



Calling out Zheng Xuan (127–200 CE) at the crossroads of ritual, maths, sport and classical commentary

Daniel Patrick Morgan

► To cite this version:

Daniel Patrick Morgan. Calling out Zheng Xuan (127–200 CE) at the crossroads of ritual, maths, sport and classical commentary . 2017. halshs-01343877v4

HAL Id: halshs-01343877

<https://shs.hal.science/halshs-01343877v4>

Preprint submitted on 29 Jan 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial - ShareAlike 4.0 International License

Calling out Zheng Xuan (127–200 CE) at the crossroads of ritual, maths, sport and classical commentary

Daniel Patrick Morgan*
(Laboratoire SPHERE, CNRS & Paris 7 – Paris 1)

Abstract It is Zheng Xuan 鄭玄 (127–200 CE), more than any other commentator, to whom we owe our current understanding of the Confucian Classics – Zheng Xuan, of course, and the subcommentators, Kong Yingda 孔穎達 (574–648 CE) and Jia Gongyan 賈公彥 (fl. 637 CE), responsible for canonising his readings in the early Tang (618–907 CE). Zheng Xuan made mistakes, however, and this chapter offers a case study on one of the more glaring examples: the calculation of physically impossible target geometries for the royal archery meet reputedly held in the Western Zhou (1045–771 BCE), the ‘Big Shoot’ (*dashe* 大射). In transforming the ambiguous language of the Classics into an elaborate maths problem with definite quantities, Zheng Xuan neglects to account for the targets being in three dimensions, his subcommentators entering to supply mathematical proofs to numbers that are flat wrong, in both senses of the word. It is not as if they could not have known, I argue, considering that the Big Shoot was living practice in their respective days, and as preeminent commentators like Zheng Xuan and Kong Yingda were equally trained in mathematics equipped to solve this very problem. Faced with this curious epistemic dissonance between exegesis, experience, and mathematical training, I offer that we might better understand the work of the Classical commentator as reconstructing abstract, ideal forms necessarily divorced from their reality and for which mathematics suggested itself as the perfect tool.

Keywords Zheng Xuan, Kong Yingda, Jia Gongyan, Confucianism, classical commentary, ritual, archery, history of mathematics

1. Introduction

Having formed over the first millennium BCE by a desultory process of accretion, transmission, translation, loss, recovery, and redaction, the Classics (*jing* 經), as they have come down to us, are hard to read.¹ They are *old*, and to understand their very language, let alone the ‘subtle words’ of the ‘plainclothes king’s’ supposed curation thereof, one must rely on the *zhuan* 傳 – the ‘traditions’ and/or/as ‘commentaries’ – of Confucius’ intellectual successors from the fifth to second

* The research leading to these results received funding from the European Research Council under the European Union’s Seventh Framework Program (FP7/2007–2013) / ERC Grant agreement n. 269804 and was conducted in the context of the project *Mathematical Sciences in the Ancient World* (SAW). A first version of this paper was presented at the exploratory Commentaries Workshop, MPIWG, 25–27 August 2016, both versions having grown out of collaborative work with Alan J. Levinovitz leading, notably, to Levinovitz and Morgan (forthcoming). I thank Alan and the workshop organisers and participants for their encouragement, questions, and criticism of previous versions of the present chapter.

¹ For a study of this process in all its complexity, see Shaughnessy (2006); on issues of ‘translation’ and ‘domestication’ between pre-imperial (< 221 BCE) scripts, see Feng (2007).

century BCE.² And so it goes for the *zhuan* as well, for the *zhuan* came about by a similarly desultory process requiring that second-century CE scholastics fix *their* meaning with ‘chapter and verse’ (*zhangju* 章句) ‘annotation’ (*zhu* 注) – an expansive genre made possible by Ma Rong’s 馬融 (79–166 CE) invention of double-line inter-linear commentary (fig. 1). History repeats itself, but for every layer of exegesis that the Classics have since accrued, there is one name to which the question ‘how do we know that?’ consistently leads back: Zheng Xuan 鄭玄 (127–200 CE), Ma Rong’s preeminent disciple. In the words of Wang Shao 王邵 (fl. 577–605 CE), ‘One would rather say that the [Master] Kong and the Sage [Kings] were mistaken than hear that [the commentators] Zheng [Xuan] and Fu [Qian] 服虔 (fl. 189–92 CE) were wrong’.³

Our modern reading of ancient texts is similarly dependant on the likes of Zheng Xuan; the stakes, however, were considerably higher in Wang Shao’s day. As the ‘warp’ (*jing*) of civilisational order – the blueprint for returning to the golden age of the Western Zhou (1045–771 BCE) – the Classics framed one’s options as concerned political and religious state-building. Consider for example Academician Feng Gui’s 封軌 debate-opener on how the Manchurian-origin Tuoba-Wei (386–585 CE) should construct a Bright Hall⁴ ritual complex on the model of their Chinese neighbours in 477/479 CE:

明堂者，布政之宮，在國之陽，所以嚴父配天，聽朔設教，其結構之式，蓋已尚矣。故『周官·匠人職』云：「夏后氏世室，殷人重屋，周人明堂，五室、九階、四戶、八窗」。鄭玄曰：「或舉宗廟，或舉王寢，或舉明堂，互之以見同制。」然則三代明堂，其制一也。

The Bright Hall is a palace for promulgating policy. Located on the sunny (southern) side of the polis, it is that by which [the emperor] consigns his respected father to heaven, hears [the announcement of] the new moon and establishes the teachings – the model for its construction was already there in the distant past. Thus does the *Offices of Zhou* ‘Post of Artisan’ [entry] say, ‘For the kings of Xia, the Chamber of Generations, for the people of Yin (?–1046 BCE), the Double House,

² The myth goes that Confucius – Master Kong 孔子 – was a latter-day Sage (*sheng* 聖) who, instead of ruling, as would normally be his right, redacted the pre-fifth-century BCE Classics as to fill them with ‘subtle words’ (*wei yan* 微言) about proper conduct by which future generations were to live, making him a ‘plainclothes king’ (*su wang* 素王). On this myth and the *zhuan* ‘traditions’ that first arose around these texts so as to codify the meaning of his ‘subtle words’, see for example Van Auken (2016).

³ 寧道孔聖誤，諱聞鄭服非, cited in *Jiu Tang shu*, 102.3181. I thank Zhu Yiwen for bringing this quote to my attention. On the Chinese commentarial tradition thereafter, see Elman (1984), Henderson (1991), and Makeham (2003).

⁴ The Bright Hall (*mingtang* 明堂) was, supposedly, a temple in the southern suburbs of the Zhou capital modelled symbolically upon the structure of heaven, earth, and time. It was there, allegedly, that the Zhou kings announced the calendar for the year and, in subsequent months, cycle through its twelve halls wearing month- and season-appropriate symbols and reading ‘monthly ordinances’ (*yueling* 月令) concerning the (mostly) agrarian activities of their subjects so as to keep heaven, earth, and man in harmonious accord. On the Bright Hall, particularly as debated and instituted in the early imperial period, see Maspero (1951), Soothill (1951), Forte (1988), Lewis (2006, 265 ff.), Tseng (2011, 17–88), and Morgan (2017, 120–125).

and for the people of Zhou, the Bright Hall – five chambers, nine stairways, four doors, and eight windows’. Zheng Xuan says ‘Some raised ancestral temples, some raised royal mausolea, and some raised Bright Halls – compare them and one sees that they are of the same construction’. As such, the Bright Halls of the Three Dynasties were of a single construction.⁵

And thus did the Tuoba-Wei erect the architectural symbol of their legitimacy in occupied Luoyang in 496 CE on the basis of Zheng Xuan’s ‘annotations’ – because it takes the second-century CE classicist to supply a single floor plan where the Classics themselves are generally silent or inconsistent on the details.

It is not of course as if ‘Three Dynasties’ institutions were self-evident or even agreed upon in Zheng Xuan’s time. Lu Lübing 盧履冰 (fl. 688–717 CE) reminds us of the confusion lingering around the most basic of rites in 717 CE in the context of yet another public debate:

三年之制，說者紛然。鄭玄以為二十七月，王肅以為二十五月。又改葬之服，鄭云服總三月，王云訖葬而除……鄭、王祖經宗傳，各有異同；荀摯采古求遺，互為損益。方知去聖漸遠，殘缺彌多。As to the institution of the ‘three-year’ [paternal mourning period], the explanations (*shuo* 說) are in disarray: Zheng Xuan took it to be twenty-seven months, and Wang Su 王肅 (195–256 CE), twenty-five. Furthermore, as to changing from one’s funeral garments, Zheng says that one wears the light hemp for three months, while Wang says that one removes it at the end of the funeral. ... Zheng and Wang treated the Classics as their forefathers, and the commentary/traditions (*zhuan*) as their ancestors, [and yet] each had their differences and commonalities; Xun [Yi] 荀顗 (d. 274 CE) and Zhi [Yu] 摯虞 (fl. 265–307 CE) picked over antiquity in search of vestiges, [and yet] the one added and subtracted from the other – it is at this that one understands our ever-increasing distance from the Sages and the surplus of deficiencies and gaps [in our understanding thereof].⁶

Wang Su’s commentaries are now lost, as are the writings of Zhi Yu, Xun Yi, and Lu Lübing, leaving us today but with one voice in a centuries-long, multi-party exchange over second-millennium BCE practice.⁷ The reason for Zheng Xuan’s modern pride of place, in other words, has everything to do with selection.

Namely, in 638 CE the emperor of the fledgling Tang empire (618–907 CE) ordered the academician Kong Yingda 孔穎達 (574–648 CE)

⁵ *Wei shu*, 32.765.

⁶ *Jiu Tang shu*, 27.1024. On the debate between Xun Yi and Zhi Yu over the commentaries of Zheng Xuan and Wang Su to which Lu Lübing is referring, see *Jin shu*, j. 19; cf. Goodman (2010, 353–362).

⁷ To be precise, all that survives of their writing is fragments cited in later sources: for Wang Su, see *Quan Sanguo wen* 全三國文, j. 23; for Xun Yi, see *Quan Jin wen* 全晉文, j. 31; for Zhi Yu, see *ibid.*, j. 76, in *Quan shanggu Sandai, Qin, Han, Sanguo, Liuchao wen*; for Lu Lübing, see *Quan Tang wen*, j. 335. Fragments of lost commentaries such as Wang Su’s can be found in *Yuhan shanfang ji yishu*. On the Zheng Xuan–Wang Su divide, see Shi Yingyong (2007).

to lead a multicultural, North–South project to canonise primary commentaries to the newly ‘fixed’ (*ding* 定) Five Classics and to compose *shu* 疏 ‘sub-’ or ‘meta-commentaries’ thereto. Of these, Zheng Xuan’s ‘annotations’ were selected for two: *Mao’s Odes* and the *Record of Rites*.⁸ After fifteen years of collaboration, negotiation, renegotiation, and reorganisation, the project would result in 653 CE in *The Correct Meaning of the Five Classics* – the canon that would serve as the pillar of the civil service examination and, thus, of the aspiring officeholder’s education for centuries to come. Tellingly, it is Kong Yingda who coined the modern term *kecheng* 課程, which we now translate as ‘curriculum’, though what it originally/still means is administrative ‘test norms’. Equally telling is that it is *the second-century commentary* that is the object of seventh-century exegesis, Zheng’s ‘annotations’ having been elevated to the scholastic (and typographic) status of base text, as we can see in fig. 1. That was the Five Classics, and Jia Gongyan 賈公彥 (fl. 637 CE), one of Kong’s collaborators, took it upon himself to produce similar editions of the lesser Classics, the *Rites of Zhou* and the *Book of Etiquette and Ceremonial*, also on the basis of Zheng Xuan’s ‘annotations’, which would eventually find their way into the expanded canon of the Southern Song (1127–1279 CE).⁹ Of the later Thirteen Classics, Zheng Xuan’s ‘annotations’ were selected for four – three for three as concerns those on ritual (*li* 禮) – thus enshrining his annotations in what has been for centuries the mainstay of the Chinese book industry: test preparation materials.¹⁰

Just because Zheng Xuan’s might be the right answer on a test, however, does not mean that he was *right*, and this chapter offers a look at how, in one case, he is not. That Zheng Xuan’s understanding of pre-first-millennium BCE state ritual is fallible, as Wang Su could have told us, is not exactly news. Indeed, the fruits of philology and archaeology have, in the last few centuries, revealed that some of what even the Classics have to say on ritual is wrong: the Bright Hall, for example, we now know to be largely a product of Warring States (475–221 BCE) metaphysics, and as to how long one mourned one’s father, as counted in months, we now know to think twice before assuming a monolithic pre-imperial ‘institution’ (*zhi* 制). Sure, some Classical-era (or Classical-text-derived) ritual *did* survive into Zheng

⁸ The other three of the Sui–Tang ‘Five Classics’ were the *Book of Changes*, *Book of Documents*, and the *Zuo Tradition of the Spring and Autumn Annals*; see Table xx in Chemla and Zhu’s contribution to the present volume. Note that the title of Zheng Xuan’s commentary to *Mao’s Odes* actually uses the label *jian* 箋 in the place of *zhu* 注, the two of which refer equally to ‘notes’, ‘annotation’, etc.

⁹ On Kong Yingda, Jia Gongyan, and *The Correct Meaning of the Five Classics* (*Wujing zhengyi* 五經正義) project, see McMullen (1988, 71–112). Note that these subcommentaries were collaborative projects, but that I shall refer to their lead authors in the singular, as is usually done, for the sake of brevity.

¹⁰ For a broader study of Zheng Xuan’s commentary to the Ritual Classics, see Yang Tianyu (2008); for more focused, Western-language studies on Zheng Xuan’s commentary-writing, see Makeham (1997) and Plaks (2010). On the civil service exam and the history of education in China, see Lee (2000).



Fig. 1 1870 reprint of 1738 Wuying dian 武英殿 edition of the *Book of Etiquette and Ceremonial with Commentary and Subcommentary* (Yili zhushu 儀禮注疏), available online at Ctext.org (<http://ctext.org/library.pl?if=en&file=80179&page=4>). Big, single-line characters: base text, followed by Zheng Xuan's 'annotations'. Small, double-line characters: Lu Deming's 陸德明 (c.550–c.630 CE) 'pronunciation and meaning' commentary, followed by the 'subcommentary' of Jia Gongyan et al. (注 'commentary', 音義 'pronunciation and meaning', and 疏 'subcommentary' marked in relief).

Xuan's day, but a lot of it was introduced in fits and starts only a few centuries earlier – introduced in response to and, finally, as amalgamated with the contemporary institutions that were, more often than not, the true point of contention in studying the past.¹¹ Still, in as much as

¹¹ On the historical origins of the Bright Hall, see the references cited in Note 4. On the three-year mourning period, see for example Yang Hua (2007, 311–312). On the

the ‘Three Ritual [Classics]’ (*Sanli* 三禮) were Classics, and in as much as their ‘Correct Meaning’ depended on a single man, it behoves us to ruminate on the things that he got wrong – on what he got *really* wrong, no less, as in on the level of describing ‘institutions’ that are physically impossible.

The interest in doing this is not to say *whether* Zheng Xuan erred; it is to say *how*. Mistakes have a way of revealing precious information about the way a given text was conceived, and all the more when mistakes occur in the context of calculation.¹² It is to that end that we will be focusing on Zheng Xuan’s peculiar, mathematical reading of Classical target archery. There is a computational glaring error in Zheng Xuan’s reading of specific target geometries in/onto the Ritual Classics, and, what is more, rather than acknowledging that, his seventh-century CE subcommentators would go on to expound upon and enshrine that error as part of ‘The Correct Meaning’ of these sacred texts. This should give us grounds for worry as concerns our continued dependence on these writers to tell us what the Classics mean, so we shall devote the first half of this study to an in-depth analysis of how the argument in question was built, botched, and ultimately justified as is. With a clearer understanding of what our three exegetes have done in their respective apparatus we will then proceed to the matter of why. This may all well be one big misunderstanding, but there are aspects of our subjects’ behaviour that stand out as suspicious upon closer examination, so we shall devote the second half of this paper to establishing intent by digging around into who and what our subjects knew. In the end, we may not be able to pin down *why* things turned out the way they did for ‘The Correct Meaning’ of Classical target specifications, but what we learn about our exegetes’ *modus operandi* along the way will prove no less valuable for our understanding of the mechanisms of commentarial writing and canonisation.

2. The mistake

The ‘Big Shoot’ (*dashe* 大射) is a royal archery meet described in more exquisite detail than your average Classical ritual. It features nowhere in the Confucian Canon other than the Ritual Classics, but there it features prominently, and it features prominently in all three. First, the *Book of Etiquette and Ceremonial* consecrates an eponymous chapter to the subject, providing step-by-step instructions for what everyone is to do, ceremony to game, set-up to clean-up. Second, the *Record of Rites* devotes a chapter to ‘The Meaning of the Shoot’ (Sheyi 射義) – its ‘meaning’ in terms of Sagetime origins and

intersection of Classical scholarship and contemporary politics in the Han, see the recent studies of Sukhu (2005–2006) and Brashier (2011).

¹² On this point, as concerns textual production and transmission in early Chinese manuscript culture, see Morgan (2015), Morgan and Chemla (2018), and Chemla and Morgan (forthcoming). Note also that the very idea to go looking for mathematical problems in Classical commentary in this chapter is inspired by the recent, ground-breaking work of Zhu Yiwen (2015; 2016a; 2016b) developed upon in Chemla and Zhu’s contribution to the present volume.

implications for contemporary factio-ethical self-reflection, the latter half of which is narrated by a famous athlete:

孔子射於矍相之圃，蓋觀者如堵牆……

[Once], when Master Kong was conducting a shoot in a vegetable garden at Juexiang, surrounded by a wall of onlookers...¹³

Third, the *Offices* or *Rites of Zhou*, in its fantastical blueprint of the Western Zhou bureaucracy, offers a description of the various officers among the rite's support staff and, in the 'Record of the Examination of Craftsmen' (Kaogong ji 考工記), an account of the fabrication of the materials involved. In as much as our focus in this chapter is a single argument as developed in Zheng Xuan's 'annotations' to these texts – and in as much as these texts have been thoroughly studied and translated elsewhere – let us content ourselves with these descriptions and turn to the matter of the target geometries.¹⁴

Before getting into the sources, I must warn the reader about what awaits in the following pages. Zheng Xuan's argument is difficult to follow. It is difficult to follow for both textual and mathematical reasons, and I shall explain here what you need to know to navigate the excerpts presented in this section and in the Appendix below.

First, due to the nature of the genre, most of Zheng Xuan's argument is built on explaining and/or equating one thing with another ('*x* is *a*', '*y* is *b*', '*z* is *c*'). Where he offers some justification for such an equation, it is usually by way of Classical precedent ('such-and-such text says...') or linguistic proximity ('word *x* = word *a*, because *x* looks and/or sounds like *a*'). Where he resorts to the latter, I provide the modern transliteration, modern character form, phonological reconstruction, and/or literal translation, in that order, so as to highlight what is proximate, e.g. that *hao* 鵠 (*kək) 'swan' means *jiao* 較 (*kək) 'frankness' thanks to the power of the pun.¹⁵ What it is important to know about this latter sort of justification is that it is more convincing in the original than in translation, and that some such equations are conventional, while others are forced. Due to the history of Chinese language and the process by which pre-imperial texts circulated in different, evolving scripts, before being collated, transcribed, and standardised over the Western Han, philologists would not bat an eye, for example, at reading *san/can* 參 (*səm/ts^həm) 'three/tripartite' as same-word-family member *san* 糝 (*səm) 'blended', whereas the

¹³ *Liji zhushu*, 46.1017a; tr. modified from Legge (1885, vol. 2, p. 449).

¹⁴ The following analysis builds off of the philological studies of target-related sources and terminology in Chen Pan (1950) and Riegel (1982). For other editions and translations of the Ritual Classics than those used here, see Loewe (1993). For a broader study of the Big Shoot and related archery contests, see Selby (2006, chaps. 4–6) and Levinovitz and Morgan (forthcoming).

