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Secular Stagnation, Liquidity Trap and Rational Asset Price Bubbles

Mathieu Boullot *

Paris School of Economics - Paris 1 and University Bielefeld

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Abstract

I build a model of liquidity trap of arbitrary duration provoked by a shortage of liquid assets and propose a new mechanism through which rational bubbles sustain investment and output in the liquidity trap. Financial frictions that hamper the well-functioning of the financial markets preclude some agents to hold equity and introduce a liquidity premium between the interest rate and the stock market returns. A negative shock on the supply of assets available to the constrained agents raises the liquidity premium, decreases the natural interest rate and pushes the economy toward the liquidity trap. At the ZLB, such a shock cannot be accommodated by a cut in the interest rate. Instead, as the interest rate gap grows, output falls below its potential to restore equilibrium between the supply and demand for liquidity. Financial bubbles, by increasing the supply of liquidity and therefore the natural interest rate, close the output gap and restore efficiency of the equilibrium, at the possible cost of a lower potential output in the long run.

Keywords: Bubbles, Monetary policy, Zero lower bound, Liquidity trap, Secular stagnation, Asset shortage, Liquidity, Financial frictions, Dynamic inefficiency

JEL Classification Numbers: E21, E24, E520.

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1 Introduction

Some, if not most, of the major financial crises were initiated by boom-burst cycles in asset prices. But the macroeconomic consequences of those “bubbly” episodes drastically differ from one to another. While the US economy quickly recovered from the dot-com crisis in the early 2000’s, it remained depressed for nearly a decade after the burst of the housing and mortgage-backed securities bubble in 2007 – despite zero nominal interest rate policy, forward guidance, fiscal stimulus, quantitative easing etc. The Japanese’ lost decade that followed the banking and stocks bubble in the 90’s is another example of a boom-bust cycle gone bad.

One usual explanation is that bad bubbly episodes are associated with credit booms, whereas the benign are not ([Jorda et al., 2015](#)). During the upward phase of a bad episode, the private agents accumulate a lot of debt. Consequently, the bursting depresses investment and output as the firms and the households enter a deleveraging process ([Mian and Sufi, 2014](#)). In this paper, I explore an alternative theory and formalize the idea advanced by [Summers \(2013\)](#) and [Krugman \(2013\)](#): under particular circumstances, large financial bubbles are necessary to sustain investment and output. Indeed, an asset bubble provides the agents with a greater supply of liquidity, allowing them to overcome shortages of “fundamental” liquidity, i.e the sum of public and private debt. In normal times, the bursting bubble can be accommodated by a drop in the nominal interest rate with virtually no aggregate consequences. But when the shortage of fundamental liquidity is severe, the natural interest rate – consistent with full employment – becomes permanently negative after the bubble bursts. If the inflation expectations are sticky and the central bank hits the zero lower bound (ZLB), the interest rate is pegged higher than its natural value: the economy enters the liquidity trap, and output falls short of its potential. Therefore, the usual explanation mixes up the cure – the bubble – for the disease – the shortage of fundamental liquidity.

My contributions are threefold: first, I uncover another force behind the liquidity trap and the demand-side secular stagnation, which stems from a shortage of liquid assets ¹, as opposed to a general shortage of assets ([Eggertsson and Mehrotra, 2014](#)) or a shortage of safe assets ([Caballero and Farhi, 2016](#)). Second, I provide a new mechanism to explain how rational bubbles sustain investment, aggregate demand and output in the liquidity trap – and therefore why the bursting bubble is likely to depress the economy. Third, I clarify how the existence and macroeconomic effects of rational bubbles are conditional on respectively the fiscal and monetary regimes.

¹[Asriyan et al. \(2015\)](#) and [Bacchetta et al. \(2015\)](#) study supply-side secular stagnation driven by shortages of liquid assets.

The model has two key ingredients: financial frictions and nominal rigidities. Households are in overlapping generations and live for two periods. Each generation features two types of households: savers and investors. The investors own the firms and manage the whole capital stock of the economy; the savers cannot access the stock market and therefore they must hold liquid assets: public debt, private debt issued by the investors or bubbles. Since the government has a fixed debt-to-GDP target and the investors face borrowing constraints, the supply of fundamental liquidity is finitely interest rate-elastic. Thus, if the financial frictions are severe and absent financial bubbles, there is a shortage of liquid assets: to balance the supply and demand, the interest rate must fall short of the rate of dividends – the rental rate of capital. This liquidity premium gives the investors incentives to issue more bonds and invest more, thereby restoring equilibrium in the financial and good markets.

However, if the firms are subject to menu costs – such that the expected inflation rate is sticky – and the central bank is subject to the ZLB constraint, the interest rate that monetary policy can implement has a lower limit. Once the natural interest rate ² crosses this lower bound – because of a very severe liquidity shortage – the central bank cannot accommodate downward fluctuations in the liquidity premium anymore. Instead, in the Keynesian liquidity trap, the interest rate gap – the gap between the current interest rate and its natural value – causes an excess demand for liquidity at full employment, or equivalently an excess supply of goods. To restore equilibrium in the financial and goods markets, output drops below its potential. This output gap reduces the wealth of the young savers and consequently their demand for liquidity. Also, potential output is negatively affected in the long run as the agents accumulate less capital – the aggregate savings rate shrinks as an output gap redistributes wealth away from the young generations.

But the economy can escape the liquidity trap and reach equilibrium in another way. Indeed, if the agents succeed in coordinating their expectations, they create asset price bubbles. By providing the savers with an additional source of liquidity, bubbly liquidity, those bubbles lower the liquidity premium and raise the natural interest rate above the interest rate floor. Thus, asset bubbles mitigate the shortage of liquidity and prevent an loss of income by closing the output gap. However, as bubbly liquidity crowds-out fundamental liquidity in the portfolio of the savers, it also reduces the share of the aggregate savings channeled toward the accumulation of capital. Depending on the severity of the fundamental shortage of liquidity and the monetary response, capital accumulation can be higher crowded in (in the liquidity trap) or out (in normal times).

²That is the interest rate consistent with equilibrium in the liquidity and goods market at full employment

Related literature Some papers have already considered demand side secular stagnation ³, and in particular [Kocherlakota \(2013\)](#) and [Eggertsson and Mehrotra \(2014\)](#), who study general asset shortages, and [Caballero and Farhi \(2016\)](#), who study safe asset shortages. The present paper builds extensively on, and complement, theirs by introducing another type of asset shortage: liquid. This new type of asset shortage emphasizes the importance of the distribution of wealth and investment opportunities within a generation, rather than life-cycle savings or differences in risk-aversion. Also, as the accumulation of capital is endogenous in my model, it allows to characterize not only the dynamic of the output gap, but also potential output, in a demand side secular stagnation. Thereby, it may explains the drop in measured potential output after the Great Recession ([Bosworth, 2015](#)) ⁴.

Building on the seminal papers by [Samuelson \(1958\)](#) and [Tirole \(1985\)](#), a flourishing literature has studied the macroeconomic effects of rational bubbles in the Neoclassical regime ⁵ (e.g [Santos and Woodford \(1997\)](#), [Caballero and Krishnamurthy \(2006\)](#), [Kocherlakota \(2010\)](#), [Arce and Lopez-Salido \(2011\)](#), [Kunieda et al. \(2014\)](#), [Martin and Ventura \(2014\)](#), [Clain-Chamosset-Yvrard and Seegmuller \(2015\)](#), [Ikeda and Phan \(2015\)](#)); in particular, a few papers provided conditions for bubbles to be expansionary in environments with heterogeneous agents and binding financial constraints (e.g [Kocherlakota \(2009\)](#), [Farhi and Tirole \(2012\)](#), [Martin and Ventura \(2012\)](#), [Miao and Wang \(2015a\)](#), [Miao and Wang \(2015b\)](#)): bubbles, by reallocating wealth across agents, increase either the aggregate TFP or savings rate. Unlike those papers, I focus on the Keynesian regime: bubbles, by reallocating wealth across agents, close the output gap.

