



HAL
open science

Strategic Choices in Polygamous Households: Theory and Evidence from Senegal

Pauline Rossi

► **To cite this version:**

Pauline Rossi. Strategic Choices in Polygamous Households: Theory and Evidence from Senegal. 2016. halshs-01340567v2

HAL Id: halshs-01340567

<https://shs.hal.science/halshs-01340567v2>

Preprint submitted on 11 Oct 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



PARIS SCHOOL OF ECONOMICS
ÉCOLE D'ÉCONOMIE DE PARIS

WORKING PAPER N° 2016 – 14

**Strategic Choices in Polygamous Households:
Theory and Evidence from Senegal**

Pauline Rossi

JEL Codes: C72, D13, J13, J16, O15, O55.

Keywords : Fertility, Polygamy, Africa, Noncooperative models, Duration models.



PARIS-JOURDAN SCIENCES ÉCONOMIQUES

48, Bd JOURDAN – E.N.S. – 75014 PARIS
TÉL. : 33(0) 1 80 52 16 00=

www.pse.ens.fr

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE – ÉCOLE DES HAUTES ÉTUDES EN SCIENCES SOCIALES
ÉCOLE DES PONTS PARISTECH – ÉCOLE NORMALE SUPÉRIEURE
INSTITUT NATIONAL DE LA RECHERCHE AGRONOMIQUE – UNIVERSITÉ PARIS 1

Strategic Choices in Polygamous Households: Theory and Evidence from Senegal

Pauline Rossi*

August, 2018

Abstract

This paper proposes a strategic framework to account for fertility choices in polygamous households. It uses unique data on fertility histories of a representative sample of co-wives in Senegal to estimate a duration model of birth intervals with individual baseline hazards. Exploiting entries and exits of co-wives as well as gender of births, empirical tests show that children are strategic complements. One wife raises her fertility in response to an increase by the other wife, because children are the best claim to resources controlled by the husband. This result is the first quantitative evidence that the competition between co-wives drives fertility upwards. It suggests that polygamy undermines the fertility transition in Sub-Saharan Africa by incentivizing women to want many children. This paper is also one of the few attempts to open the black box of non-nuclear families, placing strategic interactions at the heart of household decision-making.

Keywords: Fertility, Polygamy, Africa, Noncooperative models, Duration models.

JEL Codes: C72, D13, J13, J16, O15, O55.

*p.rossi@uva.nl, University of Amsterdam, Roeterstraat 11, 1018WB Amsterdam, the Netherlands. I am grateful to Sylvie Lambert and Xavier d'Haultfoeuille for their advice and guidance, and to Michele Tertilt, four anonymous referees, Chris Udry, Pramila Krishnan and Catherine Guirkinger for their careful reading of the paper and their numerous suggestions. This paper benefited from discussions with Philippe Aghion, Josh Angrist, Nava Ashraf, Yann Bramoullé, Denis Cogneau, Pascaline Dupas, Sergei Guriev, Véronique Hertrich, Marie Laclau, Hessel Oosterbeek, Erik Plug, Jérôme Pouyet, Birendra Rai, Marcos Rangel, Rohini Somanathan, Michelle Sovinsky, Hannes Schwandt, Diego Ubfal, Liam Wren Lewis and Roberta Ziparo. For their helpful comments, I also thank participants to Journées LAGV, Annual Meeting of AFSE, EEA Congress, EDP Jamboree, OxDev Workshop, NEUDC, Novafrika, PopPov, Essen Health Conference and to seminars at CREST, PSE, Collège de France, Université de Namur, HEC Lausanne, University of Amsterdam, Stanford and UCLA.

1 Introduction

In the past 50 years, the developing world has experienced a sharp decline in fertility from six or seven down to two or three children per woman. One noteworthy exception is Sub-Saharan Africa, where an average woman still gives birth to five children today (World Development Indicators). In the general view, this slow fertility transition primarily reflects women's lack of control over births.¹ Women have supposedly more children than they want, either because they lack access to modern contraceptives or because they obey men. This paper investigates another explanation: what if women themselves had a vested interest in large families? I claim that a specific feature of African family systems, namely the prevalence of polygamy,² generates incentives for women to want many children. The main idea is that co-wives in polygamous unions compete for more children, because their status and their access to resources are determined by their relative number of children.

Substantiating this claim is a challenge due to puzzling empirical results and lacking theoretical models. Existing evidence on the relationship between polygamy and fertility is at first sight contradictory. On the one hand, anthropologists have qualitatively documented that a strong reproductive rivalry exists between co-wives (Jankowiak, Sudakov, and Wilreker 2005). On the other hand, demographers have established that, at the micro level, women engaged in polygamous unions tend to have fewer children than other women (Lesthaeghe 1989). The economics literature is of little help to rationalize the evidence, because the focus is on nuclear families i.e. two parents and their children. All theoretical models assume that fertility is a joint decision of only two spouses, and that the husband and the wife necessarily have the same number of children.³ Since men typically report wanting more children than women, these models emphasize that spouses disagree, and that the realized fertility increases with husband's bargaining power. In a polygamous context, this is often irrelevant or even misleading. Indeed, the discrepancy in preferences does not necessarily translate into a conflict because men and women can realize their fertility individually, as already pointed out by Field, Molitor, Tertilt, and Schoonbroodt (2016). Moreover, it is not clear that men's preferences drive the fertility of each wife upwards. For all these reasons, a specific framework is needed to think about fertility choices in polygamous societies.

¹For instance, the United Nations (2015) recommend that, to curb fertility in Africa "it is essential to invest in reproductive health and family planning, so that women can achieve their desired family size".

²Polygamy is a marriage that includes more than two partners. It encompasses both polygyny, in which a man has several wives, and polyandry, in which a woman has several husbands. Throughout this paper, I use the term polygamy to refer to the former situation, which is by far the most practiced form of polygamy.

³See Doepke and Kindermann (2017) for a review of the literature.

This paper provides the first quantitative evidence that co-wife rivalry raises fertility. This has implications for aggregate demographic trends and public health in Sub-Saharan Africa. Throughout the continent, polygamy is not a marginal phenomenon limited to a small elite. Arthi and Fenske (2018) recently estimated that roughly 25% of married women in Sub-Saharan Africa are in polygamous marriages. As shown by Figure 1, this proportion ranges between one third and one half in the so-called "polygamy belt", an area between Senegal and Congo (Jacoby 1995). In total, approximately 75 million women, and hundreds of millions of people if husbands and children are included, currently live in polygamous families.⁴ This paper relates polygamy to slow fertility transitions, which are a matter of concern for policy makers. The United Nations (2015) predict that the population in Sub-Saharan Africa will be multiplied by four by 2100, reaching 4.4 billions of people. This generates a range of social, economic, and environmental challenges described in Dasgupta (1995). A particular worry is that frequent pregnancies put women's lives in jeopardy.⁵ Maternal mortality is the second main cause of female excess mortality, leading to a large number of "missing women" (Anderson and Ray 2010). Deaton (2013) claims that these health inequalities are one of the great injustices of the world today. My results suggest that women trade the health costs of child-bearing off for a safe future because they have no option but to rely on their children to protect their economic security. Population policies should therefore aim at improving not only women's control over fertility, but also more generally improving their autonomy.

This paper further contributes to the literature by emphasizing the role of household bargaining for fertility decisions, as advocated by Doepke and Tertilt (2018). I build upon recent theoretical and empirical studies to rule out cooperative approaches. The main argument is that inefficiencies are likely to arise in polygamous households due to coordination, information, monitoring and commitment problems. Beyond polygamy, non-cooperative approaches may also be relevant to analyze step-families, a living arrangement that gains momentum in the Western world as marriage practices evolve towards serial monogamy (divorce and remarriage). To the best of my knowledge, there is no theoretical attempt to model strategic interactions in non-nuclear families. This paper aims at opening the way.

The theoretical section organizes three potential mechanisms through which polygamy might have an impact on fertility. First, a competition effect; it reflects the rivalry between

⁴From a longitudinal perspective, there is no global estimate, but numbers would be much higher. For instance, the polygamous rate in my Senegalese data is only 1/3 for women below age 30 compared to 2/3 for women above age 60.

⁵In Sub-Saharan Africa, the lifetime risk of maternal death is 1 in 38. Each year, 180,000 women die from causes related to childbirth or pregnancy (WHO, UNICEF, UNFPA and The World Bank 2014). It amounts to approximately half of the deaths related to AIDS in the same age group (aidsinfo.unaids.org).

co-wives, who care about the *relative* number of children, driving fertility upwards. Second, a substitution effect; it comes from the side of the husband, who cares about his *total* number of children, imposing some limits on the fertility of each wife. Third, a natural effect; by limiting the frequency of intercourse, polygamy has traditionally been a way to lengthen birth spacing. The last effect exists independently of the fertility of the other wife, whereas the first two effects have a strategic component. Since these forces go in opposite directions, it is a priori not clear whether polygamy should increase or decrease fertility, nor whether children should be strategic complements or substitutes.

The model provides some guidance to solve these questions empirically using a difference in difference with duration data. First, to identify the total impact of polygamy on fertility, I look at the change in first wives' birth spacing before and after the arrival of the second wife. Second, to identify the type of strategic interactions, I test if first wives react differently depending on who they face as a competitor. I exploit the variation in the timing of the second marriage, in particular in the husband's and wife's ages at marriage, which are strong predictors of how many children the couple will have. Formally, I estimate a duration model of birth intervals with a baseline hazard specific to each woman. The empirical strategy thus compares the birth rates of the same woman in the monogamous stage and in the polygamous stage, not monogamous and polygamous wives, nor senior and junior wives. The main advantage is to deal properly with endogeneity issues related to time-invariant unobserved heterogeneity. The key identification assumption is a common trend assumption. It states that, in the absence of the second marriage, the evolution of birth spacing over the life-cycle should have been the same for all women. I test this assumption on births that occurred before the second marriage, and show that it holds. Finally, I provide two supplementary pieces of evidence exploiting (i) the opposite transition, from polygamous to monogamous, and (ii) the gender of births, boys being more valuable than girls in this context.

Implementing such a strategy requires information that is rarely available. In particular I need to observe all spouses in a given union, and to know the dates of successive marriages as well as the birth dates of all children. I exploit original data from a Senegalese household survey that provides information on a husband and all co-wives, even if they do not live in the same household, and detailed information on the timing of marriages and births.

My main result brings together demographers' and anthropologists' findings: polygamy has, in total, a negative impact on fertility *and* co-wife reproductive rivalry is a strong upward force. I find that first wives lengthen birth spacing in the polygamous stage, but less when the second wife's reproductive period is longer. It implies that the competition

effect dominates the substitution effect, leading to a positive strategic response. Yet, the response is not large enough to offset the negative natural effect. In accordance with this result, I find that, after the exit of a co-wife, fertility increases, and increases more if the exiting wife had more children. There is also suggestive evidence that second wives shorten birth spacing when they face a more fertile co-wife, too. Last, both wives react to the gender composition of their own births and the other's births: they intensify fertility when they have no son, all the more so as the co-wife has a son. An important finding is that indicators of socio-economic development like (male) wealth and female education are associated with a stronger strategic response. It implies that, when wives have more incentives to compete and more control over births, strategic interactions gain momentum, thus undermining the fertility transition.

The outline of the paper is as follows. Section 2 provides interdisciplinary background on fertility in polygamous unions. Section 3 presents the data and some statistics. Section 4 sets up the model and derives a test for strategic interactions. Section 5 describes the empirical strategy, and Section 6 reports the results. Section 7 deals with robustness tests and alternative models. Section 8 discusses policy implications and concludes.

2 Literature review

This section reviews the main insights into the relationship between polygamy and fertility from the literature in anthropology, demography and economics.

2.1 Anthropology: co-wife reproductive rivalry

Conflicts between co-wives are pervasive in polygamous societies: co-wife rivalry is a recurring theme in African novels⁶ and it has been thoroughly studied by anthropologists and sociologists working on polygamous ethnic groups. Jankowiak, Sudakov, and Wilreker (2005) gathered information on co-wife interactions in 69 polygamous systems from all over the world (among which 39 are in Africa) to identify the determinants of co-wife conflict and cooperation. They conclude that conflict is widespread and primarily caused by competing reproductive interests. They note that "reproductive vitality, women's age in the marriage, and the presence or absence of children influence a woman's willingness to enter in or avoid

⁶See for instance books by Chinua Achebe, Sefi Atta, Mariama Ba, Fatou Diome, Buchi Emecheta, Aminata Saw Fall and Ousmane Sembene.

forming some kind of pragmatic cooperative relationship with another co-wife.” Thus, conflict is less prevalent when one wife cannot have children.

In the African context, various studies have documented that wives in polygamous unions overbid for children. Women commonly resort to marabouts to increase their own chances to get pregnant and to cause infertility or stillbirths for the co-wife (Fainzang and Journet 2000). In most extreme cases, aggressions may jeopardize children’s lives. Child mortality is found to be higher in polygamous households and co-wife rivalry is considered as one important risk factor (Strassman 1997, Areny 2002). Anthropologists further argue that women are ready and able to control births: ”most demographers have failed to perceive women’s efforts to plan their childbearing and how the timing, pacing and termination depends not only on her own fertility status but that of other co-resident women” (Madhavan and Bledsoe 2001).⁷

There are many reasons why wives care so much about their number of children, relative to the number of children of co-wives. This difference defines social status, authority over co-wives and husband’s respect (Fainzang and Journet 2000). It may also be interpreted as a sign of husband’s sexual and emotional attention, which clearly matters for the wife’s well-being when jealousy is rife (Jankowiak, Sudakov, and Wilreker 2005). In an economic perspective, the relative number of children directly determines the wife’s share in husband’s resources (Bledsoe 1990). It is particularly important in case of widowhood because in most African countries, a man’s bequest is to be shared among his children. The surviving wives generally have little control over inheritance other than through their own children (United Nations 2001, Lambert and Rossi 2016). In polygamous unions, each wife needs thus to ensure that enough children are born to secure current and future access to husband’s assets.

2.2 Demography: lower fertility in polygamous unions

Although the reproductive rivalry has been qualitatively documented, quantitative evidence is very scarce. Demographers have been working on the relationship between polygamy and fertility for a long time, but their theoretical framework does not take into account strategic behaviors. They assume that a regime of natural fertility prevails in Sub-Saharan Africa.

⁷Women use traditional methods to slow down or speed up births in the absence of modern contraceptives (Bledsoe, Banja, and Hill 1998, Caldwell and Caldwell 1977, Castle 2001). First, they can vary the extent of breastfeeding, a practice that substantially inhibits subsequent fertility (cf. Jayachandran and Kuziemko (2011) for a comprehensive summary of the medical literature). Second, they have some control over abstinence. They ”enjoy a measure of autonomy allowing them to resist their husband’s sexual advances, an autonomy which is increased when a co-wife is available to share conjugal duties” (Lesthaeghe 1989). In Mali, Madhavan (2002) reports that some women voluntarily give up their assigned nights with their husbands, allowing them to sleep with co-wives.

It implies that a woman's fertility is mostly determined by external factors influencing the length and the intensity of her exposure to pregnancy risk: men's and women's ages at marriage, imperatives for widows or divorcees to remarry, breastfeeding norms, post-partum taboos etc. This framework emphasizes the importance of biological constraints and social norms, leaving little room for individual choices.

Demographers working on Sub-Saharan Africa have established a negative correlation between polygamy and women's fertility at the micro level (see Chapter 7 in Lesthaeghe (1989) or Garenne and van de Walle (1989) and Lardoux and van de Walle (2003) for specific studies on Senegal). This is first explained by a composition effect. Infertility is more prevalent in polygamous unions because husbands of infertile women commonly take another wife, and because widowed or divorced women commonly join a polygamous marriage. The second explanation is the difference in the timing of marriage: junior wives tend to get married older, and to an older husband. These couples have shorter reproductive periods, hence fewer children. The last explanation is about the frequency of intercourse, which is lower in polygamous unions. Indeed, a rule of rotation between the wives for marital duties is implemented and highly monitored. Wives have to share the husband's bed time, which lengthens the period of time before getting pregnant. This is especially true when spouses do not co-reside and the husband pays his wives regular visits. Also, the non-susceptible period following a birth is longer in polygamous unions because the availability of alternative partners makes it easier for husbands to observe the post-partum abstinence.

Interestingly, the correlation between polygamy and women's fertility is no longer negative in the most recent data collected in many African countries, including Senegal. As far as I know, this reversal has not been documented by demographers yet. My strategic framework offers an explanation, as will be discussed in section 8.⁸

2.3 Economics: inefficiencies in non-nuclear families

To my knowledge, there is no empirical study providing quantitative evidence that the fertility behavior of a wife impacts the choices of her co-wife. This paper attempts to fill in this

⁸Moreover, strategic choices help explain some results in the study by Lardoux and van de Walle (2003) that cannot be rationalized in the natural fertility framework. First, the authors find a strong positive association between each wife's probability of child-bearing during a given year. It is inconsistent with husbands optimizing post-partum abstinence periods, and consistent with wives keeping pace with each other. Second, the presence of a wife who is past her fecund years impacts positively the fertility of younger wives. The authors had hypothesized the opposite relationship assuming that the older wife would claim her share of bed time and enforce the compliance with intercourse taboos. I interpret this as a hint that junior wives are catching up with an older wife who already gave birth to many children.

gap by adopting an economic approach. However, I cannot rely on standard economic models to formalize the decision process in polygamous families. The unitary approach abstracts from potential disagreement between members. The collective approach, although validated by influential articles examining the intra-household allocation of resources in OECD countries,⁹ does not stand up well to empirical testing in contexts where nuclear families do not prevail. For instance in Sub-Saharan Africa, a growing set of papers has evidenced behaviors that are incompatible with a key ingredient of collective models: efficiency.¹⁰ Regarding fertility, only two papers test the efficiency of outcomes in developing countries, and both provide evidence of inefficiencies. In Malaysia, Rasul (2008) finds that spouses bargain without commitment. Fertility is too high at equilibrium when wives are able to obtain a greater share of the marital surplus in renegotiation by having more children. In Zambia, Ashraf, Field, and Lee (2014) find that fertility is too high due to moral hazard. The authors totally abstract from polygamy.

There are good reasons to believe that inefficiencies are even more likely to arise in polygamous unions. In a recent survey of the literature, Baland and Ziparo (2017) question the relevance of the collective model for the analysis of polygamous households because polygamy exacerbates coordination, information and monitoring problems and undermines altruism. A lab experiment in Nigeria shows that spouses' willingness to cooperate to maximize household gains is lower in polygamous unions than in monogamous ones (Barr, Dekker, Janssens, Kebede, and Kramer 2017). More specifically on investment in children, Kazianga and Klonner (2009) reject the efficiency of resource allocation in polygamous households in Mali and point to co-wife rivalry as the main explanation. Arthi and Fenske (2016) provide descriptive evidence of strategic time mis-allocation in child care using historical data from Nigeria. They indicate that "this siloed approach to the affairs of matrifocal units, even where these units are part of a larger household, is a feature in anthropological work on Igbo polygamy".

Theoretically, inefficiencies can be explained by risk and uncertainty, limited ability to sustain cooperation due to power imbalances and low discount factors, endogenous outside options, information asymmetries and social norms (Baland and Ziparo 2017). In my context,

⁹Browning and Chiappori (1998), Bourguignon, Browning, Chiappori, and Lechene (1993), Chiappori, Fortin, and Lacroix (2002).

¹⁰They document a sub-optimal allocation of resources in agricultural production (Udry 1996, Goldstein and Udry 2008, Peterman, Quisumbing, Berhman, and Nkonya 2011, Kazianga and Wahhaj 2013, Akresh, Chen, and Moore 2016); partial risk-sharing and insurance (Dercon and Krishnan 2000, Duflo and Udry 2004); strategic appropriation of resources (Anderson and Baland 2002); lying and hiding (Baland, Guirkinger, and Mali 2011, Castilla and Walker 2013, de Laat 2014, Jakiela and Ozier 2016, Boltz, Marazyan, and Villar 2015); inefficient outcomes in public good games (Hoel 2015).

commitment is further prevented by a finite horizon: given age differences, the husband is likely to die first, after which co-wives are no longer co-wives. Children represent an irreversible investment that determines each wife’s outside option: ”traditional systems of inheritance make it plausible that polygynously married women rely on their own children for support in old-age, not the children of their co-wives, inviting competition among co-wives to get the best for their own children” (Mammen 2004).

For all these reasons, I chose a non-cooperative framework to model fertility choices in polygamous unions. In Appendix C, I show that a collective model would predict that children of co-wives are substitutes, not complements as found in the data. Intuitively, an efficient allocation of children requires that the fertility of one wife relative to the other reflects relative preferences and bargaining powers. So the share allocated to one wife is predicted to decrease with the determinants of the other wife’s fertility.

3 Data and Descriptive Statistics

3.1 Data

Poverty and Family Structure (PSF) Most empirical tests use the PSF survey conducted in Senegal in 2006-2007.¹¹ It is a nationally representative survey of 1,800 households spread over 150 primary sampling units drawn randomly among the census districts.

Standard household surveys only gather information on co-residing spouses. So the sample of unions with information on all spouses is usually selected. Instead, PSF surveys all spouses of the household head, even if they do not co-reside. I therefore have information on a representative sample of husband and *all* co-wives. This is important because roughly one fourth of women do not live with their husbands. My sample consists of 1,317 unions: 906 monogamous unions and 411 polygamous unions, among which 321 with two wives, 66 with three wives and 24 with four wives. Roughly one half of women are engaged in a polygamous union, among which 80% are bigamous unions. These proportions are in line with demographers’ estimations (Antoine 2002).

In addition to the usual information on individual characteristics, the PSF survey obtains a comprehensive description of the family structure. In particular it registers the dates of

¹¹Detailed description in Vreyer, Lambert, Safir, and Sylla (2008). Momar Sylla and Matar Gueye of the Agence Nationale de la Statistique et de la Démographie of Senegal (ANSD), and Philippe De Vreyer (University of Paris-Dauphine and IRD-DIAL), Sylvie Lambert (Paris School of Economics-INRA) and Abba Safir (World Bank) designed the survey. The data collection was conducted by the ANSD.

birth for all living children below 25, even if they do not live with their parents. Children who died are also reported but there is no information on the timing of deaths. As a result, a woman's complete birth history for surviving children is available if all her children are under 25 years old. Moreover, detailed information is collected on the marital history of all spouses: age at first marriage, date of current union, having or not broken unions, date of termination of latest union. Therefore, I am able to retrace the timing of marriages and births. One weakness of PSF is that there is no information on fertility preferences.

Demographic and Health Surveys (DHS) To get complementary information, I exploit the DHS collected in Senegal in 1992, 1997, 2005 and 2010.¹² They contain stratified samples of households in which all mothers aged 15 to 49 are asked about their reproductive history, including children who left the household or who are dead at the time of the survey. Male questionnaires are further applied to all eligible men in a sub-sample of these households.¹³ I obtain a sample of 5,226 unions.

