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Calling out Zheng Xuan (127–200 CE) at the crossroads of ritual, maths, sport and classical commentary

Daniel Patrick Morgan*
(Laboratoire SPHERE, CNRS & Paris 7 – Paris 1)

Abstract It is Zheng Xuan 鄭玄 (127–200 CE), more than any other commentator, to whom we owe our current understanding of the Confucian Classics – Zheng Xuan, of course, and the subcommentators, Kong Yingda 孔穎達 (574–648 CE) and Jia Gongyan 賈公彥 (fl. 637 CE), responsible for canonising his readings in the early Tang (618–907 CE). Zheng Xuan made mistakes, however, and this chapter offers a case study on one of the more glaring examples: the calculation of physically impossible target geometries for the royal archery meet reputedly held in the Western Zhou (1045–771 BCE), the ‘Big Shoot’ (*dashe* 大射). In transforming the ambiguous language of the Classics into an elaborate maths problem with definite quantities, Zheng Xuan neglects to account for the targets being in three dimensions, his subcommentators entering to supply mathematical proofs to numbers that are flat wrong, in both senses of the word. It is not as if they could not have known, I argue, considering that the Big Shoot was living practice in their respective days, and as preeminent commentators like Zheng Xuan and Kong Yingda were equally trained in mathematics equipped to solve this very problem. Faced with this curious epistemic dissonance between exegesis, experience, and mathematical training, I offer that we might better understand the work of the Classical commentator as reconstructing abstract, ‘Platonic forms’ necessarily divorced from their reality and for which mathematics suggested itself as the perfect tool.

Keywords Zheng Xuan, Kong Yingda, Jia Gongyan, Confucianism, classical commentary, ritual, archery, history of mathematics

Introduction

Having formed over the first millennium BCE by a desultory process of accretion, transmission, translation, loss, recovery, and redaction, the Classics (*jing* 經), as they have come down to us, are hard to read.¹ They are *old*, and to understand their very language, let alone the ‘subtle words’ of the ‘plainclothes king’s’ supposed curation thereof, one must rely on the *zhuan* 傳 – the ‘traditions’ and/or/as ‘commentaries’ – of Confucius’ intellectual successors from the fifth to second

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¹ For a study of this process in all its complexity, see Shaughnessy (2006); on issues of ‘translation’ and ‘domestication’ between pre-imperial (< 221 BCE) scripts, see Feng (2007).

century BCE.² And so it goes for the *zhuan* as well, for the *zhuan* came about by a similarly desultory process requiring that second-century CE scholastics fix *their* meaning with ‘chapter and verse’ (*zhangju* 章句) ‘annotation’ (*zhu* 注) – an expansive genre made possible by Ma Rong’s 馬融 (79–166 CE) invention of double-line inter-linear commentary (fig. 1). History repeats itself, but for every layer of exegesis that the Classics have since accrued, there is one name to which the question ‘how do we know that?’ consistently leads back: Zheng Xuan 鄭玄 (127–200 CE), Ma Rong’s preeminent disciple. In the words of Wang Shao 王邵 (fl. 577–605 CE), ‘One would rather say that the [Master] Kong and the Sage [Kings] were mistaken than hear that [the commentators] Zheng [Xuan] and Fu [Qian] 服虔 (fl. 189–92 CE) were wrong’.³

Our modern reading of ancient texts is similarly dependant on the likes of Zheng Xuan; the stakes, however, were considerably higher in Wang Shao’s day. As the ‘warp’ (*jing*) of civilisational order – the blueprint for returning to the golden age of the Western Zhou (1045–771 BCE) – the Classics framed one’s options as concerned political and religious state-building. Consider for example Academician Feng Gui’s 封軌 debate-opener on how the Manchurian-origin Tuoba-Wei (386–585 CE) should construct a Bright Hall⁴ ritual complex on the model of their Chinese neighbours in 477/479 CE:

明堂者，布政之宮，在國之陽，所以嚴父配天，聽朔設教，其經構之式，蓋已尚矣。故『周官 匠人職』云：「夏后氏世室，殷人重屋，周人明堂，五室、九階、四戶、八窗」。鄭玄曰：「或舉宗廟，或舉王寢，或舉明堂，互之以見同制。」然則三代明堂，其制一也。

The Bright Hall is a palace for promulgating policy. Located on the sunny (southern) side of the polis, it is that by which [the emperor] consigns his respected father to heaven, hears [the announcement of] the new moon and establishes the teachings, and the model for its construction was already there in the distant past. Thus does the *Offices of Zhou* ‘Post of Artisan’ [entry] say, ‘For the kings of Xia, the Chamber of Generations, for the people of Yin (?–1046 BCE), the Double House,

² The myth goes that Confucius – Master Kong 孔子 – was a latter-day Sage (*sheng* 聖) who, instead of ruling, as would normally be his right, redacted the pre-fifth-century BCE Classics to as to fill them with ‘subtle words’ (*wei yan* 微言) about proper conduct by which future generations were to live, making him a ‘plainclothes king’ (*su wang* 素王). On this myth and the *zhuan* ‘traditions’ that first arose around these texts so as to codify the meaning of his ‘subtle words’, see for example Van Auken (2016).

³ 寧道孔聖誤，諱聞鄭服非, cited in *Jiu Tang shu*, 102.3181. I thank Zhu Yiwen for bringing this quote to my attention. On the Chinese commentarial tradition thereafter, see Elman (1984), Henderson (1991), and Makeham (2003).

⁴ The Bright Hall (*mingtang* 明堂) was, supposedly, a temple in the southern suburbs of the Zhou capital modelled symbolically upon the structure of heaven, earth, and time. It was there, allegedly, that the Zhou kings announced the calendar for the year and, in subsequent months, cycle through its twelve halls wearing month- and season-appropriate symbols and reading ‘monthly ordinances’ (*yueling* 月令) concerning the (mostly) agrarian activities of their subjects so as to keep heaven, earth, and man in harmonious accord. On the Bright Hall, particularly as debated and instituted in the early imperial period, see Maspero (1951), Soothill (1951), Forte (1988), Lewis (2006, 265 ff.), Tseng (2011, 17–88), and Morgan (2017, 120–125).

and for the people of Zhou, the Bright Hall – five chambers, nine stairways, four doors, and eight windows’. Zheng Xuan says ‘Some raised ancestral temples, some raised royal mausolea, and some raised Bright Halls – compare them and one sees that they are of the same construction’. As such, the Bright Halls of the Three Dynasties were of a single construction.⁵

And thus did the Tuoba-Wei erect the architectural symbol of their legitimacy in occupied Luoyang in 496 CE on the basis of Zheng Xuan’s ‘annotations’ – because it takes the second-century CE classicist to supply a single floor plan where the Classics themselves are generally silent or inconsistent on the details.

It is not of course as if Sages’ institutions were self-evident or even agreed upon in Zheng Xuan’s time. Lu Lübing 盧履冰 (fl. 688–717 CE) reminds us of the confusion lingering around the most basic of rites in yet another public debate of 717 CE:

三年之制，說者紛然。鄭玄以為二十七月，王肅以為二十五月。又改葬之服，鄭云服緦三月，王云訖葬而除。...鄭、王祖經宗傳，各有異同；荀摯采古求遺，互為損益。方知去聖漸遠，殘缺彌多。As to the institution of the ‘three-year’ [paternal mourning period], the explanations are in disarray: Zheng Xuan took it to be twenty-seven months, and Wang Su 王肅 (195–256 CE), twenty-five. Furthermore, as to changing from one’s funeral garments, Zheng says that one wears the light hemp for three months, while Wang says that one removes it at the end of the funeral. ... Zheng and Wang treated the Classics as their forefathers, and the commentary/traditions as their ancestors, [and yet] each had their differences and commonalities; Xun [Yi] 荀顛 (d. 274 CE) and Zhi [Yu] 摯虞 (fl. 265–307 CE) picked over antiquity in search of vestiges, [and yet] the one added and subtracted from the other – it is at thus that one understands our ever-increasing distance from the Sages and the surplus of deficiencies and gaps [in our understanding thereof].⁶

Wang Su’s commentaries are now lost, as are the writings of Zhi Yu, Xun Yi, and Lu Lübing, leaving us today but with one voice in a centuries-long, multi-party exchange over second-millennium BCE practice.⁷ The reason for Zheng Xuan’s modern pride of place, in other words, has everything to do with selection.

Namely, in 631 CE the emperor of the fledgling Tang empire (618–907 CE) ordered the imperial academician Kong Yingda 孔穎達 (574–

⁵ *Wei shu*, 32.765.

⁶ *Jiu Tang shu*, 27.1024. On the debate between Xun Yi and Zhi Yu over the commentaries of Zheng Xuan and Wang Su to which Lu Lübing is referring, see *Jin shu*, j. 19; cf. Goodman (2010, 353–362).

⁷ To be precise, all that survives of their writing is fragments cited in later sources: for Wang Su, see *Quan Sanguo wen* 全三國文, j. 23; for Xun Yi, see *Quan Jin wen* 全晉文, j. 31; for Zhi Yu, see *ibid.*, j. 76, in *Quan shanggu Sandai, Qin, Han, Sanguo, Liuchao wen*; for Lu Lübing, see *Quan Tang wen*, j. 335. Fragments of lost commentaries such as Wang Su’s can be found in *Yuhan shanfang ji yishu*. On the Zheng Xuan–Wang Su divide, see Shi Yingyong (2007).



Fig. 1 1870 reprint of 1738 Wuying dian 武英殿 edition of the *Book of Etiquette and Ceremonial with Commentary and Subcommentary* (Yili zhushu 儀禮注疏), available online at Ctext.org (<http://ctext.org/library.pl?if=en&file=80179&page=4>). The text of the Classic is in large, single-line characters, followed in small, double-line characters by Zheng Xuan's 'annotations' (zhu 注) and 'pronunciation and meaning' (yinyi 音義) and, thereafter, the 'subcommentary' (shu 疏) of Jia Gongyan et al.

648 CE) to lead a multicultural, North–South project to produce standardised editions and subcommentaries (shu 疏) for the Five Classics, two of which would incorporate Zheng Xuan's 'annotations' – Mao's *Odes* and the *Record of Rites*. After fifteen years of collaboration, negotiation, renegotiation, and reorganisation, the project would result in 653 CE in *The Correct Meaning of the Five Classics*, which would

come to serve as a pillar of the civil service examination and, thus, of the aspiring officeholder's education. Tellingly, it is Kong Yingda who coined the modern term *kecheng* 課程, which we now translate as 'curriculum', though what it originally/still means is administrative 'test norms'. Jia Gongyan 賈公彥 (fl. 637 CE), one of Kong's collaborators, further took it upon himself to do the same for the lesser Classics, the *Rites of Zhou* and the *Book of Etiquette and Ceremonial*, also on the basis of Zheng Xuan's 'annotations', which would eventually find their way into the expanded canon of the Southern Song (1127–1279 CE).⁸ Of the later Thirteen Classics, Zheng Xuan's 'annotations' were selected for four – three for three as concerns those on ritual (*li* 禮) – thus canonising his interpretations in what has been for centuries the mainstay of the Chinese book industry: test preparation materials.⁹

Just because Zheng Xuan's might be the right answer on a test, however, does not mean that he was *right*, and this chapter offers a look at how, in one case, he is not. That Zheng Xuan's understanding of pre-first-millennium BCE state ritual is fallible, as Wang Su could have told us, is not exactly news. Indeed, the fruits of philology and archaeology have, in the last few centuries, revealed that some of what even the Classics have to say on ritual is wrong: the Bright Hall, for example, we now know to be largely a product of Warring States (475–221 BCE) metaphysics, and as to how long one mourned one's father, as counted in months, we now know to think twice before assuming a monolithic pre-imperial 'institution' (*zhi* 制). Sure, some Classical-era (or Classical-text-derived) ritual *did* survive into Zheng Xuan's day, but a lot of it was introduced in fits and starts only a few centuries earlier – introduced in response to and, finally, as amalgamated with the contemporary institutions that were, more often than not, the true point of contention in studying the past.¹⁰ Still, in as much as the 'Three Ritual [Classics]' (*Sanli* 三禮) were believed infallible, and in as much as their 'Correct Meaning' depended on a single man, it behoves us to ruminate on the things that he got wrong – on what he got *really* wrong, no less, as in on the level of describing 'institutions' that are physically impossible.

The interest in doing this is not to say *whether* Zheng Xuan erred; it is to say *how*. Mistakes have a way of revealing precious information about the way a given text was conceived, and all the more when mistakes occur in the context of calculation, so it is to that end that we

⁸ On Kong Yingda, Jia Gongyan, and *The Correct Meaning of the Five Classics* (*Wujing zhengyi* 五經正義) project, see McMullen (1988, 71–112). Note that these subcommentaries were collaborative projects, but that I shall refer to their lead authors in the singular, as is usually done, for the sake of brevity.

⁹ For a broader study of Zheng Xuan's commentary to the Ritual Classics, see Yang Tianyu (2008); for more focused, Western-language studies on Zheng Xuan's commentary-writing, see Makeham (1997) and Plaks (2010). On the civil service exam and the history of education in China, see Lee (2000).

¹⁰ On the historical origins of the Bright Hall, see the references cited in Note 4. On the three-year mourning period, see for example Yang Hua (2007, 311–312). On the intersection of Classical scholarship and contemporary politics in the Han, see the recent studies of Sukhu (2005–2006) and Brashier (2011).

will focus on Zheng Xuan's treatment of target geometries in the 'Big Shoot' (*dashe* 大射).¹¹ There is a glaring, unequivocal error in Zheng Xuan's commentaries on this subject – an error, nonetheless, that his seventh-century CE subcommentators would enshrine as the 'Correct Meaning' of the respective Classics. At that, in legal terms, what looked like an accident now looks like criminal conspiracy, and it is thus, in legal terms, that we shall conduct an investigation of the events in question and those suspected of possible wrongdoing. First, we will reconstruct the actions of each party as witnessed by his testimony to better understand what happened, what went wrong, and how it was concealed. Then, we will conduct a background check to establish what (and *who*) each of them knew so as to build a case for foreknowledge and, from there, for intent. The purpose of this investigation, once again, is not to establish *guilt*, let alone the vagaries of *motive*; what interests us is the *modus operandi* and what we might learn from it, moving forward, when reading a canon moulded by their hands.

The mistake

The Big Shoot is a royal archery meet described in more exquisite detail than your average Classical ritual. The *Book of Etiquette and Ceremonial* consecrates an eponymous chapter to providing step-by-step instructions for what everyone is to do, ceremony to game, set-up to clean-up. The *Record of Rites*, in turn, devotes a chapter to 'The Meaning of the Shoot' (Sheyi 射義) in terms of Sagetime origins and its implications for contemporary factio-ethical self-reflection. The latter half is narrated by a famous athlete:

孔子射於矍相之圃，蓋觀者如堵牆...

