By Process of Elimination: Further Remarks on the Operation chú in Early Imperial Mathematical Astronomy

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By Process of Elimination:
Further Remarks on the Operation *chú* 除 in Early Imperial Mathematical Astronomy

(and other things not originally foreseen when asked months ago for a title)

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1 Problématique: Translation & the History of Science

Lì 曆 literature is hard to read. It shouldn't be, from all I've been told. We used to translate *lì* as 'calendrics' or 'calendrical science', which, when compounded with the ancient motto of "observing the signs & granting the seasons" 觀象授時, leaves one with the distinct impression that we're dealing with something like medieval European computus.[1] The comparison is unjust. Where there is injustice in the history of science, we're keen to round up the usual suspects—'positivism', nationalism, and other twentieth-century *faux pas*—but 'calender science' is an example of a gaffe all together more thoughtlessly specific to sinology. Sadly, it is more the belief in a platonic realm of one-to-one translations, transcending a variable, incommensurable, and *living* language that allows the sinologist in particular to argue "I know what a *lì* is, I have one here on my desk."

The cool kids now call it mathematical astronomy. That sounds even harder, but we're told that it isn't. Nathan Sivin describes the *lì* procedure text as "a set of step-by-step instructions worked out so that a minor functionary with limited mathematical skills could calculate the annual ephemeris."[2] Christopher Cullen goes so far as to call them "systems of mathematical software," suggesting that they practically run themselves.[3] In a way, it's not hard to see their point: procedure texts *do* give the reader all the *shù* 數 'numbers', *lì* 'sequence/tables', and *shù* 術 'procedures' that they theoretically need to compute, "step-by-step," their quarter-moons, eclipses, and retrogradations. That said, as a "minor functionary with limited mathematical skills," I still find these texts vexingly obstinate.

I suspect that I am not alone in this, considering how my forebearers read *lì*. Our first inclination was to extract from them the relevant 'numbers'—astronomical constants like the mean synodic month—that we may calculate with them freely, without the noise of all the repetitive, antiquated, and self-evident 'procedures'. We can point to the essentializing tables of

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[1] For 'calendar science', see for example Nakayama Shigeru 中山茂, "Characteristics of Chinese Calendrical Science," *Japanese Studies in the History of Science* no. 4 (1965): 124–31.

[2] *Granting the Seasons: The Chinese Astronomical Reform of 1280, with a Study of Its Many Dimensions and a Translation of Its Records* (New York: Springer, 2009), 21.

[3] "Actors, Networks, and 'Disturbing Spectacles' in Institutional Science: 2nd Century Chinese Debates on Astronomy," *Antiqvorvm Philosophia* 1 (2007): 237–67, p. 244.

scholars like Zhū Wénxīn 朱文鑫 (1883–1939), Joseph Needham (1900–1995), Chén Zūnguī 陳遵嫣 (1901–1991), or Yabuuti Kiyosi 藪内清 (1906–2000) in this regard, but let us not forget that the tradition of reducing a *lì* in this way to "its essentials" 其要 was begun long ago by the astronomer and historian of astronomy Lǐ Chúnfēng 李淳風 (602–670).[4] It is recent and rare that historians return to the details of the original 'procedures'. When we do so now, we tend do so through symbolic algebra for the sake of understanding the shortcuts and roundabouts that our subjects took in practice. We now have a number studies that take us step-by-step through *shù* 'procedures' in this way, most notably the works of Liú Hóngtāo 劉洪濤, Christopher Cullen, Yabuuti Kiyosi & Nakayama Shigeru 中山茂, Nathan Sivin, Qū Ānjīng 曲安京, and Jean-Claude Martzloff.[5] Cullen has gone the extra mile of translating procedure texts into Excel code, realizing their ultimate potential as "mathematical software"—tools that require the user to input only a single variable and make meaning from the tables and intermediary steps that appear instantaneously before him.[6]

Inspired by Cullen's approach to reading, understanding, and literally animating a text, I took it upon myself in the summer of 2010 to translate the four earliest procedure texts into both English and code. The process was more laborious than I had imagined. The vocabulary is simple and repetitive, the problem is that the *idiom* of the primary source is alien to the point of incomprehensibility, even with an understanding of the precise phenomenon being modeled. One inevitably turns to translations into modern terms and algebra like Liú Hóngtāo's seminal *Gǔdài lìfǎ jìsuànfǎ* 古代曆法計算法, which treats all *lì* from 91 BCE – 605 CE.[7] Much is lost in translation, and it takes painstaking concentration to reconcile what is written in symbolic algebra with what is written in classical Chinese. Ultimately, you tinker with your own formulae until they give you the results that you expect (or those that Liú Hóngtāo is sometimes kind enough to give you), and only then can you be sure that you have read the text correctly. In other words, you have to understand astronomy to understand an astronomical text, and that alone is not enough.

The paramount task of the Western sinologist is translation—the translation, to be specific, of unreadable Chinese into unreadable English, French or German. Sinology is a guild comprised of a federation of master-disciple lineages, membership to which is signaled first and foremost by one's reference to an inherited canon of translation conventions. Whether you render *tiān xià* 天下 (lit. 'under heaven') as "the world," "the empire," or "the sub-celestial realm" communicates your academic ancestry, as do decisions to capitalize words from a language without capitals (天 as 'Heaven' vs. '*ciel*'). However we learned to translate or typeset the word 天, whether, when faced with some truly incommensurable term we create a new word or borrow one from a

[4] See Zhū Wénxīn, *Lìfǎ tōngzhì* 曆法通志 (Shànghǎi: Shāngwù yìnshūguǎn, 1934); Joseph Needham, *Science and Civilisation in China, vol.3: Mathematics and the Sciences of the Heavens and the Earth* (Cambridge: Cambridge University Press, 1959); Chén Zūnguī, *Zhōngguó tiānwénxué shǐ* 中國天文學史, 2d ed. (Shànghǎi: Shànghǎi rénmín chūbǎnshè, 2006); Yabuuti, "Astronomical Tables in China, from the Han to the T'ang Dynasties," in *Chūgoku chūsei kagaku gijutsushi no kenkyū* 中國中世科學技術史の研究, ed. idem. (Tōkyō: Kadokawa shoten, 1963), 445–92; Yabuuti, "Astronomical Tables in China—from the Wutai to the Ch'ing Dynasties," *Japanese Studies in the History of Science* no. 2 (1963): 94–100. For Lǐ Chúnfēng's historical monographs, see *Jìn shū* 晉書 (Zhōnghuá shūjú 中華書局 ed.), *j.* 16–18; *Suí shū* 隋書 (Zhōnghuá shūjú ed.), *j.* 16–18. On the "essentials" of various systems left to us by Lǐ Chúnfēng, see Liú Hóngtāo 劉洪濤, *Gǔdài lìfǎ jìsuànfǎ* 古代曆法計算法 (Tiānjīn: Nánkāi dàxué chūbǎnshè, 2003), 235–40, 614–20.

[5] Liú Hóngtāo, *Gǔdài lìfǎ jìsuànfǎ*; Cullen, "The First Complete Chinese Theory of the Moon: The Innovations of Liu Hong c. A.D. 200," *Journal for the History of Astronomy* 33 (2002): 21–39; Yabuuti and Nakayama, *Jujireki: yakuchū to kenkyū* 授時曆：譯注と研究 (Kawasaki: I. K. Corporation, 2006); Sivin, *Granting the Seasons*; Qū Ānjīng, *Zhōngguó shùlǐ tiānwénxué* 中國數理天文學 (Běijīng: Kèxué chūbǎnshè, 2008); Martzloff, *Le calendrier chinois: structure et calculs, 104 av. JC–1644: indétermination céleste et réforme permanente: la construction chinoise officielle du temps quotidien discret à partir d'un temps mathématique caché, linéaire et continu* (Paris: Champion, 2009). For a recent criticism of the current state of the field in this regard, see Cullen, "Translating Ancient Chinese Calendars," *Revue de Synthèse* 131, no. 4 (2010): 605–12.

[6] On methodology, see Cullen, "Translating Ancient Chinese Astronomical Systems with EXCEL: How Not to Stew the Strawberries?," *Journal for the History of Astronomy* 36, no. 3 (2005): 336–38. Cullen's automated Excel translations of *lì* procedure texts are available online at http://www.nri.org.uk/lifa.html. Xíng Gāng 邢鋼 has also done extensive work on the automation of early *lì* procedure texts, see his "Zhōngguó zǎoqī lìfǎ de jìsuànjī mónǐ fēnxī yú zōnghé yánjiū" 中國早期歷法的計算機模擬分析與綜合研究 (Ph.D. diss., Zhōngguó kèxué jìshù dàxué, 2005).

[7] Liú Hóngtāo's magnum opus is by far the most complete and helpful resource for these centuries; for other resources, see Note 5.

Table 1: The vocabulary of *lì* (origins to 8th cent. CE)

曆/歷/歷/厤 *lì*: (1)* a celestial 'sequence' (*Documents*, 11.13b); (2)* a/the 'calendar' or 'astronomical table' (*Zuo Tradition*, Xiang 27; Ai 12); (3) a/the 'system manual' from which these are generated; (4) 'tables' and 'sequences' within such a manual (Supernal Emblem system [#10], in *Jìn shū*, *j.* 17); (5) a/the 'calendro-astronomical system' from which a manual is distilled; (6) to 'calculate' or 'sequence' any of the above (*Documents*, 1.8b); (7) the study of any of the above.

曆數 *lì shù*: 'calendro-astronomical numbers' (1)* numbers inherent in Heaven and reproduced by man (*Documents*, 11.13b); (2)* the term limit of the Mandate of Heaven (*Documents*, 3.12a, *Analects* XX.1); (3) the numbers forming the body of a calendro-astronomical system/manual; (4) by synecdoche, a calendro-astronomical system/manual (*Hàn shū*, 6.212); (5) the study of calendro-astronomy (*Hàn shū*, 58.2634); (6) 'calendro-astronomy & mathematics' (*Suí shū*, 34.1026).

天曆 *tiān lì*: 'celestial *lì*' (1) the state astronomical system (*Shǐjì*, 130.3285; uncommon); (2) abbreviation of 'the *li* numbers of Heaven' 天之曆數, i.e. the term limit of the Mandate of Heaven (*Tàipíng jīng héjiào*, 137.707).

曆術 *lì shù*: 'calendro-astronomical technique(s)' (1) a calendro-astronomical system/manual (*Shǐjì*, *j.* 26); (2) a 'sequence technique' for computing lunar latitude or equation of center (Luminous Inception and Epochal Excellence systems, in *Sòng shū*, *j.* 12 & 13); (3) the study of calendro-astronomy (*Wèi shū*, 48.1068).

曆法 *lì fǎ*: 'calendro-astronomical method(s)' (1) calendro-astronomical system/manual (*Sòng shū*, 12.230).

曆算 *lì suàn*: 'calendro-astronomical calculation' (1) the study of calendro-astronomy (*Hàn shū*, 12.258); (2) to perform calendro-astronomical calculations (*Suí shū*, 18.479); (3) 'calendro-astronomy & mathematics' (*Jiù Táng shū*, 47.2039).

星曆 *xīng lì*: 'star/planet *lì*' (1) stellar/planetary sequences inherent in nature (*Guǎnzǐ*, 41.703); (2) an undefined responsibility of the Prefect Grand Clerk (*Hàn shū*, 99.4170); (3) undefined mysterious knowledge (*Hàn shū*, 62.2732; common); (4) the study of calendro-astronomy (*Jiù Táng shū*, 66.2463).

年曆 *nián lì*: 'annual calendar' (1) a/the civil calendar (*Zhōnglùn*, B.13a); (2) an annals (*Jiù Táng shū*, 149.4030).

曆日 *lì rì*: (1) 'sequence day', i.e. the number of days entered into the lunar speed or latitude sequence (Supernal Emblem system (#10), in *Jìn shū*, *j.* 17); (2) a civil calendar (*Wùlǐ lùn*, in *Yiwen leiju*, 5.97; uncommon).