DHS contain relevant data on fertility preferences of men and women. Respondents are asked how many children they would like to have, or would have liked to have, in their whole life, irrespective of the number they already have. The main drawback is that the DHS sample of husbands and wives is not representative of all Senegalese unions. A spouse is not surveyed if (i) she does not co-reside, or (ii) she is above the age limit, 50 for women, 60 for men in the last two waves. As a result, among surveyed women, only one in five has a match with a male questionnaire. In polygamous unions, all wives are not systematically observed; e.g. in bigamous unions, both wives are found in the sample in only half of the cases.

3.2 Descriptive statistics

Fertility in current union In Table 1, I regress the number of children born in the current union on the union type, using PSF data on women over 45 years old. Women in polygamous unions have, on average, one child fewer than women in monogamous unions. The whole gap is driven by junior wives: they have, on average, 2.5 children fewer, whereas senior wives have the same number as monogamous ones. In line with demographers' findings, roughly half of the gap between junior wives and the others is explained by infertile unions. When I restrict the sample to women having at least one child with the current husband,

¹²Three other surveys were conducted in Senegal but there is no information on men (1986, 2012-13) and the quality of data does not meet the criteria of standard DHS (1999).

¹³In 1992 and 1997, men should be older than 20 to be eligible, whereas in 2005 and 2010, they should be between 15 and 59 years old. The proportion of households selected to administer male questionnaires was 33%, except in 1997, when it was 75%.

the difference decreases down to 1.4 children.¹⁴ To investigate the role of the length of the couple’s reproductive period, I construct a proxy $T = \min(45 - \text{wife’s age at marriage}; 60 - \text{husband’s age at marriage})$ reflecting the difference in age at fertility decline between men and women. As expected, it is an important driver of women’s completed fertility.¹⁵ After controlling for T , there remains a gap of approximately one child. This last figure is unchanged if I control for having children from previous unions. When I disentangle the results by mother’s rank in polygamous unions, I find that fertility decreases as the number of wives increases. Table OA.2 in Appendix A shows that the total number of births, as well as the frequency, decreases with the mother’s rank.

Turning to men’s total fertility, it is more complicated to avoid censoring issues because older men may still have the opportunity to take another fertile wife. When I restrict the sample to unions in which all wives are above 45, and the husband is above 60, figures are consistent with statistics computed on women’s side. On average, monogamous men have 5 children, men with two wives 7.6 children, and men with three wives 9.7 children.¹⁶

Fertility preferences Preferences of African men and women differ considerably (Westoff 2010). In the Senegalese DHS, women reporting wanting on average 5.7 children, whereas men would like to have 9 children. Medians are respectively 5 and 7. Within couples, a husband wants on average 3.1 children more than his wife. Preferences are aligned in only 13% of the cases. Moreover, being childless turns out to reflect an inability to conceive rather than a choice. Less than 1% of husbands and wives report wanting no child at all.

I exploit DHS to identify the predictors of the ideal number of children of men and women. Since there is no information on preferences in PSF, I will use this set of predictors when empirical tests require controlling for preferences. Appendix A provides a detailed discussion of the predictors. Two results are worth highlighting here. On the wife’s side, there is no difference between the preferences of women in monogamous and polygamous unions. On the husband’s side, the ideal number of children is much larger for polygamous men than for monogamous ones: 12 vs. 7.5 children on average. The gap goes down to 2.7 children when I control for the whole set of predictors, and it remains significant. So there is a positive

¹⁴30% of junior wives have no child with the current husband. Those women are probably engaged in a kind of safety net union. The proportion of childless women is similar for monogamous and first wives.

¹⁵In Appendix A, Table OA.1, I test whether husband’s and wife’s age at marriage are negatively correlated to the number of children, differentiating between spouses with a small and a large age difference. If the age difference is small, the length of the reproductive period should be driven by the wife’s age at marriage, not by the husband’s age at marriage, because the wife reaches the end of her reproductive life sooner than the husband. If the age difference is large, it should be the opposite. This is exactly what I find.

¹⁶I do not report the average for unions with four wives because there are too few data points.

correlation between a man's taste for a large family and polygamy.

Timing of unions The timing of unions plays a key role in my framework. Figure 2 provides some descriptive statistics on ages at the husband's first and second marriages. Women tend to marry much older husbands: the median age at marriage is 17 for first wives against 28 for their husbands. The second marriage generally takes place around 12 years later: the husband is 40, the first wife 29, and the second wife 22. But the variation in the timing of the second marriage is large: it ranges from within 6 years in the first quartile, to after 16 years in the last quartile. First wives may be still very young when the rival arrives (below 24 in the first quartile) or already quite old (above 36 in the last quartile). The situation of second wives is even more diverse. Around one third have already been married, which explains why the age at marriage is so high in the last quartile (31 years old), while others are very young (below 17 in the first quartile). Empirical tests exploit this variation.

Note that in the vast majority of polygamous unions, the wives' reproductive periods overlap. In 70% of the cases, both wives were below 35 as the second marriage took place; in 83% they were below 40. That is why I focus on unions with fecund wives in the core of the paper. In Appendix B, I consider cases in which the first wife is past her fecund years as the second wife arrives.

I do not report statistics on the third and fourth marriages because there are too few observations. In the theoretical part, I consider a model with two players, and in the empirical part, I am interested in the impact of the second marriage. I could easily extend the model to three or four wives, but I would lack power to perform empirical tests on the third and fourth marriages.

4 Model

4.1 Key insights

The goal of the theory section is to put some structure on the different forces at play and to provide some guidance on the identification strategy. Building upon the literature review, I consider three mechanisms through which polygamy might affect fertility: strategic complementarity, strategic substitutability and non-strategic reduction.

The first mechanism is the reproductive rivalry documented by anthropologists. I call it the competition effect. It generates a positive correlation between co-wives' fertility. It can be obtained by assuming that, for women, the marginal utility of children is larger when the

co-wife has more children. This can be further micro-founded by a tournament allocating status and/or old age security based on women’s relative number of children.

The second mechanism comes from an economic, unitary view of the household. When fertility choices are driven by men’s preferences or shaped by a household budget constraint, what matters is the total number of children, not the allocation between wives. I call it the substitution effect. It generates a negative correlation between co-wives’ fertility. It can be obtained by assuming that, for men, the marginal utility of children is decreasing and hence smaller when more children are already born to the other wife. Alternatively, one can consider a household budget constraint with convex costs of children.

The third mechanism reflects the biological and social constraints described by demographers. I call it the natural effect in reference to their concept of natural fertility. It generates a negative correlation between fertility and polygamy, irrespective of the co-wife’s behavior. It can be obtained by assuming that external factors limit births more in polygamous unions than in monogamous ones.

I propose to organize these mechanisms in a simple non-cooperative framework to explore how to empirically disentangle them and, in particular, isolate the reproductive rivalry.

4.2 Formalization

As explained before, standard economic models are inadequate here. Modeling extensively the interactions between household members and the intertemporal evolution of fertility choices and outcomes is beyond the scope of this paper. A dynamic stochastic model in which competition and substitution effects arise from optimizing behaviors would best reflect the true decision process. But there is a trade-off between realism and tractability. Existing dynamic stochastic models of fertility choices cannot easily be adapted to polygamous unions. Even in a monogamous context, they do not generate closed-form solutions and the predictions derived from comparative dynamics vary from model to model (see the survey by Arroyo and Zhang (1997)). I propose a simple way to formalize the problem by (i) relating each mechanism to one stylized driver of fertility: how many children the wife wants, how many children the husband wants and what is feasible in terms of biological constraints and social norms; and (ii) writing the objective function as a weighted sum of distances.

First, the wife’s objective is determined by her own innate preferences ($n_{w,i}^{id}$) and by the number of children born to the co-wife (n_{-i}) if any. I assume that the wife targets ($n_{w,i}^{id} + \gamma_i n_{-i}$) children, where $\gamma_i \geq 0$ is a woman-specific parameter capturing the intensity of co-wife rivalry. Appendix B1 provides micro-foundations for this functional form. Second,

I assume that the husband targets a total number of children (n_h^{id}) and is indifferent to the allocation between wives. For him, all children are perfect substitutes. So his objective with wife i is ($n_h^{id} - n_{-i}$). Third, I introduce a natural birth rate λ^{nat} and a cost to deviate from this level. The idea is that birth intervals have to be in a given feasible range. If women choose a very short interval, they incur a health cost as well as a psychological cost for transgressing social norms.¹⁷ If they choose a very long interval, they have to switch from traditional to modern birth control methods, and this may entail an economic cost, an opportunity cost of time and a psychological cost to hide contraceptives from the husband or the community (Ashraf, Field, and Lee 2014). As per demographers, I further assume that the frequency of intercourse is lower in polygamous unions, hence $\lambda_p^{nat} < \lambda_m^{nat}$.

Next, to combine the three elements, I propose a model in which a woman chooses the birth rate λ minimizing a weighted sum of the distance to her objective, the distance to the husband's objective, and the distance to the natural rate. At date T , the couple reaches the end of the reproductive period with $n = \lambda T$ children.¹⁸ This framework can be viewed as a reduced form of more general, structural models of the whole decision-making process. For example, (a) the wife is the only decision-maker and the husband's preferences enter her utility function be it because of altruism, fear of a punishment or participation constraint; (b) the wife is the only decision-maker and the total number of children enters the problem through a household budget constraint; (c) the wife and the husband jointly decide by maximizing a weighted sum of their utilities. Appendix B2 discusses how similar predictions can be obtained with more general functional forms.

Monogamous unions Let me first illustrate the model with the case of monogamous unions ($n_{-i} = 0$). The wife chooses the birth rate λ maximizing:

$$u(n) = -(n - n_w^{id})^2 - \theta^h (n - n_h^{id})^2 - \theta^n ((\lambda - \lambda_m^{nat}) \cdot T)^2 \text{ s.t. } n = \lambda T. \quad (1)$$

$\theta^h \geq 0$ and $\theta^n \geq 0$ capture the intensity of marital and natural constraints, respectively. The third term corresponds to the accrual of instantaneous deviations from the natural rate during the whole reproductive period, $\int_0^T (\lambda - \lambda_m^{nat}) dt$. It can be rewritten $(n - n^{nat})$ where $n^{nat} = \lambda_m^{nat} T$ is the number of children that would be born in a natural fertility regime. Payoffs are paid at the end of the reproductive period to ensure time consistency. The first

¹⁷Fainzang and Journet (2000) document that pregnancies in quick succession are frowned upon in West Africa. The mother is despised for giving in to her husband at the expense of the youngest child's health.

¹⁸The number of children is therefore not necessarily an integer. Leung (1991) proposed to consider the number of children as a flow of child services in efficiency units when a continuous measure is needed. Note that there is no uncertainty in this framework: λ is a frequency and λT is the realized number of children.

order condition gives an optimal number of children, which is a weighted average of the wife's and husband's preferences, and a natural number proportional to marriage duration:¹⁹

$$n^{NS} = \frac{n_w^{id} + \theta^h n_h^{id} + \theta^n n^{nat}}{(1 + \theta^h + \theta^n)}.$$

In section 7.1, I check that this simple formalization is empirically relevant in monogamous unions. It fails to account for fertility in polygamous unions, though.

Polygamous unions Modeling the polygamous case requires thinking about the timing of events. Senior wives in polygamous unions today used to be monogamous for many years. One option is to focus on one period, the polygamous stage.²⁰ This modeling choice makes it hard to account for unobserved heterogeneity in empirical tests. Instead, I propose to introduce two periods in the model: the monogamous stage and the polygamous stage. This allows the empirical strategy to leverage the timing in order to get rid of the time-invariant component of unobserved heterogeneity.

Consider a game with two players, wife 1 and wife 2, characterized by their preferences and their reproductive periods. The husband has a type, monogamous (m) or polygamous (p). At $t = 0$, a couple is formed between the husband and wife 1. If the husband is of type m , the union remains monogamous and wife 2 never enters the game. If the husband is of type p , wife 2 enters at date S . At $t = 0$, the first wife only knows $n_{w,1}^{id}$, n_h^{id} and T_1 . She has some beliefs about the risk of polygamy, and in case of polygamy, about $n_{w,2}^{id}$, T_2 and S . In particular, she believes that her husband is of type p with probability π . As long as the second marriage has not taken place, the first wife does not know her husband's type yet. She chooses λ_0 taking into account her expectations. When the second marriage takes place, the type p is revealed and all information become public: S , T_2 and $n_{w,2}^{id}$.²¹ The two wives play a simultaneous, non-cooperative game. The first wife chooses λ_1 and the second wife chooses λ_2 . Payoffs are paid when both reproductive periods are over.

Formally, the utility function of wife i is:

$$u(n_i, n_{-i}) = -(n_i - \gamma_i n_{-i} - n_{w,i}^{id})^2 - \theta_i^h (n_i + n_{-i} - n_h^{id})^2 - \theta_i^n (n_i - n_i^{nat})^2. \quad (2)$$

where $n_1^{nat} = \lambda_m^{nat} S + \lambda_p^{nat} (T_1 - S)$ and $n_2^{nat} = \lambda_p^{nat} T_2$.

¹⁹I use the superscripts NS for Non-Strategic because there is no strategic interaction here.

²⁰This is the choice previously made in the economics literature: papers compare women in polygamous unions and women in monogamous unions, and/or senior wives and junior wives (Mammen 2004, Kazianga and Klønner 2009, Akresh, Chen, and Moore 2016).

²¹In Appendix B3, I relax the assumption that $n_{w,2}^{id}$ is observed by the first wife.

In the polygamous stage, the second wife chooses λ_2 maximizing:

$$u(n_2, n_1) = -(n_2 - \gamma_2 n_1 - n_{w,2}^{id})^2 - \theta_2^h (n_2 + n_1 - n_h^{id})^2 - \theta_2^n (T_2 \cdot (\lambda_2 - \lambda_p^{nat}))^2 \text{ s.t. } n_2 = \lambda_2 T_2.$$

λ_2^* is the optimal birth rate of second wives, and $n_2^* = \lambda_2^* T_2$, their optimal number of children.

Turning to the first wife, λ_0 is the birth rate chosen in the monogamous stage. At date S , the first wife has $\lambda_0 S$ children, and she is able to update her choice. Her final number of children is given by $n_1 = \lambda_0 S + \lambda_1 (T_1 - S)$. So she maximizes over λ_1 :

$$u(n_1, n_2) = -(n_1 - \gamma_1 n_2 - n_{w,1}^{id})^2 - \theta_1^h (n_1 + n_2 - n_h^{id})^2 - \theta_1^n (S(\lambda_0 - \lambda_p^{nat}) + (T_1 - S)(\lambda_1 - \lambda_p^{nat}))^2 \\ \text{s.t. } n_1 = \lambda_0 S + \lambda_1 (T_1 - S).$$

$\lambda_1^*(\lambda_0)$ is the optimal birth rate of first wives in the polygamous stage. At $t = 0$, the first wife chooses λ_0 that maximizes her expected utility given her beliefs about S , $n_{w,2}^{id}$ and T_2 :

$$(1 - \pi) \times u(\lambda_0 T_1, 0) + \pi \times \mathbb{E}[u(\lambda_0 S + \lambda_1^*(\lambda_0)(T_1 - S), n_2)].$$

One important assumption in this setting is that the union type, the date of the second marriage and the characteristics of the second wife are given ex-ante. They may well be correlated with $n_{w,1}^{id}$, n_h^{id} and T_1 , e. g. polygamous husbands may want more children. Therefore, they may be correlated with λ_0 . But the model rules out any reverse causality: the occurrence and characteristics of the second marriage should not be caused by fertility choices made during the monogamous stage.²² I provide qualitative and quantitative support for this assumption in section 7.1.

Solution I solve the problem by backward induction. I determine the best response of each wife and compute the equilibrium in the polygamous stage, and then I determine the optimal initial birth rate in the monogamous stage. Details are provided in the appendix. A noteworthy feature of the solution is that expectations about the second period do not influence choices in the first period.²³

²²An extreme case is when the first wife turns out to be infertile. I exclude infertile unions (2% of the population) from the theoretical and empirical analysis.

²³This is verified in the data (cf. section 7.1). This is the consequence of three specific assumptions: (i) the second marriage is not caused by fertility choices, (ii) the cost to deviate from natural fertility levels is based on the average birth rate and ignores variations throughout the reproductive period, and (iii) first wives have some time left to update their number of children during the simultaneous game. Without these assumptions, expectations would play a key role. Women would have incentives to adjust fertility in order to influence the second marriage, smooth births over time, or ensure that enough children are born in case of a late second marriage (cf. extension of the model discussed in Appendix B4).

Formally, the Nash equilibrium of the polygamous stage is given by:

$$n_i^* = (n_i^{NS} + n_{-i}^{NS} B_i) \times \frac{1}{1 - B_1 B_2} \text{ for } i = 1, 2. \quad (3)$$

where $n_i^{NS} = \frac{n_w^{id} + \theta_i^h n_h^{id} + \theta_i^n n_i^{nat}}{1 + \theta_i^h + \theta_i^n}$ is the optimal choice in the absence of strategic interactions, and $B_i = \frac{\gamma_i - \theta_i^h}{1 + \theta_i^h + \theta_i^n}$ is the strategic response of wife i . B_i will play a key role in the empirical analysis because its sign determines if children are strategic complements or substitutes. The sign of B_i is given by the difference between γ_i which captures the intensity of co-wife rivalry, and θ_i^h which is the weight given to husband's preferences. If $\gamma_i > \theta_i^h$, then $B_i > 0$ and n_i is increasing in n_{-i} .

The key quantity of interest is the difference in first wives' optimal birth rates before and after the second marriage, because time-invariant unobservable characteristics of spouses cancel out. We have:

$$\lambda_1^* - \lambda_0^* = \frac{B_1}{T_1 - S} \times n_2^* - \frac{\theta_1^n}{1 + \theta_1^h + \theta_1^n} \times (\lambda_m^{nat} - \lambda_p^{nat}). \quad (4)$$

The term can be decomposed into (i) a natural effect, $-\frac{\theta_1^n}{1 + \theta_1^h + \theta_1^n} \times (\lambda_m^{nat} - \lambda_p^{nat})$, that is always negative; and (ii) a strategic effect, $\frac{B_1}{T_1 - S} \times n_2^*$, that might be positive or negative depending on the sign of B_1 . The strategic effect is further split into a positive competition effect, driven by $(\gamma \times n_2^*)$, and a negative substitution effect, driven by $(-\theta^h \times n_2^*)$.

4.3 Predictions

The main takeaway of the theory section is that empirical tests should examine the change in first wives' fertility to estimate the relative magnitude of each effect. The sign of $(\lambda_1^* - \lambda_0^*)$ indicates if the positive force (competition) is large enough to compensate the sum of negative forces (substitution and natural). Next, $\frac{\partial \lambda_1^* - \lambda_0^*}{\partial T_2}$ has the same sign as B_1 . It is positive iff the competition effect dominates the substitution effect.

Figure 3 provides an illustration of the potential scenarios. At one extreme, the competition effect prevails: birth rates are predicted to increase after the second marriage, and to increase more when the second wife's fertility is expected to be higher. At the other extreme, the substitution effect prevails: birth rates are predicted to decrease after the second marriage, and to decrease more when the second wife's fertility is expected to be higher. The last scenario is an intermediate situation in which the competition effect dominates the

substitution effect, but is not large enough to dominate the combination of the substitution effect and the natural effect. Birth rates are predicted to decrease after the second marriage, and to decrease less when the second wife’s fertility is expected to be higher.

To find out which scenario is observed in the data, the main empirical tests investigate how first wives react to entries of second wives. Additionally, I exploit exits of co-wives to circumvent concerns about the endogeneity of the second marriage. The findings should mirror the entry case. The natural effect now drives fertility upwards because the natural birth rate increases from λ_p^{nat} to λ_m^{nat} . The strategic effect depends on the change in the expected number of rivals. In the polygamous stage, births reflect the best response to n_{-i}^* . Assume that the co-wife leaves after giving birth to n_{-i} children, and this was not fully anticipated by the remaining wife - if it were, I would observe no response. If $n_{-i} = n_{-i}^*$, there is no change in the strategic motive. On the contrary, if $n_{-i} = 0$, the change in strategic motive is very strong. So the remaining wife should react differently depending on how many children are already born to the exiting wife.²⁴

Last, I derive predictions on the second wife’s birth spacing and the equilibrium number of children. The birth rate of second wives should increase with drivers of first wives’ fertility - duration of the monogamous period (S) and time left before the end of her reproductive period ($T_1 - S$) - iff the strategic effect is positive. The final number of children of both wives should always increase with the wife’s own reproductive period, and increase with the co-wife’s reproductive period iff the strategic effect is positive. I use the tests on second wives and completed fertility to provide supplementary, suggestive evidence of strategic interactions, but the empirical strategy focuses on the change in first wives’ birth spacing.

5 Empirical Strategy

The goal of the empirical strategy is to (i) identify the total impact of polygamy on fertility and (ii) identify the sign of the strategic effect.

5.1 Preliminary evidence on raw data

Before turning to the econometric specifications, Table 2 presents some descriptive statistics on the average birth intervals of first wives before and after the second marriage. I restrict

²⁴The ideal test would use the age of the exiting wife to infer how much reproductive time she had left. However, I only know the date of exit and number of children, not the ex-wife’s age or birthdate. So n_{-i} is the best available proxy for $(n_{-i}^* - n_{-i})$.

the analysis to first wives younger than 45 because I do not know the complete birth history of older women, as explained in Section 3.1. The first column reports statistics for the whole sample: birth intervals increase by six months, from 37.6 months in the monogamous stage up to 43.6 months in the polygamous stage. The sample is further split on the median T_2 into those facing a less fertile co-wife (short T_2 , column 2) and those facing a very fertile co-wife (long T_2 , column 3). Birth intervals rise by almost 10 months for the former, whereas the magnitude is halved and the increase is no longer significant for the latter. Importantly, both groups display similar birth intervals in the monogamous stage: 37.8 and 37.1 months, not significantly different from each other. The analysis of raw data provides a first hint that polygamy reduces fertility differentially according to the second wife’s expected fertility, and is not correlated with the history of past births.

These preliminary results will be confirmed and strengthened by the identification strategy presented below. I consider two specifications: a linear model with fixed effects and a duration model with individual baseline hazards. Through the explicit modeling of unobserved heterogeneity, they allow a causal interpretation under credible identification assumptions.

5.2 Econometric specifications

The main idea is to take advantage of the panel structure of the data. Indeed, I observe several birth intervals for a given woman i , some of them occurring before and others after the second marriage.²⁵ The dependent variable is the duration between births j and $(j + 1)$, denoted $t_{i,j}$ and measured in months. The vector $x_{i,j}$ contains observed time-varying explanatory variables: woman’s age and age squared at birth j . I also include a dummy for each birth rank $j \in [2, J]$ where J is the highest parity observed; the reference category consists of intervals after the first birth. The term ν_i is meant to capture all the determinants of birth intervals that may vary across women, but not across birth ranks for a given woman. I use robust standard errors clustered at the woman level to account for the correlation between the error terms related to the different birth intervals of the same woman.

