta^{j}}{m^{j}}=\alpha_{V}^{j} \tag{326}
\end{equation*}
$$

When a vacancy and a job-seeking worker meet, a rent is created which is equal to $\xi^{j}+\gamma^{j}$, where $\xi^{j}$ is the value of an additional job and $\gamma^{j}$ is the value of an additional worker and $w^{j}$ is the subsidy. The division of the rent between the worker and the firm is determined by generalized Nash bargaining over the wage rate:

$$
\begin{equation*}
\max _{w^{j}}\left(\xi^{j}\right)^{\alpha_{W}^{j}}\left(\gamma^{j}+x^{j}\right)^{1-\alpha_{W}^{j}}, \quad \alpha_{W}^{j} \in(0,1) \tag{327}
\end{equation*}
$$

where $\alpha_{W}^{j}$ and $1-\alpha_{W}^{j}$ correspond to the bargaining power of the worker and the firm, respectively.

Solving for (327), the product wage $w^{j}$ is defined as a weighted sum of the labor marginal revenue of labor and the reservation wage:

$$
\begin{equation*}
w^{j}=\alpha_{W}^{j}\left(\Xi^{j}+r^{\star} x^{j}\right)+\left(1-\alpha_{W}^{j}\right) \frac{\left(F^{j}\right)^{1 / \sigma_{L}^{j}}}{\bar{\lambda}} \tag{328}
\end{equation*}
$$

An increase in the marginal revenue of labor, which exerts an upward pressure on labor demand, or a rise in the labor market tightness, by raising the reservation wage (see eq. $(315 b))$, pushes up the product wage. ${ }^{61}$

[^33]
## J. 4 Government

The final agent in the economy is the government. Unemployed benefits $R^{T} U^{T}+R^{N} U^{N}$ and hiring subsidies $x^{T} \dot{L}^{T}+x^{N} \dot{L}^{N}$ are covered by lump-sum taxes $T$ according to the following balanced budget constraint: ${ }^{62}$

$$
\begin{equation*}
T=\left(R^{T} U^{T}+R^{N} U^{N}\right)+\left(x^{T} \dot{L}^{T}+x^{N} \dot{L}^{N}\right) \tag{329}
\end{equation*}
$$

## J. 5 Solving the Model

In this section, we characterize the equilibrium dynamics and then discuss the steady-state.

## Market Clearing Conditions

To begin with, we have to impose the market clearing condition for the non traded good according to which non traded output is only consumed domestically:

$$
\begin{equation*}
Y^{N}(t)=C^{N}(t) \tag{330}
\end{equation*}
$$

Using the definition of the stock of financial wealth $A(t) \equiv B(t)+\gamma^{T}(t) L^{T}(t)+\gamma^{N}(t) L^{N}(t)$, differentiating with respect to time, substituting the accumulation equations of labor (313) and financial wealth (314) together with the dynamic equation for the shadow value of an additional worker (321b), using the government budget constraint (329) and the market clearing condition for the non traded good market (330), the accumulation equation for foreign assets is:

$$
\begin{equation*}
\dot{B}(t)=r^{\star} B(t)+A^{T} L^{T}(t)-C^{T}(t)-\kappa^{T} V^{T}(t)-\kappa^{N} V^{N}(t) \tag{331}
\end{equation*}
$$

Note that we assume that hiring costs in the non traded sector are paid by the traded sector to avoid unnecessary complications due to measure units for the cost per vacancy.

## Short-Run Static Solutions

In an open economy model with a representative agent having perfect foresight, a constant rate of time preference and perfect access to world capital markets, we impose $\beta=r^{\star}$ in order to generate an interior solution. This standard assumption made in the literature implies that the marginal utility of wealth, $\lambda$, will undergo a discrete jump when individuals receive new information and must remain constant over time from then on, i.e., $\lambda=\bar{\lambda}$. Equation (315a) can be solved for consumption:

$$
\begin{equation*}
C=C(\bar{\lambda}, P) \tag{332}
\end{equation*}
$$

A rise in the shadow value of wealth induces agents to cut their real expenditure (i.e., $C_{\bar{\lambda}}<$ 0 ) while an increase in the consumption price index triggered by an appreciation in the relative price of non-tradables $P$ drives down consumption (i.e., $C_{P}<0$ ). Inserting (332) into $C^{T}=\left(1-\alpha_{C}\right) P_{C} C$ and non-tradables $P C^{N}=\alpha_{C} P_{C} C$ allows us to solve for consumption in tradables and non tradables, i.e., $C^{T}=C^{T}(\bar{\lambda}, P)$ and $C^{N}=C^{N}(\bar{\lambda}, P)$ with $C_{\bar{\lambda}}^{j}<0, C_{P}^{T} \gtrless 0$ depending on whether $\phi \gtrless \sigma_{C}$ and $C_{P}^{N}<0$.

Substituting first the short-run static solution for consumption in non tradables $C^{N}=$ $C^{N}(\bar{\lambda}, P)$, the market clearing condition for the non traded good (330) can be solved for the
formally, using the fact that $\xi^{j}=\frac{\alpha_{W}^{j}}{1-\alpha_{W}^{j}} \gamma^{j}, \gamma^{j}+x^{j}=\kappa^{j} / f^{j}, m^{j} / f^{j}=\theta^{j}$, we have $\left(F^{j}\right)^{1 / \sigma_{L}^{j}} / \bar{\lambda}=\frac{\alpha_{W}^{j}}{1-\alpha_{W}^{j}} \kappa^{j} \theta^{j}+$ $R^{j}$. Plugging this term into the Nash bargaining wage (328), we have:

$$
w^{j}=\alpha_{W}^{j}\left(\Xi^{j}+r^{\star} x^{j}\right)+\left(1-\alpha_{W}^{j}\right)\left[\frac{\alpha_{W}^{j}}{1-\alpha_{W}^{j}} \kappa^{j} \theta^{j}+R^{j}\right]=\alpha_{W}\left(\Xi^{j}+\kappa^{j} \theta^{j}+r^{\star} x^{j}\right)+\left(1-\alpha_{W}^{j}\right) R^{j}
$$

[^34]relative price of non tradables as follows:
\[

$$
\begin{equation*}
P=P\left(L^{N}, \bar{\lambda}, A^{N}\right) \tag{333}
\end{equation*}
$$

\]

where $P_{L^{N}}=\partial P / \partial L^{N}=A^{N} / C_{P}^{N}<0, P_{\bar{\lambda}}=-C_{\bar{\lambda}}^{N} / C_{P}^{N}<0$, and $P_{L^{N}}=\partial P / \partial L^{N}=$ $L^{N} / C_{P}^{N}<0$.

## Saddle-Path Stability

In this subsection, we analyze saddle-path stability; hence, we first derive the system of differential equations. To determine the dynamic equation for labor market tightness $\theta^{j}$ in sector $j$, differentiate (321a) w. r. t. time, insert (321b), and eliminate $\gamma^{j}$ by using (321a):

$$
\begin{equation*}
\dot{\theta}^{j}(t)=\frac{\theta^{j}(t)}{\left(1-\alpha_{V}^{j}\right)}\left\{\left(s^{j}+r^{\star}\right)-\frac{f^{j}\left(\theta^{j}(t)\right)\left(1-\alpha_{W}^{j}\right) \Psi^{j}}{\kappa^{j}}\right\}, \tag{334}
\end{equation*}
$$

where we set the overall surplus from hiring in sector $j$ denoted by $\Psi^{j}$ :

$$
\begin{equation*}
\Psi^{j} \equiv\left(\Xi^{j}+r^{\star} x^{j}\right)+\frac{v_{F}^{j}}{\bar{\lambda}} . \tag{335}
\end{equation*}
$$

Differentiating first (315b) w. r. t. time and substituting (315d) yields the dynamic equation for job seekers:

$$
\begin{equation*}
\frac{\left(F^{j}\right)^{\frac{1}{\sigma_{L}^{j}}-1}}{\sigma_{L}^{j} \bar{\lambda}} \dot{U}^{j}=\left[\frac{\left(F^{j}\right)^{1 / \sigma_{L}^{j}}}{\bar{\lambda}}-R^{j}\right]\left[\left(s^{j}+r^{\star}\right)+\alpha_{V}^{j} \frac{\dot{\theta}^{j}}{\theta^{j}}\right]-m^{j}\left(\theta^{j}\right) \alpha_{W}^{j} \Psi^{j}-\frac{\left(F^{j}\right)^{\frac{1}{\sigma_{L}^{j}}-1}}{\sigma_{L}^{j} \bar{\lambda}} \dot{L}^{j} \tag{336}
\end{equation*}
$$

where we used the fact that $w^{j}-w_{R}^{j}=\alpha_{W}^{j} \Psi^{j}$.
Due to imperfect mobility of labor across sectors, hiring and search decisions in the traded and non traded labor markets are independent which implies that the Jacobian matrix is block recursive; hence, the saddle-path stability condition in the traded and non traded sectors can be explored separately. Inserting first appropriate short-run static solutions, linearizing in the neighborhood of the steady-state, the dynamic system for the traded (non traded) sector which comprises three equations, i.e., the accumulation equation for employment (313), the dynamic equation for labor market tightness (334) and the dynamic equation for job seekers (336), we find that the determinant of the Jacobian matrix for the traded (non traded) sector is negative. ${ }^{63}$ Hence, the linearized dynamic system possesses one negative eigenvalue denoted by $\nu_{1}^{j}$ and two positive eigenvalues denoted by $\nu_{2}^{j}$ and $\nu_{3}^{j}$. Assuming the Hosios condition holds, i.e., setting $\alpha_{W}^{j}=\left(1-\alpha_{V}^{j}\right)$, eigenvalues satisfy $\nu_{1}^{j}<0<r^{\star}<\nu_{2}^{j}$, with $\nu_{2}^{j}=r^{\star}-\nu_{1}^{j}>0$, and $\nu_{3}^{j}=s^{j}+r^{\star}>0$. Note that when considering the traded sector, the negative and the positive eigenvalues reduce to $\nu_{1}^{T}=-\left(s^{T}+\tilde{m}^{T}\right)<0$ and $\nu_{2}^{T}=\left(s^{T}+r^{\star}+\tilde{m}^{T}\right)>0$, respectively.

Denoting the long-term values with a tilde, the stable paths for employment, labor market tightness, and job seekers are given by: ${ }^{64}$

$$
\begin{aligned}
L^{T}(t)-\tilde{L}^{T} & =D_{1}^{T} e^{\nu_{1}^{T} t} \quad \theta^{T}(t)-\tilde{\theta}^{T}=\omega_{21}^{T} D_{1}^{T} e^{\nu_{1}^{T} t}, \quad U^{T}(t)-\tilde{U}^{T}=\omega_{31}^{T} D_{1}^{T} e^{\nu_{1}^{T} t}, \quad(337 \mathrm{a}) \\
L^{N}(t)-\tilde{L}^{N} & =D_{1}^{N} e^{\nu_{1}^{N} t}, \quad \theta^{N}(t)-\tilde{\theta}^{N}=\omega_{21}^{N} D_{1}^{N} e^{\nu_{1}^{N} t}, \quad U^{N}(t)-\tilde{U}^{N}=\omega_{31}^{N} D_{1}^{N} e^{\nu_{1}^{N}}(3,37 \mathrm{~b})
\end{aligned}
$$

[^35]where we have normalized $\omega_{11}^{j}$ to unity and it can be proven formally that $\omega_{21}^{T}=0, \omega_{31}^{T}=-1$, $\omega_{21}^{N}<0, \omega_{31}^{N}<0$.

Two features of the two-sector economy's equilibrium dynamics deserve special attention. First, the dynamics for labor market tightness in the traded sector $\theta^{T}(t)$ degenerate as reflected by $\omega_{21}^{T}=0$. Unlike, because the relative price of non tradables must adjust sluggishly to clear the non traded good market (because $L^{N}$ is a state variable), $\theta^{N}(t)$ exhibits transitional dynamics; since $\omega_{21}^{N}<0, L^{N}$ and $\theta^{N}$ move in opposite directions. Second, in both sectors, the number of job seekers $U^{j}$ falls as employment $L^{j}$ builds up.

Inserting first the short-run static solution for the relative price of non tradables (333) into $C^{T}=C^{T}(\bar{\lambda}, P)$, linearizing (331) around the steady-state, substituting the solutions (337), and invoking the transversality condition, yields the stable solution for traded bonds $B(t)-\tilde{B}=\Phi^{T}\left(L^{T}(t)-\tilde{L}^{T}\right)+\Phi^{N}\left(L^{N}(t)-\tilde{L}^{N}\right)$ consistent with the intertemporal solvency condition: ${ }^{65}$

$$
\begin{equation*}
\tilde{B}-B_{0}=\Phi^{T}\left(\tilde{L}^{T}-L_{0}^{T}\right)+\Phi^{T}\left(\tilde{L}^{N}-L_{0}^{N}\right) \tag{338}
\end{equation*}
$$

Because $\Phi^{T}<0$ and $\Phi^{N}<0$, the current account is negatively related to changes in sectoral employment.

## J. 6 Steady-State

We now describe the steady-state of the economy which comprises six equations. First, setting $\dot{\theta}^{j}=0$ into eq. (334), we obtain the vacancy creation equation:

$$
\begin{equation*}
\frac{\kappa^{j}}{f^{j}\left(\tilde{\theta}^{j}\right)}=\frac{\left(1-\alpha_{W}^{j}\right)}{s^{j}+r^{\star}} \tilde{\Psi}^{j}, \quad \tilde{\Psi}^{j} \equiv\left(\Xi^{j}+r^{\star} x^{j}\right)-w_{R}^{j}, \quad j=T, N \tag{339}
\end{equation*}
$$

The LHS term of eq. (339) represents the marginal cost of recruiting in sector $j=T, N$. The RHS term represents the marginal benefit of an additional worker which is equal to the share, received by the firm, of the rent created by the encounter between a vacancy and a job-seeking worker. A rise in labor productivity raises the surplus from hiring $\tilde{\Psi}^{j}$; as a result, firms to post more job vacancies which raises the labor market tightness $\tilde{\theta}^{j}$.

Second, using the fact that $\tilde{\xi}^{j}=\frac{\alpha_{W}^{j}}{1-\alpha_{W}^{j}} \tilde{\gamma}^{j}, \tilde{\gamma}^{j}=\frac{\kappa^{j}}{\tilde{f}^{j}}, \frac{\tilde{m}^{j}}{\tilde{f}^{j}}=\tilde{\theta}^{j}$, to rewrite the reservation wage, the decision of search equation reads as:

$$
\begin{equation*}
\tilde{L}^{j}=\frac{\tilde{m}^{j}}{\tilde{m}^{j}+s^{j}}\left[\bar{\lambda}\left(\frac{\alpha_{W}^{j}}{1-\alpha_{W}^{j}} \kappa^{j} \tilde{\theta}^{j}+R^{j}\right)\right]^{\sigma_{L}^{j}}, \quad j=T, N \tag{340}
\end{equation*}
$$

where $\left(\frac{\alpha_{W}^{j}}{1-\alpha_{W}^{j}} \kappa^{j} \tilde{\theta}^{j}+R^{j}\right)$ corresponds to the reservation wage $\tilde{w}_{R}^{j}$ reflecting the marginal benefit from search; note that we have eliminated $\tilde{U}^{j}$ from (315b) by using the fact that in the long-run the number of unemployed agents who find a job $\tilde{m}^{j} \tilde{U}^{j}$ and workers who lose their job $s^{j} \tilde{L}^{j}$ must equalize. According to (340), higher labor market tightness increases labor $\tilde{L}^{j}$ by the probability of hiring and thus the employment rate $\frac{\tilde{m}^{j}}{\tilde{m}^{j}+s^{j}}$. Moreover, for given $\bar{\lambda}$, the rise in the reservation wage $\left(\frac{\alpha_{W}^{j}}{1-\alpha_{W}^{j}} \kappa^{j} \tilde{\theta}^{j}+R^{j}\right)$ induces agents to supply more labor.

$$
\begin{aligned}
& { }^{65} \text { The terms } \Phi^{T} \text { and } \Phi^{N} \text { are negative and given by: } \\
& \qquad \Phi^{T} \equiv \frac{\Lambda^{T}}{\nu_{1}^{T}-r^{\star}}=-\frac{\left(A^{T}+\kappa^{T} \tilde{\theta}^{T}\right)}{\left(s^{T}+\tilde{m}^{T}+r^{\star}\right)}<0, \quad \Phi^{N} \equiv \frac{\Lambda^{N}}{\nu_{1}^{N}-r^{\star}}<0 .
\end{aligned}
$$

where $\Lambda^{N} \equiv-C_{L^{N}}^{T}-\kappa^{N} \tilde{U}^{N}\left(1-\alpha_{V}^{N}\right) \omega_{21}^{N}-\frac{\kappa^{N} \tilde{\theta}^{N}\left(s^{N}+\nu_{1}^{N}\right)}{\tilde{m}^{N}}>0$.

Third, setting $\dot{B}=0$ into eq. (331), we obtain the market clearing condition for the traded good:

$$
\begin{equation*}
r^{\star} \tilde{B}+A^{T} \tilde{L}^{T}-\tilde{C}^{T}-\kappa^{T} \tilde{U}^{T} \tilde{\theta}^{T}-\kappa^{N} \tilde{U}^{N} \tilde{\theta}^{N}=0, \tag{341}
\end{equation*}
$$

where $\tilde{C}^{T}=C^{T}\left(\tilde{L}^{N}, \bar{\lambda}, A^{N}\right)$.
The system comprising eqs. (339)-(341) can be solved for the steady-state labor market tightness, employment, job seekers, and traded bonds. All these variables can be expressed in terms of the labor productivity index $A^{j}$ and the marginal utility of wealth, i.e. $\tilde{\theta}=\theta\left(A^{T}\right)$, $\tilde{L}^{T}=L^{T}\left(\bar{\lambda}, A^{T}\right), \tilde{\theta}^{N}=\theta^{N}\left(\bar{\lambda}, A^{N}\right), \tilde{L}^{N}=L^{N}\left(\bar{\lambda}, A^{N}\right)$, and $\tilde{B}=B\left(\bar{\lambda}, A^{T}, A^{N}\right)$. Inserting first $\tilde{B}=B\left(\bar{\lambda}, A^{T}, A^{N}\right)$, and $\tilde{L}^{j}=L^{j}\left(\bar{\lambda}, A^{N}\right)$, the intertemporal solvency condition (338) can be solved for the equilibrium value of the marginal utility of wealth:

$$
\begin{equation*}
\bar{\lambda}=\lambda\left(A^{T}, A^{N}\right) . \tag{342}
\end{equation*}
$$

## J. 7 Rewriting the Steady-State

To build intuition regarding steady-state changes, we investigate graphically the long-run effects of a rise in the the ratio of sectoral productivity. To do so, it is convenient to rewrite the steady-state as follows:

$$
\begin{gather*}
\frac{\tilde{C}^{T}}{\tilde{C}^{N}}=\frac{\varphi}{1-\varphi} \tilde{P}^{\phi},  \tag{343a}\\
\frac{\tilde{L}^{T}}{\tilde{L^{N}}}=\frac{\tilde{m}^{T}}{\tilde{m}^{N}} \frac{\left(s^{N}+\tilde{m}^{N}\right)}{\left(s^{T}+\tilde{m}^{T}\right)} \frac{\left[\bar{\lambda} \tilde{w}_{R}^{T}\right]^{\sigma_{L}^{T}}}{\left[\bar{\lambda} \tilde{w}_{R}^{N}\right]^{\sigma_{L}^{N}}},  \tag{343b}\\
\frac{\kappa^{T}}{f^{T}\left(\tilde{\theta}^{T}\right)}=\frac{\left(1-\alpha_{W}^{T}\right) \tilde{\Psi}^{T}}{\left(s^{T}+r^{\star}\right)},  \tag{343c}\\
\frac{\kappa^{N}}{f^{N}\left(\tilde{\theta}^{N}\right)}=\frac{\left(1-\alpha_{W}^{N}\right) \tilde{\Psi}^{N}}{\left(s^{N}+r^{\star}\right)},  \tag{343d}\\
\frac{\tilde{Y}^{T}\left(1+v_{B}-v_{V}^{T}-v_{V}^{N}\right)}{\tilde{Y}^{N}}=\frac{\tilde{C}^{T}}{\tilde{C}^{N}} . \tag{343e}
\end{gather*}
$$

We denote by by $v_{B} \equiv \frac{r^{*} \tilde{B}}{\tilde{Y}^{T}}$ the ratio of interest receipts to traded output, by $v_{V^{j}}^{\tilde{Y}^{T}} \frac{\kappa^{j} \tilde{V}^{j}}{\tilde{Y}^{T}}$ the share of hiring cost in sector $j=T, N$ in traded output. Remembering that $\tilde{Y}_{\tilde{T}}^{T}=A^{T} \tilde{L}^{T}$ and $\tilde{Y}^{N}=A^{N} \tilde{L}^{N}$, the system (343) can be solved for $\tilde{C}^{T} / \tilde{C}^{N}, \tilde{L}^{T} / \tilde{L}^{N}, \tilde{\theta}^{T}, \tilde{\theta}^{N}$, and $\tilde{P}$, as functions of $A^{T}, A^{N},\left(1+v_{B}-v_{V}^{T}-v_{V}^{N}\right)$. Inserting these functions into $\tilde{Y}^{N}=\tilde{C}^{N}$, and $\tilde{B}-B_{0}=\Phi^{T}\left(\tilde{L}^{T}-L_{0}^{T}\right)+\Phi^{T}\left(\tilde{L}^{N}-L_{0}^{N}\right)$ (see eq. (338)), the system can be solved for $\tilde{B}$ and $\bar{\lambda}$ as functions of $A^{T}$ and $A^{N}$. Hence, when solving the system (343), we assume that the stock of foreign bonds and the marginal utility of wealth are exogenous which allows us to separate the static reallocations (or intratemporal) effects from the dynamic (or intertemporal) effects.

Because we restrict ourselves to the analysis of the long-run effects, the tilde is suppressed for the purposes of clarity. To characterize the steady-state, we focus on the goods market which can be summarized graphically by two schedules in the in the $\left(y^{T} / y^{N}, p\right)$-space, where we denote the logarithm of variables with lower-case letters.

To begin with, we characterize the goods market equilibrium. Using the fact that $v_{B}-$ $v_{V}^{T}-v_{V}^{N} \equiv-v_{N X}$ and inserting (343a) into the market clearing condition (343e) yields:

$$
\begin{equation*}
\frac{C^{T}}{C^{N}}=\frac{\varphi}{1-\varphi} P^{\phi}=\frac{Y^{T}\left(1-v_{N X}\right)}{Y^{N}} . \tag{344}
\end{equation*}
$$

The ratio of traded output to non traded output is:

$$
\begin{equation*}
\frac{Y^{T}}{Y^{N}}=\frac{1}{\left(1-v_{N X}\right)} \frac{\varphi}{1-\varphi} P^{\phi} \tag{345}
\end{equation*}
$$

Totally differentiating and denoting the percentage deviation from its initial steady-state by a hat yields the goods market equilibrium-schedule (GME henceforth).:

$$
\begin{equation*}
\left.\left(\hat{Y}^{T}-\hat{Y}^{N}\right)\right|^{G M E}=\phi \hat{P}-\mathrm{d} \ln \left(1-v_{N X}\right) \tag{346}
\end{equation*}
$$

According to (346), the GME-schedule is upward-sloping in the $\left(y^{T} / y^{N}, p\right)$-space and the slope of the $G M E$-schedule is equal to $1 / \phi$.

We now characterize the labor market equilibrium. To do so, we totally differentiate the decision of search-schedule (henceforth $D S$ ) given by eq. (343b); we have:

$$
\begin{equation*}
\left.\left(\hat{L}^{T}-\hat{L}^{N}\right)\right|^{D S}=\left(\sigma_{L}^{T}-\sigma_{L}^{N}\right) \hat{\bar{\lambda}}+\left[\alpha_{V}^{T} \tilde{u}^{T}+\sigma_{L}^{T} \tilde{\chi}^{T}\right] \hat{\theta}^{T}-\left[\alpha_{V}^{N} \tilde{u}^{N}+\sigma_{L}^{N} \tilde{\chi}^{N}\right] \hat{\theta}^{N} \tag{347}
\end{equation*}
$$

where we denote by $\chi^{j}=\frac{\frac{\alpha_{W}^{j}}{1-\alpha_{W}^{j}} \kappa^{j} \theta^{j}}{w_{R}^{j}}$, the share of the surplus associated with a labor contract in the marginal benefit of search and we computed the following expressions:

$$
\begin{aligned}
\mathrm{d} \ln \left(\frac{m^{j}}{s^{j}+m^{j}}\right) & =\alpha_{V}^{j} u^{j} \hat{\theta}^{j}, \\
\hat{w}_{R}^{j} & =\chi^{j} \hat{\theta}^{j} .
\end{aligned}
$$

Totally differentiating the vacancy creation-schedule (henceforth $V C j$ with $j=T, N$ ) in the traded and non traded sectors, given by eqs. (343c) and (343d), yields:

$$
\begin{align*}
\left.\hat{\theta}^{T}\right|^{V C T} & =\frac{A^{T}}{\left[\left(1-\alpha_{V}^{T}\right) \tilde{\Psi}^{T}+\tilde{\chi}^{T} w_{R}^{T}\right]} \hat{A}^{T},  \tag{348a}\\
\left.\hat{\theta}^{N}\right|^{V C N} & =\frac{P A^{N}\left(\hat{P}+\hat{A}^{N}\right)}{\left[\left(1-\alpha_{V}^{N}\right) \tilde{\Psi}^{N}+\tilde{\chi}^{N} w_{R}^{N}\right]} \tag{348b}
\end{align*}
$$

Inserting (348) into (347), and using the production functions to eliminate sectoral labor, i.e., $\hat{L}^{T}=\hat{Y}^{T}-\hat{A}^{T}$ and $\hat{L}^{N}=\hat{Y}^{N}-\hat{A}^{N}$, gives the labor market equilibrium schedule:

$$
\begin{align*}
\left.\left(\hat{Y}^{T}-\hat{Y}^{N}\right)\right|^{G M E}= & -\frac{P A^{N}\left[\alpha_{V}^{N} \tilde{u}^{N}+\sigma_{L}^{N} \tilde{\chi}^{N}\right]}{\left[\left(1-\alpha_{V}^{N}\right) \tilde{\Psi}^{N}+\tilde{\chi}^{N} w_{R}^{N}\right]} \hat{P}+\left(\sigma_{L}^{T}-\sigma_{L}^{N}\right) \hat{\bar{\lambda}} \\
+ & \left\{1+\frac{A^{T}\left[\alpha_{V}^{T} \tilde{u}^{T}+\sigma_{L}^{T} \tilde{\chi}^{T}\right]}{\left[\left(1-\alpha_{V}^{T}\right) \tilde{\Psi}^{T}+\tilde{\chi}^{T} w_{R}^{T}\right]}\right\} \hat{A}^{T} \\
& -\left\{1+\frac{P A^{N}\left[\alpha_{V}^{N} \tilde{u}^{N}+\sigma_{L}^{N} \tilde{\chi}^{N}\right]}{\left[\left(1-\alpha_{V}^{N}\right) \tilde{\Psi}^{N}+\tilde{\chi}^{N} w_{R}^{N}\right]}\right\} \hat{A}^{N} . \tag{349}
\end{align*}
$$

According to (349), the $L M E$-schedule is downward-sloping in the $\left(y^{T} / y^{N}, p\right)$-space and the slope of the $L M E$-schedule is equal to $-\frac{\left[\left(1-\alpha_{V}^{N}\right) \tilde{\Psi}^{N}+\tilde{\chi}^{N} w_{R}^{N}\right]}{P A^{N}\left[\alpha_{V}^{N} \tilde{u}^{N}+\sigma_{L}^{N} \tilde{\chi}^{N}\right]}$. Imposing $\sigma_{L}^{T}=\sigma_{L}^{N}$ implies that the $L M E$-schedule remains unaffected by the marginal utility of wealth. Assuming that the labor market parameters are similar across sectors, a productivity differential between tradables and non tradables shifts the $L M E$-schedule to the right, as in a model with limited substitutability of hours worked.

## J. 8 Relative Price Effect of a Productivity Differential

Equating (346) and (349) to eliminate $\hat{Y}^{T}-\hat{Y}^{N}$ yields the percentage deviation of the relative price of non tradables from its initial steady-state following a productivity differential between tradables and non tradables:

$$
\begin{equation*}
\hat{P}=\frac{\hat{A}^{T}\left(1+\Theta^{T}\right)-\hat{A}^{N}\left(1+\Theta^{N}\right)-\mathrm{d} v_{N X}}{\left(\phi+\Theta^{N}\right)} \tag{350}
\end{equation*}
$$

where we set

$$
\begin{align*}
\Theta^{T} & \equiv \frac{A^{T}\left[\alpha_{V}^{T} \tilde{u}^{T}+\sigma_{L}^{T} \tilde{\chi}^{T}\right]}{\left[\left(1-\alpha_{V}^{T}\right) \tilde{\Psi}^{T}+\tilde{\chi}^{T} \tilde{w}_{R}^{T}\right]}>0  \tag{351a}\\
\Theta^{N} & \equiv \frac{\tilde{P} A^{N}\left[\alpha_{V}^{N} \tilde{u}^{N}+\sigma_{L}^{N} \tilde{\chi}^{N}\right]}{\left[\left(1-\alpha_{V}^{N}\right) \tilde{\Psi}^{N}+\tilde{\chi}^{N} \tilde{w}_{R}^{N}\right]}>0 . \tag{351b}
\end{align*}
$$

In order to get further insight on the relative price effect of a productivity differential, it is useful to assume that labor markets display similar features across sectors so that $\Theta^{j}=\Theta$ (with $j=T, N$ ). In this case, eq. (351) reduces to

$$
\begin{equation*}
\hat{P}=\frac{(1+\Theta)}{(\phi+\Theta)}\left(\hat{A}^{T}-\hat{A}^{N}\right)-\frac{\mathrm{d} v_{N X}}{(\phi+\Theta)} \tag{352}
\end{equation*}
$$

Eq. (352) looks familiar as it is similar to the relative price effect described by eq. (27) in the main text when considering a model with limited substitutability of hours worked across sectors and abstracting from capital accumulation. In the latter framework, the term $\Theta$ is replaced with $\epsilon$ which measures the ease with which worked hours can be substituted for each other and thereby captures the degree of labor mobility across sectors. As long as $\phi>1$, a productivity differential between tradables and non tradables of $1 \%$ appreciates the relative price of non tradables by less than $1 \%$. The reason is as follows. Higher productivity in tradables and the labor inflow in that sector increases output of tradables relative to nontradables. For the market clearing condition to hold, the relative price of non-tradables must rise. With an elasticity of substitution $\phi$ greater than one, the demand for tradables rises more than proportionally so that $p$ must increase less than the productivity differential.

This configuration is depicted in Figure 6(a). Higher productivity in tradables relative to non tradables shifts the LME-schedule along the GME-schedule which is flatter than the 45 degree line. Hence, the relative price increases from $p_{0}$ to $p^{\prime}$ which is below than $p_{1}^{B S}$. As $\Theta$ is higher, the more the relative price increases following a productivity differential. The reason is that $\Theta$ captures the response of sectoral employment to a rise in labor market tightness triggered by the productivity shock. Because traded labor and thereby $Y^{T}$ rises more as $\Theta$ is higher, the relative price $P$ must appreciate by a larger amount to clear the goods market. Note that $\Theta$ is higher when labor supply at the extensive margin is more responsive to a rise in the reservation wage and the unemployment rate is initially higher.

When $\phi<1$, the relative price of non tradables must increase more than proportionately than the productivity differential to clear the the goods market. As depicted in Figure 6(b), because the $G M E$-schedule is steeper than the 45 degree line, the intersect of the two schedules (i.e., $p^{\prime}$ ) is above $p_{1}^{B S}$.

Because we restrict ourselves to the analysis of the long-run effects, the tilde is suppressed for the purposes of clarity. Introducing labor market frictions modifies the relative price equation in two respects:

- First, the term $\Theta$ depends on a set of sectoral parameters:

$$
\begin{equation*}
\Theta \equiv \frac{\Xi\left[\alpha_{V} u+\sigma_{L} \chi\right]}{\left[\left(1-\alpha_{V}\right) \Psi+\chi w_{R}\right]} . \tag{353}
\end{equation*}
$$



Figure 6: Effects of a Productivity Differential when $\phi>1$ and $\phi<1$.

These parameters are: the job destruction rate $s^{j}$ (because $u=\frac{s}{s+m}$ ), the elasticity of labor supply at the extensive margin $\sigma_{L}$, firing costs $x$ (because $\Psi \equiv \Xi+r^{\star} x-w_{R}$ ), unemployment benefits $R$ (because $\chi=\frac{\frac{\alpha_{W}}{1-\alpha_{W}} \kappa \theta}{w_{R}}$ ), and the worker's bargaining power $\alpha_{W}\left(\chi=\frac{\left(\frac{\alpha_{W}}{1-\alpha_{W}} \kappa \theta+R\right)}{w_{R}}\right.$ and $w_{R} \equiv \frac{\alpha_{W}}{1-\alpha_{W}} \kappa \theta+R$ is the reservation wage $)$.
We find that $\Theta$ decreases with: i) firing costs $x$ as the labor market tightness increases less (because firms post less job vacancies), and ii) the replacement rate as firms are less willing to post job vacancies because higher unemployment benefits lowers the surplus from hiring. ${ }^{66}$
We find that $\Theta$ increases with: i) the elasticity of labor supply at the extensive margin $\sigma_{L}$ as labor supply becomes more responsive to a rise in labor market tightness, ii) with the job destruction rate $s$ by raising the sensitivity of the employment rate to a rise in the labor market tightness (as reflected by the term $\alpha_{V} u$ ) because a higher $s$ reduces the initial employment rate and amplifies mechanically its growth rate.
We find that $\Theta$ may increase or decrease as the worker's bargaining power rises: on the one hand, by raising the share of the surplus associated with a labor contract in the marginal benefit of search $\chi$, a higher worker's bargaining power $\alpha_{W}$ implies that agents are more willing to search for a job (as reflected by a rise in the term $\sigma_{L \chi}$ ) which in turn raises further labor mobility across sectors; on the other hand, by lowering the surplus from hiring (as reflected by a rise in the term $\chi w_{R}$ ), higher $\alpha_{W}$ exerts a negative impact on labor demand and thus on job flows. Hence, higher worker's bargaining power $\alpha_{W}$ may raise or lower labor mobility across sectors. We find numerically that raising the worker bargaining power $\alpha_{W}$ tends to lower $\Theta$. The calibration and numerical results are available upon request.

