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► **To cite this version:**

Mathieu Boullot. Secular Stagnation, Liquidity Trap and Rational Asset Price Bubbles. 2016. halshs-01295012v1

HAL Id: halshs-01295012

<https://shs.hal.science/halshs-01295012v1>

Preprint submitted on 30 Mar 2016 (v1), last revised 29 May 2017 (v3)

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Secular Stagnation, Liquidity Trap and Rational Asset Price Bubbles

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First draft: 30/03/2016

Abstract

I build a model of liquidity traps and secular stagnation of arbitrary duration caused by local shortages of assets and propose a new mechanism through which rational bubbles stimulate investment and output. As opposed to global shortages of assets or safe asset shortages, local shortages of assets emphasize the macroeconomic importance of microeconomic portfolio constraints and liquidity differentials across markets. Financial frictions that hamper the well-functioning of financial markets preclude some agents to invest in capital and introduce a premium between the interest rate and the stock market returns. A negative shock on the supply of assets available to those constrained agents rises the liquidity premium, decreases the interest rate and moves the economy toward the liquidity trap. Once at the ZLB, such a shock cannot be accommodated anymore by a drop in the interest rate, output falls below its potential to restore equilibrium between the supply and demand for liquidity. Bubbles that increase the supply of assets available to the constrained agents, prevent such a bad outcome and restore efficiency of the equilibrium, at the possible cost of a drop in potential output.

Keywords: Bubbles, Monetary Policy, Zero lower bound, Liquidity trap, Secular stagnation, Financial frictions, Dynamic inefficiency

JEL Classification Numbers: E21, E24, E520.

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1 Introduction

Since 2007, it has often been advanced that US GDP growth during the years that predates the sub-prime crisis has been sustained by rising levels of private debt and housing prices due to an overwhelmingly lax monetary policy. As the central bank eventually recognized the unsustainable nature of this “speculative” growth, the theory goes, it “leaned against the wind” and increased its policy rate, which burst the bubble and provoked the Great Recession that followed. A similar chain of events may be thought of as the origins of the Japanese malaise since the 90’s. In both cases, the burst of the bubble that triggered the crisis – on the US housing market, Japanese housing and stock markets, has let the economy with falling inflation rates and rising output gap despite zero nominal interest rates for near to a decade in the US and more than two in Japan. However, this seemingly simple theory forgets that the US GDP growth during the 2000’s was barely exceptional, but rather modest; and also that inflation was under control and did not exceed the central bank’s target ¹.

In this paper, I propose a different approach. I follow [Summers \(2013\)](#) and argue that the large financial bubble, instead of putting the economy on an unsustainable growth path, was necessary to maintain GDP at its potential. I build a simple macroeconomic model of secular stagnation ([Hansen, 1939](#)), that is a depression induced by a chronic shortfall in aggregate demand of arbitrary duration, and I identify conditions under which (i) bubbles exist; (ii) bubbles are expansionary. Roughly speaking, the existence of rational bubbles is conditional on fiscal policy while their macroeconomic effects are conditional on monetary policy. My contributions are threefold: first, I uncover another force behind the liquidity trap and the secular stagnation that comes with, which stems from a local shortage of assets as opposed to a global shortage of assets ([Eggertsson and Mehrotra, 2014](#)) or a shortage of safe assets ([Caballero and Farhi, 2016](#)). Second, I provide a new mechanism to explain how rational bubbles sustain investment, aggregate demand, and output in the liquidity trap. Third, I clarify how the existence and macroeconomic effects of rational bubbles are conditional on respectively the fiscal and monetary regimes.

As in the standard New Keynesian model, the output gap – the gap between current and potential output – is an increasing function of the interest rate gap – the gap between the natural

¹[Gali \(2014\)](#) questions from a theoretical point of view, and [Gali and Gambetti \(2015\)](#) provides empirical evidences against, the “conventional” view that “leaning against the wind” monetary policy could cause the bubble to burst.

interest rate and the interest rate set by the central bank ². In turns, the natural interest rate is determined by the ratio of the supply to the demand of liquidity. Away from the zero lower bound (ZLB), monetary policy accommodates a drop in the supply or an increase in the demand of liquidity by adjusting the policy rate downward so as to maintain the interest rate gap nil. However, if fundamental liquidity – the sum of public and private debt – is sufficiently scarce, the natural interest rate becomes negative: the central bank hits the ZLB and the economy enters the liquidity trap. Since the interest rate is fixed at its lower bound, the output gap grows as the natural interest rate falls.

If, when the interest rate is equal to the growth rate of the economy, the demand of liquidity exceeds its supply, this excess demand can be channeled to create rational bubbles. Outside of the liquidity trap, bubbles absorb some savings that would have otherwise been invested in capital accumulation: output and potential output are reduced. But in the liquidity trap, bubbles also contribute to sustain aggregate demand as they rise the natural interest rate. Which of the two effects, supply- or demand-side, dominates the other depends on the degree to which monetary policy is constrained at the fundamental equilibrium, i.e on the interest rate gap without bubbles.

A negative natural interest rate is symptomatic of a global shortage of assets (Caballero, 2006), which is akin to dynamic inefficiency: in equilibrium, agents want to hold more assets than the economy can produce. Despite being theoretically possible, a permanent global shortage of assets is usually thought of as unrealistic and empirically irrelevant (Abel et al., 1989) ³. This automatically rules out the possibility of permanent liquidity trap and rational bubbles altogether ⁴, since both exist only if the equilibrium is dynamically inefficient.

However, if the agents face heterogeneous investment opportunities and borrowing constraints, one may re-conciliate dynamic efficiency and permanently negative interest rates (Martin and Ventura, 2012): only a subset of agents face a shortage of asset, i.e the shortage of assets is *local* but not *global*. If for example participation in the stock market is restricted as I assume in the paper, under some conditions, the rental rate of capital carries a liquidity premium over the interest rate ⁵. Recent empirical evidences confirm that illiquid assets pay a participation or

²Except stated otherwise, the interest rate refers to the *real* interest rate, not to the *nominal* interest rate.

³This last point has been recently challenged by Geerolf (2013).

⁴Dynamic inefficiency arises, and bubbles are possible, when $r^* < g$, where r^* is the natural interest rate and g the growth rate of the economy. A liquidity trap arises when $r^* < r$, where $r \leq 1$ is the real lower bound that depends on the ZLB and inflation expectations.

⁵“According to the last [2013] Survey of Consumer Finances, fewer than 15 percent of U.S. households own stocks directly, and only about 50 percent of households own stocks either directly or indirectly through mutual

liquidity premium over near-money assets (Krishnamurthy and Vissing-Jorgensen, 2012). Furthermore, the equity premium as well as the level of the real interest rate on safe assets have proven exceedingly difficult, if not impossible, to rationalize in terms of risk aversion (Mehra and Prescott (1985), Weil (1989)) and households' portfolio choices are barely explained without relying on financial constraints (Ragot, 2014).

The model has three key ingredients: an OLG demographic structure that allows the natural interest rate to deviate permanently from agents' discount factor; financial frictions that introduce a wedge between the rental rate of capital and the interest rate; nominal rigidities and the ZLB that create a gap between the natural interest rate and the current interest rate, or output and potential output, and open the door to liquidity traps. Households live for two periods. Young investors, who have access to the stock market, sell inside liquidity (debt) to the young savers, who don't have access to the stock market, in order to finance purchases of capital that they rent to the firms. Inside liquidity must be backed by imperfectly pledgeable future returns – dividends and profits. Because the capital market is illiquid relative to the debt market, the rental rate of capital carries a liquidity premium over the interest rate. As inside liquidity becomes scarcer, the natural interest rate falls and the liquidity premium rises. Facing a lower cost of debt, investors choose a higher level of leverage, they accumulate more capital and create more assets.

As long as the interest rate can absorb changes in the scarcity of liquidity, the financial frictions and sticky prices are irrelevant on aggregate: their effects are purely distributional. But once the economy enters the liquidity trap, that is once the ZLB binds, the central bank cannot adjust the interest rate downward anymore after a shock on the natural interest rate. To restore equilibrium in the financial markets, an output gap endogenously emerges, which reduces the demand for liquidity, rises its supply, and acts as a substitute for a fall in the interest rate. This alternative equilibrating mechanism has an efficiency cost: output is lower than its potential. In the long run, potential output also drops because households accumulate less capital.

If agents succeed in coordinating their expectations, rational bubbles are another way to restore equilibrium in the liquidity trap. Since rational bubbles rise the supply of assets available to the young savers and transfer wealth from the young agents to the old, they increase the natural interest rate and decrease the liquidity premium. Bubbles have benefits:

_____ funds or retirement accounts.” (Bricker et al., 2014).

they minimize the efficiency loss associated to the output gap, but they also have costs: they reduce the amount of savings channeled toward capital accumulation and as a consequence potential output shrinks. Even if the same trade-off applies to public debt, either in the form of bonds or money, macro-prudential policies that directly transfer wealth from the young savers to the young investors represent a pure free lunch in the liquidity trap.