日曆 *rì lì*: (1) a civil calendar (*Lùnhéng*, 70.994).

曆書 *lì shū*: (1) a monograph on calendro-astronomy (*Shǐjì*, *j.* 26; uncommon); (2) an/the almanac (Song and later).

具注曆 *jù zhù lì*: 'annotated calendar' (1) a/the almanac (excavated examples from Dunhuang, see Déng Wénkuān [1996]).

NOTE: Appended to each usage is a citation of its earliest unambiguous instance and a note concerning its subsequent ubiquity. Usages evident in the pre-Qín classics are marked with an asterisk.

fashionable European language, at the core of our guild is the belief that the perfect equivalent of every word lies out there somewhere in the realm of platonic forms. It is rare that we allow ourselves to transliterate (e.g. yin-yang & *qi*), and rarer still to translate the same word *differently* according to context, but sometimes our metaphysics of translation can no longer support to do otherwise.

Due to the traditional foci of Western sinology, technical vocabulary like *lì* is particularly maltreated, and it is up to historians of science to compose their own lexicons, as Karine Chemla has done for the *Nine Chapters of Mathematical Procedures* (*Jiǔ zhāng suàn shú* 九章算術).[8] Sometimes we must even make a case for transliteration or contextual translation where colleagues in more mainstream sub-fields may not notice the difference. In my dissertation, I used an assortment of text databases to collect every occurrence of the character *lì* 曆 in received literature from the Pre-Qín 前秦 (< 221 BCE) to the eighth century CE, which I sorted according

[8] Chemla and Guō Shūchūn 郭書春, *Les neuf chapitres: le classique mathématique de la Chine ancienne et ses commentaires* (Paris: Dunod, 2004), 897–1035.

to century, compound, and contextual sense to reveal specific patterns of usage.[9] This analysis revealed that *lì* occurs in eleven common compounds and, by itself, may carry no less than seven different senses (see Table 1). Not only did this analysis reveal the variety of possible senses and combinations that the word may take at any one time, it revealed how they appeared, changed, and disappeared *over time*. For a language where *wén* 聞 evolved from 'to hear' into 'to smell', it comes as no great surprise that people also spoke differently about astronomy a thousand years apart.

Now, when it comes to translating the procedure text, we practice a very specialized form of sinological obscurantism. If we're being honest, we first translate the language into Arabic numerals and symbolic algebra (or code). In this state, the meaning of our text is clearer than any scholar of literature, religion, philosophy or history could ever dream of from his/her sources. We then translate short, simple numbers back into words, the longer and clumsier the better:

Quantity	char
一億三千四百八萬二千二百九十七	15
134,082,297	11
one hundred thirty-four million eighty-two thousand two hundred ninety-seven	76
一丈九寸五分四分分之三	11
10.9575 *chi*	11
one *zhāng* 丈, nine *cùn* 寸, five *fēn* 分, and three quarters of a *fēn*	65

We do this to reproduce the effect of our primary source—that numbers are expressed not in decimal place value notation but in the *words* of everyday life—so as to better understand the nuances of how our subjects dealt with numbers.[10] In the case of Chinese word-numbers, the contrast with their English equivalents strikes me as exaggerated, since the Chinese alternates between *monosyllabic* characters for quantity and decimal unit (e.g., $1_y\ 3_q4_b\square_s8_w\ 2_q2_b9_s7$). It seems also illusory, considering that our subjects essentially (though not always) calculated by converting word-numbers into decimal place value notation on the space of the counting rod grid. In practical terms, lastly, the extra 60 characters per number eat though a publisher's word limit like crap through a goose.

Whatever the pros and cons of our nuanced and/or/as obscurantist approach to translating numbers and units, we pay, by contrast, very little attention to the language of the operations performed thereupon. In the process of translating *lì*, for example, one encounters a large variety of expressions (in a seemingly endless variety of combinations) that, in Chemla's *Nine Chapters* lexicon, for example, are all rendered as '*diviser*': (1) *chú* 除 (lit. 'eliminate'), (2) *ér yī* 而一 (lit. 'and then one[s]'), (3) *mǎn* 滿 (lit. 'to fill'), (4) *rú* x *ér yī* 如 *x* 而一 (lit. 'and then one[s] as per *x*'), (5) *rú* x *dé* y 如 *x* 得 *y* (lit. 'obtain *y* as per *x*'), and (6) *rú* x *dé yī* 如 *x* 得一 (lit. 'obtain one[s] as per *x*').[11] One also encounters terms signifying division that appear in her lexicon as standing for different operations: (1) *jiǎn* 減 (lit. 'diminish'; Chemla: '*soustraire*'), (2) *qù* 去 (lit. 'remove'; Chemla: '*soustraire*'), and (3) *yuē* 約 (lit. 'constrain'; Chemla: '*simplifier*').[12] Nor does Chemla's lexicon deal at all with what is the most common operation of all in the context of *lì*—*modulo*—which is proscribed by (1) *chú*, (2) *qù*, (3) *chú-qù*, and (4) *jiǎn* alike. The effect is like wading through quicksand: haltingly disorienting, one suspects that there may be some *terra firma* beneath it all, but all that matters is getting to the other side before drowning. Homogenizing and modernizing the operational terminology, the task of the historian of Chinese mathematics is complete: he has transformed counts into words and eliminated the words that count.

[9] See Daniel Patrick Morgan, "Knowing Heaven: Astronomy, the Calendar, and the Sagecraft of Science in Early Imperial China" (Ph.D. diss., University of Chicago, 2013), 55–61.

[10] Someone please direct me to readings on this topic; sorry, I wrote this paper in a hurry.

[11] *Les neuf chapitres*, 911, 918, 959, 973, 978–79.

[12] Ibid., 937, 971, 1028–29.

Under the framework of the ERC project SAW's mission to explore pluralities within the mathematical cultures of an ancient civilization like 'China', Karine Chemla and I have begun a collaborative project on the language of mathematical operations, the first results of which we presented at our *Reading Mathematical Texts* seminar on January 23. Our ultimate goal, akin to my aforementioned work with *lì*, is to map patterns in word choice synchronically across genres and diachronically across time to reveal, for example, how it is that *chú* evolved from 'subtract' to 'divide' and how *lì* and math texts, excavated and received, may use the term differently. In this paper, I will develop upon our previous findings on numbers and vocabulary, and I will introduce the results of preliminary statistics begun the weekend of March 22 as they speak to linguistic patterns within early imperial *lì* literature.

This paper will begin by circumscribing the time frame and textual corpus under consideration as well as the corpus to which I aspire to extend such analysis in the future. We move from there to an overview of the 'numbers & procedures' (*shù shù*) at the foundation of this corpus. I offer first a threefold typology of astronomical 'numbers' to clarify their respective natures, functions, and manipulations in the context of 'procedure'. I then excerpt and explain 'procedures' selected to illustrate the disorienting redundancy and polysemy of actors' vocabulary of operations. In Section 5, we turn to data that I have recently compiled on word-usage in *lì* procedure texts from the first to sixth century CE. While still in the preliminary days of analysis, I offer a number of tentative interpretations of this data, which, I argue, highlight historical ruptures and transitions in the mathematical terminology of *lì*. Having made the case here for diachronic diversity, plurality, or at least dynamism in *one* Chinese mathematical culture, I then discuss the prospects of extending such analysis to other mathematical genres to establish evidence of *synchronic* diversity as well. Finally, I offer several thoughts about how we might better render the language of these sources in translation.

2 Sources

What is our historical scope? This study will be treating *lì* from 221 BCE to 600 CE, to which I shall vaguely refer as 'early imperial'. I had originally intended to carry my analysis through to the end of the Suí 隨 (581–618), to include also Liú Zhuō's 劉焯 Sovereign Pole system (*Huángjí lì* 皇極曆) of *circa* 605, but this procedure text marks such a break with the genre up to that point that I have not had time to fully enter its language into consideration.

Where do we find these texts? We know of 52 *lì* authored in this period by name.[13] Of these, nine survive (apparently) in full as preserved in the "Lǜ-lì zhì" 律曆志 (Monograph on Standards & Sequencing) of various standard histories (see Table 2).[14] The only complete Western-language translation to date of such a procedure text is Sivin's rendering of the Season Granting system (*Shòushí lì* 授時曆) of 1280, though Christopher Cullen, Guān Yúzhēn 關瑜楨, and myself are currently working on translations of *lì* from the period in question.[15]

Can we speak of "the early imperial procedure text genre"? Yes. The nine extant texts (and those beyond the scope of this paper) are so imitative in terms of subject matter, language, structure, etc. as to meaningfully speak of them as belonging to a coherent literary genre. This modern impression can not only be qualified, it can be *quantified*: where each text deals with the calculation of the self-same lunar phenomena in the self-same order, for example, they generously appropriate expressions, paragraphs, and whole numerical tables from their antecedents, the percentage of which can be measured and tends to outweigh what is new.[16] More importantly,

[13] For a full list, see Morgan, "Knowing Heaven," 20–22, Table 2.

[14] Extant *lì* preserved in the standard histories' "Lǜ-lì zhì" are collected in *Lìdài lǜ-lì zhì jiàozhèng* 歷代律曆志校證, ed. Chén Měidōng 陳美東 (Běijīng: Zhōnghuá shūjú, 2008). I, however, for the sake of convenience, will be citing the Zhōnghuá shūjú editions of the standard histories containing these *lì*, having used the searchable digital version of these texts available at Scripta Sinica (http://hanchi.ihp.sinica.edu.tw/ihp/hanji.htm).

[15] See Sivin, *Granting the Seasons*. Note that I will happily supply anyone who is interested with my translation of the Luminous Inception system of 237, prepared for the SAW *Reading Mathematical Texts* seminar in winter 2014.

[16] An excellent example of intertextual overlap between procedure texts is the pool of tables they shared at their core and updated in pell-mell fashion. On this, see Zhāng Péiyú 張培瑜 et al., *Zhōngguó gǔdài lìfǎ* 中國古代曆法 (Běijīng: Zhōngguó kèxué jìshù chūbǎnshè, 2008), 1–90.

their titles, context of preservation, bibliographic classification, their authors' positioning, and their subsequent historiographic framing all affirm that our historical subjects also considered these texts as forming an integral tradition. The titles of lost works preserved in early imperial bibliographies suggest that there were *other* genres within the rubric of *lì*, such as treatises on waterclocks, gnomon shadows, and general *shù* 'techniques', but it is prudent to reserve comments on these genres until archaeology one day furnishes us with examples of them.

What is the procedure text genre like? A single table of contents, as I have provided in Table 3, is sufficient to adumbrate its common features *circa* 221 BCE – 600 CE. The *lì* procedure text is organized into three rubrics: (1) mean luni-solar elements, or 'the calendar'; (2) true luni-solar elements, or 'eclipse prediction'; (3) planetary visibility and east-west motion. Each rubric is furthermore comprised of three textual elements: (1) *shù* 數 'numbers' (2) *shù* 術 'procedures' (3) *lì* 曆 'sequence-tables'.[17] The block quotes and numbers featured in the following sections should provide a sufficient picture of what these elements look like up close to omit description here.

What is the disadvantage of these sources? First, we have no alternative to the "Lǜ-lì zhì" procedure texts until very late, excepting archaeologically-recovered civil calendars presumably calculated therefrom and a handful of values predicted via named *lì* sprinkled throughout other sources.[18] Second, as *received* sources recorded in the context of retrospective political histories, our procedure texts passed through a certain process of later selection, editing, copying, and possibly abridgment and corruption. With no alternative editions available to us, it is difficult to know to what degree these texts have been abridged, but it is worth noting, for example, that bibliographies record Supernal Emblem systems in three and five *juàn* 卷 ('rolls'), whereas *our* version occupies only a single *juàn* of the *Book of Jìn*.[19] Lastly, the texts that we possess have suffered varying degrees of print-related corruption and have been emended by Qīng 清 (1644–1912) and twentieth-century editors.