- Second, while the dynamics degenerate in a model with imperfect mobility of labor across sectors along the lines of Horvath [2000], introducing labor market frictions restore transitional dynamics. Households hold financial wealth which consists of foreign assets and the value of firms in the traded and the non traded sector. Because labor $L^{j}$ becomes a state variable and hiring costs $\kappa^{j} V^{j}$ act like a cost of entry, profits are no longer driven

[^36]down to zero as shown by eq. (323), even in the long-run. More precisely, households own shares on domestic firms and receive dividends $\gamma^{j}$ per worker (see eq. (322)). Revenues from holding shares on domestic firms are equal to $\gamma^{j} L^{j}$ which correspond to the present discounted value of profits $\pi^{j}$.
By raising the marginal product of labor and thus the value of firm above the rate of return on traded bonds, a productivity shock triggers the accumulation of labor. Because the economy has perfect access to external borrowing, hiring of additional workers can be financed by running a current account deficit along the transitional path. For the intertemporal solvency condition to hold, the country must run a trade balance surplus in the long run. Increased net exports (i.e., $d v_{N X}>0$ ) raise the demand for tradables which in turn impinges on the relative price and the relative wage. Hence, a productivity differential affects $P$ through a labor accumulation channel captured by $\frac{\mathrm{d} v_{N X}}{(\phi+\Theta)}>0$.

## J. 9 Relative Wage Effect of a Productivity Differential

In order to build intuition about the effects of a productivity differential on sectoral wages and thereby on $\ln \left(w^{N} / w^{T}\right)$, we find it useful to characterize the labor market equilibrium both graphically and analytically. Because we restrict ourselves to the analysis of the long-run effects, the tilde is suppressed for the purposes of clarity.

Figure 7(a) depicts the labor market equilibrium in the traded sector can be summarized by two schedules: ${ }^{67}$

$$
\begin{equation*}
L^{T}=\frac{m^{T}}{m^{T}+s^{T}}\left(\bar{\lambda} w_{R}^{T}\right)^{\sigma_{L}^{T}}, \quad \frac{\kappa^{T}}{f^{T}}=\frac{\left(1-\alpha_{W}^{T}\right)\left[\left(A^{T}+r^{\star} x^{T}\right)-w_{R}^{T}\right]}{s^{T}+r^{\star}} \tag{354}
\end{equation*}
$$

where $w_{R}^{T} \equiv\left(\frac{\alpha_{W}^{T}}{1-\alpha_{W}^{T}} \kappa^{T} \theta^{T}+R^{T}\right)$ is the reservation wage in the traded sector. The first equality in (354) represents the decision of search schedule in the traded sector (henceforth $D S T$ ) which is upward-sloping in the $\left(\theta^{T}, L^{T}\right)$-space. The reason is that a rise in the labor market tightness raises the probability of finding a job and thus increases employment $L^{T}$ by reducing the number of job seekers. Moreover, because we consider an endogenous labor force participation decision, the consecutive increase in the reservation wage induces agents to supply more labor. The second equality in (354) represents the vacancy creation schedule (henceforth $V C T$ ) which is a vertical line in the $\left(\theta^{T}, L\right)$-space. ${ }^{68}$

By raising the surplus from hiring, a rise in labor productivity in the traded sector $A^{T}$ shifts the $V C T$-schedule to the right from $V C T_{0}$ to $V C T_{1}$. Because traded firms post more job vacancies, the labor market tightness $\theta_{1}^{T}$ exceeds its initial level $\theta_{0}^{T} .{ }^{69}$ While increased labor market tightness raises traded employment by pushing up the reservation wage and reducing unemployment, the positive wealth effect moderates the expansionary effect on labor supply. Graphically, the fall in $\bar{\lambda}$ shifts to the right the $D S T$-schedule. The new steady state is $E_{1}^{T}$.

Since we are interested in the movement of sectoral wages, it is useful to explore the long-run adjustment in the traded wage following a rise in labor productivity $A^{T}$. The labor market in the traded sector can alternatively be summarized graphically in the ( $\theta^{T}, w^{T}$ )-space as shown in Figure $8(\mathrm{a})$. Using the fact $\left(1-\alpha_{W}^{T}\right) \Psi^{T}=A^{T}-w^{T} s^{T}-w^{T}$, the $V C T$-schedule is downward sloping and convex toward the origin, reflecting diminishing returns in vacancy
${ }^{67}$ Totally differentiating the $D S T$ - and $V C T$-schedule yields:

$$
\hat{L}^{T}=\sigma_{L}^{T} \hat{\bar{\lambda}}+\left[\alpha_{V}^{T} u^{T}+\sigma_{L}^{T} \chi^{T}\right] \hat{\theta}^{T}, \quad \hat{\theta}^{T}=\frac{A^{T}}{\left[\left(1-\alpha_{V}^{T}\right) \tilde{\Psi}^{T}+\tilde{\chi}^{T} w_{R}^{T}\right]} \hat{A}^{T} .
$$

The slope of the $D S T$-schedule in the $\left(\theta^{T}, L^{T}\right)$-space is given by $\left[\alpha_{V}^{T} u^{T}+\sigma_{L}^{T} \chi^{T}\right]>0$.
${ }^{68}$ Note that Figure 7(a) depicts the logarithm form of the system (354).
${ }^{69}$ Note that $\theta^{T}$ jumps immediately to its new higher steady-state level while traded employment builds up over time along the isocline $\dot{\theta}^{T}=0$ until the economy reaches the new steady-state.
creation. ${ }^{70}$ The wage setting-schedule ( $W S T$ henceforth) is upward sloping (see eq. (328)). ${ }^{71}$ A rise in $A^{T}$ shifts the $V C T$-schedule to the right by stimulating labor demand which exerts an upward pressure on the the traded wage. Because workers get a fraction $\alpha_{W}^{T}$ of the increased surplus, the productivity shock shifts the $W S T$-schedule to the left. ${ }^{72}$ Hence, the new steady-state at $F_{1}^{T}$ is associated with a higher traded wage. The higher the worker bargaining power, the larger the shift of the $W S T$ curve and thereby the more $w^{T}$ increases. To see it formally, totally differentiating the Nash bargaining traded wage and eliminating $\theta^{T}$ by using the vacancy creation schedule (i.e., the second equality (354)) yields the deviation in percentage of the traded wage from its initial steady state: ${ }^{73}$

$$
\begin{equation*}
\hat{w}^{T}=\Omega^{T} \hat{A}^{T}>0, \quad \Omega^{T}=\frac{\alpha_{W}^{T}\left[\left(1-\alpha_{V}^{T}\right)\left(s^{T}+r^{\star}\right)+m^{T}\right]}{\left[\left(1-\alpha_{V}^{T}\right)\left(s^{T}+r^{\star}\right)+\alpha_{W}^{T} m^{T}\right]} \frac{A^{T}}{w^{T}}, \tag{355}
\end{equation*}
$$

where $\Omega^{T}>0$ represents the sensitivity of the traded wage to a change in the labor productivity index $A^{T}$.

We now turn to the non traded labor market equilibrium depicted in Figure 7(b) which is summarized by two schedules: ${ }^{74}$

$$
\begin{equation*}
L^{N}=\frac{m^{N}}{m^{N}+s^{N}}\left(\bar{\lambda} w_{R}^{N}\right)^{\sigma_{L}^{N}}, \quad \frac{\kappa^{N}}{f^{N}}=\frac{\left(1-\alpha_{W}^{N}\right)\left[\left(P\left(L^{N}, \bar{\lambda}, A^{N}\right) A^{N}+r^{\star} x^{N}\right)-w_{R}^{N}\right]}{s^{N}+r^{\star}}, \tag{356}
\end{equation*}
$$

where we have inserted the short-run static solution for the relative price of non tradables (333) and $w_{R}^{N} \equiv\left(\frac{\alpha_{W}^{N}}{1-\alpha_{W}^{N}} \kappa^{N} \theta^{N}+R^{N}\right)$ is the reservation wage in the non traded sector. While the first equality in (356) represents the decision of search schedule (henceforth $D S N$ ) which is upward-sloping in the $\left(\theta^{N}, L^{N}\right)$-space, the second equality in (356) corresponds to the decision of search schedule in the non traded sector (henceforth $D S N$ ). ${ }^{75}$ As depicted in Figure $7(\mathrm{~b})$, the $V C N$-schedule is downward-sloping in the $\left(\theta^{N}, L^{N}\right)$-space. The reason is as follows. Because an increase in non traded labor raises output of this sector, the relative price

```
    \({ }^{70}\) The slope of the \(V C T\)-schedule in the \(\left(\theta^{T}, w^{T}\right)\)-space is:
        \(\left.\frac{\mathrm{d} w^{T}}{\mathrm{~d} \theta^{T}}\right|^{V C T}=-\frac{\left(s^{T}+r^{\star}\right) \kappa^{T}\left(1-\alpha_{V}^{T}\right)}{f^{T} \theta^{T}}=-\frac{\left(1-\alpha_{W}^{T}\right) \Psi^{T}\left(1-\alpha_{V}^{T}\right)}{\theta^{T}}<0\).
    \({ }^{71}\) Using the fact that \(\left(F^{T}\right)^{1 / \sigma_{L}^{T}} / \bar{\lambda}=w_{R}^{T}\), the \(W S T\)-schedule is \(w^{T}=\alpha_{W}^{T}\left(A^{T}+r^{\star} x^{T}\right)+\left(1-\alpha_{W}^{T}\right) w_{R}^{T}\) with a slope in the \(\left(\theta^{T}, w^{T}\right)\)-space given by:
```

$$
\left.\frac{\mathrm{d} w^{T}}{\mathrm{~d} \theta^{T}}\right|^{W S T}=\frac{\left(1-\alpha_{W}^{T}\right) \chi^{T} w_{R}^{T}}{\theta^{T}}=\alpha_{W}^{T} \kappa^{T}>0
$$

[^37]$$
\hat{L}^{N}=\sigma_{L}^{N} \hat{\bar{\lambda}}+\left[\alpha_{V}^{N} u^{N}+\sigma_{L}^{N} \chi^{N}\right] \hat{\theta}^{N},
$$

The slope of the $D S N$-schedule in the $\left(\theta^{N}, L^{N}\right)$-space is given by $\left[\alpha_{V}^{N} u^{N}+\sigma_{L}^{N} \chi^{N}\right]>0$. Totally differentiating $V C N$-schedule gives:

$$
\hat{\theta}^{N}\left[\left(1-\alpha_{V}^{T}\right) \tilde{\Psi}^{T}+\tilde{\chi}^{T} w_{R}^{T}\right]\left[\left(1-\alpha_{C}\right) \phi+\alpha_{C} \sigma_{C}\right]=-P A^{N}\left\{\hat{L}^{N}+\sigma_{C} \hat{\bar{\lambda}}+\left\{1-\left[\left(1-\alpha_{C}\right) \phi+\alpha_{C} \sigma_{C}\right]\right\} \hat{A}^{N}\right\} .
$$

The slope of the $V C N$-schedule is negative and given by $-\left[\left(1-\alpha_{V}^{T}\right) \tilde{\Psi}^{T}+\tilde{\chi}^{T} w_{R}^{T}\right]\left[\left(1-\alpha_{C}\right) \phi+\alpha_{C} \sigma_{C}\right] /\left(P A^{N}\right)<$ 0.
${ }^{75}$ Note that whether we consider the traded or the non traded sector, the same logic applies to explain the positive relationship between the employment and the labor market tightness along the $D S j$-schedule (with $j=T, N)$.


Figure 7: Effects of a Productivity Differential and the Stable Adjustment


Figure 8: Long-Run Sectoral Wage Effects of a Productivity Differential
of non tradables must depreciate for the market clearing condition (330) to hold. The fall in $P$ drives down the surplus from hiring an additional worker in the non traded sector which results in a decline in labor market tightness $\theta^{N}$ as firms post less job vacancies.

Imposing $\sigma_{C}=1$, a rise in $A^{N}$ raises the surplus from hiring if and only if the elasticity of substitution $\phi$ between traded and non traded goods is larger than one. The reason is that only in this case, the share of non tradables in total expenditure rises which results in an expansionary effect on labor demand in the non traded sector. In Figure 7(b), we assume that $\sigma_{C}=\phi=1$, so that the productivity shock does no impinge on the vacancy creation decision because the share of non tradables remains unchanged. Yet, by producing a positive wealth effect, higher labor productivity of non tradables shifts the VSN-schedule to the right by inducing agents to consume more which in turn raises $P$ and thereby the surplus from hiring. The fall in the shadow value of wealth also shifts the $D S N$-schedule to the right as agents are induced to supply less labor. While $\theta^{N}$ is unambiguously higher at the new steady-state $E_{1}^{N}$, the positive wealth effect exerts two conflictory effects on $L^{N}$. In Figure 7(b), non traded employment falls; note that $L^{N}$ may rise as well (but less than $L^{T}$ ).

We now explore the long-run adjustment in the non traded wage which is depicted in Figure 8(b). As for the traded sector, the $W S N$-schedule is upward sloping while the $V C N$-schedule is downward sloping. ${ }^{76}$ Before analyzing in more details the effects of a productivity shock on the non traded wage, it is convenient to determine analytically the long-run response of $w^{N}$. Totally differentiating the wage setting decision in the non traded sector, and eliminating $\hat{\theta}^{N}$ by using the vacancy creation decision equation yields the deviation in percentage of the non traded wage from its initial steady state: ${ }^{77}$

$$
\begin{equation*}
\hat{w}^{N}=\Omega^{N}\left(\hat{P}+\hat{A}^{N}\right), \quad \Omega^{N}=\frac{\alpha_{W}^{N}\left[\left(1-\alpha_{V}^{N}\right)\left(s^{N}+r^{\star}\right)+m^{N}\right]}{\left[\left(1-\alpha_{V}^{N}\right)\left(s^{N}+r^{\star}\right)+\alpha_{W}^{N} m^{N}\right]} \frac{P A^{N}}{w^{N}} . \tag{357}
\end{equation*}
$$

According to (357), the combined effects of higher labor productivity $A^{N}$ and the appreciation of the relative price of non tradables pushes up the non traded wage in the long-run. Inserting the long-run change in the equilibrium value of the relative price of non tradables (i.e., eq. (350)) and using the fact that $\chi^{N} w_{R}^{N}=m^{N} \frac{\alpha_{N}^{N} \Psi^{N}}{s^{N}+r^{\star}}$ allows us to rewrite (357) as follows:

$$
\begin{equation*}
\hat{w}^{N}=\frac{\Omega^{N}}{\left(\phi+\Theta^{N}\right)}\left[\hat{A}^{T}\left(1+\Theta^{T}\right)+\hat{A}^{N}(\phi-1)-\mathrm{d} v_{N X}\right] . \tag{358}
\end{equation*}
$$

Assuming an elasticity of substitution $\phi$ equal to one, labor productivity in the non traded sector does no longer impinge on $w^{N}$. In this case, the change in the non traded wage is only driven by $\hat{A}^{T}>0$ which appreciates the relative price of non traded goods and thereby stimulates labor demand in that sector. Further assuming that labor market parameters are similar across sectors so that $\Theta^{j} \simeq \Theta$ and $\Omega^{j} \simeq \Omega$ (with $j=T, N$ ), we find that the non traded wage is equal to $\Omega\left(\hat{A}^{T}-\mathrm{d} v_{N X}\right)$. By producing a long-run improvement in the trade
${ }^{76}$ Using the fact $\left(1-\alpha_{W}^{N}\right) \Psi^{N}=P A^{N}+r^{\star} x^{N}-w^{N}$, the wage setting and vacancy creation decisions are described by the following equalities:

$$
w^{N}=\alpha_{W}^{N}\left(P A^{N}+r^{\star} x^{N}\right)+\left(1-\alpha_{W}^{N}\right) w_{R}^{N}, \quad w^{N}=\left(P A^{N}+r^{\star} x^{N}\right)-\frac{\kappa^{N}\left(s^{N}+r^{\star}\right)}{f^{N}} .
$$

${ }^{77}$ To get (357), we first solve for the change in the labor market tightness $\hat{\theta}^{N}=\frac{P A^{N}\left(\hat{P}+\hat{A}^{N}\right)}{\left[\left(1-\alpha_{V}^{N}\right) \Psi^{N}+\chi^{N} w_{R}^{N}\right]}$. Totally differentiating the Nash bargaining non traded wage and plugging $\hat{\theta}^{N}$, we get:

$$
\hat{w}^{N}=\frac{P A^{N}}{w^{N}}\left\{\frac{\alpha_{W}^{N}\left(1-\alpha_{V}^{N}\right) \Psi^{N}+\chi^{N} w_{R}^{N}}{\left(1-\alpha_{V}^{N}\right) \Psi^{N}+\chi^{N} w_{R}^{N}}\right\}\left(\hat{P}+\hat{A}^{N}\right) .
$$

Using the fact that $\chi^{N} w_{R}^{N}=m^{N} \frac{\alpha_{N_{N}^{N}}^{N} \Psi^{N}}{s^{N}+r^{\star}}$, the above equation educes to eq. (357).
balance $N X$ and thereby stimulating the demand for tradables, a productivity shock exerts a negative impact on the relative wage $w^{N} / w^{T}$. As depicted in Figure 8(b), due to the labor accumulation effect, a productivity shock biased toward the traded sector induces smaller shifts in the $V C N$ - and the $W S N$-schedule. ${ }^{78}$

To analyze the change in the relative wage $\omega \equiv w^{N} / w^{T}$, it is useful to subtract eq. (355) from eq. (357) by inserting first the change in the relative price given by eq. (350) in order to break down the change in percentage of the relative wage into three components:

$$
\begin{equation*}
\hat{\omega}=-\left(\Omega^{T} \hat{A}^{T}-\Omega^{N} \hat{A}^{N}\right)+\Omega^{N}\left[\frac{\hat{A}^{T}\left(1+\Theta^{T}\right)-\hat{A}^{N}\left(1+\Theta^{N}\right)}{\left(\phi+\Theta^{N}\right)}\right]-\frac{\mathrm{d} v_{N X}}{\left(\phi+\Theta^{N}\right)} \tag{359}
\end{equation*}
$$

In a model abstracting from labor market frictions, $\Theta^{j}$ tends toward infinity while $\Omega^{j}$ reduces to one. In this case, we have $\hat{w}^{T}=\hat{A}^{T}$ and $\hat{w}^{N}=\hat{P}+\hat{A}^{N}$ where $\hat{P}=\hat{A}^{T}-\hat{A}^{N}$ so that the relative wage $\omega$ remains unaffected by a productivity differential (i.e., $\hat{\omega}=0$ ). Put otherwise, the assumption of perfect mobility of labor across sectors implies that agents are willing to devote their whole time in the sector that pay highest wages so that sectors must pay the same wage. By producing imperfect mobility of labor across sectors, sectoral wages do no longer equalize so that the relative wage may change in the long-run. In order to have a sense of the change in the relative wage that a productivity differential might generate, it is useful to set $\Omega^{j} \simeq \Omega$ and $\Theta^{j} \simeq \Theta$; in this case, eq. (359) reduces to:

$$
\begin{equation*}
\hat{\omega}=-\Omega\left[\left(\frac{\phi-1}{\phi+\Theta}\right)\left(\hat{A}^{T}-\hat{A}^{N}\right)+\frac{\mathrm{d} v_{N X}}{(\phi+\Theta)}\right] \tag{360}
\end{equation*}
$$

where $\Theta \equiv \frac{\Xi\left(s+r^{\star}\right)}{\Psi}\left[\frac{\alpha_{V} u+\sigma_{L} \chi}{\left(1-\alpha_{V}\right)\left(s+r^{\star}\right)+\alpha_{W} m}\right]>0$. According to the labor market frictions effect captured by $-\Omega\left(\frac{\phi-1}{\phi+\Theta}\right)\left(\hat{A}^{T}-\hat{A}^{N}\right)$, a productivity differential between tradables and non tradables lowers the relative wage $w^{N} / w^{T}$ only if the elasticity of substitution $\phi$ between traded and non traded goods is larger than one. As mentioned above, in this configuration, a productivity shock biased toward the traded sector raises the share of tradables in total expenditure and thereby has an expansionary effect on labor demand in the traded sector relative to the non traded sector. Consequently, the productivity differential pushes up the traded wage relative to the non traded wage; furthermore, $\omega$ falls less as $\Theta$ gets larger; inspection of $\Theta$ indicates that a higher unemployment rate $u$, a smaller firing cost (which reduces the surplus from hiring $\Psi$ ), a more responsive labor supply captured by $\sigma_{L}$, or lower unemployment benefits reflected by a higher $\chi$ moderates the labor market frictions effect by raising $\Theta$ that captures the extent of labor mobility across sectors.

The second term on the RHS of eq. (360), i.e., $-\Omega \frac{\mathrm{d} v_{N X}}{(\phi+\Theta)}<0$, captures the labor accumulation effect. By raising the demand for tradables in the long-run, a productivity differential drives down the relative wage, less so the larger $\phi$ and $\Theta$.

To conclude, we have to consider three cases depending on whether the elasticity of substitution between traded and non traded goods is equal, larger or smaller than one:

- If $\phi=1$, the share of non tradables remains unchanged so that a productivity differential yields similar effects to those found in the standard BS effect. However, due to the imperfect mobility of labor across sectors, increased demand for tradables triggered by the long-run trade balance exerts a negative impact on $P$ and $\omega$; hence, along the labor accumulation channel the relative price increases less than proportionately while the relative wage declines.
- If $\phi>1$, keeping unchanged net exports, the increased share of tradables produces a labor market frictions effect which stimulates labor demand in the traded sector and

[^38]thereby drives further the relative wage while the relative price appreciates less. Hence, the labor market frictions effect reinforces the labor accumulation channel.

- When $\phi<1$, the long-run adjustments in the relative price and the relative wage are the result of two conflictory effects. On the one hand, the increased share of non tradables has an expansionary effect on labor demand in the non traded sector which impinges positively on the relative wage and the relative price, in contradiction with our evidence. On the other hand, the long-run improvement in the balance of trade yields opposite effects on $P$ and $\omega$.

Because in the latter case (i.e., $\phi<1$ ), the labor accumulation effect counteracts the labor market frictions effect, we have to determine numerically if the former is large enough to produce a decline in the relative wage and a less than proportional increase in the relative price following a productivity differential between tradables and non tradables. Our preliminary numerical results confirm that across all scenarios, the labor accumulation effect more than offsets the labor market frictions effect.

## K Degree of Substitutability of Hours Worked across Sectors $\epsilon$ : Empirical strategy

In this section, we detail our empirical strategy to estimate the extent of substitutability of hours worked $\epsilon$ which captures the degree of labor mobility across sectors.

## K. 1 Limited Substitutability of Hours Worked across Sectors and the Derivation of the Testable Equation

To determine the equation we explore empirically, we follow closely Horvath [2000]. The representative agent is endowed with one unit of time, supplies a fraction $L(t)$ as labor, and consumes the remainder $1-L(t)$ as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Assuming that the felicity function is additively separable in consumption and labor, the representative household maximizes the following objective function:

$$
\begin{equation*}
U=\int_{0}^{\infty}(1-\gamma) \ln C(t)+\gamma \ln (1-L(t)) e^{-\rho t} \mathrm{~d} t \tag{361}
\end{equation*}
$$

subject to

$$
\begin{equation*}
\dot{A}(t)=r^{\star} A(t)+W(t) L(t)-P_{C}(P(t)) C(t) \tag{362}
\end{equation*}
$$

For the sake of clarity, we drop the time argument below when this causes no confusion. First-order conditions are:

$$
\begin{gather*}
\frac{1-\gamma}{C}=\left(P_{C} \lambda\right)  \tag{363a}\\
\frac{\gamma}{1-L}=W \lambda  \tag{363b}\\
\dot{\lambda}=\lambda\left(\beta-r^{\star}\right) \tag{363c}
\end{gather*}
$$

The economic system consists of $M$ distinct sectors, indexed by $j=0,1, \ldots, M$ each producing a different good. Along the lines of Horvath [2000], the aggregate leisure index is assumed to take the form:

$$
\begin{equation*}
1-L(.)=1-\left[\sum_{j=1}^{M}\left(L^{j}\right)^{\frac{\epsilon+1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon+1}} \tag{364}
\end{equation*}
$$

The agent maximizes (364) subject to

$$
\begin{equation*}
\sum_{j=1}^{M} W^{j} L^{j}=X \tag{365}
\end{equation*}
$$

where $L^{j}$ is labor supply in sector $j, W^{j}$ the wage rate in sector $j$ and $X$ total labor income. Applying standard methods, we obtain labor supply $L^{j}$ in sector $j$ :

$$
\begin{equation*}
L^{j}=\left(\frac{W^{j}}{W}\right)^{\epsilon} L \tag{366}
\end{equation*}
$$

where we used the fact that $X=W L$.
Combining (363a) and (363b), the aggregate wage index is:

$$
\begin{equation*}
W=\frac{\gamma}{1-\gamma} \frac{P_{C} C}{1-L} \tag{367}
\end{equation*}
$$

which allows us to rewrite (366) as follows:

$$
\begin{equation*}
L^{j}=\left(W^{j}\right)^{\epsilon} L\left(\frac{\gamma}{1-\gamma} \frac{P_{C} C}{1-L}\right)^{-\epsilon} \tag{368}
\end{equation*}
$$

A quantity $Q^{j}$ of good $j$ is produced by combining capital, $K^{j}$, labor devoted to the sector, $L^{j}$, and intermediate inputs, $I M^{j}$, in a production process described by:

$$
\begin{equation*}
Q^{j}=Z^{j}\left(L^{j}\right)^{\xi^{j}}\left(K^{j}\right)^{\gamma^{j}}\left(I M^{j}\right)^{1-\xi^{j}-\gamma^{j}} \tag{369}
\end{equation*}
$$

where $\xi^{j}\left(\gamma^{j}\right)$ is the share of labor (capital) income in gross output of sector $j$.
We assume that labor is imperfectly mobile across sectors, while imposing perfect capital mobility. Perfectly competitive firms in sector $j$ seek to maximize the profit function given by:

$$
\begin{equation*}
\Pi^{j}=P^{j} Q^{j}-W^{j} L^{j}-R K^{j}-P_{I M} I M^{j}, \tag{370}
\end{equation*}
$$

where $P^{j}$ is the price of gross output, $R$ is the user capital cost, $W^{j}$ the wage rate in sector $j$, and $P_{I M}$ the price of intermediate inputs. Firms take the wage rate (capital rental cost) as given and equate the labor's (capital's) marginal product to the wage (rental rate cost) to determine demand. First-order conditions are:

$$
\begin{equation*}
P^{j} \frac{\xi^{j} Q^{j}}{L^{j}}=W^{j}, \quad P^{j} \frac{\gamma^{j} Q^{j}}{K^{j}}=R, \quad P^{j} \frac{\left(1-\xi^{j}-\gamma^{j}\right) Q^{j}}{I M^{j}}=P_{I M} . \tag{371}
\end{equation*}
$$

Eliminating the sectoral wage $W^{j}$ into (368) by using labor demand given by (371), the equilibrium condition for labor is given by:

$$
\begin{equation*}
L^{j}=\left(\xi^{j} P^{j} Q^{j}\right)^{\frac{\epsilon}{\epsilon+1}} L^{\frac{1}{1+\epsilon}}\left(\frac{\gamma}{1-\gamma} \frac{P_{C} C}{1-L}\right)^{-\frac{\epsilon}{\epsilon+1}} \tag{372}
\end{equation*}
$$

Summing over the $M$ sectors and using (364), we get:

$$
\left(\frac{\gamma}{1-\gamma} \frac{P_{C} C}{1-L}\right)=\frac{\sum_{j=1}^{M} \theta^{j} P^{j} Q^{j}}{L}
$$

Plugging this equation into (372) yields:

$$
\begin{equation*}
L^{j}=\left(\frac{\xi^{j} P^{j} Q^{j}}{\sum_{j=1}^{M} \xi^{j} P^{j} Q^{j}}\right)^{\frac{\epsilon}{\epsilon+1}} L . \tag{373}
\end{equation*}
$$

As in Horvath [2000], we denote by $\beta^{j}$ the fraction of labor's share of aggregate output accumulating to labor in sector $j$ :

$$
\begin{equation*}
\beta^{j}=\frac{\xi^{j} P^{j} Q^{j}}{\sum_{j=1}^{M} \xi^{j} P^{j} Q^{j}} . \tag{374}
\end{equation*}
$$

We introduce the time subscript to avoid confusion. Expressing (373) in percentage changes and adding an estimation error term $\nu$ results in the $M$ estimation equations:

$$
\begin{equation*}
\hat{l}_{t}^{j}-\hat{l}_{t}=\frac{\epsilon}{\epsilon+1} \hat{\beta}_{t}^{j}+\nu_{t}^{j}, \quad j=1, \ldots, M, \tag{375}
\end{equation*}
$$

where

$$
\begin{equation*}
\hat{l}_{t}=\sum_{j=1}^{M} \beta_{t-1}^{j} \hat{l}_{t}^{j} \tag{376}
\end{equation*}
$$

To derive (376), we proceed as follows. Because we consider a traded and a non traded sectors, the labor index (364) can be rewritten as follows:

$$
\begin{equation*}
L\left(L_{t}^{T}, L_{t}^{N}\right)=\left[\left(L_{t}^{T}\right)^{\frac{\epsilon+1}{\epsilon}}+\left(L_{t}^{N}\right)^{\frac{\epsilon+1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon+1}} . \tag{377}
\end{equation*}
$$

Approximate changes in aggregate labor with differentials, we get:

$$
\begin{equation*}
\mathrm{d} L_{t} \equiv L_{t}-L_{t-1}=\left(L_{t-1}^{T}\right)^{\frac{1}{\epsilon}}\left(L_{t-1}\right)^{-\frac{1}{\epsilon}} \mathrm{~d} L_{t}^{T}+\left(L_{t-1}^{N}\right)^{\frac{1}{\epsilon}}\left(L_{t-1}\right)^{-\frac{1}{\epsilon}} \mathrm{~d} L_{t}^{N} . \tag{378}
\end{equation*}
$$

Expressing into (378) in percentage changes and inserting (373), i.e., $\left(\frac{L^{j}}{L}\right)^{\frac{\epsilon+1}{\epsilon}}=\beta^{j}$, we have:

$$
\begin{align*}
\hat{l}_{t} \equiv \frac{L_{t}-L_{t-1}}{L_{t-1}} & =\left(\frac{L_{t-1}^{T}}{L_{t-1}}\right)^{\frac{\epsilon+1}{\epsilon}} \hat{l}_{t}^{T}+\left(\frac{L_{t-1}^{N}}{L_{t-1}}\right)^{\frac{\epsilon+1}{\epsilon}} \hat{l}_{t}^{N} \\
& =\beta_{t-1}^{T} \hat{l}_{t}^{T}+\beta_{t-1}^{N} \hat{l}_{t}^{N} . \tag{379}
\end{align*}
$$

According to eq. (379), the percentage change in total hours worked, $\hat{l}_{t}$, can be approximated by a weighted average of changes in sectoral hours worked $\hat{l}_{t}^{j}$ (in percentage), the weight being equal to $\beta_{t-1}^{j}$.

Combining optimal rules for labor supply and labor demand, we find that the change in employment in sector $j$ is driven by the change of the fraction $\beta^{j}$ of the labor's share of aggregate output accumulating to labor in sector $j$. We use panel data to estimate (375). Including country fixed effects captured by country dummies, $f_{i}$, and common macroeconomic shocks by year dummies, $f_{t}$, (375) can be rewritten as follows:

$$
\begin{equation*}
\hat{l}_{i, t}^{j}-\hat{l}_{i, t}=f_{i}+f_{t}+\gamma_{i} \hat{\beta}_{i, t}^{j}+\nu_{i, t}^{j}, \tag{380}
\end{equation*}
$$

where $\gamma_{i}=\frac{\epsilon_{i}}{\epsilon_{i}+1}$ and $\beta_{i, t}^{j}$ is given by (374); $j$ indexes the sector, $i$ the country, and $t$ indexes time. When exploring empirically (380), the coefficient $\gamma$ is alternatively assumed to be identical, i.e., $\gamma_{i}=\gamma$, or to vary across countries. The LHS term of (380), i.e., $\hat{l}_{i, t}^{j}-\hat{l}_{i, t}$, gives the percentage change in hours worked in sector $j$ driven by the pure reallocation of labor across sectors.

## K. 2 Data Description

Data are taken from EU KLEMS database (the March 2011 data release). EU KLEMS data provide yearly information for the period 1970-2007 (except for Japan: 1974-2007) for the 14 countries of our sample (BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, KOR, NLD, SWE and USA). To classify employment and gross output as traded or non traded, we adopt the classification described in section A.1. We provide more details below about the data used to estimate equation (380) (we indicate the code in EU KLEMS in parentheses)

- Sectoral labor $L_{t}^{j}(j=T, N)$ : total hours worked by persons engaged in sector $j$ (H_EMP).
- Sectoral nominal gross output $P_{t}^{j} Q_{t}^{j}(j=T, N)$ : gross output at current prices in millions of national currency in sector $j$ (GO).
- Sectoral share of labor income in gross output $\xi^{j}$ for $j=T, N$ : labor compensation in sector $j$ (LAB) over gross output at current prices in that sector (GO) averaged over the period 1970-2007 (1974-2007 for Japan).

By combining $\xi^{j}$ and $P_{t}^{j} Q_{t}^{j}$, we can construct time series $\beta_{t}^{j}$ defined by (374).

## K. 3 Exogeneity of the Regressor

By using optimal rules for both labor supply (366) and labor demand (371), we avoid any endogeneity problem. To see it more clearly, when restricting our attention to the optimal labor supply schedule without using firms' first order conditions, eq. (366) in percentage changes is:

$$
\begin{equation*}
\hat{l}_{t}^{j}-\hat{l}_{t}=\epsilon\left(\hat{w}_{t}^{j}-\hat{w}_{t}\right) \tag{381}
\end{equation*}
$$

where $\hat{l}_{t}$ is given by (379). An endogeneity problem may arise because to construct time series for sectoral wages $W_{t}^{j}$, we have to divide the labor compensation $W_{t}^{j} L_{t}^{j}$ in sector $j$ by sectoral hours worked $L_{t}^{j}$; likewise, we have to divide the overall labor compensation $W_{t} L_{t}$ by total hours worked $L_{t}$ to construct time series for the aggregate wage index $W_{t}$. A way to circumvent any endogeneity problem is to use labor demand $\frac{\xi^{j} P_{t}^{j} Q_{t}^{j}}{L_{t}^{j}}=W_{t}^{j}$ to eliminate the sectoral wage from eq. (381), and $W_{t}=\frac{\sum_{j} \xi^{j} P_{t}^{j} Q_{t}^{j}}{L_{t}}$ to eliminate the aggregate wage index; we get $L_{t}^{j} / L_{t}=\left(\frac{\xi^{j} P_{t}^{j} Q_{t}^{j}}{L_{t}^{j}} / \frac{\sum_{j} \xi^{j} P_{t}^{j} Q_{t}^{j}}{L_{t}}\right)^{\epsilon}$. Isolating $L_{t}^{j} / L_{t}$ and differentiating yields (375). Because wages do not show up in eq. (381) as we use the labor income share which is constant over time and gross output (at current prices), we avoid any endogeneity problem. More precisely, the labor's share in gross output $\xi^{j}$ in sector $j$ is defined as the ratio of the compensation of employees to gross output in the $j$ th sector, averaged over the period 1970-2007 so that the explanatory variable (i.e., the RHS term in eq. (380)) is constructed independently from the dependent variable (i.e., the LHS term in eq. (380)).