Related literature Despite the flourishing literature on rational bubbles ⁶ ⁷ that builds on the seminal papers by [Samuelson \(1958\)](#) and [Tirole \(1985\)](#), to the best of my knowledge, the macroeconomic consequences of rational bubbles on aggregate demand have never been investigated ⁸. As numerous recent papers (e.g. [Santos and Woodford \(1997\)](#), [Caballero and Krishnamurthy \(2006\)](#), [Caballero et al. \(2006\)](#), [Kocherlakota \(2010\)](#), [Arce and Lopez-Salido \(2011\)](#), [Matsuoka and Shibata \(2012\)](#), [Miao and Wang \(2012\)](#), [Wang and Wen \(2012\)](#), [Kunieda et al. \(2014\)](#), [Martin and Ventura \(2014\)](#), [Clain-Chamosset-Yvrard and Seegmuller \(2015\)](#), [Ikeda and Phan \(2015a\)](#), [Ikeda and Phan \(2015b\)](#)), I consider an environment with financial frictions and heterogeneous agents. A few papers already provide conditions for bubbles to be expansionary in such an environment (e.g. [Kocherlakota \(2009\)](#), [Farhi and Tirole \(2012\)](#), [Martin and Ventura \(2012\)](#), [Miao and Wang \(2015a\)](#), [Miao and Wang \(2015b\)](#)): bubbles help to reallocate wealth toward productive investment, rising the stock of capital and potential output. Unlike those papers, I focus on the demand-side effects of rational bubbles in the liquidity trap.

Other papers have studied the possibility of secular stagnation, (e.g. [Kocherlakota \(2013\)](#), [Eggertsson and Mehrotra \(2014\)](#), [Caballero and Farhi \(2016\)](#) and [Bacchetta et al. \(2015\)](#)). Like me, the three first papers consider an OLG economy with nominal rigidities. One difference lies in the roots of the liquidity trap: global shortages of assets in [Kocherlakota \(2013\)](#) and [Eggertsson and Mehrotra \(2014\)](#); safe assets shortages in [Caballero and Farhi \(2016\)](#). I already discussed why local shortages of assets are more realistic than global shortages of assets; local and safe assets shortages are not mutually exclusive but rather complementary: as well as

⁶See [Miao \(2014\)](#) for a good introduction to that literature.

⁷Alternatives theories based on asymmetric information, agency problems and behavioral biases have been developed. A very limited list includes [Allen and Gorton \(1993\)](#), [Allen and Gale \(2000\)](#), [Abreu and Brunnermeier \(2003\)](#), [Scheinkman and Xiong \(2003\)](#), [Barlevy \(2014\)](#) and [Wigniolle \(2014\)](#). See [Brunnermeier and Oehmke \(2013\)](#) for a literature review.

⁸The only exception is [Caballero and Farhi \(2016\)](#). They show that bubble creation shocks are expansionary in a liquidity trap. This is somewhat artificial, as constant bubble creation shocks induce a wealth effect on each new generation; “standard” rational bubbles of the type I consider would have no effects in their model. I discuss further differences below.

differences in risk aversion, segmented markets may prevent some agents to invest in risky assets. My work borrows heavily from theirs. One further difference with respect to [Kocherlakota \(2013\)](#) and [Caballero and Farhi \(2016\)](#) is that since I model a production economy whereas they consider an endowment economy, I can identify the trade-off between efficiency loss and lower potential output and provide clear microfoundations for the supply of assets. [Bacchetta et al. \(2015\)](#) study the possibility of a supply-side secular stagnation, where money crowds out capital in the liquidity trap. I point to one limit of their analysis and explain why, conditional on a fiscal policy that targets some real variables, one can consider the cashless limit ⁹.

My paper also relates to the literature on liquidity traps (e.g [Krugman et al. \(1998\)](#), [Eggertsson and Woodford \(2003\)](#), [Christiano et al. \(2011\)](#), [Correia et al. \(2013\)](#)). While this literature usually relies on an exogenous shock to a structural parameter ¹⁰ to generate a temporary global shortage of asset and negative interest rates, e.g a discount rate shock, tightened borrowing constraints ([Lorenzoni and Guerrieri \(2011\)](#), [Eggertsson and Krugman \(2012\)](#), [Korinek and Simsek \(2016\)](#)) or overbuilding with irreversible investment ([Rognlie et al. \(2015\)](#)), my model underlines how a non-fundamental shock, a shock on the stock of bubbles, can induce a liquidity trap of arbitrary duration.

The macro-finance literature (e.g [Woodford \(1990\)](#), [Holmstrom and Tirole \(1998\)](#), [Bernanke et al. \(1996\)](#), [Kiyotaki and Moore \(1997\)](#), [Matsuyama \(2004\)](#), [Kiyotaki and Moore \(2012\)](#), [Gertler and Kiyotaki \(2013\)](#)) as well as the New Monetarist literature (e.g [Lagos and Wright \(2005\)](#), [Williamson \(2012\)](#), [Andolfatto and Williamson \(2015\)](#)) also stress the the role of fundamental stores of value and liquidity to support respectively investment and consumption in environments with financial frictions. Unlike them, I focus on the useful role of bubbly liquidity in an environment with both financial frictions and nominal rigidities.

Finally, my model is reminiscent of the literature on disequilibrium (e.g [Barro and Grossman \(1971\)](#), [Benassy \(1976\)](#), [D'Autume and Michel \(1985\)](#)), which emphasizes how, given rigid prices, an excess demand in one market translates into an excess supply in another. In the liquidity trap, the interest rate is fixed at its lower bound – the ZLB minus expected inflation: an excess demand of liquidity implies an excess supply of goods. An output gap is necessary to restore equilibrium, at the cost of a drop in the labor supply.

⁹[Asriyan et al. \(2015\)](#) show that bubbles creation shocks are expansionary in such an environment.

¹⁰One exception is [Schmitt-Groh and Uribe \(2012\)](#), where expectations of low inflation drive the economy into the liquidity trap.

The paper is organized as follows. Section 2 exposes the basic model and characterizes the equilibrium. Section 3 compares the bubbly and fundamental steady-states. Section 4 considers fiscal policy. Section 5 introduces inflation.

2 A model of liquidity shortages and bubbles

2.1 The basic environment and equilibrium

The basic setup is an overlapping generations model with a single consumption good, two factors of production, capital and labor, and three assets: private bonds, capital and bubbles. Time is discrete, $t \in \{0, 1, \dots\}$, and the horizon infinite.

Demography and preferences At each date t a mass 1 of households indexed by $j \in [0, 1]$ is born. In each generation, a fraction $\omega \in (0, 1)$ of households are investors, whereas the remaining $1 - \omega$ are savers. Investors own the firms and have access to all the financial markets, whereas savers can trade only in bubbles and real bonds. Each agent lives for two periods and has preferences over consumption and labor,

$$U(c_{j,t}^y, c_{j,t+1}^o, n_{j,t}) = u(c_{j,t}^y - v(n_{j,t})) + \beta u(c_{j,t+1}^o) \quad (1)$$

Here, $\beta \in (0, 1)$ is the discount factor, $c_{j,t}^y$ denotes young-age consumption, $c_{j,t+1}^o$ old-age consumption and $n_{j,t}$ is the labor supply. Such a non-separable utility function shuts down the wealth effect on the labor supply (Greenwood et al., 1988). In particular, the optimal labor solves the static problem,

$$e_{j,t} \equiv \max_{n_{j,t}} w_t n_{j,t} - v(n_{j,t}) \quad (2)$$

Which does not depend on consumption. Here, w_t denotes the real wage and $e_{j,t}$ denotes net labor income¹¹. Similarly, I define $\tilde{c}_{j,t}^y \equiv c_{j,t}^y - v(n_{j,t})$ as net young-age consumption. For the sake of tractability, I will assume that $u(c) = \log(c)$ and $v(n) = \frac{n^{1+\eta}}{1+\eta}$, where $\eta > 0$ is the Frish elasticity.

Assets, budget and borrowing constraints Bubbles are intrinsically worthless as-

¹¹With standard additively separable preferences, the labor supply of the savers would be inelastic in equilibrium, as their wealth consists entirely in labor income.

sets akin to Ponzi-schemes: household j buys $q_{j,t}$ units of bubbles today at a price b_t because he expects to re-sell tomorrow at a price b_{t+1} . The returns on riding bubbles, $\frac{b_{t+1}}{b_t}$ are pure capital gains. The supply of bubbles is exogenous and normalized to 1. Real bonds are promises to pay one unit of the consumption good the next period. The price of a real bond in period t is $\frac{1}{1+r_{t+1}}$, where r_{t+1} is the interest rate. Household j 's net supply of bonds at date t is denoted by $d_{j,t+1}$, that is he is a net borrower in period t if $d_{j,t+1} > 0$. Real bonds are a form of inside liquidity, whereas bubbles are a form of outside liquidity (Holmstrom and Tirole, 1998). Debt issuance is subject to a standard borrowing constraint,

$$d_{j,t+1} \leq \phi (\Pi_{j,t+1} + R_{t+1}k_{j,t}), \quad \phi \in (0, 1) \quad (3)$$

Here, $\Pi_{j,t+1}$ denotes the profit received by household j from the firms described below, R_{t+1} is the rental rate of capital – I assume full capital depreciation – and $k_{j,t+1}$ is the stock of capital of household j . Each agent j has access to the bubbles and debt markets at each date t , however participation in the capital market is restricted. Specifically, I assume that households have access to the following technology to transform the consumption good into capital,

$$k_{j,t+1} = i_{j,t}\theta_{j,t} \quad (4)$$

Where $i_{j,t}$ is investment made out at date t and $\theta_{j,t}$ indexes whether the households has an investment opportunity, with $Pr(\theta_{j,t} = 1) = \omega$ and $Pr(\theta_{j,t} = 0) = 1 - \omega$ ¹². Households know their type before taking any decision in t . Since those shocks are *i.i.d* across agents, ω households receive an investment opportunity at each date. I will call the households with an investment opportunity the investors or borrowers and the remaining households the savers or lenders.

