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WHAT WAS FAIR IN *ACTUARIAL FAIRNESS*?

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Abstract

The concept of actuarial fairness stems from an Aristotelian tradition in which fairness requires equality between the goods exchanged. When dealing with aleatory contracts, this principle evolved, among medieval scholars, into *equality in risk*: benefits and losses should be proportional to the risks undertaken. The formalization of this principle gave rise to the concept of mathematical expectation, first implemented in the calculation of the fair price of gambles. The concept of an actuarial fair price was first theoretically articulated in the 17th century as an implementation of this same Aristotelian principle in the field of life insurance. For a practical estimation of fair actuarial prices it was necessary to build mortality tables, assuming that the major risk factor was age. Yet, in the 18th and 19th centuries, we find no agreement among proto-actuaries about the proper construction of these tables. Among the obstacles they found, we want to highlight their early awareness of the possibility of adverse selection: buyers and sellers could manipulate the risk assessment for their own private interests, in a way that would either make fair companies collapse or fair customers be cheated. The paradox in the concept of actuarial fairness is that as soon as it was formally articulated, markets made clear it could never be implemented in actual pricing.

Keywords: actuarial fairness, mathematical expectation, life insurance, annuity, risk.

1. *Actuarial fairness revisited*

The advent of big data raises concern about insurability, as the minimization of classes of risk may ultimately dissolve mutuality and cause market failure –see e. g. (Charpentier, Denuit, & Elie, 2015). In this respect, fair pricing seems to contradict insurability: this opposition is especially challenging for actuaries, since they draw on an old intellectual tradition about actuarial fairness. In the last few years, we have witnessed a growing debate on what is a fair actuarial price (Johnson, 2015; Landes, 2015; Lehtonen & Liukko, 2011). The discussion is motivated partly by the impact of *big data* on the assessment of individual risks, and partly by the changing regulation of insurance in the wake of the latest financial crisis. Unlike other fields in finance, actuaries draw on an old intellectual tradition within their own discipline regarding fairness: a fair premium is understood, by default, as the expected value of the insured quantity. We want to examine this historical tradition in order to articulate a precise response to the question in our title: what was fair in the concept of *actuarial fairness*? Then we want to propose a conjecture about why, despite its clear conceptual articulation, actuarial fairness was never implemented in the calculation of actuarial prices in the almost two centuries elapsed between 1671 (Jan De Witt's foundational text) and 1829 (the British issue of life annuities after the Napoleonic wars).

From a historical standpoint, the concept of actuarial fairness articulates two different principles about the fair pricing of an insurance contract (section 2): *equality in exchange* and *equality in risk*. *Equality in exchange* is an Aristotelian principle about fairness in trade: in an exchange, both parties should leave even. *Equality in risk* is a variation of the former, stemming from a Scholastic debate on contracts with uncertain outcomes (e.g., insurance): the distribution of costs and benefits would only be fair if it was proportional to the risks each of the contracting parties took. If the parties were taking the same risks, equality in exchange applied: they should have equal costs and benefits.

Despite their successful posterity, both principles are inevitably vague: what should count as equal or proportional? How to measure the risks? We will follow a mathematical thread: a significant part of the debate on what these principles entailed was developed through formal analogies. Aristotle exemplified equality in exchange in terms of *arithmetical means*. More crucially for our analysis, early in the 17th century, equality in risk was spelled out in terms of *expected values* (a probability weighted mean). This is where our current debates on actuarial fairness originate.

The expected value of an insurance contract is usually considered its fair actuarial price (its expected value). The fairness of this price, we contend, is grounded on the equality in risk principle. In sections 3-4, we will study how expected values captured this principle, without an explicit definition of mathematical probabilities. Risks were subjectively estimated by the contracting parties, under the assumption that they all knew the same about the uncertain outcome on which their contract hinged. If they agreed on a given distribution of risks, expected values would allow them to calculate the fair price of the contract.

Actuarial justice proper started with the calculation of fair prices for life annuities (section 5). Implementing equality of risk required estimates of an individual's risk of death. Whereas initially this estimate was based on a simple agreement between the contracting parties about their chances of death, in the late 17^{th} century we find the first attempts at estimating these risk empirically on the basis of mortality tables (section 6). Age became the most prominent risk factor to calculate actuarial prices and equality in risk was implemented as membership in the relevant age rank: people within the same rank were charged equal prices.

Despite such a clear articulation, and the wide range of mortality tables that soon became available, equality in risk was never put to use in the calculation of actuarial prices in the 18^{th} and 19^{th} centuries (section 7). Among the many factors that may explain it, we want to highlight how adverse selection undermined, in a number of ways, the very possibility of actuarial fairness. Fair actuarial prices were for disinterested traders, who would never exploit them for their personal benefit. Such prices could not provide the basis for a sustainable economic model. Therefore, in the actual buying and selling of life insurance, the ideal of an objective assessment of the equality in risk principle was never fulfilled. We close (section 8) with a brief thought about why the old Aristotelian sense of fairness we have here discussed is now definitely gone.

2. *The two principles of actuarial fairness*

What we call *equality in exchange* is just the principle of *commutative justice* canonically presented in the fifth book of the *Nichomachean Ethics* (*EN* 1131b25-1132b20). Here Aristotle addressed the problem of the justice of contracts through a mathematical analogy. Suppose that you have two parties with equal claims on a given good, but they have received unequal shares of it (a, b): the fair division of this good is the arithmetical mean of those unequal shares (a+b)/2. Although Aristotle did not discuss in detail what would count as equal in actual exchanges, his intuition was enormously influential and it reached almost *verbatim* his medieval commentators (Fleischacker, 2004). E.g., for Aquinas a fair exchange is one in which the quantities traded do not deviate from the arithmetical mean of the total amount exchanged:

> [I]f, at the start, both persons have 5, and one of them receives 1 out of the other's belongings, the one that is the receiver, will have 6, and the other will be left with 4: and so there will be justice if both be brought back to the mean, 1 being taken from him that has 6, and given to him that has 4, for then both will have 5 which is the mean. (*ST* II-II, q61, a2) (Aquinas, Province, & Publishing, 2014)

With this Aristotelian background, the Schoolmen extensively discussed the fairness of the so-called *aleatory contracts*, in which the benefits and losses depended on an uncertain event (Ceccarelli, 2001). Here emerged the principle of *equality in risk*, of which we will present a particular version by Domingo de Soto (1494-1560). Soto was a Dominican theologian who systematized centuries of legal controversies among the Schoolmen in his monumental *De Iustitia et Iure* (Soto, Carro, & Instituto de Estudios Políticos (Madrid), 1967). In the 6th and 7th question of its 6th book, Soto discussed the fair distribution of benefits and losses in partnership formed through an aleatory contract. His major claim is that an Aristotelian division (an arithmetical mean) would only be fair *if the partners were taking equal risks in their contribution* (beit capital or labor). If the risks they undertake are different, they should divide the total amount *proportionally to those risks*. This is what we call the *equality in risk* principle, which incorporates uncertainty to the standard of fairness set by *equality in exchange*.