My paper also relates to the literature on liquidity traps (e.g [Krugman \(1998\)](#), [Eggertsson and Woodford \(2003\)](#), [Christiano et al. \(2011\)](#), [Correia et al. \(2013\)](#)). While this literature usually relies on an exogenous shock on a structural parameter ⁶ to generate a temporary shortage of assets and negative interest rates, e.g a discount rate shock, tightened borrowing constraints ([Lorenzoni and Guerrieri \(2011\)](#), [Eggertsson and Krugman \(2012\)](#), [Korinek and Simsek \(2016\)](#)) or overbuilding with irreversible investment ([Rognlie et al., 2015](#)) ⁷, my model underlines how a non-fundamental shock, a bursting bubble, can provoke a liquidity trap of

³[Bacchetta et al. \(2015\)](#) study supply side secular stagnation driven by a liquid asset shortage: in the liquidity trap, deflationary pressures induce the government to supply more public debt, crowding capital accumulation out – [Asriyan et al. \(2015\)](#) show that bubbles creation shocks are expansionary in such an environment. Adding nominal rigidities may reverse the logic: a higher supply of public debt crowds capital accumulation in.

⁴At least with respect to labor market participation and capital accumulation. The present model is silent about TFP growth.

⁵See [Miao \(2014\)](#) for a good introduction to that literature.

⁶One exception is [Schmitt-Groh and Uribe \(2012\)](#), where expectations of low inflation drive the economy into the liquidity trap.

⁷If we interpret capital in the present model as housing, bubbles provide an explanation for the initial overbuilding in [Rognlie et al. \(2015\)](#).

arbitrary duration.

Finally, my model is reminiscent of the old literature on disequilibrium ⁸ (e.g Barro and Grossman (1971), Benassy (1976), D’Autume and Michel (1985)), which explains how an excess supply in one market translates into an excess demand in another – because of rigid prices or more generally demand constraints. In the liquidity trap, the interest rate is fixed at its lower bound – the ZLB minus expected inflation: an excess demand for liquidity implies an excess supply of goods: the supply of goods as well as the demand for liquidity are rationed.

The paper is organized as follows. Section 2 exposes the basic model and characterizes the equilibrium. Section 3 studies the bubbly and fundamental steady states. Section 4 introduces fiscal policy.

2 The basic environment and equilibrium

The basic setup is an overlapping generations model with a single consumption good, two factors of production, capital and labor, and three assets: bonds, capital and bubbles. Time is discrete, $t \in \{0, 1, \dots\}$, and the horizon infinite.

2.1 Households

At each date t a mass 1 of agents is born who live for two periods, young and old. A fraction $\lambda \in (0, 1)$ of those households are investors, indexed by $\delta_t = i_t$, whereas the remaining $1 - \lambda$ are savers, indexed by $\delta_t = s_t$. The savers cannot own firms nor hold equities: the investors manage the firms as well as the entire capital stock.

A agent $j \in [0, 1]$ is endowed with the following discounted utility,

$$\mathcal{U}(j, \delta_t) \equiv \log \left(C_1(j, \delta_t) - \frac{N(j, \delta_t)^{1+\psi}}{1+\psi} \right) + \beta \log C_2(j, \delta_t) \quad (1)$$

Here, $C_1(j, \delta_t)$ and $C_2(j, \delta_t)$ denote respectively young- and old-age consumptions of an agent born in period t of type δ , $N(j, \delta_t)$ is the labor supply. Although not crucial for the results, the GHH preferences shut down the wealth effect on the labor supply, allowing to aggregate the labor supply across types easily.

A young household consumes, works and chooses asset holdings – bonds, $L(j, \delta_t)$ (expressed

⁸I thank my PhD advisor, Bertrand Wigniolle, who directed me to this literature.

in real terms), bubbles ⁹, $X(j, \delta_t)$, and equities, $I(j, \delta_t)$. His per-period budget constraint is:

$$C_1(j, \delta_t) + \frac{1}{R_t}L(j, \delta_t) + B_t X(j, \delta_t) + I(j, \delta_t) = W_t N(j, \delta_t) \quad (2)$$

Here, W_t is the real wage, R_t is the (gross real) interest rate and B_t is the price of the bubbly asset.

In his second period of life, the retiree collects profits from the firms – described below, $Z(j, \delta_t)$, interests on its bonds and equity holdings, sells his stock of bubbly asset and finally consumes all his wealth,

$$C_2(j, \delta_t) = Z(j, \delta_t) + L(j, \delta_t) + B_{t+1} X(j, \delta_t) + Q_{t+1} \zeta(\delta_t) I(j, \delta_t) \quad (3)$$

Here, Q_{t+1} is the rate of dividends and $\zeta(\delta_t) \in [0, 1]$ is a type-specific transaction cost on the stock market returns.

The agents also face financial frictions: a borrowing constraint on bonds and short-sales constraints on bubbles and equities.

$$X(j, \delta_t), I(j, \delta_t) \geq 0, \quad L(j, \delta_t) \geq -\rho(Z(j, \delta_t) + Q_{t+1} \zeta(\delta_t) I(j, \delta_t)) \quad (4)$$

In the basic model, I'll make two further assumption: $\zeta(i_t) = 1 > \zeta(s_t) = 0$: the savers never hold equities in equilibrium; $Z(j, s_t) = 0$ and $Z(j, i_t) = \frac{Z_t}{\lambda}$: the aggregate dividends are distributed evenly between the investors ¹⁰.

Each household maximizes (1) under (2), (3), (4), taking the prices and taxes as given. As all agents within one type are symmetric, I'll drop the j index. First, thanks to the GHH preferences, all households have the same labor supply curve:

$$W_t = N(\delta_t)^\psi \quad (5)$$

But the portfolio choices of the households will generally differ. Indeed, on the contrary to the investors, the savers cannot arbitrage between the equity and the other assets: the equity-market is segmented. If the financial markets were functional enough, trade in the financial markets would lead to the equalization of valuations across assets and types, i.e the following

⁹That is, an asset which does not generate any productive dividends but pure capital gains.

¹⁰This last assumption simplifies the analysis but bears no consequences for the results.

condition would hold as an equality for all $t \geq 0$:

$$Q_{t+1}\Lambda(i_t) \geq R_t\Lambda(s_t) = \frac{B_{t+1}}{B_t}\Lambda(s_t) = 1 \geq R_t\Lambda(i_t) = \frac{B_{t+1}}{B_t}\Lambda(i_t) \quad (6)$$

Where $\Lambda(\delta_t) \equiv \beta \frac{U_{C_2}(\delta_t)}{U_{C_1}(\delta_t)}$ is the stochastic discount factor of an agent of type δ_t . Here, however, I will focus on liquidity-scarce equilibria where condition (6) holds as a strict inequality, or equivalently: $Q_{t+1} > R_t = \frac{B_{t+1}}{B_t}$. Because the rate of dividends is strictly greater than the rate of returns on the other assets, there is an arbitrage opportunity in equilibrium: issue liquidity to accumulate stocks. The first-order conditions for a representative agent of type i_t are then:

$$L(i_t) = - (Z_j(i_t) + Q_{t+1}I_j(i_t)) \quad (7)$$

$$X(i_t) = 0 \quad (8)$$

$$(1 - \rho)\varphi_t Q_{t+1}\Lambda(i_t) = 1, \quad \varphi_t \equiv \left(1 - \frac{\rho Q_{t+1}}{R_t}\right)^{-1} \quad (9)$$

As they want to benefit from the arbitrage window, the investors do not buy any bubbly asset and they borrow up to the limit in order to invest in stocks. Being financially-constrained, they value one additional unit of equity not only for its dividends but also as a collateral – φ_t is the equilibrium leverage.