In a first step, I estimate the impact of a change in union status, from monogamous to polygamous. The covariate of interest is the dummy $After_{i,j}$, which is equal to one if the second wife had arrived when the child $(j + 1)$ was conceived by woman i . This specification allows me to identify the total impact of polygamy on fertility. In a second step, I test if

²⁵The impact of the second marriage on the first wife’s fertility is identified on women with at least two births before the second marriage and at least one birth after the second marriage. It is the case for roughly 40% of first wives. In the end, 330 birth intervals contribute to the identification.

the impact depends on the co-wife by interacting $After_{i,j}$ with $T_{2,i}$, the reproductive period of the second wife faced by woman i . The coefficient on the interaction term gives the sign of the strategic effect. I include monogamous wives in the sample to improve the accuracy of the estimators. They do not contribute to estimating the impact of polygamy, because $After$ is always equal to zero for them, but they help estimating the coefficients on $x_{i,j}$ and on the birth rank dummies. As a robustness test, I check that estimates are very similar if I exclude monogamous wives (cf. Table OD.1, columns 5 and 6, in Appendix D).

A linear model with fixed effects gives a sense of magnitude. Specifications are:

$$t_{i,j} = \alpha_0 After_{i,j} + \beta x_{i,j} + \eta_j + \nu_i + \epsilon_{i,j},$$

$$t_{i,j} = \alpha_1 After_{i,j} + \alpha_2 After_{i,j} \times T_{2,i} + \beta x_{i,j} + \eta_j + \nu_i + \epsilon_{i,j},$$

where $\epsilon_{i,j}$ is an idiosyncratic error term, and η_j is equal to one for birth rank j . In these specifications, I only consider non-censored durations (i.e. closed intervals). ν_i is treated as a woman fixed effect. I estimate α_0 , α_1 and α_2 using the within estimation method.

Under the identification assumptions specified below, the linear model predicts that polygamy causes an average change in birth intervals by α_0 months. In the theoretical model, this quantity is given by $(1/\lambda_1^* - 1/\lambda_0^*)$. If $\alpha_0 > 0$, birth spacing lengthens in the polygamous stage. Moreover, the change in case of $T_2 = 0$ is predicted to be α_1 ; each additional year in T_2 translates into α_2 additional months. If $\alpha_2 < 0$, birth spacing lengthens less (if $\alpha_0 > 0$) or shorten more (if $\alpha_0 < 0$) when T_2 is longer.

A duration model of birth intervals with individual hazards is closer in spirit to the theoretical model and better suited to the nature of the dependent variable. They make it possible to exploit information from right-censored durations (i.e. time intervals between the birth of the latest child and the date of the survey). Another advantage of duration models is that $After_{i,j}$ may vary within a spell j , and not only across spells. It captures more precisely the date of the change than in the linear model.²⁶ Formally, I consider a mixed proportional hazard model with multi-spell data. The hazard functions satisfy:

$$\theta(t|x_{i,j}, v_i) = \theta_0(t, \nu_i) \times \exp(\alpha'_0 After_{i,j} + \beta' x_{i,j} + \eta'_j),$$

$$\theta(t|x_{i,j}, v_i) = \theta_0(t, \nu_i) \times \exp(\alpha'_1 After_{i,j} + \alpha'_2 After_{i,j} \times T_{2,i} + \beta' x_{i,j} + \eta'_j).$$

²⁶An alternative identification strategy would be to rely on the timing-of-events methodology developed by Abbring and van den Berg (2003) that would exploit the variation in the union status occurring within a spell. Unfortunately, I only observe the year of the second marriage so there are large measurement errors when I split the spell into two intervals measured in months.

In these specifications, the baseline hazard θ_0 is specific to each woman. There is no restriction on the interaction of ν_i with the elapsed duration t in the hazard function. Moreover, no assumption on the tail of the distribution of the unobservables is needed. The main technical assumption is the proportional hazard assumption that I test and fail to reject (cf. Figure OD.1 in Appendix D). I estimate α'_0 , α'_1 and α'_2 using a stratified partial likelihood.²⁷

In proportional hazard models, coefficients are interpreted as hazard ratios. $\exp(\alpha'_0)$ measures the hazard ratio between births occurring after and births occurring before the second marriage. In the theoretical model, it corresponds to λ_1^*/λ_0^* . If $\exp(\alpha'_0) < 1$, birth spacing lengthens in the polygamous stage. The hazard ratio after-before also satisfies $\exp(\alpha'_1) \times \exp(\alpha'_2 T_2)$. If $\exp(\alpha'_2) > 1$, the ratio increases with T_2 . It means that birth spacing lengthens less (if $\exp(\alpha'_0) < 1$) or shortens more (if $\exp(\alpha'_0) > 1$) when T_2 is longer.

The duration model predicts hazard rates while the linear model predicts durations. The higher the hazard rate, the shorter the duration. So the predictions of both models are consistent if their estimates have opposite signs, i.e. $\alpha > 0$ if and only if $\exp(\alpha') < 1$.

5.3 Identification assumptions

Both specifications allow the explanatory variables and ν to be dependent. It means that fertility choices and the timing of the second marriage may be jointly determined by some unobserved characteristics, provided that these characteristics are fixed over time.²⁸

The key identification assumption is a strict exogeneity condition: the idiosyncratic error term ϵ should be uncorrelated with *After* and T_2 of all past, current and future spells of the same individual. It corresponds in part to the assumption of the theoretical model that the second marriage is not caused by fertility choices in the monogamous stage. It also implies that first wives do not change their fertility in anticipation of the second marriage.²⁹ The scope for this concern is somewhat limited in my context because first wives are often unaware of the second marriage until it happens (Madhavan and Bledsoe 2001). Moreover, anticipation would go against finding a strategic response. Last, exogeneity entails a condition on the evolution of fertility during the woman's life. In the model, I abstract from such dynamics by assuming that natural birth rates are constant in time. In fact, birth rates

²⁷The method is described in more details in Appendix E. In the same way as in linear models, it is based on a suitable transformation that eliminates the ν .

²⁸Time-invariant characteristics available in the data already explain 45% of the variance in T_2 . The main drivers are ethnic and regional variations, features of the husband's childhood (polygamy of his father, child fostering) and characteristics of the first marriage (age of the spouses, having broken unions). The analysis uses non-barren first wives older than 45 years old (124 observations).

²⁹Ashenfelter (1978) discusses the limitations of before-after comparisons in presence of anticipations.

evolve with the life-cycle. Using a fixed effect specification, I find that the same woman (i) at a given age, has longer intervals if the birth rank is higher, and (ii) at a given rank, has shorter birth intervals if she is older, up to a certain age above which the relationship is reversed. Since *After* is mechanically correlated with birth ranks and woman’s age, I need to control for such life-cycle effects to prevent my tests from capturing spurious dynamics. Therefore I include woman’s age and age squared at birth and dummies for birth rank as controls.³⁰ This solves the issue as long as the effect of the life-cycle does not differ between polygamous and monogamous wives, nor among polygamous wives depending on the timing of the second marriage.

I call the key identification assumption the common life-cycle assumption. It is close to the common trend assumption in a standard difference-in-difference framework. In the same way, it is testable on births that occurred *before* the second marriage. Table 3 shows that the evolution of birth spacing is the same for all women during the monogamous stage. In the first column, I interact birth rank dummies with "Future first wives" to compare the effect of the life-cycle between monogamous wives and first wives. I find that the hazard ratio between birth j and the reference category (first birth) is lower than one and decreases as j rises, meaning that birth intervals lengthens with rank, as expected. What is important for me is that the pattern is the same no matter the union type. Coefficients on the interaction terms are close to one, and never significantly different from one. In the second column, I restrict the sample to future first wives in the monogamous stage and I interact birth rank dummies with T_2 . Again, there is no evidence that birth rank effects depend on the characteristics of the second marriage. In Section 7.1, I also conduct placebo tests to check that the difference after-before exactly coincides with the second marriage.

6 Results

6.1 Change in first wives’ birth spacing after the second marriage

Table 4 reports estimates of the linear model. They are in line with the raw data. Polygamy causes a significant increase in birth intervals, by seven months. The impact is highly heterogeneous: 14 months for first wives facing a less fertile co-wife against zero for those facing a very fertile co-wife. The econometric model further predicts that intervals lengthen by approximately two years when $T_2 = 0$, and that an increase by one year in T_2 reduces this

³⁰The impact of birth ranks and the impact of *After* are separately identified because the second marriage takes place at different parities for different women.

change by one month. So intervals lengthen less when the second wife has more time ahead to give birth to children. The coefficient on the interaction term is significant at 5%.

Can these magnitudes be explained by changes in breastfeeding and abstinence? In developing countries, breastfeeding inhibits fertility for 12 to 24 months after a birth (Jayachandran and Kuziemko 2011). In Senegal, the insusceptibility period due to amenorrhea or abstinence is 15 months on average and more than two years for one fourth of women (DHS 2005). Like in other West African countries, breastfeeding is universal and long: 96% of children are breastfed and the median duration is 20 months, with a large variation from a few months to more than three years. So women have some margin up and importantly some margin down to reduce breastfeeding without jeopardizing children’s health.³¹ Table OD.2 in Appendix D shows that women are less likely to use modern contraceptives, to be breastfeeding, to have amenorrhea or to abstain when their co-wives want more children, holding own and husband’s ideal family sizes constant. This provides direct, suggestive evidence that the use of modern and traditional birth control methods is related to the co-wife’s fertility.

Estimates of the duration model are consistent with the linear specification. Columns 1 and 2 in Table 5 report $\exp(\alpha')$. The hazard ratio after-before is lower than one, although not significant.³² It suggests that birth rates are lower after the second marriage, i.e. intervals are longer. Next, the hazard ratio on the interaction term is larger than one, and significant at 5%, meaning that birth spacing lengthens less when T_2 is longer. These empirical results imply that $\lambda_1^* < \lambda_0^*$ while $B_1 > 0$, which corresponds to the third scenario in Figure 3.³³

Heterogeneity by wealth and education In which couples are strategic behaviors expected to be more salient? One source of variation is husband’s wealth. If children are a claim to male resources, then co-wife rivalry should be stronger when the stakes are higher. In the model, B_1 increases with γ_1 . In columns 3 to 5 of Table 5, I disaggregate the results by husband’s income. The ratio on the interaction term increases as we move upwards in the income distribution. It is significantly different from one only for the richest quartile.

Another source of variation is wives’ control over pregnancies. Competition can only be detected if wives have at least some say in fertility decisions and some means to implement

³¹Note that the zero increase for women facing a young co-wife does not imply that children are not breastfed. It implies no change compared to the monogamous period, when the average birth interval is 36 months, leaving at least two years of breastfeeding after a birth before the mother gets pregnant again.

³²The p-value is 0.16. The ratio becomes significant at conventional levels when the specification is slightly modified (cf. Table OD.1 in Appendix D discussed in the robustness checks).

³³Going back to Equation 4, it is only possible when $n_2^* < \frac{\theta^n (\lambda_m^{nat} - \lambda_p^{nat})(T_1 - S)}{\gamma_1 - \theta_h^1}$. Using estimates from Table OD.4 in Appendix D for $\theta^h \approx 1/2$, $\theta^n \approx 3$, $\lambda_m^{nat} \approx 1/3$, and under the additional assumption that $\lambda_p^{nat} = \frac{\lambda_m^{nat}}{2}$ and $\gamma = 1$, the condition rewrites $n_2^* < (T_1 - S)$. This is verified in 90% of households.

them. In the model, B_1 is higher when θ_1^h and θ_1^n are lower. Female education is often considered as a proxy for women’s ability to choose the timing of births. In some contexts, it also implies financial independence but this is not the case in Senegal: educated wives are far less likely to work than non-educated ones. I compare educated and non-educated wives in columns 6 and 7 of Table 5. I find that the strategic response is indeed much stronger and only significant for educated wives. Results are similar if I consider area of residence as a proxy for cost of birth control: urban households display a stronger strategic response.

Modernization factors are thus associated with a stronger impact of co-wife rivalry on fertility behaviors, because they raise both women’s incentives to compete and their control over births. This is an important result as it challenges the general view that these factors drive the fertility transition. In polygamous societies, the relationship between socio-economic development and fertility is more complex, as will be discussed in section 8.

6.2 Exits of co-wives, second wives and completed fertility

Exits of co-wives Another way to disentangle natural and strategic effects is to exploit the opposite transition, from polygamous to monogamous. I use PSF data on husband’s marital life, in particular date of broken unions and number of children born in these unions, to reconstruct exits. Those transitions result from death (25% of the cases) or divorce. To be consistent with the results exploiting entries, I should observe that, after the exit of a co-wife, fertility increases and increases more if she had more children. The natural effect is positive; the change in strategic motive is close to zero when the exiting wife had already many children, while the change is very negative when she left before having any child.

This is indeed what is found in the data. In Table 6, I estimate a similar duration model replacing the entry dummy by an exit one. On average, the remaining wife intensifies her fertility when another wife leaves the union. Moreover, birth spacing shortens more when more children were born to the co-wife at the date of the exit. In particular, when the co-wife had no child, birth rate remains the same: changes in incentives (negative) and changes in natural birth rate (positive) offset each other.

Second wives There is suggestive evidence that second wives behave strategically too. However, the test is less conclusive because I cannot account for time-invariant unobserved determinants of fertility. One might be worried that some of those are correlated to the type of marriage the woman ends up in. To mitigate this issue, I control for a wide range of characteristics, including the predictors of fertility preferences identified in Table OA.3 in Appendix A. I estimate a Cox model with a common baseline hazard, including durations

between marriage and first birth, because they convey useful information on second wives' reactions. If the second wife's strategic response is also positive, I should observe that birth intervals are shorter when the first wife's reproductive period is longer. I can also look at the response to the number of children that the first wife had at the time of entry when I know her complete birth history.³⁴

Table 7 summarizes the test using S in column 1 and number of children in column 2. Signs are in line with expectations. The hazard rate is positively correlated with all predictors of first wife's fertility: $(T_1 - S)$, S and number of children. If I break down the effects by birth ranks, they are particularly strong between marriage and first birth. All this suggests that second wives also intensify their fertility when they face a more fertile rival.

Completed fertility To test the predictions on the final number of children, I consider the sub-sample of women over 45 years old. Under the same caveat as above, looking at completed fertility provides circumstantial evidence that the impact of strategic interactions on birth spacing patterns translates into sizeable differences in terms of total number of children. Results are summarized by Table A.1 in the appendix.

Starting with first wives, Panel A shows that the number of children increases with the duration of their own reproductive period, with the length of the monogamous stage and with the duration of the co-wife's reproductive period. First wives are predicted to eventually have two children fewer when the second wife has a short time left before fertility decline (less than 10 years). Turning to second wives in Panel B, they have three children fewer when the monogamous stage is short (below the median), leaving little time for the first wife to have many children; and two children fewer when the first wife has no time left to react after the second marriage. Everything is consistent with a positive strategic response.

To sum up, all empirical tests point to children being strategic complements: characteristics raising the fertility of one wife intensify the fertility of her co-wife.

6.3 More empirical evidence based on the gender of children

So far, I have considered only the quantity of children, putting quality aside. Nonetheless, it might be argued that co-wife rivalry is also about children's characteristics such as educational achievement, social success or responsibility taken in the family welfare. According to the literature on Africa, one characteristic plays a key role: gender. Having sons substantially

³⁴There are pros and cons of using the actual fertility rather than a determinant. The number of children is more informative than the length of the monogamous stage S because it captures the optimal birth rate of the first wife. But the omitted variable issue is more serious because the fertility of both wives is related to the husband's preferences and fecundity.

improves women’s status and security (Lesthaeghe 1989). It is particularly true in patrilineal ethnic groups, and where the influence of the Islamic law is strong, like in Senegal where 95% of the population is Muslim (Kane 1972). A previous work on monogamous unions shows that Senegalese women have a stronger preference for sons when the current husband already has children with ex-wives, either divorced or deceased (Lambert and Rossi 2016). The explanation rests on the rivalry for inheritance between the husband’s children. In presence of children from ex-wives, current wives need a son to secure access to their late husbands’ resources in case of widowhood. The same rationale is at play in polygamous households, and even exacerbated by the rivalry for *current* resources, be it material or emotional.

In this section, I test if the gender of children matters. Ideally, I would like to predict how the birth of a boy vs. a girl impacts the subsequent optimal birth rate of both wives, and whether this effect depends on the gender composition of the other wife. However, my model of fertility choices lacks a true time dimension to adequately account for the uncertainty related to a child’s gender. Therefore, I build on the above-mentioned work on the rivalry with ex-wives to derive the following predictions regarding co-wives. On the one hand, the arrival of a second wife should exacerbate the preference for sons of first wives. Indeed, they move from a situation in which no other child can compete with their own offspring, to a situation with rivals. So the necessity to have a son should be stronger in the polygamous stage. This is particularly true when the second wife gives birth to boys. On the other hand, the behavior of the second wife should also depend on the gender composition of the first wife’s children, boys representing a more serious threat than girls. The second wife should therefore hurry more if the first wife already has a son.

To test the prediction on the change in son preference of first wives, I modify the main specification by introducing a dummy *No son* equal to one if the woman had no son at the time of the index birth. This variable varies across births and captures the impact of having only daughters vs. at least one son on the next interval, holding birth rank and age at birth constant. If the hazard ratio is larger than one, meaning that having only daughters decreases the expected interval, one can infer the existence of son preference.³⁵ Next I interact the gender composition with the dummy *After* to test whether the arrival of the second wife has an impact on son preference. Results are reported in the last column of Table 5. Son preference exists in the monogamous stage: the hazard ratio on *No son* is significantly greater than one. But it is substantially exacerbated by the second marriage: the hazard ratio on the interaction term, capturing the difference in son preference before

³⁵Cf. Rossi and Rouanet (2015) on how to identify son preference using duration models of birth intervals.

and after, is equal to two. Controlling for life-cycle dynamics, *the same woman* is predicted to display stronger son preference once her rival has arrived than she used to do.

An additional test exploits the gender of children born during the polygamous stage: how do first wives react to the first birth of second wives, by gender? Among first wives in my sample, 87 face a birth by the co-wife. The proportion of boys among those births (51.7%) is close to the biological level, which supports the assumption that, conditional on birth, gender is random. Table 8 shows that first wives facing a male rival give birth to 5.6 children against 4.6 for those facing a female rival; the difference is significant at 10%. Interestingly, the gap is driven by first wives with no son. They *all* have at least one more child after the birth of a male rival, whereas 1/4 of those facing a female rival and 1/3 of those who already have sons stop childbearing.

As for second wives, the last column of Table 7 show that their birth rate is divided by two if the first wife has no son at the time of entry. They wait significantly longer to have a first child and longer between births when they join a marriage without boys.

These findings are consistent with co-wife competition for sons, and further illustrate that potential and realized fertility outcomes of a wife influence the behavior of the other.

7 Robustness

7.1 Robustness checks and placebo tests

Estimating alternative specifications Table OD.1 in Appendix D shows that the main estimates are stable when I change the specification of the duration model. In columns 1 and 2, I estimate the impact of polygamy on the very next birth interval rather than the impact on all subsequent intervals. Coefficients are larger in magnitude and more significant than in the baseline specification. One interpretation is that the difference between λ_m^{nat} and λ_p^{nat} would tend to die out as spouses get older. The natural effect would be strong at the beginning of the polygamous stage, and would gradually disappear over time.³⁶ In columns 3 and 4, I focus on bigamous unions, and in columns 5 and 6, I exclude monogamous wives. In columns 7 and 8, I impose that $After = 1$ for the whole spell during which the second marriage takes place, which is the approximation made in the linear model. Coefficients are very similar to the baseline results in terms of magnitude and, in general, more precisely

³⁶Another explanation is based on learning effects. In a more complex dynamic game, preferences would be private information and both wives would update their beliefs as and when the other wife gives birth. Such a setting is beyond the scope of this paper, but it could help explaining dynamic reactions.

estimated. Another concern is that I do not observe deceased children, which might lead me to overestimate the true duration between successive births. I check in columns 9 and 10 that the main findings still hold when I restrict the sample to women who lost no child. Estimates are even slightly larger in magnitude and more significant.

Examining life-cycle effects Next, I run a placebo test to ensure that the ratio after-before is not capturing a spurious decline in fertility over time. I replace the true length of monogamous stage, S , by other arbitrary durations: the mean, the first quartile and the last quartile taken from the distribution of S in the sample. I consider two specifications: the baseline with all intervals, and the specification using only the very next birth interval. If the coefficient on *After* was purely driven by a decline in the biological fertility of women, it should remain below one whatever the cut-off. Table OD.3 in Appendix D reports the new ratios: they are much closer to one than the baseline ratio, and never significantly different from one. Therefore, the placebo test suggests that the change in first wives' birth spacing precisely coincides with the second marriage.³⁷ Another test provides some level of reassurance that the heterogenous effect along T_2 does not capture the variation in other related timing variables, T_1 and S . In fact, the ratio on the interaction term with T_2 remains unchanged when I control for *After* interacted with S and with T_1 . It is equal to 1.08 and significant at 10%. It proves that the different reactions of first wives cannot be explained by the different moments at which the second marriage took place in their own lives. They are really driven by the variation in the second wife's age at marriage.

Checking the validity of the model Last, I provide empirical support for the model by testing key assumptions and implications. First, I show that the final number of children of a couple is indeed a weighted average of the wife's and husband's preferences and a natural number proportional to marriage duration using DHS. In monogamous couples, the three drivers n_w^{id} , n_h^{id} and T are significantly correlated to total fertility, accounting for 36% of the variance (cf. Table OD.4 in Appendix D). This is virtually the largest share in variance that can be explained by a linear probability model when the dependent variable is an integer. The constant is not significantly different from zero. The estimate of the natural birth rate is sensible: one birth every three years.³⁸ Natural forces constrain fertility choices more than

³⁷Another placebo test focuses on monogamous wives. It assigns to them a virtual date of second marriage and checks that the ratio after-before is equal to one. For instance, assuming that the second marriage would take place 10 years after the first one (the average length of S), I get a ratio of 1.07 which is not significantly different from one (p-value=0.64, table available upon request).