¹⁵ This is to say that the two words here may have been homophones in Zheng Xuan's days, but that they are graphologically unrelated (鵠/較, vs the example of 參/糝 below), and thus that the two likely belong to different etymological word families (1039 and 1166, respectively, in Karlgren 1957). All phonological reconstructions used in this chapter are given as per Schuessler's (2007) Later Han Chinese, which best reflects matters of homophony in Zheng Xuan's day.

equation of ‘swan’ with ‘frankness’ is an association that is less common, less obvious, and less befitting the title of philology.¹⁶ The outside reader may not be able to tell the difference, but it is important for him/her to keep in mind that some of the links in the commentator’s chain of reasoning are bound to be weaker than some of the others. Like the Neo-Platonists studied in Harari’s chapter, the goal in Classical exegesis was often to reveal (i.e., impose) an underlying harmony between dissimilar texts, which, in places, may require a certain degree of hermeneutic forcing and flexibility. On the subject of language, lastly, the principal task of ‘annotation’ was to render an archaic and laconic base text readable in the idiom of the commentator’s day, and the reader should note that I attempt to reproduce the inherent archaisms and ambiguities of the base text in my translation so as to reflect that.

Zheng Xuan builds an argument from ‘*x* is *a*’, ‘*y* is *b*’ lemmata; what makes it difficult to follow is that that argument does not unfold linearly so much as it runs every direction at once through a redundancy-laden web of cross-references stretching across a dozen passages in separate works. Why? This is an inevitable consequence of the genre: his argument is built on glosses, and his glosses build to an argument, but the *order* of those glosses is determined by the base text (and the expectation that interlinear ‘annotations’ treat the base text chapter by chapter, verse by verse). To render what I understand to be Zheng Xuan’s train of thought more linear, therefore, I will have to hop, skip, and elide myself through the base text. I do this to facilitate the reader’s comprehension of Zheng Xuan’s argument; to facilitate his/her feeling for the *genre*, however, I must draw the line at disrupting the order the apparatus surrounding what is effectively a single line of text. If one finds the block quotes difficult to follow on their own, that is good: they are, and it is important that we not lose sight of how the argument actually reads.

Second, as concerns the numbers, the calculations to follow are fairly simple, but there are two pieces of presumably basic knowledge that we must first introduce to the modern reader for those calculations to make any sense – knowledge underlying the commentary and subcommentary’s every transformation of numerical values but that neither feels requires an explanation. The first is that the equivalency between the civil measuring units employed is as follows:¹⁷

$$1 \text{ zhang 丈} = 10 \text{ chi 尺} = 100 \text{ cun 寸} = 1000 \text{ fen 分} (= 231 \text{ cm})$$

¹⁶ See word family 647 in Karlgren (1957). For an excellent English-language introduction to the complexities of early Chinese scripts, their evolution, their early-imperial standardisation, and the rules of glossing, see Qiu Xigui (2000).

¹⁷ Note that the equivalency 1 *chi* = 23.1 cm holds for the late Warring States up to the Western Han (206 BCE–9 CE), when the Ritual Classics were redacted, canonised, and, likely, written, but that the civil *chi* of Zheng Xuan’s day was more like 23.5 cm; see Qiu Guangming (1992) and Goodman (2010, chap. 4).

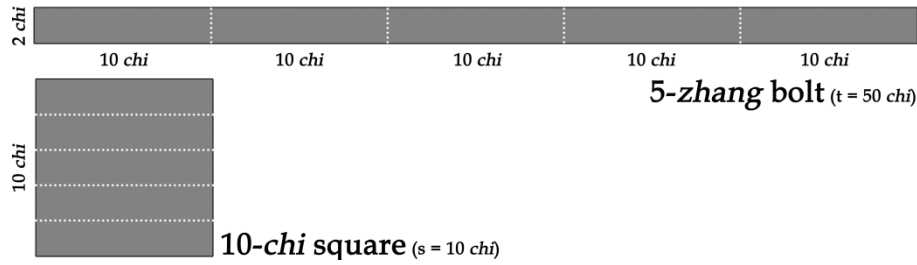


Fig. 2 Visualisation of how cloth-bolt-length converts to area, using the example of a square 10 *chi* to a side, *s*, using a total length, *t*, of 5 *zhang*. In symbolic terms, $t = \frac{s^2}{2}$.

Our subjects operate with and convert between these units without explanation – much as we do with metres, centimetres, and kilometres – so it goes without saying, for example, that ‘one *zhang*’ is the same as ‘ten *chi*’, and that, divided by three, that gives you ‘three *chi* three *cun* and one-third *cun*’ ($10 \text{ chi} \div 3 = 3\frac{1}{3} \text{ chi}$). The second piece of basic knowledge required to understand the following argument is that, on the target, these are measures of *area* as much as they are measures of *length*. Namely, the targets are made of cloth, and when measuring cloth it is generally assumed that the usable width of a bolt (*fu* 幅) is 2 *chi*. As such, it goes without saying, for example, that single bands like the ‘stalks’, ‘tongues’, and ‘frames’ below are not *lines* of a given length but *rectangles* 2 *chi* in height. It also goes without saying that a square 10 *chi* to a side ‘uses five *zhang* of cloth’ 用布五丈, for example, because where *t* is the total length of 2-*chi*-wide cloth, and *s* is the side, $t = \frac{s^2}{2}$. The reasoning is material, and I offer fig. 2 to help the reader visualise it as such.¹⁸

Third, and most importantly, everything in this section leads to the configuration illustrated in fig. 3. I offer this first, at the risk of spoiling the surprise, such that the reader may refer back to it any time he/she feels lost in the text, summary, and analysis to follow.

2.1 The commentary

The Big Shoot uses three targets at different distances, the smallest of which commentators equate with that used in the District Shoot (*xiangshe* 鄉射). As the ‘District Shoot’ chapter comes *first* in the *Book of Etiquette and Ceremonial*, it is there where any discussion of the Big Shoot must begin. With the Classic in SMALL CAPS, and Zheng Xuan’s interlinear commentary thereafter, this is what we learn about the butt or support-structure of the target:

鄉侯，上个五尋。上个，謂最上幅也。八尺曰尋。上幅用布四丈。中十尺。方者也，用布五丈。今官布幅廣二尺二寸，旁削一寸。『考工記』曰：「梓人為侯，廣與崇方，」謂中也。侯道五十弓，弓二寸以為侯中。言侯中所取數也。量侯道以狸步，而云弓者，侯之所取數宜於躬器也。正二寸較中之博也。今文改弓為肱也。倍中以為躬。躬，身也，謂中之上

¹⁸ On length measures and their use as areas, see Li Jimin (1998, 768–778).

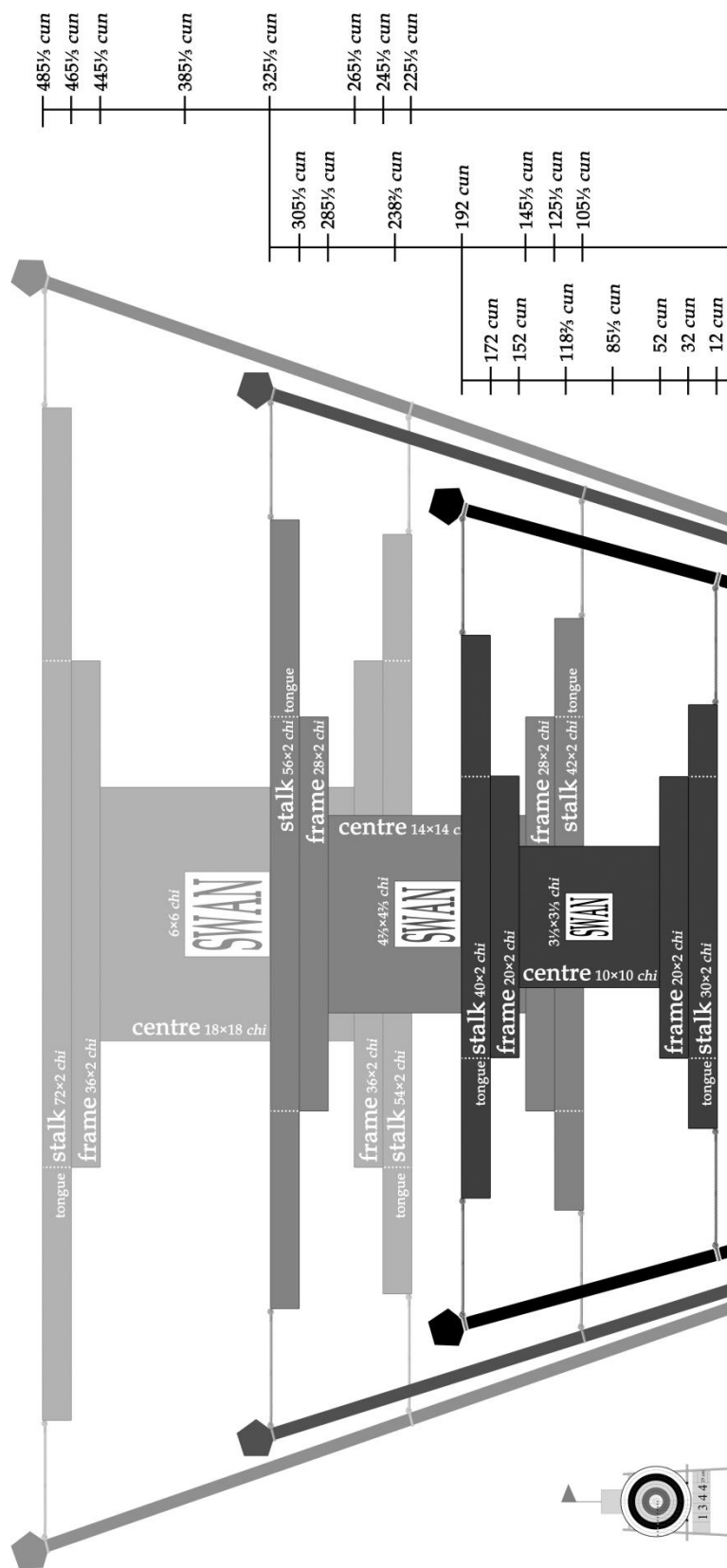


Fig. 3 Author's reconstruction of Zheng Xuan's Big Shoot target dimensions and heights as detailed in Kong Yingda and Jia Gongyan's subcommentaries (1 *chi* = 10 *cun*). Note that the World Archery Federation regulation 70 m (\approx 50 racoon-dog pace) butt is provided in the lower left corner for the sake of comparison (scaled at 1 *chi* = 23.1 cm) and that the heights of individual components are provided on the right for the sake of visualising Kong Yingda and Jia Gongyan's respective proofs in Section 2.2. Note that the dimensions of the 'stalks' include those of the protruding 'tongues'.

下幅也，用布各二丈。倍躬以為左右舌。謂上个也。居兩旁，謂之个左右，出謂之舌。下舌，半上舌。半者，半其出於射者也，用布三丈。所以半上舌者，侯，人之形類也：上个象臂，下个象足。中人張臂八尺，張足六尺，五八，四十，五六，三十，以此為衰也。凡鄉侯用布十六丈；數起侯道，五十弓以計：道七十弓之侯，用布二十五丈二尺；道九十弓之侯，用布三十六丈。

FOR THE DISTRICT TARGET, THE UPPER STALK IS FIVE ARM-SPANS. ‘Upper stalk’ (*gan* 个) refers to the bolt of cloth at the very top. Eight *chi* is called an ‘arm-span’. The upper bolt [thus] uses four *zhang* of cloth.¹⁹ THE CENTRE IS TEN *CHI*. This is ‘the square’; it uses five *zhang* of cloth. The modern official cloth bolt is two *chi* two *cun* wide, from which one cuts one *cun* from [each] side. The ‘Record of the Examination of Craftsmen’ says ‘The Worker of Precious Woods makes the target square in breadth and loftiness’, referring to the ‘centre’.²⁰ THE TARGET PATH IS FIFTY BOWS, BOW TWO *CUN* MAKES THE TARGET-CENTRE. This is talking about the dimensions of the target-centre. One measures the target path in racoon-dog paces, the reason for saying ‘bow’ here being that the target’s dimensions are adapted to the shooting instrument, the ‘two *cun*’ [per bow] at the target corresponding to the width of its grip.²¹ [Note that] the modern-text [version] changes *gong* 弓 (*kuŋ) ‘bow’ to *gong* 肱 (*kuəŋ) ‘forearm’. DOUBLE THE CENTRE FOR THE FRAME(S). ‘Frame’ (*gong* 躬) means body (*shen* 身), referring to the bolts of cloth above and below the centre, each of which uses two *zhang* of cloth. DOUBLE THE FRAME FOR THE LEFT AND RIGHT TONGUES. This refers to the ‘upper stalk’: [the lengths] that occupy its two sides are referred to as ‘left and right’, and because these stick out they are referred to as ‘tongues’. FOR THE LOWER TONGUES, HALVE THE UPPER TONGUES. ‘Halve’ means halving that which sticks out from what you shoot, [so] it uses three *zhang* of cloth.²² The reason you halve the upper tongues is because the target is an analogy for the human form. The upper stalk is in effigy of arms, and the lower stalk in effigy of legs; the average human arm-span is eight *chi*, and his leg-span is six *chi*; five eights, forty, five sixes, thirty – use these as the rates of unequal sharing.²³ In total, the district target uses sixteen *zhang* of cloth; this number arising from the target path, we calculate from fifty bows. The target whose path is seventy

¹⁹ I.e., 1 ‘arm-span’ equals 8 *chi*, so 8 ‘arm-spans’ equal $5 \times 8 = 40$ *chi*. Note that Zheng Xuan has likely arrived at this equivalency by working backwards from the information to follow: that the centre is 10 *chi* to a side, *s*; that the frame, *f*, equals 2*s*; and that the upper stalk, *u*, equals 2*f*, making $u = 2(2(10)) = 40$ *chi*.

²⁰ I.e., the ‘centre’ is a square 10 *chi* to a side comprising 5 *zhang* of cloth (see fig. 2).

²¹ I.e., ‘bow’ = ‘racoon-dog pace’ in terms of measuring the distance from target to stand along the ‘target path’, and the ‘two *cun*’ of the formulation ‘take bow two *cun* for the target-centre’ is a symbolic reference to the width of the bow’s grip. The significance of this is explained in further detail below.

²² Note that Zheng Xuan includes the left and right ‘tongues’ in the dimensions of the ‘stalks’. For the district target, with the 10-*chi* centre, the ‘frame’ is 20 *chi*, the ‘upper stalk’ is 40 *chi*, and the ‘tongues’ are the two 10-*chi* segments by which the ‘stalk’ protrudes from the ‘frame’; halving this, one gets left and right ‘tongues’ of 5 *chi* each, and a lower ‘stalk’ of $5 + 20 + 5 = 30$ *chi* (see fig. 3).

²³ *Zhouli zhushu*, 41.639a–b (cf. Biot 1851, vol. 2, p. 545), cited in part below, features a similar passage – ‘THE UPPER TWO STALKS JOINED TO THE BODY MAKE THREE; THE LOWER TWO STALKS HALVE THAT’ 上兩個與其身三，下兩個半之 – in which Zheng Xuan glosses the ‘stalks’ as ‘tongues’, citing the present passage, and computes identical lengths based on ‘all of this taking its image from man, and taking its symbolic *lǜ* 率 (ratio-elements) from his eight-*chi* arm-span and six-*chi* leg-span’ 蓋取象於人也，張臂八尺張足六尺，是取象率焉。

bows uses twenty-five *zhang* two *chi* of cloth. The target whose path is ninety bows uses thirty-six *zhang* of cloth.²⁴

In this passage, the base text lists the dimensions of the district target's components. Three of these are absolute – 5 arm-spans for the 'upper stalk' (*u*), 10 *chi* for centre's sides (*s*), and 50 bows for the target's distance – while the other three are proportional. As to proportions, the frame *f* is twice the centre ($f = 2s$), the upper stalk *u* is twice the frame ($u = 2f$), and the lower stalk *l*, in Zheng Xuan's reading, is six eighths the length of the upper tongue ($l = \frac{6}{8}u$).

This is the 'District Shoot' chapter, so the base text concerns itself solely with the district-level target. Zheng Xuan, however, would make this about the Big Shoot. Why? He has to: the *Book of Etiquette and Ceremonial* tells us nothing about *those targets* in the 'Big Shoot' chapter, so to say anything about them, he must establish here that what is true of the one is true of the other three. To that end, he affirms that the 'fifty-bow' District Shoot target *is* the 'fifty-raccoon-dog pace' Big Shoot target and, in conclusion, that the latter's seventy- and ninety-bow targets (below) can be scaled from the *cui* 衰 'unequal sharing rates' provided here. Scaling, he gives us the total length of cloth used in the fabrication of each target as derived by calculation (*ji* 計): 160 *chi*, 252 *chi*, and 360 *chi*, respectively. He does not show his work, but one notes that his values are consistent with the proportional relationships described thus far. In symbolic terms, where *c* is the total length of cloth, *t* the total used for the centre, and *f*, *u*, and *l* as per above:

$$c = t + f + f + u + l$$

Given that $f = 2s$, $u = 2f$, $l = \frac{6}{8}u$, and $t = \frac{s^2}{2}$ (fig. 2), we may reduce this in terms of the centre's *side* to

$$c = \frac{s^2}{2} + 2s + 2s + 4s + \frac{6 \times 4s}{8}$$

Or

$$c = \frac{s^2}{2} + 11s$$

Starting with a centre-side *s* of 10 *chi*, this indeed produces a total *c* of 160 *chi*, and the totals *c* of 252 *chi* and 360 *chi* correspond respectively to sides *s* of 14 *chi* and 18 *chi*, which we encounter elsewhere in Zheng Xuan's annotations.

To understand where Zheng Xuan gets the figures of 10 *chi*, 14 *chi*, and 18 *chi* for the sides *s* of the Big Shoot target-centres, we must turn to the *Rites of Zhou*:

²⁴ *Yili zhushu*, 5.151a–b; cf. Steele (1917, vol. 1, p. 120).

司裘……王大射，則共虎侯、熊侯、豹侯，設其鵠……凡此侯道，虎九十弓，熊七十弓，豹、麋五十弓……鄭司農云：「鵠，鵠毛也。方十尺曰侯，四尺曰鵠，二尺曰正，四寸曰質，」玄謂侯中之大小取數於侯道。『鄉射記』曰：「弓二寸以為侯中」，則九十弓者，侯中廣丈八尺；七十弓者，侯中廣丈四尺；五十弓者，侯中廣一丈，尊卑異等此數明矣。『考工記』曰：「梓人為侯，廣與崇方，參分其廣，而鵠居一焉」，然則侯中丈八尺者，鵠方六尺；侯中丈四尺者，鵠方四尺六寸大半寸；侯中一丈者，鵠方三尺三寸少半寸。謂之鵠者，取名於鵠鵠。鵠鵠小鳥而難中，是以中之為鵠。亦取鵠之言較。較者，直也，射所以直己志。

THE MASTER FURRIER: ... FOR THE ROYAL BIG SHOOT, HE PROVIDES THE TIGER TARGET, BEAR TARGET, AND LEOPARD TARGET, SETTING THEIR SWANS. ... As to their target paths, the tiger[-skin target] is at ninety bows, the bear at seventy bows, and the leopard and elk at fifty bows. ... Minister of Agriculture Zheng [Zhong 鄭眾 (d. 83 CE)] says 'The "swan" (*hao* 鵠) is a swan fur (i.e., pelt). As to the [concentric] squares, the ten-*chi* one is called the "target" (*hou*), the four-*chi* one the "swan" (*hao*), the two-*chi* one is the "surface" (*zheng* 正), and the four-*cun* one the "substance" (*zhi* 質)', but [I, Zheng] Xuan, say that the dimensions of the target-centres are taken from the target paths. The 'Record of the District Shoot' (cited above) says 'bow two *cun* makes the target-centre', so, at ninety bows, the target-centre is a *zhang* and eight *chi* broad; at seventy[, it] is a *zhang* and four *chi* broad; and at fifty[, it] is one *zhang* broad – the different ranks of noble and base made apparent by these numbers. The 'Record of the Examination of Craftsmen' says 'The Worker of Precious Woods makes the target, square in breadth and loftiness; dividing its breadth in three, the swan occupies one thereof',²⁵ and, as such, the swan on the one with the *zhang*-eight-*chi* target-centre is a square six *chi* [to a side] ($18\ chi \div 3 = 6\ chi$); [that] on the one with the *zhang*-four-*chi* target-centre is a square four *chi* six *cun* and a big-half ($\frac{2}{3}$) *cun* [to a side] ($14\ chi \div 3 = 4\frac{2}{3}\ chi$); and [that] on the one with the one-*zhang* target-centre is a square three *chi* three *cun* and a small-half ($\frac{1}{3}$) *cun* [to a side] ($10\ chi \div 3 = 3\frac{1}{3}\ chi$). It is called a *hao* 'swan' because it takes its name from the *ganhao* 鵠鵠 'swallow-swan' (i.e., magpie), the swallow-swan being a small bird that is difficult to hit, and that is why hitting it is considered outstanding. It is also taken from *hao*'s (**kouk*) meaning *jiao* 較 (**kək*) 'frank': 'frankness' is being straight (*zhi* 直), and shooting is that by which one 'straightens' one's ambitions.²⁶

Zheng Xuan's annotations here have very little to do with the base text, but they do, ironically, give us a clearer idea of how he reads the previous base text cited from the *Book of Etiquette and Ceremonial*: 'bow two *cun* makes the target-centre' 弓二寸以為侯中 is to be read '[for every] bow [counted on the target path, count] two *cun* [towards the centre's side *s* to] make the target-centre'. At 2 *cun* per bow, the 50-, 70-, and 90-bow targets give us target-centres 10 *chi*, 14 *chi*, and 18 *chi*, respectively, to a side – the values from which he was evidently working in the previous block quote.

