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Do Misperceptions about Demand Matter? Theory and Evidence

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Do Misperceptions about Demand Matter? Theory and Evidence*

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Abstract

We assess theoretically and empirically the consequences of demand misperceptions. In a New Keynesian model with dispersed information, agents receive noisy signals about both supply and demand. Firms and consumers have an asymmetric access to information, so aggregate misperceptions of demand by the supply side can drive economic fluctuations. The model's predictions are used to identify empirically fundamental and noise shocks on supply and demand. We exploit survey nowcast errors on both GDP growth and inflation, fundamental and noise shocks affecting the errors with opposite signs. We show that demand-related noise shocks have a negative effect on output and contribute substantially to business cycles. Additionally, monetary policy plays a key role in the transmission of demand noise.

JEL classification: **E32, D82, C32, E31.**

Keywords: Business cycles, information frictions, noise shocks, SVARs with sign restrictions.

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1 Introduction

It is commonly accepted that expectations on economic activity can be important drivers of fluctuations by generating waves of optimism and pessimism.¹ This very old idea, which dates back to Pigou (1927) and Keynes (1936), has been revived recently through the concept of “news”, “animal spirits” or “sentiments”. Agents are imperfectly informed not only about the future, but also about what is currently going on in the economy, and have “misperceptions” about the current state of fundamentals. This is apparent in Figure 1, which displays values of annualized output growth and inflation rate and their associated expectation at different horizons for the period 2007q2-2011q4.² Forecasters make mistakes in their predictions at all horizons, and even contemporaneously.

The recent literature exploring the effects of noise shocks has been mostly focused on misperceptions of total factor productivity (TFP). These misperceptions have been rationalized by noise shocks affecting a common signal about TFP. Several authors argue that these noise shocks related to supply resemble demand shocks, and account for a significant share of short-term and medium-term output fluctuations (see Blanchard et al., 2013; Forni et al., 2013; Enders et al., 2015; Déés and Zimic, 2016).³ However, misperceptions related to demand signals have been largely neglected.

Yet, misperceptions about demand exist as well, as Figure 1 shows. Consider for instance the Great Recession. It is striking to observe that forecasters strongly underestimated *both* the recession and the large slowdown in inflation at the peak of the crisis in late 2008. Excessive optimism about the supply side of the economy cannot explain these two facts simultaneously: forecasters would have underestimated the drop in output but they would also have expected a reduction in inflation larger than what has been finally observed. Could noisy signals about demand explain this misperception during the crisis? More generally, do they drive business

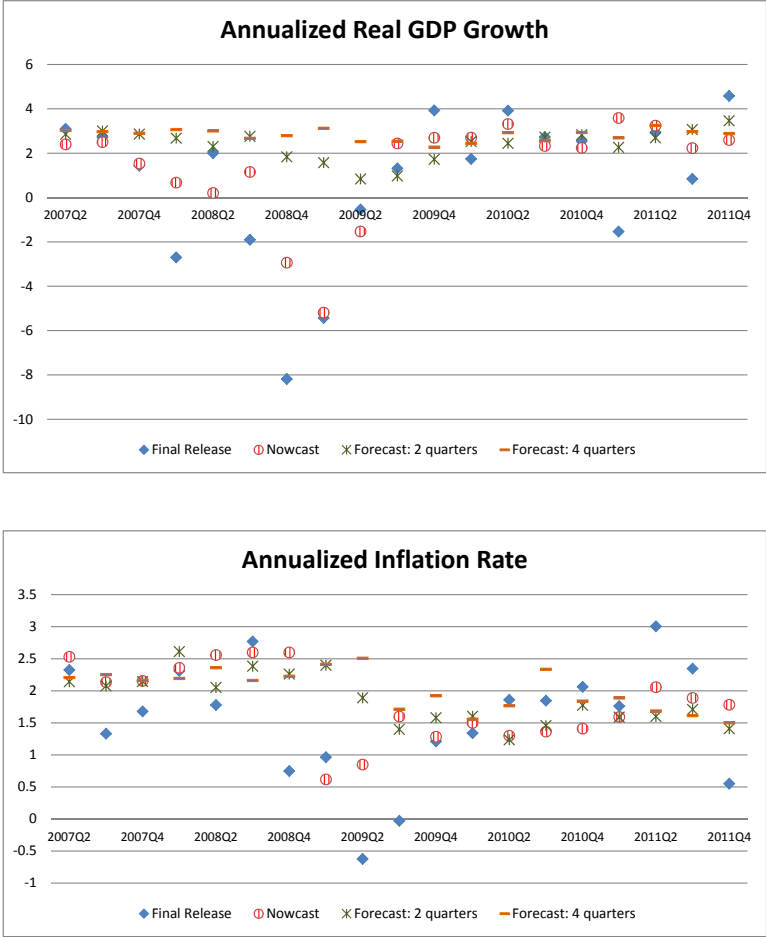
¹See for instance Beaudry and Portier (2004), Lorenzoni (2009, 2010), Jaimovic and Rebelo (2009), Barsky and Sims (2012), Christiano et al. (2010), Beaudry et al. (2011), Angeletos and La’O (2013), Benhabib et al. (2015), among others.

²Real GDP growth expressed in annualized rate is extracted from BEA and annualized inflation rate is from BLS. Forecasts are provided by the Survey of Professional Forecasters (SPF) of the Fed of Philadelphia.

³More generally, the literature has not reached a consensus regarding the contribution of non-fundamental-driven expectation shifts on the business cycle. Barsky and Sims (2012) disentangle fundamental news and non-fundamental noise shocks by estimating a structural model with a measure of consumer confidence. They argue that “animal spirit shocks” account for a negligible part of output volatility. Angeletos et al. (2014) model confidence through higher-order uncertainty and they show that these type of shocks explain half of the fluctuations. Fève and Guay (2016) or Levchenko and Pandalai-Nayar (2016) designate “sentiments shocks” as shocks that explain the largest share of confidence volatility in the US. Levchenko and Pandalai-Nayar (2016) argues that this form of demand shocks explain a substantial part of high-frequency business cycle while Fève and Guay (2016) claim that, instead, news shocks on productivity are the main driver of the fluctuations.

cycles? In this paper, we investigate the theoretical implications of noise shocks affecting common signals about demand and empirically quantify their contribution to business cycle fluctuations. To this end, we use a sign restriction approach that relies crucially on the survey expectation errors on both GDP growth and inflation.

Figure 1: GDP growth and inflation - Final releases and expectations



Note: Annualized percentage change of the median response of the nowcast prediction for real GDP (upper panel) and GDP deflator (lower panel) at different forecast horizons. The nowcast data are from the SPF of the Fed of Philadelphia.

