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Remarks on the Mathematics and Philosophy of Space-time in Early Imperial China

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Abstract This paper addresses two contradictory axioms of sinology: that (1) Chinese thought is unique in recognising the interconnectedness of space and time, and that (2) Chinese astronomy is a ‘calendar science’ dealing uniquely with the latter. These propositions cannot both be true, and a brief examination of the workings of *li* 曆 will reveal how neither is. In short, time is a function of the spatial displacement of celestial objects, and vice versa, their correlation being bound by natural velocities. In as much as ‘astronomy’ is the mathematical study of these velocities, the difference between astronomical cultures is not *space vs time* but the apparatus by which one transforms the one into the other. What little *is* unique about the Chinese case is the mathematical culture informing this operation: *li* rests upon sets of integer constants (*shu* 數), which Sivin (1969, 13) invites us to think of as cogs in a ‘gear-train’; *functionally*, however, the procedures (*shu* 術) reveal these to be *lǜ* 率, a mundane element of *suan* 算 presented in terms of grain-conversion. Examining how methods for converting between rice and beans were extended to time and space, we will consider commentators’ *Changes*-like reflections upon the metaphysics of *lǜ*, supplying actor’s categories where observer’s categories fall short of explaining how Chinese astronomy worked. In the conclusion, we will reflect upon the difference here between philosophical reflections and the contents of practice, asking why it is that the quantitative study of time and space is written out of the history of ‘Chinese thought’, ‘cosmology’ and *shushu* 數術.

Keywords *shushu* 數術, cosmology, *Yijing* 易經, history of astronomy, history of mathematics, Li Chunfeng 李淳風 (602–670), Liu Hui 劉徽 (fl. 263), Marcel Granet (1884–1940)

Contradictory axioms

There are things you learn early in your academic training that stick with you, and that you hear repeated, but that you are longer sure how you know. They are Truths, until otherwise proven, and even then they are difficult to unlearn. Two of the Truths that I picked up from somewhere, and have heard repeated since, are that, first, Chinese thought is somehow unique in recognising the interconnectedness of space and time, and that, second, Chinese astronomy is a ‘calendar science’ that deals exclusively with the latter. These Truths, it took me some time to realise, cannot both be *true*, if for the simple reason that the one, *a priori*, excludes the other. I forget how I came to know these things, and I worried that I had made them up, so I went back through the literature to familiarise myself with their

* I would like to thank Garret Olberding and Vincent Leung for organising this fabulously interdisciplinary workshop and for inviting me all the way from Paris to contribute what I can thereto. I would also like to thank Karine Chemla, whose work, heavily cited below, has helped me completely re-think my own subject over the last few years.

genesis. Both go back to at least the early twentieth century, it turns out, and both have been problematised since long before I learned to read, but both of them, for whatever reason, flutter still like forgotten prayer flags generating merit in the wind.¹

Remembering how time works

The problem, as any historian of astronomy would tell you, is that *space is time when space is turning at a constant rate against a static point of reference*, or, for short, *space is time when space is turning*.²

That – whether we are aware of it or not – is what is happening outside at this very moment: the sky is turning on its axis, east to west, carrying the sun from dawn to noon to dusk and back again in twenty-four hours just as reliably as the hour-hand of an analogue clock returns to twelve. The sun too is moving, albeit slower and in the opposite direction, charting its annual course through the ecliptic. In Cancer – where it was when I was born – it stands some twenty degrees north of the celestial equator, rising earlier, further north, and mounting higher in the sky over the course of a day; in Scorpio – where it was when Vincent was born – it’s dropped just as far below the equator, rising later, towards the south, and culminating right where you can see it in your window. *All of these* – twelve o’clock, dawn and dusk, Cancer and Scorpio, summer and winter – *all of these* are *positions* (*fang* 方) as much as they are *times* (*shi* 時). Space is time when space is turning.³

This... *fact* is at the core of ancient correlative matrices of *yin* and *yang*, the five agents, and so on, and so too is at the core of modern discussions of ‘Chinese thought’, but there is nothing particularly ‘Chinese’ about it – not as such.⁴ The Chi-

¹ I have collected a wide range of statements to this effect and later criticism thereof in the Appendices, below, for the reader’s consideration.

² Needless to say, this reductionist formula is offered not because it is *true* from the perspective of *modern knowledge* but because it is *sufficiently true* to think with in modern times (as do bees, below) and, more importantly, because it is reflective of the *pre-modern knowledge* that it is the historian’s duty to understand. In reality, we now know that it is not *space* that is turning on its axis but *the earth* (see Copernicus, Kepler, etc.), and that the earth’s diurnal and annual motion actually experience minute fluctuations and secular change over time (see for example Stephenson 1997). If there is one place where this formula really requires qualification it is that the motions of the ‘seven luminaries’ (*qiyao* 七曜) – the sun, moon and five naked-eye planets (Jupiter, Mars, Saturn, Venus and Mercury) – are not ‘constant’ *per se*, but that they experience mathematically replicable fluctuations that render them *chang* 常 (‘regular/constant’) in contemporary actors’ eyes. On the shifting line of *chang* as concerns the luminaries, see Morgan (2016b).

³ For an excellent introduction to the basics of astronomy aimed at elucidating how pre-modern peoples conceived of such matters, see Evans (1998) for Western Eurasia, Aveni (1980) for the Americas, and Cullen (1996) for Eastern Eurasia.

⁴ As my use of the term ‘Chinese thought’ intimates, this paper is largely aimed at Marcel Granet’s *La pensée chinoise* (1934) and its continued influence in our field some 83 years later. It is not my intention to deny that *La pensée chinoise* is beautifully written – it is indeed the inspiration for my rhetorical style in this paper – nor that its insights are not of continued value to this day; it is rather, in keeping with Needham (1956, esp. 288–9), Hen-

nese divided the ‘circumference of heaven’ (*zhoutian* 周天) into $365\frac{1}{4}$, one *du* 度 (‘spans’) for each *day* of the solar year.⁵ The Mesopotamians divided *theirs* into 360 UŠ for much the same reason, approximating an ‘ideal year’ to suit their sexagesimal numbers.⁶ And so it was in Greece, in Rome, in India and in the caliphates and empires in between that we, to this day, count 60 minutes to an hour, 60 seconds to a minute, and 360° to a circle – because it’s in 60s that the Mesopotamians counted and because space is time when space is turning.

We uses *hours*, of course, but so too did the Chinese, and what is an ‘hour’, anyways, except 15° of arc?⁷ When the sun stands due south, on the median, at its highest point in the sky, that is what we call noon; an hour later – ‘post meridian’ (PM) – it has moved 15 degrees towards the west as measured in the plane of the celestial equator. In modern times, we call these ‘hour angles’: an *angle* measured in *hours* that, if added to 12:00, gives you *the time of day*.⁸ In Greece, they called them *χρόνοι ἰσημερινοί* (‘equatorial times’) or simply *χρόνοι* (‘times’); there were twenty-four and they counted them in letters.⁹ In China, they called them *jiashi* 加時 (‘added times’) or simply *chen* 辰 (‘times’); there were twelve, or twenty-four, and they counted them in ‘earthly branches’ (fig. 1). Still today, *zhongwu* 中午 (‘due *wu*._{B07}’) is noon, and it is time to eat *wufan* 午飯 (lit. ‘*wu*._{B07} rice’, or ‘lunch’), but it is also the position of the sun, opposite where it is at midnight (*zi*._{B01}). Space is time when space is turning.

Sure, our ‘numbers’, coordinates, and usages are different, but *what we are counting is the same*, and *the way we do it is mathematically commensurable*.¹⁰ The same goes for bees. Bees navigate by a combination of methods, one of which, for dead reckoning in the absence of salient landmarks, scientists call a ‘sun compass’. In short, to get from a given location back

derson (1984), Graham (1986), Bodde (1991), etc., to insist that what it is he’s describing is neither *unique* to China, *exclusive* of other ‘thought’ therein, nor *timeless*.

⁵ More precisely, *zhoutian* is usually translated ‘circuits of heaven’ to reflect the concept’s origin in the number of ‘complete circuits’ (*zhou* 周) the sun/day must make from one coincidence of winter solstice._{Q22} and midnight to another (see Cullen 2017, 26 ff.). That said, because the number of *du* to a ‘complete circuit’ around the circumference of heaven is, by definition, equivalent to the number of days in a year, *zhoutian* can for our purposes here be read as ‘the circumference of heaven’.

⁶ See D. Brown (2000). On Mesopotamian sexagesimal place value notation, specifically, see Neugebauer (1969, 15–23) and Ifrah & Bellos (2000).

⁷ If this equation does not leap to the reader’s mind, rest assured that astronomers, historians thereof, and engineers regularly convert between the units without thinking: $1h = 15^\circ$, $4m = 1^\circ$, and that is that. Consider fig. 1: there we have a circle of, by definition, 360° that is divided into 24 hours; $360^\circ \div 24 \text{ hours} = 15^\circ/\text{hour}$, and $15^\circ \div 60 \text{ minutes} = 0.25^\circ/\text{minute}$.

⁸ On the hour angle, see Smart (1977, 25–32).

⁹ On *χρόνοι* ‘times’ as measures of arc, see Toomer (1998, 23). On Greek alphabetic numbers, see Neugebauer (1969, 9–13) and Ifrah & Bellos (2000, 182–262).

¹⁰ There are, as always, interesting exceptions to such absolutist statements. The *du* and UŠ, for example, are not actually measures of *angle*, like the degree, but linear measures of ‘arc’ taken around the circumference of a circle (rather than from its centre). For some of the more interesting ramifications of this in Chinese astronomy, see Huang Yi-long (1992).

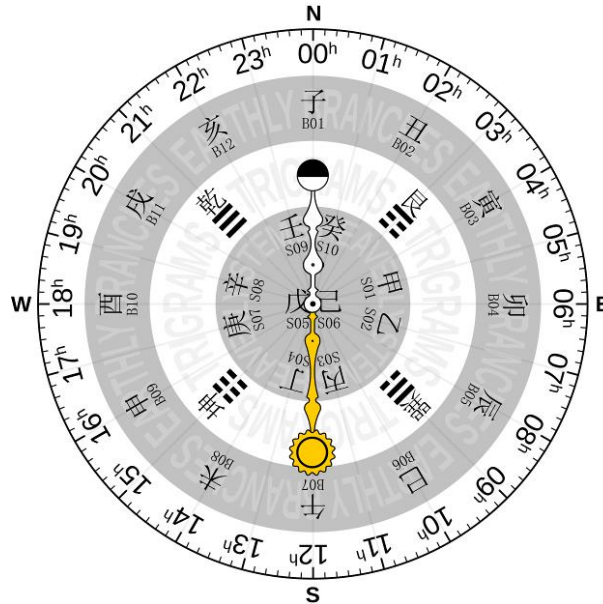


Fig. 1 Early medieval 24 *chen* or *jiashi* typical of an armillary equatorial ring, as per Qu Anjing (1994) and Morgan (2016a). The 24 *chen/jiashi* are an extension of the 12 ‘earthly-branch’ (*dizhi* 地支) ‘double-hours’, *zi*·B01, *chou*·B02, *yin*·B03, *mao*·B04... *hai*·B12, between which are inserted the eight non-medial ‘heavenly stems’ (*tiangan* 天干), *jia*·S01, *yi*·S02, *bing*·S03, *ding*·S04... *gui*·S10, and, in the ‘four corners’ (*siwei* 四維), the *Book of Changes* trigrams Qian ☰, Kun ☷, Gen ☶, and Xun ☴. In later geomancy, or *fengshui*, this configuration is known as the ‘twenty-four mountains’ (*ershisi shan* 二十四山). Note that I have flipped this diagram upside-down that the modern reader might feel more comfortable reading this with north/midnight at the top; the ‘clock hands’ represent the position of the sun and moon at noon (local apparent time), full moon day.

to the hive, bees allocate familiar places with a *direction* and a *distance* home, the direction being relative to the sun. Distance, of course, they measure in time, and the same goes for direction, because while bees do not have rulers, they are equipped with clocks.¹¹ The sky is always turning, and the same circadian clock by which they measure a distance flown they use to adjust the remembered vector in compensation for the sun’s displacement. Because the sun is south at noon, because between south and north is east, and because bees are not afraid of a little astronomy, bees can always find their way right back home. They can, that is, unless you anaesthetise them for several hours so as to disable those clocks while the heavens keep on turning. Then, as studies like Cheeseman et al. (2014) show, funny things begin to happen: they wake up and fly in the wrong direction, roughly as many degrees in error as the amount of time they were unconscious (fig. 2). Space is time when space is turning, and ‘Chinese thought’ is hardly unique for recognising this as the foundation of our cosmos and the key to finding our way therein.