²⁵ Citing *Zhouli zhushu*, 41.639a (cf. Biot 1851, vol. 2, p. 545). Zheng Xuan's commentary to said passage reiterates that the swan is 'made of hide' 以皮為之 and that big target's 18 *chi* target-centre would make the swan 'a square of 6 *chi* [to a side]' 鵠方六尺.

²⁶ *Zhouli zhushu*, 7.108a; cf. Biot (1851, vol. 1, p. 138). Note that Zheng Xuan offers the same gloss for *hao* 'swan' in *Yili zhushu*, 7.188a–b, elided in the relevant block quote below.

At the centre of the centre is the ‘swan’ – the point-scoring area. The base text here does no more than mention its existence as a component of the targets, the details of its dimensions – one third the breadth and height of the ‘target’ (*hou*) – coming at the other end of the *Rites of Zhou* (cited next). Zheng Xuan’s contention here with Zheng Zhong is revealing. Zheng Xuan demands that we be exact: the ‘Worker of Precious Woods’ entry says to *fen* 分 ‘divide’ the target’s breadth, so one mustn’t ignore the *fen* ‘fractional remainder’ that that implies. Ten divided by three doesn’t get you *four*, he chastises Zheng Zhong, it gets you $3\frac{1}{3}$. Were the swan *four chi* to a side, he implies, it would be more a *swan* than a *swallow-swan*, and where would be the sense in calling it a ‘swan’, by which the base text actually means a swallow-swan?²⁷

Zheng Xuan is silent about his predecessor’s division of the swan into further concentric squares – how/if to scale the ‘substance’ and the ‘surface’ thereto at 1:5:10. Under the ‘Marksman’ (*sheren* 射人) entry, however, he makes it clear that substance–[surface]–swan would indeed be the proper order:

今儒家云：「四尺曰正，二尺曰鵠」……此說失之矣。

Modern scholastics say ‘The four-*chi* [square] is called the “surface,” and the two-*chi* one is called the “swan”’... this explanation misses the mark.²⁸

Kong Yingda identifies this ‘off-mark’ position with ‘the *Rites of Zhou* annotated by Zheng Zhong and Ma Rong’,²⁹ the former conflicting with Zheng Xuan’s testimony, above, and the latter posing a conflict of interest. Alternatively, Yang Jian 楊簡 (1141–1226 CE) attributes this quotation to ‘Jia Kui’s 賈逵 (30–101 CE) annotated *Rites of Zhou*’.³⁰

As to the *Rites of Zhou* ‘Worker of Precious Woods’ entry that Zheng Xuan has repeatedly cited thus far, there we find there one last specification: the ‘mainstay’ cords and tie loops.

梓人為侯，廣與崇方，參分其廣，而鵠居一焉……上綱與下綱出舌尋，緝寸焉。綱所以繫侯於植者也……鄭司農云：綱，連侯繩也。緝籠綱者。緝讀為竹中皮之緝舌，維持侯者。

THE WORKER OF PRECIOUS WOODS MAKES THE TARGET, SQUARE IN BREADTH AND LOFTINESS; DIVIDING ITS BREADTH IN THREE, THE SWAN OCCUPIES ONE THEREOF (cited above). ... THE UPPER MAINSTAY AND LOWER MAINSTAY PROTRUDE AN ARM-SPAN FROM THE TONGUES. THE TIE LOOPS ARE A *CUN* THERETO. The ‘mainstays’

²⁷ A swan, obviously, could not be more different than a magpie: the one is white, the other black; the one has an adult wingspan of around 3.1 m, and the other about 60 cm (a contrast rather more stark than a target-swan of 4 vs $3\frac{1}{3}$ *chi*). Note that the use of the term *ganhao* 乾鵠 ‘swallow-swan’ to denote the magpie is attested in Xu Shen’s 許慎 (c.55–c.149 CE) commentary to the *Huainanzi* 淮南子 (139 BCE), *Huainan honglie jie*, 13.11a, but that Zheng Xuan’s reading of *hao* ‘swan’ as an abbreviation for an unrelated species seems forced.

²⁸ *Zhouli zhushu*, 30.462b (comm.).

²⁹ 周禮鄭眾馬融注, *Maoshi zhushu*, 21.492a (subcomm.).

³⁰ 賈逵周禮注, *Cihu Shi zhuan*, 15.14a.

(*gang* 綱) are that by which one ties the target to the poles. ... Minister of Agriculture Zheng [Zhong] says ‘The “mainstays” are the ropes connected to the target, and the “tie loops” (*yun* 纒) [are that which] hoop the mainstay’. *Yun* here is read like the *yun*-tongue skin in a bamboo (?), being that which supports and holds the target.³¹

At that, we have exhausted everything there is to glean from the Classics about target construction.

For Zheng Xuan, all of this is leading to ‘Big Shoot’ chapter of the *Book of Etiquette and Ceremonial*. The ‘Big Shoot’, as already mentioned, tells us next to nothing about the targets employed therein. It is up to the exegete, therefore, to harmonise what the base text *does* say with the various passages cited up to this point so that he may read his analyses there onto the base text here.

司馬命量人量侯道與所設乏以狸步，大侯九十，參七十，干五十……狸之伺物，每舉足者，止視遠近為發必中也。是以量侯道取象焉。『鄉射記』曰：「侯道五十弓」。『考工記』曰：「弓之下制六尺」，則此狸步六尺明矣。大侯、熊侯謂之大者，與天子熊侯同。參讀為糝，糝雜也。雜侯者，豹鵠而麋飾下天子大夫也。干讀為豸，豸侯者，豸鵠，豸飾也……

THE MASTER-AT-ARMS COMMANDS THE MEASURERS TO MEASURE THE TARGET PATHS AND THE POSITIONS IN WHICH THE SCREENS ARE TO BE SET UP IN RACCOON-DOG PACES: NINETY FOR THE BIG TARGET, SEVENTY FOR THE THREE, AND FIFTY FOR THE POLE. ... The raccoon-dog examines things at the raise of each foot, [as the archer must] stop and look at the distance so that his release be on-target, and it is thus that the measuring of the target paths takes the image thereof. The ‘Record of the District Shoot’ (cited above) says ‘The target path is fifty bows’, and the [*Rites of Zhou*] ‘Record of the Examination of Craftsmen’ says ‘The inferior model bow is six *chi*’, so it is evident that this ‘raccoon-dog pace’ (*li bu* 狸步) is six *chi*.³² As to the ‘big target’, the bear[-skin] target is referred to as the ‘big’ – this is none other than the son of heaven’s ‘bear[-skin] target’. *San/can* 參 (*ssam/ts^həm) ‘three’ is read as *san* 糝 (*səm) ‘blended’, ‘blended’ [meaning] ‘mixed’ (*za* 雜); the ‘mixed target’ has a leopard[-skin] swan beneath which it is decorated with elk [skin, around its edges] – this is for the son of heaven’s grandees. *Gan* 干 (*kan) ‘pole’ is read as *an* 豸 (*ŋan) ‘wild dog’, the ‘dog[-skin] target’ having a dog[-skin] swan and dog[-skin] decorations. ...

遂命量人，巾車張三侯。大侯之崇，見鵠於參；參見鵠於干，干不及地武……崇，高也。高必見鵠，鵠所射之主……『考工記』曰：「梓人為侯，廣與崇方，參分其廣，而鵠居一焉，」則大侯之鵠方六尺，糝侯之鵠方四尺六寸大半寸，豸侯之鵠方三尺三寸少半寸。及至也。武，迹也。中人之足長尺二寸，以豸侯計之，糝侯去地一丈五寸少半寸，大侯去地二丈二尺五寸少半寸。

³¹ Zhouli zhushu, 41.639b; cf. Biot (1851, vol. 2, pp. 545–546).

³² Note that ‘The inferior model bow’ 弓之下制 is but one of three models listed in the ‘Record of the Examination of Craftsmen’: ‘The six-*chi*-six-*cun*-long bow is called the superior model, for use by superior gentlemen; the six-*chi*-three-*cun*-long bow is called the average model, for use by average gentlemen; the six-*chi*-long bow is called the inferior model, for use by inferior gentlemen’ 弓長六尺有六寸，謂之上制，上士服之；弓長六尺有三寸，謂之中制，中士服之；弓長六尺，謂之下制，下士服之 (Zhouli zhushu, 42.662a; cf. Biot 1851, vol. 2, p. 596).

HE THEN ORDERS THE MEASURERS AND THE FABRIC-AND-CHARIOT OFFICER TO STRETCH THE THREE TARGETS. THE LOFTINESS OF THE BIG TARGET IS SUCH THAT IT SHOWS ITS SWAN VIS-À-VIS THE THREE, THE THREE SHOWS ITS SWAN VIS-À-VIS THE POLE, AND THE POLE FALLS SHORT OF THE GROUND MILITARY. ... ‘Loftiness’ (*chong* 崇) means ‘height’ – its height must be such that one sees the swan, the swan being the principal thing at which one shoots. ... The ‘Record of the Examination of Craftsmen’ says ‘The worker of precious woods makes the target, square in breadth and loftiness; dividing its breadth in three, the swan occupies one thereof’, and, as such, the big target’s swan is a square six *chi* [to a side], the blended target’s swan is [a square] four *chi* six *cun* and a big-half ($\frac{2}{3}$) *cun* [to a side], and the dog target’s swan is [a square] three *chi* three *cun* and a small-half ($\frac{1}{3}$) *cun* [to a side]. ‘Reach’ (*ji* 及) means ‘arrive at’ (*zhi* 至), and ‘military’ (*wu* 武) means ‘[by] a footstep’ (*ji* 迹), the length of the average human foot being a *chi* and two *cun*.³³ Calculating from the dog target, the blended target is one *zhang* five *cun* and one third *cun* from the ground, and the big target is two *zhang* two *chi* five *cun* and one third *cun* from the ground.³⁴

It takes some effort to reconcile the first line of the base text with what we read elsewhere in the *Book of Etiquette and Ceremonial* and *Rites of Zhou*. First, it is hardly apparent that the targets here speak to that in the ‘District Shoot’ chapter of the former: they have different names, they are different in number, and their target paths are measured in different units. Zheng Xuan has already established that the royal targets are scaled imitations of the district target earlier, in the ‘District Shoot’, so *that* can be taken here for granted. Instead, he focuses on the question of why the two use different units, which, he explains, they do not: ‘raccoon-dog pace’ is just another term for ‘bow’, which, in reality, refers to neither raccoon-dogs nor bows but to a half-double-pace (*bu* 步) of exactly 6 *chi*. It is no more apparent that the three royal-level targets here speak those listed in ‘Master Furrier’ entry of the *Rites of Zhou*: ‘big’, ‘three’, and ‘pole’ is, after all, a wholly different bag of things than is ‘tiger’, ‘bear’, and ‘leopard’. Zheng Xuan, however, is able to bring the one into alignment with the other by arguing that ‘three’ and ‘pole’ are to be read as animal-pelt-related synonyms. At that, all three passages can now be read as describing the same thing in different words, and that single, underlying target-form can be assumed in reading the second line.

The second line of the base text describes a configuration: each target is suspended higher than the one in front so that its ‘swan’ – its point-scoring area – is visible to the shooter. Zheng Xuan, for no apparent reason, calculates how high that would be: with the 50-unit target being ‘military’, i.e. ‘a footstep’, i.e. 1.2 *chi* off the ground, that places the 70-unit target at 10.533 *chi* and the 90-unit target at 22.533 *chi*. He doesn’t show his work, so we can’t know how this *ji* 計 ‘calculation’ was *performed*, but we can work backwards from

³³ Zheng Xuan offers the same gloss in *Yili zhushu*, 5.110a. See Jia Gongyan, below, on how the equivalency 1 footstep = 1.2 *chi* is derived.

³⁴ *Yili zhushu*, 7.187b–188b; cf. Steele (1917, vol. 1, pp. 150–151).

the numbers as illustrated in fig. 3 (and explained in the subcommentary, below) to say something about how Zheng Xuan structured the problem. Before we get to that, however, we should note that he seems to have structured it incorrectly.

In short, Zheng Xuan is calculating in *two dimensions*, as illustrated in fig. 3, as if the 50-, 70-, and 90-unit targets were on a single plane. Laid out in *three dimensions*, as illustrated laterally in fig. 4, one sees that the medium- and long-range targets need to be suspended considerably higher than Zheng Xuan's figures to meet the *Book of Etiquette and Ceremonial's* criteria of point-zone visibility. This is a problem, if at the very least because Zheng Xuan is contradicting himself, and we moderns are not the first to see it.³⁵ Were it anyone else – a Zheng Zhong, a Wang Su, or an Aristotle (see Chapter xx) – one might agree that he was wrong, but this is *Zheng Xuan*, so that is obviously not what happened.

2.2 What to do about the commentary

As concerns reception, the above passages fall within the purview of the lesser-known Jia Gongyan's subcommentaries. Jia Gongyan was working under and after the model of Kong Yingda, so it behoves us to ask how Kong Yingda might have dealt with this first. We need not guess, luckily, because Kong offers up these numbers unsolicited in *The Correct Meaning of the Record of Rites* where the base text moralises, quite simply, about how 'each shooter shoots at the swan respective his station'.³⁶ After sorting through the text as written, Kong begins rehashing equivalencies and the proportional relationship between bow-lengths, 'centres', 'frames', 'stalks', 'tongues', and the quantity of cloth employed, bringing us ultimately to the problem of the targets' heights:

其張三侯之躰，同道位之近者最下，遠者漸高。故『大射』云：「大侯之崇，見鵠於參，參見鵠於干，干不及地武。」以此計之，豸侯下畔去地尺二寸，豸侯之體上下躬及舌揔有四幅，凡廣八尺；侯中方一丈，是豸侯揔高一丈九尺二寸。

That which is positioned the closest in stretching the body of the three targets on the same path is the lowest down, and those more distant

³⁵ The first to point out Zheng Xuan's mistake here appears to have been Liu Chang 劉敞 (*jinshi* 1046), who complains that 'these [figures] are hard to take seriously, [as] they do not even agree with [the commentator's own] words!' 此之難信，不俟言矣，referring to the inadequacy of Zheng Xuan's proposed heights, and asking 'Why did he never try finding this via *gou-gu*?' 胡不嘗試以勾股求之 (*Gongshi Qijing xiaozhuan*, 14b). Note that the sample solution offered in fig. 4 is based on Liu Chang's suggestion, the example of *gou-gu* 勾股 'base and height' (i.e., similar triangles) problems cited from problem 9.22 of the *Nine Chapters*, below, and Liu's tentative figure that the height of the bottom edge of its swan from the ground, E'G' in fig. 4, should be $24\frac{4}{50}$ instead of 19.2 *chi*. Hao Jing 郝敬 (1558–1639) likewise describes Zheng Xuan's figures here as 'especially wrong' 尤非, citing similar charges of miscalculation (*Yili jiejie*, 7.3b).

³⁶ 射者各射己之鵠, *Liji zhushu*, 46.1017b; cf. Legge (1885, vol. 2, p. 451).

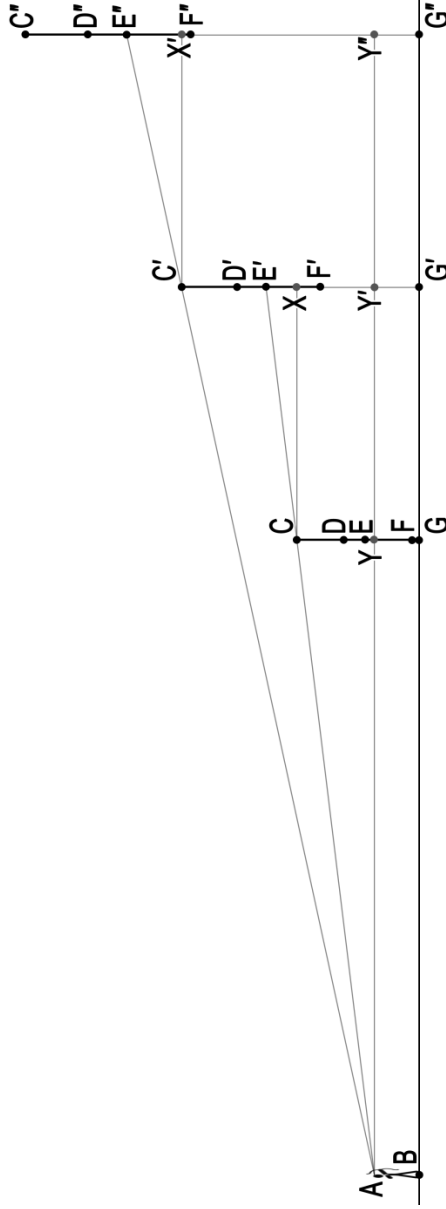


Fig. 4 Zheng Xuan's target geometries, lateral view, with targets (correctly) set at the appropriate distances (note that the y-axis is expanded 300% for the sake of legibility). Point a represents the archer's eye, and b the point directly beneath his eye on the ground. Points c and f represent the upper and lower edges of the target, d and e the upper and lower edges of the swan, and g the point directly beneath each target on the ground. Line AY'' runs parallel to the ground through the archer's eye, intersecting the plane of the respective targets at y , y' , and y'' . Line CX and CX' run parallel to the upper edge of the forward target to the plane of the target behind it. In racoon-dog paces, $BG = AY = 50$, $BG' = AY' = 70$, $BG'' = AY'' = 90$, and $G'G'' = 20$. As per problem 9.22 of the *Nine Chapters of Mathematical Procedures* (below), we assume $AB = YG = Y'G' = Y''G'' = 7$ *chi*. Zheng Xuan gives $CG = 19.2$ *chi* (fig. 2), so $CY = XY' = 19.2 - 7 = 12.2$ *chi*. ACY and $CE'X$ are similar triangles, where $E'X/CX = CY/AY$, so $E'X = 20 \times \frac{12.2}{50} = 4\frac{22}{25}$ *chi*, and $E'G' = 19\frac{1}{5} + 4\frac{22}{25} = 24\frac{2}{25}$ *chi*. As per fig. 2, $E'C' = 13\frac{1}{3}$ *chi*, and $E'F' = 8\frac{2}{3}$ *chi*, so $C'G' = 24\frac{2}{25} + 13\frac{1}{3} = 37\frac{31}{75}$ *chi*, and the distance $F'G'$ from the medium-range target to the ground is $24\frac{2}{25} - 8\frac{2}{3} = 15\frac{51}{75}$ *chi*. For the long-range target, $X'Y'' = C'Y' = 37\frac{31}{75} - 7 = 30\frac{31}{75}$ *chi*, $AY' = 70$ racoon-dog paces, $C'X' = 20$ racoon-dog paces, and since $AC'Y'$ and $C'E'X'$ are similar triangles, $E'X'/C'X' = C'Y'/AY'$, so $E'X' = 20 \times \frac{30\frac{31}{75}}{70} = 8\frac{362}{525}$ *chi*, and $E'G'' = 37\frac{31}{75} + 8\frac{362}{525} = 46\frac{54}{525}$ *chi*. As per fig. 2, $E''C'' = 16$ *chi*, and $E''F'' = 10$ *chi*, so the distance $F''G''$ from the long-range target to the ground is $46\frac{54}{525} - 10 = 36\frac{54}{525}$ *chi*. Q.E.D.

are progressively higher. Thus it is that the ‘Big Shoot’ says ‘The loftiness of the big target is such that it shows its swan vis-à-vis the three, the three shows its swan vis-à-vis the pole, and the pole falls short of the ground military’ (cited above). Calculated from this, the dog target’s lower edge is a *chi* and two *cun* from the ground; the upper and lower bodies and tongues of the dog target’s body comprise a total of four bolts, for a total width of eight *chi* ($2 \text{ chi/bolt} \times 4 \text{ bolts} = 8 \text{ chi}$); the target-centre is a square one *zhang* [to a side]; and thus is the dog target’s total height one *zhang* nine *chi* two *cun* ($12 + 80 + 100 = 192 \text{ cun}$).