We first build a model with dispersed information which is a tractable generalization of a bare-bones New-Keynesian model à la Galí with imperfect common knowledge. In this model, crucially, consumers and firms are distinct agents, so the aggregate expectations of consumers differ from those of firms, because firms and consumers have an asymmetric access to information about the demand- and the supply-side of the economy. Following the literature on TFP noise shocks, we assume that consumers do not directly observe aggregate productivity

that works as a supply shifter.⁴ Our originality lies in the assumption that firms on their side do not directly observe the aggregate preference shock that works as a demand shifter. Therefore, they make their production decisions subject to an incomplete information regarding the demand they face. All agents receive noisy signals regarding the contemporaneous true state of the economy: a signal about TFP (supply signal) and a signal about preferences (demand signal). Hence, we can study the implications of “demand noise shocks” along with the more typical supply noise shocks.

Our theoretical model shows that demand-noise shocks generate an increase in inflation and, if firms make only pricing decisions, a decrease in output. Indeed, an excessive optimism of firms on demand leads them to set higher prices. This pushes the central bank to react by setting a higher interest rate, which drives down aggregate demand. However, if firms make also quantity decisions, and in particular, use an intermediate output, then the response of output is ambiguous. The reason is that aggregate demand depends on firms’ expectations on consumption as well as actual consumption. As previously, the rise in inflation depresses household’s demand through the monetary policy channel. However, firms increase their demand for intermediate goods since they interpret the shock as a fundamental demand shock, i.e. an excess demand that they have to meet. The final effect on output depends on the share of intermediate goods in aggregate demand.

Consistently with Lorenzoni (2009), an excessive optimism of consumers about TFP leads to both an increase in output and inflation, just like fundamental demand shocks. As explained above, the demand-noise shocks has an ambiguous effect on output and so, depending on parameters, it behaves either like a (negative) supply shock or like (a positive) demand shock. Accordingly, fundamental and noise shocks cannot be distinguished based solely on inflation and output. We argue that expectation errors on output and inflation can be used to fix this identification problem. Precisely, we add external observers called surveyors whose expectation errors on output and inflation will be driven by the fundamental and noise shocks. For example, surveyors typically overestimate output following a supply-noise shock while they underestimate it following a fundamental demand shock. Similarly, surveyors overestimate inflation in the case of a demand-noise shock while they underestimate it in the case of a fundamental demand shock or a supply-noise shock. These predictions are derived analytically from the model under exogenous information and the absence of persistence, and the model is simulated under more general assumptions.⁵ Other types of demand shocks are considered (government spending

⁴See Lorenzoni (2009) and Blanchard et al. (2013). In these papers, consumers and firms are the same agents and do observe aggregate productivity, but they still need to infer its permanent component, which they do not directly observe.

⁵In that respect, our model shows the crucial role played by private information to generate non-trivial effect of fundamental and noise shocks on expectation errors. To understand this point, suppose that all information

shocks and monetary policy shocks) and the case where prices can be used as a source of information, but our results remain robust.

The contribution of noise shocks to the business cycle is then measured by estimating a Structural VAR (SVAR) model on US data, using our model-based long-run and sign restrictions to identify the shocks. Except the fundamental supply shock which can be identified through long-run restrictions, fundamental and noise shocks are identified through sign restrictions on output and inflation and their respective expectation errors. While we let the data speak regarding the effect of demand noise shocks on output, we find that they generate a persistent recession. As we impose a positive response of inflation, this makes them look like a negative fundamental supply shock. Second, noise shocks in general explain about 30% of output fluctuations on impact and about 20% after one year, in our most conservative specification, which is our baseline. Looking more carefully at our results, we observe that a substantial part of this contribution is attributable to demand-noise shocks. In other terms, we claim that misperceptions about the true state of demand emanating from firms is a key and neglected driver of the business cycle. This result reinforces the idea supported by a growing literature that expectations-driven (or sentiments) shocks matter for the understanding of the business cycle. In a series of robustness, we use alternative identification assumptions.

Our methodological contribution consists in showing how the sign restriction methodology can be extended to expectation errors in order to estimate multiple noise shocks. We exploit the fact that fundamental and noise shocks affect errors with opposite signs. While these sign properties have already been exploited in the context of noise shock identification by Dees and Zimic (2016), we are the first to combine expectation errors on more than one variable. By backing up our results with those of the literature, we show that, when using the nowcast errors for identification, omitting the demand side of the economy typically leads to underestimate the share of noise as a whole, and overestimate the supply noise in particular.

The mechanism leading to a recessionary effect of demand noise is based on the reaction of monetary policy. Indeed, the central bank adopts a Taylor rule based on its inflation expectations. It therefore reacts positively to the demand signal. This reaction to the signal is beneficial to the economy in the case of an actual demand shock, as the rise in interest rate mitigates inflation and stabilizes the economy, but it is detrimental in the case of a noise shock.⁶

is public. Then surveyors, who also observe public information, can perfectly infer aggregate variables, which are conditional on the same public information. We therefore assume that all agents receive private signal on top of the public signals that are commonly observed. In this respect, our model enables us to highlight the key role of private information in generating expectation errors that can be exploited for identification.

⁶Note that this effect does not hinge on a particular information structure of the central bank. As we show in an extension to the model, these effects remain even if the central bank observe inflation perfectly, because inflation itself reacts to the demand signal.

We test this mechanism by evaluating the effect of the structural shocks on the Fed Funds rate. Our results show that the interest rate does react positively to a demand-noise shock, which is consistent with our mechanism.

Our paper belongs to the literature that studies, both theoretically and empirically, the effect of fundamental and non-fundamental shocks on the business cycle. On the theoretical front, expectations-driven shocks have been explored from different perspectives. A strand of the literature interprets those non-fundamental shocks as waves of optimism and pessimism, or alternatively, as shift in sentiments/confidence. Agents' perception about economic conditions might be altered by informational frictions either through sunspot-like mechanisms (Angeletos and La'O, 2013, Benhabib et al., 2015) or through noisy signal about technology (Lorenzoni, 2009).⁷ We enrich the noisy signal's approach by explicitly modelling the noise-ridden signals received by the production sector about demand.

On the empirical front, we contribute to the research on the identification of noise shocks. In particular, we are the first to estimate multiple noise shocks. To do so, we have to circumvent the non-invertibility issue that plagues the identification through SVARs, as explained in Blanchard et al. (2013). This issue is simply due to the fact that, contemporaneously to a given shock, if the agents cannot identify the shock, neither can the econometrician. Some identification strategies, tightly linked to theoretical models, such as the GMM, as in Blanchard et al. (2013), or minimum-distance estimation, as in Barsky and Sims (2011), can be used to overcome that issue. Another solution is to use information that is not contemporaneously available to the agents, in which case, SVARs can be used. This is the route followed by Forni et al. (2013), Enders et al. (2015), Déés and Zimic (2016) and Masolo and Paccagnini (2015) and which we follow as well, by using data on realized output and inflation that is released in subsequent periods. Our paper is closest to Enders et al. (2015) in the methodology since we exploit expectation errors based on GDP nowcast to identify fundamental and noise shocks *ex post*. However, unlike them (and also other papers using the SVAR strategy), we expand the identification strategy by using expectation errors on inflation which allows us to identify demand-noise shocks as well.