What, by contrast, are the advantages of these sources? First, simply put, the homogeneity and historical spread of *lì* procedure texts make the genre an ideal medium within which to observe change, like watching a bacterial culture sealed within a Petri dish. Second, they afford us a window onto true change within a body of practice that we may juxtapose with actors' *claims* of change to better understand actors' concept(s) of change and the political and rhetorical culture of the field. How new *really* is the hot 'new' system? What, by contrast, lies beneath modest claims of 'modification' or 'expansion'? And how do significant ruptures in the genre align with ancient and modern understandings of *zhuǎn* 轉 'revolutions' in practice? Most importantly, *lì* literature is not our only Petri dish.

What survives of early *suàn* 算 'mathematics' literature accumulated over the same centuries. Unlike the case of *lì*, however, we posses examples of multiple genres or byways within 'mathematics': (a) the treatises of Lǐ Chúnfēng's eighth-century canon, *The Ten Mathematical Classics*, (b) commentary thereto, (c) non-canonical received treatises, and (d) excavated manuscripts potentially reflecting a different function, social milieu, and culture(s) of practice. Of the classics, two are anonymous and undated but attested by **the Hàn 漢** (206 BCE – 220 CE)—the (1) *Nine Chapters* and (2) *Gnomon of the Zhōu* (*Zhōu bì* 周髀). **The third century** saw the composition of (1) Xú Yuè's 徐岳 *non-canonical Shǔshù jìyí* 數術記遺, (2) Zhào Shuǎng's 趙爽 commentary to the *Gnomon of Zhōu*, as well as (3) Liú Huī's 劉徽 classic, *Hǎidǎo suànjīng* 海島算經, and (4) his commentary to the *Nine Chapters*. **The fourth century** saw the composition of (5) Zhāng Qiūjiàn's 張邱建 eponymous classic and (6) the anonymous classic *Sūnzǐ suànjīng* 孫子算經. **The sixth century** saw the composition of (7) Zhēn Luán's 甄

[17] Note that the term *lì* actually only appears as regards tables of equation of center and latitude. Somewhat atypically for the Chinese language, the title of other tables do not announce what category of thing they are. Not also that 'numbers' and 'sequence-tables' are not always embedded in relevant procedures but may come elsewhere ('numbers' earlier, 'sequence-tables' later) in the text.

[18] See Morgan, "Knowing Heaven."

[19] The *Suí shū* "Jīng-jí zhì" 經籍志 records a Supernal Emblem system in three *juàn* as part of the Suí imperial holdings. It also notes that Liáng 梁 (502–557) bibliographers recorded a five-*juàn* edition with commentary by Liú Hóng, et al., a five-*juan* edition with commentary by Kàn Zé 闞澤 (3rd cent.), and a *Supernal Emblem Planetary Magic* (*Qianxiang wuxing huanshu* 乾象五星幻術) in one *juàn*, all of which were lost by the Suí (*Suí shū*, 34.1022). The *Xīn Táng shū* 新唐書 confirms that a three-*juàn* edition would have still been available to the *Jìn shū*'s Táng compiler, Lǐ Chúnfēng (*Xīn Táng shū* [Zhōnghuá shūjú ed.], 59.1546).

Table 2: *Lì* extant in full from 221 BCE – 600 CE

no.	System	Lead Author	Dynasty	Creation	Implementation
1	三統曆 Triple Concordance system	劉歆 Liú Xīn	W. Hàn	c. 5	
2	四分曆 Quarter-remainder system	編訢 Biān Xīn 李梵 Lǐ Fàn	E. Hàn	85/86	Han: 85 Cao-Wei: 221 Shǔ Hàn: 221
3	乾象曆 Supernal Emblem system	劉洪 Liú Hóng	E. Hàn	206	Sūn-Wú: 223
4	景初曆 Luminous Inception system 泰始曆 Grand Beginning system 永初曆 Perpetual Inception system	楊偉 Yáng Wěi	Cáo-Wèi	237	Cáo-Wèi: 237 N. Wei: ? Jìn: 265 (renamed) Liú-Sòng: 420 (renamed)
5	元嘉曆 Epochal Excellence system	何承天 He Chengtian	Liú-Sòng	445	Liú-Sòng: 445 Qí: 479 Liáng: 502
6	大明曆 Great Enlightenment system 甲子元曆 *Jiǎzǐ*$_{01}$-origin system	祖沖之 Zǔ Chōngzhī	Liú-Sòng	462	Liú-Sòng: slated, failed 465 Liáng: test. 509, impl. 510 Chen: impl. 557
7	神龜曆 Divine Tortoise system 正光曆 Orthodox Glory system	李業興 Lǐ Yèxìng 張龍祥 Zhāng Lóngxiáng	N. Wèi	518/520	N. Wèi: 518/520
8	興和曆 Ascendant Harmony system 甲子元曆 *Jiǎzǐ*$_{01}$-origin system	李業興 Lǐ Yèxìng	E. Wèi	539	E. Wèi: 540
9	大業曆 Great Patrimony system	張冑玄 Zhāng Zhòuxuán	Sui	597	Suí: 608

Table 3: Supernal Emblem System (#10) table of contents (*Jìn shū*, 17.504–531)

Section 1: [Lunisolar astronomy/calendrics]

Numbers: elements of mean solar year, mean synodic month, and eclipse month

Methods: (1) 推入紀 "Calculate entry into current era"; (2) 推朔 "Calculate dates of new moons"; (3) 推冬至 "Calculate date of winter solstice"; (4) 求二十四氣 "Find the dates of the 24 *qi*"; (5) 推閏月 "Calculate the intercalary month"; (6) 推弦望 "Calculate the dates of quarter and full moon"; (7) 推沒 "Calculate disappearances"; (8) 推日度 "Calculate the position of the sun (at midnight on any given day)"; (9) 推月度 "Calculate the position of the moon (at midnight on any given day)"; (10) 推合朔度 "Calculate the position of conjunction (and position of subsequent lunar phases)"; (11) 推月蝕 "Calculate month of lunar eclipse"; (12) 推卦用事日 "Calculate the management of affairs by the hexagrams"; (13) 推五行用事 "Calculate the management of affairs by the five agents"; (14) 推加時 "Calculate the hour (of any lunar phase)"; (15) 推漏刻 "Calculate the clepsydra marks (of any lunar phase)."

Section 2: 月行三道術 "Technique for the three roads of lunar motion"

Table: 遲疾曆 "Speed sequence" (daily lunar equation of center and interpolation)

Numbers: elements of the sidereal and anomalistic month

Methods: (1) 推合朔入曆 "Calculate entry of conjunction into sequence"; (2) 求弦望定大小餘 "Calculate fixed date of quarter and full moon"; (3) 求朔弦望加時定度 "Calculate fixed hour and position of new, quarter, and full moon"; (4) 推月行夜半入曆 "Calculate entry of lunar motion into sequence at midnight"; (5) 求月夜半定度 "Find fixed position of moon at midnight"; (6) 求變衰法 "Find interpolation method" ; (7) 求次曆 "Find subsequent sequence[-entries]"; (8) 求次日夜半定度 "Find the fixed position at midnight on subsequent day"; (9) 求次日夜半盈縮 "Find equation of center at midnight on subsequent day"; (10) 求昏明月度 "Find lunar position at dusk and dawn"; (11) 求月行遲疾 "Find speed of lunar motion."

Table: 陰陽曆 "Latitude sequence"

Numbers: elements of the nodical month

Methods: (1) 推朔入陰陽曆 "Calculate entry of new moon into latitude sequence"; (2) 求次月 "Find subsequent months"; (3) 求朔望定數 "Find fixed numbers for new and full moon"; (4) 推夜半入曆 "Calculate sequence-entry at midnight"; (5) 求夜半定日 "Find fixed date at midnight"; (6) 求昏明數 "Find dusk and dawn numbers"; (7) 求月去極度 "Find lunar latitude."

Section 3: 推五星 "Calculate the five stars (planets)"

Numbers: elements of the planets' mean synodic and visibility periods

Methods: (1) 推星合月 "Calculate month of planetary conjunction"; (2) 推入月日 "Calculate the date"; (3) 推星合度 "Calculate position of planetary conjunction"; (4) 求後合月 "Find month of next conjunction"; (5) 求後合朔日 "Find new-moon day of said month"; (6) 求後入月日術 "Find date of next conjunction"; (7) 求後度 "Find next position"; (8) 五星曆步術 "Method for planetary sequence-pacing."

Motion-degree models: (models of planetary behavior over one synodic period)

鸞 classic *Wǔjīng suànshū* 五經算書, (8) his second classic *Wǔcáo suànjīng* 五曹算經, (9) his commentary to the *Gnomon of Zhōu*, (10) his commentary to the non-canonical *Shǔshù jìyí*, and (11) his commentary to Zhāng Qiūjiàn's classic. **The seventh century**, finally, saw the (12) canonization of the aforementioned 'classics' and (13) Lǐ Chúnfēng et al.'s subcommentaries thereto. Parallel to the expository treatises and commentaries of the received tradition, we also have later paper manuscripts from the Dūnhuáng Mogao grottoes and a growing number of third- and second-century BCE bamboo manuscripts recovered from tombs and the Hong Kong antiquities market.[20]

Karine Chemla and I are in the early stages of a project to map the synchronic variety and historical stratigraphy of the language of 'division' across genre, geography, and time. This project, in conjunction with Chemla and Lǐ Liàng's 李亮 work on progressions, will hopefully do more to reveal the plurality of Chinese mathematical cultures than anything previously attempted.[21] In the meantime, I will report here on my contribution to this project to date vis-à-vis mapping simple diachronic changes within the *lì* procedure text genre.

3 *Shù* 數 'Numbers' in Early Imperial *lì*

Numbers help us understand what operations mean, since *operands* and answers bear obvious testament to the nature of the transformation proscribed by our historical subjects. If we are to start again from zero with operations, therefore, we must then start again from zero as concerns numbers as well (or 'one' or a blank space, as the case may be, since 'zero' here is an anachronism).

Lì texts feature three types of *shù* 'number': *lǜ* 率, quantities, and civil coordinates. ***Lǜ*** are integers defined in relation to one another; they are 'abstract' in the sense that they refer to proportional relationships rather than quantities that might take measuring units. We might say of early *suàn* 'mathematics' that "$\pi = 3$" or "the relation between the circumference and diameter of a circle is 3 : 1," but the Chinese would read, "circumference *lǜ* 3, diameter *lǜ* 1," and it would mean the same thing. *Lǜ* are primarily used in the conversion of one quantity of thing into the equivalent quantity of a second thing using the *jīn yǒu shù* 今有術 'now we have' or 'suppose procedure', i.e. the rule of three.[22] In the context of *lì*, *lǜ* appear as the constituents of what we call in the history of astronomy 'resonance periods'.[23] To differentiate *lǜ* from 'quantities' and 'coordinates', I shall render theme hereafter in SMALL CAPS. Easily recognizable to the historian of astronomy, the following are the luni-solar *lǜ* of the Triple Concordance system (c. 5 CE):[24]

Rule (*zhāng* 章): coincidence of XI-1 (syzygy, month XI) with WS (winter solstice)

RULE YEARS 章歲 : 章月 RULE MONTHS
19 : 235

[20] On early calendars, see Yoshimura Masayuki 吉村昌之, "Shutsudo kandoku shiryō ni mirareru rekihu no shūsei" 出土簡牘資料にはみれる暦譜の集成, in *Henkyō shutsudo mokkan no kenkyū* 邊疆出土木簡の研究, ed. Tomiya Itaru 冨谷至 (Kyōto: Hōyū shoten, 2003), 459–516; Morgan, "Knowing Heaven," 173–271. For the later Dūnhuáng calendars, see *Dūnhuáng tiānwén lìfǎ wénxiàn jíjiào* 敦煌天文暦法文獻輯校, ed. Dèng Wénkuān 鄧文寬 (Nánjīng: Jiāngsū gǔjí chūbǎnshè, 1996); Dèng Wénkuān 鄧文寬, *Dūnhuáng-Tǔlǔfān tiānwén lìfǎ yánjiū* 敦煌吐魯番天文暦法研究 (Lánzhōu: Gānsù jiàoyù chūbǎnshè, 2002).