[Once], when Master Kong was conducting a shoot in a vegetable garden at Juexiang, surrounded by a wall of onlookers...¹²

The *Offices or Rites of Zhou*, lastly, offers a description of the various officers among the rite's support staff and, in the 'Record of the Examination of Craftsmen' (Kaogong ji 考工記), an account of the fabrication of the materials involved. All of these texts have been thoroughly studied and translated into multiple languages, so let us content ourselves with these descriptions and turn straight to the matter at hand.¹³

¹¹ On this point, as concerns textual production and transmission in early Chinese manuscript culture, see Morgan (2015), Morgan and Chemla (2018), and Chemla and Morgan (forthcoming). Note also that the very idea to go looking for mathematical problems in Classical commentary in this chapter is inspired by the recent, ground-breaking work of Zhu Yiwen (2015; 2016a; 2016b).

¹² *Liji zhushu*, 46.1017a; tr. modified from Legge (1885, vol. 2, p. 449).

¹³ The following analysis builds off of the philological studies of target-related sources and terminology in Chen Pan (1950) and Riegel (1982). For other editions and translations of the Ritual Classics than those used here, see Loewe (1993). For a broader study of the Big Shoot and related archery contests, see Selby (2006, chaps. 4–6) and Levinovitz and Morgan (forthcoming).

The matter at hand is target geometry, and to get there we must also explain two things. The first is that target geometry is a matter of measurement, and that the units employed are as follows:¹⁴

$$1 \text{ zhang 丈} = 10 \text{ chi 尺} = 100 \text{ cun 寸} = 1000 \text{ fen 分} = 231 \text{ cm}$$

These are measures of *length*, but they are also, essentially, measures of *area*. Namely, when measuring cloth it is generally assumed that the bolt's usable width is 2 *chi*, by which we know a square 'using five *zhang* of cloth' 用布五丈, for example, to be $\sqrt{50 \times 2} = 10 \text{ chi}$ to a side. In other words, five *zhang* of cloth gets you five 2×10 *chi* segments, which, stacked one atop the other, makes a 10×10 *chi* square.¹⁵ The second thing is that the Big Shoot uses three targets at different distances, the smallest of which commentators equate with that used in the 'District Shoot' (*xiangshe* 鄉射), and as the District Shoot comes *first* in the *Book of Etiquette and Ceremonial*, it is there where any discussion of the Big Shoot must begin.

With the Classic in SMALL CAPS, and Zheng Xuan's interlinear commentary thereafter, this is what we learn from the 'District Shoot' chapter of the *Book of Etiquette and Ceremonial*:

鄉侯，上个五尋。上个，謂最上幅也。八尺曰尋。上幅用布四丈。中十尺。方者也，用布五丈。今官布幅廣二尺二寸，旁削一寸。『考工記』曰：「梓人為侯，廣與崇方，」謂中也。侯道五十弓，弓二寸以為侯中。言侯中所取數也。量侯道以狸步，而云弓者，侯之所取數宜於躬器也。正二寸較中之博也……倍中以為躬。躬，身也，謂中之上下幅也，用布各二丈。倍躬以為左右舌。注：謂上个也。居兩旁，謂之个左右，出謂之舌。下舌，半上舌。半者，半其出於射者也，用布三丈。所以半上舌者，侯，人之形類也：上个象臂，下个象足。中人張臂八尺，張足六尺，五八，四十，五六，三十，以此為衰也。凡鄉侯用布十六丈；數起侯道，五十弓以計：道七十弓之侯，用布二十五丈二尺；道九十弓之侯，用布三十六丈。

FOR THE DISTRICT TARGET, THE UPPER STALK IS FIVE ARM-SPANS. 'Upper stalk' (*gan* 个) refers to the bolt of cloth at the very top. Eight *chi* is called an 'arm-span'. The upper bolt [thus] uses four *zhang* of cloth. THE CENTRE IS TEN *CHI*. This is 'the square'; it uses five *zhang* of cloth. The modern official cloth bolt is two *chi* two *cun* wide, from which one cuts one *cun* from [each] side. The 'Record of the Examination of Craftsmen' says 'The Worker of Precious Woods makes the target, square in breadth and height', which refers to the 'centre'. THE TARGET PATH IS FIFTY BOWS, TAKE BOW TWO *CUN* FOR THE TARGET-CENTRE. This is talking about the dimensions of the target-centre. One measures the target path in racoon-dog paces, the reason for saying 'bow' here being that the target's dimensions are adapted to the shooting instrument, the 'two *cun*' [per bow] at the target corresponding to the width of its grip. ... DOUBLE THE CENTRE FOR THE FRAME. 'Frame' (*gong* 躬) means body (*shen* 身), referring to the bolt of cloth above and below the centre, each of which uses two *zhang* of cloth. DOUBLE THE FRAME FOR THE LEFT AND RIGHT TONGUES. This refers to the 'upper stalk': [the lengths] that occupy its two sides are referred to as

¹⁴ Note that the equivalency 1 *chi* = 23.1 cm holds for the late Warring States up to the Western Han (206 BCE–9 CE), when the Ritual Classics were redacted, canonised, and, likely, written, but that the civil *chi* of Zheng Xuan's day was more like 23.5 cm; see Qiu Guangming (1992) and Goodman (2010, chap. 4).

¹⁵ Chemla something on lengths as areas.

‘left and right’, and because these stick out they are referred to as ‘tongues’. FOR THE LOWER TONGUE, HALVE THE UPPER TONGUE. ‘Halve’ means halving that which sticks out from what you shoot, which uses three *zhang* of cloth. The reason you halve the upper tongue is because the target is an analogy for the human form. The upper stalk is in effigy of arms, and the lower stalk in effigy of legs; the average human arm-span is eight *chi*, and his leg-span is six *chi*; five eights, forty, five sixes, thirty – use this for the rate of unequal sharing.¹⁶ In total, the district target uses sixteen *zhang* of cloth; this number arising from the target path, we calculate from fifty bows: the target whose path is seventy bows uses twenty-five *zhang* two *chi* of cloth; the target whose path is ninety bows uses thirty-six *zhang* of cloth.¹⁷

In symbolic terms, where c is the total length of cloth, comprising the length used for the target (t), two frames (f), and upper (u) and lower (l) tongues,

$$c = t + f + f + u + l$$

The lower tongue is six eighths the length of the upper tongue ($l = 6u/8$), which, in turn, is twice the length of a frame ($u = 2f$), which, in turn, is twice the length of the side s of the target ($f = 2s$). In *chi*, since side $s = \sqrt{2t}$ (above), we may reduce this to

$$c = \frac{s^2}{2} + 2s + 2s + 4s + \frac{6 \times 4s}{8}$$

Or

$$c = \frac{s^2}{2} + 11s$$

Thus does a target 10 *chi* to side require ‘sixteen *zhang* of cloth’, the figures ‘twenty-five *zhang* two *chi*’ and ‘thirty-six *zhang*’ corresponding to targets at 14 and 18 *chi* to a side, respectively, at 70 and 90 bow-lengths from the shooter.

For the origin of Zheng Xuan’s figures here one must turn to the *Rites of Zhou*:

司裘...王大射，則共虎侯、熊侯、豹侯，設其鵠.....凡此侯道，虎九十弓，熊七十弓，豹、麋五十弓.....鄭司農云：「鵠，鵠毛也。方十尺曰侯，四尺曰鵠，二尺曰正，四寸曰質，」玄謂侯中之大小取數於侯道。『鄉射記』曰：「弓二寸以為侯中」，則九十弓者，侯中廣丈八尺；七十弓者，侯中廣丈四尺；五十弓者，侯中廣一丈，尊卑異等此數明矣。『考工記』曰：「梓人為侯，廣與崇方，參分其廣，而鵠居一焉」，然則侯中丈八尺者，鵠方六尺；侯中丈四尺者，

¹⁶ *Zhouli zhushu*, 41.639a–b (cf. Biot 1851, vol. 2, p. 545), features a similar passage – ‘THE UPPER TWO STALKS JOINED TO THE BODY MAKE THREE; THE LOWER TWO STALKS HALVE THAT’ 上兩個與其身三，下兩個半之 – in which Zheng Xuan glosses the ‘stalks’ as ‘tongues’, citing the present passage, and computes identical lengths based on ‘all of this taking its image from man, and taking its symbolic *li* 率 (ratio-elements) from his eight-*chi* arm-span and six-*chi* leg-span’ 蓋取象於人也，張臂八尺張足六尺，是取象率焉。

¹⁷ *Yili zhushu*, 5.151a–b; cf. Steele (1917, vol. 1, p. 120).

鵠方四尺六寸大半寸；侯中一丈者，鵠方三尺三寸少半寸。謂之鵠者，取名於鴝鵒。鴝鵒小鳥而難中，是以中之為鵠。亦取鵠之言較。較者，直也，射所以直己志。

THE MASTER FURRIER: ... FOR THE ROYAL BIG SHOOT, HE PROVIDES THE TIGER TARGET, BEAR TARGET, AND LEOPARD TARGET, SETTING THEIR SWANS. ... As to their target paths, the tiger[-skin target] is at ninety bows, the bear at seventy bows, and the leopard and elk at fifty bows. ... Minister of Agriculture Zheng [Zhong 鄭眾 (d. 83 CE)] says ‘The “swan” (*hao* 鵠) is a swan fur. As to the [concentric] squares, the ten-*chi* one is called the “target” (*hou*), the four-*chi* one the “swan” (*hao*), the two-*chi* one is the “surface” (*zheng* 正), and the four-*cun* one the “substance” (*zhi* 質)’, but [I, Zheng] Xuan, say that the dimensions of the target-centre are taken from the target paths. The ‘Record of the District Shoot’ (cited above) says ‘the target-centre constitutes bow two *cun*’, so, at ninety bows, the target-centre is a *zhang* and eight *chi* broad; at seventy[, it] is a *zhang* and four *chi* broad; and at fifty[, it] is one *zhang* broad – the different ranks of noble and base made apparent by these numbers. The ‘Record of the Examination of Craftsmen’ says ‘The Worker of Precious Woods makes the target, square in breadth and height; dividing its breadth in three, the swan occupies one thereof’,¹⁸ and, as such, the swan on the one with the *zhang*-eight-*chi* target-centre is a square six *chi* [to a side] ($18\ chi \div 3 = 6\ chi$); [that] on the one with the *zhang*-four-*chi* target-centre is a square four *chi* six *cun* and a big-half ($\frac{2}{3}$) *cun* [to a side] ($14\ chi \div 3 = 4\frac{2}{3}\ chi$); and [that] on the one with the one-*zhang* target-centre is a square three *chi* three *cun* and a small-half ($\frac{1}{3}$) *cun* [to a side] ($10\ chi \div 3 = 3\frac{1}{3}\ chi$). It is called a *hao* ‘swan’ because it takes its name from the *ganhao* 鴝鵒 ‘swallow-swan’ (i.e., magpie), the swallow-swan being a small bird that is difficult to hit, and that is why hitting it is considered outstanding. It is also taken from *hao*’s (**kouk*) meaning *jiao* 較 (**kək*) ‘frank’: ‘frankness’ is being straight (*zhi* 直), and shooting is that by which one ‘straightens’ one’s ambitions.¹⁹

Clearly, Zheng Xuan is interpreting ‘take bow two *cun* for the target-centre’ 弓二寸以為侯中, above, to mean ‘take [each] bow [on the target path to make] two *cun* [width] at the target-centre’ such that 70 bows \times 2 *cun*/bow = 140 *cun*. What is also clear is that he demands such measures be exact. The ‘Worker of Precious Woods’ entry says to *fen* 分 ‘divide’ the target’s breadth, so one mustn’t ignore the *fen* ‘fractional remainder’ that that implies. Ten divided by three doesn’t get you *four*, he chastises Zheng Zhong, it gets you $3\frac{1}{3}$. Indeed, were the swan *four chi* to a side, he implies, it would be more a *swan* than a *swallow-swan*, and where would be the sense in that?²⁰

¹⁸ Citing *Zhouli zhushu*, 41.639a (cf. Biot 1851, vol. 2, p. 545). Zheng Xuan’s commentary to said passage reiterates that the swan is ‘made of hide’ 以皮為之 and that big target’s 18 *chi* target-centre would make the swan ‘a square of 6 *chi* [to a side]’ 鵠方六尺。

¹⁹ *Zhouli zhushu*, 7.108a; cf. Biot (1851, vol. 1, p. 138). Note that Zheng Xuan offers the same gloss for *hao* ‘swan’ in *Yili zhushu*, 7.188a–b, elided in the relevant block quote below. All phonetic reconstructions are of Later Han Chinese as per Schuessler (2007).

²⁰ A swan, obviously, could not be more different than a magpie: the one is white, the other black; the one has an adult wingspan of around 3.1 m, and the other about 60 cm (a contrast rather more stark than a target-swan of 4 vs $3\frac{1}{3}\ chi$). Note that the

Zheng Xuan is silent about his predecessor's division of the swan into further concentric squares – how/if to scale the ‘substance’ and the ‘surface’ thereto at 1:5:10. Under the ‘Marksman’ (*sheren* 射人) entry, however, he makes it clear that ‘substance’–‘surface’–‘swan’ would indeed be the proper order:

今儒家云：「四尺曰正，二尺曰鵠」……此說失之矣。

Modern scholastics say ‘The four-*chi* [square] is called the “surface,” and the two-*chi* one is called the “swan”... this explanation misses the mark.²¹

Kong Yingda identifies this position with ‘the *Rites of Zhou* annotated by Zheng Zhong and Ma Rong’,²² the former conflicting with Zheng Xuan’s testimony, above, and the latter posing a conflict of interest. Alternatively, Yang Jian 楊簡 (1141–1226 CE) attributes this quotation to ‘Jia Kui’s 賈逵 (30–101 CE) annotated *Rites of Zhou*’.²³

As to the ‘Worker of Precious Woods’ entry that Zheng Xuan has already quoted twice, we find there one last specification:

梓人為侯……上綱與下綱出舌尋，緝寸焉。綱所以繫侯於植者也……鄭司農云：綱，連侯繩也。緝籠綱者。緝讀為竹中皮之緝舌，維持侯者。

THE WORKER OF PRECIOUS WOODS MAKES THE TARGET... THE UPPER MAINSTAY AND LOWER MAINSTAY PROTRUDE AN ARM-SPAN FROM THE TONGUES. THE TIE LOOPS ARE A *CUN* THERETO. The ‘mainstays’ (*gang* 綱) are that by which one ties the target to the poles. ... Minister of Agriculture Zheng [Zhong] says ‘The “mainstays” are the ropes connected to the target, and the “tie loops” (*yun* 緝) [are that which] hoop the mainstay’. *Yun* here is read like the *yun*-tongue skin in a bamboo (?), being that which supports and holds the target.²⁴

At that, we know all there is to know about target construction and can return, finally, to the Big Shoot.