Our paper is also related to the literature that studies more specifically the effect of expectations on demand. Fiscal and monetary news, in particular, have been the focus of attention.⁸ These studies focus on foresighted demand shocks, not demand misperceptions. To the best

⁷There is another road to formalize sentiments shocks which have been adopted for instance by Melosi (2014) and Milani (2014) who assume near-rational agents.

⁸Yang (2005), Perotti (2011), Leeper et al. (2013) and Forni and Gambetti (2014) have studied the effect of fiscal news, while Campbell et al. (2012), Milani and Treadwell (2012), Ben Zeev et al. (2016) study the effect of monetary news. Milani and Rajbhandari (2012) estimate the effect of news about a large array of shocks, using a workhorse New Keynesian model and the SPF estimates of a wide range of variables.

of our knowledge, we are the first to quantify the contribution to business cycles of *noise* shocks on demand, and to show their recessionary effect. Fève and Pietrunti (2016) identify the amount of noise in fiscal policy signaling by using survey data for several countries. Ricco (2015) evaluates the impact of “misexpectations” about fiscal policy. These papers however do not quantify the contribution of these noise shocks to the business cycle. Our approach is also more comprehensive to the extent that we try to include all types of demand shocks.

The paper is structured as follows. Section 2 lays down a simple New Keynesian model with dispersed information, and Section 3 derives the predictions of the model that will be used in Section 4 for the SVAR estimation. Section 5 concludes.

2 A Model with Dispersed Information

We model the economy as a continuum of islands, each populated by a household and a continuum of firms producing differentiated goods. The economy is hit by permanent aggregate technology (supply) and transitory preference (demand) shocks. We include two sources of uncertainty. First, the households do not observe the current technology. Second, firms do not observe the current preference shock. Instead, agents receive one public signal about the technology shock and one public signal about the preference shock. These two categories of signals are affected by aggregate noise shocks, which we refer to respectively as the supply and demand-noise shocks. Additionally, each island receives one private signal about the technology shock and one private signal about the preference shock.

Each household visits other islands to shop and work. In this set-up, prices are island-specific, so they do not fully reveal aggregate shocks. As a result, noise shocks will have an impact on the economy. We derive from this setup a two-equation New-Keynesian model that extends Galí (2008) under dispersed information. In particular, we allow firms’ information to differ from the households’. As a result, both noise on demand and on supply will affect aggregate outcomes.

We also introduce surveyors (one per island), some external observers of the economy who play no role in the model but collect information on their island and produce survey expectations. Therefore, we distinguish the “survey” expectations from the “agents” expectations. Agents expectations are the expectations of the agents of the economy that are relevant for their decision-making. Survey expectations, which are conditional on the surveyor information, are empirically relevant as they will enable us to make predictions on the effect of noise shocks.

2.1 Preferences, Technology and Policy

The economy is composed of a continuum of islands indexed by $i \in [0, 1]$, each populated by a representative household and a surveyor, also indexed by i . The household owns a continuum of monopolistic firms each producing a differentiated good indexed by $j \in [0, 1]$ on island i . The household buys goods produced in another island and supplies labor in yet another island. Namely, each period nature randomly selects an island $l^c(i, t) \in [0, 1]$ visited by household i to shop and independently selects an island $l^w(i, t) \in [0, 1]$ visited by household i to work. Similarly, firms of island i are visited by a household $k^c(i, t) \in [0, 1]$ to shop and by a household $k^w(i, t) \in [0, 1]$ to work.

Fundamental shocks (technology, preferences) are identical across islands. Only information can differ from one island to another. We will specify later the exact structure of information in this economy.

Preferences and technology The household's utility is given by

$$U_{it} = E_t \sum_{s=0}^{\infty} B_{t+s} \left\{ \log(C_{it+s}) - \frac{1}{1+\zeta} N_{it+s}^{1+\zeta} \right\}, \quad (1)$$

where B_t is the coefficient of time preference for date t defined by $B_t = \beta B_{t-1} e^{-u_t^b}$, with $B_0 = 1$. β is the average factor of time preference and u_t^b is an intertemporal time preference shifter following the process:

$$u_t^b = \rho_b u_{t-1}^b + \epsilon_t^b, \quad (2)$$

with ϵ^b a gaussian i.i.d. shock with mean zero and standard deviation σ_b . Household i decides both consumption C_{it} for period t , and the labor supply N_{it} by maximizing (1), but at different sub-periods.

In each island i , a competitive final good firm combines a continuum of intermediate goods produced on i in quantities Y_{ijt} , with $j \in [0, 1]$ to produce the final good Y_{it} , following the typical production function

$$Y_{it} = \left(\int_0^1 Y_{ijt}^{(\gamma-1)/\gamma} dj \right)^{\gamma/(\gamma-1)}, \quad (3)$$

where γ is the input demand elasticity, with $\gamma > 1$. The final good is then sold at price P_{it} on island i .

Each type- j good is produced by firm j and sold on island i at price P_{ijt} . Firm j produces using a quantity of labor $N_{k^w(i,t)jt}$ supplied by household $k^w(i, t)$ with the production function

$$Y_{ijt} = A_t N_{k^w(i,t)jt}, \quad (4)$$

where A_t is a productivity shifter. Let $A_t = \bar{A}e^{u_t^a}$ where u_t^a follows a random walk:

$$u_t^a = u_{t-1}^a + \epsilon_t^a, \quad (5)$$

with ϵ_t^a a gaussian i.i.d. shock with mean zero and standard deviation σ_a . Notice that technology shocks have a permanent component while preference shocks are transitory, see Equations (2) and (5). Nominal rigidities in price-setting follow Calvo (1983): each period, a fraction $1 - \theta$ of island i firms are able to re-optimize their prices.

Monetary policy The central bank forms expectations on inflation and sets the interest rate on one-period-maturity nominal deposits according to the rule

$$i_t = \bar{i} + \varphi E_t^g(\pi_t), \quad (6)$$

where $\pi_t = p_t - p_{t-1}$ with $p_t = \log(P_t)$ the average price across islands ($P_t = \int_0^1 P_{it} di$), and $E_t^g(\cdot)$ is the expectation of the central bank.