³⁸It corresponds to approximately eight children for the average couple ($T = 26$ years). These numbers are consistent with estimates produced by demographers. Using data from various populations in the world, Henry (1961) concludes that a woman married at age 20 years old has between six and eleven children. In

husband's and wife's preferences, which both weigh in the same way. In accordance with the model, the fit is not as good in polygamous unions ($R^2 = 0.14$) because a key driver - the co-wife's fertility - is missing.³⁹

Second, I back up the assumption that a wife cannot influence the likelihood of polygamy through her own fertility choices using anthropologists' studies on West Africa. Antoine and Nanitelamio (1995) distinguish four motives for marrying a second wife: forcedly accept a bride given by relatives; improve prestige; assert African identity - be a good Muslim, reject Western values; and capture female wealth. They do not mention having more children. In the vast majority of cases, first wives are not involved at all in this decision. Madhavan and Bledsoe (2001) report "a unanimous agreement that the arrival of a co-wife is inevitable because it is a man's prerogative [... and that] satisfactory childbearing would not stop a man from getting another wife". This is further supported by recent qualitative surveys in which I asked why men become polygamous.⁴⁰ 40% of people mention one of the first three motives (family pressure, prestige, religion) as the main reason. Love comes fourth with 18%. The vast majority of people say that, to prevent polygamy, first wives can either do nothing, or behave "well" meaning being submissive. Less than 1% answer having many children. Quantitatively, I provide empirical support by regressing λ_0 on the union type, S and T_2 , controlling for $n_{w,1}^{id}$, n_h^{id} and T_1 . If there is no reverse causality, the correlation should be nil. In the appendix, Table A.2 column 1 compares the birth rates of first wives in the monogamous stage to the birth rates of monogamous wives. Controlling specifically for marriage duration and predictors of preferences, I find that first wives do behave like monogamous wives as long as the second wife has not arrived. In column 2, I consider first wives in the monogamous stage and I test if T_2 and S are systematically correlated with birth rates. I find that choices in the monogamous stage are not related to the timing of the second marriage. So there is no evidence that some specific fertility patterns would drive the occurrence and characteristics of polygamy.

Third, I test if expectations about polygamy matter for choices in the monogamous period. Specifically, does the probability of polygamy (π) influence the initial birth rate? In my framework, women with different beliefs take the same decision in the monogamous stage, holding everything else constant. In the appendix, Table A.3, I regress the union

Africa, Hertrich (1996) reports that the most widespread norm advocates birth intervals of three years.

³⁹Ideally, I would add the preferences and the length of reproductive period of the co-wife as additional drivers but these variables are only observed in a very small sub-sample in DHS (bigamous unions in which both wives are found and are in their first union i.e. 39 observations).

⁴⁰Interviews and focus groups carried out in autumn 2016 with 725 men and women from various villages in Burkina Faso, where the organization and prevalence of polygamy are similar to Senegal.

status on potential predictors of polygamy using the sample of monogamous and first wives older than 45 years old. I use this regression to come up with an estimate of π for women younger than 45 years old. Table A.4 shows that the predicted probability has no significant impact on birth spacing.⁴¹ My framework is therefore supported by the absence of empirical link between the probability of polygamy and birth spacing in the monogamous period.

7.2 Alternative models without strategic interactions

Other frameworks without strategic interactions can potentially be used to model fertility choices in polygamous unions. However, they fail to account for empirical patterns.

Unitary model with endogenous income The first alternative model emphasizes the relationship between polygamy and income. To structure ideas, I wrote a simple unitary model with endogenous income in Appendix C. Under the assumptions that (i) children are normal goods, (ii) the second marriage is caused by a positive income shock, and (iii) the income shock is positively correlated with the second wife’s reproductive period because, for instance, only men with a large increase in wealth can afford a young second wife, the model indeed predicts that the fertility of first wives decreases less when the second wife is younger - because both are related to the household income, not because of strategic interactions. I derive three testable implications that are inconsistent with the data, though.

First, fertility should increase in response to any positive income shock. I show that this is not the case exploiting data in the PSF survey about self-reported shocks.⁴² Using my estimation sample, I run a placebo test replacing the year of a second marriage by the year of a positive income shock. Table A.5 Panel A in the appendix shows an insignificant impact, if anything negative on fertility. This holds both in monogamous and in polygamous unions. Another placebo test examines how monogamous couples react to the death of the husband’s father, which generates a positive income shock if he left some bequest. Again, I find no significant impact on birth rates. These findings are consistent with the idea that, in developing countries, children are best modeled as insurance rather than normal goods.

Second, the fertility of first wives should decrease less when the positive income shock is larger. I cannot measure changes in husband’s wealth because income is only reported for the survey year. Instead, I exploit PSF data on how much the husband paid for the second wife,

⁴¹Note that the power of this test is somewhat limited by the low R^2 (0.16) of the first stage.

⁴²People are asked if they have experienced a good year, i.e. a year when income was substantially higher than usual, in the past 5 years. 34% of households answer yes, and good years are equally spread throughout the 5-year period. The most common types are good harvest and good agricultural prices in rural areas and good sales and new job in urban areas.

using bride price as a proxy for increase in husband's wealth. I find that younger brides are indeed more expensive.⁴³ But bride price is negatively correlated with first wife's fertility as shown by Table A.5 Panel B in the appendix. Birth spacing lengthens more when the second wife is more expensive. Consequently, when I control for bride price, the coefficient on the interaction with reproductive period is higher and more significant. Overlooking changes in income leads to underestimating, not overestimating, the effect of reproductive period.

Third, after the exit of a co-wife, fertility should increase more if fewer children are already born to the co-wife. The intuition is that the remaining wife compensates for the children who are not born yet (cf. Appendix C). However, the opposite pattern is observed in Table 6. In particular, there is no increase when the co-wife left before having any child. This is inconsistent with the unitary model and consistent with strategic interactions.

Unitary model with endogenous polygamy In the second model, husbands make all decisions about fertility, consumption and marriage. Details are provided in Appendix C. I assume that female fertility has some upper bound. This generates a motive for polygamy for husbands who cannot reach their optimal number of children with a sole wife. Polygamy occurs when permanent income or health shocks lead to a discrepancy between male preferences and female reproductive capacities. I discuss two situations on the marriage market, depending on whether men can or cannot choose who they marry. All variants of this model fail to predict a positive correlation between co-wives' fertility. The intuition is that the second wife's fertility aims at supplementing the first wife's fertility and this generates some substitutability. Moreover, the model fails to account for all patterns observed after the second marriage. It can either predict a decrease in the first wife's fertility when polygamy is driven by a health shock, or an increase in the husband's fertility when polygamy is driven by an income shock, but not both at the same time.

What if the health shock leading to polygamy is only temporary? Suppose that the first wife's fertility then regresses to the mean, and that a more fertile second wife is chosen in case of a more severe health problem. This would predict a positive correlation between the fertility of both wives. However, this story is inconsistent with results in Table 3 - there is no stronger decline in fertility during the monogamous stage for women who eventually end up with a younger co-wife - and Table 7 - second wives who join a union with fewer children have a lower fertility. Moreover, there is no empirical evidence that the age of second wives is related to bad fertility shocks on first wives. Using DHS, I find an insignificant, and

⁴³This is true for all wives and specifically for second wives. Still, there is a lot of variation in a given age category, so there are many observations with same age but different prices.

if anything negative, relationship between T_2 and the likelihood of terminated pregnancies (miscarriages, stillbirths and abortions).

Natural fertility model The last model ignores couple's preferences and assumes that the only driver of fertility is the natural birth rate. Lardoux and van de Walle (2003) hypothesize that husbands play favorites, hence generating differential birth rates between co-wives. They further explain that husbands are more likely to favor the youngest wife so that the natural rate of a given wife would increase with the age of her co-wife. Empirical findings show the opposite. In the same vein, birth rates might simply reflect living arrangements. According to PSF data, husbands are less likely to live with their second wives when the reproductive period T_2 is shorter, because older second wives might live in the house of a deceased husband or with an adult son. T_2 has no significant impact on the probability to live with the first wife. All in all, husbands tend to spend more time with the first wife when T_2 is shorter. This generates, again, a negative correlation between the first wife's exposure to pregnancy risk and the co-wife's reproductive period.

In conclusion, the empirical results are hardly explained by income shocks, health shocks, or sexual favoritism. If they were indeed at play, these mechanisms would only attenuate the power of my test for strategic behavior.

8 Discussion

Drivers of the fertility transition can be divided into supply factors influencing access to birth control methods (Bongaarts 1990), and demand factors affecting desired family size (Pritchett 1994). This paper combines both and further distinguishes between men's and women's preferences. Although the relative contribution of each factor is controversial, it is generally agreed that a better access to contraceptives and a greater say of women in fertility decisions spur fertility decline (Sen 1999). I argue that this is not necessarily true in Africa, because those downward forces are offset by co-wife rivalry. By incentivizing women to want many children, the institution of polygamy undermines fertility transitions.

Empirically, polygamy is indeed correlated with slow fertility transitions. At the macro level, within Sub-Saharan Africa, the decline in fertility is slower in countries where polygamy is more prevalent. They end up with roughly one more birth per woman nowadays, as shown by Figure 4. Similarly at the micro level, within highly polygamous countries, the decline in fertility is slower in polygamous unions than in monogamous ones. These different trends led to a recent reversal of the correlation between polygamy and fertility in nine out of eleven

countries represented in Figure 5. In Senegal, in the latest DHS wave, completed fertility is higher by 0.35 child for women in polygamous unions.

As far as I know, my framework is the first to rationalize these stylized facts.⁴⁴ At an early stage of the fertility transition, natural forces dominates strategic forces, and fertility is lower in polygamous unions. As women's control over births progressively improves, the natural and marital constraints lose momentum. The relative magnitude of co-wife competition rises, and polygamy is more likely to have an overall positive impact on fertility. This is consistent with the fact that, in cross-section, factors usually associated with small families - wealth, female education and urbanization - are correlated with a more positive strategic response. Therefore, the role of polygamy in limiting fertility decline becomes more salient in the course of the fertility transition.

Policy-makers are concerned by the impact of slow fertility transitions on public health and population growth. Lowering the level of fertility is a target for a vast majority of African governments (United Nations 2013). My paper cautions them against importing policies from monogamous societies without taking into account the specific structure of African households. Family planning programs alone will not be enough as long as the root causes of co-wife reproductive rivalry persist. Tackling the emotional dimension is probably a long-term endeavor, but policy makers could start by reducing women's economic reliance on children. One way ahead is to give more opportunities for self-support to women. Concretely, it means easing labor market restrictions and constraints stemming from social norms to improve female labor force participation. Another recommendation would be to reform family law in order to improve women's status in terms of property rights and inheritance rights; for instance by entitling wives to a significant share of the husband's bequest irrespective of the number of children.⁴⁵

⁴⁴Macro analyses have related polygamy to high fertility levels, but not to slow fertility declines. Demographers argue that this institution requires early marriages and quick remarriages of women, which maximizes the length of women's exposure to births (Lesthaeghe 1989). Tertilt (2005) proposes another explanation: by raising the value of women on the marriage market, polygamy leads to high bride prices, which makes it profitable for parents to have many daughters.

⁴⁵Policy-makers could also take advantage of the reproductive externalities in polygamous societies. If they manage to curb the fertility of one wife, it would reduce the fertility of the other wife. For instance, raising girls' age at marriage, by passing a bill or extending schooling, would reduce the length of their reproductive period, and therefore the number of children they will give birth to. It would also change the behavior of their future co-wives, because they would face less fertile junior wives. Thus, keeping girls in school can potentially impact all women, even those who have already started giving birth.

In a nutshell, this paper combines insights from anthropology, demography and economics to study fertility choices in polygamous unions. It brings non-cooperative games, duration models and a unique representative sample of polygamous families to show that co-wives compete for more children. My results highlight the challenges for public policies aiming at decreasing fertility in Sub-Saharan Africa. Polygamy deeply impacts intra-household dynamics and makes comparative statics far from straightforward. The main takeaway is that promoting women’s empowerment in a very broad sense - not only to improve women’s control over births, but also to disassociate their status and economic security from their offspring - is a prerequisite for curbing fertility.

More generally, the non-cooperative approach might be a relevant lead to follow to understand the behaviors of non-nuclear families. Existing economic models assume that spouses only have children together. This is not true in polygamous families, and beyond the African context, in step-families. Many societies are nowadays characterized by serial monogamy, where people can divorce and remarry (De La Croix and Mariani 2015). Multiple marriages have been found to affect men and women differently, and it is still unclear why.⁴⁶ My paper suggests that examining strategic interactions between current and ex-spouses could be insightful. When the current couple does not coincide with children’s parents, more than two adults are involved in decisions related to fertility, allocation of resources, investment in children and labor supply. Are children used as a claim on (ex-)spouses also in contemporary Western societies, at what cost and who bears it? Opening the black box of step-families is necessary to answer these questions, and more broadly, to explore the new rivalries arising between step-siblings, and between the surviving spouse and step-children in case of widowhood. These dynamics have barely been studied by economists yet, although they certainly have implications for investment in human capital and wealth transmission.

References

ABBRING, J., AND G. VAN DEN BERG (2003): “The Non-Parametric Identification of Treatment Effects in Duration Models,” *Econometrica*, 71, 1491–1517.

⁴⁶In the UK and the US, one fourth to one third of adults have had more than one partner, and this is correlated with improved reproductive success and mental health for males but not for females (Willitts, Benzeval, and Stansfeld 2004, Jokela, Rotkirch, Rickard, Pettay, and Lummaa 2010). In the economics literature, a working paper by Browning and Bonke (2006) finds that children from previous unions have a large effect on the allocation of resources between spouses in Denmark. When wives have children from previous unions, they are worse off. When husbands do, they are better off. The authors conclude that “this asymmetry in the effects of having a previous child await a theoretical explanation”. In their textbook on family economics, Browning, Chiappori, and Weiss (2011) state that this result “defies easy rationalization”.

- AKRESH, R., J. CHEN, AND C. MOORE (2016): “Altruism, Cooperation, and Efficiency: Agricultural Production in Polygynous Households,” *Economic Development and Cultural Change*, 64(4), 661–696.
- ANDERSON, S., AND J.-M. BALAND (2002): “The Economics of Roscas and Intrahousehold Resource Allocation,” *The Quarterly Journal of Economics*, 117(3), 963–995.
- ANDERSON, S., AND D. RAY (2010): “Missing Women: Age and Diseases,” *Review of Economic Studies*, 77, 1262–1300.
- ANTOINE, P. (2002): “Les complexités de la nuptialité: de la précocité des unions féminines à la polygamie masculine en Afrique,” in *Démographie : analyse et synthèses. Volume II : Les déterminants de la fécondité*, ed. by G. Caselli, J. Vallin, and G. Wunsch. INED.
- ANTOINE, P., AND J. NANITELAMIO (1995): “Peut-on échapper à la polygamie à Dakar?,” *Les dossiers du CEPED*, 32, 1–31.
- ARENY, F. K. (2002): “Polygyny and child survival in sub-Saharan Africa,” *Social Biology*, 49, 74–89.
- ARROYO, C., AND J. ZHANG (1997): “Dynamic microeconomic models of fertility choice: A survey,” *Journal of Population Economics*, 10, 23–65.
- ARTHI, V., AND J. FENSKE (2016): “Intra-Household Labor Allocation in Colonial Nigeria,” *Explorations in Economic History*, 60, 69–92.
- (2018): “Polygamy and child mortality: Historical and modern evidence from Nigeria’s Igbo,” *Review of Economics of the Household*, forthcoming.
- ASHENFELTER, O. (1978): “Estimating the Effect of Training Programs on Earnings,” *Review of Economics and Statistics*, 60, 47–57.
- ASHRAF, N., E. FIELD, AND J. LEE (2014): “Household Bargaining and Excess Fertility: An Experimental Study in Zambia,” *American Economic Review*, 104(7), 2210–2237.
- BALAND, J., C. GUIRKINGER, AND C. MALI (2011): “Pretending to be poor: Borrowing to Escape Forced Solidarity in Cameroon,” *Economic Development and Cultural Change*, 60(1), 1–16.
- BALAND, J.-M., AND R. ZIPARO (2017): “Intra-household bargaining in poor countries,” *WIDER Working Paper Series*.
- BARR, A., M. DEKKER, W. JANSSENS, B. KEBEDE, AND B. KRAMER (2017): “Cooperation in polygynous households,” *IFPRI Discussion Papers*.
- BLEDSON, C. (1990): “Transformations in Sub-Saharan African Marriage and Fertility,” *The Annals of the American Academy of Political and Social Science*, 510, 115–125.

- BLEDSON, C., F. BANJA, AND A. G. HILL (1998): "Reproductive mishaps and western contraception: An African challenge to fertility theory," *Population and Development Review*, 24(1), 15–57.
- BOLTZ, M., K. MARAZYAN, AND P. VILLAR (2015): "Income Hiding and Informal Redistribution: A Lab-in-the-Field Experiment in Senegal," *PSE Working Papers*.
- BONGAARTS, J., W. M. J. P. (1990): "The demographic impact of family planning programs," *Studies in Family Planning*, 21(6), 299–310.
- BOURGUIGNON, F., M. BROWNING, P. A. CHIAPPORI, AND V. LECHENE (1993): "Intra Household Allocation of Consumption: a Model with some evidence from French data.," *Annales D'Economie et de Statistique*, 29, 137–156.
- BROWNING, M., AND P.-A. CHIAPPORI (1998): "Efficient Intra-household Allocations: A General Characterization and Empirical Tests.," *Econometrica*, 66(6), 1241–1278.
- BROWNING, M., P.-A. CHIAPPORI, AND Y. WEISS (2011): "Family economics.," *Unpublished textbook manuscript*.
- BROWNING, R., AND J. BONKE (2006): "Allocation within the household: direct survey evidence.," *University of Oxford Economics Series Working Papers*.
- CAIN, M. (1984): *Women's Status and Fertility in Developing Countries: Son Preference and Economic Security*. World Bank.
- CALDWELL, J., AND P. CALDWELL (1977): "The role of marital sexual abstinence in determining fertility: a study of the Yoruba in Nigeria.," *Population Studies*, 31(2), 193–217.
- CASTILLA, C., AND T. WALKER (2013): "Is Ignorance Bliss? The Effect of Asymmetric Information between Spouses on Intra-Household Allocations.," *The American Economic Review*, 103(3), 263–268.
- CASTLE, S. (2001): "The tongue is venomous: perception, verbalisation and manipulation of mortality and fertility regimes in rural Mali.," *Social Science and Medicine*, 52, 1827–1841.
- CHIAPPORI, P.-A., B. FORTIN, AND G. LACROIX (2002): "Marriage Market, Divorce Legislation, and Household Labor Supply.," *Journal of Political Economy*, 110(1), 37–72.
- DASGUPTA, P. (1995): "The Population Problem: Theory and Evidence," *Journal of Economic Literature*, 33, 1879–1902.
- DE LA CROIX, D., AND F. MARIANI (2015): "From Polygyny to Serial Monogamy: A Unified Theory of Marriage Institutions.," *The Review of Economic Studies*, 82(2), 565–607.
- DE LAAT, J. (2014): "Household Allocations and Endogenous Information.," *Journal of Development Economics*, 106, 108–117.

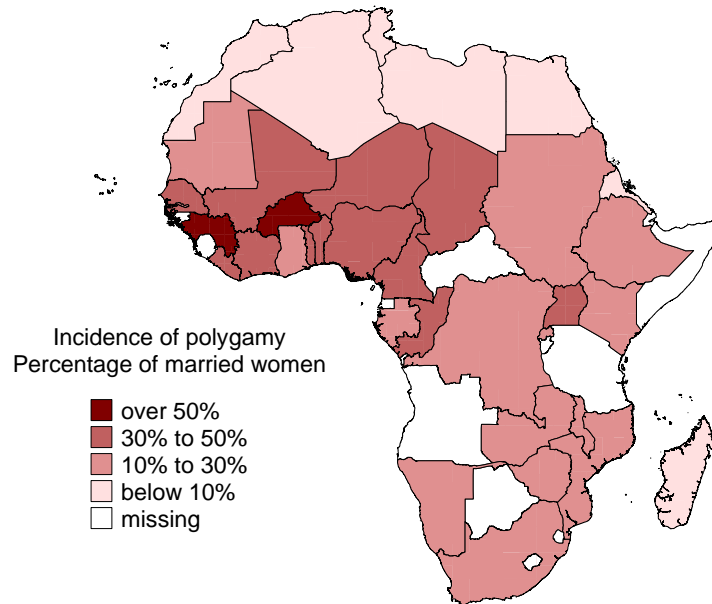
- DEATON, A. (2013): *The great escape. Health, wealth, and the origins of inequality*. Princeton University Press.
- DERCON, S., AND P. KRISHNAN (2000): “In Sickness and in Health: Risk Sharing within Households in Rural Ethiopia,” *Journal of Political Economy*, 108, 688–727.
- DOEPKE, M., AND F. KINDERMANN (2017): “Intrahousehold Decision Making and Fertility,” in *Demographic Change and Long-Run Development*, ed. by M. Cervellati, and U. Sunde. MIT Press.
- DOEPKE, M., AND M. TERTILT (2018): “Women’s Empowerment, the Gender Gap in Desired Fertility, and Fertility Outcomes in Developing Countries,” *American Economic Association, Papers and Proceedings*, forthcoming.
- DUFLO, E., AND C. UDRY (2004): “Intrahousehold Resource Allocation in Cote d’Ivoire: Social Norms, Separate Accounts and Consumption Choices,” *NBER Working Papers 10498*.
- FAINZANG, S., AND O. JOURNET (2000): *La femme de mon mari: étude du mariage polygamique en Afrique et en France*. L’Harmattan.
- FIELD, E., V. MOLITOR, M. TERTILT, AND A. SCHOONBROODT (2016): “Gender Gaps in Completed Fertility,” *Journal of Demographic Economics*, 82(2).
- GARENNE, M., AND E. VAN DE WALLE (1989): “Polygyny and Fertility Among the Sereer of Senegal,” *Population Studies*, 43(2), 267–283.
- GOLDSTEIN, M., AND C. UDRY (2008): “The Profits of Power: Land Rights and Agricultural Investment in Ghana,” *Journal of Political Economy*, 116(6), 981–1022.
- GRAMBSCH, P., AND T. THERNEAU (1994): “Proportional hazards tests and diagnostics based on weighted residuals,” *Biometrika*, 81(3), 515–526.
- HECKMAN, J., AND E. LEAMER (2001): *Handbook of Econometrics, Volume 5*. Elsevier.
- HENRY, L. (1961): “La fécondité naturelle. Observations-théories-résultats,” *Population*, 16(4), 625–636.
- HERTRICH, V. (1996): *Permanences et changements de l’Afrique rurale*. CEPED.
- HOEL, J. (2015): “Heterogeneous Households: A Within-Subject Test of Asymmetric Information between Spouses in Kenya,” *Journal of Economic Behavior and Organization*, 118, 123–135.
- JACOBY, H. (1995): “The economics of polygyny in sub-saharan africa: Female productivity and the demand for wives in cote d’ivoire,” *Journal of Political Economy*, 103(5), 938–971.

