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Chapter 5

Numbers with Histories: Li Chunfeng on Harmonics and Astronomy

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Howard L. Goodman*

Abstract Focusing on Li Chunfeng’s 李淳風 (602–670) *Sui shu* 隋書 “Lü-li zhi” 律曆志 (Monograph on Harmono-metrology & Mathematical Astronomy) and its predecessor in the *Han shu* 漢書, this chapter questions the universality of the marriage between these fields in Chinese thought, arguing instead that it is the product of specific compilers’ grappling with generic conventions and the messy course of history. The joint monograph appears first in the *Han shu*, a natural consequence of its reliance on Liu Xin’s 劉歆 (46 BCE – 23 CE) synthetic writings, and culminates in Li Chunfeng’s monographs, after which subsequent histories abandon the model. Whatever the initial strength of this marriage, we argue, the common impulse to accuracy and empirical testing began to drive the two fields apart from both one another and from the promise of an elegant universal order as early as the Eastern Han. Framing his presentation on Liu Xin’s, and drawing heavily from his and others’ work, we attempt to show how Li Chunfeng’s editorial hand acts to address the unravelling of this order by imposing a *telos* upon the history of harmono-metrology and shifting the basis of the two fields from *Zhouyi* 周易 number symbolism to *Jiuzhang suanshu* 九章算術 number ratios, freeing number from the realm of timeless, petrified truth to allow it its own history in the face of progress.

5.1 Introduction

The third and last distinction between heavenly movements, I mean the lateral, should be related to modulations between *tonoi*. ... Among them we should compare the Dorian *tonos*, which is right in the middle of the others, with the middle positions of their lateral movements... the Mixolydian and the Hypodorian, as being the extremes, with the most northerly and southerly positions, conceived in the guise of tropics; and the remaining four *tonoi*, which are between the ones mentioned, with those falling on the parallels between the tropics and the celestial equator, these being themselves four in number, because of the division of the slantwise circle [i.e. the ecliptic] into twelve, corresponding to the twelve parts of the zodiac.

– Ptolemy, *Harmonics* iii/12¹

At the earliest moment of Greek thinking about the cosmos, the pre-Socratics offered not one but several theories of harmony—one dealing with the special sets and ratios of numbers, one being, merely oppositional tension, and another, a grand apportionment of all things in nature. To Plato, these numerical concordances amounted to the world’s ultimate structure, discernible to man via their embedded nu-

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¹ Tr. Barker (1984–1989, vol. 2, 386).

merical truths; it was a worldview, we might say, reminiscent of the *Zhouyi* 周易. Hellenistic ideas moved on from Plato's ultimate structure, however, to mathematical proofs of grand harmonies, which sought to perfectly reconcile the world of musical ratios and straight-line canonicity, on one hand, with the regularities of the superlunary. Instilling Plato's old music of the spheres with a new grammar, Claudius Ptolemy (c. 90 – c. 168), in Book III of his *Harmonics*, hammered out the structures by which the ratios of music inform the mechanics of the cosmos in all of their minutiae, positing a *logos*—a pure logic or truth—undergirding harmonics and astronomy. And so it was, down to the day of Kepler, that the heavens churned in a cosmic symphony of sound and light.

In China, the institution of the 'Lü-li zhi' 律曆志 (Monograph on Harmono-metrology & Mathematical Astronomy) would seem to suggest that there is something universal about this *logos*. Compare the vision of Ptolemy's *Harmonics*, for example, with that of the *Han shu* 漢書 (≤ 92 CE) monograph, as framed by the preface to Liu Xin's 劉歆 (46 BCE – 23 CE) *Santong li* 三統曆 (Triple Concordance *li*):

三統合於一元，故因元一而九三之以為法，十一三之以為實。實如法得一。黃鐘初九，律之首，陽之變也。因而六之，以九為法，得林鐘初六，呂之首，陰之變也。皆參天兩地之法也。

The three [astronomical] concordances coalesce with a single origin.² Thus, because the origin is one, treble it nine times for the divisor, treble it eleven times to make the dividend, and get one for each divisor in the dividend: the HUANGZHONG (DO) [pitch pipe] is a 'nine in the first'; it is the head of the [yang] *lü* 律-regulator [series] and is the mutating of yang (: 9).³ Sextuple it and take nine as your divisor to get the LINZHONG (SOL) [pitch pipe], which is a 'six in the first line'; it is the head of the [yin] *lü* 呂-regulator [series] and the mutating of yin (: 6)⁴—all of this derives from the three : heaven :: two : earth method.

上生六而倍之，下生六而損之，皆以九為法。九六，陰陽夫婦子母之道也。律娶妻而呂生子，天地之情也。六律六呂，而十二辰立矣。五聲清濁，而十日行矣。傳曰「天六地五，數之常也。」

For upward generation, sextuple and double it, and for downward generation sextuple and decrease it—all with nine as the divisor.⁵

Nine : six is the *dao* of yin : yang, husband : wife, son : mother. [Yang] *lü*-regulators take wives and [yin] *lü*-regulators bear children—it is the innate tendency of heaven & earth. With the six *lü* 律 & six

² In *li*_{MA}, an 'origin' (*yuan* 元) refers to the coincidence of new moon, winter solstice, midnight, and day *jiazi*.₀₁ of the sexagenary day count as well as the period of its reoccurrence. Liu Xin uses the value of 4617 years, dividing the 'origin' into three 1539-year 'concordances' (*tong* 統)—'heaven,' 'earth,' and 'man'—marking the coincidence of new moon, winter solstice, and midnight on different sexagenary days. For an explanation of these periods, see Sivin (1969, 12–13).

³ In other words, $(1 \times 3^{11}) \div (1 \times 3^9) = 3^2$. Liu Xin's operation gets us to the quotient 9, which is the canonical *cun*-length of the HUANGZHONG pitch pipe and a yang number in *Zhouyi* numerology, whence 'nine in the first,' referring to a solid yang-line in the first line of a hexagram which changes to a broken yin-line.

⁴ In other words, $(9 \times 6) \div 9 = 6$, six being the canonical *cun*-length of the LINZHONG pitch pipe, and the value of a changing yin line in *Zhouyi* numerology.

⁵ In other words, the pitch pipe length of the next tone 'up' (in the circle of fifths) is 'generated' from pitch pipe length x as $(x \times 6 \times 2) \div 9$, while, one octave 'down,' it is 'generated' by 'decreasing' $x \times 6 \div 9$. These formulae, which we may express as $x(3 + 1)/3$ and $x(3 - 1)/3$, respectively, are known in *li*_{HM} as *sanfen sunyi* 三分損益 (decrease/increase [one of] three parts).

lü 呂, the twelve chronograms are established.⁶ With the five notes, high and low, the ten [heavenly stem] days are set in motion. Tradition states, ‘heaven : six :: earth : five—these are constants among numbers’.⁷

天有六氣，降生五味。夫五六者，天地之中合，而民所受以生也。故日有六甲，辰有五子，十一而天地之道畢，言終而復始。太極中央元氣，故為黃鐘，其實一龠，以其長自乘，故八十一為日法，所以生權衡度量，禮樂之所繇出也。

Heaven has six *qi* which descend to generate the five flavours. Five & six is the medial blending of heaven & earth—that which the populace must receive to live. Thus, the day has six *jia*._{S01} and the chronograms have five *zi*._{B01}, and at eleven the *dao* of heaven & earth is complete, and when all is said and done, one begins again.⁸ The central original *qi* of the Grand Culmen is thus the HUANGZHONG [pitch pipe], whose capacity is one *yue* 龠 (≈ 4 centilitres). Multiply its length by itself (i.e., $9 \times 9 = 81$), thus 81 is the day divisor—that which produces [all standards of] weights, balances,⁹ lengths, and capacities, and that from which ritual and music arose.

Liu’s train of thought is no less difficult to follow than Ptolemy’s, stringing, as he does, arcane terminology from mathematical astronomy, the calendar, pitch-regulator pipes, metrology, and the *Zhouyi* into a cat’s cradle of number progressions and correlative sets. The effect is dizzying—as he may well have intended it to be—sending the reader’s mind racing in several directions at once as he is led along the florid tendrils by which an inexorable order underlying our phenomenal world insinuates itself into (and beyond) every corner of our ken.

It is tempting to see the *logos* coupling harmonics to astronomy in the ancient mind as universal, but we must resist the allure of universality. First of all, we are not dealing here, in the case of China, with ‘harmonics’ and ‘astronomy’ but *lü* 律 and *li* 曆—categories that encompassed various and dynamic domains of activity incongruent with modern (or ancient Greek) taxonomies. *Lü* harmono-metrics (*lü*_{HM}), in an extension of the word’s broader sense of ‘regulation’ or ‘standard’, is the study of the standards governing harmonics (‘regulator’-tube pitch-pipes) and metrology (the ‘legal’ foot rule, etc.).¹⁰ *Li* mathematical astronomy (*li*_{MA}), in an extension of the word’s original sense of ‘sequence’, refers to the indoor, textual, and computational ‘sequencing’ of celestial phenomena as distinct from *tianwen* 天文 ‘heavenly patterns’, the outdoor, observational, and interpretive side of the astral sciences.¹¹ In terms of administration, both *lü*_{HM} and *li*_{MA} were, in

⁶ The ‘six *lü* & six *lü*’ refer to the six tones/pitch pipes of the chromatic scale, HUANGZHONG (DO) – YINGZHONG (ALT.DO). The ‘twelve chronograms’ (*chen* 辰) refer to the heavenly branches (*zi*._{B01}–*hai*._{B12}) as used as counters of astronomical time and space.

⁷ Cit. *Guoyu*, 3.5a.

⁸ In other words, in the sexagenary cycle, the ten heavenly stems (*jia*._{S01}–*gui*._{S10}) repeat six times, each series beginning at *jia*._{S01}; and the twelve earthly branches (*zi*._{B01}–*hai*._{B12}) repeat five times, each series beginning at *zi*._{B01}; add these heavenly (5) and earthly (6) numbers together, and one gets 11.

⁹ *Han shu*, 21A.980–981. In the Triple Concordance *li*, the ‘day divisor’ is one component in Liu Xin’s value for the length of the synodic month: $2392/81 = 29\frac{43}{81}$ days. This value is taken from the *Taichu li* (see below).

¹⁰ On the history of harmonics and metrology in China, see Cheung Sai-beng (1974, vol. 1, 52–62), Qiu Guangming (1992), Keightley (1995), and Bagley (2005).

¹¹ On the categories *li* and *tianwen*, see Morgan (2017a) and the other contributions to this volume.

early imperial times, under the purview of the Ministry of Rites, reminding us that the ideology behind the standard histories would see the two as sharing a common purpose: maintaining standards for the correct performance of state rituals—their pitch and rhythm, if you will. There is a sense to this pairing, but there is a sense too in pairing *lü*_{HM} with music, and *li*_{MA} with heavenly patterns, and a sense to giving each its own monograph in the standard histories.

If their coupling seems inevitable, one must remember too that not every standard history has a ‘Lü-li zhi’. The *Shiji* 史記 (1st cent. BCE) and *Song shu* 宋書 (492/493) devote independent monographs to the two topics, for example, while the *Jiu Tang shu* 舊唐書 (945) and *Xin Tang shu* 新唐書 (1060) move matters of *lü*_{HM} to the ‘Yinyue zhi’ 音樂志 (Music Monograph). The joint ‘Lü-li zhi’ is indeed the product of but six men: Ban Gu 班固 (32–92), in the *Han shu*; Cai Yong 蔡邕 (133–192) and Liu Hong 劉洪 (fl. 167–206), in the *Hou Han shu* 後漢書; Wei Shou 魏收 (506–572), in the *Wei shu* 魏書; Li Chunfeng 李淳風 (602–670), in the *Sui shu* 隋書 and *Jin shu* 晉書; and Toqto’a 脫脫 (1314–1356), in the *Song shi* 宋史.¹² With the sole exception of the latter, in fact, no standard history from the seventh century on maintains this pairing.¹³ Very little of the *lü*_{HM}-*li*_{MA} synthesis survives beyond these monographs, but that is probably because there was very little there in the first place. Of the titles listed under the rubric ‘*Li*_{MA} & Genealogies’ (*li pu* 曆譜) or ‘*Li*_{MA} & Mathematics’ (*li shu* 曆數, *li suan* 曆算) in the bibliographic monographs of the *Han shu*, *Sui shu*, *Jiu Tang shu*, and *Xin Tang shu*, we see only six titles dealing explicitly with *lü*_{HM}.¹⁴ Furthermore, what little literary production there ever was in this genre dries up by the seventh century, disappearing entirely from the ‘*Li*_{MA} & Mathematics’ rubric by the *Song shi*.

If the marriage of ‘harmonics’ and ‘astronomy’ is neither a trans-cultural nor transhistorical *a priori* of pre-modern civilization, it behoves us to treat this union as the product of particular knowledge cultures operating in, and in response to, their own particular times. Our question then is not ‘how are harmonics and astronomy connected in Chinese thought?’ but ‘how and why did certain thinkers act to forge, maintain, rethink, and eventually reject that connection?’ What

¹² On Cai Yong and Liu Hong’s authorship of the *Hou Han shu* monograph, see Mansvelt Beck (1990, 56–63). Modern critical editions of the *Song shu* also possess a ‘Lü-li zhi’, but this is nothing more than a product of modern harmonisation, as pre-modern editions universally devote distinct monographs to the two subjects as per the model of the *Shiji*. We thank Li Liang 李亮 for opening our eyes to this point.

¹³ The *Song shi*’s resurrection of the ‘Lü-li zhi’ is a natural consequence, it seems, of the brief flurry of *lü*_{HM} activity inspired by the renewed classicist orientation of Song 宋 (960–1279) intellectual culture. Prior to this, the monograph tells us, the field seems to have dried up: ‘From [627/649] to [954], five dynasties rose and fell, and 300 years passed; gentlemen of broad learning, for their part, set about meticulously collecting [the remnants] of what had been lost, but the *lü*_{HM} monographs were all lacking. At the beginning of the Song, all the realm was united, gentleman experts were gathered, and the systems of state and monarchy were all restored to the ancient *dao*’ 暨唐貞觀迄周顯德，五代隆替，踰三百年，博達之士頗亦詳緝廢墜，而律志皆闕。宋初混一寓內，能士畢舉，國經王制，悉復古道 (*Song shi*, 68.1493).