Equality in risk allowed Soto to distinguish between insurance contracts and loans. In this latter, the owner of the money does not bear any risk at lending it: the recipient should return it, independently of the success of his venture, plus interests. Thereby the shadow of usury and, therefore, unfairness. In an insurance contract, both parties bear risk instead. The insured party will lose the insurance fee if no adversity occurs. The insuring party will cover the insured capital if there is an adversity. Hence, the premium is a compensation for covering this risk. However, we may wonder what the fair premium would be for an insurance contract. Soto notices here that there is no universal valuation of risks, the contracting parties should reach an agreement on their own:

> For some, the man who covers the danger of the ship of a merchant, perhaps worth twenty or thirty thousand, in the expectation of earning a hundred or a thousand, may seem a fool. (Soto et al., 1967, p. 580)

In the 16th century, equality in risk was formalized in early probability theory, when Pascal and Huygens articulated the concept of mathematical expectation in order to analyze distribution problems in a particular kind of aleatory contracts: gambles (Teira, 2006). In his *Treatise on the arithmetical triangle* (1665), Pascal addressed the so-called Problem of Points: how to distribute the bets in an interrupted gamble. According to Pascal, "it should be strictly proportional to what they might rightfully expect from chance" (Pascal & Lafuma, 1963, p. 57). How could anyone quantify this fair expectation? In *De ratiociniis in ludo aleae* (1657), Huygens provided an algorithm. Let us assume a gamble in which two players may either earn a if they win or b if they lose:

> If I may expect either a or b and either could equally easily fall to my lot, then my expectation should be said to be worth $(a+b)/2$. (J. Bernoulli & Sylla, 2006, p. 133)

Here we have an arithmetical mean, according to the Aristotelian principle of commutative justice. But the risk at which values are put is now *implicitly* quantified. Today we would read Huygens formula as a mathematical expectation: a probability weighted average, in which both outcomes (a, b) are equiprobable. For Huygens though, ½ is not an independent mathematical entity (a probability). As we will see in the following section, it can only be grasped through the gambling contract between both players.

This formalized *equality in risk* solved the Problem of Points: if the game is interrupted, each gambler should receive an amount equal to the expectation of the game. And this would be the fair price to pay for betting in such a gamble: for those who take the same risks, the price should be the same.

3. *Measuring risks through contracts*

Huygens did not quantify risks directly through probabilities. Following a standard procedure in commercial mathematics (Sylla, 2003), he studied gambling contracts and sought equivalences between them. To the extent that various contracts entailed the same risks, they would have the same expected value. An implicit quantification of probabilities arose therein.

Here is the argument Huygens used to articulate the concept of mathematical expectation. Imagine a simple game with equal chances for the players to get outcomes a or b (where $a<b$). The game is interrupted and two players join in, each one of them paying an amount x to gamble (establishing x is the Problem of Points). They agree that the winner of this second gamble will earn $2x$ and the loser will still get a. The two gambles will be equivalent if the winner of both the first and second gamble gets the same prize, b. Hence, $2x$-a should be equal to b. The amount x that the players of the second gamble should pay to join in is just equal to (a+b)/2. This is the expected value of this gamble, and it will be the fair price of the original (interrupted) gamble. The ½ weight in the formula arises from 2 being the number of players betting in the gamble, not from a separate quantification of equiprobability.

Jan De Witt (1625-1672) used this same approach to quantify the fair price of an insurance contract, as we will discuss in section 5 below. Like Huygens, he articulated his approach on an analysis of gambling contracts:

> I presuppose that the real value of certain expectations or chances of objects, of different value, must be estimated by that which we can obtain from several expectations or chances, dependent on one or several equal contracts. (De Witt, 1671/1995, p. *)

Let us imagine a player (John) who has the same chance of obtaining three different amounts of money (e.g., 2000, 3000, 4000 florins). How can he calculate how much this expectation is worth? For this, argues De Witt, John has to set up a contract with two other players, Peter and Paul. Each of them will contribute 3000 florins to a joint fund. These 9000 florins will be awarded to one of them through a lottery, in which they will have equal chances to win. John should sign two separate contracts now with Peter and Paul: If either John or Peter wins the lottery, he will compensate his partner with 2000 florins; if either John or Paul wins, the compensation for the loser will be 3000 florins. The three contracts give John an equal expectation of winning either 4000 florins (the joint fund lottery prize, minus the compensations to his partners), 2000 or 3000 florins. This expectation will be worth 3000 florins which is the amount required to join the initial fund[1].

For De Witt, the example illustrates a general principle: the value of an expectation is equal to the addition of the quantities at stake divided by the sum of the chances of obtaining them: in the previous example 9000/3. Like Huygens, in De Witt the number of partners implicitly stands for the number of chances, provided that their agreement assumes they all have equal chances. When there is no equiprobability, for his algorithm to hold, we need as many partners in the initial contract as the sum of the chances of obtaining the different outcomes: e.g., if the chances are 6, 4 and 3, there should be 13 partners.

Probabilities do not exist separately from the bilateral contracts between them: expectation is the primitive concept for Huygens and De Witt. Therefore, a mathematician could infer, once the contracts had been signed, that they entailed equal risks to the extent that their expected price was the same. But how can the contracting parties appraise their mutual risks at the time of signing them? The equality in risk principle can be applied to contracts: same risks, same prices. We may wonder, however, how the gamblers in each individual contract assess their risks, before they sign them.

4. *Equal risks*

Applying the equality in risk principle in the analysis of equivalent contracts somehow presupposes that in each of these contracts, the signing parties have used this same principle for their agreement. But how could have they estimated their risks fairly without a definition of probability? Again, our best source about the conceptual foundations of such an agreement is legal theory. Among Pascal's closest friends, we find the jurist Jean Domat (1625-1696), author of a systematic treatise on *The Civil Law in its Natural Order* (1689), usually considered the first attempt at a rational systematization of French law.