此豸侯上畔去地一丈九尺二寸。以此，豸侯上畔，則是糝侯之鵠去地之數也。其糝侯下舌及躬，凡有四尺，糝侯之中，鵠下有三分之一。糝侯中方丈四，其三分之一得四尺六寸三分寸之二，是糝侯自鵠以下凡有八尺六寸三分寸之二，皆為豸侯所掩。豸侯上畔去地一丈九尺二寸，去其八尺六寸三分寸之二，是糝侯下畔去地一丈五寸三分寸之一。故鄭注『大射』云：「糝侯去地一丈五寸少半寸」也。云「少半寸」，則三分寸之一也。

This means that the dog target’s upper edge is one *zhang* nine *chi* two *cun* from the ground. Following this, the dog target’s upper edge is then the number [of *chi*] that the blended target’s swan is from the ground. The blended target’s lower tongue and body comprise a total of four *chi*, and as to [its] centre, one third is below the swan. The blended target centre is a square a *zhang* and four *chi* [to a side], one third of which gets you four *chi* six *cun* and two thirds *cun*, and thus, beneath [its] swan, is there a total of eight *chi* six *cun* and two thirds *cun*, all of which is covered up by the dog target. The dog target’s upper edge is one *zhang* nine *chi* two *cun* from the ground; removing eight *chi* six *cun* and two thirds *cun*, the blended target’s lower edge is thus one *zhang* five *cun* and one third *cun* from the ground ($192 - 86\frac{2}{3} \text{ cun} = 105\frac{1}{3} \text{ cun}$). Thus does Zheng’s commentary to the ‘Big Shoot’ say ‘the blended target is one *zhang* five *cun* and a small-half *cun* from the ground’. When he says ‘a small-half *cun*’, this is one third *cun*.

豸侯既去地一丈九尺二寸，更加糝侯之中三分之二，九尺三寸三分寸之一，又加糝侯上躬與个四尺，則糝侯上畔去地三丈二尺五寸三分寸之一，則是大侯鵠下之數也。其大侯，下舌及躬并侯中三分之一，摠一丈，為糝侯所掩。於三丈二尺五寸三分寸之一減其一丈，則是大侯下畔去地之數。故鄭注『大射』云：「大侯去地二丈二尺五寸少半寸」也。

Since the dog target is one *zhang* nine *chi* two *cun* from the ground, one adds to this the other two thirds of the blended target’s centre, nine *chi* three *cun* and one third *cun*, and one furthermore adds the four *chi* of the frame and stock above [it], [finding] thus that the blended target’s upper edge is three *zhang* two *chi* five *cun* and one third *cun* from the ground ($192 + 2 \times 46\frac{2}{3} + 40 = 325\frac{1}{3} \text{ cun}$), which, in turn, is the number for [everything] below the big target’s swan. For the big target, the bottom tongue and frame are combined with one third of the target-centre, totalling one *zhang* ($4 + 18 \div 3 = 10 \text{ chi}$), which is covered up by the blended target. Diminishing three *zhang* two *chi* five *cun* and one third *cun* by one *zhang*, this [gives you] the number that the big target’s lower edge is from the ground ($325\frac{1}{3} -$

100 = 225⅓ *cun*). Thus does Zheng's commentary to the 'Big Shoot' say 'the big target is two *zhang* two *chi* five *cun* and a small-half *cun* from the ground'.³⁷

Kong Yingda is attempting to show Zheng Xuan's work. More specifically, he is appending a mathematical proof for Zheng Xuan's numbers from the *Book of Etiquette and Ceremonial* onto what is the nearest passage that he could find in the *Record of Rites*. This does not need to be here, so there is clearly something at stake, and, whatever it is, Kong uses words like *shi* 是 'is/truly' and *gu* 故 'thus' to transform Zheng Xuan's mistaken sums by way of arithmetic performance into 'The Correct Meaning' of this other Classic.³⁸

Returning to the *Book of Etiquette and Ceremonial*, one notes that Jia Gongyan's subcommentary is a little more circumspect about these numbers. Rather than what 'is' 'thus' 'truly' so, Jia tends to frame his questions in terms of how it is that Zheng Xuan 'knows' (*zhi* 知), e.g.:

云「中人之足長尺二寸」者，無正文，以目驗而知。

As to 'The length of the average human foot is a *chi* and two *cun*' (cited above), this is not in the scripture/s, so it is known by visual inspection.³⁹

Jia Gongyan goes once again through the maths to explain why 'thus is it said' 故云 and, more importantly, 'thus is it known' 故知 of every step from the length of the range to the fatuous target-heights. He could have copy-and-pasted from Kong Yingda, one imagines, but instead he started over. This we can say because Jia Gongyan works in a different order through the numbers and the citations than does Kong Yingda's proof, and because each has their own way of dividing and subtracting compound numbers.⁴⁰ Kong Yingda, as shown in the bracketed equations above, would appear to operating directly on quantities reduced to an integer and fractional number of *cun*; Jia Gongyan, by contrast, separates out the units to deal with them one by one. Take for example his treatment of the 70-bow target:

糝侯，侯中丈四尺，中上、中下各四尺，得八尺，并之，二丈二尺也。鵠居侯中三分之一，則鵠下亦有四尺六寸大半寸，通躬身四尺為八尺六寸三分寸之二矣。張法，糝鵠下畔與豻侯之上綱齊，所謂「見鵠於豻」，自餘糝侯鵠下畔八尺六寸大半寸，在掩豻侯，亦如之，豻侯上豻本去地丈九尺二寸，直掩八尺，上有一丈一尺二寸，在復掩六寸上有一丈六寸，在復掩三分寸二，唯有一丈五寸三分寸一，在少半寸者即三分寸一也。言大半寸者，即三分寸二也。故知糝侯下綱去地一丈五寸少半寸也。

³⁷ *Liji zhushu*, 46.1018b (subcomm.).

³⁸ In this sense, Zheng Xuan's subcommentators are engaging in a practice of 'proof' reminiscent of – though distinct in orientation from – that with which commentators to the mathematical canon are primarily concerned, on which see Chemla and Zhu's contribution to the present volume.

³⁹ *Yili zhushu*, 7.188b (subcomm.).

⁴⁰ On the different cultures of computation witnessed both within and between Classical commentary and *suan* 算 mathematical writings, see Chemla and Zhu's contribution to the present volume.

For the blended target, the target-centre is a *zhang* and four *chi* [to a side]. Both above the centre and below the centre are four *chi*, which gets you eight *chi*. Combine these, and that is two *zhang* two *chi* ($14 + 2 \times 4 = 22$ *chi*). The swan occupies one third of the target-centre, so below the swan there is also four *chi* six *cun* and a big-half *cun* (cited above), which, connected with the four *chi* of the frame-body, makes eight *chi* six *cun* and two thirds *cun* (4 *chi* $6\frac{2}{3}$ *cun* $+ 4$ *chi* $= 8$ *chi* $6\frac{2}{3}$ *cun*). As to the rule for [target-]stretching, the blended swan's bottom edge is [to be] level with the upper mainstay of the dog[-skin] target – what is referred to as ‘showing its swan vis-à-vis the dog’ (cited above). The remaining eight *chi* six *cun* and a big-half *cun* from the lower edge of the blended target's swan [on down] is covered up by the dog[-skin] target. Likewise as such (?) is the dog[-skin] target's upper dog (sic.) a *zhang* and nine *chi* two *cun* from the ground. This directly covers up eight *chi*, [so] there is still (lit. ‘above’) one *zhang* one *chi* two *cun* (19 *chi* 2 *cun* $- 8$ *chi* $= 11$ *chi* 2 *cun*). Here, it covers up another six *cun*, [so] there is one *zhang* six *cun* (11 *chi* 2 *cun* $- 6$ *cun* $= 10$ *chi* 6 *cun*) above. Here, it covers up another two thirds *cun*, [so] there is only one *zhang* five *cun* and one third *cun* (10 *chi* 6 *cun* $- \frac{2}{3}$ *cun* $= 10$ *chi* $5\frac{1}{3}$ *cun*), considering, [that is], that a ‘small-half *cun*’ is one third *cun*, and that the expression ‘big-half *cun*’ means two thirds *cun*. Thus is it known that the blended target's lower mainstay is ‘one *zhang* five *cun* and a small-half *cun* from the ground’ (cited above).⁴¹

They may have got there by different means, but Jia Gongyan and Kong Yingda ultimately arrive at the same values as does Zheng Xuan – heights that assume that a target at seventy bow-lengths is on a plane with the one at fifty. Where Kong Yingda seems to take this for granted, however, Jia Gongyan does conclude his calculations with a bit of remove:

故注於此數也。

And thus it is that the commentary arrives at these numbers.⁴²

In Chapter **xx**, Chemla and Zhu offer, in general terms, drawing on a wider selection of base texts, exegetes, and genres, that where *shu* 疏 subcommentary on mathematical and Classical literature is equally concerned with proving the primary commentary correct, the focus of the former is the *shu* 術 ‘procedure’ invoked to solve a given problem, while that of the latter is the *shu* 數 ‘numbers’ of the result. That may be, but the aim of my case study in this chapter is to observe what happens when things go wrong – to see which of his children, so to speak, the exegete runs first to save from the burning building – and, in that sense, Kong Yingda and Jia Gongyan emerge from this looking more like individuals with different underlying goals. Kong Yingda goes out of his way to insist that Zheng Xuan is right about these target heights – out of the way as in inserting his proof of this into a Classic in which neither the base text nor primary commentary actual-

⁴¹ *Yili zhushu*, 7.188b–189a (subcomm.).

⁴² *Yili zhushu*, 7.189a (subcomm.).

ly broach the subject. Jia Gongyan, on the other hand, would seem to be shirking from his duty: unlike Kong Yingda, he is *forced* to deal with these numbers, and where he is, he seems to skirt around the question of their absolute truth.

That at least is how things seem – *suspicious* – but there is more to be said about what is going on between Jia Gongyan, Kong Yingda, Zheng Xuan, and the base text if we know the right questions to ask of our sources. It may seem trite, but one of the most fruitful questions we could probably think to ask is how well we can expect our subjects to have known what they were talking about. Zheng Xuan’s error would be one thing, for example, if the Shoot was but a Classicist fantasy with no bearing on contemporary reality; it would be quite another, however, if it were a celebrated annual event centred on Zheng Xuan’s place of birth. Likewise, his canonisers’ treatment of this error would be one thing if we knew them to be the barely-numerate products of a specialist ‘humanities’ commentarial culture and, thus, ill-equipped to identify Zheng Xuan’s calculations here as problematic; it would be quite another, however, if it turned out that everyone we’ve met so far was an avid student of higher-level mathematics. We are at a distinct advantage in terms of the context we are able to supply to our Chinese subjects’ exegetical work compared, for example, to Sanskrit authors like Pṛthūdaka (fl. 860 CE) and Brahmagupta (c.598–665 CE) in Chapter [xx](#), so allow us to luxuriate a little in their world and see to what degree it resembled the one of which they are writing.

3. The irrelevance of practice

Zheng Xuan’s target geometries are unrealistic, and it is not just that they fail to take a target’s distance into account. The numbers invoked are absolutely enormous. At the equivalent of 17 m², 27 m², and 38 m², respectively, we are talking here about *sails*, the largest of which is to be foisted 11 m in the air. Wind would make it nigh impossible to keep the targets still, let alone erect, let alone *overnight*, as the *Book of Etiquette and Ceremonial* dictates.⁴³ The numbers are, at the same time, infinitesimally small. To get that perfect $3\frac{1}{3} \times 3\frac{1}{3}$ *chi* ‘swan’, for example, one would need to shave $6\frac{2}{3}$ *cun* off the edge of the ‘modern official cloth bolt’. That would be optimistic, considering that contemporary *chi*-rules were graduated *decimally*, in *cun* and *fen* ($\frac{1}{10}$ *cun*), and that even the makers of armillary spheres tended to round such precious fractions off completely.⁴⁴ If Zheng Xuan were able to ascertain the length of the average human foot by ‘visual inspection’, as Jia

⁴³ According to the Classic, one not only erects the targets *three days in advance*, ‘THE BOTTOM LEFT MAINSTAY IS LEFT UNAFFIXED, TIED [to the opposite corner] SUCH THAT THE CENTRE IS COVERED UP’ 不繫左下綱，中掩束之 (*Yili zhushu*, 5.110a, 7.188a), it being only just before the shooting commences that ‘THE DISCIPLES UNDO THE TIE AND THEN AFFIX THE BOTTOM RIGHT MAINSTAY’ 弟子說束，遂繫左下綱 (*ibid.*, 5.118b). Tellingly, the Classic does not mention the disciples using ladders or climbing the poles to accomplish this.

⁴⁴ For a catalogue of historical *chi*-rules, see Qiu Guangming (1992). On armillary sphere graduation, see Morgan (2017, 77–85).

Gongyan alleges, one wonders if there might have been yet other things ‘not in the scripture/s’ by which he may have familiarised himself with the practicalities of a real-world target range. Indeed, one wonders more specifically if the *Big* and *District Shoot* might not still/ever have been held in Zheng Xuan’s day...

They were. Sima Biao’s 司馬彪 (c.240–c.360 CE) ‘Monograph on the Hundred Offices’ (Baiguan zhi 百官志) includes the Big Shoot under the responsibilities of the Eastern Han (25–220 CE) Ministry of Rites (Taichang 太常), the Bright Hall, whose Circular Moat in which it was held, falling under the administration of its subsidiary, the Office of the Grand Clerk (Taishi ling 太史令).⁴⁵ Sadly, the Circular Moat was abandoned along with the capital in 190 CE, when Zheng Xuan was sixty-three, following its sack in Dong Zhuo’s 董卓 (d. 192 CE) coup d’état of the previous year. Prior to that, however, we have dated historical records of Big and District Shoots having been held at the Luoyang Circular Moat going back to its opening in 59 CE:

永平二年，三雍初成，拜榮為五更。每大射、養老禮畢，帝輒引榮及弟子升堂，執經自為下說。

Eternal Tranquillity, year 2, the Three Moats (i.e., the Bright Hall, Circular Moat, and Observatory) were finally finished, and [Huan] Rong 桓榮 (d. 93 CE) was inducted into the Fifth Watch.⁴⁶ At the end of every Big Shoot and Elder-care rite [held here in the spring], the Emperor would invariably conduct Rong and his/the disciples to ascend with him into the hall, where, Classic in Hand, he would personally explain it to [the audience] below.⁴⁷

The architect of these particular rites was Cao Bao 曹褒 (d. 102 CE), a then young *boshi* 博士 ‘academician’ chosen to fix the ceremonial appropriate the new ‘Three Moats’ complex upon its commission in 56 CE. The Imperial Academy itself having been rebuilt only in 29 CE, Cao’s task would have been difficult given all that had been lost in the wake of the agrarian rebellions, civil war, and the sacking of the old capital at Chang’an in the later years of Wang Mang 王莽 (r. 9–23). That said, Cao was a master marksman, an expert in the Ritual Classics, and, most importantly, his father’s son.⁴⁸

Cao’s father, Chong 充 (fl. 25/56 CE), was a disciple of Qing Pu 慶普, who, in turn, studied the rites from Hou Cang 后蒼 (fl. 70 BCE) alongside Dai De 戴德 and his nephew, Sheng 聖. It is Qing Pu’s co-

⁴⁵ *Hou Han shu*, zhi 25, 3571–3572.

⁴⁶ The ‘Fifth Watch’ 五更 is a moniker for the venerable elders paid homage alongside the ‘Thrice Venerable’ 三老 at the Elder-care 養老 rites associated with the Han Big and District Shoots, the term being a reference to the fifth of the five watches into which the night was divided – someone, in modern terms, in the ‘last hour’ of the human life cycle.

⁴⁷ *Hou Han shu*, 37.1253; cf. *ibid.*, 2.102, 69A.2545, zhi 4, 3108, and *Jin shu*, 21.670. One finds further references to Big and District Shoots having been held at the Luoyang Circular Moat in connection with the Elder-care rites as coincide with imperial visits in 65 CE (*Hou Han shu*, 2.111), 97 CE (*ibid.*, 50.1668), 102 CE (*ibid.*, 4.189), and 132 CE (*ibid.*, 6.260). To this one may reasonably add the non-site-specific reference to a another Big Shoot held in 112 CE (*ibid.*, 44.1504).

⁴⁸ For Cao Bao’s biography, see *Hou Han shu*, 35.1201–1205.

hort to whom we owe the *Record of Rites*, the uncle–nephew pair having compiled its modern ‘Big’ and ‘Little Dai’ recensions from free-floating texts in connection with Liu Xiang 劉向 (79–8 BCE) and Liu Xin’s 劉歆 (c.50 BCE–23 CE) famous overhaul of the Imperial Library in 26–6 BCE.⁴⁹ The *Rites of Zhou* and *Book of Etiquette and Ceremonial* also first appear in the written record at around the time of Hou Cang.⁵⁰ The origins of the former are somewhat nebulous, being as it first appears in King Xian of Hejian’s 河間獻王 (r. 155–129 BCE) round-up of ‘old ancient-text pre-Qin (< 221 BCE) writings’,⁵¹ having survived the supposed bibliocaust of the Han’s imperial predecessors. The ‘Seventeen Chapters’ 十七篇 of what would become the latter, on the other hand, are reported by contemporaneous scholars as having been transmitted into Han times by Hou Cang’s master’s master’s master, Gaotang Sheng 高堂生.⁵² There may have been a several-decade interruption in the royal rites, therefore, but the Big Shoot held in 59 CE goes back father–son, master–disciple, to the very men who committed the Ritual Classics to their present written form.⁵³

It is important to remember, as concerns the Bright Hall, that the Ritual Classics took shape in a context in which the Han court was still sorting out its ritual programme. Its ‘institution’ (*zhi*) was oft debated, as we saw in the Introduction, but the first to actually *build* one was Emperor Wu (r. 141–87 BCE), and this went rather poorly. The first ever Bright Hall was planned for the southern suburbs of Chang’an in 141 BCE. The project soon met with allegations of corruption and resistance from the empress dowager’s clique, however, and Emperor Wu only managed to get one built some thirty years later as a sort of support structure for the *feng-shan* 封禪 complex at Mount Tai – a complex 800 km east of Chang’an built by the hated First Emperor of Qin (r. 221–210 BCE). The site saw little use during Emperor Wu’s reign, and it was abandoned soon thereafter amid his predecessors’ budgetary roll-backs. It is in the decades to follow, coincidentally, that Ritual Classics describing how one is to build a *real* Bright Hall emerge in scholastic discourse, and it is lead-editor Liu Xin, coincidentally, who is ultimately charged with its realisation. This one is built for the *regent*, Wang Mang, who would poison the child-emperor un-

⁴⁹ On the process by which a free-floating text extant in late-fourth-century BCE tombs like the *Ziyi* 緇衣 was transformed in the process of its first-century BCE incorporation into the *Record of Rites*, see Shaughnessy (2006, 1–130) and Wang E (2007).

⁵⁰ The first solid evidence of the *Offices* or *Rites of Zhou* is a citation in Sima Qian’s 司馬遷 (c.145–c.86 BCE) *Shiji*, 28.1357, and that of the *Book of Etiquette and Ceremonial* is a manuscript from Mozuizi 磨嘴子 tomb 6, sealed *terminus post quem* 28 BCE, for which see *Wuwei Han jian*.

