



HAL
open science

Trade Liberalisation and Optimal R&D Policies in a Model of Exporting Firms Conducting Process Innovation

Thanh Le, Cuong Le Van

► **To cite this version:**

Thanh Le, Cuong Le Van. Trade Liberalisation and Optimal R&D Policies in a Model of Exporting Firms Conducting Process Innovation. 2016. halshs-01314650

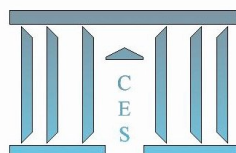
HAL Id: halshs-01314650

<https://shs.hal.science/halshs-01314650v1>

Submitted on 11 May 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



**Trade Liberalisation and Optimal R&D Policies in a
Model of Exporting Firms Conducting Process Innovation**

Thanh LE, Cuong LE VAN

2016.28



Trade Liberalisation and Optimal R&D Policies in a Model of Exporting Firms Conducting Process Innovation

Thanh Le^a, Cuong Le Van^b

8th March 2016

^a University of Queensland and Flinders University. Corresponding author.
Tel: +61 7 33467053. Fax: + 61 7 33657299. Email: t.le2@uq.edu.au.

^b IPAG Business School, Paris School of Economics, French National Centre for Scientific Research, and Vietnam Centre for Research in Economics, Management and Environment

Abstract

This paper discusses the impact of trade liberalisation and R&D policies on exporting firms' incentive to innovate and social welfare. Key factors determining the government's optimal policy are the strength of R&D spillover effect and the toughness of firm competition. When firms only compete in an overseas market, the optimal policy is to tax R&D. Trade liberalisation in the overseas market induces a higher R&D tax rate to be imposed on firms. When firms also conduct business in the home market, the government should financially support firms' R&D. Trade liberalisation always increases firms' output sales, R&D investments, and social welfare.

Keywords: Trade, R&D spillovers, subsidies, welfare, process innovation.

JEL classification: F12, F13, F15, O31.

1 Introduction

Over the last few decades, there has been an increasing number of countries that adopt export promoting trade strategy for their economic development path. Following this strategy, firms are encouraged by their national governments to export their products to an overseas market. Expected gains of this trade policy are many of which the most visible ones include generating foreign exchange revenue, increasing employment and improving production efficiency. However, the successful implementation of this policy is not always compelling. On the one hand, it depends on external factors such as demand and regulations in the international market. On the other hand, it is subject to the competitiveness of exporting firms. Hence, in order to survive and develop in such a competitive market place, firms need to improve their productivity and in that process, innovation is essential. From a policy standpoint, a government can support their domestic exporting firms by providing them with either export or R&D subsidies. However, as export subsidies are often restricted due to international agreements, providing subsidies to firms' R&D activities become the most effective policy tool of any national governments nowadays. Several studies such as Spencer and Brander (1983), Bagwell and Staiger (1994), Brander (1995), Neary and Leahy (2000), and Leahy and Neary (2001) even find that subsidising R&D is more powerful than subsidising exports.

Clearly, trade liberalisation and R&D policies are closely related. While trade liberalisation affects factors impacting innovation activities such as market size and toughness of competition, R&D investment determines the benefits of undertaking the trade. It is surprising that not much has been done to examine the links between these two policy factors although there exists rich branches of literature studying each factor separately. Filling this gap will be the main task of this paper. In doing so, this paper considers the issue of exporting duopoly¹ in a basic model of strategic R&D with trade liberalisation occurring in an exporting market.² Here, firms produce hori-

¹This market structure fits well with some special industries such as civil aviation. In these industries, due to technical or safety requirements, they are highly regulated by the government. In addition, in order to start production in these industries, it may require a substantial amount of initial investment. All these create barriers deterring firms from entering the market.

²Although it is arguable that exports and imports are highly connected, for the purpose of focusing on public policies towards supporting exporting firms, this paper does not

zontally differentiated products and invest in R&D to reduce their marginal cost of production. Unlike existing studies, in this paper, R&D investment has a special feature that it benefits both its own investor and other firms (through an R&D spillover process). Government policies include providing a subsidy to the exporting firms to stimulate their R&D activity. However, it should be noted that the main aim of the government policies is not only to expand firms' output sales (in the overseas and/or home market) but also to maximise domestic welfare. This environment creates a two-stage game which can be solved by backward induction. In the first stage, the government decides on how much to subsidise R&D activity of firms in order to maximise domestic welfare. In the second stage, firms maximise their profits by choosing export volumes and/or domestic sales as well as levels of R&D investment optimally taking into account the subsidy rate provided by the government and the other firm's action. The result at the end of the second stage is a Cournot-Nash equilibrium. Depending on the setting environment, the strategic behaviours of the government and firms are different and convey different implications for the optimal R&D subsidy. However, overall, common findings are that trade liberalisation is always welfare enhancing as it helps firms further expand their output sales, both overseas and at home. Trade liberalisation encourages firms to undertake more cost-reducing R&D by enlarging their profit margins. This, in turn, improves firms' and industry productivity.

The first results are developed in a simple setting with two domestic exporting firms competing in an overseas niche market. For simplicity, foreign firms are assumed either non-existent or too small to count on (i.e. they hold a negligible market share or operate in a completely different market segment).³ A typical example is the rivalry between two Australian airlines - Qantas and Virgin Australia (in partnership with Delta Air Lines) - in offering direct flights to American tourists from Los Angeles to Sydney.⁴

consider imports.

³This simplifying assumption allows us to focus better on the strategic behaviour of the exporting firms.

⁴According to Freed (2015a), these are the two dominant carriers in the US - Australia route, with Qantas holding of 54 per cent and Virgin (combined with Delta) having 25 per cent of market share respectively. In June 2015, American Airlines, currently having code-sharing arrangement with Qantas, announced that it would re-enter Australia (after its withdrawal in 1992) through forming a joint venture with the Australian biggest airline (Airline Leader, 2015).

The result indicates that factors that shape the government's policy action are the strength of R&D spillover effect (a social benefit) and the degree of firm rivalry (a social cost). This result is new since existing studies in this related literature are silent about R&D spillover effect. Under some certain conditions involving these two factors, the government's optimal policy turns out to be taxing firms' R&D activity instead of subsidising it. This is because too much competition between domestic firms in the foreign market will erode the power that the home country as a whole can exercise in the foreign market. This optimal R&D tax increases when trade liberalisation in the foreign market occurs. This trade liberalisation is also found to induce a higher level of R&D investments of firms, their productivity and export sales, and social welfare.

Results on optimal R&D subsidy turn out to be significantly different when exporting firms also conduct business at home.⁵ The first-best policy is always to subsidise R&D of firms. This is due to consumer-surplus motive of subsidising R&D as domestic consumers will gain very much from having access to different varieties. Trade liberalisation implemented by the foreign market does not always induce a higher optimal R&D subsidy level because the extra gain from undertaking further R&D may be smaller than its additional cost. In fact, the monotonicity of this R&D subsidy in the trade cost depends on the comparison between the R&D spillover effect and the degree of substitutability between the goods, and to some extent, on the features of the R&D marginal cost. If R&D investment is not so costly, the marginal benefit from doing R&D is greater than its cost so the optimal R&D subsidy increases. By contrast, if R&D investment is an extremely costly activity, the optimal R&D policy should discourage R&D investment through cutting down the level of R&D subsidy.