In the book, Domat discussed at various points the role of uncertainty in the fairness of an agreement. Consider, for instance, those *covenants* concerning an uncertain event, in which one of the contracting parties may, e.g., renounce all profit, and free himself from all loss. Domat claims that their justice is founded upon this:

> [O]ne party prefers a certainty, whether of profit or loss, to an uncertain expectation of events; and the other party, on the contrary, finds it his

[1] Although De Witt does not mention it, for this arrangement to be fair, there should be an additional contract between Peter and Paul establishing that if any of them wins 9000 florins, the loser will receive

> advantage to hope for a better condition. Thus, there is made up between them a sort of equality in their bargains, which renders their agreement just. (Domat & Cushing, 1850, p. 186)

If the contracting parties have complementary expectations about the outcome, the agreement is fair. Their expectation depends, of course, on their subjective estimation of their chances of suffering from a certain adversity. For Domat, this subjective estimate provides good enough grounds for a fair agreement inasmuch as the contracting parties are equally uncertain about the outcome.

E.g. (Domat & Cushing, 1850, pp. 354-355), in a "universal partnership", if only one of the partners happens to have a daughter, he is nonetheless entitled to take her dowry from the "joint stock", since all the partners were "under the same uncertainty of the event, and with the same right, having rendered their condition equal, it made also their agreement just". Rather than objective risks (their equal chances of having a daughter), Domat emphasizes their equal ignorance regarding those risks as the grounds for a fair agreement.

For a contract to be valid, the parties do not need to quantify their risks. They should just be able to grasp their commitments. If, for instance, an inheritance is accepted, but the heirs are all ignorant of some attached debts, they won't be able to reject it later for "want of knowledge".

> For it was not upon an exact and perfect knowledge of all the particular rights and charges of the inheritance that his engagement was founded; but it suffices to confirm it, and to make it irrevocable, that he knew that an inheritance consists of rights and of charges which are often unknown even to the most clear-sighted heirs; and that under the uncertainty of more or less which could not be known, he has taken his chance of losing or gaining in a thing that was altogether uncertain. (Domat & Cushing, 1850, p. 495)

A contract hinging on uncertain events is valid inasmuch as the parties are all in equal conditions regarding the risks involved, even without a precise quantification. This latter will emerge through the contract.

5. *Actuarial fairness at work*

When Jan de Witt wrote "Value of life annuities in proportion to redeemable annuities" (De Witt, 1671/1995), in 1671, he was the political leader of the United Provinces of Holland. The piece was written as a report addressed to the States-General of Holland, since the country was considering the issuance of life annuities at a fixed price (i. e. regardless of age), which De Witt wanted to compare to the price of perpetual annuities In exchange for this price, a life annuity would provide a series of equal payments for the remainder of the buyers' lifetime. Throughout centuries different institutions had been financing themselves with the sale of annuities, but

De Witt's is considered the first author to compute the value of a life annuity as the sum of expected discounted future payments.

It is not so often noticed that in this calculation De Witt operated in the normative framework presented above, exploiting the analogy between life annuities and gambling contracts. On the one hand, the prize of the gamble corresponded to the income the annuity buyer may obtain depending on the duration of his life. On the other hand, the chances of each outcome in the gamble correspond to the chances of the buyer dying at any particular point in time. Hence, using Huygens' approach, De Witt could calculate the expected value (a_x) of a life annuity for a person of age x as follows. In our current notation (Hald, 1990, p. 128), the formula for his algorithm would be:

$$a_x = \frac{1}{l_x} \sum_{t=1}^{w-x-1} a_{\overline{t}|} d_{x+t}$$

The outcomes of the gamble are $a_{\overline{t}|}$, where $a_{\overline{t}|}$ is the current value of an annuity paid every six months (at a 4% interest rate) over t half-years. The number of deaths in each period $x+t$ is d_{x+t}. The sum of the number of deaths from age x onwards is $l_x = d_x + d_{x+1} + \ldots + d_{w-1}$, where w is the maximum number of half-years in which the annuity will be paid. As (Hald, 1990, p. 128) observes, d_{x+t}/l_x is a probability distribution. But this is not how De Witt estimated the chances of dying. He first divides a person's life in four intervals: (3, 53), (53, 63), (63, 73) and (73,80). The chances of anyone dying in any of these four intervals are estimated as follows:

> [T]aking for example two persons of equal constitution, one aged 40 years, and the other 58 years, if these two persons made such a contract, that in case the person of 58 years should happen to die in less than 6 months, the one aged 40 were to inherit a sum of 2000 florins from the property of the defunct; but that if, on the other hand, the person aged 40 years should die in less than 6 months, the other aged 58 years were to have 3000 florins from the property of the deceased; such a contract cannot be considered disadvantageous for the person who would have the 3000 florins, if the event were favourable to him, and who, in the contrary event, would only lose 2000 florins. (De Witt, 1671/1995, p. *)

The chances of anyone dying in these two intervals are inferred from the fairness of the contract: for De Witt, the proportion between the chances of dying of a person whose age is the range (53-63) and the chances of one in the (3-53) range are 3 to 2, because if the older person dies, the amount to be paid is only 2/3 of the amount due if the younger one dies. If the contracting parties do not consider the agreement "disadvantageous", they will implicitly agree on the chances[2].

Hence, in buying a life annuity at a price calculated according with De Witt's formula, the contracting parties implicitly agree on a given proportionality of chances for each age range: taking (3, 53) as the baseline, the proportion will be 2/3 for (53-63), 1/2 for (63,73) and 1/3 for (73,80). De Witt's formula will then comply with the equality in risk principle if the buyers of age within a given interval take themselves to have the same chances of dying, and the same given proportion regarding the buyers in the other intervals. In accepting this distribution of chances, they acknowledge that they all were "under the same uncertainty of the event": otherwise they would exploit someone else's ignorance.

6. *Mortality tables*

De Witt's formula established actuarial fairness in the now canonical form: *same risk* (of death), *same price* (for the life annuity). The most significant innovation thereafter was the estimation of these risks from actual mortality data. The contracting parties should now agree on a given mortality table as the relevant estimate of the risks of the buyer. The relevant risk factor in these tables is, of course, age: the older the applicant, the higher the premium. In the coming two centuries, equality in risk would hinge entirely upon mortality tables. Edmund Halley and Nicolas Bernoulli provided the initial articulation of this approach.