The budget constraint are

$$c_{j,t}^y + i_{j,t} + q_{j,t+1}b_t - \frac{d_{j,t+1}}{1+r_{t+1}} = w_t n_{j,t} \quad (5)$$

$$c_{j,t+1}^o + d_{j,t+1} = \Pi_{j,t+1} + R_{t+1}k_{j,t+1} + q_{j,t+1}b_{t+1} \quad (6)$$

Firms and sticky prices As in the standard New Keynesian model, a continuum of

¹²I have provided elsewhere (...) a micro-foundation for $\theta_{j,t}$ as a minimum investment requirement in an economy with an unequal distribution of wealth.

firms indexed by $v \in [0, 1]$ produce each a differentiated intermediate good. Each firm is a monopolist on its variety. A competitive final good sector aggregates the intermediate varieties to produce the final good according to the Dixit-Stiglitz technology,

$$y_t = \left(\int y_{v,t}^{\frac{\epsilon-1}{\epsilon}} dv \right)^{\frac{\epsilon}{\epsilon-1}} \quad (7)$$

Where $\epsilon > 1$ is the elasticity of substitution between varieties. Each monopolist $v \in [0, 1]$ produces according to a standard constant-returns-to-scale production function,

$$y_{v,t} = F(k_{v,t}^d, n_{v,t}^d) \quad (8)$$

I will consider the Cobb-douglas production function, $F(k, n) = k^\alpha n^{1-\alpha}$, $\alpha \in (0, 1)$. For now, I make the extreme assumption that each monopolist has a preset and constant nominal price, $P_{v,t} = P$. Each monopolist $v \in [0, 1]$ maximizes profits under a demand constraint and a price rigidity constraint,

$$\begin{aligned} \check{\Pi}_t &= \max_{P_{v,t}, k_{v,t}^d, n_{v,t}^d} \frac{P_{v,t}}{P_t} F(k_{v,t}^d, n_{v,t}^d) - (1 - \tau(n_t)) (R_t k_{v,t}^d + w_t n_{v,t}^d) \\ \text{s.t. } & F(k_{v,t}^d, n_{v,t}^d) \leq \left(\frac{P_{v,t}}{P_t} \right)^{-\epsilon} y_t, \quad P_{v,t} = P \end{aligned} \quad (9)$$

Monopolistic competition introduces a distortion with respect to the efficient allocation: the price includes a markup over the marginal cost. To preserve the efficiency of the equilibrium with an unconstrained monetary policy, I assume that the government subsidizes both labor and capital, $\tau(n_t) = \frac{1}{\epsilon}$ if $n_t \leq n_t^*$ and $\tau(n_t) = 0$ otherwise¹³, where n_t^* is the efficient level of employment described below. The subsidies are financed by lump-sum taxes levied on the investors, $\Gamma_t \equiv \int \Gamma_{j,t} dj = \int \tau(n_t)(w_t n_{v,t} + R_t k_{v,t}) dv$. The aggregate profits net of taxes, $\Pi_t \equiv \int \check{\Pi}_{v,t} dv - \Gamma_t$, are redistributed to the investors, $\Pi_{j,t} = \theta_{j,t} \frac{\Pi_t}{\omega}$ ¹⁴.

Since monopolists face the same marginal cost and price, they are symmetric and choose the same capital-labor ratio and output. Furthermore, the aggregate price level is constant, $P_t = \left(\int P_{v,t}^{1-\epsilon} dv \right)^{\frac{1}{1-\epsilon}} = P = 1$ and normalized to one. The representative monopolist in equilibrium

¹³As [Korinek and Simsek \(2016\)](#) notes, with GHH preferences, another equilibrium would exist if the firms also receive the subsidy while producing more than the efficient level.

¹⁴If profits were equally divided across all households, my results would qualitatively not change but the analysis would be slightly more complicated because the borrowers and lenders would not discount profits at the same rate in equilibrium.

faces an aggregate demand constraint, $F(k_t^d, n_t^d) \leq y_t$. In turns, aggregate demand is the sum of investment and consumption, $y_t = c_t + i_t$, where $c_t \equiv \int (c_{j,t}^y + c_{j,t}^o) dj$ is aggregate consumption and $i_t \equiv \int i_{j,t} dj$ is aggregate investment. Output in this economy is demand-determined.

Definition 1 (Competitive equilibrium) *For given initial conditions $\{b_0, k_{j,0}, d_{j,0}, q_{j,0}\}$, any specified policy process $\{r_{t+1}\}$ and sequence of idiosyncratic shocks $\{\theta_{j,t}\}$, a competitive equilibrium is a sequence of prices $\{w_t, R_t, \Pi_t\}$ and quantities $\{y_t, k_t^d, n_t^d, n_{j,t}, e_{j,t}, i_{j,t}, k_{j,t+1} d_{j,t+1}, q_{j,t+1}, c_{j,t}^y, \tilde{c}_{j,t}^y, c_{j,t}^o\}$ such that each household maximizes (1) under (3) - (6) and the labor supply solves (2), each monopolist solves (9) under (8), final output is given by (7), and all markets clear.*

Monetary policy Since the prices are fully rigid, I follow [Rognlie et al. \(2015\)](#) and assume that monetary policy replicates the efficient allocation. Given the stock of capital, we can define the efficient level of output, $y_t^* \equiv s(k_t)$, as:

$$s(k_t) = F(k_t, n_t^*), \quad \text{where} \quad n_t^* = \arg \max_n F(k_t, n) - v(n) \quad (10)$$

Here, $k_t \equiv \int k_{i,t} di$ is the aggregate stock of capital and n_t^* is the efficient level of employment, which equalizes the marginal disutility of work to the marginal product of labor. $s(k_t)$ can be thought of as the supply-determined level of output, i.e the level of output that would prevail in an economy with flexible prices. Therefore, the natural interest rate, r_{t+1}^* , is defined as the unique interest rate such that output is efficient at the competitive equilibrium in period t , $y_t = y_t^*$. Given the natural interest rate, monetary policy sets the nominal interest rate according to the following rule ¹⁵,

$$r_{t+1}^n = \max\{r_{t+1}^*, \underline{r}\}, \quad \underline{r} = \underline{r}^n \leq 0 \quad (11)$$

Because the inflation rate is nil at each period, $\pi_{t+1} \equiv \frac{P_{t+1}}{P_t} - 1 = 0$, the ZLB constraint on the nominal interest rate, $r_{t+1}^n \geq \underline{r}^n$, translates into a constraint on the equilibrium interest rate, $r_{t+1} = r_{t+1}^n \geq \underline{r} = \underline{r}^n$. The recent experiences in some OECD countries with negative nominal interest rates have shown that the money demand does not explode at $r^n = 0$, strictly speaking the zero lower bound (ZLB) is somewhat lower than 0 ¹⁶. However, major central banks have

¹⁵The central bank has rational expectations and my model is deterministic, thus $E_t r_{t+1}^* = r_{t+1}^*$.

¹⁶See [Rognlie \(2015\)](#) for theoretical results on optimal monetary policy with negative nominal interest rates.

been quite reluctant to implement negative nominal interest rates; furthermore, there is great deal of uncertainty surrounding the “true” ZLB. Thus one may think of \underline{r}^n as resulting from either an exploding money demand at $r_{t+1}^n < \underline{r}^n$, or a self-induced paralysis of the central bank (Krugman et al. (1998), Bernanke (1999)). I will say that the economy is in the liquidity trap when the ZLB constraint in (11) binds, i.e when $r_{t+1} = \underline{r} > r_{t+1}^*$. Even though in most of the paper I consider the cashless limit, I introduce money and public bonds in subsection 4.2: conditional on a fiscal policy that targets a constant debt-to-GDP ratio, money is irrelevant.

It is well-known since Diamond (1965) that the equilibrium of the standard OLG model with production is Pareto-efficient if and only if the steady-state rental rate of capital is weakly higher than the growth rate of the economy, $R \geq g$. As long as $r_{t+1} > r$, the aggregate dynamic of the present model is the same as that of the standard frictionless OLG model. In particular, the sole effect of borrowing constraints is to redistribute consumption within a generation. Intuitively, the equilibrium in my model is Pareto-efficient if and only if $y_t = y_t^*$ for each t and $R \geq g = 1$. As I assume the latter, the monetary rule in (11) is constrained efficient ¹⁷.

2.2 Characterizing the equilibrium

In this subsection I derive the main equations that govern the dynamic of the model. To begin with, we will have a look at the supply side of the economy. In particular, we will discuss how the liquidity trap provokes an endogenous drop in output. Then we will focus on the portfolio choices and the determination of the natural interest rate. Finally, we will study the roots of the liquidity trap and analyze how rational bubbles can alleviate shortfalls in aggregate demand induced by local shortages of assets.

Output gap and factors’ shares in income To understand the macroeconomic consequences of price rigidities and the ZLB, i.e of the liquidity trap, consider the representative monopolist’s optimal factor demands for problem (9),

$$(1 - \xi_t)F_k(k_t^d, n_t^d) = R_t \quad \text{and} \quad (1 - \xi_t)F_n(k_t^d, n_t^d) = w_t \quad (12)$$

Here, $\xi_t \equiv \frac{\delta_t}{y_t} \frac{1}{1-\epsilon}$ is the labor wedge, where $\delta_t \geq 0$ is the Lagrange multiplier on the price

¹⁷If the equilibrium is dynamically inefficient, monetary policy faces a trade-off between aggregate demand management and consumption smoothing. As this is behind the scope of this paper, I rule this case out by assumption.

rigidity constraint in (9). The labor wedge is endogenously determined in equilibrium as the difference between the price, $P = 1$, and the marginal cost of production, Λ_t ,

$$\xi_t = (1 - \Lambda_t), \quad \Lambda_t \equiv \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{w_t}{1 - \alpha}\right)^{1-\alpha} \quad (13)$$

When monetary policy is unconstrained, $r_{t+1}^n = r_{t+1}^*$, it drives aggregate demand such that the price rigidity constraint in (9) never binds: firms would not set another price even if they could. Output is efficient and supply-determined, $y_t = y_t^*$ and $\xi_t = 0$.