In characterising R&D subsidies, a majority of existing studies (e.g. Brander, 1995; Neary and Leahy, 2000; Leahy and Neary, 2001) only focus on business-stealing motive and pay little attention to the welfare motive of R&D subsidisation. This is because they do not consider any welfare analysis. Collie (2002) is among a few exceptions looking at welfare effect of subsidies but it addresses production subsidies rather than R&D subsidies. Spencer and Brander (1983) and Haaland and Kind (2008) are studies most closely related

⁵Here, the example of the rivalry between Qantas and Virgin Australia can also be applied as they are also main competitors in the Australian domestic civil aviation market (Freed, 2015b).

to our paper in terms of studying R&D subsidisation. However, they only restrict their attention to competition between a home firm and a foreign firm rather than that of two exporting firms as presented in our paper. Long et al. (2011), while studying the impact of trade liberalisation on R&D, do not consider the subsidisation issue. Similar to Neary and O’Sullivan (1999) and Leahy and Neary (2004), that paper looks at R&D cooperation/competition between firms rather than R&D coordination by the government at the policy stage. To some extent, our paper is also related to Long and Stahler (2009) in terms of considering strategic behaviour of firms under different scenarios. Nevertheless, their paper focuses on firm ownership and trade policy, not R&D investment and trade policy as our paper does. Our paper also considers the impact of R&D investment externality, an issue that has not been fully explored in the R&D-trade related literature.

The rest of the paper is structured as follows. Section 2 introduces a basic model of competition between exporting firms in an overseas market. In Section 3, exporting firms are additionally allowed to trade in their home market. For each case, the existence as well as uniqueness of an optimal R&D subsidy and its key characteristics is analysed. The impacts of trade liberalisation on firms’ output sales, their cost-reducing R&D investments and productivity, and social welfare are also examined. Section 4 ends the paper with some concluding remarks.

2 The model

Consider two domestic firms i and j whose products are entirely exported to a foreign country that does not produce these goods.⁶ ⁷The utility function of an overseas representative consumer is:

$$u = \alpha q_i + \alpha q_j - \left(\frac{q_i^2}{2} + \frac{q_j^2}{2} + bq_i q_j \right), \quad b \in [0, 1), \alpha > 0 \quad (1)$$

⁶In this paper, the exporting country is referred to as the home country.

⁷This modelling assumption also fits well with the case of firms operating in export processing zones (EPZs) where all of the firm’s products are to be sold in a foreign market. Sargent and Matthew (2009), in citing statistics provided by The International Labour Organisation, indicate that by 2002, there had been 116 countries establishing EPZs to promote their exports. China is often considered as a successful country in this policy direction. For a more detailed review of EPZs around the world and the China’s EPZ success, see Sargent and Matthews (2009).

where q_i and q_j are consumption of the goods produced by the two firms respectively; b denotes the degree of substitution between the two goods (the higher the value of b , the higher the degree of substitutability). When $b = 0$, the goods are completely independent and when b tends to its limit of 1, the goods are identical. This quadratic utility function is standard and has been used by Haaland and Kind (2008). For simplicity, assume the population size in the foreign market is equal to 1.

Let p_i and p_j denote the prices of the two goods in the foreign country. The consumer surplus of the foreign country can be expressed as:

$$CS = u - p_i q_i - p_j q_j$$

As the consumer maximises his surplus with respect to the quantity of each good, the (inverse) demand function for good i (and similar for good j) can be derived as the following:

$$p_i = \alpha - (q_i + bq_j)$$

Assume that the firms' products are subject to a trade cost (e.g. import tariff, transportation or service cost) of rate τ per unit of goods they export to the foreign market ($\tau > 0$). By *trade liberalisation* in the overseas market, it is meant an exogenous *fall* in τ . In the absence of R&D, firms face a same unit cost of production, c . These imply that in order to sell their products in the foreign market, firms have to bear the exporting cost of $c + \tau$. To allow firms to be able to export even when no R&D activity is conducted, assume that

Assumption 1 $c + \tau < \alpha$

Firms invest in R&D to reduce their cost of production so that the cost of production after R&D is $c - x_k$ where x_k ($c \geq x_k \geq 0$, $k = i, j$) is the amount of R&D effort expended by firms. The R&D cost function $r(x_k)$ takes the standard form with the following assumptions:

Assumption 2 *The R&D cost function $r(x_k)$:*

- *is positively valued: $r(x_k) > 0$, $\forall x_k \geq 0$;*
- *is strictly increasing: $r'(x_k) > 0$, $\forall x_k > 0$; $r'(0) = 0$; and*
- *is strictly convex with curvature $r''(x_k) > \max \left[1; \frac{(b+5)(\lambda+1)^2}{(b+2)^2} \right]$, $\forall x_k > 0$.*

These assumptions, as will be shown later in the Appendix, are necessary for fulfilling sufficient conditions of maximisation problems.⁸ Based on these assumptions, the R&D cost function is positively valued, strictly increasing and strictly convex in the level of R&D effort conducted by firms.

Also assume that the government helps each firm by providing an R&D subsidy of rate s_k ($k = i, j$) per unit of R&D investment. Hence, the profit function for firm i (and similar for firm j) is:

$$\pi_i = [p_i - (c - x_i - \lambda x_j) - \tau] q_i - r(x_i) + s_i x_i \quad (2)$$

where $\lambda \in [0, 1]$ captures the degree of R&D spillovers between firms (when $\lambda = 0$, there are no spillovers and when $\lambda = 1$, there are perfect spillovers). An assumption that is maintained throughout this paper is that firms obtain non-negative profits when they enter the production stage of the market. Each firm will maximise its profit while the domestic government will maximise total welfare. Because all goods are exported and not consumed in domestic market, domestic consumer surplus is zero. Hence, total welfare is equal to total firms' profits less R&D subsidy costs:

$$W = \sum_{k=i,j} \pi_k - \sum_{k=i,j} s_k x_k \quad (3)$$

In this paper, we follow Long et al. (2011) in using Merlitz (2003)'s definition of productivity. Here, firm i 's productivity (and similar for firm j), z_i , is the inverse of its marginal production cost:

$$z_i = \frac{1}{c - x_i - \lambda x_j}, \quad (4)$$

and the industry productivity, Z , is the inverse of the average marginal production cost of that industry:

$$Z = \frac{2}{(c - x_i - \lambda x_j) + (c - x_j - \lambda x_i)} \quad (5)$$

The above setting provides us with a two-stage game. In the first stage, the government chooses how much to subsidise firms' R&D efforts to maximise social welfare. In the second stage, the firms choose the R&D investment levels and export volumes to maximise their corresponding profits taking into account the R&D subsidy rates given in the first stage.⁹ We will solve this

⁸A typical example of such an R&D cost function is $r(x_k) = Ax_k^2 + f$ where $f \geq 0$ is the fixed cost for setting up an R&D project and $A > \frac{5}{2}$ is a constant.

⁹Generally speaking, firms need to set up R&D projects first before conducting any production. However, for simplicity, in this paper, we assume that firms make these two decisions at the same time.

game using backward induction.

Lemma 1 *Any solution equilibrium is symmetric. Consider a symmetric equilibrium where $s_i = s_j = s$, $q_i = q_j = q$, $x_i = x_j = x$. If it is interior,¹⁰ then*

$$s = \frac{(2\lambda - b)q}{b + 2} \quad (6)$$

Proof. See Appendix.

This result indicates that in this symmetric equilibrium, firms receive a same amount of subsidy from the government, undertake a same amount of R&D investment, and exports a same quantity of goods to the foreign market. It also shows the relationship between the R&D subsidy and the export volume. Although the government subsidises the firms' R&D investment, this indirectly affects the quantity of goods that firms want to sell in the foreign market.

Remark 1 *The relationship between λ and s is the following:*

- If $\lambda = \frac{b}{2}$ then $s = 0$.
- If $\lambda < \frac{b}{2}$ then $s < 0$.
- If $\lambda > \frac{b}{2}$ then $s > 0$.