¹⁴ The six *lü*_{HM} titles listed under these headings are as follows: (1) anon., *Lü-li shu fa* 律曆數法 in 3 *juan* (*Han shu*, 30.1766); (2) anon., *Lü-li zhu jie* 律曆注解 in 1 *juan* (*Sui shu*, 34.1024); (3) anon., *Huangzhong suan fa* 黃鍾算法 in 38 and 40 *juan* (*Sui shu*, 34.1026; *Xin Tang shu*, 59.1546); (4) anon., *Suan lü-lü fa* 算律呂法 in 1 *juan* (*Sui shu*, 34.1026); (5) anon., *Tui Han shu Lü-li zhi shu* 推漢書律曆志術 in 1 *juan* (*Sui shu*, 34.1024; *Xin Tang shu*, 59.1546); and (6) Xindu Fang 信都芳 (6th cent.), *Qi zhun* 器準 in 3 *juan* (*Xin Tang shu*, 59.1546).

is at stake here is the prospect of sidestepping cumbersome anachronisms like ‘harmonics’ and ‘astronomy’ (let alone ‘science’) by ‘moving forward’ (Harrison 1998, 8–9) from the practices and categories of our historical subjects to trace how the very real ambiguities and distinctions that they themselves insisted upon were adapted to face new challenges.

To this end, the present chapter examines the ‘Lü-li zhi’ written at both ends of the genre. We begin, as did the genre, with the *Han shu* monograph, introducing its circumstances of composition, textual and philosophical framework, and the compelling synthetic vision of the $l\ddot{u}_{HM}-li_{MA}$ union that it presents. We then move to what is, in our opinion, the culmination of the genre with Li Chunfeng in the *Sui shu*. We begin with a careful look at Li’s style, assessing the extent to which we may discern his authorial/editorial voice in his quotation, appropriation, juxtaposition, emendation, and, occasionally, overt framing of his sources. Like his predecessors in the intervening centuries, Li Chunfeng was faced with a vexing problem: the fact that, driven by a burgeoning culture of empirical investigation, the two fields had since the late second century careened in messy and uncertain directions away from both one another and the Han promise of an elegantly interconnected cosmos. We deal, in other words, with the problem of progress. We argue that where Li’s voice reveals itself most clearly is in his editorial work—his choice to synthesize the Liu/Ban framework with the chronicles of his predecessors, his catalogue of archaeometry, and his emphasis of the concept of LÜ 率—and that where it does, it does so in response to the problem of progress, attempting to repair the $l\ddot{u}_{HM}-li_{MA}$ rupture and return some semblance of order to the history of science and the world order that is its object.

5.2 Liu Xin, Ban Gu, and the *Han shu* Monograph

Ban Gu presents his *Han shu* ‘Lü-li zhi’ as little more than a collage of Liu Xin’s writings. At the outset, before launching into $l\ddot{u}_{HM}$, Ban makes the following announcement:

至元始中王莽秉政，欲耀名譽，徵天下通知鐘律者百餘人，使羲和劉歆等典領條奏。言之最詳，故刪其偽辭，取正義，著于篇。
In the Yuanshi 元始 reign (1–5 CE), Wang Mang held control of the government, and wishing to burnish his reputation, he gathered together more than a hundred experts on bells and pitch-regulators from all around the empire, ordering Xi-He 羲和 Liu Xin et al. to preside over and write up and submit [their findings] to the court. [Liu’s] are the most detailed words on the matter, thus have I excised his more spurious claims, extracted its contents that are correct, and written them into the current piece.¹⁵

Where $l\ddot{u}_{HM}$ transitions into li_{MA} , Ban Gu once reminds us of what we are reading:

至孝成世，劉向總六曆，列是非，作『五紀論』。向子歆究其微眇，作三統曆及譜以說『春秋』。推法密要，故述焉。
In the generation of Xiao-Cheng[di] 孝成帝 (r. 32–7 BCE), Liu Xiang 劉向 (79–8 BCE) assembled the six [ancient] astronomical systems and enumerated their pros & cons, authoring the *Wuji lun* 五紀論. Xiang’s son Xin exhausted the minutiae and subtitles of the subject,

¹⁵ *Han shu*, 21A.955.

authoring his Triple Concordance astronomical system & genealogy (i.e. his *Shijing* 世經, *Canon of the Ages*) in order to explain the *Chunqiu* 春秋. His calculations are tight (*mi* 密) & essential, thus it is that I reproduce them here.¹⁶

As to its contents, as summarized in the Appendix, two features of this new, hybrid ‘Lü-li zhi’ stand out. The first is that it truly is little more than a collection of Liu Xin’s work, his *Santong li* and *Shijing* taking up a good half of the *juan*. The second is that, however organic their supposed union, *lü*_{HM} and *li*_{MA} contents are segregated into distinct textual units. Each is introduced with a separate historical précis, which is exceptional within the monograph genre, and the *lü*_{HM} and *li*_{MA} halves of the text are structured differently. The *lü*_{HM} half, redacted from Liu Xin, is organized *typologically* under five rubrics—(1) numbers/counting rods, (2) pitches, (3) lengths, (4) capacities, and (5) weights—each of which introduces units, their basis in HUANGZHONG pitch-pipe dimensions, their etymologies, and, for (3)–(5), instrument standards and millet-seed dimensions. The *li*_{MA} half, by contrast, is organized *chronologically* as a record of Han-time events, to which Ban Gu appends Liu Xin’s aforementioned works.

The *Han shu* ‘Lü-li zhi’ is a collage of Liu Xin’s writings, and the reason that Ban Gu opts for a joint monograph over the ‘Lü shu’ 律書 + ‘Li shu’ 曆書 model established by Sima Qian 司馬遷 (c. 145 – c. 86 BCE) is probably because the two are impossible to separate in Liu Xin’s work. Liu’s *li*_{MA}, of which we got a taste in the introduction, blurs the line between harmonic, astronomical, and numerological values, and it is to the self-same cosmic order of ‘Number’ (*shu* 數) to which his *lü*_{HM} appeal. To Liu, everything is inextricably connected, and at the centre of it all is the HUANGZHONG, the first in the series of twelve which the Yellow Thearch commanded Linglun 伶倫/伶倫 attune to the cries of the phoenix. Its span, 9 *cun*, is the root of all lengths, its volume, 1 *yue*, is the root of all capacities, and its heft, 12 *zhu* 銖, is the root of all weights, each respective unit of which is counted off in factors of the twos, threes, fives, etc. into which the myriad things of the phenomenal world are naturally divided. So too do the twelve pitches (of the octave) spiral out from it in a circle of fifths determined by simple ratios of twos and threes underpinning celestial motions and *Zhouyi* combinatorics, the HUANGZHONG bringing us back full circle in a sequence of twelve as we go through the months, chronograms, and earthly branches of civil time. And as these cycles coincide, so too do their nodes, each pitch-standard correlating to a particular month as if keys on a pipe organ through which the annual fluctuation of yin & yang *qi* blow up and down a musical scale. *Everything* is interconnected, or it was, at least, after Liu got done with it.¹⁷

Such is ‘Chinese thought’, if we listen to Granet (1934, 127–248), but this particular ‘Chinese thought’, we mustn’t forget, had an author, a context, and a history. Liu Xin brought the philosophy of ‘Number’ to a new level of sophistication and rhetorical power, but here he is building upon his own predecessors’ work. In *lü*_{HM}, he is elaborating upon harmonic series and correlative schemes that can be traced

¹⁶ *Han shu*, 21A.979.

¹⁷ On Liu Xin’s *lü*_{HM} work, see Vogel (1994). On his *Santong li*, see Nōda and Yabuuti (1947), Teboul (1983), Kawahara (1996, 148–195), and Cullen (2017, xx). On his archaeoastronomical work on the *Zuozhuan* 左傳 via the *Santong li* in his *Shijing*, see Cullen (2001). On his life and times, see Loewe (2000, 383–386), and Xu Xingwu (2005).

through the *Shiji* (91 BCE) and *Huainanzi* 淮南子 (139 BCE) back to third-century BCE sources like the *Lishi chunqiu* 呂氏春秋 and the bamboo daybook (manuscript B) from Fangmatan 放馬灘 tomb 1.¹⁸ The premise that $l\ddot{u}_{HM}$ bears some relation to li_{MA} is already there in the early Han, but it is largely limited to hemerological correlations and the symbolism of the civil calendar. The idea of introducing $l\ddot{u}_{HM}$ numbers into the actual calculation of li_{MA} —both for the calendar and for non-calendrical matters of eclipse prediction and planetary astronomy—is one that first appears in Deng Ping 鄧平 et al.’s *Taichu li* 太初曆 (Grand Inception *li*) of 104 BCE, whose origins Ban Gu describes thus:

其法以律起曆，曰：「律容一龠，積八十一寸，則一日之分也... 夫律陰陽九六，爻象所從出也。故黃鐘紀元氣之謂律。律，法也，莫不取法焉。」

Its method started li_{MA} from $l\ddot{u}_{HM}$. [Deng Ping’s team explained,] ‘The [HUANGZHONG] pitch-standard holds 1 *yue*, and its area is 81 [square] *cun*—thus is this the number by which we divide a single day. ... Now, the pitch-standards’ [characteristic of] yin : 6 :: yang : 9 is the origin of the line-images [in the *Zhouyi*]. Thus, the HUANGZHONG’s demarcation of primal *qi* is what we refer to as *lü*, [because] *lü* (regulation) = *fa* 法 (standard); there is nothing that does not take its standard (*fa*) from this.¹⁹

The *Taichu li* represents the first step towards a $l\ddot{u}_{HM}$ - li_{MA} synthesis, and it is upon its ‘day standard/divisor’ (*ri fa* 日法) that Liu Xin built the numerical superstructure of the *Santong li*, but there is nothing inevitable about dividing the day into 81. The 104 BCE reform, one notes, took place during an explosion of activity surrounding ‘Chinese thought’—now, since the 1980s, generally labelled ‘cosmology’—begun by thinkers like Dong Zhongshu 董仲舒 (c. 179 BCE – c. 104 BCE) and followed through into the turn of the Common Era by figures like Liu Xin’s father, Xiang, and his contemporary, Yang Xiong 揚雄 (53 BCE – 18 CE).²⁰ At the centre of this intellectual current were age-old questions of good governance and political legitimacy, and in Yang Xiong’s *Taixuan jing* 太玄經 of 7/1 BCE, for example, we see how the $l\ddot{u}_{HM}$ - li_{MA} synthesis has become part of the mix: ‘ $L\ddot{u}_{HM}$ completes the creatura (*wu* 物), and li_{MA} partitions seasons/time; when the *dao* of $l\ddot{u}_{HM}$ & li_{MA} cross, the sage can (qua leaders) then plot’.²¹ And plotting there was, because the work of this synthesis, particularly as conducted by Liu Xin while serving as palace attendant (*shizhong* 侍中) and Xi-He 羲和 from 5 CE on, fed directly into

¹⁸ See Kalinowski (2011).

¹⁹ *Han shu*, 21A.975–976.

²⁰ On the political and ideological context of the *Taichu li* and the reform of 104 BCE, see Cullen (1993). For a broader study of ‘cosmology’ in this period, see Wang Aihe (2000). On the very particular problem posed by the use of ‘cosmology’ as a stand-in for ‘Chinese/primitive thought’ from the perspective of a historian of astronomy interested in cosmology as we would use the term in a non ‘Chinese/primitive’ setting, see Morgan (2017b) and Chapter 7.

²¹ 律則成物，曆則編時，律曆交道，聖人以謀，*Taixuan jing*, 7.6b; cf. the first-century BCE ‘Zengzi tianyuan’ 曾子天圓, *Da Dai Liji*, 5.17b–18b. Note that the *Song shu* 宋書 monograph posits that Yang wrote the *Taixuan jing* on the basis of Liu Xin’s work: ‘Liu Xin’s Triple Concordance system was particularly & repeatedly loose and wide (inaccurate)... Yang Xiong’s heartmind was confused by its rhetoric and so adopted it for the *Taixuan*’ 劉歆三統法尤復疏闊...揚雄心惑其說，采為太玄 (*Song shu*, 12.231).

Wang Mang's 王莽 (c. 45 BCE – 23 CE) eventual usurpation of the Han throne.²²

That, however, did not end well, and Ban Gu's family were close enough to the events to remember what came of 'sages plotting': Liu Xin was driven to commit suicide in 23 CE before the 'New' dynasty (9–23 CE) of his creation ended in famine and civil war. Writing his 'Lü-li zhi' in the new Han capital of Luoyang between 62 and 92 CE, Ban Gu could hardly write Liu Xin out of the history of harmonometrics and mathematical astronomy in the previous decades, but it is only fitting that he felt the need to 'excise his more spurious claims'. The table had turned in politics and ideology, and so too had it begun to turn in *lǚ_{HM}* and *lǐ_{MA}*.

5.3 Li Chunfeng and the *Sui shu* Monograph

Five centuries later, and three monographs wiser, Li Chunfeng began his work on a 'Lü-li zhi' for the *Jin shu* and the *Wudai shi* 五代史 projects, onto both of which he was recruited in the 640s.²³ A quick glance at the latter—now found in the *Sui shu*—reveals the degree to which its organization is modelled upon that of the Liu/Ban monograph (see the Appendix). Like his predecessors, Li Chunfeng segregates *lǚ_{HM}* and *lǐ_{MA}*, introducing each with its own historical précis. Like the *Hou Han shu*, he in fact segregates into *separate juan*, returning, one might say, to the model of the *Shiji* and *Song shu*.²⁴ His *lǐ_{MA} juan* (j. 17–18) also proceed in the form of a chronicle, his detailing the events of Liang 梁 (502–557), Chen 陳 (557–589), Northern Qi 北齊 (550–577), Northern Zhou 北周 (557–581), and Sui 隋 (581–618).