In 1693, Edmund Halley published in the *Philosophical Transactions* of the Royal Society, his "estimate of the degrees of mortality of mankind", based on the records of the city of Breslaw. For Halley, "the price of insurance upon lives ought to be regulated" (Halley, 1693, p. 602) on these mortality tables, since they provided an empirical estimate of the chances of people dying at a certain age. Whereas De Witt had estimated these chances on the basis of a contract, Halley now argues on death frequencies as follows: if we want to ensure the lives of two men of 20 and 50: "It being 100 to 1 that a Man of 20 dies not in a year, and but 38 to 1 for a Man of 50 Years of Age" (Halley, 1693, p. 602). For Halley, "it is plain that the Purchaser ought to pay for only such a apart of the value of the Annuity, as he has Chances that he is living" (Halley, 1693, p. 602). I.e., what is plain for Halley is the equality in risk principle: same risks, same price. Except that now the risks are inferred from statistical records, without any moral presumption. Nonetheless, adopting Breslaw's mortality table as the source of every risk estimate is objectionable, as Halley himself acknowledges: "it may be objected, that the different Salubrity of places does hinder this proposal from being universal; nor can it be denied" (Halley, 1693, p. 619). For Halley, this is an empirical matter, subject to further investigation. In a way, this approach naturalizes

[2] We also find in De Witt an ambivalence between physical and epistemological considerations: in discussing the symmetry between the chances of dying in each half-year within one interval, he considers it similar "to the case of a tossed penny, where there is an absolute equality of likelihood or chance that it will fall *head* or *tail*" (De Witt, 1671/1995, p. *)

the estimation of risks, but it does not eliminate the necessity of agreement. Equality in risk demands the annuity buyer's agreement to risk estimate on which the annuity price is calculated.

Nicolas Bernoulli concurred in this point with Halley. In 1711, he published a summary of his doctoral dissertation on the use of the *Ars conjectandi* in Law (N. Bernoulli & Meusnier, 1992). A couple of years later, Nicholas published his uncle's unfinished *Ars Conjectandi*, a landmark establishing the foundations of modern probability theory. The framework of Nicholas' dissertation is still normative, though. In chapter IV, Nicolas discusses the legal foundations of the pricing of life annuities. The only foundation for these prices is the reason of the contracting parties (*ratione contrahentium*), since it should be fixed at the time of their agreement (and not when the outcome on which the contract hinges happens). Again, these contracts are licit to the extent that the duration of life is equally uncertain to both parties. The calculation of a fair price for an annuity depends thus on the estimation of the risks involved. Here we find again equality in risk:

> It is clear that the price cannot be established without taking the buyer's age and health into consideration, of which we should have the best knowledge in order to set the price of a life annuity. The same annuity cannot be sold indifferently to men of all ages. (N. Bernoulli & Meusnier, 1992, p. 62)

Drawing on a summary of John Graunt's 1662 mortality table, Nicolas proceeds to estimate the length of the human life. However, he observes that some further data from an unidentified Swiss city disagree with his estimates. These should remain "hypothetical" (*hypothesi*), he observes, inviting, like Halley, further research on the topic.

Both Halley and Bernoulli argued as if actuarial justice could be objectively appraised: the more accurate the mortality table, the better the implementation of the equality in risk principle. By the beginning of the 17th century this objectivist approach to actuarial justice was then firmly established on purely theoretical grounds.

7. *Actuarial justice in the actual markets*

Even if the concept of actuarial justice had been clearly articulated, during the two following centuries, we find no unified standard for pricing neither life insurance nor annuities in Europe (Clark, 1999, p. 115). On the one hand, there was a background of more or less widespread social conventions: in Northern Europe, life annuities were often sold at one half of perpetual (or redeemable) annuities (Dafforne & Malynes, 1636); in Roman law countries, Ulpian's table might be used for reckoning annuities (which were in fact an usufruct from an estate). On the other hand, we find an even wider variety of pricing schemes in the emerging market of private mutualities. These were societies created in order to provide annuities for the surviving widows of their members: in the Low Countries, we find the so-called *Bossen* from the 1650s onwards

(Algemeene Maatschappij van Levensverzekering en Lijfrente, 1898, pp. 242-262); in Britain a number of companies emerged, most of them basing their prices on simple sales agreements.

As table 1 below shows, during these 200 hundred years a growing number of mortality tables were published. Yet, none of them succeeded at standardizing the calculation of prices for life annuities: the discrepancies between their estimated life expectancies are significant and led to substantially different actuarial prices (tables 2 and 3 below). This was partly due to a lack of methodological consensus on the construction of these tables, but, in our view, the discrepancies can also be interpreted in terms of a deeper conceptual disagreement about how to handle the problem of *adverse selection* in actuarial markets[3]. We are going to argue that both insurance buyers and sellers can manipulate the principle of equality in risk for their own private interests. The diversity in mortality tables just illustrates the lack of a consensual solution to this problem, other than State intervention, which arrived at the end of the Napoleonic wars.

But let us begin with the purely technical issues. For a start, calculating the mathematical expectation of an annuity was beyond the grasp of most proto-actuaries (Poitras, 2000). Moreover, the methodology behind these tables remained in constant development for two centuries. As we just saw (section 6), Edmund Halley published the first life table really derived from mortality bills (although he smoothed the data, see (Bellhouse, 2011)) establishing a successful method, taken over by most subsequent studies. Later on, in the 1720s, De Moivre and de Graaf provided some analytical approximations for Graunt's and Halley's tables. (Simpson, 1742) addressed the (so far, implicit) problem of assuming a constant population in constructing a life table from mortality data –when cities like London obviously experienced population growth, partly due to migrant inflows. In the following 100 years, the subsequent tables use larger samples and more sophisticated methodologies (Murray, 2016).

However, the discrepancies between all these mortality tables are not just a matter of sampling and methodology. The tables were also constructed with different goals, exhibiting a clear awareness that not all of them were fit for insurance valuation. On the one hand, there were tables designed for purely descriptive purposes. For instance, Graunt, Buffon or Moheau were addressing issues in demography: in Graunt's own words, "it may now be asked, to what purpose tends all this laborious bustling and groping to know, 1. The number of people? 2. The number of male and female? 3. How many married and single?" (Birch, 1759, p. 35). Some other tables cared instead for proto-epidemiological questions, namely the increase in life

[3] When buyers or sellers cannot directly determine the quality of a good or service, adverse selection can lead to the elimination of all trade in a market (Wilson 2008): putting it very simply, if consumers know that there is a percentage of cheap (bad quality) insurance, they will be reluctant to pay the price requested for more expensive good quality insurance and the issuers of these latter may end up leaving the market.

expectancy resulting from inoculation of infants (D. Bernoulli, 1765/1982)(Lambert, 1772).