However, if monetary policy is constrained by the ZLB, $r_{t+1}^n = r > r_{t+1}^*$, aggregate demand is inefficiently low. In this case, the price rigidity constraint binds: firms must keep an inefficiently high level of markup and produce less than potential, $y_t < y_t^*$. In equilibrium,

$$\int n_{j,t} dj = n_t^d, \quad \text{and} \quad \int k_{j,t} dj = k_t^d \quad (14)$$

This induces a shift in the demand curves for capital and labor, which reduces the productive factors' share in income but increases the share of rent,

$$\frac{w_t n_t}{y_t} = (1 - \alpha)(1 - \xi_t) \quad \text{and} \quad \frac{R_t k_t}{y_t} = \alpha(1 - \xi_t) \quad (15)$$

The labor wedge acts as a markup between the price and the marginal cost. If the labor supply is endogenous, $\eta < \infty$, a shift in the labor demand curve induces households to work less in equilibrium,

$$n_t = w_t^{\frac{1}{\eta}} = ((1 - \alpha)(1 - \xi_t)k_t^\alpha)^{\frac{1}{\alpha + \eta}} \quad \text{and} \quad e_t = \frac{\eta}{1 + \eta} n_t w_t \quad (16)$$

Which creates a gap between actual and potential output,

$$\frac{y_t}{y_t^*} = (1 - \xi_t)^{\frac{1-\alpha}{\alpha+\eta}} \quad (17)$$

I will call ξ_t the labor wedge or output gap indifferently.

When monetary policy let aggregate demand fall below potential output, firms facing rigid prices must cut their demand of factors and production relative to the first-best. Consequently, the share of rent increases, labor supply goes down and output is inefficiently low. We know from (11) that this happens if and only if monetary policy is constrained by the ZLB. The next

question is: what determines the natural interest rate?

Individual consumption, savings and Euler equation The natural interest rate is determined by the savings and investment decisions of the households. A standard non-arbitrage equation implies that in equilibrium, $\frac{b_{t+1}}{b_t} = 1 + r_{t+1} \leq R_{t+1}$: as the capital market is much less liquid than the two others, the rental rate of capital may carry a liquidity premium over the interest rate. Taking this into account as well as the static labor choice (2), we can rewrite the problem of household j as:

$$\begin{aligned} & \max_{i_{j,t}, q_{j,t+1}, \tilde{c}_{j,t}^y, c_{j,t+1}^o} u(\tilde{c}_{j,t}^y) + \beta u(c_{j,t+1}^o) \\ \text{s.t. } & \tilde{c}_{j,t}^y + \frac{c_{j,t+1}^o}{1 + r_{t+1}} = e_{j,t} + \frac{\Pi_{j,t+1}}{1 + r_{t+1}} + \left(\frac{R_{t+1}}{1 + r_{t+1}} \theta_{j,t} - 1 \right) i_{j,t} \\ & i_{j,t} \leq (1 + \theta_{j,t} (\varphi_{t+1} - 1)) \left(e_{j,t} - \tilde{c}_{j,t}^y + \frac{\phi \Pi_{j,t+1}}{1 + r_{t+1}} \right) \end{aligned} \quad (18)$$

Where $\varphi_{t+1} \equiv \frac{1}{1 - \frac{\phi R_{t+1}}{1 + r_{t+1}}}$ is the leverage. A sort of distorted Euler equation holds for each household,

$$\frac{1}{\tilde{c}_{j,t}^y} = \beta R_{j,t+1}^W \frac{1}{c_{j,t+1}^o} \quad (19)$$

Here, $R_{j,t+1}^W$ is the rate of returns household j obtains on his savings,

$$R_{j,t+1}^W \equiv \theta_{j,t} R_{t+1} \varphi_{t+1} (1 - \phi) + (1 - \theta_{j,t}) (1 + r_{t+1}) \quad (20)$$

Given the discounted logarithmic preferences, young households consume a constant fraction of their discounted wealth,

$$\tilde{c}_{j,t}^y = \frac{1}{1 + \beta} \left(e_{j,t} + \frac{\Pi_{j,t+1}}{R_{t+1}} \right) \quad (21)$$

And old-age consumption is given by the Euler equation (19).

Demand and supply of assets Households are always indifferent with regards to debt or bubbles. However, they may or may not hold capital in equilibrium, depending on

their type. Assuming that $R_{t+1} > 1 + r_{t+1}$, the borrowing constraint (3) binds for productive households,

$$\frac{d_{j,t+1}}{r_{t+1}} = \begin{cases} \frac{\phi R_{t+1}}{1+r_{t+1}}(k_{j,t+1} + \frac{\Pi_{j,t+1}}{R_{t+1}}) > 0 & \text{if } \theta_{j,t+1} = 1 \\ -\frac{\beta}{1+\beta}e_{j,t} + q_{j,t+1}b_t < 0 & \text{otherwise} \end{cases} \quad (22)$$

And

$$k_{j,t+1} = i_{j,t} = \begin{cases} \varphi_{t+1} \frac{\beta}{1+\beta} \left(e_{j,t} + \frac{\Pi_{j,t+1}}{R_{t+1}} \right) - \frac{\Pi_{j,t+1}}{R_{t+1}} & \text{if } \theta_{j,t+1} = 1 \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

Investors face an arbitrage opportunity: the rental rate of capital, R_{t+1} is greater than the cost of debt, $1 + r_{t+1}$. Thus, they issue bonds in order to finance capital purchases until their borrowing constraint binds. Savers have no choice but to buy bubbles and inside liquidity issued by the leveraged borrowers.

While the demand of liquidity by the savers is inelastic, the supply of liquidity is not: a higher interest rate reduces the leverage of the investors and their investment. Bubbles, which are a form of outside liquidity, crowd out the demand of inside liquidity by the savers.

Bubbles, borrowing limits and the interest rate Inside liquidity is in zero net supply and the supply of bubbles is exogenous and normalized to one, the financial markets clearing condition is then:

$$\int d_{j,t+1} dj = 0 \quad \text{and} \quad \int q_{j,t+1} di = 1 \quad (24)$$

Given binding borrowing constraints (3), we can compute the real interest rate consistent with equilibrium in the assets markets,(24),

$$1 + r_{t+1} = \min \left\{ \frac{\phi (R_{t+1} i_t + \Pi_{t+1}) + b_{t+1}}{(1 - \omega) \frac{\beta}{1+\beta} e_t}, R_{t+1} \right\} \quad (25)$$

The interest rate is determined by the supply of liquidity relative to the demand. The binding borrowing constraints put an upper limit on the creation of inside liquidity by the

leveraged investors, which must be backed by pledgeable returns. As a consequence, the agents who are not active on the capital market, the savers, may face a shortage of assets in equilibrium, a liquidity shortage.

When inside liquidity is scarce, the rental rate of capital carries a liquidity premium over the interest rate. In comparison to the standard frictionless model, financial frictions break the link between dynamic efficiency of the equilibrium and the interest rate. Throughout the paper, I only consider equilibria where inside liquidity is scarce, i.e $r_{t+1} < R_{t+1}$.

Using the factors' share in income, (15),

$$1 + r_{t+1} = \frac{\phi(\alpha(1 - \xi_{t+1}) + \xi_{t+1}) + \bar{b}_{t+1} y_{t+1}}{(1 - \omega) \frac{\beta}{1+\beta} \frac{\eta}{1+\eta} (1 - \alpha)(1 - \xi_t) y_t} \quad (26)$$

Here, $\bar{b}_t \equiv \frac{b_t}{y_t}$ is the ratio of the stock of bubbles to output. The natural interest rate obtains in (26) when the current output gap is nil, $\xi_t = 0$ and $y_t = y_t^*$.

Bubbles increase the supply of assets available to the savers, and as a consequence they reduce the liquidity premium and rise the natural interest rate. By non-arbitrage pricing, the returns on riding the bubbles must be equal to interest rate, $\bar{b}_{t+1} = \frac{1+r_{t+1}}{g_{t+1}} \bar{b}_t$, where $g_{t+1} \equiv \frac{g_{t+1}}{g_t}$ is the growth rate of output.

$$\bar{b}_{t+1} = \frac{\phi(\alpha(1 - \xi_{t+1}) + \xi_{t+1})}{(1 - \omega) \frac{\beta}{1+\beta} \frac{\eta}{1+\eta} (1 - \alpha)(1 - \xi_t) - \bar{b}_t} \bar{b}_t \quad (27)$$

ZLB, interest rate and output gaps We have already computed the interest rate that clears the asset markets, (26). The nominal interest rate in the economy is set by central bank according to (11). In equilibrium, the Fisher equation must hold: $r_{t+1} = r_{t+1}^n = \max\{r_{t+1}^*, \underline{r}\}$ ¹⁸. If the shortage of asset is sufficiently severe, the central bank hits the ZLB and the economy enters the liquidity trap. In this case, the output gap ξ_t is proportional to the interest rate gap, and endogenously determined in equilibrium by setting $r_{t+1} = \underline{r}$ in (26),

$$1 - \xi_t = \min \left\{ \left(\frac{1 + r_{t+1}^*}{1 + \underline{r}} \right)^{\frac{\alpha+\eta}{1+\eta}}, 1 \right\} \quad (28)$$

The economy has a powerful tool to restore equilibrium in the financial markets: adjustments in the interest rate that increase the leverage, stimulate aggregate investment, and hence liquidity creation. However, once the central bank hits the ZLB, the price mechanism cannot

¹⁸It is a standard non-arbitrage condition between liquidity and some nominal asset in zero net supply.

operate anymore. The fall in the labor share in income and the rise in the share of rent that come with the labor wedge acts as a substitute for a fall in the interest rate: the wealth of the savers decreases, thus they reduce their supply of savings; the pledgeable returns of the investors increases, thus they supply more liquidity.