Clearly, if $\lambda = \frac{b}{2}$, the right hand side (RHS) of (6) is equal to zero so its left hand side (LHS) must be equal to zero as well. In other words $s = 0$. Similarly, when $\lambda < \frac{b}{2}$ we have $s < 0$ and when $\lambda > \frac{b}{2}$, we have $s > 0$.

This is an important result that links the externality of R&D investment, λ , with the degree of substitutability between the goods, b . From the whole society's point of view, λ represents the social benefit of undertaking R&D because R&D is not only good for its own investor but also others in the market. By contrast, b is somewhat a social cost to the exporting country as it reflects the rivalry between the exporting firms in the overseas market.¹¹ When $\lambda = \frac{b}{2}$, the social benefit of undertaking R&D investment cancels out

¹⁰Only in Proposition 1 below, under some further assumptions, the solution is actually interior.

¹¹According to Haaland and Kind (2008), an increase in b implies a decrease in market demand. In other words, the size of the market gets smaller when goods become less differentiated.

with the corresponding social cost so the government has no incentive to finance firms' R&D activity.

A special case that satisfies the condition $\lambda = \frac{b}{2}$ is when $\lambda = 0$ and $b = 0$. In this case, there will be no R&D spillovers between firms and goods are absolutely different (firms are independent monopolies in their product lines and facing no competition from each other). Hence, we can state the following:

Corollary 1 *When exporting firms are independent monopolies in their own market product lines (i.e. $b = 0$) and there are no R&D spillovers between them (i.e. $\lambda = 0$), the optimal policy action for the government is to provide no subsidy to the firms (i.e. $s = 0$).*

The normal wisdom is that due to absence of competition ($b = 0$), there is no need for the government to help the firms further exploit their monopoly power in the overseas market. That is true but not enough since Proposition 1 points out that, additionally, there must be no R&D spillovers between the firms. Even when firms are monopolies but if there are R&D investment spillovers, the social benefit of undertaking R&D is high (firms benefit from each other's R&D investment implementation), the government has an incentive to support the firms because this action is welfare enhancing. However, if there is no R&D investment externality, the government is willing to leave the firms untouched. In this case, each firm's marginal export revenue and its marginal R&D spending cost cancel out each other. Any firm's extra profit will be equal to the value of R&D subsidy it receives from the government. Consequently, the government cannot use R&D subsidy to increase the exporting firms' profit net of R&D subsidy cost for the welfare. This indicates that the optimal policy for the government is to withhold any R&D subsidy to the firms.

Remark 2 *Results obtained in Remark 1 and Corollary 1 can be generalised to the case of $N \geq 2$ exporting firms that are Cournot rivals in an overseas market.*

From the lemma above, we obtain results that can be summarised in the proposition below:

Proposition 1 *When exporting firms only compete in an overseas market and $\lambda \neq \frac{b}{2}$, if additionally $r' \left[\frac{c}{\lambda+1} \right] > \frac{2(\lambda+1)(\alpha-\tau)}{(b+2)^2}$, then:*

- *The social optimum can be achieved as an interior Nash equilibrium with the government taking action towards firms' R&D activities:*

$$q = \frac{(b+2)s}{2\lambda-b}$$

$$x = \frac{(b+2)^2 s}{(2\lambda-b)(\lambda+1)} - \frac{(\alpha-c-\tau)}{\lambda+1}$$

- *If $\lambda > \frac{b}{2}$, it is optimal to subsidise firms' R&D investment. Otherwise, an optimal R&D tax is required.*
- *Trade liberalisation in the foreign market induces a higher level of optimal R&D subsidy provided (optimal R&D tax imposed) if there has been such a subsidy (tax) in place.*

Proof. See Appendix.

As shown in the Appendix, the condition $r' \left[\frac{c}{\lambda+1} \right] > \frac{2(\lambda+1)(\alpha-\tau)}{(b+2)^2}$ is required for obtaining a unique interior optimal solution to the welfare maximisation problem. This condition means that the marginal cost of R&D investment once reaching its upper limit is so large that it outweighs all marginal benefit of this activity. Under this condition, there are two important results. The first result is quite interesting. The socially optimal policy turns out to be that the government may need to tax firms' R&D activity instead of subsidising it. This can be explained on the following ground. When firms conduct R&D and then compete with each other in a foreign market, there are two important factors affecting welfare of the entire economy. While the R&D spillovers effect (captured by λ), a positive externality, enhances domestic welfare, the rivalry of firms (reflected through b), a negative externality, reduces it. In particular, when the R&D spillover intensity is relatively small as compared to the degree of competition between firms ($\lambda < \frac{b}{2}$), the competition of firms result in a net effect in which the home country as a whole fails to fully exploit its potential monopoly power in that foreign market. Too much R&D conducted will lead to the situation of over-production for the two domestic exporting firms. To avoid this situation, the home government should impose an R&D tax, at the same rate, on both firms. This optimal R&D tax guarantees that social welfare will be maximised and firms will have no incentive to do less or more R&D and, hence, to produce less or more exported products. By contrast, when $\lambda > \frac{b}{2}$, the benefit of increasing

R&D is greater than its cost, providing an R&D subsidy is the optimal policy action that the government should pursue.

The second result says that when there is a reduction in the trade cost, the optimal action of the home government is to tax the firms' R&D investments more heavily if there is already a tax or to provide the firms with more financial support if there is already a subsidy in place. This is because lower trade cost expands firms' export volumes and thus raises firms' willingness to invest in cost-reducing R&D. If the social benefits of conducting more R&D is larger than its associated social costs (through fiercer firms' competition), a reduction in the trade cost induces a higher level of optimal R&D subsidy. However, in case an R&D tax is needed, to reduce firms' excessive R&D spending so that over-production, which erodes the home country's monopoly power in the foreign market, can be avoided, the government needs to raise the R&D tax rate. This action will result in an improvement in social welfare because (i) when there is an R&D tax and a higher tax rate is imposed, firms obtain more profits from exports (even though no more R&D investments occur) and the government collects more R&D tax revenues; and (ii) when there is an R&D subsidy, the extra profits obtained by the firms exceed the R&D subsidy costs expended by the government.

We now examine the economic impact of trade liberalisation on the home country. To derive the comparative static effects of a reduction in τ , we differentiate the obtained equilibrium conditions with respect to τ . The results can be summarised in the proposition below:

Proposition 2 *When exporting firms only compete in a foreign market and assuming $r'(\frac{c}{\lambda+1}) > \frac{2(\lambda+1)(\alpha-\tau)}{(b+2)^2}$, at the optimal policy action conducted by the government, trade liberalisation in the foreign market raises firms' cost-reducing R&D spending, their productivity and the industry productivity. It also enhances domestic welfare.*

Proof. See Appendix.

Again, the condition $r'[\frac{c}{\lambda+1}] > \frac{2(\lambda+1)(\alpha-\tau)}{(b+2)^2}$ is imposed to guarantee the existence of a unique interior solution to the government's welfare maximising problem. Under this condition, the results obtained can be explained as follows. Basically, trade liberalisation in the foreign market entails two different effects: a *direct effect* and an *indirect effect*. The direct effect of a fall in the trade cost, as explained under Proposition 1, encourages firms to conduct more cost-reducing R&D. By contrast, the indirect effect influence firms' R&D efforts through changing the optimal R&D policy instrument.

In case of an optimal R&D subsidy, the two effects complement each other. However, in case of an optimal R&D tax, although the two effects work in opposite directions, the direct effect dominates the indirect one resulting in a net positive effect of an increase in R&D investments for the firms. Hence, there will be an improvement in firms' and industry's productivity as well as export volumes (because the whole exporting cost is lower). This sale expansion allows firms to enjoy higher profits and the increment in profits is more than required to offset for the increase in government's subsidy expenditure. In the case of tax, the government gets more revenue through its higher R&D taxation program. All this leads to a higher level of domestic welfare.