Where Li's *Sui shu* 'Lǚ_{HM} & lǐ_{MA} Monograph' is unique is in its dealing with the *lǚ_{HM} juan* (j. 16). This, Li Chunfeng organizes around the *Han shu*'s section headings, which, in the *Jin shu*, he explicitly identifies with Wang Mang's reign.²⁵ Li is the first historiographer since the *Han shu* to revive this framework. True, the *Hou Han shu* features a theoretical subsection on the generation of pitch-regulator series via proportionality, but, for the most part, his predecessors had remade the *lǚ_{HM} juan* in the image of its counterpart: a chronicle recounting events in the history of the field in the order that they occurred. In general, these chronicles tend to focus on instrument research and development, e.g. archaeological finds, grain-measure experiments, the commission of government standards, and the loss, recovery, analysis, storage, and transfer of *realia*.

Li Chunfeng covers the *history* of *lǚ_{HM}* as well, the difference being that he divvies it up according to the Liu/Ban headings, to which he adds the rubrics 'Pitch-pipe Millet Capacity' 律管圍容黍, 'Observing the *Qi*' 候氣, and 'Pipe Day-Assignments' 律直日. At the head of each rubric, he distils (and sometimes hones) the theoretical language of the Liu/Ban monograph by way of introduction. Let us

²² On Han office titles, see Bielenstein (1980). On the history and function of the short-lived office of Xi-He, see Yoshino (2003). On the legitimation of Wang Mang's ascendancy to the Han throne, see Sukhu (2005–2006).

²³ On the life, times, and work of Li Chunfeng see Chapters 2 and 3.

²⁴ Note that the *Wei shu* follows the *Han shu* in this sense, if only for the reason that its harmono-metric contents—a précis and two historical anecdotes—are too sparse to constitute their own *juan*

²⁵ See *Jin shu*, 17.474.

compare their respective treatment of ‘Completed Numbers’ (*bei shu* 備數), beginning with the *Han shu*:

數者，一、十、百、千、萬也，所以算數事物，順性命之理也。書曰：「先其算命。」本起於黃鐘之數，始於一而三之，三三積之，歷十二辰之數，十有七萬七千一百四十七，而五數備矣。其算法用竹，徑一分，長六寸，二百七十一枚而成六觚，為一握。徑象乾律黃鐘之一，而長象坤呂林鐘之長。其數以易大衍之數五十，其用四十九，成陽六爻，得周流六虛之象也。夫推曆生律制器，規圓矩方，權重衡平，準繩嘉量，探賾索隱，鉤深致遠，莫不用焉。度長短者不失豪釐，量多少者不失圭撮，權輕重者不失黍象。紀於一，協於十，長於百，大於千，衍於萬，其法在算術。宣於天下，小學是則。職在太史，羲和掌之。

The [five] numbers are ones, tens, hundreds, thousands, and myriads—that by which one counts the numbers of [both] events (*shi* 事) & creatura (*wu*) and follows the pattern of nature (*xing* 性) & fate (*ming* 命). The [*Shang*]*shu* 尚書 says, ‘prioritize one’s calculations for ordering (*ming*) [affairs]’.²⁶ [All these] originate from the number of the HUANGZHONG: begin with one and treble it, trebling and trebling cumulatively through the numbers of the twelve chronograms for 177 147, and at that the five numbers are complete.²⁷ Their method of calculation requires bamboo rods of 1 *fen* in diameter & 6 *cun* in length, 271 of which form a hexagon—this is one handful. Their diameter symbolizes one part [of nine] of the Qian 乾 ☰ tubal regulator HUANGZHONG (9 *cun*), and the length symbolizes the length of the Kun 坤 ☷ tubal regulator LINZHONG (6 *cun*). Its numbers follow the *Zhouyi*’s great expansion number, 50, of which one uses 49 to complete the six yang lines [of the Qian ☰ hexagram], a symbol of circulation through the six voids (i.e. line positions).²⁸ Now, one fabricates devices by calculating *li*_{MA} & generating *lü*_{HM} [pitch-standards]. The compass round & the T-square square, the weights heavy & the balance level, the level, plumb, and capacity measures—in exploring the recondite & seeking out the hidden, in snaring the deep & eliciting the far away, one can never be without [these devices]. [In fabricating them thus,] the length of linear measures (*du* 度) will not miss [a single] *hao* 豪 or *mao* 釐; the capacities will not miss [a single] *gui* 圭 or *cuo* 撮, nor will the heaviness of one weight miss [a single] *shu* 黍 or *lei* 象.²⁹ Marked by ones, joined by tens, matured by hundreds, enlarged by thousands, and expanded by myriads, their methods lie in the art of calculation. Their promulgation through the sub-celestial realm and [the integration of] these principles in primary education—

²⁶ This citation does not appear in the *Shangshu* as we currently have it. Yan Shigu 顏師古 (581–645) explains that ‘this is [from] a lost document; it means to say that in mustering his patrimony, the king should *first* establish the *calculation* of numbers to *command* the hundred affairs’ 逸書也；言王者統業，先立算數以命百事也 (*Han shu*, 21A.957 (commentary)).

²⁷ In modern terms, $zi_{B01} = 1$; $chou_{B02} = 3 \times 1 = 3$; $yin_{B03} = 3^2 \times 1 = 9$... $hai_{B12} = 3^{11} \times 1 = 177147$.

²⁸ Described in the ‘Xici zhuan’ 繫辭傳 commentary, the ‘great expansion’ is the combinatory method used to produce hexagram line numbers through the manipulation of milfoil sticks. For an explanation of the divinatory procedure of the *Zhouyi*, see Nielsen (2003, 39–43). For an explanation of the significance of the number, shape, and geometric configuration of counting rods as presented in the Li Chunfeng and Liu/Ban monographs, see Li Yan (1955). On the use of positive and negative counting rods, see Zhu Yiwen (2010). We thank Zhu Yiwen 朱一文 for sharing with us his understanding of the counting-rod-related passages in Li and Liu’s monographs and directing us to the studies cited here (personal communication, October, 2013).

²⁹ These are very small units of measure.

that duty lies with the Grand Clerk[’s office] and is handled by the Xi-He.³⁰

The *Sui shu* version proceeds in three paragraphs:

五數者，一、十、百、千、萬也。傳曰：「物生而後有象，滋而後有數。」是以言律者，云數起於建子，黃鍾之律，始一，而每辰三之，歷九辰至酉，得一萬九千六百八十三，而五數備成，以為律法。又參之，終亥，凡歷十二辰，得十有七萬七千一百四十七，而辰數該矣，以為律積。以成法除該積，得九寸，即黃鍾宮律之長也。此則數因律起，律以數成，故可歷管萬事，綜覈氣象。其算用竹，廣二分，長三寸，正策三廉，積二百一十六枚，成六觚，乾之策也。負策四廉，積一百四十四枚，成方，坤之策也。觚方皆經十二，天地之大數也。是故探賾索隱，鈎深致遠，莫不用焉。一、十、百、千、萬，所同由也。律、度、量、衡、歷、率，其別用也。故體有長短，檢之以度，則不失毫釐。物有多少，受之以器，則不失圭撮。量有輕重，平之以權衡，則不失黍絲。聲有清濁，協之以律呂，則不失宮商。三光運行，紀以曆數，則不差晷刻。事物糅見，御之以率，則不乖其本。故幽隱之情，精微之變，可得而綜也。

The five numbers are ones, tens, hundreds, thousands, and myriads. The [*Zuo*] Tradition states that ‘creatura (*wu*) exhibit form only upon birth, and number, only upon reproduction’.³¹ This is why those who discuss *li_{MA}* say that numbers start from establishment at *zi*._{B01} (the month of winter solstice), that the HUANGZHONG [tubal] regulator begins at 1: every chronogram trebles it, arriving at *you*._{B10} through 9 chronograms yields 19 683, and at that the five numbers are complete—this is the ‘pitch-standard divisor’—trebled again to end at *hai*._{B12}, the sequence through the 12 chronograms yields 177 147, and at that the chronogram numbers are replete—this is the ‘pitch-standard product’. Dividing the replete product by the complete divisor yields 9 *cun*, which is the length of the HUANGZHONG-DO regulator.³² This is how numbers arise from *li_{HM}*, and how *li_{HM}* is completed through numbers. Thus is it possible to successively manage the myriad matters and comprehensively examine [the piped] *qi* & [observed] signs (*xiang* 象). Their calculation requires bamboo: 2 *fen* wide & 3 *cun* long, positive rods are three-sided and add up to 216, forming a hexagon—the rods of Qian ☰—while negative rods are four-sided and add up to 144, forming a square—the rods of Kun ☷. The polygon (of 216 rods) and square (of 144 rods) are both divisible by 12, the great number of heaven & earth.³³ Thus it is that in exploring the recondite & seeking out the hidden, in snaring the deep & eliciting the far away, [counting rods/numbers] can never be done without. Ones, tens, hundreds, thousands, and myriads derive alike from them, and pitches, lengths, capacities, weights, *li_{MA}*, and LÜ 率 are [simply] their distinct applications. Thus, bodies can be long or short, but if one examines them with a ruler, then one will not miss [a single] *hao* or *li* 釐; creatura (*wu*) can be numerous or few, but if one places them in a vessel, then one will not miss [a single] *gui* or *cuo*; amounts can be light or heavy, but if one levels them with a weight and balance, then one will not miss [a single] *shu* or *si* 絲; tones can be high or low, but if one tunes them to the [twelve] pitch-standards, then one will not miss DO and RE; the three luminaries (sun, moon & stars/planets) travel in revolutions, but if one marks them with *li_{MA}* numbers, then one will not

³⁰ *Han shu*, 21A.956.

³¹ Here the *Zuo Tradition* actually reads, ‘creatura (*wu*) exhibit form only upon birth, reproduction, only upon form, and number, only upon reproduction’ 物生而後有象，象而後有滋，滋而後有數 (*Chunqiu Zuo zhuan zhushu*, 13.17a).

³² In other words, $3^{11} \div 3^9 = 3^2 = 9$; see Note 27.

³³ See Note 28.

err in [timing]; matters (*shi*) & things (*wu*) can appear jumbled together, but if one takes charge of them with LÜ, then one will not pervert their bases. Thus, even dark and hidden natures & fine and subtle shifts can be successfully synthesized.³⁴

夫所謂率者，有九流焉：一曰方田，以御田疇界域。二曰粟米，以御交質變易。三曰衰分，以御貴賤廩稅。四曰少廣，以御積畧方圓。五曰商功，以御功程積實。六曰均輸，以御遠近勞費。七曰盈朒，以御隱雜互見。八曰方程，以御錯糅正負。九曰句股，以御高深廣遠。皆乘以散之，除以聚之，齊同以通之，今有以貫之。則算數之方，盡於斯矣。

Now, as for that which we refer to as ‘LÜ’, there are nine [divisions of mathematics] that flow from it: (1) ‘Rectangular Field’, for dealing with the boundaries (area) of cultivated fields; (2) ‘Unhulled and Hulled Grain’, for dealing with exchange and conversion; (3) ‘Parts Weighted According to Degrees’, for dealing with [price fluctuation] in grain distribution and taxation; (4) ‘Reducing the Width’, for dealing with areas of squares and circles; (5) ‘Discussion of works’, for dealing with earthworks and capacities; (6) ‘Tax Payment on an Equal Basis with Transportation’, for dealing with labour and expenditures according to distance; (7) ‘Surplus and Deficit’, for dealing with how hidden and mixed things mutually appear; (8) ‘Fangcheng’, for dealing with the intermingling of positive and negative; and (9) ‘Base & Height’, for dealing with things high, deep, broad, and distant.³⁵ All of them multiply to disaggregate them (LÜ), simplify to assemble them, homogenize and equalize to make them communicate, [apply the procedure of] ‘Suppose’ (i.e., the rule of three) to link them together—and thus are the methods of calculating numbers replete therein (in LÜ).³⁶

古之九數，圓周率三，圓徑率一，其術疏舛。自劉歆、張衡、劉徽、王蕃、皮延宗之徒，各設新率，未臻折衷。宋末，南徐州從事史祖冲之，更開密法，以圓徑一億為一丈，圓周盈數三丈一尺四寸一分五釐九毫二秒七忽，朒數三丈一尺四寸一分五釐九毫二秒六忽，正數在盈朒二限之間。密率，圓徑一百一十三，圓周三百五十五。約率，圓徑七，周二十二。又設開差冪，開差立，兼以正圓參之。指要精密，算氏之最者也。所著之書，名為綴術，學官莫能究其深奧，是故廢而不理。

In the ancient Nine Numbers/Procedures, the circle-circumference LÜ was 3, and the circle-diameter LÜ was 1 (i.e., $\pi = 3$). This procedure (i.e. value) is loose & errant. With the likes of Liu Xin, Zhang Heng 張衡 (78–139), Liu Hui 劉徽 (fl. 263), Wang Fan 王蕃 (228–266), and Pi Yanzong 皮延宗 (fl. 443), each posited new LÜ, but they had yet to arrive at a compromise. [Then], at the end of the [Liu-]Song 劉宋 (420–479), Southern Xuzhou Attendant Clerk Zu Chongzhi 祖冲之 (429–500) introduced a tight (*mi*) method, taking a circle diameter of 100000 as 1 *zhang*, [he found] a maximum circle circumference of 3 *zhang* 1 *chi* 4 *cun* 1 *fen* 5 *li* 9 *hao* 2 *miao* 7 *hu*, and a minimum value of 3 *zhang* 1 *chi* 4 *cun* 1 *fen* 5 *li* 9 *hao* 2 *miao* 6 *hu*, the correct number

³⁴ The last few lines of this paragraph are modified from *Hou Han shu*, *zhi* 1, 2999, the main difference being that Li Chunfeng has inserted LÜ 率 into the list of ‘applications’.

³⁵ Here Li Chunfeng is citing the chapter headings of the *Jiuzhang suanshu* 九章算術 and the purport of each chapter as summarized in Liu Hui’s commentary thereto; see Chemla and Guo (2004).