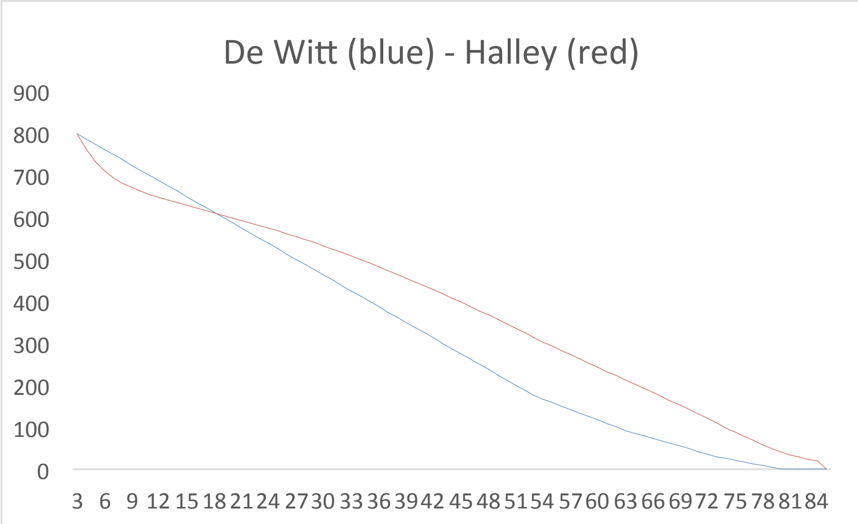
It is obviously possible to calculate fair actuarial prices for life annuities with these descriptive tables. After all, if the sole relevant factor for assessing equality in risk is age, the better the estimation of the life expectancy, the fairer the price. But De Witt himself noticed that in actuarial markets customers do not care just about justice, but also for their own private interests. While the Low Countries, already at war with Britain, were on the verge of being attacked by the French, De Witt wanted to establish whether it was more costly for the state to borrow with life annuities at 8% (12.5 years' purchase) or perpetuities at 4% (25 years' purchase). De Witt argued that 8% for life annuities was too generous since most purchasers placed them on the heads of those healthy children who presumably had the longest life expectation.

> [T]he said life annuities are oftenest purchased and sunk upon the lives of young and healthy children of 3, 4, 5, 6, 7, 8, 9, 10 years, or thereabouts. During that time, and for some years ensuing, these young lives, having become more robust, are less subject to mortality than about 50 years afterwards, and than for some years anterior to these 50 years; and so much the more, as during the first aforesaid years they either are not, or are but little, exposed to external accidents and extraordinary causes of death, such as those from war, dangerous voyages, debauch, or excess of drink, of the sex, and other dangers (De Witt, 1671/1995, p. 15)

Indeed, according to (Hup, 2011), between 1662 and 1713 in the Low Countries, only 20% of the life annuities actually sold provided insurance for adults against future poverty; the remaining 80% were placed on healthy children with a view to maximizing the expected return of the annuity.[4] In other words, if the life annuity was sold at its fair price, simply taking into account a mortality table describing the risks for every age class in the general population, the issuer of the annuity may simply go

[4] As life annuities were sold at a constant price irrespective of life expectancy, the expected return of buying a life annuity is increasing in life expectancy, which is maximal around 6 years.

bankrupt for sheer *adverse* *selection*.



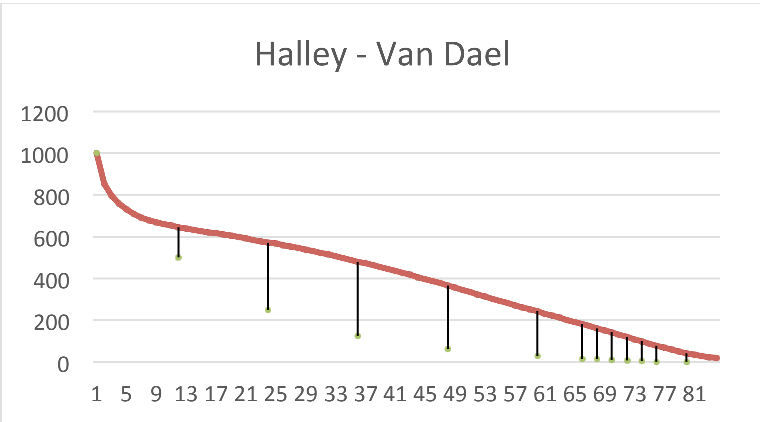
By way of protection against adverse selection, De Witt suggested instead a price closer to 6.67% (16 years' purchase).[5] If we take Halley's table as a purely descriptive benchmark, we see de Witt's underestimation of the mortality of the first age rank (3-18) and overestimation of mortality at later age raises the value of annuities placed on young lives only. In this view, the mortality table was not purely descriptive and *solvency*, as we would now call it, appeared as a feature distinguished from *fairness*. There was not much opportunity for implementing solvency in a context of war. A year later, the City of Amsterdam issued life annuities at a price decreasing with the age of the annuitant: (Hebrard, 2004) has shown that the prices were related to mortality data compiled by Hudde, the mayor of Amsterdam with a 8.13% discount rate (12 years' purchase). That very high rate prevailed because the pending war had begun. Fortunately for the Dutch it did not last but the annuities issued to provide for the urgent needs of the war were so expensive nobody would reasonably borrow at that price if not forced to (by the war, in this case).

In 1740 Nicolaas Struyck, an enlightened *dilettante*, made De Witt's strategy methodologically explicit, constructing mortality tables organized by gender and sampling from annuitants lives only. As Struyck wrote (Struyck & Vollraf, 1912, p. 217): "human life will be [...] on average a little shorter [than the figures derived from the table] as the heads on which an insurance is bought are chosen. We can be sure that they were not very ill when their lives were insured". We might interpret Struyck as refining the equality in risk principle: the self-selection of annuitants in buying insurance somehow guarantees that they all see themselves in the same risk class – i.e., not just of the same age, but equally healthy. Whereas in the purely descriptive tables, the reference class usually is the general population, in tables constructed in order to price life insurance, the reference class should just be the accepted applicants.

[5] Struyck and Eneström later demonstrated this Price to be erroneous.