- JAKIELA, P., AND O. OZIER (2016): “Does Africa Need a Rotten Kin Theorem? Experimental Evidence from Village Economies,” *The Review of Economic Studies*, 83(1), 231–268.
- JANKOWIAK, W., M. SUDAKOV, AND C. WILREKER (2005): “Co-wife Conflict and Cooperation,” *Ethnology*, 44(1), 81–98.
- JAYACHANDRAN, S., AND I. KUZIEMKO (2011): “Why Do Mothers Breastfeed Girls Less Than Boys? Evidence and Implications for Child Health in India,” *The Quarterly Journal of Economics*, 126, 1485–1538.
- JOKELA, M., A. ROTKIRCH, I. J. RICKARD, J. PETTAY, AND V. LUMMAA (2010): “Serial monogamy increases reproductive success in men but not in women,” *Behavioral Ecology*, 21(5), 906–912.
- KANE, M. (1972): “The Status of Married Women under Customary Law in Senegal,” *The American Journal of Comparative Law*, 20(4), 716–723.
- KAZIANGA, H., AND S. KLONNER (2009): “The Intra-household Economics of Polygyny: Fertility and Child Mortality in Rural Mali,” *MPRA Paper*.
- KAZIANGA, H., AND Z. WAHHAJ (2013): “Gender, Social Norms and Household Production in Burkina Faso,” *Economic Development and Cultural Change*, 61(3), 539–576.
- LAMBERT, S., AND P. ROSSI (2016): “Sons as Widowhood Insurance: Evidence from Senegal,” *Journal of Development Economics*, 120, 113–127.
- LARDOUX, S., AND E. VAN DE WALLE (2003): “Polygyny and Fertility in Rural Senegal,” *Population*, 58(6), 717–743.
- LESTHAEGHE, R. (1989): *Reproduction and Social Organization in Sub-Saharan Africa*. Berkeley: University of California Press.
- LEUNG, S. (1991): “A Stochastic Dynamic Analysis of Parental Sex Preferences and Fertility,” *Quarterly Journal of Economics*, 106(4), 1063–1088.
- MADHAVAN, S. (2002): “Best of Friends and Worst of Enemies: Competition and Collaboration in Polygyny,” *Ethnology*, 41(1), 69–84.
- MADHAVAN, S., AND C. BLEDSOE (2001): “The compound as a locus of fertility management: The case of the Gambia,” *Culture, Health and Sexuality*, 3(4), 451–468.
- MAMMEN, K. (2004): “All for one or each for her own: Do polygamous families share and share alike?” *Unpublished manuscript*.
- PETERMAN, A., A. QUISUMBING, J. BERHMAN, AND E. NKONYA (2011): “Understanding the Complexities Surrounding Gender Differences in Agricultural Productivity in Nigeria and Uganda,” *Journal of Development Studies*, 47(10), 1482–1509.

- PRITCHETT, L. (1994): “Desired Fertility and the Impact of Population Policies.,” *Population and Development Review*, 20(1), 1–55.
- RASUL, I. (2008): “Household Bargaining Over Fertility: Theory and Evidence From Malaysia.,” *Journal of Development Economics*, 86, 215–241.
- RIDDER, G., AND I. TUNALI (1999): “Stratified partial likelihood estimation,” *Journal of Econometrics*, 92, 193–232.
- ROSSI, P., AND L. ROUANET (2015): “Gender Preferences in Africa: A Comparative Analysis of Fertility Choices,” *World Development*, 72, 326–345.
- SEN, A. (1999): *Development as Freedom*. Oxford : Oxford University Press.
- STRASSMAN, B. (1997): “Polygyny as a Risk Factor for Child Mortality among the Dogon,” *Current Anthropology*, 38, 688–695.
- TABUTIN, D., AND B. SCHOUMAKER (2004): “La démographie de l’Afrique au sud du Sahara des années 1950 aux années 2000. Synthèse des changements et bilan statistique,” *Population*, 59(3-4), 521–622.
- TERTILT, M. (2005): “Polygyny, fertility, and savings,” *Journal of Political Economy*, 113(6), 1341–1374.
- UDRY, C. (1996): “Gender, Agricultural Production, and the Theory of the Household,” *Journal of Political Economy*, 104, 1010–1046.
- UNITED NATIONS (2001): “Widowhood: invisible women, secluded or excluded,” *United Nations publication, DAW/DESA*.
- (2013): “World Population Policies.,” *United Nations publication, ST/ESA/SER, A/341*.
- (2015): “World Population Prospects: The 2015 Revision.,” *United Nations publication, ESA/P, WP/241*.
- VREYER, P. D., S. LAMBERT, A. SAFIR, AND M. B. SYLLA (2008): “Pauvreté et structure familiale, pourquoi une nouvelle enquête?,” *Stateco*, 102, 261–275.
- WESTOFF, C. (2010): “Desired Number of Children: 2000-2008,” *DHS Comparative Reports*, 25.
- WHO, UNICEF, UNFPA AND THE WORLD BANK (2014): “Trends in Maternal Mortality: 1990 to 2013.,” *WHO publication*.
- WILLITTS, M., M. BENZEVAL, AND S. STANSFELD (2004): “Partnership history and mental health over time.,” *Journal of Epidemiology and Community Health*, 58, 53–58.

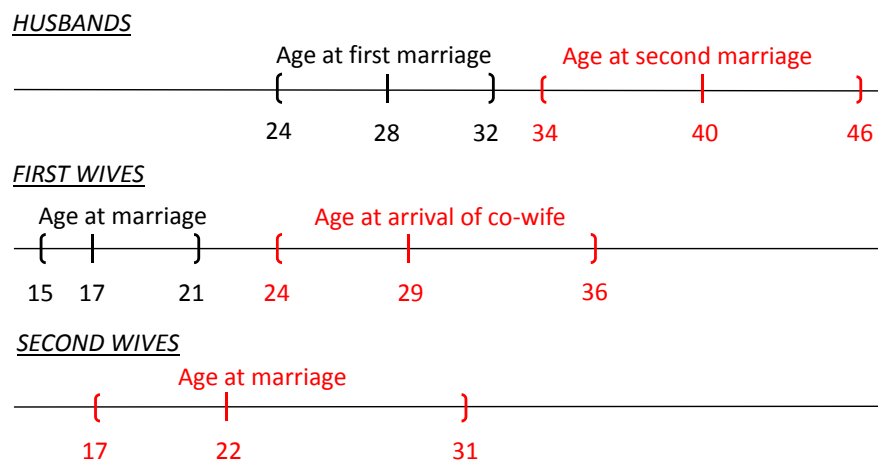
Figures and Tables

Figure 1: Polygamy in Africa



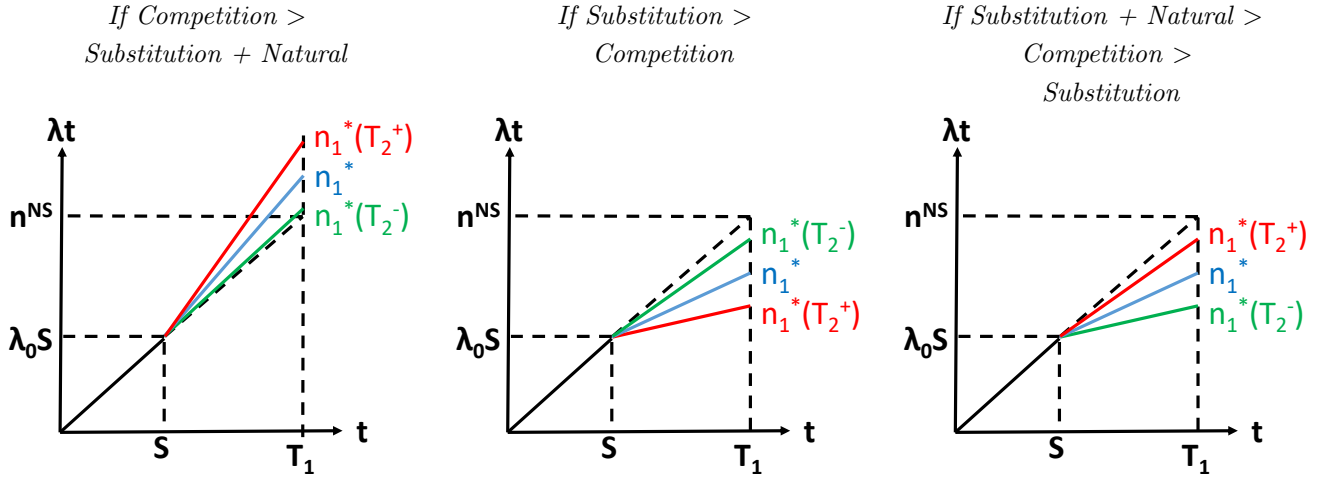
The map shows the percentage of married women, aged 15-49, with a polygamous husband in the early 2000s (the exact year depends on the country). Source: Tabutin and Schoumaker (2004) and Tertilt (2005).

Figure 2: Timing of successive unions



The graph represents the distribution of ages at the husband's first (in black) and second (in red) marriages. Median ages are indicated by vertical dots, while braces indicate the first and last quartiles. Data: PSF. Sample: polygamous unions (411 unions).

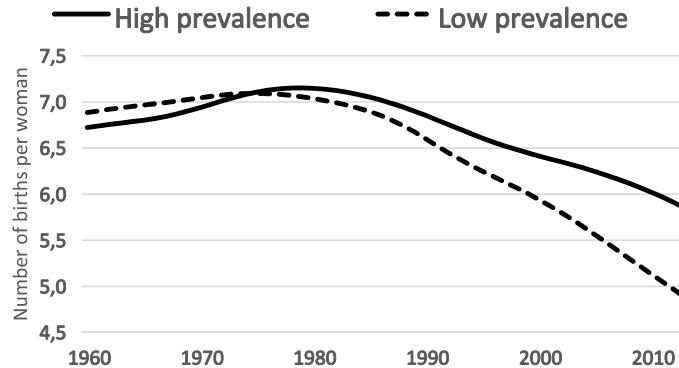
Figure 3: Theoretical change in first wives' birth rate after the entry of second wives



The graphs plot the first wife's realized number of children λt as a function of time in three different scenarios. Between $t = 0$ and $t = S$, in the monogamous stage, the wife targets n^{NS} and hence chooses λ_0 . At date S , the polygamous stage starts. The wife updates her choice and chooses λ_1 in order to reach n_1^* at the end of her reproductive period T_1 . According to the model, the change in birth rate depends on the sign and magnitude of the strategic effect. The blue line is the average response, the red line is the response when the second wife's reproductive period is long (T_2^+) and the green line is the response when the second wife's reproductive period is short (T_2^-).

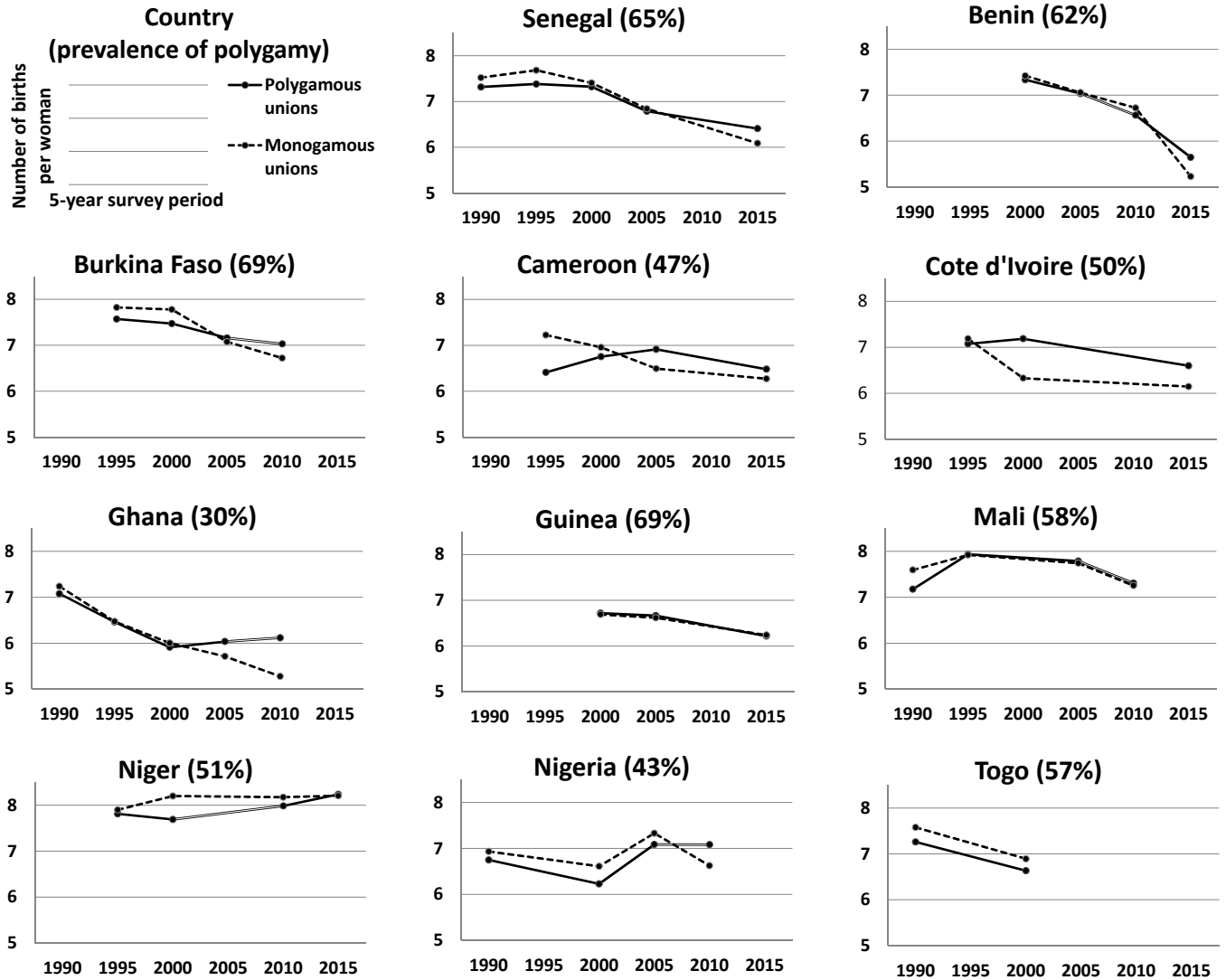
In the first scenario, the competition effect dominates. The birth rate increases after the second marriage, and it increases more when the second wife's fertility is expected to be higher. In the second scenario, the prevailing force is the substitution effect. The birth rate decreases after the second marriage, and it decreases more when the second wife's fertility is expected to be higher. In the third scenario, the competition effect dominates the substitution effect, but it is not large enough to dominate the combination of the substitution effect and the natural effect. The birth rate decreases after the second marriage, and it decreases less when the second wife's fertility is expected to be higher.

Figure 4: Macro level: evolution of fertility rate, by prevalence of polygamy



The graph shows the evolution of the total fertility rate (weighted by population size) in two groups of countries: those with a high and those with a low prevalence of polygamy (over and below 30%). See Figure 1 above for a list of the countries. Source: author's calculations using World Development Indicators.

Figure 5: Micro level: evolution of completed fertility, by union type



Each graph shows the evolution of the total number of births per woman for women older than 40 married to a polygamous (solid line) or a monogamous (dotted line) husband, in a given country. The title indicates the country and the proportion of women in the sample married to a polygamous husband circa 2000. Only countries with a high prevalence of polygamy are represented. Source: author's calculations using DHS waves between 1986 and 2013.

Table 1: Number of children in current union, by union type

Sample	All	All	At least one child	
Constant (Monogamous union)	5.074*** (0.183)	5.074*** (0.170)	5.477*** (0.157)	3.687*** (0.338)
Polygamous union	-1.071*** (0.232)			
Senior wife		0.188 (0.250)	0.138 (0.230)	-0.130 (0.228)
Junior wife		-2.512*** (0.260)	-1.370*** (0.270)	-0.933*** (0.279)
Length of reproductive period				0.081*** (0.013)
Observations	674	674	568	551

Data: PSF. Sample: women over 45 years old. In the last two columns, I restrict the sample to women having at least one child with the current husband. Dep. Var.: number of surviving children in current union. The length of the couple's reproductive period, in years, is proxied by min (45- wife's age at marriage; 60-husband's age at marriage). OLS regression. Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

Table 2: Birth intervals before and after the second marriage, by type of co-wife

First wives' birth intervals (in months)	Whole sample	Less fertile co-wife	Very fertile co-wife
Before	37.6 (21.0)	37.8 (21.9)	37.1 (19.0)
nb obs.	216	147	69
After	43.6 (26.2)	47.4 (31.1)	41.8 (23.6)
nb obs.	184	58	126
Difference after-before	6.0** (2.4)	9.6** (3.8)	4.7 (3.3)

Data: PSF. Sample: first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union. The sample is further split on the median co-wife's reproductive period into those facing a less fertile co-wife in column 2 (reproductive period below 20 years) and those facing a very fertile co-wife in column 3 (reproductive period above 20 years). Co-wife's reproductive period proxied by min (45-second wife's age at marriage; 60-husband's age at second marriage). Statistics: average birth intervals in months, non-censored durations, standard deviations are in parentheses. The last line reports the difference after-before, standard errors are in parentheses, significance levels: * p<0.10, ** p<0.05, *** p<0.01.

Table 3: Testing the common life-cycle assumption

First wives' birth rates (hazard ratios)	Between polygamous and monogamous wives	Among polygamous wives
Birth rank 2	0.237*** (0.034)	0.128*** (0.071)
Birth rank 3	0.066*** (0.016)	0.018*** (0.017)
Birth rank 4	0.015*** (0.006)	0.005** (0.011)
Future first wives * Rank 2	0.842 (0.168)	
Future first wives * Rank 3	0.844 (0.227)	
Future first wives * Rank 4	1.121 (0.414)	
Co-wife's reproductive period * Rank 2		0.995 (0.028)
Co-wife's reproductive period * Rank 3		1.004 (0.034)
Co-wife's reproductive period * Rank 4		0.946 (0.091)
Controls	Mother's age and age ² at birth	
Baseline hazard	Woman-specific	Woman-specific
Observations	1411	207
Clusters	571	94

Data: PSF. Sample: in column 1, monogamous and senior wives *before the second marriage*, below 45 years old, for whom the complete birth history is known, having at least one child from current union. In column 2, senior wives *before the second marriage*, below 45 years old, for whom the complete birth history is known, having at least one child from current union. Dep. var.: duration between births j and $(j + 1)$. Birth ranks higher than 4 are excluded because there are not enough observations in each cell. The reference category is rank 1. "Future first wives" is equal to 1 if the woman is in a polygamous union at the time of the survey. Co-wife's reproductive period proxied by min (45-second wife's age at marriage; 60-husband's age at second marriage). Stratified partial likelihood estimation with baseline hazards specific to each woman; Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1): * p<0.10, ** p<0.05, *** p<0.01.

Table 4: Change in first wives' birth spacing: linear model

First wives' birth intervals (in months)	Whole sample	Less fertile co-wife	Very fertile co-wife	Linear interaction
After	6.988** (2.815)	14.000*** (4.011)	0.698 (3.422)	25.613*** (8.288)
After * Co-wife's reproductive period				-0.994** (0.414)
Controls	Birth rank dummies, mother's age and age ² at birth			
Woman FE	Yes	Yes	Yes	Yes
Observations	1715	1509	1508	1715
Clusters	597	525	532	597

Data: PSF. Sample: monogamous and first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union. First wives are further split on the median co-wife's reproductive period into those facing a less fertile co-wife in column 2 (reproductive period below 20 years) and those facing a very fertile co-wife in column 3 (reproductive period above 20 years). Co-wife's reproductive period proxied by $\min(45 - \text{second wife's age at marriage}; 60 - \text{husband's age at second marriage})$. Dep. var.: duration between births j and $(j + 1)$. Non-censored durations. Extreme values are excluded (larger than seven years, top 5%); results are qualitatively similar when I include them. *After* is a time-varying variable indicating if the second wife had arrived when the child $(j + 1)$ was conceived. Linear estimation with woman fixed effects; robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 5: Change in first wives' birth spacing: duration model

First wives' birth rates (hazard ratios)	Total effect	Natural vs. strategic effect	By husband's income			By wife's education		Adding gender
	(1)	(2)	Poor (3)	Middle (4)	Rich (5)	Yes (6)	No (7)	
After	0.770 (0.143)	0.201** (0.140)	0.636 (0.549)	0.297 (0.310)	0.003*** (0.005)	0.00003** (0.00014)	0.293 (0.231)	0.226** (0.161)
After * Co-wife's reproductive period No son		1.077** (0.036)	1.034 (0.051)	1.063 (0.053)	1.265*** (0.084)	1.517** (0.264)	1.055 (0.041)	1.067* (0.039)
After * No son								1.381*** (0.164)
Controls								2.197* (0.919)
Baseline hazard								
Observations	2483	2483	543	1140	450	814	1591	2397
Clusters	716	716	150	322	143	257	441	695

46

Data: PSF. Sample: monogamous and first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union. Dep. var.: duration between births j and $(j + 1)$. Co-wife's reproductive period proxied by min (45-second wife's age at marriage; 60-husband's age at second marriage). *After* is a time-varying variable indicating if the second wife has arrived. *No son* is a dummy equal to one if the woman had no son among her first j births. Income, education and gender are missing for some observations. Columns 1, 2 and 8 include the whole sample. Columns 3-5 present the main estimates for different categories of the husband's income distribution: poor = bottom 25% (column 3), middle class = between 25% and 75% (column 4) and rich = top 25% (column 5). The next two columns split the sample into educated (column 6) and non-educated wives (column 7). Stratified partial likelihood estimation with baseline hazards specific to each woman; Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1): * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. The p-value of the difference between the interaction terms of two categories is 0.71 for poor vs. middle; 0.12 for middle vs. rich; 0.06 for poor vs. rich and 0.04 for educated vs. non-educated.

Table 6: Change in birth spacing after the exit of a co-wife

Women's birth rates (hazard ratios)	Whole sample		Co-wife has children alive:	
			No	Yes
After exit	1.761*	1.473	0.886	2.223**
	(0.601)	(0.776)	(0.605)	(0.805)
After exit * co-wife's number of children		1.119		
		(0.201)		
Controls	Birth rank dummies, mother's age and age ² at birth			
Baseline hazard	Woman-specific		Woman-specific	
Observations	1738	1738	1627	1705
Clusters	516	516	490	504

Data: PSF. Sample: wives in monogamous unions at the date of the survey, below 45 years old, for whom the complete birth history is known, having at least one child from current union. 38 women out of 516 experienced the exit of a co-wife, among these 12 had no child alive and 26 had at least one child alive. Dep. var.: duration between births j and $(j + 1)$. *After* is a time-varying variable indicating if the co-wife has left the union. Stratified partial likelihood estimation with baseline hazards specific to each woman; Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1): * p<0.10, ** p<0.05, *** p<0.01.