³⁶ Here Li Chunfeng is citing Liu Hui’s commentary to the *Jiuzhang suanshu*, which, after introducing the procedure for adding fractions, opines, ‘Multiply to disaggregate them, simplify to assemble them, homogenize and equalize to make them communicate, how could those not be the key-points (*gangji*) of computations/mathematics (*suan* 算)?’ 乘以散之，約以聚之，齊同以通之，此其算之綱紀乎 (*Jiuzhang suanshu*, 1.9 (commentary); tr. Chemla (2010, 277)).

being between the two limits of maximum & minimum. The tight (*mi*) LÜ were circle diameter 113 : circle circumference 355 (= 3.1415929...). The simplified LÜ were circle diameter 7 : circle circumference 22 (= 3.1428571...). He further posited [methods of] square and cubic extraction with difference, both of which he examined (*can*) with correct circles. The idea is fine (*jing*) & tight (*mi*), the most superlative amongst calculators. The book that he wrote is titled *Zhui shu* 綴術; [unfortunately,] no education official was able to fathom its profundity, which is why they abandoned & paid no attention to it.³⁷

Li Chunfeng's introductory paragraph is drawn topic-by-topic from the Liu/Ban monograph: he introduces the fundamental units ('the five numbers'), identifies their origin as the HUANGZHONG pitch-standard, explains the make-up and symbolism of normative instrumentation (counting rods), and affirms the interpenetration of harmonic, astronomical, and metrological numbers. Where Li does not reproduce the language of the *Han shu* verbatim, he reiterates its contents more-or-less faithfully through abridgment and rearrangement; what little he does add is mostly drawn from canonical sources, be his references explicit (the *Zuozhuan* 左傳) or implicit (the *Hou Han shu*). That said, we can see Li honing his predecessor's philosophy of Number through subtle editorial intervention. On his way to the HUANGZHONG pitch-standard product (177 147), for example, he stops at the chronogram *you*._{B10} for a five-digit value (19 683) more suggestive of the primacy of the 'five numbers', which end with the ten-thousands place. Likewise, his account of the apparatus of number includes positive and negative counting rods, out of which he forms two regular geometrical figures, which, furthermore, he makes correlate with *Zhouyi* milfoil counting (positive : Qian ☰ : round : 216 :: negative : Kun ☷ : square : 144).³⁸ Li furthermore posits a reciprocal relationship between harmonics and numbers: 'This is how numbers arise from *lü*_{HM}, and how *lü*_{HM} is completed through numbers'. The most conspicuous difference between the two, of course, is that Li has added two paragraphs to what the Liu/Ban monograph has to say: the first, a collage of citations on LÜ mathematics from the *Jiuzhang suanshu* 九章算術 (*Nine Chapters of Mathematical Procedures*) and Liu Hui's 263 commentary thereto; the second, an account of Zu Chongzhi's work on determining (in modern terms) the value of π , the source of which is unclear.

The example here is representative of Li Chunfeng's authorial/editorial style throughout the *Sui shu*'s *lü*_{HM} *juan*. At the core is the *Han shu* framework: 'here', he tells us, 'I follow the example of the [Liu/]Ban monograph in compiling matters of Five Dynasties harmonics and metrology'.³⁹ Under each rubric, he then presents a (static) distillation of Liu/Ban metrosophy followed by a (dynamic) chronicle of events cobbled from extant sources. Some of his sources are explicitly identified, particularly those from the classics, Han and pre-Han philosophy, epigraphic materials, and recent literature, but he also lifts whole paragraphs from the *Hou Han shu* and *Song shu* monographs without a word of explanation.

The extent of citation and textual appropriation between such monographs is considerable. The case of the 'Tianwen zhi' 天文志 (Heav-

³⁷ *Sui shu*, 16.387–388.

³⁸ See Note 28.

³⁹ 今依班志編錄五代聲律度量, *Sui shu*, 16.386.

only Patterns Monograph), examined in Chapter 7, gives us a clear idea of what we may expect from Li Chunfeng's writing in this regard. Here, as to *lǚ*_{HM} and *li*_{MA}, we offer Li's *Jin shu* monograph as an example, its chronological overlap with other extant histories making it a better indicator of borrowing. From the 254 characters on watching the *qi* in the *Hou Han shu*, for example, 204 appear verbatim in the *Song shu* (89 per cent of its 229-character paragraph), and 225 of the *Song shu*'s account appear likewise in the *Jin shu* (77 per cent of its 291-character paragraph).⁴⁰ In fact, more than half of the *Jin shu*'s *lǚ*_{HM} *juan* overlaps with that of the *Song shu*, including a 2745-character section on Xun Xu 荀勗 (c. 222–289) that is nearly word for word the same.⁴¹ The *Jin shu li*_{MA} *juan* exhibit similar borrowing from the *Song shu*, e.g. the ≈ 9000 characters of Yang Wei's 楊偉 *Jingchu li* 景初曆 (Luminous Inception *li*). In this case, however, it is the *Song shu* that reads like an abridged version of the *Jin shu*, hinting at some third sources from which they both drew.⁴² Naturally, where sources such as the numerous Six Dynasties (220–589) forerunners to the *Jin shu* no longer survive we can only guess at how much of any one monograph's contents have been cut and pasted from elsewhere. There is less to say about the *Sui shu* 'Lü-li zhi' in this regard, but the extent to which the 'Completed Numbers' section draws upon Liu Xin and Liu Hui's writing is telling.

Li describes his modest role vis-à-vis the contents of his monographs in terms of 'following' (*yi* 依), 'editing' (*bian* 編), 'recording' (*lu* 錄), 'outlining' (*lie* 略), 'picking out' (*cai* 采), 'continuing' (*xu* 續), and 'arranging' (*lie* 列).⁴³ His claims comport with Confucius' dictum of 'transmitting without innovating' 述而不作 (*Lunyu* 論語 VII:1) as it had been internalized by historiographers such as Sima Qian, who insists that 'what I call 'transmitting (*shu*) ancient events and putting their genealogies and biographies into proper order' is not what we refer to as 'innovation' (*zuo*)'.⁴⁴ The objective of this historiographic method/ rhetorical strategy is, we might say, to disassociate creator and creation and let the facts speak for themselves. What little we see of the compiler is but jetsam on a sea of others' words.

Li Chunfeng is still there, of course, behind his selection and presentation of sources, and the occasional transitions and nudges that he slips between them. For better or for worse, the genre provides the perfect backdrop of anonymous homogeneity against which to highlight the compiler's acts of 'innovation'.⁴⁵ There are, in our opinion,

⁴⁰ *Hou Han shu*, *zhi* 1, 3016; *Song shu*, 11.208–209; *Jin shu*, 16.489–490.

⁴¹ *Song shu*, 11.212–220; *Jin shu*, 16.480–485.

⁴² The *Jingchu li*, preface and procedure text, is found in *Song shu*, 12.232–258, and *Jin shu*, 18.535–562. For an excellent comparison and synthetic treatment of the *Song shu* and *Jin shu li*_{MA} chronicles, see Hasebe (1991). As to Li Chunfeng's source, Morgan (2017a, chap. 6) suggests that it may also have been He Chengtian.

⁴³ *Sui shu*, 16.386, 16.397, 17.416; *Jin shu*, 17.498, 17.503.

⁴⁴ 余所謂述故事，整齊其世傳，非所謂作也，*Shiji*, 130.3299–3300. By the 170s–180s and on through the Southern Dynasties, the politicization of historiographical writing had become acute, and participants, though not creating new genres per se, pushed their compilational and editing agendas in the spirit of a certain type of literary creativity; on arguments over chronology and editing in the third century CE, see Goodman (2010, 334–346).

⁴⁵ Particularly remarkable in this regard is Li's conspicuous failure to mention Shen Yue 沈約 (441–513) and He Chengtian's *Song shu* monograph(s) for his new *whatsoever*, though it is the single source from which he seems to have appropriated the majority of his *Jin shu* monograph; see Morgan (2017a, chap. 6). Note that Li does

three in particular that stand out to us in Li's *Sui shu* 'Lü-li zhi': his synthesis of the Liu/Ban framework with the later chronological approach to $l\ddot{u}_{HM}$, his catalogue of archaeometry therein, and his consistent emphasis on $L\ddot{U}$ 率. In the remaining half of this chapter, we will argue that these innovations are Li's response to a singular problem in what one might call the history and historiography of science in his day: the rise of a culture of empirical investigation and the 'cumulative' change that it engendered.

5.4 The *Changes* and Historical Change

Texts predating the second-century Cai-Liu monograph (the classics, weft texts, *Lishi chunqiu*, *Huainanzi*, and the Sima and Liu/Ban monographs) tend to present $l\ddot{u}_{HM}$ as a matter of transhistorical fact. Liu Xin's work is a case in point: his is not a medley of opinions and developments; he states facts and invokes scripture, tracing a singular compelling order—a hypostasis of Number—out of which the fabric of harmonics, metrology, the astral sciences, the *Zhouyi*, and the *Chunqiu* is mutually woven to provisional ends. Yang : 9 : sun : HUANGZHONG :: yin : 6 : moon : LINZHONG, so how, for example, could the astronomical value of the 'day divisor' be anything other than 81, the square of the HUANGZHONG's *cun*-length and the veritable epitome of yang? Perfect and static, such knowledge *does* have a history, but it is one merely of revelation and loss to mankind according to Ban Gu's introductory précis:

『虞書』曰「乃同律度量衡」，所以齊遠近立民信也。自伏戲畫八卦，由數起，至黃帝、堯、舜而大備。三代稽古，法度章焉。周衰官失，孔子陳後王之法，曰：「謹權量，審法度，修廢官，舉逸民，四方之政行矣。」漢興，北平侯張蒼首律曆事，孝武帝時樂官考正。至元始中王莽秉政，欲燿名譽，...

The *Yu shu* 虞書 says that 'then [Emperor Shun] made uniform the standard tubes, the measures of length and of capacity, and the balances', and so he unified the near & distant and establishing the confidence of the people. [Thus,] what began when Fuxi first drew the Eight Trigrams arose from Number, and by the time of the Yellow Emperor, Yao, and Shun, [these systems] were all complete. When the Three Dynasties (Xia, Shang, and Zhou) searched out the ancient [ways], regulations (*fadu*) were [already] clearly displayed in them. When the Zhou dynasty declined and its offices went amiss, Confucius explained the methods [of proper government] of the later kings by saying: 'Pay careful attention to [standard] weights and capacity measures, investigate regulations (*fadu*), re-establish official posts fallen into disuse, and raise men, who have withdrawn from society, and government measures will be enforced everywhere'. When the Han rose, Zhang Cang 張蒼, Marquis of Beiping 北平, initiated the matters of harmonics and calendro-astronomy. [Harmonics] were [thereafter] investigated and corrected by the music office during the reign of Xiao-wudi 孝武帝 (r. 140–87 BCE). In the Yuanshi reign (1–5 CE), Wang Mang held control of the government, and wishing to burnish his reputation... (continued above).⁴⁶

however refer to He's *ideas* in his *Sui shu Lü_{HM} juan* (*Sui shu*, 16.389), and that there were lacunae in the transmission of He's private writings soon after his death in 447 CE; see Goodman (2015).

⁴⁶ *Han shu*, 21A.955

Looking back on it, even from the late second century, experts would have found it difficult to reconcile the simplicity of this vision with the complexities of practice and recent history. If the cosmos conspires to produce a ‘day divisor’ of 81, why then, in practice, does a mean synodic month of $29\frac{43}{81}$ days seem to lag behind observation? Why is it that later astronomers fared better with numerologically arbitrary values like $29\frac{499}{940}$ and $29\frac{773}{1457}$?⁴⁷ And just after the end of the Eastern Han, why is it that a flurry of newly discovered foot rules and regulators *differed*, leading some to declare the Eastern Han court’s pitch-standards to have been flat? And, most importantly, why is it that statements of transhistorical fact never seem to agree (below)?

The world was changing, and our historical subjects’ ability to recognize and accommodate such change far exceeds most modern expectations of them. The thing is that the ‘Chinese thought’ or ‘cosmology’ of yin-yang, five-agents, and *Zhouyi* correlative matrices had a history *after* Liu Xin as well, and it was a history, in John Henderson’s words, of ‘decline’. Redressing the rareness with which the scholars in the 1980s ‘explored the possibility that the cosmological conceptions that dominated the intellectual world of the Han era ever came to be generally superseded before modern times’, Henderson (1984, xiv) narrates this ‘decline’ through premodern expressions of critical thinking. He shows how individuals early on like Wang Chong 王充 (27 – c. 100) expressed scepticism towards the extension of this thinking to particular questions as well as how correlative sets were either abandoned or emptied of their cosmological content as they became overcomplicated, forced, or irrelevant to contemporary circumstances. It was not until the seventeenth century, he concludes, that people were willing to throw the baby out with the bathwater, since ‘medieval skeptics and critics of correlative thought were generally constrained to take it quite seriously’ (Henderson 1984, 95), but here, we would argue, it is important to move away from a global view defined by philosophical literature and narrow our focus to specific fields of inquiry—specific ‘scientific cultures’. When you do that, and when, instead of supposing ‘Chinese thought’ you ask where, exactly, it *is* expressed, you find actors in certain fields begin to compartmentalize and move away from it in their epistemic and rhetorical strategies as early as the late-Eastern Han.⁴⁸

⁴⁷ $29\frac{43}{81} = 29.53086$, cf. the modern value at epoch of 29.53059 days. $29\frac{499}{940} = 29.53085$ days is the value adopted by Bian Xin 編訢 and Li Fan 李梵 in the *Sifen li* 四分曆 of 85/86 CE, and $29\frac{773}{1457} = 29.53054$ days is that of Liu Hong’s *Qianxiang li* 乾象曆 of circa 206 CE.¹⁴⁵⁷ Note that the mean synodic month is but one element determining the predictive accuracy of an astronomical system, and that the *Sifen li*’s perceived success in predicting lunar phenomena over the *Santong li* in the late first century CE had more to do with accumulated error from system origin than this particular value. Note also that while the *Qianxiang li* maintains the ‘day divisor’ as the denominator of the mean synodic month, the *Sifen li* names this instead the ‘obscuration months’ (*bu yue* 部月), having reappropriated the term ‘day divisor’ as the denominator of the solar year. For historical values of the mean synodic month, see Chen Meidong (1995, 212–232).

