



HAL
open science

UNFOLDING PARALLEL REASONING IN ISLAMIC JURISPRUDENCE (I). Epistemic and Dialectical Meaning within Abū Ishāq al-Shīrāzī's System of Co-Relational Inferences of the Occasioning Factor

Shahid Rahman, Muhammad Iqbal

► **To cite this version:**

Shahid Rahman, Muhammad Iqbal. UNFOLDING PARALLEL REASONING IN ISLAMIC JURISPRUDENCE (I). Epistemic and Dialectical Meaning within Abū Ishāq al-Shīrāzī's System of Co-Relational Inferences of the Occasioning Factor . 2016. halshs-01265206v4

HAL Id: halshs-01265206

<https://shs.hal.science/halshs-01265206v4>

Preprint submitted on 16 Nov 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

UNFOLDING PARALLEL REASONING IN ISLAMIC JURISPRUDENCE (I)

Epistemic and Dialectical Meaning within

Abū Ishāq al-Shīrāzī's System of Co-Relational Inferences of the Occasioning Factor

Shahid Rahman

shahid.rahman@univ-lille3.fr ,

Univ. Lille, CNRS, UMR 8163 - STL
- Savoirs Textes Langage, F-59000 Lille, France)

Muhammad Iqbal

iqbal.ego@gmail.com

(Univ. Lille, CNRS, UMR 8163 - STL
- Savoirs Textes Langage, F-59000 Lille, France ;
and Islamic University Antasari, Banjarmasin,
Indonesia)

To the memory of my late Grand-Uncle *Yahya Ansari*

S. Rahman

Abstract: One of the epistemological results emerging from this initial study, is that the different forms of *co-relational inference*, known in the Islamic jurisprudence as *qiyās* represent an innovative and sophisticated form of reasoning that not only provide new epistemological insights of legal reasoning in general but they also furnish a fine-grained pattern for *parallel reasoning* that can be deployed in a wide range of problem-solving contexts and that does not seem to reduce to the standard forms of analogical argumentation studied in contemporary philosophy of science. More specifically the main claim is that a dialectical framework provides the right instrument to stress three of the most salient features of this form of inference: (1) the interaction of heuristic with logical steps, (2) the dynamics underlying the meaning-explanation of the terms involved (3) the unfolding of parallel reasoning as similarity in action. They display what we take to be the main epistemological idea behind the *qiyās*, namely: the open texture of the extension of normative statements subject to this kind of parallel reasoning. In the present paper we will only discuss the case of so-called *co-relational inferences of the occasional factor*. The other kinds of inferences will be studied in a second paper.

I Introduction

Uṣūl al-fiqh (أصول الفقه), that is, the science of Islamic jurisprudence, is deeply rooted on the notion of rational knowledge and understanding. Indeed, *fiqh* constitutes the body of knowledge and methods of reasoning that the Islamic jurists deploy in order to provide solutions to legal problems based on the juridical understanding of the sources. The point is that, according to *uṣūl¹ al-fiqh* legal knowledge is achieved by rational endeavour, the

¹ Cf. Weiss (1998).

intellectual effort of human being: this is what is meant when the term *ijtihād* (اجتهاد), endeavour of the intellect, is attached to *fiqh*. Let us quote the beautiful paragraph on *ijtihād* by Wael B. Hallaq in his landmark work *A History of Islamic Legal Theories* (1997).

In his Mustasfā Ghazali depicts the science of legal theory in terms of a tree cultivated by man. The fruits of the tree represent the legal rules that constitute the purpose behind planting the tree; the stem and the branches are the textual materials that enable the tree to bear the fruits and to sustain them. But in order for the tree to be cultivated, and to bring it to bear fruits, human agency must play a role. [...] We shall now turn to the "cultivator," the human agent whose creative legal reasoning is directed toward producing the fruit, the legal norm. The jurist (faqīh) or jurisconsult (muftī) who is capable of practicing such legal reasoning is known as the mujtahid, he who exercises his utmost effort in extracting a rule from the subject matter of revelation while following the principles and procedures established in legal theory. The process of this reasoning is known as ijihād the effort itself. Hallaq (1997, p. 117).

One of the most remarkable features of the practice of *ijtihād* is that it presupposes that *fiqh* is dynamic in nature. Indeed, since the ultimate purpose of such kind of rational endeavour is to achieve decisions for new circumstances or cases not already established by the juridical sources, the diverse processes developed within Islamic jurisprudence presuppose that juridical meaning is subject to changes.

This dynamic feature animates Walter Edward Young's (2016) main thesis as developed in his book *The Dialectical Forge: Juridical Disputation and the Evolution of Islamic Law*. In fact the main claim underlying the work of Young – that originates in the work of Hallaq, his PHD-advisor – is that the dynamic nature of *fiqh* is put into action by means of a dialectical understanding and practice of legal reasoning. Young (2016, chapter 2.2) acknowledges and discusses his debt to the work of his mentor in many sections of his book. The following lines of Hallaq, quoted by Young (2017) in the second chapter of his monography, are particularly interesting to the present study:

In one sense, dialectic constituted the final stage in the process of legal reasoning, in which two conflicting opinions on a case of law were set against each other in the course of a disciplined session of argumentation with the purpose of establishing the truthfulness of one of them. The aim of this exercise, among other things, was to reduce disagreement (ikhtilāf) among legists by demonstrating that one opinion was more acceptable or more valid than another. Minimizing differences of opinion on a particular legal question was of the utmost importance, the implication being that truth is one, and for each case there exists only one true solution. Hallaq (1997, p. 136-137).

Similar applies to the following remark of Hallaq also quoted by Young:

The most common method of exposition—though exceptions are many—is that of the question-answer: 'If someone says such and such, we reply with such and such.' But this method is versatile, and an analysis of the logical structure of questions, and especially answers, shows that theorists adopted for their use the entire gamut of arguments we nowadays subsume under logic and rhetoric. Hallaq (1997, p. 137).

According to this perspective, the practice of *ijtihād* takes the form of an interrogative enquiry where the intertwining of giving and asking for reasons features the notion of meaning that grounds legal rationality.² More precisely the conception of legal reasoning developed by

² See too Hallaq (1987a,b, 2004, 2009a,b). Another early study that stressed this point is Larry Miller's (1984) PHD-thesis of 1984 on the development of dialectic in Islam. Hassan Tahiri discusses the crucial role of dialectical reasoning for astronomy (2008) and for the development of sciences in general – Tahiri/Rahman/Street (2008), Tahiri (2014, 2015 pp. 4-5, 2016).

Islamic jurisprudence is that it is a combination of deductive rules moves with hermeneutic and heuristic ones deployed in an epistemic frame. Let us once more quote Hallaq:

Armed with the knowledge of hermeneutical principles, legal epistemology and the governing rules of consensus, the mujtahid is ready to undertake the task of inferring rules. Inferring rules presupposes expert knowledge in hermeneutics because the language of the texts requires what may be called verification; namely, establishing, to the best of one's ability, the meaning of a particular text as well as its relationship to other texts that bear upon a particular case in the law. For this relationship, as we have seen, may be one of particularization, corroboration or abrogation. Before embarking on inferential reasoning, the mujtahid must thus verify the meaning of the text he employs, and must ascertain that it was not abrogated by another text. Knowledge of the principles of consensus as well as of cases subject to the sanctioning authority of this instrument is required to ensure that the mujtahid's reasoning does not lead him to results contrary to the established consensus in his school. This knowledge is also required in order to ensure that no case that has already been sanctioned by consensus is reopened for an alternative rule. Hallaq (1997, p. 82).

In fact, out of the dissatisfaction with the efficiency of the standard post-Aristotelian notion of syllogism in jurisprudence emerged an ambitious dialectical frame for argumentation by parallelisms, (including exemplification, symmetry and analogy) which should offer a new unifying approach to epistemology and logic.³

The most perspicuous outcome of this approach within *fiqhi* is the notion of *qiyās* (قياس), known as *co-relational inference* (Young (2017)).⁴

The aim of co-relational inferences is to provide a rational ground for the application of a juridical ruling to a given case not yet considered by the original juridical sources. It proceeds by combining heuristic (and/or hermeneutic) moves with logical inferences. The simplest form follows the following pattern:

In order to establish if a given juridical ruling applies or not to a given case, we look for a case we already know that falls under that ruling – the so-called source-case. Then we search for the property or set of properties upon which the application of the ruling to the source-case is grounded. If that grounding properties are known we ponder if they can also be asserted of the new case under consideration. In the case of affirmative answer it is inferred that the new case also falls under the specific juridical ruling at stake and so the range of its application is extended.

Complications arrive when the grounds behind a given juridical ruling are not explicitly known or even not known at all. In such a case other devices are put into action. The latter, as discussed in the next sections, yields a system of different types of *qiyās* that are hierarchically organized in relation to the epistemic strength achieved by their inferential procedures.

With regard to the kind of dynamics created by the practice of the *qiyās* it is fair to say that it constitutes a system of juridical reasoning that is in the middle of two other more radical (and not infrequently contested) forms of rational juridical change called respectively, *the doctrine of rational juridical preference* or *istiḥsān* (استحسان), that might produce the withdrawal of a conclusion achieved by a *qiyās*, and *the theory of public welfare* or *maṣlaḥa* (مصلحة), that can trigger the production of a new juridical ruling. We will not deal with these forms *ijtihād* in

³ Cf. *Ibn Taymiyya against the Greek Logicians*, edited and translated by Hallaq (1993).

⁴ Cf. Young (2016, chapter 4.3). The term has quite often a broader meaning encompassing legal reasoning in general. However, Young's translation, renders a narrower sense that stems from al-Shīrāzī's approach.

the present study but, as the title suggest, we will propose a framework that allows making explicit some of the formal traits of the construction of meaning that result from the dialectical practice of co-relational inferences.

More precisely, the main claim of our paper is that the dialectical understanding of the *qiyās* also displays the logical and epistemic features of this form of inference if casted in a formal system for rational interaction able to express content-based reasoning.⁵ Furthermore, according to our view, the dialogical conception of Per Martin-Löf's *Constructive Type Theory* provides both a natural understanding and a fine-grained instrument for the analysis of:

- 1) The dialectical processes by the means of which the conclusion is inferred by relating it to a reason or occasional factor that allows to infer the legality of a case already acknowledged by the sources.
- 2) The type-bounded notion of “case” within *fiqh*. The ultimate purpose of *fiqh* is to achieve a decision in relation to the determination of the range of abstract instantiations or exemplifications of a given a type of juridical ruling, not to decide about the application of a ruling to the action of some individual.
- 3) The meaning-structure of the general rule of jurisprudence on the basis of which the legal meaning of the root-case is grounded. This structure shows that the meaning of the ruling results from its dependence upon-the occasional factor.
- 4) The move that yields an epistemic assumption as some strengthened form of *ra'y* (رأى) or *pondered perspective*.⁶
- 5) The dynamics underlying the meaning-explanation of the notion of juridical ruling.
- 6) The hypothetical form of those co-relational inferences that are drawn in absence of knowledge about the grounds for a juridical ruling.
- 7) The notion of epistemic priority that introduces degrees of evidence and that structures the typology of the *qiyās*.
- 8) The dynamic deployment of *ijtihad* by means of a *jadal*-process in order to achieve a rational decision concerning a new case not yet established by the sources acknowledged by *uṣūl al-fiqh*.

In other words, we claim that a dialectical framework provides the right instrument to stress three of the hallmarks of this form of inference: (a) the interaction of heuristic with logical steps, (b) the dynamics underlying the meaning-explanation of the terms involved, (3) the unfolding of parallel reasoning as similarity in action. They display what we take to be the main epistemological idea behind the *qiyās*, namely: the open texture of the extension (or range of application) of normative statements. To put it more generally:

- The notion of co-relational inference suggests that every form of parallel reasoning that shares the formal structure of the *qiyās* presupposes that the extension of the predicates involved is open to contextual changes⁷. This strongly suggests that the whole process deployed is intrinsically dialectic.

⁵ In fact there is ongoing work on deploying the dialogical setting in order to reconstruct logical traditions in ancient philosophy (see Castelnérac/Marion (2009), Marion/Rückert (2015) and medieval logical theories (C. Dutilh Novaes (2007), Popek (2012)).

⁶ Young (2016, chapter 2.2) translated *ra'y* as *considered opinion*. Young's translation has the advantage that it links this notion with Aristotle's *endoxa* (ἐνδοξα). In our translation, we made the choice to link *ra'y* with it's linguistic root, namely the verb *seeing*.

⁷ In fact, sometimes parallel reasoning might take other forms, the objective of which is to compare structural properties rather than to complete the extension of a predicates: take the example of linguistic comparative methods as applied to the syntax or phonology of different languages, where the target of parallel reasoning is

Our study and reconstruction is focused on Abū Ishāq al-Shīrāzī's (1986, Kuwait ed.) classification of *qiyās* as discussed in his *Mulakhkhaṣṣī'l-Jadal (Epitome on Dialectical Disputation)*.⁸

The leading idea of our reconstruction is that al-Shīrāzī's classification seems to identify three main forms of co-relational inferences, namely:

- Exemplification,
- Symmetry, and
- Resemblance.

Let us point out that, though our reconstruction is grounded on the textual sources as thoroughly worked out by scholars such as Hallaq (1987a,b, 1997, 2004, 2009a,b) and Young (2017). In fact all of our textual references stem from Young's text.

It is important to consider that we are not claiming (yet) that the formalization we develop here is a literal description of the *jadal*-disputation-form in which the *qiyās* is carried out.

Our systematic reconstruction provides, so we claim, a *dialectical meaning-explanation* of the notion of co-relational inference relevant for the development of Al-Shīrāzī's (1986, Kuwait ed.) classification of *qiyās*

However,

- we think that our work can be further developed into a system for actual juridical disputation that provides a full reconstruction of *jadal* (جدل) as deployed in *uṣūl al-fiqh*.⁹ There is some work in progress on this part of the project. Moreover,
- while developing the rules and example for the underlying dialogical framework we engage in a systematic discussion on crucial moves of the classical *jadal*, such as *mu'ārada* (counter indication), *naqḍ* (inconsistency), *kasr* (breaking apart), *fasād al-waḍ'* (invalidity of the occasioned status) and *'adam al-ta'thīr* (lack of efficiency).

One of the main epistemological results emerging from this initial study is that the different forms of *qiyās* as developed in the context of *fiqh* represent an innovative approach that does not only provide new epistemological insights of legal reasoning in general but they also furnish a fine-grained pattern for *parallel reasoning*¹⁰ that can be deployed in a wide range of problem-solving contexts where degrees of evidence and inferences by drawing parallelisms are relevant.

not to determine the meaning of an expression but rather to determine its phonology or syntactic function. It very much looks as if the use of comparative reasoning in linguistics constitutes a field of its own. We owe this remark to Promise Dodzi Kpoglu (UMR 8163: STL, Université de Lille3).

⁸ Actually, Al-Shīrāzī, who was follower of the *Shāfi'ī* school of jurisprudence, endorsed the mistrust of the *Shāfi'ī* in relation to what they considered *subjective features* of *istiḥsān* and *maṣlaḥa*. Indeed, though Al-Shīrāzī accepted that the extension of the scope of a juridical ruling is necessary, he was convinced that extensions should result from a rational process such as the one deployed by a *qiyās*.

⁹ Worth mentioning is also the fact that, to the best of our knowledge there is no systematic study yet comparing the theory of juridical argumentation as developed within the Islamic tradition with the dialectical form of medieval disputations known as *Obligationes*. Such a study that will fill up some flagrant gaps in the history of the development of rational argumentation is certainly due.

¹⁰ We borrowed the term *parallel reasoning* from Bartha (2010).

A dialectical genealogy of Abū Ishāq al-Shīrāzī's system of *qiyās*

Michel Crubellier stresses in several writings that the epistemic point of the syllogism is a heuristic one.¹¹ According to this thoroughly documented insight the main epistemic objective of a syllogism is to find a pair of suitable premises that allow to link subject and predicate of the conclusion by the means of a third term involved in the premises with the help of which the conclusion can be inferred. In other words, the epistemic aim and gain of an inference in the context of syllogism is to find some epistemically suitable and fruitful way to relate the terms of the conclusion by inferential means. Moreover, Crubellier suggests that the heuristic move of finding the premises that provided the searched link is rooted in the dialectical work of Aristotle.

If we keep in mind this background the conceptual innovation of the co-relational inference-processes deployed by the *qiyās* will become apparent. Indeed, the classical studies on juridical argumentation or *jadāl* (جدل) by Abū al-Ḥusayn al-Baṣrī (...436H/1044 CE) in his *Kitāb al-Qiyās al-Sharʿī* (*Book of Correlational Inference Consonant to God's Law*, edited 1964-65) and by Abū Ishāq al-Shīrāzī (393-476 H/1003-1083 CE) in his *Mulakhkhaṣ fī al-Jadal* (*Epitome on Dialectical Disputation*, 688 H/1289 CE), recorded, commented and worked out by Young (2016, chapter 4.3) yield the following description of the *qiyās*:

- The aim of a *qiyās*, in its more general form, is to provide a rational ground to the ascription of some *juridical ruling* or *ḥukm* (حكم) such as (forbidden, allowed, obligatory) to a given case not yet considered by the sources acknowledged by *uṣūl al-fiqh* (for short, *juridical sources*).

In fact the thesis of a *qiyās* expresses the claim that a specific *ḥukm* applies to some case and the point is to ground this claim by relating it to the ruling of an already juristically acknowledged instantiation of such a ruling.

Accordingly, the grounding is carried out in two main steps (involving subsidiary ones).

1. It starts by bringing forward a case, known as *al-aṣl* or *the root-case* (الأصل), of which the juridical sources have already established that it falls under the scope of the same specific juridical ruling as the one claimed to apply to the new case, called *al-farʿ* (الفرع), *the branch-case*.¹² What qualifies something as being a case subject of juridical inquiry is that a parallel case in the sources can be indicated. Thus a case is a branch-case iff there is a root-case purported to be parallel to the former.

- 2.1 It proceeds by positing the assumption that the property established by a specific juridical source as *ground* or *occasioning factor*, called *al-ʿilla* (العلة) that founds the

¹¹Crubellier (2008, 2014).

¹²The Arabic terminology makes use of the botanic metaphor of, respectively, *root* and *branch* in order to express the relation between the case established by the juridical sources, *al-aṣl*, and the case under consideration, *al-farʿ*. The idea is not that the *farʿ* is a subcase of the *aṣl*, but that the ruling claimed to apply to the *farʿ* is rooted on the one of the *aṣl*. Hence root and branching seem to apply to assertions of the form $\text{ḥukm}_i(a)$ and $\text{ḥukm}_i(f)$, where ḥukm_i expresses some specific juridical ruling i that applies to both the root-case a and the branch-case f .

ruling of the root-case,¹³ also applies to the branch-case. The proceeding, assumes that the precise occasioning factor brought forward is to be found in the juridical sources.¹⁴

Qiyās is the linking of a branch-case with a source-case by way of an occasioning factor, and the application of the ruling of the source-case to the branch-case. Al-Shīrāzī, *Mulakhkhaṣ*, Niyāzī MA ed., vol. 1b, p. 22, quoted by Young (2016, 4.3).

والقياس حمل فرع على أصل بعلة واجراء حكم الأصل على الفرع

Al-Shīrāzī calls this form of *qiyās*, *qiyās al-'illa* (*co-relational inference of the occasioning factor*) and distinguishes three cases classified by the strength on the evidence for the 'illa: either (1): 1.1 the evidence for the determination of the 'illa stems from the juridical authority (i.e., sources and consensus of the experts), or 1.2 from a generalization of the description by the experts of the occasional factor (2): 2.1 it stems from some hermeneutic process (*zāhir*) or 2.2 from some acknowledged report (3) the 'illa is specified by positing some suitable hypothesis.¹⁵ Sometimes the process, is achieved by elimination, that is, the scholar counts several properties in the root-case that might count as the occasional factor and then eliminates those that seem unsuitable. The latter has some relation to Aristotle's *argument from example* (*paradeigma*) described in the *Rhetoric* (1402b15) and the *Prior Analytics* (*Pr. An.* 69a1).¹⁶ However, as mentioned above and more discussed below there is a difference concerning the notion of "case" in use.

¹³The term 'illa is also translated into English as *effective cause*, *operative cause*, *ratio legis* and *ratio decidendi*. Some of these translations do not seem to bear the causal significance of the term. The term 'illa is derived from ancient Syriac, where it means a "fault" or "blame" constituting the cause for returning articles or property. The term penetrated from Syriac into the lexicon of rational thought even before Aristotelianism penetrated Arabic culture (we owe the remark on the etymology of term 'illa to David Joseph (2010, 2014).