However, this refinement of the equality in risk principle presupposes that the insurance provider is somehow able to correctly estimate their risks, independently of his own commercial interests. Yet, the admission procedure in most mutualities was less than transparent. In Britain, for instance, the Equitable Company pioneered the systematic use of mortality tables for the calculation of premiums since 1762. But it did not present its estimates to the applicants. They were just questioned about their age and health and, if accepted, they were given a premium that they could only take or leave (Ogborn, 1962, pp. 252-253).

Producing the table was not in itself a warrant of impartiality: in the 1670s, some Dutch cities issued tontine bonds under the supervision of Jacob van Dael (Kopf, 1927, p. 244), advertising them with a survival table. It is unclear whether this represented the potential mortality of annuitants or a commitment to pay, as in a tontine, the value of each annuity grows with the death of each member, since its share is devolved to the other participants. But if we take again Halley's data as a benchmark, it seems as if Van Dael's table overestimated the mortality of the first age ranks in order to make the investment more attractive



To what extent was Van Dael overselling his product? After all, the overestimation of mortality makes the investment more attractive for anybody who hoped to reach the latest age ranks. And how could any buyer tell? The asymmetries of information between buyers and sellers subverted the very possibility of actuarial fairness. If the sellers estimated their mortality tables on the general population, buyers with a better than average life expectancy may exploit it to their own advantage. If the sellers corrected their mortality tables adjusting the life expectancy to the actual risks factors of the buyers, how could these latter ascertain whether their risks were correctly assessed? The proliferation of mortality tables during the 18th and 19th centuries put the ideal of actuarial fairness in question. The discrepancies between the tables were so significant that the accumulation of data, on its own, did not yield any benchmark for calculating fair actuarial prices. Table 2 shows how, over 200 years, life expectancies at 6 ranged from 19 to 48 years; at 20 from 19 to 40; and at 50 from 10 to 20. Table 3 displays the corresponding price of life annuities: although the variance is less, one can still find 80% difference between the lower and higher price at 6, 60% at 20, 50% at 50…

In hindsight, the equality in risk principle, plausible as it may be on purely theoretical grounds, cannot stand the corrosive force of adverse selection. However, in the 18th century, most mortality tables were constructed under the opposite assumption (Behar, 1976): despite the evidence showing that mortality was not the same everywhere, the tables were estimated as if they provided a universal representation of mortality –upon which equality in risk could safely rest. Halley was already concerned with "mankind"[6]. In the same vein, Buffon wrote: "Mrs Halley, Graunt, Kersboom, Sympson [*sic*], etc. also gave tables of mortality of mankind [...] but it seems that their research, though very detailed and extensively worked out, can give only distant approximations of mortality of *mankind in general* (e. a.)" (Buffon, 1749, pp. 383-384). Later, Moheau (1778), who investigated enough samples to find undeniable differences, concluded: "By bringing together several parishes, and by comparing a collection to another, the differences are much less significant; so that the eight parishes of the *Generalité* de Rouen, appear those who can give the most fair idea of the *common lot of humanity* (e. a.) in France"(Moheau, 1778, p. 178). Moheau even suggests that the "common lot" would be readily found by "operating on larger number of parishes, with their climate and situation chose so that the result would be the average term" (p. 194). In a more theological vein, De Moivre states: "*altho' Chance produces Irregularities, still the Odds will be infinitely great, that in process of Time, those Irregularities will bear no proportion to the recurrency of that Order which naturally results from* ORIGINAL DESIGN" (de Moivre, 1756, p. 251). Even Struyck, despite his awareness of adverse selection, believed in the objectivity of his own approach[7].

This objectivist approach to mortality and actuarial justice collapsed, for all practical purposes, in the aftermath of the Napoleonic wars. Without the financial pressure of funding military expenses, France and Britain started to issue life annuities based on conservative life tables that became a default benchmark for actuarial markets in terms of solvency. In 1829, the British Treasury issued very large amounts of life annuities in order to repay government debt, choosing John Finlaison's mortality tables to price them. Finlaison had earned himself an appointment as *Actuary of the National Debt Office*, showing the money his estimates could save to the Treasury. The crucial difference was not in Finlaison's method, but in the situation: he followed methods already implemented by Struyck (gender classification) and Duvillard (population growth). But, in times of peace, there was no need to "squeeze the money

[6] The title of (Halley, 1693) features the word, which is repeated: "From these Considerations I have formed the *adjoyned Table*, whose Uses are manifold, and give a more just *Idea* of the *State* and *Condition* of *Mankind*, than any thing yet extant that I know of." (Halley, 1693, p. 600)

[7] "I think if unbiased people were taking data on annuities from other accounts in other countries, considering all the people who bought insurance around the same time, dividing them into classes and noting the number of years during which they drew their pensions, the same way I did above, they would arrive at a nearly identical result." (Struyck & Vollraf, 1912, p. 222)

out[8]" of the public by offering high returns, in competition with private insurance companies. The British Treasury could now offer lower returns, grounded on Finlaison's tables, but with fewer prospects of bankruptcy than any private seller.

The French had been the champions of universalism during the 18th century, but they had no sustainable life insurance business. Life insurance had been forbidden by Colbert's *Ordonnance de Marine* (1681) and the Hospitals were later denied the right to issue life annuities after the bankruptcy of the Paris *Hôtel-Dieu* in 1690 (Pradier, 2016). There was an attempt at creating a life insurance company in 1786, but it was fruitless at a time where the State sold life annuities at a flat 10% in order to consolidate the debt resulting from the American war (Thiveaud, 1989). Only at the end of the Napoleonic wars, in 1818, were life insurance companies authorized in France. Laplace already had a theory of probability ready to manage the new businesses. In contrast with the eighteenth century doctrine of actuarial fairness, the Laplace model assumed that insurance companies should *load* their premiums in order to guarantee solvency. The loading also provided an unintentional protection against adverse selection...

So far, adverse selection had undermined any public consensus on the relevant mortality tables. By adopting one for their annuities, European States created national benchmarks in terms of solvency: in principle, no private company could reliably undertake more risks than they did. The actuarial fairness of the prices of public annuities became a secondary concern: while in theory it was possible to calculate it, for all practical purposes the assessment of risk factors became a social convention, rather than an objective fact of nature. Buyers and sellers had a default risk assessment in public mortality tables, independently of how well it captured their individual circumstances. If they wanted to undertake more risks, it was their private decision[9].