⁵¹ 古文先秦舊書, *Han shu*, 53.2410.

⁵² See *Shiji*, 121.3126; cf. Ban Gu’s 班固 (32–92 CE) *Han shu*, 30.1710; cf. Wang Chong’s 王充 (27–c.100 CE) *Lunheng*, 12.560–561. Note that a different, ‘old-text’ version later attributed to King Xian’s book collecting also pops up in Liu Xiang and Xin’s overhaul of the Imperial Library in 26–6 BCE.

⁵³ On the Gaotang Sheng–Xiao Fen 丘蕭–Meng Qing 孟卿–Hou Cang lineage and the transmission and codification of the Rites in Han times, see *Shiji*, 121.3126, *Han shu*, 30.1710, 88.3615, *Hou Han shu*, 79B.2576; cf. Loewe (1993, 25–27, 237–239, 293–295).

der his care in 5 CE to preside *as king* over its opening ceremony in the following year, formally deposing Emperor Ping's (b. 9 BCE; r. 1 BCE–5 CE) successor, 'Baby Ying' (b. 5CE), in 9 CE. This, the first 'Classical' Bright Hall would soon find itself converted into a fantastical ancestral temple illustrating Wang Mang to have descended from ancient god-kings and, in 23 CE, the year its regretful architect committed suicide, razed to the ground by one several warring rebel armies.⁵⁴

Such were the political realities of the Bright Hall when the Ritual Classics took their present written form, and it is difficult to imagine that they failed to impress themselves upon the memory, selection, and editorial decisions of Cao Bao's intellectual processors. The same could be said of the Shoot, though the circumstances there were wholly different. The Big Shoot, for one, was considerably older and less contested a symbol of imperial legitimacy than was its ultimate venue. It may have been Wang Mang, who first conducted the Big Shoot and Elder-care rites in their (newly) Classically-appointed venue in 6 CE,⁵⁵ but all he really did was move them from where they were previously being held. As to where that was, Jia Shan's 賈山 memorial to Emperor Wen (r. 179–157 BCE) mentions how 'one cares for the Thrice Venerable at the Imperial Academy',⁵⁶ and the record of a bird-omen in 19 BCE has 'the academicians conducting the Big Shoot rite when a flying pheasant [suddenly] alit upon the courtyard, mounted the stairs one by one, and crowed from atop the hall'.⁵⁷ This would seem to place the Big Shoot *at* the Imperial Academy, but later commentators cite instead Bend Terrace as its official venue prior to 6 CE.⁵⁸ Wherever it was held, it was *being held*, and it was sufficiently important for Yuqiu Shouwang 吾丘壽王 to raise it as the centrepiece of his argument against bow control legislation in 124 BCE:

禮曰：「男子生，桑弧、蓬矢以舉之，明示有事也」。孔子曰：「吾何執？執射乎？」大射之禮，自天子降及庶人，三代之道

⁵⁴ On the history of Bright Hall construction, see the sources listed in Note 4.

⁵⁵ Namely, 'In Occupying Regent year 1, month I (17 Feb–19 Mar 6 CE), [Wang] Mang offered sacrifice to the Thearch on High in the southern suburbs, welcomed the spring in the eastern suburbs, conducted the Big Shoot rite at the Bright Hall, and left after the rites of caring for the Thrice Venerable and Fifth Watch were finished' 居攝元年正月，莽祀上帝於南郊，迎春於東郊，行大射禮於明堂，養三老五更，成禮而去 (*Han shu*, 99A.4082). The only connection of Emperor Wu's Bright Hall of c.110 BCE with archery is that the emperor was to shoot the sacrificial cow to be used in the larger *feng-shan* sacrifices and, in preparation, 'commanded various scholastics to practice shooting cows' 令諸儒習射牛 (*Shiji*, 12.473, 28.1397).

⁵⁶ 養三老於大學, cited in *Han shu*, 51.2330; cf. *Shiji*, 24.1230.

⁵⁷ 博士行大射禮，有飛雉集于庭，歷階登堂而雉, *Han shu*, 27.1417.

⁵⁸ Bend Terrace (Qutai 曲臺) is variously identified as the Qin imperial palace in Xianyang (*Han shu*, 51.2338), a publically-accessible roadside terrace in Chang'an (ibid., 51.2364 (comm.)), a hall in the Han imperial palace at Chang'an (ibid.), and the son of heaven's shooting palace (ibid., 30.1710 (comm.)). In *Han shu*, 30.1710 (comm.), Ru Chun 如淳 (third century CE) tells us that 'ritual archery was conducted at Bend Terrace' 行禮射於曲臺, citing Hou Cang's *Qutai ji* 曲臺記 and Wang Long's 王隆 (fl. 25/56 CE) *Han guan* 漢官, both of which are now lost, while Jin Zhuo 晉灼 (third/fifth century CE), oddly, adds that '[when?] the Western Capital had no Imperial Academy, the rites were conducted here' 於此行禮也.

也……竊以為無益於禁姦，而廢先王之典，使學者不得習行其禮，大不便。

The Rites say, ‘When a male child is born, one hangs up a mulberry bow and fleabane arrows as clear notice that such is his task’. Confucius says, ‘What shall I pursue? Shall I pursue archery?’ The rite of the Big Shoot reaches from the son of heaven down to the commoner and is the *dao* of the Three Dynasties (Xia, Shang, and Zhou). ... It is my humble opinion that [bow control] would do nothing to prohibit wickedness and that to abolish the former kings’ canons of state would make it impossible for scholars to practice and conduct the relevant rites, posing a major inconvenience.⁵⁹

As to the canons of state, one notes, the *Records of the Grand Clerk* (91 BCE) tell us that the Big Shoot was canonised into Han imperial ceremonial at the very beginning of the dynasty by the surrendered Qin ritual specialist Shusun Tong 叔孫通 (d. c.188 BCE).⁶⁰ As to how Shusun knew to conduct this rite, lastly, Shusun, like Gaotang Sheng, Hou Cang, Qing Pu, Cao Chong, and his son Bao, was a man of Lu, and it is there, the *Records of the Grand Clerk* tell us, that this particular rite had survived from some higher antiquity:

魯世世相傳以歲時奉祠孔子冢，而諸儒亦講禮鄉飲、大射於孔子冢。孔子冢大一頃。故所居堂弟子內，後世因廟藏孔子衣冠琴車書，至于漢二百餘年不絕。

[The people of] Lu continued generation upon generation the tradition of holding annual and seasonal cult at Confucius’ tomb, [where] scholars, for their part, held lectures on the rites of the District Libation and Big Shoot. Confucius’ tomb was one *qing* 頃 (6.7 hectares) in size. Later generations of his disciples converted the hall in which he formerly lived into a temple wherein were stored Confucius’ robes, caps, zither, chariot, and books. This [cult] continued uninterrupted for more than two hundred years up to the Han.⁶¹

And that is what marks the Shoot as different from other ‘institutions’ described in the later Confucian Classics: the Shoot was *historical practice*, and, *historically*, this royal right wasn’t bound to a single venue or even the presence of a temporal king.⁶² Nor for that matter was the Shoot even bound to a single ideology. The *Book of Han*, in fact, identifies the practice with Confucius’ principle rival:

墨家者流，蓋出於清廟之守。茅屋采椽，是以貴儉；養三老五更，是以兼愛；選士大射，是以上賢。

The Mohist school likely emerged from the guardians of the Pure Temple (to King Wen of Zhou (r. 1099/1056–1050 BCE)). They [lived

⁵⁹ *Han shu*, 34A.2796–2797; tr. modified from Sanft (2008, 155–158).

⁶⁰ See *Shiji*, 121.3117; cf. *Han shu*, 88.3592. For Shusun Tong’s biography, see *Shiji*, 99.2720–2727, cf. *Han shu*, 43.2124–2131.

⁶¹ *Shiji*, 47.1945. On archery and archery rites as evidenced in even earlier sources, see Adamski (2012) and Yuan Junjie (2013).

⁶² As a dukedom under nominal fealty to the Eastern Zhou (771–256/249 BCE) prior to its annexation by Chu in 249 BCE and Qin in 223 BCE, Lu was beyond its rights to host ceremonies reserved for the Zhou kings except, perhaps, for the spiritual presence of the ‘plainclothes king’ (Note 2) at the site of his post-mortem cult.

in] reed huts with oak rafters, and that is how they practised ‘the veneration of poverty’. They cared for the Thrice Venerable and Fifth Watch, and that is how they practised ‘universal love’. They selected gentleman-knights at the Big Shoot, and that is how they practised ‘the exaltation of the virtuous’.⁶³

The relative diffusion and transferability of these rites is all the more apparent as concerns the District Shoot, of course, as the District Shoot was normally a local, community-sponsored event. Some thirty years before the Luoyang Bright Hall opened its doors, for example, we read about governor of Lu Commandery in 28/30 CE ‘gathering everyone to conduct the rite of the District Shoot’ in a plot to lure local rebels into custody,⁶⁴ while next door, in 76 CE, the governor of Shanyang ‘held a District Shoot every spring and autumn’ because he believed in ‘teaching people through the rites rather than by resort to punishment and fines’.⁶⁵ On the other side of the empire, in modern Chengdu, we read about Yang Hou’s 楊厚 (72–153 CE) disciples dedicating a temple to his memory, where ‘the commandery instructor and clerk held a District Shoot in the spring and autumn to offer him eternal cult’.⁶⁶ And after Luoyang was sacked in 189 CE, and the Big Shoot subsequently abandoned for several centuries, one continues to see District Shoot pop up in places as far apart as Wu, on the verdant Southeast Seaboard, and Dunhuang, in the lifeless sands leading out into Central Asia.⁶⁷

Zheng Xuan too was a man of Lu; he was an educated, well-travelled man with a marked interest in ritual who had studied at the Imperial Academy (below) with the very men who hosted and participated in the royal Shoot. True, we have no roster attesting his to personal attendance thereat, but though he might write like it, it is hard to imagine that he was totally oblivious to contemporary archery culture. True, it is equally unclear from our sources how the Big and District Shoot as practiced c.406 BCE–200 CE may have varied from the Ritual Classics, let alone from one time and from one place to another. Pictorial evidence from Eastern Zhou (771–256/249 BCE) bronzeware does confirm the use of targets of roughly the same shape as those described in the Ritual Classics, of course, but the scale appears a little more reasonable than that in fig. 3 and so too does the apparent use of some sort of backstop (fig. 5).⁶⁸ Still, for everything that we do not

⁶³ *Han shu*, 30.1738. Note that ‘the veneration of poverty’ 貴儉, ‘universal love’ 兼愛, ‘the exaltation of the virtuous’ 尚賢, etc., are the chapter titles and central tenants of the *Mohist Canon*. On *jia* 家 ‘families’ and/as ‘schools’ as read onto pre-Qin philosophy by later sources like the *Book of Han*, see Csikszentmihalyi and Nylan (2003) and Brashier (2011, 6–34).

⁶⁴ 會人眾，修鄉射之禮，*Hou Han shu*, 29.1019.

⁶⁵ 以禮訓人，不任刑罰……每春秋饗射，*Hou Han shu*, 76.2467.

⁶⁶ 門人為立廟，郡文學掾史春秋饗射常祠之，*Hou Han shu*, 30A.1050.

⁶⁷ See *Song shu*, 57.1583, and *Jin shu*, 94.2449, 109.2826.

⁶⁸ ‘Proficiency Ordinance no 45’ 功令第卅五 of the Han code prescribes the use of *zhun* 埴 ‘embankments’ or ‘butts’ in the annual archery proficiency test for military officers, for which see Loewe (1967, vol. 1, p. 118, vol. 2, pp. 50–60, and Li Junming (2009, 209–210). I thank Thies Staack for bringing these sources to my attention. Later versions of the Shoot likewise mention the use of *lie* 埴 ‘embankments’,

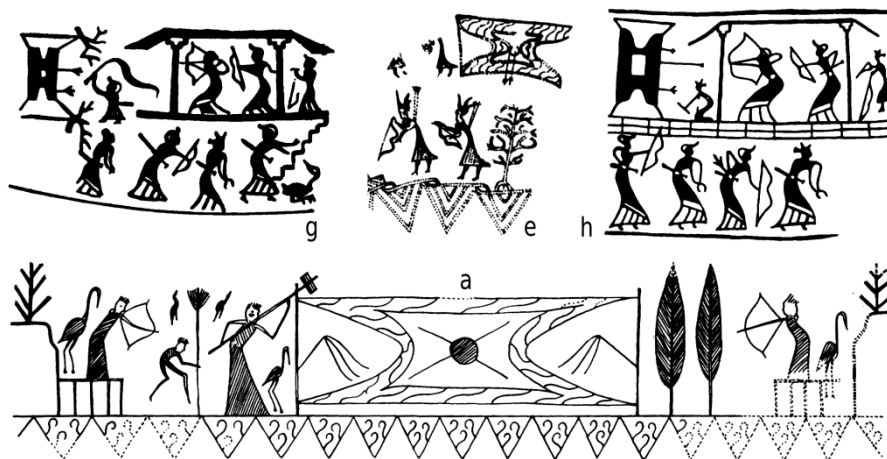


Fig. 5 Archery targets as illustrated in Eastern Zhou (771–256/249 BCE) bronzeware. Source: Weber (1968, fig. 77).

know for certain, there is one thing that we do: no real-world measurer, furrier, woodworker, or fabric-and-chariot officer could possibly realise Zheng Xuan’s specifications – not to a precision of $\frac{1}{3}$ *cun*, not without one target blocking the other’s swan, and not without wind posing a noteworthy challenge. If Zheng Xuan’s numbers are physically impossible, then they clearly aren’t the product of ‘visual inspection’, but neither, if we recall, are they anywhere to be found in the Classics.

Zheng Xuan’s problem is of his own creation, as too are most of the values featured therein. If the grounds for ‘swallow-swans’ seemed tenuous, as insinuated earlier, how much more so the leap from ‘the three shows its swan vis-à-vis (i.e., over) the pole’ 參見鵠於干 to ‘the blended swan’s bottom edge is [to be] level with the upper mainstay of the dog[-skin] target’ 糝鵠下畔與豸侯之上綱齊 (Jia Gongyan, above)? Why is *that* how anyone would read this passage? As to the numbers, most of what the base text gives us are vague, anthropomorphic, and object-specific measures linked by such ambiguous phrases as ‘bow two *cun* makes the target-centre’. The base text’s instructions are most likely linked to real-world practice: it is probably not a coincidence, for example, that it is the rope that is measured in arm-spans and that it is distances on (and a little off) the ground that are measured with one’s feet. Such units are contingent, variable, imprecise, and incommutable, but they tend to suffice for the purposes of shooting: one still counted *paces* in nineteenth-century pistol duels, for instance, rather than the two sides carrying tape measures. Zheng Xuan, however, would clearly prefer that everyone used a tape measure, because he converts *everything* into commensurable units of *zhang*, *chi*, and *cun*. He is not converting here between day lengths measured in 1/360s (UŠ) vs 1/6s (MA.NA), as examined in Chapter xx, but treating ‘bows’, ‘arm-spans’, ‘raccoon-dog paces’, and ‘militaries’ as if they were distinct, convertible units from which the

e.g. *Sui shu*, 8.165–166. For a speculative reconstruction of the backstop featured in fig. 4, see Selby (2006, 110–111).

geometer might squeeze precise values upon which to operate. As such, if we are to understand how Zheng Xuan's commentary arrives at the target heights it does, it should now be clear that ancient Classics and contemporary sports are the wrong places to be looking.

4. The imperative of maths

The precise height at which the mid- and long-range targets must be suspended for the lower edges of their respective swans to align perfectly with the upper edge of the preceding target is not a question with which the Ritual Classics think to bother. And why would they? In practice, the World Archery Federation allows for a slop of ± 5 cm (2.16 *cun*) in height for a $\varnothing 122$ cm ($\varnothing 52.81$ *cun*) target at 70 m (50.5 racoon-dog paces), because, in practice, nothing is *perfect*.⁶⁹ And in setting up the Big Shoot, one imagines, it would simply suffice to shout 'a little bit higher' to the men downfield until the mid- and long-range targets fell into proper visual alignment from the stand. Still, Zheng Xuan wants to know *how high*, to the precision of $\frac{1}{3}$ *cun* (7.7 mm), and his canonisers want us to know how Zheng Xuan knows. At that, we have stepped with Zheng Xuan beyond the wind-blown world of the here-and-now and into a higher plane. This other world, whither we have been fittingly conveyed in the name of ritual, is one in which perfection alone may be assumed – a world of forms outside of time around which the dimensions of space may be losslessly expanded and collapsed. This other world is one in which animals render themselves to customs posts to discuss and pay taxes on the respective value of their furs;⁷⁰ it is one in which a district may conscript a gang of $135 \frac{11637}{12175}$ corvée labourers,⁷¹ and it is one in which rope could just as easily be measured in feet, and a walking distance in hands. This – the Alice-in-Wonderland world of *suan* 算 mathematics – is the *only world* in which an entire dimension of space can go accidentally missing from a shooting range.⁷²

This, Zheng Xuan's mistake, is by no means due to the limitations of contemporary knowledge. Indeed, problem 9.22 of the *Nine Chapters of Mathematical Procedures* gives us a procedure eminently suited to this category of problem:

⁶⁹ World Archery Federation interactive rulebook, book 2, p. 31 (<https://worldarchery.org/Rules>; accessed 26 June 2017).

⁷⁰ 'A fox, a racoon dog, and a dog pass through a customs post; they are taxed a hundred and eleven cash. The dog says to the racoon dog, and the racoon dog says to the fox, "Your skin is worth twice mine; you should pay twice as much tax!"' 狐、狸、犬出關，租百一十一錢。犬謂狸、（狸）謂狐：「而（爾）皮倍我，出租當倍戔（哉）！」, *Suan shu shu* 算數書 (MS, *terminus ante quem* c.186 BCE), slip 34, in *Zhangjiashan Han mu zhujian (ersiqi hao mu)*; tr. modified from Cullen (2004, 45).

⁷¹ See problem 3.5 of the *Nine Chapters of Mathematical Procedures* in Chemla and Guo (2004, 288–290).

⁷² On recognising abstraction, generality, and philosophical reflections in a mathematical tradition often labelled 'practical' (in contrast to its 'theoretical' Western counterpart), see Chemla (2003; 2010) and Note 38.

今有山居木西，不知其高。山去木五十三里，木高九丈五尺。人立木東三里，望木末適與山峰斜平。人目高七尺。問山高幾何？

Suppose there is, to the west of a tree, a mountain of unknown height, that the distance from the tree to the mountain is fifty-three *li* 里, that the height of the tree is nine *zhang* five *chi*, and that a person standing three *li* east of the tree sees the tip of the tree and peak of the mountain as exactly level on the same oblique. If the height of the person's eye is seven *chi*, we ask what is the height, then, of the mountain?

答曰：一百六十四丈九尺六寸、太半寸。

Answer: one hundred and sixty-four *zhang* nine *chi* six *cun* and a big-half *cun* ($1649\frac{2}{3}$ *chi*).

術曰：置木高減人目高七尺，餘，以乘五十三里為實。以人去木三里為法。實如法而一，所得，加木高，即山高。

Procedure: set down the height of the tree and diminish it by the seven *chi* of the height of the person's eye ($95 - 7 = 88$). Take what remains and multiply it by the fifty-three *li* to make the dividend ($88 \times 53 = 4664$). Take the three-*li* distance from person to tree as the divisor. Divide the dividend by the divisor, and what you get is added to the height of the tree to give you the height of the mountain ($4664 \div 3 + 95 = 1649\frac{2}{3}$ *chi*).⁷³

This too is describing a world wholly apart from that of physical reality – a world of infinite precision at incredible distances, where a three-*li* stretch of mountain road is perfectly flat and the height of a mountain depends on the height of the observer.⁷⁴ Indeed, it could just as well be a seven-*chi* mountain looking over the tree at a $1649\frac{2}{3}$ *chi* man, for all the realism demanded of such a text, and either could just as easily be swapped for the ‘tongues’ and ‘swans’ of windless sails as we have done above in fig. 3.