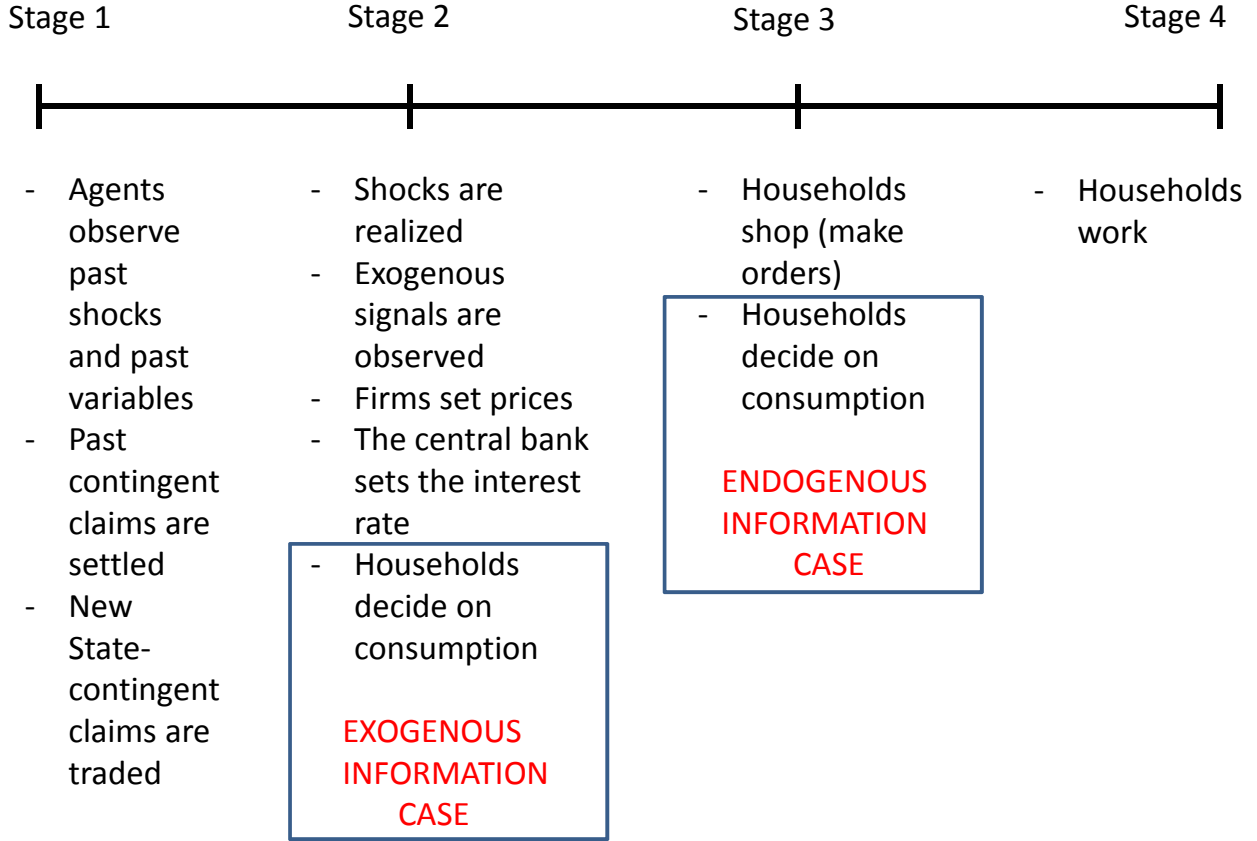
Timing and trading We follow a timing and a trading structure similar to Lorenzoni (2009). The central bank works as an account keeper for households. Each household holds an interest-bearing deposit denominated in nominal terms at the central bank. This account is credited and debited by the agents of the household whenever they make a purchase or receive payments. A period is divided in four stages, as described in Figure 2: the contingent claims trading stage, the price-setting stage, the shopping stage and the production stage. In the first stage, households trade contingent claims in a centralized market. These claims are paid in the next period's first stage. The market for contingent claims closes in the next three stages. In the second stage, shocks are realized. Firms set their prices, the central bank sets the interest rate. In the third stage, the consumer visits an island and participates to the local good market, making orders to firms. In the fourth stage, the household goes to another island to work and production takes place. At the end of period, the household receives firms' profits and the remaining resources are left in the central bank's account. We will discuss later in details how information unfolds along this timeline.

The representative household faces then the following budget constraint:

$$(1 + i_t)D_{it+1} + P_{l^c(i,t)t}C_{it} + \int Q(\omega_{it})Z_{it+1}(\omega_{it})d\omega_{it} = D_{it} + W_{l^w(i,t)t}N_{it} + \int_0^1 P_{ijt}Y_{ijt}dj + Z_{it}(\omega_{it-1}), \quad (7)$$

where D_{it+1} denotes the one-period-maturity nominal deposits, $P_{l^c(i,t)t}$ is the price of the final consumption good in island $l^c(i, t)$, $W_{l^w(i,t)t}$ is the nominal wage in island $l^w(i, t)$. ω_{it} denotes the state, which depends on the set of aggregate and idiosyncratic shocks that occur in the second

Figure 2: Timeline



stage. $Q(\omega_{it})$ is the unit price of a contingent claim that delivers 1 in state ω_{it} . $Z_{it+1}(\omega_{it})$ are the quantities of contingent claims bought by the household.

Each household starts with zero deposits so $D_{it} = 0$. Since households face idiosyncratic shocks, their ex post deposits may evolve over time. However, since they have access to state-contingent claims and since they face identical shocks ex ante, they can fully insure against those shocks. Since in equilibrium we have $\int_0^1 D_{it} di = 0$, their ex post net position therefore stays equal to zero over time. This eliminates ex post heterogeneity across households, which greatly simplifies the problem.

2.2 Information

At the first stage of date t , agents (households, firms and surveyors) observe past variables: u_{t-1}^n , $n = a, b$ and past prices p_{t-1} and p_{it-1} , $i \in [0, 1]$. In the second stage, firms also learn their productivity u_t^a and the households learn the households' preferences u_t^b . Additionally, the households, the firms, the central bank and the surveyors all receive exogenous public signals

about the fundamentals ϵ_t^a and ϵ_t^b .

We denote by s_t^n , $n = a, b$, the public signal received by all the agents at date t regarding shock ϵ_t^n , so that we have, for $n = a, b$:

$$s_t^n = \epsilon_t^n + e_t^n, \quad (8)$$

where e_t^n , is a gaussian i.i.d. shock with mean zero and a standard deviation equal to σ_{0n} . e_t^a and e_t^b correspond respectively to the productivity and preference noise shocks while ϵ_t^a and ϵ_t^b are the corresponding fundamental shocks.

Besides, on island i , the household, firms and the surveyors receive the following private signals about productivity and preferences:

$$x_{it}^n = \epsilon_t^n + \lambda_{it}^n, \quad (9)$$

for $n = a, b$, where λ_{it}^n , is a gaussian i.i.d. shock with mean zero and a standard deviation equal to σ_{1n} and that satisfies $\int_0^1 \lambda_{it}^n di = 0$.

Agents expectations are defined as follows: $E_{it}^m(\cdot) = E(\cdot | I_{it}^m)$, $m = c, w, f, s$, where c , w , f and s denote respectively households at stage 3 (shopping stage), households at stage 4 (production stage), firms and surveyors, and I_{it}^m is the information set of agent m . $E_t^g(\cdot) = E(\cdot | I_t^g)$ is the expectation of the central bank.

Notice that the firms and the households share common signals, but they have an unequal access to ϵ_t^a and ϵ_t^b . Namely, firms have a privileged access to the information on technology while households have the knowledge on their preferences.

