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Demand shocks, new keynesian model and supply effects of monetary policy

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ABSTRACT

Demand shocks likely play a key role in driving business cycles. However, in the standard new keynesian model, the monetary policy reaction to these shocks have a supply side effect. The change in real rate affects the marginal utility of consumption generating an income effect on labor supply. Wages, inflation and through monetary policy, aggregate demand will increase. This supply side effect have a surprising importance for the model, especially when the sensitivity of aggregate demand to interest rate is low. A demand shock will have a large impact (close to one) on output, but a very small one on the output gap. The limited monetary policy movement induced by the taylor rule remains very close to the natural rate of interest. There are nearly no differences between the sticky price and the flexible price model. It represents a very disappointing result, the entire purpose of sticky prices being to generate inefficiencies when the aggregate demand is hit. Coupled with very tiny empirical support for this supply side effect of monetary policy, it suggests to explore the theoretical possibilities to kill this effect. First, we review the two ways the literature have proposed, nonseparable preferences and sticky wages. The main drawback is a strong reliance on very specific assumption for the labor market. We explore an alternative approach. We attempt a radical departure from traditionnal assumption about the optimizing behavior of the representative agent. Instead of optimizing simulatneously with respect to hours, consumption and saving, the household decomposes the problem in two steps. First, the agent chooses between labor income and hours. Second, he optimizes between consumption and saving. The interest is to disentangle the income effect which affects the labor equation and those affecting the intertemporal choice. Thus it is possible to reduce the wealth effect on labor supply whereas keeping a low sensitivity of consumption to interest rate. This flexible approach also allows to challenge the effect of interest rate on wealth offering a potential explanation for small effects of interest rate on both labor supply and consumption whereas keeping large income effects.

JEL Classification: D01,D03,E21,E24,E32

Keyword: Demand shock, comovement, labor supply, elasticity of intertemporal substitution, wealth effect, hand-to-mouth consumers, sticky wages

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1 Introduction

In keynesian and new keynesian views, business cycles are mainly driven by changes in aggregate demand or "demand shocks". The intuition is well understood. Prices and wages are sticky and firm output is demand determined. A rise in aggregate demand will lead to an increase in aggregate supply instead of increasing prices. This idea has been backed by a variety of empirical evidences. Blanchard and Quah (1989) show in a VAR model with long run restrictions that demand shocks accounts for two third of output variance at three years horizon and more than 85% of the employment variance. The recent wave of Bayesian DSGE models also support a prominent role for demand, especially investment. Smets and Wouters (2007) model suggest that markup wage shock drives output variation, but historical decomposition shows the result is mainly due to the 70-80 period of inflation and disinflation. True recessions are mostly generated by demand shocks. In following models like Justiniano-Primiceri-Tambalotti (2011) or Christiano Motto Rostagno (2014), demand shocks play even a larger role. Business cycles is mostly driven by investment fluctuation explained either by a generic investment specific technologic shock or by fluctuations in financial conditions. Beside these macroeconomic evidences, the new keynesian mechanism have also received some empirical support. Prices and wage stickiness have been supported by Bils and Klenow (2014), Kehoe and Midrigan (2010), and Barattieri, Basu and Gottschalk (2014). Bils and Klenow (2013) also found evidences for a keynesian labor demand. The value of these very stylized evidences should not be overestimated but they make the case for studying demand driven business cycles.

An increase in aggregate demand have several effects in the new keynesian model. First, there is the direct effect on output. Firms increase their production when their demand increases. Beside this direct effect, there is an indirect channel through monetary policy. This channel can be decomposed in two effects, one demand side effect and one less known supply side effect.

The demand side is well understood. The rise in real rate creates an incentive to postpone consumption and increase savings. The aggregate demand falls, dampening the initial impulse of the investment shock. This is the well known crowding out effect. This dampening effect is a concern for the NK model. because it reduces the effect of demand shocks, It makes much harder for them to be a credible candidate for generating business cycles. Limiting the sensitivity of aggregate demand to interest rate is an obvious solution. The elasticity of intertemporal substitution (EIS thereafter) is lowered. When the EIS approaches zero, the consumption becomes unresponsive to real rate moves and the crowding out effect collapses. Perhaps surprisingly, this solution have not attracted a lot of interest in modern theory. Whereas keynes itself was quite skeptical about very large effects of monetary policy, small effect of real rate and thus monetary policy on aggregate demand have not been very popular. Yet such assumption would not be unreasonable on empirical ground. There are actually very few evidences for a large impact of interest rate and monetary policy on consumption choice. There is a wide literature aiming

to test euler equation since the seminal paper of Hall (1988). Campbell and Mankiw (1991) rejects the permanent income hypothesis and suggest that a substantial fraction of the population does not behave intertemporally whereas the other does not substitute intertemporally. More recently, Canzoneri et al. (2007) finds that consumption growth is negatively correlated with the FED funds rate for many specifications of the euler equation. Yogo (2004) finds an EIS no significantly different from zero for eleven countries. Evidences for a strong sensitivity of corporate investment to interest rate or asset prices are also not very convincing (see for example Fazzari, Hubbard and Petersen (1988) or Chirinko (1993)). It is true that other literature have found strong effects of monetary policy to interest rate. Romer and Romer (1990,2003) , Barakchian and Crowe (2011), Gertler and Karadi (2015) have found relatively large effects of monetary shocks using "unconventional" identification method. They do not use only macro information but either narrative information using FOMC reports or high frequency financial information. A second bunch of literature have extensively studied transmission channels and especially the credit channel. They have shown a very large sensitivity of housing and small business investment to interest rate. It suggests credit conditions offered by banks reacts strongly and even overreact to changes in short term interest rate. Whereas I am aware of these facts, I want to focus on more "traditional" component of aggregate demand. If the effects of lower real interest rate are strongly concentrated on a small part of aggregate demand, they can exhaust quickly. Moreover, problems in banking system may also limit the transmission for example in case of financial crisis. Thus, the low sensitivity of overall aggregate demand is not an illegitimate subject of study. Nearly a decade of zero or negative real rates suggest that their effects could be much more limited than previously believed. I think that the hypothesis of small effects of real interest rate on aggregate demand deserves deeper investigation.

However when considering the basic new keynesian model with low sensitivity of aggregate demand to interest rate, a striking issue emerges. Beside the crowding out effect, monetary policy have a supply side effect. Higher interest rate reduces wealth which creates an incentive to work more. This effect works in three steps. First, real interest rate changes have a one for one impact on the marginal utility of consumption growth rate. The changes in marginal utility of consumption creates an income effect on labor supply. The shift in the labor supply curve modifies wages, marginal cost and inflation triggering a monetary policy reaction and a change in aggregate demand. The EIS does not affect the supply effect because the marginal utility of consumption is independent from him. Thus, even when the demand side effect of monetary policy is cancelled, this supply side effect remains. Whereas this effect has a limited impact on the output reaction to demand shock, it has a considerable one on the **output gap** reaction. With this effect, the response of the output in the sticky price case remains surprisingly very close to the reaction of the flexible price model, whatever the EIS is. The output gap generated by a demand shock remains very limited. The real interest rate generated by the taylor rule remains very close to the natural interest rate which is bounded below. In a nutshell, with this effect, demand driven fluctuations are stronger but remains near optimal and a taylor rule

monetary policy remains very close to the natural rate of interest. By contrast, if you suppress this effect by removing the marginal utility of consumption term in the labor supply equation, you get very different results. When the consumption remains strongly sensitive to interest rate, the crowding out effect plays. Demand shocks are partially dampens. Output gap is equal to the output reaction because response to demand shock under flexible prices is zero without that effect. But when the EIS becomes very small, the output gap approaches one meaning that the whole shock translates into output gap. The monetary policy is no longer close to the natural interest rate which approaches minus infinity. These results suggest that the supply side effect of monetary policy plays a much more important role in the new keynesian model than the literature have acknowledged until now. It is one of the two effect with the crowding out effect which keeps the new keynesian model very close to the real business cycles model. Whereas the new keynesian model have succeeded in showing that nominal disturbances can play a role, he does not present a considerable change compared to the other key characteristic of Real Business Cycles, the optimality of fluctuations. Crowding out can be easily scaled by manipulating the EIS, but the supply effect is independant from this parameter.

Given its importance for the now standard macroeconomic model, the supply side effect of monetary policy should have been the object of a very large empirical literature. This is not really the case. At our knowledge, there is nothing in the monetary polciy transmission channel literature about that effect. On the labor economic side, the effects of interest rate are not taken very seriously. According to the theory, wages and interest rate have nearly similar impact on labor supply at business cycles frequencies. To give an idea, there are two entries for "interest rate" in the Cahuc "labor economics" index (about unrelated effects). The occurence of the word "wages" fill a whole page of the index.

In this paper, my goal is to propose a solution to kill this supply side effect. The supply side effect can be decomposed in three steps and each step can be cancelled. Some possibilities can be found in the literature. Sticky wages means that labor supply have no effect on output. Non separable preferences challenges the relation between marginal utility of consumption or wealth and the income effect on labor. I review these solutions in the third section and highlight some drawbacks. I look at a more intuitive solution. I think that the first step, the link between interest rate and marginal utility of consumption or wealth is the main issue. Real interest rate have an effect on labor supply in the model because they modify the perceived labor wealth of households. Labor wealth is the discounted sum of labor income. Whereas it does not appear explicitly in the model, it is very important. It represents the household true intertemporal budget, the income in the sense of the consumer microeconomic theory. So, a temporary fall in real interest rate have an immediate positive impact on the income of households. It relaxes the intertemporal budget constraint, inducing more consumption and more leisure. However, this effect requires that households compute their labor by using the product of future real interest rate as a discount factor. This is the more rational behavior but a very impractical one. The household should guess the whole path of future interest rate. But, assume now they have more

limited capabilities and use a constant discount rate, for example their rate of time preference. You can still have a wealth effect on labor supply. For example, labor tax changes still have an income effect on labor supply. But the temporary fall in real interest rate will not result in a lower discount rate. Perceived wealth remains unchanged and hours and consumption are not stimulated. The interest of that approach would be to provide an explanation for both limited effects on leisure and on consumption. However, discounting with interest rate is implied by the standard intertemporal optimization behavior.