A contrario, the savers do not have access to the stock market and therefore cannot arbitrage between stocks on the one hand and liquid assets on the other. Thus, they will accumulate bubbles and the bonds issued by the leveraged investors. The first-order conditions for a representative agent of type s_t are:

$$R_t\Lambda(s_t) = 1 \quad (10)$$

$$B_{t+1}\Lambda(s_t) = B_t \quad (11)$$

$$I(s_t) = 0 \quad (12)$$

The two types of market frictions, i.e transaction costs and borrowing constraints, are crucial for the existence of liquidity-constrained equilibria. Indeed, the first ensures that the demand for liquidity remains bounded from below – as the savers must hold liquid assets, while the second ensures that the supply remains bounded from above – as the investors cannot borrow as much as they want to – such that there can be an arbitrage opportunity in equilibrium.

2.2 Firms and nominal rigidities

The final good is produced by a competitive sector that aggregates a continuum of intermediate varieties according to the Dixit-Stiglitz technology,

$$Y_t = \left(\int Y_t(v)^{\frac{\iota-1}{\iota}} dv \right)^{\frac{\iota}{\iota-1}}$$

Here, Y_t is the supply of the final good, or output, and $Y_t(v)$ is the quantity of intermediate good $v \in [0, 1]$. In turn, the for each individual variety is:

$$Y_t(v) = \left(\frac{P_t(v)}{P_t} \right)^{-\iota} Y_t \quad (13)$$

Where $P_t(v)$ is the price of the variety v , and the aggregate price level satisfies:

$$P_t = \left(\int P_t(v)^{1-\iota} dv \right)^{\frac{1}{1-\iota}}$$

The intermediate producers are monopolistically competitive. They have access to a Cobb-Douglas technology to produce each a particular variety $v \in [0, 1]$ out of capital and labor,

$$Y_t(v) = K_t(v)^\alpha N_t(v)^{1-\alpha} \quad (14)$$

Minimizing the cost leads to the marginal cost of production,

$$MC_t(v) = \left(\frac{Q_t}{\alpha} \right)^\alpha \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \quad (15)$$

And the conditional factor demands,

$$Q_t = \alpha \frac{Y_t(v)}{K_t(v)} MC_t(v), \quad W_t = (1-\alpha) \frac{Y_t(v)}{N_t(v)} MC_t(v) \quad (16)$$

Firms rise and fall with their owners, the old investors. Thereby, the flexible-price firm sets its price so as to maximize the current flow of profits under the demand and technology constraints,

$$Z_t^*(v) = \max_{P_t^*(v)} Y_t(v) \left(\frac{P_t^*(v)}{P_t} - MC_t(v) \right)$$

s.t (13)

Which yields the usual optimal pricing condition,

$$\frac{P_t^*(v)}{P_t} = \mathcal{M}^* MC_t(v) \quad (17)$$

Here, $\mathcal{M}^* \equiv \frac{\iota}{\iota-1}$ is the optimal mark-up. Plugging (13) in the demand constraint, the optimal supply of firm v is then:

$$Y_t^*(v) = (\mathcal{M}^* MC_t(v))^{-\iota} Y_t$$

However, each firm faces a price-setting friction: to charge the optimal price, it must pay a menu cost, $\phi \geq 0$. A firm that refuses to pay the menu cost follows a rule-of-thumb,

$$\frac{P_t(v)}{P_t} = \frac{\Pi^\#}{\Pi_t} \quad (18)$$

And produces what is demanded at the given price,

$$Y_t(v) = \left(\frac{\Pi^\#}{\Pi_t} \right)^{-\iota} Y_t$$

Knowing its technology and demand constraints, each firm chooses the pricing policy that maximizes its profits,

$$Z_t(v) = \max \left\{ \left(\frac{\Pi^\#}{\Pi_t} \right)^{-\iota} \left(\frac{\Pi^\#}{\Pi_t} - MC_t(v) \right) Y_t, \quad (\mathcal{M}^* MC_t(v))^{1-\iota} \left(1 - \frac{1}{\mathcal{M}^*} \right) Y_t - \phi \right\}$$

To simplify, I will make two further assumptions. First, the aggregate menu costs are rebated lump-sum to the investors,

$$Z_t = \int Z_t(v) dv + \int \mathbb{1}_*(v) \phi dv$$

Second, the inflation rate in the rule-of-thumb is equal to the central bank's inflation target, $\Pi^\# = \Pi^{CB}$: we can think of the firms as having well-anchored inflation expectations. Those firms will change their expectations and pricing policy/production plans if and only if the current inflation rate deviates sufficiently from this target.

2.3 Monetary policy

The nominal interest is set according to a Taylor rule with a time-varying intercept,

$$R_t^n = \max \left\{ R_t^* \Pi^{CB} \left(\frac{\Pi_t}{\Pi^{CB}} \right)^{\phi_\pi}, \underline{R}^n \right\}, \quad \underline{R}^n \leq 1 \quad (19)$$

Here, $\Pi^{CB} \geq 1$ is the inflation target, R_t^* is the natural interest rate and \underline{R}^n is the ZLB. The central bank follows the Taylor principle, $\phi_\pi > 1$.

2.4 Equilibrium

Aggregate supply block From (5), the aggregate labor supply curve is:

$$W_t = N_t^\psi \tag{20}$$

As all firms face the same marginal cost, (15), they are symmetric in equilibrium. In particular, the aggregate factor demand curves obtain in (16),

$$(1 - \alpha) \frac{Y_t}{N_t} MC_t = W_t, \quad \alpha \frac{Y_t}{K_t} MC_t = Q_t$$

Equalizing the supply and demand for labor and using the production function, (14), to express in terms of output, the labor market equilibrium writes as:

$$(1 - \alpha) K_t^{\alpha(1+\gamma)} MC_t = Y_t^\gamma, \quad \gamma \equiv \frac{\alpha + \psi}{1 - \alpha}$$

The equilibrium marginal cost, and the corresponding level of production, depend on the pricing policy of the firms, which can be modeled as the solution of a dynamic and forward-looking game. Since this is beyond the scope of this paper, I'll adopt a simple shortcut and focus on one extreme case, full price-stickiness at the firm-level.

Assumption 1 $\phi \rightarrow \infty$

Under assumption 1, the firms always prefer to follow the indexation rule. Nevertheless, it is useful to characterize the flexible-price case as it is the efficient benchmark. If each firm individually decides to pay the menu cost, symmetry across firms implies that the aggregate price level is equal to the optimal reset price, $P_t = P_t^o$. Therefore, the optimal equilibrium mark-up obtains in (17) as:

$$MC_t^* = \frac{1}{\mathcal{M}^*}$$

Plugging in the labor market equilibrium, we recover a long-run aggregate supply (LRAS) curve,

$$Y_t^* = \left(\frac{1 - \alpha}{\mathcal{M}^*} \right)^{\frac{1}{\gamma}} K_t^{\alpha \frac{1+\gamma}{\gamma}} \tag{21}$$

Given the stock of capital, output is supply-determined and efficient ¹¹: the LRAS curve is

¹¹Apart from the static distortion related to monopolistic competition.

vertical. The equilibrium inflation rate, Π_t^* , will be demand-determined in general equilibrium such that the goods market clears, i.e aggregate demand is consistent with the efficient level of production, $C_t + I_t = Y_t^*$ – here $C_t \equiv (1 - \lambda)(C_1(s_t) + C_2(s_{t-1})) + \lambda(C_1(i_t) + C_2(i_{t-1}))$ and $I_t = (1 - \lambda)I(s_t) + \lambda I(i_t)$ denote respectively aggregate consumption and investment.