Table 7: Second wives' birth spacing

Second wives' birth rates (hazard ratios)	Using length of monogamous stage	Using number of children born	Adding gender
Own reproductive period (T_2)	0.977 (0.034)	0.988 (0.064)	0.992 (0.066)
Length of polygamous stage ($T_1 - S$)	1.018 (0.022)	1.206*** (0.053)	1.168*** (0.043)
Length of monogamous stage (S)	1.027* (0.015)		
Number of children born to the first wife		1.305*** (0.101)	1.173* (0.101)
First wife has no son			0.527** (0.162)
Specific controls	Predictors of preferences		
Additional controls	Yes	Yes	Yes
Baseline hazard	Common to all women		
Observations	473	233	233
Clusters	146	84	84

Data: PSF. Sample: second wives, below 45 years old, for whom the complete birth history is known. In columns 2 and 3, I focus on unions in which the number and gender of children born to the first wife at the time of the second marriage is known. Dep. var.: duration between births j and $(j + 1)$. Duration between marriage and first birth is also included. $T_2 = \min(45 - \text{second wife's age at marriage}; 60 - \text{husband's age at second marriage})$; $S = (\text{husband's age at second marriage} - \text{husband's age at first marriage})$; $(T_1 - S) = \min(45 - \text{first wife's age at marriage}; 60 - \text{husband's age at first marriage}) - S$. Predictors of husband's and wife's preferences: religion, ethnic group, education, rural dummy, income (husband), employment status (wife), birth cohort, region of residence, age at marriage, be in first marriage. Additional controls: co-residence status, work in public sector (husband), at least one child from previous union, having at least one dead child from current union, wife's age and age² at birth j , a dummy for each j . Cox estimation with a baseline hazard common to all women. Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1): * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 8: First wives' fertility, by gender of second wives' children

	Co-wife's first born is:		pvalue of difference
	a boy	a girl	
Total number of births	5.58 (2.31)	4.62 (2.86)	0.09
Number of dead children	1.18 (1.64)	1 (1.41)	0.6
Number of children born after the co-wife's first birth	1.78 (1.54)	1.31 (1.32)	0.13
First wives with a son	1.33 (1.22)	1.12 (1.23)	0.51
First wives with no son	3.18 (1.6)	2.13 (1.46)	0.08
At least one child born after the co-wife's first birth	76%	64%	0.25
First wives with a son	67%	62%	0.59
First wives with no son	100%	75%	0.24
Number of observations	45	42	

Data: PSF. Sample: first wives, below 45 years old, for whom the complete birth history is known, having at least one child from current union. I restrict to unions in which the second wife has at least one birth. Standard deviations are in parentheses.

Appendix

1. Detailed solution of the model

Time consistency

Consider the case of monogamous unions. At date 0, the woman maximizes Equation 1 and chooses λ^* . Suppose that she can update her choice at date t . She already has λ^*t children, and maximizes over λ' :

$$-(n - n_w^{id})^2 - \theta^h(n - n_h^{id})^2 - \theta^n(t \cdot (\lambda^* - \lambda_m^{nat}) + (T - t) \cdot (\lambda' - \lambda_m^{nat}))^2 \text{ s.t. } n = \lambda^*t + \lambda'(T - t)$$

It is equivalent to maximizing Equation 1 over $\lambda = \frac{\lambda^*t + \lambda'(T-t)}{T}$. Therefore $\lambda = \lambda^* = \lambda'$.

Polygamous stage: best responses and Nash equilibrium

Taking the first-order condition for wife i when the other plays n_{-i} , I find:

$$n_i^* = n_i^{NS} + n_{-i}B_i \text{ for } i = 1, 2,$$

where $n_i^{NS} = \frac{n_w^{id} + \theta_i^h n_h^{id} + \theta_i^n n_i^{nat}}{1 + \theta_i^h + \theta_i^n}$ is the optimal choice in the absence of strategic interactions, and $B_i = \frac{\gamma_i - \theta_i^h}{1 + \theta_i^h + \theta_i^n}$ is the strategic response.

The Nash equilibrium is the intersection of both best responses. I get:

$$n_i^* = (n_i^{NS} + n_{-i}^{NS} B_i) \times \frac{1}{1 - B_1 B_2} \text{ for } i = 1, 2.$$

I further impose that $\gamma_i \in [0, 1]$ so that $B_i \in [-1, 1]$ and $(1 - B_1 B_2) \geq 0$. If $(1 - B_1 B_2) = 0$, there is no equilibrium. It happens when $\gamma_i = 1$, $\theta_i^h = \theta_i^n = 0$ for $i = 1, 2$, meaning that the rivalry effect is not offset by any kind of marital or biological constraint. The number of children of both wives is pushed to infinity. Another extreme case is when $B_i \rightarrow -1$ for $i = 1, 2$. It happens when $\gamma_i = \theta_i^n = 0$ and θ_i^h is very large, meaning that only the husband's objective is driving fertility choices. Here, there is an infinite number of equilibria: (n_1, n_2) s.t. $n_1 + n_2 = n_h^{id}$. Both cases are easily ruled out by the fact that fertility choices are not free from any biological constraints, so θ_i^n is never equal to zero.

Note that the equilibrium of the static game is fully determined by n_1^{NS} , n_2^{NS} , B_1 and B_2 . Whatever λ_0 , first wives adjust their birth rate after the second marriage; they choose $\lambda_1^*(\lambda_0)$ such that $\lambda_0 S + \lambda_1^*(\lambda_0)(T_1 - S) = n_1^* = (n_1^{NS} + n_2^{NS} B_1) \times \frac{1}{1 - B_1 B_2}$. As a consequence,

choices in the first period are not influenced by expectations about the second period.

Monogamous stage: back to $t = 0$

At $t = 0$, first wives maximize over λ_0 :

$$(1 - \pi) \times u(\lambda_0 T_1, 0) + \pi \times \mathbb{E}[u(n_1^*, n_2^*)].$$

Since $u(n_1^*, n_2^*)$ does not depend on λ_0 , the maximization problem boils down to the problem in monogamous societies. The optimal initial birth rate is:

$$\lambda_0^* = \frac{n_w^{id} + \theta^h n_h^{id} + \theta^n n_0^{nat}}{(1 + \theta^h + \theta^n) \cdot T_1} = \frac{n_0^{NS}}{T_1},$$

where $n_0^{nat} = \lambda_m^{nat} T_1$. In Appendix B5, I compare the equilibrium and the outcome maximizing total welfare. Consistently with most non-cooperative models, I find that household members are unable to reach an optimal allocation.

We can prove that an equilibrium exists as soon as $B_i \geq 0$ for $i = 1, 2$. If we come back to the Nash equilibrium described above, it exists if and only if $\lambda_i^* \geq 0$ for $i = 1, 2$. For the second wife, it is the case when $B_2 \geq 0$. For the first wife, it is the case when $n_1^*(S) \geq \lambda_0^* S$, which is true when $B_1 \geq 0$.⁴⁷ In other words, whatever the length of the monogamous period, the first wife always wants more children than she currently has at the time of the second marriage.

2. Additional empirical tests

⁴⁷ $f(S) = n_1^*(S) - \lambda_0^* S$ is monotonic, and $f(0)$ and $f(T_1)$ are both non-negative, so $f(S) \geq 0$ for all S .

Table A.1: Testing for strategic interactions using completed fertility

Dep. var.	Total number of children n^*	
Panel A: first wives		
Own reproductive period (T_1)	0.220 (0.198)	0.245 (0.189)
Length of monogamous stage (S)	0.056 (0.069)	0.114** (0.048)
Co-wife's reproductive period (T_2)	0.023 (0.073)	
Short T_2		-2.098*** (0.761)
Long T_2		0.580 (0.684)
Observations	101	101
Panel B: second wives		
Own reproductive period (T_2)	0.074 (0.244)	0.334* (0.160)
Length of monogamous stage (S)	0.040 (0.107)	
Length of polygamous stage ($T_1 - S$)	0.013 (0.157)	
Short S		-3.354** (1.013)
Short ($T_1 - S$)		-2.307 (1.753)
Observations	48	48
Specific controls	Predictors of preferences	
Additional controls	Yes	Yes
Woman FE	No	No

Data: PSF. Sample: first wives (in Panel A) and second wives (in Panel B) over 45 years old, having at least one child from current union. Dep. var.: number of children in current union. S = (husband's age at second marriage - husband's age at first marriage); T_1 = min (45-first wife's age at marriage; 60-husband's age at first marriage); T_2 = min (45-second wife's age at marriage; 60-husband's age at second marriage); Short T_2 : T_2 is below Q_1 (10 years) and Long T_2 : T_2 is above Q_3 (22 years). Short S : S is below the median (9 years). Short ($T_1 - S$): $T_1 - S = 0$. Predictors of preferences: cf. Table 7. Additional controls: co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union. OLS estimation. Significance levels : * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A.2: Testing if polygamy is caused by choices in the monogamous stage

First wives' birth rates (hazard ratios)	Testing the occurrence	Testing the timing
Future first wives	0.953 (0.154)	
Co-wife's reproductive period (T_2)		0.907 (0.083)
Length of monogamous stage (S)		0.908 (0.086)
Specific controls	T_1 and predictors of preferences	
Additional controls	Yes	Yes
Baseline hazard	Common to all women	
Observations	476	209
Clusters	106	75

Data: PSF. Sample: in column 1, monogamous and senior wives *before the second marriage*, between 40 and 45 years old, for whom the complete birth history is known, having at least one child from current union. I restrict the sample to women over 40 to ensure that most women in the reference category "monogamous wives" will remain the sole wife in this union. In column 2, senior wives *before the second marriage*, below 45 years old, for whom the complete birth history is known, having at least one child from current union; I restrict the analysis to bigamous unions. Dep. var.: duration between births j and $(j + 1)$. "Future first wives" is equal to 1 if the woman is in a polygamous union at the time of the survey. $T_i = \min(45 - \text{age at marriage of wife } i; 60 - \text{husband's age at marriage with wife } i)$. $S = (\text{husband's age at second marriage} - \text{husband's age at first marriage})$. Predictors of preferences: religion (husband and wife), ethnic group (husband and wife), education (husband and wife), rural dummy (husband and wife), income (husband), employment status (wife), birth cohort (husband and wife), region of residence, age at marriage (husband and wife), be in first marriage (husband and wife). Additional controls: co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union, mother's age and age² at birth j , a dummy for each j . Cox estimation with a baseline hazard common to all women. Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1): * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A.3: Predictors of polygamy

Sample	Polygamous unions
Husband's father is polygamous	0.079 (0.065)
Fostered before age 15 (wife)	-0.029 (0.087)
Fostered before age 15 (husband)	0.092 (0.078)
Age at first marriage (wife)	0.002 (0.007)
Age at first marriage (husband)	-0.009 (0.006)
Income (husband)	0.006 (0.010)
Work in public sector (husband)	0.091 (0.096)
No education (wife)	-0.055 (0.090)
No education (husband)	0.143* (0.080)
Rural household	-0.006 (0.089)
Children from previous unions (wife)	-0.308 (0.247)
Children from previous unions (husband)	-0.144 (0.135)
Being in first marriage (wife)	-0.128 (0.235)
Being in first marriage (husband)	-0.242* (0.130)
Never worked (wife)	0.013 (0.076)
Christian (wife)	-0.004 (0.342)
Christian (husband)	0.005 (0.347)
Cohort dummies	Yes
Region dummies	Yes
Ethnic groups dummies	Yes
Observations	244
Pseudo R2	0.16

Data: PSF. Sample: monogamous and first wives, older than 45 years old, having at least one child from current union. Dep. var: being in a polygamous union at the time of the survey. Probit estimation. Marginal effects are reported. The average predicted probability of polygamy is 45%. Significance levels : * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A.4: Testing if the risk of polygamy influences birth spacing

First wives' birth rates (hazard ratios)	In the monogamous stage
Predicted risk of polygamy ($\hat{\pi}$)	0.901 (0.660)
Specific controls	T and predictors of preferences
Additional controls	Yes
Baseline hazard	Common to all women
Observations	983
Clusters	298

Data: PSF. Sample: monogamous and senior wives *before the second marriage*, below 45 years old, for whom the complete birth history is known, having at least one child from current union. Dep. var.: duration between births j and $(j + 1)$. $\hat{\pi}$ is the predicted probability of a second marriage (cf. Probit estimation in Table A.3). Predictors of preferences: religion (husband and wife), ethnic group (husband and wife), education (husband and wife), rural dummy (husband and wife), income (husband), employment status (wife), birth cohort (husband and wife), region of residence, age at marriage (husband and wife), be in first marriage (husband and wife). Additional controls: co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union, mother's age and age² at birth j , a dummy for each j . Cox estimation with a baseline hazard common to all women. Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1): * p<0.10, ** p<0.05, *** p<0.01.

Table A.5: Ruling out change in income as a potential confounder

Panel A: does fertility increase after any positive income shock?

Women' birth rates (hazard ratios)	All wives	Monogamous	First wives
After a good year	0.8 (0.135)	0.82 (0.148)	0.558 (0.282)
Controls	Birth rank dummies, mother's age and age ² at birth		
Baseline hazard	Woman-specific	Woman-specific	Woman-specific
Observations	2427	1906	521
Clusters	697	562	135

Panel B: is the differential response by co-wife's age driven by bride price?

First wives' birth rates (hazard ratios)	Reproductive period alone	Bride price alone	Reproductive period and bride price
After the second marriage	0.175* (0.161)	1.19 (0.353)	0.236 (0.216)
After * Co-wife's reproductive period	1.082* (0.046)		1.103** (0.047)
After * Co-wife's bride price		0.997 (0.002)	0.996** (0.002)
Controls	Birth rank dummies, mother's age and age ² at birth		
Baseline hazard	Woman-specific	Woman-specific	Woman-specific
Observations	2349	2349	2349
Clusters	683	683	683

Data: PSF. Sample: monogamous and first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union. A few observations are missing compared to my main specification (716 women) because they have missing information for income shock (in Panel A) or bride price (in Panel B). Dep. var.: duration between births j and $(j + 1)$. In Panel A, *After a good year* is a time-varying variable indicating if the household had an exceptionally good year in terms of income (self-reported). 34% of households report a good year in the past 5 years. In Panel B, *After* is a time-varying variable indicating if the second wife has arrived. Stratified partial likelihood estimation with baseline hazards specific to each woman; Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1): * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

For online publication

Online appendix A: Additional descriptive statistics

A1. Realized fertility

Table OA.1: Impact of ages at marriage on total fertility

Threshold for large age difference	15 years	18 years
Wife's age at marriage	-0.078*** (0.027)	-0.087*** (0.022)
Husband's age at marriage	-0.011 (0.027)	0.001 (0.021)
Wife's age at marriage * Large age difference	0.023 (0.048)	0.043 (0.059)
Husband's age at marriage * Large age difference	-0.050 (0.044)	-0.105* (0.055)
Specific controls	Union type	Union type
Additional controls	Yes	Yes
Nb obs	564	564
F-test (p-val)		
Wife's age + Wife's age * Large=0	0.177	0.434
Husband's age + Husband's age * Large=0	0.088*	0.045**

Data: PSF. Sample: women over 45 years old. Dep. var.: number of children in current union. Union type: monogamous, senior wife, junior wife; Large age difference is dummy equal to 1 if the age difference between spouses is larger than 15 years in column 1 or 18 years in column 2. These thresholds correspond roughly to the difference in ages at fertility decline between men and women. Additional controls: co-residing with husband, education, area of residence, at least one child from previous union, being in first marriage, employment status, ethnic group. OLS estimation. Significance levels : * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table OA.2: Fertility in current union, by mother's rank

Dep. var.	Number of children	Birth rates
Estimation	OLS	Cox (hazard ratios)
rank1	-0.134 (0.226)	0.889** (0.046)
rank2	-0.870*** (0.294)	0.783*** (0.041)
rank3	-1.222** (0.533)	0.696*** (0.075)
rank4	-1.467 (1.065)	0.561** (0.149)
Controls	T and children from previous unions	
Observations	550	3717

Data: PSF. Sample: in column 1, women over 45 years old having at least one child with current husband. In column 2, women below 45 years old, having at least one child with current husband, and for whom the complete birth history is known. Reference category: monogamous wives. Dep.var: in column 1, number of children in current union. In column 2, duration between births j and $(j + 1)$. $T = \min(45 - \text{wife's age at marriage}; 60 - \text{husband's age at marriage})$. In column 1: OLS estimation; the unit of observation is the woman. In column 2: Cox estimation with birth rank dummies; the unit of observation is the birth; Breslow method to handle ties among non-censored durations; robust standard errors clustered at the woman level. Significance levels (for hazard ratio= 1 in the Cox estimation): * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

In both regressions, the coefficient on *rank1* is significantly different from the coefficients on other ranks, but the coefficients on *rank2*, *rank3*, and *rank4* are not significantly different from one another (pair-wise F-tests). Since there are few women of rank 3 and 4, the estimates are very imprecise.

A2. Fertility preferences

I compute the descriptive statistics on fertility preferences using the most recent DHS waves (2005 and 2010), which were conducted in the same years as the PSF survey (2006-2007). Table OA.3 below reports the predictors of the ideal number of children, for men and women separately. First, socio-economic status is clearly negatively correlated with the ideal family size: educated and wealthier men and women, as well as urban men, want fewer children. This is also the case for younger cohorts. Then, marital history matters: men and women who got married younger, and men who got married more than once, want more children. Household heads and their wives display on average the same preferences as other members. Last, there is some variation across religions – the ideal family size is smaller for Christians – ethnic groups and regions.

Note that a sizeable proportion of the respondents gave a non-numerical answer to the question "How many children would you like to have, or would you have liked to have?". 23% of women and 31% of men answered "I don't know" or any non-numerical statement such as "It is up to God". As a result, I know the ideal number of children for both spouses in roughly half of the couples (55%). The selection is not random: those couples are younger, more urban, richer and more educated.

One may wonder if respondents rationalize ex-post their fertility behavior, and report that they would have wanted exactly the same number of children as they actually have had. This would be an issue when I use the reported ideal number as a proxy for innate preferences to estimate the model in Table OD.4. To assess the validity of such a concern, I restrict the sample to older couples (all wives above 40, husbands above 50) and I compare the ideal family size to the realized one. Ideal and realized numbers coincide for only 10% of husbands and 17% of wives. Over one third of men and one half of women declare that they would have wanted fewer children than they actually have had. Such figures provide some level of assurance that the scope for ex-post rationalization is limited.

Table OA.3: Predictors of the ideal number of children

Sample	Husbands	Wives
Christian	-1.731*** (0.481)	-0.935*** (0.209)
Other religion	-0.437 (1.311)	0.351 (0.650)
Serer	-0.259 (0.402)	0.434** (0.198)
Fula	-0.891** (0.352)	0.224* (0.131)
Mandinka	-0.337 (0.417)	-0.000 (0.240)
Soninke	-1.908* (1.024)	0.290 (0.509)
Jola	-0.728 (0.550)	-0.146 (0.235)
Other ethnic group	-0.356 (0.512)	0.138 (0.202)
No education	0.821*** (0.295)	0.421*** (0.119)
Rural	0.937*** (0.341)	0.187 (0.138)
Wealth index	-0.555*** (0.138)	-0.246*** (0.052)
Employed	-0.265 (0.419)	0.158 (0.110)
Head or head's spouse	0.295 (0.289)	-0.187 (0.114)
Age at first marriage	-0.073*** (0.022)	-0.041*** (0.014)
Being in first marriage	-0.727** (0.352)	-0.047 (0.146)
Monogamous union	-2.709*** (0.479)	-0.123 (0.117)
Constant	14.739*** (1.286)	7.055*** (0.419)
Cohort Fixed Effect	Yes	Yes
Region Fixed Effect	Yes	Yes
Observations	1928	2523
R2	0.26	0.15

Data: DHS, waves 2005 and 2010. Weights. Sample: respondents who gave a numerical answer to the question "How many children would you like to have, or would you have liked to have, in your whole life?" Dep. var: ideal number of children. Reference categories are "Muslims" for the religion and "Wolof" for the ethnic group. OLS estimation. Significance levels : * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Online appendix B: Extensions of the model

B1. Determining the wife's objective

Following Jayachandran and Kuziemko (2011), I assume that women face a standard economic trade-off modeled by a function $v(n) = q(n) - c(n)$ where $q(n)$ and $c(n)$ measure respectively the benefits and costs of having n children for the mother. $v(n)$ captures the total net gain and is assumed to display an inverted u-shape. Several factors shape the trade-off: the wife's individual characteristics and, in a context of polygamy, the number of children of her co-wives, n_{-i} . According to the literature review, wives care about their relative number of children, compared to the co-wives. It must therefore be the case that co-wives' fertility enters the trade-off. More precisely, I assume that n_{-i} raises the marginal benefit of children for woman i . It reflects the idea that children are more valuable when co-wives have themselves many children. I consider the following parametric forms:

$$c(n_i) = \sigma_i n_i^2 \text{ and } q(n_i, n_{-i}) = n_i(\alpha_i + \beta_i n_{-i}).$$

The parameters $\sigma_i > 0$, $\alpha_i > 0$ and $\beta_i \geq 0$ are specific to each woman. The marginal benefit of children is assumed to be a linear function of n_{-i} . This assumption enables a simple closed form solution for the Nash equilibrium. It will be relaxed in Appendix B2.

The first-order condition gives the optimal number of children of woman i :

$$\frac{\alpha_i + \beta_i n_{-i}}{2\sigma_i}.$$

I denote $n_{w,i}^{id} = \alpha_i/2\sigma_i$ the wife's ideal number of children in case of monogamy. Further, $\gamma_i = \beta_i/2\sigma_i$ measures by how much the wife's objective increases when a co-wife has an additional child; it captures the intensity of co-wife rivalry. In the end, wife i targets $(n_{w,i}^{id} + \gamma_i n_{-i})$ children.

B2. Relaxing the assumptions on functional forms

In this section, I investigate whether the predictions of the model remain valid with more general functional forms. Instead of a weighted sum of distances, the wife's utility is assumed to satisfy:

$$u(n_i, n_{-i}) = v(n_i, n_{-i}, n_{w,i}^{id}) - \theta_i^h H(n_i, n_{-i}, n_h^{id}) - \theta_i^n N(n_i, n_i^{nat}).$$

$v(\cdot)$ is the total net gain of children for the wife. Consistently with Appendix B1, I assume that it is concave in n_i , and that the cross-derivative with respect to n_i and n_{-i} is positive. $H(\cdot)$ is the cost to deviate from the husband's objective. I assume that it is convex in n_i , and that the cross-derivative with respect to n_i and n_{-i} is positive. $N(\cdot)$ is the cost to deviate from the natural level. I assume that it is convex in n_i and does not depend on n_{-i} .

In such a general framework, there is no closed form for the key dependent variables, but I can still predict that children are strategic complements iff:

$$b_i = \frac{\partial^2 v}{\partial n_i \partial n_{-i}} - \theta_i^h \frac{\partial^2 H}{\partial n_i \partial n_{-i}} > 0.$$

In the basic model, $\frac{\partial^2 v}{\partial n_i \partial n_{-i}} = 2\gamma_i$ and $\frac{\partial^2 H}{\partial n_i \partial n_{-i}} = 2$ so the condition boils down to $\gamma_i - \theta_i^h > 0$. When considering more general forms, one needs to compare the impact of an additional child born to the co-wife on the marginal net gain from children to the mother and the impact on the marginal net cost related to the husband's constraint.