[21] Chemla & Lǐ, "Progressions, Motions and Changes in the Astral Sciences of Ancient China," presented at the conference Mathematical Practices in Relation to the Astral Science, Université Paris Diderot, March 27, 2015.

[22] On *lǜ* and the 'suppose procedure' in *suàn* 'mathematics', see Chemla & Guō, *Les neuf chapitres*, esp. 199–219 940–41, 956–59; Chemla, "Mathematics, Nature and Cosmological Inquiry in Traditional China," in *Concepts of Nature: A Chinese-European Cross-Cultural Perspective*, ed. Günter Dux, Hans Ulrich Vogel, and Mark Elvin (Leiden: Brill, 2010), 255–84.

[23] For an explanation of 'resonance periods' in various traditions, see for example Needham, *Science and Civilisation in China, vol.3*, 406–08; Noel M. Swerdlow, *The Babylonian Theory of the Planets* (Princeton: Princeton University Press, 1998), 57–64.

[24] For a Western-language explanation of the following *lǜ*, see Sivin, "Cosmos and Computation in Early Chinese Mathematical Astronomy," *T'oung Pao* 2d ser., 55, no. 1/3 (1969): 1–73; Michel Teboul, *Les premières théories planétaires chinoises* (Paris: Collège de France, 1983), 1–7.

Concordance (*tǒng* 統): coincidence of XI-1, WS + midnight

CONCORDANCE DIVISOR 統法 : CONCORDANCE MONTHS : 周天 CIRCUITS OF HEAVEN		
(years)	(months)	(days)
1 539	: 19 035 :	562 120

Origin (*yuán* 元): coincidence of XI-1, WS, midnight + sexagenary day-count

ORIGIN DIVISOR 元法 : 元月 ORIGIN MONTHS	
(years)	: (months)
4 617	: 57 105

[**Mean synodic**] **month (*yuè* 月):** the mean period (in days) between successive syzygies

MONTH DIVISOR 月法 : 日法 DAY DIVISOR	
2 392	: 81
$\therefore$ $29\frac{43}{81}$ days in a month	

[**Mean tropical-sidereal**] **year (*suì* 歲) & circuits of heaven (*zhōu tiān* 周天):** the mean period (in days) between successive WS *and* the distance (in *dù*) traveled by the sun in the same period—by definition, the mean sun travels 1 *dù*/day.

CIRCUITS OF HEAVEN 周天 : 統法 CONCORDANCE DIVISOR	
562 120	: 1 539
$\therefore$ $365\frac{385}{1\,539}$ days in a year	
$\therefore$ $365\frac{385}{1\,539}$ *dù* in a circuit	

'Quantities' are integers or mixed numbers that either refer directly to units of time and/or/as space or serve as intermediaries in the calculation of such units. Such 'quantities' typically appear as the operands and end-products of 'suppose' (rule of three) procedures involving *lǜ̃* and the *modulo* operation (below). 'Quantities' usually imply measuring units, whether or not such units are explicitly given; the interchangeability of spatial and temporal units, however, allow quantities in the one to act as or operate directly on the other. Actors express 'quantities' in integers and remainders, the later of which implies the numerator of a fraction whose denominator is generally the divisor of the previous operation. Exact terminology depends on the *shù* 'procedure', as we will see in the examples given in the following section.

Lastly, **'civil coordinates'** are 'quantities' transformed into the conventions of daily usage. In society, people count daytime in ordinal *chén* 辰 'double-hours' and/or cardinal waterclock *kè* 刻 'notches' (12 *chén* = 100 *kè* = 1 nychthemeron); they count days in ordinal sexagenary *gān-zhī* 干支 pairs (*jiǎ-zǐ*$_{01}$ – *guǐ-hài*$_{60}$); they count months in ordinal numbers, starting from X, XI, XII or *zhèng* 正 'correct' (I), depending on the calendar; and they count luni-solar civil years (*nián* 年) in ordinal numbers from a propagandistic *nián hào* 年號 'reign period' chosen by the throne. Together, this is called the *rì* 日 'day/date' and *jiā shí* 加時 'appended hour' ("appended" in the sense of being time passed since the beginning of said day, at midnight). Right ascension and longitude are expressed in *xiù dù* 宿度 'lodge-*dù*' comprised of the lodge (Horn.$_{L01}$ – Baseboard.$_{L28}$) and the cardinal *rù xiù dù* 入宿度 'degrees entered into lodge' (*chū* 初 'beginning', *yī* 一 'one' ... *sān-shí-sān* 三十三 'thirty-three'). In Tables 4 & 5, I provide examples of how these coordinates are rendered in Chinese, in translation, and, lastly, in simplified notation.

Table 4: Civil date & time coordinates

年號 reign period	年 civil year	月 civil month	日 *gān-zhī* day	加時 *chén*-hour	刻 *kè* 'notches'
元和	二年	三月	丁亥	加午時	二十五刻五
Epochal Harmony	2[nd] year	3[rd] month	*dīng-hài*	added *wǔ* hour	25 *kè* 5[/10]
Epochal Harmony	2 –	III –	$dīng\text{-}hài._{24/60}$,	$wǔ._{\mathrm{B7/12}}$,	25.5 *kè*

Table 5: Civil RA/longitude coordinates

宿 Lodge	度 *dù*	分度 *dù* parts
井	十二度	三十五分度
Well	12 *dù*	35 *dù* parts [/ appropriate divisor]
$\text{Well.}_{\mathrm{L22/28}}$	12 ;	35

4 *Shù* 術 'procedures' & Operations in Early Imperial *lì*

What do procedure texts tell us to do with these numbers? Generally speaking, the reader is instructed to operate on integer *lǜ* and 'quantities' for the ultimate end of producing 'civil coordinates' via what we would call addition, subtraction, multiplication, division, and *modulo*—simple operations frequently cached in complex 'if/then' structures. One is also asked to operate directly on mixed-number 'quantities', but only by addition (frequent), subtraction (uncommon), and multiplication (rare). One *never* operates directly on civil dates; it is however, common, to perform simple addition and subtraction on civil RA/longitude coordinates. The reader is not actually asked to 'subtract', 'divide', or '*modulo*', of course, because our texts have their own vocabulary. To reflect upon this vocabulary, we will examine in this section extracts from several procedure texts, the operational language of which will be rendered in **bold** with literal one-to-one translations.

Let us begin with the first procedure of the Triple Concordance system: "To calculate the luni-solar origin & concordance" 推日月元統. This procedure has us find the current 'concordance' and the years elapsed since 'concordance head'. Each concordance begins with a coincidence of syzygy, month XI, (XI-1) + winter solstice (WS) + midnight (00:00), but on a *different* sexagenary day. Knowing the sexagenary day of this WS, the years elapsed since then, and the length of a year, the text later instructs the user how to find the date of WS for the year sought. The procedure reads as follows:

> 置太極上元以來，外所求年。
>
> Set out the [number of years elapsed] since grand culmen upper origin (143 231 BCE XI-1 00:00, WS), excluding the year sought.

year sought = 2015
the 'since high origin' = 145 245 (= 2015 + 143 230)

> **盈**元法**除**之。
>
> [If/by] **overflowing (*yíng*)** the ORIGIN DIVISOR (4617), **eliminate (*chú*)** it.[25]

[25] *Hàn shū* 漢書 (Zhōnghuá shūjú ed.), 21B.1000.

Repeated exposure to sentences like this teaches one to recognize that, in conjunction with *chú* and other operations, the words (1) *yíng* 盈 'overflow', (2) *mǎn* 滿 'fill' (3) *rú* 如 'as per', and (4) *yǐ* 以 'with' are auxiliaries that function like 'by' to identify the second operand; 'it' (*zhī* 之), on the other hand, always refers to the first operand. In other words, we are meant here to "eliminate 145 245 by 4 617" or, perhaps, "eliminate 4 617 *from* 145 245." So, what does "eliminate" mean? Knowing the *purpose* of this procedure, we can safely say that it means "to eliminate all multiples of" or "*modulo*" to find the number of years elapsed since the last 'origin' (coincidence of XI-1 + WS + 00:00 + sexagenary cycle):

$$\text{eliminate } 4\,617 \text{ from } 145\,245 = 2118\ (= 145\,245 - 31 \times 4\,617)$$

The aim of this *chú* operation is clear: the *yú* 餘 'remainder/difference'. What is unclear is how it would have been carried out in practice—be it by repeated subtraction, long division, or subtraction from a table of factors—but let us leave that for another day.

The procedure continues (gray indicates repetition):

> 餘不**盈**統者，則天統甲子以來年數也；
>
> Any remainder that does not **overflow** a concordance (1539 years) is 'the number of years since Heaven concordance at *jiǎ-zǐ*$_{01/60}$'.
>
> **盈**統，**除**之，餘則地統甲辰以來年數也。
>
> [If] **overflowing** a concordance, **eliminate** it, and the remainder is 'the number of years since Earth concordance at *jiǎ-chén*.$_{41/60}$.
>
> 又**盈**統，**除**之，餘則人統甲申以來年數也。
>
> [If] **overflowing** yet another concordance, **eliminate** it, and the remainder is 'the number of years since Man concordance at *jiǎ-shēn*.$_{21/60}$.
>
> 各以其統首日為紀。
>
> Each [concordance] takes the (sexagenary) head day of the concordance as the mark (from which to count subsequent sexagenary dates in the procedures to follow).[26]

Here, 'elimination' in the sense of *modulo* would give us 579 (= 2118 − 1 × 1539), but that is not what the procedure is instructing us to do. Instead, we are being told to 'eliminate' 1539 *by subtraction*, noting each time whether the difference is sufficient to keep 'eliminating'. In one procedure, curiously, the Triple Concordance system deploys the word *chú* 'eliminate' in two mutually exclusive senses.

***

Now let us turn to the Supernal Emblem system (extant version *terminus post quem* 206 CE) and the procedure "extrapolating the sun's *dù*" 推日度 for midnight, XI-1 of the year sought. The procedure uses the integer 'accumulated days' of XI-1 00:00 from epoch (*jì* 紀 'era' rather than 'concordance' in this case) as calculated in a prior procedure ("extrapolating new moons" 推朔), which, for the year 2015, is 128 546.[27] The procedure reads:

> **以**紀法**乘**積日，**滿**周天**去**之。
>
> **Mount** (multiply) the accumulated days **by (*yǐ*)** the ERA DIVISOR (589). [If/as] **filling (*mǎn*)** CIRCUITS OF HEAVEN (215 130), **remove (*qù*)** it.
>
> 餘**以**紀法**除**之，所得為度。
>
> As to the remainder, **eliminate (*chú*)** it **by (*yǐ*)** the ERA DIVISOR, and what is obtained is the *dù*.[28]

[26] Ibid, 21B.1000–01.
[27] Note that something is funny about my LibreOffice Calc file, so this might be wrong.
[28] *Jìn shū*, 17.507.

In the Supernal Emblem system, the *lǜ* CIRCUITS OF HEAVEN and ERA DIVISOR give us the number of *dù* in a circuit *and* days in a year: $215\,130 : 589 = 365\frac{145}{589}$. Since we want to find the *dù* traveled from winter solstice (below), it is clear that the text is instructing us to 'remove' full circuits of heaven from the 'accumulated days'—which for the sun we freely treat as *dù*—via a *modulo* operation. Because the number of *dù* in a circuit of heaven is fractional, however, rather than deal with mixed numbers, we convert the accumulated *dù* into 'parts' (1/589 *dù*) and 'remove' (*modulo*) full circuits in the same unit. When we have finished, we convert the 'remainder/difference' back into *dù* by 'elimination', *which here signals division*:

$$\begin{aligned} \text{mount } 128\,546 \text{ by } 589 &= 75\,713\,594 && (= 28\,546 \times 589) \\ \text{remove } 215\,130 \text{ from } 75\,713\,594 &= 202\,964 && (= 75\,713\,594 - 351 \times 215\,130) \\ \text{eliminate } 589 \text{ from } 202\,964 &= 344; 348 && (= 202\,964 \div 589) \end{aligned}$$

We begin to see here just how confusing the terminology is. Both *chú* 'eliminate' and *qù* 'remove' can order a *modulo* operation. But the object of *chú* 'eliminate' not only the 'remainder/difference' of subtraction (above) and repeated subtraction (*modulo*), it can also be the integer quotient of division. Here, in fact, the procedure makes no mention of the remainder; it is only in the following lines that it becomes clear that it should be noted and kept.