All that remains for the exegete to do in the ‘Big Shoot’ chapter of the *Book of Etiquette and Ceremonial* is to tie the targets there with that described in the ‘District Shoot’ and to account for how *three targets* are to be arranged on a single field:

司馬命量人量侯道與所設乏以狸步，大侯九十，參七十，干五十。……狸之何物，每舉足者，止視遠近為發必中也。是以量侯道取象焉。【鄉射記】曰：「侯道五十弓」。【考工記】曰：「弓之下制六尺」，則此狸步六尺明矣。大侯、熊侯謂之大者，與天子熊侯同參讀為穆糝雜也。雜侯者，豹鵠而麋飾下天子大夫也。干讀為豸，豸侯者，豸鵠，豸飾也。……

THE MASTER-AT-ARMS COMMANDS THE MEASURERS TO MEASURE THE TARGET PATHS AND THE POSITIONS IN WHICH THE SCREENS ARE TO BE SET UP IN RACCOON-DOG PACES: NINETY FOR THE BIG TARGET,

use of the term *ganhao* 乾鵠 ‘swallow-swan’ to denote the magpie is attested in Xu Shen’s 許慎 (c.55–c.149 CE) commentary to the *Huainanzi* 淮南子 (139 BCE), *Huainan honglie jie*, 13.11a, but that Zheng Xuan’s reading of *hao* ‘swan’ as an abbreviation for an unrelated species seems forced.

²¹ *Zhouli zhushu*, 30.462b (comm.).

²² 周禮鄭眾馬融注, *Maoshi zhushu*, 21.492a (subcomm.).

²³ 賈逵周禮注, *Cihu Shi zhuan*, 15.14a.

²⁴ *Zhouli zhushu*, 7.108a; cf. Biot (1851, vol. 2, pp. 545–546).

SEVENTY FOR THE THREE, AND FIFTY FOR THE POLE. ... The racoon-dog examines things at the raise of each foot, [as the archer must] stop and look at the distance so that his release be on-target, and it is thus that the measuring of the target paths takes the image thereof. The ‘Record of the District Shoot’ (cited above) says ‘The target path is fifty bows’, and the [Rites of Zhou], and the ‘Record of the Examination of Craftsmen’ says ‘The inferior model bow is six *chi*’, so it is evident that this ‘racoon-dog pace’ (*li bu* 狸步) is six *chi*.²⁵ As to the ‘big target’, the bear[-skin] target is referred to as the ‘big’ – this is none other than the son of heaven’s ‘bear[-skin] target’. *San* 參 ‘three’ is read as *san* 糝 ‘blended’, ‘blended’ [meaning] ‘mixed’ (*za* 雜); the ‘mixed target’ has a leopard[-skin] swan beneath which it is decorated with elk [skin, around its edges] – this is for the son of heaven’s grandees. *Gan* 干 ‘pole’ is read as *an* 豸 ‘wild dog’, the ‘dog[-skin] target’ having a dog[-skin] swan and dog[-skin] decorations. ...

遂命量人，巾車張三侯。大侯之崇，見鵠於參；參見鵠於干，干不及地武。……崇，高也。高必見鵠，鵠所射之主……『考工記』曰：「梓人為侯，廣與崇方，參分其廣，而鵠居一焉，」則大侯之鵠方六尺，糝侯之鵠方四尺六寸大半寸，豸侯之鵠方三尺三寸少半寸。及至也。武，迹也。中人之足長尺二寸，以豸侯計之，糝侯去地一丈五寸少半寸，大侯去地二丈二尺五寸少半寸。

HE THEN ORDERS THE MEASURERS AND THE FABRIC-AND-CHARIOT OFFICER TO STRETCH THE THREE TARGETS. THE LOFTINESS OF THE BIG TARGET IS SUCH THAT IT SHOWS ITS SWAN VIS-À-VIS THE THREE, THE THREE SHOWS ITS SWAN VIS-À-VIS THE POLE, AND THE POLE FALLS SHORT OF THE GROUND MILITARY. ... ‘Loftiness’ (*chong* 崇) means ‘height’ – its height must be such that one sees the swan, the swan being the principal thing at which one shoots. The ‘Record of the Examination of Craftsmen’ says ‘The worker of precious woods makes the target, square in breadth and height; dividing its breadth in three, the swan occupies one thereof’, and, as such, the big target’s swan is a square six *chi* [to a side], the blended target’s swan is [one] four *chi* six *cun* and a big-half ($\frac{2}{3}$) *cun* [to a side], and the dog target’s swan is [one] three *chi* three *cun* and a small-half ($\frac{1}{3}$) *cun* [to a side]. ‘Reach’ (*ji* 及) means ‘arrive at’ (*zhi* 至), and ‘military’ (*wu* 武) means ‘[by] a footstep’ (*ji* 迹), the length of the average human foot being a *chi* and two *cun*.²⁶ Calculating from the dog target, the blended target is one *zhang* five *cun* and one third *cun* from the ground, and the big target is two *zhang* two *chi* five *cun* and one third *cun* from the ground.²⁷

In sum, ‘pole’ = ‘dog’ = the district target, ‘three’ = ‘blended’, and ‘big’ = ‘tiger’, the strange names, *excepting the ‘swan’*, all referring to

²⁵ Note that ‘The inferior model bow’ 弓之下制 is but one of three models listed in the ‘Record of the Examination of Craftsmen’: ‘The six-*chi*-six-*cun*-long bow is called the superior model, for use by superior gentlemen; the six-*chi*-three-*cun*-long bow is called the average model, for use by average gentlemen; the six-*chi*-long bow is called the inferior model, for use by inferior gentlemen’ 弓長六尺有六寸，謂之上制，上士服之；弓長六尺有三寸，謂之中制，中士服之；弓長六尺，謂之下制，下士服之 (*Zhouli zhushu*, 42.662a; cf. Biot 1851, vol. 2, p. 596).

²⁶ Zheng Xuan offers the same gloss in *Yili zhushu*, 5.110a. See Jia Gongyan, below, on how the equivalency 1 footstep = 1.2 *chi* is derived.

²⁷ *Yili zhushu*, 7.187b–188b; cf. Steele (1917, vol. 1, pp. 150–151)..

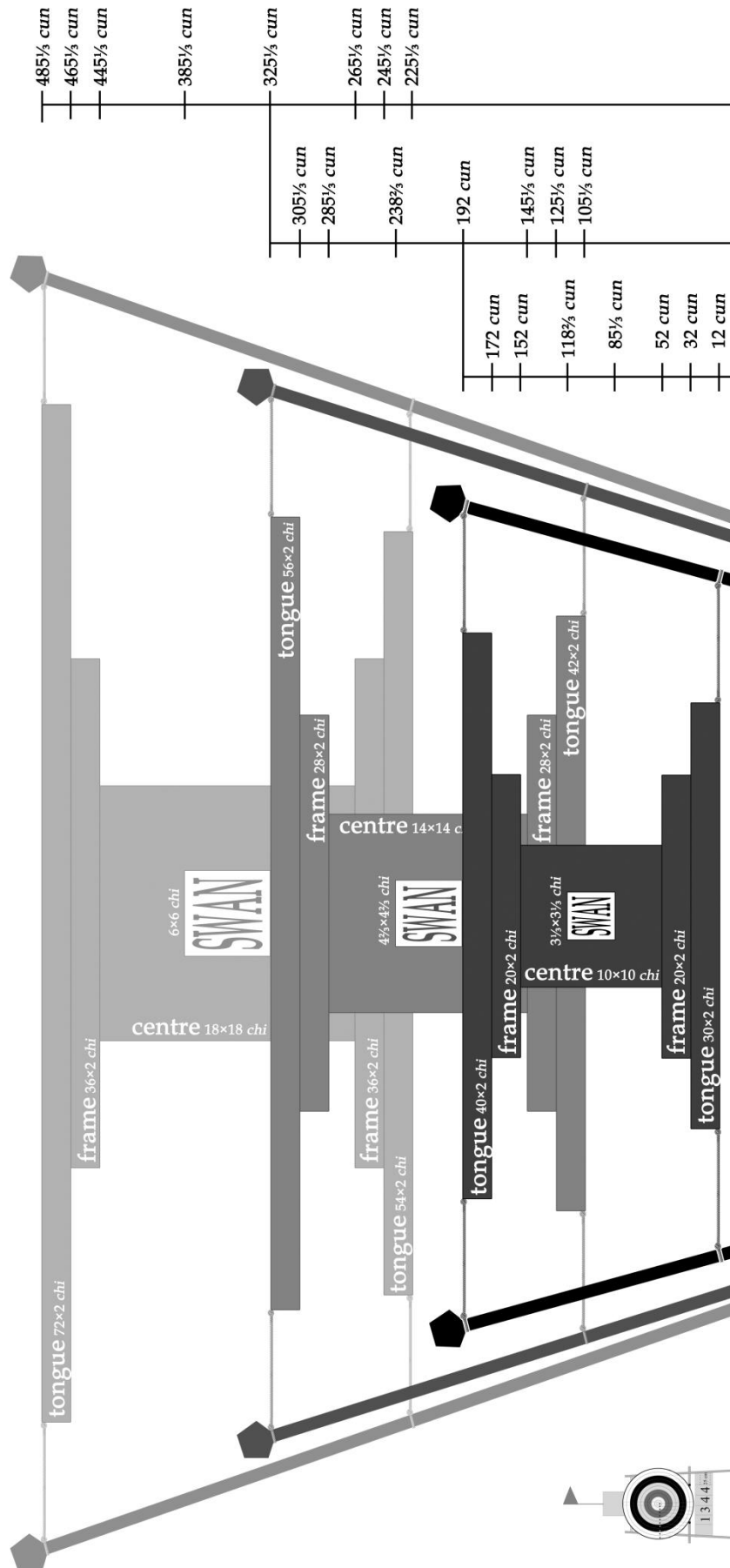


Fig. 2 Reconstruction of Zheng Xuan's Big Shoot target dimensions and heights as corroborated in Kong Yingda and Jia Gongyan's subcommentaries (1 *chi* = 10 *cun*). The World Archery Federation regulation 70 m (\approx 50 racoon-dog pace) butt is provided in the lower left corner for sake of comparison (scaled at 1 *chi* = 23.1 cm).

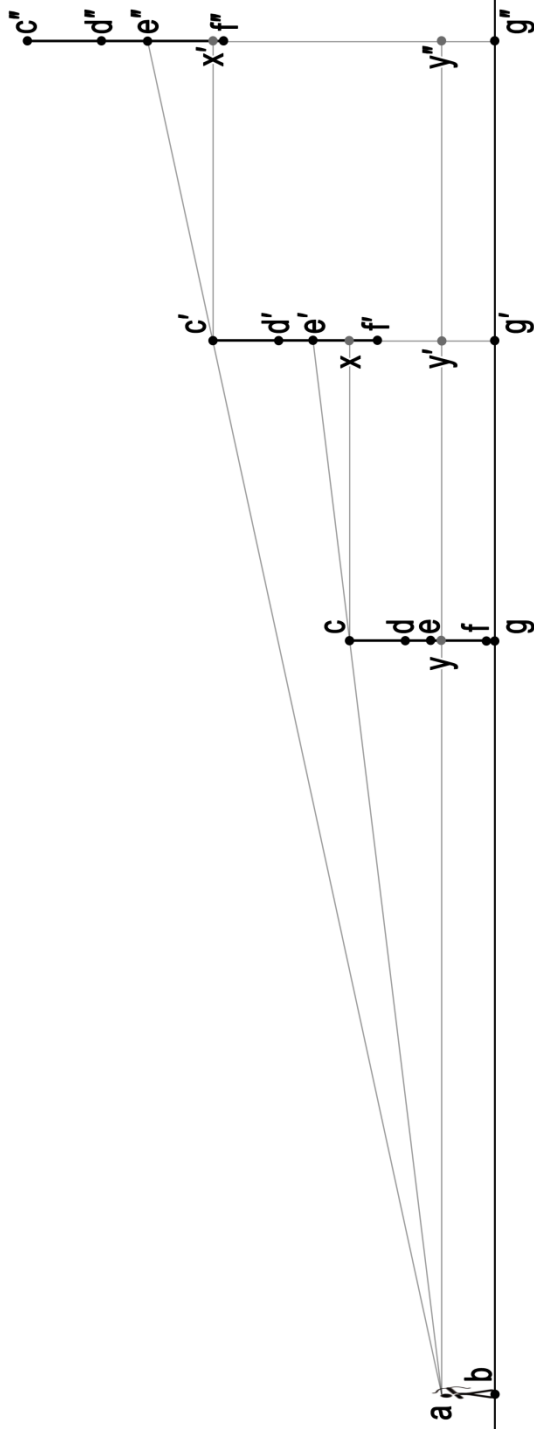


Fig. 3 Zheng Xuan's target geometries in three dimensions (note that the y-axis is expanded 300% for the sake of legibility). Point a represents the archer's eye, and b the point directly beneath his eye on the ground. Points c and f represent the upper and lower edges of the target, d and e the upper and lower edges of the swan, and g the point directly beneath each target on the ground. Line AY'' runs parallel to the ground through the archer's eye, intersecting the plane of the respective targets at y , y' , and y'' . Line CX and CX' run parallel to the ground from the upper edge of the forward target to the plane of the target behind it. In racoon-dog paces, $BG = AY = 50$, $BG' = AY' = 70$, $BG'' = AY'' = 70$, and $G'G'' = CX = C'X' = YY' = Y'Y'' = 20$. As per problem 9.22 of the *Nine Chapters of Mathematical Procedures* (below), we assume $AB = YG = Y'G'' = 7$ *chi*. Zheng Xuan gives $CG = 19.2$ *chi* (fig. 2), so $CY = XY' = 19.2 - 7 = 12.2$ *chi*. ACY and $CE'X$ are similar triangles, where $E'X/CX = CY/AY$, so $E'X = 20 \times \frac{12.2}{50} = 4\frac{22}{25}$ *chi*, and $E'G' = 19\frac{1}{5} + 4\frac{22}{25} = 24\frac{2}{25}$ *chi*. As per fig. 2, $E'C' = 13\frac{1}{3}$ *chi*, and $E'F' = 8\frac{2}{3}$ *chi*, so $C'G' = 24\frac{2}{25} + 13\frac{1}{3} = 37\frac{31}{75}$ *chi*, and the distance $F'G'$ from the medium-range target to the ground is $24\frac{2}{25} - 8\frac{2}{3} = 15\frac{51}{75}$ *chi*. For the long-range target, $X'Y'' = C'Y' = 37\frac{31}{75} - 7 = 30\frac{31}{75}$ *chi*, $AY' = 70$ racoon-dog paces, $C'X' = 20$ racoon-dog paces, and since $AC'Y'$ and $CE'X'$ are similar triangles, $E'X'/C'X' = C'Y'/AY'$, so $E'X' = 20 \times \frac{30\frac{31}{75}}{70} = 8\frac{362}{525}$ *chi*, and $E''G'' = 37\frac{31}{75} + 8\frac{362}{525} = 46\frac{54}{525}$ *chi*. As per fig. 2, $E''C'' = 16$ *chi*, and $E''F'' = 10$ *chi*, so the distance $F''G''$ from the long-range target to the ground is $46\frac{54}{525} - 10 = 36\frac{54}{525}$ *chi*. Q.E.D.

animal pelts as per the *Rites of Zhou* (above). More importantly, ‘raccoon-dog pace’ = ‘bow’ = 6 *chi*, and ‘military’ = ‘footstep’ = 1.2 *chi*, which allows Zheng Xuan to provide the exact height at which each target is to be suspended such that the ‘[the one] shows its swan vis-à-vis the [other]’: 1.2 *chi*, 10.533 *chi*, and 22.533 *chi*, respectively.