To check that endogeneity is not a major issue in eq. (380), we test for strict exogeneity of the regressor with respect to the dependent variable. Engle et al. [1983] refer to a variable $x_{t}$ as strongly exogenous with respect to the variable $y_{t}$ if $y_{t}$ does not Granger-cause $x_{t}$ (see Granger [1969]). Formally, $y_{t}$ Granger causes $x_{t}$ if its past value can help to predict the future value of $x_{t}$ beyond what could have been done with the past value of $x_{t}$ only. To implement the test of whether $\left(\hat{l}_{i, t}^{j}-\hat{l}_{i, t}\right)$ (i.e., the LHS term in eq. (380)) Granger-causes $\hat{\beta}_{i, t}^{j}$ (i.e., the RHS term in eq. (380)) we run the following regression:

$$
\begin{equation*}
\hat{\beta}_{i, t}^{j}=\alpha_{i}^{j}+\sum_{i=1}^{p} a_{i, p}^{j} \hat{\beta}_{i, t-p}^{j}+\sum_{i=1}^{p} b_{i, p}^{j}\left(\hat{L}_{i, t}^{j}-\hat{L}_{i, t}\right)+u_{i, t}^{j}, \tag{382}
\end{equation*}
$$

where $p$ is the autoregressive lag length and $u_{i, t}^{j}$ the error term. With respect to (382), in country $i$ sector $j$, the test of the null hypothesis that $\left(\hat{l}_{i, t}^{j}-\hat{l}_{i, t}\right)$ does not Granger cause $\hat{\beta}_{i, t}^{j}$ is a $F$ test of the form: $H_{0}: b_{i, 1}^{j}=b_{i, 2}^{j}=\cdots=b_{i, p}^{j}=0$. By not rejecting the null, one may conclude that the regressor in (380) is strictly exogenous to the dependent variable $\left(\hat{l}_{i, t}^{j}-\hat{l}_{i, t}\right)$.

The results of causality tests for $p=1,2,3$ from the change in hours worked in sector $j$ driven by the pure reallocation of labor across sectors $\left(\hat{l}_{i, t}^{j}-\hat{l}_{i, t}\right)$ to the fraction of labor's share of aggregate output accumulating to labor in sector $j\left(\hat{\beta}_{i, t}^{j}\right)$ are displayed in Table 17. The

Table 17: Granger Causality Test (p-values)

| Country | Sector | $p=1$ | $p=2$ | $p=3$ | Country | Sector | $p=1$ | $p=2$ | $p=3$ |
| :--- | :---: | :---: | :---: | :---: | :--- | :---: | :---: | :---: | :---: |
| BEL | T | 0.263 | 0.934 | 0.206 | ITA | T | 0.481 | 0.303 | 0.054 |
| BEL | N | 0.655 | 0.962 | 0.176 | ITA | N | 0.362 | 0.262 | 0.022 |
| DEU | T | 0.168 | 0.439 | 0.236 | JPN | T | 0.049 | 0.019 | 0.051 |
| DEU | N | 0.378 | 0.779 | 0.529 | JPN | N | 0.130 | 0.070 | 0.112 |
| DNK | T | 0.172 | 0.494 | 0.006 | KOR | T | 0.947 | 0.103 | 0.397 |
| DNK | N | 0.230 | 0.491 | 0.015 | KOR | N | 0.962 | 0.138 | 0.415 |
| FIN | T | 0.191 | 0.120 | 0.160 | NLD | T | 0.239 | 0.533 | 0.703 |
| FIN | N | 0.341 | 0.153 | 0.107 | NLD | N | 0.285 | 0.426 | 0.615 |
| FRA | T | 0.727 | 0.844 | 0.796 | ESP | T | 0.015 | 0.024 | 0.022 |
| FRA | N | 0.951 | 0.535 | 0.362 | ESP | N | 0.018 | 0.020 | 0.021 |
| GBR | T | 0.216 | 0.508 | 0.505 | SWE | T | 0.344 | 0.218 | 0.204 |
| GBR | N | 0.087 | 0.247 | 0.399 | SWE | N | 0.133 | 0.111 | 0.096 |
| IRL | T | 0.470 | 0.511 | 0.819 | USA | T | 0.958 | 0.459 | 0.634 |
| IRL | N | 0.252 | 0.535 | 0.798 | USA | N | 0.832 | 0.632 | 0.885 |

Notes: the null hypothesis that $\left(\hat{L}_{i, t}^{j}-\hat{L}_{i, t}\right)$ does not Granger-cause ( $\hat{\beta}_{i, t}^{j}$ ) is rejected if p-value $\leq 0.05$ at a $5 \%$ significance level.
results for $p=1$ show that, with the exception of Japan (sector $T$ ) and Spain (both sectors), there is no causality running from $\left(\hat{l}_{i, t}^{j}-\hat{l}_{i, t}\right)$ to $\hat{\beta}_{i, t}^{j}$ at the $5 \%$ level of significance. Setting $p=2$ and $p=3$ leads to similar qualitative results. By and large, these results show that one can consider the regressor in eq. (380) as exogenous with respect to the dependent variable.

## K. 4 Panel Data Estimations of $\epsilon$

The parameter we are interested in, the degree of substitutability of hours worked across sectors, is given by $\epsilon_{i}=\gamma_{i} /\left(1-\gamma_{i}\right)$. In the regressions that follow, the parameter $\gamma_{i}$ is alternatively assumed to be identical across countries when estimating for the whole sample ( $\gamma_{i}=\gamma_{i^{\prime}} \equiv \gamma$ for $i \neq i^{\prime}$ ) or to be different across countries when estimating for each economy $\left(\gamma_{i} \neq \gamma_{i^{\prime}}\right.$ for $\left.i \neq i^{\prime}\right)$. The sample is running from 1971 to 2007 but we run regression (380) over two sub-periods 1971-1989 and 1990-2007 as well in order to have a larger sample when shedding light on the determinants of $\epsilon$ (we get 28 observations instead of 14).

Empirical results reported in Table 18 are consistent with $\epsilon>0$. For the whole sample, we find $\hat{\gamma}=0.378$ over the period 1971-2007. Using the fact that $\epsilon=\frac{1}{1-\gamma}$, we find empirically that an increase by 1 percentage point of the labor's share of aggregate output accumulating to labor in sector $j$ shifts employment by 0.607 percentage point towards that sector. When estimating $\epsilon$ for each economy of our sample over the period 1971-2007, all coefficients are statistically significant, as shown in Table 18, except for Denmark. Excluding Denmark, we find that the degree of substitutability of hours worked across sectors ranges from a low of 0.216 for the Netherlands to a high of 1.800 for the United States. As shown in the last line of Table 18, the panel data estimations of $\epsilon$ for the whole sample are similar whether the sample is running from 1971 to 2007 or is split into two sub-periods.

## K. 5 Determinants of the Degree of Substitutability of Hours Worked across Sectors $\epsilon$

While in the model costs of switching sectors are utility losses (compared with a model with perfect substitutability of hours worked) and thus capture workers' psychological costs, these costs may also capture other barriers to mobility that are not included in the model such as geographic mobility costs, sector-specific human capital, or labor market regulation like firing and hiring costs. In the following, in order to investigate if the degree of labor mobility

Table 18: Panel Data Estimation of $\epsilon$ (eq. (380))

|  | 1971-2007 |  | 1971-1989 |  | 1990-2007 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\hat{\gamma}_{i}$ | $\hat{\epsilon}_{i}$ | $\hat{\gamma}_{i}$ | $\hat{\epsilon}_{i}$ | $\hat{\gamma}_{i}$ | $\hat{\epsilon}_{i}$ |
| BEL | $0_{(2.82)}$ | $\underset{(2.14)}{0.320^{b}}$ | ${\underset{(2.11)}{0.258}}^{b}$ | $\begin{aligned} & 0.347 \\ & (1.57) \end{aligned}$ | $\underbrace{0.226^{b}}_{(1.85)}$ | $\underset{(1.43)}{0.292}$ |
| DEU | ${ }_{(4.27)}^{0.423}{ }^{a}$ | $\underbrace{0.733^{b}}_{(2.47)}$ | $\underset{(2.27)}{0.377^{b}}$ | $\underset{(1.41)}{0.604}$ | ${\underset{(3.69)}{0.450^{a}}}^{a}$ | ${\underset{(2.03)}{0.819}{ }^{b}}^{0}$ |
| DNK | $\underset{(1.18)}{0.106}$ | $\underset{(1.05)}{0.119}$ | $\underset{(0.98)}{0.119}$ | $\underset{(0.86)}{0.135}$ | $\underset{(0.64)}{0.088}$ | $\underset{(0.59)}{0.097}$ |
| ESP | $\underbrace{0.623^{a}}_{(6.92)}$ | $\underset{(2.61)}{1.649^{a}}$ | ${\underset{(5.25)}{0.830^{a}}}^{a}$ | $\underset{(0.89)}{4.886}$ | $\underset{(4.73)}{0.516^{a}}$ | $\underset{(2.29)}{1.066^{b}}$ |
| FIN | $0_{(4.72)}^{0.344^{a}}$ | $\underset{(3.09)}{0.525^{a}}$ | $0_{(3.88)}^{0.485^{a}}$ | ${\underset{(2.00)}{0.941 b}}^{b}$ | $0_{(2.95)}^{0.265^{a}}$ | $\underbrace{0.361}_{(2.16)}{ }^{b}$ |
| FRA | ${\underset{(4.77)}{0.558}{ }^{a}}^{a}$ | ${\underset{(2.11)}{1.262^{b}}}^{b}$ | ${\underset{(3.43)}{0.558}{ }^{a}}^{a}$ | $\begin{aligned} & 1.265 \\ & (1.51) \end{aligned}$ | $\underset{(3.30)}{0.557^{a}}$ | $\begin{aligned} & 1.259 \\ & (1.46) \end{aligned}$ |
| GBR | $0_{(6.56)}^{0.498^{a}}$ | ${\underset{(3.29)}{0.994^{a}}}^{a}$ | $\underbrace{0.390^{a}}_{(4.06)}$ | $\underset{(2.48)}{0.638^{b}}$ | $0_{(5.47)}^{0.716^{a}}$ | $\underset{(1.55)}{2.522}$ |
| IRL | $0_{(3.32)}^{0.199^{a}}$ | $0_{(2.66)}^{0.249^{a}}$ | $\underset{(0.32)}{0.033}$ | $\underset{(0.31)}{0.034}$ | $0_{(3.98)}^{0.294^{a}}$ | $\underset{(2.81)}{0.417^{a}}$ |
| ITA | ${\underset{(4.39)}{0.434}{ }^{a}}^{a}$ | $\underset{(2.48)}{0.768^{b}}$ | $0_{(3.53)}^{0.4699^{a}}$ | $\underset{(1.87)}{0.885^{c}}$ | ${\underset{(2.53)}{0.385^{a}}}^{a}$ | $\underset{(1.56)}{0.625}$ |
| JPN | $\underbrace{0.499^{a}}_{(5.04)}$ | $\underbrace{0.994^{b}}_{(2.53)}$ | ${\underset{(3.96)}{0.534}{ }^{a}}^{a}$ | $1_{(1.84)}^{1.147^{c}}$ | ${\underset{(3.00)}{0.451}{ }^{a}}^{a}$ | $\underbrace{0.820^{c}}_{(1.65)}$ |
| KOR | ${ }_{(8.56)}^{0.642^{a}}$ | ${ }_{(3.06)}^{1.795^{a}}$ | $0_{(6.44)}^{0.760^{a}}$ | $\begin{aligned} & 3.175 \\ & (1.54) \end{aligned}$ | ${\underset{(5.64)}{0.553}}^{a}$ | $1_{(2.52)}$ |
| NLD | ${\underset{(2.02)}{0.177^{a}}}^{a}$ | ${ }_{(1.66)}^{0.216^{c}}$ | $\begin{gathered} 0.098 \\ (0.91) \end{gathered}$ | $\underset{(0.82)}{0.108}$ | $\underset{(2.30)}{0.356^{b}}$ | $\underset{(1.48)}{0.553}$ |
| SWE | $0_{(4.34)}^{0.295^{a}}$ | $\underbrace{0.419^{a}}_{(3.06)}$ | $\underset{(2.61)}{0.254^{a}}$ | $\underbrace{0.340^{c}}_{(1.95)}$ | ${\underset{(3.49)}{0.342^{a}}}^{a}$ | ${\underset{(2.30)}{0.519^{b}}}^{b}$ |
| USA | $\begin{gathered} 0.6433^{a} \\ (5.14) \\ \hline \end{gathered}$ | $\begin{gathered} 1.800^{c} \\ (1.84) \end{gathered}$ | $\begin{gathered} 0.678^{a} \\ (4.11) \\ \hline \end{gathered}$ | $\begin{array}{r} 2.110 \\ (1.32) \end{array}$ | ${ }_{(3.05)}^{0.592^{a}}$ | $\begin{aligned} & 1.454 \\ & (1.24) \\ & \hline \end{aligned}$ |
| R-squared <br> Number of observations <br> Number of countries <br> Number of sectors | $\begin{gathered} 0.270 \\ 992 \\ 14 \\ 2 \end{gathered}$ |  | $\begin{gathered} 0.284 \\ 488 \\ 14 \\ 2 \end{gathered}$ |  | $\begin{gathered} 0.280 \\ 504 \\ 14 \\ 2 \end{gathered}$ |  |
| Whole sample | $\begin{aligned} & \hline 0.378^{a} \\ & (16.43) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0.607^{a} \\ & (10.22) \\ & \hline \end{aligned}$ | $\begin{gathered} 0.368^{a} \\ (10.81) \\ \hline \end{gathered}$ | $\begin{gathered} 0.581^{a} \\ \hline(6.84) \\ \hline \end{gathered}$ | $\begin{aligned} & 0.388^{a} \\ & (12.12) \\ & \hline \end{aligned}$ | $\begin{gathered} 0.634^{a} \\ (7.42) \\ \hline \end{gathered}$ |
| R-squared <br> Number of observations <br> Number of countries <br> Number of sectors | $\begin{gathered} \hline 0.224 \\ 992 \\ 14 \\ 2 \end{gathered}$ |  | $\begin{gathered} \hline 0.202 \\ 488 \\ 14 \\ 2 \end{gathered}$ |  | $\begin{gathered} 0.243 \\ 504 \\ 14 \\ 2 \\ \hline \end{gathered}$ |  |

Notes: ${ }^{a},{ }^{b}$ and ${ }^{c}$ denote significance at $1 \%, 5 \%$ and $10 \%$ levels; t-statistics are reported in parentheses.
captured by $\epsilon$ in our model is correlated with the switching costs put forward by the literature related to the labor market, we run the regression of $\epsilon$ on a number of indicators like union density, unemployment benefits, the share of young workers in total employment, the share of low-skill workers in total labor force, and employment protection legislation.

## Data

We use observable characteristics of individual workers (age and education) and labor market regulation variables (union density, unemployment benefits and employment protection legislation) as determinants of labor switchings costs across sectors. Before discussing our empirical strategy and estimation results, we provide a detailed description of the data used in this analysis. Mnemonics are given in parentheses. Summary statistics of the data used in the empirical analysis are displayed in Table 19.

- Union Density (UD): net union membership as a proportion wage and salary earners in employment. This variable is constructed as: total union membership (minus union members outside the active, dependent and employed labour force) over wage and salary earners in employment. Source: ICTWSS (Jelle Visser [2009]). Data coverage: 19712007 for BEL, DEU, DNK, FIN, FRA, GBR, IRL, ITA, NLD, SWE and USA, 1979-2007 for ESP, 1971-2006 for JPN (data are not available for KOR).
- Unemployment Benefit Replacement Rate (UBRR): net unemployment replacement rate for an average production worker (single person). This measure is defined as: $(\text { Cash Benefits - Taxes })_{\text {out of work }} /(\text { Wages - Taxes })_{\text {in work }}$, where taxes include net social charges (compulsory contributions to social insurance program less cash transfers). The calculations assume a worker, aged 40, who earns the average production worker wage. Source: Van Vliet and Caminada [2012]. Data coverage: 1971-2007 for BEL, DEU, DNK, FIN, FRA, GBR, IRL, ITA, JPN, NLD, SWE and USA, 1979-2007 for ESP (data are not available for KOR).
- Employment Protection Legislation (EPL): this index, developed by the OECD, is designed as a multi-dimensional indicator of the strictness of legal protection against dismissals for permanent as well as temporary workers. The higher is EPL, the more restricted is a country's employment protection regulation. Source: OECD labour market statistics database. Data coverage: 1990-2007 for all countries of our sample.
- Young Workers (Young): share of young workers (15-34 years) in total employment. Source: OECD Labour Force Survey database. Data coverage: 1971-2007 for DEU, FIN, ITA, JPN, SWE and USA, 1972-2007 for ESP, 1980-2007 for KOR, 1983-2007 for BEL, DNK, FRA and IRL, 1984-2007 for GBR, and 1987-2007 for NLD.
- Low Skilled Workers (Low skilled) : share of workers with primary education in total labor force. Source: World Bank World Development Indicators. Data coverage: 19922007 for BEL, DEU, DNK, ESP, GBR, IRL, ITA and KOR, 1993-2007 for FRA, 19952007 for FIN and SWE, 1996-2007 for JPN and NLD, and 1997-2001 for USA.
- High Skilled Workers (High skilled) : share of workers with secondary or tertiary education education in total labor force. Source: World Bank World Development Indicators. Data coverage: 1992-2007 for BEL, DEU, DNK, ESP, GBR, IRL, ITA and KOR, 19932007 for FRA, 1995-2007 for FIN and SWE, 1996-2007 for JPN and NLD, and 1997-2001 for USA.


## Predictions and Estimates

According to our two-sector model with labor market frictions developed in section J, we expect countries with more stringent employment protection legislation (EPL), higher union density, and more generous unemployment benefit scheme to display lower labor mobility (i.e., the parameter $\epsilon$ takes smaller values). We provide more details below:

Table 19: Summary Statistics per Country

| Countries | Variables |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | UD | UBRR | EPL | Young | Low skilled |
| BEL | 0.52 | 0.64 | 2.55 | 0.41 | 0.31 |
| DEU | 0.30 | 0.63 | 2.58 | 0.40 | 0.49 |
| DNK | 0.74 | 0.69 | 1.75 | 0.40 | 0.22 |
| ESP | 0.16 | 0.64 | 3.17 | 0.43 | 0.32 |
| FIN | 0.71 | 0.53 | 2.10 | 0.40 | 0.24 |
| FRA | 0.13 | 0.65 | 3.01 | 0.39 | 0.17 |
| GBR | 0.40 | 0.29 | 0.66 | 0.41 | 0.33 |
| IRL | 0.54 | 0.39 | 0.98 | 0.37 | 0.36 |
| ITA | 0.41 | 0.22 | 2.74 | 0.34 | 0.37 |
| JPN | 0.27 | 0.62 | 1.60 | 0.35 | 0.32 |
| NLD | 0.28 | 0.78 | 2.42 | 0.45 | 0.56 |
| SWE | 0.79 | 0.79 | 2.50 | 0.37 | 0.20 |
| USA | 0.17 | 0.62 | 0.21 | 0.43 | 0.14 |
| KOR | no data | no data | 2.35 | 0.41 | 0.29 |

Notes: UD is Union Density, UR is Unemployment Benefits Remplacement Rate, EPL is Employment Protection Legislation, Young is the share of young workers (15-34 years) in total employment, Low skilled is the share of workers with primary education in total labor force.

- Labor mobility across sectors decreases with firing costs because firms post less job vacancies which reduces job flows. This prediction is in accordance with recent empirical findings. Kambourov [2009] constructs an annual intersectoral reallocation index in the lines of Wacziarg and Wallack [2004]. He finds that countries with low firing costs experience larger sectoral reallocation of workers in the years following their trade reforms.
- While sector-specific skills lower labor mobility across sectors as emphasized by Lee and Wolpin [2006], drawing on Tang [2012], we expect specific skills to reduce mobility more in countries with stringent employment protection legislation. Lee and Wolpin [2006] find empirically that the cost of moving between sectors within the same occupation is estimated to be significantly larger than moving between occupations within the same sector; more specifically, workers who shift from one sector to another experience a cost ranging from $50 \%$ to $75 \%$ of annual earnings due to sector-specific skills. Drawing on Tang's [2012] article, we also expect that in countries where labor laws are more protective, workers expect a more stable relationship with their employers and obtain higher bargaining power vis-a-vis their employers. Thus, they have more incentives to acquire firms specific skills relative to general skills on the job and thus are less prone to change jobs. Because educated workers may be more prone to accumulate specific skills which are not perfectly transferable across occupations or sectors, we expect countries with a higher share of skilled workers and stringent employment protection legislation to experience lower labor mobility.
- Labor mobility across sectors decreases with the unemployment benefit replacement rate as firms are less willing to post job vacancies because higher unemployment benefits lowers the surplus from hiring.
- The effect of an increase in the worker's bargaining power on labor mobility is ambiguous. On the one hand, by raising the share of the surplus associated with a labor contract in the marginal benefit of search $\chi$, a higher worker's bargaining power $\alpha_{W}$ implies that agents are more willing to search for a job which in turn raises further labor mobility across sectors; on the other hand, by lowering the surplus from hiring, an increase in the
worker's bargaining power reduces labor demand and thus job flows. Hence, a higher worker's bargaining power $\alpha_{W}$ may raise or lower labor mobility across sectors. We find numerically that raising the worker bargaining power, $\alpha_{W}$, tends to reduce labor mobility, although the effect is small (since it is the result of two opposite forces). In the empirical literature, the worker bargaining power is commonly captured by the unionization rate. Hence, we expect countries with larger union density to experience smaller job flows and thus lower mobility of labor across sectors.

We can also provide more information about the nature of switching costs captured by $\epsilon$ by drawing on the labor market literature. In particular, estimates documented by Kambourov and Manovskii [2009] reveal that educated workers exhibit lower occupational mobility than their less educated counterparts. The reason is that investment in human capital is not perfectly transferable across all occupations, and thus it is expected to reduce workers' occupational mobility. Moreover, the authors find that mobility increased significantly for workers younger than 40 in the USA. Building on these results, we expect countries with higher shares of young employees and low-education workers to experience larger labor mobility, as these workers' groups accumulate relatively less specific human capital and thus should be more prone to switch jobs.

Our estimates across countries are reported in (20. The dependent variable is $\epsilon$ while explanatory variables are shown in the first column. We use two sample for panel data estimations of $\epsilon$ : those over the period 1971-2007 (14 observations) and those over two subperiods 1971-1989 and 1990-2007 (28 observations) in order to have more observations. In all cases, variables have an expected sign but several determinants like employment protection legislation, the unemployment benefit replacement rate and the interaction term between EPL and the share of skilled-workers are not significant at conventional levels exhibit low statistic significance (remember that the time horizon is short). By and large, our estimates reported in Table 20 reveal that in countries where unemployment benefits are more generous (as captured by the replacement rate), firing costs are higher (as captured by EPL), and/or worker bargaining power is large (as captured by union density), the labor mobility across sectors captured by $\epsilon$ is lower. When turning to workers' characteristics, we find that in countries where the share of young workers or low-skilled workers is higher, the mobility of labor captured by $\epsilon$ is larger. Moreover, we find that in countries with stringent employment protection legislation, high-skilled workers are more prone to accumulate specific skills and thus exhibit lower mobility across sectors. While these findings should give us confidence about the ability of $\epsilon$ to reflect the degree of labor mobility across sectors and allow us to discuss the nature of switching costs, the evidence are based on small sample ( 14 observations over the period 1971-2007 or 28 observations when considering two sub-periods).

## K. 6 Dealing with Correlated Errors across Sectors

In this subsection, we address the issue of correlated errors across sectors within each country by relaxing our assumption of independence of the errors and replacing it with the assumption of independence between clusters while the errors are allowed to be correlated within clusters.

When running the regression (380), we allow the coefficient $\gamma_{i}$ (with $\gamma_{i}=\frac{\epsilon_{i}}{\epsilon_{i}+1}$ ) to vary across countries. Hence, we assume that the coefficient is the same across sectors within each country. While this assumption $\gamma_{i}^{T}=\gamma_{i}^{N}=\gamma_{i}$ does not raise some issues, one difficulty arises because the workers' inflow in one sector is necessarily correlated with the workers' outflow from the other sector. ${ }^{79}$ We thus relax the homoscedasticity assumption and allow the error terms to be heteroscedastic and correlated within clusters. Put otherwise, to take into account the correlation between dependent variables within each country (i.e., across

[^39]Table 20: Determinants of $\epsilon$

| Time period | 1971-2007 |  | 1971-1989 \& 1990-2007 |  |
| :---: | :---: | :---: | :---: | :---: |
| Specification | (1) | (2) | (3) | (4) |
| Union Density | $\underset{(-2.92)}{-0.886^{a}}$ | $\underset{(-3.19)^{a}}{-0.94]^{a}}$ | $\underset{(-3.84)}{-1.003^{a}}$ | $\underset{(-4.25)}{-1.059^{a}}$ |
| Replacement rate |  | $\underset{(-1.35)}{-0.686}$ |  | $\underset{(-1.92)}{-0.607^{c}}$ |
| Young workers | $\underbrace{1.531^{a}}_{(4.01)}$ | $\begin{gathered} 2.04 .85) \\ \hline(3.85 \end{gathered}$ | ${ }_{(5.06)}^{1.697^{a}}$ | $\begin{array}{r} 2.188^{a}{ }^{a}(5.36) \end{array}$ |
| Observations | 13 | 13 | 26 | 26 |
| R -squared | 0.38 | 0.42 | 0.37 | 0.44 |


| Time period | 1990-2007 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Specification | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
| Union Density | $\begin{gathered} -0.738^{a} \\ (-2.88) \end{gathered}$ | $\begin{gathered} -0.728^{a} \\ (-3.00) \end{gathered}$ | $\underset{(-2.69)}{-0.695^{a}}$ | $\underset{(-2.26)}{-0.630^{b}}$ | $\underset{(-2.06)}{-0.612^{b}}$ | $\underset{(-2.75)}{-0.674^{a}}$ | $\underset{(-2.60)}{-0.681^{a}}$ |
| Replacement rate |  | $\underset{(-1.49)}{-0.657}$ |  |  | $\begin{gathered} -0.145 \\ (-0.30) \end{gathered}$ |  |  |
| EPL |  |  | $\underset{(-1.05)}{-0.260}$ |  |  | $\underset{(-2.08)}{-0.456^{b}}$ |  |
| EPL $\times$ High skilled |  |  |  |  |  |  | $\underset{(-1.02)}{-0.264}$ |
| Young workers | $\underbrace{1.329^{a}}_{(3.80)}$ | $\underset{(4.73)^{a}}{1.73)^{a}}$ | $1_{(2.131)}^{139^{a}}$ |  |  |  | $1.227^{a}{ }_{(3.38)}$ |
| Low skilled workers |  |  |  | $\begin{gathered} 0.872^{a} \\ (3.06) \end{gathered}$ | $0_{\left(2.922^{a}\right.}{ }^{a}$ | $0_{\left(3.766^{a}\right.}^{0_{(3.7}}$ |  |
| Observations | 13 | 13 | 13 | 13 | 13 | 13 | 13 |
| R-squared | 0.38 | 0.44 | 0.39 | 0.22 | 0.15 | 0.40 | 0.38 |

Notes: all variables enter in regression in logarithms. ${ }^{a},{ }^{b}$ and ${ }^{c}$ denote significance at $1 \%, 5 \%$ and
$10 \%$ levels. t-statistics are reported in parentheses.
sectors within each country), standard errors are clustered at the country level. To do so, we re-estimate equation (380) by relaxing the homoscedasticity assumption and allowing the error terms to be heteroscedastic and correlated within clusters. We follow Huber [1967] and White [1980] and use a cluster-robust covariance matrix that assumes no particular kind of within-cluster correlation nor a particular form of heteroscedasticity. This specification allow us to run a robustness check regarding the statistic significance while coefficients are identical. More specifically, the parameter $\gamma_{i}$ is consistently estimated and asymptotically equivalent to its OLS counterpart (see Wooldridge [2003]). This implies that our previous OLS estimates of $\epsilon_{i}$ are unbiased. Moreover, the clustering method provides robust standard errors and allows us to infer in the panel model. Table 21 contrasts OLS standard errors $\hat{\sigma}_{\hat{\epsilon}_{i}}^{\text {ols }}$ (i.e., when cluster effects are left in the error term) with standard errors obtained with the cluster-robust variance matrix $\hat{\sigma}_{\epsilon_{i}}^{\text {custer }}$ for three alternative periods: 1971-2007, 1971-1989, 1990-2007. For 10 countries, standard errors that are clustered at the country level are larger, as one would expect if the error term $\nu_{i, t}^{j}$ in eq. (380) is correlated across sectors. However, the estimates of $\epsilon$ remain statistically significant at the $10 \%$ significance for 11 countries of our sample, except for Denmark, Ireland and Korea. By contrast, $\epsilon$ is significant for 13 countries when using OLS standard errors at the $10 \%$ significance, with Denmark the sole exception. Note that for the three time periods, the whole sample estimate of $\epsilon$ is still highly significant when standard errors are clustered at the country level (the t-statistics are 6.91, 5.05 and 4.57 for the periods 1971-2007, 1971-1989 and 1990-2007 respectively). By and large, these results suggest that allowing errors to be correlated within each country merely influences the statistical significance of our estimates of the degree of labor mobility across sectors for the whole sample and the majority of the countries.

Table 21: Standard Errors for $\epsilon$ Estimates

|  | $1971-2007$ |  |  | $1971-1989$ |  |  | $1990-2007$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\hat{\epsilon}_{i}$ | $\hat{\sigma}_{\hat{\epsilon}_{i}}^{\text {ols }}$ | $\hat{\sigma}_{\hat{\epsilon}_{i}}^{\text {cluster }}$ | $\hat{\epsilon}_{i}$ | $\hat{\sigma}_{\hat{\epsilon}_{i}}^{\text {ols }}$ | $\hat{\sigma}_{\hat{\epsilon}_{i}}^{\text {cluster }}$ | $\hat{\epsilon}_{i}$ | $\hat{\sigma}_{\hat{\epsilon}_{i}}^{\text {ols }}$ | $\hat{\sigma}_{\hat{\epsilon}_{i}}^{\text {cluster }}$ |
| BEL | 0.320 | $0.150^{b}$ | $0.183^{c}$ | 0.347 | 0.221 | 0.227 | 0.292 | 0.204 | 0.289 |
| DNK | 0.119 | 0.113 | 0.144 | 0.135 | 0.157 | 0.207 | 0.097 | 0.165 | 0.189 |
| DEU | 0.733 | $0.297^{b}$ | $0.393^{c}$ | 0.604 | 0.427 | 0.378 | 0.819 | $0.404^{b}$ | 0.616 |
| ESP | 1.649 | $0.632^{a}$ | $0.800^{b}$ | 4.886 | 5.474 | 7.067 | 1.066 | $0.465^{b}$ | $0.512^{b}$ |
| FIN | 0.525 | $0.170^{a}$ | $0.216^{b}$ | 0.941 | $0.471^{b}$ | $0.554^{c}$ | 0.361 | $0.167^{b}$ | $0.204^{c}$ |
| FRA | 1.262 | $0.598^{b}$ | $0.419^{a}$ | 1.265 | 0.836 | $0.405^{a}$ | 1.259 | 0.862 | 0.796 |
| GBR | 0.994 | $0.302^{a}$ | $0.318^{a}$ | 0.638 | $0.258^{b}$ | $0.215^{a}$ | 2.522 | 1.625 | 2.121 |
| IRL | 0.249 | $0.094^{a}$ | 0.153 | 0.034 | 0.110 | 0.147 | 0.417 | $0.149^{a}$ | 0.301 |
| ITA | 0.768 | $0.309^{b}$ | $0.425^{c}$ | 0.885 | $0.472^{c}$ | 0.746 | 0.625 | 0.401 | 0.446 |
| JPA | 0.994 | $0.394^{b}$ | $0.282^{a}$ | 1.147 | $0.622^{c}$ | $0.452^{b}$ | 0.820 | $0.497^{c}$ | $0.331^{b}$ |
| KOR | 1.795 | $0.586^{a}$ | 1.461 | 3.175 | 2.056 | 4.514 | 1.235 | $0.490^{b}$ | 1.294 |
| NLD | 0.216 | $0.130^{c}$ | $0.086^{b}$ | 0.108 | 0.133 | 0.076 | 0.553 | 0.374 | $0.280^{b}$ |
| SWE | 0.419 | $0.137^{a}$ | $0.159^{a}$ | 0.340 | $0.174^{c}$ | 0.235 | 0.519 | $0.226^{b}$ | $0.127^{a}$ |
| USA | 1.800 | $0.980^{b}$ | $0.737^{b}$ | 2.110 | 1.596 | $1.190^{c}$ | 1.454 | 1.168 | 0.891 |
| All sample | 0.607 | $0.059^{a}$ | $0.088^{a}$ | 0.581 | $0.085^{a}$ | $0.115^{a}$ | 0.634 | $0.085^{a}$ | $0.139^{a}$ |
| Notes: $^{a},{ }^{b}$ and ${ }^{c}$ denote significance at $1 \%, 5 \%$ | and $10 \%$ | levels. |  |  |  |  |  |  |  |

## L Elasticity of Substitution $\phi$ between Traded and Non Traded Goods: Empirical Strategy

In this section, we detail our empirical strategy to estimate the elasticity of substitution between traded and non traded goods $\phi$. While the ability of the two-sector model with imperfect mobility of labor across sectors to accommodate the data related to sectoral productivity shocks relies heavily upon the size of the elasticity of substitution between traded and non traded goods, estimates of the elasticity of substitution $\phi$ by the existing literature are rather diverse. The cross-section studies report an estimate of $\phi$ ranging from 0.44 to 0.74 , see e.g., Stockman and Tesar [1995] and Mendoza [1995], respectively. ${ }^{80}$ The literature adopting the Generalized Method of Moments and the cointegration methods, see e.g. Ostry and Reinhart [1992] and Cashin and Mc Dermott [2003], respectively, reports a value in the range $[0.75,1.50]$ for developing countries and in the range $[0.63,3.50]$ for developed countries. Since existing empirical studies do not unanimously report an elasticity of substitution larger than one, we explore this assumption empirically for the whole sample and each economy.