Another way to think of the preceding analysis is through the lens of the good market. By Walras' law, an excess demand of liquidity is equivalent to a shortfall in the aggregate demand for goods (investment).

$$y_t = \min\{y_t^*, c_t + i_t\} \quad (29)$$

Given $r_{t+1} = r$, we can compute the aggregate investment, i_t , and aggregate consumption, c_t . If aggregate demand is inefficiently low, the output gap grows so as to reduce production and ensure equilibrium on the good market, (29).

Aggregate dynamic and potential output So far we have considered the static properties of the equilibrium, taking the capital and potential output, as given. But the capital stock at date $t + 1$ is built on investment at date t ,

$$k_{t+1} = i_t = \frac{\beta}{1 + \beta} \left(e_t + \frac{\Pi_{t+1}}{R_{t+1}} \right) - \frac{\Pi_{t+1}}{R_{t+1}} - b_t \quad (30)$$

Bubbles and profits not only transfer wealth *infra*-generation, but also *inter*-generations from the young to the old agents, which reduces the incentives to save. Since profits depend on the stock of capital in $t + 1$,

$$k_{t+1} = \frac{\frac{\beta}{1+\beta} \frac{\eta}{1+\eta} (1-\alpha)(1-\xi_t) - \bar{b}_t}{1 + \frac{1}{1+\beta} \frac{\xi_{t+1}}{\alpha(1-\xi_{t+1})}} y_t \quad (31)$$

In a production economy, a (static) labor wedge translates into a (dynamic) investment wedge: an aggregate demand shortfall has supply-side effects in the long run through the accumulation of capital. In an aggregate demand-constrained economy, bubbles contribute to close the output gap, but have a negative impact on potential output. Without nominal rigidities and the ZLB, the former effect would disappear and output would unambiguously shrink with rational bubbles.

Given initial conditions, $\{k_0, b_0\}$, the dynamic of the economy can be summarized by a

three dimensional system, (27), (28) and (31), in $\{k_{t+1}, \bar{b}_{t+1}, \xi_t\}$, where \bar{b}_{t+1} and k_{t+1} are state variables, and ξ_t is a forward-looking variable.

3 Expansionary bubbles in the liquidity trap

In this section, I consider the steady-state of the basic model. I focus only on dynamically efficient constrained equilibria, that is equilibria where $R > \max\{1+r, 1\}$. First, I analyze the fundamental roots of the liquidity trap and the secular stagnation. Second, I study the conditions under which bubbles exist and their macroeconomic effects.

3.1 Scarcity of fundamental liquidity, liquidity trap and secular stagnation

Without bubbles, $\bar{b}_t = \bar{b} = 0$, the only source of liquidity is the debt supplied by the borrowers, i.e fundamental liquidity. Since the output gap and the interest rate are constant over time, the economy is either in the liquidity trap or at full employment forever. Thus the “long run” natural interest rate is given by (26) with $\xi_{t+1} = \xi_t = \xi = 0$, and $y_{t+1} = y_t = y^*$,

$$1 + r^* = \frac{\phi}{1 - \omega} \frac{1}{\frac{\beta}{1+\beta} \frac{\eta}{1+\eta} \frac{1-\alpha}{\alpha}} \quad (32)$$

In contrast to standard models featuring infinitely-lived agents, the natural interest rate in steady-state is not equal to the time discount rate but it is a function of the aggregate supply and demand of assets.

Assumption 1 [*Local shortage of fundamental assets*] $\frac{\phi}{1-\omega} < 1 < \frac{1}{\frac{\beta}{1+\beta} \frac{\eta}{1+\eta} \frac{1-\alpha}{\alpha}}$

The borrowing constraints put an upper limit on the supply of inside liquidity, whereas the portfolio constraints put a lower limit on its demand. If both are sufficiently severe, $\phi + \omega < 1$, inside liquidity is scarce in equilibrium. The second inequality is equivalent to $R^* > 1$, or no global shortage of assets in equilibrium.

The interest rate in steady-state is given by the monetary rule (11) and the Fisher equation, $r^n = r = \max\{r^*, \underline{r}\}$. The labor wedge solves (28) for $\xi_{t+1} = \xi_t = \xi$,

$$\xi = \max \left\{ \frac{\frac{1+r}{1+r^*} - 1}{\frac{1+r}{1+r^*} + \frac{1-\alpha}{\alpha}}, 0 \right\} \quad (33)$$

A severe local shortage of assets depresses the long run natural real interest rate, forcing the central bank to hit the ZLB. Consequently, monetary policy becomes impotent: it cannot

accommodate further drops in the natural interest rate, nor manage aggregate demand efficiently any more. As the interest rate gap expands, a growing output gap is necessary to restore equilibrium in the financial and good markets. The liquidity trap and the output gap that comes with are permanent: this is the secular stagnation.

From the law of motion of capital, (31), and the production function, (8), the capital-labor ratio is strictly decreasing in the labor wedge,

$$\kappa^\xi \equiv \frac{k^\xi}{n^\xi} = \left(\frac{\frac{\beta}{1+\beta} \frac{\eta}{1+\eta} (1-\alpha)(1-\xi)}{1 + \frac{1}{1+\beta} \frac{\xi}{\alpha(1-\xi)}} \right)^{\frac{1}{1-\alpha}} \quad (34)$$

This is true even if labor supply is fully inelastic, $\eta \rightarrow \infty$: a higher share of rent transfers wealth from the old to the young households, which reduces the incentives to save. And since households generally supply less labor, (16), the steady-state stock of capital shrinks with ξ .

$$k^\xi = ((1-\alpha)(1-\xi))^{\frac{1}{\eta}} \kappa^\xi \frac{\eta+\alpha}{\eta} \quad (35)$$

As potential output, $y^{*\xi} \equiv s(k^\xi)$, is an increasing function of the stock of capital, an aggregate demand shortage has negative effects on potential output in the long run. Given the structural and policy parameters, there exists a unique fundamental steady-state.

Proposition 1 [*Fundamental steady-state*] *There exists a unique (stable) fundamental steady-state. Furthermore,*

- (i) *The natural interest rate, r^* , is increasing in ϕ and ω .*
- (ii) *Output is efficient if and only if the economy is outside of the liquidity trap, $r^* > \underline{r}$.*
- (iii) *The efficient level of output, y^* , does not depend on ϕ or ω , neither do the stock of capital, labor supply or capital-labor ratio.*
- (iv) *In the liquidity trap, the output gap, $1 - \xi$, expands with the interest rate gap, $\frac{\underline{r}}{r^*}$.*
- (v) *Output, the capital-labor ratio, as well as the stocks of capital and labor are decreasing in the labor wedge, ξ .*

Proposition 1 summarizes our findings about the steady-state in the fundamental or bubbleless economy. Through the natural interest rate, aggregate demand is conditional on the

distribution of wealth between and within generations. If it is inefficiently low, the central bank cuts the policy rate to stimulate aggregate investment.

However, once the central bank hits the ZLB, the interest rate is fixed at its lower bound. The equilibrating mechanism previously discussed cannot operate anymore. An output gap emerges that decreases the labor income of the savers and increases the pledgeable rent of the investors, which restores equilibrium in the financial markets.

In this case the economy reaches equilibrium at the cost of an efficiency loss: as employment is lower than the efficient level, output is lower than its potential. Furthermore, potential output also decreases because the stock of capital is down. In the following subsection, we will see that bubbles provide the economy with another equilibrating mechanism, which minimizes the efficiency loss but shrinks potential output.

3.2 Bubbly liquidity, current and potential output

In any bubbly steady-state, the interest rate must be equal to the growth rate of output, (27),

$$r = r^{*b} = 0 > r \quad (36)$$

In turns, the bubbly steady-state is unique with $\xi = 0$ and $y^b = y^{*b} \equiv s(k^b)$: output is always equal to its potential, and the economy outside of the liquidity trap. The size of the bubble in steady-state is increasing in the scarcity of fundamental liquidity:

$$b = \hat{b} \equiv (1 - \omega) \frac{\beta}{1 + \beta} \frac{\eta}{1 + \eta} (1 - \alpha) - \phi\alpha \quad (37)$$

The maximum bubble equalizes the demand of liquidity to the supply when $r = 1$ in (27). Note that $\phi\alpha$ is the supply of inside liquidity as a share of output: a higher supply of fundamental liquidity – inside liquidity here, I also introduce public debt in 4, crowds out one-for-one the supply of bubbly liquidity.

Lemma 1 *[Existence of bubbles] Bubbles are possible, $\hat{b} > 0$, if and only if the natural interest rate is negative in the fundamental economy, $r^* < 0$.*

On the contrary to the standard model of bubbles (Tirole, 1985), the existence of bubbles is not conditional on the dynamic inefficiency of the fundamental equilibrium, or on a global

shortage of assets. If the liquidity premium is sufficiently high, that is if the shortage of asset is pronounced, the interest rate is negative in an otherwise dynamically efficient economy.

In the bubbly economy, bubbles absorb the excess of savings over investment, and not the output gap as in the fundamental economy. The capital-labor ratio depends on the severity of the financial frictions through the size of the bubble, \hat{b} ,

$$\kappa^b = \left(\frac{\beta}{1 + \beta} \frac{\eta}{1 + \eta} (1 - \alpha) - \hat{b} \right)^{\frac{1}{1-\alpha}} \quad (38)$$

Savers buy the bubble, thus they reduce their lending to the investors. The supply of productive savings shrinks, which tends to decrease both the capital-labor ratio and the stock of capital,

$$k^b = (1 - \alpha)^{\frac{1}{\eta}} \kappa^b \frac{\eta + \alpha}{\eta} \quad (39)$$

As a consequence, potential output in the bubbly economy is lower than in the fundamental economy away from the liquidity trap, $y^{*b} < y^*$.