8. *Concluding remarks*

We have shown how the concept of actuarial fairness stems from an Aristotelian tradition in which fairness requires *equality in exchange*. When dealing with aleatory contracts, this principle evolved, among medieval scholars, to *equality in risk*. The formalization of this principle gave rise to the concept of mathematical expectation, quantifying the fair price of aleatory contracts. Among these, the quantification of equal risks in annuities and life insurance led to the development of mortality tables, upon which it was possible to calculate actuarial fair prices. Yet, in the two following centuries, we find no agreement about the proper quantification of the risks associated with age. Among the obstacles, the most prominent for our purposes, is the early awareness of the possibility of adverse selection. Buyers and sellers could

[8] "Faire rentrer l'argent", as wrote an employee of the *Contrôle Général* in 1688 or 1689 (during the war of the League of Augsburg, hence), see AN, G7, 1593, mémoires, 4 p.

[9] If they were allowed to make it: life insurance regulation in France insists on prudence, and actuaries can certify life tables to justify a higher price (than the legal table), not a lower one.

manipulate the risk assessment for their own private interests, in a way that would either make fair companies collapse or fair customers be cheated.

In our view, the principle of equality in risk collapsed at this point: if there was no objective assessment of individual risks through universal mortality tables, it did not make any sense to hold a standard of fairness based on risk equality. At most, it could be equality in uncertainty, following Domat's intuition. But in life insurance, equality in uncertainty has never been more than a fiction: even with State-backed mortality tables, the seller knew more than the buyer about his own health and the seller understood more the mortality tables than the average buyer. Rather than fair (in an Aristotelian sense), it was a mutually convenient agreement at most. Does it make sense then to keep referring to expected values as a standard of actuarial fairness? In our view, the Aristotelian intuition that once sustained it conceptually is now definitely lost. No arithmetical mean, on its own, can capture our contemporary intuitions about fairness and it is, indeed, about time to rethink actuarial fairness on completely different grounds.

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Author	*Date*	*Data type*	*Hypo thetical*	*Fact-based*	*Actuarial notations*	*Notes | Sources*
Ulpianus	ca. 225	Legal value of (life) annuity in years' purchase	—	No	a_n	Can be interpreted as a_n.
John Graunt	1662	Survival table for 10-years intervals	Yes	Allegedly	l_7, l_{17}, l_{27}, … l_{77}	(Birch, 1759) Disputed authorship: Petty (Le Bras, 1998)
Hôtel-Dieu, Paris	1668	Mean life, median life, life annuity price table.	—	No	a_n	(Pradier, 2016) Life annuity price table
C. & L. Huygens	1669	Life annuity price table			e_n, a_n	(Rohrbasser & Véron, 1999) Data from Graunt's 1662 table (Birch, 1759)
Jacob van Dael	1670	Unclear: survival table or income stream.	Yes	No	Doubtful	(Hald, 1990, p. 121)
Johan de Witt	1671	Mortality rates, survival table.	Yes	No	$a_{\bar{x}\|4\%}, \frac{q_{x+n}}{q_x}$, a_x for some x, n.	(De Witt)
Johan Hudde	1672	Compilation of mortality data, life annuity price table	(likely)	Yes	a_n	(van Ham, 2005) Data from annuitants' lives, with new annuitants from 1 to 50 years.
Edmund Halley	1693	Survival table	(likely)	Yes	l_x, v^x, a_x.	(Bellhouse, 2011; Halley, 1693)
Abraham de Moivre	1725	Analytical simplification	Yes	No	$l_{x+h} = l_x - \frac{h}{n}(l_x - l_{x+n})$	"De Moivre's law" = linear survival function to fill missing values in Graunt's 1662 table (Birch, 1759)
Isaac de Graaf	1729	Analytical simplification	Yes	No	$p_x = 1 - \left(\frac{x}{92}\right)^n$ $with\ n = 5$	(Struyck & Vollraf, 1912, p. 203)
John Smart	1738	Survival table	Yes	unclear	l_x, d_x.	(Smart, 1738)
Nicolaas Struyck	1740	Survival tables	Yes	Yes	l_x, a_x.	(Struyck & Vollraf, 1912) Data from annuitants' lives. Separate tables for male and female.
Kersseboom	1738,	Survival table		Yes	l_x, a_x.	(Kersseboom, 1742)

	1742					Data from annuitants' lives.
Johann Süssmilch	1740 1761	Survival table		Yes	l_x, a_x.	(Süssmilch & Baumann, 1761) Compilation of data from previous works.
Thomas Simpson	1742	Survival table	Yes	Yes	l_x, v^x, a_x.	Computed from (Smart, 1738) correcting for migration using (Halley, 1693).
Antoine Deparcieux	1746	Survival table	Yes	Yes	l_x, a_x.	(Deparcieux, Behar, Gallais-Hamonno, Rietsch, & Berthon, 1746/2003) Data from annuitants' lives to control adverse selection. De-clustering of 5 years clusters.
Georges-Louis Leclerc de Buffon	1749	Median life		Yes	n. a.	(Buffon, 1749)
James Dodson	1756	Survival table		Yes	l_x, d_x, a_x.	(Dodson, 1756)
Corbyn Morris	1759	Survival table		Yes	l_x.	(Morris, 1759)
Leonhard Euler*	1760					(Euler, 1760) Refined the computation of life annuities using data from (Kersseboom, 1738, 1742)
Daniel Bernoulli	1765	Hypothetical survival table	Yes	Yes	l_x, e_x.	(D. Bernoulli, 1765/1982) Table deduced from (Halley, 1693)with assumptions about mortality from small pox.
Pehr Wargentin	1765	Survival table		Yes	l_x, d_x.	(Wargentin, 1995) From census data.
Richard Price	1769	Survival table		Yes	l_x, d_x.	(Price, 1773) Usually known as "Northampton table"
Louis Messance	1766	Survival table		Yes	l_x for 0, 5, 10, 15…	(Messance, 1766)
Jean-Henri Lambert	1772	Hypothetical survival table	Yes	Yes	l_x, d_x, e_x.	(Lambert, 1772) Table deduced from (Süssmilch & Baumann, 1761) with assumptions about mortality from small pox.
Jean-Baptiste Moheau	1778	Mean life			l_x, d_x, e_x.	(Moheau, 1778)
Johann Augustin Kritter	1781	Survival tables		Yes	l_x, d_x, e_x.	(Kritter, 1780)
Adrien Duvillard	1806	Survival table	Yes	Yes		(Duvillard De Durand, 1806)

						Compilation of data from (Moheau, 1778) and from Geneva. Adjustment for non-stationarity.
Joshua Milne	1815					(Milne, 1815) Usually known as “Carlisle table”
James Finlayson	1829					(Finlaison, 1829)
Auguste Demonferrand	1830					(Demonferrand, 1835)