The predictions on how the reproductive period of one wife should impact fertility choices of her co-wife are still valid if we replace the condition $B_i > 0$ by the more general one stated above.⁴⁸ In particular, I find that $\frac{\partial n_1^*}{\partial T_2}$ and $\frac{\partial \lambda_1^* - \lambda_0^*}{\partial T_2}$ have the same sign as b_1 while $\frac{\partial n_2^*}{\partial S}$, $\frac{\partial n_2^*}{\partial (T_1 - S)}$, $\frac{\partial \lambda_2^*}{\partial S}$ and $\frac{\partial \lambda_2^*}{\partial (T_1 - S)}$ have the same sign as b_2 . Overall, the empirical tests for strategic interactions are not driven by specific functional forms.

B3. Relaxing the assumption that information on preferences is complete

When I model the interaction between both wives as a non-cooperative, simultaneous game, I implicitly assume that each player knows the payoff of the other player. In my context, it means that each wife knows the ideal number of children of the other. From the second wife's perspective, it might seem reasonable to assume that she infers $n_{w,1}^{id}$ from the behavior of the first wife in the monogamous stage. She observes S and the number of children as she enters the household, so she is able to deduce the optimal initial birth rate of the first wife, and hence her preferences. Things are not as straightforward from the first wife's perspective, who only observes T_2 , but has no piece of evidence to infer $n_{w,2}^{id}$.

In this extension, I consider that $n_{w,2}^{id}$ is private information of the second wife. To simplify the notations, let me call $l = n_{w,2}^{id}$ the type of the second wife, and $f(l)$ the density, defined on an interval $I \in \mathbb{R}^+$. The first wife knows the distribution of types in the population of second wives. I denote $n_2(l)$ the strategy played by a second wife of type l . From section 8,

⁴⁸One difficulty is that b_i may vary with the values of n_i and n_{-i} .

the best response of a second wife of type l when the first wife plays n_1 is:

$$n_2(l) = n_2^{NS}(l) + B_2 n_1, \quad (5)$$

where $n_2^{NS}(l) = \frac{l + \theta_2^h n_h^{id} + \theta_2^n n_2^{nat}}{1 + \theta_2^h + \theta_2^n}$.

What is the best response of first wives when second wives of type l play $n_2(l)$? First wives maximize their expected utility:

$$\mathbb{E}[u(n_1, n_2(l))] = \int_I u(n_1, n_2(l)) f(l) dl.$$

When $n_2(l)$ is considered as given, $\frac{\partial u}{\partial n_1}$ is linear in n_1 and in $n_2(l)$, so the FOC gives:

$$n_1 = n_1^{NS} + B_1 \int_I n_2(l) f(l) dl. \quad (6)$$

The Nash equilibrium is the intersection of all best responses. Plugging the expression of $n_2(l)$ from Equation 5 into Equation 6, I get:

$$n_1^* = (n_1^{NS} + B_1 \mathbb{E}[n_2^{NS}]) \times \frac{1}{1 - B_1 B_2},$$

$$n_2^*(l) = (n_2^{NS}(l) + B_2 n_1^{NS} + B_1 B_2 (\mathbb{E}[n_2^{NS}] - n_2^{NS}(l))) \times \frac{1}{1 - B_1 B_2},$$

where $\mathbb{E}[n_2^{NS}] = \frac{\int_I l f(l) dl + \theta_2^h n_h^{id} + \theta_2^n n_2^{nat}}{1 + \theta_2^h + \theta_2^n}$.

Under incomplete information, comparative statics described in the paper are still valid. The first wife responds to the preferences of the average second wife.⁴⁹ This framework can be easily extended to relax the assumption that $n_{w,1}^{id}$ is known by the second wife.

B4. Relaxing the assumption that the game is simultaneous

So far, I have considered that either the first wife always remains in a monogamous union, or a second wife arrives at date S and both wives play a simultaneous game. In fact, when the first wife is relatively old as the second marriage takes place, the game is not simultaneous, but sequential. Indeed, the first wife has already given birth to $n_1 = \lambda_0 T_1$ children, and she

⁴⁹In theory, I could investigate whether the assumption of complete information is likely to hold by testing if n_1^* depends on $n_{w,2}^{id}$. In practice, the DHS sample of complete bigamous unions is too small to perform a credible test.

can no longer update this quantity. Then the second wife chooses her best response to n_1 , and payoffs are paid when the reproductive period of the second wife is over. Therefore, if $T_1 \leq S$, the situation is best described by a Stackelberg leadership model.⁵⁰

To account for the possibility of a late second marriage, I create a new husband's type, late polygamous. The first wife's belief about the probability of polygamy, π , is split into π_a the probability of late polygamy, and π_b the probability of early polygamy. At $t = 0$, first wives consider three scenarios: no strategic interaction, sequential game, and simultaneous game. Using notations of section 8, first wives maximize their expected utility over λ_0 :

$$(1 - \pi) \times u(\lambda_0 T_1, 0) + \pi_a \times \mathbb{E}[u(\lambda_0 T_1, n_2(\lambda_0 T_1))] + \pi_b \times \mathbb{E}[u(n_1^*, n_2^*)],$$

where $n_2(\lambda_0 T_1)$ is the best response of the second wife when she faces a first wife with $\lambda_0 T_1$ children. As already noted above, $u(n_1^*, n_2^*)$ does not depend on λ_0 .

To find the subgame perfect Nash equilibrium, I solve the game by backward induction. I start by considering the last stage of the sequential game. Building on section 8, I know that $n_2(n_1) = n_2^{NS} + B_2 n_1$. First wives anticipate the reaction of second wives. Let me compute the best strategy of first wives for a given T_2 (which determines n_2^{NS}). They maximize $u(n_1, n_2^{NS}(T_2) + B_2 n_1)$. The FOC gives:

$$n_1^{st}(T_2) = \frac{n_w^{id}(1 - B_2 \gamma_1) + \theta_1^h n_h^{id}(1 + B_2) + \theta_1^n n_0^{nat} + n_2^{NS}(T_2)(\gamma_1(1 - B_2 \gamma_1) - \theta_1^h(1 + B_2))}{(1 - B_2 \gamma_1)^2 + \theta_1^h(1 + B_2)^2 + \theta_1^n}. \quad (7)$$

Note that n_1^{st} increases with T_2 if and only if $B_1^{st} = \gamma_1(1 - B_2 \gamma_1) - \theta_1^h(1 + B_2) \geq 0$. In the simultaneous game, I found that n_1^* increases with T_2 if and only if $\gamma_1 - \theta_1^h \geq 0$. To understand the difference, one needs to consider the cross-derivative of $u_1(\cdot)$ with respect to n_1 and n_2 . When n_2 is taken as exogenous, I have $\frac{\partial^2 u_1}{\partial n_1 \partial n_2} = 2(\gamma_1 - \theta_1^h)$. When n_2 depends on n_1 in such a way that $\frac{\partial n_2}{\partial n_1} = B_2$, the expression is:

$$\frac{\partial^2 u_1}{\partial n_1 \partial n_2} = 2(\gamma_1 \times (1 - \gamma_1 B_2) - \theta_1^h \times (1 + B_2)).$$

When n_2 is fixed, having one more child for the first wife means that her relative number of children increases by one unit, and that the total number of children of the husband increases by one unit. Whereas when n_2 depends on n_1 , having one more child for the first wife means that the second wife will have B_2 additional children. So her relative number of

⁵⁰I use the superscripts *st* to denote the equilibrium quantities in this version of the model.

children increases by $(1 - \gamma_1 B_2)$, and the total number of children of the husband increases by $(1 + B_2)$.⁵¹

Now, first wives do not know T_2 before the second marriage takes place. But they have some information about the timing of events that can be exploited to refine the expectation about T_2 . Indeed, it can be shown that $\mathbb{E}[T_2|T_1, T_h]$ is increasing in $(T_h - T_1)$, where T_h is the length of the husband's reproductive period at $t = 0$. The intuition is that the expected value of T_2 depends on S : the later the second marriage takes place in the husband's life, the shorter the time left to the second wife to have children. T_2 cannot be larger than $(T_h - S)$. Moreover, $S \geq T_1$ in the sequential scenario so that T_2 is bounded above by $(T_h - T_1)$.⁵²

First wives maximize on n_1 their expected utility, $\mathbb{E}[u(n_1, n_2^{NS}(T_2) + B_2.n_1)]$. The derivative of $u(\cdot)$ is linear in T_2 so, using Equation 7, the equilibrium number of children writes:

$$n_1^{st} = n_1^{st}(\mathbb{E}[T_2|T_1, T_h]).$$

Since $\mathbb{E}[T_2|T_1, T_h]$ is increasing in $(T_h - T_1)$, n_1^{st} is also increasing in $(T_h - T_1)$ as long as $B_1^{st} \geq 0$.

Let me come back to $t = 0$ and consider the optimal initial birth rate under the three scenarios that I mentioned above: (i) no strategic interaction: $\lambda_0 = \frac{n_0^{NS}}{T_1}$; (ii) sequential game: $\lambda_0 = \frac{n_1^{st}}{T_1}$; and (iii) simultaneous game: indifferent between any $\lambda_0 \geq 0$. The first-order condition is a weighted average of the first-order condition under no strategic interaction (weight $(1 - \pi)$) and the first-order condition of the sequential game (weight π_a). As result, the optimal initial birth rate λ_0^{st} lies between $\frac{n_0^{NS}}{T_1}$ and $\frac{n_1^{st}}{T_1}$.

⁵¹I rewrite $B_1^{st} = B_1(1 + \theta_1^h + \theta_1^n) - B_2(\theta_1^h + \gamma_1^2)$. It may be the case that $B_1^{st} < 0$ even if $B_1 \geq 0$, for instance when B_2 is much larger than B_1 . The intuition is that, when the second wife reacts very strongly to an increase in n_1 , the increase in the relative number is small compared to the increase in the total number of children. On the other hand, when $B_1 \geq B_2 \geq 0$, then $B_1^{st} \geq 0$. In other words, when the strategic response of the second wife is not stronger than the one of the first wife, the first wife is always better off raising her number of children when she faces a more fertile rival.

⁵²The formal proof is as follows. Denote $L = T_h - T_1$ the length of the time period between the end of the first wife's reproductive life and the end of the husband's reproductive life. In the sequential game, this is the length of the second stage, when the second wife arrives and has children. Denote μ a random variable representing the entry date of the second wife, and ν a random variable such that $(\mu + \nu)$ is the exit date of the second wife. The only assumption on their distributions $f(\mu)$ and $g(\nu)$ is that both have a positive support. Using these notations, I can rewrite $T_2 = \min(\nu, \max(L - \mu, 0))$. T_2 can never be larger than L . When L increases, it enlarges the widow of opportunity for a second wife to enter and exit after a substantial period of time. Intuitively, an increase in L can only raise the expected value of T_2 . Formally, we have:

$$\mathbb{E}[T_2|L] = \int_0^{+\infty} \int_0^{+\infty} \min(\nu, \max(L - \mu, 0))g(\nu)f(\mu)d\nu d\mu.$$

Let $L' \geq L$, then $\min(\nu, \max(L' - \mu, 0)) \geq \min(\nu, \max(L - \mu, 0)) \forall (\mu, \nu)$. So $\mathbb{E}[T_2|L'] \geq \mathbb{E}[T_2|L]$.

How does n_1^{st} compare to n_0^{NS} ? It depends on the sign of B_1^{st} and on the relative magnitude of $n_{w,1}^{id}$ and n_h^{id} . The case that seems the most consistent with empirical evidence is $B_1 \approx B_2 \geq 0$ (implying that $B_1^{st} \geq 0$) and $n_{w,1}^{id} \leq n_h^{id}$. In this case, $n_1^{st} \geq n_0^{NS}$. When the strategic reaction is similar for both wives, and the husband wants more children than the first wife, the likelihood of a sequential game raises the initial birth rate. The first wife intensifies her fertility to improve her position in the event of a late second marriage.

It is possible to test for such a strategic overshooting by comparing the choices of women more or less exposed to the risk of a late second marriage. The idea is to exploit the variation in $(T_h - T_1)$, which is driven by the age difference between the first wife and the husband. When they have a very large age difference, then $(T_h - T_1) = 0$ because the length of the reproductive period of the couple (T_1) is determined by the length of the husband's period (T_h). It is very unlikely that the husband is still alive and fertile when the first wife is past her own reproductive years. On the contrary, when the age difference is low, then $(T_h - T_1)$ may be as large as 15 years, which leaves time for a potential rival to have many children.

In Table OB.1 below, I test whether the fertility of monogamous wives is indeed higher if the age difference with the husband is lower. It is crucial in this test to control for the preferences of each spouse because their age difference is correlated to their ideal family size.⁵³ On the other hand, I do not need information on the timing of unions since I consider anticipations. This is why I perform the test on DHS instead of PSF.

Regarding completed fertility, the prediction is verified. The first column shows that, controlling for T_1 and the preferences of each spouse, $(T_h - T_1)$ has a positive and significant impact on the final number of children. In column 2, I investigate whether the effect is truly linear or driven by the difference between women not exposed at all and the others. I create three categories of women depending on their exposure to the risk of a late second marriage: not exposed if $(T_h - T_1) = 0$, weakly exposed if $(T_h - T_1)$ is below the median; strongly exposed if $(T_h - T_1)$ is above the median. I find that women not exposed have significantly fewer children than the others. Also, among exposed women, the degree of exposure matters: strongly exposed women have more children than weakly exposed ones.

Regarding birth spacing, the effect of $(T_h - T_1)$ is of predicted sign, although not significant. In the last column, I interact $(T_h - T_1)$ with different ranks of birth; the impact is all

⁵³Since I consider only the monogamous stage, I can not rely on a specification with fixed effects. So there could be other omitted variables such as the wife's bargaining power. Using a cross-section of nations, Cain (1984) shows that the median age difference between spouses is positively correlated to total fertility rate. If age difference is a proxy for women empowerment at the household level, then a low $(T_h - T_1)$ would be correlated with a high θ_1^h , and hence with a large n_1^{NS} . This would create an attenuation bias.

the larger as the rank is high, and it is significant after birth 5. One interpretation is that wives update upwards the relative likelihood of a late marriage as time passes by.⁵⁴

Table OB.1: Testing strategic overshooting

Dep. var. Estimation	Total number of births		Birth intervals	
	OLS		Cox (hazard ratios)	
Wife's ideal number (n_w^{id})	0.195** (0.095)	0.229** (0.092)	1.022** (0.011)	1.022** (0.011)
Husband's ideal number (n_h^{id})	0.119** (0.059)	0.145*** (0.052)	1.026*** (0.005)	1.026*** (0.005)
Couple's reproductive period (T_1)	0.281*** (0.054)	0.612*** (0.114)	1.008 (0.015)	1.009 (0.016)
Husband's extra fertile years ($T_h - T_1$)	0.131** (0.056)		1.009 (0.006)	
$T_h - T_1$ below median (ref: $T_h - T_1 = 0$)		1.585** (0.710)		
$T_h - T_1$ above median (ref: $T_h - T_1 = 0$)		2.783** (1.073)		
$(T_h - T_1) \times \{\textit{birth rank} = 1, 2\}$				1.005 (0.007)
$(T_h - T_1) \times \{\textit{birth rank} = 3, 4\}$				1.004 (0.011)
$(T_h - T_1) \times \{\textit{birth rank} \geq 5\}$				1.030*** (0.011)
pval test below=above		0.141		
Controls	No	Yes	Yes	Yes
Observations	109	109	3959	3959
Clusters	na	na	1089	1089

Data: DHS. Weights. Sample: women in first union, monogamous, at least one child. In columns 1 and 2, I restrict to women over 40. Dep.var: in column 1 and 2, number of births; in column 3 and 4, duration between births j and $(j+1)$. $T_1 = \min(45 - \text{first wife's age at marriage}; 60 - \text{husband's age at marriage})$. $T_h - T_1 = (60 - \text{husband's age at marriage} - T_1)$. The median is 7 years. Controls: in column 2, husband's and wife's age at marriage; in columns 3 and 4, wife's age at marriage and a dummy for each j . In columns 1 and 2: OLS estimation; the unit of observation is the woman. In columns 3 and 4: Cox estimation; the unit of observation is the birth; baseline hazard common to all women; Breslow method to handle ties among non-censored durations; robust standard errors clustered at the woman level. Significance levels (for hazard ratio = 1 in the Cox estimation): * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

⁵⁴In the model, to keep things simple, I assume that the ratio $\frac{\pi_a}{1-\pi}$ remains constant over the monogamous stage. The idea is that, even if the probability of an early second marriage (π_b) decreases as time goes by, it does not change the relative likelihood of a late marriage compared to the likelihood of no second marriage. In fact, first wives seem to consider that $(1 - \pi)$ is fixed, and that the decreasing risk of an early second marriage is fully converted into a rising risk of late marriage.

B5. Welfare analysis

In this section, I compare the Nash equilibrium and the outcome maximizing total welfare. I focus on two quantities of interest: the total number of children ($N = n_1 + n_2$) and the relative number of children ($\Delta = n_1 - n_2$). I consider the simple case when parameters are the same for both wives: $\theta_1^h = \theta_2^h = \theta^h$, $\theta_1^n = \theta_2^n = \theta^n$ and $\gamma_1 = \gamma_2 = \gamma$.

From equation 3, I derive the quantities at equilibrium:

$$N^* = n_1^* + n_2^* = \frac{N^{id} + 2\theta^h n_h^{id} + \theta^n N^{nat}}{1 - \gamma + 2\theta^h + \theta^n},$$

$$\Delta^* = n_1^* - n_2^* = \frac{\Delta^{id} + \theta^n \Delta^{nat}}{1 + \gamma + \theta^n},$$

where $N^{id} = n_1^{id} + n_2^{id}$, $N^{nat} = n_1^{nat} + n_2^{nat}$, $\Delta^{id} = n_1^{id} - n_2^{id}$ and $\Delta^{nat} = n_1^{nat} - n_2^{nat}$.

I further define the total welfare function as $W(n_1, n_2) = u_1(n_1, n_2) + u_2(n_2, n_1)$, where $u_i(n_i, n_{-i})$ is the utility of wife i . By maximizing $W(n_1, n_2)$ over n_1 and n_2 , I find:

$$N^{Opt} = n_1^{Opt} + n_2^{Opt} = \frac{(1 - \gamma)N^{id} + 4\theta^h n_h^{id} + \theta^n N^{nat}}{(1 - \gamma)^2 + 4\theta^h + \theta^n},$$

$$\Delta^{Opt} = n_1^{Opt} - n_2^{Opt} = \frac{(1 + \gamma)\Delta^{id} + \theta^n \Delta^{nat}}{(1 + \gamma)^2 + \theta^n},$$

The same elements drive N and Δ when I maximize total welfare and when I compute the Nash equilibrium. But the weights given to each element differ. The table below summarizes the drivers and their weights in both cases.

Weight on each driver	Nash equilibrium	Welfare-maximizing outcome
Drivers of N		
$N^{id}/(1 - \gamma)$	$\frac{(1 - \gamma)}{(1 - \gamma) + 2\theta^h + \theta^n}$	$\frac{(1 - \gamma)^2}{(1 - \gamma)^2 + 4\theta^h + \theta^n}$
n_h^{id}	$\frac{2\theta^h}{(1 - \gamma) + 2\theta^h + \theta^n}$	$\frac{4\theta^h}{(1 - \gamma)^2 + 4\theta^h + \theta^n}$
N^{nat}	$\frac{\theta^n}{(1 - \gamma) + 2\theta^h + \theta^n}$	$\frac{\theta^n}{(1 - \gamma)^2 + 4\theta^h + \theta^n}$
Drivers of Δ		
$\Delta^{id}/(1 + \gamma)$	$\frac{(1 + \gamma)}{(1 + \gamma) + \theta^n}$	$\frac{(1 + \gamma)^2}{(1 + \gamma)^2 + \theta^n}$
Δ^{nat}	$\frac{\theta^n}{(1 + \gamma) + \theta^n}$	$\frac{\theta^n}{(1 + \gamma)^2 + \theta^n}$

Compared to the welfare-maximizing outcome, the total number of children at equilibrium depends too much on the preferences of the wives, $N^{id}/(1-\gamma)$, and too little on the preferences of the husband, n_h^{id} . The relative number of children depends too much on the difference in wives' natural fertility, Δ^{nat} , and too little on the difference in wives' preferences, Δ^{id} .

To sum up, if wives were to agree on maximizing total welfare instead of maximizing their own utility, the total number of children would be closer to the husband's preferences. The relative number of children would reflect more the difference in wives' preferences and less the difference in natural fertility.

Are there too many or too few children at equilibrium? We have:

$$N^* - N^{Opt} = \frac{(\frac{N^{id}}{1-\gamma} - n_h^{id})2\theta^h(1-\gamma^2) + (\frac{N^{id}}{1-\gamma} - N^{nat})\theta^N\gamma(1-\gamma) + (N^{nat} - n_h^{id})2\theta^h\theta^n}{((1-\gamma)^2 + 4\theta^h + \theta^n)(1-\gamma + 2\theta^h + \theta^n)}.$$

The comparison between N^* and N^{Opt} depends on the relative values of $\frac{N^{id}}{1-\gamma}$, n_h^{id} and N^{nat} . In particular, if $\frac{N^{id}}{1-\gamma} \geq N^{nat} \geq n_h^{id}$, then $N^* \geq N^{Opt}$; whereas if $\frac{N^{id}}{1-\gamma} \leq N^{nat} \leq n_h^{id}$, then $N^* \leq N^{Opt}$. The first statement is all the more likely to hold as γ rises. In the empirical tests, I found that γ is large enough to induce a positive strategic reaction. One plausible value for γ is therefore its upper bound. When $\gamma = 1$, we have:

$$N^* - N^{Opt} = \frac{N^{id}(4\theta^h + \theta^n) + (N^{nat} - n_h^{id})2\theta^h\theta^n}{(2\theta^h + \theta^n)(4\theta^h + \theta^n)}.$$

Using the values of the parameters estimated on monogamous unions in Table OD.4 ($\theta^h \approx 1/2$ and $\theta^n \approx 3$), it means that $N^* \geq N^{Opt}$ if and only if $N^{id} \geq \frac{3}{5}(n_h^{id} - N^{nat})$. This condition is likely to hold in the vast majority of cases given that preferences in polygamous unions are on average 5 or 6 children for each wife, and 12 children for the husband. To sum up, a deficit of children would be observed at equilibrium only when n_h^{id} reaches uncommonly high values. In general, the non-cooperative model leads to a surplus of children with respect to the outcome maximizing total welfare.

Online appendix C: Alternative models

This appendix presents three alternative models of fertility choices in polygamous households, building upon the literature survey by Doepke and Kindermann (2017):

1. A unitary model with endogenous income.

2. A unitary model with endogenous polygamy.
3. A collective model.

For each model, I predict how one wife’s fertility should change when another wife joins or exits the household and I explain how these predictions differ from my non-cooperative model. In a nutshell, the unitary model with endogenous polygamy and the collective model fail to explain empirical patterns observed after the entry of a second wife. In particular, they predict that children of co-wives are substitutes. As for the unitary model with endogenous income, it is not consistent with empirical patterns observed after the exit of a co-wife.