To solve the issue, I explore a radical departure from the traditional optimization framework. Our goal is to separate completely the labor choice and the consumption choice. To do so, the representative agent will choose in two steps. First, he will choose labor supply by comparing disutility of labor with the marginal utility of labor wealth or income. In a second step, given the chosen level of hours, he will choose between present consumption and future one. This approach is equivalent to the traditional one if the marginal utility of labor wealth is equal to the marginal utility of present consumption. But, we do not impose such restriction. It allows us to separate completely the parameters governing the intertemporal choice and those governing the labor supply choice. this method introduces several ways to kill the wealth effect on labor supply whereas keeping something close to an euler equation. but, we can freely parametrize the effect of wealth on labor supply without affecting the consumption equation.

This framework also allows us to directly challenge the effect on interest rate on the perceived wealth. unlike the standard model, wealth appears explicitly. Instead of discounting using the product of real rates induced by monetary policy, agents will use a constant discount factor to compute it. It cancels the effect of interest rate on wealth offering an explanation for the low impact of interest rate on both hours and consumption.

In a first part, we show that under separable preferences and with a wealth effect on labor supply, the economy behaves in a very similar way conditionnally to demand shocks for both flexible and sticky prices. In the second one, we present our optimization framework and how it affects the macroeconomic model. In a third part, we review the other solutions like sticky wages or nonseparable preferences and I discuss some robustness issues.

2 Demand shocks, low elasticity of intertemporal substitution and new keynesian model

In this section, I explore in more details the effects of demand shock in a standard new keynesian model. I detail the analytics of the demand side and the supply side effect of interest rate. I show that the supply side effect have a limited impact on the output reaction to demand shock but a considerable one on the output gap reaction. Eventually, I review the possible empirical evidences for this effect.

2.1 Demand shock in flexible prices model

First, I start with usual flexible price benchmark. The features of the model are well known. An infinite horizon representative agent maximizes its lifetime utility with respect to both consumption and labor supply. We make the critical assumption that preferences are separable between leisure (or hours) and consumption

$$Max \quad \frac{\sigma C_t^{\frac{\sigma-1}{\sigma}}}{\sigma-1} - \frac{\theta N_t^{\frac{\theta+1}{\theta}}}{1+\theta} + \beta V_{t+1}(A_{t+1}) \quad (1a)$$

$$w.r.t \quad (1+r_t)A_t = W_t N_t + C_t + A_{t+1} \quad (1b)$$

It leads to the two First Order conditions.

$$C_t^{-\frac{1}{\sigma}} W_t = N_t^{\frac{1}{\theta}} \quad (2a)$$

$$C_t^{-\frac{1}{\sigma}} = \beta(1+r_{t+1})C_{t+1}^{-\frac{1}{\sigma}} \quad (2b)$$

On the good market, the output is used either as investment or consumption

$$Y_t = C_t + I_t \quad (3)$$

The production function is

$$Y_t = N_t^{1-\alpha} \quad (4)$$

We assume that investment is simply exogenous, and evolves according a random AR(1) process. We also abstract from supply side effect of investment on productive capacity. These two assumptions may seem very weird. We discuss it more thoroughly in the robustness section. The spirit is reminiscent of the ultra simple old keynesian model of investment multiplier.

Thanks to these tricks, we remain in line with the standard treatment of new keynesian model as exposed in Gali(2001) or in the first chapters of Woodford(2001). Expansion in private investment and government spending have similar effects. So, the results of the two following section and the results of the fiscal multiplier literature will be the same (see for example Woodford 2010).

In order to have a clearer idea, we write the whole linear model with flexible price.

$$y_t^{flex} = (1 - \alpha)n_t^{flex} \quad (5a)$$

$$y_t^{flex} = w_t^{flex} + n_t^{flex} \quad (5b)$$

$$\theta w_t^{flex} - \frac{\theta}{\sigma_n} c_t^{flex} = n_t^{flex} \quad (5c)$$

$$y_t^{flex} = c_t^{flex} + i_t^{flex} \quad (5d)$$

$$c_t^{flex} = -\sigma E_t r r_{t+1}^{flex} + E_t c_{t+1}^{flex} \quad (5e)$$

$$i_t^{flex} = \rho i_{t-1}^{flex} + \epsilon_t \quad (5f)$$

Here, we express the two components of aggregate demand in steady state GDP percentage to simplify further notations. As a consequence, parameters σ and σ_n are the product of a deep behavioral parameter $\tilde{\sigma}$ and $\tilde{\sigma}_n$ and a steady state value $\frac{C}{Y}$.

$$\sigma = \tilde{\sigma} \frac{C}{Y}$$

$$\sigma_n = \tilde{\sigma}_n \frac{C}{Y}$$

We are particularly interested by the effects of the σ and σ_n parameters. <under separable preferences, the two are the same $\sigma_n = \sigma$. This parameter governs the marginal utility of consumption and thus intervenes twice in the model. It controls both the wealth effect in the labor supply equation $\frac{\theta}{\sigma_n} c_t^{flex}$ and the sensitivity of consumption to interest rate in the euler equation. In order to better understand the role of these different channels, we differentiate the value of the parameter in the LS equation σ_n and in the euler equation σ .

When prices are flexible, a rise in investment increases aggregate demand. This increase has no direct effect on output which is determined by the production function and the equilibrium on the labor market. To clear the good market, the equilibrium or natural interest rate has to move. The rise of the interest rate has a demand side effect and a supply side effect. On the demand side, it lowers consumption. On the supply side, it reduces wealth and consumption increasing desired hours worked. The higher marginal utility of consumption leads household to increase the marginal utility of leisure and thus hours. At equilibrium, hours, output and interest rate are higher whereas wages are lower.

The only state variable of the model is the shock i_t which represents the investment. The solution of the model consist in expressing all other variables with respect to i_t . We will compute the multiplier, the coefficient of i_t for each variable. For example, the investment multiplier of output for the standard model with flexible prices is equal to $\psi_y^{flexstand}$

$$y_t = \psi_y^{flexstand} i_t \quad (6)$$

We compute multiplier for output and interest rate after an investment shock. These multipliers are denoted ψ

$$\begin{aligned}\psi_y^{flexstand} &= \frac{(1-\alpha)\theta}{\sigma_n(1+\alpha\theta) + (1-\alpha)\theta} \\ \psi_r^{flexstand} &= \frac{1-\rho}{\sigma} \frac{\sigma_n(1+\alpha\theta)}{\sigma_n(1+\alpha\theta) + (1-\alpha)\theta}\end{aligned}\tag{7}$$

The output multiplier depends from the determinants of the reaction of hours to consumption. It increases with the frisch labor supply elasticity θ and decreases with the elasticity of intertemporal substitution. It is positive but inferior to one. When σ approaches zero, the reaction of output is close to one.

Interestingly, it is σ_n which affects output whereas σ only impacts interest rate.

2.2 Demand shock with sticky prices and supply side effect

I now analyze the effects of the investment shock in a standard new keynesian model. I assume standard calvo pricing and I display the full linear model like in the flexible price section.

Aggregate Supply Block

$$\pi_t = \kappa p_t^z + \beta \pi_{t+1} \tag{8a}$$

$$y_t = (1 - \alpha)n_t \tag{8b}$$

$$p_t^z + y_t = w_t + n_t \tag{8c}$$

$$\theta w_t - \frac{\theta}{\sigma_n} c_t = n_t \tag{8d}$$

Aggregate demand block

$$y_t = c_t + i_t \tag{8e}$$

$$c_t = -\sigma E_t r r_{t+1} + E_t c_{t+1} \tag{8f}$$

$$r r_{t+1} = n r_t - \pi_{t+1} \tag{8g}$$

$$n r_t = \phi_\pi \pi_t \tag{8h}$$

$$i_t = \rho i_{t-1} + \epsilon_t \tag{8i}$$

Unlike some misconceptions, the logic of the new keynesian model is radically different from the real business cycles. Prices are sticky and the output is determined by the aggregate demand. The increases in demand has a direct effect on output.

Like in the flexible price model, I compute multipliers. I detail the steps of the computations to highlight the role of demand in determining output in the sticky price model.

First, the market clearing condition gives

$$\psi_y = \psi_c + 1 \tag{9}$$

The consumption multiplier itself is a function of the real rate reaction

$$\psi_c = \frac{\sigma}{\rho - 1} \psi_r \quad (10)$$

Using philips curve equation and taylor rule, we find a relation between the real rate and the marginal cost

$$\psi_r = \frac{\kappa(\phi_\pi - \rho)}{1 - \beta\rho} \psi_{pz} \quad (11)$$

The marginal cost have two components, a wage component and an output component

$$\psi_{pz} = \psi_w + \frac{\alpha}{1 - \alpha} \psi_y \quad (12)$$

And the wage coefficient depends from both the labor supply equation and the labor demand curve which is equal here to the production function as the model is demand determined

$$\psi_w = \frac{1}{\theta(1 - \alpha)} \psi_y + \frac{1}{\sigma_n} \psi_y - \frac{1}{\sigma_n} \quad (13)$$

Actually, the investment multiplier for sticky prices is

$$\psi_y^{stand} = \frac{(1 - \rho)(1 - \beta\rho)(1 - \alpha)\theta + \frac{\sigma}{\sigma_n}(\phi_\pi - \rho)\kappa(1 - \alpha)\theta}{(1 - \rho)(1 - \beta\rho)(1 - \alpha)\theta + \sigma(\phi_\pi - \rho)\kappa(1 + \alpha\theta) + \theta(1 - \alpha)\kappa(\phi_\pi - \rho)\frac{\sigma}{\sigma_n}} \quad (14)$$

We can also rewrite the result more clearly by considering the separable preferences case

$$\psi_y^{stand} = 1 - \frac{\sigma(\phi_\pi - \rho)\kappa(1 + \alpha\theta)}{(1 - \rho)(1 - \beta\rho)(1 - \alpha)\theta + \sigma(\phi_\pi - \rho)\kappa(1 + \alpha\theta) + \theta(1 - \alpha)\kappa(\phi_\pi - \rho)} \quad (15)$$

These formula may appear complicated but it allows to understand the three channels by which a demand shock affects output, the direct effect, the crowding out effect, and the supply effect of real rate.