## L. 1 Empirical Strategy to Estimate $\phi$

## Using Time Series by Industry Provided by EU KLEMS

To estimate $\phi$, we adopt the following strategy. To determine an empirical relationship, we combine the optimal rule for intra-temporal allocation of consumption (15) (that we repeat for clarity purposes)

$$
\begin{equation*}
\frac{C^{T}}{C^{N}}=\left(\frac{\varphi}{1-\varphi}\right) P^{\phi} \tag{383}
\end{equation*}
$$

with the goods market equilibrium

$$
\begin{equation*}
\frac{C^{T}}{C^{N}}=\frac{Y^{T}-N X-G^{T}-I^{T}}{Y^{N}-G^{N}-I^{N}} \tag{384}
\end{equation*}
$$

[^40]where we used the fact that $\dot{B}-r^{\star} B=Y^{T}-C^{T}-G^{T}-I^{T} \equiv N X$. Inserting (383) into (384) leads to
\[

$$
\begin{equation*}
\frac{Y^{T}-N X-G^{T}-I^{T}}{Y^{N}-G^{N}-I^{N}}=\left(\frac{\varphi}{1-\varphi}\right) P^{\phi} . \tag{385}
\end{equation*}
$$

\]

According to the market clearing condition, we could alternatively use data for consumption or for sectoral value added along with times series for its demand components to estimate $\phi$. Unfortunately, nomenclatures for valued added by industry and for consumption by items are different and thus it is most likely that $C^{T}$ differs from $Y^{T}-N X-G^{T}-I^{T}$, and $C^{N}$ from $Y^{N}-G^{N}-I^{N}$ as well. Because time series for traded and non traded consumption display a short time horizon for half countries of our sample while data for sectoral value added and net exports are available for the 14 OECD countries of our sample over the period running from 1970 to 2007 (except for Japan: 1974-2007), we find appropriate to estimate $\phi$ by computing $Y^{T}-N X-G^{T}-I^{T}$ and $Y^{N}-G^{N}-I^{N}$. Yet, an additional difficulty shows up because the classification adopted to split government spending and investment expenditure into traded and non traded items is different from that adopted to break down value added into traded and non traded components. Moreover, the time horizon is short at a disaggregated level (for $I^{j}$ and $G^{j}$ ) for most of the countries, especially for time series of $G^{j}$. To overcome these difficulties, we proceed as follows. Denoting by $v_{G^{T}}=\frac{P^{T} G^{T}}{P^{T} Y^{T}-P^{T} N X}$ and $v_{I^{T}}=\frac{P^{T} I^{T}}{P^{T} Y^{T}-P^{T} N X}$ the ratio of government and investment expenditure on tradables to traded value added adjusted with net exports at current prices, respectively, and by $v_{G^{N}}=$ $\frac{P^{N} G^{N}}{P^{N} Y^{N}}$ and $v_{I^{N}}=\frac{P^{N} I^{N}}{P^{N} Y^{N}}$ the ratio of government and investment expenditure on non tradables to non traded value added at current prices, the goods market equilibrium can be rewritten as follows:

$$
\frac{\left(P^{T} Y^{T}-P^{T} N X\right)\left(1-v_{G^{T}}-v_{I^{T}}\right)}{P^{N} Y^{N}\left(1-v_{G^{N}}-v_{I^{N}}\right)}=\left(\frac{\varphi}{1-\varphi}\right) P^{\phi-1},
$$

or alternatively

$$
\begin{equation*}
\frac{\left(Y^{T}-N X\right)\left(1-v_{G^{T}}-v_{I^{T}}\right)}{Y^{N}\left(1-v_{G^{N}}-v_{I^{N}}\right)}=\left(\frac{\varphi}{1-\varphi}\right) P^{\phi} . \tag{386}
\end{equation*}
$$

Setting

$$
\begin{equation*}
\alpha \equiv \ln \frac{\left(1-v_{G^{N}}-v_{I^{N}}\right)}{\left(1-v_{G^{T}}-v_{I^{T}}\right)}+\ln \left(\frac{\varphi}{1-\varphi}\right), \tag{387}
\end{equation*}
$$

and taking logarithm, eq. (386) can be rewritten as follows:

$$
\begin{equation*}
\ln \left(\frac{Y^{T}-N X}{Y^{N}}\right)=\alpha+\phi \ln P . \tag{388}
\end{equation*}
$$

Indexing time by $t$ and countries by $i$, and adding an error term $\mu$, we estimate $\phi$ by exploring the following empirical relationship:

$$
\begin{equation*}
\ln \left(\frac{Y^{T}-N X}{Y^{N}}\right)_{i, t}=f_{i}+f_{t}+\alpha_{i} t+\phi_{i} \ln P_{i, t}+\mu_{i, t} . \tag{389}
\end{equation*}
$$

$f_{i}$ captures the country fixed effects, $f_{t}$ are time dummies, and $\mu_{i, t}$ are the i.i.d. error terms. Because the term (387) may display a trend over time, we add country-specific trends, as captured by $\alpha_{i} t$.

Because data to construct time series for traded $\left(I^{T}\right)$ and non traded investment $\left(I^{N}\right)$ are available for ten countries over the fourteen in our sample over a time horizon varying between 37 years (1970-2007) and 27 years (1980-2007), we computed time series $Y^{T}-N X-I^{T}$ and $Y^{N}-I^{N}$. In this case, eq. (386) can be rewritten as follows:

$$
\begin{equation*}
\frac{\left(Y^{T}-N X-I^{T}\right)\left(1-v_{G^{T}}\right)}{\left(Y^{N}-I^{N}\right)\left(1-v_{G^{N}}\right)}=\left(\frac{\varphi}{1-\varphi}\right) P^{\phi} . \tag{390}
\end{equation*}
$$

Denoting by

$$
\begin{equation*}
\kappa \equiv \ln \frac{\left(1-v_{G^{N}}\right)}{\left(1-v_{G^{T}}\right)}+\ln \left(\frac{\varphi}{1-\varphi}\right) \tag{391}
\end{equation*}
$$

where $v_{G^{T}}=\frac{P^{T} G^{T}}{P^{T}\left(Y^{T}-N X-I^{T}\right)}$ and $v_{G^{N}}=\frac{P^{N} G^{N}}{P^{N}\left(Y^{N}-I^{N}\right)}$ and taking logarithm, we explore alternatively the following relationship to estimate $\phi$ :

$$
\begin{equation*}
\ln \left(\beta^{T} / \beta^{N}\right)_{i, t}=f_{i}+f_{t}+\alpha_{i} t+\phi_{i} \ln P_{i, t}+\nu_{i, t} \tag{392}
\end{equation*}
$$

where $\beta^{T}=\left(Y^{T}-N X-I^{T}\right)$ and $\beta^{N}=\left(Y^{N}-I^{N}\right)$.
When determining (388), we can alternatively make use of first-order conditions equating the marginal revenue of labor and the sectoral wage:

$$
\begin{equation*}
\frac{\theta^{j} P^{j} Y^{j}}{L^{j}}=W^{j} \tag{393}
\end{equation*}
$$

where $\theta^{j}$ is labor's share in value added in sector $j=T, N$. Using (393) to eliminate the nominal sectoral value added $P^{j} Y^{j}$, the goods market clearing condition can be rewritten as follows:

$$
\begin{equation*}
\frac{\left(W^{T} L^{T}-\theta^{T} P^{T} N X\right) \frac{\theta^{N}}{\theta^{T}}\left(1-v_{G^{T}}-v_{I^{T}}\right)}{W^{N} L^{N}\left(1-v_{G^{N}}-v_{I^{N}}\right)}=\left(\frac{\varphi}{1-\varphi}\right) P^{\phi-1} \tag{394}
\end{equation*}
$$

We first set

$$
\begin{equation*}
\eta \equiv \ln \frac{\left(1-\varrho_{G^{N}}-\varrho_{I^{N}}\right)}{\left(1-\varrho_{G^{T}}-\varrho_{I^{T}}\right)}+\ln \left(\frac{\theta^{T}}{\theta^{N}}\right)+\ln \left(\frac{\varphi}{1-\varphi}\right) \tag{395}
\end{equation*}
$$

where $\varrho_{G^{T}}=\frac{P^{T} G^{T}}{\left(W^{T} L^{T}-\theta^{T} P^{T} N X\right)}$ and $v_{G^{N}}=\frac{P^{N} G^{N}}{W^{N} L^{N}}, \varrho_{I^{T}}=\frac{P^{T} I^{T}}{\left(W^{T} L^{T}-\theta^{T} P^{T} N X\right)}$ and $v_{I^{N}}=\frac{P^{N} I^{N}}{W^{N} L^{N}}$. Denoting by $\gamma^{T}=\left(W^{T} L^{T}-\theta^{T} P^{T} N X\right)$ and $\gamma^{N}=W^{N} L^{N}$, and taking logarithm, eq. (395) can be rewritten as follows:

$$
\begin{equation*}
\ln \left(\frac{\gamma^{T}}{\gamma^{N}}\right)=\eta+(\phi-1) \ln P \tag{396}
\end{equation*}
$$

Indexing time by $t$ and countries by $i$, and adding an error term $\zeta$, we estimate $\phi$ by exploring the following empirical relationship:

$$
\begin{equation*}
\ln \left(\gamma^{T} / \gamma^{N}\right)_{i, t}=g_{i}+g_{t}+\sigma_{i} t+\rho_{i} p_{i, t}+\zeta_{i, t} \tag{397}
\end{equation*}
$$

Because $\eta_{i}$ (see eq. (395)) is composed of both preference (i.e., $\varphi$ ) and production (i.e., $\theta^{j}$ ) parameters, and (logged) ratios which may display trend over time, we introduce country fixed effects $g_{i}$ and add country-specific trends, as captured by $\sigma_{i} t$. Once we have estimated $\rho_{i}$, we can compute $\hat{\phi}_{i}=\hat{\rho}_{i}+1$ where a hat refers to point estimate in this context.

## Using Time Series for Consumption by Purpose Provided by COICOP

The cross-section studies by Stockman and Tesar [1995] and Mendoza [1995] estimate $\phi$ by running a regression of the (logged) ratio of consumption in non tradables to consumption in tradables on the (logged) relative price of non tradables:

$$
\begin{equation*}
\ln \left(\frac{C^{N}}{C^{T}}\right)=\ln \left(\frac{1-\varphi}{\varphi}\right)-\phi \ln P \tag{398}
\end{equation*}
$$

Note that when exploring the relationship (398) empirically, we abstract from the goods market clearing condition. Indexing time by $t$ and countries by $i$, and adding an error term $\iota$, we explore the following relationship empirically by using panel data:

$$
\begin{equation*}
\ln \left(C^{N} / C^{T}\right)_{i, t}=d_{i}+d_{t}+\zeta_{i} t-\phi_{i} \ln P_{i, t}^{C}+\iota_{i, t} \tag{399}
\end{equation*}
$$

where $P_{i, t}^{C}=P_{i, t}^{C, N} / P_{i, t}^{C, T}$ is the ratio of the price deflator for consumption in non tradable goods $\left(P_{i, t}^{C, N}\right)$ to the price deflator for consumption in tradable goods $\left(P_{i, t}^{C, T}\right) ; d_{i}$ captures the
country fixed effects; $d_{t}$ are time dummies; $\iota_{i, t}$ are the i.i.d. error terms. Because preferences may not be homothetic, there might be income effects in the relative demand for tradable and non-tradable goods. Cross-section studies by Stockman and Tesar [1995] and Mendoza [1995] include GDP per capita in the regression to capture the wealth effect. Because it is likely that GDP per capita is correlated with the relative price of non tradables, we capture the wealth effect by time trend, i.e., $\zeta_{i} t$.

Estimating $\phi$ by running the regression (398) has two drawbacks. First, it does not take into account the goods market clearing condition. Second, time series for consumption by purpose provided by COICOP are available over a short time horizon for most of the countries of the sample. Consequently, to estimate $\phi$, we restrict ourselves to eqs. (389), (392) and (397).

## L. 2 Data Construction and Source

Our dataset covers the fourteen OECD countries in our sample over the period 1970-2007 (except Japan: 1974-2007). We provide more details below on the construction of data employed to estimate equations (389), (392) and (397) (codes in EU KLEMS are reported in parentheses):

- Sectoral value-added deflator $P_{t}^{j}(j=T, N)$ : value added at current prices (VA) over value added at constant prices (VA_QI) in sector $j$. Source: EU KLEMS database. The value added relative price of non tradables, $P_{t}$, corresponds to the ratio of the non traded value added deflator to the traded value added deflator: $P_{t}=P_{t}^{N} / P_{t}^{T}$.
- Sectoral output $Y_{t}^{j}(j=T, N)$ : value added at constant prices in sector $j$ (VA_QI). Source: EU KLEMS database.
- Net exports $N X_{t}$ : net exports deflated by the traded value added deflator, $P_{t}^{T}$. Net exports correspond to the external balance of goods and services at current prices. Source: OECD Economic Outlook Database.
- Sectoral investment $I_{t}^{j}(j=T, N)$ : Real investment in sector $j, I_{t}^{j}$, is investment expenditure in sector $j$ deflated by the value added price index $P_{t}^{j}$ defined above. Investment expenditure are gross capital formation at current prices; to split aggregate investment expenditure into tradables and non tradables, we use the methodology presented in section F. 8 of the Technical Appendix. Source: OECD Input-Output database [?].
- Sectoral labor income $W_{t}^{j} L_{t}^{j}(j=T, N)$ : labor compensation in sector $j$ (LAB). Source: EU KLEMS database.
- Sectoral labor income share $\theta^{j}$ for $j=T, N$ : labor compensation in sector $j$ (LAB) over value added at current prices (VA) averaged over the period 1970-2007 (1974-2007 for Japan). Source: EU KLEMS database.
Data mentioned above are used to construct time series for $\frac{Y_{t}^{T}-N X_{t}}{Y_{t}^{N}}, \frac{\beta_{t}^{T}}{\beta_{t}^{N}}, \frac{\gamma_{t}^{T}}{\gamma_{t}^{T}}$, and $P_{t}$. When estimating equations (389), (392) and (397), all variables are converted into index $1995=100$ and are expressed in log levels.


## L. 3 Empirical Results

Since the set of variables of interest in regressions (389), (392), (397) display trends, we first run panel unit root tests, see Table 22. By and large, all tests show that non stationarity is pervasive, making it clear that pursuing a cointegration analysis is appropriate. We thus implement the seven Pedroni's [2004] tests of the null hypothesis of no cointegration, see Table 23. Across almost all cases the null hypothesis of no cointegration is rejected but only
at the $10 \%$ level. In small samples, Pedroni's [2004] simulations reveal that the group-mean parametric $t$-stat is the most powerful. Based on this result, in the three specifications, the null hypothesis of no cointegration is strongly rejected at the $5 \%$ level.

To estimate the cointegrating vector, we use the group-mean fully modified dynamic OLS estimator of Pedroni [2001]. Given our relatively limited time and cross section dimensions, we restrict our attention here on the FMOLS estimator. Indeed, the group-mean DOLS estimator adds leads and lags of the explanatory as additional regressors. This correction allows to take care of a possible endogeneity of the regressors and to correct for correlation in the residuals. However, this correction reduces sizeably the number of degrees of freedom, even in the case of a DOLS estimator with one lead and lag. Accordingly, we will only consider the group-mean FMOLS when estimating cointegration relationships.

Table 24 reports panel dates estimations of the coefficient $\phi$ for the panel as a whole and for each country, when running the regression (389), (392), (397), respectively; these empirical relationships are are derived by taking into account the goods market equilibrium. Moreover, exploring alternatively the relationship (389) or (397) empirically has the advantage of allowing us to use time series for sectoral value added or labor compensation which are available over the period 1970-2007 for all countries of our sample (except Japan: 1974-2007).

Panel data estimations of $\phi$ when running the regression (389) where the dependent variable is $\left(Y^{T}-N X\right) / Y^{N}$, are shown in column 1 of Table 24. The regressor in this case (and for the rest of the analysis) is the log of the non traded value added deflator to the traded value added deflator. The sample covers all countries we are interested in. For the whole sample, the FMOLS estimate gives a significant value of $\phi$ of 0.66 . The vast majority ( 9 out of 14) of the individual estimated coefficients are statistically significant. In addition, we find that $\phi$ is larger than one in only two countries (Finland and Germany), the estimated value for Korea being not statistically significant.

Column 2 of Table 24 shows panel data estimations of $\phi$ when running the regression (392)) which explicitly takes into account investment expenditures. This, however, reduces the size of the sample: the series for investment are not available for Belgium and Korea, and Sweden and Ireland are excluded from the sample due to data limitation. Among the 10 countries, we find that 6 have positive and statistically significant $\phi$ coefficients, ranging from a low of 0.25 (the United Kingdom) to a high of 2.02 (Japan). Note that the coefficient $\phi$ is found to be larger than one in 4 countries (Germany, Finland, Japan and the Netherlands). Two estimated coefficients are negative (Denmark and the United States), none of them are statistically significant. Due to data limitations and inconsistent estimates (i.e., negative or statistically insignificant at conventional level for several countries), we find that including investment expenditure does not improve the precision of our estimates, likely due to classification issues as the nomenclature for investment by items differs from that for value added by industry; different nomenclatures are most likely to result in different classification to treat investment items or value added by industry as traded or non traded goods.

The last column of Table 24 gives panel data estimations of $\phi$ when running the regression (397); the dependent variable is the (logged) ratio of the labor income in tradables adjusted with net exports at current prices to labor income in non tradables, i.e., $\left(W^{T} L^{T}-\theta^{T} P^{T} N X\right) / W^{N} L^{N}$. By and large, estimates are similar to those shown in column 3 of Table 24.

When calibrating the model, our strategy is as follows. Our reference model is (389). We thus take estimates of $\phi$ when running the regression (389) shown in column 1 of Table 24. When estimates of $\phi$ are not statistically significant at conventional levels, we take values given in column 3. Two estimates are problematic: those for Ireland and Italy which are either negative or not statistically significant. In this case, we take the panel data estimation for the whole sample, i.e., $\hat{\phi}=0.66$.

Table 22: Panel Unit Root Tests (p-values)

|  | LLC <br> $($ t-stat $)$ | Breitung <br> $(\mathrm{t}$-stat $)$ | IPS <br> $(\mathrm{W}$-stat $)$ | MW <br> $(\mathrm{ADF})$ | MW <br> $(\mathrm{PP})$ | Hadri <br> $\left(Z_{\mu}\right.$-stat $)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\ln \left(P^{V A, N} / P^{V A, T)}\right.$ | 0.840 | 0.730 | 1.000 | 1.000 | 0.982 | 0.000 |
| $\ln \left(Y^{T}-N X / Y^{N}\right)$ | 0.409 | 0.017 | 0.408 | 0.155 | 0.199 | 0.000 |
| $\ln \left(Y^{T}-N X-I^{T}\right) /\left(Y^{N}-I^{N}\right)$ | 0.878 | 0.169 | 0.991 | 0.986 | 0.977 | 0.000 |
| $\ln \left(W^{T} L^{T}-\theta^{T} P^{T} N X\right) /\left(W^{N} L^{N}\right)$ | 0.238 | 0.021 | 0.069 | 0.013 | 0.245 | 0.000 |

Notes: For all tests, except for Hadri [2000], the null of a unit root is not rejected if p-value $\geq 0.05$ at a $5 \%$ significance level. For Hadri [2000], the null of stationarity is rejected if p-value $\leq 0.05$ at a $5 \%$ significance level.

Table 23: Panel Cointegration Tests (p-values)

| Dependent variable | $\frac{Y^{T}-N X}{Y^{N}}$ | $\frac{Y^{T}-N X-I^{T}}{Y^{N}-I^{N}}$ | $\frac{W^{T} L^{T}-\theta^{T} P^{T} N X}{W^{N} L^{N}}$ |
| :--- | :---: | :---: | :---: |
|  | $P^{V A, N} / P^{V A, T}$ | $P^{V A, N} / P^{V A, T}$ | $P^{V A, N} / P^{V A, T}$ |
| Explanatory variable | 0.060 | 0.000 | 0.002 |
| Panel tests | 0.009 | 0.102 | 0.014 |
| Non-parametric $\nu$ | 0.001 | 0.035 | 0.005 |
| Non-parametric $\rho$ | 0.067 | 0.002 | 0.061 |
| Non-parametric $t$ |  |  |  |
| Parametric $t$ | 0.257 | 0.026 | 0.299 |
| Group-mean tests | 0.007 | 0.007 | 0.028 |
| Non-parametric $\nu$ | 0.017 | 0.001 | 0.028 |
| Non-parametric $t$ |  |  |  |
| Parametric $t$ |  |  |  |

Notes: the null hypothesis of no cointegration is rejected if the p-value is below 0.05 (0.10 resp.) at $5 \%$ ( $10 \%$ resp.) significance level.
Table 24: FMOLS Estimates of $\phi$

| Dependent variable | $\left(Y^{T}-N X\right) / Y^{N}$ |  | $\left(Y^{T}-N X-I^{T}\right) /\left(Y^{N}-I^{N}\right)$ |  | $\left(W^{T} L^{T}-\theta^{T} P^{T} N X\right) / W^{N} L^{N}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sectoral prices | value-added |  | value-added |  | value-added |  |
|  | (1) | Data coverage | (2) | Data coverage | (3) |  |
| BEL | 1970-2007 | $\begin{gathered} 0.205 \\ \hline(0.90) \end{gathered}$ | no data for $I^{T} / I^{N}$ |  | 1970-2007 | ${ }_{(4.78)^{a}}^{0.7}$ |
| DEU | 1970-2007 | $1_{(3.98)}^{\left(.236^{a}\right.}$ | 1970-2007 | $\underset{(6.25)}{1.797^{a}}$ | 1970-2007 | ${\underset{(4.96)}{1.581^{a}}}^{a}$ |
| DNK | 1970-2007 | $\begin{aligned} & 0.493 \\ & (0.63) \end{aligned}$ | 1970-2006 | $\underset{(-0.41)}{-0.323}$ | 1970-2007 | $\begin{aligned} & \left(4.416^{a}\right. \\ & { }_{(2.69)}^{(4.90} \end{aligned}$ |
| ESP | 1970-2007 | ${\underset{(5.29)}{0.779^{a}}}^{(0)}$ | 1980-2007 | $\begin{aligned} & 0.335 \\ & (0.53) \end{aligned}$ | 1970-2007 | $0_{(1.90)}^{0.284^{c}}$ |
| FIN | 1970-2007 | ${\underset{(8.99)}{ }{ }^{1.041^{a}}}^{a}$ | 1970-2007 | ${\underset{(4.04)}{1.092^{a}}}^{a}$ | 1970-2007 | $1_{(7.71)}$ |
| FRA | 1970-2007 | $0_{(4.74)^{a}}^{0.74}$ | 1978-2006 | $\begin{gathered} (4.04)^{a} \\ 0.847^{a} \\ (7.67) \end{gathered}$ | 1970-2007 | $0_{\left(4.8444^{a}\right.}$ |
| GBR | 1970-2007 | ${\underset{(8.50)}{0.482^{a}}}^{a}$ | 1970-2007 | ${\underset{(2.30)}{0.2488^{b}}}^{6}$ | 1970-2007 | ${ }_{(16.87)}$ |
| IRL | 1970-2007 | $\begin{aligned} & 0.133 \\ & (0.52) \end{aligned}$ | 1990-2007 |  | 1970-2007 | $\underset{(-0.27)}{-0.147}$ |
| ITA | 1970-2007 | $\underset{(-0.05)}{-0.006}$ | 1970-2006 | $\underset{(1.10)}{0.272}$ | 1970-2007 | $\begin{aligned} & 0.091 \\ & (0.88) \end{aligned}$ |
| JPN | 1974-2007 | $0_{(4.816)}$ | 1980-2007 | ${\underset{(4.75)}{ }}_{2.018^{a}}$ | 1974-2007 | ${ }_{(6.60)}^{0.785^{a}}$ |
| KOR | 1970-2007 | $\underset{(1.58)}{1.580}$ | no data for $I^{T} / I^{N}$ |  | 1970-2007 | $1_{(2.99)}$ |
| NLD | 1970-2007 | $0_{(2.96)}^{0.927^{b}}$ | 1970-2007 | ${\underset{(6.42)}{1.229^{a}}}^{a}$ | 1970-2007 | $\mathrm{O}_{\left(1.786^{c}\right.}^{0^{c}}$ |
| SWE | 1970-2007 | $0_{(2.215)}^{0_{2}}$ | 1993-2007 |  | 1970-2007 | $0_{(5.87)}^{0^{0}}$ |
| USA | 1970-2007 | $\begin{gathered} 0_{(3.57)^{a}} \\ \hline \end{gathered}$ | 1977-2006 | $\begin{gathered} -0.043 \\ (-0.13) \\ \hline \end{gathered}$ | 1970-2007 | $\underbrace{0.593^{c}}_{(1.73)}$ |
| All sample |  | $\begin{gathered} 0.660^{a} \\ (12.61) \\ \hline \end{gathered}$ |  | $\begin{array}{r} 0.747^{a} \\ (10.28) \\ \hline \end{array}$ |  | $\begin{gathered} 0.871^{a} \\ (16.84) \\ \hline \end{gathered}$ |
| Number of countries |  | 14 |  | 10 |  | 14 |
| Number of observations |  | 529 |  | 341 |  | 529 |
| Country fixed effects |  | yes |  | yes |  | yes |
| Time dummies |  | yes |  | yes |  | yes |
| Time trend |  | yes |  | yes |  | yes | Notes: all variables enter in regression in logarithms. ${ }_{\text {ticity and and antocorrelation consistent } t \text {-statistics are reported in parentheses. }}$

## M Skill-Biased Technological Change

In this section, we investigate to which extent our analytical results are sensitive to biased technological change. As opposed to the model explored in the paper where technological change is assumed to be Hicks-neutral, we first allow for labor- and capita-augmenting productivity which differ across sectors. Then, we consider low-skilled and high-skilled labor while abstracting from physical capital which allows us to explore the role of skill-bias technological change.

We first analyze the long-run effect of a productivity differential between tradables and non tradables by considering that: i) traded and non traded goods are produced with capital and labor according to CES technologies instead of Cobb-Douglas technologies, and ii) labor- and capital-augmenting productivity allowing for biased technological change. We draw heavily on Alvarez-Cuadrado, Van Long and Poschke [2014]. We show that whether CES technologies are considered and technological change is biased toward labor, as recent empirical evidence suggest, our main conclusions hold. Note that when introducing labor- and capital-augmenting productivity, technological change cannot be computed by the standard method and must be estimated empirically along with the elasticity of substitution between capital and labor, both in the traded and the non traded sector.

Drawing on Alvarez-Cuadrado, Van Long and Poschke [2014] who develop a closed economy dynamic general equilibrium model with two sectors, i.e., manufacturing and services, we consider a small open economy model with traded and non traded goods produced by means of CES technologies. For the sake of simplicity, we consider perfect mobility of capital and labor across sectors.

In a second step, we analyze the implications of skilled biased technological change by developing an open economy model with traded and non traded goods produced by perfectly competitive firms using both high- and low-skilled labor. While assuming perfect labor mobility across sectors, we find that the relative price of non tradables appreciates by a smaller amount than the productivity differential between tradables and non tradables as long as: i) the share of labor income paid to low-skilled labor in sectoral value added is higher in the traded sector than that in the non traded sector, and ii) the skill premium increases.

## M. 1 Labor- and Capital-Augmenting Productivity

## Households

At each instant of time $t$, the representative agent consumes traded goods and non-traded goods denoted by $C^{T}(t)$ and $C^{N}(t)$, respectively, which are aggregated by a constant elasticity of substitution function:

$$
\begin{equation*}
C\left(C^{T}(t), C^{N}(t)\right)=\left[\varphi^{\frac{1}{\phi}}\left(C^{T}(t)\right)^{\frac{\phi-1}{\phi}}+(1-\varphi)^{\frac{1}{\phi}}\left(C^{N}(t)\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{400}
\end{equation*}
$$

The agent is endowed with a unit of time and supplies a fraction $L(t)$ of this unit as labor, while the remainder, $1-L(t)$, is consumed as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Households decide on consumption and worked hours by maximizing lifetime utility:

$$
\begin{equation*}
U=\int_{0}^{\infty}\left\{\frac{1}{1-\frac{1}{\sigma_{C}}} C(t)^{1-\frac{1}{\sigma_{C}}}-\frac{1}{1+\frac{1}{\sigma_{L}}} L(t)^{1+\frac{1}{\sigma_{L}}}\right\} e^{-\beta t} \mathrm{~d} t \tag{401}
\end{equation*}
$$

where $\beta$ is the consumer's discount rate, $\sigma_{C}>0$ is the intertemporal elasticity of substitution for consumption, and $\sigma_{L}>0$ is the Frisch elasticity of labor supply.

The representative household chooses consumption, decides on labor supply, and investment that maximizes his/her lifetime utility (401) subject to the flow budget constraint:

$$
\begin{equation*}
\dot{B}(t)=r^{\star} B(t)+R(t) K(t)+W(t) L(t)-P_{C}(P(t)) C(t)-P(t) I(t) \tag{402}
\end{equation*}
$$

and capital accumulation which evolves as follows:

$$
\begin{equation*}
\dot{K}(t)=I(t)-\delta K(t), \tag{403}
\end{equation*}
$$

where $I(t)$ corresponds to investment expenditure which are assumed to be non traded and $0 \leq \delta_{K}<1$ is a fixed depreciation rate. For the sake of clarity, we drop the time argument below when this causes no confusion.

Denoting the co-state variables associated with (402) and (403) by $\lambda$ and $\psi$, respectively, the first-order conditions characterizing the representative household's optimal plans are:

$$
\begin{gather*}
C=\left(P_{C} \lambda\right)^{-\sigma_{C}}  \tag{404a}\\
L=(W \lambda)^{\sigma_{L}}  \tag{404b}\\
\dot{\lambda}=\lambda\left(\beta-r^{\star}\right),  \tag{404c}\\
\frac{R}{P}-\delta+\frac{\dot{P}}{P}=r^{\star}, \tag{404d}
\end{gather*}
$$

and the transversality conditions $\lim _{t \rightarrow \infty} \bar{\lambda} B(t) e^{-\beta t}=0$ and $\lim _{t \rightarrow \infty} P(t) K(t) e^{-\beta t}=0$; to derive (404d), we used the fact that $\psi(t)=\lambda P(t)$.

## Firms

Traded and non traded goods are produced according to CES technologies:

$$
\begin{align*}
Y^{T} & =\left[\gamma^{T}\left(B^{T} K^{T}\right)^{\frac{\sigma-1}{\sigma}}+\left(1-\gamma^{T}\right)\left(A^{T} L^{T}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}  \tag{405a}\\
Y^{N} & =\left[\gamma^{N}\left(B^{N} K^{N}\right)^{\frac{\sigma-1}{\sigma}}+\left(1-\gamma^{N}\right)\left(A^{N} L^{N}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{405b}
\end{align*}
$$

where $\sigma$ is the elasticity of substitution between capital and labor; for clarity purposes, we abstract from sector-specific elasticities of substitution. As opposed to the model explored in the paper where technological change is assumed to be Hicks-neutral, we allow for labor- and capita-augmenting productivity $A_{i}$ and $B_{i}$ which differ across sectors.