In a general context a distinction is drawn between providing a *ground* ('illa) and providing a *factual cause* or *reason* (*sabab*): while grounding is a rational endeavor, providing a *sabab* might be bounded to an empirical task. It seems to be related to the St. Thomas' (*Summa Theologiae* I.2.2c:) distinction between *propter quid* and *quia* that stems from Aristotle's distinction in *Posterior Analytics* I.13) (for a discussion in the context of CTT see Granström (2011), p. 157). In the context of the *qiyās* the notion of *sabab* seems to allude to the justification underlying the choice of one specific occasional factor. This use is witnessed by al-Shīrāzī denomination of the second subtype of *qiyā sal-'illa* as *qiyās plainly evident by reported reason* (*Wāḍiḥbi'l-Sabab*). That is, those *qiyās* where the 'illa is not being found in the sources but specified because of the report of some juridical acknowledged authority. In fact we should also mention the notion *ḥikma* that stands for the underlying higher purpose of the 'illa. Hence the *ḥikma* is a crucial part of the meaning of the 'illa and bestows the link between the occasional factor and the ruling. Moreover, the notion of *ḥikma*, underlies the *doctrine of rational juridical preference* or *istiḥsān*, and the *theory of public welfare* or *maṣlaḥa* mentioned afore. However, this notion does not seem to play a role in the inferential processes deployed by the use of a *qiyās*.

¹⁴See al-Shīrāzī, *Mulakhkhaṣ*, Niyāzī MA ed., vol. 1b, p. 22, quoted by Young (2016, 4.3): *Qiyās is the linking of a branch-case with a source-case by way of an occasioning factor, and the application of the ruling of the source-case to the branch-case.*

والقياس حمل فرع على أصل بعلة واجراء حكم الأصل على الفرع

¹⁵See al-Shīrāzī, *Mulakhkhaṣ*, Niyāzī MA ed., vol. 1b, p. 76, quoted by Young (2016, 4.3). Al-Baṣrī, distinguishes a positive inferential process (*Qiyās al-Tard*, *correlational inference of co-presence*), covered by the description above – from a negative one (*Qiyās al-'Aks*, *correlational inference of the opposite*). The result of the negative one is to deny that some designated juridical ruling that applies to the root case also applies to the branch-case, on the grounds that the occasioning factor does not apply to the branch-case – see al-Baṣrī, *Mu'tamad*, Ḥamīd Allāh ed., vol. 2, pp. 697-699.; and *K. al-Qiyās al-Shar'ī*, pp. 1031-3 (trans. of the latter in Hallaq, "Treatise," pp. 207-9), quoted by Young (2016, 4.3).

¹⁶The references on Aristotle (are taken from Barnes (1984).

2.2 It proceeds by finding some way to relate the branch-case to the branch-case *in absence of knowledge of the occasioning factor* by developing a parallel reasoning based on some kind of similarity: similarity in relation to

- 2.2.1 because both the root-case and the branch-case share some other juridical ruling, or
- 2.2.2 because, in absence of the similarities between the root-case and the branch case, it can nevertheless be established that there is some parallelism between a pair of source-cases and a pair of branch-cases such that the if some specific juridical ruling applies to the pair of source-cases it also applies to the pair of branch-cases.
- 2.2.3 Because both the root-case and the branch-case share some properties.

Al-Shīrāzī calls these three forms of *qiyās*, *qiyās al-dalāla*, *correlational inference of indication* (قياس الدلالة) also known as *qiyās al-shabah*, *correlational inference of resemblance* (قياس الشبه) – though it might be perhaps useful to restrict the denomination *qiyās al-shabah* for the last form of *qiyās al-dalāla*.¹⁷ The *qiyās al-shabah* based on the resemblance of the branch-case to the root-case in relation to a set of properties is considered to be epistemically speaking the less strong and is very close to what is known in other traditions as analogical argumentation by similarity or agreement. By contrast the *qiyās* based on the resemblance of the branch-case to the root-case in relation to a set of juridical rulings is considered to be epistemically the strongest form of inference of the type *al-shabah*. The form of inference-form of *qiyās al-shabah* based on double parallelisms constitutes a generalization and a deeply innovative approach to what is known as *proportionality-based analogical reasoning*.¹⁸ In relation to its epistemic strength it is placed between the former two.

- The result of carrying out both steps, if successful, is extending the scope of the juridical ruling to new case. In fact, one way to put it is that the extension of the juridical ruling under consideration is constituted (or so-to-say *updated*) during a dialectical process. For short the extension of juridical rulings is dynamic in its very nature.

Remarks:

- One way to express the rationale behind Al-Shīrāzī's typology (not shared by all of the other authors) is that he is viewing *qiyās* as a system of parallel reasoning that deploys arguments by
 - a) exemplification (of a general law): *qiyās al-'illa*
 - b) symmetry between structures (established by either chains of rulings or pairs of parallel rulings) (the two first forms of *qiyās al-dalāla*).
 - c) resemblance between the root-case and the branch-case (*qiyās al-shabah*)

¹⁷See al-Shīrāzī, *Mulakhkhaṣ*, Niyāzī MA ed., vol. 1b, p. 80, quoted by Young (2017, 4.3).

¹⁸Cf. Cellucci (2013, pp. 340-41). Moreover it looks seem to be very close to Bartha's (2010) own model.

- Al-Shīrāzī's text seems to differentiate between three types rather than two. This fits nicely with the three main forms of parallel reasoning just described. Certainly, there are good arguments, as those brought forward by Young(2016, chapter 4.3) to describe two rather than three.
- The ultimate purpose of *fiqh* is to achieve a decision in relation to the determination of the range of abstract instantiations or exemplifications of a given a type of juridical ruling, not to decide about the application of a ruling to the action of some individual. This particularly applies to the *qiyās* and that makes it radically different from the precedent-cases- argument so prominent in American Law for example. Thus, from the perspective of the *qiyās* the consideration of a case does not involve, as in the reasoning by precedent-cases of American law, some individual action, like *Donald reads the e-mails of someone else*, but it involves the more general question if the case of *reading the emails-of someone else is or not an instance the type of cases that violate the privacy of a citizen*: it is about tokens of types rather than about individuals.

Islamic jurisprudence is foremost about the reasons underlying the meaning and logical bindings of juridical rulings not about the application of those rulings to individual cases: it is the science of Law after all.

Hence, the ruling *everything that violates privacy should be forbidden*, does in principle have as instance not individuals but instantiations of violations of privacy such as *peeking at the e-mails, entering to a house without permission, inspecting the cloths of an individual*, and so on.

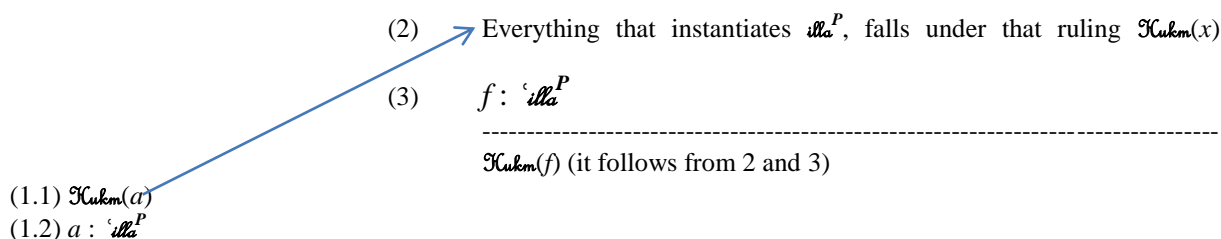
- As mentioned above, the *qiyās* constitutes a system of juridical reasoning that is in the middle of two other more radical (and sometimes contested) forms of rational juridical change deployed in *fiqh* called respectively, the doctrine of rational *juridical preference* or *istiḥsān* (استحسان) and the theory of *public welfare* or *maṣlaḥa* (مصلحة). Indeed, while the use of a *qiyās* might extend the scope of application of a specific juridical ruling by extending its meaning, it does not really refute neither the ruling nor that occasioning factor that the juridical source explicitly declare as ground for that ruling. However, on one hand the doctrine of *istiḥsān* might lead to change of a given ruling and of an occasional factor even if they have been explicitly established by the juridical sources. On the other, the theory of public welfare can be invoked to produce a new juridical ruling. The changes possible by the use of *qiyās* are, in some sense, of a more logical and semantic nature.

Before delving into this logical structure let us motivate the underlying dialectical processes with help of two informal diagrams. These diagrams present the most general forms of the *qiyās al-illa*, without (for the moment) drawing a distinction between subdivisions inside each type of co-relational inference. The graphical presentation follow the following notational conventions:

- $\mathcal{K}_{ukm}(a)$ expresses that the root-case falls under the scope of the juridical ruling $\mathcal{K}_{ukm}(x)$. Similarly, $\mathcal{K}_{ukm}(f)$ expresses that the branch-case f falls under the scope of the juridical ruling.
- $illa_j$ expresses a specific property j known as the occasioning factor for H_i .
- $a : illa^P$ expresses that the root-case instantiates the *occasioning factor* P . Similar applies to the branch-case: $f : illa^P$

*Qiyās al-‘illa*¹⁹

(schema 1)

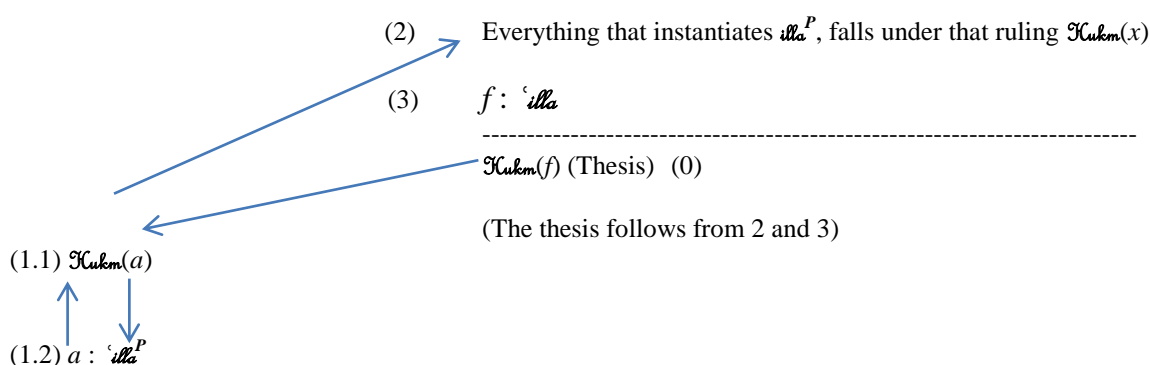


It is crucial to see that the method deployed by the *al-‘illa*-form of co-relational inference is not to establish a resemblance between the branch and the source-case. The point is to find a general law and a property, shared by both the branch- and the source case, which allows inferring the ruling we are looking to ground. It is not really a case of analogy by resemblance, but a kind of what is nowadays called *deductive parallel reasoning*, since it combines some kind of *symmetric* reasoning with inferential moves. Notice that neither 1.1. nor 1.2 are premises for the last inferential step. Indeed, steps 1.1 and 1.2 have the heuristic role of leading to the required general rule.

Now though this graphic does suggest that there is a move by the means of which the general rule (3) is extracted from *only* one example (namely, the example constituted by 1.1 and 1.2, it shows neither that this process is preceded by an heuristic move that finds that example (1.1, 1.2) as the relational term for the branch-case, nor the move by the means of which the branch-case is taken to have the property identified as the occasioning factor. Some improvement offers the following diagram.

Qiyās al-‘illa

(schema 2)



Now notice that this pattern shows that the reasoning-pattern is a combination of asserted *propositions* plus *actions* – the latter are represented by arrows. It is the combination of

¹⁹ The graphic has been adapted from Bartha’s (2010, p. 36) figure for Aristotle’s reasoning by *paradeigma*. Further on in the text we will come back briefly to the comparison between the Aristotelian and the *qiyās*-based patterns. However recall our remark on the general notion of case. Aristotle starts the process with an individual case, but the process of a co-relational inference starts with some particular instance of a type.

assertions and actions (arrows) that grounds Young's dialectical interpretation of the *qiyās*, according to which this combination is the result of the interaction of questions and answers. In fact this second graphic does not show all of those interactions. For example, the graphic does not express the fact that (2) is the result of both accepting that the occasioning factor applies to the branch-case and making use of this acceptance in order to ground the searched ruling.

These considerations strongly suggest that we should move away from mono-logical patterns and take seriously the dialectical interaction that structures co-relational reasoning. This is the systematic advantage of an underlying *jadāl* structure.

The dialectical structure of the two other subtypes of *qiyās al-‘illa* require some additional moves. Particularly so the last one, where there is no knowledge stemming from the sources of an explicit occasioning factor but some hypothetical one. We will deal with these cases in the next sections.

Now, before delving into the dialectical structure let us motivate the use of a notation inspired by Constructive Type Theory. In fact, we only deploy very basic features of the CTT-framework. A deep and thorough development is due.

II. Motivating the deployment of a CTT-framework

The expressive power of Per Martin Löf's Constructive Type Theory²⁰ allows expressing at the object language level the following features of both, the theory of meaning and the logical structure underlying the *qiyās*:

- The meaning-explanation of juridical rulings and the type-theoretical conception of *juridical study-case*
- The intensional rather than extensional understanding of sets as types. Since the drawing of an co-relational inference presupposes that the extension is of the type of juridical rulings is not closed, a non extensional framework for the meaning constitution of those types is required.
- Hypothetical reasoning within *qiyās* and degrees of epistemic strength
- The restrictive form of the substitution rules

Let us discuss each point separately the first three. The fourth point relates to the co-relational inferences by indication that will not be discussed in the present paper.

II.1 The meaning explanation of juridical rulings and its type-theoretical conception

Per Martin-Löf's CTT-framework allows distinguishing between two main meanings underlying predicative "is"-expressions in natural language, namely

²⁰ For a systematic presentation of CTT see Martin-Löf ((1971, 1975a,b, 1982, 1984, 1992, 1996, 2006, 2011), Nordström/ Petersson/Smith (1990, 2000), Thompson (1991), Ranta (1994), Granström (2011)). For philosophical and historic insights into CTT see Ranta (1988), Primiero (2008), Sundholm (1997, 1998, 2001, 2006, 2009, 2012, 2013, 2016). For the use of CTT-framework in order to study arabic logic see Rahman/Granström/Salloum (2014).

recognizing that something exemplifies a type (let us take the case of the type *set*).

predicating over a domain (e.g. of the type *set*) in such a way that the predicate is defined over that domain (that is, the predicate is said to hold or not of elements of the domain),

Indeed

Take the example

a is B

We can read it as establishing that

a exemplifies (instantiates) some type B

(formally $a : B$, or, $a \in B$), that can also be read as *a is an element of B*

or

as establishing that

the predicate $B(x)$ can be asserted from a, such that a is an element of set, say D, that provides the universe of discourse.

In other words predicates are defined in such way that they yield a proposition (are of the type *prop*) if the elements that substitute the variable x in the predicate are chosen from a given domain or *set*, upon which its meaning depends. This we express in the following way:

$B(x) : \text{prop } (x : D)$,

that reads: the predicate $B(x)$ renders a proposition if x is an element of D (or if x exemplifies D)

So the most fundamental element for the formation of a predicate, that what provides its basic meaning is its *dependence upon* another type:

In other words, while the first form of predication involves the exemplification of an independent type the second involves a *type-dependency*. A striking example of the use of this difference is the analysis of Aristotle's example,

Some shoemakers are good

It would be preposterous to analyze the sentence with standard first-order means and render the formulation

For some elements of the universe of discourse it holds that they are good and that they are shoemakers:

Some x ($\text{Good}(x) \ \& \ \text{Shoemaker}(x)$)

This makes no sense. *Good* is meant here as the predicate $\text{Good}(x)$ defined over (dependent upon) the domain of *Shoemakers*: there can be bad persons that are good shoemakers after all.

The meaning expressed by the sentence is that we can find at least one exemplar of the universe of shoemakers of which it can be predicated that they are good (shoemakers):

(Some x : *Shoemakers*) *Good*(x)

To put it in traditional terms,

the predicate Good is said from the subject Shoemakers.

Now, according to CTT the number of types is infinite, the introduction of them requires some specific conditions, but the point is that the elements exemplifying a type can be certainly interpreted as quite abstract instances. Let us come back to the subject of our study.

Hallaq²¹ (1985, pp. 88-91, 1987b, pp-50-58) points out three conditions that a property must satisfy in order to be identified as the occasioning factor:

1. Efficiency (*ta'thīr*).
2. Co-extensiveness (*tard*) – the presence of the occasioning factor when the judgment is present.
3. Co-exclusiveness (*'aks*) – the absence of the occasioning factor when the judgment is absent.

Indeed, as we discussed in the next sections, arguments for endorsing or rejecting some proposed property as constituting the occasioning factor are based on either showing that the property is present (*wujūd*) when the ruling at stake is, and that when the ruling is absent (*raf'*) the property is absent. It is quite often the case that an argument for endorsing a property as constitutive of the occasioning factor ends with the formulation: *Therefore, the presence of the hukm is due to the presence of the 'illa, and the absence of the hukm is due to its absence.*

II.1.2 On *ta'thīr*

Given this background we can understand the branch-case of the first example deployed in the next main section the present paper as an element of a set, that is, as instantiating some type. The type, we are targeting is the set identified by means of a property as its occasioning factor, namely the property of *violating privacy*. In other words we propose to take the branch-case

reading the e-mails of someone else

to instantiate a certain subset of the set of occasioning factors. Namely the subset determined by all those cases of occasioning factors that have the property of *Violating privacy*. This subset can be exemplified by instances such as *reading the e-mails of someone else, inspecting the e-mails of someone else*, and so on.

²¹ See too Young (2017, chapter 4).

Over the set *Privacy-Violation* we can then define the specific juridical ruling relevant to this occasioning factor, let us call it the predicate

$$\mathcal{K}_{ukm}(x)$$

that expresses a juridical interdiction to any instance of the type *Privacy-Violation*. So the meaning of the juridical ruling $\mathcal{K}_{ukm}(x)$ is here specific and made dependent upon the set of instances of *violating privacy*. In other words the ruling $\mathcal{K}_{ukm}(x)$ is according to our analysis a predicate that applies over instances of the set *Privacy-Violation*, and the latter is a subset of the set of occasioning factors:

- $\mathcal{K}_{ukm}(x) : \text{prop } (x : \text{Privacy-Violation}),$

where *Privacy-Violation* is a property that selects a subset of the set of occasioning factors. That is the predicate that expresses the property *Privacy-Violation*, defines the set

- $\{x : \mathcal{U} \mid \text{Privacy-Violation } (x)\}$

This set underlies the meaning of the relevant ruling $\mathcal{K}_{ukm}(x)$: *the meaning of this ruling is dependent upon the set of occasioning factors*:

- $\mathcal{K}_{ukm}(x) (\{x : \mathcal{U} \mid \text{Privacy-Violation } (x)\})$

In other words, the specific ruling *prohibition against unauthorized access to the information kept by someone else* has been constituted has been occasioned by the property *Privacy-Violation*.

By these means two types result: (1) a specific *type* of ruling, namely the *ruling specific to Privacy-Violation* (2) the type of occasioning factor *Privacy-Violation*.

The efficiency stems from the fact that the second type depends upon the first. It should be clear that establishing this dependence is not a question of logic but a question of juridical and epistemological meaning presupposed by the application of the ruling.

Furthermore this is the meaning that underlies the juridical law.

- Instances of the type of *Privacy-Violation* are also instances of the type *Prohibition against unauthorized access to the information kept by someone else*

Such an affirmation requires for its verification some piece of evidence that renders the proposition true upon with the hypothetical depends. Likewise, a refutation requires a piece of evidence for the consequent of the hypothetical, given one for its antecedent. Such a requirement has been explicitly formulated by the Muslim jurists in the context of those assertions linking an occasional factor with a specific ruling. Indeed, the presence of a property purported to be the occasioning factor with the absence of the specific ruling attached to it does not constitute *per se* a refutation, unless there is *evidence* for absence of the ruling (Hallq, 1987b, p. 54).

Logically seen, this yields to formulate the above hypothetical as the following restricted universal quantification:

- *Prohibition against unauthorized access to the information kept someone else* applies to everything that *violates privacy*

We could thus deploy the juridical law as a hypothetical verified by a function. However for perspicuity we use the explicit universal-quantification form.

Notice that the universal law that allows the application of the law to a new case presupposes the meaning explanation of the semantic bounds linking occasioning factor and ruling. But from the former we cannot derive the universal. The former only displays the meaning of the juridical ruling, it does not determine its quantification (or to put it in the traditional terminology it does not determine the form of distribution of the subject). In fact the same meaning construction is presupposed by the expression *Something that violates privacy is forbidden*. Notice too that if we formulate this law as $(\forall x : \{x : 'illa \mid \text{Violates-privacy}(x)\})$ $\mathcal{K}_{ukm}(x)$ does not assure that the conditions *ṭard* and *'aks* are satisfied.