Neither can we attribute Zheng Xuan's mistake to his ignorance of mathematics, nor even of the *Nine Chapters* in particular. Consider the order of subjects he learned in his youth as presented in the opening of his *Book of Later Han* biography (numbers added by the author):

鄭玄字康成，北海高密人也。八世祖崇，哀帝時尚書僕射。玄少為鄉嗇夫，得休歸，常詣學官，不樂為吏，父數怒之，不能禁。遂造太學受業，師事京兆第五元，先始通『京氏易』、『公羊春秋』、『三統歷』、『九章算術』。又從東郡張恭祖受『周官』、『禮記』、『左氏春秋』、『韓詩』、古文『尚書』。以山東無足問者，乃西入關，因涿郡盧植，事扶風馬融。

Zheng Xuan, styled Kangcheng 康成, was a man of Gaomi in Beihai [Commandery]. His eighth-generation ancestor Chong 崇 was vice director of the Imperial Secretariat in the time of Emperor Ai (r. 6–

⁷³ Tr. modified from Chemla and Guo (2004, 740–741). As mentioned in Note 35, the eleventh-century scholar Liu Chang would likewise identify the sort of *gou-gu* ‘base and height’ similar triangles as the appropriate solution to Zheng Xuan's problem.

⁷⁴ The fact that the height of the observer's eye is given last – after his/her predetermined distance from the tree – is, to this author, reminiscent of the famous procedure in *Gnomon of Zhou* whereby the height of heaven depends on the length of the gnomon used for observation. See Cullen (1996, esp. 78–82).

1 BCE). Xuan served as a district manager in his youth, but he obtained a discharge to return [home at 11 or 12 years of age]. He paid frequent visit to instructional officers, taking no pleasure from serving office himself, and [though] his father reprimanded him numerous times, he was unable to prohibit [his son from doing as he desired]. [Zheng Xuan] later went to the Imperial Academy to receive his patrimony, serving Diwu Yuan 第五元 of the Capital as his disciple, [under whom] he first mastered (1) *Mr Jing [Fang's Book of] Changes*, (2) the *Gongyang Tradition of the Spring and Autumn Annals*, (3) the *Triple Concordance li*, and (4) the *Nine Chapters of Mathematical Procedures*. He then became a follower of Zhang Gongzu 張恭祖 of Dong Commandery, [from whom] he received (5) the *Offices of Zhou*, (6) the *Record of Rites*, (7) the *Zuo Tradition of the Spring and Autumn Annals*, (8) *Han's Odes*, and (9) the ancient-text *Book of Documents*.⁷⁵ As there were not enough [masters] to question East of the Mountains, he therefore entered the Passes to follow Lu Zhi 盧植 (d. 192 CE) of Zhuo Commandery and serve Ma Rong of Fufeng.⁷⁶

It was normal for an educated man in this day to learn mathematics, so it is hardly surprising to find the *Nine Chapters* on his CV.⁷⁷ One finds it also on that of Ma Rong's elder brother:

續字季則，七歲能通『論語』，十三明『尚書』，十六治『詩』，博觀羣籍，善『九章算術』。

[Ma] Xu 馬續 (fl. 111–41 CE), styled Jize 季則, was able to master the *Analects* at seven years of age; he understood the *Book of Documents* by thirteen and was studying the *Odes* by sixteen. He was broadly read (lit. 'observed') in a diversity of works, and he was good at the *Nine Chapters of Mathematical Procedures*.⁷⁸

Returning to Zheng Xuan, we read that it is this very subject that would in fact link his destiny with that of the Brothers Ma (numbering continued from above):

融門徒四百餘人，升堂進者五十餘生。融素驕貴，玄在門下，三年不得見，乃使高業弟子傳授於玄。玄日夜尋誦，未嘗怠倦。會融集諸生考論圖緯，聞玄善算，乃召見於樓上，玄因從質諸疑義，問畢辭歸。融喟然謂門人曰：「鄭生今去，吾道東矣。」

[Ma] Rong's disciples numbered more than four-hundred, of which only some fifty [were permitted to] ascend the hall and approach [the master]. [Ma] Rong was proud and elitist of nature, and [so Zheng] spent three years beneath his gate without seeing him before [Ma Rong] dispatched an advanced disciple to transmit [his teachings] to [Zheng] Xuan. [Zheng] Xuan researched and recited night and day, [succumbing] never once to idleness or exhaustion. One day [Ma] Rong gathered his various students to study and discuss (10) the charts

⁷⁵ On the 'old-' and 'new-text' Classics, see Dull (1966) and Nylan (1994).

⁷⁶ *Hou Han shu*, 35.1207. Note that insertions are as per Li Xian's 李賢 (654–684 CE) commentary in *ibid.* and the rather exhaustive study of Zheng Xuan's life and times in Wang Liqi (1983).

⁷⁷ On the prevalence of mathematical training among Han intellectuals, see Cullen (2009). On polymathy, more broadly, see Goodman (2005; 2010).

⁷⁸ *Hou Han shu*, 24.862.

and wefts (i.e. the apocrypha),⁷⁹ and having heard that [Zheng] Xuan was good at maths (*shan suan* 善算) he thereupon summoned him to appear upstairs. [Zheng] Xuan obeyed, testifying to them on doubtful points [of interpretation] and, when the interrogation was finished, politely announced his return [home]. [Ma] Rong sighed, saying to his disciples, ‘With Student Zheng’s departure today my *dao* [now travels] east’.⁸⁰

As in the Neo-Platonist curriculum studied in Chapter xx, maths played a critical role in Zheng Xuan’s training, networking, and ultimate success as a scholar. Stepping back, one notes that they also happen to come *first*: at the Imperial Academy, at least, Zheng Xuan studied (4) mathematics immediately prior to the (5) *Rites of Zhou* and (6) *Record of Rites*; and so too did he study Liu Xin’s work on mathematical astronomy – the (3) Triple Concordance *li* – prior to the (7) *Zuo Tradition* of the *Spring and Autumn Annals*, whose chronology it was written to explain.⁸¹

Not surprisingly, Zheng Xuan’s subsequent commentarial activities would be clustered around the same things he studied in his youth: Classics, weft-texts, and the astral and mathematical sciences (Table 1). This is not surprising, because commentary went hand in hand with education. Commentary, as written by a ‘master’ (*shi* 師), served as the textual embodiment and transmission vector of an academic ‘family’ (*jia* 家) – its ‘master’s method’ (*shi fa* 師法), or ‘family method’ (*jia fa* 家法). Commentary, as in later times, also served as the basis of *test norms*. In a precursor to the civil service exam, Emperor Wu of Han (r. 140–87 BCE) established a ‘test’ (*shi* 試) at the Imperial Academy for ‘classifying’ (*ke* 科) recommendees on the basis of their knowledge of the Classics. The test – a game of ‘shoot-strips’ (*shece* 射策) – involved ‘making bamboo-strip with challenging questions, arranging them atop a desk, and having examinees randomly throw-shoot (cast lots) to select and answer [questions]’.⁸² There was a lot at stake in this glorified parlour game, and reading Xu Fang’s 徐防 (fl. 75–107 CE) policy paper of 103/104 CE one gets an idea of just how important commentaries and ‘family methods’ were to preparing for and refereeing the event:

臣聞詩書禮樂，定自孔子；發明章句，始於子夏。其後諸家分析，各有異說。漢承亂秦，經典廢絕，本文略存，或無章句。收拾缺遺，建立明經，博徵儒術，開置太學。孔聖既遠，微旨將絕，故

⁷⁹ On ‘chart and weft’ apocrypha, see Dull (1966) and Lü Zongli (2003). On Zheng Xuan’s later commentaries to the apocrypha, see Lü Kai (2011).

⁸⁰ *Hou Han shu*, 35.1207.

⁸¹ On Liu Xin’s archaeoastronomical approach to the *Zuo Tradition* through the Triple Concordance *li* and appended *Canon of Ages* (*Shijing* 世經), see Cullen (2001).

⁸² 作簡策難問，列置案上，任試者意投射取而答之，*Hou Han shu*, 6.260 (comm.). For more on ‘shoot-strips’ and the abstraction of archery as a model of empirical testing in this period, see Levinovitz and Morgan (forthcoming). On the Imperial Academy and civil service recruitment in the Han, see Bielenstein (1980), 132–42.

| No | Text annotated | Rolls | |
|---------------------------------------|--|-------|----|
| | | ZX | MR |
| 經 CLASSICS | | | |
| 1 | 周易 <i>Book of Changes</i> | 9 | 10 |
| 2 | 尚書 <i>Book of Documents</i> | 9 | 10 |
| 3 | 尚書音 <i>Book of Documents Pronunciations</i> | 1 | |
| 4 | 尚書釋問 <i>Explanations and Questions on the Documents</i> | 4 | |
| 5 | 尚書大傳 <i>Great Comm. to the Book of Documents</i> | 3 | |
| 6 | 毛詩 <i>Mao's Odes</i> | 20 | 10 |
| 7 | 毛詩音 <i>Mao's Odes Pronunciations</i> | 15 | |
| 8 | 毛詩譜 <i>Mao's Odes Tables</i> | 2 | |
| 9 | 周官禮 <i>Offices & Rites of Zhou</i> | 12 | 12 |
| 10 | 周官音 <i>Offices of Zhou Pronunciations</i> | 1 | |
| 11 | 儀禮 <i>Book of Etiquette & Ceremonial</i> | 17 | |
| 12 | 儀禮音 <i>Book of Etiquette & Ceremonial Pronunciations</i> | 2 | 2 |
| 13 | 喪服經傳 <i>Mourning Garments, Classic and Commentary</i> | 1 | 1 |
| 14 | 喪服變除 <i>Mourning Garments, Transformation and Removal</i> | 1 | |
| 15 | 喪服譜 <i>Mourning Garment Tables</i> | 1 | |
| 16 | 禮記 <i>Record of Rites</i> | 20 | |
| 17 | 禮記 <i>Record of Rites Pronunciations</i> | 1 | |
| 18 | 三禮圖 <i>Diagrams of the Three Ritual [Classics]</i> | 9 | |
| 19 | 春秋左傳 <i>Mr Zuo's Spring and Autumn Annals</i> | ? | |
| 20 | 春秋左傳音 <i>Mr Zuo's S&A Annals Pronunciations</i> | ? | |
| 21 | 春秋左氏膏肓 <i>Mr Zuo's S&A Annals, Vital Organs</i> | 10 | |
| 22 | 發公羊墨守 <i>Exposing Mohist Defenses in Gongyang</i> | 2 | |
| 23 | 釋穀梁廢疾 <i>Explaining Arrogations and Illnesses in Guliang</i> | 3 | |
| 24 | 孝經 <i>Classic of Filial Piety</i> | 1 | 2 |
| 25 | 論語 <i>Analects</i> | 10 | ? |
| 26 | 論語音 <i>Analects Pronunciations</i> | ? | |
| 27 | 孟子 <i>Mencius</i> | 7 | |
| 緯 WEFT-TEXTS | | | |
| 28 | 易緯 <i>Changes Weft(s)</i> | 8 | |
| 29 | 尚書緯 <i>Documents Weft(s)</i> | 6 | |
| 30 | 尚書中候 <i>Book of Documents On-target [Omen] Observations</i> | 5 | |
| 31 | 禮緯 <i>Rites Weft(s)</i> | 3 | |
| 32 | 禮記默房 <i>Record of Rites Silent Chamber</i> | 3 | |
| 33 | 河圖洛書 <i>Yellow River Diagram and Luo River Book</i> | 24 | |
| 天文曆算 ASTRAL AND MATHEMATICAL SCIENCES | | | |
| 34 | 乾象曆 <i>Supernal Icon li</i> | ? | |
| 35 | 日月交會圖 <i>Lunar & Solar Crossing Diagrams</i> | 1 | |
| 刑法職官 LAW AND ADMINISTRATION | | | |
| 36 | 漢律 <i>Han Regulations</i> | ? | |
| 37 | 漢官香方 <i>Han Offices Incense Workshop</i> | ? | |
| 五行 FIVE AGENTS DIVINATION | | | |
| 38 | 九宮經 <i>Classic of the Nine Palaces</i> | 3 | |
| 39 | 九宮行棊經 <i>Classic of the Nine Palace Moving Chess Pieces</i> | 3 | |

Table 1 Zheng Xuan's principal commentarial writings (*zhu* 注, *jian* 箋, *shi* 釋, *zhen* 箴) as gathered in Wang Liqi (1983, 327–368). Note that this list excludes 'authored' monographs (*zhuan* 撰, *zhu* 著) and that the number of rolls (*juan* 卷) is given as per the earliest source. Grey indicates the commentaries selected for canonisation in the seventh century CE. The 'Rolls' row is divided into Ma Rong (MR) and Zheng Xuan (ZX) to indicate where and to what degree the latter's output overlaps with his master's, Ma Rong's titles and roll-numbers being gathered principally from the bibliographic monographs of the *Book of Sui* and *Old Book of Tang*.

立博士十有四家，設甲乙之科，以勉勸學者，所以示人好惡，改敝就善者也。

Your humble servant has heard that the [Classics of] *Odes* and *Documents* and *Rites* and *Music* were fixed from the time of Master Kong, and that the revelation and illumination of their chapter and verse (*zhangju*) began with [his disciple] Zixia 子夏; thereafter, the various ‘families’ split apart, each possessed of different sayings. The Han inherited the mess of Qin, and the Classics and canons [of state] were abandoned and lost; their original text survived in part, some devoid of chapter and verse (*zhangju*) [commentary]. We collected and tidied what was defective and lost, establishing and erecting the Classics Competency [Exam] (*mingjing* 明經), and we broadly recruited scholastics and technicians for the opening and institution of the Imperial Academy. Sage [Master] Kong is already far away, and his subtle directives on the verge of dissolution, and thus did we establish ten-and-four academicians (*boshi*) and introduce their A and B classification (*ke*) to encourage and reward scholastics – that by which to show people good and bad and correct the faults of those wont to go to goodness.

伏見太學試博士弟子，皆以意說，不修家法，私相容隱，開生姦路。每有策試，輒興諍訟，論議紛錯，互相是非臣以為博士及甲乙策試，宜從其家章句，開五十難以試之。解釋多者為上第，引文明者為高說；若不依先師，義有相伐，皆正以為非。五經各取上第六人，論語不宜射策。雖所失或久，差可矯革。

Prostrate [before You, I] see that the Imperial Academy’s testing (*shi*) of the academicians’ disciples is [now conducted] solely by wayward rhetoric, cultivated not of family models (*jiafa*); by accommodating one another’s sensibilities by private accord, they open the road for students’ jealousies. Every time there is a strip-exam, it immediately provokes argument and accusation; discourse and opinions entangle and divide, calling right and wrong on one another. ... Your humble servant thinks that the academicians as well as the A and B [class] strip-exam had best follow their family (*jia*) of chapter and verse (*zhangju*) [commentary] and open fifty challeng[ing questions] by which to test them. Those who amply analyse and explain should be made ‘upper-rank’, and those who clearly cite the texts should be made ‘advanced exegetes’; any [answer] that does not rely on a former master, [and/or] that militates against itself in its meaning, [should] in all cases be considered wrong [even] if it is correct. For each the Five Classics one [should] take six people from the upper-rank, [while/but] the *Analects* is inappropriate for shoot-strips. It may be that some of [these] missteps are persistent, but they are nonetheless amenable to prideful reform.⁸³

Zheng Xuan would never pursue a career in the civil service. Instead, he retired early, so to speak, to become a private school master after the model of Ma Rong.⁸⁴ He may not have played ‘shoot-strips’,

⁸³ *Hou Han shu*, 44.1500–1501.

⁸⁴ Note that Ma Rong was equally hesitant to pursue a career in the civil service in his youth, having devoted himself to *youxue* 遊學 ‘wandering studies’ until the age of thirty, when he ran into a war zone to escape from a job offer made by Empress Dowager Deng Sui’s 鄧綏 (b. 81; r. 105–121 CE) older brother. After suffering hun-

therefore, but he was still very much in the business of transmitting ‘family methods’. As illustrated in Table 1, Zheng Xuan’s commentarial output is not only clustered around those areas that he studied in his youth, it also overlaps with that of his master. Both produced a twelve-roll commentary to the *Offices* or *Rites of Zhou*, for example, and the *Book of Later Han* gives us a glimpse of how textual production articulates with the process of transmission in this particular case:

中興，鄭眾傳『周官』經，後馬融作『周官』傳，授鄭玄，玄作『周官』注。

During the Revival (25–57 CE), Zheng Zhong transmitted (*chuan*) the *Offices of Zhou* Classic. Later, Ma Rong made the *Offices of Zhou* commentary-tradition (*zhuan*), which he bestowed upon Zheng Xuan, and Xuan made the *Offices of Zhou* annotations (*zhu*).⁸⁵

One sees a similar pattern in the way that Zheng Xuan, later in life, learned the Supernal Icon *li* 乾象曆 (written c.178–206 CE) from its author, then Governor of Shanyang, Liu Hong 劉洪 (fl. 167–206 CE). Having already manifested an interest in mathematical astronomy, and happening to pass by Liu’s Commandery seat in 196 CE,

鄭玄受其法，以為窮幽極微，又加注釋焉。

Zheng Xuan received [Liu Hong’s] method (*fa*), and considering it to plumb all that is recondite and crown all that is subtle, he further added annotations (*zhu*) and explanations (*shi*) thereto.⁸⁶

Clearly, one wrote commentaries in part to bestow (*shou* 授) one’s teachings, but one wrote them too in the process of reception (*shou* 受). One wrote them to ancient Classics, and one wrote them also to cutting-edge astronomical procedure texts. Commentary-writing was simply a part of learning in Zheng Xuan’s world, and it’s not particularly strange that some of his annotations to the *Rites of Zhou* look like those we might expect him to have added to the tables and algorithms of the Supernal Icon *li* – the two commentaries were written by the same person, after all.

ger and exhaustion behind Qiang 羌 lines, he finally caved in, accepting an appointment to the Eastern Observatory (*Dongguan* 東觀) upon its founding as an academic institution in 110 CE. Formerly a sort of palace archives, the Eastern Observatory was transformed by the Empress Dowager into a think-tank of eminent scholars whose task was to ‘check and collate the commentary/traditions and records’ 讎校傳記 and ‘instruct the palace women’ 教授宮人 (*Hou Han shu*, 10A.424). With the Imperial Academy having suffered a long decline in elite men’s education under competition from the Circular Moat, the Eastern Observatory quickly became the empire’s preeminent centre for textual scholarship, and it was there, as a disciple of Ban Gu’s sister, Zhao 昭 (44/49–118/121 CE), that Ma Rong made his name as a commentator and served most of his career in higher education before retiring ‘within the Passes’ to open his own school. For Ma Rong’s biography, see *Hou Han shu*, 60A.1953–1973. On the Eastern Observatory, see Goodman (2005).

⁸⁵ *Hou Han shu*, 79B.2557.

⁸⁶ *Jin shu*, 17.498. For more on this event and the transmission of Liu Hong’s work in the second and third century CE, see Chen (1986) and Morgan (2015). Sadly, Zheng Xuan’s commentary to the Supernal Icon *li* is lost, but for an idea of what he was commentating, see the complete English translation in Cullen (2017, 235–355).

Given what we know of his background, we know that Zheng Xuan could have easily arrived at the ‘right answer’ for the appropriate height of the Big Shoot targets, in fig. 3, and the fact that he did not probably points to a lapse of memory. If he could understand the Supernal Icon *li*, he could definitely understand problem 9.22 of the *Nine Chapters of Mathematical Procedures*. Mistakes happen; what is curious about this one is how it was received – how it became part of the ‘Correct Meaning’ of the Ritual Classics in the seventh century CE. It is curious because his subcommentators were, if anything, better steeped in mathematics than was Zheng Xuan.

I say this, in part, because mathematics itself would see considerable innovation and expansion over the intervening centuries. Indeed, of the *Ten Mathematical Classics*, canonised by Li Chunfeng 李淳風 (602–670 CE) et al. in 656 CE, seven date to the period of disunion, between Han and Tang, as do three of the commentaries included therein.⁸⁷ As concerns this specific category of problem, Liu Hui 劉徽 would expand upon the theoretical dimensions of procedures like those in problem 9.22 (above) in his 263 CE *zhu* ‘annotation’ of the *Nine Chapters*, a portion of which would be canonised as a separate Classic on its own right: the *Sea Island Mathematical Classic*.⁸⁸ So too, lastly, would scholastics develop the mathematical exegesis of the Confucian canon into its very own genre by the end of this period, the most noteworthy examples of which being Xindu Fang’s 信都芳 (d. 543/550 CE) *Ancestry of the Five Classics* and Zhen Luan’s 甄鸞 (fl. 535–570 CE) *Mathematical Procedures for the Five Classics*.⁸⁹ All of these were extant in Kong Yingda and Jia Gongyan’s day. All but the *Ancestry of the Five Classics*, moreover, were being subcommentated in the parallel canonisation project lead by Li Chunfeng within the same few years.⁹⁰ Whatever their training, therefore, the resources were definitely there for Kong Yingda and Jia Gongyan to realise Zheng Xuan’s mistake.