The direct demand effect on output Because prices are sticky and output demand determined, the demand shock has a direct effect equal to one. It can also be associated to the term $(1 - \rho)(1 - \beta\rho)(1 - \alpha)\theta$. If we follow an out of equilibrium reasoning, the demand shock will increase output by a one for one factor, but it will increase wages and inflation. The monetary policy will react and the real interest rate will rise. This rise have two effects.

The monetary policy effect: demand channel The first and well known effect is the crowding out of private expenditure, here consumption. It is the term $\sigma(\phi_\pi - \rho)\kappa(1 + \alpha\theta)$. This term depends from σ_c which represents the sensitivity of aggregate demand to interest rate. More this effect is important, more consumption will be crowd out by the investment shock. However, the interest rate has another and hidden effect which goes through supply.

The monetary policy effect: supply channel The supply side effect of monetary policy is not very intuitive and differs from the flexible price effect. The rise in interest rate will generate a lower consumption, increasing desired hours worked. The labor supply curve will shift and the equilibrium wage will decrease dampening inflation pressure and the monetary policy reaction. Thus the supply side effect is an expansionary effect conditional to demand shocks.

We could believe that the supply side effect in the new Keynesian model is similar to its flexible price counterpart. This is a mistake and it can be seen in the multiplier formula. The supply effect was entirely responsible for the rise of output in the flexible price case and depends from σ_n . This effect remains important in the model but more significantly, it does not depend from σ_n but from the ratio $\frac{\sigma}{\sigma_n}$ which is equal to one in case of separable preferences. Indeed, in the flexible price case, the output is determined by supply factor. A rise in the exogenous investment lowers consumption. Through the σ parameter, it increases the marginal utility of consumption which leads to an increase in hours worked. As there is only a supply side effect

When prices are sticky, the supply side effect has a different impact on the multiplier. He adds both $\frac{\sigma}{\sigma_n}\theta(1-\alpha)\kappa(\phi_\pi - \rho)$ to both numerator and denominator. Crucially, the mechanism is different from the flexible price case *it does not depend of σ* . The mechanism is the following. When a positive exogenous demand shock occurs, the output increases because firms served the demand. Firms increase their labor demand and thus wages. It increases the marginal cost, and inflation. This inflation generates a reaction of monetary policy. The real rate increases triggering a fall in the **the marginal utility of consumption** which has a direct effect on the labor supply which is independent from σ . The increase in labor supply lowers real wages, marginal cost, inflation and thus a **fall** of the real interest rate. This fall stimulates consumption and thus output. This stimulation effect is obviously limited by the sensitivity of consumption to interest rate.

The effect of a low EIS We are particularly interested by the limit case where σ , the elasticity of intertemporal preferences approaches zero. In that case, consumption becomes unresponsive to interest rate change. The crowding out effect disappears. Demand shocks are no more dampened by the monetary policy reaction. Intuitively, they should create pretty large inefficiencies.

Under separable preferences, that means that σ_n also approaches zero. So, the supply side effect term $\frac{\sigma}{\sigma_n}(\phi_\pi - \rho)\kappa(1-\alpha)\theta$ remains unchanged. The crowding out effect term $\sigma(\phi_\pi - \rho)\kappa(1+\alpha\theta)$ also approaches zero and the multiplier approaches one. A very counter intuitive result is that it is also the case for the flexible price multiplier. As σ_n approaches zero, the term $\sigma_n(1+\alpha\theta)$ follows and the limit of $\psi^{flexstand}$ is equal to 1.

The output gap The previous result should seem very counter intuitive. The purpose of introducing sticky prices is to allow aggregate demand change to have an impact on output.

In the new keynesian model however, the effect seems not so different from the flexible price case. This result is not a consequence of the low EIS. I compute the output gap multiplier. the output gap here is the difference between the sticky price output and the flexible price one. This is simply the difference $\psi^{stand} - \psi^{flexstand}$

$$\psi_y^{gapstand} = \frac{\sigma_n(1 + \alpha\theta)\kappa(\phi_\pi - \rho)}{(1 - \alpha)\theta\kappa(\phi_\pi - \rho) + \sigma_n(1 + \alpha\theta)\kappa(\phi_\pi - \rho)} - \frac{\sigma(1 + \alpha\theta)\kappa(\phi_\pi - \rho)}{(1 - \alpha)\theta\kappa(\phi_\pi - \rho)\frac{\sigma}{\sigma_n} + \sigma(1 + \alpha\theta)\kappa(\phi_\pi - \rho) + \theta(1 - \alpha)(1 - \rho)(1 - \beta\rho)} \quad (16)$$

Surprisingly, If $\sigma = \sigma_n$, The only difference between the value of output multiplier under flexible prices and under sticky prices is the term $\theta(1 - \alpha)(1 - \rho)(1 - \beta\rho)$ at the denominator. This is not very large. It is striking that the value of σ which is supposed to govern the scale of the crowding out effect, have a very small impact on the output gap multiplier. We display in figure 1 the multiplier values with respect to σ , the other parameters being calibrated using standard values. Whereas the two multipliers converge toward one when σ approaches zero, they remain very close for all values of σ . Actually, the output gap response is larger when the crowding out effect on consumption is large. The output gap remains very low. The economy reacts nearly perfectly to demand disturbances even when prices are sticky.

2.3 Removing the supply side effect

When the EIS is very low, a demand shock have a one for one effect on output. But in the new keynesian model, it does not imply a large difference between output under sticky and flexible prices. **This result is entirely due to the supply side effect of interest rate.** Indeed, consider the model without it. I assume $\sigma_n \rightarrow +\infty$. This hypothesis kills the wealth effect on labor supply. The labor supply equation takes a very simple form

$$\theta w_t = n_t \quad (17)$$

This is the only change in the model.

The output multiplier becomes

$$\psi_y = \frac{(1 - \rho)(1 - \beta\rho)(1 - \alpha)\theta}{(1 - \rho)(1 - \beta\rho)(1 - \alpha)\theta + \sigma_c(\phi_\pi - \rho)\kappa(1 + \alpha\theta)} \quad (18)$$

The term $\frac{\sigma}{\sigma_n}\theta(1 - \alpha)\kappa(\phi_\pi - \rho)$ have now disappeared. Moreover, a demand shock have no impact on output if prices are flexible.

$$\psi_y^{flex} = 0$$

These two formulas allows to see more clearly what components of the multiplier are related to the supply

As a consequence, the output gap is equal to the output multiplier the output multiplier. It also reacts one for one to demand shocks for an EIS close to zero.

$$\psi_y^{gap} = \frac{(1-\rho)(1-\beta\rho)(1-\alpha)\theta}{(1-\rho)(1-\beta\rho)(1-\alpha)\theta + \sigma_c(\phi_\pi - \rho)\kappa(1+\alpha\theta)} \quad (19)$$

When supply side effect does not exist in the model, demand driven fluctuation translate into output gap fluctuations because the flexible price output have does not react to the shock. This is much more in line the concept of output gap used in practical decision making. Most decision maker seems simply using the deviation from trend output as proxy for the output gap. This is for example the concept used by Taylor in its original paper estimating the Taylor rule. It is perfectly consistent with a model without supply side effect from interest rate. Moreover, this supply side effect is completely inexistent in public debate whereas according to the standard model, seven years of negative real rate should have had a very strong effect on desired labor supply right in 2008.

In figure 2, I display the output gap response with respect to the value of the EIS. In accordance with intuition, the response is a decreasing function of σ which scales the crowding out effect.

2.4 The real equilibrium interest rate

We could believe that a low EIS means that monetary policy have a limited importance for the model. This is radically wrong. Monetary policy keeps a very strong importance even if its impact on output approaches zero.

Indeed, the supply effect of monetary policy does not depend from σ because it is the marginal utility of consumption which is moved. Thus, the labor supply curve is shifted by monetary policy. This effect remains even if the impact on output of this shift is low.

As a consequence, slight moves of monetary policy closely mimic the optimal monetary policy.

With flexible prices and supply side effect, the interest rate multiplier is

$$\psi_r^{flexstand} = \frac{1-\rho}{\sigma} \frac{\sigma_n(1+\alpha\theta)}{\sigma_n(1+\alpha\theta)\frac{C}{Y} + (1-\alpha)\theta}$$

This value is bounded below by $\frac{(1-\rho)(1+\alpha\theta)}{(1-\alpha)\theta}$ when $\sigma = \sigma_n \rightarrow 0$ When prices are sticky, the response is not very different.

$$\psi_r^{stand} = \frac{(1-\rho)(1+\alpha\theta)}{\frac{(1-\rho)(1-\beta\rho)(1-\alpha)\theta}{\kappa(\phi_\pi - \rho)} + \sigma(1+\alpha\theta) + \theta(1-\alpha)}$$

The main change compare to the flexible price reaction is the term $\frac{(1-\rho)(1-\beta\rho)(1-\alpha)\theta}{\kappa(\phi_\pi - \rho)}$. Both the natural real rate and the sticky price rate are bounded below and remains very close as we can see on the graphic [insert graphic] This near optimality of monetary policy contrast with the results without supply side effect. When prices are flexible, the reaction of interest rate is very simple.

$$\psi_r^{flex} = \frac{1-\rho}{\sigma} \quad (20)$$

This response is not bounded below. Complying with intuition, natural interest rate now approaches minus infinity when the EIS approaches zero. The sticky price response introduces the same monetary policy term

$$\psi_r = \frac{(1 - \rho)(1 + \alpha\theta)}{\frac{(1-\rho)(1-\beta\rho)(1-\alpha)\theta}{\kappa(\phi_\pi - \rho)} + \sigma(1 + \alpha\theta)}$$

Here monetary policy introduces a bound to the response of real interest rate to the demand shock when the EIS becomes low, creating a complete divergence between the monetary policy rate and the natural interest rate.