II.1.3 On *ṭard* and *'aks*

In the context of *jadal* and dialectical frameworks, there are moves aimed at refusing to accept that the selected property is the one occasioning the juridical ruling. On our view, they are challenges on the formation-rules underlying the universal law. Let us take the very discussed example of the *prohibition of consuming wine* – we will discuss the example in detail further on. Let us further assume that, the property selected as relevant was *being red*. The, refusal to accept *being a red drink* as determining the occasioning factor, is not only a refusal to endorse the universal generalization *Every red drink is to be forbidden*. The refusal lies deeper in the structure. It is about denying that the notion *prohibition of consuming wine* is constituted by *being a red drink*.²²

The latter considerations suggest that the universal binding the occasioning factor with the ruling should be more complex in such a way its structure makes apparent the binding force of the relevant occasioning factor. One possibility is the following.

Let us abbreviate the notation of

$$\begin{array}{ll} \{x : 'illa \mid \text{Toxic Drink}(x)\} \text{ true} & \text{as } \mathbf{T} \text{ and} \\ \{x : 'illa \mid \text{Toxic Drink}(x)\} \text{ false} & \text{as } \sim\mathbf{T} \end{array}$$

Let us further assume a set of substances \mathbf{D} and single out those substances y that are either toxic drinks or not: $(y : \mathbf{T} \vee \sim\mathbf{T})$ – so that while x is an arbitrary element of \mathbf{D} , y one of those substances of which it can be said that they are or not a toxic drink. A counter example is a piece of wood, it simply does not apply to those kind of substances.

²² We borrowed the example from Hallaq (1985, pp. 88-89).

Let us call, $left(y)$, some substance x from \mathbf{D} that is indeed toxic. In other words, from this substance it is not only the case that is one of the substances y of which it can be said that they are or not toxic drinks, but of this substance it can be said that it is indeed a toxic drink. Thus $x = left(y)$ (x verifies the *left* side of the disjunction). Similarly, let us call, $right(y)$, some drink x from \mathbf{D} that is a toxic substance. In other words $x = right(y)$ (x verifies the *right* side of the disjunction)

If we spell out the precise formulation of the occasioning factor, the point is that

tard: If it y is a *toxic drink* (i.e. if the drink $x = left(y)$) then its consumption is forbidden.

'aks: If y is not a toxic drink (i.e. if the drink $x = right(y)$) then its consumption is not forbidden.

This yield the general law:

Consumption is forbidden for any instance x of the set \mathbf{D} equal to an instance of the type *toxic drink*; and *Consumption is not forbidden* if the former is not the case (provided, the property of *being or not a toxic drink* applies to x).

Or expressed as universal

(For any substance x), if it is (equal to) a *toxic drink its consumption is forbidden* and if it is *not* (equal to) a *toxic drink its consumption is not forbidden*, (provided, the property of *being or not a toxic drink* applies to x).

$$(\forall x : \mathbf{D}) (x = left(y) \supset \text{Consumption-prohibition}(x)) \wedge (x = right(y) \supset \sim \text{Consumption-prohibition}(x)) (y : \mathbf{T} \vee \sim \mathbf{T})$$

II.1.4 Building counterexamples

An instantiation that verifies the interdiction on wine formulated in the preceeding section is the following

Given

$$d : \mathbf{T} \vee \sim \mathbf{T} \text{ (} d \text{ is a drink that is either toxic or not)}$$

$$wine : \mathbf{D}$$

$$wine = left(d) : \mathbf{T} \vee \sim \mathbf{T} \text{ (} wine \text{ is a substance that is (equal to) a toxic drink, that is the occasional factor recorded by the sources)}$$

We obtain

$$wine = left(d) \supset \text{Consumption-prohibition}(wine) \wedge (wine = right(y) \supset \sim \text{Consumption-prohibition}(x))$$

and from this and $wine : \mathbf{D}$, and $wine = left(d) : \mathbf{T} \vee \sim \mathbf{T}$, it follows

$$\text{Consumption-prohibition}(wine)$$

A conclusion known from the sources.

A counterexample might come from a substance that is recorded both to be toxic and not to be forbidden for consumption. However, in the practice of Islamic jurisprudence the way to

produce a counterexample to the general law that expresses the links between the occasional factor and the ruling:

$$(\forall x : \mathbf{D}) (x = \text{left}(y) \supset \mathcal{H}_{ukm}(x)) \wedge (x = \text{right}(y) \supset \sim \mathcal{H}_{ukm}(x)) (y : \mathbf{P} \vee \sim \mathbf{P})$$

was based on bringing forward a counterexample to either the claim

For everything where the property P is present, the ruling applies

$$(\forall x : \mathbf{P}) \mathcal{H}_{ukm}(x)$$

Or

For everything where the property P is absent, the ruling does not apply

$$(\forall x : \sim \mathbf{P}) \sim \mathcal{H}_{ukm}(x)$$

Or even the stronger claim (that assumed double negation)

For everything to which the ruling applies, the presence of property P is a fact

$$(\forall x : \mathcal{H}_{ukm}) \mathbf{P}(x)$$

We will follow this practice in our rules. Moreover, on our view those moves that challenge the occasioning factor, challenge in fact the formation of the predicated purported to specify this factor.

Let point out that the deployment of a co-relational inference assumes that the extension of the set of those objects to which the ruling applies is not closed. This means that the determination of the set D underlying the whole structure must be dynamic by nature. This is why on one side those sets should not be defined by its extensions (that is why we use a CTT-framework) and on the other, a dialectical method is needed that makes it possible to determine if a case falls or not under a given rule. Let us delve further in the structure just studied.

II.2 Hypotheticals and further remarks on *ta'thīr*, *ṭard* and *'aks*

What is a categorical judgement from the point an epistemological point of view? Well, it is a judgement by the means of which the proposition involved is asserted to be true. In other words it is a judgement backed by some evidence that renders true the asserted proposition. A hypothetical judgement in contrast is one that its truth is made dependent on the truth of other propositions or more generally when its truth is achieved by elements of given set. This is what we deployed when we formulated in the section above a predicate of juridical ruling dependent upon of a set.

So, let us take that we do not have evidence yet to categorically claim that the property P is the property that specifies the relevant factor. In such a case we rather make the *assumption* @ that P is such a property. If we apply it to our previous example we have

$$\{x : \mathcal{H}_{ukm} | \text{Privacy-Violation}(x)\} (@)$$

In general the assumption is not one but a sequence of them, even not closed

$$\{x : \text{Privacy-Violation}(x)\} (\mathcal{Q}_1, \dots, \mathcal{Q}_n)$$

and most importantly, the arguments must show the proposed property satisfies efficiency, co-extensiveness and co-exclusiveness.

The open-end feature of \mathcal{Q} makes that all type of co-relational inferences based on such kind of assumption is in principle open to further specification and contextualization.

In the context of the *qiyās* the sequence can be understood as a sequence of arguments that supports selecting the property, say, P , as relevant for the juridical ruling of the root-case at stake (see below our discussion of examples of dialogues for the development of a *qiyās al-illa al-khafī*).

At this point of the discussion we can already distinguish different degrees of epistemic strength. Let briefly discuss the issue that will naturally lead to the dialectical interpretation.

II.3 Some consideration on authority and epistemic strength

II.3.1 Epistemic assumptions within CTT and its Dialogical Interpretation

Per Martin-Löf, in recent lectures, deployed the dialogical perspective on epistemic assumptions to get out of a certain circle that threatens the explanation of the notions of inference and demonstration. A demonstration may be explained as a chain of (immediate) inferences starting from no premisses. That an inference

$$\frac{J_1 \dots J_n}{J}$$

is valid means that the conclusion J can be made evident on the assumption that J_1, \dots, J_n are known. The notion of epistemic assumption thus enters in the explanation of valid inference. We cannot, however, in this explanation understand 'known' in the sense of *demonstrated*, for then we are explaining the notion of inference in terms of demonstration, whereas demonstration has been explained in terms of inference. Martin-Löf suggests that we here understand 'known' in the sense of *asserted*, so that epistemic assumptions are judgements others have made, judgements for which others have taken the responsibility; that the inference is valid then means that, given that others have taken responsibility for the premisses, I can take responsibility for the conclusion:

The circularity problem is this: if you define a demonstration to be a chain of immediate inferences, then you are defining demonstration in terms of inference. Now we are considering an immediate inference and we are trying to give a proper explanation of that; but, if that begins by saying: Assume that J_1, \dots, J_n have been demonstrated – then you are clearly in trouble, because you are about to explain demonstration in terms of the notion of immediate inference, hence when you are giving an account of the notion of immediate inference, the notion of demonstration is not yet at your disposal. So, to say: Assume that J_1, \dots, J_n have already been demonstrated, makes you accusable of trying to explain things in a circle. The solution to this circularity problem, it seems to me now, comes naturally out of this dialogical analysis. [...]

The solution is that the premisses here should not be assumed to be known in the qualified sense, that is, to be demonstrated, but we should simply assume that they have been asserted, which is to say that others have taken responsibility for them, and then the question for me is whether I can take responsibility for the conclusion. So, the assumption is merely that they have been asserted, not that they have been demonstrated. That seems to me to be the appropriate definition of epistemic assumption in Sundholm's sense.²³

Indeed, one of the main features of the dialogical framework is the so-called *copy-cat Rule*, nowadays more aptly the *Socratic Rule*, by Marion / Rückert (2015), or copy-cat rule by the means of which:

the Proponent is entitled to use the Opponent's moves in order to develop the defence of his own thesis.²⁴

According to this perspective the Proponent takes the assertions of the Opponent as *epistemic assumptions* (to put it into Sundholm's happy terminology), and this means that the Proponent trusts them only because of its force, just because she claims that she has some grounds for them.²⁵

- In the context of the present paper the idea is that the deployment of different degrees of authoritative evidence for a claim as brought forward in an argumentation is the means that the Islamic jurists conceived in order to put into action the dialectical process for the foundations of *uṣūl al-fiqh* involved in the use of *qiyās* (see Young (2017, chapter 8).

More precisely in the context of a dialectical process underlying the *qiyās* the Socratic-rule needs to be refined and leveled:

1. If a player backs his claim with a reference to the sources, it has the maximal authoritative force.
2. If the Proponent backs his claim by appealing only to the Opponent's own concessions during the dialectical process, then it has a logical force. *Logical force* underlies the logical fragments of a *qiyās*- process.
3. If the Proponent backs his claim by the Opponent's endorsements during the dialectical process, because of some contentual or material circumstances, then it has a *contextual force*. Contextual force, admits also degrees: either the Proponent produces some *direct factual evidence* for a claim or *indirectly* by developing a chain of arguments for the selection of a particular occasioning factor. Direct factual evidence will produce a direct endorsement. Indirect evidence might trigger a new proposal of the Opponent
4. In the latter case, if the Opponent wishes to bring forward a *constructive criticism* he might contribute to the *specification* of an initial assumption

²³ Transcription by Ansten Klev of Martin-Löf's talk in May 2015.

²⁴ In fact, Martin-Löf's discussion is a further development of Sundholm's (2013, p. 17) – see too Sundholm (1997, 1998, 2012) proposal of linking some pragmatist tenets with inferentialism . According to this proposal those links emerge from the following insight of J. L Austin (1946, p. 171):

If I say "S is P" when I don't even believe it, I am lying: if I say it when I believe it but am not sure of it, I may be misleading but I am not exactly lying. When I say "I know" ,I give others my word: I give others my authority for saying that "S is P".

²⁵ Let us point out that one of the main philosophical assumptions of the constructivist school of Erlangen was precisely the tight interconnection between logic and ethics, see among others: Lorenzen (1969) and Lorenzen/Schwemmer (1975). In a recent paper, Dutilh Novaes (2015) undertakes a philosophical discussion of the normativity of logic from the dialogical point of view.

concerning the occasioning factor by bringing forward a new proposal. In such a case the more specific determination of the occasioning factor will have priority to a less specific one.

5. The deployment of concessions based on similarities and/or resemblances without any appeal to the occasioning factor, have less authoritative and epistemic force than all the previous ones. This form reliability involves the deployment of *qiyās al-dalāla* (not to be discussed in the present paper).
6. If the Opponent brings forward some form of *non-cooperative criticism*, showing that the property cannot be the one that determines the relevant occasioning factor, then the Proponent will not have any endorsement upon which he can back his claims. So he has either to find another property or give up the main thesis.

These degrees provide the structure of the development of a juridical disputation. The main technical aim of our paper is to implement such notions within a dialogical framework. Let us recall once more that Islamic jurist developed their theory of co-relational inference within a dialectical structure of *jadal*. Indeed, the following lines of Young (2017, chapter 1.1) sets the motivations for the development of a dialogical framework as the one we are aiming at in the present paper.

The primary title of this monograph is “The Dialectical Forge,” and its individual terms provide a suitable launching point for discussing the current project as a whole. As for the first, the most common Arabic terms for “dialectic” are jadal and munāẓara, both denoting formal disputation between scholars in a given domain, with regard to a specific thesis. When one encounters the term “dialectical” in the present work, one should think foremost of procedure-guided debate and the logic inherent to this species of discourse. A dialectical confrontation occurs between two scholars, in question and answer format, with the ultimate aims of either proving a thesis, or destroying it and supplanting it with another. A proponent-respondent introduces and attempts to defend a thesis; a questioner-objector seeks (destructively) to test and undermine that thesis, and (constructively) to supplant it with a counter-thesis. Through progressive rounds of question and response the questioner endeavours to gain concession to premises which invalidate the proponent’s thesis, justify its dismantling, and provide the logical basis from which a counter-thesis necessarily flows.

Ultimately, and most importantly, a truly dialectical exchange—though drawing energy from a sober spirit of competition—must nevertheless be guided by a cooperative ethic wherein truth is paramount and forever trumps the emotional motivations of disputants to “win” the debate. This truth-seeking code demands sincere avoidance of fallacies; it views with abhorrence contrariness and self-contradiction. This alone distinguishes dialectic from sophistical or eristic argument, and, in conjunction with its dialogical format, from persuasive argument and rhetoric. And to repeat: dialectic is formal—it is an ordered enterprise, with norms and rules, and with a mutually-committed aim of advancing knowledge.

III. A dialogical framework for *Co-Relational Inferences of the Occasioning Factor*

In order to develop our analysis of the dialectical structure of the *qiyās* we make use of the dialogical framework for logic. The dialogical approach to logic is not a specific logical system but rather a framework rooted on a rule-based approach to meaning in which different forms of inferences can be developed, combined and compared. More precisely, in a dialogue two parties argue about a thesis respecting certain fixed rules. The player that states the thesis is called Proponent (**P**), his rival, who contests the thesis is called Opponent (**O**). Dialogues are designed in such a way that each of the plays end after a finite number of moves with one

player winning, while the other loses. Actions or moves in a dialogue are often understood as speech-acts involving *declarative utterances or posits and interrogative utterances or requests*. The point is that the rules of the dialogue do not operate on expressions or sentences isolated from the act of uttering them. The rules are divided into rules for *local meaning* including the rules for the logical constants (*Partikelregeln*) and *structural rules* (*Rahmenregeln*) that set the *global meaning*. The structural rules determine the general course of a dialogue game, whereas the particle rules regulate those moves (or utterances) that are requests and those moves that are answers (to the requests).

In fact as explained below, in the context of the present paper we introduce a distinction between *challenges, requests, defences* (to the *challenges*), *answers* (to the *requests*), *posits* and *assertions*, in order to render some dialectical features specific of the *qiyās*. We focus here in the rules that extend those for standard classical logic (in the appendix we provide a short overview of standard dialogical logic with profuse literature on the subject).

Let us first describe informally the overall argumentative structure of a dialogue for *qiyās al-‘illa*.

The overall development of a dialogue of the form *qiyās al-‘illa*

1. A dialogical play starts with the Proponent asserting that some specific legal ruling applies to a certain branch-case.
2. After agreement on the finiteness of the argument to be development the Opponent will launch a challenge to the assertion by asking for justification.
3. The Proponent's strategy will try to develop an argument in such a way that if forces the Opponent to concede the challenged assertion.
4. In order to develop the argument of the previous step the Proponent will start by choosing a (by the best of his juridical knowledge) suitable root-case from the sources for which the ruling at stake has been applied. The move consists in the Proponent forcing the Opponent to acknowledge this fact.
5. Since the evidence comes from the sources the Opponent is forced to concede it – assuming the reference to the sources is correct. If not the play stops or the Proponent finds another root-case.
6. Once conceded the Proponent will start by choosing a (by the best of his juridical and epistemological knowledge) suitable property (that will later on characterize the relevant occasioning factor). The move consists in the Proponent forcing the Opponent to acknowledge that this property applies to the root-case
7. The Opponent might accept or reject it. If he rejects it the Proponent will search for a new property, until the Opponent concedes or until the Proponent gives up.
8. Once the Opponent conceded that the ruling applies to the selected ruling. The Proponent starts with the crucial task of extracting from the set of possible occasioning factors the one relevant for the root-case under discussion. More precisely, the Proponent will ask the Opponent to concede that the property just selected is the one that identifies the relevant occasioning factor.
9. The Opponent might accept it, ask for justification or *strongly* reject it.
10. If the Opponent ask for a justification the occasioning link between the property and the ruling, the Proponent either can bring some evidence from the sources that the selected property is the relevant occasioning factor, or (by switching to the development of a dialogue of the form *qiyās al-‘illa khaṭf*) he will develop an argument for it. The Opponent accepts or challenges the argument by proposing a new property for the constitution of the occasioning factor. This counterattack of the Opponent is a

mu'arāḍa move, profusely discussed in the jadal-literature. Young (2017, chapters 4 and 5) calls it *constructive criticism* – we come back to it further on in the text. It is opposed to the *destructive criticism* or *naqḍ* displayed in the following step..

11. If the Opponent rejects it *strongly*. It is him, the Opponent, who has to bring a counterexample from the sources. If he succeeds the Proponent must start with another property or give up the thesis of the dialogue. This corresponds to the move of jadal known as *naqḍ* or *destructive criticism* – to be discussed further on.
12. If the Opponent concedes that the property is indeed the occasioning factor for the ruling of the root-case, the Proponent will start by asking the Opponent to formulate the general juridical rule, exemplified by the root-case.
13. Once the universally quantified rule has been spelled out the Proponent will start by asking the Proponent to acknowledge that the property also applies to the branch-case. If the Opponent rejects this, the Proponent might bring some evidence in favour of the claim that the relevant property also applies to the branch-case.
14. After the Opponent conceded that the property does apply to the branch case, and since the Opponent also conceded that the property is the one that characterizes the relevant occasioning fact, the Proponent will ask the Opponent to acknowledge that the branch-case also exemplifies the general rule. This move forces the Opponent to concede the challenged thesis. In fact the dialogue will end (if successful) by the Proponent indicating that the Opponent finished by conceding the thesis under scrutiny.
15. If at stage 10 the Proponent does not find the source backing his claim that the selected property is the relevant occasioning factor, the Proponent will develop an argument for that claim – based on a shared epistemological background – and switch to the development of a dialogue of the form *qiyās al-'illa khafī*.
16. The Opponent can either accept the argument and then the dialogue will proceed as described by stages 11 to 14, or he might reject that the property is the occasioning factor for the ruling of the root-case.
17. If the Opponent rejects the argument he must develop a new argument by proposing a new property as the occasioning factor. The Proponent must accept it if the new property is either a specification of the property leading to a more precise formulation or to a suitable contextualization. Once the new property has been settled the dialogue proceeds to the steps 11 to 14. If the new property has not been settled then the dialogue stops or goes back to the first property proposed and once more that dialogue will follow the steps 11 to 14 and end.

We proceed not to a systemic presentation of the rules that prescribe the development of such a form of dialogues.

III.1 Local Rules for the *Qiyās al-'illa*

The rules below make use of the following specific terminological conventions based on a simplified form of Constructive Type Theory.²⁶

Terminology

Abbreviated Expression	Dialogical	Stands for	Type-theoretical expression
\mathcal{S}		Set of cases recorded by <i>Nass</i> (= <i>Quran</i> + <i>Sunna Authority</i>) or some <i>linguistic variation of Nass</i> . The set is called $\mathcal{S}_{sources}$. If necessary we distinguish between	$\mathcal{S}_{sources} : set$ $\mathcal{S}_{sources}^Q : set$

²⁶ For a full-presentatio of CTT in the terms of the dialogical framework for logic see Clerbout/Rahman (2015).