However, under assumption 1, each individual firm chooses the indexation rule. Thereby, the short run aggregate supply (SRAS) curve is horizontal:

$$\Pi_t = \Pi^{CB} \quad (22)$$

The logic is reversed with respect to the flexible-price case: the equilibrium level of output is demand-determined in general equilibrium, $Y_t = C_t + I_t$. Indeed, each individual firm does not get to choose its price, nor its mark-up or its level of production, but rather produces what is demanded at the given price. Therefore, the aggregate production is equal to the level of aggregate demand at the given inflation rate, $\Pi_t = \Pi^{CB}$. Hence, when the prices are sticky, the model has a standard Keynesian cross representation.

Given the level of aggregate demand, we can use (21) to express the marginal cost as a function of the output gap, ξ_t ,

$$MC_t = \frac{1}{\mathcal{M}^*} \xi_t^\gamma$$

Where $\xi_t \equiv \frac{Y_t}{Y_t^*}$

If $\xi_t < 1$, the economy suffers from an aggregate demand shortages, which, according to the Keynesian cross mechanism, depresses aggregate output; of course, if $\xi_t > 1$, the economy is booming and produces currently more than its potential.

Substituting back in the factor demand curves, (16), we can observe that the factorial distribution of income is conditional on the output gap:

$$\frac{W_t N_t}{Y_t} = \frac{1 - \alpha}{\mathcal{M}^*} \xi_t^\gamma, \quad \frac{Q_t K_t}{Y_t} = \frac{\alpha}{\mathcal{M}^*} \xi_t^\gamma, \quad \frac{Z_t}{Y_t} = 1 - \frac{1}{\mathcal{M}^*} \xi_t^\gamma \quad (23)$$

Intuitively, a negative output gap ($\xi_t < 1$) occurs when the sticky-price firm, facing an aggregate demand shortage, produces less than the efficient level and therefore cuts its demand for factors – the flexible-price firm would rather cuts its price until the efficient level of demand is restored. As all firms behave symmetrically, the aggregate factor demand curves shift to the left, putting downward pressures on the wage and the rental rate of capital. If the supply of factors is price-elastic, and finitely so – as it is the case in this model, the equilibrium is characterized by an

under-utilization of factors, lower factor shares in income and a higher share of rent.

Except for stabilizing policies, the sticky-price decentralized economy has not any inherent tool to guarantee that the level of aggregate demand coincides with the efficient level of output, $\xi_t = 1$. Of course, that is the task of the central bank: by adjusting the policy rate so as to target an interest rate gap nil, $\frac{R_t}{R^*} = 1$, the central bank seeks to ensure full employment in equilibrium. And as long as monetary policy is not constrained, it works well: the sticky-price equilibrium reproduces the flexible-price's; prices are sticky at the micro-level, but flexible at the macro-level – i.e R_t is flexible even through Π_t is not. However, if the ZLB binds in (19), that is if $R_t^* < \underline{R} \equiv \frac{\mathbf{R}^n}{\Pi^{CB}}$, the central bank loses its control over aggregate demand and therefore cannot preserve full employment, $\xi_t < 1$: this is the liquidity trap.

The next obvious questions is then: what determines the natural interest rate?

Aggregate demand block As the bubble is safe, the first-order conditions of the savers, (11) and (12), implies the following non-arbitrage condition between bubbles and bonds,

$$\frac{B_{t+1}}{B_t} = R_t \quad (24)$$

The aggregate demand for liquidity obtains by aggregating the savers' Euler equation, (10). Using the above non-arbitrage condition, we can write it as:

$$\frac{L_t^d}{R_t} + B_t^d = (1 - \lambda) \left(\frac{L(s_t)}{R_t} + B_t H(s_t) \right) = (1 - \lambda) \frac{\beta}{1 + \beta} \frac{\psi}{1 + \psi} W_t N_t$$

With a logarithmic utility function, the substitution and income effects perfectly offset each other: the marginal propensity to save out of wealth is independent of the interest rate. Thus, the aggregate demand for liquidity, $\frac{L_t^d}{R_t} + B_t^d$, is fully inelastic with respect to the interest rate. Note, however, that through the aggregate labor income, it is output- and output gap-elastic.

Turning to the aggregate supply of liquidity, there are two cases: either liquidity is not scarce, $Q_{t+1} = R_t$, the borrowing constraints does not bind and therefore the supply is infinitely elastic – it is horizontal; or liquidity is scarce, $Q_{t+1} > R_t$, such that the investors are financially-constrained and their supply of liquidity is given by (7) and (8):

$$\frac{L_t^s}{R_t} = -\lambda L(i_t) = \rho \frac{Z_{t+1} + Q_{t+1} I_t}{R_t}$$

The supply of bubbly liquidity is exogenous and normalized to one, $H = 1$. If liquidity is scarce, the aggregate supply of liquidity, $\frac{L_t^s}{R_t} + B_t$, remains interest rate elastic (but finitely so): a lower

interest rate increases the price of the bonds, allowing the investors to leverage more.

Imposing bubble market clearing, $(1 - \lambda)H(s_t) = 1$, the liquidity market equilibrium is:

$$\min \left\{ \frac{\rho(Z_{t+1} + I_t Q_{t+1})}{(1 - \lambda) \frac{\beta}{1 + \beta} \frac{\psi}{1 + \psi} W_t N_t - B_t}, Q_{t+1} \right\} = R_t \quad (25)$$

As liquidity becomes scarcer – i.e. ρ , B_t or λ decreases: there is a lower supply / higher demand – the interest rate falls short of the rental rate of capital. This liquidity premium is necessary to provide the investors with incentives to raise their leverage and issue more debt, thereby restoring equilibrium. Thus, the model is consistent with dynamic efficiency from an aggregate point of view, $Q_{t+1} > 1$, but dynamic inefficiency for particular assets, $R_t < 1$.

Capital accumulation As the savers do not hold equity, (12), the aggregate investment function obtains from the aggregate Euler equation of the investors, (9), in which we can substitute the liquidity market equilibrium, (25), and re-organize:

$$K_{t+1} = I_t = \frac{\beta}{1 + \beta} \frac{\psi}{1 + \psi} W_t N_t - \frac{1}{1 + \beta} \frac{Z_{t+1}}{Q_{t+1}} - B_t \quad (26)$$

In equilibrium, the aggregate savings are channeled toward either the accumulation of either capital or bubbly assets. Naturally, since the households are smoothing consumption, the aggregate savings rate increases (decreases) with the share of non-financial income that accrues to them when they are young (old), i.e it decreases (increases) with the share of rent.

Policy The model has multiple equilibria, each corresponding to a different path for the nominal interest rate $\{R_t^n\}$. In the New Keynesian tradition, the central bank follows a Taylor rule, (19) that picks one particular path. Given inflation expectations, the interest rate must satisfy the Fisher equation,

$$R_t = \frac{\max \left\{ R_t^* \Pi^{CB} \left(\frac{\Pi_t}{\Pi^{CB}} \right)^{\phi_\pi}, \underline{R}^n \right\}}{E_t \Pi_{t+1}} \quad (27)$$

An equilibrium is a sequence $\{R_t^*, R_t, Q_t, W_t, Z_t, Y_t^*, Y_t, \xi_t, N_t, K_{t+1}, B_t, \Pi_t\}$ such that (20)-(27) hold for all $t \geq 0$, $\xi_t = \frac{Y_t}{Y_t^*}$ and the natural interest rate, R_t^* , obtains in (25) when $\xi_t = 1$.