As to their training, it is difficult to say much about Jia Gongyan without his having left us a biography. Given the way that he handles things like square root extraction in his subcommentaries, however, it is clear that this isn’t his first rodeo.⁹¹ Kong Yingda is a different matter. Kong’s biography tells us that he was good at maths (below), re-

⁸⁷ For a list and further specificities, see Chemla and Zhu’s contribution to the present volume. On the history of mathematics from the Three Kingdoms (220–280 CE) to Sui (581–618 CE), see Martzloff (1997) and Wu Wenjun (1998–2004, vols. 3–4).

⁸⁸ For Liu Hui’s commentary to this category of problem, see Chemla and Guo (2004, 661–745). On the *Sea Island Mathematical Classic* (*Haidao suanjing* 海島算經), see Swetz (1992).

⁸⁹ On the *Ancestry of the Five Classics* (*Wujing zong* 五經宗) and *Mathematical Procedures of the Five Classics* (*Wujing suanshu* 五經算術), see Wu Wenjun (1998–2004, vol. 4, pp. 165–172). For a comparison of the latter with the sort of mathematics witnessed in Kong Yingda and Jia Gongyan’s subcommentaries to the same Classics, see Chemla and Zhu’s contribution to the present volume. For Xindu Fang and Zhen Luan’s biographies, see the relevant entries in Martin and Chaussende (forthcoming).

⁹⁰ On Li Chunfeng and the broader context in which the *Ten Mathematical Classics* came together, see Siu and Volkov (1999) and Goodman (forthcoming).

⁹¹ See Zhu Yiwen (2016b).

counting how in c.633 CE he ‘joined various scholars for a debate on mathematical astronomy and the Bright Hall, and everyone ended up following Yingda’s exposition’.⁹² At another such debate in 640 CE our canonisers crossed paths, Kong ‘requesting that [the court] follow [Li] Chunfeng’ as concerned the latter’s proposal to replace calendro-astronomical policy of the time with his own.⁹³ It is not particularly surprising to see Kong Yingda throw his political and academic weight behind Li Chunfeng at this occasion. Both were in the same business, so to speak. Li would go on to work with certain of Kong’s collaborators on other projects.⁹⁴ The two, moreover, shared a special bond over a common figure: Liu Zhuo 劉焯 (544–610 CE).

Liu Zhuo was an independent scholar primarily active under the Sui (581–618 CE) who was forced into the life of a provincial school master after a fruitless, decade-long struggle for recognition in court astronomy and tono-metrology.⁹⁵ Li Chunfeng never got to meet him, but Liu Zhuo was nonetheless a major presence in his life. In 665 CE, when Li Chunfeng’s astronomical work *was* finally admitted into the annals of public policy, it was with the Unicorn Virtue *li* 麟德曆, which ‘followed Liu Zhuo’s Sovereign Pole *li* 皇極曆 with minor modifications as appropriate’.⁹⁶ Stymied by factional politics within the Sui astronomical office, Liu Zhuo himself was denied the opportunity in life to see his ground-breaking *li* enter government service. In 665 CE, Li Chunfeng vindicated his work, and that this was his goal is evident in Li Chunfeng’s earlier history-writing. Invited to write a chapter on tono-metrology and mathematical astronomy for a history of the ‘Five Dynasties’ (502–618 CE) in the 640s, Li would violate the rules of the *zhi* 志 genre to make his a history foremost of the underdog, preserving Liu Zhuo’s procedure text in a spot normally reserved for ‘winners’ and foregrounding his tribulations at court. Later scholars were divided about this intensely personal subversion of what was a chapter of state history;⁹⁷ if there was anyone who could appreciate his passion for Liu Zhuo, however, it was probably Kong Yingda. Liu Zhuo was Kong Yingda’s teacher.

孔穎達字沖遠，冀州衡水人也……穎達八歲就學，日誦千餘言。及長，尤明『左氏傳』、鄭氏『尚書』、王氏『易』、『毛詩』、『禮記』，兼善算曆，解屬文。同郡劉焯名重海內，穎達造其門，焯初不之禮，穎達請質疑滯，多出其意表，焯改容敬之。穎達固辭歸，焯固留，不可。還家，以教授為務。隋大業初，舉明經高第，授河內郡博士。

⁹² 與諸儒議曆及明堂，皆從穎達之說, *Jiu Tang shu*, 73. 2602.

⁹³ 請從淳風, *Xin Tang shu*, 25.536.

⁹⁴ In the 640s, for example, Li Chunfeng would be invited aboard the literary compilation *Wensi boyao* 文思博要 alongside Kong Yingda’s collaborator on the *Correct Meaning of Changes of Zhou* and *Spring and Autumn Annals*, Ma Jiayun 馬嘉運 (d. 645). See *Jiu Tang shu*, 73.2603, 79.2718.

⁹⁵ For Liu Zhuo’s biography, see *Sui shu*, 75.1718–1719, *Bei shi*, 82.2762–2763, cf. the relevant entries in Martin and Chaussende (forthcoming).

⁹⁶ 餘因劉焯皇極曆法，增損所宜, *Xin Tang shu*, 26.559.

⁹⁷ See Li Liang (forthcoming).

Kong Yingda, styled Chongyuan 沖遠, was a man of Hengshui, in Jizhou Commandery. ... Yingda went to school at eight years of age, making a recitation of over a thousand words every day. By the time he was grown, he had become particularly conversant in the *Zuo Tradition*, Mr Zheng [Xuan's] *Book of Documents*, Mr Wang [Bi's] *Changes*, *Mao's Odes*, and the *Record of Rites*; he was equally good at both mathematics and mathematical astronomy, explaining and composing text. Liu Zhuo, of his same commandery, had a name that carried weight [all across the land] within the seas, and Yingda delivered himself to his gates (i.e., as a 'below-the-gates' disciple). Zhuo did not at first treat him with due ritual propriety, but Yingda asked good and stubborn questions, greatly surpassing his expectations, so Zhuo changed his countenance and [began] to respect him. Yingda long/firmly bid his leave, and Zhuo long/firmly insisted that he must stay. [Eventually, Kong Yingda] returned home to take up the profession of teaching. At the beginning of the Sui Grand Patrimony reign (605–617 CE), [in his early thirties], he tested into the upper ranks of the Classics Competency [exam] as was named an academician of Henei Commandery.⁹⁸

In astronomy, Liu Zhuo was ahead of his time; the 'name that carried' had more to do with the local polytechnic school he opened with his childhood friend, Liu Xuan 劉炫 (c.546–c.613 CE), and their respective contributions to Classical exegesis. It was this school that launched Kong Yingda down the path to a career in the Academy, and twenty-six years later, as its chancellor, tasked with deciding The Correct Meaning of the Classics for the national curriculum (and generations to come), Kong turned first and foremost to his 'family method'. In his preface to the *Correct Meaning of the Book of Documents*, for example, Kong foregrounds his teachers' respective commentaries thereto, declaring that 'Liu Zhuo and Liu Xuan's are truly the most detailed and elegant' of recent times.⁹⁹ He means these adjectives respectively – 'Xuan, begrudging Zhuo's prolixity, set himself to cutting things out of [his commentary]',¹⁰⁰ but Xuan's, in turn, was too elegantly succinct – and Kong Yingda frames his project around finding a middle ground between the two. Interestingly, one of his complaints about his late master is...

若其言必託數，經悉對文，斯乃鼓怒浪於平流，震驚飆於靜樹，使教者煩而多惑，學者勞而少功，過猶不及，良為此也。

... the way, for example, that [Liu Zhuo's] words necessarily fall back on numbers to which the text of the Classic is [invoked] only by way

⁹⁸ *Jiu Tang shu*, 73.2601; cf. *Xin Tang shu*, 198.5643. One finds another anecdote involving Liu Zhuo playing the role of 'master' (*shi*) or 'headmaster' (*shishou* 師首) at a sort of conference panel featuring Kong Yingda, Gai Wenda 蓋文達 (d. 644 CE), and his own teacher, Liu Guisi 劉軌思, in *Jiu Tang shu*, 189A.4951, and *Xin Tang shu*, 198.5651.

⁹⁹ 惟劉焯、劉炫最為詳雅, *Shangshu zhusu*, 0.3b. Now lost, the *Old Book of Han* bibliographic monograph records a 'Book of Documents, Meaning and Subcommentary' in twenty rolls, authored by Liu Zhuo 尚書義疏二十卷, 劉焯撰 and a 'Book of Documents, Narrating the Meaning' in twenty rolls, authored by Liu Xuan 尚書述義二十卷, 劉炫撰 in *Jiu Tang shu*, 46.1970.

¹⁰⁰ 炫嫌焯之煩雜，就而刪焉, *Shangshu zhusu*, 0.4a.

of complement – this is really beating a level current into a tidal wave, or jolting a quiescent tree into a whirlwind; it leaves the teacher vexed and full of doubts, and the student toiling with little reward, going too far while at the same time falling short. He was really good at that.¹⁰¹

If at any point it has crossed the reader's mind to ask why a Confucian scholar would employ mathematics towards Classical exegesis we must realise now that that question is out of place. Maths were a given. Pushed to become the Classicist that he was by a visionary astronomer who saw the *Book of Documents* as a book of numbers, the question for Kong Yingda, as we see here, was not *if* or *why* mathematics might be useful to this end – the question was how it might be kept down to a level that the average student would understand.

5. Conclusion

To recapitulate the findings of our investigation, we can say for certain that, though 'one would rather say that the [Duke of] Zhou and [Master] Kong were mistaken', Zheng Xuan *was* wrong about something: the feasible heights and dimensions of the targets used in the Classical archery rite of the 'Big Shoot'. His numbers are physically impossible, and to understand why he would project them on the ancient past, we went through his annotations of the relevant passages of the Ritual Classics to diagnose their source. The problem, we learned, is that Zheng Xuan has forcibly transformed a set of practical instructions into an abstract geometrical puzzle and, in doing so, left out one dimension – depth – from a three-dimensional problem. This happened, we were able to confirm, not for want of exposure to living practice: the Shoot originated in his region, spread therefrom throughout the empire, and continued to be held without major interruption to his day. This happened, we were also able to confirm, not for want of exposure to geometry: Zheng Xuan had a life-long history of mixing commentary and mathematics, and the very text needed to solve this imaginary problem appears on his CV. This was probably an accident. It became something more, however, in the hands of his canonisers, Kong Yingda and Jia Gongyan, who would induct his faulty figures into the 'Correct Meaning' of the Confucian canon, supplying worked mathematical proofs with nary a word of doubt. As to whether *these two* knew what they were doing, the answer is a definitive 'yes': Kong Yingda, who goes out of his way to insist that these figures 'are' 'thus' 'truly' so, we have caught implicated in his own life-long history with mathematical astronomy, while from Jia Gongyan, who we see elsewhere performing square root extractions, we detect a hint of evasion in his emphasis on addressing only 'how the commentary knows'.

What on earth were they thinking? The charitable response would be to assume that Zheng Xuan had made a mistake and that his sub-

¹⁰¹ Ibid. Note that Kong Yingda's prefaces to the *Correct Meaning of Mao's Odes* and the [*Zuo Tradition of the*] *Spring and Autumn Annals* make similar reference to the two, his subcommentary to the later frequently citing Liu Xuan.

commentators were simply doing their job the best way that they knew how, ensuring that ‘the subcommentary does not break the commentary’ 疏不破注 (see Chapter xx). ‘What were they thinking?’, however, is not a question that we can answer, so I leave that to the reader’s imagination. Far more important is what we can glean about *how* they were thinking. Viewed through the lens of modern academic specialisation, modern Sinologists tend to lump the men in this story into the box of ‘Confucianism’, ‘Classical studies’ (*jingxue* 經學), or ‘philology’, as wholly separate, say, from the history of sport, astronomy, or mathematics. They do not go in these boxes, so they must be forced. We do this by ignoring and, ultimately, *redacting* the things that we don’t feel belong in their writing, personal story, or milieu, regardless of how tightly woven therein it may be. Mathematics can be particularly ‘vexing’ in this regard, as Kong Yingda complains of his incorrigible master, so it tends to lead to the most extreme expressions of denial. For example,

L’idée de quantité ne joue autant dire aucun rôle dans les spéculations philosophiques des Chinois. Les Nombres, cependant, intéressent passionnément les Sages de l’ancienne Chine. Mais, – quelles qu’aient pu être les connaissances arithmétiques ou géométriques de certaines corporations (arpenteurs, charpentiers, architectes, charrons, musiciens...), – nul Sage n’a accepté de les utiliser, si ce n’est dans la mesure où, sans jamais contraindre à des opérations dont le résultat ne se pût commander, ce savoir facilitait des jeux numériques. ... A côté d’une valeur quantitative qui les distingue, mais qu’on tend à négliger, les Nombres possèdent une valeur symbolique beaucoup plus intéressante, car, n’offrant aucune résistance au génie opératoire, elle les laisse se prêter à une sorte d’alchimie. Les Nombres sont susceptibles de *mutations*. Ils le sont en raison de l’efficacité multiple dont ils paraissent dotés et qui dérive de leur fonction principale ; ils servent et valent en tant que *Rubriques emblématiques* (Granet 1934, 149–151).

But the Ritual Classics are *riddled* with quantities, and mathematics, as we have seen, was as indispensable a tool in the commentator’s toolbox as it was for any of the lesser trades in Granet’s list. In a way, the very story of how these texts came to read the way that they now do begins with a professor looking for help with the numbers in an ancient text. That someone happened to be Zheng Xuan, and it is to him, thanks to Ma Rong’s backing, that modern professors still turn for help. Zheng Xuan was no stranger to Granet’s *Rubriques emblématiques*, mind you, but our investigation has caught him playing *jeux numériques* of a different order with a cabal of Confucians, Classicists, and philologists who were also very openly, in their own words, ‘good at maths’. We might not be able to say *what* Zheng Xuan, Kong Yingda, or Jia Gongyan were thinking at a given point in their commentary, therefore, but we are able to say *how*: they were thinking, at least here, like mathematicians.

Suan ‘mathematics’, like most theory-oriented traditions, offers us a world apart from and infinitely surpassing the real – a realm, populated by absurdities, of absolute perfection and infinite possibility be-

yond time, space, and physical constraints. It is only here that the physically impossible can become The Correct Meaning of ancient rites, and those rites are only here because it is here that Zheng Xuan et al. have brought them, preferring and even *forcing* readings that make the base text into a geometrical problem that it was clearly never intended to be. These are not Granet's 'surveyors, carpenters, architects, wheelwrights, [or] musicians' – they are *philosophers*, or the next closest thing, and we must assume that there is a philosophy to why they have done this. Consider Zheng Zhong, Zheng Xuan's predecessor, who suggests that that one third of ten is four for the purposes of the short-range target's swan. Zheng Zhong knew how to work with fractions – *he too* came from a family tradition in mathematical astronomy – his are simply dimensions that take things like the two-*chi* cloth bolt and decimal *chi*-rule into practical consideration.¹⁰² And maybe *that* is precisely why this story is not his: because we are talking here about the perfect institutions of perfect kings in a perfect past deserving of better than a contemporary clothier's approximations.

Ironically, the result of Zheng Xuan and his seventh-century canonisers thinking like mathematicians in this case is that these unsolicited figures for the Big Shoot target heights would be repeated uncritically ever after as a historical fact about pre-imperial China. Yes, Liu Chang 劉敞 (*jinshi* 1046) would eventually complain of these figures that 'they are hard to take seriously, [as] they do not even agree with [the commentator's own] words', but by then the die was already cast.¹⁰³ I have done my best thus far to avoid generalising from this one case about the writers, genres, or processes of canonisation we see at work therein. They are immense, and because it is difficult for this author to say for certain that similar problems don't play out differently even in the thousands of pages of Kong Yingda and Jia Gongyan's extant subcommentaries alone. Maybe they do, but allow me to leave you on a more terrifying note: What if they don't? How could we then trust *anything* that we thought we'd understood about the Classics having read them for some thirteen centuries through Zheng Xuan, Kong Yingda, and Jia Gongyan's eyes?

¹⁰² More specifically, Zheng Zhong's biography describes him as 'conversant in the Triple Concordance *li*' 明三統歷, which his father, Xing 興 (fl. 14–33 CE), had personally learned along with a number of Classical commentaries from its author, Liu Xin (*Hou Han shu*, 36.1217, 1224).

¹⁰³ For Liu Chang's eleventh-century criticism of Zheng Xuan's mistake, see Note 35. This gets picked up in later works like Hao Jing's (1558–1639), *Yili jiejie*, 7.3b, and Sheng Shizuo's 盛世佐 1747 *Yili jibian*, 16.15a–16b, but the later exegetical tradition mostly regurgitates the figures given in *The Correct Meaning of the Five Classics* without a second thought to their epistemic status. See for example Li Rugui's 李如圭 (1167–c.1233), *Yili jishi*, 9.3b, Ao Jigong's 敖繼公 1301 *Yili jishuo* 儀禮集說, 7.4a–6a, Huang Zongxi's 黃宗羲 (1610–1695), *Nanlei wen'an* 南雷文案, 1.5a–6b, Zhang Erqi's 張爾岐 (1612–1678), *Yili Zheng zhu judu* 儀禮鄭註句讀, 7.2b, Li Guangpo's 李光坡 (1651–1723), *Yili shushu* 儀禮述註卷七, 7.4a–5b, Jiang Yong's 江永 (1681–1762), *Lishu gangmu* 禮書綱目, 12.2a–b, and Qin Huitian's 秦蕙田 (1702–1764), *Wuli tongkao* 五禮通考, 171.4b–5a.