	$\mathcal{S}_{sources}^Q$, $\mathcal{S}_{sources}^S$, $\mathcal{S}_{sources}^{LN}$ in order to identify the precise source: either Quran or Sunna or a linguistic/logical variation of some text in <i>Nass</i>	$\mathcal{S}_{sources}^S : set$ $\mathcal{S}_{sources}^{LN} : set$
$'illa$	Set of <i>'illa</i>	$'illa : set$
<i>far</i> <i>aṣl</i>	branch case of which a specific <i>ḥukm</i> is claimed to apply, by relating it to a precisely identified root-case <i>aṣl</i> of which the relevant <i>ḥukm</i> has been sanctioned	the description of the precise form of the <i>far</i> is the objective of the <i>qiyās</i>
$\mathcal{H}_{ukm}(x)$	propositional function <i>ḥukm</i>	$\mathcal{H}_{ukm}(x) : prop(x : 'illa^P)$ The ruling applies to the element x such that x is one instance of the occasioning factor specified by the property P .
$\mathcal{H}_{ukm}_{\mathcal{S}}(x)$	Propositional function over \mathcal{S} identifying those cases from the sources of which a <i>ḥukm</i> has been sanctioned	$\mathcal{H}_{ukm}(x) : prop(x : \mathcal{S}_{sources})$ The ruling applies to the element x such that x is one of the cases included in the set of cases recorded by the sources.
$P_{\mathcal{S}}(x)$	Set of those cases recorded by the sources as having the property P	$\{x : \mathcal{S}_{sources} \mid P(x)\} : set$ Subset defined over the set of cases recorded by the sources. The subset is separated by means of the property P
$'illa^P$	Set of those cases from the set $'illa$ that instantiate property P . In other words, the subset results by selecting of the set of all possible occasioning factors, those having property P .	$\{x : 'illa \mid P(x)\} : set$
$'illa^{P_{\mathcal{S}}}$	Set of those cases from the set $'illa$ recorded by the sources as having property P . In other words, the subset results by selecting of the set of all possible occasioning factors, those cases the sources record as having property P .	$\{x : 'illa \mid P_{\mathcal{S}}(x)\} : set$
$\mathcal{H}_{ukm}(x) (x : 'illa^P)$	Identification of the (set) $'illa$ specific to the set of <i>ḥukm</i> under discussion: the propositional function <i>ḥukm</i> is defined over the set of those cases that have property P .	$f(x) : \mathcal{H}_{ukm}(x) (x : 'illa^P)$ Remark: Function $f(x)$, explicitly displays the dependence of the legal ruling upon the occasioning factor. Thus, in order for a player to assert that a certain ruling applies to a case, he has to deploy a method that for any instance of the occasioning factor produces some evidence that the ruling applies to that instance. For the sake of notational simplicity we will not make explicit this function during the development of a play. However, the dialogical local rule for the universal quantifier implements the use of a such a function.
$\mathcal{H}_{ukm}(x) (x : 'illa^{P_{\mathcal{S}}})$	Identification of the (set) $'illa$ specific to the set of <i>ḥukm</i> under discussion: the	$f(x) : \mathcal{H}_{ukm}(x) (x : 'illa^{P_{\mathcal{S}}})$

	propositional function <i>hukm</i> is defined over the set of those cases recorded by the sources as having property <i>P</i>	
SF $(\forall x : 'illa^{P(\mathcal{Q})}) \mathfrak{I}(x)$ or SF $(\forall x : 'illa^{P(\mathfrak{S})}) \mathfrak{I}(x)$	Simplified form of the juridical law $(\forall x : D) (x = left(y) \supset \mathfrak{I}(x)) \wedge (x = right(y) \supset \sim \mathfrak{I}(x)) (y : 'illa^{P(\mathcal{Q})} \vee \sim 'illa^{P(\mathcal{Q})})$ (or with the subscript $'illa^{P(\mathfrak{S})}$) (For any case <i>x</i>), if it is (equal to) a case of the type $'illa^{P(\mathcal{Q})}$, the juridical ruling \mathfrak{I} applies; and, if is not of the type $'illa$ then the ruling does not apply, (provided <i>P</i> applies or not to <i>D</i>).	$(\forall x : D) (x = left(y) \supset \mathfrak{I}(x)) \wedge (x = right(y) \supset \sim \mathfrak{I}(x)) (y : 'illa^P \vee \sim 'illa^{P(\mathcal{Q})})$ true Provided $\{x : 'illa P(x)\} (\mathcal{Q}_1, \dots, \mathcal{Q}_n)$ true or $\{x : 'illa \sim P(x)\} (\mathcal{Q}_1, \dots, \mathcal{Q}_n)$ true
X ! ϕ	Player X asserts ϕ	ϕ true
X ! $far : 'illa^P$ X ! $a\mathfrak{I}l : 'illa^P$	Player X asserts that the branch-case has the property specific to the relevant <i>'illa</i> Player X asserts that the root-case has the property specific to the relevant <i>'illa</i>	$'illa^P$ true ²⁷
X ! $e : P(far)$	Player X produces some evidence <i>e</i> in support of <i>P(far)</i>	<i>P(far)</i> true
P ! $\phi[a\mathfrak{I}l]$ (or $\phi[a\mathfrak{I}l]$) ... O Why? ... P ! <i>sic n</i> Where ϕ is an elementary expression of one of the forms: $\mathfrak{H}_{ukm}(a\mathfrak{I}l), \mathfrak{H}_{ukm}(far), a\mathfrak{I}l : 'illa$ (with or without exponent), $a\mathfrak{I}l : 'illa,$ $P(a\mathfrak{I}l)$ $P(far)$	“You (player O) conceded in move <i>n</i> the posit $\phi[a\mathfrak{I}l]$ (or $\phi[a\mathfrak{I}l]$) you are asking for”	$a\mathfrak{I}l : \phi(a\mathfrak{I}l : \phi)$ $far : \phi(far : \phi)$
$'illa^{P(\mathcal{Q})}$ $'illa^{P(\mathcal{Q}_1, \dots, \mathcal{Q}_n)}$	Set of those cases <i>assumed</i> to be elements of the set $'illa$ that determines the ruling at stake because of having property <i>P</i> Set of those cases assumed on the basis of arguments (hypotheses) $\mathcal{Q}_1, \dots, \mathcal{Q}_n$ to be elements of the set $'illa$ that determines the ruling at stake because of having property <i>P</i>	$\{x : 'illa P(x)\} : set(\mathcal{Q})$ $\{x : 'illa P(x)\} : set(\mathcal{Q}_1, \dots, \mathcal{Q}_n)$
X ! $\forall! a\mathfrak{I}l^* : 'illa^{P(\mathfrak{B})}$	Launching of a <i>constructive criticism</i> upon Y 's proposal $a\mathfrak{I}l : 'illa^{P(\mathcal{Q})}$. X commits himself to develop some arguments $\mathfrak{B}_1, \dots, \mathfrak{B}_n$ in support of the juridical bond between the occasional factor and ruling by bringing forward both a new root-case $a\mathfrak{I}l^*$, by proposing an alternative property <i>P'</i> such as an more	$\{x : 'illa P'(x)\} : set(\mathfrak{B}_1, \dots, \mathfrak{B}_n)$ $a\mathfrak{I}l^* : \{x : 'illa P'(x)\} (\mathfrak{B}_1, \dots, \mathfrak{B}_n)$

²⁷ Here and in similar expressions we deploy the following eliminations rules from CTT (cf. Ranta (1994), p. 35) as applied to *a*, standing for *a\mathfrak{I}l* or *far*, and *A* for some set such as the set of *'illa* etc. :

$$\frac{a : \{x : A | P(x)\}}{a : A} \qquad \frac{a : \{x : A | P(x)\}}{P(a) \text{ true}}$$

	accurate specification of P	
$Y ! F (\forall x : P^{(a)}) \mathfrak{I}(x)$	Launching (of some of the forms) of a <i>non-cooperative criticism</i> : X commits himself to bring forward some form of counterexample to $SF (\forall x : \text{'illa}^{P^{(a)}}) \mathfrak{I}(x)$	Counter-example to the claim $(\forall x : D) (x = \text{left}(y) \supset \mathfrak{I}(x) \wedge (x = \text{right}(y) \supset \sim \mathfrak{I}(x)) (y : \text{illa}^{P^{(a)}} \vee \sim \text{illa}^{P^{(a)}})$

In dialogical logic, the particle rules are said to state the *local semantics*: what is at stake is only the request/challenge and the answer/defence corresponding to the utterance of a given expression, rather than the whole context where the logical constant is embedded.

The following rules are to be thought as extending the local rules for standard dialogical logic.

In the context of the present paper

- We distinguish between *assertions* and *positis*: the latter and not the former are moves where of proposition is claimed to hold provided some hypotheses.
- An *expression* is whatever is claimed by an assertion/posit-move. *Elementary expressions* are moves of the form $\mathfrak{I}_{ukm}(a\mathfrak{I})$, $\mathfrak{I}_{ukm}(far)$, $a\mathfrak{I} : \text{'illa}$ (with or without exponent), $a\mathfrak{I} : \text{'illa}, P(a\mathfrak{I}), P(far)$

We also distinguish between *challenges* and *requests* in the following way:

- By bringing forward the request $Y \pi ?$, the challenger asks X to bring forward the expression π .
- The response to a challenge on π , a defence of π , is an expression π' of the form prescribed by the rule.

We further distinguish *Defensive-Requests* (D-Req.) from *Challenging-Requests* (C-Req)

- *Defensive-Requests* (D-Req.) are responses to a previous challenge
- *Challenging-Requests* (C-Req) challenge a previous assertion/posit.

Local Rules for the *Qiyās al- ʿilla*

	Local Meaning		Local <i>qiyās</i> -rules
	Table I		
Assertion	Challenge	Defence	LQR1
$X ! \mathfrak{I}_{ukm}(far)$	Y Why?	$X ! \text{sic } n$	
		-----Or-----	
		D-Req. 1	
		$X \mathfrak{I}_{ukm}(a\mathfrak{I}) ?$	
		D-Req. 2	

		<p>X ! P(aşl) ?</p> <p>Description</p> <p>Player X has the choice to ask Y to concede, both $\mathfrak{H}ukm(aşl)$ and $P(aşl)$ before defending the move that launched the dialogue. However both requests have to be developed in the same play.</p>	
<p>D-Request 1</p> <p>X $\mathfrak{H}ukm(aşl)$?</p>	<p>Answer</p> <p>Y $\mathfrak{H}ukm(aşl)$</p>	<p>Description</p> <p>Player X forces Y to accept that the <i>aşl</i> at stake has been sanctioned as forbidden by bringing forward evidence from the sources.</p>	LQR2
<p>D-Request 2</p> <p>X ! P(aşl) ?</p>	<p>Answers</p> <p>Y ! P(aşl) / Y ! ~ P(aşl)</p>	<p>Description</p> <p>Player Y has the choice between conceding or denying that <i>P</i> can be asserted of the <i>aşl</i></p> <p>Rationale: This rule involves one of the most important dialectical moves of the <i>qiyās</i>. The point is that since a ruling is known to have been sanctioned in relation to the root-case, there must be some occasioning factor for this ruling. The main heuristic objective of the <i>qiyās</i> is to find out which of all the possible occasioning factors is the relevant one. The choice of the property has the aim of selecting the type of the relevant occasioning factor. The choice requires not only dialectical skill but also expertise in jurisprudence and more generally in epistemology.</p>	LQR3
<p>Assertion</p> <p>X ! ~P(aşl)</p>	<p>D-Req. Challenge</p> <p>Y ! P'(aşl) ? / Y ! P_g(aşl)</p>	<p>Description</p> <p>If player X denies that <i>P</i> applies to <i>aşl</i>, then Y can either try with launching a new defensive request with the new property <i>P'</i> or Y can challenge the negation by producing evidence from \mathfrak{S} that $P(aşl)$ is the case.</p> <p>The counterattack involved in the second form of challenge to the negation requires the challenger to assert the positive form. It represents one of the most salient dialectical features of a logical connective: it amounts to the challenger contributing with a constructive argument (in this case from the sources). It is the dual of a kind of denial profusely discussed in the context of <i>jadal</i> namely <i>mu'āraḍa</i>. Indeed in <i>mu'āraḍa</i> it is the one who asserts the negation who has to bring up the argument (see rule LQR10), not the challenger as with this rule.²⁸</p>	LQR4
<p>Assertion</p>	<p>C-Req</p>	<p>Answers</p>	LQR5

²⁸ As we will discuss in our comments to the wine-example, , from the point of view of contemporary dialogical logic, this type of negation seems to display the play-level counterpart to a winning strategy for the assertion of negation.

<p>X ! $P(a\bar{s}l)$</p>	<p>Y ! $P(far)$?</p>	<p>X ! $P(far)$ / X ! $\sim P(far)$</p> <p style="text-align: center;">Description</p> <p>Given that X conceded $P(a\bar{s}l)$, the challenger asks X to acknowledge that the same property applies to the branch-case. Player X might concede or refuse to endorse the claim.</p> <p>Rationale: This move complements the heuristic process underlying the choice of the property P, commented above. This request assumes the <i>open texture</i> of the extension of the sets involved by the dialectical process of the <i>qiyās</i>.²⁹ Indeed, since the extension of the set \mathcal{H}_{akm} is not closed, it is necessary to provide some method in order to decide if the ruling should or not apply to the branch-case. This presupposes that it has to be decided first if the branch-case has or not the property that characterizes the occasioning factor relevant for the ruling specific to the root-case.</p>	
<p style="text-align: center;">Assertion</p> <p>X ! $\sim P(far)$</p>	<p style="text-align: center;">Challenge</p> <p>Y ! $e : P(far)$</p>	<p style="text-align: center;">Description</p> <p>Given the refusal of X to endorse $P(far)$, Y challenges the refusal to concede $P(far)$ by producing some factual evidence e in support of $P(far)$ – here again we are in presence of the same type of negation as the one described in LQR4.</p> <p>If Y cannot produce that evidence the play will stop. This will trigger a new play with a new property. In practice we assume that the Proponent has found eventually a suitable property P that allows the play to continue. Perhaps sometimes guided by the refutations of the challenger that might help narrowing down the set of properties until finding one that applies both to the root- and the branch-case (see the <i>mu'arāda</i>-move in rule LQR10). If that is not the case, there is no <i>qiyās al-'illa</i> procedure to be developed for the main thesis of the dialogue.</p>	LQR6
<p style="text-align: center;">Assertion</p> <p>X ! $P(a\bar{s}l)$</p> <p>(Or Y ! $P_{\bar{s}}(a\bar{s}l)$)</p> <p>...</p> <p>X ! $\mathcal{H}_{akm}(a\bar{s}l)$</p>	<p style="text-align: center;">Challenge</p> <p>Y ! $a\bar{s}l : 'illa^P$</p>	<p style="text-align: center;">Defence Challenge</p> <p>X ! $\mathcal{H}_{akm}(a\bar{s}l) (a\bar{s}l : 'illa^P) / \mathbf{X}$ justify $'illa^P$</p> <p style="text-align: center;">Description</p> <p>If any of the players asserted $P(a\bar{s}l)$ (or $P_{\bar{s}}(a\bar{s}l)$), and X also asserted $\mathcal{H}_{akm}(a\bar{s}l)$ then Y can ask X to concede that P is the $'illa^P$</p>	LQR7

²⁹ For the notion *open-texture* in the context of analogical reasoning see Hart (1961, pp. 121–32), and Bartha (2010, pp. 9-11).

		<p>$(illa^P)$ that grounds $\mathcal{H}_{ukm}(a\mathcal{S}l)$.</p> <p>X can choose to defend or counterattack $a\mathcal{S}l$: $'illa^P$. Once the challenge has been responded (see next rule) he might come back to make use of his defence option.</p> <p>Rationale: Notice, that the defence to the challenge involves the set \mathcal{H}_{ukm} and not the \mathcal{H}_{ukm_s}. The point is that by deploying such a rule X forces Y to endorse a kind of generalization that might later on allow concluding that the relevant ruling applies to the branch-case. This requires that the extension of the relevant set of rulings is not closed (as the set of sources is).</p>	
<p>Assertion</p> <p>$\mathbf{X} ! a\mathcal{S}l : 'illa^P$</p>	<p>Challenge</p> <p>Y justify !</p>	<p>Defences</p> <p>$\mathbf{X} ! a\mathcal{S}l : 'illa^{Ps} / \mathbf{X} ! \mathcal{H}_{ukm}(a\mathcal{S}l) (a\mathcal{S}l : 'illa^{P(\text{a})})$</p> <p>Description</p> <p>Player X can either justify or not with the sources that the $'illa-P$ applies to the $a\mathcal{S}l$. If the defender cannot produce a source as evidence then, he <i>posits</i> that he <i>assumes</i> that P is the $'illa$ for that $Hukm$ sanctioned for the root-case. In such a case, a new type of <i>qiyās</i> starts. Namely, <i>qiyās al-'illa khafī</i>.</p>	LQR8
<p>Posit</p> <p>$\mathbf{X} ! \mathcal{H}_{ukm}(a\mathcal{S}l) (a\mathcal{S}l : 'illa^{P(\text{a})})$</p>	<p>Challenge1</p> <p>$\mathbf{Y} ! \mathbf{F} (\forall x : P(x)^{\text{(a)}} \mathcal{H}_{ukm}(x))$</p> <p>Challenge2</p> <p>Y Justify @ !</p>	<p>Response to Challenge 1</p> <p>See the second table below:</p> <p><i>Non-cooperative criticism</i></p> <p>Description</p> <p>This includes a set of non-cooperative forms of challenges where the antagonist objects that the property P determines the occasioning factor. In fact the challenger states that he can show that the property has not the generalizing power to define the occasioning factor by bringing forward some form of counter example . We will describe some of the main forms of counter-example in a separate second table</p> <p>Defence to challenge 2</p> <p>$\mathbf{X} ! \mathcal{H}_{ukm}(a\mathcal{S}l) (a\mathcal{S}l : 'illa^{P(\text{a}1 \dots \text{a}n)})$</p> <p>Description</p> <p>The challenger asks X to bring up the reasons behind his assumption. This triggers a new argument where X makes explicit the arguments backing his assumption.</p>	LQR9 LQR9.1 LQR9.2

Assertion	Challenge:	Response: Counterattack		LQR10
<p>X! $\mathcal{H}_{ukm}(a\mathcal{S}l)$ ($a\mathcal{S}l : \text{'illa}^{P(\mathcal{A}^1 \dots \mathcal{A}^n)}$)</p>	<p>$mu'ara\mathcal{d}a$ or <i>constructive criticism</i></p> <p>Y! $\mathcal{V} a\mathcal{S}l^* : \text{'illa}^{P(\mathcal{B})}$</p>	<p>Challenge by Y</p>	<p>Counterattack by X</p>	
		<p>! $\mathcal{V} a\mathcal{S}l^* : \text{'illa}^{P(\mathcal{B})}$</p>	<p>Justify !</p>	
		<p>Y deploys a $mu'ara\mathcal{d}a$-move in support of P'...</p> <p>Sub-play</p> <hr/> <p>! $\mathcal{H}_{ukm}(a\mathcal{S}l^*)$ ($a\mathcal{S}l : \text{'illa}^{P(\mathcal{B}^1 \dots \mathcal{B}^n)}$)</p>	<p>Justify $\mathcal{B}!$</p>	
		<p>...</p> <p>Justification of $\mathcal{H}_{ukm}(a\mathcal{S}l^*)$ ($a\mathcal{S}l : \text{'illa}^{P(\mathcal{B}^1 \dots \mathcal{B}^n)}$)</p> <p>Y's answer to the request</p> <p>see LQR5</p>	<p>...</p> <p>Request by X conceding the justificatoin</p> <p>(provided the arguments of Y in support of 'illa^P comply with : <i>ta'thūr</i>, <i>ṭard</i> and <i>'aks</i></p> <p>! $P'(far) ?$ (back to the main play)</p>	
		<p>Description</p>		
		<p>The challenger might take exception of the assumption leading to the specification of the <i>'illa</i> by means of P. The challenger becomes now the defender in a sub-play where he is committed to bring forward a new argument that either specifies some of the arguments within \mathcal{Q} or extends them by adding some new argument and propose a new property. He conceded after all that there is evidence for an explicit ruling on the root-case. So there must be some occasioning factor for it.</p> <p>This challenge is a $mu'ara\mathcal{d}a$-move, profusely discussed in the jadal-literature. Young (2017, chapters 4 and 5) calls it <i>constructive criticism</i>. It is opposed to the <i>non-cooperative criticism</i> displayed in LQR9).</p> <p>Now, since this move will introduce a new argument some priority device will be required.³⁰ The structural rules to be specified below include the introduction of such a priority device. For the sake of simplicity we force the defender to accept the new property. In practice it might lead to a series of counterattacks until one property that identifies the <i>'illa</i> at stake has been settled. Our rules assume that such a process</p>		