In a nutshell, with the supply side effect, monetary policy closely mimic the response of a flexible price economy to a demand shock. The business cycles driven by this shock is nearly optimal. When the supply side effect disappear and the crowding out effect vanishes, the demand shock translates completely into output gap variation. [Insert graphic for natural interest rate]

2.5 The empirical foundation for the supply side effect of interest rate

The supply side effect of monetary policy works through several step. Let's summarize them. First, the interest rate have a one for one impact on the growth of the marginal utility of consumption. Through persistence of shocks, it translates into a lower marginal utility of consumption level. This marginal utility of consumption have a wealth effect on the labor supply. The labor supply curve shifts which alter the output either directly (flexible prices case) either through wages, marginal cost, inflation, real rate and eventually consumption and output. The supply side effect of interest rate have a considerable importance in the standard new keynesian model. It should have been thoroughly investigated by the empirical literature. Surprisingly, it is not the case. Actually, relevant empirical literature are completely silent about this effect. There is simply no empirical work at our knowledge on the topic in labor economics in a considerable amount of literature. For example at business cycles frequencies, conditionnal to demand shocks, the multiplier of hours worked can be written as

$$\psi_n = \theta\psi^w + \frac{\theta}{\sigma_n} \frac{\sigma_c}{\rho - 1} \psi_r \quad (21)$$

With separable preferences, the effects of interest rate should be similar to those of wages. The word interest rate appears twice in the index of "labor Economics" by Cahuc and Zylberberg and for an unrelated purpose (i.e job creation of financially constrained firms, and interest rate effects on value function in matching model). By comparison, the occurrence of the word "wages" fill a whole page of the index. The closer empirical work are study on wealth effect on labor supply. These papers are often based on the changes of labor supply of lottery winners. They find a significant but not considerable effect whereas they considered a very large and well perceived shock. Whereas lower interest rate should have theoretically a similar effect

under rational expectations assumption, it is not clear that in reality lottery wins and interest change are the same phenomenon for a given individual. Thus, these limited evidences cannot be used as arguments for interest rate effects on labor supply. The other interested branch of the literature is the literature on transmission channels of monetary policy. At our knowledge, no paper have tried to assess such an effect of monetary policy on hours and central banks (despite the very large human capital available to them) seems not eager to investigate the topic further. this effect seems entirely absent from public debate on macroeconomic policy. For example at our knowledge, nobody argues that the zero interest rate policy of the FED was behind the decline of the labor force participation in the US whereas it is actually what the standard model would predict¹.

There are (debated) credentials for the existence of income effect on labor supply, especially when computing long run impact of labor tax change. It is not clear in what extent these long run elasticities are relevant for business cycles. More importantly, whereas you can reasonably think that there exist income effect on labor supply, the real important factor for the new keynesian model is an effect of interest rate on marginal utility of consumption and from marginal utility of consumption to hours. It is possible that the problem lies in the first part instead of the second.

3 An alternative solution

I look at a completely different solution. Instead of making a particular assumption about preferences or wage negotiations, my goal is to completely disentangle the parametrs governing the two behavioral equation of households. To do that, the way households solve their optimization problem should be reconsidered. My idea is that the household choice takes place in two steps. First, households choose between leisure and labor income, then between between consumption and asset accumulation. Actually, the classical household program can be decomposed between the same steps, but the two steps will be linked by the envelope theorem and the Roy identity. In my problem, the two choice are completely separated. To justify this point, it would be possible to tell a story in which some household member chooses the labor supply before giving the money to another member who chooses consumption. This is a very frequent story in macroeconomics. In financial frictions literature for example, the household is both composed by traditionnal workers-consumers and by entrepreneurs (also called bankers sometimes since the financial crisis, the term capitalist having lost its popularity), the latter transferring regularly a fraction of its gains to the former. However, we preferred another story less in line with usual optimization methods but probably more in line with "reality". The

¹interestingly, such explanation would not not be entirely absurd on the extensive margin if very low interest rate allows for earlier retirement because asset prices are going up or delay the entry of students on labor market with more affordable student loans. Even if such a channel is not entirely implausible, we seriously doubt it can be as large as predicted by standard separable preferences with low sensitivity of consumption to interest rate

agent choosing labor supply and consumption is the same but he takes the two decisions separately. In usual optimization, households need to form expectations about the whole path of wages and interest rate. Given these two paths, he implicitly computes simultaneously both individual hours and consumption conditionally to its information for its whole life duration which is infinite. This is a very complex tasks requiring a considerable amount of information and computationnal capabilities. Choosing variables two by two is much simpler and let more space for economic intuition at each stage. It does not necessarily requires less information but it creates more ground to reduce information set. In another work, I develop in more details the advantages and drawbacks of that approach. The cost is that the choice made by agents does not necessarily comply with the revealed preferences axiom. It means that an household cannot compare two by two all bundles. Preference are not complete over the whole space of bundles. In infinite horizon optimization, the space of bundle is of infinite dimension (this is the space of sequences). Having a complete preorder over such a space is largely beyond capabilities of human being. Moreover this form of irrationality is probably of second order compare to some other assumption of the new keynesian model especially about firm behavior.

3.1 A reformulation of the classical problem with separable preferences

Before having a closer look at our behavioral assumption, we rewrite the traditionnal problem in two steps.

The agent chooses leisure l_t and consumption c_t at each period. Its income have two components, the wage and the assets previously accumulated. We write the budget constraint in an intertemporal fashion. a_t here is the amount of an asset which gives one unit of consumption good in $t + 1$. The price of this asset is q_t . It is the inverse of the interest rate factor.

Instead of having only the wage, we treat the time endowment of the agent like an asset whose value ω_t is equal to the discounted sum of wages. ω_t is the total labor wealth of the agent, i.e the discounted value of the labor income stream which would be received by the agent if he dedicates its whole time endowment to labor

$$\omega_t = E_t \sum_{T=t}^{+\infty} w_T \prod_{k=t}^{T-1} \frac{1}{R_{k+1}} = w_t + E_t \frac{1}{R_{t+1}} \omega_{t+1} \quad (22)$$

Under separable preferences, you can write

$$\max_{l_t, \chi_t} v(l_t) + \tilde{V}_t(\chi_t) \quad (23a)$$

$$w.r.t \quad a_t + \omega_t = w_t l_t + \chi_t \quad (23b)$$

$a_t + \omega_t$ is the total wealth of the household. It divides this wealth between leisure cost $w_t l_t$ and what remains χ_t . with

$$\tilde{V}(\chi_t) = \max_{c_t, a_{t+1}} u(c_t) + \beta E_t V_{t+1}(a_{t+1} + \omega_{t+1}) \quad (24a)$$

$$w.r.t \ \chi_t = q_t a_{t+1} + E_t \frac{1}{R_{t+1}} \omega_{t+1} + c_t \quad (24b)$$

$$V_{t+1}(a_{t+1} + \omega_{t+1}) = \max_{l_{t+1}, \chi_{t+1}} v(l_{t+1}) + \tilde{V}_t(\chi_{t+1}) \quad (25a)$$

$$w.r.t \ a_{t+1} + \omega_{t+1} = w_{t+1} l_{t+1} + \chi_{t+1} \quad (25b)$$

Solving the two program leads to first order condition

$$\begin{aligned} v'(l_t) &= w_t * \tilde{V}'(\chi_t) \\ u'(c_t) &= q_t * V'(a_{t+1} + \omega_{t+1}) \end{aligned}$$

Using envelope theorem, you verify that

$$\begin{aligned} \tilde{V}'(\chi_t) &= u'(c_t) \\ V'(a_{t+1} + \omega_{t+1}) &= \tilde{V}'(\chi_{t+1}) = u'(c_{t+1}) \end{aligned}$$

It is possible to decomposed the problem solved by the agent in successive steps in which at each point he chooses between leisure (resp. consumption) and the income which will allow him to buy consumption and assets (resp. leisure). However, in practice the choice is still the choice of a sequence of vectors (c_k, l_k) . The marginal utility of wealth remains equal to the marginal utility of consumption.

3.2 Subjective indirect utility function

The supply side effect of interest rate is precisely an issue created by the marginal utility of consumption whose growth rate is equal to interest rate. So, a first step to cancel the supply side effect would be to disentangle the marginal utility of wealth from the marginal utility of consumption. The idea is V and \tilde{V} function becoming subjective utility function instead of indirect one. Instead of being defined over the infinite sequence of vector (c_k, l_k) , preferences are defined over a vector of one real variable and of a budget set.

To see more concretely what it means, the program becomes the following First, the agent will solve

$$\max_{l_t, \chi_t} u(l_t, \chi_t) \quad (26a)$$

$$w.r.t \ a_t + \omega_t = w_t l_t + \chi_t \quad (26b)$$

The agent chooses its amount of work by comparing the utility of leisure plus the utility provided by the labor income. Here, we use ω_t which is equal to the financial wealth of the

agent plus the discounted sum of labor income streams. ω_t is also equal to the discounted value of consumption stream. The first order condition for the labor supply is

$$w_t \frac{\partial u}{\partial \chi}(l_t, \chi_t, q_t) = \frac{\partial u}{\partial l}(l_t, \chi_t) \quad (27)$$

In a second step, knowing his labor income $y_t^n = w_t l_t$ he will choose consumption given his choice of hours

$$\max_{c_t, a_{t+1}} E\tilde{u}(c_t, a_{t+1} + \omega_{t+1}) \quad (28a)$$

$$w.r.t \ \chi_t = c_t + E_t \frac{1}{R_{t+1}} \omega_{t+1} + q_t a_{t+1} \quad (28b)$$

Solving this problem leads to the first order condition

$$\frac{\partial E\tilde{u}}{\partial \omega}(c_t, a_{t+1} + \omega_{t+1}) = q_t \frac{\partial E\tilde{u}}{\partial c}(c_t, a_{t+1} + \omega_{t+1}) \quad (29)$$

3.3 Comparison

In this section, we summarize in what aspect our method differ from the classical optimization problem. A detailed analysis is provided by a companion paper which have yet to be completed.

In the traditionnal infinite horizon optimization problem, the agent defines its preferences over a sequence of vector. The space of sequence is of infinite dimension. Beside the strong assumption which are needed to keep the problem tractable, the agent have to defined a complete and transitive order over this set of infinite dimension which seems a very strong assumption. In that respect, our approach is much more reasonable. At each step, the agent defined its preferences over a vector of two variables, without links with the future way he will define utility. This does not mean however that future will be unimportant for the agent. But, it will be through intertemporal asset price q_t or income through ω_t .

The immediate consequence is that the marginal utility of the wealth relevant for the labor choice and those relevant for the consumption choice are no more equal to the present or future marginal utility of consumption. It could seem irratiounal because wealth can translate into consumption, so the marginal utility of the two should be equal. However, utility function are just a convenient step between the preference structure of the agent and demand function. Marginal utility does not exist. What is important are prices and how agents react to price change. In our view, utility are just a convenient way to rationalize these reactions.