The information set of the agents derives from the timing assumption and from the island structure of the economy. Consider firms of island i first. Firms know technology, but do not observe preferences, which they try to infer. Firms set their price in the second stage, before the good and labor markets open. Their pricing decisions are then conditional on their private and public signals about the preference shock, and their assessment of the preference shock is imperfect, which will generate excessive optimism or pessimism about demand. What is crucial in this respect is that firms do not observe the marginal cost before they set their prices. This assumption is a natural one. First, in the New-Keynesian literature, prices are typically predetermined. Second, marginal costs are notoriously difficult to measure. Note that labor decisions, by contrast, are taken conditional on the nominal wage observed on the island, in the fourth stage. Since the household from island $k^c(i, t)$ at this stage knows her consumption, nominal price, nominal wage and labor supply, the nominal wage (and hence the marginal cost) will perfectly reflect the nominal marginal rate of substitution of the working household.

Consider households from island i at their consuming stage now. They know the preference shock but not the technology shock, so they try to infer its value from their exogenous signals,

but also from market signals. Indeed, while shopping, they observe the price of the final good on island $l^c(i, t)$, which is a source of information on technology. However, because prices are conditional on the island's private information, they are imperfect signals of technology. As a result, households are not able to fully disentangle the fundamental shock from the noise. Errors on the technology shock will then drive excessive optimism or pessimism about supply among households.

Finally, when setting the interest rate in stage two, the central bank observes the public signals but does not have access to the agents' private information. Thus, despite being observed by households at the shopping stage, the interest rate does not convey any additional information.⁹

For expositional purposes, we first assume that the households' consumption decisions are not conditional on the price observed on island $l^c(i, t)$, where they shop. As a result, consumption depends only on the exogenous signals and on the interest rate (observed in the second stage). This would imply that consumption decisions are made before the third stage, that is, before participating to the good market, as described in Figure 2. Besides, as a result of our setup where firms set prices before participating to any market, pricing decisions are conditional on exogenous signals only. Therefore, information is mostly exogenous: the only endogenous signal is the interest rate, observed by the households, but it does not improve their information. The agents' information sets are defined precisely in the following assumption:

Assumption 1 (Exogenous information) Define $I_t = \left\{ (s_t^n)_{n=a,b}, (u_{t-1}^n)_{n=a,b}, p_{t-1}, (p_{it})_{i \in [0,1]} \right\}$ as the information set common to the whole economy and $I_{it} = \{x_{it}^a, x_{it}^b, I_t\}$ as the information set common to island i . We have $I_{it}^f = \{\epsilon_t^a, I_{it}\}$, $I_{it}^c = \{\epsilon_t^b, i_t, I_{it}\}$, $I_{it}^w = \{W_{l^w(i,t),t}, P_{l^c(i,t),t} \epsilon_t^b, i_t, I_{it}\}$, $I_{it}^s = \{I_{it}\}$ and $I_t^g = I_t$.

In the exogenous information case, households cannot use prices to infer the technology shock. While not realistic, this assumption is useful to derive closed-form solutions, as the household's consumption decisions are not conditional on endogenous variables. In an extension, we will allow consumers to use their endogenous information to form expectations and show

⁹This assumption reflects the idea that the central bank communication is transparent: upon setting its interest rate, the central bank communicates whatever private information it has, so the central bank information becomes public. It is subsumed in the public signals s_t^a and s_t^b . This assumption also implies that the central bank does not perfectly observe inflation when setting its interest rate, which is realistic. However, in an extension, we show that our results do not hinge on a particular information structure of the central bank.

that our results are still valid.^{10,11}

In the endogenous information case, we assume that consumption decisions are taken as the consumer participates to the good market, as described in Figure 2. In that case, the agents have access to the same exogenous information as described in Assumption 1, but household i , as he participates to the final good market in island $l^c(i, t)$, observes $P_{l^c(i, t), t}$ while deciding how much to shop. We formulate this case in through the following assumption:

Assumption 2 (Endogenous information) *Define I_t and I_{it} as in Assumption 1. We have $I_{it}^f = \{\epsilon_t^a, I_{it}\}$, $I_{it}^s = \{I_{it}\}$, $I_{it}^c = \{P_{l^c(i, t), t}, \epsilon_t^b, i_t, I_{it}\}$, $I_{it}^w = \{W_{l^w(i, t), t}, P_{l^c(i, t), t}, \epsilon_t^b, i_t, I_{it}\}$ and $I_t^g = I_t$.*

2.3 Model's Summary

Except for the information structure, this model is close to Galí (2008). Small-case letters denote variables in log-deviation from their steady-state value. From the households perspective, the Euler equation on consumption and bonds yields:

$$c_{it} = E_{it}^c \{c_{it+1}\} + E_{it}^c \{\pi_{it+1}\} - i_t + u_t^b. \quad (10)$$

where $\pi_{it+1} = p_{l^c(i, t+1), t+1} - p_{l^c(i, t), t}$. The Euler equation depends the expected real interest rate $i_t - E_{it}^c \{\pi_{it+1}\}$, on the expectation about future consumption and on the preference disturbance.

A firm j that is part of the portion $1 - \theta$ of firms who reset their price in period t on island i sets the following price that depends on the expected marginal cost in the period and on the future optimal price (see the Appendix for details):

$$p_{ijt}^* = p_{it}^* = (1 - \beta\theta)[E_{it}^f(w_{it}) - u_t^a] + \beta\theta E_{it}^f(p_{it+1}^*) \quad (11)$$

where w_{it} is determined as follows:

$$w_{it} - p_{l^c(k^w(i, t), t), t} = c_{k^w(i, t), t} + \zeta n_{k^w(i, t), t}. \quad (12)$$

Indeed, production and labor hiring take place in stage 4, when households know their consumption and its price, and obviously know how much they work. Therefore, the competitive

¹⁰In this context, there is asymmetric information between firms and households, and among firms and households as well, which generates higher-order beliefs. Note that the assumption that agents learn past variables reduces the dimensionality issue that is typical of higher-order beliefs. See Woodford (2003), Nimark (2008) and Melosi (2014).

¹¹Note that we do not keep track of past values of u_t^n and x_{it}^n beyond u_{t-1}^n and x_{it-1}^n , $n = a, b$. This is because agents learn the past value u_{t-1}^n , which is enough to summarize the current state of the economy. Keeping track of past shocks would not be relevant in that case.

wage satisfies exactly their labor supply equation. Since it is household $k^w(i, t)$ who works in island i in period t , the wage in i depends on the price in island $l^c(k^w(i, t))$, and the consumption and labor of household $k^w(i, t)$.

The price of the final good on island i is $P_{it} = \left(\int_0^1 (P_{ijt})^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$. The log-linearization of this equation gives us $p_{it} = \int_0^1 p_{ijt} dj$. Since firms are identical, p_{ijt} depends only on the last date where j has reset its price, following (11). We can then show that p_{it} on island i is defined by

$$p_{it} = \theta p_{it-1} + (1 - \theta) p_{it}^* \quad (13)$$

Additionally, the production functions and the resource constraints in island i respectively read:

$$n_{k^w(i,t)t} = y_{it} - u_t^a. \quad (14)$$

$$y_{it} = c_{k^c(i,t)t}. \quad (15)$$

Finally, the central bank follows the simple rule (6).