³⁰ As we will discuss in our remarks to the wine example it amounts contributing to the refinement of occasioning property.

		is finished. After the successful defence by Y of $\mathcal{H}_{ukm}(a\mathcal{S}l)$ ($a\mathcal{S}l : 'illa^{P^{(\text{a}1 \dots \text{a}n)}}$), X can ask Y to endorse that also the branch case has the new property. The response and rationale behind is the one of LQR5.	
<p style="text-align: center;">Assertion</p> <p style="text-align: center;">X ! $\mathcal{H}_{ukm}(a\mathcal{S}l)$ ($a\mathcal{S}l : 'illa^P$)</p> <p style="text-align: center;">Posit</p> <p style="text-align: center;">X ! $\mathcal{H}_{ukm}(a\mathcal{S}l)$ ($a\mathcal{S}l : 'illa^{P^{(\text{a}1 \dots \text{a}n)}}$)</p>	<p style="text-align: center;">C-Requests</p> <p style="text-align: center;">Y Law^{<i>Uṣūl al-fiqh</i>} ?</p> <p style="text-align: center;">Y Law^{<i>Uṣūl al-fiqh</i>} ?</p> <p>If the posit involves a new property P', then the request Y ! $P'(far)$? (see rule LQR10) has to be deployed before asking for the law</p>	<p style="text-align: center;">Answers</p> <p style="text-align: center;">X ! SF ($\forall x : 'illa^P$) $\mathcal{H}_{ukm}(x)$</p> <p style="text-align: center;">X ! SF ($\forall x : 'illa^P$) $\mathcal{H}_{ukm}(x)$ $'illa^{P^{(\text{a}1 \dots \text{a}n)}}$</p> <p style="text-align: center;">Description</p> <p>The challenger asks X to assert (posit) the general <i>Uṣūl al-fiqh</i>-law exemplified by the assertion/posit. The answer is a generalization that takes the form of the assertion of a universally quantified proposition.</p>	LQR11
<p style="text-align: center;">Assertion</p> <p style="text-align: center;">X ! SF ($\forall x : 'illa^P$) $\mathcal{H}_{ukm}(x)$</p> <p style="text-align: center;">X ! $P(far)$ (or Y ! $e : P(far)$)</p>	<p style="text-align: center;">Challenge</p> <p style="text-align: center;">Y ! $far : 'illa^P$</p>	<p style="text-align: center;">Defense</p> <p style="text-align: center;">X ! $\mathcal{H}_{ukm}(far)$</p> <p style="text-align: center;">Description</p> <p>Once X has asserted the general law, and he further conceded $P(far)$ (or the challenger himself asserted $e : P(far)$); then the challenger can launch a challenge to that law, asserting the antecedent. The defender must then assert the consequent. Similar happens if the general law has been posited rather than asserted.</p>	LQR12

Four forms of Non-cooperative criticism

The antagonist might launch a non-cooperative criticism against the assertion $\mathcal{H}_{ukm}(a\mathcal{S}l)$ ($a\mathcal{S}l : 'illa^{P^{(\text{a}1 \dots \text{a}n)}}$) by proposing a counterexample. In fact the challenger states that he can show that the property has not the generalizing power to define the occasioning factor. More precisely, as discussed in II.1.4, the antagonist claims that he is able to bring up from the sources a counterexample to the possibility of generalizing the property P as specifying the occasioning factor. The development of this criticism involves the development of a sub-play where the player who asserted $\mathcal{H}_{ukm}(a\mathcal{S}l)$ ($a\mathcal{S}l : 'illa^{P^{(\text{a}1 \dots \text{a}n)}}$) is now committed to assert $(\forall x : P) \mathcal{H}_{ukm}(x)$ and $(\forall x : \sim P) \sim \mathcal{H}_{ukm}(x)$, or $(\forall x : \sim \mathcal{H}_{ukm}) \sim \sim P(x)$. The challenger brings up a counterexample (a new case root-case) from the sources, showing that the ruling under discussion does not apply, despite the fact that it instantiates the property P , or that if the property is absent, the ruling still applies. Moreover, in relation to the objection to the co-presence; it has been conceded before that P applies to the original root-case. Therefore the conclusion is that the purported property cannot be the occasioning factor. In other words, if we assume P then it follows both that the juridical ruling applies and that it not applies. Therefore P is not the case – similarly for the absence. Notice that since the counterexample comes from the sources, neither defence nor counterattack is

possible (unless there is some mistake concerning the reference to the sources).³¹ This might trigger the play to start again with another property or the proponent must withdraw his main thesis.

We will restrict ourselves to only four main forms of non-cooperative criticism that are distinguished by their form to building the counterexample to the universal. Namely, by

- Bringing forward a root-case of which it is recorded that exactly the opposite of the claimed ruling applies, despite the fact that the property does. It is called *fasād al-waḍ'* (invalidity of the occasioned status). The counterexample undermines the *tard*-condition of the purported property – the property applies but the opposite of ruling is the case.
- Bringing forward a root-case of which it is recorded that a ruling different to the claimed ruling applies and that it has been acknowledged that both rulings are incompatible, despite the fact that the property does. It is called, *naqḍ (inconsistency)*. The counterexample can be seen as undermining the '*aks*-condition of the purported property – the absence of the ruling does not stem from the absence of the property – it also undermines the *tard*-condition (provided both rulings are incompatible).
- Bringing forward a root-case of which it is recorded that a ruling different to the claimed ruling applies despite the fact that the property does, and this shows that the proposed occasioning factor unifies cases that must be kept apart. It is called, *kasr (breaking apart)*. The counterexample can be seen as undermining both the '*aks*- and the *tard*-condition of the purported property – the absence of the ruling does not lead to the absence of the property and the presence of the property does not necessarily lead to the presence of the ruling. It could be also understood as a particular form of *naqḍ*.
- Bringing forward a root-case of which it is recorded that the claimed ruling applies despite the absence of the property claimed to specify the occasional factor. It is called, '*adam al-ta'thīr (lack of efficiency)*'. The counterexample undermines the *ta'thīr* condition of the purported property –the occasioning factor for the ruling is not specified by the proposed property (is not dependent upon the property). This also undermines the other two conditions.

Examples:

- 1) *fasād al-waḍ'*. We will develop it in detail below a dialogue for it.

Claim

Ruling: Saliva of beast of prey is impure. *Occasional factor:* Beasts of prey have canine teeth.

Counterexample: Cats have canine teeth but their saliva is not impure.

- 2) *naqḍ*

Claim

Ruling: Punishment with jail. *Occasional factor:* Homicide.

Counterexample: Some forms of homicide do not lead to jail but to certain specific social duties.

- 3) *kasr*

Claim:

Ruling: Interdiction of transaction. *Occasional factor:* Establishing a contract with someone in such a way that the benefactor has no access to object of the contract.

Counterexample: Contract-Marriages closed before the members of the couple have acquaintance with each other are not forbidden. .

- 4) '*adam al-ta'thīr*

Claim:

Ruling: Interdiction of consumption of wine. *Occasional factor:* Presence of *euphoric intensity* and red-colour.

Counterexample: White wine is forbidden, despite the fact that it is not red. .

Let us now render the suitable rules. In fact we will render with the rules the development of those:

	Local meaning Table II Non-cooperative criticism	LQR9.1
Assertion launching challenges of the form <i>fasād al-waḍ', naqḍ, kasr</i>	Challenge Sub-play	Development 1 Sub-play -----Ⓣ ₁

³¹ Cf. Young (2017, chapter 4 and 5).

<p>$X \text{ ! } F (\forall x : P) \mathfrak{K}_{ukm}(x)$</p> <p style="text-align: center;">Description</p> <p>With this assertion that arises as the first possible challenge in LQR9 to $\mathfrak{K}_{ukm}(a\mathfrak{S}l)$ ($a\mathfrak{S}l : 'illa^{P(\mathfrak{Q})}$) X asserts that the antagonist will not be able to win the assertion:</p> <p>“$\mathfrak{K}_{ukm}(x)$), applies to anything instantiating $P(x)$</p> <p>This amounts to the refutation that $P(x)$ is the relevant property for the determination of the occasioning factor</p>	<p style="text-align: center;">-----\mathfrak{D}_1</p> <p style="text-align: center;">$Y \text{ ! } (\forall x : P) \mathfrak{K}_{ukm}(x)$</p> <p style="text-align: center;">Description</p> <p>The challenger opens a sub-play where he asserts that he can indeed defend the universal.</p> <p>Notice the change of roles. See local rule for F in the appendix.</p>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 50%; text-align: center;">X</th> <th style="width: 50%; text-align: center;">Y</th> </tr> </thead> <tbody> <tr> <td></td> <td style="text-align: center;">$! (\forall x : P) \mathfrak{K}(x) \mathbf{0}$</td> </tr> <tr> <td> <i>fasād al-waḍ'</i> 1 ! $a^* : P$ [0] </td> <td style="text-align: center;">$\mathfrak{K}_a^* 2$</td> </tr> <tr> <td>3 ? $\sim \mathfrak{K}_{\mathfrak{S}a}^*$ [2]</td> <td style="text-align: center;">$\sim \mathfrak{K}_{\mathfrak{S}a}^* 4$</td> </tr> <tr> <td>5 ! $\mathfrak{K}_{\mathfrak{S}a}^*$ [4]</td> <td style="text-align: center;">Gives up</td> </tr> <tr> <td colspan="2">(In] indicates the line challenged)</td> </tr> </tbody> </table> <p style="text-align: center;">Description</p> <p>X brings a root case of which it is recorded that P is present. Y is then forced to assert that the ruling applies. However, it is recorded by the sources that it does not apply and Y is forced to assert that too. The counterargument leads Y to contradict himself.</p> <p style="text-align: center;">Development 2</p> <p style="text-align: center;">Sub-play</p> <p style="text-align: center;">-----\mathfrak{D}_1</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 50%; text-align: center;">X</th> <th style="width: 50%; text-align: center;">Y</th> </tr> </thead> <tbody> <tr> <td></td> <td style="text-align: center;">$! (\forall x : P) \mathfrak{K}(x) \mathbf{0}$</td> </tr> <tr> <td> <i>naqḍ</i> 1 ! $a^* : P$ [0] </td> <td style="text-align: center;">$\mathfrak{K}_a^* 2$</td> </tr> <tr> <td>3 ? $\mathfrak{K}_{\mathfrak{S}a}^*$</td> <td style="text-align: center;">$\mathfrak{K}_{\mathfrak{S}a}^* 4$</td> </tr> <tr> <td>5 ? $\sim (\mathfrak{K}_{\mathfrak{S}a}^* \wedge \mathfrak{K}_a^*)$ [2, 4]</td> <td style="text-align: center;">$\sim (\mathfrak{K}_{\mathfrak{S}a}^* \wedge \mathfrak{K}_a^*) 5$</td> </tr> <tr> <td>6 ! $(\mathfrak{K}_{\mathfrak{S}a}^* \wedge \mathfrak{K}_a^*)$ [5]</td> <td style="text-align: center;">Gives up</td> </tr> </tbody> </table> <p style="text-align: center;">Description</p> <p>X brings a root case of which it is recorded that P is present. Y is then forced to assert that the ruling applies. However, it is recorded by the sources that, in fact a different ruling \mathfrak{K}^* applies, and that the application of this new ruling to the new case is incompatible with the application of the former ruling. This leads Y to contradict himself.</p> <p style="text-align: center;">Development 3</p> <p style="text-align: center;">Sub-play</p> <p style="text-align: center;">-----\mathfrak{D}_1</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 50%; text-align: center;">X</th> <th style="width: 50%; text-align: center;">Y</th> </tr> </thead> <tbody> <tr> <td></td> <td style="text-align: center;">$! (\forall x : P) \mathfrak{K}(x) \mathbf{0}$</td> </tr> <tr> <td> <i>kasr</i> 1 ? { $x : P \mid T(x)$ } [0] </td> <td style="text-align: center;">$! (\forall x : \{ x : P \mid T(x) \}) \mathfrak{K}(x) 2$</td> </tr> </tbody> </table>	X	Y		$! (\forall x : P) \mathfrak{K}(x) \mathbf{0}$	<i>fasād al-waḍ'</i> 1 ! $a^* : P$ [0]	$\mathfrak{K}_a^* 2$	3 ? $\sim \mathfrak{K}_{\mathfrak{S}a}^*$ [2]	$\sim \mathfrak{K}_{\mathfrak{S}a}^* 4$	5 ! $\mathfrak{K}_{\mathfrak{S}a}^*$ [4]	Gives up	(In] indicates the line challenged)		X	Y		$! (\forall x : P) \mathfrak{K}(x) \mathbf{0}$	<i>naqḍ</i> 1 ! $a^* : P$ [0]	$\mathfrak{K}_a^* 2$	3 ? $\mathfrak{K}_{\mathfrak{S}a}^*$	$\mathfrak{K}_{\mathfrak{S}a}^* 4$	5 ? $\sim (\mathfrak{K}_{\mathfrak{S}a}^* \wedge \mathfrak{K}_a^*)$ [2, 4]	$\sim (\mathfrak{K}_{\mathfrak{S}a}^* \wedge \mathfrak{K}_a^*) 5$	6 ! $(\mathfrak{K}_{\mathfrak{S}a}^* \wedge \mathfrak{K}_a^*)$ [5]	Gives up	X	Y		$! (\forall x : P) \mathfrak{K}(x) \mathbf{0}$	<i>kasr</i> 1 ? { $x : P \mid T(x)$ } [0]	$! (\forall x : \{ x : P \mid T(x) \}) \mathfrak{K}(x) 2$
X	Y																															
	$! (\forall x : P) \mathfrak{K}(x) \mathbf{0}$																															
<i>fasād al-waḍ'</i> 1 ! $a^* : P$ [0]	$\mathfrak{K}_a^* 2$																															
3 ? $\sim \mathfrak{K}_{\mathfrak{S}a}^*$ [2]	$\sim \mathfrak{K}_{\mathfrak{S}a}^* 4$																															
5 ! $\mathfrak{K}_{\mathfrak{S}a}^*$ [4]	Gives up																															
(In] indicates the line challenged)																																
X	Y																															
	$! (\forall x : P) \mathfrak{K}(x) \mathbf{0}$																															
<i>naqḍ</i> 1 ! $a^* : P$ [0]	$\mathfrak{K}_a^* 2$																															
3 ? $\mathfrak{K}_{\mathfrak{S}a}^*$	$\mathfrak{K}_{\mathfrak{S}a}^* 4$																															
5 ? $\sim (\mathfrak{K}_{\mathfrak{S}a}^* \wedge \mathfrak{K}_a^*)$ [2, 4]	$\sim (\mathfrak{K}_{\mathfrak{S}a}^* \wedge \mathfrak{K}_a^*) 5$																															
6 ! $(\mathfrak{K}_{\mathfrak{S}a}^* \wedge \mathfrak{K}_a^*)$ [5]	Gives up																															
X	Y																															
	$! (\forall x : P) \mathfrak{K}(x) \mathbf{0}$																															
<i>kasr</i> 1 ? { $x : P \mid T(x)$ } [0]	$! (\forall x : \{ x : P \mid T(x) \}) \mathfrak{K}(x) 2$																															

III. 2 Global meaning for the *Qiyās al- ʿilla*

The global rules

As mentioned above global meaning is defined by means of *structural rules* that determine the general development of the plays, by specifying who starts, what are the allowed moves and in which order, when does a play end and who wins.

Before providing the structural rules (**SRQI.n**) for the *Qiyās al- ʿilla* let us precise the following notions:

Play: A *play* is a legal sequence of moves, i.e., a sequence of moves which observes the game rules. Particle rules are not the only rules which must be observed in this respect. In fact, it can be said that the second kind of rules, namely, the *structural rules* are the ones giving the precise conditions under which a given sequence is a play.

Dialogical game or dialogue: The *dialogical game* for φ , written $D(\varphi)$, is the set of all plays with φ being the *thesis* (see the Starting rule below).

Terminal play: A play is called *terminal* when it cannot be extended by further moves in compliance with the rules.

X-terminal: We say it is **X-terminal** when the last move in the play is an **X**-move.

SQL.0 Starting rule:

Every dialogue for a *qiyās al- ʿilla* starts with a thesis of the form **P ! $\mathcal{K}_{ukm}(far)$**

After the setting of the thesis, players each choose a positive integer called *repetition rank*.

- The *repetition rank* of a player bounds the number of challenges he can play in reaction to a same move.

Justification of the repetition-restriction: A *qiyās al- ʿilla*-procedure is deployed in order to take a decision and the decision cannot run to the infinite and avoid unnecessary repetitions of the same situation

SQL.1

Players move alternately. After the repetition ranks have been chosen, each move is a challenge or a defence in reaction to a previous move and in accordance with the local rules.

SQL.2 General Socratic rule:

The Opponent can challenge an elementary expression iff he did not assert (posit) it before.³²

The Proponent can answer to a challenge upon an elementary expression following the suitable local rules.

Other challenges on elementary expressions follow the local rules

Justification of the rule: This relates to "logical degree" of *logical* authoritative force we mentioned at the end of section II. It might also lead to a *contextual force*, when the Opponent concedes (or does not

³² This, rule is one of the most salient characteristics of dialogical approaches to logic and it at the roots of every formal reasoning by the interaction of arguments. As discussed by Marion / Rückert (2016), it can be traced back to Aristotle's reconstruction of the Platonic Dialectics: the main idea is that, when an elementary proposition is challenged then, from the purely argumentative point of view – that is, without making use of an authority beyond the moves brought forward during an argumentative interaction-, the only possible response is to appeal to the concessions of the challenger. The idea behind can be expressed by the following argumentative move:

*my grounds for the proposition you are asking for are exactly the same as the ones you bring forward when you conceded the same proposition.*³²

Cf. Clerbout/Rahman (2015), Rahman/Clerbout/Keiff (2009) and Rahman/Keiff (2010).

launch a counterattack) an expression of the Proponent because of some contextual circumstances, such as: being convinced by the arguments displayed by the Proponent in favor of selecting the property P as relevant for the determination of the occasioning factor.

SQL2.1 Socratic rule for the *Qiyās*:

Authority moves

Moves of the form $! P_{\mathfrak{g}}(a\mathfrak{s}l)$ and of the form $! a\mathfrak{s}l : 'illa^{P_{\mathfrak{g}}}$ cannot be challenged.

Moves of the form $\mathbf{P} ! e : P(far)$, or $\mathbf{P} ! far : 'illa^P$ (if $\mathbf{P} ! e : P(far)$, or $\mathbf{O} ! P(far)$ has been asserted before) cannot be challenged either.

Requests for endorsement of an assertion coming from the sources or from an established evidence must positively responded

Challenges of the form $\mathbf{Y} ! P_{\mathfrak{g}}(a\mathfrak{s}l)$ and defences of the form $\mathbf{X} ! a\mathfrak{s}l : 'illa^{P_{\mathfrak{g}}}$ are the product of an heuristic/hermeneutic procedure by the means of which it is required that the player finds the source backing his move. The play assumes that when these moves have been brought forward the heuristic/hermeneutic procedure leading to those moves have been closed and settled.

Assertions of the form $\mathbf{P} ! e : P(far)$ bring forward the piece of (definitive) evidence e for the truth of $P(far)$.

- *Justification of the rule*: since the sources are considered to constitute the canonical evidence, they are considered to be the maximal authority.³³ The heuristic procedure that brings out the source backing an affirmation is considered to be closed at that moment of the development of the *qiyās*. In other words at the moment when the move mentioned afore are asserted it is assumed that the heuristic-hermeneutic discussion on or about the authenticity of the source has finished. Nevertheless, notice that the deployment of a dialectical frame does not assume that the relevant sources should be added as initial concessions (premises) before the dialogue starts. It is rather the development of the dialogue that brings up the necessary source.
Direct factual evidence such as $e : P(far)$ also provide some backing (but from a lesser degree than the sources). It has a *contextual authoritative force*.

Mu'āraḍa-moves and the priority criterium

Moves of the form $\mathbf{O} ! \mathfrak{K}ulm(a\mathfrak{s}l)$ ($a\mathfrak{s}l : 'illa^{P'((\mathfrak{g}1 \dots \mathfrak{g}n))}$) can be endorsed by the Proponent, iff the property P' leads to a **more specific formulation** of the relation between the ruling and the occasioning factor.

Justification of the rule: The point of the rule is that with this kind of rule the Opponent brings forward a constructive criticism. It is constructive since he proposes a refinement of the determination of the occasioning factor by offering a more specific formulation. This amounts establishing a preference of those P' over P that result from specification elaborations.