This is all no doubt somewhat mysterious to someone who’s never really studied these sorts of things, but it is, let me tell you, nowhere near as mysterious as the idea that Chinese astronomy could somehow function independently of space. Say-

¹¹ On insect circadian ‘clocks’, see Tomioka & Matsumoto (2010).

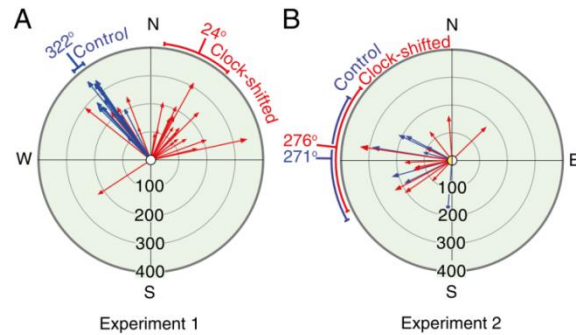


Fig. 2 Initial vector flight components of captured and released bees from Cheeseman et al. (2014, 8951, fig. 3); clock-shift = 6 hours, clock-shifted bees in red and control bees in blue. (A) Experiment 1: bees released in an open field; clock-shifted mean vector angle 24°, (95% CI 7°, 41°); control mean vector angle 322°, (95% CI 319°, 325°). (B) Experiment 2: bees released next to a familiar hedge; clock-shifted mean angle 276°, (95% CI 244°, 308°); control mean vector angle 271°, (95% CI 240°, 301°). Scale in metres.

ing this, as we were taught to do, is the equivalent of saying that this people could only (or, indeed, *could*) build one-dimensional clocks – an unmoving needle without a face. This might make for an excellent riddle or meditational aide, like the sound of one hand clapping, but bees know better, and so too should we. There is a reason behind this riddle, of course, but it is not one that has anything to do with physics.

Li 曆: not your father's calendar

The study of these things, which I just described, was called *li* 曆, and it was called that starting from at least the second century BCE.¹² It was called that when Buddhists brought *jyotiṣa* into the mix, the Nestorians *iṣṭrunumiia*, the Muslims *ilm al-nujuum*, and the Catholics, finally, *astronomia* – each time, some Chinese expert took a look at this foreign thing and said, ‘I know that: that is *li*’.¹³ We have been less kind: *li* is the one among these that does not go back to the Greeks, and in as much as Greekness was a necessary condition for ‘science’, ‘theory’, ‘proof’ and ‘abstraction’ in nineteenth-century Europe, it was there decided that *li* is nothing other (i.e. more) than the

¹² For a more nuanced explanation of the meaning of *li* 曆 than that offered here, see Martzloff (2009, esp. 367–72), Sivin (2009, 38–40), Sivin (2011), Cullen (2017, esp. 7–25) and Morgan (2017, chap. 1).

¹³ On the Buddhist transmission of Meso-Greco-Indian *jyotiṣa* into East Asia under the Tang 唐 (618–907 CE), see Yabuuti (1979), Niu Weixing (2004) and Mak (2015). On the Nestorian transmission of Meso-Greco-Syriac *iṣṭrunumiia*, also under the Tang, see Lai Swee Fo (2003) and Mak (2016). On the Muslim transmission of Meso-Greco-(and sometimes Perso- / Indo-)Arabic *ilm al-nujuum* into China, under the Yuan 元 (1271–1368), see Yabuuti (1997), van Dalen (2002) and Li Liang (2016). On the Jesuit transmission of Meso-Greco-Arab-Latin *astronomia*, in the sixteenth century, see Jami (2012). I thank Isahaya Yōichi 諫早庸一 for helping me sort out the correct actors' categories to use here for ‘astronomy’ in a Syriac and Arabic context.

‘practical’, ‘concrete’ art of ‘calendar-making’.¹⁴ This was decided, ironically, on a continent where the transmission of Arabic-language astronomy had, some centuries earlier, undermined and marginalised the indigenous tradition of *computus* on the same charge.¹⁵ Regardless, *this* and the momentum of convention is why we learn that *li* is ‘calendars’ and that it is independent of space.

Li, however, are *not* calendars, not in any normal sense of the word, and anyone who doubts this is free to read one or to look for a single quote in Chinese history to the effect of ‘Lunch tomorrow? Let me check my *li*’.¹⁶ *That thing* – the table of dates, and months, with holidays and the sort – *that thing* is called many things, and none of them is *li*: it is called *ri* 日 (‘day/s’), *zhiri* 質日 (‘duty day/s?’), and, yes, even *liri* 曆日 (‘*li* day/s’; see fig. 3), but a *liri* is no more a *li* than a *matong* 馬桶 is a *ma* 馬.¹⁷ To a Jean-Baptiste Biot (1862, 268), however, a *matong* is precisely where *li* belonged:

On s’est plu souvent à remarquer que les Chinois, dans leurs idées, dans leurs usages, leurs préjugés même, offrent un singulier et perpétuel contraste avec les peuples européens [...] Leur astronomie ne fait pas exception à cette règle. Elle n’a jamais constitué chez eux, une **science spéculative**, apanage spécial d’un petit nombre d’esprits. Dans tous les siècles, elle a été une œuvre de gouvernement. Son principal office consiste à préparer chaque année, plusieurs mois à l’avance, le calendrier impérial. [...] Elle est chargée, en outre, de l’avertir personnellement des phénomènes extraordinaires qui arrivent dans le ciel, pour en tirer les présages, favorables ou défavo-

¹⁴ Ironically, the Mesopotamians, from which the Greeks learned much of their astronomy, had similar charges brought against them in the last two centuries. For a critique of the discourse *Occident : theoretical : science :: Orient : practical : non-science* from an Assyriological perspective equally relevant to China, see Rochberg (2004, esp. 14–43).

¹⁵ On the tension and transition between Latin *computus* and Arabic astronomy in mediaeval Europe, see Haskins (1924, chap. 5) and Moreton (1995; 1999). I thank Philipp Nothaft for directing me to these sources and for disabusing me of several ideas on the subject in a personal communication of 6 March 2017.

¹⁶ If the reader is curious to learn from first-hand experience about the nature of primary sources that self-identify with *li*, I suggest starting with the expertly annotated translations of the *Santong li* 三統曆 (c.5 CE), *Sifen li* 四分曆 (85/86 CE) and *Qianxiang li* 乾象曆 (c.206 CE) in Cullen (2017) or that of the *Shoushi li* 授時曆 (1280 CE) in Sivin (2009).

¹⁷ The *Oxford English Dictionary* offers the following definition of ‘calendar’: ‘A table showing the division of a given year into its months and days, and referring the days of each month to the days of the week; often also including important astronomical data, and indicating ecclesiastical or other festivals, and other events belonging to individual days. Sometimes containing only facts and dates belonging to a particular profession or pursuit, as Gardener’s Calendar, Racing Calendar, etc. Also a series of tables, giving these facts more fully; an almanac’ (definition 2.a, accessed 16 Jan 2017, <http://www.oed.com>). When labelled, documents matching this description recovered from early imperial archaeological sites identify themselves as *ri*, *zhiri*, *liri*, etc., but not *li* (not, at least, as of 2017). For an overview of ‘calendars’, properly speaking, see Arrault (2002), Yoshimura (2003), and Morgan (2017, chap. 3); for the ‘calendars’/‘almanacs’ from the later Dunhuang cave library, see Deng Wenkuan (1996). For a detailed look at how one produces a *liri* from a *li*, see Martzloff (2009) and Morgan (2017, chap. 3).

rables, qui concernent son gouvernement. Aussi, mus par ces deux intérêts **purement pratiques**, a-t-on vu, de tout temps ...¹⁸

What is at stake in translating *li* any differently is admitting that ‘Chinese thought’ was capable of *science* or *spéculation* on its own, and that, in 1862, was a little much for ‘Western thought’ to handle.¹⁹ *All of this*, however, is a problem of observer’s categories, and observer’s categories are beside the point.

Li, befitting the early bibliographic classification that it heads, is comprised of ‘numbers and procedures’ (*shushu* 數術).²⁰

¹⁸ Note that in the opening pages of his ‘Précis de l’histoire de l’astronomie chinoise’, the only source that Biot (1862, 263–74) cites as concerns this history is the *Rites of Zhou* and his translation of eleven years earlier: *Le Tcheou-li, ou Rites des Tcheou* (1851). Coincidentally, Biot’s description of how astronomy in China was by ‘singulier et perpétuel contraste’ to Europe, ‘dans tous les siècles’, ‘de tout temps’, etc., comes nearly word-for-word from the *Rites of Zhou*: ‘Grand annaliste (*Ta-ssé*): [...] il rectifie l’année moyenne et l’année lunaire, pour régler l’ordre des divers travaux ; et il distribue le *calendrier régulier* aux officiers de la cour, ainsi qu’aux chefs d’apanages et domaines affectés’ 大史[...]正歲年以序事, 頒之于官府及都鄙 (*Zhouli zhushu* 26.401a–b; tr. Biot 1851, vol. 2, 104–5); ‘Officier chargé de préserver et d’éclaircir (*pao-tchang-chi*): [...] Il divise les territoires des neuf régions de l’empire, en terres *dépendantes* d’astérismes spéciaux. Les limites de toutes les principautés d’investiture, ont des astérismes distincts. pour reconnaître les pronostics extraordinaires qui les concernent’ 保章氏[...]以星土辨九州之地, 所封封域皆有分星, 以觀妖祥 (*Zhouli zhushu*, 26.405b–406a; tr. Biot 1851, vol. 2, 113–4), etc. The history of astronomy in China, however, does not stop at the *Rites of Zhou*, and for a study of how things changed and how actors spoke about that change over the early imperial period, see Morgan (2017).