⁴⁸ For further criticism of the equation of yin-yang, five-agents, and *Zhouyi* analogical reasoning as the one, true ‘Chinese thought,’ see Graham (1986, 8–11), Saussy (2000), Brown (2006), Martzloff (2009, 38–41), and Nylan (2010). For an excellent example of compartmentalization, see the example of Ge Hong’s 葛洪 (283–343) argument against correlative arguments for ‘umbrella heaven’ 蓋天 cosmology by appeal to experiential falsification from shadows, motions, apparent sizes, and optics as well as prophecy and more sophisticated metaphysics: *Sui shu*, 19.509–511, and *Jin shu*, 11.281–284, tr. Ho Peng Yoke (1966, 55–58); cf. Morgan (2017a, chap. 6).

In $l\ddot{u}_{HM}$, we see the rise of a ‘politics of precision’ (Goodman 2010) pegged in a large way to the mission of the Eastern Han Dongguan 東觀. Originally a palace archive, where Ban Gu went to write the *Han shu*, Empress Dowager Deng Sui 鄧綏 (b. 81; r. 105–121) ordered the Dongguan transformed into a centre for advanced studies under the dual charter of scholastic policy research and palace women’s education in 110, staffing its offices with the brightest minds of a generation. The Empress was *obsessed* with learning, for women, for men, in scripture, and in the sciences, and the Dongguan, for its part, proved a fitting legacy. Charged first with collating the classics and compiling historical monographs, Dongguan faculty like Ban Zhao 班昭 (44/49–118/121), Ma Rong 馬融 (79–166), and, in the 170s–180s, our very own Cai Yong and Liu Hong extended the Dongguan’s program from text into technologies, ritual, and $l\ddot{u}_{HM}$. Cai Yong expanded the pitch-standards from 12 to 60, Ma Rong produced the likes of Zheng Xuan 鄭玄 (127–200), who he favoured for his skill in math, and so did the effects of its program ripple beyond the Dongguan’s physical destruction in 189. And with these ripples came more, in 280, when the opening of the Ji Tomb 汲冢 unveiled a horde of new third-century BCE texts and, importantly, a set of late-Eastern Zhou 東周 (770–256 BCE) bells confirming suspicions of the historical inflation of the civil *chi*. In the wake of these two developments the course of $l\ddot{u}_{HM}$ would take a radical departure from that established by Liu Xin, the following centuries seeing a turn towards *realia*, the expansion of pitch-standards to 360, the tempering of pitch to create new scales and polyphonies, and physical experiments to test the validity of ‘watching the ethers’ and ancient metrological standards counted in seeds.⁴⁹

The same period witnesses a similar upheaval in li_{MA} . Up to Ban Gu’s day, the debate in li_{MA} was between the $l\ddot{u}_{HM}$ - li_{MA} synthesis of Deng Ping and Liu Xin’s day and the revival of a ‘quarter-remainder *li*’ (*sifen li* 四分曆) based on antiquarian precedence, but the following decades would see an explosion of new activity. Starting in the second century, we see increasing mentions of observation programs, instrument-commission, competitive testing, private education, textual transmission, and innovation. Numbers alone here speak volumes over words: the 425-year Han-Xin period sees 10 new li_{MA} systems; the 398 years after, leading up to the Tang, sees 27.⁵⁰ One of the main motors for this innovation, Morgan (2017a) argues, was the institution of the *ke* 課 live trial and *yi* 議 public debate, which encouraged private practitioners to bring ideas to the government’s door in exchange for a fair chance at winning definitive public recognition in a competition run by the rules of archery.⁵¹ Much of the work of ‘ li_{MA} reform’ (*gai li* 改曆) was indeed conducted by individuals on their personal time and out of personal interest, He Chengtian’s 何成天 (370–447) 80-year family gnomonics program and Zhang Zixin’s 張子信 (fl. 526–576) 30-year observational retreat to a ‘sea isle’ 海島中 being two rather conspicuous examples.⁵² Observation/testing programs

⁴⁹ See Goodman (2005), Goodman (2010), and, on the Ji Tomb, Shaughnessy (2006, 131–184).

⁵⁰ This is according to the master list in Qu Anjing (2008, 629–633); cf. Yabuuti (1963) and Sivin (2009, 42–53, Table 2.1). Note that these tables tend not include *every li* ‘astronomical system’ mentioned in received sources, focusing on those leading to government reforms; by our count, the middle period between the Han and Tang produced something more like 43 new *li*.

⁵¹ On public debate, see also Cullen (2007).

⁵² For He Chengtian, see *Song shu*, 12.261; for Zhang Zixin, see *Sui shu*, 20.561.

grew not only in number and duration but in scope: compare, for example, the 3-year set of 5 eclipses deployed *circa* 226 to the 1251-year set of 48 eclipses, 2 syzygies, and 25 gnomon shadows deployed in 597.⁵³ One of the reasons this happened, Morgan (2017a) emphasizes, is because there were things that one could do in 597 that no one thought possible in 226. The ‘subtle discrepancies’ 微差 of heaven that mysteriously limited any one li_{MA} system to a window of no more than 300 years in 226, for example, were only decades later *quantified* and *given a name*—*sui cha* 歲差 (lit. ‘year difference’), the precession of the equinoxes—which allowed practitioners, through subsequent refinement, to finally ‘put past and present in communication’ 通古今.

As it expanded and improved after Ban Gu’s day, importantly, Chinese astronomy quickly left ‘Chinese thought’ behind. Consider the attempt by one Zhang Long 張隆 to reproduce Liu Xin’s success as criticized by Jia Kui 賈逵 (30–101) in the year of Ban Gu’s death, in 92 CE:

永平中，詔書令故太史待詔張隆以四分法署弦、望、月食加時。隆言能用易九、六、七、八爻知月行多少。今案隆所署多失。臣使隆逆推前手所署，不應，或異日，不中天乃益遠，至十餘度。During the Yongping 永平 reign (57–75), there was an edict ordering Zhang Long, former expectant appointee to the grand clerk, to (predictively) note the added hour of [lunar phases] & lunar eclipse according to the Quarter-remainder method. [Zhang] Long said that he was able to use the nine, six, seven, and eight lines from the *Zhouyi* to know the extent of lunar motion.⁵⁴ We now know Long’s notes to have missed the mark (*shi*) in most cases. I, Your servant, made Long retrodict [the hour of syzygy] noted by former hands, and they did not correspond (*ying*), sometimes [even] falling on different days; he was even further off (*yuan*) in failing to hit the mark (*zhong*) in heaven, [being off target by] up to more than ten *du*.⁵⁵

Yes, the *Zhouyi* was an inviolable ‘classic’ (*jing* 經), but it was not the most effective tool for predicting the hour and right ascension of a lunar eclipse. It is not as if Liu Xin proposed *divining* data from milfoil sticks, oracles, and magic numbers, of course, Teboul (1983, xi) reminds us, it is the other way around:

En tout état de cause il serait faux de croire que la théorie de Liú Xīn, du fait qu’elle semble basée sur des a priori numériques, est sans rapport avec l’expérience. C’est exactement le contraire qui est vrai et il faut considérer les justifications numériques... comme des justifications a posteriori rendues possibles par le fait que Liú Xīn n’y opérait que sur des nombres entiers.

So-called ‘Chinese thought’, where it *is* present in Chinese astronomy, plays only supporting role to the tools of the trade, imbuing values wrought from mundane data collection and number-crunching the promise of Metaphysical Truth.⁵⁶ And for whatever reason, people

⁵³ The former—the first extant example of the sort of datasets applied to the ranking of competing li_{MA} systems—occurs in the context of a failed reform at the Cao-Wei court *circa* 226; see *Jin shu*, 17.500–502; cf. Morgan (2017a, chap. 5). The latter occurs in the context of debate surrounding the late-Sui *Daye li* 大業曆; see *Sui shu*, 17.429–434.

⁵⁴ On the *Zhouyi* hexagram line numbers ‘nine, six, seven, and eight’, see Nielsen (2003, 294).

⁵⁵ *Hou Han shu*, *zhi* 2, 3030

⁵⁶ Chen Meidong (2007, 551–565).

lost interest in this tactic rather abruptly after the Han, as Martzloff (2009, 40) reminds us in discussing the ‘Completed Numbers’ metrosophy presented in the *Han shu*:

Alors que le *Hanshu* s’intéresse seulement au système décimal, d’autres modes de représentation des nombres fondamentalement non-décimaux et fondés implicitement sur des fractions et des fractions de fractions de diverses manières, sont en effet apparus dès les Han mais sans faire l’objet de quelque présentation que ce soit, dans les C.A.O. (canons astronomiques officiels) ou ailleurs. Leur histoire est donc plus difficile à appréhender qu’elle le paraît *a priori*. De même, l’aspect numérolgique du nombre est lui aussi porteur d’une complexité insoupçonnée mais il a tendu à prendre de moins en moins d’importance et même à s’estomper au fil du temps : à partir des Song, les C.A.O. ne s’occupent à peu près jamais du sujet, même s’ils continuent de s’appeler assez artificiellement *lǐli* (litt. « tubes musicaux *lǐ* et canons astronomiques *li* ») alors qu’ils ne les mentionnent qu’en termes vagues et généraux.

$lǐ_{HM}$ and $lǐ_{MA}$ were eventually freed of the *Zhouyi*. And with little more holding the two together, they were free to move apart.

5.5 The Historiography of Change

Things had changed in $lǐ_{HM}$ and $lǐ_{MA}$, and experts looking back from the sixth and seventh centuries were well aware. The question is how that change was understood.

In the case of $lǐ_{MA}$, as Henderson (2006) argues, progress was a given, the contentious point in historiographical debate being whether modern man had surpassed the ancient sage.⁵⁷ As to how this was expressed, Morgan notes that the actors themselves tend to describe this ‘progress’ in terms of historical ‘accumulation’ (*ji* 積 or *lei* 累), ‘turning’ points (*zhuan* 轉), and ‘awakenings’ (*wu* 悟) leaving $lǐ_{MA}$ better than it used to be.⁵⁸ It was construed as ‘better’, that is to say, in the sense of *accuracy*, as framed in the same terms used in the quote from Jia Kui in the previous section: ‘hitting the mark’ (*zhong* 中), vs. ‘missing’ (*shi* 失); ‘accordance’ (*he* 合, *ying* 應), vs. ‘error’ (*cha* 差); ‘tightness’ (*mi* 密), vs. ‘looseness’ (*shu* 疏, *kuo* 闊); ‘finess’ (*jing* 精), vs. ‘coarseness’ (*cu* 粗); and being ‘close’ (*jin* 近), vs. ‘far off’ (*yuan* 遠). It is not enough to simply *claim* these qualities, as we likewise learn from Jia Kui’s criticism, they must instead be ‘proven’ (*yan* 驗) via empirical examination (*ji* 稽, *kao* 考, *jiao* 校, *xiao* 效, *jian* 檢, *cha* 查, *shen* 審, *can* 參, *ke* 課, etc.). The model, once again, is archery, and the Cai Yang-Liu Hong monograph puts its demands rather elegantly:

以是言之，則術不差不改，不驗不用。天道精微，度數難定，術法多端，曆紀非一，未驗無以知其是，未差無以知其失。失然後改之，是然後用之，此謂允執其中。

For procedures, one doesn’t revise what isn’t in error (*cha*), and one doesn’t use what isn’t proven (*yan*). The way of heaven is perfect & subtle, its *du* & numbers being hard to fix; there is a variety of procedures & methods, the net-threads of *li* being not [just] one. Without proof (*yan*), there is no way to know that it is correct (*shi* 是); without

⁵⁷ Cf. Sivin (1986).

⁵⁸ The following analysis is taken from Morgan (2017a, chaps. 2, 6). On the conceptual language of accuracy, see Hashimoto (1979).

erring (*cha*), there is no way to know that it is amiss (*shi*). If it is amiss (*shi*), you then revise it; if it is correct (*shi*), you then use it—this is called ‘holding truly to the middle way’ (*Lunyu* XX.1).⁵⁹

What ‘proves’ ‘correct’ at one point in history, however, may be later revealed to be ‘amiss’. Where this happens, later writers tend either to dismiss their forbearers’ ideas as ‘absurd’ (*miu* 謬), ‘obscene’ (*wei* 猥), ‘preposterous’ (*wang* 妄), etc., or apologize for ‘what the ancients had yet to learn’.⁶⁰ Whatever the ambiguous value of ‘antiquity’ (*gu* 古), the word ‘old’ (*jiu* 舊) is used as a pejorative, the assumption in *li*_{MA} being that what is ‘new’ (*xin* 新) is necessarily better. All the same, there are certain ‘scientific ancestors’ to which one must pay homage, despite their failings, and such is the way that Li Chunfeng, for example, writes about Liu Hong before him:

其為之也，依易立數，遁行相號，潛處相求，名為乾象曆。又創制日行遲速，兼考月行，陰陽交錯於黃道表裏，日行黃道，於赤道宿度復有進退。方於前法，轉為精密矣。

What he did was establish numbers based on the *Zhouyi* [such that] they called out to one another in hidden motion and sought each other out from secret parts—and [at this he] named it the *Qianxiang li* 乾象曆 (Supernal Icon *li*). Also, [he] created the solar/daily motion slow-fast (speed correction) while concurrently investigating (*kao*) lunar motion, [concluding that] yin & yang (i.e. negative and positive latitude) cross inside & outside of the yellow road, and that the sun travels on the yellow road, experiencing advance & retreat in terms of red-road lodge-*du*—and only with this was there a turn (*zhuan*) towards the fine (*jing*) & tight (*mi*) relative to prior methods.⁶¹

Returning to *li*_{HM}, one finds a similar rhetoric of ‘accuracy’ and ‘testing’. Under ‘Completed Numbers’, cited above, we read Li Chunfeng describe Zu Chongzhi as having ‘introduced a tight (*mi*) method’ for calculating the circumference of a given circle from its diameter. In the *Jin shu*, he likewise reports that ‘Xun Xu constructed a new set of bells & pitch-standards which harmonized perfectly with the ancient instruments [from which he was working], and the people of his time lauded his fineness (*jing*) & tightness (*mi*)’.⁶²

The question of what counts as ‘fine & tight’ in harmono-metrics is a little harder to intuit than in the case of eclipse prediction, so let us briefly recall the axiomatic foundations of harmono-metrics:

- (1) the Ur HUANGZHONG pitch-regulator was created (*zuo*) by sages according to immutable natural principles;
- (2) passed down through subsequent ages, and described in the classics, the Ur regulator-series is the one correct standard to which we must aspire;
- (3) the HUANGZHONG pitch-regulator is the basis of all metrological standards, spanning 9 *cun* in length, holding 1 *yue* in capacity, and weighing 12 *zhu*;

⁵⁹ *Hou Han shu*, *zhi* 2, 3041; tr. modified from Sivin (1986, 158).