The new formulation of the Socratic leads to the formulation of the following rule – that recalls our discussion at the end of section II, lead to the formulation of the following rule::

SQL4 Degrees of authoritative force:

³³ Thus, insofar as deciding a new case is concerned, the Muslim jurist can operate on two levels which are determined by the nature of these two sources. On the first level, the jurist is bound by the explicit textual statements and commands. What determines the judgment in the new case is solely the explicit ratio in the original text, i.e., the precedent. There is little latitude for deciding the case in light of current exigencies. On the second level, however, the jurist is allowed a certain, although limited, freedom of interpretation in deciding the new case, due to the ambiguous nature of the textual precedents. Hallaq (1985, p. 88)

1. If a player backs his claim with a reference to the sources, it has the maximal authoritative force.
2. If the Proponent backs his claim with the Opponent's own concessions during the dialectical process, because of following the optimal strategy circumstances; then it has a logical force. *Logical force* underlies the logical fragments of a *qiyās*- process.
3. If the Proponent backs his claim by the Opponent's endorsements during the dialectical process, because of some contentual or material circumstances, then it has a *contextual force*. Contextual force, admits also degrees: either the Proponent produces some *direct factual evidence* for a claim or *indirectly* by developing a chain of arguments for the selection of a particular occasioning factor. Direct factual evidence will produce a direct endorsement. Indirect evidence might trigger a new proposal of the Opponent
4. In the latter case, if the Opponent wishes to bring forward a *constructive criticism* he might contribute to the *specification* of an initial assumption concerning the occasioning factor by bringing forward a new proposal. In such a case the more specific determination of the occasioning factor will have priority to a less specific one. Provided he brings forward arguments that comply with efficiency; co-presence and co-absence.

SQL3 (Winning rule).

Player **X** wins a play only if that play is **X**-terminal.

X-winning-strategy: An *X-strategy* is *winning* if playing according to it leads to a **X**-terminal play no matter how **Y** plays.

Remark: In standard dialogical approaches to logic a winning strategy amounts to a winning strategy for **P** consisting of a tree-shaped structure such that each branch is a sequence of terminal plays for **P**. In the context of the *qiyās* from the logical point of view we do not need to consider the options of **P**, since a winning strategy is defined when **P** wins with one of the options. However, since the *qiyās* is aimed at establishing material truth rather than logical truth, it is useful sometime to display the different options available for **P**.

III.3 Examples

Let us develop some examples, but some terminology first:

Terminological convention and strategy procedure:

We follow the usual notation of the dialogical framework with the addition of some further indications specific to the deployment of the local rules for the *qiyās*. More precisely

- **Standard Notation:**

1. Proponent's moves are numbered with even numbers starting from **0**. Those, moves are recorded at the outmost right column.

2. Oponent's moves are numbered with odd numbers starting from **0**. Those, moves are recorded at the outmost left column.
 3. A challenge upon a move placed at the horizontal line **h** is to be recorded on **a line below h**.
 4. A defence to a challenge placed at the horizontal line **h** is to be recorded on **the same line**.
 5. If move **n** by **P** challenges **O**'s move **m**, the challenged move **m** is to be recorded in the interior column at the left of the central column where **P** records his moves.
- **Addenda specific to the *qiyās*:**
 D-Requests that are responses to a challenge are to be considered as a kind of defensive move. Accordingly they do not record a challenge. The numeration of a D-request move has the form $n [m]$, where n is the number where the request has been formulated and m the number of the move (the challenge) that triggered the request.
 C-Requests follow the notation of challenges.

The star “*” in moves 5, 9 and 13 of the first example indicates that **O** has another option. So a winning strategy for **P** requires running further plays where **O** takes the alternative options prescribed by the local rules. Now the alternative option to move 5 might lead to the engagement into a *qiyās al-‘illa khafī*. This type is discussed in the second example.

More precisely,

a **O**-decision is a move in a play where **O** decided for one of two available options. Let us mark with a star such a move.

Once the play is finished and if it is won by **O**. The opponent might ask to run a new play, where he takes another decision. In order to do so, scan the play bottom up. Take the last decision taken by **O** (the first star bottom up), say at move n , and redo the play with the other option. The new play includes all the moves before move n and delete all those after the decision of move n . Once finished and if **O** is loses again; start a new play until the last decision (if any) of the new play. Continue until all the options have been explored.

The mark # in move 8 of the first example indicates that **P** had an option. However, if **P** can bring forward the required evidence; this will only produce a variant of the play very close to the one that follow the former option.

After the informal formulation we added the abbreviated formal notation with the number of the local rule applied.

The importance of this form of this *qiyās al-‘illa*, despite its simplicity, is that it has the canonical form of a *qiyās al-‘illa*. Moreover it is closely related to Aristotle's *reasoning by exemplification* or *paradigmatic inference* (cf. Aristotle, *Pr. An.* 69a1, Bartha (2010), pp. 36-40)³⁴.

³⁴ Unfortunately Bartha does not seem to be aware of the long and deep discussions that took place in the Arabic tradition. This is patent not only in relation to this form of *qiyās* but to further more complex ones, that come very close to his own model.

For the sake of notational simplicity we did not include the moves related to the repetition rank. We assume that the repetition rank for **O** is 1 and it is 2 for **P**. That is why, a new option of **O** triggers a new play but not the options of **P**. He can take care of the second option in the same play. It is patent that the example does not come from the traditional texts. We deploy it to stress the dynamic in relation to the scope of applicability of a old-rule. In other; words to its adaptability to deal with new cases.

Example of a qiyās al- 'illa Jalī

O			P	
			Reading (without permission) the mail of someone else is forbidden ! $\mathcal{H}_{ukm}(far)$	0
1	Why? [LQR1]	0	because of your own words in move 15 <i>sic</i> 15 [LQR1]	16
3	Yes ! $\mathcal{H}_{ukm}(a\mathcal{S}l)$ [LQR2]		Entering (without permission) into a house of someone else is forbidden by the Quran isn't it? ³⁵ $\mathcal{H}_{ukm}(a\mathcal{S}l)$? [LQR1]	2 [1]
5	Yes* ! $P(a\mathcal{S}l)$ [LQR3]		Does entering (without permission) into a house of someone else violate privacy? ! $P(a\mathcal{S}l)$? [LQR1]	4 [1]
9	I do # $\mathcal{H}_{ukm}(a\mathcal{S}l)$ ($a\mathcal{S}l : 'illa^P$) [LQR7]	3	Given 3 and 5 you must concede that violation of privacy is the 'illa of that <i>hukm</i> . ! $a\mathcal{S}l : 'illa^P$ [LQR7]	6
7	How do you justify this illa? Justify ! [LQR8]	6	According to the sources, the relevant <i>hukm</i> sanctions that entering (without permission) into a house of someone else is forbidden because of violation of privacy ³⁶ # $a\mathcal{S}l : 'illa^{Ps}$) [LQR8]	8
11	Indeed: Everything that violates the privacy of a person is forbidden.	9	Moreover, based on this source, you must see that your last concession (9) exemplifies the general <i>Uṣūl al-fiqh-law</i> : <i>everything that violates the</i>	10

³⁵In fact this interdiction is explicitly sanctioned in the Quran:

يَا أَيُّهَا الَّذِينَ ءَامَنُوا لَا تَدْخُلُوا بُيُوتًا غَيْرَ بُيُوتِكُمْ حَتَّى تَسْتَأْذِنُوا وَتُسَلِّمُوا عَلَى أَهْلِهَا

(O believers! Do not enter houses other than your own until you have sought permission and said greetings of peace to the occupants) [Q.S. *An Nur*:27].

³⁶The 'illa for this *hukm* has been explicitly formulated in the prophetic tradition: إِنَّمَا جُعِلَ الْإِسْتِئْذَانُ مِنْ أَجْلِ الْبَصَرِ (Verily! The order of taking permission to enter has been enjoined because of the sight, (that one should not look unlawfully at the state of others). See *Shahih Bukhari*, Ch. 8, p. 54

				<i>privacy of a person is forbidden. Doesn't it?</i>	
	! SF ($\forall x : 'illa^P$) $\mathfrak{H}_{ukm}(x)$ [LQR11]			Law $U_{s\tilde{u}l} al-fiqh$? [LQR11]	
13	Yes, it does *		3	Does reading (without permission) the mail of someone else violate the privacy of that person?	12
	$P(far)$ [LQR5]			$P(far)$? [LQR5]	
15	Indeed, I endorse this interdiction too		11	So, since reading (without permission) the mail of someone else violate the privacy of that person, it should also be forbidden. Right?	14
	! $\mathfrak{H}_{ukm}(far)$ [LQR12]			! $far : 'illa^P$ [LQR12] [according to structural rule SQI.2.1, this move cannot be challenged, because of move 14]	

In fact, as already mentioned there are some other alternatives to the development of the example.

First, in move 5 the Opponent can in principle deny that the selected property applies to the root-case ($\sim P(a\mathfrak{s}l)$). Then the Proponent can either try with launching a new defensive request with a new property ($P'(a\mathfrak{s}l)$) or challenge the denial by producing evidence in the source to force the Opponent to accept that the property applies to the root-case and withdraw his refusal. If the Proponent cannot produce evidence from the sources or some factual evidence then the play will stop. In our particular example it seems implausible that someone could refuse to accept that *entering (without permission) into a house of someone else* is not a *violation of privacy*. One might think of cases where violation of privacy should not be forbidden (in case of the house under fire for example). However this in fact is the refusal to accept that violation of privacy alone is the occasioning factor for its interdiction. This is what can happen in the next alternative.

The second alternative in fact is dependent upon the Proponent's moves, that is why we indicated the move 9 with "#" rather than with "*". So it is not really an option of the Opponent within a play for a *qiyās al-'illa*. Indeed, if the Proponent has no source backing his claim that *violating privacy* is the occasioning factor for the ruling of the root-case, then the Proponent can be taken to the task of developing an argument. However, this move triggers the switch to a *qiyās al-'illa al-khafī*, to be discussed in our next example. In our particular example, since the Proponent can back his claim with a reference to the sources, this alternative does not occur.

Third, in the move 13 the Opponent can in principle refuse to concede that the selected property applies to the branch-case. Again, in our particular example it seems implausible that someone could refuse to accept that *reading the email of someone*

else, is an instance of *violation of privacy*. Be that as it may, if the Opponent refuses, the Proponent must produce some evidence (*e*) supporting presence of the property in the branch-case (*e*: $P(\textit{far})$).

Remark: It is important to recall that if we are building a winning strategy by constituting a sequence of plays, we must start with exploring the last possibility and then go up.

The following example is a reconstruction from different fragments brought forward by *Al-Shīrāzī*' (1986, Kuwait ed., p. 112) and from the discussion by Young (2017, chapter 4). It gives us the chance to display the deployment of *destructive criticism* or *fasād al-waḍ'* and delve into its logical structure.

Examples of qiyās al- 'illa Khaft

On beasts of prey, impure saliva and the deployment of fasād al-waḍ'

O			P		
				The saliva of the beast of prey qualifies as impurity (<i>najāsa</i>) ! $\mathfrak{K}_{ukm}(far)$	0
1	Why? [LQR1]	0			
3	Yes it does ! $\mathfrak{K}_{ukm}(a\mathfrak{s}l)$ [LQR2]			Do the saliva of pigs qualify as impurity (<i>najāsa</i>)? ! $\mathfrak{K}_{ukm}(a\mathfrak{s}l)$? [LQR1]	2
5	Yes ! $P(a\mathfrak{s}l)$ [LQR3]			Do pigs have canine teeth (<i>dhū nābīn</i>)? ! $P(a\mathfrak{s}l)$? [LQR1]	4
			3, 5	Given 3 and 5 you must concede that possessing canine is the 'illa of that <i>ḥukm</i> ! $a\mathfrak{s}l: 'illa^P$ [LQR7]	6
7	I have a counterexample ! $F(\forall x : P) \mathfrak{K}(x)$ [LQR9.1, <i>fasād al-waḍ'</i>] START OF THE SUBPLAY -----	6		START OF THE SUBPLAY -----	
			7	I insist: Possesing canine is the 'illa of that <i>ḥukm</i> ($\forall x : P) \mathfrak{K}(x)$	8
9	Cats poses canine teeth. Thus, according to your assertion, its saliva is impure.	8		Indeed I have to concede this $\mathfrak{K}(cat)$	10
11	<i>e</i> : $P(cat)$	10			12

13	<p>We know (from the sources) that the saliva of cats is not impure. Do you see this? $? \sim \mathcal{K}_s(cat)$.</p> <p>But you asserted before that according to your view on the occasioning factor, it follows that the saliva of cats is impure. You contradict yourself!</p> <p>$\mathcal{K}(cat)$</p> <p>Therefore that possessing canine teeth is not be the occasioning factor of saliva's impurity.</p>	12	<p>I must. It comes from the sources $\sim \mathcal{K}_s(cat)$.</p> <p>I concede. I give up</p>	
----	---	----	---	--

What the Opponent is doing is displaying a winning strategy for a claim that denies that P determines the relevant occasioning factor. Notice that it is stronger than the rejection of endorsing a claim: The opponent is changing the roles and defending that he has a winning strategy in order to reject P as determining occasioning factor: it is not only the assertion of the negation at the play-level but the assertion that the negation holds from the strategic point of view. In the context of contemporary natural-deduction in principle it might be linked to the introduction-rule for negation. However, it is the dialectical rendering of the natural deduction rule that is at stake here and its real meaning is a dialectical one: *the switch of roles* pointed out by scholars as Hallaq (1985) and Young (2017). Our dialogical framework allows too to stress its strategic feature. In fact in our view the rule should be also studied in the context of the dialectical interpretation of universal quantifiers. Indeed, the local rule for such quantifiers (see appendix to our paper) require that the challenger chooses and instance of the antecedent and the defender must show that it satisfies the property expressed in the main body of the quantified expression. In our context, the point is that the counterexample builds a counter-example to the claim that the root-case exemplifies a universal law. Accordingly, no generalization can be constituted by binding P with the relevant ruling. Notice that since the counterexample comes from the sources, neither defence nor counterattack is possible (unless there is some mistake concerning the reference to the sources).

The following example is one that has received very much attention in the specialized literature. It is not only the subject (wine-interdiction) that makes it so important but because it represents both, the most difficult and the most deployed form of *qiyās al-'illa*.

The Wine-example

O	P
---	---

			(Consuming) Date-wine (<i>nabīdh</i>) is forbidden (<i>ḥarām</i>) ³⁷ . ! $\mathcal{H}_{ukm}(far)$	0
1	Why? [LQR1]	0	Because of your own words in move 19. <i>sic</i> 19 [LQR1]	20
3	Yes, it is forbidden. ! $\mathcal{H}_{ukm}(a\mathcal{S}l)$ [LQR2]		Is drinking grape-wine (<i>khamr</i>) not forbidden by the Quran? ³⁸ ! $\mathcal{H}_{ukm}(a\mathcal{S}l)$? [LQR1]	2 [1]
5	Yes ! $P(a\mathcal{S}l)$ [LQR3]		Is grape-wine a drink in which there is a <i>euphoric intensity</i> (<i>shiddat muṭriba</i>)? $P(a\mathcal{S}l)$? [LQR1]	4 [1]
13	I concede. According to argumentation @ deployed it seems reasonable to conclude that the presence of euphoric intensity occasions the proscription of the grape-wine. * ! $\mathcal{H}_{ukm}(a\mathcal{S}l)$ ($a\mathcal{S}l : 'illa^{P(@)}$) [LQR9]	3,	So, according to your moves 3 and 5, the presence of euphoric intensity occasions the proscription of consuming grape-wine. Right? ! $a\mathcal{S}l : 'illa^P$ [LQR7]	8
9	How can you conclude that? Justify ! [LQR8]	8	There are good reasons for doing so ! $\mathcal{H}_{ukm}(a\mathcal{S}l)$ ($a\mathcal{S}l : 'illa^{P(@)}$) [LQR8]	10
11	Can you develop those reasons ? Justify @ ! [LQR9.2]	10	@1 : Before the occurrence of the [euphoric] intensity, the lawfulness of consuming pressed-juice is the object of consensus @2 : After the euphoric intensity occurs [i.e., when it becomes wine] and nothing else occurs its proscription is object of consensus. @3 : When its euphoric intensity falls away [i.e., when it becomes vinegar] and nothing else falls away it is object of consensus that it should not be forbidden. @4 : @1, @2, @3, lead us to the claim that if we evaluate the force of the euphoric intensity, then we have evaluated the intensity of the proscription In other words, the proscription is dependent upon the presence or not of the euphoric intensity. Therefore, the presence of the <i>ḥukm</i> is due to the presence of the <i>'illa</i> , and the absence of the <i>ḥukm</i> is due to its absence. ! $\mathcal{H}_{ukm}(a\mathcal{S}l)$ ($a\mathcal{S}l : 'illa^{P(@4)}$ (@1, @2, @3)) [LQR9]	12
15	Indeed	13	Are you assuming then that everything containing 'euphoric intensity' is forbidden?	14

³⁷ The original text deploys the word *ḥarām*. This notion, the opposite of *ḥalāl*, refers (in this context) to the interdiction of consuming certain food.

³⁸ It is sanctioned in the Quran that wine is *ḥarām* (forbidden [to be consumed]) :

يَا أَيُّهَا الَّذِينَ آمَنُوا إِنَّمَا الْخَمْرُ وَالْمَيْسِرُ وَالْأَنْصَابُ وَالْأَزْلامُ رِجْسٌ مِّنْ عَمَلِ الشَّيْطَانِ فَاجْتَنِبُوهُ لَعَلَّكُمْ تُفْلِحُونَ

(O you believe! Wine, gambling, altars and divining arrows are filth, made up by Satan. Therefore, refrain from it, so that you may be successful). [Q.S: 5: 90]

helps to a *more accurate specification* of the occasional factor that yields the ruling. This new argument, must yield a better understanding of the conditions *ta'thīr*, *ṭard* and *'aks* at stake.

The example below, that stems from Tahiri, makes use of *nutmeg*, that strictly speaking does not come from the source. Thus, in fact it is not a new root-case. However, for the sake of developing the argument; we might take this as coming from consensus or even, for the purist reader, he might take *nutmeg* to stand for some place-holder for a real root-case.

The Wine-example and the deployment of mu'āraḍa.
for the refinement of the occasional factor.

O			P	
			(Consuming) Date-wine (<i>nabīdh</i>) is forbidden (<i>ḥarām</i>) ⁴⁰ . ! $\mathfrak{K}(far)$	0
1	Why? [LQR1]	0	Because of your own words in move 29. <i>sic</i> 29 [LQR1]	30
3	Yes, it is forbidden. ! $\mathfrak{K}_s(a\mathfrak{S}l)$ [LQR2]		Is drinking grape-wine (<i>khamr</i>) forbidden? ! $\mathfrak{K}_s(a\mathfrak{S}l)$? [LQR1]	2 [1]
5	Yes ! $P(a\mathfrak{S}l)$ [LQR3]		Is grape-wine a drink in which there is a <i>euphoric intensity</i> (<i>shiddat muṭriba</i>)? $P(a\mathfrak{S}l)$? [LQR1]	4 [1]
		3,	So, according to your moves 3 and 5, the presence of euphoric intensity occasions the proscription of consuming grape-wine. Right? ! $a\mathfrak{S}l : 'illa^P$ [LQR7]	8
9	How can you conclude that? Justify ! [LQR8]	8	There are good reasons for doing so ! $\mathfrak{K}(a\mathfrak{S}l) (a\mathfrak{S}l : 'illa^{P(@)})$ [LQR8]	10
11	Can you develop those reasons ? Justify @ ! [LQR9]	10	@1 : Before the occurrence of the [euphoric] intensity, the lawfulness of consuming pressed-juice is the object of consensus @2 : After the euphoric intensity occurs [i.e., when it becomes wine] and nothing else occurs its proscription is object of consensus. @3 : When its euphoric intensity falls away [i.e., when it becomes vinegar] and nothing	12

⁴⁰ The original text deploys the word *ḥarām*. This notion, the opposite of *ḥalāl*, refers (in this context) to the interdiction of consuming certain food.