The problem, as illustrated in fig. 2, is that Zheng Xuan is calculating in *two dimensions*, as if the 50-, 70-, and 90-unit targets were on a single plane. Laid out in *three dimensions*, as illustrated in fig. 3, one sees that the medium- and long-range targets need to be suspended considerably higher than this to meet the *Book of Etiquette and Ceremonial*’s criteria. Were it anyone else – anyone at all – one might simply say that he was wrong, but this is *Zheng Xuan*, so that is obviously not what happened.

The cover-up

As concerns reception, the above passages fall within the purview of the lesser-known Jia Gongyan’s subcommentaries. Jia Gongyan was working under and after the model of Kong Yingda, so it behoves us to ask how Kong Yingda might have dealt with this first. We need not guess, luckily, because Kong offers up these numbers unsolicited in *The Correct Meaning of the Record of Rites* where the Classic moralises, quite simply, about how ‘EACH SHOOTER SHOOTS AT THE SWAN RESPECTIVE HIS STATION’.²⁸ After sorting through the text as written, Kong begins rehashing equivalencies and the proportional relationship between bow-lengths, target-centres, ‘bodies’, upper and lower ‘tongues’, and the quantity of cloth employed, bringing us ultimately to the problem of the targets’ heights:

其張三侯之躰，同道位之近者最下，遠者漸高。故『大射』云：「大侯之崇，見鵠於參，參見鵠於干，干不及地武。」以此計之，豸侯下畔去地尺二寸，豸侯之體上下躬及舌摠有四幅，凡廣八尺；侯中方一丈，是豸侯摠高一丈九尺二寸。

That which is positioned the closest in stretching the body of the three targets on the same path is the lowest down, and those more distant are progressively higher. Thus it is that the ‘Big Shoot’ says ‘The loftiness of the big target is such that it shows its swan vis-à-vis the three, the three shows its swan vis-à-vis the pole, and the pole falls short of the ground military’ (cited above). Calculated from this, the dog target’s lower edge is a *chi* and two *cun* from the ground; the upper and lower bodies and tongues of the dog target’s body comprise a total of four bolts, for a total width of eight *chi* (2 *chi*/bolt × 4 bolts = 8 *chi*); the target-centre is a square one *zhang* [to a side]; and thus is the dog target’s total height one *zhang* nine *chi* two *cun* (12 + 80 + 100 = 192 *cun*).

此豸侯上畔去地一丈九尺二寸。以此，豸侯上畔，則是糝侯之鵠去地之數也。其糝侯下舌及躬，凡有四尺，糝侯之中，鵠下有三分之一。糝侯中方丈四，其三分之一得四尺六寸三分寸之二，是

²⁸ 射者各射己之鵠, *Liji zhushu*, 46.1017b; cf. Legge (1885, vol. 2, p. 451).

糝侯自鵠以下凡有八尺六寸三分寸之二，皆為豨侯所掩。豨侯上畔去地一丈九尺二寸，去其八尺六寸三分寸之二，是糝侯下畔去地一丈五寸三分寸之一。故鄭注『大射』云：「糝侯去地一丈五寸少寸半」也。云「少半寸」，則三分寸之一也。

This means that the dog target's upper edge is one *zhang* nine *chi* two *cun* from the ground. Following this, the dog target's upper edge is then the number [of *chi*] that the blended target's swan is from the ground. The blended target's lower tongue and body comprise a total of four *chi*, and as to [its] centre, one third is below the swan. The blended target centre is a square a *zhang* and four *chi* [to a side], one third of which gets you four *chi* six *cun* and two thirds *cun*, and thus, beneath [its] swan, is there a total of eight *chi* six *cun* and two thirds *cun*, all of which is covered up by the dog target. The dog target's upper edge is one *zhang* nine *chi* two *cun* from the ground; removing eight *chi* six *cun* and two thirds *cun*, the blended target's lower edge is thus one *zhang* five *cun* and one third *cun* from the ground ($192 - 86\frac{2}{3} \text{ cun} = 105\frac{1}{3} \text{ cun}$). Thus does Zheng's commentary to the 'Big Shoot' say 'the blended target is one *zhang* five *cun* and a small-half *cun* from the ground'. When he says 'a small-half *cun*', this is one third *cun*.

豨侯既去地一丈九尺二寸，更加糝侯之中三分之二，九尺三寸三分寸之一，又加糝侯上躬與个四尺，則糝侯上畔去地三丈二尺五寸三分寸之一，則是大侯鵠下之數也。其大侯，下舌及躬并侯中三分之一，摠一丈，為糝侯所掩。於三丈二尺五寸三分寸之一減其一丈，則是大侯下畔去地之數。故鄭注『大射』云：「大侯去地二丈二尺五寸少半寸」也。

Since the dog target is one *zhang* nine *chi* two *cun* from the ground, one adds to this the other two thirds of the blended target's centre, nine *chi* three *cun* and one third *cun*, and one furthermore adds the four *chi* of the frame and stock above [it], [finding] thus that the blended target's upper edge is three *zhang* two *chi* five *cun* and one third *cun* from the ground ($192 + 2 \times 46\frac{2}{3} + 40 = 325\frac{1}{3} \text{ cun}$), which, in turn, is the number for [everything] below the big target's swan. For the big target, the bottom tongue and frame are combined with one third the target-centre, totalling one *zhang* ($4 + 18 \div 3 = 10 \text{ chi}$), which is covered up by the blended target. Diminishing three *zhang* two *chi* five *cun* and one third *cun* by one *zhang*, this [gives you] the number that the big target's lower edge is from the ground ($325\frac{1}{3} - 100 = 225\frac{1}{3} \text{ cun}$). Thus does Zheng's commentary to the 'Big Shoot' say 'the big target is two *zhang* two *chi* five *cun* and a small-half *cun* from the ground'.²⁹

Kong Yingda is showing Zheng Xuan's work. More specifically, he is inserting a mathematical proof for Zheng Xuan's numbers from the *Book of Etiquette and Ceremonial* into what is clearly the nearest passage that he could find in the *Record of Rites*. There is clearly something at stake here, and, whatever it is, Kong uses words like *shi* 是

²⁹ *Liji zhushu*, 46.1018b (subcomm.).

‘is/truly’ and *gu* 故 ‘thus’ to transform his mistaken sums by way of arithmetic performance into The Correct Meaning of this other text.³⁰

Returning to the *Book of Etiquette and Ceremonial*, one notes that Jia Gongyan’s subcommentary is a little more circumspect about these numbers. Rather than what ‘is’ ‘thus’ ‘truly’ so, Jia tends to frame his questions in terms of how it is that Zheng Xuan ‘knows’ (*zhi* 知), e.g.:

云「中人之足長尺二寸」者，無正文，以目驗而知。

As to ‘The length of the average human foot is a *chi* and two *cun*’ (cited above), this is not in the scripture/s, so it is known by visual inspection.³¹

Jia Gongyan goes once again through the maths to explain why ‘thus is it said’ 故云 and, more importantly, ‘thus is it known’ 故知 of every step from the length of the range to the fatuous target-heights. He could have copy-and-pasted from Kong Yingda, one imagines, but instead he started over. This we can say because Jia Gongyan works in a different order through the numbers and the citations than does Kong Yingda’s proof, and because each has their own way of dividing and subtracting compound numbers.³² Kong Yingda, as shown in the bracketed equations above, would appear to operating directly on quantities reduced to an integer and fractional number of *cun*; Jia Gongyan, by contrast, separates out the units to deal with them one by one. Take for example his treatment of the 70-bow target:

糝侯，侯中丈四尺，中上、中下各四尺，得八尺，并之，二丈二尺也。鵠居侯中三分之一，則鵠下亦有四尺六寸大半寸，通躬身四尺為八尺六寸三分寸之二矣。張法，糝鵠下畔與豸侯之上綱齊，所謂「見鵠於豸」，自餘糝侯鵠下畔八尺六寸大半寸，在掩豸侯，亦如之，豸侯上豸本去地丈九尺二寸，直掩八尺，上有一丈一尺二寸，在復掩六寸上有一丈六寸，在復掩三分寸二，唯有一丈五寸三分寸一，在少半寸者即三分寸一也。言大半寸者，即三分寸二也。故知糝侯下綱去地一丈五寸少半寸也。

For the blended target, the target-centre is a *zhang* and four *chi* [to a side]. Both above the centre and below the centre are four *chi*, which gets you eight *chi*. Combine these, and that is two *zhang* two *chi* ($14 + 2 \times 4 = 22$ *chi*). The swan occupies one third of the target-centre, so below the swan there is also four *chi* six *cun* and a big-half *cun* (cited above), which, connected with the four *chi* of the frame-body, makes eight *chi* six *cun* and two thirds *cun* (4 *chi* $6\frac{2}{3}$ *cun* + 4 *chi* = 8 *chi* $6\frac{2}{3}$ *cun*). As to the rule for [target-]stretching, the blended swan’s bottom edge is [to be] level with the upper mainstay of the dog[-skin] target – what is referred to as ‘showing its swan vis-à-vis the dog’ (cited above). The remaining eight *chi* six *cun* and a big-half *cun* from the lower edge of the blended target’s swan [on down] is covered up

³⁰ In this sense, Zheng Xuan’s subcommentators are engaging in a practice of ‘proof’ reminiscent to that with which commentators to the mathematical canon are primarily concerned, for recent studies of which see Chemla (2010a; 2012).

³¹ *Yili zhushu*, 7.188b (subcomm.).

³² On the different cultures of computation witnessed both within and between Classical commentary and *suan* 算 mathematical writings, see Zhu Yiwen (2015; 2016a; 2016b).

by the dog[-skin] target. Likewise as such (?) is the dog[-skin] target's upper dog (sic.) a *zhang* and nine *chi* two *cun* from the ground. This directly covers up eight *chi*, [so] there is one *zhang* one *chi* two *cun* ($19 \text{ chi } 2 \text{ cun} - 8 \text{ chi} = 11 \text{ chi } 2 \text{ cun}$) above [on the calculating surface?]. Here, it covers up another six *cun*, [so] there is one *zhang* six *cun* ($11 \text{ chi } 2 \text{ cun} - 6 \text{ cun} = 10 \text{ chi } 6 \text{ cun}$) above. Here, it covers up another two thirds *cun*, [so] there is only one *zhang* five *cun* and one third *cun* ($10 \text{ chi } 6 \text{ cun} - \frac{2}{3} \text{ cun} = 10 \text{ chi } 5\frac{1}{3} \text{ cun}$), considering, [that is], that a 'small-half *cun*' is one third *cun*, and that the expression 'big-half *cun*' means two thirds *cun*. Thus is it known that the blended target's lower mainstay is 'one *zhang* five *cun* and a small-half *cun* from the ground' (cited above).³³

They may have got there by different means, but Jia Gongyan and Kong Yingda ultimately arrive at the same number as does Zheng Xuan for the blended target – a height that assumes that a target at seventy bow-lengths is on a plane with the one at fifty. Where Kong Yingda takes this for granted, however, Jia Gongyan at least concludes his calculations with a bit of remove:

故注於此數也。

And thus it is that the commentary arrives at these numbers.³⁴

The irrelevance of practice

Zheng Xuan's target geometries are unrealistic, and it is not just that they fail to take a target's distance into account. First, the numbers invoked are enormous. At 17, 27, and 38 m² respectively, we are talking about *sails*, the largest of which is to be foisted 11 m in the air. Wind would make it neigh impossible to keep these still, let alone erect, let alone *overnight*, as the *Book of Etiquette and Ceremonial* dictates.³⁵ The numbers are, at the same time, infinitesimally small. To get that perfect $3\frac{1}{3} \times 3\frac{1}{3}$ *chi* 'swan', for example, one would need to shave $6\frac{2}{3}$ *cun* off the edge of the 'modern official cloth bolt'. That would be optimistic, considering that contemporary *chi*-rules were graduated *decimally*, in *cun* and *fen* ($\frac{1}{10}$ *cun*), and that even the makers of armillary spheres tended to round such precious fractions off completely.³⁶ If Zheng Xuan were able to ascertain the length of the average human foot by 'visual inspection', as Jia Gongyan alleges, then one imagines that there might have been yet other things 'not in the

³³ *Yili zhushu*, 7.188b–189a (subcomm.).

³⁴ *Yili zhushu*, 7.188b–189a (subcomm.).

³⁵ According to the Classic, one not only erects the targets *three days in advance*, 'THE BOTTOM LEFT MAINSTAY IS LEFT UNAFFIXED, TIED [to the opposite corner] SUCH THAT THE CENTRE IS COVERED UP' 不繫左下綱，中掩束之 (*Yili zhushu*, 5.110a, 7.188a), it being only just before the shooting commences that 'THE DISCIPLES UNDO THE TIE AND THEN AFFIX THE BOTTOM RIGHT MAINSTAY' 弟子說束，遂繫左下綱 (ibid., 5.118b). Tellingly, the Classic does not mention the disciples using ladders or climbing the poles to accomplish this.

³⁶ For a catalogue of historical *chi*-rules, see Qiu Guangming (1992). On armillary sphere graduation, see Morgan (2017, 77–85).

scripture/s’ by which he may have familiarised himself with the practicalities of target archery.

Indeed, it is not as if the Big and District Shoot weren’t living practice. Sima Biao’s 司馬彪 (c.240–c.360 CE) ‘Monograph on the Hundred Offices’ (Baiguan zhi 百官志) includes the Big Shoot under the responsibilities of the Eastern Han (25–220 CE) Ministry of Rites (Taichang 太常), the Bright Hall, whose Circular Moat in which it was held, falling under the administration of its subsidiary, the Office of the Grand Clerk (Taishi ling 太史令).³⁷ Sadly, the Circular Moat was abandoned along with the capital in 190 CE, when Zheng Xuan was sixty-three, following its sack in Dong Zhuo’s 董卓 (d. 192 CE) coup d’état of the previous year. Prior to that, however, we have dated historical records of Big and District Shoots having been held at the Luoyang Circular Moat going back to its opening in 59 CE:

永平二年，三雍初成，拜榮為五更。每大射、養老禮畢，帝輒引榮及弟子升堂，執經自為下說。

Eternal Tranquility, year 2, the Three Moats (i.e., the Bright Hall, Circular Moat, and Observatory) were finally finished, and [Huan] Rong 桓榮 (d. 93 CE) was inducted into the Fifth Watch.³⁸ At the end of every Big Shoot and Elder-care rite [held here in the spring], the Emperor would invariably conduct Rong and his/the disciples to ascend with him into the hall, where, Classic in Hand, he would personally explain it to [the audience] below.³⁹

The architect of these particular rites was Cao Bao 曹褒 (d. 102 CE), a then young *boshi* 博士 ‘academician’ chosen to fix the ceremonial appropriate the new ‘Three Moats’ complex upon its commission in 56 CE. The Imperial Academy itself having been rebuilt only in 29 CE, Cao’s task would have been difficult given all that had been lost in the wake of the agrarian rebellions, civil war, and the sacking of the old capital at Chang’an in the later years of Wang Mang 王莽 (r. 9–23). That said, Cao was a master marksman, an expert in the Ritual Classics, and, most importantly, his father’s son.⁴⁰

Cao’s father, Chong 充 (fl. 25/56 CE), was a disciple of Qing Pu 慶普, who, in turn, studied the rites from Hou Cang 后蒼 (fl. 70 BCE) alongside Dai De 戴德 and his nephew, Sheng 聖. It is Qing Pu’s cohort to whom we owe the *Record of Rites*, the uncle–nephew pair having compiled its modern ‘Big’ and ‘Little Dai’ recensions from free-

³⁷ *Hou Han shu*, zhi 25, 3571–3572.