3 Adding domestic sales

In the previous section, firms are assumed to sell all of their products overseas. Because there is no domestic consumption of firms' product, only firms' export sales and government expenditure/revenue matter for the social welfare. If this assumption is relaxed, i.e. if firms are allowed to sell their products in the home market, the strategic behaviours of firms and the home government are expected to change significantly. This is because firms will now weigh out between selling products at home and overseas. In addition, the government will now need to take into account consumer surplus in calculating the social welfare. To examine this interesting case, we slightly restructure our model below.

In addition to the competition in the foreign market as described in Section 2, we now further assume that competition between two exporting firms also takes place in the home market.¹² As there are now two markets, we need to make some small changes in notation. Define the home market as Country 1 and the foreign market as Country 2. Assume the population size in each country is equal to 1 and consumers everywhere have the same preferences for simplicity. The representative consumer in home country derives utility from consuming goods supplied by the firms:

$$u_1 = \alpha q_{i1} + \alpha q_{j1} - \left(\frac{q_{i1}^2}{2} + \frac{q_{j1}^2}{2} + bq_{i1}q_{j1} \right), b \in [0, 1), \alpha > 0 \quad (7)$$

¹²The setting in this section not only accommodates well the competition between Qantas and Virgin Australia as documented in Section 1 but also reflects a recent move of several countries in additionally allowing EPZ firms to sell a certain fraction of their products to their corresponding domestic markets.

and similar for the consumer in the foreign country. Here, q_{i1} and q_{j1} denote the consumption of goods produced by the firms. The first subscript is used to indicate the firm producing the consumption good and the second subscript refers to the country of consumption. The domestic consumer surplus is:

$$CS_1 = u_1 - p_{i1}q_{i1} - p_{j1}q_{j1}$$

From this, the inverse demand function for firm i 's product (and similar for firm j 's product) is:

$$p_{i1} = \alpha - (q_{i1} + bq_{j1})$$

Using these results, the maximised domestic consumer surplus can be calculated as:

$$CS_1 = \frac{1}{2} (q_{i1}^2 + q_{j1}^2) + bq_{i1}q_{j1}$$

The inverse demand functions for goods in the overseas market are the same as previously described in Section 2. Hence, the profit function for firm i is:

$$\pi_i = [p_{i1} - (c - x_i - \lambda x_j)] q_{i1} + [p_{i2} - (c - x_i - \lambda x_j) - \tau] q_{i2} - r(x_i) + s_i x_i \quad (8)$$

and similar for firm j . In this profit function, the first two terms capture the firm's domestic sales revenue and export sales revenue respectively while the last two terms are R&D investment spending and financial support from the government.

Welfare of the home country will be:

$$W = \pi_i + \pi_j + CS_1 - s_i x_i - s_j x_j \quad (9)$$

A slight difference between this welfare function and the one defined in Section 2 is the inclusion of consumer surplus. Any R&D policies should now also take this component into account.

Lemma 2 *Any solution equilibrium is symmetric. Assuming interior solution, a symmetric equilibrium outcome implies that $s_i = s_j = s$, $q_{i1} = q_{j1} = q_1$, $q_{i2} = q_{j2} = q_2$, and $x_i = x_j = x$ where:*

$$q_1 = \frac{(b+2)s}{(b+5)\lambda + 1 - b} + \frac{(2\lambda - b)\tau}{(b+2)[(b+5)\lambda + 1 - b]} \quad (10)$$

$$q_2 = \frac{(b+2)s}{(b+5)\lambda + 1 - b} - \frac{[(b+3)\lambda + 1]\tau}{(b+2)[(b+5)\lambda + 1 - b]} \quad (11)$$

$$x = \frac{(b+2)^2 s}{(\lambda+1)[(b+5)\lambda+1-b]} + \frac{(2\lambda-b)\tau}{(\lambda+1)[(b+5)\lambda+1-b]} - \frac{\alpha-c}{\lambda+1} \quad (12)$$

$$[(b+5)\lambda+1-b](\alpha-c) - (2\lambda-b)\tau < s < \frac{[(b+5)\lambda+1-b]\alpha - (2\lambda-b)\tau}{(b+2)^2} \quad (13)$$

Proof. See Appendix.

This lemma provides us with interior equilibrium levels of domestic sale, export sale and R&D investment of the firms. It also spells out the condition on the equilibrium R&D subsidy following which a unique optimal solution to the firms' maximisation problem is obtained. Given this setting and conditions, we can derive the following:

Proposition 3 *When firms compete in both home and foreign markets, if $r'(\frac{c}{\lambda+1}) > \frac{(\lambda+1)[(b+5)\alpha-2\tau]}{(b+2)^2}$, then:*

- *The welfare maximising R&D subsidy expended by the government to each firm exists and is positively valued and uniquely determined.*
- *Trade liberalisation in the foreign market induces an increase in this optimal R&D subsidy level only if $\lambda \geq \frac{b}{2}$ or $\lambda < \frac{b}{2}$ and $\frac{(b+5)(\lambda+1)^2}{(b+2)^2} < r''(x) < \frac{(b+1)(\lambda+1)^2}{(b-2\lambda)(b+2)}$.*

Proof. See Appendix.

It should be noted that, similar to the condition required for the case of no domestic sale (under Proposition 1), it is necessary that $r'(\frac{c}{\lambda+1}) > \frac{(\lambda+1)[(b+5)\alpha-2\tau]}{(b+2)^2}$. This means that marginal cost of R&D investment evaluating at the upper threshold must be higher than its marginal benefit so that there is more incentive for firms to undertake further R&D. Otherwise, the production cost would fall below zero (not a sensible scenario). The results obtained deserved some comments. Unlike the results obtained under Proposition 1 where an R&D tax might be imposed, when firms also trade in the home market, the government's optimal policy is always to subsidise R&D. This is very much because of the consumer surplus motive. In this case, the gain in consumer surplus due to R&D subsidy, which lowers the product prices by lowering firms' marginal production cost, is more than sufficient to compensate for the associated costs (incurred through R&D subsidy expenditure) so the government has an incentive to grant R&D subsidy to the firms.

Another difference is that the effect of trade liberalization in the foreign market on optimal R&D subsidy, to some extent, is also dependent on the curvature of the R&D cost function.¹³ When the intensity of R&D spillovers is relatively large as compared to the degree of substitutability of goods ($\lambda > \frac{b}{2}$), an improvement in terms of trade cost always encourages the government to subsidize more firms' R&D investment. By contrast, when the intensity of R&D spillovers is not so large relatively to the degree of substitutability of goods, whether trade liberalisation increases or decreases the subsidy rate depends on the curvature of the R&D cost function. As we know, when trade liberalisation occurs, firms enjoy more profits even if R&D spending is held fixed. If the R&D cost function is highly convex (R&D investment is a very costly activity), holding R&D investments fixed or even a slight decrease in R&D efforts will allow firms to save a great deal of R&D spending. In terms of welfare, the society will be better off if firms do not change or conduct less R&D because the savings (of R&D spending and R&D subsidy) obtained from doing so more than outweighs any reduction in firms' profits and/or consumer surplus. To discourage firms from doing any further R&D, the government reduces its R&D subsidy extended to firms. However, when the R&D cost function is not so convex, the marginal benefit from implementing an R&D project is greater than its corresponding cost, the government should encourage firms to do more R&D by increasing the R&D subsidy level in the face of trade liberalisation.