To assess the "rationality" of our proposed behavior, we should look at preferences. The relevant criteria is the degree of compliance with revealed preferences axiom. The revealed preference axiom is itself a very strong one, supposing a complete and transitive preordering of bundles by economic agents to hold. In a related work, I have studied the problem in a three good case. Instead of choosing a bundle (x_1, x_2, x_3) , the agent chooses (x_1, R) and then (x_2, x_3) using an exogenous income. We assumed the two problems being fully independant. The result

is that you can find in some case two price and income vector for which the two chosen bundles are mutually available. Prices and income should move in the same direction. It is important to stress that some prices or income change should be large to violate axioms. In practice, it means it is often impossible to derive a complete order of preferences over (x_1, x_2, x_3) using preferences over (x_1, R) and (x_2, x_3) . In what extent it should be considered as irrational remains unclear for us. Completeness and transitivity are very strong assumption. Frequent violations have been highlighted by experimental evidences. The new keynesian model also postulates that firms adjust their supply to the demand, a key mechanism which reduces markup in expansion and increase it in recession, even if it is not beneficial to them.

What are the consequences of our assumption for demand curve ? The difference between the demand curves obtained under the classical approach are pretty low. There are just less parameter restrictions. Consider a matrix of partial derivative of demand with respect to prices. The biggest consequences is to impose much less structure. Under the classical method, the effects of a given price on the different good should be consistent. Our method let the possibility to freely parametrize the effects.

These elements of comparison are for the moment a first look at the difference between the two methodology. To be clear, I have yet to complete a rigorous axiomatic analysis of my approach. As indicated before, however, in another work in progress, I define preferences and have studied the ability of my approach to comply or not with revealed preferences axiom.

3.4 Macroeconomic application

The linearized version fo the FOC are

$$\frac{U_{xx}\chi}{U_x}\chi_t + \frac{U_{xl}L}{U_x}l_t + w_t = \frac{U_{ll}L}{U_l}l_t + \frac{U_{xl}\chi}{U_l}\chi_t \quad (30)$$

$$\frac{\tilde{U}_{\omega\omega}\Omega}{\tilde{U}_\omega}\omega_{t+1} + \frac{\tilde{U}_{\omega c}C}{\tilde{U}_\omega}c_t = \frac{\tilde{U}_{cc}C}{\tilde{U}_c}c_t + \frac{\tilde{U}_{\omega c}\Omega}{\tilde{U}_c}\omega_{t+1} + q_t \quad (31)$$

Note that the a_{t+1} parameter disappear in the log lienarized version because it is supposed to be equal to zero at steady state [verify] We construct the four parameters

$$\begin{aligned} \theta &= \frac{U_{xl}L}{U_x} - \frac{U_{ll}L}{U_l} \\ \sigma &= \frac{U_{xl}\chi}{U_l} - \frac{U_{xx}\chi}{U_x} \\ \gamma &= \frac{\tilde{U}_{\omega c}C}{\tilde{U}_\omega} - \frac{\tilde{U}_{cc}C}{\tilde{U}_c} \\ \varphi &= \frac{\tilde{U}_{\omega c}\Omega}{\tilde{U}_c} - \frac{\tilde{U}_{\omega\omega}\Omega}{\tilde{U}_\omega} \end{aligned}$$

The two first order conditions can be rewritten

$$\theta l_t + w_t = \sigma \hat{\chi}_t \quad (33a)$$

$$\gamma c_t = \varphi \omega_{t+1} + q_t \quad (33b)$$

These two equations replace equations (6d) and (6f) in the model of the first section. We also need the two definitions of χ and ω to close the system. In the previous section, households income was provided by labor or bonds. In the new keynesian model, they receive profit and should finance investment without receiving earnings for it. Thus, we should add profits and deduce investment. There are several ways to define the wealth of the household, using either the discounted sum of income or the discounted sum of consumption. We will use the second one as it is clearly easier to compute. The log linearized equation is

$$\hat{\omega}_t = \frac{R-1}{R} \left(\frac{W w_t - I i_t + Y y_t - W N(w_t - n_t)}{Y + W(1-N)} \right) + \frac{1}{R}(\omega_{t+1} - r_{t+1}) \quad (34)$$

$$\hat{\chi}_t = \frac{\omega}{\omega - wl} \hat{\omega}_t - \frac{wl}{\omega - wl} (w_t + l_t) \quad (35)$$

4 Interest rate and wealth: killing the wealth effect of interest rate

The indirect approach of preferences allows to completely disentangle the effects of interest rate on leisure and on consumption. A solution to our problem could be to both reduce the wealth effect on labor supply and the effects of interest rate on consumption whereas keeping a simple equation for consumption. The advantage over non separable preferences is that we do not rely on a very high level of substitutability between leisure and consumption.

But indirect preferences also allows an alternative strategy which is completely impossible under the traditionnal optimization framework. The supply side effect of monetary policy works in three steps. First, interest rate have a strong impact on marginal utility of wealth. The change in the marginal utility of wealth creates an income effect on labor supply which will affect output either directly if prices are flexible or through inflation and monetary policy if they are sticky. Rigid wages cancels the last channel between labor supply and output. Non separable preferences and low wealth effect with indirect preferences challenges the second one. But indirect preferences also allows to disturb the first channel, the link between monetary policy and wealth.

Indeed, a major effect of the monetary policy in theory is to increase the labor wealth, i.e the discounted sum of wages. The discounting factor is the product of future real rates. The wealth formula is for example

$$\omega_t = \sum_{T=t}^{+\infty} w_T \prod_{k=t}^{T-1} \frac{1}{R_{k+1}} \quad (36)$$

This effect would be entirely killed if agents use a constant discount rate instead of using this very complicated formula. Concretely, we redefine the wealth as

$$\omega_t = \sum_{T=t}^{+\infty} w_T \beta^{T-t} \quad (37)$$

This change have no impact in the traditionnal optimization framework. The problem can be solved without using the intertemporal budget constraint. The marginal utility of wealth is solved using the envelope theorem which makes it necessarily equal to the marginal utility of consumption. By contrast, in our framework, the new definition of wealth can be used in the parametrized indirect utility function.

Concretely, the only change to our equation is the labor wealth definition equation (43) which becomes

$$\hat{\omega}_t = \frac{R-1}{R} \left(\frac{Ww_t - Ii_t + Yy_t - WN(w_t - n_t)}{Y + W(1 - N)} \right) + \frac{1}{R} \omega_{t+1}^{\hat{}} \quad (38)$$

This change suppressed the supply side effect and dramatically reduced the effects of interest rate on consumption. A very large γ , or a very low σ are no longer required. [insert impulse response from dynare] There are many arguments in favor of that approach. First, it would be an explanation for the limited effect of interest rate on both consumption and leisure. Second, we kept wealth effect on labor supply and consumption. So, for example, permanent tax change will continue to have a substantial income effect on labor supply. Third, there is some ground

4.1 Results

I simulate the behavior of the model after a demand shock using Dynare. I compare the result of the model with interest rate discounting and those with constant rate discounting. Results strongly confirm the importance of the discounting assumption. Usual parameters are calibrated in a very standard way (see table in appendix). I parametrize the behavioral parameters to let interest rate having a reasonable influence over consumption. The leisure elasticity is equal to 0.5 and the wealth effect σ equal to one. The elasticity of consumption with respect to interest rate is equal to 3 which allows the interest rate to have a reasonable influence over consumption. The coefficient of future wealth on consumption is equal to 1.

When the agent computes wealth using real interest rate as discount factor, the effect of demand shocks are very closed to the new keynesian model. I display result in figure 6. When prices are flexible, income reacts strongly to the shock and consumption decrease a little bit. The natural interest rate moves by 0.3 percent for a one GDP percentage of stimulus. The reaction of the different variables are not really different in the sticky price case.

This result is radically changed when households discount using a constant rate. The flexible price output remains nearly unchanged. The fall in consumption compensates the increase in private investment. To generate this fall, the interest rate has to increase by 3 percentage

points, ten times more than previously. A five percent decrease in output would lead to a natural rate interest rate drop by fifteen percent. When prices are sticky, the increase in real rate is much more moderated, around 0.5 percent. Consumption drops only by 0.2 percent. As a result, national income increases by 0.8 percent for an initial investment expansion of 1 percent in GDP term.

5 Solutions in the literature

The existence of the wealth effect on labor supply have long been recognized as a problem by papers emphasizing demand shocks. Greenwood, Hercowitz and Hufmann (GHH) kills this wealth effect in their real business cycles model by adopting non separable preferences. This way of research have been continued later by the news shocks literature (Jaimovich and Rebelo 2009) and studied in depth by Bilbiie (2009,2011) and Furlanetto and Seneca (2013).

A popular alternative in the literature, but largely for unrelated purpose, is wage stickiness. Wage stickiness does not kill the wealth effect on labor supply but largely cancels the effects of labor supply on output. These two solutions certainly are interesting lines of research, but they raise several issues. The more prominent one is their reliance on very specific assumptions for the labor market. These assumptions seems very far away from the reality. It can be justified by a better tractability, it would be more careful to avoid as much as possible to have results relying on these assumptions.

5.1 Rigid Wages

Rigid or sticky wages have interesting consequences for our model. They make labor supply unimportant, and thus largely alleviate the supply effect of real rate on output. As a consequence, output multiplier are very close to the model without wealth effect. However, the results of output gap becomes more difficult to interpret. Because the outcome of the flexible price flexible wages model is similar, the output gap and the spread between the natural and the monetary policy interest rate remain bounded. The welfare effect can be high but is displaced toward hours rather than consumption. In recession, I consume and work nearly as much as in the flexible price case and I would work more for the value of the wage.

5.1.1 Rigid wages in the linear model

When wages are rigid, the outcome is very close to a model with a walrasian labor market but without wealth effect.

Let's consider the simplest approach to wage stickiness. Suppose that real wages remains at their steady state levels across the business cycles.