The procedure continues:

> 命度以牛前五度起，宿**次除**之，不**滿**宿，即天正朔夜半日所在。
>
> Count from 5 *dù* prior to $\text{Ox.}_{\text{L09/28}}$ (the winter solstice), **sequentially (*cì*) eliminating (*chú*)** it [by] the lodges. [When you] cannot **fill (*mǎn*)** [another] lodge, that is the position of the sun at midnight of the new moon of astronomical [month] I (civil month XI).[29]

This step requires the table of lodge-widths provided later in the text, for which we may handily substitute Figure 1. The procedure is ordering the reader to convert a 'quantity'—344 *dù*, the computed travel from winter solstice—into the 'civil coordinates' of 'lodge-*dù*', i.e. the lodge and the 'lodge-entry *dù*'. To do this, one must 'eliminate' the lodges counted from winter solstice 'in succession' *cì* 次 by *subtraction*:

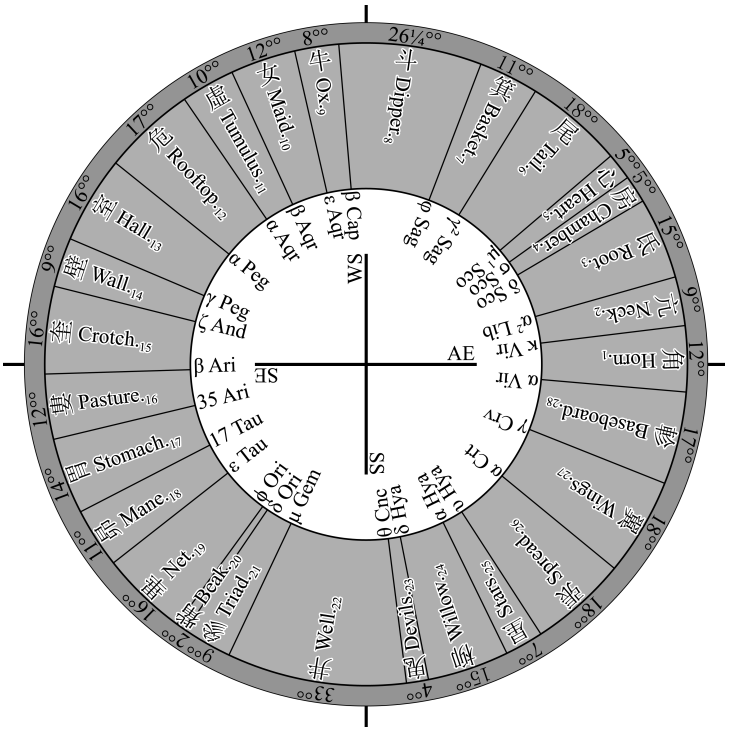
$$\begin{aligned} \text{Distance from WS at } \text{Ox.}_{\text{L09/28}} - 5 &= 344[;348] \\ \text{Eliminate } 5 &= 339[;348] \\ \text{Eliminate } \text{Ox.}_{\text{L09/28}}\ 8 &= 336[;348] \\ \text{Eliminate Maid.}_{\text{L10/28}}\ 12 &= 324[;348] \\ \text{Eliminate Tumulus.}_{\text{L11/28}}\ 10 &= 314[;348] \\ \text{Eliminate Rooftop.}_{\text{L12/28}}\ 17 &= 297[;348] \\ &\ldots\ldots \quad \ldots\ldots \\ \text{Eliminate Root.}_{\text{L03/28}}\ 15 &= 39[;348] \\ \text{Eliminate Chamber.}_{\text{L04/28}}\ 5 &= 34[;348] \\ \text{Eliminate Heart.}_{\text{L05/28}}\ 5 &= 29[;348] \\ \text{Eliminate Tail.}_{\text{L06/28}}\ 18 &= 11[;348] \\ \text{Eliminate Basket.}_{\text{L07/28}}\ 11 &= 0[;348] \\ \therefore &= \text{'beginning' of Dipper.}_{\text{L08/28}}\ (0; 348\ \textit{dù}\ \text{entered}) \end{aligned}$$

We are no longer in the realm of *lǜ*; we are operating on mixed number 'quantities' in *dù* (the distance traveled from winter solstice) to convert them into 'civil coordinates' ('lodge-*dù*'). The former—344[;348]—is technically a mixed number, but we are 'eliminating' only from its integer component.

The final step has us operate on *both* components:

[29] Ibid.

Figure 1: The twenty-eight Hàn-era lodges



求次日，**加**一度。經斗**除**分；分少，**損**一度為紀法，**加**焉。

To find [the position of the sun at midnight] the next day, **augment** [by] 1 *dù*. Having passed through Dipper.$_{\text{L08/28}}$, **eliminate (*chú*)** the [DIPPER] PARTS (145). If the parts (remainder/denominator) are too few, **reduce (*sǔn*)** 1 *dù* into an ERA DIVISOR (589) and **augment** thereby.[30]

To calculate a position in 'lodge-*dù*', the beginning of the procedure had us convert the integer number of days-as-*dù* accumulated from the era head into ERA DIVISOR parts (1/589 *dù*) to cast out full circuits *in that unit* before converting back to *dù*. The reason for this conversion is to deal with the fractional part of a *dù*/day at the end of the circuit/year: here, $\frac{145}{589}$. The numerator of this fraction is referred to as the DIPPER PARTS (*dǒu fēn* 斗分), since the fraction is added to Dipper.$_{\text{L08/28}}$ (see Figure 1). Why is Dipper.$_{\text{L08/28}}$ the only lodge of non-integer size? Dipper.$_{\text{L08/28}}$ has 'parts' for the sake of computational ease: with the winter solstice at "5 *dù* prior to Ox.$_{\text{L09/28}}$," placing the fraction in Dipper.$_{\text{L08/28}}$ puts it at the *end* of all the lodges. As such, it is only when we pass once again through Dipper.$_{\text{L08/28}}$ (width $\approx 26\frac{1}{4}$) into Ox.$_{\text{L09/28}}$ that we have to worry ourselves about this fraction. Let us follow the example here for 2015:

.....		
XI-23 00:00	Dipper.$_{\text{L08/28}}$ 22; 348	+1
XI-24 00:00	Dipper.$_{\text{L08/28}}$ 23; 348	+1
XI-25 00:00	Dipper.$_{\text{L08/28}}$ 24; 348	+1
XI-26 00:00	Dipper.$_{\text{L08/28}}$ 25; 348	+1
		−0; 145
XI-27 00:00	Ox.$_{\text{L09/28}}$ _; 203	+1
XI-28 00:00	Ox.$_{\text{L09/28}}$ 1; 203	+1

[30] *Jìn shū*, 17.507.

In this line, *chù* 'eliminate' can only mean subtraction. After repeatedly adding 1 to the sun's 'lodge-*dù*' position, it would make no sense to suddenly divide or *modulo* its position by $\approx \frac{1}{4}$ *dù*.

***

So, is the problem specific to the context of a specific procedure or, perhaps, the ambiguity of the word *chú*? No. Consider the "parts problem" as it appears in the Epochal Excellence system of 444 CE. Due to precession, DIPPER PARTS is now HALL PARTS, and Hé Chéngtiān's 何承天 (c. 370–447) choice of parameters have changed these from $\frac{145}{589}$ to $\frac{150}{608}$ *dù*; the procedure, however, remains the same. Hé Chéngtiān expresses this procedure three different ways:

> 推日所在度法：... 求次日，日**加**一度，經室**去**度分。
>
> Method for extrapolating the *dù*-position of the sun: ... To find [the position of the sun at midnight] the next day, **augment** [by] 1 *dù*. Having passed through Hall.$_{\text{L13/28}}$, **remove (*qù*)** the [HALL] DÙ PARTS.[31]
>
> 推合朔度：..... 經室**除**度分。
>
> To extrapolate the *dù* of syzygy-conjunction: ... To find [the position of the mean moon at syzygy,] the next month, **augment** [by] 29 to the *dù*, 161 to the big parts, & 14 to the small parts. ... Having passed through Hall.$_{\text{L13/28}}$, **eliminate (*chú*)** the [HALL] DÙ PARTS.[32]
>
> 推五星法：... 經室**去**分，不足**減**者，**破**全度。
>
> Method for extrapolating the five stars (planets): ... Having passed through Hall.$_{\text{L13/28}}$, **remove (*qù*)** the [HALL] PARTS; if insufficient to **diminish (*jiǎn*)**, **break apart (*pò*)** [a] whole *dù* (into the appropriate number of parts, to be added to the denominator and subtracted from).[33]

Is all this ambiguity confusing? Yes. Rest assured though that our historical subjects found it confusing too. Consider the following definition that Liú Hóng 劉洪 (c. 135 – c. 210) feels it necessary to provide the reader near the end of his Supernal Emblem system:

> 凡言**如**、**盈**、**約**、**滿**，皆求實之**除**也；**去**及**除**之，取盡之**除**也。
>
> Any talk of 'as per' (*rú*), 'overflow' (*yíng*), 'simplify' (*yuē*), and 'fill' (*mǎn*) all [refers to] **elimination (*chú*)** to seek the solid (integer quotient); 'remove' (*qù*) as well as 'eliminate (*chú*) it' [refer to] **elimination (*chú*)** to take the exhausted (remainder).[34]

Not only does Liú Hóng equate multiple terms, and define one word by itself, he makes no mention the *third* use of 除 (subtraction), nor are his definitions entirely consistent with his usages. Particularly odd is the fact that Liú does not actually use the word *yíng* in the sense that he defines it, nor had *anyone* done so since the Triple Concordance system (c. 5 CE), two centuries earlier. Liú Hóng's Supernal Emblem system (*terminus post quem* 206 CE) is the only *lì* of the early imperial period to define its terms. For what it's worth, this quotation is the best and only written explanation of operational terminology that we possess.

5 Preliminary Statistical Analysis

If there is any intelligible pattern to the vocabulary of division, subtraction, and *modulo* in *lì* literature, it does not leap to the modern reader's eye at first glance. There is simply no hard and fast correspondence between *word* and *intended operation*, nor is there any clear pattern that we might ascribe *beyond the word* to auxiliary verbs, the type of operand & number sought, procedural context, or author. When one reads at once through a 600-year block of the tradition

[31] *Sòng shū* 宋書 (Zhōnghuá shūjú ed.), 13.276.
[32] Ibid., 13.279.
[33] Ibid., 13.283.
[34] *Jìn shū*, 17.528.

in a single weekend, however, one *is* left with the vague impression/caffeine-addled delusion of there being certain patterns to the distribution and development of its language. This author's impressions, claims to authority, and carefully chosen examples are hardly sufficient evidence of order. To know for certain what is going on with this terminology, we must collect every instance of a particular word, identify that instance's intended operational meaning, and then sort these correspondences vis-à-vis one another (and other variables) until order either emerges or recedes. I began this work only on March 22, 2015, but I should like to present here the findings of my preliminary analyses.

5.1 Methodology

How does one find all instances of a word in the procedure text corpus? This is easy part. One simply downloads the standard Zhōnghuá shūjú 中華書局 edition of these texts as digitalized and made publicly available on Academia Sinica's Scripta Sinica website and runs a 'find & replace' in the word processor of one's choice to highlight the lexical items in question. An additional reading of the text then reveals any expressions one may have missed. When known terms for division like 'eliminate' and 'remove' are typically followed by the phrase "is x, while the remainder is y" 爲 x，餘則 y, for example, one begins to note *other terms* that proceed this formula. It is in this manner that I came to realize that the auxiliary verbs *yíng* 'overflow', *mǎn* 'fill', and *rú* 'as per' often initiate division on their own, independent of the more familiar vocabulary of division.