As for the impacts of trade liberalisation on the home economy, we can show that:

Proposition 4 *When firms compete in both home and foreign markets and $r' \left(\frac{c}{\lambda+1} \right) > \frac{(\lambda+1)[(b+5)\alpha-2\tau]}{(b+2)^2}$, at the optimal R&D subsidy, trade liberalisation in the foreign market: (i) increases a firm's R&D spending; (ii) increases the firm's export volumes, its domestic sales and, hence, its total sales; (iii) improves the firm's and industry productivity; and (iv) raises social welfare.*

Proof. See Appendix.

Similarly, the condition $r' \left(\frac{c}{\lambda+1} \right) > \frac{(\lambda+1)[(b+5)\alpha-2\tau]}{(b+2)^2}$ is required here for having an optimal R&D subsidy. The results that trade liberalisation in the overseas market induces higher R&D spending of firms and, hence, lead to the improvement of their productivity as well as the industry productivity

¹³Once firms are allowed to make their domestic sale, the R&D cost function matters as it affects the price perceived by the consumer (and, hence, the consumer surplus).

are, in general, similar to the case of no domestic sales investigated under Proposition 2. Trade liberalisation in the export market is not only welcome by exporting firms as they can expand their output but also by their host country. This is because it makes the domestic economy as a whole become more efficient and reap more welfare.

4 Conclusion

In this paper we have considered different scenarios of exporting firm competition to explore the effect of trade liberalisation in the foreign market and R&D policy on firms' incentive to innovate and social welfare. In particular, we study in details the international setting in which firms invest in R&D and sell their differentiated products in a foreign market. Here, R&D investment contains a positive externality. The government uses R&D subsidy as a policy tool to maximise the social welfare. We show that the magnitude of the R&D externality is an important factor, alongside the degree of substitutability between goods, that shapes the government's optimal policy behaviour. In particular, under some certain conditions involving these two factors, it might be optimal for the government to tax R&D instead of subsidising it. With this R&D tax put in place, trade liberalisation in the foreign market induces the government to tax R&D more heavily as this policy response improves the domestic welfare.

In the next step, we examine if there are any changes in the results when firms also sell their products in the home market. It is found that the optimal policy for the government in this case is always to provide financial support to firms' R&D activity (positive R&D subsidy). The impact of trade liberalisation on this optimal subsidy depends on the comparison between the R&D spillover effect and the degree of substitutability between goods and, to some extent, on the convexity of the R&D cost function.

Although the settings explored change from foreign market to both home and foreign markets, all in all, we find that trade liberalisation in the overseas market is always welfare enhancing as it induces higher output sales, both at home and overseas, of firms. It also entails a higher level of cost-reducing R&D spending which then leads to an improvement of firms' and industry productivity.

Overall, the results of our model are broadly in line with the literature stressing the complementarity between innovation and export: firms are more

likely to export if they innovate and are more likely to innovate if they find good export opportunities (e.g. Lileeva and Treffer, 2010; Bustos, 2011). Although the attention in this paper is restricted to the competition of only two firms, the model can easily be extended to a multiple firm setting. With regards to future research, it would be interesting to investigate the case in which firms are to form R&D joint ventures to strengthen their collective competitiveness and export volumes. Another direction is to ask the question of to what extent empirical evidence confirms the theoretical predictions obtained in this paper.

Appendix

Proof of Lemma 1

Conditional on the government's decision made regarding R&D subsidies in the first stage, each firm chooses how much to invest in R&D and how much to export to maximise its profit defined in (2). Observe that $\{x_i, x_j, q_i, q_j\}$ must satisfy

$$\begin{aligned} x_i &\geq 0, q_i \geq 0, x_i + \lambda x_j \leq c, q_i + bq_j \leq \alpha \\ x_j &\geq 0, q_j \geq 0, x_j + \lambda x_i \leq c, q_j + bq_i \leq \alpha \end{aligned}$$

Hence $\{x_i, x_j, q_i, q_j\}$ belongs to a compact set. The maximization problem has a solution which is symmetric and can be written as

$$x_i = X(s_i, s_j), x_j = X(s_j, s_i), q_i = Q(s_i, s_j), q_j = Q(s_j, s_i)$$

The total welfare can be written as

$$W = \Pi(s_i, s_j) + \Pi(s_j, s_i)$$

The problem $\max\{W : (s_j, s_j)\}$ will yield a symmetric solution $s_i = s_j$.

When the solution is interior, the first order necessary conditions for firm i 's profit maximisation problem give:

$$(\alpha - c - \tau) + x_i + \lambda x_j - bq_j - 2q_i = 0 \quad (14)$$

$$q_i + s_i - r'(x_i) = 0 \quad (15)$$

and similar for firm j . The Hessian matrix of the second order sufficient conditions for firm i is $H = \begin{pmatrix} -2 & 1 \\ 1 & -r''(x_i) \end{pmatrix}$ and similar for firm j . It

can be seen that $|H_1| = -2 < 0$ and $|H_2| = 2r''(x_i) - 1 > 0$ according to Assumption 1. Hence, the second order sufficient conditions are satisfied for a maximum.

In the first stage, the government, having known the firms' strategic response functions in (14) and (15), chooses R&D subsidy rates (s_i, s_j) to grant to firms in order to maximise the social welfare defined in (3) which can now be rewritten as:

$$W = q_i^2 - r(x_i) + q_j^2 - r(x_j)$$

Setting $\frac{\partial W}{\partial s_i} = 0$ and $\frac{\partial W}{\partial s_j} = 0$ yields the following:

$$2q_i \cdot \frac{\partial q_i}{\partial s_i} - r'(x_i) \cdot \frac{\partial x_i}{\partial s_i} + 2q_j \cdot \frac{\partial q_j}{\partial s_i} - r'(x_j) \cdot \frac{\partial x_j}{\partial s_i} = 0$$

$$2q_i \cdot \frac{\partial q_i}{\partial s_j} - r'(x_i) \cdot \frac{\partial x_i}{\partial s_j} + 2q_j \cdot \frac{\partial q_j}{\partial s_j} - r'(x_j) \cdot \frac{\partial x_j}{\partial s_j} = 0$$

where q_i and x_i (and, similarly, q_j and x_j) are given in (14) and (15). It can be seen that the first order conditions yield a symmetric outcome at which $s_i = s_j = s$, $q_i = q_j = q$, and $x_i = x_j = x$.

From (14) and (15), the following is obtained:

$$q = \frac{\alpha - c - \tau + (\lambda + 1)x}{b + 2}$$

Using this result to recalculate the social welfare we have:

$$W = 2 \left[\left(\frac{\alpha - c - \tau + (\lambda + 1)x}{b + 2} \right)^2 - r(x) \right]$$

In what follows, we assume interior solutions for both profit maximisation of firms and welfare maximisation of the government. After differentiating the above welfare function with respect to s , setting it to zero and using (14) and (15), we obtain:

$$s = \frac{(2\lambda - b)q}{b + 2}$$

Proof of Proposition 1

We will prove this proposition in two parts. In the first part, we prove the existence of a unique value of s . We then indicate that s can either be positive (i.e. an optimal subsidy) or negative (i.e. an optimal tax) depending

on values of relevant parameters. In the last part, we examine the comparative statics on this policy variable with regard to a decrease in τ (trade liberalisation).

When $\lambda \neq \frac{b}{2}$, for any given level of subsidy provided from the government, the equilibrium export volume is:

$$q = \frac{(b+2)s}{2\lambda - b} \quad (16)$$

Inserting the result in (16) into (14) and (15) under symmetry delivers:

$$x = \frac{(b+2)^2 s}{(2\lambda - b)(\lambda + 1)} - \frac{(\alpha - c - \tau)}{\lambda + 1} \quad (17)$$

Because export volume and R&D investment are non-negative, we must have $\frac{s}{2\lambda - b} > 0$. This implies either $s > 0$ when $\lambda > \frac{b}{2}$ or $s < 0$ when $\lambda < \frac{b}{2}$.