⁶⁰ 古人所未達，Yixing 一行 (683–727), cited in *Xin Tang shu*, 27A.593. For similar historical criticism and apology in the astral sciences, see Morgan (2016).

⁶¹ *Jin shu*, 17.498; cf. *Song shu*, 12.231–232. On Liu Hong’s *Qianxiang li*, see (2002) and Cullen (2017, xx). Note that Liu Hong’s use of *Zhouyi* number symbolism marks the last attempt of its kind until Yixing in the eighth century.

⁶² 荀勗造新鍾律，與古器諧韻，時人稱其精密，*Jin shu*, 16.491; cf. Goodman (2010, 204–205) (worded differently).

- (4) the exact dimensions of each metrological unit can also be empirically measured in terms of small, regular bodies like millet seeds according to the numbers provided in ancient texts;
- (5) each tube in the twelve-regulator series (octave) correlates to a particular month of the year, e.g. HUANGZHONG : month XI;
- (6) incorrect pitches and intervals ‘just sound wrong’ to the trained ear.⁶³

These are the facts as Liu Xin understood them, and he hints at six ways in which one can empirically verify whether one’s *li*_{HM} standards are ‘tight’ in this regard:

參五以變，錯綜其數，稽之於古今，効之於氣物，和之於心耳，考之於經傳，咸得其實，靡不協同。

Mutating in threes & fives, synthesize its (the present *li*_{HM} system’s) numbers;⁶⁴ investigate (*ji*) it in the past & present, compare (*xiao*) it to *qi* & creatura (*wu*), harmonize it with the heart-mind & ears, and research (*kao*) it in the classics & their commentaries—all of this will reveal its truth (*shi* 實), for nothing does not accord.⁶⁵

The first way would seem to be **archaeometrology**: the investigation of *realia* ‘past and present’ (see Axiom 2). The second presumably refers to **qi-testing**, the practice known as ‘watching the ethers’ (*hou qi* 候氣; see Axiom 5), followed by **seed-stacking** in accordance with classical seed-dimensions (see Axioms 2 & 4). The fourth is **ear-testing**, which requires no further explanation (see Axiom 6). The fifth is **textual scholarship**: scouring ‘the classics and their commentaries’ for evidence of correct Western Zhou 西周 (1045–771 BCE) practice (see Axiom 2). The sixth, lastly, is **consistency-checking** across all of the previous criteria.

Whatever Liu Xin had in mind by this statement, these at least are what later generations understood their options to be in terms of *li*_{HM} testing, and testing quickly revealed their axioms to be in contradiction. Li Chunfeng tells us for example how Mao Shuang 毛爽 et al.’s court-ordered experiment with *qi*-testing in 589 produced mixed results that they tried to pass off as omens, winning them the ire of a first curious then sceptical emperor.⁶⁶ The perfect-length bell did not always sound right, nor hold the correct amount of millet, and after citing no less than seven different early- and pre-imperial sources on the origin of length-measures, Li Chunfeng can only conclude thus:

此皆起度之源，其文舛互

These are all the sources there are on the origin of measures, and their words contradict one another.⁶⁷

Faced with the impossible challenge of meeting all six of Liu Xin’s criteria, scholars were forced to *prioritize among them*, leading to a

⁶³ Axiom 6 we deduce from the strategy of ear-testing mentioned in the following quotation from Liu Xin; for the others, see above.

⁶⁴ Here, Liu Xin is citing the ‘Xici zhuan’ commentary to the *Zhouyi*, which reads ‘The three and five operations are undertaken in order to obtain a change. Divisions and combinations of the numbers are made’ 參伍以變錯綜其數 (*Zhouyi zhushu*, 11.35a; tr. Wilhelm (1967, 314)).

⁶⁵ *Han shu*, 21A.956.

⁶⁶ *Sui shu*, 16.394.

⁶⁷ *Sui shu*, 16.402.

diversity of research programs across time and warring kingdoms. *Qi*-testing, for example, is not really taken seriously as an empirical strategy until the Song 宋 (960–1279),⁶⁸ and archaeometry and seed-stacking, while treated in earlier texts, come to the fore only in the Wei-Jin and Six Dynasties.

They may have been motivated by ‘tightness’, but practitioners of *li_{HM}* do not talk about contemporary knowledge as a product ‘accumulation’, and it is easy to understand why. Practitioners of *li_{HM}* were split into a multiplicity of irreconcilable approaches, whereas the institution of a public contest and reward structure in *li_{MA}* provided a simple answer to who was ‘tightest’. Tonal and metrological benchmarks are also *social* and *aesthetic conventions* that differ inconsequentially across time and culture, the fact of which accommodates more different ‘right’ answers there than in predicting the hour and position of an eclipse. Nor, importantly, could the object of study be any more different: that of *li_{MA}* is as unchanging sky, manifest to all; that of *li_{HM}* is a nebulous antiquity mediated by human artefacts that are ultimately contestable, ephemeral, and limited in number and accessibility. The sky, in other words, does not get hoarded, lost, or melted down. One might say that *li_{HM}* and *li_{MA}* began moving in different directions once freed from the *Zhouyi*, but that would imply that *li_{HM}* was headed some one direction in particular.

How then did monograph compilers then make sense of these changes in *li_{HM}* without, in this case, a single standard by which to arbitrate the winners and losers of history? Mostly they did not. From the late second-century Cai-Liu monograph on, compilers simply listed relevant developments in chronological order. The result is, by contrast, considerably messier than the typical *li_{MA} juan*. Unlike its counterpart, which is comprised of episodic variations on a single sequence of events (observe, create, submit, test, debate, institute, repeat), the *li_{HM} juan* commingles very different matters without any clear sense of direction: theoretical developments, archaeological finds, the temperament of musical instruments and scales, and variations discovered between official and market foot rules. The challenge for an ambitious historian like Li Chunfeng was double: how to give the history of *li_{HM}* a clear direction and how to reconcile it with that of *li_{MA}* so as to return us to the elegance of something like the Han dream of an universal order.

5.5.1 Archaeology: the Past as the Way Forward

Under the ‘Examined Lengths’ heading of the *Sui shu* monograph, Li Chunfeng adds something hitherto unprecedented in the genre: an itemized instrument catalogue.⁶⁹ This catalogue bears no title or subject heading, but is announced clearly by Li: ‘Now, I summarize all the foot rule measures throughout the dynasties into 15 classes, and also include discussions about the differences or sameness [among them], as follows’.⁷⁰ More specifically, he divides historic foot rules

⁶⁸ On the practice of *hou qi*, see Bodde (1959) and Huang and Chang (1996). Suffice it to say that pre-Song sources explicitly describe the practice as a method for generating omens, e.g. ‘if it matches: harmony; otherwise: omen’ 效則和，否則占 (*Hou Han shu*, *zhi* 1, 3024), which comports with the long-standing connection between harmonics and divination. On harmonics divination, see Kalinowski (2011).

⁶⁹ He does something similar in the ‘Tianwen zhi’, on which see Chapter 7.

⁷⁰ 今略諸代尺度一十五度，并異同之說如左，*Sui shu*, 16.402.

into 15 categories determined by their absolute length vis-à-vis the standard of Xun Xu's Jin Former Foot Rule (*chi*), e.g.:

三、梁表尺實比晉前尺一尺二分二釐一毫有奇

III. Liang-era gnomon foot rule. In actuality, 1 *chi* 2 *fen* 2 *li* 1 *hao* & a smidgen (1.221... *chi*) compared against the Jin Former *chi*.⁷¹

Under each category is listed one or more historical objects matching said dimensions, their provenance, and a history of contestations.⁷²

The core of Li's catalogue is built chiefly upon another inventory mentioned earlier in his monograph: the 12 named foot rules utilized in Niu Hong 牛弘 et al.'s experiment of 589 to determine the millet-capacity of a HUANZHONG (diameter: 3 *fen*; length: 9 *cun*) constructed according to historical foot rules. The purpose of this experiment, Li tells us, was to 'research (*can kao*) ancient pitch-regulator dimensions', and no overall judgment is rendered from their data.⁷³ Shortly afterward (< 601), the Sui court 'issued an edict that the 'water *chi*' be used for pitch-regulators and music, and that the metal and stone [instruments] of previous dynasties all be melted down and destroyed to cease *realia*-based debate',⁷⁴ which suggests that Li Chunfeng's access to the items listed in his catalogue may have been textual or, at least, through his and others' private collections.

Li Chunfeng's approach could not be more different from Niu's, considering that Li begins his own catalogue with a methodological statement about the dangers of seed-stacking:

後之作者，又憑此說，以律度量衡，並因秬黍，散為諸法。其率可通故也。黍有大小之差，年有豐耗之異，前代量校，每有不同。The works of later ages, further relying upon this (the *Han shu*'s) explanation, based pitch-regulators, lengths, capacities, and weights all upon black millet, which led to a scattering of various standards (*fa*). The reason [that this could happen] is that their LÜ (i.e. the definition of *li*_{HM} standards via proportional relation to one another, see below) can communicate with one another. Millet grains are discrepant in size, and harvests vary in quality, [and so it is that if you] compare the capacities of previous dynasties every one of them is different.⁷⁵

In one stroke, Li has swept the seeds from the table, leaving us but one recourse for the empirical study of *li*_{HM}: archaeological *realia*. Under the first heading of his catalogue of foot rules—'I. The Zhou *chi*' 一，周尺—Li recounts Xun Xu's third-century work to reconstruct the 'Zhou' *chi*, from which Xun had concluded, based in part on textual and antiquarian scholarship, that the 'Zhou' value had been lengthened at some point in the Eastern Han. The verification of Xun's 'Zhou' *chi* came in the form of a spectacle that would meet the demands of any astronomical trial: the unexpected archaeological recovery in 280 of a set of late-Warring States (480–222 BCE) bells from

⁷¹ *Sui shu*, 16.404.

⁷² For studies of Li Chunfeng's instrument catalogue, see Ma Heng's 馬衡, 'Suishu Lüli zhi shiwu deng chi' 隋書律曆志十五等尺, translated in Ferguson (1941); see also Qiu Guangming et al. (2001, 305–317) and Goodman (2010, 197–206), which examine the back-stories of certain of Li's instruments. To compare with ancient Greek instrument catalogues of this nature, see Creese (2010, 50 ff., esp. 118–122).

⁷³ 參考古律度, *Sui shu*, 16.392–394.

⁷⁴ 詔施用水尺律樂，其前代金石，並鑄毀之，以息物議, *Sui shu*, 16.392.

⁷⁵ *Sui shu*, 16.402.

the Ji Tomb that perfectly confirmed his new (ancient) standard. Li Chunfeng takes this, Xun's 'Jin Former *chi*', as the standard for further comparison and cites additional *realia* and historical accounts corroborating Xun's hypothesis.⁷⁶

Citing historical texts, giving the antiquarian arguments, and jumping in and out of quasi-legend—e.g. Cai Yong's flute, vague stories about the uncovering of Xun and his critics' respective smoking-gun evidence—Li mythologizes what Goodman has described as Xun Xu's 'prisca Zhou' ritual scholarship of the 270s, which sought a return to Zhou qualities and quantities through empirical investigation.⁷⁷ The point of this exercise for Li Chunfeng, we might surmise, was to rescue Xun Xu from his contemporaries' accusations that he had destroyed the Western Jin 西晉 (266–317) through unseemly court interventions as a grasping factionalist—and so too to rescue the Jin itself, an aim that would have resonated with Tang Taizong's historical sensibilities.⁷⁸ Li's approach to the history of metrology resembles Ptolemy's review, reanalysis, and approval of his ancient predecessor Archytas, in which he laid out the latter's numbers (ratios in this case), placing them in a table, and announcing that, above all, Archytas 'followed the principles of reason (*logos*)', even though he is later pronounced as wrong.⁷⁹ Li Chunfeng did not pass such explicit judgment on Xun Xu in the *Sui shu*, but he does reveal his feelings in the corresponding section of his *Jin shu* monograph:

推百代之法，度數既宜，聲韻又契，可謂切密，信而有徵也。
As to his inference of the standards (*fa*) of the hundred dynasties, not only were his metrics appropriate, his tones matched tally-like (*qi*)—it was, we might say, exact (*qie*), tight (*mi*), credible (*xin*), and well-evidenced (*you zheng*).⁸⁰

Li Chunfeng does not stop there, however, because there is a history of archaeometrology *after* Xun Xu, even if in Li's opinion there shouldn't be—he does have to write *something* for the Five Dynasties after all. Having in his opinion already solved the problem of *li*_{HM} in category I, Li keeps moving forward through time as the one true 'Zhou' standard gives way to the 'scattering of various standards' wrought of seed-stacking through fourteen additional categories. And though these later developments are, in his opinion, misguided, he devotes to them the same degree of seriousness—the same degree of mathematical, textual, and archaeological rigor—that he does to the

⁷⁶ Goodman (2010, 189–190, 198–200) analyses comments in several sources that cast doubt on the discovery of bells or chimes in the Ji Tomb, or, at least, that Xun Xu or his son ever used them in any tests. In fact Xun Xu had already begun, even before the Ji Tomb discovery, to 'call in' (or hunt down) ancient foot-rules and bells in others' collections around the country.