³⁸ The ‘Fifth Watch’ 五更 is a moniker for the venerable elders paid homage alongside the ‘Thrice Venerable’ 三老 at the Elder-care 養老 rites associated with the Han Big and District Shoots, the term being a reference to the fifth of the five watches into which the night was divided – someone, in modern terms, in the ‘last hour’ of the human life cycle.

³⁹ *Hou Han shu*, 37.1253; cf. *ibid.*, 2.102, 69A.2545, zhi 4, 3108, and *Jin shu*, 21.670. One finds further references to Big and District Shoots having been held at the Luoyang Circular Moat in connection with the Elder-care rites as coincide with imperial visits in in 65 CE (*Hou Han shu*, 2.111), 97 CE (*ibid.*, 50.1668), 102 CE (*ibid.*, 4.189), and 132 CE (*ibid.*, 6.260). To this one may reasonably add the non-site-specific reference to a another Big Shoot held in 112 CE (*ibid.*, 44.1504).

⁴⁰ For Cao Bao’s biography, see *Hou Han shu*, 35.1201–1205.

floating texts in connection with Liu Xiang 劉向 (79–8 BCE) and Liu Xin’s 劉歆 (c.50 BCE–23 CE) famous overhaul of the Imperial Library in 26–6 BCE.⁴¹ The *Rites of Zhou* and *Book of Etiquette and Ceremonial* also first appear in the written record at around the time of Hou Cang.⁴² The origins of the former are somewhat nebulous, being as it first appears in King Xian of Hejian’s 河間獻王 (r. 155–129 BCE) round-up of ‘old ancient-text pre-Qin (< 221 BCE) writings’⁴³ having survived the supposed bibliocaust of the Han’s imperial predecessors. The ‘Seventeen Chapters’ 十七篇 of what would become the latter, on the other hand, are reported by contemporaneous scholars as having been transmitted into Han times by Hou Cang’s master’s master’s master, Gaotang Sheng 高堂生.⁴⁴ There may have been a several-decade interruption in the royal rites, therefore, but the Big Shoot held in 59 CE goes back father–son, master–disciple, to the very men who committed the Ritual Classics to their present written form.⁴⁵

It is important to remember, as concerns the Bright Hall, that the Ritual Classics took shape in a context in which the Han court was still sorting out its ritual programme. Its ‘institution’ (*zhi*) was oft debated, as we saw in the Introduction, but the first to actually *build* one was Emperor Wu (r. 141–87 BCE), and this went rather poorly. The first ever Bright Hall was planned for the southern suburbs of Chang’an in 141 BCE. The project soon met with allegations of corruption and resistance from the empress dowager’s clique, however, and Emperor Wu only managed to get one built some thirty years later as a sort of support structure for the *feng-shan* 封禪 complex at Mount Tai – a complex 800 km east of Chang’an built by the hated First Emperor of Qin (r. 221–210 BCE). The site saw little use during Emperor Wu’s reign, and it was abandoned soon thereafter amid his predecessors’ budgetary roll-backs. It is in the decades to follow, coincidentally, that Ritual Classics describing how one is to build a *real* Bright Hall emerge in scholastic discourse, and it is lead-editor Liu Xin, coincidentally, who is ultimately charged with its realisation. This one is built for the *regent*, Wang Mang, who would poison the child-emperor under his care in 5 CE to preside *as king* over its opening ceremony in the following year, formally deposing Emperor Ping’s (b. 9 BCE; r. 1 BCE–

⁴¹ On the process by which a free-floating text extant in late-fourth-century BCE tombs like the *Ziyi* 緇衣 was transformed in the process of its first-century BCE incorporation into the *Record of Rites*, see Shaughnessy (2006, 1–130) and Wang E (2007).

⁴² The first solid evidence of the *Offices* or *Rites of Zhou* is a citation in Sima Qian’s 司馬遷 (c.145–c.86 BCE) *Shiji*, 28.1357, and that of the *Book of Etiquette and Ceremonial* is a manuscript from Mozuizi 磨嘴子 tomb 6, sealed *terminus post quem* 28 BCE, for which see *Wuwei Han jian*.

⁴³ 古文先秦舊書, *Han shu*, 53.2410.

⁴⁴ See *Shiji*, 121.3126; cf. Ban Gu’s 班固 (32–92 CE) *Han shu*, 30.1710; cf. Wang Chong’s 王充 (27–c.100 CE) *Lunheng*, 12.560–561. Note that a different, ‘old-text’ version later attributed to King Xian’s book collecting also pops up in Liu Xiang and Xin’s overhaul of the Imperial Library in 26–6 BCE.

⁴⁵ On the Gaotang Sheng–Xiao Fen 丘蕭–Meng Qing 孟卿–Hou Cang lineage and the transmission and codification of the Rites in Han times, see *Shiji*, 121.3126, *Han shu*, 30.1710, 88.3615, *Hou Han shu*, 79B.2576; cf. Loewe (1993, 25–27, 237–239, 293–295).

5 CE) successor, ‘Baby Ying’ (b. 5CE), in 9 CE. This, the first ‘Classical’ Bright Hall would soon find itself converted into a fantastical ancestral temple illustrating Wang Mang to have descended from ancient god-kings and, in 23 CE, the year its regretful architect committed suicide, razed to the ground by one several warring rebel armies.⁴⁶

Such were the real-world circumstances surrounding the Bright Hall when the Ritual Classics took their present written form, and it is difficult to imagine that they failed to impress themselves upon the memory, selection, and editorial decisions of Cao Bao’s intellectual processors. The same could be said of the Shoot, though the circumstances there were wholly different. The Big Shoot, for one, was considerably older and less contested a symbol of imperial legitimacy than was its ultimate venue. It may have been Wang Mang, who first conducted the Big Shoot and Elder-care rites in their (newly) Classically-appointed venue in 6 CE,⁴⁷ but all he really did was move them from where they were previously being held. As to where that was, Jia Shan’s 賈山 memorial to Emperor Wen (r. 179–157 BCE) mentions how ‘one cares for the Thrice Venerable at the Imperial Academy’,⁴⁸ and the record of a bird-omen in 19 BCE has ‘the academicians conducting the Big Shoot rite when a flying pheasant [suddenly] alit upon the courtyard, mounted the stairs one by one, and crowed from atop the hall’.⁴⁹ This would seem to place the Big Shoot *at* the Imperial Academy, but later commentators cite instead Bend Terrace as its official venue prior to 6 CE.⁵⁰ Wherever it was held, it was *being held*, and it was sufficiently important for Yuqiu Shouwang 吾丘壽王 to raise it as the centrepiece of his argument against bow control legislation in 124 BCE:

禮曰：「男子生，桑弧、蓬矢以舉之，明示有事也」。孔子曰：「吾何執？執射乎？」大射之禮，自天子降及庶人，三代之道也。...竊以為無益於禁姦，而廢先王之典，使學者不得習行其禮，大不便。

⁴⁶ On the history of Bright Hall construction, see the sources listed in Note 4.

⁴⁷ Namely, ‘In Occupying Regent year 1, month I (17 Feb–19 Mar 6 CE), [Wang] Mang offered sacrifice to the Thearch on High in the southern suburbs, welcomed the spring in the eastern suburbs, conducted the Big Shoot rite at the Bright Hall, and left after the rites of caring for the Thrice Venerable and Fifth Watch were finished’ 居攝元年正月，莽祀上帝於南郊，迎春於東郊，行大射禮於明堂，養三老五更，成禮而去 (*Han shu*, 99A.4082). The only connection of Emperor Wu’s Bright Hall of c.110 BCE with archery is that the emperor was to shoot the sacrificial cow to be used in the larger *feng-shan* sacrifices and, in preparation, ‘commanded various scholastics to practice shooting cows’ 令諸儒習射牛 (*Shiji*, 12.473, 28.1397).

⁴⁸ 養三老於大學, cited in *Han shu*, 51.2330; cf. *Shiji*, 24.1230.

⁴⁹ 博士行大射禮，有飛雉集于庭，歷階登堂而雉, *Han shu*, 27.1417.

⁵⁰ Bend Terrace (Qutai 曲臺) is variously identified as the Qin imperial palace in Xianyang (*Han shu*, 51.2338), a publically-accessible roadside terrace in Chang’an (ibid., 51.2364 (comm.)), a hall in the Han imperial palace at Chang’an (ibid.), and the son of heaven’s shooting palace (ibid., 30.1710 (comm.)). In *Han shu*, 30.1710 (comm.), Ru Chun 如淳 (third century CE) tells us that ‘ritual archery was conducted at Bend Terrace’ 行禮射於曲臺, citing Hou Cang’s *Qutai ji* 曲臺記 and Wang Long’s 王隆 (fl. 25/56 CE) *Han guan* 漢官, both of which are now lost, while Jin Zhuo 晉灼 (third/fifth century CE), oddly, adds that ‘[when?] the Western Capital had no Imperial Academy, the rites were conducted here’ 於此行禮也.

The Rites say, ‘When a male child is born, one hangs up a mulberry bow and fleabane arrows as clear notice that such is his task’. Confucius says, ‘What shall I pursue? Shall I pursue archery?’ The rite of the Big Shoot reaches from the son of heaven down to the commoner and is the *dao* of the Three Dynasties (Xia, Shang, and Zhou). ... It is my humble opinion that [bow control] would do nothing to prohibit wickedness and that to abolish the former kings’ canons of state would make it impossible for scholars to practice and conduct the relevant rites, posing a major inconvenience.⁵¹

As to the canons of state, one notes, the *Records of the Grand Clerk* (91 BCE) tell us that the Big Shoot was canonised into Han imperial ceremonial at the very beginning of the dynasty by the surrendered Qin ritual specialist Shusun Tong 叔孫通 (d. c.188 BCE).⁵² As to how Shusun knew to conduct this rite, lastly, Shusun, like Gaotang Sheng, Hou Cang, Qing Pu, Cao Chong, and his son Bao, was a man of Lu, and it is there, the *Records of the Grand Clerk* tell us, that this particular rite had survived from some higher antiquity:

魯世世相傳以歲時奉祠孔子冢，而諸儒亦講禮鄉飲、大射於孔子冢。孔子冢大一頃。故所居堂弟子內，後世因廟藏孔子衣冠琴車書，至于漢二百餘年不絕。

[The people of] Lu continued generation upon generation the tradition of holding annual and seasonal cult at Confucius’ tomb, [where] scholastics, for their part, held lectures on the rites of the District Libation and Big Shoot. Confucius’ tomb was one *qing* 頃 (6.7 hectares) in size. Later generations of his disciples converted the hall in which he formerly lived into a temple wherein were stored Confucius’ robes, caps, zither, chariot, and books. This [cult] continued uninterrupted for more than two hundred years up to the Han.⁵³

And that is what marks the Shoot as different from other ‘institutions’ described in the later Confucian Classics: the Shoot was *historical practice*, and, *historically*, this royal right wasn’t bound to a single venue or even the presence of a temporal king.⁵⁴ Nor for that matter was the Shoot even bound to a single ideology. The *Book of Han*, in fact, identifies the practice with Confucius’ principle rival:

墨家者流，蓋出於清廟之守。茅屋采椽，是以貴儉；養三老五更，是以兼愛；選士大射，是以上賢……

The Mohist school likely emerged from the guardians of the Pure Temple (to King Wen of Zhou (r. 1099/1056–1050 BCE)). They [lived in] reed huts with oak rafters, and that is how they practised ‘the veneration of poverty’. They cared for the Thrice Venerable and Fifth

⁵¹ *Han shu*, 34A.2796–2797; tr. modified from Sanft (2008, 155–158).

⁵² See *Shiji*, 121.3117; cf. *Han shu*, 88.3592. For Shusun Tong’s biography, see *Shiji*, 99.2720–2727, cf. *Han shu*, 43.2124–2131.

⁵³ *Shiji*, 47.1945. On archery and archery rites as evidenced in even earlier sources, see Adamski (2012) and Yuan Junjie (2013).

⁵⁴ As a dukedom under nominal fealty to the Eastern Zhou (771–256/249 BCE) prior to its annexation by Chu in 249 BCE and Qin in 223 BCE, Lu was beyond its rights to host ceremonies reserved for the Zhou kings except, perhaps, for the spiritual presence of the ‘plainclothes king’ (Note 2) at the site of his post-mortem cult.

Watch, and that is how they practised ‘universal love’. They selected gentleman-knights at the Big Shoot, and that is how they practised ‘the exaltation of the virtuous’. ...⁵⁵

The relative diffusion and transferability of these rites is all the more apparent as concerns the District Shoot, of course, as the District Shoot was normally a local, community-sponsored event. Some thirty years before the Luoyang Bright Hall opened its doors, for example, we read about governor of Lu Commandery in 28/30 CE ‘gathering everyone to conduct the rite of the District Shoot’ in a plot to lure local rebels into custody,⁵⁶ while next door, in 76 CE, the governor of Shanyang ‘held a District Shoot every spring and autumn’ because he believed in ‘teaching people through the rites rather than by resort to punishment and fines’.⁵⁷ On the other side of the empire, in modern Chengdu, we read about Yang Hou’s 楊厚 (72–153 CE) disciples dedicating a temple to his memory, where ‘the commandery instructor and clerk held a District Shoot in the spring and autumn to offer him eternal cult’.⁵⁸ And after Luoyang was sacked in 189 CE, and the Big Shoot subsequently abandoned for several centuries, one continues to see District Shoot pop up in places as far apart as Wu, on the verdant Southeast Seaboard, and Dunhuang, in the lifeless sands leading out into Central Asia.⁵⁹

Zheng Xuan too was a man of Lu, but whatever his presumed exposure to contemporary practice it does not seem to have informed his treatment of Classical target archery. True, it is unclear from our sources how the Big and District Shoot as practiced c.406 BCE–200 CE may have varied from the Ritual Classics, let alone from one time and from one place to another. Pictorial evidence from Eastern Zhou (771–256/249 BCE) bronzeware does confirm the use of targets of roughly the same shape as those described in the Ritual Classics, of course, but the scale appears a little more reasonable than that in fig. 2 and so too does the apparent use of some sort of backstop (fig. 4).⁶⁰ That said, no real-world measurer, furrier, woodworker, or fabric-and-chariot officer could possibly realise Zheng Xuan’s specifications – not to a precision of $\frac{1}{3}$ *cun*, not without one target blocking the oth-

⁵⁵ *Han shu*, 30.1738. Note that ‘the veneration of poverty’ 貴儉, ‘universal love’ 兼愛, ‘the exaltation of the virtuous’ 尚賢, etc., are the chapter titles and central tenants of the *Mohist Canon*. On *jia* 家 ‘families’ and/as ‘schools’ as read onto pre-Qin philosophy by later sources like the *Book of Han*, see Csikszentmihalyi and Nylan (2003) and Brashier (2011, 6–34).