To simplify notation, let $\theta = \frac{s}{2\lambda - b} > 0$. We next identify conditions that need to be imposed on θ to make sure that the firms' profit maximisation problem yield interior solutions. More specifically, we need:

$$0 < (1 + b)q < \alpha$$

$$0 < (\lambda + 1)x < c$$

While the first condition guarantees positive quantities and prices of the goods, the second one is necessary for having plausible positive R&D investments. Using the result in (16), the double inequalities $0 < (1 + b)q < \alpha$ imply $0 < \theta < \frac{\alpha}{(b+1)(b+2)}$. Using (17), the double inequalities $0 < (\lambda + 1)x < c$ imply $\frac{(\alpha - \tau)}{(b+2)^2} > \theta > \frac{(\alpha - c - \tau)}{(b+2)^2}$. Combining these two results, the range of value for θ is $\frac{(\alpha - \tau)}{(b+2)^2} > \theta > \frac{(\alpha - c - \tau)}{(b+2)^2}$.

It can be seen that the function W is strictly concave in x . Indeed, we have $\frac{\partial^2 W}{\partial x^2} = 2 \left[\frac{2(\lambda+1)^2}{(b+2)^2} - r''(x) \right]$. Since $r''(x) \cdot (b+2)^2 > (b+5)(\lambda+1)^2 > 2(\lambda+1)^2$, $\forall b \in (0, 1)$, $\forall \lambda \in [0, 1]$ following Assumption 2 then $\frac{\partial^2 W}{\partial x^2} < 0$. Because x is affine in s according to (17), W is also strictly concave in s . We will next show that there exists a unique interior solution to the government's welfare maximising problem.

Substituting the obtained results into (15) gives:

$$2(\lambda + 1)\theta - r'(x) = 0 \quad (18)$$

We consider the LHS of (18) which is a function of θ : $f(\theta) = 2(\lambda + 1)\theta - r'(x)$. Differentiating this function with respect to θ yields:

$$f'(\theta) = 2(\lambda + 1) - r''(x) \cdot \frac{\partial x}{\partial \theta} = \frac{2(\lambda+1)^2 - r''(x) \cdot (b+2)^2}{\lambda+1}$$

Because $r''(x) \cdot (b+2)^2 > 2(\lambda+1)^2$, $\forall b \in (0, 1)$, $\lambda \in [0, 1]$ based on Assumption 1, $f'(\theta) < 0$ meaning that the LHS of (18) is a decreasing function of θ while its RHS is equal to zero. At $\theta = \frac{(\alpha-c-\tau)}{(b+2)^2}$, $f(\theta) = \frac{2(\lambda+1)(\alpha-c-\tau)}{(b+2)^2} > 0$. When $\theta = \frac{(\alpha-\tau)}{(b+2)^2}$, $f(\theta) = \frac{2(\lambda+1)(\alpha-\tau)}{(b+2)^2} - r'(\frac{c}{\lambda+1}) < 0$ because $r'(\frac{c}{\lambda+1}) > \frac{2(\lambda+1)(\alpha-\tau)}{(b+2)^2}$ as per our above stated assumption (inside the statement of the proposition). Hence, there exists a unique positive value of θ that solves (18).

Note that the first and second derivatives of the welfare function with respect to s are:

$$\frac{\partial W}{\partial s} = \frac{2(b+2)^2}{(2\lambda-b)(\lambda+1)} [2(\lambda+1)\theta - r'(x)]$$

$$\frac{\partial^2 W}{\partial s^2} = \frac{2(b+2)^2}{(2\lambda-b)^2(\lambda+1)^2} [2(\lambda+1)^2 - r''(x)(b+2)^2]$$

Clearly, the condition $\frac{\partial W}{\partial s} = 0$ is equivalent to that of (18) and $\frac{\partial^2 W}{\partial s^2}$ has a same sign as $2(\lambda+1)^2 - r''(x)(b+2)^2$. With $\frac{(\alpha-\tau)}{(b+2)^2} > \theta > \frac{(\alpha-c-\tau)}{(b+2)^2}$, the welfare function is concave over its domain and has a maximum. Therefore, $s = (2\lambda - b)\theta$ is the unique optimal R&D policy measure that should be applied by the government to the firms' R&D efforts in order to maximise the social welfare. When $\lambda > \frac{b}{2}$, $s > 0$, there is an optimal R&D subsidy conducted. However, when $\lambda < \frac{b}{2}$, $s < 0$, it is optimal to have an R&D tax instead.

The condition $r'(\frac{c}{\lambda+1}) > \frac{2(\lambda+1)(\alpha-\tau)}{(b+2)^2}$ carries some special economic meaning. Clearly, $\frac{c}{\lambda+1}$ is the upper limit on firms' cost reducing R&D investment at which their production cost will be driven down to zero. This means that $r'(\frac{c}{\lambda+1})$ is the marginal cost of R&D investment once reaching this upper limit. Meanwhile, $\frac{2(\lambda+1)(\alpha-\tau)}{(b+2)^2}$ is the marginal benefit of R&D investment evaluating at this threshold level. It is required that marginal cost of R&D investment is greater than its marginal benefit at this point as otherwise there would be incentive for firms making further R&D investment and production cost would be negative (which is implausible).

Regarding the impact of trade liberalisation in the overseas market, differentiating both sides of (18) with respect to τ and rearranging we get:

$$\left[\frac{2(\lambda+1)^2 - r''(x) \cdot (b+2)^2}{\lambda+1} \right] \frac{\partial \theta}{\partial \tau} = \frac{r''(x)}{\lambda+1}$$

Because the term in the square bracket on the LHS is negative while the RHS is always positive, we have $\frac{\partial \theta}{\partial \tau} < 0$. Now, translating that into the relationship between s and τ , this implies $\frac{\partial s}{\partial \tau} = (2\lambda - b) \frac{\partial \theta}{\partial \tau}$. Clearly, if $\lambda > \frac{b}{2}$, $s > 0$, and $\frac{\partial s}{\partial \tau} < 0$. As this is the case of an optimal R&D subsidy, other things equal, trade liberalisation (a smaller τ) induces a higher level of optimal R&D subsidy provided to firms. By contrast, if $\lambda < \frac{b}{2}$, $s < 0$, and $\frac{\partial s}{\partial \tau} > 0$. In this case, a decrease in τ leads to a corresponding decrease in s (s becomes more negative). In other words, a higher level of optimal R&D tax should be levied.

Proof of Proposition 2

The proof of this proposition is quite straightforward. Indeed, making use of (16) and (17) and the result of $\frac{\partial \theta}{\partial \tau}$ obtained in the proof of Proposition 1, we get:

$$\frac{\partial x}{\partial \tau} = \frac{(b+2)^2}{(\lambda+1)} \cdot \frac{\partial \theta}{\partial \tau} + \frac{1}{\lambda+1} = \frac{2(\lambda+1)}{2(\lambda+1)^2 - r''(x) \cdot (b+2)^2} < 0$$

$$\frac{\partial q}{\partial \tau} = (b+2) \cdot \frac{\partial \theta}{\partial \tau} < 0$$

These mean that trade liberalisation (lower τ) leads to an expansion of both R&D investments and export volumes of firms at the optimal policy measure that the government conducts.