¹⁹ Note that the formula *Europe : astronomy : theoretical :: China : calendars : practical* was not there from the beginning. In *Observations mathématiques, astronomiques, géographiques...* (1729–32), for example, the Jesuit missionary Antoine Gaubil (1689–1759) habitually translates the word *li* in historical sources as *l’Astronomie* or, in the case of works like the *Kaihuang li* 開皇曆 (584 CE), *une Astronomie*. The equation *li* = ‘calendar(s)’ is something, rather, that sinologists actively invented, cultivated and codified c.1732–1968 (see Appendix).

²⁰ Namely, the bibliographic heading ‘Shushu’ 數術 in the *Han shu* 漢書 ‘Yiwen zhi’ 藝文志 (c.92 CE), based on Prefect Grand Clerk Yin Xian’s 尹咸 ‘numbers & procedures in six types’ 數術六種 in Liu Xiang 劉向 (79–8 BCE) and Liu Xin’s 劉歆 (c.50 BCE–23 CE) now lost *Qi lie* 七略 of 26/1 BCE. Modelled on the hierarchy of functions at the Han 漢 (206 BCE–220 CE) Clerk’s Office, the Grand Clerk’s ‘six types’ are, in order, (1) *tianwen* 天文, ‘heavenly patterns’; (2) *li pu* 曆譜, ‘li & chronology’; (3) *wuxing* 五行, ‘five agents’ divination; (4) *shi gui* 蓍龜, ‘milfoil & tortoise’ divination; (5) *za zhan* 雜占, ‘misc. omen-reading’; and (6) *xingfa* 形法, ‘morphoscopy’. The astral and mathematical sciences (*tianwen*, *li* and *suan* 算) head this category through subsequent instantiations from 26/1 BCE to Wang Jian’s 王儉 (452–489 CE) *Qizhi* 七志 of 473 CE, which swaps ‘Numbers & Procedures’ for ‘Yin and Yang’ 陰陽, and the *Sui shu* 隋書 ‘Jingji zhi’ 經籍志 of 656 CE, which divorces *tianwen* and *li suan* 曆算 from *wuxing* and places the three under ‘Masters’ 子. From this point on, in terms of bibliography, *tianwen*, *li* and *suan* are no longer ‘Shushu’, which re-emerges by the time of the *Siku quanshu* 四庫全書 (1782) as an exclusive repository of *yin-yang*, five-agents numerology and personal fate divination. On the evolution of the bibliographic category, see Drège (1991, esp. 87–142) and Kalinowski (2003, 11–17; 2004b, esp. 224–8). As to why this c.26 BCE–472 CE bibliographic category is called *shushu*, scholars such as Li Ling (2008, 403–4) and Chao Fulin (2010, 44) consider mathematics to be the origin and model of the various forms of divination in categories 3–5 (particularly hemerology and *shi* 式-board divination), and the prevalence of actual, self-identified ‘numbers’ (*shu*) and

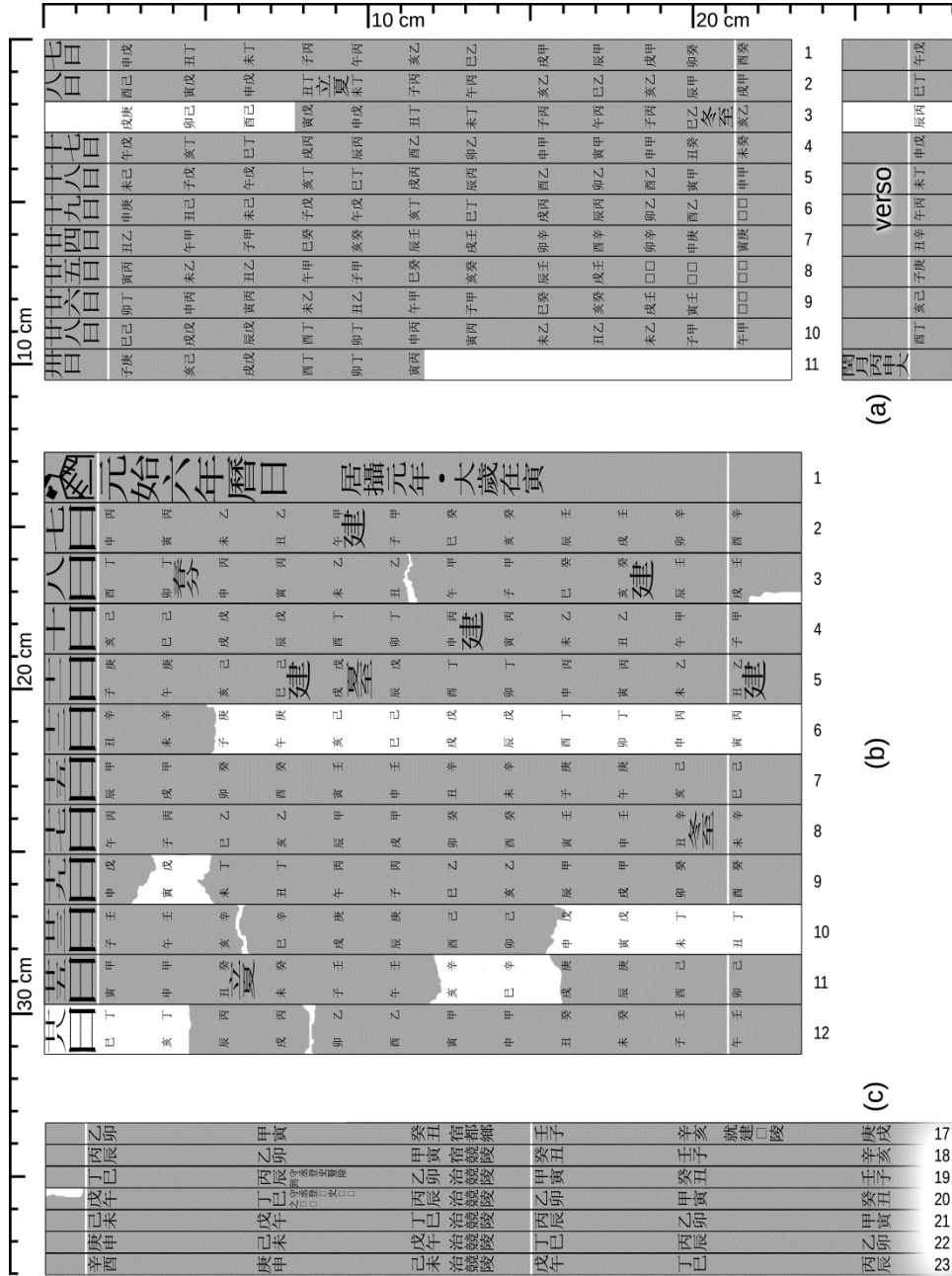


Fig. 3 Wood bookmat *liri* 曆日 -style day calendars. Right (a): Dunhuang 敦煌 T6b Shenjue year 3 (59 BCE) *liri*. Centre (b): Jinguan 金關 T23 ‘Yuanshi liu nian *liri*’ 元始六年曆日 (5 CE). Left (c): slips from the Zhoujiaitai 周家臺 M30 Shihuang year 34 (213 BCE) *zhiri* 質日 for comparison. Note that the two *liri* are incomplete. Source: Morgan (2017, chap. 3, fig. 3.3).

A *li*, like the Han *Sifen li* 四分曆 (85/86 CE), more specifically, is a chain of numbers, tables and algorithms; it reads fortuitously like computer code,²¹ and *translating it into computer code* is, fortuitously, one of the best ways to learn it.²² This no doubt sounds very abstruse – a world away from the pre-modern Chinese with which the reader is familiar – so allow me to show you what this looks like.

The *Sifen li*, authored by *li* workers (*zhili* 治曆) Bian Xin 編訢 and Li Fan 李梵 and preserved in *Hou Han shu* 後漢書, *zhi* 3, opens with the following ‘numbers’ (for the moment, don’t worry yourself with ‘why?’, just pretend that you’re watching a command-line process run):

當漢高皇帝受命四十有五歲[...]冬十有一月甲子夜半朔旦冬至，日月閏積之數皆自此始[...]

In the 45th year after Han Emperor Gao[zu] 高祖 (r. 206–196 BCE) received the mandate [...], in winter, month XI, day *jiazi*.₀₁ 甲子, at midnight, [at the coincidence of] new moon and winter solstice.^{Q22} (00:00, 25 December 162 BCE) – all the numbers (*shu*) of solar and lunar intercalation and accumulation start from this point. ...

又上兩元，而月食五星之元，並發端焉。

Two origins (2 × 4,560 years) further up (00:00, 25 December 9282 BCE), and that is the origin for lunar eclipses and the five stars (planets), which all start from this point. ...

元法	Origin divisor	4,560
紀法	Era divisor	1,520
紀月	Era months	18,800
蔀法	Obscuration divisor	76
蔀月	Obscuration months	940
章法	Rule divisor	19
章月	Rule months	235
周天	Circuits of heaven	1,461
日法	Day divisor	4
蔀日	Obscuration days	27,759
...		
日餘	Day remainder	168
中法	Medial [<i>qi</i>] divisor	32
大周	Big circuits	343,335
...		

Now, the only datum the user need enter for things to begin is ‘the [number of years from] high origin to the year sought’, which, for 2017, would be 2017 – –9,281 = 11,298 years.²⁴ Let’s plug this into procedure 2:

‘procedures’ (*shu*) in the literatures of *li* and *suan* would seem to corroborate this idea.

²¹ On thinking about the algorithms comprising *suan* 算 ‘mathematics’ procedure texts through the lens of computer language, see Chemla (1987).

²² This, as recommended by Cullen (2005), is at least how the author taught himself the subject over the summer of 2011.

²³ *Hou Han shu*, *zhi* 3, 3057–60. Note that here and what follows are my own translations and explanations, but that for more serious purposes one is advised to refer to those in Cullen (2017, chap. 3), whose procedure numbers I have adopted, or Liu Hongtao (2003, chap. 3).

²⁴ There is no 0 BCE/CE, so 9282 BCE is, for the sake of calculation, –9281.

Enter years from high origin: **11,298**.

推入藝術曰：

[2.] Procedure for calculating [the sexagenary year number of] the obscuration entered:

以元法除去上元，

Eliminate (*modulo*) the origin divisor (4,560) from the distance from high origin.²⁵

$$11,298 \div 4,560 \rightarrow 2; \text{ rem. } 2,178$$

其餘以紀法除之，所得數從天紀，筭外則所入紀也。

With the remainder, one eliminates (divides) it by the era divisor (1,520), and one counts off exclusively by [the integer quotient] obtained from ‘heaven origin’ for the era entered.²⁶

$$2,178 \div 1,520 \rightarrow 1; \text{ rem. } 658, \\ \text{i.e., ‘filled’ 1 era, 658 years into era no. 2:} \\ 1. \text{ ‘heaven’}, 2. \text{ ‘earth’}, 3. \text{ ‘man’}.$$

不滿紀法者，入紀年數也。

Any [remainder] that does not fill an era divisor (1,520) is the number of years entered into the [current] era.

Retrieve: **658 years** into the ‘earth’ era.

以蔀法除之，所得數從甲子蔀起，筭外，所入【蔀】歲名命之，[...]