Bibliography

Primary sources

- Bei shi* 北史, Li Yanshou 李延壽, 643/659 CE; edition Beijing: Zhonghua shuju, 1974 (10 vols.).
- Cihu Shi zhuan* 慈湖詩傳, Yang Jian 楊簡 (1141–1226); reprint from *Wenyuange Siku quanshu* 文淵閣四庫全書, 1782; edition Taipei: Taiwan shangwu yinshuguan, 1983–1986.
- Gongshi Qijing xiaozhuan* 公是七經小傳, Liu Chang 劉敞 (*jinsi* 1046); edition *Sibu congkan* 四部叢刊, Shanghai: Shangwu yinshuguan, 1919–1936.
- Han shu* 漢書 (Book of Han), Ban Gu 班固 et al., 111, commentary by Yan Shigu 顏師古 (581–645 CE); edition Beijing: Zhonghua shuju, 1962 (8 vols.).
- Hou Han shu* 後漢書 (Book of Later Han), Fan Ye 范曄 (398–445) et al., commentary by Li Xian 李賢 (654–684); edition Beijing: Zhonghua shuju, 1965 (12 vols.).
- Huainan honglie jie* 淮南鴻烈解, Liu An 劉安 et al., 139 BCE, commentary by Xu Shen 許慎 (c.55–c.149 CE); edition *Sibu congkan* 四部叢刊, Shanghai: Shangwu yinshuguan, 1919–1936.
- Jin shu* 晉書 (Book of Jin), Fang Xuanling 房玄齡 et al., 648 CE; edition Beijing: Zhonghua shuju, 1974 (10 vols.).
- Jiu Tang shu* 舊唐書 (Old Book of Tang), Liu Xu 劉昫 (887–946 CE), 945 CE; edition Beijing: Zhonghua shuju, 1975 (16 vols.).
- Liji zhushu* 禮記注疏 (Record of Rites), pre-Qin (< 221 BCE), commentary by Zheng Xuan 鄭玄 (127–200 CE), subcommentary by Kong Yingda 孔穎達 (574–648 CE), 653 CE; critical edition by Ruan Yuan 阮元 (1764–1849), *Chongkan Song ben Shisanjing zhushu* 重刊宋本十三經注疏, 1815; reprint Taipei: Yiwen yinshuguan, 1965.
- Lishu gangmu* 禮書綱目, Jiang Yong 江永 (1681–1762); reprint from *Wenyuange Siku quanshu* 文淵閣四庫全書, 1782; edition Taipei: Taiwan shangwu yinshuguan, 1983–1986.
- Lunheng jiaoshi* 論衡校釋, Wang Chong 王充 (27–c.100 CE), 70/80 CE; critical edition with annotations by Huang Hui 黃暉; edition *Xinbian zhuzi jicheng* 新編諸子集成 Beijing: Zhonghua shuju, 1990 (4 vols.).
- Maoshi zhushu* 毛詩注疏 (Mao's Odes), pre-Qin (< 221 BCE), commentary by Zheng Xuan 鄭玄 (127–200 CE), subcommentary by Kong Yingda 孔穎達 (574–648 CE), 653 CE; critical edition by Ruan Yuan 阮元 (1764–1849), *Chongkan Song ben Shisanjing zhushu* 重刊宋本十三經注疏, 1815; reprint Taipei: Yiwen yinshuguan, 1965.
- Nanlei wen'an* 南雷文案, Huang Zongxi 黃宗羲 (1610–1695); edition *Sibu congkan* 四部叢刊, Shanghai: Shangwu yinshuguan, 1919–1936.
- Quan Tang wen* 全唐文, Dong Gao 董誥 et al., 1819; edition Beijing: Zhonghua shuju, 1983 (11 vols.).
- Quan shanggu Sandai, Qin, Han, Sanguo, Liuchao wen* 全上古三代秦漢三國六朝文, compiled by Yan Kejun 嚴可均 (1762–1843); edition Beijing: Zhonghua shuju, 1958 (4 vols.).
- Shangshu zhushu* 尚書正義 (Book of Documents), pre-Qin (< 221 BCE), commentary by Kong Anguo 孔安國, subcommentary by Kong Yingda 孔穎達 (574–648), 653; edition *Shisanjing zhushu* 重刊宋本十三經注疏, 1815; reprint Taipei: Yiwen yinshuguan, 1965.
- Shiji* 史記. Sima Qian 司馬遷 (c.146–c.86 BCE) et al.; edition Beijing: Zhonghua shuju, 1959 (10 vols.).
- Song shu* 宋書 (Book of Song), Shen Yue 沈約 (441–513 CE), 492/493 CE; edition Beijing: Zhonghua shuju, 1974 (8 vols.).

- Wei shu* 魏書 (Book of Wei), Wei Shou 魏收, 554 CE; edition Beijing: Zhonghua shuju, 1974 (8 vols.).
- Wuli tongkao* 五禮通考, Qin Huitian 秦蕙田 (1702–1764); reprint from *Wenyuange Siku quanshu* 文淵閣四庫全書, 1782; edition Taipei: Taiwan shangwu yinshuguan, 1983–1986.
- Wuwei Han jian* 武威漢簡, ed. Gansu-sheng bowuguan 甘肅省博物館 and Zhongguo kexueyuan kaogu yanjiusuo 中國科學院考古研究所, Beijing: Wenwu chubanshe, 1964.
- Xin Tang shu* 新唐書 (New Book of Tang), Ouyang Xiu 歐陽修 (1007–72) and Song Qi 宋祁 (998–1061), 1060; edition Beijing: Zhonghua shuju, 1975 (20 vols.).
- Yili jibian* 儀禮集編, Sheng Shizuo 盛世佐, 1747; reprint from *Wenyuange Siku quanshu* 文淵閣四庫全書, 1782; edition Taipei: Taiwan shangwu yinshuguan, 1983–1986.
- Yili jishi* 儀禮集釋, Li Rugui 李如圭 (1167–c.1233); reprint from *Wenyuange Siku quanshu* 文淵閣四庫全書, 1782; edition Taipei: Taiwan shangwu yinshuguan, 1983–1986.
- Yili jishuo* 儀禮集說, Ao Jigong 敖繼公, 1301; reprint from *Wenyuange Siku quanshu* 文淵閣四庫全書, 1782; edition Taipei: Taiwan shangwu yinshuguan, 1983–1986.
- Yili jiejie* 儀禮節解, Hao Jing 郝敬 (1558–1639); edition Jingshan Haoshi 京山郝氏 (?), 1617; rpt. *Xuxiu Siku quanshu* 續修四庫全書, Shanghai: Shanghai guji chubanshe, 2002.
- Yili shushu* 儀禮述註, Li Guangpo 李光坡 (1651–1723); reprint from *Wenyuange Siku quanshu* 文淵閣四庫全書, 1782; edition Taipei: Taiwan shangwu yinshuguan, 1983–1986.
- Yili Zheng zhu judu* 儀禮鄭註句讀, Zhang Erqi 張爾岐 (1612–1678); reprint from *Wenyuange Siku quanshu* 文淵閣四庫全書, 1782; edition Taipei: Taiwan shangwu yinshuguan, 1983–1986.
- Yili zhushu* 儀禮注疏 (Book of Etiquette and Ceremonial), pre-Qin (< 221 BCE), commentary by Zheng Xuan 鄭玄 (127–200 CE), subcommentary by Jia Gongyan 賈公彥 (seventh century CE); critical edition by Ruan Yuan 阮元 (1764–1849), *Chongkan Song ben Shisanjing zhushu* 重刊宋本十三經注疏, 1815; reprint Taipei: Yiwen yinshuguan, 1965.
- Yuhan shanfang ji yishu* 玉函山房輯佚書, compiled by Ma Guohan 馬國翰 (1794–1857), edition Changsha Langhuanguan 榔嬛館, 1888; rpt. *Xuxiu Siku quanshu* 續修四庫全書, Shanghai: Shanghai guji chubanshe, 2002.
- Zhangjiashan Han mu zhujian (ersiqi hao mu)* 張家山漢墓竹簡 (二四七號墓), ed. Zhangjiashan ersiqi hao Han mu zhujian zhengli xiaozu 張家山二四七號漢墓整理小組. Beijing: Wenwu chubanshe, 2001.
- Zhouli zhushu* 周禮注疏 (Rites of Zhou), pre-Qin (< 221 BCE), commentary by Zheng Xuan 鄭玄 (127–200 CE), subcommentary by Jia Gongyan 賈公彥 (seventh century CE); critical edition by Ruan Yuan 阮元 (1764–1849), *Chongkan Song ben Shisanjing zhushu* 重刊宋本十三經注疏, 1815; reprint Taipei: Yiwen yinshuguan, 1965.

Secondary sources

- Adamski, Susanne Jenny. 2012. Die Darstellung des Bogenschießens in Bronzeinschriften der West-Zhōu-Zeit (1045-771 v.Chr.): Eine philologische Quellenanalyse. Veröffentlichungen des Ostasien-Instituts der Ruhr-Universität Bochum. Wiesbaden: Harrassowitz Verlag.
- Auken, Newell Ann Van. 2016. The commentarial transformation of the Spring and Autumn. SUNY Series in Chinese Philosophy and Culture. Albany: State University of New York Press.

- Bielenstein, Hans. 1980. *The bureaucracy of Han times*. Cambridge: Cambridge University Press.
- Biot, Édouard Constant. 1851. *Le Tcheou-li, ou Rites des Tcheou*, traduit pour la première fois du chinois par feu Édouard Biot. Edited by Jean-Baptiste Biot. 3 vols. Paris: Imprimerie Nationale.
- Brashier, Kenneth E. 2011. *Ancestral memory in early China*. Cambridge, Mass.: Harvard University Asia Center.
- Chemla, Karine. 2003. Generality above Abstraction: The General Expressed in Terms of the Paradigmatic in Mathematics in Ancient China. *Science in Context* 16. doi:10.1017/S0269889703000905.
- Chemla, Karine. 2010. Mathematics, nature and cosmological inquiry in traditional China. In *Concepts of nature: a Chinese-European cross-cultural perspective*, ed. Günter Dux, Hans Ulrich Vogel, and Mark Elvin, 255–284. Leiden: Brill.
- Chemla, Karine, and Guo Shuchun 郭書春. 2004. *Les neuf chapitres: le classique mathématique de la Chine ancienne et ses commentaires*. Paris: Dunod.
- Chemla, Karine, and Daniel Patrick Morgan. forthcoming. Math lessons: towards an interdisciplinary approach. *Manuscript Cultures*.
- Chen Meidong 陳美東. 1986. Liu Hong de shengping, tianwenxue chengjiu he sixiang 劉洪的生平、天文學成就和思想. *Ziran kexue shi yanjiu* 自然科學史研究 5: 129–142.
- Chen Pan 陳槃. 1950. “Hou” yu “she hou” 「侯」與「射侯」. *Zhongyan yanjiuyuan lishi yuyan yanjiusuo jikan* 中央研究院歷史語言研究所集刊 22: 121–128.
- Csikszentmihalyi, Mark, and Michael Nylan. 2003. Constructing lineages and inventing traditions through exemplary figures in early China. *T’oung pao* 2nd ser., ser.89: 59–99.
- Cullen, Christopher. 1996. *Astronomy and mathematics in ancient China: the Zhou bi suan jing*. Cambridge: Cambridge University Press.
- Cullen, Christopher. 2001. The birthday of the old man of Jiang County and other puzzles: work in progress on Liu Xin’s Canon of the Ages. *Asia Major* 3rd ser., ser.14: 27–60.
- Cullen, Christopher. 2004. *The Suàn shù shū 算數書 “Writings on reckoning”*: a translation of a Chinese mathematical collection of the second century BC, with explanatory commentary. Cambridge: Needham Research Institute.
- Cullen, Christopher. 2009. People and numbers in early imperial China: locating “mathematics” and “mathematicians” in Chinese space. In *Oxford handbook of the history of mathematics*, ed. Eleanor Robson and Jacqueline A. Stedall, 591–618. Oxford: Oxford University Press.
- Cullen, Christopher. 2017. *The Foundations of celestial reckoning: Three ancient Chinese astronomical systems*. New York: Routledge.
- Dull, Jack L. 1966. A historical introduction to the apocryphal (ch’an-wei) texts of the Han Dynasty. Ph.D. diss., University of Washington.
- Elman, Benjamin A. 1984. *From philosophy to philology: intellectual and social aspects of change in late imperial China*. Cambridge, MA: Harvard University Press.
- Feng Shengjun 馮勝君. 2007. *Guodian jian yu Shangbo jian duibi yanjiu* 郭店簡與上博簡對比研究. Beijing: Xianzhuang shuju.
- Forte, Antonino. 1988. *Mingtang and Buddhist utopias in the history of the astronomical clock: the tower, statue and armillary sphere constructed by Empress Wu*. Serie Orientale Roma v. 59. Rome and Paris: Istituto italiano per il Medio ed Estremo Oriente and École française d’Extrême-Orient.
- Goodman, Howard L. forthcoming. The life and intellectual world of Li Chunfeng (602–670). In *Monographs in Tang official history: Per-*

- spectives from the technical treatises of the Book of Sui, ed. Damien Chaussende, Daniel Patrick Morgan, Daniel Patrick Morgan, and Karine Chemla. Dordrecht: Springer.
- Goodman, Howard L. 2005. Chinese polymaths, 100–300 AD: the Tung-kuan, Taoist dissent, and technical skills. *Asia Major* 3rd ser., ser.18: 101–174.
- Goodman, Howard L. 2010. *Xun Xu and the politics of precision in third-century AD China*. Leiden: Brill.
- Granet, Marcel. 1934. *La pensée chinoise*. Paris: La Renaissance du livre.
- Henderson, John B. 1991. *Scripture, canon, and commentary: a comparison of Confucian and Western exegesis*. Princeton: Princeton University Press.
- Karlgren, Bernhard. 1957. *Grammata serica recensa*. Stockholm: Museum of Far Eastern Antiquities.
- Lee, Thomas H. C. 2000. *Education in traditional China: a history*. Handbuch Der Orientalistik. Vierte Abteilung, China 13. Leiden: Brill.
- Legge, James. 1885. *The Lî Kî*. 2 vols. *The Sacred Books of the East* XXVII–XXVIII. Oxford: Clarendon Press.
- Levinovitz, Alan Jay, and Daniel Patrick Morgan. forthcoming. *Virtue on trial: Ritual archery and astronomical testing in early China*. In *Ritual, Mathematics, and the Astral Sciences*, ed. Hirose Shō 廣瀨匠, Matthieu Husson, Agathe Keller, and Daniel Patrick Morgan, xx. Xx. Dordrecht: Springer.
- Lewis, Mark Edward. 2006. *The construction of space in early China*. Albany: State University of New York.
- Li Jimin 李繼閔. 1998. *Jiuzhang suanshu daodu yu yizhu* 九章算術導讀與譯註. Xi'an: Shaanxi renmin jiaoyu chubanshe.
- Li Junming 李均明. 2009. *Qin-Han jian du wenshu fenlei jijie* 秦漢簡牘文書分類輯解. Beijing: Wenwu chubanshe.
- Li Liang 李亮. forthcoming. *The compilation of the calendrical portion of the Book of Sui “Monograph on Tono-metrology and Mathematical Astronomy” and its Impacts*. In *Monographs in Tang official history: Perspectives from the technical treatises of the Book of Sui*, ed. Damien Chaussende, Daniel Patrick Morgan, and Karine Chemla. Dordrecht: Springer.
- Loewe, Michael. 1967. *Records of Han administration*. 2 vols. Cambridge: Cambridge University Press.
- Loewe, Michael, ed. 1993. *Early Chinese texts: a bibliographical guide*. Berkeley: Institute of East Asian Studies, University of California.
- Lü Kai 呂凱. 2011. *Zheng Xuan zhi chenweixue* 鄭玄之讖緯學. 2nd edn. (1st 1982). Taipei: Taiwan shangwu yinshuguan.
- Lü Zongli 呂宗力. 2003. *Power of the words: chen prophecy in Chinese politics, AD 265–618*. Oxford: Peter Lang.
- Makeham, John. 1997. *The earliest extant commentary on Lunyu: Lunyu zheng shi zhu*. T'oung Pao, 2nd ser. 83: 260–299.
- Makeham, John. 2003. *Transmitters and creators: Chinese commentators and commentaries on the Analects*. Cambridge, MA: Harvard University Asia Center: Distributed by Harvard University Press.
- Martin, François, and Damien Chaussende, ed. forthcoming. *Dictionnaire des individus et groupes humains dans la Chine du haut Moyen Âge*. Paris: Les Belles Lettres.
- Martzloff, Jean-Claude. 1997. *A history of Chinese mathematics*. Translated by Stephen S. Wilson. Berlin: Springer.
- Maspero, Henri. 1951. *Le ming-t'ang et la crise religieuse chinoise avant les Han*. *Mélanges chinois et bouddhiques* 9: 1–71.
- McMullen, David. 1988. *State and scholars in T'ang China*. Cambridge: Cambridge University Press.

- Morgan, Daniel Patrick. 2015. What good's a text? Textuality, orality, and mathematical astronomy in early imperial China. *Archives internationales d'histoire des sciences* 65: 549–572.
- Morgan, Daniel Patrick. 2017. *Astral sciences in early imperial China: observation, sagehood and the individual*. Cambridge: Cambridge University Press.
- Morgan, Daniel Patrick, and Karine Chemla. 2018. Writing in turns: an analysis of scribal hands in the bamboo manuscript *Suan shu shu* 算數書 (Writings on Mathematical Procedures) from Zhangjiashan tomb no. 247. *Bamboo and Silk* 1.
- Nylan, Michael. 1994. The chin wen/ku wen controversy in Han times. *T'oung pao*, 2nd ser. 80: 83–145.
- Plaks, Andrew H. 2010. Zheng Xuan's commentary on the Zhouli. In *Statecraft and classical learning the Rituals of Zhou in East Asian history*, ed. Benjamin A. Elman and Martin Kern, 155–178. Leiden: Brill.
- Qiu Guangming 丘光明. 1992. *Zhongguo lidai du liang heng kao* 中國歷代度量衡考. Beijing: Kexue chubanshe.
- Qiu Xigui 裘錫圭. 2000. *Chinese writing 文字學概要*. Translated by Gilbert L. Mattos and Jerry Norman. Early China Special Monograph Series, No. 4. Berkeley: Society for the Study of Early China and the Institute of East Asian Studies, University of California.
- Riegel, Jeffrey K. 1982. Early Chinese target magic. *Society for the Study of Chinese Religions Bulletin* 10: 1–18.
- Sanft, Charles. 2008. Bow control in Han China: Yuqiu Shouwang on self-defense. *Journal of Asian History* 42: 143–164.
- Schuessler, Axel. 2007. *ABC etymological dictionary of old Chinese*. Honolulu: University of Hawai'i Press.
- Selby, Stephen. 2006. *Chinese archery*. Hong Kong: Hong Kong University Press.
- Shaughnessy, Edward L. 2006. *Rewriting early Chinese texts*. Albany: State University of New York Press.
- Shi Yingyong 史應勇. 2007. *Zheng Xuan tongxue ji Zheng Wang zhi zheng yanjiu* 鄭玄通學及鄭王之爭研究. Chengdu: Ba Shu shushe.
- Siu Man Keung 蕭文強, and Alexei Volkov. 1999. Official curriculum in traditional Chinese mathematics: how did candidates pass the examinations? *Historia Scientiarum* 9: 85–99.
- Soothill, William Edward. 1951. *The hall of light: a study of early Chinese kingship*. London: Lutterworth Press.
- Steele, John Clendinning. 1917. The "I li", or book of etiquette and ceremonial. *Probsthain's Oriental Series* 8. London: Probsthain.
- Sukhu, Gopal. 2005–2006. Yao, Shun, and prefiguration: the origins and ideology of the Han imperial genealogy. *Early China* 30: 91–151.
- Swetz, Frank J. 1992. *The sea island mathematical manual: surveying and mathematics in ancient China*. University Park, Pa.: Pennsylvania State Univ. Press.
- Tseng, Lillian Lan-ying. 2011. *Picturing heaven in early China*. Cambridge: Harvard University Asia Center.
- Wang E 王鏐. 2007. *Liji chengshu kao* 『禮記』成書考. *Nanjing daxue gudian wenxian yanjiu congkan* 南京大學古典文獻研究叢刊. Beijing: Zhonghua shuju.
- Wang Liqi 王利器. 1983. *Zheng Kangsheng nianpu* 鄭康成年譜. Jinan: Qi Lu shushe.
- Weber, Charles D. 1968. Chinese pictorial bronze vessels of the late Chou period. part IV. *Artibus Asiae* 30: 145–236.
- Wu Wenjun 吳文俊, ed. 1998–2004. *Zhongguo shuxueshi daxi* 中國數學史大系. 10 vols. Beijing: Beijing shifan daxue chubanshe.
- Yang Hua 楊華. 2007. *Xinchu jianbo yu lizhi yanjiu* 新出簡帛與禮制研究. Taipei: Taiwan guji chuban youxian gongsi.

- Yang Tianyu 楊天宇. 2008. Zheng Xuan Sanli zhu yanjiu 鄭玄三禮注研究. Guojia sheke jijin chengguo wenku 國家社科基金成果文庫. Beijing: Zhongguo shehui kexue chubanshe.
- Yuan Junjie 袁俊傑. 2013. Liangzhou sheli yanjiu 兩周射禮研究. Beijing: Kexue chubanshe.
- Zhu Yiwen 朱一文. 2015. Ruxue jingdian zhong de shuxue zhishi chutan: yi Jia Gongyan dui Zhouli “Kaogong ji” “Li Shi wei liang” de zhushu wei li 儒學經典中的數學知識初探：以賈公彥對『周禮·考工記』「桌氏為量」的注疏為例. Ziran kexue shi yanjiu 自然科學史研究 34: 131–141.
- Zhu Yiwen 朱一文. 2016a. Zailun Zhongguo gudai shuxue yu ruxue de guanxi: yi Tangdai xuezhe dui lishu de butong zhushu wei li 再論中國古代數學與儒學的關係——以唐代學者對禮數的不同注疏為例. Ziran bianzhengfa tongxun 自然辯證法通訊 33: 81–87.
- Zhu Yiwen 朱一文. 2016b. Different cultures of computation in seventh century China from the viewpoint of square root extraction. Historia Mathematica 43: 3–25.