⁵⁶ 會人眾，修鄉射之禮，*Hou Han shu*, 29.1019.

⁵⁷ 以禮訓人，不任刑罰……每春秋饗射，*Hou Han shu*, 76.2467.

⁵⁸ 門人為立廟，郡文學掾史春秋饗射常祠之，*Hou Han shu*, 30A.1050.

⁵⁹ See *Song shu*, 57.1583, and *Jin shu*, 94.2449, 109.2826.

⁶⁰ ‘Proficiency Ordinance no 45’ 功令第卅五 of the Han code prescribes the use of *zhun* 埠 ‘embankments’ or ‘butts’ in the annual archery proficiency test for military officers, for which see Loewe (1967, vol. 1, p. 118, vol. 2, pp. 50–60, and Li Junming (2009, 209–210). I thank Thies Staack for bringing these sources to my attention. Later versions of the Shoot likewise mention the use of *lie* 埒 ‘embankments’, e.g. *Sui shu*, 8.165–166. For a speculative reconstruction of the backstop featured in fig. 4, see Selby (2006, 110–111).

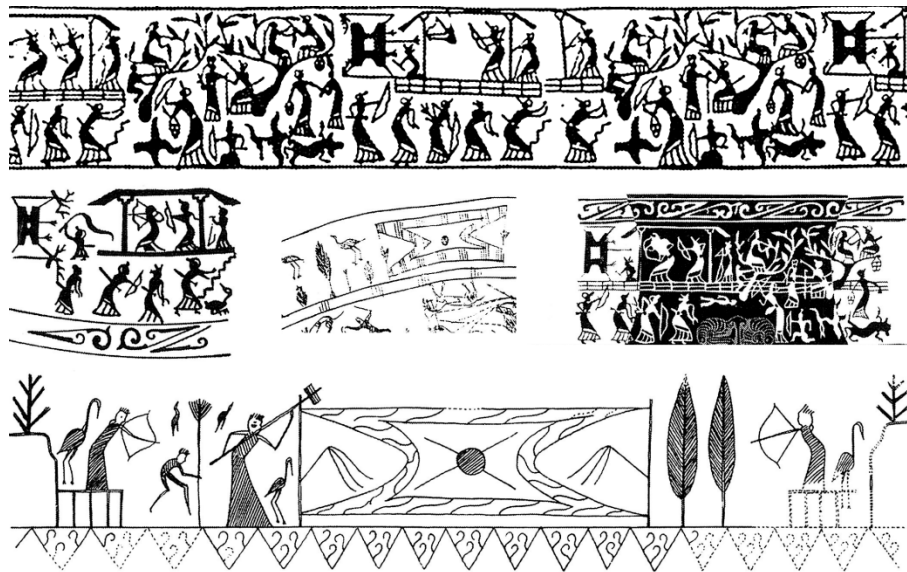


Fig. 4 Archery targets as illustrated in Eastern Zhou (771–256/249 BCE) bronzeware. Source: Hsu Ya-hwei (2002, figs. 9a, 13, 17, 40g, 43b).

er’s swan, and not without wind posing a serious challenge to their stability. If Zheng Xuan’s numbers are physically impossible, then they clearly aren’t the product of ‘visual inspection’, but neither, if we recall, are they anywhere to be found in the Classics.

Zheng Xuan’s numbers are his own creation. All the Classics give us are here vague, anthropomorphic, and object-specific measures like ‘bows’, ‘arm-spans’, ‘raccoon-dog paces’, and ‘militaries’ – measures linked by such vaguely-worded ratios as ‘take bow two *cun* for the target centre’ and ‘dividing its breadth in three, the swan occupies one thereof’. These instructions are clearly linked to practice. It is not a coincidence, one imagines, that it is the rope that is measured in arm-spans and that it is distances on (and a little off) the ground that are measured with one’s feet. Such units are contingent, variable, imprecise, and incommutable, but they tend to suffice for the purposes of shooting: one still counted *paces*, for example, in nineteenth-century pistol duels, rather than the two sides carrying tape measures. Zheng Xuan, however, would clearly prefer that everyone used a tape measure, because he converts *every measure* into commensurable units of *zhang*, *chi*, and *cun*. He does this so that he can operate upon them – so that he can transform practical ‘rules of thumb’ into an abstract geometrical problem. To understand how his commentary arrives at the numbers it does, therefore, one must set aside ancient Classics and contemporary practice to focus Zheng Xuan et al.’s relationship with geometry.

The imperative of maths

The precise height at which the mid- and long-range targets must be suspended for the lower edges of their respective swans to align perfectly with the upper edge of the preceding target is not a question

with which the Ritual Classics think to bother. And why would they? In practice, the World Archery Federation allows for a slop of ± 5 cm (≈ 2 *cun*) in height for a $\varnothing 122$ cm ($\approx \varnothing 53$ *cun*) target at 70 m (≈ 50 racoon-dog paces), because, in practice, nothing is *perfect*.⁶¹ And in setting up the Big Shoot, one imagines, it would simply suffice to shout ‘a little bit higher’ to the men downfield until the mid- and long-range targets fell into proper visual alignment from the stand. Still, Zheng Xuan wants to know *how high*, to the precision of $\frac{1}{3}$ *cun* (7.7 mm), and his subcommentators want us to know how Zheng Xuan knows. At that, we have stepped with Zheng Xuan beyond the wind-blown world of the here-and-now and into a higher plane. This other world, whither we have been fittingly conveyed in the name of ritual, is one in which perfection alone may be assumed – a world of forms outside of time around which the dimensions of space may be losslessly expanded and collapsed. This other world is one in which animals render themselves to customs posts to discuss and pay taxes on the respective value of their furs;⁶² it is one in which a district may conscript a gang of $135 \frac{11637}{12175}$ corvée labourers,⁶³ and it is one in which rope can just as easily be measured in feet, and a walking distance in hands. This – the Alice-in-Wonderland world of *suan* 算 mathematics – is the *only world* in which an entire dimension of space can go accidentally missing from a shooting range.⁶⁴

This, Zheng Xuan’s folly, is by no means due to the limitations of contemporary knowledge. Indeed, problem 9.22 of the *Nine Chapters of Mathematical Procedures* gives us a procedure eminently suited to this category of problem:

今有山居木西，不知其高。山去木五十三里，木高九丈五尺。人立木東三里，望木末適與山峰斜平。人目高七尺。問山高幾何？

Suppose there is, to the west of a tree, a mountain of unknown height, that the distance from the tree to the mountain is fifty-three *li* 里, that the height of the tree is nine *zhang* five *chi*, and that a person standing three *li* east of the tree sees the tip of the tree and peak of the mountain as exactly level on the same oblique. If the height of the person’s eye is seven *chi*, we ask what is the height, then, of the mountain?

答曰：一百六十四丈九尺六寸、太半寸。

⁶¹ World Archery Federation interactive rulebook, book 2, p. 31 (<https://worldarchery.org/Rules>; accessed 26 June 2017).

⁶² ‘A fox, a racoon dog, and a dog pass through a customs post; they are taxed a hundred and eleven cash. The dog says to the racoon dog, and the racoon dog says to the fox, “Your skin is worth twice mine; you should pay twice as much tax!”’ 狐、狸、犬出關，租百一十一錢。犬謂狸、（狸）謂狐：「而（爾）皮倍我，出租當倍我（哉）！」, *Suan shu shu* 算數書 (MS, *terminus ante quem* c.186 BCE), slip 34, in *Zhangjiashan Han mu zhujian (ersiqi hao mu)*; tr. modified from Cullen (2004, 45).

⁶³ See problem 3.5 of the *Nine Chapters of Mathematical Procedures* in Chemla and Guo (2004, 288–290).

⁶⁴ On recognising abstraction, generality, and philosophical reflections in a mathematical tradition often labelled ‘practical’ (in contrast to its ‘theoretical’ Western counterpart), see Chemla (2003; 2010b) and Note 30.

Answer: one hundred and sixty-four *zhang* nine *chi* six *cun* and a big-half *cun* ($1649\frac{2}{3}$ *chi*).

術曰：置木高減人目高七尺，餘，以乘五十三里為實。以人去木三里為法。實如法而一，所得，加木高，即山高。

Procedure: set down the height of the tree and diminish it by the seven *chi* of the height of the person's eye ($95 - 7 = 88$). Take what remains and multiply it by the fifty-three *li* to make the dividend ($88 \times 53 = 4664$). Take the three-*li* distance from person to tree as the divisor. Divide the dividend by the divisor, and what you get is added to the height of the tree to give you the height of the mountain ($4664 \div 3 + 95 = 1649\frac{2}{3}$ *chi*).⁶⁵

This too is describing a world wholly apart from that of physical reality – a world of infinite precision at incredible distances, where a three-*li* stretch of mountain road is perfectly flat and the height of a mountain depends on the height of the observer.⁶⁶ Indeed, it could just as well be a seven-*chi* mountain looking over the tree at a $1649\frac{2}{3}$ *chi* man, for all the realism demanded of such a text, and either could just as easily be swapped for the ‘tongues’ and ‘swans’ of windless sails as we have done above in fig. 3.

Neither can we attribute Zheng Xuan's folly to his ignorance of mathematics, nor even of the *Nine Chapters* in particular. Consider the order of subjects he learned in his youth as presented in the opening of his *Book of Later Han* biography (numbers added by the author):

鄭玄字康成，北海高密人也。八世祖崇，哀帝時尚書僕射。玄少為鄉嗇夫，得休歸，常詣學官，不樂為吏，父數怒之，不能禁。遂造太學受業，師事京兆第五元，先始通『京氏易』、『公羊春秋』、『三統歷』、『九章算術』。又從東郡張恭祖受『周官』、『禮記』、『左氏春秋』、『韓詩』、古文『尚書』。以山東無足問者，乃西入關，因涿郡盧植，事扶風馬融。

Zheng Xuan, styled Kangcheng 康成, was a man of Gaomi in Beihai [Commandery]. His eighth-generation ancestor Chong 崇 was vice director of the Imperial Secretariat in the time of Emperor Ai (r. 6–1 BCE). Xuan served as a district manager in his youth, but he obtained a discharge to return [home at 11 or 12 years of age]. He paid frequent visit to instructional officers, taking no pleasure from serving office himself, and [though] his father reprimanded him numerous times, he was unable to prohibit [his son from doing as he desired]. [Zheng Xuan] later went to the Imperial Academy to receive his patrimony, serving Diwu Yuan 第五元 of the Capital as his disciple, [under whom] he first mastered (1) *Mr Jing [Fang's Book of] Changes*, (2) the *Gongyang Tradition of the Spring and Autumn Annals*, (3) the *Triple Concordance li*, and (4) the *Nine Chapters of Mathematical Procedures*. He then became a follower of Zhang Gongzu 張恭祖 of Dong Commandery, [from whom] he received (5) the *Offices of Zhou*,

⁶⁵ Tr. modified from Chemla and Guo (2004, 740–741).

⁶⁶ The fact that the height of the observer's eye is given last – after his/her predetermined distance from the tree – is, to this author, reminiscent of the famous procedure in *Gnomon of Zhou* whereby the height of heaven depends on the length of the gnomon used for observation. See Cullen (1996, esp. 78–82).

(6) the *Record of Rites*, (7) the *Zuo Tradition of the Spring and Autumn Annals*, (8) *Han's Odes*, and (9) the ancient-text *Book of Documents*.⁶⁷ As there were not enough [masters] to question East of the Mountains, he therefore entered the Passes to follow Lu Zhi 盧植 (d. 192 CE) of Zhuo Commandery and serve Ma Rong of Fufeng.⁶⁸

It was normal for an educated man in this day to learn mathematics, so it is hardly surprising to find the *Nine Chapters* on his CV.⁶⁹ One finds it also on that of Ma Rong's elder brother:

續字季則，七歲能通『論語』，十三明『尚書』，十六治『詩』，博觀羣籍，善『九章算術』。

[Ma] Xu 馬續 (fl. 111–41 CE), styled Jize 季則, was able to master the *Analecets* at seven years of age; he understood the *Book of Documents* by thirteen and was studying the *Odes* by sixteen. He was broadly read (lit. 'observed') in a diversity of works, and he was good at the *Nine Chapters of Mathematical Procedures*.⁷⁰

Returning to Zheng Xuan, we read that it is this very subject that would in fact link his destiny with that of the Brothers Ma (numbering continued from above):

融門徒四百餘人，升堂進者五十餘生。融素驕貴，玄在門下，三年不得見，乃使高業弟子傳授於玄。玄日夜尋誦，未嘗怠倦。會融集諸生考論圖緯，聞玄善算，乃召見於樓上，玄因從質諸疑義，問畢辭歸。融喟然謂門人曰：「鄭生今去，吾道東矣。」

[Ma] Rong's disciples numbered more than four-hundred, of which only some fifty [were permitted to] ascend the hall and approach [the master]. [Ma] Rong was proud and elitist of nature, and [so Zheng] spent three years beneath his gate without seeing him before [Ma Rong] dispatched an advanced disciple to transmit [his teachings] to [Zheng] Xuan. [Zheng] Xuan researched and recited night and day, [succumbing] never once to idleness or exhaustion. One day [Ma] Rong gathered his various students to study and discuss (10) the charts and wefts (i.e. the apocrypha),⁷¹ and having heard that [Zheng] Xuan was good at maths (*shan suan* 善算) he thereupon summoned him to appear upstairs. [Zheng] Xuan obeyed, testifying to them on doubtful points [of interpretation] and, when the interrogation was finished, politely announced his return [home]. [Ma] Rong sighed, saying to his disciples, 'With Student Zheng's departure today my *dao* [now travels] east'.⁷²

⁶⁷ On the 'old-' and 'new-text' Classics, see Dull (1966) and Nylan (1994).

⁶⁸ *Hou Han shu*, 35.1207. Note that insertions are as per Li Xian's 李賢 (654–84 CE) commentary in *ibid.* and the rather exhaustive study of Zheng Xuan's life and times in Wang Liqi (1983).

⁶⁹ On the prevalence of mathematical training among Han intellectuals, see Cullen (2009). On polymathy, more broadly, see Goodman (2005; 2010).