Eliminate (divide) this by the obscuration divisor (76), and one counts off exclusively by [the integer quotient] obtained starting from the obscuration *jiazi*.₀₁. [...]²⁷

$$658 \text{ years} \div 76 \rightarrow 8; \text{ rem. } 50, \\ \text{i.e., ‘filled’ 8 obs., 50 years into obs. 9:} \\ 1. \text{ } jiazi_{.01}, 2. \text{ } guimao_{.40}, \dots \rightarrow 9. \text{ } bingzi_{.13}.$$

We now know that the last ‘obscuration head’ (*bushou* 蔀首), 50 years prior, started with a coincidence of new moon, month XI and winter solstice._{Q22} at midnight, day *bingzi*.₁₃ 丙子 (6 January 1967).²⁸ Now, to procedures 5 and 6 for 2017:²⁹

²⁵ In the *Sifen li*, a *yuan* 元 is the coincidence of: (a) new moon, month XI; (b) winter solstice._{Q22}; (c) midnight; (d) day *jiazi*.₀₁; (e) year *gengchen*.₁₇, with 1 ‘origin’ = 4,560 years = 56,400 months = 1,665,540 days. *Modulo*, or *mod*, means dividing for the remainder.

²⁶ In the *Sifen li*, a *ji* 紀 is the coincidence of: (a) new moon, month XI; (b) winter solstice._{Q22}; (c) midnight; (d) day *jiazi*.₀₁; (e) year *gengchen*.₁₇, with 1 ‘era’ = 1,520 years = 18,800 months = 555,180 days. Each ‘era’, the sexagenary year of this coincidence shifts forward 20 places, from ‘the era of heaven’, starting year *gengchen*.₁₇ 庚辰, to ‘the era of earth’, starting year *gengzi*.₃₇ 庚子, to ‘the era of man’, starting year *gengshen*.₅₇ 庚申, and back to the ‘era of heaven’. *Hou Han shu*, *zhi* 3, 3061–2, provides a look-up table omitted here (cf. Cullen 2017, 162–3).

²⁷ In the *Sifen li*, a *bu* 蔀 is the coincidence of: (a) new moon, month XI; (b) winter solstice._{Q22}; (c) midnight; (d) day *jiazi*.₀₁; (e) year *gengchen*.₁₇, with 1 ‘obscuration’ = 76 years = 940 months = 27,759 days. Each ‘obscuration’, the sexagenary day of this coincidence shifts forward 39 places from *jiazi*.₀₁. *Hou Han shu*, *zhi* 3, 3061–2, provides a look-up table omitted here (cf. Cullen 2017, 162–3).

²⁸ Note that the true winter solstice fell on 22 December 1966, and true syzygy on 12 December 1966. The discrepancy is due in part to the omission of 10 days from the Occidental calendar following the Gregorian reform of 1582 and, more importantly, to accumulated errors to be expected when using the *Sifen li* centuries beyond its intended lifespan. In 85/86 CE,

推天正術：

[5.] Procedure for calculating astronomical month I:

置入蔀年減一，

Set out the years entered into the [current] obscuration and diminish (subtract) by 1.³⁰

Retrieve: **50 years** into obs. no. 9, *bingzi*.¹³,
 $50 - 1 \rightarrow$ **49 ac. years.**

以章月乘之，滿章法得一，名為積月，不滿為閏餘，
Multiply this by the rule months (235), and get 1 [for each time it] fills the rule divisor (19) – this is called the ‘months accumulated [into current obscuration]’, and that which does not fill [the divisor] is the ‘intercalary remainder’. [...]³¹

$49 \text{ ac. years} \times 235/19 \rightarrow$ **606 ac. months**; rem. 1.

推天正朔日，

[6. Procedure for] calculating new moon day, ast. month I:

置入蔀積月，

Set out the [integer] months accumulated into [current] obscuration.

Retrieve: **606 ac. months**; rem. 1.

以蔀日乘之，滿蔀月得一，名為積日，不滿為小餘，
Multiply this by the obscuration days (27,759), and get 1 [for each time it] fills the obscuration months (940) – this is called the ‘days accumulated [into current obscuration]’, and that which does not fill [the divisor] is the ‘little remainder’.³²

$606 \text{ ac. months} \times 27,759/940 \rightarrow$ **17,895 ac. days**; rem. 654.

積日以六十除去之，其餘為大餘，以所入蔀名命之，筭盡之外，則前年天正十一月朔日也。...

With the ‘accumulated days’, remove [all] 60[s] by elimination (*modulo*) from it – the remainder is the ‘big remainder’. Name this according to the name (sexagenary day) [heading] the obscuration entered, counting exclusively, and this is the new moon day of astronomical month I – [civil] month XI – of the previous year. ...³³

$17,895 \text{ ac. days} \div 60 \rightarrow 298$; **rem. 15**,
i.e., 15 days past sex. date at obs. head.

the best of *li* were not expected to function beyond 300 years (Sivin 1986; cf. Morgan 2017, chap. 5).

²⁹ Procedure 3 has to do with eclipses and can be skipped for our purposes here.

³⁰ ‘Astronomical month I’ = civil month XI = the month containing winter solstice._{Q22}, so for 2017 we need to count from winter solstice._{Q22} of the year prior (2016), thus the need to subtract 1.

³¹ In the *Sifen li*, a **zhang** 章 is the coincidence of: (a) new moon, month XI; (b) winter solstice._{Q22}; (c) ~~midnight~~; (d) ~~day jiazi~~₋₀₁; (e) ~~year gengchen~~₋₁₇, with 1 ‘rule’ = 19 years = 235 months = 6,939³/₄ days. Each ‘rule’, the hour of this coincidence shifts forward ³/₄ day. To convert from years to months, one multiplies by 235/19 (i.e. 12 ⁷/₁₉ months/year).

³² An ‘obscuration’ = 76 years = 940 months = 27,759 days (Note 27), so to convert from months to days, one multiplies by 27,759/940 (i.e. 29 ⁴⁹⁹/₉₄₀ or ≈ 29.53085 days per month).

³³ *Hou Han shu*, *zhi* 3, 3062.

Retrieve sex. date at obs. head: *bingzi.13.*

‘Name’ 15 places from *bingzi.13.*, exclusively:
1. *bingzi.13.*, 2. *dingchou.14.*, ... → 16. *xinmao.28.*

For 2017, new moon, month XI, thus falls (654/940 day past midnight) on day *xinmao.28* 辛卯 (4 January 2017).³⁴ On to procedure 8 for winter solstice.Q22:

推二十四氣術曰：

[8.] Procedure for calculating the twenty-four *qi*:

置入蔀年減一，

Set out the years entered into the [current] obscuration and diminish (subtract) by 1.³⁵

Retrieve: 50 years into obs. 9,
50 - 1 → 49 ac. years.

以日餘乘之，滿中法得一，名曰大餘，不滿為小餘，大餘滿六十除去之，其餘以蔀名命之，筭盡之外，則前年冬至之日也。...

Multiply this by the day remainder (168), and get 1 [for each time it] fills the medial divisor (32) – this is called the ‘big remainder’, and that which does not fill [the divisor] is the ‘little remainder’. With the [big] remainder, name it according to the name (sexagenary day) [heading] the obscuration entered, counting exclusively, and this is the winter solstice.Q22 day of the previous year. ...³⁶

49 ac. years × 168/32 → 257 ac. days, rem. 8,
257 ÷ 60 → 4; rem. 17.

Retrieve sex date at obs. head: *bingzi.13.*

‘Name’ 17 places from *bingzi.13.*, exclusively:
1. *bingzi.13.*, 2. *dingchou.14.*, ... → 18. *guisi.30.*

For 2017, winter solstice.Q22 thus falls (8/32 day past midnight) on day *guisi.30* 癸巳 (6 January 2017), two days after new moon.³⁷ For subsequent new moons and *qi*, the elided text of procedures 5 and 6 instructs the user to simply add 29 499/940 and 15 7/32 days, respectively, up to the end of said year. And at that, we have produced the luni-solar framework of a calendar like that in Appendix 1.³⁸

How did this happen? Nathan Sivin (1969) long ago invited us to think of the numbers on p. 9 as intricately connected gears.

³⁴ Note that true syzygy prior to winter solstice, 2016, actually fell on 29 November; see the caveat in Note 28.

³⁵ See Note 30.

³⁶ *Hou Han shu*, *zhi* 3, 3063. In the *Sifen li*, 168/32 (= 5¼) is the number of days by which one solar year (= 365¼ days) exceeds six sexagenary cycles (= 360 days). As such, multiplying this by the years accumulated, from obscuration head gives us the days in the sexagenary cycle advanced therefrom.

³⁷ Note that true winter solstice, 2016, actually fell on 21 December; see the caveat in Note 28.

³⁸ In the above presentation, I have elided matters of intercalation, otherwise treated in procedure 5 (‘Procedure for calculating astronomical month I’) and procedure 9 (‘Calculating the position of the intercalary month’推閏月所在), because the *Sifen li* does not give 2017 an intercalary month.

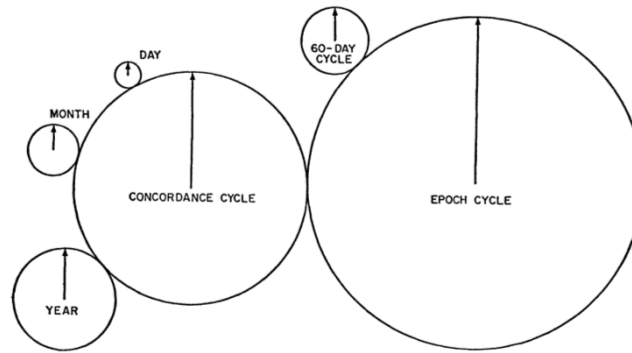


Fig. 4 Sivin's 'gear train', from Sivin (1969, 13, fig. 1). Original caption: System of calendrical constants in the Triple Concordance treatise [*Santong li*]. In a scale model, circumference would be proportional to length of cycle. The rotating arrows all point upward at the same time only once every 4617 years.

Describing the (different) values used by the *Santong li* 三統曆 (c.5 CE), he tells us:

We can look at this set of constants as a complex of circles turning upon each other (Figure [4]). The Epoch Cycle [元] simply specifies what motion of the integral system is needed to return all cycles simultaneously to their original orientations. In such a system, if we know the original orientation and the number of revolutions any one circle has passed through at a given moment, we can predict the orientation of any other circle (Sivin 1969, 13).

... an astronomical system [i.e. *li*] was meant to be like the gear train of a well-functioning machine, requiring no human intervention (ibid., 58).³⁹

It's an elegant metaphor, but it's one – contrary to Sivin's intentions – that subsequent scholars cite as evidence that 'the history of astronomy in ancient China was largely a history of calendar making' (Sun Xiaochun 2015, 2059).⁴⁰ Think about it: what other kind of 'gear train' could this be other than that you find in a clock? And is not a *clock*, like a 'calendar', (just) for telling time?

One could bring in *astronomical clocks* like the Antikythera mechanism to buttress Sivin's metaphor, but instead I would like to propose another – to *return*, if I may, to that with which contemporary actors conceived of what they were doing. Nothing's wrong with a little anachronism, of course, if it helps us grasp a difficult subject, but there are three points where the 'gear train' metaphor falls short.