We assume that factors are fully utilized:

$$
\begin{gather*}
L^{T}+L^{N}=L,  \tag{406a}\\
k^{T} L^{T}+k^{N} L^{N}=K, \tag{406b}
\end{gather*}
$$

where $k^{j} \equiv K^{j} / L^{j}$.
The traded good is the numeraire and we denote the price of non traded goods in terms of traded goods by $P \equiv P^{N} / P^{T}$. We assume perfect mobility of capital and labor across sectors and denote the capital rental cost by $R$ and the wage rate by $W$. First order conditions are:

$$
\begin{gather*}
\frac{\partial Y^{T}}{\partial K^{T}}=\gamma^{T}\left(B^{T}\right)^{\frac{\sigma-1}{\sigma}}\left(K^{T}\right)^{-\frac{1}{\sigma}}\left(Y^{T}\right)^{\frac{1}{\sigma}}=R,  \tag{407a}\\
\frac{\partial Y^{T}}{\partial L^{T}}=\left(1-\gamma^{T}\right)\left(A^{T}\right)^{\frac{\sigma-1}{\sigma}}\left(L^{T}\right)^{-\frac{1}{\sigma}}\left(Y^{T}\right)^{\frac{1}{\sigma}}=W,  \tag{407b}\\
P \frac{\partial Y^{N}}{\partial K^{N}}=P \gamma^{N}\left(B^{N}\right)^{\frac{\sigma-1}{\sigma}}\left(K^{N}\right)^{-\frac{1}{\sigma}}\left(Y^{N}\right)^{\frac{1}{\sigma}}=R,  \tag{407c}\\
P \frac{\partial Y^{N}}{\partial L^{N}}=P\left(1-\gamma^{N}\right)\left(A^{N}\right)^{\frac{\sigma-1}{\sigma}}\left(L^{N}\right)^{-\frac{1}{\sigma}}\left(Y^{N}\right)^{\frac{1}{\sigma}}=W, \tag{407d}
\end{gather*}
$$

Dividing the marginal revenue of labor by the marginal revenue of capital, the capital-labor ratios in the traded and the non traded sectors, respectively, are:

$$
\begin{align*}
k^{T} & \equiv\left(\frac{\gamma^{T}}{1-\gamma^{T}}\right)^{\sigma}\left(\frac{B^{T}}{A^{T}}\right)^{\sigma-1}\left(\frac{W}{R}\right)^{\sigma}  \tag{408a}\\
k^{N} & \equiv\left(\frac{\gamma^{N}}{1-\gamma^{N}}\right)^{\sigma}\left(\frac{B^{N}}{A^{N}}\right)^{\sigma-1}\left(\frac{W}{R}\right)^{\sigma} \tag{408b}
\end{align*}
$$

Equations (408) lead to the following positive relationship between the capital-labor in the non traded sector and the capital-labor ratio in the traded sector:

$$
\begin{equation*}
k^{N}=\left(\frac{\gamma^{N}}{\gamma^{T}} \frac{1-\gamma^{T}}{1-\gamma^{N}}\right)^{\frac{1}{\sigma}}\left[\left(\frac{B^{N}}{B^{T}}\right)\left(\frac{A^{T}}{A^{N}}\right)\right]^{\sigma-1} k^{T} \equiv \rho k^{T} . \tag{409}
\end{equation*}
$$

Using the fact that marginal revenues of labor equalize across sectors, i.e., (407b) and (407d), leads to:

$$
\begin{equation*}
P=\frac{1-\gamma^{T}}{1-\gamma^{N}}\left(\frac{A^{T}}{A^{N}}\right) \frac{\sigma-1}{\sigma}\left(\frac{Y^{T}}{Y^{N}}\right)^{\frac{1}{\sigma}}\left(\frac{L^{N}}{L^{T}}\right)^{\frac{1}{\sigma}} . \tag{410}
\end{equation*}
$$

Dividing the production function (405) by labor $L^{j}$, we get per worker output denoted by $y^{j}$ :

$$
\begin{equation*}
y_{j} \equiv \frac{Y^{j}}{L^{j}}=\left[\gamma^{j}\left(B^{j} k^{j}\right)^{\sigma-1} \sigma+\left(1-\gamma^{j}\right)\left(A^{j}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{411}
\end{equation*}
$$

and plugging this equation into (410) implies that the relative price of non tradables is given by:

$$
\begin{equation*}
P=\frac{1-\gamma^{T}}{1-\gamma^{N}}\left(\frac{A^{T}}{A^{N}}\right) \frac{\sigma-1}{\sigma}\left\{\frac{\gamma^{T}\left(B^{T} k^{T}\right)^{\sigma-1} \sigma+\left(1-\gamma^{T}\right)\left(A^{T}\right)^{\frac{\sigma-1}{\sigma}}}{\gamma^{N}\left(B^{N} \rho k^{T}\right)^{\sigma-1} \sigma+\left(1-\gamma^{N}\right)\left(A^{N}\right)^{\frac{\sigma-1}{\sigma}}}\right\}^{\frac{1}{\sigma-1}} \tag{412}
\end{equation*}
$$

Eq. (412) can be rewritten in a more interpretable form by calculating first the unit cost for producing $c^{j}$ in sector $j=T, N$. To do so, we have to determine the conditional demands for both inputs. Using (407), we have:

$$
\begin{align*}
L^{j} & =K^{j}\left(\frac{1-\gamma^{j}}{\gamma^{j}}\right)^{\sigma}\left(\frac{A^{j}}{B^{j}}\right)^{\sigma-1}\left(\frac{W}{R}\right)^{\sigma},  \tag{413a}\\
K^{j} & =L^{j}\left(\frac{\gamma^{j}}{1-\gamma^{j}}\right)^{\sigma}\left(\frac{B^{j}}{A^{j}}\right)^{\sigma-1}\left(\frac{W}{R}\right)^{-\sigma} . \tag{413b}
\end{align*}
$$

Inserting (413) in the CES production function (405), isolating $L^{j}$ and $K^{j}$, we have the conditional demand for labor and capital:

$$
\begin{equation*}
L^{j}=Y^{j}\left(A^{j}\right)^{\sigma-1}\left(\frac{1-\gamma^{j}}{W}\right)^{\sigma}\left(X^{j}\right)^{\frac{\sigma}{1-\sigma}}, \quad K^{j}=Y^{j}\left(B^{j}\right)^{\sigma-1}\left(\frac{\gamma^{j}}{R}\right)^{\sigma}\left(X^{j}\right)^{\frac{\sigma}{1-\sigma}} \tag{414}
\end{equation*}
$$

where

$$
\begin{equation*}
X^{j}=\left(1-\gamma^{j}\right)^{\sigma}\left(A^{j}\right)^{\sigma-1} W^{1-\sigma}+\left(\gamma^{j}\right)^{\sigma}\left(B^{j}\right)^{\sigma-1} R^{1-\sigma} . \tag{415}
\end{equation*}
$$

Owing to the constant returns to scale property of our CES production function, we can further simplify these conditional demand equations, by introducing the unit cost $c^{j}$. Total cost is equal to the sum of the labor and capital cost:

$$
\begin{equation*}
C^{j}=W L^{j}+R K^{j} . \tag{416}
\end{equation*}
$$

Inserting conditional demand for inputs (414) into total cost (416), we find $C^{j}$ is homogenous of degree one with respect to the level of production

$$
\begin{equation*}
C^{j}=c^{j} Y^{j}, \quad \text { with } \quad c^{j}=\left(X^{j}\right)^{\frac{1}{1-\sigma}}, \tag{417}
\end{equation*}
$$

where $X^{j}$ is given by (415). Substituting the capital-labor ratio in the traded sector given by (408a) into the formal expression for the relative price of non tradables (410) and rearranging terms leads to an alternative and more interpretable formal expression for the relative price:

$$
\begin{equation*}
P \equiv \frac{P^{N}}{P^{T}}=\frac{c^{N}}{c^{T}}=\left\{\frac{\left(\gamma^{N}\right)^{\sigma}\left(B^{N}\right)^{\sigma-1} R^{1-\sigma}+\left(1-\gamma^{N}\right)^{\sigma}\left(A^{N}\right)^{\sigma-1} W^{1-\sigma}}{\left(\gamma^{T}\right)^{\sigma}\left(B^{T}\right)^{\sigma-1} R^{1-\sigma}+\left(1-\gamma^{T}\right)^{\sigma}\left(A^{T}\right)^{\sigma-1} W^{1-\sigma}}\right\}^{\frac{1}{1-\sigma}} \tag{418}
\end{equation*}
$$

Before linearizing (418), it is useful to determine formal expressions for the labor and capital shares in sectoral output. Using the fact that $\left(c^{j}\right)^{1-\sigma}=X^{j}$, conditional demand for labor (414) can be rewritten as $L^{j}=Y^{j}\left(A^{j}\right)^{\sigma-1}\left(\frac{1-\gamma^{j}}{W}\right)\left(c^{j}\right)^{\sigma}$ which gives the labor share denoted by $s_{L}^{j}$ :

$$
\begin{equation*}
s_{L}^{j}=\frac{W L^{j}}{P^{j} Y^{j}}=\left(1-\gamma^{j}\right)^{\sigma}\left(A^{j}\right)^{\sigma-1}\left(c^{j}\right)^{\sigma-1} W^{1-\sigma} \tag{419}
\end{equation*}
$$

where $P^{j}=c^{j}$ (remembering that the price of tradable goods $P^{T}$ is normalized to one). An alternative formal expression for the labor share can be obtained from the first-order condition (407b) (or (407d)):

$$
\begin{equation*}
s_{L}^{j}=\frac{\left(1-\gamma^{j}\right)\left(A^{j} L^{j}\right)^{\frac{\sigma-1}{\sigma}}}{\gamma^{j}\left(B^{j} K^{j}\right)^{\frac{\sigma-1}{\sigma}}+\left(1-\gamma^{j}\right)\left(A^{j} L^{j}\right)^{\frac{\sigma-1}{\sigma}}}=\frac{\left(1-\gamma^{j}\right)\left(A^{j}\right)^{\frac{\sigma-1}{\sigma}}}{\left(y^{j}\right)^{\frac{\sigma-1}{\sigma}}} \tag{420}
\end{equation*}
$$

where $y^{j}$ is given by (411).

## The Relative Price Effect of a Productivity Differential with Biased Technological Change

As it will be useful later, differentiating the wage rate (407b), i.e., $W=\left(1-\gamma^{T}\right)\left(A^{T}\right)^{\frac{\sigma-1}{\sigma}}\left(y^{T}\right)^{\frac{1}{\sigma}}$, and denoting the percentage deviation from its initial steady-state by a hat, we have:

$$
\begin{equation*}
\hat{W}=\left(\frac{\sigma-1}{\sigma}\right) \hat{A}^{T}+\frac{1}{\sigma} \hat{y}^{T} . \tag{421}
\end{equation*}
$$

Differentiating the output per worker (411) in the traded sector and plugging the labor share $s_{L}^{T}$, we get:

$$
\begin{equation*}
\hat{y}^{T}=\left(1-s_{L}^{T}\right)\left(\hat{B}^{T}+\hat{k}^{T}\right)+s_{L}^{T} \hat{A}^{T} \tag{422}
\end{equation*}
$$

Differentiating the capital-labor ratio in the traded sector $k^{T}$ given by (408a) leads to:

$$
\begin{equation*}
\hat{k}^{T}=(\sigma-1)\left(\hat{B}^{T}-\hat{A}^{T}\right)+\sigma(\hat{W}-\hat{R}) \tag{423}
\end{equation*}
$$

Plugging (423) into (422), inserting the resulting expression into (421), we can solve for the wage rate:

$$
\begin{equation*}
\hat{W}=\hat{A}^{T}+\left(\frac{1-s_{L}^{T}}{s_{L}^{T}}\right) \hat{B}^{T}-\left(\frac{1-s_{L}^{T}}{s_{L}^{T}}\right) \hat{R} \tag{424}
\end{equation*}
$$

Differentiating the relative price of non tradables given by (418) and using the labor share $s_{L}^{T}$ given by (420) yields:

$$
\begin{align*}
\hat{P}=\hat{c}^{N}-\hat{c}^{T}= & \left(s_{L}^{N}-s_{L}^{T}\right)(\hat{W}-\hat{R})+\left[\left(1-s_{L}^{T}\right) \hat{B}^{T}-\left(1-s_{L}^{N}\right) \hat{B}^{N}\right] \\
& +\left(s_{L}^{T} \hat{A}^{T}-s_{L}^{N} \hat{A}^{N}\right) \tag{425}
\end{align*}
$$

Because we analyze the long-run changes and thus we estimate the steady-change of the relative price of non tradables, we first evaluate the return on domestic capital by setting $\dot{P}=0$ into (404d) which gives $R=P\left(r^{\star}+\delta_{K}\right)$; differentiating yields $\dot{R}=\dot{P}$. Inserting (424) into (425) and plugging $\dot{R}=\dot{P}$, isolating $\hat{P}$ and rearranging terms leads to the long-run change of the relative price of non-tradables:

$$
\hat{P}=s_{L}^{T}\left(\hat{A}^{T}-\hat{A}^{N}\right)+\left(1-s_{L}^{T}\right) \hat{B}^{T}-\frac{s_{L}^{T}}{s_{L}^{N}}\left(1-s_{L}^{N}\right) \hat{B}^{N}
$$

By rearranging terms, the equation above can be rewritten in a more familiar form:

$$
\begin{equation*}
\hat{P}=\left[s_{L}^{T} \hat{A}^{T}+\left(1-s_{L}^{T}\right) \hat{B}^{T}\right]-\frac{s_{L}^{T}}{s_{L}^{N}}\left[s_{L}^{N} \hat{A}^{N}+\left(1-s_{L}^{N}\right) \hat{B}^{N}\right] \tag{426}
\end{equation*}
$$

When the labor-augmenting productivity $A^{j}$ rises at the same speed as the capital-augmenting productivity $B^{j}$, and denoting by $\hat{Z}^{j}=\hat{A}^{j}=\hat{B}^{j}$ the Hicks-neutral technological change in sector $j=T, N$, eq. (426) leads to the usual strict proportional relationship between the relative price of non tradables and the productivity differential between tradables and non tradables:

$$
\begin{equation*}
\hat{P}=\hat{Z}^{T}-\frac{s_{L}^{T}}{s_{L}^{N}} \hat{Z}^{N} \tag{427}
\end{equation*}
$$

where $s_{L}^{j}$ corresponds to the labor share in sector $j=T, N$. However, recent empirical evidence documented by Antràs [2004] and Alvarez-Cuadrado et al. [2014] suggest positive labor-augmenting technological change and negative capital-augmenting technical change. Both growth rates are larger in the manufacturing sector. The fact that evidence indicate that $\hat{A}^{j} \neq \hat{B}^{j}$ suggests the presence of a technological change biased toward labor. While Hicks-neutral technological change does not conform to the data, the relationship (427) could remain an acceptable approximation if the weighted average of labor- and capital-augmenting productivity growth, $s_{L}^{j} \hat{A}^{j}+\left(1-s_{L}^{j}\right) \hat{B}^{j}$, is close to the estimates of total factor productivity growth, $\hat{Z}^{j}$.

## M. 2 Skill Biased Technological Change

In this subsection, we analyze the implications of skill biased technological change for the relative price effect of a technological change biased toward the traded sector. In the lines of Acemoglu [2002], we suppose that there are $L$ unskilled (low-education) workers and $H$ skilled (high-education) workers, supplying labor inelastically. We further assume that labor markets are competitive. For the sake of simplicity, we abstract from physical capital and impose perfect labor mobility so that the wage rates must equalize across sectors for both types of labor (i.e., for high- and low-skilled workers, respectively).

Traded and non traded goods are produced according to CES technologies:

$$
\begin{align*}
Y^{T} & =\left[\gamma^{T}\left(B^{T} H^{T}\right)^{\frac{\sigma-1}{\sigma}}+\left(1-\gamma^{T}\right)\left(A^{T} L^{T}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}  \tag{428a}\\
Y^{N} & =\left[\gamma^{N}\left(B^{N} H^{N}\right)^{\frac{\sigma-1}{\sigma}}+\left(1-\gamma^{N}\right)\left(A^{N} L^{N}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{428b}
\end{align*}
$$

where $L^{j}$ and $H^{j}$ are low- and high-skilled labor, respectively, used in sector $j=T, N$ to produce $Y^{j} . B^{j}$ and $A^{j}$ (with $j=T, N$ ) are factor-augmenting technology terms and $\sigma$ is the elasticity of substitution between high- and low-skilled labor; for clarity purposes, we abstract from sector-specific elasticities of substitution; as Acemoglu, we refer to high- and low-skilled workers as gross substitutes when the elasticity of substitution $\sigma>1$.

We assume that factors are fully utilized:

$$
\begin{gather*}
L^{T}+L^{N}=L  \tag{429a}\\
H^{T}+H^{N}=H \tag{429b}
\end{gather*}
$$

where low-skilled labor $L$ and high-skilled labor $H$ are taken to be given for the sake of simplicity.

The traded good is the numeraire and we denote the price of non traded goods in terms of traded goods by $P \equiv P^{N} / P^{T}$. We assume perfect mobility of high- and low-skilled labor across sectors and denote the high-skilled wage by $W^{H}$ and the low-skilled wage by $W^{L}$. First
order conditions are:

$$
\begin{gather*}
\frac{\partial Y^{T}}{\partial H^{T}}=\gamma^{T}\left(B^{T}\right)^{\frac{\sigma-1}{\sigma}}\left(H^{T}\right)^{-\frac{1}{\sigma}}\left(Y^{T}\right)^{\frac{1}{\sigma}}=W^{H},  \tag{430a}\\
\frac{\partial Y^{T}}{\partial L^{T}}=\left(1-\gamma^{T}\right)\left(A^{T}\right)^{\frac{\sigma-1}{\sigma}}\left(L^{T}\right)^{-\frac{1}{\sigma}}\left(Y^{T}\right)^{\frac{1}{\sigma}}=W^{L},  \tag{430b}\\
P \frac{\partial Y^{N}}{\partial H^{N}}=P \gamma^{N}\left(B^{N}\right)^{\frac{\sigma-1}{\sigma}}\left(H^{N}\right)^{-\frac{1}{\sigma}}\left(Y^{N}\right)^{\frac{1}{\sigma}}=W^{H},  \tag{430c}\\
P \frac{\partial Y^{N}}{\partial L^{N}}=P\left(1-\gamma^{N}\right)\left(A^{N}\right)^{\frac{\sigma-1}{\sigma}}\left(L^{N}\right)^{-\frac{1}{\sigma}}\left(Y^{N}\right)^{\frac{1}{\sigma}}=W^{L}, \tag{430d}
\end{gather*}
$$

Dividing the marginal revenue of low-skilled labor by the marginal revenue of high-skilled labor, the skilled-unskilled ratios (denoted by $h^{j}$ ) in the traded and the non traded sectors, respectively, are:

$$
\begin{align*}
h^{T} & \equiv\left(\frac{\gamma^{T}}{1-\gamma^{T}}\right)^{\sigma}\left(\frac{B^{T}}{A^{T}}\right)^{\sigma-1}\left(\frac{W^{L}}{W^{H}}\right)^{\sigma},  \tag{431a}\\
h^{N} & \equiv\left(\frac{\gamma^{N}}{1-\gamma^{N}}\right)^{\sigma}\left(\frac{B^{N}}{A^{N}}\right)^{\sigma-1}\left(\frac{W^{L}}{W^{H}}\right)^{\sigma} . \tag{431b}
\end{align*}
$$

Dividing the production function (428) by labor $L^{j}$, we get per low-skilled worker output denoted by $y^{j}$ :

$$
\begin{equation*}
y^{j} \equiv \frac{Y^{j}}{L^{j}}=\left[\gamma^{j}\left(B^{j} h^{j}\right)^{\frac{\sigma-1}{\sigma}}+\left(1-\gamma^{j}\right)\left(A^{j}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{432}
\end{equation*}
$$

In order to determine a useful form for the relative price of non tradables, we first calculate the unit cost for producing $c^{j}$ in sector $j=T, N$. To do so, we have to determine the conditional demands for both inputs. Using (431), we have:

$$
\begin{align*}
L^{j} & =H^{j}\left(\frac{1-\gamma^{j}}{\gamma^{j}}\right)^{\sigma}\left(\frac{A^{j}}{B^{j}}\right)^{\sigma-1}\left(\frac{W^{L}}{W^{H}}\right)^{\sigma}  \tag{433a}\\
H^{j} & =L^{j}\left(\frac{\gamma^{j}}{1-\gamma^{j}}\right)^{\sigma}\left(\frac{B^{j}}{A^{j}}\right)^{\sigma-1}\left(\frac{W^{L}}{W^{H}}\right)^{-\sigma} \tag{433b}
\end{align*}
$$

Inserting (433a) ((433a) resp.) in the CES production function (428), isolating $L^{j}$ ( $K^{j}$ resp.), we have the conditional demand for low-skilled labor (high-skilled labor resp.):

$$
\begin{equation*}
L^{j}=Y^{j}\left(A^{j}\right)^{\sigma-1}\left(\frac{1-\gamma^{j}}{W^{L}}\right)^{\sigma}\left(X^{j}\right)^{\frac{\sigma}{1-\sigma}}, \quad H^{j}=Y^{j}\left(B^{j}\right)^{\sigma-1}\left(\frac{\gamma^{j}}{W^{H}}\right)^{\sigma}\left(X^{j}\right)^{\frac{\sigma}{1-\sigma}} \tag{434}
\end{equation*}
$$

where

$$
\begin{equation*}
X^{j}=\left(1-\gamma^{j}\right)^{\sigma}\left(A^{j}\right)^{\sigma-1}\left(W^{L}\right)^{1-\sigma}+\left(\gamma^{j}\right)^{\sigma}\left(B^{j}\right)^{\sigma-1}\left(W^{H}\right)^{1-\sigma} . \tag{435}
\end{equation*}
$$

Owing to the constant returns to scale property of our CES production function, we can further simplify these conditional demand equations, by introducing the unit cost $c^{j}$. Total cost is equal to the sum of the labor and capital cost:

$$
\begin{equation*}
C^{j}=W^{L} L^{j}+W^{H} H^{j} . \tag{436}
\end{equation*}
$$

Inserting conditional demand for inputs (434) into total cost (436), we find $C^{j}$ is homogenous of degree one with respect to the level of production

$$
\begin{equation*}
C^{j}=c^{j} Y^{j}, \quad \text { with } \quad c^{j}=\left(X^{j}\right)^{\frac{1}{1-\sigma}} \tag{437}
\end{equation*}
$$

where $X^{j}$ is given by (435). As shown in section M.1, the price of non traded goods in terms of traded goods $P^{N} / P^{T}$ must equalize the ratio of units cost for producing $c^{N} / c^{T}$ :

$$
\begin{equation*}
P \equiv \frac{P^{N}}{P^{T}}=\frac{c^{N}}{c^{T}}=\left\{\frac{\left(\gamma^{N}\right)^{\sigma}\left(B^{N}\right)^{\sigma-1}\left(W^{H}\right)^{1-\sigma}+\left(1-\gamma^{N}\right)^{\sigma}\left(A^{N}\right)^{\sigma-1}\left(W^{L}\right)^{1-\sigma}}{\left(\gamma^{T}\right)^{\sigma}\left(B^{T}\right)^{\sigma-1}\left(W^{H}\right)^{1-\sigma}+\left(1-\gamma^{T}\right)^{\sigma}\left(A^{T}\right)^{\sigma-1}\left(W^{L}\right)^{1-\sigma}}\right\}^{\frac{1}{1-\sigma}} \tag{438}
\end{equation*}
$$

Before linearizing (438), it is useful to determine the formal expression for the low-skilled labor share in sectoral output. Using the fact that $\left(c^{j}\right)^{1-\sigma}=X^{j}$, conditional demand for low-skilled labor (434) can be rewritten as $L^{j}=Y^{j}\left(A^{j}\right)^{\sigma-1}\left(\frac{1-\gamma^{j}}{W}\right)\left(c^{j}\right)^{\sigma}$ which gives the low-skilled labor income share denoted by $s_{L}^{j}$ :

$$
\begin{equation*}
s_{L}^{j}=\frac{W^{L} L^{j}}{P^{j} Y^{j}}=\left(1-\gamma^{j}\right)^{\sigma}\left(A^{j}\right)^{\sigma-1}\left(c^{j}\right)^{\sigma-1}\left(W^{L}\right)^{1-\sigma} \tag{439}
\end{equation*}
$$

where $P^{j}=c^{j}$. An alternative formal expression for the low-skilled labor income share can be obtained from the first-order condition (430b) (or alternatively (430d)):

$$
\begin{equation*}
s_{L}^{j}=\frac{\left(1-\gamma^{j}\right)\left(A^{j} L^{j}\right)^{\frac{\sigma-1}{\sigma}}}{\gamma^{j}\left(B^{j} H^{j}\right)^{\frac{\sigma-1}{\sigma}}+\left(1-\gamma^{j}\right)\left(A^{j} L^{j}\right)^{\frac{\sigma-1}{\sigma}}}=\frac{\left(1-\gamma^{j}\right)\left(A^{j}\right)^{\frac{\sigma-1}{\sigma}}}{\left(y^{j}\right)^{\frac{\sigma-1}{\sigma}}} \tag{440}
\end{equation*}
$$

where $y^{j}$ is given by (432).

## The Relative Price Effect of a Productivity Differential with skill-biased Technological Change

As it will be useful later, we first differentiate the unit cost for producing $c^{j}=\left(X^{j}\right)^{\frac{1}{1-\sigma}}$ where $X^{j}$ is given by (435); denoting the percentage deviation from its initial steady-state by a hat, we have:

$$
\begin{equation*}
\hat{c}^{j}=\left(1-s_{L}^{j}\right)\left(-\hat{B}^{j}+\hat{W}^{H}\right)+s_{L}^{j}\left(-\hat{A}^{j}+\hat{W}^{L}\right) \tag{441}
\end{equation*}
$$

where we have inserted the low-skilled labor income share denoted by $s_{L}^{j}$ given by (439).
Differentiating the relative price of non tradables given by (438) and using the labor share $s_{L}^{j}$ given by (440) yields:

$$
\begin{align*}
\hat{P}= & \hat{c}^{N}-\hat{c}^{T}=-\left(s_{L}^{N}-s_{L}^{T}\right)\left(\hat{W}^{H}-\hat{W}^{L}\right)+\left[\left(1-s_{L}^{T}\right) \hat{B}^{T}-\left(1-s_{L}^{N}\right) \hat{B}^{N}\right] \\
& +\left(s_{L}^{T} \hat{A}^{T}-s_{L}^{N} \hat{A}^{N}\right) . \tag{442}
\end{align*}
$$

Totally differentiating the skilled-unskilled ratio given by (431a) and assuming that the relative supply of skilled rises at the rate $\hat{H}^{T}-\hat{L}^{T}$ in the traded sector, we are able to determine a formal expression for the percentage change in the skill premium $\hat{W}^{H}-\hat{W}^{L}$ :

$$
\begin{equation*}
\hat{W}^{H}-\hat{W}^{L}=\left(\frac{\sigma-1}{\sigma}\right)\left(\hat{B}^{T}-\hat{A}^{T}\right)-\frac{1}{\sigma}\left(\hat{H}^{T}-\hat{L}^{T}\right) . \tag{443}
\end{equation*}
$$

According to (443), the change in the skill premium is driven by exogenous forces: the percentage change in relative productivity of skilled labor and the percentage change in the relative supply of skills in the traded sector. The skill premium falls when skilled workers become more abundant, i.e., $\hat{H}^{T}-\hat{L}^{T}>0$. Additionally, as equation (443) shows, the elasticity of substitution, $\sigma$, is important for the behavior of the skill premium. If $\sigma>1$, then an improvement in the productivity of skilled workers, $B^{T}$, relative to the productivity of unskilled workers, $A^{T}$, stimulates the relative demand for skilled labor and thus raises the skill premium. As documented by Acemoglu [2002], most estimates report an elasticity of substitution between skilled and unskilled workers greater than 1 so that the skill premium increases when skilled workers become relatively more productive.

Inserting (443) into (442), we find that skill-biased technological change modifies the response of the relative price of non tradables to a productivity differential between tradables and non tradables:

$$
\begin{align*}
\hat{P} & =\left[s_{L}^{T} \hat{A}^{T}+\left(1-s_{L}^{T}\right) \hat{B}^{T}\right]-\left[s_{L}^{N} \hat{A}^{N}+\left(1-s_{L}^{N}\right) \hat{B}^{N}\right] \\
& +\left(s_{L}^{T}-s_{L}^{N}\right)\left[\left(\frac{\sigma-1}{\sigma}\right)\left(\hat{B}^{T}-\hat{A}^{T}\right)-\frac{1}{\sigma}\left(\hat{H}^{T}-\hat{L}^{T}\right)\right] . \tag{444}
\end{align*}
$$

Denoting the weighted average of high-skilled and low-skilled labor-augmenting productivity growth, i.e., the term $s_{L}^{j} \hat{A}^{j}+\left(1-s_{L}^{j}\right) \hat{B}^{j}$, by $\hat{Z}^{j}$, eq. (444) can be rewritten in a more compact form:

$$
\begin{equation*}
\hat{P}=\left(\hat{Z}^{T}-\hat{Z}^{N}\right)+\left(s_{L}^{T}-s_{L}^{N}\right)\left(\hat{W}^{H}-\hat{W}^{L}\right) . \tag{445}
\end{equation*}
$$

where $s_{L}^{j}$ is the low-skilled labor's share in value added in sector $j=T, N$. As shown by the first term of the RHS of eq. (445), a productivity differential between tradables and non tradables of $1 \%$ appreciates the relative price of non tradables by $1 \%$, as in the standard BS model which assumes that labor is homogenous. As captured by the second term of the RHS of eq. (445), a rise in the skill premium, i.e., $\hat{W}^{H}-\hat{W}^{L}>0$, amplifies the appreciation in the price of non traded goods in terms of traded goods following a productivity differential as long as the low-skilled labor income share in the traded sector is higher than that in the non traded sector, i.e., $s_{L}^{T}-s_{L}^{N}>0$. Table 25 shows the ratio of labor compensation to value added for the whole economy and for the traded and non traded sector as well. Column 10 of Table 25 gives the difference (i.e., $s_{L}^{T}-s_{L}^{N}$ ) between the low-skilled labor income share in the traded sector, $s_{L}^{T}$, and the low-skilled labor income share in the non traded sector, $s_{L}^{N}$. The difference $s_{L}^{T}-s_{L}^{N}$ is positive in all countries of our sample, except Sweden. Hence, data reveal that the traded sector is relatively more intensive in unskilled labor than the non traded sector. Regarding the change in the skill premium, using data on the employment of production and nonproduction workers in manufacturing from twelve developed countries Berman, Bound and Machin [1998] find that the relative wage of nonproduction workers typically declined in the 1970s and increased in the 1980s. In the 1980s, estimates by Berman, Bound and Machin [1998] indicate that relative wages of nonproduction workers rose by an average of 4 percent in these developed countries in the 1980s. The U.S. increase of 7 percent was above average. When estimating the (percentage) change of the relative wage of high-skilled workers over the period 1970-2005, we find that the skill premium decreases in most of the OECD countries in our sample, except for Germany, Ireland and the United States. By and large, columns 10 and 11 of Table 25 reveal that the term $\left(s_{L}^{T}-s_{L}^{N}\right)\left(\hat{W}^{H}-\hat{W}^{L}\right)$ is negative for most of the countries of our sample. Hence, technological change biased toward skilled labor tends to moderate the appreciation in the relative price of non tradables following a productivity differential between tradables and non tradables captured by $\hat{Z}^{T}-\hat{Z}^{N}>0$.

## The Relative Wage Effect of a Productivity Differential with skill-biased Technological Change

We now derive the change in the ratio of non traded wage $W^{N}$ to traded wage $W^{T}$ following a productivity differential. According to (436), total cost is equal to the sum of the low- and high-skilled labor cost, i.e., $C^{j}=W^{L} L^{j}+W^{H} H^{j}$ where $C^{j}=c^{j} Y^{j}$ (see (437)) witg $C^{T}=Y^{T}$ and $C^{N}=P Y^{N}$. The average wage rate in sector $j$ is determined by dividing labor compensation $C^{j}$ by total labor used in sector $j$ :

$$
\begin{equation*}
W^{j}=\frac{W^{L} L^{j}+W^{H} H^{j}}{L^{j}+H^{j}}=\frac{c^{j} Y^{j}}{L^{j}+H^{j}}=\frac{c^{j} y^{j}}{1+h^{j}}, \tag{446}
\end{equation*}
$$

where $h^{j} \equiv \frac{H^{j}}{L^{j}+H^{j}}$ is the share of high-skilled workers employed by sector $j=T, N$ and $y^{j} \equiv Y^{j} / L^{j}$ is given by (432). Totally differentiating (446) shows that the percentage change
in the wage rate in sector $j$ is driven by percentage changes in the unit cost for producing, the per low-skilled worker output and the share of low-skilled labor:

$$
\begin{equation*}
\hat{W}^{j}=\hat{c}^{j}+\hat{y}^{j}-\frac{h^{j}}{1+h^{j}} \hat{h}^{j} \tag{447}
\end{equation*}
$$

where $\hat{c}^{T}=0$ and $\hat{c}^{N}=\hat{P}$ and $\hat{y}^{j}=s_{L}^{j} \hat{A}^{j}+\left(1-s_{L}^{j}\right) \hat{B}^{j}+\left(1-s_{L}^{j}\right) \hat{h}^{j}$. Inserting $\hat{y}^{j}$, the percentage change in the wage rate in sector $j$ can be rewritten as follows:

$$
\begin{equation*}
\hat{W}^{j}=\hat{c}^{j}+\left[\left(1-s_{L}^{j}\right)-\frac{h^{j}}{1+h^{j}}\right] \hat{h}^{j}+s_{L}^{j} \hat{A}^{j}+\left(1-s_{L}^{j}\right) \hat{B}^{j} \tag{448}
\end{equation*}
$$

Denoting the weighted average of high-skilled and low-skilled labor-augmenting productivity growth, i.e., the term $s_{L}^{j} \hat{A}^{j}+\left(1-s_{L}^{j}\right) \hat{B}^{j}$, by $\hat{Z}^{j}$, and denoting by $\Omega \equiv W^{N} / W^{T}$ the ratio of non traded wage to traded wage and totally differentiating, making use of (448), the percentage change in the relative wage of non tradables is given by:

$$
\begin{align*}
\hat{\Omega}=\hat{W}^{N}-\hat{W}^{T} & =\hat{P}+\hat{Z}^{N}-\hat{Z}^{T} \\
& +\left[\left(1-s_{L}^{N}\right)-\frac{h^{N}}{1+h^{N}}\right] \hat{h}^{N}-\left[\left(1-s_{L}^{T}\right)-\frac{h^{T}}{1+h^{T}}\right] \hat{h}^{T} \tag{449}
\end{align*}
$$

Substituting $\hat{P}$ given by (445), the percentage change in the relative wage of non tradables reduces to:

$$
\begin{align*}
\hat{\Omega}=\hat{W}^{N}-\hat{W}^{T} & =\left(s_{L}^{T}-s_{L}^{N}\right)\left(\hat{W}^{H}-\hat{W}^{L}\right) \\
& +\left[\left(1-s_{L}^{N}\right)-\frac{h^{N}}{1+h^{N}}\right] \hat{h}^{N}-\left[\left(1-s_{L}^{T}\right)-\frac{h^{T}}{1+h^{T}}\right] \hat{h}^{T} \tag{450}
\end{align*}
$$

where the skill premium change $\left(\hat{W}^{H}-\hat{W}^{L}\right)$ is given by (443) and depends on skilled biased technological change captured by $\hat{B}^{T}-\hat{A}^{T}>0$. We discuss below the effects of a productivity differential between tradables and non tradables when technological change is skill-biased.

The first line of eq. (450) shows that the change in the skill premium $\left(\hat{W}^{H}-\hat{W}^{L}\right)$ is positively related to the change in the relative wage $\hat{\Omega}$ of non tradables because data reported in column 10 of Table 25 indicate that the non traded sector is relatively more high-skilled intensive than the traded sector, i.e., $s_{L}^{T}>s_{L}^{N}$ or alternatively $\left(1-s_{L}^{N}\right)>\left(1-s_{L}^{T}\right)$. Hence, a rise in the skill premium would increase the non traded wage relative to the traded wage as the non traded sector uses relatively more intensively high-skilled workers. As shown in the last column of Table 25, the skill premium tends to decline, except for the United States, France and Finland. By and large, for two-third of the countries of our sample, the term $\left(s_{L}^{T}-s_{L}^{N}\right)\left(\hat{W}^{H}-\hat{W}^{L}\right)$ is negative, so that biased technological change exerts a negative impact on the relative wage of non tradables for these countries.

The second line of eq. (450) shows the impact of a rise in the ratio of $h^{j}$ on the sectoral wage and thus on $W^{N} / W^{T}$. According to (448), the average wage $W^{j}$ is increasing in $h^{j}=H^{j} / L^{j}$ as long as the skill premium is positive (i.e., $W^{H} / W^{L}>1$, see Acemoglu [2002]). Intuitively, as the skill composition of the labor force improves, wages will increase. As shown in columns 11 and 12 of Table 26 , the term $\left[\left(1-s_{L}^{j}\right)-\frac{h^{j}}{1+h^{j}}\right]$ is positive in both sectors (but small); while a rise in $h^{j}$ raises the average wage $W^{j}$, by assuming that $\left[\left(1-s_{L}^{j}\right)-\frac{h^{j}}{1+h^{j}}\right]$ is almost identical across sectors, it is the speed at which sectors increase the share of high-skilled workers in total labor, as captured by the term $\hat{h}^{N}-\hat{h}^{T}$, that determines the change in the ratio $W^{N} / W^{T}$. Column 15 of Table 26 reveals that $\hat{h}^{N}-\hat{h}^{T}$ is negative in all countries of our sample. As the sectoral average wage is positively related with the share of skilled labor,

Table 25: The Labor Income Share by Country, Sector and Skill

| Country | Aggregate |  |  |  | Sector $T$ |  |  |  | Sector $N$ |  |  | Differences |  | Data coverage |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ | $(6)$ | $(7)$ | $(8)$ | $(9)$ | $(10)$ | $(11)$ | $(12)$ |  |  |
|  | $s$ | $s_{H}$ | $s_{L}$ | $s^{T}$ | $s_{H}^{T}$ | $s_{L}^{T}$ | $s^{N}$ | $s_{H}^{N}$ | $s_{L}^{N}$ | $s_{L}^{T}-s_{L}^{N}$ | $\hat{W}^{H}-\hat{W}^{L}$ |  |  |  |
| BEL | 0.66 | 0.12 | 0.54 | 0.66 | 0.09 | 0.57 | 0.66 | 0.13 | 0.52 | 0.04 | -0.28 | $1980-2005$ |  |  |
| DEU | 0.69 | 0.09 | 0.61 | 0.77 | 0.07 | 0.70 | 0.65 | 0.10 | 0.55 | 0.00 | -0.80 | $1970-2005$ |  |  |
| DNK | 0.68 | 0.06 | 0.62 | 0.66 | 0.03 | 0.63 | 0.69 | 0.07 | 0.62 | 0.04 | -1.25 | $1980-2005$ |  |  |
| ESP | 0.64 | 0.16 | 0.48 | 0.59 | 0.09 | 0.51 | 0.67 | 0.21 | 0.46 | 0.10 | -1.11 | $1980-2005$ |  |  |
| FIN | 0.71 | 0.24 | 0.46 | 0.66 | 0.17 | 0.49 | 0.74 | 0.30 | 0.44 | 0.15 | 0.30 | $1970-2005$ |  |  |
| FRA | 0.69 | 0.13 | 0.56 | 0.71 | 0.08 | 0.63 | 0.68 | 0.15 | 0.53 | -0.10 | 0.49 | $1980-2005$ |  |  |
| GBR | 0.72 | 0.12 | 0.61 | 0.70 | 0.08 | 0.63 | 0.74 | 0.15 | 0.59 | 0.12 | -0.02 | $1970-2005$ |  |  |
| IRL | 0.59 | 0.13 | 0.45 | 0.47 | 0.07 | 0.40 | 0.69 | 0.19 | 0.50 | 0.01 | -0.43 | $1988-2005$ |  |  |
| ITA | 0.71 | 0.06 | 0.65 | 0.75 | 0.03 | 0.72 | 0.68 | 0.08 | 0.60 | 0.18 | -1.48 | $1970-2005$ |  |  |
| JPN | 0.65 | 0.17 | 0.48 | 0.60 | 0.12 | 0.48 | 0.68 | 0.20 | 0.47 | -0.06 | -0.89 | $1973-2005$ |  |  |
| KOR | 0.78 | 0.34 | 0.44 | 0.75 | 0.22 | 0.53 | 0.82 | 0.47 | 0.35 | 0.04 | -0.47 | $1970-2005$ |  |  |
| NLD | 0.68 | 0.08 | 0.60 | 0.61 | 0.04 | 0.56 | 0.73 | 0.11 | 0.62 | -0.01 | -0.27 | $1979-2005$ |  |  |
| SWE | 0.69 | 0.13 | 0.56 | 0.64 | 0.08 | 0.56 | 0.72 | 0.16 | 0.56 | 0.03 | -0.88 | $1981-2005$ |  |  |
| USA | 0.62 | 0.22 | 0.41 | 0.64 | 0.18 | 0.46 | 0.61 | 0.24 | 0.38 | 0.08 | 0.53 | $1970-2005$ |  |  |
| Average | 0.68 | 0.15 | 0.53 | 0.66 | 0.10 | 0.56 | 0.70 | 0.18 | 0.51 | 0.05 | -0.47 |  |  |  |

Notes: $s_{H}$ is the labor income share for high-skilled workers and $s_{L}$ the labor income share for low-skilled workers, $W^{H}$ is the high-skilled wage rate while $W^{L}$ is the low-skilled wage rate; hence $\hat{W}^{H}-\hat{W}^{L}$ is the percentage change of the skill premium.
because the traded sector increases more rapidly the share of skilled labor than the non traded sector, this movement exerts a negative impact on the relative wage.