For given functions k^c , k^w , l^c and l^w , given past values p_{t-1} , p_{it-1} , $i \in [0, 1]$, u_{t-1}^a and u_{t-1}^b , given shocks ϵ_t^n , e_t^n , $\{\lambda_{it}^n\}_{i \in [0,1]}$, for $n = a, b$, laws of motion (2) and (5) and given information sets as defined by Assumptions 1 or 2, a period- t equilibrium is defined by quantities $\{c_{it}, y_{it}, n_{it}\}_{i \in [0,1]}$ and prices $\{p_{it}^*, p_{it}, w_{it}\}_{i \in [0,1]}$ and i_t satisfying Equations (6) and (10)-(15).

3 The Model's Predictions

We first derive the model's prediction in our benchmark specification, under exogenous information as described by Assumption 1 and with i.i.d. demand shocks: $\rho_b = 0$. These two assumptions enable us to derive closed-form results. We then relax these assumptions using numerical simulations, and consider several extensions.

3.1 Benchmark case

In the benchmark case, Assumption 1 holds, and $\rho_b = 0$. Denote by $\bar{E}_t^m(\cdot) = \int_0^1 E_{it}^m(\cdot) di$, $m = c, w, f, s$, the aggregate expectations.

Consider the Euler equation (10). Aggregating across households and using (6) and (15), we obtain (see the Appendix for details):

$$y_t = \bar{E}_t^c \{y_{t+1} + \pi_{t+1}\} - \varphi \bar{E}_t^g \{\pi_t\} + u_t^b. \quad (16)$$

Equation (16) corresponds to the aggregate Euler equation, or the New IS. Unlike the traditional New IS, it does not depend on a homogenous expectation of future output and inflation, but on the average of households' expectations.

Using Equations (11)-(15) and aggregating across firms and islands, we obtain the aggregate Phillips curve (see the Appendix for details):

$$\begin{aligned} \pi_t = & \kappa \left(\bar{E}_t^f \{y_t\} - u_t^a \right) + \beta \bar{E}_t^f \{ \pi_{t+1} \} \\ & + \frac{1-\theta}{\theta} \left[\bar{E}_t^f \{ \pi_t \} - \pi_t \right] \\ & + (1-\theta) \beta \bar{E}_t^f \{ p_{it+1}^* - p_{t+1}^* \}. \end{aligned} \tag{17}$$

where $\kappa = (1 + \zeta)(1 - \theta)(1 - \beta\theta) / \theta$. It is determined by firms' expectations. The first term depends on the average firms' expected output gap and the second term depends on the average expectations by firms of future inflation. So far, the system is similar to the bare-bones New Keynesian model.

Dispersed information introduces some additional terms to the Phillips curve. The third term depends on the difference between the average inflation expectations and actual inflation. Indeed, because of dispersed information, there are higher-order beliefs. The firms' expectations about other firms' expectations differ from their own. Thus, the average price will differ from the expected average price. This term represents the fact that, if firms expect that other firms set higher prices, strategic complementarities in price-setting leads them to set higher prices. The fourth term represents the average expected difference between the individual optimal price and the average one. Again, because of dispersed information, firms might expect their optimal price to differ from the average one. This term simply represents the fact that firms are more concerned about their individual future optimal price when setting their current price, as apparent in Equation (11).

Consider now u_t^a and u_t^b . They appear here respectively as a supply shifter (it corresponds to capacity output) and a demand shifter. While supply u_t^a shifts (17), the aggregate Phillips curve, demand u_t^b shifts the aggregate Euler equation (16).

Notice that the demand shifter, everything else equal, has a positive effect on current output. Notably, a shift in households' expectations on future output has a similar effect. This is consistent with Lorenzoni (2009), who shows that supply-noise shocks (i.e. overly optimistic expectations on future output) have an effect on the economy that is observationally equivalent to demand shocks. Similarly, the supply shifter has a negative effect on inflation, while a shift in firms' expectations on current aggregate demand has a positive effect. Consistently, we will show that positive demand-noise shocks (i.e. overly optimistic expectations on demand) have an effect that is observationally equivalent to negative supply shocks.

Output and inflation We first solve for the equilibrium output and inflation y_t and π_t , and then infer the corresponding expectation errors $y_t - E_t^s(y_t)$ and $\pi_t - E_t^s(\pi_t)$. We derive the following Lemma:

Lemma 1 *Under Assumption 1 and $\rho_b = 0$, the equilibrium output and inflation are*

$$\begin{aligned}
y_t &= u_{t-1}^a + \frac{\delta_{a0} + \kappa\varphi\delta_{a0}^g(1-\delta_{a1})}{1+\kappa\varphi} S_t^a + \delta_{a1}\epsilon_t^a \\
&\quad - \frac{\kappa\varphi(\delta_{0b} + \theta\delta_{1b}\delta_{0b}^g)}{(1+\kappa\varphi)[1-(1-\theta)\delta_{1b}]} S_t^b + \epsilon_t^b \\
\pi_t &= \kappa \left[\frac{\delta_{a0} + \kappa\varphi\delta_{a0}^g(1-\delta_{a1})}{1+\kappa\varphi} S_t^a - (1-\delta_{a1})\epsilon_t^a \right] \\
&\quad + \frac{\kappa}{1-(1-\theta)\delta_{1b}} \left[\frac{\delta_{0b} - \theta\kappa\varphi\delta_{1b}\delta_{0b}^g}{1+\kappa\varphi} S_t^b + \theta\delta_{1b}\epsilon_t^b \right]
\end{aligned} \tag{18}$$

with $\delta_{0j} = (\sigma_{j0})^{-2}/[(\sigma_j)^{-2} + (\sigma_{j0})^{-2} + (\sigma_{j1})^{-2}]$, $\delta_{1j} = (\sigma_{j1})^{-2}/[(\sigma_j)^{-2} + (\sigma_{j0})^{-2} + (\sigma_{j1})^{-2}]$ and $\delta_{0j}^g = (\sigma_{j0})^{-2}/[(\sigma_j)^{-2} + (\sigma_{j0})^{-2}]$ for $j = a, b$.

Proof. See the Appendix. ■

It is useful to define the following condition.

Condition 1 $\theta\kappa\varphi\sigma_{1b}^{-2} < \sigma_b^{-2} + \sigma_{0b}^{-2}$.

The effect of shocks on output and inflation is then summarized by the following proposition.