Proposition 2 [*Bubbly steady-state*] *There exists a unique (unstable) bubbly steady-state with $b = \hat{b}$. Furthermore,*

- (i) *The natural interest rate, r^{*b} , is constant and equal to the growth rate of output.*
- (ii) *The economy is outside of the liquidity trap and output efficient.*
- (iii) *The size of the bubble, \hat{b} , is decreasing in ϕ and ω .*
- (iv) *Output, the capital-labor ratio, as well as the stocks of capital and labor are decreasing in the size of the bubble, \hat{b} .*

Proposition 2 summarizes our findings about the bubbly steady-state. When agents coordinate their expectations to create bubbles, the economy is always away from the liquidity trap, output supply-determined and the equilibrium efficient. The capital-labor ratio and the stock of capital are decreasing with the stock bubbles, since bubbles divert savings away from productive investment: potential output is negatively affected. The steady-state stock of bubbles is just enough to close the gap between the natural interest rate and the growth rate of the economy. If the shortage of asset is severe, either through a low supply – low pledgeability, ϕ – or a high demand – low participation in the capital market, ω – the steady-state stock of bubbles is large.

If monetary policy is unconstrained at the fundamental steady-state, i.e the fundamental steady-state is not in the liquidity trap, bubbles have unambiguously negative effects on the stock of capital and output: the equilibrium with or without bubbles is efficient. Because bubbles absorb some productive savings, they must reduce capital accumulation and potential output. However, in an economy which suffers from an aggregate demand shortage, bubbles also contribute to eliminate the output gap and increase the labor supply.

Proposition 3 [*Expansionary bubbles*] *There exist $r_y^*(\underline{r})$ and $r_k^*(\underline{r})$ such that,*

- (i) *Bubbles expand aggregate demand and output if and only if $r^* < r_y^*(\underline{r})$.*
- (ii) *Bubbles expand the stock of capital and potential output if and only if $r^* < r_k^*(\underline{r})$.*
- (iii) *$r_y^*(\underline{r})$ and $r_k^*(\underline{r})$ satisfy $r_k^*(\underline{r}) < r_y^*(\underline{r}) < \underline{r}$ and $r_x^*(\underline{r}) > 0, x = k, y$.*

If the output gap is large but not too large in the fundamental economy, case (i), output is higher at the bubbly steady-state, despite a lower stock of capital. The positive demand-side effect of rational bubbles on the labor supply overcomes the negative supply-side effect on capital accumulation.

If the output gap is very large absent bubbles, case (ii), the stock of capital and potential output are even higher in the bubbly economy. As we have already discussed, the static labor wedge also translates into a dynamic investment wedge which reduces the rental rate of capital. As a consequence, borrowers invest less and accumulate less capital. Bubbles have a positive supply side effect in this case.

In any case, output is lower in the bubbly economy than in the economy with efficient production, as was first noted by [Tirole \(1985\)](#). But once we depart from the neoclassical framework with immediate price adjustment, bubbles may actually expand output and the stock of capital in the liquidity trap.

Note that, on the contrary to popular beliefs, the existence of rational bubbles has nothing to do with monetary policy. Their macroeconomic effects, however, crucially depend on whether or not the economy is in the liquidity trap, i.e whether or not monetary policy is constrained. In the next section, I will discuss the existence of rational bubbles more in details, and show that fiscal policy is the right tool to prevent the formation of rational bubbles. By rising the supply of liquidity available to the savers, public debt can prevent the economy from falling into the liquidity trap and bubbles to emerge.

4 Public bonds, money and local shortages of assets

I introduce public debt – money and bonds – in the basic framework, and study how it affects the results of the previous section. On the contrary to monetary policy, fiscal policy has a direct control on the supply of liquidity available to the savers and can cure the local shortage of assets. The results in propositions 1, 2 and 3 hold, with the addition that the debt-to-GDP ratio increases the natural interest rate and crowds out one-for-one the stock of bubbles.

This extension also helps to understand the monetary - fiscal nexus in a world with permanently negative natural interest rates. On the contrary to the conventional view, e.g [Sargent and Wallace \(1981\)](#), an *insufficient* supply of public debt may threaten the ability of the central bank to manage aggregate demand.

4.1 Public debt, the supply of liquidity and liquidity trap

Let us consider fiscal policy. The government issues real bonds at each date t , l_{t+1} , which are perfect substitutes for the private bonds issued by the borrowers, d_{t+1} . The government budget constraint is given by

$$\frac{l_{t+1}}{1 + r_{t+1}} = T_t + l_t \quad (40)$$

Here, $T_t^y \equiv \int T_{j,t}^y dj$ is an aggregate lump-sum transfer from the government to young households. I assume that fiscal policy targets a constant debt-to-GDP ratio at each period, $l_{t+1} = \bar{l}y_{t+1}$, $\bar{l} \in [0, 1]$. The lump-sum transfers adjust to ensure that the government always meets his per-period budget constraint.

Now, the savers buy bubbles, private and public bonds. The equilibrium condition on the assets markets is then,

$$\int q_{j,t+1} dj = 1 \quad \text{and} \quad \int l_{j,t+1} dj = l_{t+1} \quad \text{and} \quad \int d_{j,t+1} dj = 0 \quad (41)$$

Assuming binding borrowing constraint, (3), we can compute the equilibrium interest rate:

$$1 + r_{t+1} = \min \left\{ \frac{\phi(R_{t+1}i_t + \Pi_{t+1}) + l_{t+1} + b_{t+1}}{(1 - \omega)\frac{\beta}{1+\beta}e_t + \frac{\beta}{1+\beta}T_{L,t}^y}; R_{t+1} \right\} \quad (42)$$

Where $T_{L,t}^y \equiv \int (1 - \theta_{j,t})T_{j,t}^y dj$ is the aggregate transfer to the savers. Public debt rises the supply of liquidity by providing the savers with another asset, but it also increases their wealth

because of the lump-sum transfers that the public debt finances. If we take the government budget constraint, (40), and the factors' share in income, (15),

$$1 + r_{t+1} = \frac{\phi(\alpha(1 - \xi_{t+1}) + \xi_{t+1}) + \frac{1+\beta\tau_\omega}{1+\beta}\bar{l} + \bar{b}_{t+1}}{(1 - \omega)\frac{\beta}{1+\beta}\frac{\eta}{1+\eta}(1 - \alpha)(1 - \xi_t) - \frac{\beta}{1+\beta}(1 - \tau_\omega)\bar{l}} \frac{y_{t+1}}{y_t} \quad (43)$$

Here, $\tau_\omega \equiv 1 - \frac{T_{L,t}}{T_t}$ is the fraction of the aggregate lump-sum transfer that accrues to the investors. The interest rate is an increasing function of the supply of public debt, \bar{l} , and of τ_ω . Intuitively, public debt affects the interest rate through two channels: first, it increases the supply of liquidity available to the savers; second, it redistributes wealth between and within generations.

As rational bubbles, given the level of output, public debt crowds out capital as it divers some savings away from capital accumulation,

$$k_{t+1} = i_t = \frac{\beta}{1 + \beta} \left(e_t + T_t^y + \frac{\Pi_{t+1}}{R_{t+1}} \right) - \frac{\Pi_{t+1}}{R_{t+1}} - \frac{l_{t+1}}{1 + r_{t+1}} - b_t \quad (44)$$

Or equivalently,

$$k_{t+1} = \frac{\frac{\beta}{1+\beta}\frac{\eta}{1+\eta}(1 - \alpha)(1 - \xi_t) - \frac{\frac{g_{t+1}}{1+r_{t+1}} + \beta}{1+\beta}\bar{l} - \bar{b}_t}{1 + \frac{1}{1+\beta}\frac{\xi_{t+1}}{\alpha(1-\xi_{t+1})}} y_t \quad (45)$$

Note that if $g_{t+1} = 1 + r_{t+1}$ and $\tau_\omega = 1$, public debt has exactly the same macroeconomic effects as rational bubbles. Indeed, public debt *is* a rational bubble in this case, as the government perpetually rolls over its debt. Public debt rises the same trade-off as rational bubbles: from the one hand, a higher supply of public debt stimulate investment and output in the liquidity trap; on the other hand, it decreases capital accumulation and potential output out of the trap.

Proposition 4 [*Public debt and liquidity trap*]

- (i) *The natural interest rate, r^* , is increasing in \bar{l} and τ_ω .*
- (ii) *Public liquidity crowds out bubbly liquidity one-for-one.*
- (iii) *The results in proposition 3 also apply to public debt.*

A higher stock of public debt implies a lower stock of bubbles in steady-state: fiscal policy can eliminate the bubbly steady-state by supplying the savers with enough liquidity.

$$b = \hat{b}^{\bar{l}} \equiv (1 - \omega) \frac{\beta}{1 + \beta} \frac{\eta}{1 + \eta} (1 - \alpha) - \phi\alpha - \bar{l} \quad (46)$$

Note that one big difference between local and global shortages of asset is that the latter are conditional on the distribution of wealth between generations, whereas the former are conditional on the distribution of wealth within generation. A fiscal policy that redistributes wealth from the young savers to the young investors can prevent the economy to fall in a liquidity trap induced by a local shortage of assets, without any cost in terms of potential output.