In conclusion, both the change in the skill premium and the skill composition of the labor force in each sector matter in determining the responses of the relative price and relative wage of non tradables to technological change biased toward the traded sector. We find that a declining skill premium moderates the response of the relative price to a productivity differential between tradables and non tradables and exerts a negative impact on the relative wage, even when imposing perfect mobility of labor across sectors. We believe that more work must be done in this direction. We leave a further analysis of these issues for future research.

## Data Construction and Source

The EU KLEMS database (the March 2008 data release) provides details data on labor compensation and the number of hours worked by industry and by skill level (low, medium and high) for the fourteen countries of our sample. Note that in the March 2011 data release of the EU KLEMS, which is our baseline database, sectoral disaggregation of employment and labor compensation between skilled and unskilled labor is not available. In detail, the 2008 EU KLEMS database gives information, at the sectoral level, on the share of labor compensation devoted to each skill level in total labor compensation and on the share of hours worked by each skill level in total hours worked. Having pinned down those ratios, we are able to decompose the aggregate labor compensation and hours worked in the March 2011 data release of the EU KLEMS using the same proportions that we observe in the March 2008 data release of the EU KLEMS. The EU KLEMS categorization of labor by skill provides data on three types (low, medium and high skills). We group the low and medium levels into a single category that corresponds to our definition of unskilled workers, and the remaining data coincides with our definition of high-skilled workers. The EU KLEMS defines high-skilled workers as those workers that hold a Bachelor degree (or equivalent) or above. The period begins earliest in 1970 and ending at the latest in 2005 (see the last column of Table 25). No country is followed for less than 25 years with the exception of Ireland (1988-2005). We now describe the construction for the data shown in Table 25 and 26 (mnemonics are given in parentheses).

- Labor compensation of high-skilled workers $W_{t}^{H} H_{t}^{j}$ in sector $j=T, N$ : share of high-
Table 26: The Share of Skilled and Low-Skilled Workers in Total Employment by Country and Sector

| Country | Aggregate |  | Sector $T$ |  | Sector $N$ |  | High-skilled |  | Low-skilled |  | Effect of $h^{j}$ on $W^{j}$ |  | (13) | (14) | (15) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) |  | (3) | (4) | (5) | (6) | (7) |  | (9) |  | (11) | (12) |  |  |  |
|  | $h$ | $1-h$ | $h^{T}$ | $1-h^{T}$ | $h^{N}$ | $1-h^{N}$ | $T$ | $N$ | $T$ | $N$ | $\left(s_{H}^{T}-\frac{h^{T}}{1+h^{T}}\right)$ | $\left(s_{H}^{N}-\frac{h^{N}}{1+h^{N}}\right)$ | $\hat{h}^{T}$ | $\hat{h}^{N}$ | $\hat{h}^{N}-\hat{h}^{T}$ |
| BEL | 0.11 | 0.89 | 0.08 | 0.92 | 0.13 | 0.87 | 0.23 | 0.77 | 0.36 | 0.64 | 0.02 | 0.01 | 3.83 | 2.36 | -1.47 |
| DEU | 0.07 | 0.93 | 0.05 | 0.95 | 0.08 | 0.92 | 0.30 | 0.70 | 0.43 | 0.57 | 0.02 | 0.03 | 2.83 | 1.46 | -1.37 |
| DNK | 0.05 | 0.95 | 0.03 | 0.97 | 0.07 | 0.93 | 0.16 | 0.84 | 0.35 | 0.65 | 0.00 | 0.00 | 6.01 | 3.34 | -2.67 |
| ESP | 0.14 | 0.86 | 0.08 | 0.92 | 0.19 | 0.81 | 0.19 | 0.81 | 0.42 | 0.58 | 0.02 | 0.05 | 6.51 | 2.51 | -4.00 |
| FIN | 0.24 | 0.76 | 0.18 | 0.82 | 0.29 | 0.71 | 0.32 | 0.68 | 0.48 | 0.52 | 0.02 | 0.08 | 3.48 | 2.12 | -1.36 |
| FRA | 0.10 | 0.90 | 0.05 | 0.95 | 0.13 | 0.87 | 0.19 | 0.81 | 0.37 | 0.63 | 0.03 | 0.03 | 4.77 | 2.91 | -1.86 |
| GBR | 0.09 | 0.91 | 0.06 | 0.94 | 0.11 | 0.89 | 0.25 | 0.75 | 0.38 | 0.62 | 0.02 | 0.05 | 5.24 | 4.70 | -0.54 |
| IRL | 0.13 | 0.87 | 0.09 | 0.91 | 0.16 | 0.84 | 0.24 | 0.76 | 0.41 | 0.59 | -0.01 | 0.05 | 8.07 | 2.06 | -6.01 |
| ITA | 0.07 | 0.93 | 0.02 | 0.98 | 0.10 | 0.90 | 0.16 | 0.84 | 0.47 | 0.53 | 0.01 | -0.01 | 3.05 | 3.03 | -0.02 |
| JPN | 0.17 | 0.83 | 0.13 | 0.87 | 0.20 | 0.80 | 0.31 | 0.69 | 0.42 | 0.58 | 0.00 | 0.03 | 3.09 | 2.41 | -0.68 |
| KOR | 0.27 | 0.73 | 0.20 | 0.80 | 0.36 | 0.64 | 0.39 | 0.61 | 0.60 | 0.40 | 0.05 | 0.21 | 4.11 | 2.42 | -1.68 |
| NLD | 0.08 | 0.92 | 0.04 | 0.96 | 0.09 | 0.91 | 0.15 | 0.85 | 0.34 | 0.66 | 0.00 | 0.03 | 6.97 | 3.53 | -3.44 |
| SWE | 0.13 | 0.87 | 0.08 | 0.92 | 0.16 | 0.84 | 0.21 | 0.79 | 0.36 | 0.64 | 0.01 | 0.02 | 3.38 | 2.32 | -1.06 |
| USA | 0.24 | 0.76 | 0.18 | 0.82 | 0.27 | 0.73 | 0.23 | 0.77 | 0.34 | 0.66 | 0.03 | 0.03 | 2.66 | 1.19 | -1.46 |
| Average | 0.1 | 0.86 | 0.09 | 0.91 | 0.17 | 0.83 | 0.24 | 0.76 | 0.41 | 0.59 | 0.02 | 0.03 | 4.57 | 2.60 | -1.97 |

Notes: $h=H /(H+L)$ is the high-skilled workers as a share of total employment; $h^{j}$ is high-skilled workers as a share of total employment in sector $j$; $\hat{h}^{j}$ is the average annual change in percentage of the ratio of high-skilled workers to total employment in sector $j$; the data coverage for each country is reported in the last coly)
1981-2005, respectively
skilled labor compensation in total labour compensation (LABHS) times labor compensation in sector $j$ (LAB). Source: EU KLEMS database (the March 2008 for LABHS and the March 2011 for LAB).

- Labor compensation of low-skilled workers $W_{t}^{L} L_{t}^{j}$ in sector $j=T, N$ : share of lowskilled and medium-skilled labor compensation in total labour compensation (LABMS and LABLS) times labor compensation in sector $j$ (LAB). Source: EU KLEMS database (the March 2008 for LABMS and LABLS and the March 2011 for LAB).
- Hours worked by high-skilled persons engaged $H_{t}^{j}$ in sector $j=T, N$ : share of hours worked by high-skilled persons engaged in total hours (H_HS) times total hours worked by persons engaged in sector $j$ (H_EMP). Source: EU KLEMS database (the March 2008 for H_HS and the March 2011 for H_EMP).
- Hours worked by low-skilled persons engaged $L_{t}^{j}$ in sector $j=T, N$ : share of hours worked by low-skilled and medium-skilled persons engaged in total hours (H_MS and H_LS) times total hours worked by persons engaged in sector $j$ (H_EMP). Source: EU KLEMS database (the March 2008 for H_MS and H_LS and the March 2011 for H_EMP).

Using time series for $W_{t}^{H} H_{t}^{j}, W_{t}^{L} L_{t}^{j}, H_{t}^{j}$ and $L_{t}^{j}$, we construct the low-skilled labor income share $s_{L}^{j}$ and the share of low-skilled labor in total labor $\kappa^{j}$ in sector $j=T, N$.

## N Endogenous Markups and Technological Change Biased toward the Traded Sector

The framework builds on Jaimovich and Floetotto [2008]. We assume that two imperfectly competitive sectors produce a traded good denoted by the superscript $T$ and a non-traded good denoted by the superscript $N$. Following Yang and Heijdra [1993] and Jaimovich and Floetotto [2008], we depart from the usual practice by assuming that the number of firms is large enough so that we can ignore the strategic effects but not so large that the effect of entry on the firm's demand curve is minuscule. Consequently, the price elasticity of demand faced by a single firm is no longer constant and equal to the elasticity of substitution between any two varieties, but rather a function of the number of firms $M^{j}$ in sector $j$.

The final output, $Y^{j}$, is produced in a competitive retail sector with constant-returns-to-scale production which aggregates a continuum measure one of goods. We denote the elasticity of substitution between any two different goods by $\omega>0$. In each industry, there are $M^{j}>1$ firms producing differentiated goods that are aggregated into an intermediate good. The elasticity of substitution between any two varieties within an industry is denoted by $\rho>0$, and we assume that this is higher than the elasticity of substitution across industries, i.e. $\epsilon>\omega$ (see Jaimovich and Floetotto [2008]). Within each industry, there is monopolistic competition; each firm that produces one variety is a price setter. Output $\mathcal{X}_{i, s}^{j}$ of firm $i$ in industry $s$ in sector $j$ is produced using labor, i.e. $\mathcal{X}_{i, s}^{j}=A^{j} \mathcal{L}_{i, s}^{j}$.

## N. 1 Households

At each instant the representative household consumes traded and non traded goods denoted by $C^{T}(t)$ and $C^{N}(t)$, respectively, which are aggregated by means of a CES function:

$$
\begin{equation*}
C(t)=\left[\varphi^{\frac{1}{\phi}}\left(C^{T}(t)\right)^{\frac{\phi-1}{\phi}}+(1-\varphi)^{\frac{1}{\phi}}\left(C^{N}(t)\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{451}
\end{equation*}
$$

where $0<\varphi<1$ is the weight of the traded good in the overall consumption bundle and $\phi$ corresponds to the elasticity of substitution between traded goods and non traded goods.

The representative agent supplies inelastically labor $L$ that we normalize to one. At any instant of time, households derive utility from their consumption $C(t)$. The representative household maximizes the following objective function:

$$
\begin{equation*}
U=\int_{0}^{\infty} \frac{1}{1-\frac{1}{\sigma_{C}}} C(t)^{1-\frac{1}{\sigma_{C}}} e^{-\beta t} \mathrm{~d} t \tag{452}
\end{equation*}
$$

where $\beta$ is the discount rate, $\sigma_{C}>0$ corresponds to the intertemporal elasticity of substitution for consumption.

Labor income is derived by supplying labor at a wage rate $W(t)$. In addition, households accumulate internationally traded bonds, $B(t)$, that yield net interest rate earnings of $r^{\star} B(t)$. The flow budget constraint is equal to households' income less consumption expenditure:

$$
\begin{equation*}
\dot{B}(t)=r^{\star} B(t)+W(t) L-P_{C}(P(t)) C(t) . \tag{453}
\end{equation*}
$$

where $L=1$.
Denoting the co-state variable associated with eq. (453) by $\lambda$ the first-order conditions characterizing the representative household's optimal plans are:

$$
\begin{gather*}
C(t)=\left[P_{C}(P(t)) \lambda(t)\right]^{-\sigma_{C}},  \tag{454a}\\
\dot{\lambda}(t)=\lambda(t)\left(\beta-r^{\star}\right), \tag{454b}
\end{gather*}
$$

and the transversality conditions $\lim _{t \rightarrow \infty} \lambda B(t) e^{-\beta t}=0$. In an open economy model with a representative agent having perfect foresight, a constant rate of time preference and perfect access to world capital markets, we impose $\beta=r^{\star}$ in order to generate an interior solution. Setting $\beta=r^{\star}$ into (454b) yields $\lambda=\bar{\lambda}$. This standard assumption made in the literature implies that the marginal utility of wealth, $\lambda$, will undergo a discrete jump when individuals receive new information and must remain constant over time from then on.

For the sake of clarity, we drop the time argument below when this causes no confusion.
The homogeneity of $C($.$) allows a two-stage consumption decision: in the first stage,$ consumption is determined, and the intratemporal allocation between traded and non-traded goods is decided at the second stage. Applying Shephard's lemma gives $C^{N}=P_{C}^{\prime} C$ with $P_{C}^{\prime}=\frac{\partial P_{C}}{\partial P}$; denoting by $\alpha_{C}$ the share of non-traded goods in the consumption expenditure, we have $C^{N}=\alpha_{C} P_{C} C / P$ and $C^{T}=P_{C} C-P C^{N}=\left(1-\alpha_{C}\right) P_{C} C$. Intra-temporal allocation of consumption follows from the following optimal rule:

$$
\begin{equation*}
\frac{C^{T}}{C^{N}}=\left(\frac{\varphi}{1-\varphi}\right) P^{\phi} . \tag{455}
\end{equation*}
$$

## N. 2 Firms

The final output in sector $j=T, N, Y^{j}$, is produced in a competitive retail sector using a constant-returns-to-scale production function which aggregates a continuum measure one of intermediate goods:

$$
\begin{equation*}
Y^{j}=\left[\int_{0}^{1}\left(\mathcal{Q}_{s}^{j}\right)^{\frac{\omega-1}{\omega}} \mathrm{~d} s\right]^{\frac{\omega}{\omega-1}}, \tag{456}
\end{equation*}
$$

where $\omega>0$ represents the elasticity of substitution between any two different sectoral goods and $\mathcal{Q}_{s}^{M^{j}}$ stands for intermediate consumption of industry's variety in sector $j=T, N$ (with $\left.s \in\left[0, M^{j}\right]\right)$. The final good producers behave competitively, and the households use the final good for consumption.

In each of the $s$ industries, there are $M^{j}>1$ firms producing differentiated goods that are aggregated into an intermediate good by a CES aggregating function. The intermediate
good output of industry $s$ is given by: ${ }^{81}$

$$
\begin{equation*}
\mathcal{Q}_{s}^{j}=\left(M^{j}\right)^{-\frac{1}{\rho-1}}\left[\int_{0}^{M^{j}}\left(\mathcal{X}_{i, s}^{j}\right)^{\frac{\rho-1}{\rho}} \mathrm{~d} i\right]^{\frac{\rho}{\rho-1}} \tag{457}
\end{equation*}
$$

where $\mathcal{X}_{i, s}^{j}$ stands for output of firm $i$ in industry $s$ and $\rho$ is the elasticity of substitution between any two varieties.

Denoting by $P^{j}$ and $\mathcal{P}_{s}^{j}$ the price of the final good and of the $s$ th variety of the intermediate good, respectively, the profit the final good producer is written as follows:

$$
\begin{equation*}
\Pi^{j}=P^{j}\left[\int_{0}^{1}\left(\mathcal{Q}_{s}^{j}\right)^{\frac{\omega-1}{\omega}} \mathrm{~d} s\right]^{\frac{\omega}{\omega-1}}-\int_{0}^{1} \mathcal{P}_{s}^{j} \mathcal{Q}_{s}^{j} \mathrm{~d} s \tag{458}
\end{equation*}
$$

Total cost minimizing for a given level of final output gives the (intratemporal) demand function for each input:

$$
\begin{equation*}
\mathcal{Q}_{s}^{j}=\left(\frac{\mathcal{P}_{s}^{j}}{P^{j}}\right)^{-\omega} Y^{j} \tag{459}
\end{equation*}
$$

and the price of the final output $P^{j}$ is given by:

$$
\begin{equation*}
P^{j}=\left(\int_{0}^{1}\left(\mathcal{P}_{s}^{j}\right)^{1-\omega} \mathrm{d} s\right)^{\frac{1}{1-\omega}} \tag{460}
\end{equation*}
$$

where $\mathcal{P}_{s}^{j}$ is the price index of industry $s$ and $P^{j}$ is the price of the final good.
Within each sector, there is monopolistic competition; each firm that produces one variety $\mathcal{X}_{i, s}^{j}$ is a price setter. One variety $\mathcal{X}_{i, s}^{j}$ is produced using labor $\mathcal{L}_{i, s}^{j}$ as the sole input in a linear (constant returns to scale) technology:

$$
\begin{equation*}
\mathcal{X}_{i, s}^{j}=A^{j} \mathcal{L}_{i, s}^{j} \tag{461}
\end{equation*}
$$

where labor productivity is assumed to be symmetric across producers of differentiated goods.
Denoting by $\mathcal{P}_{i, s}^{j}$ the price of good $i$ in industry $s$ in sector $j$, the profit function for the $s$ th good producer denoted by $\pi_{s}^{j}$ is:

$$
\begin{equation*}
\pi_{s}^{j} \equiv \mathcal{P}_{s}^{j}\left(M^{j}\right)^{-\frac{1}{\rho-1}}\left(\int_{0}^{M^{j}}\left(\mathcal{X}_{i, s}^{j}\right)^{\frac{\rho-1}{\rho}} \mathrm{~d} i\right)^{\frac{\rho}{\rho-1}}-\int_{0}^{M^{j}} \mathcal{P}_{i, s}^{j} \mathcal{X}_{i, s}^{j} \mathrm{~d} i \tag{462}
\end{equation*}
$$

The demand faced by each producer $\mathcal{X}_{i, s}^{j}$ is defined as follows:

$$
\begin{equation*}
\mathcal{X}_{i, s}^{j}=\left(\frac{\mathcal{P}_{i, s}^{j}}{\mathcal{P}_{s}^{j}}\right)^{-\rho} \frac{\mathcal{Q}_{s}^{j}}{M^{j}}, \tag{463}
\end{equation*}
$$

and the price index of industry $s$ in sector $j$ is given by:

$$
\begin{equation*}
\mathcal{P}_{s}^{j}=\left(M^{j}\right)^{-\frac{1}{1-\rho}}\left(\int_{0}^{M^{j}}\left(\mathcal{P}_{i, s}^{j}\right)^{1-\rho} \mathrm{d} i\right)^{\frac{1}{1-\rho}} \tag{464}
\end{equation*}
$$

Combining (459) and (463), the demand for variety $\mathcal{X}_{i, s}^{j}$ can be expressed in terms of the relative price of the final good:

$$
\begin{equation*}
\mathcal{X}_{i, s}^{j}=\left(\frac{\mathcal{P}_{i, s}^{j}}{\mathcal{P}_{s}^{j}}\right)^{-\rho}\left(\frac{\mathcal{P}_{s}^{j}}{P^{j}}\right)^{-\omega} \frac{Y^{j}}{M^{j}} . \tag{465}
\end{equation*}
$$

[^41]In order to operate, each imperfectly competitive producer must pay a fixed cost denoted by $\phi^{j}$ measured in terms of the final good which is assumed to be symmetric across firms. Each firm $i$ chooses labor to maximize profits. The profit function for the $i$ th producer in industry $s$ denoted by $\pi_{i, s}^{j}$ is:

$$
\begin{equation*}
\pi_{i, s}^{j} \equiv \mathcal{P}_{i, s}^{j} A^{j} \mathcal{L}_{i, s}^{j}-W^{j} \mathcal{L}_{i, s}^{j}-P^{j} \phi^{j} \tag{466}
\end{equation*}
$$

The demand for hours worked is given by the equality of the markup-adjusted marginal revenue of labor to the producer wage $W^{j}$, respectively:

$$
\begin{equation*}
\frac{\mathcal{P}_{i, s}^{j} A^{j}}{\mu^{j}}=W^{j} \tag{467}
\end{equation*}
$$

where the markup $\mu^{j}$ is a decreasing function of the price-elasticity of demand $e^{j}$ :

$$
\begin{equation*}
\mu^{j} \equiv \frac{e^{j}}{e^{j}-1} \tag{468}
\end{equation*}
$$

We consider a symmetric equilibrium where all imperfectly competitive producers within one industry $s$ produce the output level $\mathcal{X}_{i, s}^{j}=\mathcal{X}^{j}$ with the same quantities of labor $\mathcal{L}_{i, s}^{j}=\mathcal{L}^{j}$. Hence, total hours worked are $L^{j}=M^{j} \mathcal{L}^{j}$. They also set the same price $\mathcal{P}_{i, s}^{j}=\mathcal{P}^{j}$. Hence, eqs. (460) and (464) imply that $\mathcal{P}^{j}=P^{j}$. The first-order condition (467) reduces to:

$$
\begin{equation*}
\frac{P^{j} A^{j}}{\mu^{j}}=W^{j} \tag{469}
\end{equation*}
$$

We follow Yang and Heijdra [1993] and Jaimovich and Floetotto [2008] by taking into account that output of one variety does not affect the general price index $P$, but influences the intermediate good price level; in a symmetric equilibrium, the resulting price elasticity of demand is:

$$
\begin{equation*}
e\left(M^{j}\right) \equiv-\frac{\partial \mathcal{X}_{i, s}^{j}}{\partial \mathcal{P}_{i, s}^{j}} \frac{\mathcal{P}_{i, s}^{j}}{\mathcal{X}_{i, s}^{j}}=\epsilon-\frac{(\epsilon-\omega)}{M^{j}}, \quad M^{j} \in(1, \infty) \tag{470}
\end{equation*}
$$

where we used the fact that $\frac{\partial P_{s}}{\partial \mathcal{P}_{i, s}^{j}} \frac{\mathcal{P}_{i, s}^{j}}{P_{s}}=\frac{1}{M^{j}}$. Note that we assume that the elasticity of substitution among intermediate goods $s$ captured by $\omega$ and among varieties $i$ captured by $\rho$ are identical across sectors $j=T, N$. Assuming that $\epsilon>\omega$, the price elasticity of demand faced by one single firm is an increasing function of the number of firms $M^{j}$ within an industry $s$. Henceforth, the markup $\mu^{j}=\frac{e^{j}}{e^{j}-1}$ decreases as the number of competitors increases:

$$
\begin{equation*}
\mu^{j}=\mu\left(M^{j}\right), \quad \frac{\partial \mu^{j}}{\partial M^{j}}<0 . \tag{471}
\end{equation*}
$$

We further assume that free entry drives profits down to zero in all industries of sector $j$ at each instant of time. Aggregating, the zero profit condition implies in sector $j$ :

$$
\begin{equation*}
P^{j} Y^{j}-W^{j} L^{j}-P^{j} M^{j} \phi^{j}=0 . \tag{472}
\end{equation*}
$$

where $Y^{j}=M^{j} \mathcal{X}^{j}=A^{j} L^{j}$. Inserting (461) and (469), the zero-profit condition can be rewritten as follows:

$$
\begin{equation*}
A^{j} L^{j}\left(1-\frac{1}{\mu^{j}}\right)=M^{j} \phi^{j} . \tag{473}
\end{equation*}
$$

Remembering that $\mu^{j}=\mu\left(M^{j}\right)$, eq. (470) can be solved for the number of firms:

$$
\begin{equation*}
M^{j}=M^{j}\left(L^{j}, A^{j}, \phi^{j}\right) \tag{474}
\end{equation*}
$$

Totally differentiating (473) and denoting by a hat a percentage deviation from the initial steady-state, we have:

$$
\begin{equation*}
\hat{M}^{j}=\frac{M^{j} \phi^{j}\left(\hat{A}^{j}+\hat{L}^{j}-\hat{\phi}^{j}\right)}{\left(M^{j} \phi^{j}-\frac{Y^{j}}{\mu^{j}} \frac{\partial \mu^{j}}{\partial M^{j}} \frac{M^{j}}{\mu^{j}}\right)} . \tag{475}
\end{equation*}
$$

Using the fact that $\frac{Y^{j}}{\mu^{j}}=\frac{M^{j} \phi^{j}}{\mu^{j}-1}$, the partial derivatives reduce to:

$$
\begin{equation*}
\frac{\hat{M}^{j}}{\hat{A}^{j}}=\frac{\hat{M}^{j}}{\hat{L}^{j}}=-\frac{\hat{M}^{j}}{\hat{\phi}^{j}}=\frac{\left(\mu^{j}-1\right)}{\left(\mu^{j}-1\right)+\eta_{\mu^{j}, M^{j}}}>0, \tag{476}
\end{equation*}
$$

where $\eta_{\mu^{j}, M^{j}}=-\frac{\partial \mu^{j}}{\partial M^{j}} M^{j} \mu^{j}$ is the elasticity of markup to entry. The partial derivatives (476) are increasing in $\mu^{j}$ which imply that entry reacts more to a change in $L^{j}, A^{j}$ or $\phi^{j}$ when the markup $\mu^{j}$ is initially high. Hence, a sector poor with poor competition in product markets will experience larger firm entry so that the markup will fall by a larger amount.

Since the denominator of (476) is positive (see eq. (471)), the number of firms $M^{j}$ in sector $j$ is an increasing of the productivity index $A^{j}$ and market size captured by $L^{j}$. Because fixed costs lower firm entry by reducing profit opportunities, such recurring costs act like a cost of entry. Hence, the number of firms $M^{j}$ in sector $j$ is a decreasing function of fixed costs $\phi^{j}$. In summary, we have:

$$
\begin{equation*}
\frac{\partial M^{j}}{\partial A^{j}}>0, \quad \frac{\partial M^{j}}{\partial L^{j}}>0, \quad \frac{\partial M^{j}}{\partial \phi^{j}}<0 . \tag{477}
\end{equation*}
$$

## N. 3 Market-Clearing Conditions

Aggregating labor over the two sectors gives us the resource constraint:

$$
\begin{equation*}
L^{T}+L^{N}=1, \tag{478}
\end{equation*}
$$

where we assume that agents supply inelastically labor normalized to one.
To fully describe the equilibrium, we impose good market clearing conditions. The non traded good market clearing condition requires that non traded output is equalized with consumption in non-tradables and total fixed cost:

$$
\begin{equation*}
Y^{N}=C^{N}+M^{N} \phi^{N} . \tag{479}
\end{equation*}
$$

Plugging this condition into the flow budget constraint (453) and using firms' optimal conditions yields the market clearing condition for tradables or the current account equation:

$$
\begin{equation*}
\dot{B}=r^{\star} B+Y^{T}-C^{T}-M^{T} \phi^{T}=r^{\star} B+\frac{Y^{T}}{\mu^{T}}-C^{T} . \tag{480}
\end{equation*}
$$

where we used the free entry condition in the traded sector, i.e., $Y^{T}-M^{T} \phi^{T}=\frac{Y^{T}}{\mu^{T}}$; the second term on the RHS, i.e., $\frac{Y^{T}}{\mu^{T}}-C^{T} \equiv N X$, corresponds to net exports denoted by $N X$.

## N. 4 Short-Run Static Solutions

## Short-Run Static Solutions for Consumption

We compute the short-run static solution for consumption. Static efficiency condition (454a) can be solved for consumption which of course must hold at any point of time:

$$
\begin{equation*}
C=C(\bar{\lambda}, P), \tag{481}
\end{equation*}
$$

with

$$
\begin{align*}
C_{\bar{\lambda}} & =\frac{\partial C}{\partial \bar{\lambda}}=-\sigma_{C} \frac{C}{\bar{\lambda}}<0  \tag{482a}\\
C_{P} & =\frac{\partial C}{\partial P}=-\alpha_{C} \sigma_{C} \frac{C}{P}<0 \tag{482b}
\end{align*}
$$

Inserting (481) into $C^{T}=\left(P_{C}-P P_{C}^{\prime}\right) C$ and $C^{N}=P_{C}^{\prime} C$, we can solve for consumption in tradables and non tradables:

$$
\begin{equation*}
C^{T}=C^{T}(\bar{\lambda}, P), \quad C^{N}=C^{N}(\bar{\lambda}, P), \tag{483}
\end{equation*}
$$

where partial derivatives are

$$
\begin{align*}
C_{\bar{\lambda}}^{T} & =-\sigma_{C} \frac{C^{T}}{\bar{\lambda}}<0,  \tag{484a}\\
C_{P}^{T} & =\alpha_{C} \frac{C^{T}}{P}\left(\phi-\sigma_{C}\right) \lessgtr 0,  \tag{484b}\\
C_{\bar{\lambda}}^{N} & =-\sigma_{C} \frac{C^{N}}{\bar{\lambda}}<0,  \tag{484c}\\
C_{P}^{N} & =-\frac{C^{N}}{P}\left[\left(1-\alpha_{C}\right) \phi+\alpha_{C} \sigma_{C}\right]<0, \tag{484d}
\end{align*}
$$

where we used the fact that $-\frac{P_{C}^{\prime \prime} P}{P_{C}^{\prime}}=\phi\left(1-\alpha_{C}\right)>0$.

## Short-Run Static Solutions for Labor

Inserting the short-run static solution for consumption in non tradables (483) and for the number of firms in the non traded sector (474) into the market clearing condition (479) for the non tradables yields:

$$
\begin{equation*}
A^{N} L^{N}=C^{N}(\bar{\lambda}, P)+M^{N}\left(L^{N}, A^{N}, \phi^{N}\right) \phi^{N} . \tag{485}
\end{equation*}
$$

Eq. (485) can be solved for non traded labor:

$$
\begin{equation*}
L^{N}=L^{N}\left(P, \bar{\lambda}, A^{N}, \phi^{N}\right) \tag{486}
\end{equation*}
$$

Partial derivatives are determined by totally differentiating (485) and by isolating the percentage change in non traded labor $\hat{L}^{N}$ :

$$
\begin{align*}
\hat{L}^{N} & =-\hat{A}^{N}-\frac{C^{N}}{Y^{N}}\left\{\frac{\sigma_{C}}{\Gamma^{N}} \hat{\bar{\lambda}}+\frac{\left[\left(1-\alpha_{C}\right) \phi+\alpha_{C} \sigma_{C}\right]}{\Gamma^{N}} \hat{P}\right\} \\
& +\frac{M^{N} \phi^{N}}{Y^{N}} \frac{1}{\Gamma^{N}} \frac{\eta_{\mu^{N}, M^{N}}^{\left(\mu^{N}-1\right)+\eta_{\mu^{N}, M^{N}}} \hat{\phi}^{N},}{} \tag{487}
\end{align*}
$$

where we set:

$$
\begin{equation*}
\Gamma^{N}=1-\frac{\left(\mu^{N}-1\right)}{\left(\mu^{N}-1\right)+\eta_{\mu^{N}, M^{N}}} \frac{M^{N} \phi^{N}}{Y^{N}}>0 . \tag{488}
\end{equation*}
$$

The sign of $\Gamma^{N}>0$ follows from the fact that $0<\frac{\left(\mu^{N}-1\right)}{\left(\mu^{N}-1\right)+\eta_{\mu^{N}, M^{N}}}<1$ and the share of total fixed cost in non traded output $0<\frac{M^{N} \phi^{N}}{Y^{N}}<1$.

Plugging (486) into (478), the resource constraint for labor can be solved for traded labor:

$$
\begin{equation*}
L^{T}=1-L^{N}=1-\left(P, \bar{\lambda}, A^{N}, \phi^{N}\right) \tag{489}
\end{equation*}
$$

with

$$
\begin{equation*}
\hat{L}^{T}=-\frac{L^{N}}{L^{T}} \hat{L}^{N} . \tag{490}
\end{equation*}
$$

## Short-Run Static Solutions for the Number of Firms

Inserting (486) into (474), we can solve for the number of firms in the non traded sector:

$$
M^{N}=M^{N}\left[L^{N}\left(P, \bar{\lambda}, A^{N}, \phi^{N}\right), A^{N}, \phi^{N}\right]
$$

Because partial derivatives $\frac{\hat{M}^{N}}{\hat{L}^{N}} \frac{\hat{L}^{N}}{\hat{A}^{N}}$ and $\frac{\hat{M}^{N}}{\hat{A}^{N}}$ cancel each other, short-run static solution for $M^{N}$ reduces to:

$$
\begin{equation*}
M^{N}=M^{N}\left(P, \bar{\lambda}, \phi^{N}\right) \tag{491}
\end{equation*}
$$

where partial derivatives are determined by inserting (487) into (475) and using (476)

$$
\begin{align*}
\hat{M}^{N} & =-\frac{\left(\mu^{N}-1\right)}{\left(\mu^{N}-1\right)+\eta_{\mu^{N}, M^{N}}} \frac{1}{\Gamma^{N}}\left\{\left(1-\frac{M^{N} \phi^{N}}{Y^{N}}\right) \hat{\phi}^{N}\right. \\
& \left.+\frac{C^{N}}{Y^{N}}\left\{\sigma_{C} \hat{\bar{\lambda}}+\left[\left(1-\alpha_{C}\right) \phi+\alpha_{C} \sigma_{C}\right] \hat{P}\right\}\right\} \tag{492}
\end{align*}
$$

where $\Gamma^{N}$ is given by (488) and $0<\frac{M^{N} \phi^{N}}{Y^{N}}<1$. According to (492), a rise in $P$ or an increase in the marginal utility of wealth $\bar{\lambda}$ lowers the number of firms by reducing the demand for tradables and thus profit opportunities in that sector while higher fixed cost $\phi^{N}$ acts like a cost of entry and thus reduces the number of firms $M^{N}$ as well.