			<p>else falls away it is object of consensus that it should not be forbidden.</p> <p>@4 : @1, @2, @3, lead us to the claim that if we evaluate the force of the euphoric intensity, then we have evaluated the intensity of the proscription In other words, the proscription is dependent upon the presence or not of the euphoric intensity.</p> <p>Therefore, the presence of the <i>hukm</i> is due to the presence of the '<i>illa</i>', and the absence of the <i>hukm</i> is due to its absence.</p> <p>! $\mathfrak{K}(a\mathfrak{S}l)$ ($a\mathfrak{S}l$: '<i>illa</i>^{P(@4 (@1, @2, @3))} [LQR9])</p>	
13	<p>I am not satisfied by the argumentation @. I rather think that the property should be refined to the form <i>P'</i>. That is, the property of (potentially) liberating into the body a toxic amount of euphoric intensity while ingested under normal circumstances.</p> <p>$\forall!$ $a\mathfrak{S}l$: '<i>illa</i>^{P'(@)} [LQR10]</p>	12		
	<p style="text-align: center;">START OF THE SUBPLAY</p> <p>-----</p> <p>15 ! $\mathfrak{K}(a\mathfrak{S}l^*)$ ($a\mathfrak{S}l^* : P^{(\mathfrak{B}1 \dots \mathfrak{B}n)}$)</p> <p>$\mathfrak{B}1$: Nutmeg contains euphoric intensity. Doesn't it?</p> <p>17 ? $a\mathfrak{S}l^* : P$</p> <p>19 But its consumption is not forbidden. Isn't it?</p> <p>? $\sim\mathfrak{K}(a\mathfrak{S}l^*)$</p> <p>21 $\mathfrak{B}2$: Your last assertion seems to contradict the generalization behind your assertion 12: nutmeg is not within its scope. So let us consider the following: The nutmeg does not have the property <i>P'</i> of liberating a harmful amount of euphoric intensity when consumed under normal circumstances. Does it?</p> <p>? $a\mathfrak{S}l^* : \sim P'$</p> <p>23 $\mathfrak{B}3$: Because of your own arguments it is reasonable to assume that the presence of the euphoric intensity is relevant for its interdiction. So, given the ruling of sources, I propose to decline the <i>P'</i> in two parts namely <i>P</i>₁ and <i>P</i>₂, such that the occasioning factor is constituted by substances that show both, presence of euphoric intensity (<i>P</i>₁) and the property of liberating a harmful amount of euphoric intensity when consumed under normal circumstances (<i>P</i>₂).</p> <p>So I propose the following the following</p> <p>$\mathfrak{K}(a\mathfrak{S}l^*)$ ($a\mathfrak{S}l^* : P_1 \wedge P_2^{(\mathfrak{B})}$)</p> <p>(where <i>P</i>₁ \wedge <i>P</i>₂, declines <i>P'</i>)</p> <p>[LQR10]</p>		<p>13 Justify !</p> <p style="text-align: center;">START OF THE SUB-PLAY</p> <p>-----</p> <p>15</p> <p>Justify $\mathfrak{B}!$</p> <p>16</p> <p>Yes it does</p> <p>18 $a\mathfrak{S}l^* : P$</p> <p>20</p> <p>No it is not!</p> <p>$\sim\mathfrak{K}(a\mathfrak{S}l^*)$</p> <p>22</p> <p>I agree</p> <p>$a\mathfrak{S}l^* : \sim P'$</p>	14

	END OF THE SUB-PLAY -----		END OF THE SUB-PLAY -----	
25	Yes it is * $P'(far)$ [LQR5]		23 Is <i>nabīdh</i> a drink in which there is presence of euphoric intensity in terms of P' ? $P'(far)$? [LQR10]	24
27	Indeed $!SF (\forall x : 'illa^{P'(\text{is})}) \exists(x)$ [LQR11]		15 Are you assuming then the interdiction applies to any substance that <i>liberates a harmful amount of euphoric intensity when consumed under normal circumstances</i> ? $Law^{Uṣūl al-fiqh}$? [LQR11]	26
29	Yes it should. $\exists(far)$ [LQR12]		27 So, since there is 'euphoric intensity' in date-wine in the terms of P' , consuming it should also be forbidden $!far : 'illa^{P'(\text{is})}$ [LQR12] [according to structural rule SQI.2.1, this move cannot be challenged, because of move 19]	28

Final remarks and work ahead

The meaning of *ijtihād* in Islamic jurisprudence presupposes that the notion of law is dynamic in nature. This dynamic was performed in the process of the development of *uṣūl al-fiqh* that occurred in the conceptual venue that Young (2017) calls the *dialectical forge*. In such a dialectical setting premises of legal theory were continually produced, tested and reproduced in order to yield a deeper systematization. To put it another way, it seems that the dialectical forge is not only the venue but moreover it is a dialectical engine which powered the process by which the legal theory had been continuously forged and refined. Moreover, different to other dialectical frameworks the focus of the dialectical forge is on developing methods of dialectical interaction aimed at the win of knowledge and meaning, beyond the rhetoric purposes of a legal trial or debate. This gave *jadāl* a crucial epistemological role on the pursuit of truth (see Hallaq (1987)).

In this context Islamic jurists studied and developed several instruments suitable for implementing the dialectical forge. One of the most important of these instruments is *qiyās*, that constitutes the subject of our study. The aim of this form of inference is to provide a rational ground for the application of a *ḥukm* to a given case not yet considered by the original juridical sources. As a product of legal theory shaped by the dialectical forge, it is fair to say that a dialogical framework as the one developed in the present paper provides a suitable setting in order to delve into the structure and meaning underlying the legal notion of *qiyās*. The dialogical framework displays three of the hallmarks of this form of inference.

First, the interaction of heuristic with logical steps. This interaction was displayed by two main steps

- a. finding the root-case from which the occasional factor can be inferred;
- b. linking the root-case logically with the branch-case by means of a generalization that links the occasional factor with the relevant juridical ruling.

Second, the dynamics underlying the extension of the legal terms involved. This dynamics is displayed by the intertwining of confirmations and refutations that contribute to establish the most suitable conclusion in relation to the consideration of a new case.

Third, the unfolding of parallel reasoning as similarity in action. Parallel reasoning is about unfolding the process by the means of which similarity is constituted. All in all argumentation is nothing-more and nothing-less than a collaborative enquiry into the ways of building up those symmetries that ground rationality and harmony within inquisitive interaction.

In short, these three of the hallmarks display what we take to be the main epistemological idea behind the *qiyās*, namely: the open texture of the extension (or range of application) of normative statements.

In order to complete our study about al-Shīrāzī's system of *qiyās*, our forthcoming paper will be concerned with epistemic and dialectical meaning of the two other types of this form of inference, namely: *Qiyās al-Dalāla* (Correlational Inference of Indication) and *Qiyās al-Shabah* (Correlational Inference of Resemblance).

One of the main epistemological results emerging from this initial study is that the different forms of *qiyās* as developed in the context of *fiqh* represent an innovative approach that does not only provide new epistemological insights of legal reasoning in general but they also furnish a fine-grained pattern for *parallel reasoning* that can be deployed in a wide range of problem-solving contexts where degrees of evidence and inferences by drawing parallelisms are relevant. Let us mention here the important work of Bartha (2010), that includes a dialectical device to develop his theory of parallel reasoning as applied to sciences. However, Bartha's (2010, chapter 4) *articulation-model*, is not thoroughly argumentative. The argumentative device does not really deal with the heuristic moves, but rather with the justificatory ones while searching for counterexamples. In contrast, as discussed above, the dialectical framework underlying the notion of co-relational inferences is meaning constitutive. In fact, we are convinced that a comparative study between both paradigms, Bartha's argumentative approach and the *qiyās*-approach, will be beneficial for the development of a general framework of parallel reasoning. The dialogical setting for CTT, this is our last claim, provides a bridge to launch such a study.

Acknowledgments. Herewith we would like to thank Joseph David (Sapir Academic College), Fikri Hafid (EISTI-Paris), Hassan Tahiri (CFCUL-Lisbon), Walter E. Young (Bochum) and Farid Zidani (Alger II) for very helpful comments on an earlier version of the paper.

The authors would like to thank the laboratory STL: UMR-CNRS: 8163, the MESHS-Nord-pas-de Calais, the department of Philosophy of the University of Lille3, for fostering the research in the research axes: *SEMAINO* (STL), *Argumentation* (STL) and *Argumentation, Decision and Action* (MESHS-Nord-pas-de Calais). The second author thanks to his home institution the *Islamic University Antasari* (Banjarmasin, Indonesia: and the Funding of the *Indonesian Ministry of Education* for a grant that made it possible to develop our at the university of Lille 3 and the laboratory STL.

Appendix

The Dialogical Framework: A short Overview

AI.1 Basic notions

The dialogical approach to logic is not a specific logical system but rather a framework rooted on a rule-based approach to meaning in which different logics can be developed, combined and compared. More precisely, in a dialogue two parties argue about a thesis respecting certain fixed rules. The player that states the thesis is called Proponent (**P**), his rival, who contests the thesis is called Opponent (**O**). Dialogues are designed in such a way that each of the plays end after a finite number of moves with one player winning, while the other loses. Actions or moves in a dialogue are often understood as speech-acts involving *declarative utterances or posits and interrogative utterances or requests*. The point is that the rules of the dialogue do not operate on expressions or sentences isolated from the act of uttering them. The rules are divided into particle rules or rules for logical constants (*Partikelregeln*) and structural rules (*Rahmenregeln*). The structural rules determine the general course of a dialogue game, whereas the particle rules regulate those moves (or utterances) that are requests and those moves that are answers (to the requests).⁴¹

Crucial for the dialogical approach are the following points:

1. The distinction between *local* (rules for logical constants) and *global* meaning (included in the structural rules that determine how to play)
2. The player independence of local meaning
3. The distinction between the play level (local winning or winning of a play) and the strategic level (existence of a winning strategy).
4. A notion of demonstration that amounts of building a winning strategy
5. The distinction between material dialogues, dialogues that include a rule allowing copy-cat moves, and dialogues combining both.

AI.2 The standard frame

AI.2.1 Local meaning of the logical constants

Let **L** be a first-order language built as usual upon the propositional connectives, the quantifiers, a denumerable set of individual variables, a denumerable set of individual constants and a denumerable set of predicate symbols (each with a fixed arity).

We extend the language **L** with two labels **O** and **P**, standing for the players of the game, and the two symbols `!' and `?'. When the identity of the player does not matter, we use variables **X** or **Y** (with $X \neq Y$).

A move **M** is an expression of the form `**X**-*e*', where *e* is either of the form `! φ ' for some sentence φ of **L** or of one of the forms specified by the particle rules.

⁴¹ The main original papers are collected in Lorenzen/Lorenz (1978) – see too Lorenz (1970), Lorenzen (1969), Lorenzen/Schwemmer (1975). For an historical overview of the transition from operative logic to dialogical logic see Lorenz (2001). For a presentation about the initial role of the dialogical framework as a foundation for intuitionistic logic, see Felscher (1985). Other papers have been collected more recently in Lorenz (2010a,b). An account of developments since, say, Rahman (1993), can be found in Rahman/Rückert (2001), Rahman/Keiff (2005, 2010) and Keiff (2009), Rahman/Clerbout/Keiff (2009), Beirlaen/Fontaine (2016), Cardascia (2016). For the underlying metalogic see Clerbout (2014a, b, c). For a textbook presentation: Clerbout (2014b), Redmond/Fontaine (2011) and Rückert (2011). For the key role of dialogic in regaining the link between dialectics, games and logic, see Rahman/Tulenheimo (2009), Rahman/Keiff (2010) and Marion/Rückert (2015). Clerbout/Gorisse/Rahman (2011) studied Jain Logic in the dialogical framework. Popek (2011) develops a dialogical reconstruction of medieval *obligationes*. For other books see Redmond (2010) – on fiction and dialogic – Fontaine (2013) – on intentionality, fiction and dialogues – and Magnier (2013) – on dynamic epistemic logic and legal reasoning in a dialogical framework, Nzokou (2013), on dialogic and non-monotonic reasoning in legal debates within oral traditions. The most recent work links dialogical logic and Constructive Type Theory (see, Clerbout/Rahman (2015)), Jovanovic (2015), Dango (2016), Rahman/Clerbout (2013, 2015), Rahman/Clerbout/M^cConaughy (2014), Jovanovic (2013), Rahman/Clerbout/Jovanovic (2015), Rahman/Redmond (2015, 2016)

In dialogical logic, the particle rules are said to state the *local semantics*: what is at stake is only the request and the answer corresponding to the utterance of a given logical constant, rather than the whole context where the logical constant is embedded.

- The standard terminology makes use of the terms *challenge* or *attack* and *defence*. However let us point out that at the local level (the level of the particle rules) this terminology should be devoid of strategic underpinning.

The *particle* (or local) *rules* for standard dialogical games are given in the following table:

Previous move	$\mathbf{X} ! \varphi \wedge \psi$	$\mathbf{X} ! \varphi \vee \psi$	$\mathbf{X} ! \varphi \supset \psi$	$\mathbf{X} ! \neg \varphi$
Challenge	$\mathbf{Y} ?_L$ or $\mathbf{Y} ?_R$	$\mathbf{Y} ? [\varphi, \psi]$	$\mathbf{Y} ! \varphi$	$\mathbf{Y} ! \varphi$
Defence	$\mathbf{X} ! \varphi$ resp. $\mathbf{X} ! \psi$	$\mathbf{X} ! \varphi$ or $\mathbf{X} ! \psi$	$\mathbf{X} ! \psi$	--

For the quantifiers we provide a formulation suitable for the purposes of the present paper and that deploys a simplified form of the one of Constructive Type Theory. In particular, we make explicit the domain over which the quantifiers range.

Previous move	$\mathbf{X} ! (\forall x : D) \varphi$	$\mathbf{X} ! (\exists x x : D) \varphi$
Challenge	$\mathbf{Y} ! a : D$	$\mathbf{Y} ?_{L\exists}$ or $\mathbf{Y} ?_{R\exists}$
Defence	$\mathbf{X} ! \varphi(x / a)$	$\mathbf{Y} ! a : D$ resp. $\mathbf{X} ! \varphi(x / a)$

In this table, a is an individual constant and $\varphi(x / a)$ denotes the formula obtained by replacing every free occurrence of x in φ by a . D is a set that provides the domain of discourse over which the quantification ranges. The predicate is said to hold of the elements of that set.

We distinguish conjunction from disjunction and universal quantification from existential quantification in terms of which player chooses. With conjunction and universal quantification, the **challenger** chooses which formula he asks for. With disjunction and existential quantification, it is the **defender** who can choose between various formulas. Notice that there is no defence in the particle rule for negation.

We add too rules for the operators \mathbf{F} and \mathbf{V} adapted to the purposes of our present paper. In fact these operator is very important for the reconstruction of the distinction between, what we call *cooperative* and *non-cooperative criticism*, to be developed in section three of the main text. .

The operator \mathbf{F} ⁴²

⁴² Cf. Rahman/Rückert (pp. 113-116). The main difference of the present formulation of \mathbf{F} is that here it is the defender of the operator and not the challenger who must play under the *copy-cat* rule. The changes is due to the fact that in the context of the present paper the assertion of $\mathbf{F}\varphi$ occurs only as a *challenge* to a previous move of the Proponent.

In uttering the formula $F\varphi$ the argumentation partner X claims that he can find a counterexample during a play where the antagonist Y asserts φ .

The antagonist Y challenges $F\varphi$ by asserting that φ can be challenged successfully. Thus, the challenge of Y compels Y to open a *subdialogue* where he (Y) utters φ .

$X ! F\varphi$	Challenge	Defence
	$Y ? F$	
	<i>Subdialogue \mathcal{D}_1</i>	<i>Subdialogue \mathcal{D}_1</i>
	$Y ! \varphi$ Y must play under the restriction of the <i>copy-cat rule</i> in the subdialogue	$X ?_\varphi$ (he challenges φ)

In uttering the formula $V\varphi$ the argumentation partner X claims that he can win a play where he (X) asserts φ .

The antagonist Y responds by challenging X to open a *subdialogue* where he (X) defends φ .

$X : V\varphi$	Challenge	Defence
	$Y : ?V$	
	<i>Subdialogue \mathcal{D}_1</i>	<i>Subdialogue \mathcal{D}_1</i>
	$Y ?_\varphi$ (he challenges φ) Y must play under the restriction of the <i>copy-cat rule</i>	$X ! \varphi$

Particle rules provide an abstract description of how the game can proceed locally: they specify the way a formula can be challenged and defended according to its main logical constant. In this way the particle rules govern the local level of meaning. Strictly speaking, the expressions occurring in the table above are not actual moves because they feature formula schemata and the players are not specified. Moreover, these rules are indifferent to any particular situations that might occur during the game. For these reasons we say that the description provided by the particle rules is abstract.

The expressions occurring in particle rules are all move schematas. The words "challenge" and "defence" are convenient to name certain moves according to their relation with other moves which can be defined in the following way.

- Let σ be a sequence of moves. The function ρ_σ assigns a position to each move in σ , starting with 0.
- The function F_σ assigns a pair $[m, Z]$ to certain moves M in σ , where m denotes a position smaller than $\rho_\sigma(M)$ and Z is either C or D , standing respectively for "challenge" and "defence". That is, the function F_σ keeps track of the relations of challenge and defence as they are given by the particle rules.

Because of our deployment expressions coming from Constructive-Type Theory the language contains expressions such as the following (further expressions are provided in the section on terminology in the main text)

$X ! a : B$	Player X claims that a instantiates B/a provides evidence for B .
$X ! B (A)$	Player X claims that there is evidence for B dependent upon evidence for A .

$X \vdash B(a) \ (a : A)$ Payer X claims that there is evidence for a being B given that there is evidence for a being (and element of) A

AI.2.2 Global meaning

As mentioned above global meaning is defined by means of *structural rules* that determine the general development of the plays, by specifying who starts, what are the allowed moves and in which order, when does a play end and who wins. The structural rules include the following rule on elementary expressions, i.e., expressions of one of the forms $a : B, a : B(c), A, B$:

- **P** may not utter an elementary expression unless **O** uttered it first. Elementary expressions cannot be challenged.

This, rule is one of the most salient characteristics of dialogical logic. As discussed by Marion / Rückert (2016), it can be traced back to Aristotle's reconstruction of the Platonic Dialectics: the main idea is that, when an elementary expression is challenged then, from the purely argumentative point of view – that is, without making use of an authority beyond the moves brought forward during an argumentative interaction-, the only possible response is to appeal to the concessions of the challenger. In fact, one could see the Copy-cat rule as allowing copy-cat moves such as:

*my grounds for the proposition you are asking for are exactly the same as the ones you bring forward when you conceded the same proposition.*⁴³

In previous literature on dialogical logic this rule has been called the *copy-cat rule* or *Socratic rule*. Now, if the ultimate grounds of a dialogical thesis are elementary propositions and if this is implemented by the use of the copy-cat rule, then the development of a dialogue is in this sense necessarily asymmetric. Indeed, if both contenders were restricted by the copy-cat rule no elementary proposition can ever be uttered. Thus, we implement the copy-cat rule by designing one player, called the *Proponent*, whose utterances of elementary propositions are, restricted by this rule. It is the win of the Proponent that provides the dialogical notion of validity. More precisely, in the dialogical approach validity is defined via the notion of *winning strategy*, where winning strategy for X means that for any choice of moves by Y, X has at least one possible move at his disposal such that he (X) wins:

Validity (definition): A proposition is valid in a certain dialogical system iff **P** has a winning strategy for this formula.

Before providing the structural rules let us precise the following notions:

Play: A *play* is a legal sequence of moves, i.e., a sequence of moves which observes the game rules. Particle rules are not the only rules which must be observed in this respect. In fact, it can be said that the second kind of rules, namely, the *structural rules* are the ones giving the precise conditions under which a given sequence is a play.

Dialogical game: The *dialogical game* for φ , written $D(\varphi)$, is the set of all plays with φ being the *thesis* (see the Starting rule below).

The *structural rules* are the following:⁴⁴

SR0 (Starting rule). Any dialogue starts with the Opponent positing initial concessions, if any, and the Proponent positing the thesis. After that the players each choose a positive integer called *repetition rank*.

- The *repetition rank* of a player bounds the number of challenges he can play in reaction to a same move.

⁴³ Cf. Rahman/Clerbout/Keiff (2009) and in Rahman/Keiff (2010).

⁴⁴ For a formal formulation see Clerbout (2014a,b,c)

SR1i (Classical game-playing rule). Players move alternately. After the repetition ranks have been chosen, each move is a challenge or a defence in reaction to a previous move and in accordance with the particle rules.

SR1i (Intuitionistic game-playing rule). Players move alternately. After the repetition ranks have been chosen, each move is a challenge or a defence in reaction to a previous move and in accordance with the particle rules.

Players can answer only against the *last non-answered* challenge by the adversary.⁴⁵

SR2 (Copy-cat rule).

P may not utter an elementary proposition unless **O** uttered it first. Elementary propositions cannot be challenged.

Remark: This formulation of the rule has the problem that elementary propositions cannot be set as thesis of a dialogical game. This motivated to use the following rule that will a main subject of our discussion on equality.

Modified Copy-cat rule. **O**'s elementary sentences cannot be challenged. However, **O** can challenge a **P**-elementary move. The challenge and correspondent defence is ruled by the following table.