4.2 Money and the cashless limit

As a robustness check, I consider what happens away from the cashless limit. On the contrary to most monetary models, I find that money and bubbles are not substitutes for each other: money is *not* a bubble. Furthermore, if the central bank follows a Taylor rule which takes the ZLB into account and the government targets some real variables, e.g real debt- or deficit-to-GDP ratio, money is irrelevant.

Both results cannot be understood without carefully analyzing fiscal policy, which is usually not included in monetary models. Since money is a form of government debt, it is implicitly backed by future taxes (Cochrane, 2005), whereas bubbles are not. If the government targets some real values, the composition of its debt between money, nominal or real bonds, does not matter, even at the ZLB.

I introduce a role for money to ease transactions through a cash-in-advance (CIA) in savers' program,

$$\mu(1 - \theta_{j,t})c_{j,t+1}^o \leq M_{j,t+1} \quad (47)$$

Where $\mu \in (0, 1)$ is the share of savers' old-age consumption that must be purchased with money. If the economy is away from the liquidity trap, the CIA binds for the savers: because money is a dominated asset, they hold as few money as possible. However, when the economy enters the liquidity trap, they value money not only for its transaction services but also as a store of value.

The central bank still sets the nominal interest rate according to (11), with $r = 0$ ¹⁹. Given the policy rate, the central bank adjusts the money supply to meet the demand, $M_{t+1} =$

¹⁹With a CIA, households would never choose to hold debt if $r_{t+1}^n < \bar{r}^n = 0$. Empirically, this is not observed.

$\int M_{j,t+1}dj$. The aggregate government debt is now the the sum of money and public bonds,

$$M_{t+1} + \frac{l_{t+1}}{1+r_{t+1}} = T_t + M_t + l_t \quad (48)$$

Again, the government follows a constant real debt-to-output ratio at each period, $M_t + l_t = \bar{l}y_t$. The lump-sum transfers, T_t , adjust to ensure that (48) holds at each period. The supply of real bonds by the government at each period is then:

$$l_t = \bar{l}y_t - \int M_{j,t}dj \quad (49)$$

In most OLG monetary models, the central bank sets a path for the money supply, $\{M_t\}_{t=0}^{\infty}$, the government a path for the bonds and taxes, $\{l_t, T_t\}_{t=0}^{\infty}$, the only restriction on policies being that (48) holds. This is not, however, how monetary policy is implemented in modern economies. To paraphrase [Sargent \(1989\)](#), since the central bank does not have the power to rise taxes, its only role is to manage the portfolio of debts of the government, i.e it changes the *composition* of government debt – money versus bonds, but *not its level*. Thus, most monetary models are not about monetary policy, but rather a mix between fiscal and monetary policy ²⁰. In reality, the central bank has no power over the *level* of the aggregate supply of asset, only on its *composition*.

The equilibrium condition on the assets markets is still given by (41), where the the supply of public bonds is now given by (49). The interest rate is increasing in the demand of money,

$$1+r_{t+1} = \frac{\phi(\alpha(1-\xi_{t+1})+\xi_{t+1})+\frac{1+\beta\tau_\omega}{1+\beta}(\bar{l}+r_{t+1}m_{t+1})+\bar{b}_{t+1}y_{t+1}}{(1-\omega)\frac{\beta}{1+\beta}\frac{\eta}{1+\eta}(1-\alpha)(1-\xi_t)-\frac{\beta}{1+\beta}(1-\tau_\omega)\bar{l}}y_t \quad (50)$$

Here, $m_{t+1} \equiv \frac{M_{t+1}}{y_{t+1}}$ is the stock of money as a share of output. The savers hold some of their wealth in the form of money, which reduces their demand of liquidity. Note however that the maximum bubble is still given by (46), because in steady-state $r^b = 0$. As public bonds, money crowds out capital accumulation,

$$k_{t+1} = \frac{\frac{\beta}{1+\beta}\frac{\eta}{1+\eta}(1-\alpha)(1-\xi_t)-\frac{\frac{g_{t+1}}{1+r_{t+1}}+\beta}{1+\beta}(\bar{l}+r_{t+1}m_{t+1})-\bar{b}_t}{1+\frac{1}{1+\beta}\frac{\xi_{t+1}}{\alpha(1-\xi_{t+1})}}y_t \quad (51)$$

Two things are to note: first, money and bubbles can co-exist. Intuitively, money is not a

²⁰(i) The government issues new bonds; (ii) the central creates new money and buys the bonds back. Finally, the supply of bonds is unchanged whereas the supply of money has increased and so government debt as well, which is the fiscal side of the operation.

“pure” bubble as it is implicitly backed by taxes through the government budget constraint, (48). Second, as $\mu \rightarrow 0$ and conditional on the interest rate rule, money becomes purely neutral: money is irrelevant ²¹, especially at the ZLB! Why do other monetary models (e.g. [Bacchetta et al. \(2015\)](#)) arrive at different conclusions with almost the same model? Money is irrelevant *per se*, but the supply of *real* public debt is not.

Money is important in the liquidity trap if and only if it affects the stance of fiscal policy and its supply of real debt. Consider this example taken from [Bacchetta et al. \(2015\)](#) with a flexible price level: the government keeps the supply of real bonds constant, $l_{t+1} = \bar{l}y_{t+1}$, and money grows at a constant rate, $\frac{M_{t+1}}{M_t} = \theta$. In the liquidity trap, a higher demand for money decreases the price level. Thus, the supply of real money balances rises, i.e the supply of real debt by the government rises. However, the crowding-out effect has nothing to do with money at all. Indeed, consider a government that issues only real bonds but keeps the *nominal* value of debt as a share of *real* GDP constant at each period. At the ZLB, the deflationary pressures induce the government to supply more debt in real terms, crowding out capital accumulation.

5 Inflation, liquidity trap and expansionary bubbles

In the introduction, I mentioned the paradox raised by [Summers \(2013\)](#): why did inflation remain so low during the years that predate the sub-prime crisis despite a huge housing bubble? Obviously, the basic framework with fixed prices cannot answer this question. Now, I relax the assumption of fully rigid prices and consider an economy where firms can re-set their price at each period, however subject to a downward price rigidity constraint that puts a lower limit on the rate of inflation – or deflation. The results are qualitatively similar to those obtained with fixed prices, with the novelty that rational bubbles also prevent inflation from falling short of the central bank’s target in the liquidity trap.

Monopolists can change their prices at each period, however subject to a downward rigidity constraint: the current price, $P_{v,t}$, can fall at most by a factor $1 + \pi(\xi_t) > 0$ with respect to last period’s price, $P_{v,t-1}$. I assume that prices become more flexible as the output gap increases, $\pi'(\xi_t) < 0$. Full price flexibility obtains when $\pi(\xi) = -1$, and we recover the results in the last section when prices cannot fall, $\pi(\xi) = 0$. The program of a monopolist $v \in [0, 1]$ is similar to (9) with a minor modification,

²¹Of course, the supply of money relative to public bonds determines the nominal interest rate. However, one can consider the cashless limit without further troubles.

$$\begin{aligned}\tilde{\Pi}_t &= \max_{P_{v,t}, k_{v,t}^d, n_{v,t}^d} \frac{P_{v,t}}{P_t} F(k_{v,t}^d, n_{v,t}^d) - (1 - \tau(n_t)) \left(R_t k_{v,t}^d + w_t n_{v,t}^d \right) \\ \text{s.t. } & F(k_{v,t}^d, n_{v,t}^d) \leq \left(\frac{P_{v,t}}{P_t} \right)^{-\epsilon} y_t, \quad P_{v,t} \geq (1 + \pi(\xi_t)) P_{v,t-1}\end{aligned}\tag{52}$$

The first-order conditions for labor and capital with partial price flexibility are similar to those with fully rigid prices, (12) and (13). Facing a demand shortage, firms try to cut their price until it is equal to their real marginal cost,

$$P_{v,t} = \max\{\Lambda_t P_t, (1 + \pi)(\xi_{v,t}) P_{t-1}\}\tag{53}$$

If they are prevented to do so by the binding downward price rigidity constraint in (52), the production is inefficiently low whereas the markup inefficiently high. As in Caballero and Farhi (2016), the supply side of the economy can be summarized by a Philips curve that relates the labor wedge to the inflation rate,

$$(\pi_{t+1} - \pi(\xi_{t+1})) \xi_{t+1} = 0\tag{54}$$

To understand this Philips curve, note that if we impose symmetry across firms in (53), $P_{v,t} = P_t$ for each $v \in [0, 1]$, either $\Lambda_{t+1} = 1$ or $\pi_{t+1} = \pi(\xi_{t+1})$. In the former case, firms are unconstrained: they set the price equal to the marginal cost and produce the efficient level. In the latter, monopolists are constrained and must adjust their production to the aggregate demand: an gap emerges between potential and current output.

Since the inflation rate is endogenous, I replace the monetary rule (11) by a standard Taylor rule which includes the ZLB,

$$r_{t+1}^n = \max\{r_{t+1}^* + \pi^* + \gamma(\pi_{t+1} - \pi^*), \underline{r}^n\}\tag{55}$$

Here, π^* is the inflation target of the central bank, and π_{t+1} is the the expected inflation rate. The central bank reacts more than-to-one with inflation, $\gamma > 1$, to ensure determinacy of the equilibrium according to the Taylor principle.

The interest rate is still given by (26), whereas the law of motion of capital and bubbles are given respectively by (31) and (27). In equilibrium, the labor wedge, ξ_{t+1} , the inflation rate,

π_{t+1} , and the nominal interest rate, r_{t+1}^n , are jointly determined by (54) and (55) together with the Fisher equation, $r_{t+1}^n = r_{t+1} - \pi_{t+1}$.