1. Unitary models

Here, I consider a unitary model of the household, where the husband makes all decisions about fertility, consumption and marriage. Following Doepke and Kindermann (2017), the preferences for consumption c , and fertility n , can be represented by a utility function $u(c, n)$, where both goods are normal. The budget constraint is $c + qn = I$, where q is the relative price of children and I is the household income. For the sake of clarity, I further assume that preferences are Cobb-Douglas, with a weight α on fertility.

The optimal number of children is $n^* = \alpha \frac{I}{q}$. Households devote a share α of income to children and the rest to consumption. I use subscripts $\{m, w\}$ for husbands and wives in monogamous households, and $\{p, 1, 2\}$ for husbands, first wives and second wives in polygamous households. We have $n_m = n_w = n_p = n_1 + n_2$. Holding income I and fertility preferences α constant, monogamous and polygamous men are predicted to have the same number of children. To account for the fact that polygamous men have on average more children than monogamous ones, the next two sub-sections will consider that polygamy is related to (i) income and (ii) fertility preferences.

1.1. Endogenous income

What if polygamy is related to income? Now, I assume that income in monogamous households I_m is lower than income in polygamous ones I_p . It might be the case for two reasons:

- Shock to husbands’ wealth: if men are the sole breadwinners and women are costly, only men with a positive income shock can afford a second wife.
- Productive wives: if both men and women are breadwinners, we can decompose I into a male component and a female component and total income would increase from

$(I_h + I_w)$ to $(I_h + 2I_w)$ when another wife joins the household.

We have $n_m = n_w < n_p = n_1 + n_2$. Denote s the share of children born to the first wife. As long as $s < \frac{I_m}{I_p}$, the first wife's fertility decreases when a co-wife arrives. E.g. the husband splits equally the number of children between both wives and the household income less than doubles. The model can therefore predict that more children are born in polygamous unions (income effect) and fewer children are born to the first wife (substitution effect). The model can also predict a positive relationship between co-wives' fertility if I further assume that the income effect is correlated with determinants of the second wife's fertility while the substitution effect is not. For example, the sharing rule is always half-half and I_p decreases with the bride's age because older co-wives are less productive or reflect a smaller income shock because they cost less. These predictions are therefore consistent with empirical patterns observed after the second marriage: the fertility of first wives decreases, and decreases less when the second wife is younger because both are related to the household income.

To discriminate between this model and my non-cooperative framework, I examine the exit of co-wives. As explained in the paper, the non-cooperative model predicts that fertility should increase on average, and more if more children are already born to the co-wife. On the contrary, the unitary model with endogenous income predicts that fertility should increase less if more children are already born to the co-wife. The intuition is that the remaining wife compensates for the children who are not born yet.

Formally, after the exit of a co-wife, the maximization problem of the household has an additional constraint: $n \geq \bar{n}$ where \bar{n} is the number of children already born to that wife. The fertility of the remaining wife moves from her share in the polygamous union, $s.n_p$, to the complement of \bar{n} in the monogamous union, $n_m - \bar{n}$. If the exit has no impact on income, then the optimal number of children in the household stays the same. The remaining wife compensates for the children not born to the co-wife. Therefore, the increase is weaker when \bar{n} is high. Assume now that the exit is correlated with a decrease in income, because a productive wife is lost or because a negative shock to the husband's wealth led to divorce. We have $n_m < n_p$. The remaining wife's fertility might increase or decrease, and the decrease is all the more likely as \bar{n} is high. For instance, if the co-wife leaves many children and the drop in income is large, it may well be the case that the household already has more children than they can afford, and the fertility of the remaining wife would be stopped. As shown in Table 6, empirical patterns are inconsistent with the unitary model and consistent with my framework.

1.2. Endogenous polygamy

What if polygamy is endogenous to fertility choices? Let me assume that female fertility has some upper bound, \bar{n} . This generates a motive for polygamy for husbands who cannot reach the optimal number of children with a sole wife. We have the following equations:

With one wife,

$$n_m = \begin{cases} \alpha \frac{I_m}{q}, & \text{if } \alpha \frac{I_m}{q} \leq \bar{n}_w \\ \bar{n}_w, & \text{otherwise} \end{cases}$$

With two wives,

$$n_p = \begin{cases} \alpha \frac{I_p}{q}, & \text{if } \alpha \frac{I_p}{q} \leq \bar{n}_1 + \bar{n}_2 \\ \bar{n}_1 + \bar{n}_2, & \text{otherwise} \end{cases}$$

Consider two situations on the marriage market. Both predict a negative correlation between the fertility of co-wives through different mechanisms.

Men can choose who they marry

First assume that people are matched in such a way that $\bar{n}_w = \alpha \frac{I_m}{q}$ i.e. men marry those women who can bear exactly their optimal number of children. Every man would start monogamous and switch to polygamous only in case of an income shock ($I_p > I_m$) or a health shock ($\bar{n}_1 < \bar{n}_w$) creating a discrepancy between male preferences and female reproductive capacities. To solve the discrepancy, the husband would then be matched with a second wife with $\bar{n}_2 = \alpha \frac{I_p}{q} - \bar{n}_1$.

In case of health shock, we have $\bar{n}_1 + \bar{n}_2 = \alpha \frac{I_m}{q}$ with $\bar{n}_1 < \bar{n}_w$. The first wife's fertility decreases and the second wife has more children when the first wife's fertility decreases more, in order to offset the loss. Note that the husband's fertility remains constant.

In case of income shock, we have $\bar{n}_w + \bar{n}_2 = \alpha \frac{I_p}{q}$ with $I_p > I_m$. The first wife's fertility remains constant and the second wife's fertility is negatively related to the first wife's fertility.

Men cannot choose who they marry

Next assume that people are randomly matched on the marriage market. Then men with $\bar{n}_w < \alpha \frac{I_m}{q}$ are strictly better off with two wives and polygamy happens right after the first marriage. Men with $\bar{n}_w \geq \alpha \frac{I_m}{q}$ will marry a second wife later iff health or income shocks are large enough to revert the inequality. The difference with efficient matching is that \bar{n}_2 can take any value. So now, we need a rule to allocate children between both wives when the

optimal point is an interior solution. We can imagine different rules: equal treatment (half-half), husband's sexual favoritism (more children to the younger wife), relative capacities ($\frac{\bar{n}_i}{\bar{n}_1 + \bar{n}_2}$). It is difficult to argue that the share allocated to one wife should *increase* with determinants of the co-wife's fertility.

In case of health shock, we have $n_m = \alpha \frac{I_m}{q} > \bar{n}_1$ and n_m might be larger than $(\bar{n}_1 + \bar{n}_2)$ if \bar{n}_2 happens to be low. Note that the husband's fertility may decrease in this setting. The first wife's fertility decreases and is negatively correlated with \bar{n}_2 in two ways. A low \bar{n}_2 (i) makes the corner solution, where both wives produce their maximum number of children, more likely; and (ii) increases the share allocated to the first wife in case of an interior solution.

In case of income shock: $n_m \leq \bar{n}_w < n_p$ and n_p might be equal to $\bar{n}_w + \bar{n}_2$ if \bar{n}_2 happens to be low. Here, the first wife's fertility might increase, and this is all the more likely as \bar{n}_2 happens to be low, for the reasons already mentioned above.

All in all, this model is not consistent with empirical results showing a positive correlation between the fertility of co-wives. Moreover, when polygamy is driven by an income shock, the model fails to predict a decrease in the first wife's fertility after the arrival of the second wife. And when polygamy is driven by a health shock, it fails to predict that men's fertility increases.

2. Collective model

In this section, I consider a collective model of the household, where preferences of husbands and wives over consumption (private good) and children (public good) differ. In particular, men's preferences are more strongly tilted towards children. The key assumption is that household members are able to reach an efficient allocation.

Following again Doepke and Kindermann (2017), the optimization problem of the household maximizes a weighted average of individual utilities subject to a common budget constraint. Importantly, the relative weights of husbands and wives are not constant; they depend on distribution factors such as individual incomes. In our context, the union status, polygamy or monogamy, can be considered as a key distribution factor.

In monogamous unions, people choose c_w , c_m and n to maximize:

$$\gamma u_w(c_w, n) + (1 - \gamma) u_m(c_m, n) \text{ s.t. } c_w + c_m + qn = I$$

With Cobb-Douglas preferences, the optimal allocation is $n^* = \frac{I}{q}(\gamma\alpha_w + (1 - \gamma)\alpha_m)$. The

household acts as a single entity whose preferences are a weighted average of both members' preferences.

I extend this framework to accommodate a second wife. A bigamous household chooses c_1, c_2, c_p, n_1 and n_2 to maximize:

$$\gamma_1 u_1(c_1, n_1) + \gamma_2 u_2(c_2, n_2) + (1 - \gamma_1 - \gamma_2) u_p(c_p, n_1 + n_2) \text{ s.t. } c_1 + c_2 + c_p + q(n_1 + n_2) = I$$

The optimal allocation satisfies two conditions: $n_1^* + n_2^* = \frac{I}{q}(\gamma_1 \alpha_1 + \gamma_2 \alpha_2 + (1 - \gamma_1 - \gamma_2) \alpha_p)$ and $n_i^* = (n_1^* + n_2^*) \frac{\gamma_i \alpha_i}{\gamma_1 \alpha_1 + \gamma_2 \alpha_2}$. The *total* fertility is driven by a weighted average of the three members' preferences and the *relative* fertility depends on the preferences and bargaining power of one wife compared to the other.

The sharing rule between first and second wives is given by the first-order conditions and implies that $\frac{n_1^*}{n_2^*} = \frac{\gamma_1 \alpha_1}{\gamma_2 \alpha_2}$. The share of one wife decreases when the other wife wants more children and has a higher weight.

Holding income constant, how does fertility change when the household switches from monogamous to polygamous? Let me assume that polygamy increases the Pareto weight of the husband i.e. $\gamma_1 + \gamma_2 < \gamma$. The total number of children born in polygamous unions is therefore closer to the husband's preferences. Given that men want more children than women, this is consistent with the fact that polygamous men have more children than monogamous ones.

What happens to the first wife's fertility? There are two opposite effects:

- Positive: the total number of children increases.
- Negative: the share born to the first wife decreases.

The first wife's fertility decreases in the polygamous stage iff the second effect dominates. Importantly, her fertility is negatively correlated with the second wife's preferences. Intuitively, these preferences influence more the allocation between wives than the total number of children. When the second wife wants many children, more children are born in total but this is more than offset by a lower share allocated to the first wife. So her fertility decreases more. Formally, we have $\frac{\partial n_1^*}{\partial \alpha_2} < 0$ because:

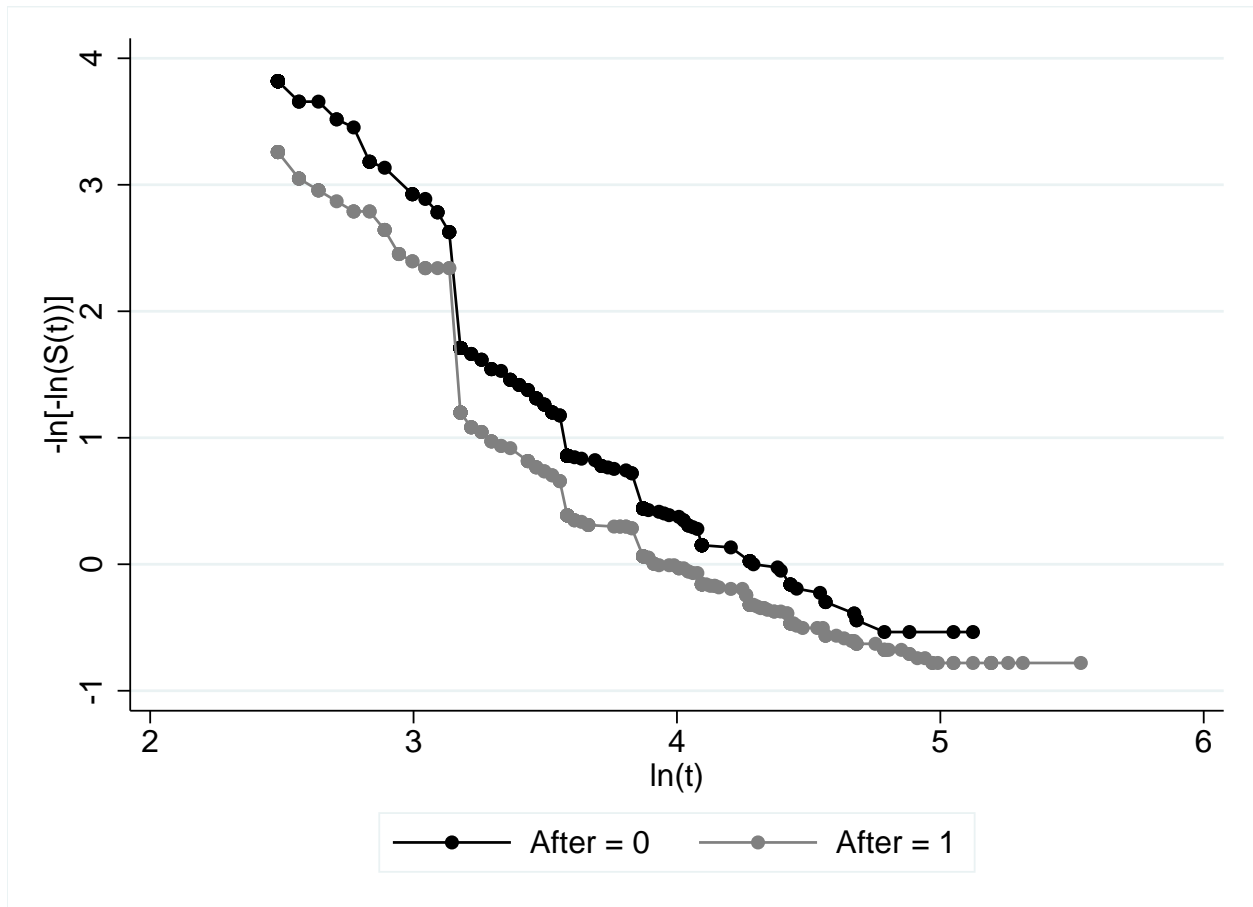
$$n_1^* = \frac{I}{q} \gamma_1 \alpha_1 \left(1 + \frac{(1 - \gamma_1 - \gamma_2) \alpha_p}{\gamma_1 \alpha_1 + \gamma_2 \alpha_2} \right)$$

Thus, a collective framework predicts that children of co-wives are substitutes. This is because an efficient allocation of children requires that the fertility of one wife relative to

the other increases with her own taste for children and decreases with the other's taste for children.

Online appendix D: Additional empirical tests

Figure OD.1: Testing the proportional hazard assumption (log-log plot)



Data: PSF. Sample: first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union. The log-log plot graphs $-\ln(-\ln(S(t)))$ against $\ln(t)$ for the category *After* = 0 (birth intervals occurring before the second marriage) and for the category *After* = 1 (birth intervals occurring after the second marriage). Estimates are adjusted for covariates: a dummy for each j , mother's age and age^2 at birth j . As shown by the graph, the curves are parallel, meaning that the proportional hazard assumption is not violated.

Another way to test the proportional hazard assumption is to follow the procedure developed by Grambsch and Therneau (1994). I fail to reject the assumption, both in the regression estimating the impact of *After* (p-value=0.28) and in the regression with the interaction term $\textit{After} \times T_2$ (p-value=0.49).

Table OD.1: Change in first wives' birth spacing: robustness

First wives' birth rates (hazard ratios)	Next interval	Bigamous	Excl. monogamous	Rough measure	No dead child
After	0.557** (0.142)	0.694* (0.145)	0.844 (0.182)	0.507*** (0.099)	0.666 (0.180)
After * Co-wife's reproductive period	1.142*** (0.044)	1.067* (0.039)	1.089** (0.038)	1.064* (0.035)	1.168*** (0.060)
Controls	Birth rank dummies, mother's age and age ² at birth				
Baseline hazard	Woman-specific	Woman-specific	Woman-specific	Woman-specific	Woman-specific
Observations	2286	2399	553	2483	1563
Clusters	699	695	144	716	468

Data: PSF. Sample: monogamous and first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union. Dep. var.: duration between births j and $(j + 1)$. *After* is a time-varying variable indicating if the second wife has arrived. Co-wife's reproductive period proxied by min (45-second wife's age at marriage; 60-husband's age at second marriage). Stratified partial likelihood estimation with baseline hazards specific to each woman; Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1): * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

In columns 1 and 2, I consider all intervals in the monogamous stage, and only the very next interval after the second marriage. I exclude first wives with more than one co-wife in columns 3 and 4, and I exclude monogamous wives in columns 5 and 6. In columns 7 and 8, I impose that *After* = 1 for the whole spell during which the second marriage takes place, which is the approximation made in the linear model, whereas in the baseline duration model, *After* varies within a spell. In columns 9 and 10, I exclude women who have lost at least one child from current union.

Table OD.2: Testing if the use of birth control methods responds to co-wife's fertility

Dep. var.	Current modern contraceptive use	Current insusceptibility: breastfeeding, amenorrhea or abstinence
Co-wife's ideal family size	-0.0168** (0.0083)	-0.0091 (0.0121)
Husband's ideal family size	-0.0076** (0.0031)	-0.0013 (0.0044)
Wife's ideal family size	-0.0131* (0.0081)	0.0119 (0.0117)
Couple reproductive period	0.0016 (0.0035)	0.0086 (0.0065)
Number of children born	0.0157 (0.0101)	-0.0125 (0.012)
Years since last birth	0.003 (0.0058)	-0.0987*** (0.0147)
Constant	0.28	0.61
Observations	261	261

Data: DHS. Weights. Sample: women in polygamous unions, having at least one child, no broken union. OLS regression. Standard errors in parentheses. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table OD.3: Placebo test: falsifying the length of monogamous period (S)

Hazard ratios	Baseline: true S		$S = 10$ years (mean)		$S = 6$ years (Q_1)		$S = 13$ years (Q_3)	
	Next int.	All int.	Next int.	All int.	Next int.	All int.	Next int.	All int.
After	0.557** (0.142)	0.770 (0.143)	0.668 (0.170)	0.848 (0.140)	0.791 (0.191)	0.992 (0.162)	0.863 (0.211)	0.920 (0.199)
Controls	Birth rank dummies, mother's age and age ² at birth							
Baseline hazard	Woman-specific		Woman-specific		Woman-specific		Woman-specific	
Observations	2286	2483	2285	2483	2165	2483	2353	2483
Clusters	699	716	703	716	688	716	712	716

Data: PSF. Sample: monogamous and first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union. Dep. var.: duration between births j and $(j + 1)$. In odd-numbered columns, I consider all intervals in the monogamous stage, and the very next interval after the second marriage. In even-numbered columns, I consider all intervals. *After* is a time-varying variable indicating if the index birth occurred after S . In column 1, S is the observed length of the monogamous period. In the last three columns, I run Placebo tests using alternative cut-offs, respectively the mean, the first quartile and the last quartile taken from the distribution of S in the sample. Stratified partial likelihood estimation with baseline hazards specific to each woman; Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1): * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table OD.4: Estimating the model on monogamous and polygamous unions

Dep. var. Sample	Total number of births	
	Monogamous	Polygamous
Wife's ideal number of children (n_w^{id})	0.232** (0.092)	-0.024 (0.096)
Husband's ideal number of children (n_h^{id})	0.128** (0.064)	0.067* (0.039)
Couple's reproductive period (T)	0.205*** (0.046)	0.188*** (0.047)
Constant	-1.083 (0.985)	1.785 (1.302)
R^2	0.36	0.14
pval $n_w^{id} = n_h^{id}$	0.36	0.39
Observations	109	151
Structural parameters		
θ^h	0.55	na
θ^n	2.77	na
λ^{nat}	0.32	na
n^{nat} (mean)	8.30	na

Data: DHS. Weights. Sample: women over 40, having at least one child. Women with broken unions are excluded because I only observe the date of first marriage, so I am able to deduce the timing of successive marriages only when all wives are in the first marriage. I also restrict the sample to husbands with no broken union. $T = \min(45 - \text{wife's age at marriage}; 60 - \text{husband's age at marriage})$. $n^{nat} = \lambda^{nat} \times \text{mean}(T)$ with $\text{mean}(T) = 26$. OLS regression. Significance levels : * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

For monogamous unions, the R^2 is close to the largest share in variance that can be explained by a linear probability model. Indeed, the dependent variable is an integer. Assuming that the true data generating process is a Poisson model of parameter μ , then a LPM could explain at most $\mathbb{V}(\mu)/[\mathbb{E}(\mu) + \mathbb{V}(\mu)]$. In my sample, the empirical counterparts of $\mathbb{E}(\mu)$ and $\mathbb{V}(\mu)$ are respectively 6.44 and 2.65, leading to an upper bound of 0.30 for the R^2 .

For polygamous unions, the model specification is not right. Since the coefficient on n_w^{id} is not significantly different from zero, I cannot compute the estimates of the structural parameters.

Online appendix E: Stratified partial likelihood estimation

This section draws on Ridder and Tunalı (1999), and on the chapter "Duration Models: Specification, Identification, and Multiple Durations" by Gerard van den Berg in Heckman and Leamer (2001).

I consider a mixed proportional hazard model with multi-spell data. It means that several durations, indexed by j , are generated by a single individual i , which is characterized by a vector of observed explanatory variables x and an unobserved heterogeneity term ν . It is possible to identify the impact of x on the hazard function under very weak conditions (in addition to the proportional hazard assumption) if x varies between spells for a given individual while ν does not. Formally, the hazard function satisfies:

$$\theta(t|x_{i,j}, v_i) = \theta_0(t, \nu_i) \times \exp(x_{i,j}\beta).$$

In this specification, the baseline hazard θ_0 is allowed to differ across individuals. There is no restriction on the interaction of ν with the elapsed duration t in the hazard function. Moreover, x and ν may be dependent. We do not need any assumption on the tail of the distribution of the unobservables.

The intuition underlying the estimation method is to construct a Cox partial likelihood within individuals (or strata) by ordering the uncensored durations. For each duration $t_{i,k}$, we can compute the probability that item k fails at $t_{i,k}$ given that exactly one item in i fails at $t_{i,k}$. It satisfies:

$$\frac{\theta(t_{i,k}|x_{i,k}, v_i)}{\sum_{j \in R_i(k)} \theta(t_{i,k}|x_{i,j}, v_i)} = \frac{\exp(x_{i,k}\beta)}{\sum_{j \in R_i(k)} \exp(x_{i,j}\beta)},$$

where $R_i(k)$ is the set of observations in i at risk when k fails. This expression can be written for each spell of each individual; the product gives the stratified partial likelihood.

The main caveat with multi-spell data is the issue of censoring. If all individuals are observed for the same period of time, then the right-censoring variable is not independent from previous durations, and therefore not independent from the current duration (because durations are jointly determined by the unobserved heterogeneity). This violates a standard assumption in duration analysis. It is not the case here: women are subject to censoring at the date of the survey, but they have different starting points corresponding to the date of first birth. So I do not follow them for a fixed period of time.