I suspect that the proscription of division by formulae like "x as per y is z, while the remainder is r" is an instance of abbreviation. *Lì* literature abounds with abbreviation. Since the incremental addition of any '*dù* traveled' (*xíng dù* 行度) inevitably leads to the "parts problem" at the end of every circuit (p. 14), for example, it is only natural that we see abbreviations like "augment & eliminate as per the previous method" 加、除如前法, "[crossing] Tumulus.$_{\text{L11/28}}$, remove [Tumulus] parts as per the above method" 虛去分如上法, and so on.[35] Comparison of how the same text talks about adding mixed numbers likewise suggests that the use of auxiliary verbs like 'overflow', 'fill', and 'as per' for division is also an act of abbreviation. Take the following two procedures from the Orthodox Glory system (518/520 CE) for example:

> 求交道所在月：... 乃**以**十一月朔小餘**加**之，**滿**日法，**除去**之，**從**日一，餘為日餘
>
> To find the month(s) in which the crossing of paths is located: ... Then, **augment** it **by (*yǐ*)** the small remainder of the syzygy of month XI. [If/as] **filling (*mǎn*)** the day divisor, **remove-by-elimination (*chú-qù*)** (÷) it and **assign (*cóng*)** one (the integer quotient) to the day; the remainder is the day remainder.
>
> 求後交月及日：... 以會數及餘加前入月日及餘，餘滿日法，從日一，如曆月大小除之，命起前蝕月，得後交月及餘。
>
> To find the next month as well as day of crossing: ... **Augment** the prior month-entry days as well as remainder **by (*yǐ*)** the coincidence number as well as [coincidence] remainder. As to the remainder, [if/as] **filling (*mǎn*)** the day divisor, **assign (*cóng*)** one (the integer quotient) to the day...[36]

So, regardless of whether we are dealing with explicit or abbreviated operations, how are we to reliably identify *words* with the *operations* they proscribe? Sometimes, as in the previous example, the immediate linguistic context gives the operation away, i.e. what produces both an integer and 'remainder/difference' is probably division. As to what is producing *only* a 'remainder/difference', however, we must rely on our understanding of ancient and modern astronomy: once we understand what the Dipper parts is, for example, we know that the procedure text means for us to *subtract* it at the end of a 'circuit' rather than *modulo*. Ultimately, though, we must work through symbolic algebra or computer code until a named procedure produces a meaningful and expected result. Again, as Clemency Montelle, Matthieu Husson, and other SAW colleagues have kindly reminded me, the operation thus deduced does not necessarily

[35] *Hòu Hàn shū* 後漢書 (Zhōnghuá shūjú ed.), *zhì* 3, 3065; *Sòng shū*, 13.296.
[36] *Wèi shū* 魏書 (Zhōnghuá shūjú ed.), 107A.2667–68.

reveal *how* it was performed—e.g. whether the 'remainder' of a *modulo* operation was sought by repeated subtraction or long division.

With this in mind, I took a three-stage approach to the correlation of *words* and *operations*. First, I identified cases of division by association with phrases like "to obtain integer x and remainder/difference y." Second, I extended identifications to words occurring in parallel procedures, e.g. the operations typical to adding or subtracting with mixed numbers. Third, I checked every instance of each word against my own automated LibreOffice spreadsheets and Liú Hóngtāo's prose and algebraic explanations. The second and third phases I performed iteratively until the intended meaning of all highlighted terms was settled and harmonized to my satisfaction.

The final step was to make these identifications speak in the aggregate. Having tagged each instance of each word, I tabulated word-operation correspondences in another LibreOffice spreadsheet. Therein, I sorted this data along two lines. First, I sorted by word to identify the semantic range and distribution of each word. Second, I sorted by meaning to assess the distribution of words used for each operation. In Section 5.2, I present the aggregate results by word to give the reader a sense of each word's semantic range and frequency of deployment in each sense. In Section 5.3, I then map this data over time to tease out potential historical trends in the meaning and choice of words.

5.2 The vocabulary of *lì* operations

減 (*jiǎn*) 'diminish'

Operational

(1) to **subtract** to 'diminish' by a quantity for an *yú* 餘 'remainder' [×233]

(2) to **divide** to 'diminish' by a quantity for integer quotient & remainder [×1]

Non-operational

(3) that which is 'diminished' vis-à-vis (i.e. less than) a given quantity [×1]

除 (*chú*) 'eliminate'

Operational

(1) to 'eliminate' a quantity by **subtraction** for a 'remainder' [×65]

(2) to **divide** to 'eliminate' a quantity for an *yi* 一 'ones' (integer) & 'remainder' [×57]

(3) to 'eliminate' all factors of a quantity by ***modulo*** for a 'remainder' [×34]

(4) *abbr.* to [sequentially] 'eliminate' lodges/months by subtraction for a 'remainder' [×21]

去 (*qù*) 'remove'

Operational

(1) ... ***modulo*** ... for 'remainder' [×110]

(2) ... **subtraction** for 'remainder' [×66]

(3) to **divide** ... for ... integer quotient & 'remainder' [×16]

(4) to 'remove' by rounding or canceling out [×3]

(5) *abbr.* ... [sequential] ... subtraction for a 'remainder' [×2]

Non-operational

(6) the 'remove' (distance) from given point in space or time [×124]

除去 (*chú-qù*) 'remove by elimination'

Operational

(1) ... ***modulo*** ... for a 'remainder' [×15]

(2) ... **subtraction** for a 'remainder' [×1]

(3) to **divide** ... for ... integer quotient & 'remainder' [×1]

約 (*yuē*) 'simplify'

Operational

(1) to **divide** to 'simplify' by a quantity for an integer quotient & remainder [×19]
(2) to place things into a lǜ 率 integer ratio (?) [×4]

【盈 • 满 • 如】X 得一 (*yíng/mǎn/rú* x *dé yī*) 'get one every full / per *x*'
Operational
(1) to **divide** by a quantity for *yī* 'ones' (integer quotient) & 'remainder' [×134]
(2) to **round up** (eliminate column and add 1 to the next column) if greater than a given quantity [×8]
Note: in definition (1), *yíng*, *mǎn* & *rú* are interchangeable and sometimes omitted auxiliaries meaning 'by'.

【盈 • 满 • 如】X 而一 (*yíng/mǎn/rú* x *ér yī*) 'every / every full *x* makes one'
Operational
(1) to **divide** by a quantity for *yī* 'ones' (integer quotient) & 'remainder' [×37]
Note: see previous entry on *yíng*, *mǎn* & *rú*.

如一 (*rú yī*) 'for-one'
Operational
(1) *abbr.* **divide** by a quantity for *yī* 'ones' (integer quotient) & 'remainder' [×5]
(2) (?) **modulo** [×1]
Note: I believe this to be an abbreviation of *rú* x *ér yī* (previous entry).

盈 *yíng* 'overflow'
Auxiliary
(1) '[if/as] overflowing', i.e. 'by' [×23]
Operational
(2) *abbr.* to **divide** by a quantity for *yī* 'ones' (integer) & 'remainder' [×6]
Non-operational
(3) 'overflow' (positive), vis-à-vis equation of center [×191]
(4) the 'overflow' or *not* 'enough' (= 'remainder') from subtraction or division [×17]
(5) if 'overflowing', i.e. if equal to or greater than [×5]

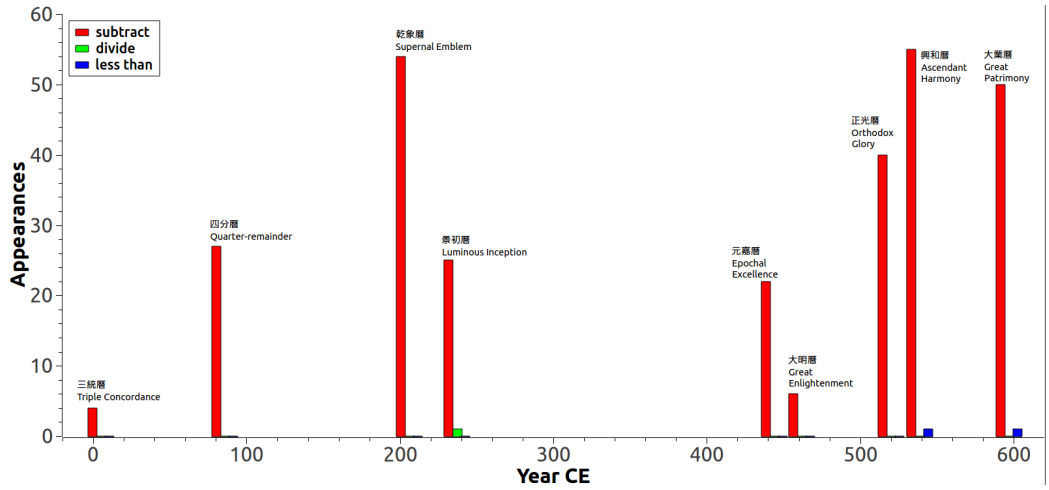
滿 *mǎn* 'fill'
Auxiliary
(1) '[if/as] filling', i.e. 'by' [×176]
Operational
(2) *abbr.* to **divide** by a quantity for *yī* 'ones' (integer) & 'remainder' [×191]
Non-operational
(3) the *not* 'enough' (= 'remainder') from subtraction or division [×20]
(5) if 'filling', i.e. if equal to or greater than [×96]

如 *rú* 'for/if/as per'
Auxiliary
(1) 'by' [×59]
Operational
(2) *abbr.* to **divide** by a quantity for *yī* 'ones' (integer) & 'remainder' [×32]

次 *cì* 'sequentially'
Auxiliary
(1) 'sequentially' or 'in order', for the progression of uneven lodge-widths (in *dù*) or civil month-lengths (in days) counted from winter solstice [×27]
Non-operational
(2) 'the next'

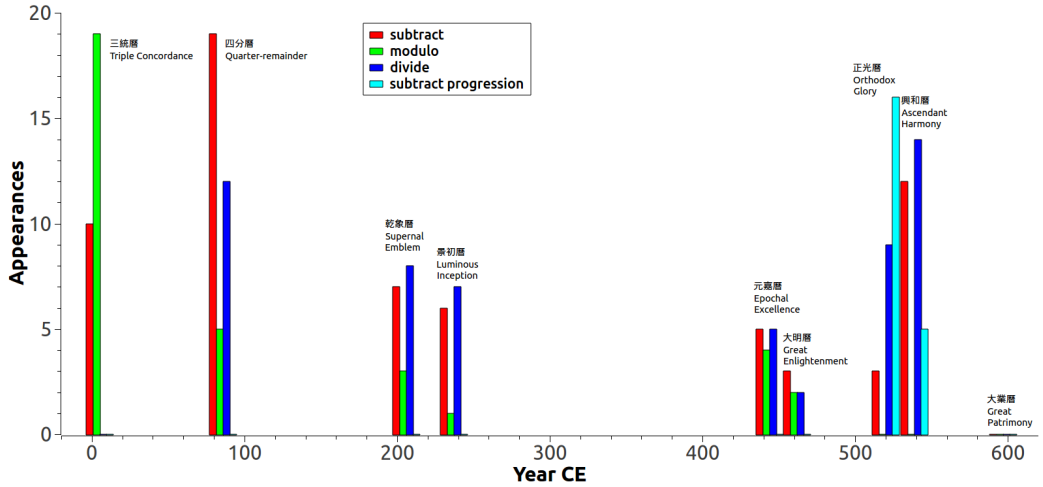
5.3 Distribution Analysis

5.3.1 減 (*jiǎn*) 'diminish'



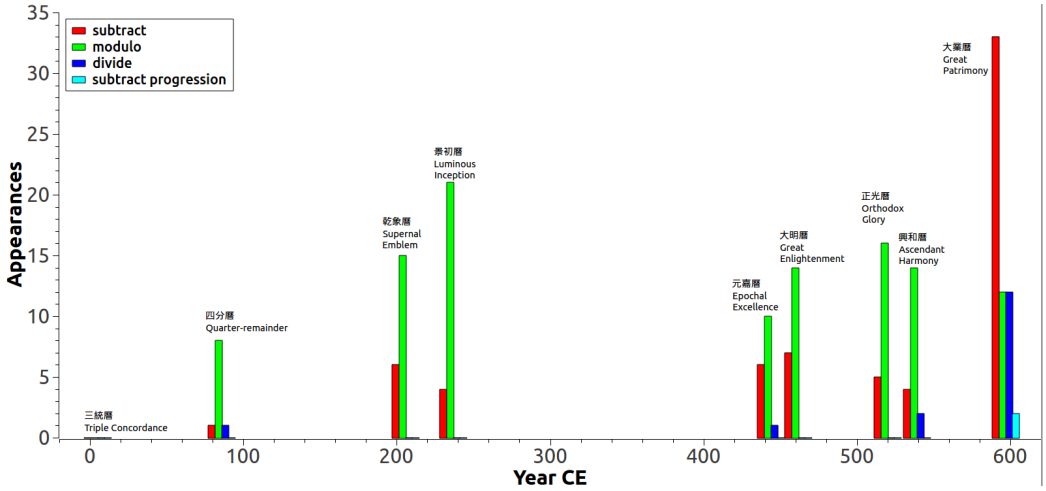
1. 'Diminish' almost exclusively for subtraction; the single instance of division (237 CE) is exceptional.