The labor supply equations includes a wedge term

$$\mu_t^w = \theta w_t - n_t - \frac{\theta}{\sigma} c_t \quad (39)$$

And a new equation is introduced

$$w_t = 0 \quad (40)$$

Indeed, the output multiplier is straightforward to compute. Indeed, as wages remains constant, marginal cost, inflation and interest rate only reacts to output change

$$\psi_y^{sticky} = \frac{(1 - \beta\rho)(1 - \rho)(1 - \alpha)}{(1 - \beta\rho)(1 - \rho)(1 - \alpha) + \sigma\alpha\kappa(\phi_\pi - \rho)} \quad (41)$$

The proximity with the result under the model with walrasian labor market but without wealth effect is striking.

Sticky wages and output gap However, it does not necessarily translate into similar results for output gap. Indeed, the result of the model without price and wage stickiness will keep the wealth effect. So, most of output and consumption moves will be along the optimal path. The real difference will be that for the given wages, households would like to work more in recession than they actually do. Thus, the actual allocation will be very different from the desired one for given wages and prices. The low output

Sticky wages in the nonlinear model Our rigid wage formulation allows a tractable analysis of the linear model but raises issue otherwise. Indeed, there is no explicit mechanism by which wages will be negotiated to reach their steady state level, whereas remaining at this level when the economy is hit by a shock. A more rigorous but not really more realistic approach is for example to suppose a calvo type setting for wages, but it adds a lot of complexity. The equation defining the wedge between wages and labor disutility will remain but the nominal wage will become a complicated function of this wedge and of future inflation.

5.2 Non separable preferences

literature The theoretical literature usually makes the simple assumption that preferences are separable between consumption and leisure. It is convenient but there is no obvious reason to make this assumption. Relaxing this assumption is a seducing solution as the structure model remains unchanged. King, Plosser and Rebelo (1985) adopts non separable preferences but mainly to ensure balanced growth path. Greenwood, Hercowitz, Huffman (1988) pioneered a more active use to explain business cycles features. News shocks literature (Jaimovich and Rebelo) got more deeper into the topic. Surprisingly, new keynesian model were late to explore the problem. Bilbiie (2009, 2011), Furlanetto and Seneca studies the more general case.

5.2.1 General case

When preferences between hours and consumption are not separable. Consumption and saving are no more determined by the choice between future and present consumption but largely by the choice between present leisure and future one and by interaction between leisure and consumption.

Let's consider a log linearized version of the first order conditions in the general case

$$\frac{U_{cc}C}{U_c}c_t + \frac{U_{cl}L}{U_c}l_t + w_t = \frac{U_{ll}L}{U_l}l_t + \frac{U_{cl}C}{U_l}c_t \quad (42)$$

$$\frac{U_{cc}C}{U_c}c_t + \frac{U_{cl}L}{U_c}l_t - r_{t+1} = \frac{U_{cc}C}{U_c}c_{t+1} + \frac{U_{cl}L}{U_c}l_{t+1} \quad (43)$$

This expression of the euler equation is not very convenient to interpret the behavior of the agent. Because hours and consumption are decided simultaneously, the euler equation does not give an easy way to interpret how the consumption respond to shock. To solve the problem, we combine the two equations. we can isolate the consumption growth which becomes a function of interest rate and wages growth. When combining the two to get a relation between present and future consumption, we get

$$\left(\frac{U_{cc}C}{U_c} + \frac{U_{cl}L}{U_c} \frac{\frac{U_{cl}C}{U_l} - \frac{U_{cc}C}{U_c}}{\frac{U_{cl}L}{U_c} - \frac{U_{ll}L}{U_l}} \right) (c_{t+1} - c_t) = -r_{t+1} - \frac{\frac{U_{cl}L}{U_c}}{\frac{U_{ll}L}{U_l} - \frac{U_{cl}L}{U_c}} (w_{t+1} - w_t) \quad (44)$$

2

The key point: wage growth in consumption growth Let's consider the model presented in the section one with a slight change in the consumption equation. Now, we have

$$c_{t+1} - c_t = \sigma_c r r_{t+1} + \gamma (w_{t+1} - w_t) \quad (48)$$

²Obviously, using production function and good market clearing condition, it is also possible to get back something which looks like an euler equation with a simpler equivalent of the EIS. Nevertheless, reintegrating general equilibrium condition in the individual first order condition is not easy to interpret and in our view cannot be truly compare to an euler equation.

It is worth noting, you can also express leisure growth in function of interest rate and wage growth. The interest of that approach is that combined with intratemporal FOC, we get a similar reduced form equations when comparing to the separable case.

Indeed, in the separable case, we have

$$\frac{U_{ll}L}{U_l}(l_{t+1} - l_t) = -r_{t+1} + (w_{t+1} - w_t) \quad (45)$$

In the non separable case, we get the system

$$c_t = -w_t \frac{1}{\frac{U_{cc}C}{U_c} - \frac{U_{cl}C}{U_l}} + l_t \frac{\frac{U_{ll}L}{U_l} - \frac{U_{cl}L}{U_c}}{\frac{U_{cc}C}{U_c} - \frac{U_{cl}C}{U_l}} \quad (46)$$

$$\left(\frac{U_{cl}L}{U_c} + \frac{U_{cc}C}{U_c} \frac{\frac{U_{ll}L}{U_l} - \frac{U_{cl}L}{U_c}}{\frac{U_{cc}C}{U_c} - \frac{U_{cl}C}{U_l}} \right) (l_{t+1} - l_t) = -r_{t+1} + (w_{t+1} - w_t) \frac{\frac{U_{cc}C}{U_c}}{\frac{U_{cc}C}{U_c} - \frac{U_{cl}C}{U_l}} \quad (47)$$

The consumption multiplier is written

$$\psi_c = \frac{\sigma_c}{\rho - 1} \psi_r + \gamma \psi_w \quad (49)$$

But

$$\psi_w = \frac{1}{\theta(1 - \alpha)} \psi_y + \frac{1}{\sigma_n} \psi_c \quad (50)$$

When combining, we get

$$\psi_c = \frac{\sigma_n}{\sigma_n - \gamma} \frac{\sigma_c}{\rho - 1} \psi_r + \frac{\gamma}{\theta(1 - \alpha)} \psi_y \quad (51)$$

We can see here the main effect of a wage growth term in the euler equation. It introduces a feedback from income to consumption, creating a classical keynesian multiplier effect.³ This feedback allows to get a comovement between a demand shock and consumption. A positive demand shock will rise wage, but will have a negative impact on wage growth. It will generate a Lucas Rapping effect, an incentive to work more and to reduce leisure when wages are high. Because leisure and consumption are substitutes, consumption will also rise amplifying the demand shock.

Killing crowding out and wealth effect through large substitutability We now examine how non separable preferences may help to alleviate the issues raised by the supply side effect of monetary policy. With nonseparable preferences, it is possible to reduce both crowding out effect and wealth effect. The key is to increase both $\frac{-U_{cc}C}{U_c}$ and $\frac{-U_{cl}C}{U_l}$. The sensitivity of consumption growth to interest rate (for a given wage growth) will reduce but the wealth effect on labor supply will also fall.

This configuration implies a large negative value for $\frac{U_{cl}C}{U_l}$ which is equivalent to a very large degree of substitutability between leisure and consumption. It seems a non trivial proposition. Leisure and consumption are two very different goods and the intuition would rather suggest a large complementarity. However, at business cycles frequencies, changes in women's participation accounts for a large fraction of changes in labor supply. The idea would be a large substitutability between domestic labor and labor market participation, leisure being mismeasured as integrating women's domestic labor.

Supply side effect of interest rate How do nonseparable preferences precisely affect the supply side effect ? We could believe in the case where the wealth effect disappear that the supply effect of monetary policy have been killed. However, that is not how the model works. The supply effect through marginal utility of consumption remains. The effect of interest rate on the marginal utility of consumption is anyway implied by the basic euler equation. What

³Second, if $\sigma_n < \gamma$, the reaction of consumption to real rate change is reversed. This second possibility seems ruled out with non separable preferences

happens is that nonseparable preferences introduces a second supply effect of interest rate which cancels the other one.

Let's go in detail. Beside the effect on marginal utility of consumption, lower interest rate have an effect on consumption. This rise in a consumption decreases the marginal utility of leisure. Even if consumption is not very responsive to interest rate, the cross elasticity is necessarily very high, meaning that this substitution effect will be roughly proportionate to interest rate. Thus for a given wages, I am willing to work more. Thus lower interest rate have now a positive effect on hours which counterbalances the negative one which exists under non separable preferences.

5.2.2 Empirical issue

Consumption wage growth correlation At macroeconomic level, the key difference between non separable and separable preferences is to introduce a wage growth term in the behavioral consumption growth equation. It is a more convenient way to understand the difference than using the more common expression with leisure growth and consumption growth. Indeed, the formulation with wage growth links an endogenous variable for an individual agent and variable which are exogenous for the individual agent. We have something closer to the true reaction of an individual agent to changes in exogenous conditions (obviously, the closest thing would be a true demand function, i.e the expression of the level of consumption with respect to the whole path of wages and interest rate, but with non separable preferences, it is not very easy to derive such curve). This expression with respect to interest rate and wage growth is also necessary to fully understand how demand shocks may be amplified in such framework. When a demand shocks occurs, interest rate rises reducing consumption but wages increases, triggering a lower consumption growth translating into higher consumption level. The intuition for the result is a very strong Lucas Rapping effect.

A strong empirical problem of that approach is that whereas datas actually shows a correlation between hours growth and consumption growth, the correlation between consumption growth and wage growth is much less striking in datas.

Theoretical issue: The rigid wage problem A very strong theoretical issue for this approach is that there are plenty of evidences for strong wage rigidity at business cycles frequencies. Rigid wages by definition completely killed the effects and wage viscosity will likely strongly dampens it.

Whereas rigid wages and involuntary unemployment would be a more accurate representation of the labor market than a perfect walrasian market, combining it with non separable preferences could be quite challenging. The labor supply equation becomes irrelevant for computing wages and hours. There is now a wedge between the marginal utility of consumption times the wage and the marginal utility of leisure and this wedge will move with business cycles.

It creates reprots effect in the words of the old disequilibrium theory from the labor market to consumption.

This report effects are complicated to manage and sensitive to particular assumption of how the labor market works.

The supply side effect of monetary policy The supply side effect of monetary policy remains with non separable preferences. However, the increase of both $\frac{-U_{cc}C}{U_c}$ and $\frac{-U_{cl}C}{U_l}$ reduces the sensitivity of both consumption and leisure growth to interest rate for a given wage path, limiting demand side effect of interest rate change. As we have seen, the effect on supply side effect is much more subtle. Whereas the effect through marginal utility of consumption, a second effect is introduced.