Due to symmetry, in equilibrium, firms' and industry productivity are the same $Z = z = \frac{1}{c - (\lambda+1)x}$. Differentiating this with respect to τ delivers:

$$\frac{\partial Z}{\partial \tau} = \frac{\partial z}{\partial \tau} = \frac{\lambda+1}{[c - (\lambda+1)x]^2} \cdot \frac{\partial x}{\partial \tau} < 0$$

A decrease in the trade cost helps strengthen firms' as well as the industry's average productivity. Regarding what happens to the whole society, the effect on welfare is:

$$\frac{\partial W}{\partial \tau} = 2 \left[2q \cdot \frac{\partial q}{\partial \tau} - r'(x) \cdot \frac{\partial x}{\partial \tau} \right] = 2 \left[(b+2)^2 \cdot \frac{2(\lambda+1)\theta - r'(x)}{\lambda+1} \cdot \frac{\partial \theta}{\partial \tau} - \frac{r'(x)}{\lambda+1} \right]$$

A close look at the first term inside the square bracket indicates that it is equal to zero according to equation (18). Hence, $\frac{\partial W}{\partial \tau} < 0$ or W is decreasing in τ . A fall in τ will increase W or welfare increases with trade liberalisation in the foreign market.

Proof of Lemma 2

The first order conditions from firm i 's profit maximisation problem are:

$$(\alpha - c) + x_i + \lambda x_j - bq_{j1} - 2q_{i1} = 0 \quad (19)$$

$$(\alpha - c - \tau) + x_i + \lambda x_j - bq_{j2} - 2q_{i2} = 0 \quad (20)$$

$$q_{i1} + q_{i2} + s_i - r'(x_i) = 0 \quad (21)$$

and similar for firm j . The Hessian matrix of second order conditions are $H = \begin{pmatrix} -2 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 1 & -r''(x_i) \end{pmatrix}$. We have $|H_1| = -2 < 0$, $|H_2| = 4 > 0$, and $|H_3| = 4[1 - r''(x_i)] < 0$ meaning the second order conditions are satisfied for a maximum.

In the first stage, the aggregate welfare is:

$$W = \frac{3q_{i1}^2}{2} + q_{i2}^2 - r(x_i) + \frac{3q_{j1}^2}{2} + q_{j2}^2 - r(x_j) + bq_{i1}q_{j1}$$

The government's welfare maximisation delivers the first order conditions:

$$3q_{i1} \cdot \frac{\partial q_{i1}}{\partial s_i} + 2q_{i2} \cdot \frac{\partial q_{i2}}{\partial s_i} - r'(x_i) \cdot \frac{\partial x_i}{\partial s_i} + 3q_{j1} \cdot \frac{\partial q_{j1}}{\partial s_i} + 2q_{j2} \cdot \frac{\partial q_{j2}}{\partial s_i} - r'(x_j) \cdot \frac{\partial x_j}{\partial s_i} + bq_{i1} \cdot \frac{\partial q_{j1}}{\partial s_i} + bq_{j1} \cdot \frac{\partial q_{i1}}{\partial s_i} = 0$$

$$3q_{i1} \cdot \frac{\partial q_{i1}}{\partial s_j} + 2q_{i2} \cdot \frac{\partial q_{i2}}{\partial s_j} - r'(x_i) \cdot \frac{\partial x_i}{\partial s_j} + 3q_{j1} \cdot \frac{\partial q_{j1}}{\partial s_j} + 2q_{j2} \cdot \frac{\partial q_{j2}}{\partial s_j} - r'(x_j) \cdot \frac{\partial x_j}{\partial s_j} + bq_{i1} \cdot \frac{\partial q_{j1}}{\partial s_j} + bq_{j1} \cdot \frac{\partial q_{i1}}{\partial s_j} = 0$$

where q_{i1} , q_{i2} , and x_i (and similar for q_{j1} , q_{j2} , and x_j) are given in (19) - (21). These equations imply a symmetric outcome where $s_i = s_j = s$, $q_{i1} = q_{j1} = q_1$, $q_{i2} = q_{j2} = q_2$, and $x_i = x_j = x$. Using this symmetric result to recalculate the social welfare we get:

$$W = (b + 3)q_1^2 + 2q_2^2 - 2r(x)$$

which in turn imply the following after re-deriving the first order condition:

$$q_1 [(3 + b)\lambda + 1] + q_2(2\lambda - b) - (b + 2)s = 0$$

Using this result, we can figure out:

$$q_1 = \frac{(b+2)s}{(b+5)\lambda+1-b} + \frac{(2\lambda-b)\tau}{(b+2)[(b+5)\lambda+1-b]}$$

$$q_2 = \frac{(b+2)s}{(b+5)\lambda+1-b} - \frac{[(b+3)\lambda+1]\tau}{(b+2)[(b+5)\lambda+1-b]}$$

$$x = \frac{(b+2)^2 s}{(\lambda+1)[(b+5)\lambda+1-b]} + \frac{(2\lambda-b)\tau}{(\lambda+1)[(b+5)\lambda+1-b]} - \frac{\alpha-c}{\lambda+1}$$

Now, we check for the second order condition:

$$\frac{\partial^2 W}{\partial s^2} = \frac{2(b+2)^2}{[(b+5)\lambda+1-b]^2} \left[b + 5 - r''(x) \cdot \frac{(b+2)^2}{(\lambda+1)^2} \right]$$

It is easy to check that $\max_{b \in [0,1]} \frac{b+5}{(b+2)^2} = \frac{5}{4}$. From Assumption 1 and given that $\lambda \in [0, 1]$ then we have $\frac{\partial^2 W}{\partial s^2} < 0$ implying that the second order condition is satisfied for a maximum.

To make sure that quantities and prices are positive, we need to impose that $0 < (\lambda + 1)x < c$, and $0 < (b + 1)q_1 < \alpha$, as well as $0 < (b + 1)q_2 < \alpha$. These lead to the following:

$$\frac{[(b+5)\lambda+1-b](\alpha-c)-(2\lambda-b)\tau}{(b+2)^2} < s < \frac{[(b+5)\lambda+1-b]\alpha-(2\lambda-b)\tau}{(b+2)^2}$$

Proof of Proposition 3

Substituting results in (10) and (11) into (21) and rearranging gives:

$$\frac{(b+5)(\lambda+1)s}{(b+5)\lambda+1-b} - \frac{(b+1)(\lambda+1)\tau}{(b+2)[(b+5)\lambda+1-b]} - r'(x) = 0 \quad (22)$$

Now define $h(s) = \frac{(b+5)(\lambda+1)s}{(b+5)\lambda+1-b} - \frac{(b+1)(\lambda+1)\tau}{(b+2)[(b+5)\lambda+1-b]} - r'(x)$. We have:

$$h'(s) = \frac{(b+5)(\lambda+1)}{(b+5)\lambda+1-b} - r''(x) \cdot \frac{\partial x}{\partial s} = \frac{(b+5)(\lambda+1)}{(b+5)\lambda+1-b} - r''(x) \cdot \frac{(b+2)^2}{(\lambda+1)[(b+5)\lambda+1-b]}$$

As $r''(x) \cdot (b+2)^2 > (b+5)(\lambda+1)^2$ as per Assumption 2 then $h'(s) < 0$ or LHS of (22) is decreasing in s . In the meantime, the RHS of (22) is constant at zero. Given the range of s specified in (13) then the range of value of $h(s)$ should be $\frac{(\lambda+1)[(b+5)\alpha-2\tau]}{(b+2)^2} - r'\left(\frac{c}{\lambda+1}\right) < h(s) < \frac{(\lambda+1)[(b+5)(\alpha-c)-2\tau]}{(b+2)^2}$.