Proposition 1 (Responses of output and inflation) *Using Lemma 1, we establish that:*

- (i) *Fundamental supply shocks ϵ_t^a have a permanent, positive effect on output y_t and a negative effect on inflation π_t .*
- (ii) *supply-noise shocks e_t^a have a temporary, positive effect on output and a positive effect on inflation.*
- (iii) *Fundamental demand shocks ϵ_t^b , have a temporary, positive effect on output and a positive effect on inflation.*
- (iv) *demand-noise shocks e_t^b , have a temporary, negative effect on output. They have a positive effect on inflation if and only if Condition 1 is satisfied.*

Proof. For results (i)-(iii) and for the first part of (iv), we use y_t and π_t as defined in Lemma 1 and use the fact that $0 < \delta_{0j} < 1$, $0 < \delta_{1j} < 1$ and $0 < \delta_{0j}^g < 1$ for $j = a, b$. The second part of (iv) derives from the fact that the effect of demand noise shocks on inflation depends on the sign of $\delta_{0b} - \theta\kappa\varphi\delta_{1b}\delta_{0b}^g$, which is of the same sign as $\sigma_b^{-2} + \sigma_{0b}^{-2} - \theta\kappa\varphi\sigma_{1b}^{-2}$. The permanent and temporary effect of shocks come from the nature of u_t^a , ϵ_t^b , e_t^a and e_t^b . ■

Result (i) is standard in New Keynesian models: a positive productivity shock has a permanent, positive effect on output and a negative effect on inflation. First, consumers increase their consumption because they receive a positive signal about productivity, which is permanent. Firms decrease their prices as a response to a lower marginal cost. The negative response of the policy rate to this deflation further stimulates demand.

Note that result (ii) is reminiscent of Lorenzoni (2009), that is supply-noise shocks behave as demand shocks. As in the case of a fundamental productivity shock, consumers increase their consumption because they receive a positive signal about productivity. Since this increase in demand is not matched by an actual increase in productivity, firms increase their prices in expectation of an increase in the marginal cost.

Result (iii), which states that fundamental demand shocks are both expansionary and inflationary, is also standard in New Keynesian models. Here, these shocks have a direct, positive effect on aggregate demand. As firms receive a positive signal about aggregate demand, they increase their prices in expectation of higher marginal costs.

Result (iv), which describes the effects of demand-noise shocks, is new. Namely, a positive noise shock provokes a decrease in output. The central bank increases the interest rate, as it anticipates a boost in demand and therefore a price increase. Consumers respond to this increase in interest rate by decreasing consumption, which generates a decrease in output. We will show below that this result is more ambiguous when firms also make quantity decisions, which will affect our identification strategy in Section 4.

The effect of demand-noise shocks on inflation is ambiguous and depends on Condition 1. When firms receive a positive public signal about the demand shock, they anticipate a positive demand shock, but they also anticipate an interest rate increase, which has a negative effect on aggregate demand. The effect on inflation is then positive if firms anticipate an overall rise in aggregate demand. This happens if the policy rate is not too responsive (φ small) and if the firms do not have too much of an advantage in detecting the noise shock as compared to the central bank, hence if the private signals received by firms are not too precise as compared to the public signal, shared by both firms and the central bank. We argue that it is likely to be the case. Consider the coefficient $\theta\kappa\varphi$. It is typically below 1.¹² Therefore, a sufficient condition for demand-noise shocks to be inflationary is that the public signal is more precise than the private signals, so that $\sigma_{1b}^{-2} < \sigma_{0b}^{-2}$. This is a natural assumption, in particular since aggregate signals in fact often arise from a partial aggregation of individual signals.¹³ We therefore assume throughout that Condition 1 is satisfied, so that demand-noise shocks are inflationary.

Note the role played by private and public signals here. To generate an effect of aggregate noise shocks on output and inflation, only public signals are needed. This can be seen by setting δ_{a1} and δ_{1b} (the weights of private signals in agents' expectations) to zero. In this case, e_t^a and e_t^b still affect y_t and π_t in the same way, through the aggregate signals s_t^a and s_t^b . This is intuitive as aggregate noise, by definition, affects only aggregate signals. However, as we will see, private signals play a key role to generate expectation errors.

¹²In our baseline parametrization, described in the Appendix, $\theta\kappa\varphi = 0.55$.

¹³For example, suppose that the public signal is the average of two randomly chosen private signals $x_{j_1t}^b$ and $x_{j_2t}^b$, so that $s_t^b = (x_{j_1t}^b + x_{j_2t}^b)/2 = \epsilon_t^b + (\lambda_{j_1t}^b + \lambda_{j_2t}^b)/2$. Then $\sigma_{0b}^2 = \sigma_{1b}^2/2$, so we do have $\sigma_{1b}^{-2} < \sigma_{0b}^{-2}$.

The role of monetary policy Note that monetary policy is a central channel to understand the effect of noise shocks on the economy. In the case of a supply-noise shock, the central bank, because it expects a deflation, decreases the interest rate, which accentuates the positive response of aggregate demand. What drives the recession after a demand-noise shock is the increase in interest rate due to the expected inflation by the central bank. This policy-driven volatility is in fact a natural result of the Taylor rule and a limited information of a central bank. This exacerbated response of output to noise shocks is the counterpart of the traditional stabilizing role of the monetary policy in presence of fundamental shocks.

Expectation errors The predictions summarized in Proposition 1 are not sufficient to identify fundamental and noise shocks on demand and supply using sign restrictions on output and inflation. While sign restrictions on output and inflation have been widely used to identify supply and demand fundamental shocks, they are not consistent in the presence of noise shocks. Indeed, as highlighted by Lorenzoni (2009), a supply-noise shock behaves like a fundamental demand shock (results (ii) and (iii)) and, as implied by our predictions, a positive demand-noise shock behaves like a negative fundamental supply shock (results (i) and (iv)).¹⁴ In order to broaden our set of identifying assumptions, we can use predictions on the expectation errors of the surveyor $E_t^s y_t - y_t$ and $E_t^s \pi_t - \pi_t$.

The effect of shocks on expectation errors is summarized in the following proposition.

Proposition 2 (Responses of errors in survey expectations) *We establish the following:*

- (i) *Fundamental supply shocks ϵ_t^a have a negative effect on the average survey expectation error on output $\bar{E}_t^s y_t - y_t$ and a positive effect on the average survey expectation error on inflation $\bar{E}_t^s \pi_t - \pi_t$.*
- (ii) *Noise supply shocks e_t^a have a positive effect on the average survey expectation error on output and a negative effect on the average survey expectation error on inflation.*
- (iii) *Fundamental demand shocks ϵ_t^b , have a negative effect on the average survey expectation error on output and a negative effect on the survey average expectation error on inflation.*
- (iv) *Noise demand shocks e_t^b , have a positive effect on the average survey expectation error on output and a positive on the average survey expectation error on inflation.*

¹⁴Note that in our identification strategy, we also use long-run restrictions to identify the fundamental supply shock, which should make the identification of the demand noise easier, based on the responses of output and inflation. However, as we show in the extensions, the negative response of output to this shock is not robust. This implies that, whatever is the identification strategy used for supply shocks, additional restrictions are needed.