3 Expansionary bubbles in the liquidity trap

In general equilibrium, the model has two regimes with drastically different properties. If the shortage of liquidity is benign, the equilibrium is in the Neoclassical regime: the interest rate

rate is the adjusting variable that restores equilibrium in the financial markets. Both the interest rate and output gaps are nil, production is efficient and bubbles crowd out capital. However, if liquidity is very scarce, the economy enters the Keynesian liquidity trap: the interest rate is fixed and therefore the output gap is the adjusting variable. By providing the agents with a greater supply of liquidity, bubbles raise aggregate demand and crowd in capital accumulation.

The temporary equilibrium – given expectations – can be represented as a single equation relating the output gap to the interest rate,

$$\min \left\{ \frac{\rho \left(1 - \frac{1-\alpha}{\mathcal{M}^*} \xi_{t+1}^\gamma\right)}{(1-\lambda) \frac{\beta}{1+\beta} \frac{\psi}{1+\psi} \frac{1-\alpha}{\mathcal{M}^*} \xi_t^\gamma - b_t}, \frac{\alpha}{\mathcal{M}^*} \xi_{t+1}^\gamma \frac{Y_t^* \xi_t}{K_{t+1}} \right\} \frac{Y_{t+1}^* \xi_{t+1}}{Y_t^* \xi_t} = \max \{R_t^*, \underline{\mathbf{R}}\} \quad (28)$$

Here $b_t \equiv \frac{B_t}{Y_t}$ is the bubble-output ratio. On the right hand side is the interest rate set by the central bank according to the Taylor rule and the Fisher equation, (27), taking into account that the SRAS is horizontal, (22). On the left hand side, which obtains by substituting the factor shares, (23), in the liquidity market equilibrium, (25), we have an IS curve that determines the output gap consistent with equilibrium in the financial and good markets at a given interest rate. The natural interest rate obtains in the IS curve at full employment,

$$R_t^* = \min \left\{ \frac{\rho \left(1 - \frac{1-\alpha}{\mathcal{M}^*} \xi_{t+1}^\gamma\right)}{(1-\lambda) \frac{\beta}{1+\beta} \frac{\psi}{1+\psi} \frac{1-\alpha}{\mathcal{M}^*} - b_t}, \frac{\alpha}{\mathcal{M}^*} \xi_{t+1}^\gamma \frac{Y_t^*}{K_{t+1}} \right\} \frac{Y_{t+1}^* \xi_{t+1}}{Y_t^* \xi_t} \quad (29)$$

Depending on the level of the natural interest rate relative to the lowest interest rate the central bank can implement, $\underline{\mathbf{R}} \equiv \frac{\mathbf{R}^n}{\Pi^{\sigma_B}}$, the equilibrium point $(\tilde{\xi}_t, \tilde{R}_t)$ may lie either in the Neoclassical regime, $(\tilde{\xi}_t, \tilde{R}_t) = (0, R_t^*)$, or in the Keynesian, $(\tilde{\xi}_t, \tilde{R}_t) = (\xi_t, \underline{\mathbf{R}})$.

3.1 Neoclassical regime: benign asset shortage

As long as the ZLB does not bind, the central bank succeeds in targeting the natural interest rate. Thus, the level of aggregate demand is equal to the efficient level of output: at the macro-level, the economy behaves "as if" prices were flexible at the micro-level¹². From (28):

$$R_t = R_t^* \geq \underline{\mathbf{R}} \quad \text{and} \quad \xi_t = 1$$

When liquidity becomes scarcer – i.e the supply shrinks: lower b_t or ρ ; the demand increases: lower λ – the liquidity premium rises and therefore the natural interest rate falls. In turn, the central bank cuts its policy rate one-for-one with the natural interest rate, keeping the interest rate gap nil. This provides the investors with incentives to increase their leverage and their

¹²In equilibrium, each individual firm has no incentives to set another price even in the absence of menu cost.

supply of liquidity until equilibrium in the financial markets is restored. Consequently, as long as the interest rate can freely adjust to meet its natural value, a liquid asset shortage is benign: it does not generate any output loss.

Plugging (29) in (24), the bubble-output ratio can be written in recursive form:

$$b_{t+1}^* = \frac{\rho \left(1 - \frac{1-\alpha}{\mathcal{M}^*}\right)}{\rho \left(1 - \frac{1-\alpha}{\mathcal{M}^*}\right) + b^* - b_t} b_t$$

Where $b^* \equiv (1 - \lambda) \frac{\beta}{1 + \beta} \frac{\psi}{1 + \psi} \frac{1 - \alpha}{\mathcal{M}^*} - \rho \left(1 - \frac{1 - \alpha}{\mathcal{M}^*}\right)$

The dynamic of the bubble is globally unstable and therefore determinate: if $b_t^* < b^*$, the bubble implodes as a share of output while it explodes if $b_t^* > b^*$; the economy converges to a bubbly steady state if and only if $b_t = b^*$, which is equivalent to $R_t^{*,b} = 1$.

Then, the dynamic of the stock of capital is obtained by substituting the factor shares, (23), in the law of motion for capital, (26),

$$K_{t+1}^* = \frac{\frac{\beta}{1+\beta} \frac{\psi}{1+\psi} \frac{1-\alpha}{\mathcal{M}^*} - b_t^*}{1 + \frac{1}{1+\beta} \frac{1-\frac{1}{\mathcal{M}^*}}{\frac{\alpha}{\mathcal{M}^*}}} \left(\frac{1-\alpha}{\mathcal{M}^*} \right)^{\frac{1}{\gamma}} K_t^{*\alpha \frac{1+\gamma}{\gamma}} \quad (30)$$

As $\alpha \frac{1+\gamma}{\gamma} < 1$, the capital stock converges to a steady state if and only if b_t^* converges as well. The economy always has a bubble-free steady state where the stock of capital is the same as that of the standard frictionless OLG model:

$$R^* = \min \left\{ \frac{\rho \left(1 - \frac{1-\alpha}{\mathcal{M}^*}\right)}{(1-\lambda) \left(\frac{\alpha}{\mathcal{M}^*} + \frac{1}{1+\beta} \left(1 - \frac{1}{\mathcal{M}^*}\right) \right)}, 1 \right\} Q^*$$

$$Q^* = \frac{\frac{\alpha}{\mathcal{M}^*} + \frac{1}{1+\beta} \left(1 - \frac{1}{\mathcal{M}^*}\right)}{\frac{\beta}{1+\beta} \frac{\psi}{1+\psi} \frac{1-\alpha}{\mathcal{M}^*}}$$

This fundamental steady state also co-exist with a bubbly steady state if and only $b^* > 0$, which is equivalent to $R^* < 1$: fundamental liquidity must be sufficiently scarce such that the savers begin to value the bubbles for the liquidity they provide. Of course, if $R^* = Q^* < 1$, the investors also value the bubbles: the economy suffers from a general asset shortage. Here, I'll focus on liquid asset shortage: $R^* < 1 < Q^*$.