³⁹ Note that the 'gear train' metaphor goes back to Needham (1959, 390–408), who describes the various cycles surrounding the civil calendar and the *Santong li*, specifically, as 'cogwheels'. Nevertheless, I cite Sivin (1969) here, because his illustrations and elaboration of this metaphor have left a stronger impact on the field.

⁴⁰ This statement goes back to Nakayama (1965, 125): 'The history of Chinese astronomy is, for the most part, the history of calendar-calculation'. On how 'the basic method of calendrical astronomy can be likened to the construction of a cogwheel system', see Note 39, Sun Xiaochun (2015, 2062) and the similar description in Kalinowski (2004a, 88–9).

First, *li* are *not* written in computer code, nor is their operation nearly as ‘intervention’-less as either metaphor implies. *People* did these calculations, and they did them *by hand*. In the Eastern Han 東漢 (25–220 CE) Clerk’s Office (*shiguan* 史官), the ‘Astronomical Bureau’, for example, this was the job of six *li* workers (*zhili*) like Bian Xin and Li Fan, who were ‘expectant appointees’ (*daizhao* 待詔) selected for their skills.⁴¹ The Northern Dynasties 北朝 (386–581 CE) and Sui 隋 (518–618 CE) furthermore saw the same office appoint ‘erudites’ (*boshi* 博士) and students (*sheng* 生) to begin training their own talent.⁴² Why? Probably because *li*-calculation is anything but automatic. Converting between cardinal and ordinal numbers can get confusing, but less so than a word like *chu* 除 ‘eliminate’, which can refer to one of four mathematical operations (subtraction, division, *modulo* or sequence subtraction); get confused, *just once*, and your mistake propagates down *all subsequent procedures*, leaving you a garbled mess for all your efforts.⁴³ Yes, *li* workers are ‘computers’, but in an older sense of the word, and a *li* is a lot less like a machine or a computer program than it is a liturgy – a liturgy to be performed.

Second, ‘origins’, ‘eras’, ‘obscurations’ and ‘rules’ are not just matters of *time*, like you might expect of clockwork, they are matters of *space-time*. The gear-train metaphor gets a little strained as we move, for example, to procedure 13:

推合朔所在度，

[13. Procedure for] calculating the *du*-position of syzygy:

置入蔀積月，

Set out [integer] months accumulated into [current] obscuration.

Retrieve: **606 ac. months**; rem. 1.

以蔀日乘之，滿大周除去之，其餘滿蔀月得一，名為積度，不盡為餘分。

Multiply this by obscuration days (27,759), remove full big circuits (343,335) by elimination (*modulo*) from it, and, with the remainder, get 1 [for each time it] fills the obscuration months (940) – this is called the ‘*du* accumulated [into current obscuration]’, and that which is not exhausted is the ‘remainder parts’.

606 ac. months × 27,759 → 16,821,954,
16,821,954 ÷ 343,335 → 48; rem. 341,874,
341,874 ÷ 940 → **363; rem. 654.**

積度加斗二十一度，加二百三十五分，

With the *du* accumulated, add Dipper._{L08} 21 *du* and add 235 parts.⁴⁴

363;654 + 21;235 → **384;889**,
or ‘Dipper._{L08} 383 *du* and 889[940] parts.’

⁴¹ On ‘expectant appointees’ in *li*, see Lai Swee Fo (2003), Cullen (2007, 246) and Morgan (2017, chap. 1).

⁴² On the composition and history of the Clerk’s Office, see Deane (1989) and Chen & Zhang (2008).

⁴³ See Morgan (2017, chap. 3). This insight derives from collaborative work with Karine Chemla arising from Chemla (2013; 2014).

⁴⁴ In the *Sifen li*, **Dipper._{L08} 21 235/940 *du*** (=Dipper._{L08} 21¼ *du*) is the right ascension – i.e. *equatorial position* – of winter solstice._{Q22}.

ratus around them.⁴⁶ *Lü* is not a word that translates, because it is not a mathematical construct that *exists* in other traditions, but it is, rest assured, one that’s easy enough to grasp.

You know *lǚ*; you have seen them and you have used them



Fig. 6 *Huilü* 匯率 ‘exchange rates’ as per the application *Huilü* 匯率 available on Android™ devices [here](#).

in their modern form. You have seen them, namely, on reader boards and mobile applications like that in fig. 6: *huilü* 匯率 ‘exchange rates’, a list of numbers, *with units*, equating one unit of something with a *decimal value* of something else. You have used them, in this form, to go from one currency to another in your head – to calculate, as per fig. 6, that if a dollar gets you 30.48 TWD, 100 gets you 304.85. Such a table is useful, when changing money, but there are limits to what such a table can do. You can’t (easily) go *backwards*, for example, or from euros to yen, but there’s no one table by which you can convert *any two things* as simply as that... right? Wrong; and welcome to the world of *lǚ*.

Lǚ, in their *pre-modern form*, allowed exactly that, and to that end let us turn to the ‘classic’ (*jing* 經) *Jiuzhang suan-shu* 九章算術 (c. first century CE), which devotes one of its ‘nine chapters’ to the topic.⁴⁷ Like the *Sifen li*, the chapter ‘Su mi’ 粟米 (Unhulled and Hulled) begins with a table of unit-less numbers:⁴⁸

粟率 [Unhulled] millet <i>lǚ</i>	50
糲米 hulled grain	30
粳米 milled grain	27
粳米 milled grain, fine	24
御米 milled grain, superior	21
小糲 small oats	13½
大糲 big oats	54
糲飯 hulled grain (cooked)	75
粳飯 milled grain (cooked)	54
粳飯 milled grain, fine (cooked)	48
御飯 milled grain, superior (cooked)	42
菽苔麻麥 soy, adzuki, hemp or wheat	45
稻 Paddy rice.....	60
豉 fermented soy	63
飧 diluted rice (cooked).....	90
熟菽 soy (cooked).....	130½
蘖 fermented grain	175

⁴⁶ The *lǚ-li* connection developed here draws heavily from Chemla (2010; forthcoming) and Goodman & Morgan (forthcoming).

⁴⁷ For a complete critical edition, translation, and study of the *Jiuzhang suan-shu*, see Chemla & Guo (2004). On the date, authorship and first appearance of the work, see *ibid.*, 43-7, 58-81.

⁴⁸ The following table is modified from Chemla & Guo (2004, 222-3).

After these numbers (*shu*) comes a procedure (*shu*):

今有術曰：以所有數乘所求率為實。以所有率為法。實如法而一。

‘Suppose you have’ procedure: Multiply the quantity of what you have by the *liü* of that which you seek to make dividend; take the *liü* of what you have as the divisor; and [divide] the dividend by the divisor.⁴⁹

Using ‘suppose you have’ – i.e., the [rule of three](#) – with an appropriate list of *liü*, one can transform *any one thing* into *any other* in *whichever direction* one desires. Take for instance problem 2.16:

今有粟七斗八升，欲為豉。問得幾何？

Suppose you have 7 *dou* 8 *sheng* (≈ 15.6 litres) of millet, and you desire to make it into fermented soy. How much do you get?

答曰：為豉九斗八升二十五分升之七。

Answer: It makes 9 *dou* 8 *sheng* and $7/25$ *sheng* (≈ 19.7 litres) of fermented soy.

術曰：以粟求豉，六十三之，五十而一。

The procedure: With millet, seeking fermented soy, multiply by [the fermented soy *liü*] 63 and divide by [the millet *liü*] 50.⁵⁰

$$\begin{aligned} 7 \text{ dou } 8 \text{ sheng} &= 78 \text{ sheng} \\ 78 \text{ sheng} \times 63 &\rightarrow 4914 \text{ sheng} \\ 4914 \text{ sheng} \div 50 &\rightarrow \mathbf{98 \text{ } 7/25 \text{ sheng}} \end{aligned}$$

One can go backwards just as easily, of course, if you simply flip the *liü* around (from 63/50 to 50/63), and either can be swapped with any other to get you something else.

This multiply–divide combo, ‘suppose you have’, is precisely the same operation we see in *li*, and it is performed there using numbers that are identical in form, function and presentation to those in *suan*. It is probably safe to call these *liü*, especially since that is what many of them call themselves: ‘circuit *liü* 周率’, ‘day *liü* 日率’, ‘coincidence *liü* 會率’, ‘discrepancy *liü* 差率’, ‘decrease–increase *liü* 損益率’, etc.⁵¹ *Liü*, moreover, is exactly how contemporaries active in *suan* speak of these numbers in philosophising upon the mathematics of *li* and tonometry (*liü* 律). Consider, for example, Li Chunfeng 李淳風 (602–70 CE). Li rose through the Tang 唐 (618–907 CE) Clerk’s Office by merit of his talent in *tianwen* 天文 and *li* from an auxiliary appointment (*zhi* 直) in 627/629 CE to its directorship

⁴⁹ Chemla & Guo (2004, 223–5, tr. mod.).

⁵⁰ Chemla & Guo (2004, 234–5, tr. mod.).

⁵¹ For ‘circuit *liü*’, see *Hou Han shu*, *zhi* 3, 3067–70; and *Jin shu*, 17.519–25. For ‘day *liü*’, see *Hou Han shu*, *zhi* 3, 3067–8; and *Jin shu*, 17.519–25. For ‘coincidence *liü*’, see *Jin shu*, 17.505, 508 18.570; *Song shu*, 12.236; and *Sui shu*, 17.422. For ‘discrepancy *liü*’, see *Jin shu*, 17.515, 517–8, 18.538–540, 545, 548, 570; *Song shu*, 12.235–7, 241–2, 245, 13.273, 287–8, 292, 296, 313; *Wei shu*, 107A.2669; and *Sui shu*, 17.453–4. For ‘decrease–increase *liü*’, see *Jin shu*, 17.510–11, 513–18, 18.547–8; *Song shu*, 12.243, 245, 13.277–8, 286, 288, 295–9; *Wei shu*, 107A.2671, 2673, 107B.2707, 2710; and *Sui shu*, 17.438–9, 441–2.