⁷⁷ Goodman (2010, *passim*). The political crises in the background of Xun's and his peers' work concerning precision in ritual arts never directly used the vocabulary or findings associated with a pure Zhou standard ('prisca Zhou'), even though the latter impacted the court's music and *li*_{HM}, nor were such empirical findings ever directly expressed in an ideology of delegitimation; Goodman (2010, 374–382) explains that Xun's peers generally rejected this ritual precision based mostly on *ad hominem* grounds.

⁷⁸ For an analysis of how Xun Xu was judged in the *Jinshu* biography written for him in the Tang, see Goodman (2010, 114–118). For Taizong's interest in the Jin as a political model, although a delicate one, see McMullen (2013).

⁷⁹ Ptolemy, *Harmonics* i.13-14; tr. Barker (1984–1989, vol. 2, 304–305).

⁸⁰ *Jin shu*, 16.491.

‘Zhou *chi*’. It is our task in the history of science, Li would seem to say, not only to impart events with a coherent *telos*, but also to order them impartially, even if the picture that emerges is less like the (supposedly) onward-and-upward march of li_{MA} than a roller coaster of gains and losses.

5.5.2 From *shu* 數 to LÜ 率: harmonics and astronomy as mathematics

Through archaeology, Li Chunfeng is able to offer us a (in his opinion) definitive solution to the vexing problem of li_{HM} —how best to know the past—but how are we to account for the unceasing history of ‘accumulation’ and ‘turns’ in a science oriented towards the ever-unfolding future like li_{MA} ? If the Sages have revealed to man all that there is to know about the hypostasis of Number, and the sage minds of the Han, for their part, have mulled over every possible extension of this plan into human experience, why then are *shu-shu* 數術 ‘numbers & technē’ possessed of such complicated histories? We acknowledge the wisdom of ‘investigating the past & present’, but how are they to be *reconciled*? It is our opinion that Li’s solution to these problems lies in his use of the concept LÜ 率, which he deploys to shift the (supposed) epistemic bases of li_{HM} and li_{MA} from number symbolism (*xiang shu* 象數) to mathematics (*suan shu* 算數).

In mathematics, the term LÜ appears foremost in the context of definitions of units of value for grains. Where millet (*shu* 黍) and unhulled rice (*dao* 稻) are exchangeable at a rate of 50 : 60, for example, they are said to have respective LÜ of 50 and 60. In the *Jiuzhang suanshu*, conversion of a given quantity from one into the other—‘how much millet is 120 units of unhulled rice equivalent to?’—is performed via the procedure *jinyou* 今有 ‘suppose’ (i.e., the rule of three), whereby one multiplies the quantity given (120) by the LÜ of what is sought (50), then divides by the LÜ of what is given (60), to arrive at the quantity sought, i.e., $(120 \times 50)/60 = 100$. In his commentary of 263, Liu Hui describes the concept thus:

凡數相與者謂之率。率知，自相與通。有分則可散，分重疊則約也。等除法實，相與率也。故散分者，必令兩分母相乘法實也。Chaque fois que des quantités (*shu*) sont données en relation les unes avec les autres, on les appelle des LÜ. Les LÜ, étant par nature donnés les uns en relation avec les autres, communiquent. S’il y a des parts, on peut désagréger ; si les parts sont des superpositions rattachées, on simplifie. Diviseur et dividende, divisés par le nombre égal, sont des LÜ mis en relation l’un avec l’autre. Par conséquent, si on désagrège les parts, c’est qu’on fait nécessairement en sorte que les deux dénominateurs multiplient l’un et l’autre diviseur et dividende.⁸¹

Returning to his treatment of ‘Completed Numbers’ in the *Sui shu* (above), we will recall that most of what Li Chunfeng has to add to Liu Xin’s *summa* concerns LÜ. In the first paragraph, he concludes with a line from the Cai-Liu monograph, into which he inserts LÜ in two different places, one identifying it, alongside pitches, lengths, capacities, weights, and li_{MA} , as a ‘distinct application’ 別用 of counting rods, the other, offering it as a solution to things ‘appearing jumbled together’ 糅見, i.e. determining the correct relative values of dif-

⁸¹ *Jiuzhang suanshu*, 1.18; tr. Chemla and Guo (2004, 167). For more on LÜ in the context of mathematics, see *ibid.*, 119–219, 956–959.

ferent but communicable things. In the second paragraph, he declares LÜ the ultimate origin of all the *Jiuzhang suanshu* and adds the (LÜ-based) ‘suppose’ procedure a description of the essence of algorithms drawn from Liu Hui’s commentary. Then, in the third paragraph, he presents us with the history of the circle-circumference and diameter LÜ (i.e., π).

By contrast, the Liu/Ban monograph devotes commensurate emphasis to *shu* 數 ‘Number’. Liu Xin tells us that *shu* is that by which one ‘follows the pattern of nature (*xing*) and fate (*ming*)’.⁸² Ban Gu likewise places *shu* as the very origin of the *Zhouyi*: ‘beginning from when Fuxi first drew the Eight Trigrams, arisen as they were from number’.⁸³ These, interestingly enough, are the very sorts of statements that Li Chunfeng emends or edits out of what he takes from their writing. Where the Liu/Ban monograph reads ‘[The five *shu*] originate from the *shu* of the HUANGZHONG’, for example, Li gives us ‘it is said that *shu* start from establishment at *zi*._{B01} (the month of winter solstice)’, balancing out the Liu/Ban proposition by adding that ‘*shu* arise from li_{HM} , and li_{HM} is completed by *shu*’ (above). It would seem from his creative editing job that Li Chunfeng is trying to hedge out *shu* in favour of LÜ.

What’s the difference? It should be clear by now what ‘Number’ meant to Liu Xin and how it functioned to connect the diverse constituents of his cosmos—it meant a *Zhouyi*-rooted worldview of corresponding sets and mysterious number progressions. LÜ, for Li Chunfeng, meant mathematics—a world of algorithm-driven mutations as embodied in the *Jiuzhang suanshu*. There is, we must stress, nothing less profound in Li’s framing of the subject. Karine Chemla has written at length, for example, about how their frequent recourse to the *Zhouyi* reveals that early imperial mathematicians expressed interest in algorithms—specifically, those involving complementary operations such as ‘suppose’—as meditations on the greater cosmic processes of transformation and change.⁸⁴ It is precisely the issue of change, we argue, with which the seventh-century historian of li_{HM} and li_{MA} found himself grappling: historical change, be it progressive or regressive. Let us remember that it is LÜ that Li Chunfeng offers as the explanation for the history of harmono-metrics: ‘the reason is that their LÜ can communicate with one another’ (above), which is to say that, because pitches, lengths, capacities, and weights are all quantities ‘données en relation les unes avec les autres’, when you move the benchmark for one, you move the other standards along with it. And when it comes to historicizing number (*shu*) through the framework of LÜ, where better to begin than the history of the LÜ of the circle?

When it comes down to it, the history of li_{HM} and li_{MA} is nothing other than a history of LÜ values in flux—of improving ratios determining the circle of fifths, the synodic month (month divisor : day divisor), and so on. And so it is that Li Chunfeng seems to be taking a cue from Liu Hui, who, in his third-century preface to the *Jiuzhang suanshu*, makes an overture at subordinating subsuming li_{HM} and li_{MA} within the genesis and program of mathematics:

⁸² 順性命之理, *Han shu*, 21A.956.

⁸³ 自伏羲畫八卦, 由數起, *Han shu*, 21A.955.

⁸⁴ See Chemla (2010).

昔在庖犧氏始畫八卦，以通神明之德，以類萬物之情，作九九之術，以合六爻之變。暨于黃帝神而化之，引而伸之，於是建曆紀，協律呂，用稽道原，然後兩儀四象精微之氣可得而效焉。

In the past, Baoxi first drew the eight trigrams to enter into communication with the virtue of the spirits-illuminant and to classify the inner tendencies (*qing*) of the myriad entities (*wu*), and he created the Nine-Nine Procedure (multiplication table) to accord with the mutations (*bian*) of the six lines (of the hexagrams). And then it was that the Yellow Thearch transformed them through spiritualization and expanded them through extension, at this establishing li_{MA} rules and harmonizing the twelve pitch-standards (*liü ü* 律呂), which were used to investigate the source of the *dao*. Thereafter was the fine and subtle *qi* of the two *yi* 儀 and four *xiang* 象 finally made manifest.⁸⁵

That harmonics and astronomy are ‘extensions’ of mathematics (or LÜ) may seem self-evident to us, but it is something of an interesting reversal in the context of historical taxonomies and hierarchies of knowledge. We would point out that, for example, early imperial thinkers had always subordinated mathematics to li_{MA} under such rubrics as ‘ Li_{MA} & Genealogies’ and ‘ Li_{MA} & Mathematics’ in the organization of imperial collections and bibliographies.⁸⁶ Li Chunfeng, however, turns the tables, uniting what once was held together by the rigid vagaries of number symbolism by the open-ended pursuit of ‘tightness’ promised by mathematics.

5.6 Conclusion

In this chapter, we have examined the institution of the ‘Lü-li zhi’ in the standard histories from both ends of the historical spectrum. At one end, we saw how the genre emerges in the early-first century as a collage of Liu Xin’s work in the previous century to synthesize the two through the framework of an all-encompassing *Zhouyi*-based hypostasis of Number. At the other end, we saw how, in the seventh century, Li Chunfeng sought to revive, hone, and integrate Liu’s typological framework with the chronicle-based approach that had since emerged in the historiography of li_{HM} . The central problem with which Li and his predecessors grappled, we argue, was that of progress—the ‘accumulation’ of knowledge driven by a culture of ‘tightness’ and empirical ‘examination’ that sent li_{HM} and li_{MA} in different directions from both one another and the promised elegance of the cosmos that we see expressed in Han thought. The two places where Li’s authorial voice is most clearly manifest in his monograph, we argue, are where he speaks to this problem. One is his Catalogue of 15 historic foot rules, wherein he both (purports to) solve the abiding problem of li_{HM} through archaeometry and impart the subsequent history of the field with a coherent sense of order. The other, drawing from the *Jiuzhang suanshu*, is Li’s substitution of the concept of number (*shu*) upon which Liu Xin’s static hypostasis is founded with that

⁸⁵ *Jiuzhang suanshu*, preface; tr. modified from Chemla and Guo (2004, 127). The connection drawn here between LÜ and li_{MA} is inspired by Chemla (forthcoming); cf. Morgan (2017a, chap. 2).

⁸⁶ Not only is *suan* 算 ‘mathematics’ either omitted from or subordinate within such headings, so too are mathematical texts like the *Jiuzhang suanshu* listed *after* titles concerning the various facets of li_{MA} ; see *Han shu*, 30.1765–1767; *Sui shu*, 34.1022–1026; *Jiu Tang shu*, 47.2037–2039. On bibliographical classification, see Chapter 13.

of LÜ, by which he establishes *mathematics* as the enduring foundation of this dynamic knowledge.

Was Li Chunfeng's the final word on the li_{HM} - li_{MA} synthesis? In a way, yes, but if only for the reason that his were the penultimate entries in this genre and that the field of li_{HM} seems to have fallen into obscurity within a decade or two of his lifetime. When picked up again some three centuries later by Song antiquarians, however, Li's *Sui shu* 'Lü-li zhi' does appear to enjoy a position of authority next to that of the Liu/Ban monograph. In a memorial to Renzong 仁宗 (r. 1022–1063) concerning the fabrication of court-commissioned tube-regulators according to the principle of seed-stacking, for example, Ding Du 丁度 (990–1053) et al. begin with summary of the points that Li Chunfeng makes in his 'Examined Lengths' section, bemoaning the contradictory state of early sources and the limitations of said approach:

雖存異說，莫可適從...然而歲有豐儉，地有礪肥，就令一歲之中，一境之內，取以校驗，亦復不齊。是蓋天物之生，理難均一，古之立法，存其大概爾。故前代制尺，非特累黍，必求古雅之器以雜校焉。

There exist [many] different doctrines, but none of them is reliable. ... Harvests vary in their bounty, and the earth varies in its fertility, so that even if, for the purpose of comparative examination (*jiao yan*), one takes [seeds] from a single harvest within a single territory they will still be at odds. This is because the principles (*li* 理) of natural entities (*wu*) are difficult to reduce to one, and [because] all that is extant on how the ancients erected their standards (*fa*) is but a rough sketch. Thus, in the matter of [studying] foot rules instituted in previous dynasties, [we] reject the exclusive [recourse] to piling millet and [demand] that one must seek out instruments of great antiquity by which to variously compare (*jiao*) them.⁸⁷

And so did the debate between archaeometry and seed-stacking live on.⁸⁸

Our intent in this chapter is not to summarize 'the harmonics-astronomy connection in China' but to address how historical figures like Liu Xin and Li Chunfeng actively made, remade, and unravelled that connection out of materials at hand and in response to the political exigencies, historical developments, and scientific cultures of their respective days. There was once something like Granet's 'Chinese thought' behind this, but it was very much the product of a time, a context, an author—and some salesmanship on the latter's part—and this one 'Chinese thought' gave way to others. Whatever connection there ever was, the two quickly drifted apart in practice, leaving the 'Lü-li zhi' an artefact from a different time. Li Chunfeng tried to revive it, as best he could, but the day when something like 'Chinese thought' could hold everything together had long since passed. li_{MA} , later historians decided, deserved its own monograph again.

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⁸⁷ *Song shi*, 71.1608.

⁸⁸ See Lam (2006).