Inserting (489) into (474), we can solve for the number of firms in the traded sector:

$$
\begin{equation*}
M^{T}=M^{T}\left[L^{T}\left(P, \bar{\lambda}, A^{N}, \phi^{N}\right), A^{T}, \phi^{T}\right] . \tag{493}
\end{equation*}
$$

A rise in $P, \bar{\lambda}, A^{N}$ lowers non traded labor $L^{N}$ by producing an excess of supply in the non traded goods market. Hence, an increase in $P, \bar{\lambda}, A^{N}$ raises $L^{T}$ and thus $M^{T}$ while an increase in $\phi^{N}$ reduces it by reallocating labor toward the non traded sector (see (487)). An increase in $A^{T}$ raises $M^{T}$ by producing profit opportunities in the traded sector while $\phi^{T}$ raises the cost of entry and thus lowers $M^{T}$.

## Short-Run Static Solutions for the relative Price of Non Tradables

We suppose that the traded good is the numeraire so that $P^{T}=1$. Assuming perfect mobility of labor across sectors the marginal revenue of labor must equalize across sectors (see eq. (469)):

$$
\begin{equation*}
\frac{A^{T}}{\mu^{T}}=\frac{P A^{N}}{\mu^{N}}=W . \tag{494}
\end{equation*}
$$

Isolating $P$, we find that the price of non traded goods in terms of traded goods depends on the ratio of productivity and the ratio of markups:

$$
\begin{equation*}
P=\frac{A^{T}}{A^{N}} \frac{\mu^{N}\left[M^{N}\left(P, \bar{\lambda}, \phi^{N}\right)\right]}{\mu^{T}\left\{M^{T}\left[L^{T}\left(P, \bar{\lambda}, A^{N}, \phi^{N}\right), A^{T}, \phi^{T}\right]\right\}}, \tag{495}
\end{equation*}
$$

where we inserted (491) and (493), using the fact that $\mu^{j}=\mu\left(M^{j}\right)$ (see eq. (471)). Eq. (495) can be solved for the relative price of non tradables:

$$
\begin{equation*}
P=P\left(A^{T}, A^{N}, \bar{\lambda}, \phi^{T}, \phi^{N}\right) . \tag{496}
\end{equation*}
$$

## N. 5 Dynamics

Linearizing (496) around the steady-state implies that the dynamics for the relative price of non tradables degenerate, i.e., $P(t)=\tilde{P}$. Inserting the short-run static solutions for labor in the traded sector, consumption in tradables, and the number of firms in the traded sector, given by (483), (489), (493) respectively, into the accumulation equation of foreign bonds (480) and linearizing around the steady-state yields:

$$
\begin{equation*}
\dot{B}(t)=r^{\star}(B(t)-\tilde{B}) . \tag{497}
\end{equation*}
$$

Solving and invoking the transversality condition $\lim _{t \rightarrow \infty} \lambda B(t) e^{-r^{\star} t}=0$ leads to:

$$
\begin{equation*}
B(t)=B_{0} . \tag{498}
\end{equation*}
$$

Hence, for the transversality condition to hold, the stock of traded bonds $B(t)$ must be equal to its initial predetermined level. Combining (498) with (480) yields:

$$
\begin{equation*}
r^{\star} B_{0}+Y^{T}=C^{T} . \tag{499}
\end{equation*}
$$

Because the stock of foreign bonds must stick to its initial value, for the sake of simplicity and without loss of generality, we set $B_{0}=0$.

## N. 6 Relative Price Effect of a Productivity Differential

Taking logarithm and differentiating (494), denoting the the percentage deviation from its initial steady-state by a hat, gives the change in the relative price of non tradables:

$$
\begin{equation*}
\hat{P}=\left(\hat{A}^{T}-\hat{A}^{N}\right)+\left(\hat{\mu}^{N}-\hat{\mu}^{T}\right) . \tag{500}
\end{equation*}
$$

Because the model is analytically untractable, a quantitative analysis would be necessary. Yet, we can state some predictions by rewriting the steady-state in relative terms.

The equilibrium is defined by the following set of equations:

$$
\begin{gather*}
\frac{C^{T}}{C^{N}}=\left(\frac{\varphi}{1-\varphi}\right) P^{\phi},  \tag{501a}\\
Y^{T}\left[1-\frac{1}{\mu^{T}\left(M^{T}\right)}\right]=M^{T} \phi^{T},  \tag{501b}\\
Y^{N}\left[1-\frac{1}{\mu^{N}\left(M^{N}\right)}\right]=M^{N} \phi^{N},  \tag{501c}\\
P=\frac{A^{T}}{A^{N}} \frac{\mu^{N}\left(M^{N}\right)}{\mu^{T}\left(M^{T}\right)},  \tag{501d}\\
\frac{Y^{T}}{Y^{N}} \frac{\mu^{N}}{\mu^{T}}=\frac{C^{T}}{C^{N}}, \tag{501e}
\end{gather*}
$$

where we used the zero profit condition, i.e., $Y^{j}-M^{j} \phi^{j}=\frac{Y^{j}}{\mu^{j}}$ to derive the goods market clearing condition (501e). Remember that we set $B_{0}=0$.

The equilibrium which comprises (501a)-(501e) can be reduced to two equations. Combining the optimal rule for intra-temporal allocation of consumption (501a) with market clearing conditions for the non traded and the traded good, i.e., (501e) yields the goods market equilibrium (henceforth GME):

$$
\begin{equation*}
\frac{Y^{T}}{Y^{N}}=\frac{\varphi}{1-\varphi} \frac{\mu^{T}}{\mu^{N}} P^{\phi} \tag{502}
\end{equation*}
$$

As it will be useful, we differentiate the zero-profit condition $Y^{j}\left[1-\frac{1}{\mu^{j}\left(M^{j}\right)}\right]=\phi^{j} M^{j}$ (501b) or alternatively (501c) (see eq. (476)):

$$
\begin{equation*}
\hat{M}^{j}=\chi^{j} \hat{Y}^{j}, \quad \chi^{j}=\frac{\left(\mu^{j}-1\right)}{\left(\mu^{j}-1\right)+\eta_{\mu^{j}, M^{j}}}>0 . \tag{503}
\end{equation*}
$$

Totally differentiating (501d) and inserting (503) yields:

$$
\begin{equation*}
\hat{P}=\left(\hat{A}^{T}-\hat{A}^{N}\right)+\left[\eta_{\mu^{T}, M^{T}} \chi^{T} \hat{Y}^{T}-\eta_{\mu^{N}, M^{N}} \chi^{N} \hat{Y}^{N}\right], \tag{504}
\end{equation*}
$$

where $\eta_{\mu^{j}, M^{j}}=-\frac{\partial \mu^{j}}{\partial M^{j}} \frac{M^{j}}{\mu^{j}}>0$. For the sake of clarity, let us assume that $\chi^{j} \simeq \chi>0$ and $\eta_{\mu^{j}, M^{j}}=\eta_{\mu, M}>0$; these approximations imply that eq. (504) reduces to:

$$
\begin{equation*}
\hat{P}=\left(\hat{A}^{T}-\hat{A}^{N}\right)+\eta_{\mu, M} \chi\left(\hat{Y}^{T}-\hat{Y}^{N}\right) \tag{505}
\end{equation*}
$$

Eq. (505) corresponds to the labor market equilibrium (henceforth $L M E$ ) which is upwardsloping in the $\left(\ln \left(Y^{T} / Y^{N}\right), \ln P\right)$-space where the slope is equal to $\eta_{\mu, M} \chi>0$.

Totally differentiating the goods market equilibrium (502) and inserting (503) yields:

$$
\hat{Y}^{T}-\hat{Y}^{N}=\phi \hat{P}-\left[\eta_{\mu^{T}, M^{T}} \chi^{T} \hat{Y}^{T}-\eta_{\mu^{N}, M^{N}} \chi^{N} \hat{Y}^{N}\right]
$$

For the sake of clarity, let us assume that $\chi^{j} \simeq \chi>0$ and $\eta_{\mu^{j}, M^{j}}=\eta_{\mu, M}>0$; these approximations imply that the $G M E$-schedule reduces to:

$$
\begin{equation*}
\hat{P}=\frac{\left(1+\eta_{\mu, M \chi)}\right.}{\phi}\left(\hat{Y}^{T}-\hat{Y}^{N}\right) \tag{506}
\end{equation*}
$$

The $G M E$-schedule is upward-sloping in the $\left(\ln \left(Y^{T} / Y^{N}\right), \ln P\right)$-space where the slope is equal to $\frac{1+\eta_{\mu, M \chi}}{\phi}>0$. The $G M E$-schedule is steeper than the $L M E$-schedule as long as the following inequality holds:

$$
\begin{equation*}
\phi<1+\frac{1}{\eta_{\mu, M \chi}} \tag{507}
\end{equation*}
$$

Since $\eta_{\mu, M} \chi>0$ is usually small for reasonable values of parameters, it is reasonable to assume that the $G M E$-schedule is steeper than the $L M E$-schedule in the $\left(\ln \left(Y^{T} / Y^{N}\right), \ln P\right)$-space.

A productivity shock biased toward the traded sector shifts the $L M E$-schedule to the right in the $\left(\ln \left(Y^{T} / Y^{N}\right), \ln P\right)$-space. As traded output increases relative to non-traded output, the relative price of non-tradables must rise to clear the goods market. Because the markup of tradables falls more than the markup of non tradables, the relative price must appreciate by a larger amount. This can be seen more formally by totally differentiating (501d):

$$
\begin{equation*}
\hat{P}=\left(\hat{A}^{T}-\hat{A}^{N}\right)+\left(\hat{\mu}^{N}-\hat{\mu}^{T}\right) \tag{508}
\end{equation*}
$$

where $\hat{\mu}^{j}<0$ with $j=T, N$. Eq. (508) allows us to draw several conclusions related to the long-run movements in the relative price of non tradables:

- Assuming that markups are fixed as in the standard BS model, the percentage change in the relative price of non tradables reduces to the first term on the RHS of eq. (508) so that a productivity differential of $1 \%$ appreciates the relative price by $1 \%$. When markups are endogenous, because traded output increases relative to non traded output, more firms enter the traded good market than the non traded good market so that the markup in that the traded sector, $\mu^{T}$, falls more than that in the non traded sector, $\mu^{N}$, thus amplifying the appreciation in the relative price of non tradables.
- Because we impose perfect mobility of labor across sectors, both sectors pay the same wage so that the ratio $\Omega \equiv \frac{W^{N}}{W^{T}}$ remains unaffected.
- While traded output increases relative to non traded output and thus $\left(\hat{\mu}^{N}-\hat{\mu}^{T}\right)>0$ should be positive, we expect the term $\left(\hat{\mu}^{N}-\hat{\mu}^{T}\right)$ to be small; we have assumed that the elasticity of the markup to entry $\eta_{\mu^{j}, M^{j}}=\eta_{\mu, M}>0$ and the elasticity of entry to sectoral output $\chi^{j} \simeq \chi>0$ were homogenous across sectors; as documented by Epifani and Gancia [2011], some sectors are more shielded than others from foreign competition; hence, the markup in the non traded sector should be initially high and thus we have $\chi^{N}>\chi^{T}$ and $\eta_{\mu^{N}, M^{N}}>\eta_{\mu^{T}, M^{T}}$. While traded output increases relative to non traded
output, for given change in sectoral output, the non traded sector should experience a larger entry of firms and thus a larger decline in the markup. As a result, we expect the term $\left(\hat{\mu}^{N}-\hat{\mu}^{T}\right)$ to be small and the relative price of non tradables to appreciate by roughly $1 \%$ following a productivity differential of $1 \%$.
- We have restricted our attention to the analysis of the effects of a productivity differential between tradables and non tradables while the last thirty years have been witnessed of a deregulation episode on an unprecedented scale in OECD countries. Such a deregulation episode has mainly affected non traded sectors and thus has contributed to lower the markup in those sectors, as documented by Bertinelli et al. [2013]. As a consequence, the deregulation episode, by reducing the markup in the non traded sector, should exert a negative impact on the relative price of non tradables. More specifically, eq. (496) shows that both labor productivity and cost of entry impinge on $P$, i.e., $P\left(A^{T}, A^{N}, \bar{\lambda}, \phi^{T}, \phi^{N}\right)$ where $\partial P / \partial \phi^{N}>0$. It would be interesting to explore the following relationship empirically:

$$
p_{i, t}=\alpha_{i}+\gamma \cdot\left[z_{i, t}^{T}-\left(\theta_{i}^{T} / \theta_{i}^{N}\right) z_{i, t}^{N}\right]+\delta \cdot \operatorname{PMR}_{i, t}+u_{i, t},
$$

where PMR represents a set of time and country varying indicators of product market regulations. We expect the coefficient $\delta$ to be positive as stringent anti-competitive product market regulation (in the non traded sector) raises $\mu^{N}$ and thus appreciates the relative price of non tradables. We leave a further analysis of these issues for future research.

## N. 7 Introducing Physical Capital and Assuming Imperfect Mobility of Labor

In this subsection, we derive the long-run adjustment of the relative price following a productivity differential by emphasizing the role of endogenous markups when introducing physical capital while labor is assumed to be imperfectly mobile across sectors. We suppose that investment is non traded.

Assuming that products markets are imperfectly competitive, capital is perfectly mobile while labor imperfectly mobile across sectors, first-order conditions from the firm's profit maximization evaluated at the steady-state are:

$$
\begin{align*}
\frac{1}{\mu^{T}} Z^{T}\left(1-\theta^{T}\right)\left(k^{T}\right)^{-\theta^{T}}=\frac{P}{\mu^{N}} Z^{N}\left(1-\theta^{N}\right)\left(k^{N}\right)^{-\theta^{N}} & \equiv R,  \tag{509a}\\
\frac{1}{\mu^{T}} Z^{T} \theta^{T}\left(k^{T}\right)^{1-\theta^{T}} & \equiv W^{T},  \tag{509b}\\
\frac{P}{\mu^{N}} Z^{N} \theta^{N}\left(k^{N}\right)^{1-\theta^{N}} & \equiv W^{N}, \tag{509c}
\end{align*}
$$

where $R$ is the capital rental cost and $W^{j}$ the labor cost in sector $j$.
Setting $\dot{P}=0$ into ( 95 d ), the capital rental cost can be written as follows:

$$
\begin{equation*}
R=P\left(r^{\star}+\delta\right) . \tag{510}
\end{equation*}
$$

Dividing the marginal product of labor by the marginal product of capital in each sector yields the sectoral capital-labor ratios:

$$
\begin{equation*}
k^{T}=\frac{1-\theta^{T}}{\theta^{T}} \frac{W}{R}, \quad k^{N}=\frac{1-\theta^{N}}{\theta^{N}} \frac{W}{R} . \tag{511}
\end{equation*}
$$

Substituting sectoral capital-labor ratios (511) into (509b)-(509c) yields an expression of the steady-state relative price of non tradables :

$$
\begin{equation*}
P=\frac{\Psi^{T}}{\Psi^{N}} \frac{Z^{T}}{Z^{N}} \frac{\left(W^{N}\right)^{\theta^{N}}(R)^{1-\theta^{N}}}{\left(W^{T}\right)^{\theta^{T}}(R)^{1-\theta^{T}}}=\frac{\Psi^{T}}{\Psi^{N}} \frac{Z^{T}}{Z^{N}} \frac{\left(W^{N}\right)^{\theta^{N}}}{\left(W^{T}\right)^{\theta^{T}}} R^{\left(\theta^{T}-\theta^{N}\right)} \tag{512}
\end{equation*}
$$

where

$$
\begin{equation*}
\Psi^{j}=\left(\theta^{j}\right)^{\theta^{j}}\left(1-\theta^{j}\right)^{1-\theta^{j}}, \quad j=T, N . \tag{513}
\end{equation*}
$$

Substituting the sectoral capital-labor ratio $k^{T}$ given by (511) into (509b), the wage rate in the traded sector can be rewritten as follows:

$$
\begin{equation*}
W^{T}=\left(Z^{T}\right)^{\frac{1}{\theta^{T}}}\left(\mu^{T}\right)^{-\frac{1}{\theta^{T}}}\left(\Psi^{T}\right)^{\frac{1}{\theta^{T}}} R^{-\left(\frac{1-\theta^{T}}{\theta^{T}}\right)} \tag{514}
\end{equation*}
$$

Multiplying and dividing the RHS of (512) by $\left(W^{T}\right)^{\theta^{N}}$ and substituting (514) yields:

$$
\begin{align*}
P & =\frac{\Psi^{T}}{\Psi^{N}} \frac{Z^{T}}{Z^{N}}\left(\frac{W^{N}}{W^{T}}\right)^{\theta^{N}}\left(W^{T}\right)^{\left(\theta^{N}-\theta^{T}\right)} R^{\theta^{T}-\theta^{N}} \\
& \left.=\frac{\left(\Psi^{T}\right)^{\frac{\theta^{N}}{\theta^{T}}}}{\Psi^{N}} \frac{\left(Z_{T}\right)^{\frac{\theta^{N}}{\theta^{T}}}}{Z_{N}}\left(\mu^{T}\right)^{\left(\frac{\theta^{T}-\theta^{N}}{\theta^{T}}\right.}\right)\left(\frac{W^{N}}{W^{T}}\right)^{\theta^{N}} R^{\frac{\theta^{T}-\theta^{N}}{\theta^{T}}} \tag{515}
\end{align*}
$$

Inserting (510) into (515) and collecting terms yields:

$$
\begin{equation*}
P=\frac{\Psi^{T}}{\left(\Psi^{N}\right)^{\theta^{T} / \theta^{N}}} \frac{Z^{T}}{\left(Z^{N}\right)^{\theta^{T} / \theta^{N}}}\left(\mu^{T}\right)^{-\left(\frac{\theta^{N}-\theta^{T}}{\theta^{N}}\right)}\left(\frac{W^{N}}{W^{T}}\right)^{\theta^{T}}\left(r^{\star}+\delta_{K}\right)^{\frac{\theta^{T}-\theta^{N}}{\theta^{N}}} . \tag{516}
\end{equation*}
$$

Taking logarithm, and denoting by $\omega=\ln \left(W^{N} / W^{T}\right)$ the (logged) relative wage yields:

$$
\begin{equation*}
p=c+\left(z^{T}-\frac{\theta^{T}}{\theta^{N}} z^{N}\right)+\theta^{T} \omega-\left(\frac{\theta^{N}-\theta^{T}}{\theta^{N}}\right) \mu^{T} \tag{517}
\end{equation*}
$$

where $c=\ln \Psi^{T}-\frac{\theta^{T}}{\theta^{N}} \ln \Psi^{N}+\left(\frac{\theta^{T}-\theta^{N}}{\theta^{N}}\right) \ln \left(r^{\star}+\delta_{K}\right)$ is a constant. According to eq. (517), imperfect mobility of labor moderates the appreciation in the relative price $p$ following a productivity differential between tradables and non tradables by reducing the relative wage $\omega$. Conversely, higher productivity triggers firm entry and thus raises labor demand which in turn pushes up the wage rate, as captured by the fall in the markup $\hat{\mu}^{T}<0$. If the labor income share is higher in the non traded sector than in the traded sector, i.e., $\theta^{N}>\theta^{T}$, in line with empirical evidence for most of the countries of our sample, the fall in the markup exerts a positive impact on the relative price of non tradables by raising wages.

## O Quantitative Analysis: Additional Numerical Results

In this section, we provide additional numerical results: i) for the relative price and relative wage responses to a productivity differential between tradables and non tradables when considering a model abstracting for physical capital, ii) for aggregate and sectoral variables when considering a model with physical capital accumulation, iii) for the relative price and relative wage responses to a productivity differential between tradables and non tradables when abstracting from government spending. In the latter case, we keep the calibration detailed in section 5.1 unchanged but set $G^{T}=G^{N}=0$. Finally, iv) we contrast predicted with observed responses for the relative price and the relative wage for each country, setting $\phi_{i}$ and $\epsilon_{i}$ to their estimated values and calibrating the model in order to target the (aggregate and non-tradable) ratios for each country $i$.

## O. 1 Relative Price and Relative Wage Responses in a Model without Physical Capital

Table 27 shows the long-run relative price and relative wage responses to a productivity differential between tradables and non tradables when considering a model abstracting for physical capital. Relative wage and relative price responses are given by (83) and (81), respectively.

## O. 2 Responses of Aggregate and Sectoral Variables: Numerical Estimates

Table 28 gives numerical results for the long-run changes of several aggregate and sectoral variables following a productivity differential between tradables and non tradables of $1 \%$. We consider three alternative scenarios (panel B, panel C, panel D): the elasticity of substitution $\phi$ between traded and non traded goods is set alternatively to one, 0.5 and 1.5. Note that we report in panel A the long-run responses of aggregate variables only when $\phi=1$ to save space.

## O. 3 Relative Price and Relative Wage Response when Abstracting from Government Spending in the Calibration

Table 29 gives numerical results for the long-run changes of several aggregate and sectoral variables following a productivity differential between tradables and non tradables of $1 \%$. We consider three alternative scenarios (panel B, panel C, panel D): the elasticity of substitution $\phi$ between traded and non traded goods is set alternatively to one, 0.5 and 1.5. The calibration is identical to that detailed in section 5.1, except that we set $G^{T}=G^{N}=0$.

We contrast the relative price and relative wage effects shown in Table 4 with those reported in Table 29. Numerical results reported in panel A of Table 29 show that, across all scenarios, the capital accumulation effect is more than three times smaller when abstracting from government spending. When $\phi=1$, the relative price of non tradables increases by $0.91 \%$ instead of $0.72 \%$ and the relative wage falls by $0.13 \%$ instead of $0.45 \%$. When $\phi<1$, the combined effect of the capital accumulation effect and capital reallocation effect is not large enough to more than offset the productivity effect so that the relative price appreciates more than proportionately than the productivity differential while the relative wage increases instead of decreasing, in contradiction with our evidence.

The explanation of these results is straightforward. The percentage deviation of the relative wage from its initial steady-state following a productivity differential between tradables and non tradables is:

$$
\begin{equation*}
\hat{\omega}=-(\phi-1)\left[\Theta^{L}+\left(\Theta^{K}-\Theta^{L}\right)\right]\left[\hat{z}^{T}-\left(\theta^{T} / \theta^{N}\right) \hat{z}^{N}\right]-\Theta^{K}\left(\mathrm{~d} v_{N X}-\mathrm{d} v_{I}\right), \tag{518}
\end{equation*}
$$

and the deviation in percentage of the relative price from its initial steady state is:

$$
\begin{equation*}
\hat{p}=(1+\epsilon)\left[\Theta^{L}+\left(\Theta^{K}-\Theta^{L}\right)\right]\left[\hat{z}^{T}-\left(\theta^{T} / \theta^{N}\right) \hat{z}^{N}\right]-\theta^{T} \Theta^{K}\left(\mathrm{~d} v_{N X}-\mathrm{d} v_{I}\right) \tag{519}
\end{equation*}
$$

where $v_{N X} \equiv\left(Y^{T}-C^{T}\right) / Y^{T}$ the ratio of net exports to traded output and $v_{I} \equiv \delta_{K} K / Y^{N}$ is the ratio of investment to non traded output.

Inspection of eqs. (518) and (519) reveals that the abstracting from government spending does not impinge on $\Theta^{L}$ and $\Theta^{K}$ which depend on parameters. Conversely, setting $G^{T}=$ $G^{N}=0$ affect the term $\mathrm{d} v_{N X}-\mathrm{d} v_{I}$ and thus the capital accumulation channel, remembering that $G^{N}$ and $G^{T}$ are set so as to yield a non-tradable share of government spending of $90 \%$, and government spending as a share of GDP of $20 \%$. When abstracting from government spending, the non tradable content of GDP falls from $63 \%$ to roughly $50 \%$ as non traded government spending raises non traded output by raising demand for non tradables. Hence, setting $G^{T}=G^{N}=0$ raises substantially $Y^{T}$ and lowers $Y^{N}$. Hence, $\mathrm{d} v_{I}>0$ becomes larger

|  | Relative Price Effect |  |  |  |  |  |  | Relative Wage Effect |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BS | Imperfect Labor Mobility |  |  |  |  |  | BS | Imperfect Labor Mobility |  |  |  |  |  |
|  | $(\epsilon=\infty)$ | $(\epsilon=0)$ | ( $\epsilon=0.5$ ) | ( $\epsilon=1$ ) | ( $\epsilon=1.5$ ) | $(\epsilon=2)$ | $(\epsilon=4)$ | $(\epsilon=\infty)$ | $(\epsilon=0)$ | $(\epsilon=0.5)$ | $(\epsilon=1)$ | ( $\epsilon=1.5$ ) | $(\epsilon=2)$ | $(\epsilon=4)$ |
| $\phi=0,5$ | 1.00 | 2.00 | 1.50 | 1.33 | 1.25 | 1.20 | 1.11 | 0.00 | 1.00 | 0.50 | 0.33 | 0.25 | 0.20 | 0.11 |
| $\phi=1$ | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $\phi=1,5$ | 1.00 | 0.67 | 0.75 | 0.80 | 0.83 | 0.86 | 0.91 | 0.00 | -0.33 | -0.25 | -0.20 | -0.17 | -0.14 | -0.09 |
| $\phi=2$ | 1.00 | 0.50 | 0.60 | 0.67 | 0.71 | 0.75 | 0.83 | 0.00 | -0.50 | -0.40 | -0.33 | -0.29 | -0.25 | -0.17 |
| $\phi=2,5$ | 1.00 | 0.40 | 0.50 | 0.57 | 0.63 | 0.67 | 0.77 | 0.00 | -0.60 | -0.50 | -0.43 | -0.38 | -0.33 | -0.23 |

between tradables and non-tradables; $\epsilon$ is the degree of imperfect substitutability in hours worked across sectors.

|  | BS | Bench | Labor supply |  | Mobility |  | $k^{N}>k^{T}$ |  | Non sep. | Traded inv. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(\epsilon=\infty)$ | ( $\epsilon=0.8$ ) | ( $\left.\sigma_{L}=0.2\right)$ | ( $\sigma_{L}=1$ ) | ( $\epsilon=0.1$ ) | $(\epsilon=2.5)$ | $(\epsilon=\infty)$ | $(\epsilon=0.8)$ | ( $\sigma=2$ ) | ( $\varphi_{I}=0.35$ ) |
|  |  |  |  |  |  |  |  |  |  |  |
| A.Aggregate |  |  |  |  |  |  |  |  |  |  |
| Consumption, $d \tilde{C}$ | 1.26 | 1.30 | 1.33 | 1.26 | 1.32 | 1.28 | 1.33 | 1.39 | 1.28 | 1.32 |
| Wage, $d \tilde{W}$ | 2.44 | 2.32 | 2.31 | 2.32 | 2.25 | 2.36 | 2.68 | 2.55 | 2.28 | 2.48 |
| Labor, $d \tilde{L}$ | -0.09 | -0.10 | -0.05 | -0.15 | -0.11 | -0.10 | -0.17 | -0.17 | -0.08 | -0.14 |
| Capital, $d \tilde{K}$ | 1.42 | 1.56 | 1.63 | 1.50 | 1.63 | 1.51 | 1.43 | 1.59 | 1.63 | 1.99 |
| GDP, $d \tilde{Y}$ | 2.37 | 2.24 | 2.29 | 2.20 | 2.16 | 2.29 | 2.48 | 2.35 | 2.24 | 2.36 |
| Foreign assets, $d \tilde{B}$ | -4.77 | -4.12 | -4.24 | -4.01 | -3.61 | -4.48 | -5.35 | -4.07 | -4.95 | -5.90 |
| B.Sectoral |  |  |  |  |  |  |  |  |  |  |
| Non traded wage, $d \tilde{W}^{N}$ | 2.44 | 2.16 | 2.15 | 2.18 | 2.02 | 2.27 | 2.68 | 2.38 | 2.11 | 2.30 |
| Traded wage, $d \tilde{W}^{T}$ | 2.44 | 2.62 | 2.63 | 2.61 | 2.72 | 2.55 | 2.68 | 2.80 | 2.66 | 2.73 |
| Traded labor, $d \tilde{L}^{T}$ | 0.13 | 0.05 | 0.07 | 0.03 | -0.01 | 0.09 | 0.09 | 0.01 | 0.08 | 0.02 |
| Non traded labor, $d \tilde{L}^{N}$ | -0.22 | -0.14 | -0.11 | -0.17 | -0.10 | -0.17 | -0.26 | -0.18 | -0.14 | -0.17 |
| sectoral labor ratio, $d \tilde{L}^{T} / \tilde{L}^{N}$ | 0.73 | 0.36 | 0.38 | 0.34 | 0.14 | 0.51 | 0.65 | 0.33 | 0.43 | 0.34 |
| Traded output, $d \tilde{Y}^{T}$ | 1.05 | 1.01 | 1.03 | 0.99 | 0.98 | 1.03 | 1.10 | 1.05 | 1.03 | 1.30 |
| Non traded output, $d \tilde{Y}^{N}$ | 0.69 | 0.77 | 0.80 | 0.74 | 0.81 | 0.74 | 0.76 | 0.85 | 0.78 | 0.68 |
| sectoral output ratio, $d \tilde{Y}^{T} / \tilde{Y}^{N}$ | 1.73 | 1.53 | 1.56 | 1.51 | 1.41 | 1.62 | 1.64 | 1.45 | 1.65 | 1.49 |
| $\phi<1$ |  |  |  |  |  |  |  |  |  |  |
| C.Sectoral |  |  |  |  |  |  |  |  |  |  |
| Non traded wage, $d \tilde{W}^{N}$ | 2.44 | 2.29 | 2.28 | 2.31 | 2.24 | 2.34 | 2.68 | 2.51 | 2.20 | 2.45 |
| Traded wage, $d \tilde{W}^{T}$ | 2.44 | 2.53 | 2.55 | 2.52 | 2.57 | 2.50 | 2.68 | 2.75 | 2.60 | 2.68 |
| Traded labor, $d \tilde{L}^{T}$ | 0.06 | 0.01 | 0.03 | -0.01 | -0.02 | 0.03 | 0.02 | -0.02 | 0.05 | -0.01 |
| Non traded labor, $d \tilde{L}^{N}$ | -0.15 | -0.10 | -0.07 | -0.13 | -0.08 | -0.12 | -0.19 | -0.15 | -0.10 | -0.13 |
| sectoral labor ratio, $d \tilde{L}^{T} / \tilde{L}^{N}$ | 0.41 | 0.19 | 0.21 | 0.17 | 0.06 | 0.28 | 0.36 | 0.19 | 0.31 | 0.18 |
| Traded output, $d \tilde{Y}^{T}$ | 0.87 | 0.79 | 0.81 | 0.77 | 0.74 | 0.82 | 1.00 | 0.95 | 0.81 | 1.12 |
| Non traded output, $d \tilde{Y}^{N}$ | 0.82 | 0.89 | 0.92 | 0.85 | 0.92 | 0.86 | 0.85 | 0.92 | 0.90 | 0.79 |
| sectoral output ratio, $d \tilde{Y}^{T} / \tilde{Y}^{N}$ | 1.40 | 1.28 | 1.31 | 1.24 | 1.19 | 1.34 | 1.35 | 1.25 | 1.47 | 1.26 |
| D.Sectoral |  |  |  |  |  |  |  |  |  |  |
| Non traded wage, $d \tilde{W}^{N}$ | 2.44 | 2.08 | 2.07 | 2.10 | 1.91 | 2.21 | 2.68 | 2.29 | 2.04 | 2.19 |
| Traded wage, $d \tilde{W}^{T}$ | 2.44 | 2.67 | 2.68 | 2.67 | 2.79 | 2.59 | 2.68 | 2.84 | 2.70 | 2.76 |
| Traded labor, $d \tilde{L}^{T}$ | 0.20 | 0.07 | 0.10 | 0.05 | -0.00 | 0.12 | 0.15 | 0.03 | 0.09 | 0.04 |
| Non traded labor, $d \tilde{L}^{N}$ | -0.29 | -0.17 | -0.14 | -0.20 | -0.11 | -0.21 | -0.32 | -0.21 | -0.17 | -0.20 |
| sectoral labor ratio, $d \tilde{L}^{T} / \tilde{L}^{N}$ | 1.01 | 0.46 | 0.48 | 0.45 | 0.17 | 0.67 | 0.93 | 0.43 | 0.52 | 0.45 |
| Traded output, $d \tilde{Y}^{T}$ | 1.22 | 1.17 | 1.20 | 1.15 | 1.14 | 1.20 | 1.20 | 1.12 | 1.19 | 1.44 |
| Non traded output, $d \tilde{Y}^{N}$ | 0.56 | 0.68 | 0.71 | 0.65 | 0.74 | 0.64 | 0.66 | 0.81 | 0.69 | 0.59 |
| sectoral output ratio, $d \tilde{Y}^{T} / \tilde{Y}^{N}$ | 2.02 | 1.69 | 1.72 | 1.67 | 1.52 | 1.82 | 1.92 | 1.59 | 1.77 | 1.65 |

[^42] aggregate and sectoral variables, respectively. Responses are scaled by initial GDP, except for long-run changes of capital, overall labor, aggregate and sectoral wage rates (percent of steady state). Responses of sectoral labor are scaled by initial steady-state overall labor while responses of sectoral outputs are scaled by initial steady-state GDP.
while $\mathrm{d} v_{N X}<0$ becomes smaller in absolute terms so that the term $\mathrm{d} v_{N X}-\mathrm{d} v_{I}$ becomes much smaller (less negative).

It is important to state that calibrating the model without government spending yields ratios at odds with the data: consumption as a share of GDP is $78 \%$ instead of $58 \%$, the non tradable content of GDP is $50 \%$ instead of $2 / 3$, the non-tradable content of labor compensation is $58 \%$ instead of $65 \%$. In brief, by abstracting from government spending and especially setting $G^{N}=0$, the non tradable content of various aggregates are reduced by such an amount that the initial steady state is not consistent with the key empirical properties of a representative OECD economy.

## O. 4 Calibration of the Model by Country: Predicted Responses for the Relative Price and Relative Wage

In section 5.3 , we compare the predicted values for $\hat{p}$ and $\hat{\omega}$ with estimates for each country and the whole sample. To do so, we keep unchanged the baseline calibration, except for the parameter capturing the degree of labor mobility across sectors (i.e., $\epsilon$ ) and the elasticity of substitution between traded and non traded goods (i.e., $\phi$ ) which play a major role in the determination of responses of $p$ and $\omega$. In this subsection, we set $\phi$ and $\epsilon$ to their estimated values for each country and calibrate the model in order to target all the ratios shown in Table 30.

Predicted values for the relative price response to a productivity differential of $1 \%$ are shown in columns 1 and 2 of Table 31 while predicted values for the relative wage are shown in columns 6 and 7 of Table 31. Columns 3 and 6 report fully modified OLS estimates of $\hat{p}$ and $\hat{\omega}$ for each country. While columns 1 and 6 of Table 31 give the predicted values $\hat{p}^{\text {predict }}$ and $\hat{\omega}^{\text {predict }}$ when cross-country heterogeneity is generated by $\phi$ and $\epsilon$ while other parameters are kept fixed, columns 2 and 7 of Table 31 give the predicted values denoted by $\hat{p}^{p r e d i c t, I}$ and $\hat{\omega}^{\text {predict,l }}$ by letting $\phi$ and $\epsilon$ to vary along with all observed ratios and parameters shown in Table 30.