(*taishi ling* 太史令) in 649 CE; in 656 CE, he also oversaw a project to commentate and canonise works like the *Jiuzhang suanshu* as part of the *Suanjing shishu* 算經十書 – the mathematical counterpart to the *Wujing zhengyi* 五經正義 (653 CE).⁵² Li knows his stuff, and this is how he presents the underpinnings of these pursuits in the *Sui shu* 隋書 ‘Lü-li zhi’ 律曆志:

探蹟索隱，鈎深致遠，莫不用焉。一、十、百、千、萬，所同由也。律、度、量、衡、歷、率，其別用也。故體有長短，檢之以度，則不失毫釐[...]三光運行，紀以曆數，則不差晷刻。事物糅見，御之以率，則不乖其本。故幽隱之情，精微之變，可得而綜也。

In exploring the recondite and seeking out the hidden, in snaring the deep and eliciting the far away, [counting rods] can never be done without. Ones, tens, hundreds, thousands and myriads derive alike from them, and pitches, lengths, capacities, weights, *li* 曆 and *lü* 率 are [simply] their distinct applications. Thus, bodies can be long or short, but if one examines them with a ruler, then one will not miss by [a single] hair; [...] the three luminaries (i.e., the sun, moon and planets) travel in revolutions, but if one marks them with *li* numbers, then one will not err in gnomon and waterclock [timing]; matters and things can appear jumbled together, but if one takes charge of them with *lü* 率, then one will not pervert their bases. Thus, even dark and hidden natures (*qing* 情) and fine and subtle transformations (*bian* 變) can be grasped and synthesised.⁵³

It is clear that these are the same *lü* 率 we saw in *suan*, because Li Chunfeng next defines his terms via the chapters of the *Jiuzhang suanshu* in words borrowed from Liu Hui’s 劉徽 263 CE commentary thereto:

夫所謂率者，有九流焉：一曰方田，以御田疇界域。二曰粟米，以御交質變易。[...]皆乘以散之，除以聚之，齊同以通之，今有以貫之。則算數之方，盡於斯矣。

Now, as for that which we refer to as ‘*lü*’, there are nine [subjects] that flow from it: [chap.] 1, ‘Rectangular Field’ 方田, for dealing with the boundaries and areas of cultivated fields; [chap.] 2, ‘Unhulled and Hulled’ 粟米, for dealing with transformations (*bian* 變) and changes (*yi* 易) of the exchange of goods; [...] All of these [subjects/chapters] multiply to disaggregate them and simplify to assemble them; [they] homogenise and equalise to make them communicate and ‘suppose you have’ to link them together – and so do the methods of calculating numbers all come down to this (i.e., to *lü* 率).⁵⁴

⁵² On Li Chunfeng, see Goodman (forthcoming).

⁵³ *Sui shu*, 16.387. On Li Chunfeng’s authorship of the *Sui shu* ‘Lü-li zhi’, see Goodman (forthcoming). Note that, in this passage, Li Chunfeng is appropriating a parallel passage in *Han shu*, 21A.956., and *Hou Han shu*, *zhi* 1, 2999, which he modifies to emphasise the centrality of *lü*; see Morgan & Goodman (forthcoming) and Zhu Yiwen (forthcoming).

⁵⁴ *Sui shu*, 16.387, tr. modified from Chemla (2010, 278). Li Chunfeng is citing Liu Hui’s *Jiuzhang suanshu zhu* 九章算術注 as concerns the six-character summaries of each chapter and, also, paraphrasing him in the conclusion: ‘Multiply to disaggregate them, simplify to assemble them, homogenise and equalise to make them communicate, how could those not be the key-points of *suan* (computations/mathematics)?’ 乘以散之，約以聚之，齊同以通之，此其筭之綱紀乎 (Chemla & Guo 2004, 158–9; tr. modified from Chemla 2010, 277).

Going from a number of months to the position of the sun in *li*, as we did above, is a ‘transformation’ (*bian*) that Li Chunfeng, for one, likens to that between millet and fermented soy in *suan* – a transformation mediated by *lü* and performed by ‘suppose you have’ (*jin you shu* 今有術).⁵⁵ Yes, the mechanical metaphor is helpful for thinking about how ‘*li* numbers’ (*li shu* 曆數) are *structured*, but a ‘gear train’ can no better explain the procedures (*shu* 術) – the numbers’ *use* – than it can the transformation of millet into soy, adzuki beans or wheat. Luckily, we need not go searching for a better metaphor, because that with which our historical subjects have left us is rather fitting.

Of milfoil, counting rods and ‘transformations’

When actors philosophise about such ‘transformations’ in mathematics, as Karine Chemla (1997; 1999; 2010) has written about at length, they tend to turn to the *Book of Changes*. This makes sense, and it makes sense on several levels.

Mythologically, both *li* and *suan* go back to the demiurge Fuxi’s 伏羲 awakening to the incipient, civilising order in heaven, earth and the ‘myriad creatures’ in between. The version of the story that *you* know, from the ‘Xici zhuan’ 系辭傳, may end with the trigrams, but the version that mathematicians told went a little different. Here, for example, is Liu Hui on the origins of maths:

昔在庖犧氏始畫八卦，以通神明之德，以類萬物之情，作九九之術，以合六爻之變。暨于黃帝神而化之，引而伸之，於是建曆紀，協律呂，用稽道原。

In the past, [Fu]xi first drew the eight trigrams to enter into communication (*tong* 通) with the virtue of the spirits-illuminant and to classify (*lei* 類) the inner tendencies of the myriad creatures; he [also] created the ‘nine-nine’ procedure (i.e., the multiplication table) to accord with the transformations (*bian* 變) of the six lines (of the hexagrams). The Yellow Thearch, in his time, transformed (*hua* 化) them through spiritualization and expanded them through extension, thereupon establishing the rules of *li* and harmonising the pitch-standards (*lü ü* 律呂), which [he] used to investigate the source of the *dao*.⁵⁶

Going even further back, the *Zhoubi suanjing* 周髀算經 has the Duke of Zhou 周公 (r. 1042–1036 BCE) ask the following of Shang Gao 商高:

竊聞乎大夫善數也，請問古者包犧立周天曆度：夫天不可階而升，地不可將尺寸而度，請問數從安出？

⁵⁵ For more on Li Chunfeng’s treatment of *lü* 率 in the *Sui shu* ‘Lü-li zhi’, see Chemla (2010, 278–79), Morgan & Goodman (forthcoming) and Zhu Yiwen (forthcoming).

⁵⁶ Tr. modified from Chemla & Guo (2004, 127) and Chemla (2010, 269). Note that elsewhere, more typically, ‘it is said that Lishou 隸首 invented numbers’ 云隸首作數 in conjunction with/service of the Yellow Emperor (*Sui shu*, 16.395; cf. *Zhonglun*, 2.26a; *Hou Han shu*, *zhi* 1, 2999; *Jin shu*, 17.497; *Song shu*, 12.233, etc.).

I have heard, sir, that you excel in numbers. May I ask how [Fu]xi laid out the *li du* of the circumference of heaven in ancient times? Heaven cannot be scaled like a staircase, and earth cannot be measured out with a footrule. Where do the numbers come from?⁵⁷

The answer, the Duke of Zhou learns, is that Fuxi derived these from the *liü* of ‘the circle and the square’ 圓方, pertaining, respectively, to heaven and earth, and ‘it was thus that Yu [the Great] was able to bring order to the sub-celestial realm – this [triumph] was brought about by numbers’.⁵⁸

Operationally, ‘numbers and procedures’ literature does not much talk about ‘calculating the corresponding number of’ but about ‘making’ 為 *a* into *b*, the algorithm-based approach typical of *li* and *suan* being more a process of ‘transformation’ (→) than of equation (=).⁵⁹ With the *Sifen li*, above, we began with a single datum – a year, 2017, 11,298 years from ‘origin’ – which was transformed before our eyes into the date of the solstice (*guisi*.₃₀), the new moon (*xinmao*.₂₈), and the position of the sun at syzygy (Dipper._{L08} 19 889/940 *du*). More to the point, people of this time did not calculate as we do, let alone in the faux computer code offered above; they did so with counting rods (*suan* 算), on a physical surface, using their own set of conventions. Within those conventions, the multiplication and division of the ‘suppose you have’ procedure present ‘opposed but complementary operations’ (Chemla 2010, 257) whose interplay vis-à-vis the object of change – the operand – is suggestive of the transformations of *yin* and *yang*. Namely, on the calculating surface, ‘the digits of the number by which one multiplies all vanish’ (ibid., 261) as they are carried into the product that they ‘mount’ (*cheng* 乘); and so too is the product physically ‘eliminated’ (*chu* 除) from the counting surface, step by step, in building therefrom the quotient (fig. 7). In this sense, achilleomancy is not the only stick-based magic act in town.⁶⁰

Terminologically, algorithms present a meditation upon ‘the capacity of realities to be transformed without being destroyed in the process’ (Chemla 1997, 243), and philosophical commentators, like Liu Hui and Li Chunfeng, bring a similar vocabulary to bear upon the subject. Whether it be milfoil or counting rods, one speaks equally of ‘transformation’ (*bian* 變 /

⁵⁷ *Zhoubi suanjing*, 1.1 a–2a; tr. modified from Cullen (1996, 174).

⁵⁸ 故禹之所以治天下者，此數之所生也，*Zhoubi suanjing*, 1.4a; tr. modified from Cullen (1996, 174). Note that the antiquity of the *Zhoubi* was not a given in early imperial times, influential textual and technical scholars like He Chengtian 何承天 (370–447 CE) and Shen Yue 沈約 (441–513 CE) having already called the authenticity of this text into question; see Morgan (2017, chap. 5) and, from a modern perspective, Cullen (1996, 138–56). Note that elsewhere, the credit for *zuo* 作 ‘inventing’ *li* goes to a variety of figures around the Five Thearchs, of which Zhuangzi 莊子 is the most often cited.

⁵⁹ Here, I am paraphrasing Chemla (1999, 90); all symbols are my own.

⁶⁰ The ritual/symbolic parallel in physical support was not lost on contemporary actors. On *Changes* symbolism in philosophical treatments of counting rods, for example, see Li Yan (1955) and Zhu Yiwen (2010). In this vein, it might indeed prove interesting to compare the relevant liturgy for milfoil counting in, say, Zhu Xi’s 朱熹 (1130–1200) *Zhouyi benyi*, j. 14, with that for counting rods in mathematics and athletic score-keeping as studied, for example, in Chemla (1996; 2010) and Morgan (2017, chap. 4).

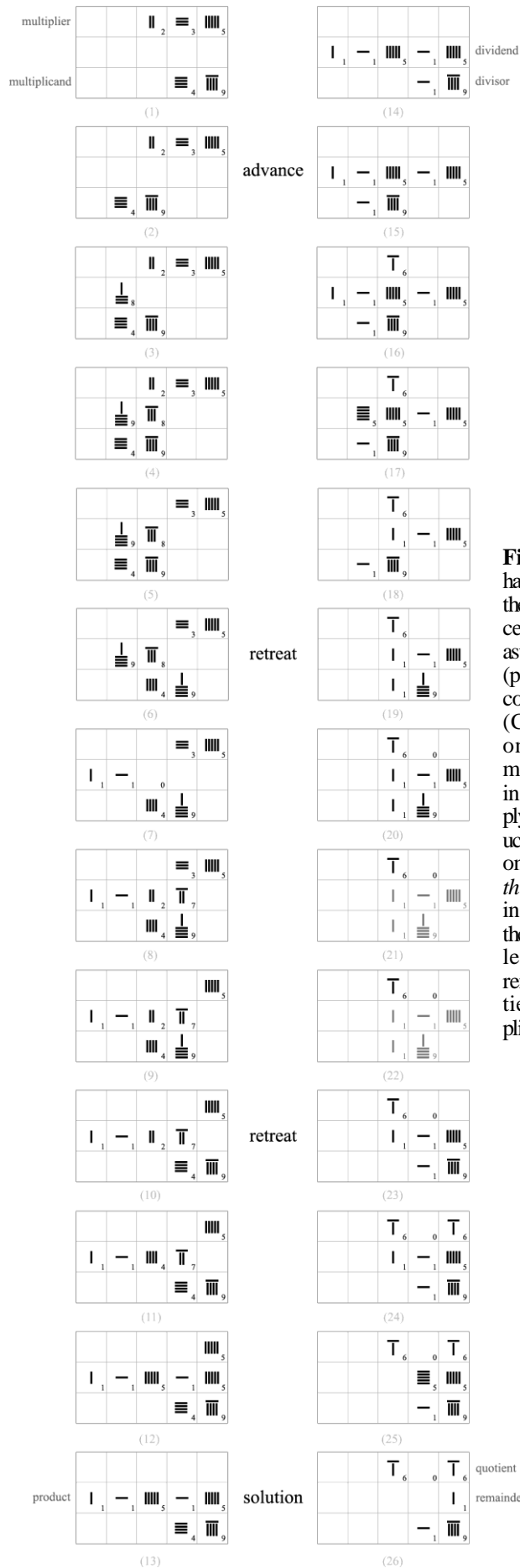


Fig. 7 ‘Suppose you have’ procedure behind the *Sifen li*’s ‘[5.] Procedure for calculating astronomical month I’ (p. 11), performed by counting rods as per (Chemla 2010). Note, on the left, how the multiplier disappears in the process of multiplying to find the product in step 13, and that, on the right, in dividing *this* to find the quotient in step 26, so too does the dividend disappear, leaving only a ‘little remainder’ and a quotient where the multiplier once sat in step 1.