Assumption 2 $\frac{\alpha}{\mathcal{M}^*} + \frac{1}{1+\beta} \left(1 - \frac{1}{\mathcal{M}^*}\right) > \max \left\{ \frac{\rho}{1-\lambda} \left(1 - \frac{1-\alpha}{\mathcal{M}^*}\right), \frac{\beta}{1+\beta} \frac{\psi}{1+\psi} \frac{1-\alpha}{\mathcal{M}^*} \right\}$

Under assumption 2, liquid assets are scarce but the economy has productive investment opportunities: only the savers value the bubbles. It is a particular form of dynamic inefficiency: the bubble-less steady state is not be dynamically inefficient *on aggregate* – a general asset

shortage, but dynamically inefficiency from *the point of view of the savers* – a liquid asset shortage.

In the bubble-less Neoclassical regime, the level of liquidity is completely neutral from a macroeconomic point of view: the interest rate absorbs any changes in the scarcity of liquidity such that it does not affect the stock of capital and potential output. However, if there is a severe shortage of liquidity, the savers have incentives to create financial bubbles, which break down the dichotomy between the financial and real sides of the economy. Instead, the equilibrium size of the bubble is decreasing in the level of fundamental liquidity. In turn, a bigger bubble reduces the flow of funds channeled toward the accumulation of capital, thereby decreasing output and potential output in the long run.

Proposition 1 *In the liquidity-constrained Neoclassical regime,*

- (i) *There is a unique fundamental steady state; there is also a unique bubbly steady state if and only if fundamental liquidity is sufficiently scarce – ρ and λ are very low;*
- (ii) *If both steady states co-exist, the fundamental is globally indeterminate whereas the bubbly is globally determinate;*
- (iii) *The level of liquidity is neutral at the fundamental steady state; scarcer liquidity implies a bigger bubble and therefore a lower stock of capital and output at the bubbly steady state.*

Finally, note that if the economy is on the saddle-path which converges to a bubbly steady state, the natural interest rate is equal to the growth rate of the economy at each period, $R_t^{*,b} = \frac{Y_{t+1}^{*,b}}{Y_t^{*,b}}$ for all $t \geq 0$. Thus, assuming that the economy is not reducing its stock of capital, the ZLB never binds in (19)¹³: if the economy converges to a bubbly steady state, it must be in the Neoclassical regime.

To summarize briefly the dynamic of the model in the neoclassical regime: output is supply-determined and efficient. If liquidity is scarce, the interest rate is the adjusting variable that restores equilibrium in the financial markets. Bubbly liquidity can be valued if and only if the economy suffers from a severe shortage of fundamental liquidity – $R^* < 1$. When this happens, the savers may begin to value bubbles as a way to smooth consumption even through their fundamental value is nil: the savers trade the bubbles for the liquidity they provide. As the interest rate rises with the supply of liquidity, the leverage of the investors plunges and therefore aggregate investment, capital accumulation and output.

¹³As $Q_{t+1}^{*,b} = Q^{*,b} \left(\frac{Q_t}{Q^{*,b}} \right)^\alpha \frac{1+\gamma}{\gamma}$, the equilibrium is also always liquidity constrained.

3.2 Keynesian regime: the liquidity trap

However, the model presents a non-linearity at the ZLB. If the shortage of liquidity is severe, the natural interest rate is lower than the minimal interest rate the central bank can deliver: the economy enters the Keynesian liquidity trap.

$$R_t = \underline{R} \geq R_t^*$$

As monetary policy cannot accommodate downward fluctuations in the liquidity premium anymore, it cannot prevent aggregate demand to fall short of the efficient level of output. The interest rate being pegged higher than its natural value, the liquidity and good markets do not clear at full employment. Instead, there is an excess demand for liquidity and an excess supply of goods: at the given interest rate, the financially-constrained investors cannot pledge enough returns to issue the quantity of debt asked by the savers; furthermore, as the aggregate savings are greater than the aggregate investment, the aggregate demand for goods is lower than the efficient level of output.

Given its fixed price and the sub-optimally low level of aggregate demand, each firm reduces its production and its demand for factors. This shifts the aggregate labor demand curve to the left and moves the equilibrium down along the labor supply curve: the wage rate as well as the hours worked both decrease. This output gap impacts the wealth of the savers through two channels: the aggregate income and also the labor share shrink. Since the savings rate of the young savers remains unaffected but their income is lower, they ask for less liquidity: equilibrium in the financial and good markets is restored.

As $R_t = \underline{R}$, we can use (28) to compute the output gap consistent with equilibrium in the financial markets:

$$\xi_t = \left(\frac{R_t^*}{\underline{R}} \right)^{\frac{1}{1+\gamma}} \in (0, 1)$$

The interest rate gap, $\frac{R_t^*}{\underline{R}}$, is a proxy for the disequilibrium in the liquidity market at full employment, that is a proxy for the severity of the liquidity trap. The greater the excess demand for liquidity at full employment, the larger the interest rate gap and therefore the bigger the output gap (in absolute value). Thus, in the Keynesian regime, the level of liquidity is not neutral as in the Neoclassical regime, but rather it determines the level of aggregate demand and thereby the output gap. In some respect, the model provides micro-foundation and formalizes the insight of [Keynes \(1936\)](#): absent of stabilizing policies, the economy has not any inherent mechanism to ensure that the level of investment, or the supply of liquidity, is

consistent with full employment of productive resources in general equilibrium ¹⁴.

Dynamically, the expected output gap also enters the IS curve through the natural interest rate, as well as the law of motion of capital accumulation – see below. Indeed, a negative output gap raises the share of rent in aggregate income at the expense of the productive factors. Therefore, if the agents expect aggregate demand to contract the next period, they expect the non-financial wealth of the old investors to be high. Thus, those – currently young – investors are able to pledge more returns and supply more liquidity as a share of output.

As the economy endogenously accumulates capital, the short-run contraction propagates over time. Indeed, even through the output gap restored the equilibrium in the liquidity market, and therefore the equality between the aggregate savings and aggregate investment, it did so at the expense of lower aggregate savings, not a greater aggregate investment – as a fall in the interest rate would have.

$$K_{t+1} = \frac{\frac{\beta}{1+\beta} \frac{\psi}{1+\psi} \frac{1-\alpha}{\mathcal{M}^*} \xi_t^{1+\gamma}}{1 + \frac{1}{1+\beta} \frac{1-\frac{1}{\mathcal{M}^*} \xi_{t+1}^\gamma}{\frac{\alpha}{\mathcal{M}^*} \xi_{t+1}^\gamma}} \left(\frac{1-\alpha}{\mathcal{M}^*} \right)^{\frac{1}{\gamma}} K_t^{\alpha \frac{1+\gamma}{\gamma}} \quad (31)$$

The expected output gap, which raises the non-financial wealth of the old investors, reduces their incentives to save and therefore their savings rate. The economy currently in the liquidity trap will start the next period with a smaller stock of capital than the economy in the Neoclassical regime: a shortage of aggregate demand causes an output gap in the short and also a "potential output gap" in the longer run. This may lead one to underestimate potential output in the liquidity trap, and in particular it is a way to rationalize the evidence of [Bosworth \(2015\)](#) that capital accumulation does not seem to recover following the bubble burst in 2007.

$$\xi = \left(\frac{\frac{R^*}{\underline{R}}}{1 - \frac{1-\alpha}{\mathcal{M}^*} \left(1 - \frac{R^*}{\underline{R}} \right)} \right)^{\frac{1}{\gamma}}$$

$$Q = \frac{\frac{\alpha}{\mathcal{M}^*} \xi^\gamma + \frac{1}{1+\beta} \left(1 - \frac{1}{\mathcal{M}^*} \xi^\gamma \right)}{\frac{\beta}{1+\beta} \frac{\psi}{1+\psi} \frac{1-\alpha}{\mathcal{M}^*} \xi^{1+\gamma}}$$

As $\frac{\partial Q}{\partial \xi} < 0$ and $\xi \in (0, 1)$, $Q \geq Q^*$ and $K \leq K^*$.