Table 2 Life expectancy at age

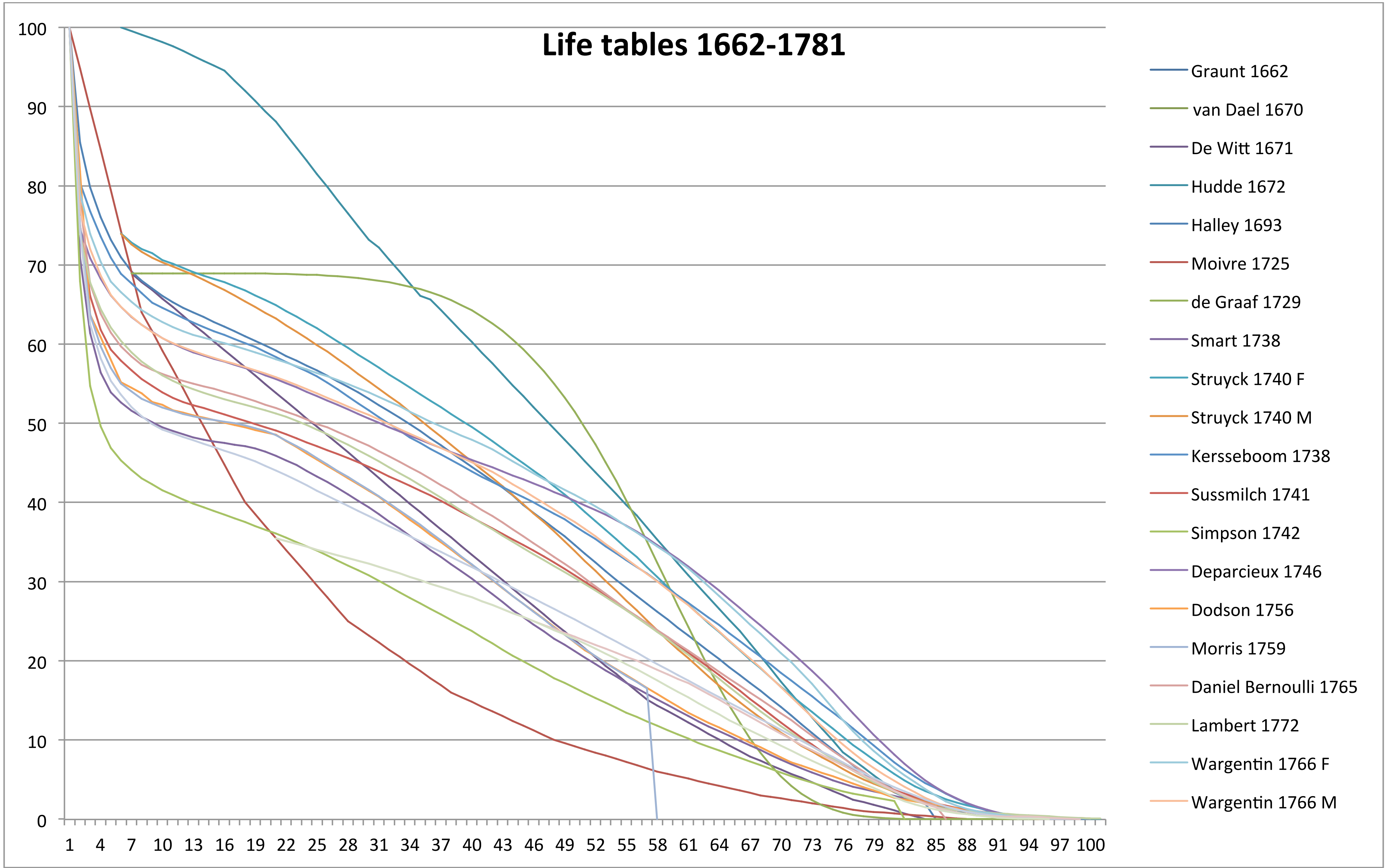
Age	Hudde 1672	Halley 1693	Moivre 1725	de Graaf 1729	Smart 1738	Struyck 1740 F	Struyck 1740 M	Sussmilch 1741	Simpson 1742	Deparcieux 1746	Dodson 1756	Wargentin 1766 F	Wargentin 1766 M	Price 1769	Kritter 1781 F	Kritter 1781 M
6,0	40,0	41,0	19,7	48,9	38,6	43,7	39,7	42,7	35,6	47,8	38,5	47,4	45,0	40,6	n/a	n/a
20,0	30,1	32,9	19,1	34,9	28,7	34,3	30,6	34,5	28,2	39,8	28,6	38,7	36,5	32,9	36,3	35,0
50,0	15,5	16,0	13,3	9,9	15,6	17,0	14,7	16,4	14,8	20,0	15,6	18,8	17,2	17,5	18,2	16,5

* Sources in Table 1 above | F: female; M: Male

Table 3 Fair price of life annuities in years' purchase for a 5% interest rate

Years purchase	Hudde 1672	Halley 1693	Moivre 1725	de Graaf 1729	Smart 1738	Struyck 1740 F	Struyck 1740 M	Sussmilch 1741	Simpson 1742	Deparcieux 1746	Dodson 1756	Wargentin 1766 F	Wargentin 1766 M	Price 1769	Kritter 1781 F	Kritter 1781 M
6	16,5	16,3	10,8	18,9	16,2	16,9	16,3	16,5	15,3	17,0	16,2	17,2	16,9	16,0	n/a	n/a
20	14,3	15,3	10,9	16,8	14,2	15,5	14,7	15,6	14,1	16,5	14,1	16,5	16,1	15,0	15,9	15,8
50	10,6	10,8	9,4	8,1	10,3	11,0	10,0	10,9	10,2	12,4	10,3	12,0	11,3	11,3	11,7	10,9

* Sources in Table 1 above | F: female; M: Male



Life tables 1662-1781
100
90
80
70
60
50
40
30
20
10
0
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97 100
Graunt 1662
van Dael 1670
De Witt 1671
Hudde 1672
Halley 1693
Moivre 1725
de Graaf 1729
Smart 1738
Struyck 1740 F
Struyck 1740 M
Kersseboom 1738
Sussmilch 1741
Simpson 1742
Deparcieux 1746
Dodson 1756
Morris 1759
Daniel Bernoulli 1765
Lambert 1772
Wargentin 1766 F
Wargentin 1766 M