5.3.2 除 (*chú*) 'eliminate'



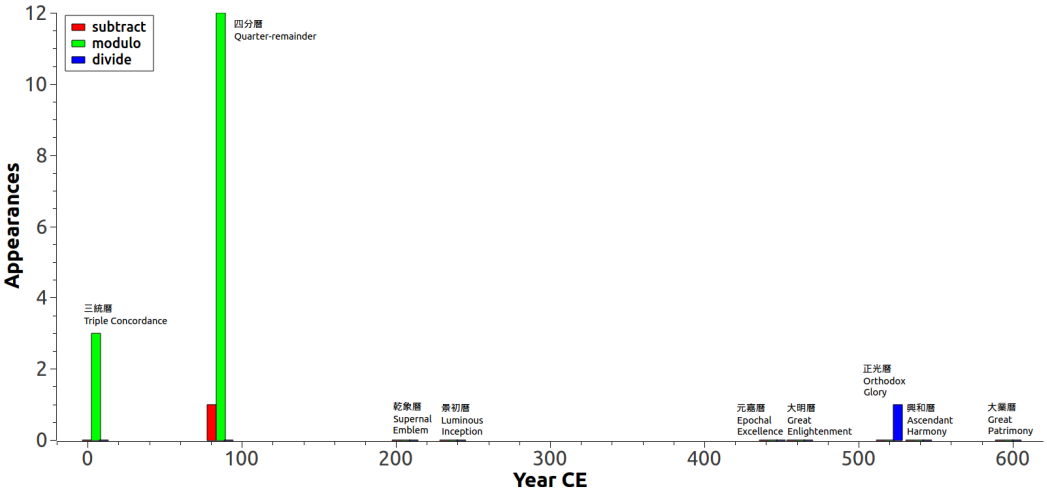
1. **5 CE:** 'eliminate' stands only for subtraction and *modulo* for 'remainder'.
2. **86 CE:** first use for division (for integer quotient *with or without* 'remainder').
3. **1st - 6th cent.:** proportionally, 'eliminate' increasingly signifies division and, decreasingly, *modulo*.
4. **6th cent.:** 'eliminate' as abbreviation of the operation 'sequentially eliminate'.
5. **597 CE:** temporary avoidance in Great Patrimony system, *why?*

5.3.3 去 (*qù*) 'remove'



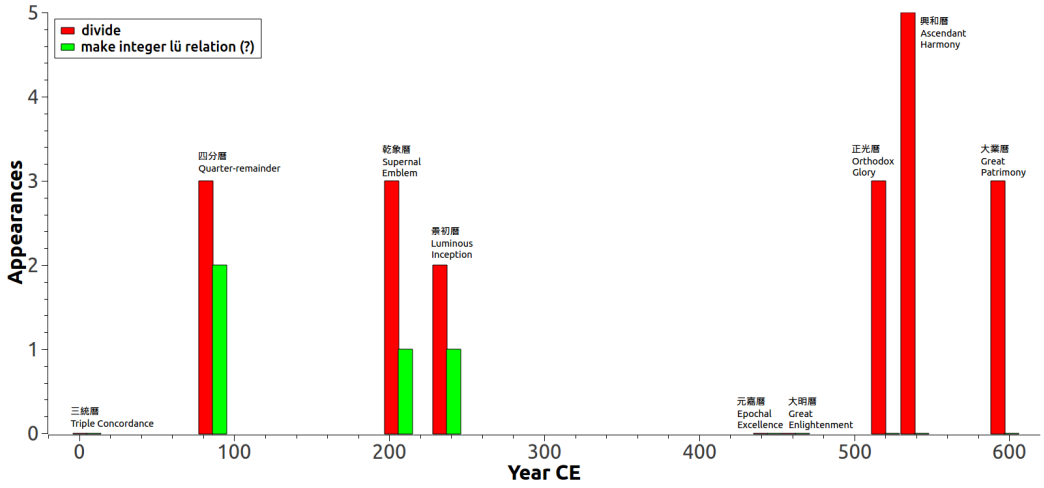
1. 'Remove' appears in 86 CE, increases in frequency thereafter.
2. Proportionally, 'remove' increasingly signifies division & subtraction and, decreasingly, *modulo*.
3. **597 CE:** the Great Patrimony system appears to compensate for dropping the word 'eliminate' by instead using 'remove' in every possible sense, including the newly coined abbreviation '[sequentially] eliminate'.

5.3.4 除去 (*chú qù*) 'remove by elimination'



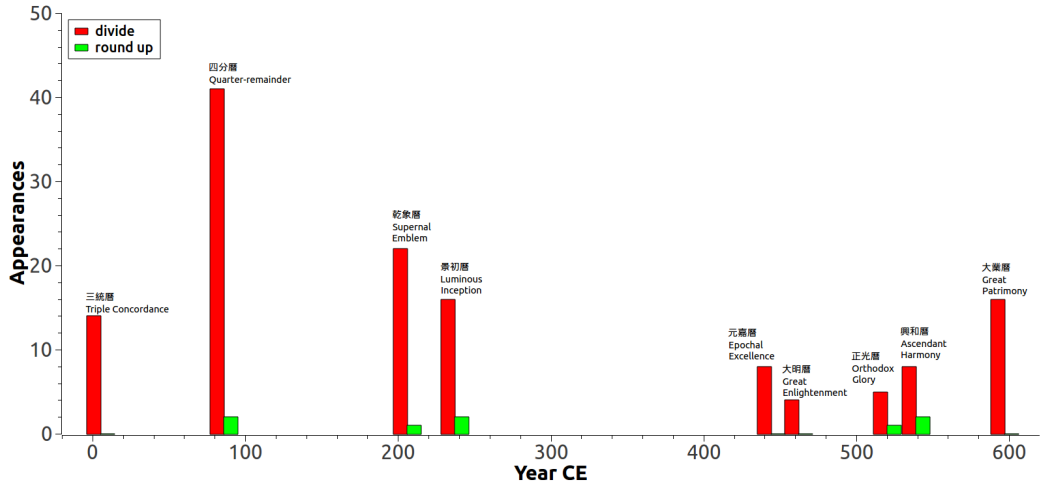
1. 'Remove by elimination' appears in three systems; *significant* use only in the 1st century.
2. The predominate sense of *modulo*, which peaks in 86 CE, replaced by 'remove' in subsequent systems (green bar, previous graph).

5.3.5 約 (*yuē*) 'simplify'



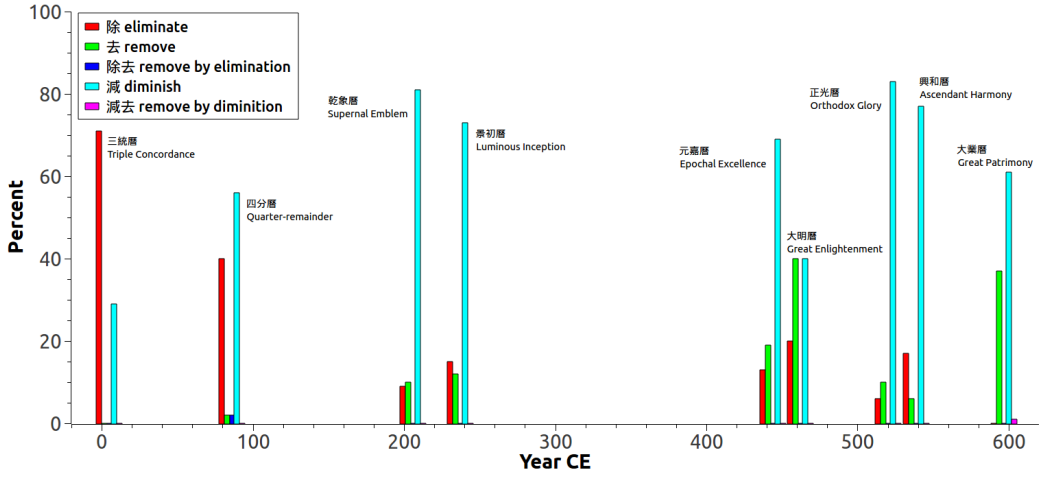
1. 'Simplify' sees relatively little use in the sense of divide.
2. Though rare, 'simplify' is the only term that deals with creating resonance-period *lǜ* 率.

5.3.6 得一 (*dé yī*) 'get one...'



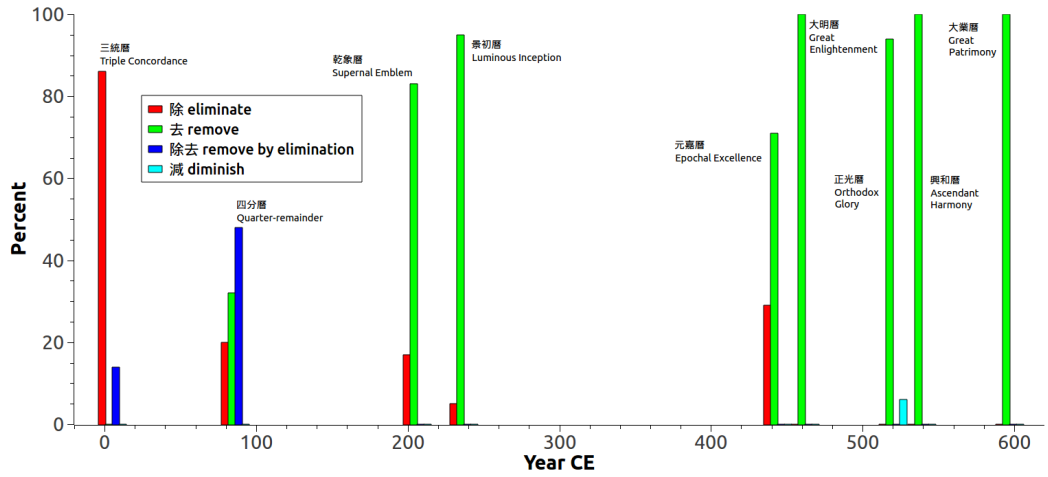
1. 'Get one...' sees extensive and nearly exclusive use for signifying division.
2. **6th cent.:** usage of 'get one' *drops* as usage of 'eliminate', 'simplify', 'for-one', etc. *increase*.