Posit	Challenge	Defence
$\mathbf{P} ! a : A$ (for elementary A)	$\mathbf{O} ?$	$\mathbf{P} \text{ sic } (n)$ (\mathbf{P} indicates that \mathbf{O} posited a : A at move n)

The last structural rule requires some additional terminology:

- **Terminal play:** A play is called *terminal* when it cannot be extended by further moves in compliance with the rules.
- **X-terminal:** We say it is *X-terminal* when the last move in the play is an **X**-move.

SR3 (Winning rule). Player **X** wins the play ζ only if it is **X**-terminal.

Strategy: A *strategy* for player **X** in $\mathbf{D}(\phi)$ is a function which assigns an **X**-move M to every non terminal play ζ having a **Y**-move as last member such that extending ζ with M results in a play.

X-winning-strategy: An *X-strategy* is *winning* if playing according to it leads to **X**-terminal play no matter how **Y** moves.

For the description of a winning strategy it is usual to deploy the following notions:

Extensive form of a dialogical game: The *extensive form* $\mathbf{E}(\phi)$ of the dialogical game $\mathbf{D}(\phi)$ is simply the tree representation of it, also often called the game-tree. Nodes are labelled with moves so that the root is labelled with the thesis, paths in $\mathbf{E}(\phi)$ are linear representations of plays and maximal paths represent terminal plays in $\mathbf{D}(\phi)$.

The extensive form of a dialogical game is thus an infinitely generated tree where each branch is of finite length. Many dialogical game metalogical results are obtained by leaving the level of rules and plays to move to the

⁴⁵ This last clause is known as the *Last Duty First* condition, and is the clause making dialogical games suitable for Intuitionistic Logic, hence the name of this rule.

level of strategies. Significant among these results are the ones concerning the existence of winning strategies for a player.

Strategy: A *strategy* for player \mathbf{X} in $D(\varphi)$ is a function which assigns an \mathbf{X} -move M to every non terminal play ζ having a \mathbf{Y} -move as last member such that extending ζ with M results in a play.

X -winning-strategy: An X -strategy is *winning* if playing according to it leads to \mathbf{X} 's victory no matter how \mathbf{Y} plays.

Also, strategies can be considered from the perspective of extensive forms:

Extensive form of an X -strategy: The *extensive form of an X -strategy* s in $D(\varphi)$ is the tree-fragment $S_\varphi = (T_s, \perp_s, S_s)$ of E_φ such that:

- i) The root of S_φ is the root of E_φ ,
- ii) Given a node t in E_φ labelled with an \mathbf{X} -move, we have $t' \in T_s$ and $tS_s t'$ whenever tSt' .
- iii) Given a node t in E_φ labelled with a \mathbf{Y} -move and with at least one t' such that tSt' , we have a unique $s(t)$ in T_s with $tS_s s(t)$ and $s(t)$ is labelled with the \mathbf{X} -move prescribed by s .

Definition 4 Let s_x be a strategy of player \mathbf{X} in $D(\varphi)$ of extensive form $E(\varphi)$. The extensive form of s_x is the fragment S_x of $E(\varphi)$ such that:

1. The root of $E(\varphi)$ is the root of S_x ,
2. For any node t which is associated with an \mathbf{X} -move in $E(\varphi)$, any immediate successor of t in $E(\varphi)$ is an immediate successor of t in S_x ,
3. For any node t which is associated with a \mathbf{Y} -move in $E(\varphi)$, if t has at least an immediate successor in $E(\varphi)$ then t has exactly one immediate successor in S_x namely the one labelled with the \mathbf{X} -move prescribed by s_x .

Here are some results pertaining to the level of strategies:⁴⁶

- **Winning P -strategies and leaves.** *Let w be a winning P -strategy in $D(\varphi)$. Then every leaf in the extensive form W_φ of w is labelled with a P elementary sentence.*
- **Determinacy.** *There is a winning \mathbf{X} -strategy in $D(\varphi)$ if and only if there is no winning \mathbf{Y} -strategy in $D(\varphi)$.*
- **Soundness and Completeness of Tableaux.** *Consider first-order tableaux and first-order dialogical games. There is a tableau proof for φ if and only if there is a winning P -strategy in $D(\varphi)$.*
The fact that existence of a winning P -strategy coincides with validity (*there is a winning P -strategy in $D(\varphi)$ if and only if φ is valid*) follows from the soundness and completeness of the tableau method with respect to model-theoretical semantics.

⁴⁶ These results are proven, together with others, in Clerbout (2014a).

References

- Aristotle (1984). In Barnes (1984).
- J. L. Austin (1946). "Other Minds". *The Aristotelian Society Supplementary Volume*, 20, pp. 148–187
- J. Barnes (1984). *The Complete Works of Aristotle. The Revised Oxford Translation*, ed. Princeton NJ: Princeton University Press.
- P. F. A. Bartha (2010). *By Parallel Reasoning. The Construction and Evaluation of Analogical Arguments*. Oxford: Oxford University Press.
- M. Beirlaen and M. Fontaine (2016). "Inconsistency-Adaptive Dialogical Logic". *Logica Universalis*. Online first, DOI 10.1007 / s11787-016-0139-y.
- B. Castelnérac and M. Marion (2009). "Arguing for Inconsistency: Dialectical Games in the Academy". In G. Primiero and S. Rahman (eds.), pp. 37-76.
- N. Clerbout (2014a). "First-order dialogical games and tableaux". *Journal of Philosophical Logic*, 43(4), 785-801.
- N. Clerbout (2014b). *Etude sur quelques sémantiques dialogiques: Concepts fondamentaux et éléments de métathéorie*. London : College Publications.
- N. Clerbout (2014c). "Finiteness of Plays and the Dialogical Problem of Decidability". *IfCoLog Journal of Logics and their Applications*, 1(1), pp. 115–130.
- N. Clerbout, M.H. Gorisse, and S. Rahman (2011). "Context-sensitivity in Jain philosophy: A dialogical study of *Siddharsiganis Commentary on the Handbook of Logic*". *Journal of Philosophical Logic*, 40(5), pp. 633–662.
- N. Clerbout and S. Rahman (2015). *Linking Game-Theoretical Approaches with Constructive Type Theory: Dialogical Strategies as CTT-Demonstrations*. Dordrecht: Springer.
- M. Crubellier (2008). "The Programme of Aristotelian Analytics". In C. Dégremont, L. Keiff, H. Rückert (eds.) *Dialogues, Logics and Other Strange Things. Essays in honour of Shahid Rahman*. London: College Publications, pp. 103-129.
- M. Crubellier (2014). *Aristote, Premiers Analytiques ; traduction, introduction et commentaire*. Paris: Garnier-Flammario.
- A. B. Dango (2016). *Approche dialogique de la révision des croyances dans le contexte de la théorie constructive des types*. London: College Publications.
- J. David (2010). "Legal Comparability and Cultural Identity: The Case of Legal Reasoning in Jewish and Islamic Traditions". *Electronic Journal of Comparative Law*, vol. 14.1, <http://www.ejcl.org>.
- J. David (2014). *Jurisprudence and Theology*. Dordrecht: Springer.
- C. Dutilh Novaes (2015). "A Dialogical, Multi-Agent Account of the Normativity of Logic". *Dialectica*, 69, N° 4 (2015), pp. 587–609.
- W. Felscher (1985). "Dialogues as a foundation for intuitionistic logic". In D. Gabbay and F. Guenther (eds.), *Handbook of Philosophical Logic*, Dordrecht: Kluwer, vol. 3, pp. 341–372.
- M. Fontaine (2013). *Argumentation et engagement ontologique. Être, c'est être choisi*. London: College Publications.
- J. Granström (2011). *Treatise on Intuitionistic Type Theory*. Dordrecht: Springer.
- W. Hallaq (1985). "The Logic of Legal Reasoning in Religious and Non-Religious Cultures: The Case of Islamic Law and Common Law". *Cleveland State Law Review*, 34, pp. 79-96.
- W. Hallaq (1987a). "A Tenth-Eleventh Century Treatise on Juridical". *The Muslim World*, 77 3-4, pp. 151-282.
- W. Hallaq (1987b). "The Development of Logical Structure in Islamic Legal Theory," *Der Islam*, 64/1, pp. 42-67.
- W. Hallaq (1993). *Ibn Taymiyya against the Greek Logicians*. Oxford: Clarendon Press.
- W. Hallaq (1997). *A History of Islamic Legal Theories: An Introduction to Sunnī Uṣūl al-Fiqh*. Cambridge, N. York: Cambridge U. Press.
- W. Hallaq (2009a). *The Origins and Evolution of Islamic Law*. Cambridge, N. York: Cambridge U. Press.
- W. Hallaq (2009b). *Sharī'a: Theory, Practice, Transformation*. Cambridge, N. York: Cambridge U. Press.
- R. Jovanovic. (2013) "Hintikkas Take on the Axiom of Choice and the Constructivist Challenge". *Revista de Humanidades de Valparaíso*, 2, pp. 135-152.
- R. Jovanovic. (2015) *Hintikkas Take on Realism and the Constructivist Challenge*. London: College Publications.
- L. Keiff (2007). *Le Pluralisme Dialogique: Approches dynamiques de l'argumentation formelle*. Lille: PhD - thesis -, Lille 3.
- L. Keiff (2009). "Dialogical Logic". In Edward N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*, URL <http://plato.stanford.edu/entries/logic-dialogical/>.
- K. Lorenz (1970). *Elemente der Sprachkritik. Eine Alternative zum Dogmatismus und Skeptizismus in der Analytischen Philosophie*. Frankfurt: Suhrkamp.

- K. Lorenz (2001). "Basic objectives of dialogue logic in historical perspective". In S. Rahman and H. Rückert (eds.), 127 (1-2), pp. 255–263.
- K. Lorenz (2010a). *Logic, Language and Method: On Polarities in Human Experience*. Berlin / New York: De Gruyter.
- K. Lorenz (2010b). *Philosophische Variationen: Gesammelte Aufsätze unter Einschluss gemeinsam mit Jürgen Mittelstraß geschriebener Arbeiten zu Platon und Leibniz*. Berlin and New York: De Gruyter.
- P. Lorenzen (1969). *Normative Logic and Ethics*, Mannheim / Zürich: Bibliographisches Institut.
- P. Lorenzen and O. Schwemmer (1975) *Konstruktive Logik, Ethik und Wissenschaftstheorie*. Mannheim: Bibliographisches Institut, second edition.
- P. Lorenzen and K. Lorenz (1978). *Dialogische Logik*. Darmstadt: Wissenschaftliche Buchgesellschaft.
- S. Magnier (2013). *Approche dialogique de la dynamique épistémique et de la condition juridique*. London: College Publications.
- M. Marion and H. Rückert (2015). "Aristotle on Universal Quantification: A Study from the Perspective of Game Semantics". *History and Philosophy of Logic*, Online first, DOI: 10.1080 / 01445340.2015.1089043.
- P. Martin-Löf (1971). "Hauptsatz for the intuitionistic theory of iterated inductive definitions. In Fenstad, J. E. (ed.), *Proceedings of the Second Scandinavian Logic Symposium*. Amsterdam: North-Holland, pp. 179–216.
- P. Martin-Löf (1975a). "About models for intuitionistic type theories and the notion of definitional equality". In Kanger, S. (ed.), *Proceedings of the Third Scandinavian Logic Symposium*. Amsterdam: North-Holland, pp. 81–109.
- P. Martin-Löf (1975b). "An intuitionistic theory of types: Predicative part". In Rose, H. E. and Shepherdson, J. C. (eds.), *Logic Colloquium '73*. Amsterdam: North-Holland, pages 73–118.
- P. Martin-Löf (1982). "Constructive mathematics and computer programming". In Cohen, J. L., Los', J., et al. (eds.), *Logic, Methodology and Philosophy of Science VI, 1979*, pages 153–175. North-Holland, Amsterdam.
- P. Martin-Löf (1984). *Intuitionistic Type Theory. Notes by Giovanni Sambin of a series of lectures given in Padua, June 1980*. Naples: Bibliopolis.
- P. Martin-Löf (1992). "Substitution calculus". Lecture notes. Available at <http://archive-pml.github.io/martin-lof/pdfs/Substitution-calculus-1992.pdf>.
- P. Martin-Löf (1996). "On the meanings of the logical constants and the justifications of the logical laws". *Nordic Journal of Philosophical Logic*, 1, pp. 11–60.
- P. Martin-Löf (2006). "100 years of Zermelo's axiom of choice: what was the problem with it?". *The Computer Journal*, 49 / 3, pp. 345–350.
- P. Martin-Löf (2011). "When did 'judgement' come to be a term of logic?". Lecture held at *Ecole Normale Supérieure* on 14 October 2011. Video recording available at <http://savoirs.ens.fr/expose.php?id=481>.
- L. B. Miller, (1985). *Islamic Disputation Theory: a Study of the Development of Dialectic in Islam from the Tenth Through Fourteenth Centuries*. Princeton: Princeton University. (Unpublished dissertation)
- B. Nordström, K. Petersson, and J. M. Smith (1990). *Programming in Martin-Löf's Type Theory: An Introduction*. Oxford: Oxford University Press.
- B. Nordström, K. Petersson, and J. M. Smith (2000). "Martin-Löf's type theory". In Abramsky, S., Gabbay, D., and Maibaum, T. S. E. (eds.), *Handbook of Logic in Computer Science. Volume 5: Logic and Algebraic Methods*. Oxford: Oxford University Press, pp. 1–37.
- C. Dutilh Novaes (2007). *Formalizing Medieval Logical Theories*. Dordrecht: Springer.
- C. Dutilh Novaes (2015). "A Dialogical, Multi-Agent Account of the Normativity of Logic". *Dialectica*, 69, N° 4 (2015), pp. 587–609.
- G. Nzokou (2013). *Logique de l'argumentation dans les traditions orales africaines*. London: College Publications.
- A. Popek (2012). "Logical dialogues from Middle Ages". In C. Barés Gómez, S. Magnier and F. J. Salguero (eds.), *Logic of Knowledge. Theory and Applications*, London: College Publications, pp. 223–244.
- G. Primiero (2008). *Information and Knowledge*. Dordrecht: Springer.
- G. Primiero and S. Rahman (eds.) (2009). *Acts of Knowledge: History, Philosophy and Logic*. London: College Publications.
- W. V. Quine (1969). *Ontological Relativity and Other Essays*. New York: Columbia University Press.
- S. Rahman (1993). *Über Dialogue, Protologische Kategorien und andere Seltenheiten*. Frankfurt / Paris / New York: P. Lang.

- S. Rahman and N. Clerbout (2013). "Constructive Type Theory and the Dialogical Approach to Meaning". *The Baltic International Yearbook of Cognition, Logic and Communication: Games, Game Theory and Game Semantics*, 8, pp. 1-72. Also online in: www.thebalticyearbook.org.
- S. Rahman and N. Clerbout (2015). "Constructive Type Theory and the Dialogical Turn – A new Approach to Erlangen Constructivism". In J. Mittelstrass and C. von Bülow (ed.), *Dialogische Logik*, Münster: Mentis, pp. 91-148.
- Rahman, S., N. Clerbout, and L. Keiff (2009). "On dialogues and natural deduction". In G. Primiero and S. Rahman (eds.) *Acts of Knowledge: History, Philosophy and Logic: Essays Dedicated to Göran Sundholm*, London: College Publications, pp. 301–336.
- S. Rahman, N. Clerbout and R. Jovanovic (2015). "The Dialogical Take on Martin-Löf's Proof of the Axiom of Choice". *South American Journal of Logic*, 1(1), pp. 179-208.
- S. Rahman, N. Clerbout, and Z. M^cConaughey (2014) "On play-objects in dialogical games. Towards a dialogical approach to Constructive Type Theory". In P. Allo / V. v. Kerkhove (ed.), *Modestly radical or radically modes . Festschrift for Jean-Paul van Bendegem*, London: College Publications, pp. 127-154.
- S. Rahman, J. Granström , Z. Salloum (2014). "Ibn Sīnā's Approach to equality and unity". *Cambridge Journal for Arabic Sciences and Philosophy*, 4(02), pp. 297-307.
- S. Rahman and L. Keiff (2005). "On how to be a dialogician". In D. Vanderveken (ed.), *Logic, Thought, and Action*, Dordrecht: Kluwer, pp. 359–408.
- S. Rahman and L. Keiff (2010). "La Dialectique entre logique et rhétorique". *Revue de Métaphysique et de Morale*, 66(2), pp. 149–178.
- S. Rahman and J. Redmond (2015). "A Dialogical Frame for Fictions as Hypothetical Objects". *Filosofia Unisinos*, 16(1), pp. 2-21.
- S. Rahman and J. Redmond (2016) . "Armonía Dialógica. Tonk, Teoría Constructiva de Tipos y Reglas para Jugadores Anónimos" (Dialogical Harmony: Tonk, Constructive Type Theory and Rules for Anonymous Players"). *Theoria*. S. Rahman/J. Redmond. (ISI), 31/1 (2016), pp. 27-53.
- S. Rahman and H. Rückert (eds.) (2001). *New Perspectives in Dialogical Logic*, special volume *Synthese* 127, Dordrecht: Springer.
- S. Rahman, T. Street and H. Tahiri (eds.) (2008). *The Unity of Science in the Arabic Tradition*. Dordrecht: Springer
- S. Rahman and T. Tulenheimo (2009). "From Games to Dialogues and Back: Towards a General Frame for Validity". In O. Majer. A. Pietarinen and T. Tulenheimo (eds.), *Games: Unifying Logic, Language and Philosophy*, Dordrecht: Springer, pp. 153–208.
- A. Ranta (1988). "Propositions as games as types". *Synthese*, 76, pp. 377–395.
- A. Ranta (1994). *Type-Theoretical Grammar*. Oxford: Clarendon Press.
- J. Redmond (2010) *Logique dynamique de la fiction: Pour une approche dialogique*. London: College Publications.
- J. Redmond and M. Fontaine (2011). *How to Play Dialogues: An Introduction to Dialogical Logic*. London: College Publications.
- H. Rückert (2011). *Dialogues as a Dynamic Framework for Logic*. London: College Publications.
- Abū Ishāq al-Shīrāzī's (1986, Kuwait ed.). *Mulakhkhasfi'l-Jadal (Epitome on Dialectical Disputation)*.
- G. Sundholm (1997) "Implicit epistemic aspects of constructive logic". *Journal of Logic, Language, and Information* 6:2, pp. 191-212.
- G. Sundholm (1998). "Inference versus Consequence". In T. Childers (ed.), *The Logica Yearbook 1997*, Prague: Filosofia, pp. 26–36.
- G. Sundholm (2001). "A Plea for Logical Atavism". In O. Majer (ed.), *The Logica Yearbook 2000*, Prague: Filosofia, pp. 151-162.
- Sundholm, B. G. (2006). "Semantic values for natural deduction derivations". *Synthese*, 148:623–638.
- Sundholm, B. G. (2009). "A century of judgement and inference, 1837-1936: Some strands in the development of logic". In Haaparanta, L. (ed.), *The Development of Modern Logic*, pages 263–317. Oxford: Oxford University Press.
- G. Sundholm (2012). " "Inference versus consequence" revisited: inference, conditional, implication" *Synthese*, 187, pp. 943-956.
- G. Sundholm (2013). " Inference and Consequence in an Interpreted Language". Talk at the Workshop PROOF THEORY AND PHILOSOPHY, Groningen, December 3-5, 2013.
- G. Sundholm (2016). " Independence Friendly Language is First Order after all?". *Logique et Analyse*, forthcoming.

- H. Tahiri (2008). "The Birth of Scientific Controversies: The Dynamic of the Arabic Tradition and its Impact on the Development of Science: Ibn al-Haytham's Challenge of Ptolemy's *Almagest*". In S. Rahman, T. Street and H. Tahiri (eds), 2008, Springer, pp. 183-225.
- H. Tahiri (2014). "Al Kindi and the Universalization of Knowledge through Mathematics". *Revista de Humanidades de Valparaíso*, 4, pp. 81-90.
- H. Tahiri (2015). *Mathematics and the Mind. An Introduction to Ibn Sīnā's Theory of Knowledge*. Dordrecht: Springer.
- H. Tahiri (2016). "When the Present Misunderstands the Past. How a Modern Arab Intellectual Reclaimed his Own Heritage". *Cambridge Journal for Arabic Sciences and Philosophy*, **accepted**.
- H. Tahiri, S. Rahman and T. Street (2008). "Introduction: The Major Breakthrough in Scientific Practice". In Rahman/Street/Tahiri (eds.) 2008, pp. 1-40.
- S. Thompson (1991). *Type Theory and Functional Programming*. Boston: Addison-Wesley.
- W. E. Young (2017). *The Dialectical Forge. Juridical Disputation and the Evolution of Islamic Law*. Dordrecht: Springer. IN PRINT.