Large substitutability and consumption leisure growth correlation To reduce both crowding out and wealth effect, we need a large substitutability between consumption and leisure, and a positive correlation between consumption growth and hours. Such a positive correlation is strongly supported by macroeconomic datas. The issue is that there is plenty ways to understand that positive correlation. The basic keynesian consumption function is highly consistent with that fact. More hours means more income and more consumption. To distinguish between the competing explanations, we should look at microeconomic evidences but they are quite hard to get on that particular problem. For example; Basu and Kimball trying to estimate long run substitution and income effect on labor supply parameatrize the level of substitutability between consumption and leisure and indicates large uncertainty about that value. Very high level of substitutability seems however implausible.

Reliance on labor market assumption Rigid wages and non separable preferences expose the issues raised by the relaince on labor market based assumption. Intuitively and empirically, the labor market seem very far away from the walrasian ideal. But we have yet to wait for a tractable and realistic representation of labor market easy to integrate into business cycles model. It seems better to keep for the moment a simple representation like walrasian market. But, it is better in that case to avoid results relying too much on labor market assumption

6 Robustness

6.1 Exogenous investment

In the previous section, we have studied the effects of a demand shock identified to an investment shock. However, the investment is purely exogenous and I have abstracted from their effect on productive capacity. The investment is not different from an increase in wasteful governement spending. This is not completely unjustifiable. We can think for example that production

requires a fixed amount of capital. This capital stock does not depreciate but need a certain amount of consumption good to be maintained at each period. investment shock are increase in maintenance cost. It would not be an unreasonable story but there are other reason to focus on demand effects of investment. First, they are probably quantitatively more important. On the demand side, rise in investment should be compared to national income. On the supply side, it increases the capital stock. The latter is between four and six times the annual value of national income. Moreover, the impact of investment on supply is delayed by around one year in average whereas the demand effect is immediate. So, from a quantitative perspective, demand effects should play a larger role than supply one if the model is demand determined like the new keynesian model. Second, supply effects of investment will add to the demand effects we have highlighted in previous section. A well known issue for investment shocks is the difficulty they have to generate comovement between consumption, leisure and investment. Reducing the crowding out effect and the effect of interest rate on leisure is not a definitive solution but seems a good start.

Conclusion

The inability of loose monetary policy to overcome the recent economic downturn suggests that real interest rate fall may have a much weaker impact on economic activity than previously believed. The conventionnal thinking is that a monetray expansion will stimulate private spending and increase aggregate demand. In the standard new keynesian model, the quantitative magnitude of the effect is given by the elasticity of intertemporal substitution. Study the model when the value of the parameter is low seems an interesting line of research. I show that when the EIS approaches zero, The output response to an aggregate demand shock approaches one But in the standard new keynesian model, this is not the only effect of the interest rate. It has also a supply effect on hours through the marginal utility of consumption term in the labor supply equation. This effect is independant from the value of the EIS. Whereas, thsi effect does not modify the output response, it changes considerably the nature of the response. With this effect, the response is nearly optimal, whereas the departure from the natural output is considerably larger without it. This very powerful effect have not been backed by any empirical evidences at our knowledge. My personal guess is that the discrepancy between the model and the reality may come from the way that agents are supposed to discount their future earnings in the model. Whereas it does not appear explicitly in the new keynesian model, the discunted sum of future earnings have a considerable importance in the model. Because the discount factor is equal to the inverse of the expected product of future real rates, this sum reacts very strongly to real rate change. If households discount their future earnings like economist discount themselves, with a constant factor, this effect should be seriously alleviate. However, in the traditionnal optimization framework in infinite horizon, the wealth does not

appear explicitly and it is not possible to modify the discounting method. We propose an alternative optimization framework where instead of choosing the whole sequence of leisure and consumption over its lifetime, the agent chooses step by step between leisure and labor income and between consumption and future assets. It allows to separate completely the labor supply choice and the consumption choice. It allows to cancel the wealth effect whereas keeping a tractable consumption equation. But, by introducing explicitly the wealth, it also allows us to modify the discount assumption. We built a macroeconomic model and verify that the supply effect of interest rate goes through the discounting assumption. This result also shows that relatively small departure from complete rationality can have a very strong macroeconomic impact. We can wonder if usual behavioral assumption used by macroeconomist can be an acceptable approximation of the reality.

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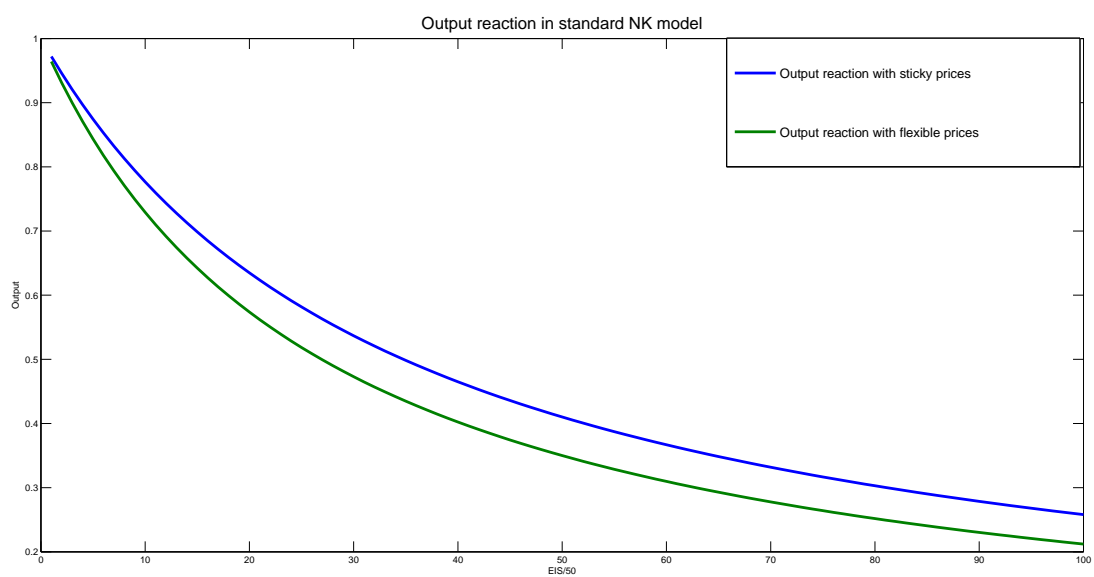