⁷⁰ *Hou Han shu*, 24.862.

⁷¹ On 'chart and weft' apocrypha, see Dull (1966) and Lü Zongli (2003). On Zheng Xuan's later commentaries to the apocrypha, see Lü Kai (2011).

⁷² *Hou Han shu*, 35.1207.

No	Text annotated	Rolls	
		ZX	MR
經 CLASSICS			
1	周易 <i>Book of Changes</i>	9	10
2	尚書 <i>Book of Documents</i>	9	10
3	尚書 <i>Book of Documents Pronunciations</i>	1	
4	尚書釋問 <i>Explanations and Questions on the Documents</i>	4	
5	尚書大傳 <i>Great Comm. to the Book of Documents</i>	3	
6	毛詩 <i>Mao's Odes</i>	20	10
7	毛詩音 <i>Mao's Odes Pronunciations</i>	15	
8	毛詩譜 <i>Mao's Odes Tables</i>	2	
9	周官禮 <i>Offices & Rites of Zhou</i>	12	12
10	周官音 <i>Offices of Zhou Pronunciations</i>	1	
11	儀禮 <i>Book of Etiquette & Ceremonial</i>	17	
12	儀禮音 <i>Book of Etiquette & Ceremonial Pronunciations</i>	2	2
13	喪服經傳 <i>Mourning Garments, Classic and Commentary</i>	1	1
14	喪服變除 <i>Mourning Garments, Transformation and Removal</i>	1	
15	喪服譜 <i>Mourning Garment Tables</i>	1	
16	禮記 <i>Record of Rites</i>	20	
17	禮記 <i>Record of Rites Pronunciations</i>	1	
18	三禮圖 <i>Diagrams of the Three Ritual [Classics]</i>	9	
19	春秋左傳 <i>Mr Zuo's Spring and Autumn Annals</i>	?	
20	春秋左傳 <i>Mr Zuo's Spring and Autumn Annals Pronunciations</i>	?	
21	春秋左氏膏肓 <i>Mr Zuo's SAA, Vital Organs</i>	10	
22	發公羊墨守 <i>Exposing Mohist Defenses in Gongyang</i>	2	
23	釋穀梁廢疾 <i>Explaining Arrogations and Illnesses in Guliang</i>	3	
24	孝經 <i>Classic of Filial Piety*</i>	1	2
25	論語 <i>Analects</i>	10	?
26	論語音 <i>Analects Pronunciations</i>	?	
27	孟子 <i>Mencius</i>	7	
緯 WEFT-TEXTS			
28	易緯 <i>Changes Weft(s)</i>	8	
29	尚書緯 <i>Documents Weft(s)</i>	6	
30	尚書中候 <i>Book of Documents On-target [Omen] Observations</i>	5	
31	禮緯 <i>Rites Weft(s)</i>	3	
32	禮記默房 <i>Record of Rites Silent Chamber</i>	3	
33	河圖洛書 <i>Yellow River Diagram and Luo River Book</i>	24	
天文曆算 ASTRAL AND MATHEMATICAL SCIENCES			
34	乾象曆 <i>Supernal Icon li</i>	?	
35	日月交會圖 <i>Lunar & Solar Crossing Diagrams</i>	1	
刑法職官 LAW AND ADMINISTRATION			
36	漢律 <i>Han Regulations</i>	?	
37	漢官香方 <i>Han Offices Incense Workshop</i>	?	
五行 FIVE AGENTS DIVINATION			
38	九宮經 <i>Classic of the Nine Palaces</i>	3	
39	九宮行碁經 <i>Classic of the Nine Palace Moving Chess Pieces</i>	3	

Table 1 Zheng Xuan's principal commentarial writings (*zhu* 注, *jian* 箋, *shi* 釋, *zhen* 箴) as gathered in Wang Liqi (1983, 327–368). Note that this list excludes 'authored' monographs (*zhuan* 撰, *zhu* 著) and that the number of rolls (*juan* 卷) is given as per the earliest source. Grey indicates the commentaries selected for canonisation in the seventh century CE. The 'Rolls' roll is divided into Ma Rong (MR) and Zheng Xuan (ZX) to indicate where and to what degree the latter's output overlaps with his master's, Ma Rong's titles and roll-numbers being gathered principally from the bibliographic monographs of the *Book of Sui* and *Old Book of Tang*.

Maths played a critical role in Zheng Xuan's training, networking, and ultimate success as a Classical exegete. Stepping back, one notes that they also happen to come *first* in his CV: at the Imperial Academy, at least, Zheng Xuan studied (4) mathematics immediately prior to the (5) *Rites of Zhou* and (6) *Record of Rites*; and so too did he study Liu Xin's work on mathematical astronomy – the (3) Triple Concordance *li* – prior to the (7) *Zuo Tradition* of the *Spring and Autumn Annals*, whose chronology it was written to explain.⁷³

Not surprisingly, Zheng Xuan's subsequent commentarial activities would be clustered around the same things he studied in his youth: Classics, weft-texts, and the astral and mathematical sciences (Table 1). This is not surprising, because commentary went hand in hand with education. Commentary, as written by a 'master' (*shi* 師), served as the textual embodiment and transmission vector of an academic 'family' (*jia* 家) – its 'master's method' (*shi fa* 師法), or 'family method' (*jia fa* 家法). Commentary, as in later times, also served as the basis of *test norms*. In a precursor to the civil service exam, Emperor Wu of Han (r. 140–87 BCE) established a 'test' (*shi* 試) at the Imperial Academy for 'classifying' (*ke* 科) recommendees on the basis of their knowledge of the Classics. The test – a game of 'shoot-strips' (*shece* 射策) – involved 'making bamboo-strip with challenging questions, arranging them atop a desk, and having examinees randomly throw-shoot (cast lots) to select and answer [questions]'.⁷⁴ There was a lot at stake in this glorified parlour game, and reading Xu Fang's 徐防 (fl. 75–107 CE) policy paper of 103/104 CE one gets an idea of just how important commentaries and 'family methods' were to preparing for and refereeing the event:

臣聞詩書禮樂，定自孔子；發明章句，始於子夏。其後諸家分析，各有異說。漢承亂秦，經典廢絕，本文略存，或無章句。收拾缺遺，建立明經，博徵儒術，開置太學。孔聖既遠，微旨將絕，故立博士十有四家，設甲乙之科，以勉勸學者，所以示人好惡，改敝就善者也。

Your humble servant has heard that the [Classics of] *Odes* and *Documents* and *Rites* and *Music* were fixed from the time of Master Kong, and that the revelation and illumination of their chapter and verse (*zhangju*) began with [his disciple] Zixia 子夏; thereafter, the various 'families' split apart, each possessed of different sayings. The Han inherited the mess of Qin, and the Classics and canons [of state] were abandoned and lost; their original text survived in part, some devoid of chapter and verse (*zhangju*) [commentary]. We collected and tidied what was defective and lost, establishing and erecting the Classics Competency [Exam] (*mingjing* 明經), and we broadly recruited scho-

⁷³ On Liu Xin's archaeoastronomical approach to the *Zuo Tradition* through the Triple Concordance *li* and appended *Canon of Ages* (*Shijing* 世經), see Cullen (2001).

⁷⁴ 作簡策難問，列置案上，任試者意投射取而答之，*Hou Han shu*, 6.260 (comm.). For more on 'shoot-strips' and the abstraction of archery as a model of empirical testing in this period, see Levinovitz and Morgan (forthcoming). On the Imperial Academy and civil service recruitment in the Han, see Bielenstein (1980), 132–42.

lastics and technicians for the opening and institution of the Imperial Academy. Sage [Master] Kong is already far away, and his subtle directives on the verge of dissolution, and thus did we establish ten-and-four academicians (*boshi*) and introduce their A and B classification (*ke*) to encourage and reward scholastics – that by which to show people good and bad and correct the faults of those wont to go to goodness.

伏見太學試博士弟子，皆以意說，不修家法，私相容隱，開生姦路。每有策試，輒興諍訟，論議紛錯，互相是非……臣以為博士及甲乙策試，宜從其家章句，開五十難以試之。解釋多者為上第，引文明者為高說；若不依先師，義有相伐，皆正以為非。五經各取上第六人，論語不宜射策。雖所失或久，差可矯革。」

Prostrate [before You, I] see that the Imperial Academy's testing (*shi*) of the academicians' disciples is [now conducted] solely by wayward rhetoric, cultivated not of family models (*jiafa*); by forgiveness of sensibilities by private accord, they open the road for students' jealousies. Every time there is a strip-exam, it immediately provokes argument and accusation; discourse and opinions entangle and divide, calling right and wrong on one another. ... Your humble servant that the academicians as well as the A and B [class] strip-exam had best follow their family (*jia*) of chapter and verse (*zhangju*) [commentary] and open fifty challeng[ing questions] by which to test them. Those who amply analyse and explain should be made 'upper-rank', and those who clearly cite the texts should be made 'advanced exegetes'; any [answer] that does not rely on a former master, [and/or] that militates against itself in its meaning, [should] in all cases be considered wrong [even] if it is correct. For each the Five Classics one [should] take six people from the upper-rank, [while/but] the *Analects* is inappropriate for shoot-strips. It may be that some of [these] missteps are persistent, but they are nonetheless amenable to prideful reform.⁷⁵

Zheng Xuan would never pursue a career in the civil service. Instead, he retired early, so to speak, to become a private school master after the model of Ma Rong.⁷⁶ He may not have played 'shoot-strips', therefore, but he was still very much in the business of transmitting 'family methods'. As illustrated in Table 1, Zheng Xuan's commen-

⁷⁵ *Hou Han shu*, 44.1500–1501.

⁷⁶ Note that Ma Rong was equally hesitant to pursue a career in the civil service in his youth, having devoted himself to *youxue* 遊學 'wandering studies' until the age of thirty, when he ran into a war zone to escape from a job offer made by Empress Dowager Deng Sui's 鄧綏 (b. 81; r. 105–121 CE) older brother. After suffering hunger and exhaustion behind Qiang 羌 lines, he finally caved in, accepting an appointment to the Eastern Observatory (*Dongguan* 東觀) upon its founding as an academic institution in 110 CE. Formerly a sort of palace archives, the Eastern Observatory was transformed by the Empress Dowager into a think-tank of eminent scholars whose task was to 'check and collate the commentary/traditions and records' 讎校傳記 and 'instruct the palace women' 教授宮人 (*Hou Han shu*, 10A.424). With the Imperial Academy having suffered a long decline in elite men's education under competition from the Circular Moat, the Eastern Observatory quickly became the empire's preeminent centre for textual scholarship, and it was there, as a disciple of Ban Gu's sister, Zhao 昭 (44/49–118/121 CE), that Ma Rong made his name as a commentator and served most of his career in higher education before retiring 'with-in the Passes' to open his own school. For Ma Rong's biography, see *Hou Han shu*, 60A.1953–1973. On the Eastern Observatory, see Goodman (2005).

tarial output is not only clustered around those areas that he studied in his youth, it also overlaps with that of his master. Both produced a twelve-roll commentary to the *Offices* or *Rites of Zhou*, for example, and the *Book of Later Han* gives us a glimpse of how textual production articulates with the process of transmission in this particular case:

中興，鄭眾傳『周官』經，後馬融作『周官』傳，授鄭玄，玄作『周官』注。

During the Revival (25–57 CE), Zheng Zhong transmitted (*chuan*) the *Offices of Zhou* Classic. Later, Ma Rong made the *Offices of Zhou* commentary-tradition (*zhuan*), which he bestowed upon Zheng Xuan, and Xuan made the *Offices of Zhou* annotations (*zhu*).⁷⁷

One sees a similar pattern in the way that Zheng Xuan, later in life, learned the Supernal Icon *li* 乾象曆 (written c.178–206 CE) from its author, then Governor of Shanyang, Liu Hong 劉洪 (fl. 167–206 CE). Having already manifested an interest in mathematical astronomy, and happening to pass by Liu’s Commandery seat in 196 CE,

鄭玄受其法，以為窮幽極微，又加注釋焉。

Zheng Xuan received [Liu Hong’s] method (*fa*), and considering it to plumb all that is recondite and crown all that is subtle, he further added annotations (*zhu*) and explanations (*shi*) thereto.⁷⁸

Clearly, one wrote commentaries in part to bestow (*shou* 授) one’s teachings, but one wrote them too in the process of reception (*shou* 受). One wrote them to ancient Classics, and one wrote them also to cutting-edge astronomical procedure texts. Commentary-writing was simply a part of learning in Zheng Xuan’s world, and it’s not particularly strange that some of his annotations to the *Rites of Zhou* look like those we might expect him have added to the tables and algorithms of the Supernal Icon *li* – the two commentaries were written by the same person, after all.

Given what we know of his background, we know that Zheng Xuan could have easily arrived at the ‘right answer’ for the appropriate height of the Big Shoot targets, in fig. 3, and the fact that he did not probably points to a lapse of memory. If he could understand the Supernal Icon *li*, he could definitely understand problem 9.22 of the *Nine Chapters of Mathematical Procedures*. Mistakes happen; what is curious about this one is how it was received – how it became part of the ‘Correct Meaning’ of the Ritual Classics in the seventh century CE. It is curious because his subcommentators were, if anything, better steeped in mathematics than was Zheng Xuan.

I say this, in part, because the field of mathematics itself would see considerable innovation and expansion over the intervening centuries. Indeed, of the *Ten Mathematical Classics*, canonised by Li

⁷⁷ *Hou Han shu*, 79B.2557.

⁷⁸ *Jin shu*, 17.498. For more on this event and the transmission of Liu Hong’s work in the second and third century CE, see Chen (1986) and Morgan (2015). Sadly, Zheng Xuan’s commentary to the Supernal Icon *li* is lost, but for an idea of what he was commentating, see the complete English translation in Cullen (2017, 235–355).

Chunfeng 李淳風 (602–670 CE) et al. in 656 CE, seven date to the period of disunion, between Han and Tang, as do three of the commentaries included therein.⁷⁹ As concerns this specific category of problem, Liu Hui 劉徽 would not only expand upon the theoretical dimensions of procedures like those in problem 9.22 (above) in his 263 CE *zhu* ‘annotation’ of the *Nine Chapters*, he would go on, exceptionally, to dedicate an entire monograph to the subject: the *Sea Island Mathematical Classic*.⁸⁰ So too, lastly, would scholastics develop the mathematical exegesis of the Confucian canon into its very own genre by the end of this period, the most noteworthy examples of which being Xindu Fang’s 信都芳 (d. 543/550 CE) *Ancestry of the Five Classics* and Zhen Luan’s 甄鸞 (fl. 535–570 CE) *Mathematical Procedures for the Five Classics*.⁸¹ All of these were extant in Kong Yingda and Jia Gongyan’s day. All but the *Ancestry of the Five Classics*, moreover, were being subcommentated in the parallel canonisation project lead by Li Chunfeng within the same few years.⁸² Whatever their training, therefore, the resources were clearly there for Kong Yingda and Jia Gongyan to realise Zheng Xuan’s mistake.