hua 化),⁶¹ ‘[ex]change’ (*yi* 易)⁶² and ‘placing in communication’ (*tong* 通).⁶³ When speaking of mathematics, moreover, commentators tend to frame these transformations in parallel prose around *yin–yang* operational pairs: ‘homogenise’ (*qi* 齊) and ‘equalise’ (*tong* 同), ‘disaggregate’ (*san* 散) and assemble’ (*ju* 聚), ‘gain’ (*de* 得) and loss’ (*shi* 失), ‘advance’ (*jin* 進) and ‘retreat’ (*tui* 退).⁶⁴ We are, of course, talking about ‘advancing’ place values, ‘disaggregating’ fractions, and ‘homogenising’ those with different denominators, but such is the magic that turns time to space and rice to beans.

Coming back to space, time, and their representation, therefore, we may think about *li* mathematical astronomy thus, from the perspective of the *Changes*. Time and space are the warp and woof of the self-same fabric – tug at one end, and the other comes mysteriously with.⁶⁵ This fabric in constant (*chang* 常), but only in so much as it is in constant flux – time is space when space is turning, but only *against a fixed background* and *at a constant rate*. Were you to stop that somehow, time and space would cease to be, because the two are only possible *in concert, in opposition* and *in flux* – without the sun by day and stars by night, one couldn’t tell north (*zi*._{B01}) from south (*wu*._{B07}), and without an unmoving horizon by which to ‘clock’ them, one could not tell midnight (*zi*._{B01}) from noon (*wu*._{B07}). Round and square, heaven and earth – space and time are *in communication* (*tong* 通), which means that you can *pass freely* (*tong* 通) from one to the other and back. You can, that is, if you know the trick, because long ago, before our day, ‘[Yao 堯] ordered Chong 重 and Li 黎 to sever the *tong* of earth and heaven that there be no more descent and ascent [between]’.⁶⁶ By what sorcery, then, are we to tame the waters, conquer the world, achieve a destiny denied us by the gods, and move in an out of time and space?

You can do this with counting rods, and ‘if one takes charge of them with *lü* 率’, Li Chunfeng tells us, then ‘even dark and hidden natures and fine and subtle transformations can be grasped and synthesised’ on the calculating surface. *Lü*, to reiterate, are unit-less integer numbers that, like time and space, *yin* and *yang*, bear meaning only in concert, in opposition and in flux – i.e., in proportional relationship to one another, in the act of ‘transformation’. To insert a modern metaphor in place of another, *lü* are not so much cogs in a machine as they are a reader board of exchange rates that one (manually) consults when wanting to ‘change’ (*yi* 易) a fistful of money, commodities, time or space into something else – everything has a price, modern economics has taught us, and so too, in the ancient world, did everything have its *lü*. You cannot just ‘make’ 為 water into wine as such, of course, you need a little imagination

⁶¹ Compare Nielsen (2003, 19, 115–16) to Chemla & Guo (2004, 902).

⁶² Compare Nielsen (2003, 301–2) to Chemla & Guo (2004, 1024).

⁶³ Compare Nielsen (2003, 234) to Chemla & Guo (2004, 994–8).

⁶⁴ On this point, see Chemla (1997, 240–1; 2010, 276–8); for a glossary of these terms, see Chemla & Guo (2004, 898–1035).

⁶⁵ On weaving metaphors in the early astral sciences, see Pankenier (2015); for an alternative, agricultural metaphor for *li*, see Morgan (2017, chap. 1).

⁶⁶ 乃命重黎絕地天通，罔有降格 (*Shangshu zhushu*, 19.297b; cf. Birrell 1999, 91–5).

to ‘suppose you have’. But on the calculating surface, at least, the numbers are physically transformed step by step via opposed but complementary operations until from ‘what you have’ 所有 nothing is left but ‘what you seek’ 所求.

Conclusion

Echoing Jean-Baptiste Biot and anthropologists of ‘primitive thought’, Marcel Granet (1934) would argue that, like astronomy, ‘Chinese thought’ was incapable of mathematics as such:

L'idée de quantité ne joue autant dire aucun rôle dans les spéculations philosophiques des Chinois. Les Nombres, cependant, **intéressent passionnément** les Sages de l'ancienne Chine. Mais, – quelles qu'aient pu être les connaissances arithmétiques ou géométriques de certaines corporations (arpenteurs, charpentiers, architectes, charrons, musiciens...), – nul Sage n'a accepté de les utiliser, si ce n'est dans la mesure où, sans jamais contraindre à des opérations dont le résultat ne se pût commander, ce savoir facilitait des jeux numériques. [...] Un symbole numérique *commande* à tout un lot de réalités et d'emblèmes ; mais, à ce même mot, peuvent être attachés divers nombres, que l'on considère, *en l'espèce*, comme *équivalents*. À côté d'une valeur quantitative qui les distingue, mais **qu'on tend à négliger**, les Nombres possèdent une valeur symbolique **beaucoup plus intéressante**, car, n'offrant aucune résistance au génie opératoire, elle les laisse se prêter à une sorte d'alchimie. Les Nombres sont susceptibles de *mutations*. Ils le sont en raison de l'efficacité multiple dont ils paraissent dotés et qui dérive de leur fonction principale ; ils servent et valent en tant que *Rubriques emblématiques*.⁶⁷

⁶⁷ Granet (1934, 127–8); italics are original, bold is added to enforce the following point about the *négligence* of ‘uninteresting’ primary sources. As to his intellectual influences, one sees an echo of Granet’s ‘interest’ in numbers as *emblèmes, symboles, aspects, ensembles*, and *groupements concrets* in E.B. Tylor’s ‘examination of the methods of numeration in use among the lower races’ (Tylor 1871, vol. 1, chap. 7): that, compared to ‘the philosopher’ and ‘our advanced system of numeration’ (ibid., 219), ‘the savage’ generally counts only in small numbers, and in concrete terms. ‘The ancient Egyptian and the still-used Roman and Chinese numeration are indeed founded on savage picture-writing, while the abacus and the swan-pan [算盤], the one still a valuable school-instrument, and the other in full **practical use**, have their germ in the savage counting by groups of objects, as when South Sea Islanders count with coco-nut stalks, putting a little one aside every time they come to 10, [...], or when African negroes reckon with pebbles or nuts, and every time they come to 5 put them aside in a little heap’ (ibid., 244–5). As to Granet’s insistence on the exclusively ‘emblematic’ (rather than quantitative) function of *les Nombres* as concerns time and space, one likewise hears an echo of Lévy-Bruhl (1923): ‘To the primitive time is not, as it is to us, a kind of intellectualized intuition, an “order of succession.” Still less is it a homogeneous quantity. It is felt as a quality, rather than represented’ (ibid., 124); and ‘These minds will not picture space as a uniform and immaterial quantum. On the contrary, to them it will appear burdened with qualities; its regions will have virtues peculiar to themselves; they will share in the mystic powers which are revealed therein. Space will not be so much imagined, as *felt*, and its various directions and positions will be qualitatively differentiated from one another’ (ibid., 95). For more on Granet’s intellectual influences and the influence of ‘primitive thought’ on twentieth-century sinology, see M. Brown (2006).

Granet is flat wrong about *l'idée de quantité*, and he got there by conflating several things. He got there, first of all, by conflating what 'interests' 'the Chinese' and what interests Marcel Granet.⁶⁸ Of the eminent philosophers and classicists (*ru* 儒) of which you may have heard, here, for example, are a few from the period that were involved in numbers in a quantitative sense: Mozi 墨子 (c.468–c.391 BCE), Sima Qian 司馬遷 (c.145–c.86 BCE), Yang Xiong 揚雄 (53 BCE–18 CE), Liu Xin 劉歆 (c.50 BCE–23 CE), Ban Zhao 班昭 (44/49–118/121 CE), Ma Rong 馬融 (79–166 CE), Zheng Xuan 鄭玄 (127–200 CE), Du Yu 杜預 (222–85 CE), Yu Xi 虞喜 (fl. 307–46 CE), Zu Chongzhi 祖沖之 (429–500 CE), Xindu Fang 信都芳 (d. c. 539 CE), Zhen Luan 甄鸞 (fl. 535–70 CE), Niu Hong 牛弘 (545–610 CE), Liu Zhuo 劉焯 (544–610 CE), Kong Yingda 孔穎達 (574–648 CE), Jia Gongyan 賈公彥 (fl. 637 CE) and Yixing 一行 (683–727 CE).⁶⁹

He also got there by conflating imagery and substance, but in this he is not the first. Commentaries and introductions to mathematical works appeal to the *Changes* and its affiliated 'cosmology/ies' as a *metaphor*, but magic numbers do find their way into *li*.⁷⁰ Indeed, when *li* emerges on the historical stage, it does so wrapped in tonometrics and the *Changes*. Liu Xin, whose *li* of c.5 CE is the earliest to survive, cites the *Changes* throughout as the supposed origin of his *li*,⁷¹ and Liu Hong 劉洪 (fl. 167–206 CE), some two centuries later, would push this about as far as it could go. Here is Li Chunfeng on the latter:

其為之也，依易立數，遁行相號，潛處相求，名為乾象曆。又創制日行遲速，兼考月行，陰陽交錯於黃道表裏，日行黃道，於赤道宿度復有進退。方於前法，轉為精密矣。

What [Liu Hong] did was establish numbers based on the *Changes* [such that] they called out to one another in hidden motion and sought each other out from secret parts – and [at this he] named it the *Qianxiang li* 乾象曆 (after Qian 乾 ☰).

⁶⁸ I thank Karine Chemla for opening my eyes to this pattern during one of our meetings at the Project SAW: that wherever Granet (1934) mentions what 'interests' or 'is interesting' to 'the Chinese', it is usually to justify the omission of an entire swath of primary and secondary sources that do not interest *him* in so much as they do not fit his monolithic discourse on 'Chinese thought'. For a similar criticism, see Needham (1956, 217).