5.3.7 Subtraction for ‘remainder/difference’



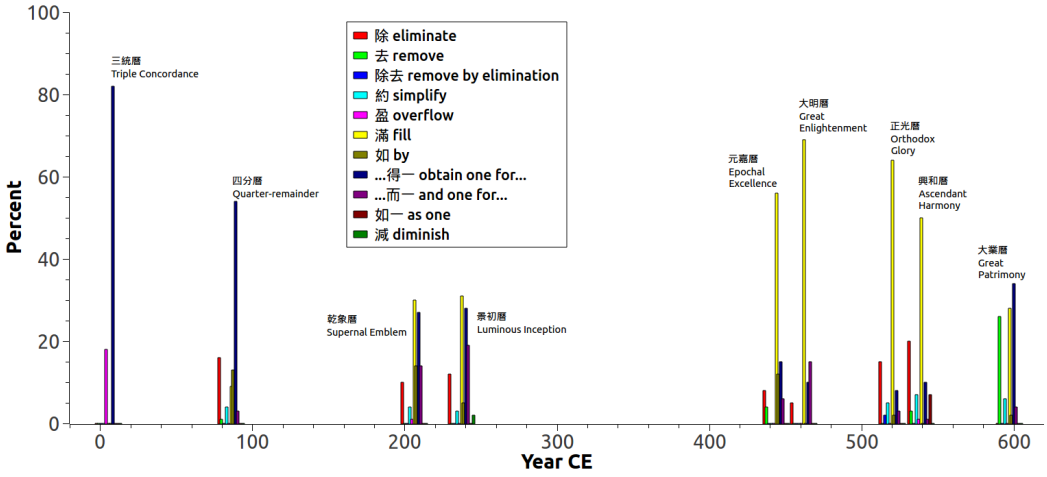
1. The use of ‘eliminate’ for subtraction falls in the first two centuries from above 70% to below 20%.
2. Trading places, ‘diminish’ takes over 60–80% of subsequent subtraction.
3. Introduced in 86 CE, ‘remove’ begins to rival ‘eliminate’s’ diminished popularity.

5.3.8 *Modulo* for ‘remainder/difference’



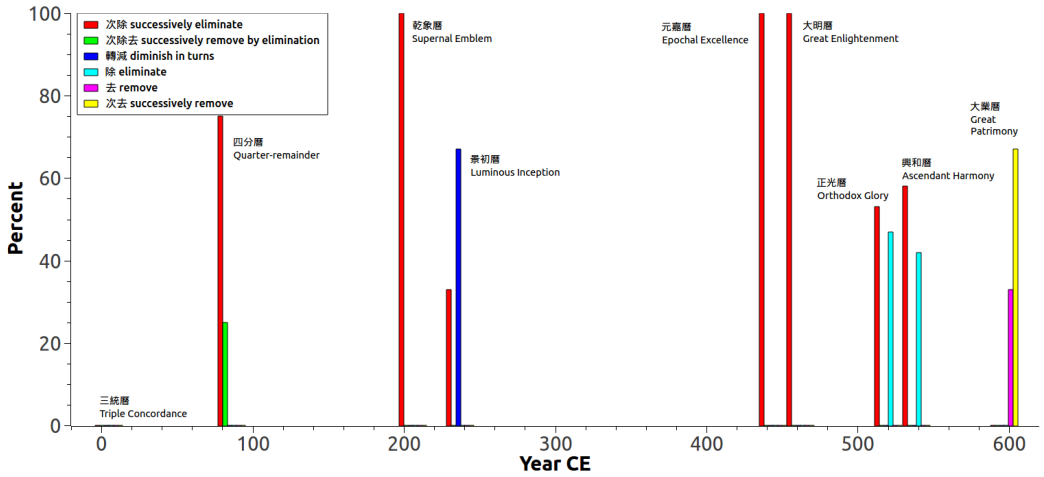
1. ‘Remove’ takes over *modulo* from ‘eliminate’ over the first two centuries as well.
2. ‘Remove by elimination’ peaks in the first two centuries and disappears.
3. The use of ‘diminish’ for *modulo* appears exceptional.

5.3.9 Division for integer quotient with or without 'remainder/difference'



1. Division is the most historically diffuse observer's category.
2. *Yíng* 'overflow's' sense of division is taken over by *mǎn* 'fill' in 85 CE to appear only twice more in the following centuries.
3. Full expressions like *dé yī* 'obtain one...' and *ér yī* '... makes one' drop steadily from 82% to 11% from the 1st to 6th cent., spiking with a spike to 38% with the At the same time, expressions that I have identify as abbreviations (above) rise from 18% to 58–69%. For whatever reason, full and abbreviated expressions spring back into parity with the Great Patrimony system at 38% & 30%, respectively.
4. Excepting the first and last *lì* system, 'eliminate' hovers around an average of 12% of division.

5.3.10 Series subtraction to find lodge/month and *dù*/days 'entered' therein



1. Abbreviations ('eliminate' & 'remove') arise to rival the standard expressions in the 6th cent.
2. Other changes are idiosyncratic.

5.4 Summary of findings

I would like to offer the following tentative speculations about the meaning of these data as arrived at on March 28, 2015. As to the ambiguity of actors' terminology, Subsection 5.2 reveals that a number of terms see consistent usage in this corpus but for a small number of exceptions: 'then one(s)' (*ér yī*) indicates division 37/37 times (100%), 'diminish' (*jiǎn*) indicates subtraction 233/235 times (99%), 'get one(s)' (*jiǎn*) indicates division 134/142 times (94%), and 'remove by elimination' (*chú-qù*) indicates *modulo* 15/17 times (88%). Other terms, we find, are considerably more flexible, i.e. 'eliminate' (*chú*) at 65 : 57 : 34 : 21, 'remove' (*qù*) at 124 : 110 : 66 : 16 : 3 : 2, and 'filling' (*mǎn*) 191 : 176 : 96 : 20. Whether or not it was *consistent*, this terminology was undeniably *redundant*. Taken as a whole, one might say of the operational vocabulary of early imperial *lì* that any number of words may be used to mean the same operation, and the same word may be used to mean any number of operations.

In Subsection 5.3, mapping the polysemy and redundancy of this vocabulary *diachronically* over the period in question adds another dimension to the problem. Some terms appear from seemingly nowhere. 'Remove' (*qù*), for example, suddenly appears in the *lì* of 85/86 CE, and 'remove by elimination' (*chú-qù*) skyrockets to prominence in the same system. Some terms drop out of use. 'Overflow' (*yíng*) disappears after the *lì* of 5 CE, and 'remove by elimination' (*chú-qù*), after its peak in that of 85/86. After disappearing, terms can also *reappear*, and in a completely different sense. 'Overflow' (*yíng*), for example, reappears by 206 CE in relation to the introduction of lunar anomaly, and 'remove by elimination' (*chú-qù*) makes an idiosyncratic appearance after four centuries of desuetude suddenly mark division. Other terms experience gradual shifts. 'Remove' (*qù*) slowly edges out 'eliminate' (*chú*) in *modulo* operations, while 'diminish' (*jiǎn*) and, to a lesser degree, 'remove' (*qù*) increasingly displace 'eliminate' (*chú*) in subtraction as well. In cases of division, finally, we see a building historical trend toward abbreviations.

6 Conclusion & Prospects

This paper set out to address two themes recurrent within the SAW project in the last several months: critical reflection on the translation of technical literature and the potential for reading *lì* and *suàn* texts side-by-side to reveal pluralities in Chinese mathematical cultures. The one, it turns out, is an effective route to the other. We began by approaching the 'numbers' (*shù*) and 'procedures' (*shù*) of *lì* literature afresh, noting the type, form, and function of different numbers and the linguistic ambiguity and redundancy of the operations performed upon them. To find some method to this madness, we then gathered and submitted *lì* vocabulary to statistical analysis from first to sixth century CE. Though my work is just begun, I nonetheless offered several observations and hypotheses about *diachronic* diversity in this one Chinese mathematical tradition.

Why is the terminology of the *lì* procedure text so confusing? At the very least, this paper has solved the author's personal conundrum: "why is this so hard?" It's hard, I now realize, because the vocabulary of subtraction, *modulo*, and division is a complete jumble. The fact that Liú Hóng felt the need to define this vocabulary—and the obscurity of his definition—suggest that, for his/her part, the ancient reader found it difficult too. Ultimately, the only sure way to know what a word like 'eliminate' is instructing you to do is to know the genre and the phenomena in question. As concerns the modern historian of science, this statement is completely banal; as concerns the ancient reader, however, this banality is, in my opinion, the strongest argument against the presumption that *lì* would have run themselves in the hands of "a minor functionary with limited mathematical skills." Then and now, you have to know astronomy before you go about 'eliminating', 'removing', or 'filling' in its name.

Why did this terminology change? Language changes, *tout simplement*, and it comes as no surprise that the language of mathematics too evolves over the course of six centuries. Some change was undoubtedly societal. The word 'overflow' (*yíng*), for example, was the given name of Hàn Huìdì 漢惠帝 (r. 195–188 BCE), and as such would have been more-or-less strictly taboo for the remainder of the Hàn dynasty (i.e. from 195 BCE – 9 CE and 23–220 CE). None of the

other operation words appear in the given names of first-millennium emperors.[37] I do, however, know of at least one example of a man named Chú in 213 BCE, so it is conceivable, for example, that the author of the Great Patrimony system avoided the term 'eliminate' out of consideration of a family or home-town taboo on the word.[38] Some tendencies might also be linked to regional or authorial habits. We might explain the relative similarity of the Epochal Excellence and Great Enlightenment systems' terminology, for example, to the fact that they were authored in *the south*, unlike the Orthodox Glory, Ascendant Harmony, and Great Patrimony systems, which issued from northern courts. The similarity between the Orthodox Glory and Ascendant Harmony systems, on the other hand, might be explained by the fact that they were authored by the same man—Lǐ Yèxìng 李業興 (484–549 CE).

One phenomenon that I am particularly keen to explore in my remaining time at SAW, in collaboration with with Karine Chemla, is how some of this terminology may have been imposed from *suàn* 'mathematics' genres. Extending the analysis presented here to the mathematical literature that came together over the same centuries should reveal, with the help of Karine's expertise, how such terminology may have moved at different rates and in different directions in each genre and/or how they spilled horizontally between them. Of particular interest to me personally is how polymaths working across these genres, like Lǐ Chúnfēng, may have either bridged *or compartmentalized* these mathematical cultures and their respective idioms in their writings.

To end back on the problem of translation, I intended this paper to bring needed attention the pitfalls of rendering actors' operational terms into modern ones as well as the potential of exploring the semantic range of those terms within, at least, the history of mathematics in China. Problematization is one thing, the bigger question is what conventions we should adopt moving forward to best respect such distinctions while guaranteeing the concision and comprehensibility of the end product. It is my naive impression, at this point in the project, that the language of operations requires more thought than the language of numbers in translating sources from this period. In my opinion, little is lost in rendering Chinese word-numbers into Arabic numerals except for precious space; nor do written numbers in English represent the structure, rhythm, length, or fundamental units of Chinese numbers (e.g. the 'myriad'). If we are to insist on avoiding modern notation, I should like to suggest that we consider instead a notation like "$1_y\ 3_q4_b\square_s\,8_w\ 2_q2_b9_s7$" for long integers and "$1_Z\square_C\,9_c\,5_f3/4_f$" for linear measures. When it comes to operations, however, I suggest that we give one-to-one translations to allow for the ambiguity and potential historical mutability of a word's sense while at once defining it in parentheses in understandable modern terms, e.g. "eliminate (divide)" or even "eliminate (÷)."

[37] See *Lìdài bìhuìzì huìdiǎn* 歷代避諱字匯典, ed. Wáng Yànkūn 王彥坤 (Zhèngzhōu: Zhōngzhōu gǔjí chūbǎnshè, 1997).

[38] The name Chú occurs in the Zhōujiātái calendar for Qín Shǐhuáng 秦始皇 34 (213 BCE), slip 19 reg. 2, published in in *Guānjǔ Qín-Hàn mù jiǎn-dú* 關沮秦漢墓簡牘, ed. Húběi-shěng Jīngzhōu-shì Zhōuliáng yùqiáo yízhǐ bówùguǎn 湖北省荊州市周梁玉橋遺址博物館 (Běijīng: Zhōnghuá shūjú, 2001).