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Appendix 1: *Han shu* 漢書 ‘Lü-li zhi’ 律曆志 Table of Contents

律曆上 Harmono-metrology & Mathematical Astronomy <i>juan A</i>
[Harmono-metrology]*
[Historical introduction]: historical précis from sage times to the 1st cent. AD; Ban states that he is working off of Liu Xin’s writings but has “deleted his outrageous comments (<i>weici</i>) and selected his correct meanings”
[備數 Completed Numbers]: list of the five numbers (i.e., decimals), they serve as bases of calculating things and fate, and their origin in the HUANGZHONG; introduction to the geometric, harmonic, and <i>Zhouyi</i> symbolism of counting rods; the interpenetration of the numbers of harmonics, astronomy, and metrology; the responsibility of the Clerk’s Office in maintaining and promoting these numbers.
[和聲 Harmonized Sounds]: (¶1) list of the five tones and eight timbres as the constituent elements of music, including folk etymologies and five-agents correspondences for the former; (¶2) inventory of the twelve pitch-standards, moving from the myth of their sagetime origin to folk etymologies and yin-yang, chronogram, and month correspondences; (¶3) the correspondence of the three integer-length pitch-standards (HUANGZHONG, LINZHONG, & TAIZU), on the basis of <i>Changes</i> numerology, with the three concordances 三統 (Heaven, Earth, & Man) in the context of cosmogonic stages; (¶4) the correspondence of said three pitch-standards with the three first months 三正 (Xia, Shang, & Zhou), followed by list of the pitch-standards in the pentatonic scale starting from HUANGZHONG and emphasis of the superiority of the latter as a mode-base; (¶5) explanation of said pitch-standards’ DIVIDEND in terms of <i>Changes</i> numbers; (¶6) The procedure for arriving at the HUANGZHONG DIVIDEND, starting from 1 and multiplying by 3 for every chronogram; (¶7) the <i>Sanfen sunyi</i> production sequence.
[審度 Reliable Length Measures]: inventory of units of length, basis in HUANGZHONG, HUANGZHONG length in millet seeds, unit conversion, instrument standards, folk etymologies.
[嘉量 Good Capacity Measures]: inventory of units of capacity, basis in HUANGZHONG, HUANGZHONG capacity in millet seeds, unit conversion, instrument standards, folk etymologies.
[權衡 Weights and Balances]: (¶1) philosophical introduction to the working of scales; (¶2) inventory of units of weight, basis in HUANGZHONG, HUANGZHONG weight in millet seeds, unit conversion, instrument standards, folk etymologies & astronomical/numerological correspondences; (¶3) the interrelationship of the five archetypes (scale, compass, t-square, level & plumb), each of which is described in terms of correspondences (directions, yin-yang phases, seasons & five agents) and folk etymologies; (¶4) <i>Shangshu</i> quote concerning Shun on harmonics, discussion of the need for standardizing metrology for the sake of political unity, and the benefits of bronze for the construction of such standards.
[Mathematical Astronomy]
[Historical introduction]: historical précis from sage times to the 3rd cent. BC
[Chronicle]: (¶1) Han inherits Qin Zhuanxu system; (¶2) 105-104 BC, Grand Inception reform; (¶3) 78-74 BC, Xianyu Wangren, et al. charged with observation program to test alternatives to the Grand Inception system; (¶4) c. 32 BC – AD 5, Liu Xiang and Liu Xin’s scholarship.

[Triple Concordance system]
<p>[Preface]: (㉑) the calendro-astronomy of the <i>Chunqiu</i> reflects celestial time, upon which human time is ordered and human fate is secured, the <i>Zhouyi</i> affirms that the ordering of time is the <i>dao</i> of harmonizing humanity; (㉒) examples of the <i>Chunqiu</i> criticizing post-Zhou lapses in calendro-astronomy and related state rites, <i>Zhouyi</i>-based numerological derivation of key harmonic and astronomical values, reaffirmation of the foundational status of the <i>Chunqiu</i> and <i>Zhouyi</i>; (㉓) continued explanation of harmonic and astronomical values in terms of <i>Zhouyi</i> numerology, <i>Chunqiu</i> citations, and three-concordance, five-agent, and yin-yang correlations.</p>
律曆下 Harmono-metrology & Mathematical Astronomy <i>juan B</i>
<p>[<i>Santong li</i> procedure text]: (1) 統母 lunisolar elements; (2) 紀母 planetary elements; (3) 五步 planetary models; (4) 統術 lunisolar algorithms; (5) 紀術 planetary algorithms; (6) 歲術 Jovian/Taisui year-count algorithms.</p>
<p>世經 <i>Shijing</i>: chronicle of astronomical records from sagetimes to the 1st cent. AD as successfully verified by the <i>Santong li</i>.</p>

Note: section headings in brackets have been added to reflect breaks in the treatise's contents.

Appendix 2: *Sui shu* 隋書 ‘Lü-li zhi’ 律曆志 Table of Contents

律曆上 Harmono-metrology & Mathematical Astronomy <i>juan</i> A [HM]
[Historical introduction]: (¶1-2) philosophical & historical précis from sage times to the 6th cent.; (¶3) organization of the treatise is according to the <i>Han shu</i> .
備數 Complete Numbers: (¶1) list of the five numbers (i.e., decimals), numbers originate from the HUANGZHONG and, conversely, numbers perfect harmonics; introduction to the geometric, harmonic, and <i>Zhouyi</i> symbolism of counting rods; the interpenetration of the numbers of harmonics, astronomy, metrology, and <i>li</i> ; (¶2) <i>li</i> is the foundation of mathematics; (¶3) chronicle: the history of π .
和聲 Harmonized Sounds: (¶1) chronicle: myth of sagetime origin of pitch-standards, the historical bases of pitch-standards in natural sounds and units of length; recap of <i>Han shu</i> account of Jing Fang’s work on pitch-generation and expansion of the pitch-standards to 60; Zu Chongzhi’s expansion to 360; He Chengtian’s solution to the Pythagorean comma; (¶2-7) excerpt from Liang Wudi’s <i>Zhong-lü wei</i> ; (¶8) chronicle: Chen court, no reform; (¶9) chronicle: Western Wei, Su Chuo’s abortive commission to correct pitch-standards according to a Liu-Song <i>chi</i> rule, archeological discovery of jade <i>dou</i> ladle in Northern Zhou (557-581) and commission of new measures; (¶10) chronicle: early Sui, Niu Hong ordered to explore fixing pitch-standards, experts and <i>realia</i> recovered from Chen, <i>qi</i> observation program, collection & testing of found <i>realia</i> , establishment of official standards; (¶11) differentiation of five-agents <i>chi</i> ; (¶12) 604, Liu Zhuo reports his findings <i>vis-à-vis</i> past scholarship on pipe-lengths; 606, edict to tune instruments to pitch-standards based on the Liang gnomon standard.
律管圍容黍 Pitch-pipe Millet Capacity: the problem: Han accounts provide values for pitch-standard diameters and circumferences, but later scholars discovered that historical variation in metrological standards has resulted in variations in tone and capacity; list of millet capacities for 12 HUANGZHONG constructed according to different <i>chi</i> rules.
候氣 Observing the Qi: (¶1) chronicle: anecdote about Xindu Fang’s virtuosity & innovations; (¶2) 589, Mao Shuang, et al. ordered to perform observation, Niu Hong explains the variations in their results in terms of omenology, and Sui Wendi expresses skepticism about the connection; (¶3-8) excerpt from Mao Shuang, et al., <i>Lü pu</i> : generation sequence, correspondence of pitch-standards with monthly <i>qi</i> , Jing Fang’s 60-standard expansion, explanation for the variation in response as independent of their harmonic correctness; (¶9) c. 617, chronicle: loss of the standards from which Mao Shuang, et al. were working into the Yangtze.
律直日 Pipe Day-Assignments: (¶1) chronicle: 5th cent., Qian Lezhi expands Jing Fang’s series to 360 pitch-standards; 6th cent., Shen Chong likewise produces a series of 360, which he correlates with the days of each of the 30-day solar months; (¶2) list of pitch-standards in Shen Chong’s series.
審度 Reliable Length Measures: (¶1) list of conflicting citations from Han and pre-Qin texts concerning the basis of the <i>chi</i> rule, of which later ages favored the <i>Han shu</i> ‘Lü-li zhi’ because of its proportional commensurability between units; problem: variation in millet seeds and human error have led to a historical variation in the length of the <i>chi</i> rule; (¶2-) list of 15 <i>chi</i> rules giving length in Xun Xu’s early Jin <i>chi</i> , the provenance of extant <i>realia</i> , citations concerning the excavation, commission, comparative testing, and/or promulgation of said <i>chi</i> rules, and the author’s assessment of previous findings.
嘉量 Good Capacity Measures: (¶1) list of conflicting citations concerning

<p>the units and linear dimensions of capacity standards, of which the <i>Han shu</i> ‘Lü-li zhi’ is correct; when tested against Zu Chongzhi’s value for π, however, Liu Xin’s standard proves somewhat imprecise; (¶2) chronicle: 263, excerpt from Liu Hui’s commentary to the <i>Jiuzhang suanshu</i> concerning discrepancies between 1st and 3rd cent. standards; (¶3) chronicle: 6th cent., the Liang & Chen courts adopt ancient standards; (¶4) chronicle: 6th cent., the Qi court re-defines the <i>dou</i>; (¶5) chronicle: the Zhou court, 560-567, excavation & millet-seed testing of ancient jade <i>dou</i> and promulgation of new standards based thereon; excerpt from the inscription on said standard; comparison in terms of cubic <i>chi</i>, weight, and seed capacity; (¶6) chronicle: Sui court redefines the <i>dou</i>, then restores the ancient standards in 605/617.</p>
<p>權衡 Weights and Balances: (¶1) definition of terms & units, basis in HUANGZHONG, unit conversion; the five models : weight & balances, compass, T-square, plumb, and level; chronicle: 336, the Zhao court excavates a Xin-era inscribed weight; 351/355, inscribed Xin-era weight submitted to the Northern Wei court, verified match with Xin-era scales; 471/499, commission of <i>dou chi</i> based on the <i>Han shu</i>; (¶2) chronicle: 6th cent., the Liang & Chen courts adopt ancient scales; (¶3) chronicle: 6th cent., the Qi court redefines the <i>jin</i>; (¶4) chronicle: 6th cent., the Zhou court redefines the <i>liang</i>; (¶5) chronicle: Sui court redefines the <i>jin</i>, the restores the ancient standards in 605/617.</p>
<p>律曆中 Harmono-metrology & Mathematical Astronomy juan B [MA]</p>
<p>[Historical introduction]: philosophical & historical précis from sage times to the 6th cent.; note on treatise composition.</p>
<p>[Chronicle, Liang]: the Liang inherited the <i>Yuanjia li</i> from the Qi; 504, Zu Geng responds to call to fix the state system, recounting the expertise of his family; 509, Zu Geng memorializes the throne concerning his new <i>li</i>, which is proven against the old one through court-ordered testing, Zu then explains the failures of the <i>Yuanjia li</i>; Zu Chongzhi’s <i>Jiazi</i>.₀₁-origin system numbers.</p>
<p>[Chronicle, Chen]: the Chen court inherited Zu Chongzhi’s system from the Liang.</p>
<p>[Chronicle, Later Qi]: 550, the Qi court commissions the <i>Tianbao li</i> from Song Jingye. [Li numbers]. 576, Dong Jun & Zheng Yuanwei author a memorial declaiming the <i>Tianbao li</i> as stodgy and inaccurate, offering up the numbers of their own solution. Attested experts Liu Xiaosun and Zhang Mengbin thereupon memorialize their own <i>li</i>. [Li numbers]. Liu & Zhang’s systems were able to successfully retrodict astronomical events, however, none of them were able to accurately predict the hour of the solar eclipse of 12 July 576. Debate was cut short by the fall of the court.</p>
<p>[Chronicle, Northern Zhou]: The Western Wei inherited the <i>Zhengguang li</i>. 557, the Zhou emperor’s call for a new system is met with Ming Kerang & Yu Jicai’s misinformed attempt at synthesis. 561/578, Zhen Luan’s <i>Tianhe li</i> is implemented. [Li numbers]. 579, Ma Xian memorializes his <i>Bingyin</i>.₀₃-origin system, recounting the political importance of astronomy and praising the marvelous and accurate nature of his system. [Li numbers]. Ma’s <i>li</i> is implemented.</p>
<p>[Chronicle, Sui]: ca. 581, Zhang Bin is commissioned to create a new <i>li</i> (in cooperation with more than 16 others) due to his role in legitimating the new court through prophecies and omenology. 584, Zhang’s team presents the throne with a modification of the <i>Yuanjia li</i>, which responds with a glowing review. [Li numbers]. After implementation, Liu Xiaosun and Liu Zhuo inveigh against Zhang’s system in six specific points as dilettante, inaccurate, and regressive, raise the now centuries-old history of models for lunar inequality to criticize their choice to use mean lunations for civil calendrics, and provide three data sets that verify the retrodictive accuracy of their own system. Zhang Bin succeeds in having his opponents dismissed from office. After his</p>

<p>death, however, Liu Xiaosun returns to office and protestations, eventually convincing the throne to test his system against Zhang Bin's. Disagreement arises between him and Grand Clerk Zhang Zhouxuan. 594, emperor orders a comparison of solar eclipse retrodictions, the result of which is that Zhang gets 25 out of 25, and Liu, more than half. Summoned to court for encouragement, Liu once again offended the throne and died soon after; Zhang, on the other hand, ingratiated himself with the throne and was commissioned to fix his method. 597, after further protest from Liu Zhuo, Zhang completes his work and memorializes it to the throne, which orders it tested. High officials then criticize it for failing to retrodict <i>Zuozhuan</i> eclipse records as well as a range of 5th cent. and contemporary lunisolar observations, frequently underperforming Zhang Bin's <i>li</i>. Confused, the emperor vacillates but eventually issues an edict accusing Zhang's detractors at the Clerk's Office of deceit, removing them from office, implementing Zhang's <i>li</i>, and granting the latter an honorary title.</p>
<p>[Daye li procedure text]: (1) elements; (2) mean lunisolar algorithms; (3) lunar inequality algorithms; (4) planetary algorithms; (5) eclipse algorithms.</p>
<p>律曆下 Harmono-metrology & Mathematical Astronomy <i>juan C</i> [MA]</p>
<p>[Chronicle, Sui, continued]: 600, upon receiving gnomonics memorial, the emperor hands astronomical affairs off to the heir apparent, who orders further gnomonics research and gathers experts in the Eastern Palace. Liu Zhuo submits his system in protest of Zhang Zhouxuan's, and the latter steps down due to illness. 604, Liu Zhuo presents the heir apparent with a list of six errors afflicting Zhang's system.</p>
<p>[Huangji li procedure text]</p>
<p>[Historical precis] (recap of events leading up to implementation)</p>
<p>[Procedure text]: (1) elements; (2) mean & true lunisolar algorithms; (3) eclipse algorithms.</p>