If the ZLB binds, the economy converges to a steady state with a low stock of capital and a negative output gap. In general, even if the menu costs are finite, the firms may have no incentives to deviate from the indexation rule: as long as the central bank does not increase its inflation target, Π^{CB} , or implement negative nominal interest rate, $\underline{R}^n < R^*$, the economy remains permanently depressed. Furthermore, and as I show in appendix [B](#), economy can be

¹⁴With more general preferences, under-consumption may also be possible.

subject to endogenous output gap fluctuations in the liquidity trap.

Proposition 2 *In the liquidity-constrained Keynesian regime,*

- (i) *There is a unique fundamental steady state, which co-exists with the bubbly steady state of the Neoclassical regime;*
- (ii) *The output gap grows as liquidity becomes scarcer;*
- (iii) *A higher inflation target or negative nominal interest rate are expansionary.*

In the liquidity trap, the central bank has no room to offset a negative shock on the supply – or a positive shock on the demand – of liquidity. Since the prices are fixed, quantities must adjust to restore equilibrium. An output gap emerges that is proportional to the interest rate gap, or scarcity of liquidity at the ZLB. More severe financial frictions go hand-in-hand with bigger output gaps and lower stocks of labor and capital: this is the secular stagnation.

Assumption 3 $(1 - \frac{1}{\mathcal{M}^*}) + \frac{\alpha}{\mathcal{M}^*} (1 - \beta \frac{1-\lambda}{\lambda}) > 0$

The economy reaches equilibrium at the cost of an efficiency loss: factors of production are underemployed. But if, given the policy parameters and the level of liquidity, secular stagnation is the unique fundamental steady state as in [Caballero and Farhi \(2016\)](#) or [Eggertsson and Mehrotra \(2014\)](#), there exists also a unique bubbly steady state since $R^* < 1$. Furthermore, as in any bubbly steady state, $R^{*,b} = 1 > \underline{R}$, the creation of bubbles help the economy to escape the liquidity trap.

Proposition 3 (Expansionary bubbles) *In the liquidity-constrained Keynesian regime,*

- (i) *Output is greater at the bubbly steady state if and only if $R^* < R_Y^*(\underline{R})$;*
- (ii) *The stock of capital is greater at the bubbly steady state if and only if $R^* < R_K^*(\underline{R})$;*
- (iii) *$R_K^*(\underline{R}) \leq R_Y^*(\underline{R}) \leq \underline{R}$ and $R_X^{*'}(\underline{R}) > 0, X = K, Y$. In particular, $R_Y^*(1) = R_K^*(1) = 1$.*

When agents coordinate their expectations to create rational bubbles, the supply of liquidity increases: the natural interest rises and the economy jumps out of the liquidity trap. As point (iii) illustrates, the macroeconomic effects of asset bubbles are conditional on the stance of monetary policy. If monetary policy is unconstrained, i.e $\underline{R} \rightarrow -1$, bubbles are always contractionary because the economy is in the Neoclassical regime. However, since the major central banks seem to be reluctant to implement negative nominal interest rates or high inflation targets, bubbles are likely to be expansionary in an asset-scarce environment.

The model therefore provides an alternative story for the sub-prime crisis: before 2007-2008, the housing and mortgage-backed securities bubble masked and “cured” the fundamental asset shortage. The bursting bubble was a great negative shock on the supply of liquidity, thus on the natural interest rate. As major central banks hit the ZLB, production had to fall to restore equilibrium in the financial markets. Importantly, this shock on the natural interest rate is permanent, and not temporary as in the usual literature on the liquidity trap.

4 Fiscal policy: no bubble, no liquidity trap

On the contrary to monetary policy, fiscal policy has a direct control over the supply of liquidity. By issuing enough public debt, the government can raise the natural interest above the ZLB or even above zero, thereby suppressing the possibility of rational bubbles and liquidity traps altogether: on the contrary to the conventional view, e.g [Sargent and Wallace \(1981\)](#), an *insufficient* supply of public debt threatens price and financial stability in an liquid-assets-scarce economy.

The government does not consume but issues bonds according to a constant debt-to-GDP target, $-L(g_t) = \bar{L}Y_{t+1}$. His budget constraint is then:

$$-L(g_{t-1}) = -\frac{L(g_t)}{R_t} + T_t$$

The lump-sum tax, T_t , adjusts at each period such that the government meets his budget, $\frac{T_t}{Y_t} = \bar{l}^g \left(1 - R_t^{-1} \frac{Y_{t+1}}{Y_t}\right)$. On aggregate, the young savers pay a fraction $\tau \in [0, 1]$ of the lump-sum tax while the young investors pay what is left.

As the tax reduces the wealth of the young savers, it lowers their demand for liquidity; the supply of liquidity has three components: private and public fundamental, plus bubbly.

$$\begin{aligned} \frac{L_t^d}{R_t} &= (1 - \lambda) \left(\frac{L(s_t)}{R_t} + B_t H(s_t) \right) = \frac{\beta}{1 + \beta} \left((1 - \lambda) \frac{\psi}{1 + \psi} W_t N_t - \tau T_t \right) \\ \frac{L_t^s}{R_t} &= -\lambda \frac{L(i_t)}{R_t} + \frac{L(g_t)}{R_t} + B_t H = \rho \frac{Z_{t+1} + Q_{t+1} I_t + L(g_t)}{R_t} + B_t \end{aligned}$$

As in equilibrium, $(1 - \lambda)H(s_t) = 1$, the liquidity market equilibrium writes as:

$$\min \left\{ \frac{\rho (Z_{t+1} + I_t Q_{t+1}) + L(g_t)}{\frac{\beta}{1 + \beta} \left((1 - \lambda) \frac{\psi}{1 + \psi} W_t N_t - \tau T_t \right) - B_t}, Q_{t+1} \right\} = R_t$$

Plugging in the factors shares, [\(23\)](#), and re-organizing – conditional on the equilibrium being

liquidity-constrained:

$$R_t^* = \frac{\rho \left(1 - \frac{1-\alpha}{\mathcal{M}^*}\right) + \left(1 - \tau \frac{\beta}{1+\beta}\right) \bar{l}^g}{\frac{\beta}{1+\beta} \left((1-\lambda) \frac{\psi}{1+\psi} \frac{1-\alpha}{\mathcal{M}^*} - \tau \bar{l}^g \right) - b_t} \frac{Y_{t+1}^*}{Y_t^*}$$

Whereas the natural interest rate is always increasing in the supply of public liquidity, \bar{l} , it depends non-linearly on the distribution of the tax burden, τ . Indeed, public debt is a way for the government to redistribute wealth both between and within generations. If the interest rate is lower than the growth rate of the economy, $R_t < \frac{Y_{t+1}}{Y_t}$, public liquidity is a bubble: the government rolls over its debt ad infinitum and the tax rate is negative: a higher τ raises the wealth of the young savers and increases their demand for liquidity. In the other case, $R_t > \frac{Y_{t+1}}{Y_t}$, however, the government must tax the households to pay the interests on its debt: a higher τ lowers the wealth of the young savers and their demand for liquidity.