Figure 1: Output response with respect to EIS with supply side effect

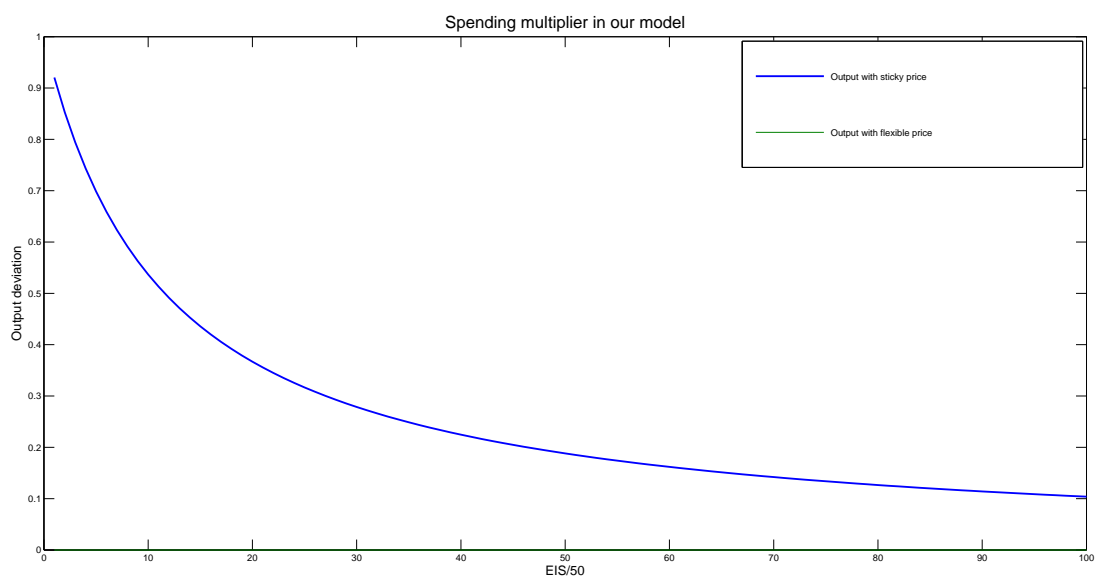


Figure 2: Output response with respect to EIS without supply side effect

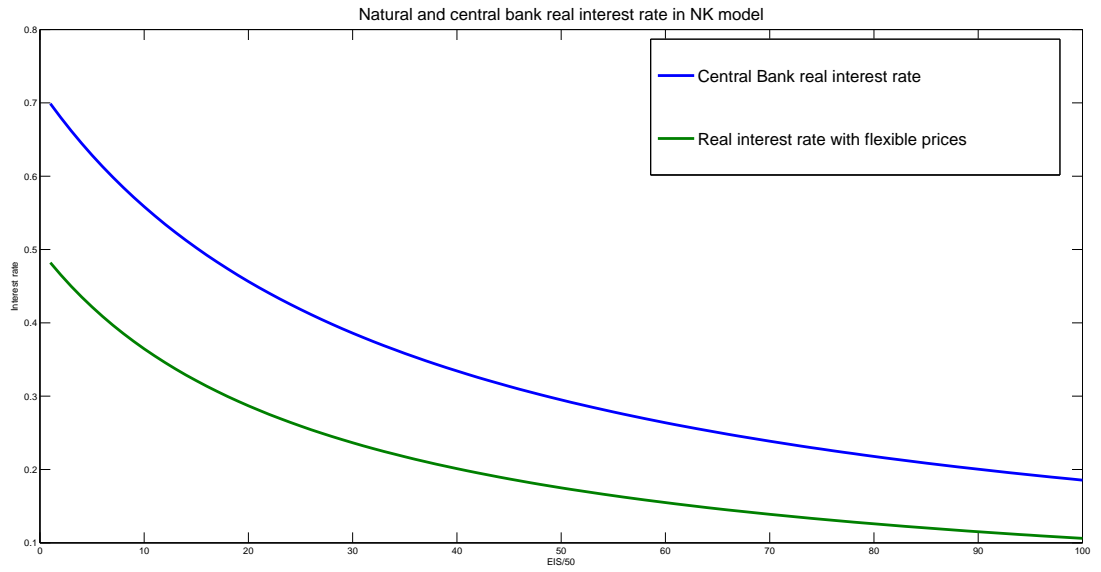


Figure 3: Real rate response with respect to EIS with supply side effect

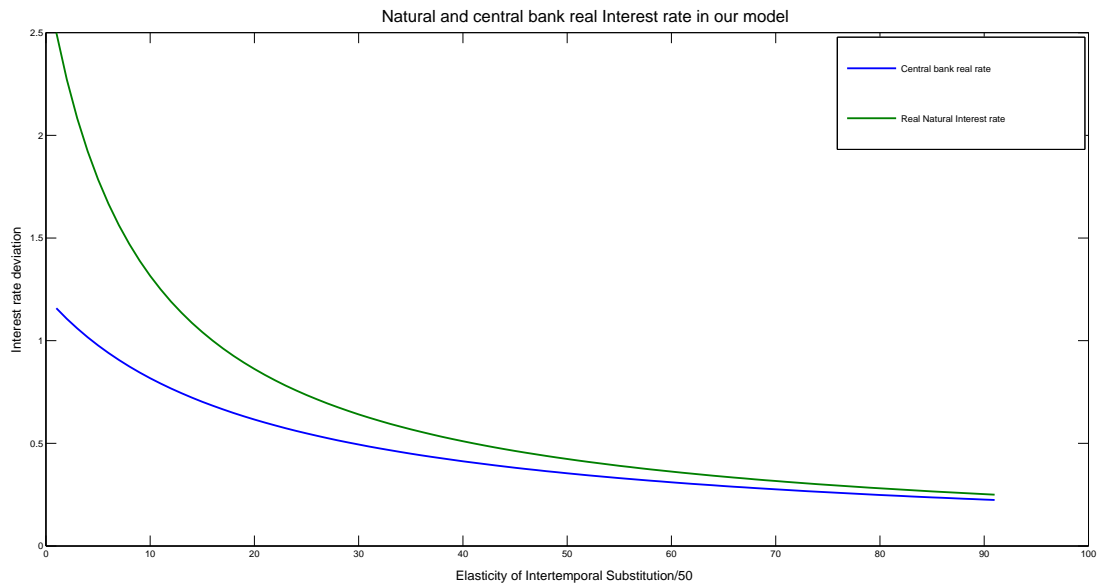


Figure 4: Real rate response with respect to EIS without supply side effect

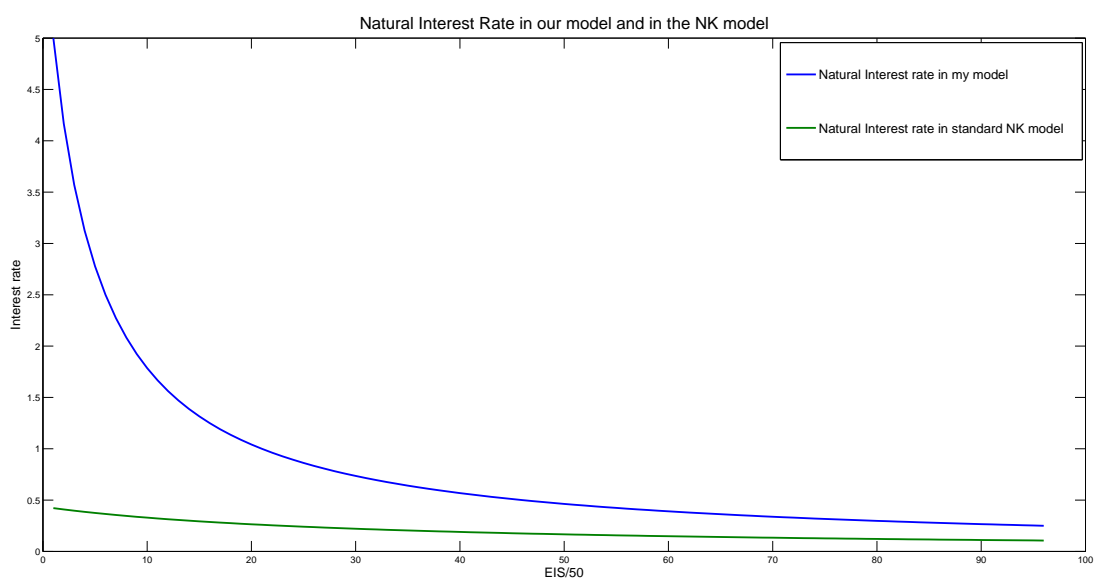


Figure 5: Natural interest rate with and without supply side effect

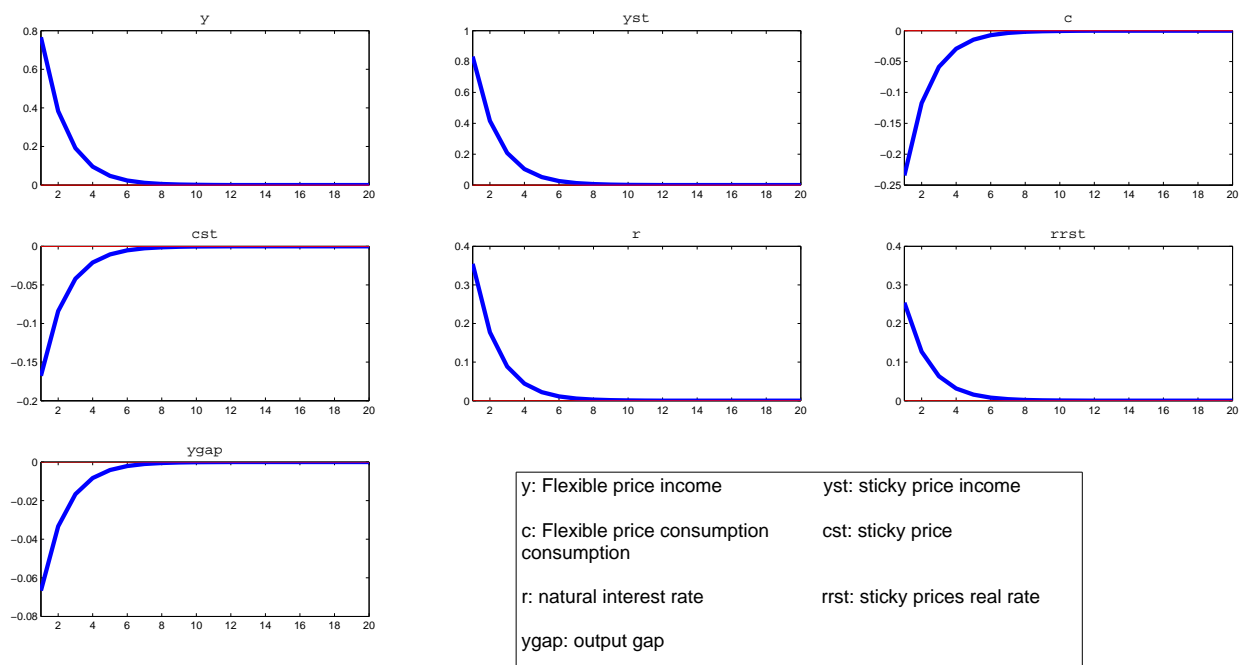


Figure 6: IRF with interest rate discounting

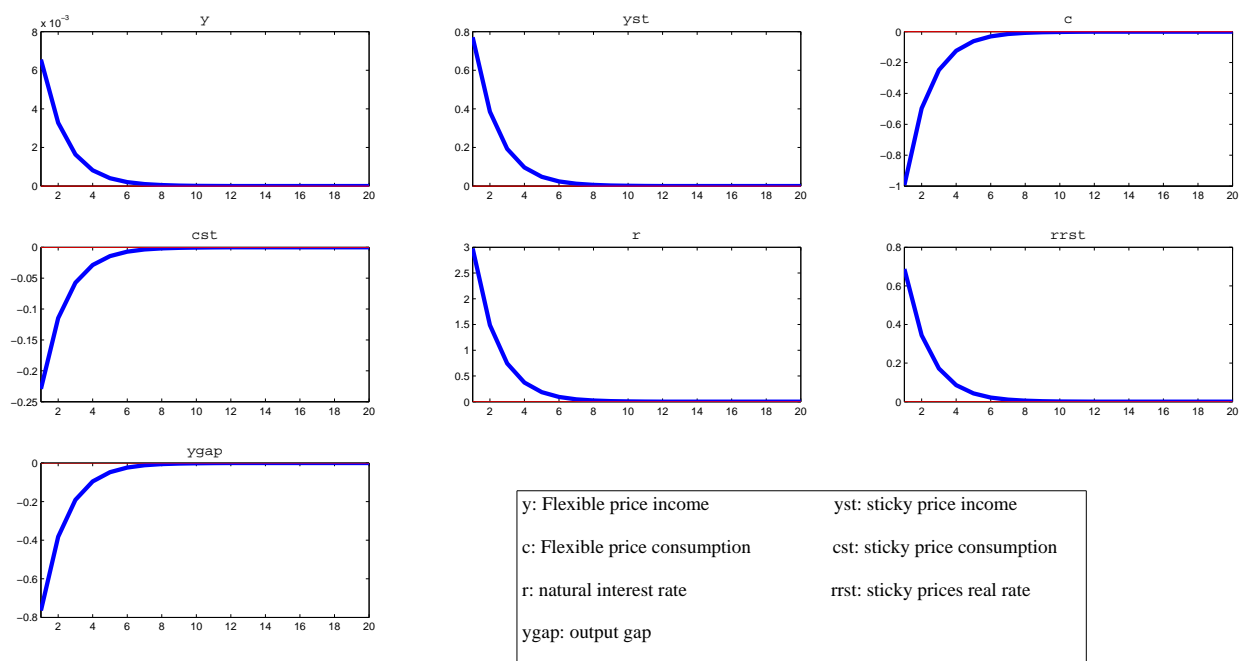


Figure 7: IRF with constant discount