Assumption 2 [*Nominal rigidities and the ZLB*] $\pi^* > \underline{\pi}(0) > \underline{r}^n$.

Assumption 2 ensures that (i) in any liquidity trap steady-state, the ZLB binds; (ii) the central bank hits the ZLB only if the natural interest rate is negative.

Proposition 5 [*Inflation target and policy-induced liquidity trap*] *There is a unique liquidity trap steady-state that exists if and only if $r^* < \underline{r}^n - \underline{\pi}(0)$. Furthermore,*

(i) *It is the unique steady-state if and only if $r^* < r^n - \pi^*$.*

(ii) *If $r^* > r^n - \pi^*$, there are two other steady-states with $\xi = 0$, and $\pi \in \{r^n - r^*, \pi^*\}$.*

(iii) *In the liquidity trap steady-state, $\pi^\xi = \underline{\pi}(\xi)$, where ξ solves (33) for $\underline{r} = \underline{r}(\xi) = \underline{r}^n - \underline{\pi}(\xi)$.*

The extension with an endogenous inflation rate does not question the conclusions related to the existence of a permanent liquidity trap, which again is possible in case of a severe local shortage of assets, but it underlines the possibility of policy-induced convergence to this particular steady-state. Indeed, by picking a too low inflation target, the central bank suppresses the two efficient steady-states and keeps only the liquidity trap steady-state.

A liquidity occurs if, at the ZLB, the central bank cannot engineer enough inflation to maintain the interest rate gap nil. In anticipation of an aggregate demand shortage, firms cut their prices. The falling inflation rate that results rises the interest gap, which further reduces aggregate demand. Etc. At one point, the firms cannot reduce their prices any further, and an output gap grows that is bigger than in the economy with fixed prices. Nominal rigidities mitigate this deflationary spiral as they reduce the volatility of the inflation rate in the short run and anchor inflation expectations, which helps to stabilize the economy and may even prevent the liquidity trap in the first place.

This possible multiplicity of equilibria is standard in New Keynesian models with a Taylor rule that includes a ZLB (Benhabib et al., 2001). In contrast to the model with fully rigid prices, sunspots can occur that induce the economy to jump from one equilibrium to another: the deflationary secular stagnation steady-state can be of arbitrary duration with endogenous inflation. This multiplicity may be desirable because it could potentially explain the divergent

long run trajectories of Japan and the US after the bubbles burst (Aruoba et al., 2013)²².

Price flexibility and the liquidity trap As Kocherlakota (2016) notices, there is a discontinuity between a vertical Philips curve, $\pi(\xi) = -1$, and a Philips curve that tends to be vertical, $\pi(\xi) \rightarrow -1$. If prices immediately adjust, the output gap is always nil, $\xi = 0$, even at the ZLB: the liquidity trap has no real effects. However, if there is a vanishingly small amount of price rigidity, $\pi(\xi) \rightarrow -1$, the economy is always in the liquidity trap, $r^* > \underline{r}^n - 1$, and the output gap becomes arbitrarily large, $\xi \rightarrow 1$.

Proposition 6 [*Bubbly steady-state and inflation*] *There is a unique bubbly steady-state that exists if and only if $r^* < 0$. Furthermore,*

- (i) *The economy is outside of the liquidity trap, $r^{*b} = 0 > \underline{r}^n - \pi(0)$.*
- (ii) *Inflation is on target, $\pi^b = \pi^*$, and the output gap nil, $\xi = 0$.*
- (iii) *The results of proposition 3 apply for $r_x^\pi(\underline{r}^n) > r_x(\underline{r}^n)$, $x = y, k$.*

The intuitions for proposition 6 with downward price rigidities are the same as for propositions 2 and 3. Bubbles, that increase the supply of assets available to the savers, rise the natural interest rate, close the interest rate gap and push the economy out of the liquidity trap. The demand-side expansionary effects of rational bubbles are magnified by price flexibility, because of the deflationary spiral that occurs in the fundamental economy. Despite the presence of bubbles, or actually thanks to them, the inflation rate is exactly on target.

6 Conclusion

I have presented a model of liquidity trap and demand-side secular stagnation driven by local shortages of assets; I also provided a new mechanism to explain expansionary bubbles. As in the standard New Keynesian model, the output gap is proportional to the interest rate gap. If some agents face a shortage of assets, a local assets shortage, the natural rate falls below 0 in an otherwise dynamically efficient economy. Because of the binding ZLB and nominal rigidities, the economy enters the liquidity trap and an output gap emerges that restores equilibrium in the financial markets. Outside of the liquidity trap, bubbles decrease output and potential

²²The US economy eventually recovered after 8 years, whereas Japan did not yet.

output as they divert some savings away from capital accumulation. However, in the liquidity trap, bubbles are expansionary as they increase the supply of assets available to the constrained agents, rise the natural interest rate and close the interest rate gap.

The predictions of the model are consistent with the broad trends of GDP, investment, interest and inflation rates during two of the recent crises triggered by the bust of financial bubbles, the US in 2007 and Japan in the 90's, as well as during the few years that predates the crash. In particular, the model explains why: (i) a large financial bubble did not create an economic boom; and (ii) the bubble's burst pushed the economy in a liquidity trap of arbitrary duration, with a growing output gap and a falling inflation rate despite a zero nominal interest rate policy.

One obvious limitation of the present model, which it shares with the rest of the literature on rational asset price bubbles, is that it cannot explain how bubbles emerge and crash. Another limitation is that it takes the financial frictions as exogenous. It could be interesting to study the coordination failures at the origin of local shortages of assets, and how agents can overcome those by creating rational bubbles. I leave an elaboration of these questions for future research.

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Appendix A Extensions

A.1 Public bonds

The young- and old-age budget constraints of household j include the public bonds and the lump-sum transfers,

$$c_{j,t}^y + i_{j,t} + q_{j,t+1}b_t + \frac{l_{j,t+1}}{r_{t+1}} - \frac{d_{j,t+1}}{r_{t+1}} = w_t n_{j,t} + T_{j,t}^y \quad (56)$$

$$c_{j,t+1}^o + d_{j,t+1} = \Pi_{j,t+1} + R_{t+1}k_{j,t+1} + q_{j,t+1}b_{t+1} + l_{j,t+1} \quad (57)$$

Otherwise the program of household j is unchanged. The asset demand functions are very slightly modified,

$$i_{j,t} = k_{j,t+1} = \theta_{j,t} \left(\varphi_{t+1} \frac{\beta}{1+\beta} \left(e_{j,t} + T_{j,t}^y + \frac{\Pi_{j,t+1}}{R_{t+1}} \right) - \frac{\Pi_{j,t+1}}{R_{t+1}} \right) \quad (58)$$

And

$$\frac{d_{j,t+1}}{r_{t+1}} = (\theta_{j,t} \varphi_{t+1} - 1) \frac{\beta}{1+\beta} \left(e_{j,t} + T_{j,t}^y + \frac{\Pi_{j,t+1}}{R_{t+1}} \right) + (1 - \theta_{j,t}) \left(q_{j,t+1}b_t + \frac{l_{j,t+1}}{r_{t+1}} \right) \quad (59)$$

A.2 Money

The budget constraints are modified to include real balances,

$$c_{j,t}^y + i_{j,t} + q_{j,t+1}b_t + \frac{l_{j,t+1}}{r_{t+1}} + M_{j,t+1} - \frac{d_{j,t+1}}{r_{t+1}} = w_t n_{j,t} + T_{j,t}^y \quad (60)$$

$$c_{j,t+1}^o + d_{j,t+1} = \Pi_{j,t+1} + R_{t+1}k_{j,t+1} + q_{j,t+1}b_{t+1} + l_{j,t+1} + M_{j,t+1} \quad (61)$$

Both the CIA constraint is binding as long as money is dominated by debt, i.e as long as the economy is away from the RLB. The asset demand functions are

$$i_{j,t} = k_{j,t+1} = \theta_{j,t} \left(\varphi_{j,t+1} \frac{\beta}{1+\beta} \left(e_{j,t} + T_{j,t}^y + \frac{\Pi_{j,t+1}}{R_{t+1}} \right) - \frac{\Pi_{j,t+1}}{R_{t+1}} \right) \quad (62)$$

And

$$\frac{d_{j,t+1}}{r_{t+1}} = (\theta_{j,t}\varphi_{j,t+1} - 1) \frac{\beta}{1 + \beta} \left(e_{j,t} + T_{j,t}^y + \frac{\Pi_{j,t+1}}{R_{t+1}} \right) + (1 - \theta_{j,t}) \left(q_{j,t+1}b_t + M_{j,t+1} + \frac{l_{j,t+1}}{r_{t+1}} \right) \quad (63)$$

Where the money demand is given by the complementary slackness conditions,

$$M_{j,t+1} \geq \mu(1 - \theta_{j,t})r_{t+1} \frac{\beta}{1 + \beta} \left(e_{j,t} + T_{j,t}^y + \frac{\Pi_{j,t+1}}{R_{t+1}} \right), r_{t+1} \geq r \quad (64)$$

Appendix B Omitted proofs

$$\frac{\frac{r}{r_y^*(r)} + \beta}{1 + \beta} (\omega + (1 - \omega)r_y^*(r)) \left(\alpha \frac{r}{r_y^*(r)} + (1 - \alpha) \right)^{1 + \frac{1 - \alpha}{\alpha(1 + \eta)}} = 1 \quad (65)$$

$$\frac{\frac{r}{r_k^*(r)} + \beta}{1 + \beta} (\omega + (1 - \omega)r_k^*(r)) \left(\alpha \frac{r}{r_k^*(r)} + (1 - \alpha) \right)^{1 + \frac{1 - \alpha}{\alpha + \eta}} = 1 \quad (66)$$