Column 4 of Table 31 gives the difference between the observed and the predicted value for the relative price in absolute terms when letting only two parameters (i.e., $\phi$ and $\epsilon$ ) to vary while column 5 gives the difference between the observed and the predicted value in absolute terms by letting all parameters to vary. The last line shows the average value of the error in absolute terms. By and large, we find that the ability of the model to predict the relative price response relies mainly upon the cross-country heterogeneity generated by $\epsilon$ and $\phi$.

Column 9 of Table 31 gives the difference between the observed and the predicted value for the relative wage in absolute terms when letting only two parameters (i.e., $\phi$ and $\epsilon$ ) to vary while column 10 gives the difference between the observed and the predicted value in absolute terms by letting all parameters to vary. The last line shows the average value of the error. The same conclusion applies for the relative wage; we find that the ability of the model to predict the relative wage response relies mainly upon the cross-country heterogeneity generated by $\epsilon$ and $\phi$.

Table 29: Long-Term Relative Price and Relative Wage Responses to a Productivity Differential between Tradables and Non Tradables by Setting $G^{T}=G^{N}=0$ (in \%)


Notes: Effects of a labor share-adjusted TFPs diffential between tradables and non tradables of $1 \%$. Panels A and B show the deviation in percentage relative to steady-state for the relative price of non tradables $p \equiv p^{N} / p^{T}$ and the relative wage of non traded workers $\omega \equiv w^{N} / w^{T}$, respectively, and break down changes in a productivity effect (keeping unchanged sectoral capital-labor ratios $k^{j}$, the overall capital stock $K$ and the stock of foreign bonds $B$ ), a capital reallocation effect (induced by changes in $k^{j}$ keeping unchanged $K$ and $B$ ), a capital accumulation effect (stemming from the investment boom causing a current account deficit in the short-run and therefore requiring a steady-state improvement in the balance of trade). While panels A and B show the results when setting $\phi$ to one, panels C and D show results for $\phi<1$ and panels E and F show results for $\phi>1 ; \phi$ is the elasticity of substitution between tradables and non tradables; $\epsilon$ captures the degree of labor mobility across sectors.
Table 30: Data to Calibrate the Two-Sector Model (1990-2007)

| Countries | Share of GDP demand component |  |  | Non tradable Share |  |  |  |  |  | $(8)$ | $(9)$ | $(10)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ | $(6)$ | $(7)$ | $(8)$ | $(11)$ |  |  |  |
|  | $C / Y$ | $I / Y$ | $G / Y$ | $N X / Y$ | Output | Consumption | Gov. Spending | Labor | Lab. comp. | $\theta^{T}$ | $\theta^{N}$ |  |
| BEL | 0.53 | 0.21 | 0.22 | 0.04 | 0.65 | 0.42 | 0.91 | 0.68 | 0.66 | 0.61 | 0.63 |  |
| DEU | 0.58 | 0.21 | 0.19 | 0.02 | 0.65 | 0.40 | 0.91 | 0.65 | 0.60 | 0.75 | 0.63 |  |
| DNK | 0.49 | 0.20 | 0.26 | 0.05 | 0.66 | 0.42 | 0.94 | 0.68 | 0.68 | 0.63 | 0.70 |  |
| ESP | 0.59 | 0.25 | 0.18 | -0.02 | 0.64 | 0.46 | 0.88 | 0.66 | 0.67 | 0.60 | 0.66 |  |
| FIN | 0.52 | 0.20 | 0.22 | 0.06 | 0.58 | 0.43 | 0.89 | 0.63 | 0.63 | 0.59 | 0.73 |  |
| FRA | 0.57 | 0.19 | 0.23 | 0.01 | 0.70 | 0.40 | 0.94 | 0.69 | 0.68 | 0.70 | 0.64 |  |
| GBR | 0.64 | 0.17 | 0.20 | -0.02 | 0.64 | 0.40 | 0.93 | 0.70 | 0.65 | 0.70 | 0.73 |  |
| IRL | 0.52 | 0.22 | 0.15 | 0.12 | 0.52 | 0.43 | 0.89 | 0.62 | 0.62 | 0.46 | 0.69 |  |
| ITA | 0.59 | 0.21 | 0.19 | 0.02 | 0.64 | 0.38 | 0.91 | 0.63 | 0.62 | 0.71 | 0.64 |  |
| JPN | 0.56 | 0.27 | 0.16 | 0.01 | 0.63 | 0.43 | 0.86 | 0.64 | 0.65 | 0.55 | 0.60 |  |
| KOR | 0.53 | 0.33 | 0.12 | 0.02 | 0.52 | 0.44 | 0.76 | 0.58 | 0.56 | 0.73 | 0.83 |  |
| NLD | 0.50 | 0.21 | 0.23 | 0.06 | 0.65 | 0.40 | 0.90 | 0.70 | 0.69 | 0.60 | 0.70 |  |
| SWE | 0.49 | 0.18 | 0.27 | 0.06 | 0.64 | 0.45 | 0.92 | 0.68 | 0.67 | 0.63 | 0.71 |  |
| USA | 0.69 | 0.19 | 0.16 | -0.03 | 0.68 | 0.51 | 0.90 | 0.73 | 0.69 | 0.61 | 0.66 |  |
| Mean | 0.56 | 0.22 | 0.20 | 0.03 | 0.63 | 0.43 | 0.90 | 0.65 | 0.65 | 0.63 | 0.68 |  |

Table 31: Comparison of Predicted Values with Empirical Estimates

|  | Relative price response |  |  |  |  | Relative wage response |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
|  | $\hat{p}^{\text {predict }}$ | $\hat{p}^{\text {predict, }}$ | $\hat{p}^{\text {FMOLS }}$ | error $\mid$ (3) - (1)\| | error \|(3) - (2)| | $\hat{\omega}^{\text {predict }}$ | $\hat{\omega}^{\text {predict, }}$ | $\hat{\omega}^{\text {FMOLS }}$ | error $\mid$ (8) - (6)\| | error $\mid$ (8) - (7)\| |
| BEL | 0.691 | 0.694 | 0.825 | 0.134 | 0.131 | -0.498 | -0.482 | -0.142 | 0.356 | 0.340 |
| DEU | 0.668 | 0.600 | 0.606 | 0.062 | 0.006 | -0.535 | -0.513 | -0.582 | 0.047 | 0.069 |
| DNK | 0.445 | 0.443 | 0.470 | 0.025 | 0.027 | -0.905 | -0.864 | -0.453 | 0.452 | 0.411 |
| ESP | 0.844 | 0.840 | 0.836 | 0.008 | 0.004 | -0.245 | -0.250 | -0.280 | 0.035 | 0.030 |
| FIN | 0.662 | 0.788 | 0.733 | 0.071 | 0.055 | -0.545 | -0.344 | -0.221 | 0.324 | 0.123 |
| FRA | 0.823 | 0.724 | 0.841 | 0.018 | 0.117 | -0.279 | -0.374 | -0.412 | 0.133 | 0.038 |
| GBR | 0.867 | 0.789 | 0.922 | 0.055 | 0.133 | -0.207 | -0.285 | -0.141 | 0.066 | 0.144 |
| IRL | 0.720 | 0.833 | 0.737 | 0.017 | 0.096 | -0.450 | -0.347 | -0.210 | 0.240 | 0.137 |
| ITA | 0.798 | 0.734 | 0.767 | 0.031 | 0.033 | -0.321 | -0.356 | -0.290 | 0.031 | 0.066 |
| JPN | 0.786 | 0.821 | 0.970 | 0.184 | 0.149 | -0.340 | -0.307 | -0.178 | 0.162 | 0.129 |
| KOR | 0.744 | 0.712 | 0.651 | 0.093 | 0.061 | -0.410 | -0.381 | -0.482 | 0.072 | 0.101 |
| NLD | 0.608 | 0.678 | 0.800 | 0.192 | 0.122 | -0.635 | -0.519 | -0.345 | 0.290 | 0.174 |
| SWE | 0.976 | 0.919 | 0.918 | 0.058 | 0.001 | -0.026 | -0.115 | -0.004 | 0.022 | 0.111 |
| USA | 0.884 | 0.816 | 0.822 | 0.062 | 0.006 | -0.178 | -0.284 | -0.034 | 0.144 | 0.250 |
| Average |  |  |  | 0.072 | 0.067 |  |  |  | 0.170 | 0.152 |

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[^0]:    *We thank Christian Bayer, David de la Croix, François Langot, Gernot Müller, Jean-Christophe Poutineau, Stefan Schubert, participants at the IRES Macroeconomics Lunch Seminar at the Université catholique de Louvain, 26th October 2010, the SIUTE seminar at the University of Lille, 25th January 2011, the GAINS seminar at the University of Le Mans, 15th February 2011, the 60th Congress of the French Association of Economics (AFSE), 8-9th September 2011, REGLES seminar at the University of Nancy, 20th September 2011, the Cournot seminar at the University of Strasbourg, 2nd December 2011, the lunch seminar at the University of Bolzano, 12th April 2012, the 14th Conference Theories and Methods of Macroeconomics, 1011th May 2012, the seminar at the University of Rennes, 21st June 2012, the Macro/Econometrics/Finance seminar at the University of Bonn, 13th June 2012, the Latin American Meeting of the Econometric Society, 1-3rd November 2012, and the 20th conference of Computing in Economics and Finance, 22-24th June 2014, for valuable comments. We are also very grateful to Luisito Bertinelli for his help. Finally, we would like to thank the editor Timothy Kehoe and two anonymous referees, for very constructive suggestions and remarks which improved both the content and exposition of a previous draft. Of course the usual disclaimers apply.
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[^1]:    ${ }^{1}$ In the new literature analyzing the BS effect, the appreciation in domestic goods relative to foreign goods operates mainly through an improvement in the terms of trade, see e.g., Corsetti et al. [2007].
    ${ }^{2}$ Using sectoral data for the U.S. over the period 1992-2002, Jensen and Kletzer [2006] find that when education is controlled for, the earnings differential between traded and non traded industries amounts to between $10 \%$ and $17 \%$.
    ${ }^{3}$ Bergin et al. [2006] simulate their model and highlight numerically the role of both the endogenous ordering of tradability and the endogenous concentration of traded goods in replicating a rising BS coefficient over time. Unlike Bergin et al. [2006], we analyze the change in the ratio of sectoral wages and focus on the relative price of non tradables instead of the real exchange rate by assuming perfectly competitive product markets.

[^2]:    ${ }^{4}$ A number of variants of the two-sector model with tradables and non tradables have been used to investigate the real exchange rate and trade balance effects of financial liberalization (see Cordoba (de) and Kehoe [2000], Bems and Hartelius [2006]), or to analyze disinflation policy transmission (see Mendoza and Uribe [2000]). See also Turnovsky [1997] who presents variants of the two-sector model.
    ${ }^{5}$ See e.g., Bouakez et al. [2009], Kim and Kim [2006] who assume that sectoral hours worked are aggregated by means of a CES function in order to account for the evidence related to the co-movement of sectoral aggregates, or Altissimo et al. [2011] who address inflation dispersion across EMU members.

[^3]:    ${ }^{6}$ Although Lane and Milesi-Ferretti's [2004] paper is particularly closely related to our analysis, our approach is different. Lane and Milesi-Ferretti [2004] investigate the impact of net foreign asset positions on the relative price of non tradables while we analyze the role of imperfect mobility of labor across sectors in determining the long-run responses of the relative price and relative wage to higher productivity growth of tradables relative to non tradables. Moreover, the authors' assumption that the output of the tradable sector is an endowment can be interpreted as the fact that labor is not mobile between the two sectors which corresponds to a polar case in our framework. Finally, they abstract from physical capital accumulation which plays a key role in our model.

[^4]:    ${ }^{7}$ Additional empirical results, in particular alternative unit root tests or cointegration methods, and more details on the model as well as the derivations of the results which are stated below, are provided in a Technical Appendix which is available at http://www.beta-umr7522.fr/productions/publications/2014/2014-16.pdf.

[^5]:    ${ }^{8}$ In the long-run, the return on domestic capital $R$ must equalize with the capital rental cost $P_{I}\left(r^{\star}+\delta\right)$ with $P_{I}$ is the investment price index. For clarity purposes, in the following, we assume that investment is non traded so that $P_{I}$ reduces to $P$.

[^6]:    ${ }^{9}$ While the assumption of a small open economy is not suited to large countries such as the United States or Japan, we decided to include these economies in our sample when conducting our empirical analysis in order to make our estimates comparable with those documented by previous studies, in particular Canzoneri et al. [1999] and Kakkar [2003] who adopt cointegration methods like us.
    ${ }^{10}$ De Gregorio et al. [1994], classify a sector as tradable if more than 10 percent of its total production is exported. This classification has been updated by Jensen and Kletzer [2006] who use locational Gini coefficients to measure the geographical concentration of different sectors and classify sectors with a Gini coefficient below 0.1 as non-tradable and all others as tradable (the authors classify activities that are traded domestically as potentially tradable internationally). Unlike De Gregorio et al. [1994] who treat "Financial intermediation" as non tradable, we classify this industry as tradable, following Jensen and Kletzer [2006]; our sensitivity analysis reveals that our conclusions hold whether "Financial intermediation" is classified as tradable or non tradable.

[^7]:    ${ }^{11}$ Because the relative price of non tradables is computed by using value added deflators and thus might be correlated with sectoral valued added shares, we alternatively construct time series for $K^{T}$ and $K^{N}$ by using capital stocks at constant-prices by industry taken from the EU KLEMS database, applying the classification for sectoral value added detailed above. Whether we adopt Garofalo and Yamarik's [2002] procedure or use disaggregated capital stocks to construct time series for $K^{T}$ and $K^{N}$, running the regression for the same panel of countries (i.e., eight instead of fourteen countries due to data availability) yields similar results.
    ${ }^{12}$ As a robustness check, we run the same regressions by using an alternative measure of the productivity differential when investment expenditures are assumed to be traded, implying $\left(\theta^{N} / \theta^{T}\right) z^{T}-z^{N}$, or both traded and non traded, leading to $\frac{z^{T}-\frac{\theta^{T}}{\theta^{N}} z^{N}}{\alpha_{I}+\left(1-\alpha_{I}\right) \frac{\theta^{T}}{\theta^{N}}}$ where $\alpha_{I}$ is the investment expenditure share on non tradable goods. Because results are very similar, to save space we do not present them and they are therefore relegated to the Technical Appendix.

[^8]:    ${ }^{13}$ The DOLS estimator adds $q$ leads and lags of $\triangle\left(z^{T}-\left(\theta^{T} / \theta^{N}\right) z^{N}\right)$ as additional regressors in (6). We set $q=1$; our results were identical for $q=2$ and $q=3$.

[^9]:    ${ }^{14}$ Following Wacziarg and Wallack [2004], we eschew year-to-year changes because of the low frequency changes in labor at that horizon and restrict our attention to differences over alternatively 2 or 3 years.

[^10]:    ${ }^{15}$ There are two alternative ways to depart from the strong assumption of wage equalization across sectors. Shi [2011] introduces intersectoral adjustment costs to produce imperfect mobility of labor across sectors along the transitional path. While the wage equalization does not hold in the short-run, wages equalize across sectors in the long-run, in contradiction with our findings. The second approach is to introduce labor market frictions in the tradition of Diamond-Mortensen-Pissarides. While this strategy could serve our purpose, the advantage of our approach is that the BS effect emerges as a special case. Moreover, we show in a Technical Appendix that a two-sector dynamic general equilibrium with search unemployment leads to very similar formal relationships between the relative price (the relative wage) change and the productivity differential. Finally, an additional difficulty is to calibrate search parameters at a sectoral level. In contrast, our specification allows us to estimate the deep parameters for each country in order to calibrate the model.
    ${ }^{16}$ While in the model, the difficulty in reallocating labor across sectors may reflect psychological costs when switching sectors, it may also capture other barriers to mobility that are not included in the model such as labor market regulation or specific human capital. When running the regression of $\epsilon$ on a number of labor market regulation indicators, we find empirically that countries with more stringent employment protection legislation, higher union density, and more generous unemployment benefit scheme display lower labor mobility (i.e., the parameter $\epsilon$ takes smaller values). Moreover, countries with higher shares of young employees and low-skilled workers experience larger labor mobility, as these two workers' groups have relatively less sector-specific skills than other types of workers and thus are more prone to switch jobs across sectors.
    ${ }^{17}$ In a Technical Appendix, we assess to what extent our results depend on the assumption of separability in preferences. Numerical results reveal that considering a more general specification for preferences by allowing for consumption and labor to be non-separable does not affect our conclusions.

[^11]:    ${ }^{18}$ Specifically, we have $\alpha_{C}=\frac{(1-\varphi) P^{1-\phi}}{\varphi+(1-\varphi) P^{1-\phi}}$. Note that $\alpha_{C}$ depends negatively on the relative price $P$ as long as $\phi>1$ and reduces to $1-\varphi$ when $\phi=1$.
    ${ }^{19}$ Specifically, we have $\alpha_{L}=\frac{(1-\vartheta)\left(W^{N}\right)^{\epsilon+1}}{\left[\vartheta\left(W^{T}\right)^{\epsilon+1}+(1-\vartheta)\left(W^{N}\right)^{\epsilon+1}\right]}$.

[^12]:    ${ }^{20}$ The cross-section studies report an estimate of $\phi$ ranging from 0.44 to 0.74 , see e.g., Stockman and Tesar [1995] and Mendoza [1995], respectively. Empirical analysis using annual time series data pooled for a group of countries by Ostry and Reinhart [1992] reports estimates ranging from 0.66 to 1.28. Adopting cointregation methods over the period 1970-2007, we find an elasticity $\phi$ of 0.66 for the whole sample while estimates vary between roughly 0.2 and 1.8 across countries. We provide more details in Appendix B.
    ${ }^{21}$ In a Technical Appendix, we relax this assumption and instead assume that investment expenditures are both traded and non traded. Numerical results reveal that the relative price and relative wage effects of a productivity differential are almost identical to those obtained when abstracting from traded investment.

[^13]:    ${ }^{22}$ Linearizing the capital accumulation equation which clears the non traded goods market $\dot{K}(t)=$ $Y^{N}(K(t), P(t))-C^{N}(P(t))-\delta_{K} K(t)$ and the dynamic equation for the relative price of non traded goods which equalizes the rates of return on domestic capital and foreign bonds $\dot{P}(t)=P(t)\left[\left(\delta_{K}+r^{\star}\right)-\frac{R(t)}{P(t)}\right]$, the (linearized) system possesses one negative eigenvalue denoted by $\nu_{1}$ and one positive eigenvalue $\nu_{2}=r^{\star}-\nu_{1}$. The stable path for the relative price is: $P(t)-P=\omega_{2}^{1}(K(t)-K)$.
    ${ }^{23}$ While we solve the steady-state keeping unchanged the capital stock and the stock of foreign bonds, these two aggregates can be determined as follows. The system consisting of (15)-(16), (30)-(31), and (28a)-(28b) together with $Y^{N}=C^{N}+\delta_{K} K, K=k^{T} L^{T}+k^{N} L^{N}$ (first inserting the solutions for $L^{T}=L^{T}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right)$ and $\left.L^{N}=L^{N}\left(\bar{\lambda}, K, P, Z^{T}, Z^{N}\right)\right)$ and (32), can be solved for $K, B$ and $\bar{\lambda}$ as functions of $Z^{T}$ and $Z^{N}$.
    ${ }^{24}$ When solving the steady-state, changes in capital stock and foreign assets as reflected by changes in $v_{I}$ and $v_{B}$ are assumed to be exogenous. Such a procedure allows us to isolate the relative price and relative wage effects stemming from capital accumulation and changes in traded bonds holding.
    ${ }^{25}$ This point can be seen formally by combining (30) and (28a).

[^14]:    ${ }^{26}$ To determine labor demand, we use the market clearing condition (31), eliminate $Y^{j}$ by using production functions (1) and eliminate $P$ by dividing the second equality of (28b) by the first equality. Then using (30) and (28a) to eliminate the sectoral capital-labor ratios, combining the market-clearing condition (31) along with the optimal rule allocating consumption into tradables and non tradables (15) and production functions (1), and taking logarithm allow us to derive the (relative) $L D$-schedule:

    $$
    \left.\left(l^{T}-l^{N}\right)\right|^{L D}=\left[1+\theta^{T}(\phi-1)\right] \omega+(\phi-1)\left(z^{T}-\frac{\theta^{T}}{\theta^{N}} z^{N}\right)-\ln \left(\frac{1+v_{B}}{1-v_{I}}\right)-\ln \Theta
    $$

    where $\Theta>0$ is a term composed of exogenous preference $(\varphi, \phi)$ and production parameters $\left(\theta^{j}, \delta_{K}, r^{\star}\right)$.
    ${ }^{27}$ Note that to derive the RHS of (33), we use a first-order Taylor approximation to rewrite $\mathrm{d} \ln \left(\frac{1+v_{B}}{1-v_{I}}\right)$ as $\mathrm{d} v_{B}+\mathrm{d} v_{I}$ which eases the discussion. Remembering that at the steady-state the traded good market clearing condition is $-N X=r^{\star} B$. Dividing the LHS and the RHS by $Y^{T}$, we get $v_{B}=-v_{N X}$.

[^15]:    ${ }^{28}$ The worker/consumer reduces private savings to avoid a reduction in consumption while she/he lowers labor supply.
    ${ }^{29}$ Using (28b) to determine the relative wage $\Omega$, inserting the optimal allocation of aggregate labor supply across sectors (16) and production functions (1), using (30) and (28a) to eliminate the sectoral capital-labor ratios, and taking logarithm yields the relative $L M E$-schedule:

    $$
    \left.\left(y^{T}-y^{N}\right)\right|^{L M E}=-\left[\epsilon+\left(\frac{1-\theta^{T}}{\theta^{T}}\right)(1+\epsilon)\right] p+\left(\frac{1+\epsilon}{\theta^{T}}\right)\left(z_{T}-\frac{\theta^{T}}{\theta^{N}} z_{N}\right)+\ln \Pi
    $$

    where $\Pi>0$ is a term composed of exogenous preference $(\vartheta, \epsilon)$ and production parameters $\left(\theta^{j}, \delta_{K}, r^{\star}\right)$.

[^16]:    ${ }^{30}$ Technically, the assumption $\beta=r^{\star}$ requires the joint determination of the transition and the steady-state. We have recourse to the standard linear approximation.
    ${ }^{31}$ Government spending on traded $G^{T}$ and non traded goods $P G^{N}$ are considered for calibration purposes. Hence, the market clearing condition for the traded good and the non traded good at the steady-state are $r^{\star} B+Y^{T}=C^{T}+G^{T}$ and $Y^{N}=C^{N}+I+G^{N}$.

[^17]:    ${ }^{32}$ Appendix B presents the empirical strategy while details of derivation of the relationship we explore empirically can be found in a Technical Appendix.
    ${ }^{33}$ As shown in Table 6, estimates of $\phi$ for Ireland and Italy are either negative or not statistically significant. Hence, column 2 of Table 5 reports only consistent estimates for the elasticity of substitution $\phi$ between traded and non traded goods which average to 0.9. The advantage of setting $\phi$ to 1 in the baseline scenario is twofold. First, the share of non traded goods in consumption expenditure $\alpha_{C}$ coincides with the weight of the non traded good $1-\varphi$ in the CES sub-utility function (9) if $\phi=1$. Second, setting $\phi=1$ implies that only the capital accumulation channel is in effect as the baseline and capital reallocation channels vanish, allowing us to highlight the intertemporal effect trigged by the investment boom.

[^18]:    ${ }^{34}$ Sectoral government spending allows us to target a non-tradable content of GDP of $63 \%$ in accordance with the mean value shown in the last line of Table 7 .

[^19]:    ${ }^{35}$ The reason is as follows. Following the productivity differential, the worker/consumer lowers labor supply

[^20]:    on impact and thus reduces private savings to avoid a decline in consumption, all else being equal. The more responsive the labor supply, the more private savings decline, and therefore the larger the current account deficit. As a result, net exports and therefore demand for tradables rise further, exerting a larger negative impact on $\omega$ and $p$, as shown in the fourth line of panel $C$ and $D$, respectively.
    ${ }^{36}$ Numerical results indicate that raising $\theta^{T}$ from 0.6 to 0.7 while reducing $\theta^{N}$ from 0.7 to 0.6 moderates the capital reallocation channel. When the non traded sector becomes more capital intensive, the capital inflow in the non traded sector is less pronounced as the traded sector is more labor intensive. Hence, decreases in $p$ and $\omega$ due to the shift of capital are moderated.

[^21]:    ${ }^{37}$ While we could calibrate the model so as to target the ratios summarized in Table 7 for each country, we only allow for $\epsilon$ and $\phi$ to vary across countries. The reason is twofold. First, we have shown analytically and numerically that both the degree of labor mobility $\epsilon$ and the elasticity of substitution play a major role in determining the responses of the relative price and the relative wage to a productivity differential. In a Technical Appendix, we contrast the prediction error allowing for $\epsilon$ and $\phi$ to vary across countries with that when the calibration of the model is consistent with the key ratios of each economy as well. While the latter strategy reduces the model's prediction error, the discrepancy between the two approaches is small. Second, our aim is to isolate the role of the degree of labor mobility $\epsilon$ and of the elasticity of substitution $\phi$ in determining the responses of $p$ and $\omega$ to a productivity differential, all else being equal. Keeping the baseline calibration unchanged except for $\epsilon$ and $\phi$ is consistent with our objective.
    ${ }^{38}$ Because we use the FMOLS procedure to estimate $\phi$, and since FMOLS and DOLS cointegration procedures give very similar estimates for the relative price and relative wage responses to a productivity differential, we compare predicted values with FMOLS estimates of $\beta$ and $\gamma$. We reach similar conclusions when using DOLS estimates.

[^22]:    ${ }^{39}$ Details of derivation of the equation we explore empirically can be found in a Technical Appendix.
    ${ }^{40}$ Like Horvath [2000], we use time series for output instead of value added so that our estimates can be compared with those documented by the author.
    ${ }^{41}$ In a Technical Appendix, we address two potential econometric issues. While $\beta_{i, t}^{j}$ (i.e., the RHS term in eq. (35)) is constructed independently from the dependent variable (i.e., the LHS term in eq. (35)), if the labor's share is (almost) constant over time and thus is close from the average $\xi^{j}$, an endogeneity problem

[^23]:    ${ }^{43}$ 'Short-run static solutions' refer to static optimality conditions (58a)-(58b) which can be solved for consumption and labor.

[^24]:    ${ }^{44}$ We do not discuss the effects of $Z^{j}$ (with $j=T, N$ ) since the interpretation is straightforward.
    ${ }^{45}$ The reason is that the traded sector experiences a substantial outflow of capital since it is costless to reallocate capital. When workers are reluctant to shift hours worked across sectors, the capital outflow is large enough to reduce $k^{T}$.

[^25]:    ${ }^{46}$ Remembering that at the steady state the traded good market clearing condition is $r^{\star} B+Y^{T}-C^{T}=0$, and rearranging terms yields $-N X=r^{\star} B$. Dividing the LHS and the RHS by $Y^{T}$, we get $v_{B}=-v_{N X}$.

[^26]:    $j=T, N$. The non tradable share of investment in Belgium and Korea are taken from Bems [2008] (see Tables 7 and 8).

[^27]:    ${ }^{49}$ For Japan the time span is 1974-2006. Belgium, France, Korea and Ireland do not provide disaggregated capital stock series, and, due to data limitation, we exclude from the econometric analysis Germany (1991-2007) and Sweden (1993-2007).

[^28]:    ${ }^{50}$ To derive $(256 \mathrm{e})$, we used the fact that $\psi(t)=\lambda P(t)$.

[^29]:    ${ }^{51}$ To derive $(274 \mathrm{c})$, we used the fact that $\psi(t)=\lambda P(t)$.
    ${ }^{52}$ Specifically, we have $\alpha_{C}=\frac{(1-\varphi) P^{1-\phi}}{\varphi+(1-\varphi) P^{1-\phi}}$. Note that $\alpha_{C}$ depends negatively on the relative price $P$ as long as $\phi>1$.

[^30]:    ${ }^{54}$ Our framework also builds upon Merz [1995], Andolfatto [1996], Shi and Wen [1999] who construct dynamic general equilibrium models with labor markets characterized by search frictions. We depart from these papers by considering a two-sector framework.
    ${ }^{55}$ The price of the traded good is determined on the world market and exogenously given for the small open economy.

[^31]:    ${ }^{56}$ First-order conditions consist of (315a) and (315c) together with $\left(F^{j}\right)^{1 / \sigma_{L}^{j}}=m^{j} \xi^{\prime, j}+R^{j} \lambda$ and $\dot{\xi}^{\prime}=$ $\left(s^{j}+\rho\right) \xi^{\prime, j}-\left[\lambda w^{j}-\left(F^{j}\right)^{1 / \sigma_{L}^{j}}\right]$. Denoting by $\xi^{j} \equiv \xi^{\prime, j} / \lambda$, using (315a) and (315c), we get (315b) and (315d). Since $\xi^{\prime, j}$ is the utility value of an additional job and $\lambda$ is the marginal utility of wealth, $\xi^{j}$ is the pecuniary value of an additional job.

[^32]:    ${ }^{57}$ More precisely, depending on the search parameters captured by $s^{j}$ and $m^{j}$, labor force is split between working time and job search. Along the transitional dynamics, using the fact that $U^{j}=F^{j}-L^{j}$, agents supply working time $L^{j}$ according to the following accumulation equation $\dot{L}^{j}=m^{j} U^{j}-s^{j} L^{j}=m^{j} F^{j}-\left(m^{j}+s^{j}\right) L^{j}$, where $F^{j}$ is labor force and $L^{j}$ corresponds to hours worked in sector $j$ supplied by the representative household.
    ${ }^{58}$ Specifically, we have $\alpha_{C}=\frac{(1-\varphi) P^{1-\phi}}{\varphi+(1-\varphi) P^{1-\phi}}$. Note that $\alpha_{C}$ depends negatively on the relative price $P$ as long as $\phi>1$.

[^33]:    ${ }^{59}$ We made use of the fact that $\pi^{j}=\Xi^{j} L^{j}-w^{j} L^{j}+x^{j} \dot{L}^{j}-\kappa^{j} V^{j}$ since production functions are linearly homogeneous in labor.
    ${ }^{60}$ Note that the flows of workers in and out of employment are equal to each other in any symmetric equilibrium, i.e., $m^{j} U^{j}=f^{j} V^{j}$. Hence equations $\dot{L}^{j}=f^{j} V^{j}-s^{j} L^{j}$ and $\dot{L}^{j}=m^{j} U^{j}-s^{j} L^{j}$ indicate that the demand for labor indeed equates the supply.
    ${ }^{61}$ Note that the Nash bargaining wage depends positively on unemployment benefits $R^{j}$. To see it more

[^34]:    ${ }^{62}$ In the numerical analysis, we consider government spending for calibration purpose. In this case, where eq. (329) can be rewritten as follows: $\left(x^{T} \dot{L}^{T}+x^{N} \dot{L}^{N}\right)+T=\left(R^{T} U^{T}+R^{N} U^{N}\right)+G^{T}+P G^{N}$ where $G^{T}$ and $G^{N}$ government spending on tradables and non tradables, respectively. When $\dot{L}^{j}<0$, government proceeds from the firing costs are redistributed back to agents as lump-sum transfers.

[^35]:    ${ }^{63}$ When focusing on the non traded sector, we have $\Xi^{N}=P A^{N}$; in this case, we have to insert the short-run stock solution for the relative price of non tradables (333) into the dynamic equation for $\theta^{N}$ and $U^{N}$.
    ${ }^{64}$ Elements $\omega_{21}^{N}$ and $\omega_{31}^{N}$ of the eigenvector (associated with the stable eigenvalue $\nu_{1}^{N}$ ) are:
    $\omega_{21}^{N}=\frac{\left(2 s^{N}+r^{\star}\right)+\left(s^{N}+r^{\star}-\nu_{i}^{N}\right)\left(\frac{s^{N}+\nu_{i}^{N}}{\tilde{m}^{N}}\right)+\tilde{m}^{N}\left(P_{L^{N}} A^{N} \frac{\bar{\lambda}}{v_{F F}^{N}}+1\right)}{\frac{\left(m^{N}\right)^{\prime} \tilde{U}^{N}}{\tilde{m}^{N}}\left(s^{N}+\tilde{m}^{N}+r^{\star}-\nu_{i}^{N}\right)}<0, \quad \omega_{31}^{N}=\left(\frac{s^{N}+\nu_{1}^{N}}{\tilde{m}}\right)-\frac{\left(m^{\prime}\right)^{N} \tilde{U}^{N}}{\tilde{m}^{N}} \omega_{21}^{N}<0$.

[^36]:    ${ }^{66}$ Note that the term in the denominator of eq. (353) $\chi w_{R}=\frac{\alpha_{W}}{1-\alpha_{W}} \kappa \theta$ does not depend on unemployment benefits; moreover, a rise in $R$ lowers the surplus form hiring $\Psi \equiv \Xi+r^{\star} x-w_{R}$ depends on $R$ by raising the reservation wage $w_{R} \equiv \frac{\alpha_{W}}{1-\alpha_{W}} \kappa \theta+R$.

[^37]:    ${ }^{72}$ Note that the shift in the $V C T$-schedule dominates the shift in the $W S T$-schedule because workers and firms have to share the surplus, i.e., $0<\alpha_{W}^{T}<1$.
    ${ }^{73}$ To get (355), we used the fact that $\chi^{T} w_{R}^{T}=m^{T} \frac{\alpha_{W}^{T} \Psi^{T}}{s^{T}+r^{\star}}$.
    ${ }^{74}$ Totally differentiating the first equality in (356) yields:

[^38]:    ${ }^{78}$ Note that, as for the traded labor market, the shift in the $V C N$-schedule dominates the shift in the $W S N$-schedule because the worker bargaining power $\alpha_{W}^{N}$ is smaller than one.

[^39]:    ${ }^{79}$ Note that the definition of the percentage change in total hours worked $\hat{l}_{t}$ implies that job flows are not perfectly correlated between sectors; more specifically, we have $L^{t} \neq L_{t}^{N}+L_{t}^{T}$ due to our assumption of limited substitutability of hours worked).

[^40]:    ${ }^{80}$ While the sample used by Stockman and Tesar [1995] covers 30 countries (including 17 developing and 13 industrialized), Mendoza [1995] uses exactly the same data set in his estimation but includes only the 13 industrialized countries. Note that the estimate of $\phi$ has been obtained by using the cross sectional dataset by Kravis, Heston and Summers for the year 1975.

[^41]:    ${ }^{81}$ By having the term $\left(M^{j}\right)^{-\frac{1}{\rho-1}}$ in (457), the analysis abstracts from the variety effect and concentrates solely on the effects of markup variation.

[^42]:    Notes: Effects of a permanent rise in the sectoral productivity ratio by $1 \%$. Panels A and B show the deviation in percentage relative to steady-state for