Given the level of output, public debt crowds out capital as it divers some savings away from capital accumulation,

$$K_{t+1} = I_t = \frac{\beta}{1+\beta} \frac{\psi}{1+\psi} W_t N_t - \frac{1}{1+\beta} \frac{Z_{t+1}}{Q_{t+1}} - B_t - \frac{L(g_t)}{R_t}$$

Or equivalently,

$$K_{t+1} = \frac{\frac{\beta}{1+\beta} \frac{\psi}{1+\psi} \frac{1-\alpha}{\mathcal{M}^*} \xi_t^\gamma - \frac{\frac{Y_{t+1}}{Y_t} R_t^{-1+\beta} \bar{l} - b_t}{1+\beta}}{1 + \frac{1}{1+\beta} \frac{1 - \frac{1}{\mathcal{M}^*} \xi_{t+1}^\gamma}{\frac{\alpha}{\mathcal{M}^*} \xi_{t+1}^\gamma}} \xi_t Y_t^*$$

Public debt raises the same trade-off as rational bubbles: from the one hand, a higher supply of public debt stimulate investment and output in the liquidity trap; on the other hand, it decreases capital accumulation and potential output outside of the trap.

Proposition 4 [*Public liquidity and liquidity trap*]

- (i) The natural interest rate, R^* , is increasing in \bar{l} .
- (ii) If $R^* < 1$, the natural interest is decreasing in τ . It is increasing otherwise.
- (iii) Public liquidity crowds out bubbly liquidity one-for-one.
- (iv) The results of proposition 3 apply to public debt.

A higher stock of public debt implies a lower stock of bubbles in steady state: fiscal policy can eliminate the bubbly steady state by supplying the savers with enough liquidity.

$$b^{*,\bar{l}} \equiv (1 - \lambda) \frac{\beta}{1 + \beta} \frac{\psi}{1 + \psi} (1 - \alpha) - \rho\alpha - \bar{l}$$

Note that one difference between liquid assets and general assets shortage is that the latter are conditional on the distribution of wealth between generations, whereas the former are conditional on the distribution of wealth within generation. A fiscal policy that redistributes wealth from the young savers to the young investors can prevent the economy to fall in a liquidity trap induced by a shortage of liquid assets, without any cost in terms of potential output.

5 Conclusion

I have presented a model of liquidity trap and demand side secular stagnation driven by liquid assets shortages; I also provided a new mechanism to explain expansionary bubbles. As in the standard New Keynesian model, the output gap is proportional to the interest rate gap. If some agents face a shortage of assets, a liquid assets shortage, the natural interest rate turns negative in an otherwise dynamically efficient economy. Because of the binding ZLB and nominal rigidities, the economy enters the liquidity trap and an output gap emerges that restores equilibrium in the financial markets. Outside of the liquidity trap, bubbles decrease output and potential output as they divert some savings away from capital accumulation. However, in the liquidity trap, bubbles are expansionary as they increase the supply of assets available to the constrained agents, raise the natural interest rate and close the interest rate gap.

The predictions of the model are consistent with the broad trends of GDP, investment, interest and inflation rates during two of the recent crises triggered by the bust of financial bubbles, the US in 2007 and Japan in the 90's, as well as during the decade that predate the crash. In particular, the model can explain why: (i) a large financial bubble did not generate an economic boom; and (ii) the burst pushed the economy in a liquidity trap of arbitrary duration, with a growing output gap and a falling inflation rate despite a zero nominal interest rate policy.

One obvious limitation of the present model, which it shares with the rest of the literature on rational asset price bubbles, is that it cannot explain how bubbles emerge and crash. Another limitation is that it takes the financial frictions as exogenous. It could be interesting to study the coordination failures at the origin of liquid assets shortages, and how agents can overcome those by creating rational bubbles. I leave an elaboration of these questions for future research.

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Appendix A Proofs

Proof of proposition 3. Using (30), (31) and the steady state value of the bubble, b^* :

$$\frac{K^{*,b}}{K} = \left(\frac{\lambda + (1 - \lambda)R^* \left(1 + \frac{1}{1+\beta} \frac{1 - \frac{1}{\mathcal{M}^*} \xi^\gamma}{\mathcal{M}^*} \right)^{\frac{1}{1-\alpha} \frac{1+\gamma}{\gamma}}}{1 + \frac{1}{1+\beta} \frac{1 - \frac{1}{\mathcal{M}^*}}{\mathcal{M}^*}} \right) \frac{1}{\xi^{1+\gamma}}$$

Plugging in the steady state output gap and re-organizing, $K^b \geq K$ if and only if:

$$LHS \equiv (\lambda + (1 - \lambda)R^*) \frac{\underline{R}}{R^*} \left(1 - \frac{1-\alpha}{\mathcal{M}^*} \right) + \beta \frac{\alpha}{\mathcal{M}^*} \geq RHS \equiv \xi^{1+\gamma}$$

As the output gap is decreasing in the interest rate gap, the RHS is increasing in R^* ; furthermore, $\lim_{R^* \rightarrow 0} RHS \rightarrow 0$ and $\lim_{R^* \rightarrow \underline{R}} RHS = 1$. The LHS is decreasing in R^* if and only if $\frac{1 - \frac{1-\alpha}{\mathcal{M}^*}}{\mathcal{M}^*} > \beta R^* \frac{1-\lambda}{\lambda} \frac{R^*}{\underline{R}}$, which is true in the liquidity trap under assumption 3 since $R^* \leq \min\{\underline{R}, 1\}$; furthermore, $\lim_{R^* \rightarrow 0} LHS = +\infty$ and $\lim_{R^* \rightarrow \underline{R}} LHS = \lambda + (1 - \lambda)\underline{R}$.

From (21) and the definition of ξ ,

$$\frac{Y^{*,b}}{Y} = \xi^{-1} \left(\frac{K^{*,b}}{K} \right)^{\alpha \frac{1+\gamma}{\gamma}}$$

As $\xi \in (0, 1)$, this completes the proof. ■

Appendix B Local dynamic in the liquidity trap

We've already studied the global dynamic of the model in the Neoclassical regime. Now, let us study the local (fundamental) dynamic around the liquidity trap steady state and show that it is indeterminate. Let $x_t \equiv \frac{\xi_t - \xi}{\xi}$ and $k_t \equiv \frac{K_t - K}{K}$. Log-linearizing (28), (29) and (31) and substituting to express the dynamic as a function of x_t and k_t only,

$$\begin{bmatrix} k_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha \frac{1+\gamma}{\gamma} \Gamma - \Upsilon & \frac{\gamma}{\alpha} \left(\alpha \frac{1+\gamma}{\gamma} \Gamma - \Upsilon \right) \\ \frac{\alpha}{\gamma} \left(1 - \alpha \frac{1+\gamma}{\gamma} \right) & 1 - \alpha \frac{1+\gamma}{\gamma} \end{bmatrix} \begin{bmatrix} k_t \\ x_t \end{bmatrix}$$

Where $\Gamma \equiv \frac{1}{1+\gamma} \frac{1-\alpha}{\mathcal{M}^*} \xi^\gamma < 1$ and $\Upsilon \equiv \frac{\alpha}{\beta \frac{\alpha}{\mathcal{M}^*} \xi^\gamma + 1 - \frac{1-\alpha}{\mathcal{M}^*} \xi^\gamma} \in (0, 1)$. One root will be equal to $\lambda_k = 0$ and the other to $\lambda_x = (1 - \Upsilon) - \alpha \frac{1+\gamma}{\gamma} (1 - \Gamma)$. $\lambda_x > 1$ is equivalent to $0 > \Upsilon + \alpha \frac{1+\gamma}{\gamma} (1 - \Gamma)$ which is never true. Therefore the dynamic is determinate if and only if $\lambda_x < -1$ or $1 - \Upsilon < \alpha \frac{1+\gamma}{\gamma} (1 - \Gamma) - 1$. But since $\frac{1}{\mathcal{M}^*} \xi^\gamma < 1$, $\alpha \frac{1+\gamma}{\gamma} (1 - \Gamma) - 1 < 0 < 1 - \Upsilon$. Thus, $\lambda_x \in (-1, 1)$.