⁶⁹ On Mozi, see Graham (1986, 8–11; 2003). On the Han-era figures listed here, see Cullen (2009); on Zheng Xuan, Kong Yingda, and Jia Gongyan, see also the early version of my forthcoming chapter 'Calling out Zheng Xuan (127–200 CE) at the crossroads of ritual, maths, sport and classical commentary' [here](#). On those figures from the Three Kingdoms 三國 (220–80 CE) to the Sui 隋 (581–618 CE), see the author's contributions to Martin & Chaussende (forthcoming), available in draft form [here](#). On Yixing, see Chen Jinhua (2000–1) and Beer et al. (1961).

⁷⁰ Note that I do not support the use of the observer's category 'cosmology' in post-1980s sinology to refer to what actors called 'yin-yang' and 'five-agents' (*wuxing* 五行) correlative thinking in an *exclusive* and *monolithic* world-view. For a taste of the primary sources that were once called 'cosmology'/宇宙論 and have, since the 1980s, been excluded from that category and forgotten, see Forke (1925), Maspero (1929), Needham (1959, 210–27), Nakayama (1969, 24–44), Loewe (1975) and Xi & Zheng (1975). On the construction of 'Chinese cosmology' since, see Saussy (2000).

⁷¹ On Liu Xin's application of *Book of Changes* numerology to *li*, tonometrics (*li-lü* 律), and their synthesis (*li-li*), see Kawahara (1991; 1996, 148–95), Teboul (1983) and Vogel (1994). On the shelf-life of this synthesis, see Morgan & Goodman (forthcoming).

Also, [he] created the solar/daily motion slow–fast while concurrently investigating lunar motion, [concluding that] *yin* and *yang* cross inside an outside the yellow road, and that the sun travels on the yellow road, experiencing advance and retreat in terms of red-road lodge *du* – and only with this was there a turn towards the fine and tight relative to prior methods.⁷²

Compare this to Jia Kui’s 賈逵 (30–101 CE) somewhat less enthusiastic appraisal of one Zhang Long 張隆:

永平中，詔書令故太史待詔張隆以四分法署弦、望、月食加時。隆言能用易九、六、七、八爻知月行多少。今案隆所署多失。臣使隆逆推前手所署，不應，或異日，不中天乃益遠，至十餘度。

In [57/75 CE], there was an edict ordering Zhang Long, former expectant appointee to the Grand Clerk, to (predictively) note the added hour of [lunar phases] and eclipses according to the *Sifen* method. [Zhang] Long said that he was able to use the nine, six, seven and eight lines from the *Changes* to know the extent of lunar motion. [We] now know [Zhang] Long’s [predictive] notes to have missed the mark in most cases. [I,] Your servant, made Long retrodict [added hours] noted by former hands, and they did not correspond, sometimes [even] falling on different days; he was even further off in failing to hit the mark in heaven, [erring a matter of] up to more than ten *du*.⁷³

One of these men *no one* has heard of, the other was later enshrined at temple among those ‘renowned from the days of yore for calculating numbers’, and you can probably guess which is which.⁷⁴ As it turns out, Liu Hong’s numerology was only impressive in so far as it capped off ‘revolutionary’ mathematical models for lunar anomaly (*chiji* 遲疾), latitude (*yinyang* 陰陽), eclipse ‘crossing’ (*jiao* 交) and solar reduction to the equator (*jintui* 進退) – models thanks to which Liu Hong was winning predictive competitions against the living several decades after his death.⁷⁵ Liu Hong, moreover, was one of only two or three to actually bother; mostly, *li* men didn’t dress up their *lǚ* as something else – as something, no less, that *didn’t work*.⁷⁶

Suan, for its part, is likewise devoid of similar contents: one does not perform a cubic root extraction with ‘fire’, nor can one operate upon ‘metal’, *yin*, Mingyi ䷗, stems or branches.⁷⁷ The

⁷² *Jin shu*, 17.498.

⁷³ Cited in *Hou Han shu*, *zhi* 2, 3030. On the *Changes* hexagram line numbers ‘nine, six, seven, and eight’, see Rutt (1996, 151–201).

⁷⁴ 自昔著名算數者, *Song shi*, 105.2552. The correct answer is Liu Hong, who was posthumously promoted to Viscount of Mengyin 蒙陰子 and whose sacrificial icon was repainted according in King Wenxuan Temple 文宣王廟, Bianjing, in 1109; see Morgan (2017, chap. 5).

⁷⁵ On Liu Hong’s technical innovations, see Cullen (2002). On his posthumous victories in live-trial testing and debate, see Morgan (2017, chap. 4).

⁷⁶ On the dearth and superficiality of *Changes* symbolism in *li* mathematical astronomy and its rupture from tonometrics, see Martzloff (2009, 38–44), Morgan (2017, chap. 1), and Morgan & Goodman (forthcoming).

⁷⁷ Excluding expressions like *yangma* 陽馬 ‘pyramid’, *jin* 金 ‘cash’, and *zhi wuxing* 置五行 ‘set out five columns’, the very words *yin*, *yang*, *wuxing*, *mu* 木 (wood), *huo* 火 (fire), *tu* 土 (earth), *jin* 金 (metal) and *shui* 水 (water) do not appear once in the base-text of the *Jiuzhang suanshu*. In the commentary, one does find four instances of ‘*yin* and *yang*’, but one of them, tellingly, is by point of criticism for bringing such matters into the calculation of

question – ‘what one seeks’ 所求 – is not formulated in terms of the ‘bane’ 凶 or ‘auspice’ 吉 of human events, let alone the colours or directions to ‘avoid’ 避, but in the sort of 1–2–3 numbers of ‘what one has’ 所有.⁷⁸ Where mathematicians *do* bring the vocabulary of the *Changes* into actual calculations, Chemla (2010, 280) notes, it is ‘linked to the practice of proof in the context of exegesis, and, more precisely, to a specific function assigned to it: identifying the “fundamental transformations” at play in all procedures’. It is deployed in establishing the correctness of an algorithm, in drawing parallels with others, and in unveiling the mathematical strategies, hidden operations and ‘reason’/‘internal constitution’ (*li* 理) behind the problems of the ‘classic’. It is heuristic framework, in other words, towards ‘a mathematical research on the rationality of change’ (ibid., 281).

And that’s the thing about ‘numbers’, and why *they* enjoyed a place among the ‘six arts’ 六藝 of the gentleman and in the normal, imperial curriculum ever after: with numbers, ‘transformations’ can be reliably reversed, repeated and explained, and with numbers, more importantly, there is *proof*. Proof is what a paperwork empire demands – proof of travel, proof of payment, proof of proficiency, and the list goes on – and there is a reason that that empire entrusted such things as taxation, censuses and market payments to *accountants*: because *quantities*, in this regard, are far more crucial to the work of sagecraft than are *Rubriques emblématiques*. And when it went about ‘observing the signs and granting the season’ 觀象授時, so too is there a reason why the Empire appointed relevant experts to the task – why they recruited talent, why they hosted debates, why they ran competitive trials, and why Zhang Long, above, is introduced as a ‘former appointee’. Time and space are commutable, and by numbers you can take command of them to literally see the future and to *prove* it, to all under heaven, all at once, in the minute the midday sun goes black.⁷⁹

They may address different questions, but the ‘numbers and procedures’ (*shushu*) of *li* and *suan* are by no means a lesser magic than the *Changes*. They are, let us recall, sister sciences revealed by the demiurge at the beginning of human time, and

the volume of the sphere: ‘La théorie de Zhang Heng 張衡 (78–139 CE) veut naturellement s’accorder avec la théorie du pair et de l’impair, du *Yin* et du *Yang*, et ne prend pas en considération la précision’ 衡說之自然，欲協其陰陽奇耦之說，而不顧疎密矣 (Chemla & Guo 2004, 383). As to the mathematical operability of the ‘heavenly branches’ and ‘earthly stems’, one notes that though Granet (1934, 129) speaks of these as ‘les nombres des séries décimale et duodécimale’, they are not ‘numbers’ that one can actually add, subtract, multiply or divide, and they appear, as such, only in the context of *li* coordinates where they are ‘named’ (*ming* 命), rather than ‘counted’ (*shu* 數), at the end of an operation carried out upon *real numbers*, as seen in the *Sifen li* above.

⁷⁸ ‘Bane’, ‘auspice’, ‘avoidance’, etc., are problematics that come up only in other genres of ‘numbers and procedures’ (*shushu*), as listed in Note 20, particularly ‘Five Agents’ hemerology and *shi*-board divination. On ‘Five Agents’ hemerology and *shi*-board divination, see Kalinowski (1983; 1986; 1996; 2013).

⁷⁹ On the subject of recruitment and *li* debates (*yi* 議), see Cullen (2007); this and the subject of competitive testing is taken up in further detail in Morgan (2017, chaps. 1 and 4).

they are, in the *Documents*, the very first thing to which Yao 堯 and Shun 舜 attended upon the throne.⁸⁰ And where the *Han shu* 漢書 ‘Lü-li zhi’ 律曆志 opens on numbers citing a lost *Document* to the effect that ‘one must prioritise one’s *suanming* 算命’,⁸¹ it is telling how Yan Shigu 顏師古 (581–645 CE) chooses to read this statement. Normally ‘fate divination’, Yan Shigu, by virtue of its context, understands *suanming* to mean something altogether greater for the fate of man:

言王者統業，先立算數以命百事也。

This means to say that, in consolidating his patrimony, he who is to be king must first establish the calculation of numbers so as to take command of the hundred affairs.⁸²

On that note – on the ‘mathematical mandate’ of the aspiring sage – I want to end on a question concerning further Truths that we might want to rethink in early China studies: Why is it, for one, that the *shushu* of *li* and *suan* are systematically excluded from *shushu* studies? How is it, furthermore, that that the *quantitative* study of space (*yu* 宇) and time (*zhou* 宙) is likewise excluded from ‘cosmology’ (*yuzhou lun* 宇宙論)? Would not the story of ‘Chinese thought’ be more *interesting* (let alone more *historically accurate*) if it were to include *all* of what the Chinese thought to do with numbers?

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⁸⁰ Namely, the ‘Yao dian’ 堯典 chapter opens with Sage King Yao ‘ordering the Xi and He [brothers], in reverent accordance with prodigious heaven, to *li* and *xiang* the sun, moon and stars and respectfully grant the seasons of man’ 乃命羲和，欽若昊天，歷象日月星辰，敬授人時 (*Shangshu zhushu*, 2.21a). And when Yao abdicated to Shun, he did so declaring ‘the *li* numbers of heaven rest in thy person, and thou shalt eventually ascend [to the throne] of the great sovereign’ 天之歷數在汝躬，汝終陟元后 (ibid., 4.55b; cf. *Analects* xx.1). Following his ascension, lastly, Shun ‘attended to the Rotating Device and Jade Traverse (i.e., Beidou 北斗; UMA) so as to order the seven governors/government matters’ 在璿璣玉衡，以齊七政 (*Shangshu zhushu*, 3.35b).

⁸¹ 書曰：先其算命，*Han shu*, 21A.956.

⁸² Ibid. (comm.). Compare this to Archytas (428–347 BCE) (B 3): ‘The invention of calculation put an end to discord and increased concord [...] A standard and a barrier to the unjust, it averts those who can calculate from injustice, persuading them that they would not be able to stay unexposed when they resort to calculation, and prevents those who cannot calculate from doing injustice by showing through calculation their deceit’ (tr. Zhmud 2006, 71).

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