Obviously, $\frac{(\lambda+1)[(b+5)(\alpha-c)-2\tau]}{(b+2)^2} > 0$ because $\alpha - c - \tau > 0$. Hence, as soon as $\frac{(\lambda+1)[(b+5)\alpha-2\tau]}{(b+2)^2} - r'\left(\frac{c}{\lambda+1}\right) < 0$ or $\frac{(\lambda+1)[(b+5)\alpha-2\tau]}{(b+2)^2} < r'\left(\frac{c}{\lambda+1}\right)$, (22) yields a positive and unique solution s (it can be verified that the lower bound on s given in (13) is greater than zero). Economically, this means that the marginal cost of R&D evaluating at its upper bound (where production cost is zero) must be greater than its benefit. This will prevent any further R&D investment from happening and the production cost does not get negative values.

Differentiating both sides of (22) with respect to τ and rearranging gives:

$$\frac{\partial s}{\partial \tau} = \frac{(b+1)(\lambda+1)^2 + (2\lambda-b)(b+2)r''(x)}{(b+2)[(b+5)(\lambda+1)^2 - (b+2)^2 r''(x)]} \quad (23)$$

It should be noted that the denominator is always negative. As for the numerator, it is positive if $\lambda \geq \frac{b}{2}$ or $\lambda < \frac{b}{2}$ and $\frac{(b+5)(\lambda+1)^2}{(b+2)^2} < r''(x) < \frac{(b+1)(\lambda+1)^2}{(b-2\lambda)(b+2)}$. In that case the whole fraction $\frac{\partial s}{\partial \tau} < 0$ or s is decreasing in τ . A decrease in τ will result in an increase in s at the optimal. When $r''(x) > \frac{(b+1)(\lambda+1)^2}{(b-2\lambda)(b+2)}$ for $\lambda < \frac{b}{2}$ the numerator is negative so $\frac{\partial s}{\partial \tau} > 0$ or s is increasing in τ . When $r''(x) = \frac{(b+1)(\lambda+1)^2}{(b-2\lambda)(b+2)}$ for $\lambda < \frac{b}{2}$, $\frac{\partial s}{\partial \tau} = 0$ implying that s is unaffected by a change in τ .

Proof of Proposition 4

Using (10) - (12) and then (23), we obtain the following partial derivatives:

$$\begin{aligned}\frac{\partial q_1}{\partial \tau} &= \frac{2(\lambda+1)^2}{(b+2)[(b+5)(\lambda+1)^2 - (b+2)^2 r''(x)]} < 0 \\ \frac{\partial q_2}{\partial \tau} &= \frac{(b+2)^2 r''(x) - (b+3)(\lambda+1)^2}{(b+2)[(b+5)(\lambda+1)^2 - (b+2)^2 r''(x)]} < 0 \\ \frac{\partial x}{\partial \tau} &= \frac{2(\lambda+1)}{[(b+5)(\lambda+1)^2 - (b+2)^2 r''(x)]} < 0\end{aligned}$$

Defining $q = q_1 + q_2$ as a firm's total sales then:

$$\frac{\partial q}{\partial \tau} = \frac{\partial q_1}{\partial \tau} + \frac{\partial q_2}{\partial \tau} < 0$$

The industry productivity is equal to firm's productivity $Z = z = \frac{1}{c - (\lambda+1)x}$. Differentiating this with respect to τ delivers:

$$\frac{\partial Z}{\partial \tau} = \frac{\partial z}{\partial \tau} = \frac{\lambda+1}{[c - (\lambda+1)x]^2} \cdot \frac{\partial x}{\partial \tau} < 0$$

As for the welfare effect, we have:

$$\frac{\partial W}{\partial \tau} = (b+3) \cdot 2q_1 \cdot \frac{\partial q_1}{\partial \tau} + 4q_2 \cdot \frac{\partial q_2}{\partial \tau} - 2r'(x) \cdot \frac{\partial x}{\partial \tau}$$

Substituting (22) and the results derived above into this equation and simplifying we get:

$$\frac{\partial W}{\partial \tau} = \frac{4\{[(b+3)\lambda+1]\tau - s(b+2)^2\}}{(b+2)^2[(b+5)\lambda+1-b]}$$

Note that the denominator of this fraction is positive. Given the range of value of s in (13), we can work out that:

$$\begin{aligned}-[(b+5)\lambda+1-b](\alpha-\tau) &\leq [(b+3)\lambda+1]\tau - s(b+2)^2 \leq \\ &-[(b+5)\lambda+1-b](\alpha-\tau-c)\end{aligned}$$

This means that $[(b+3)\lambda+1]\tau - s(b+2)^2 < 0$. Hence, we can conclude $\frac{\partial W}{\partial \tau} < 0$.

References

- [1] Airline Leader (2015), “Qantas and Virgin Australia seek to establish international strategies”, The Strategy Journal for Airline CEOs, Issue 29, July.
- [2] Bagwell, K. and Staiger, R.W. (1994), “The sensitivity of strategic and corrective R&D policy in oligopolistic industries”, Journal of International Economics, Vol. 36, pp. 133-150.
- [3] Bustos, P. (2011), “Trade liberalization, exports and technology upgrading: evidence on the impact of MERCOSUR on Argentinian firms”, American Economic Review, Vol. 101(1), pp. 304-340.
- [4] Brander, J. (1995), “Strategic trade policy”. In Grossman, G., Rogoff, K. (Eds.), Handbook of International Economics, Vol. III. Elsevier Science B.V, Amsterdam, pp. 1458-1476.
- [5] Collie, D.R. (2002), “State aid in the European Union: the prohibition of subsidies in an integrated market”, International Journal of Industrial Organization, Vol. 18, pp. 867-884.
- [6] Freed, J. (2015a), “Virgin Australia and Delta Air Lines to keep trans-Pacific flight capacity steady”, The Sydney Morning Herald, March 18.
- [7] Freed, J. (2015b), “Virgin Australia still chasing Qantas in profitability stakes”, The Sydney Morning Herald, November 5.
- [8] Haaland, J. and Kind, H. (2008), “R&D policies, trade and process innovation”, Journal of International Economics, Vol. 74, pp. 170-187.
- [9] Leahy, D. and Neary, J.P. (2001), “Robust rules for industrial policies in open economies”, Journal of International Trade and Economic Development, Vol. 10(4), pp. 393-409.
- [10] Leahy, D. and Neary, J.P. (2004), “Symmetric research joint ventures: cooperative substitutes and complements”, mimeo.
- [11] Lileeva, A. and Treffer, D. (2010), “Improved access to foreign markets raises plant-level productivity... for some plants”, Quarterly Journal of Economics, Vol. 125, pp. 1051-1099.

- [12] Long, N.V., Raff, H. and Stahler, F. (2011), “Innovation and trade with heterogeneous firms”, *Journal of International Economics*, Vol. 84, pp. 149-159.
- [13] Long, N.V. and Stahler, F. (2009), “Trade policies and mixed enterprises”, *Canadian Journal of Economics*, Vol. 42(2), pp. 590-614.
- [14] Merlitz, M.J. (2003), “The impact of trade on intra-industry reallocations and aggregate industry productivity”, *Econometrica*, Vol. 71, pp. 1695-1725.
- [15] Neary, J.P. and O’Sullivan, P. (1999), “Beat ’em or join ’em? Export subsidies versus international research joint ventures in oligopolistic markets”, *Scandinavian Journal of Economics*, Vol. 101(4), pp. 577-596.
- [16] Neary, J.P. and Leahy, D. (2000), “Strategic trade and industrial policy towards dynamic oligopolies”, *Economic Journal*, Vol. 110, pp. 484-508.
- [17] Sargent, J. and Matthews, L. (2009), “China vs. Mexico in the global EPZ industry: maquiladoras, FDI quality, and plant mortality”, *World Development*. Vol. 37(6), pp. 1069-1082.
- [18] Spencer, B. and Brander, J.A. (1983), “International R&D rivalry and industrial strategy”, *Review of Economic Studies*, Vol. 50(4), pp. 707-722.