As to their training, it is difficult to say much about Jia Gongyan without his having left us a biography. Given the way that he handles things like square root extraction in his subcommentaries, however, it is clear that this isn’t his first rodeo.⁸³ Kong Yingda is a different matter. Kong’s biography tells us that he was good at maths (below), recounting how in c.633 CE he ‘joined various scholars for a debate on mathematical astronomy and the Bright Hall, and everyone ended up following Yingda’s exposition’.⁸⁴ At another such debate in 640 CE our canonisers crossed paths, Kong ‘requesting that [the court] follow [Li] Chunfeng’ as concerned the latter’s proposal to replace calendro-astronomical policy of the time with his own.⁸⁵ It is not particularly surprising to see Kong Yingda throw his political and academic weight behind Li Chunfeng at this occasion. Both were in the same business, so to speak. Li would go on to work with certain of Kong’s collaborators on other projects.⁸⁶ The two, moreover, shared a special bond over a common figure: Liu Zhuo 劉焯 (544–610 CE).

⁷⁹ On the history of mathematics from the Three Kingdoms (220–280 CE) to Sui (581–618 CE), see Wu Wenjun (1998–2004, vols. 3–4).

⁸⁰ For Liu Hui’s commentary to this category of problem, see Chemla and Guo (2004, 661–745). On the *Sea Island Mathematical Classic* (*Haidao suanjing* 海島算經), see Swetz (1992).

⁸¹ On the *Ancestry of the Five Classics* (*Wujing zong* 五經宗) and *Mathematical Procedures of the Five Classics* (*Wujing suanshu* 五經算術), see Wu Wenjun (1998–2004, vol. 4, pp. 165–172). For a comparison of the latter with the sort of mathematics witnessed in Kong Yingda and Jia Gongyan’s sum-commentaries to the same Classics, see Zhu Yiwen (2016b). For Xindu Fang and Zhen Luan’s biographies, see the relevant entries in Martin and Chaussende (forthcoming).

⁸² On Li Chunfeng and the broader context in which the *Ten Mathematical Classics* came together, see Siu and Volkov (1999) and Goodman (forthcoming).

⁸³ See Zhu Yiwen (2016b).

⁸⁴ 與諸儒議曆及明堂，皆從穎達之說，*Jiu Tang shu*, 73. 2602.

⁸⁵ 請從淳風，*Xin Tang shu*, 25.536.

⁸⁶ In the 640s, for example, Li Chunfeng would be invited aboard the literary compilation *Wensi boyao* 文思博要 alongside Kong Yingda’s collaborator on the *Cor-*

Liu Zhuo was an independent scholar primarily active under the Sui (581–618 CE) who was forced into the life of a provincial school master after a fruitless, decade-long struggle for recognition in court astronomy and tonometry.⁸⁷ Li Chunfeng never got to meet Liu Zhuo, but Liu Zhuo was nonetheless a major presence in his life. In 665 CE, when Li Chunfeng’s astronomical work *was* finally admitted into the annals of public policy, it was with the Unicorn Virtue *li* 麟德曆, which ‘followed Liu Zhuo’s Sovereign Pole *li* 皇極曆 with minor modifications as appropriate’.⁸⁸ Stymied by factional politics within the Sui astronomical office, Liu Zhuo himself was denied the opportunity in life to see his ground-breaking *li* enter government service. In 665 CE, Li Chunfeng vindicated his work, and that this was his goal is evident in Li Chunfeng’s earlier history-writing. Invited to write a chapter on tonometry and mathematical astronomy for a history of the ‘Five Dynasties’ (502–618 CE) in the 640s, Li would violate the rules of the *zhi* 志 genre to make his a history foremost of the underdog, preserving Liu Zhuo’s procedure text in a spot normally reserved for ‘winners’ and foregrounding his tribulations at court. Later scholars were divided about this intensely personal subversion of what was a chapter of state history;⁸⁹ if there was anyone who could appreciate his passion for Liu Zhuo, however, it was probably Kong Yingda. Liu Zhuo was Kong Yingda’s teacher.

孔穎達字沖遠，冀州衡水人也……穎達八歲就學，日誦千餘言。及長，尤明『左氏傳』、鄭氏『尚書』、王氏『易』、『毛詩』、『禮記』，兼善算曆，解屬文。同郡劉焯名重海內，穎達造其門，焯初不之禮，穎達請質疑滯，多出其意表，焯改容敬之。穎達固辭歸，焯固留，不可。還家，以教授為務。隋大業初，舉明經高第，授河內郡博士。

Kong Yingda, styled Chongyuan 沖遠, was a man of Hengshui, in Jizhou Commandery. ... Yingda went to school at eight years of age, making a recitation of over a thousand words every day. By the time he was grown, he had become particularly conversant in the *Zuo Tradition*, Mr Zheng [Xuan’s] *Book of Documents*, Mr Wang [Bi’s] *Changes*, *Mao’s Odes*, and the *Record of Rites*; he was equally good at both mathematics and mathematical astronomy, explaining and composing text. Liu Zhuo, of his same commandery, had a name that carried weight [all across the land] within the seas, and Yingda delivered himself to his gates (i.e., as a ‘below-the-gates’ disciple). Zhuo did not at first treat him with due ritual propriety, but Yingda asked good and stubborn questions, greatly surpassing his expectations, so Zhuo changed his countenance and [began] to respect him. Yingda long/firmly bid his leave, and Zhuo long/firmly insisted that he must stay. [Eventually, Kong Yingda] returned home to take up the profession of teaching. At the beginning of the Sui Grand Patrimony reign (605–617 CE), [in his early thirties], he tested into the upper ranks of

rect Meaning of Changes of Zhou and Spring and Autumn Annals, Ma Jiayun 馬嘉運 (d. 645). See *Jiu Tang shu*, 73.2603, 79.2718.

⁸⁷ For Liu Zhuo’s biography, see *Sui shu*, 75.1718–1719, *Bei shi*, 82.2762–2763, cf. the relevant entries in Martin and Chaussende (forthcoming).

⁸⁸ 餘因劉焯皇極曆法，增損所宜，*Xin Tang shu*, 26.559.

⁸⁹ See Li Liang (forthcoming).

the Classics Competency [exam] as was named an academician of Henei Commandery.⁹⁰

In astronomy, Liu Zhuo was ahead of his time; the ‘name that carried’ had more to do with the local polytechnic school he opened with his childhood friend, Liu Xuan 劉炫 (c.546–c.613 CE), and their respective contributions to Classical exegesis. It was this school that launched Kong Yingda down the path to a career in the Academy, and twenty-six years later, as its chancellor, tasked with deciding The Correct Meaning of the Classics for the national curriculum (and generations to come), Kong turned first and foremost to his ‘family method’. In his preface to the *Correct Meaning of the Book of Documents*, for example, Kong foregrounds his teachers’ respective commentaries thereto, declaring that ‘Liu Zhuo and Liu Xuan’s are truly the most detailed and elegant’ of recent times.⁹¹ He means these adjectives respectively – ‘Xuan, begrudging Zhuo’s prolixity, set himself to cutting things out of [his commentary]’,⁹² but Xuan’s, in turn, was too elegantly succinct – and Kong Yingda frames his project around finding a middle ground between the two. Interestingly, one of his complaints about his late master is...

若其言必託數，經悉對文，斯乃鼓怒浪於平流，震驚飆於靜樹，使教者煩而多惑，學者勞而少功，過猶不及，良為此也。

... the way, for example, that [Liu Zhuo’s] words necessarily fall back on numbers to which the text of the Classic is [invoked] only by way of complement – this is really beating a level current into a tidal wave, or jolting a quiescent tree into a whirlwind; it leaves the teacher vexed and full of doubts, and the student toiling with little reward, going too far while at the same time falling short. He was really good at that.⁹³

Pushed to become the Classicist he was by a visionary astronomer that saw the *Book of Documents* as a book of numbers, Kong Yingda was, in his own way, adverse to maths: maths were critical, but he wanted Classical commentary to be more.

⁹⁰ *Jiu Tang shu*, 73.2601; cf. *Xin Tang shu*, 198.5643. One finds another anecdote involving Liu Zhuo playing the role of ‘master’ (*shi*) or ‘headmaster’ (*shishou* 師首) at a sort of conference panel featuring Kong Yingda, Gai Wenda 蓋文達 (d. 644 CE), and his own teacher, Liu Guisi 劉軌思, in *Jiu Tang shu*, 189A.4951, and *Xin Tang shu*, 198.5651.

⁹¹ 惟劉焯、劉炫最為詳雅, *Shangshu zhusu*, 0.3b. Now lost, the *Old Book of Han* bibliographic monograph records a ‘*Book of Documents, Meaning and Subcommentary* in twenty rolls, authored by Liu Zhuo’ 『尚書義疏』二十卷, 劉焯撰 and a ‘*Book of Documents, Narrating the Meaning* in twenty rolls, authored by Liu Xuan’ 『尚書述義』二十卷, 劉炫撰 in *Jiu Tang shu*, 46.1970.

⁹² 炫嫌焯之煩雜, 就而刪焉, *Shangshu zhusu*, 0.4a.

⁹³ Ibid. Note that Kong Yingda’s prefaces to the *Correct Meaning of Mao’s Odes* and the [*Zuo Tradition of the*] *Spring and Autumn Annals* make similar reference to the two, his subcommentary to the later frequently citing Liu Xuan.

Conclusion

To recapitulate the findings of our investigation, we can say for certain that, though ‘one would rather say that the [Duke of] Zhou and [Master] Kong were mistaken’, Zheng Xuan *was* wrong about something: the feasible heights and dimensions of the targets used in the Classical archery rite of the ‘Big Shoot’. His numbers are *physically impossible*, and to understand why he would project them on the ancient kings of Zhou, we went through his exegesis of the relevant passages of the Ritual Classics to diagnose the problem. The problem, we learned, is that Zheng Xuan has forcibly transformed a set of practical instructions into an abstract geometrical puzzle and, in doing so, left out one dimension – depth – from a three-dimensional problem. This happened, we were able to confirm, not for want of exposure to living practice: the Shoot originated in his region, spread therefrom throughout the empire, and continued to be held without major interruption to his day. This happened, we were also able to confirm, not for want of exposure to geometry: Zheng Xuan had a life-long history of mixing commentary and mathematics, and the very text needed to solve this imaginary problem appears on his CV. This is all very suspicious, but nowhere near as suspicious as the actions of his co-conspirators, who would later induct his faulty maths into the ‘Correct Meaning’ of the Confucian canon, supplying worked mathematical proofs to show how this ‘is’ ‘thus’ ‘truly’ so (Kong Yingda) or, at least, ‘[how] the commentary arrives at these numbers’ (Jia Gongyan). As to whether *they* knew better, lastly, we caught the one implicated in his own life-long history with mathematical astronomy, and the other performing square root extractions. What were they *thinking*?

They are dead – all three of them – so is difficult to prove motive; there is, however, something to be gleaned from their *modus operandi*. Viewed through the lens of modern academic specialisation, modern sinologists tend to lump the men in this story into the box of ‘Confucianism’, ‘Classical studies’ (*jingxue* 經學), or ‘philology’, as wholly separate, say, from the history of sport, astronomy, or mathematics. They do not go in these boxes, so they must be forced. We do this by ignoring and, ultimately, *redacting* the things that we don’t feel belong in their writing, personal story, or milieu, regardless of how tightly woven therein it may be. Mathematics can be particularly ‘vexing’ in this regard, as Kong Yingda complains of his incorrigible master, so it tends to lead to the most extreme expressions of denial. For example,

L’idée de quantité ne joue autant dire aucun rôle dans les spéculations philosophiques des Chinois. Les Nombres, cependant, intéressent passionnément les Sages de l’ancienne Chine. Mais, – quelles qu’aient pu être les connaissances arithmétiques ou géométriques de certaines corporations (arpenteurs, charpentiers, architectes, charrons, musiciens...), – nul Sage n’a accepté de les utiliser, si ce n’est dans la mesure où, sans jamais contraindre à des opérations dont le résultat ne se pût commander, ce savoir facilitait des jeux numériques. ... A côté

d'une valeur quantitative qui les distingue, mais qu'on tend à négliger, les Nombres possèdent une valeur symbolique beaucoup plus intéressante, car, n'offrant aucune résistance au génie opératoire, elle les laisse se prêter à une sorte d'alchimie. Les Nombres sont susceptibles de *mutations*. Ils le sont en raison de l'efficiences multiples dont ils paraissent dotés et qui dérive de leur fonction principale ; ils servent et valent en tant que *Rubriques emblématiques* (Granet 1934, 149–151).

But the Ritual Classics are *riddled* with quantities, and mathematics, as we have seen, was as indispensable a tool in the commentator's toolbox as it was for any of the lesser trades in Granet's list. In a way, the very story of how the Ritual Classics came to read the way they do begins with a professor looking for someone who was 'good at maths' to help him interpret an ancient text. That someone happened to be Zheng Xuan, and it is to him, thanks to Ma Rong's backing, that modern professors still turn for help. Zheng Xuan was no stranger to Granet's *Rubriques emblématiques*, mind you, but our investigation has caught him playing *jeux numériques* of a different order with a cabal of Confucians, Classicists, and philologists who were also very openly, in their own words, 'good at maths'.

Suan 'mathematics', as in any culture, is a world apart from and infinitely surpassing the real – a realm, populated by absurdities, of absolute perfection and infinite possibility beyond time, space, and physical constraints. It is only here that the physically impossible can become The Correct Meaning of ancient rites, and those rites are only here because it is here that Zheng Xuan et al. have brought them. These are not Granet's 'surveyors, carpenters, architects, wheelwrights, [or] musicians' – they are *philosophers*, or the next closest thing, and they have clearly done this for a reason. Consider Zheng Zhong, above, who suggests that that one third of ten is four for the purposes of the short-range target's swan. Zheng Zhong knew how to work with fractions – *he too* came from a family tradition in mathematical astronomy – his are dimensions, however, that take things like the two-*chi* cloth bolt and decimal *chi*-rule into practical consideration.⁹⁴ *And that is precisely why this story is not his* – we are talking here about the perfect institutions of perfect kings in a perfect past deserving of better than a contemporary clothier's approximations. We are talking about the quest for Goodman's '*prisca Zhou*': 'a centuries-old, disjointed search for a lost, ritual standard that was thought recoverable' (Goodman 2010, 209) through, among other things, astronomy, archaeology, and physical experimentation. In so much as the goal of Classical commentary here was to reconstruct perfect and timeless forms, it is only natural that it availed itself to those of *Les Nombres*, in both the Granetian and non-Granetian sense, and that it may lead us to a vision of the Kingdom of Zhou that resembles nothing of the fallen, three-dimensional world in which we live.

⁹⁴ More specifically, Zheng Zhong's biography describes him as 'conversant in the Triple Concordance *li*' 明三統歷, which his father, Xing 興 (fl. 14–33 CE), had personally learned along with a number of Classical commentaries from its author, Liu Xin (*Hou Han shu*, 36.1217, 1224).

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