Table 1: Sign restriction summary

	y_t	π_t	$E_t^s(y_t) - y_t$	$E_t^s(\pi_t) - \pi_t$
Supply (ϵ_t^a)	+	-	-	+
	(permanently)			
Supply noise (e_t^a)	+	+	+	-
Demand (ϵ_t^b)	+	+	-	-
Demand noise (e_t^b)	-	+	+	+

Proof. Note that, because s_t^a and s_t^b are part of the common information set, Lemma 1 implies

$$\begin{aligned}\bar{E}_t^s y_t - y_t &= \delta_{a1} (\bar{E}_t^s \epsilon_t^a - \epsilon_t^a) + \bar{E}_t^s \epsilon_t^b - \epsilon_t^b \\ \bar{E}_t^s \pi_t - \pi_t &= \kappa \left[-(1 - \delta_{a1}) (\bar{E}_t^s \epsilon_t^a - \epsilon_t^a) + \frac{\theta \delta_{1b}}{1 - (1 - \theta) \delta_{1b}} (\bar{E}_t^s \epsilon_t^b - \epsilon_t^b) \right]\end{aligned}\quad (19)$$

The surveyors' average expectation errors on output and inflation depend on their average expectation errors on fundamental shocks $\bar{E}_t^s \epsilon_t^j - \epsilon_t^j$, $j = a, b$. We have $\bar{E}_t^s \epsilon_t^j - \epsilon_t^j = -(1 - \delta_{0j} - \delta_{1j}) \epsilon_t^j + (\delta_{0j} + \delta_{1j}) e_t^j$. Since $0 < \delta_{0j} + \delta_{1j} < 1$, then the fundamental affects the average error negatively while the noise affects it positively. ■

Table 1 summarizes the sign restrictions in our benchmark specification implied by Propositions 1 and 2.

We find that fundamental shocks that induce a *direct* positive (negative) response of endogenous variables tend to generate a negative (positive) error on these variables by the surveyors. Corresponding noise shocks then generate a positive (negative) error. Said differently, the fundamental shock leads the surveyors to underestimate the actual response of the variable while the noise shock leads them to overestimate it. This implies that supply-noise shocks can be differentiated from fundamental demand shocks through the survey expectation errors. The supply-noise shock drives the surveyors to overestimate output, while the fundamental shock drives them to underestimate it. Besides, a positive demand-noise shock can be distinguished from a negative supply shock by looking at inflation: the former leads the surveyors to overestimate inflation, while the latter leads them to underestimate it. Indeed, a positive demand shock would lead to inflation, so the surveyors anticipate inflation if they get a positive signal about demand. Inflation however does not materialize fully if the signal was in fact driven by noise, which implies that they overestimate inflation. Similarly, a negative supply shock leads to inflation, which will tend to be underestimated by the surveyors.

Importantly, expectation errors arise in this model because of private information. Indeed, if all information were public, then there would be no expectation errors, as all agents would share the same – though possibly noisy – information. This can be seen by setting δ_{a1} and

δ_{1b} (the weights of private signals in agents' expectations) to zero, as would be the case in the absence of private signals x_{it}^a and x_{it}^b . As a result, fundamental supply and supply-noise shocks would have no effect on the expectation errors on output, because consumers and surveyors would share the same information on supply. Similarly, fundamental demand and demand-noise shocks would have no effect on the expectation errors on inflation, because firms and surveyors would share the same information on demand.¹⁵ The fact that firms observe productivity and that households observe preferences constitute also private information, and is at the source of the effect of fundamental supply and supply-noise shocks on the expectation errors on inflation and of the effect of fundamental demand and demand-noise shocks on the expectation errors on output.

3.2 Extensions

We further extend the model by allowing firms to make quantity decisions. We then include other types of shocks and add some persistence by setting $\rho_b > 0$. We also explore other information structures by allowing households to use prices as signals and by examining a case where the central bank is perfectly informed. Finally, we assess the robustness of our predictions in the presence of temporary supply shocks. Some of these extensions are solved numerically. The parametrization is described in the Appendix. The online Appendix details the methodology for numerical simulations.

Quantity decisions by firms In our baseline setup, demand-noise shocks have a negative effect on aggregate demand because firms set higher prices. This aggregate decline in demand hinges on the fact that firms make only pricing decisions. We study here an extension allowing firms to make quantity decisions as well. We show that, under some conditions, the effect on aggregate demand can become positive. However, the effect on inflation and expectation errors remains the same, still allowing identification. In the empirical exercise, we therefore relax the restriction on the effect of demand noise on output. Instead, we let the data determine the sign of the effect.

To introduce quantity decisions by firms, we assume that firms use a quantity X_{ijt} of final good in the individual good production function. The production function of firm j on island i (4) then becomes

$$Y_{ijt} = X_{ijt}^\alpha (A_t N_{k^w(i,t)jt})^{1-\alpha}, \quad (20)$$

with $0 < \alpha < 1$. One can think of X_{ijt} as intermediate input or as an investment that fully depreciates from period to period. Firms make plans on X_{ijt} at the second stage, at the same

¹⁵Note that the firms' and consumers' information need not be superior, as here, surveyors also receive private signals.

time when they set prices, and shop at the third stage, on island i . Hence, the log-linearized equilibrium equation for island i is modified as follows

$$y_{it} = (1 - \tau)c_{k^c(i,t)t} + \tau x_{it}. \quad (21)$$

where $\tau = X/Y$ is the steady-state share of intermediate input in aggregate demand.

Using the firms' optimal choice of intermediate input, and taking the island average, we obtain¹⁶

$$x_{it} = E_{it}^f(y_{it}). \quad (22)$$

Crucially, the demand for intermediate input depends on firms' expectation on the demand for the final good. Combining Equations (22) and (28), we get that $x_{it} = E_{it}^f(c_{k^c(i,t)t})$, so local demand now depends not only on local consumption, but also on firms' expectations on consumption. As explained in the Appendix, under Assumption 1, aggregate demand then follows

$$y_t = (1 - \tau)c_t + \tau \bar{E}_{it}^f(c_t), \quad (23)$$

Aggregate demand depends on aggregate consumption and on the firms' expectations on aggregate consumption.

We can show that the aggregate Euler equation and the aggregate Phillips curve can be written as a function of c_t and π_t only:

$$c_t = \bar{E}_t^c \{c_{t+1} + \pi_{t+1}\} - \varphi E_t^g \{\pi_t\} + u_t^b. \quad (24)$$

$$\begin{aligned} \pi_t &= \kappa(1 - \alpha) \left(\bar{E}_t^f \{c_t\} - u_t^a \right) + \beta \bar{E}_t^f \{\pi_{t+1}\} \\ &\quad + \frac{1-\theta}{\theta} [1 - \alpha(1 - \beta\theta)] \left[\bar{E}_t^f \{\pi_t\} - \pi_t \right] \\ &\quad + (1 - \theta)\beta \bar{E}_t^f \{p_{it+1}^* - p_{t+1}^*\}. \end{aligned} \quad (25)$$

then we use (23) to determine y_t . The Euler equation is the same as before, while the Phillips curve is slightly different. As the share of labor $1 - \alpha$ is lower than one, inflation reacts less to the expected marginal cost of labor $\bar{E}_t^f \{c_t\} - u_t^a$. Notice that when $\alpha = 0$, this system boils down to (16)-(17), with $y_t = c_t$. It is useful to define the following set of conditions:

Condition 2 .

- (i) $\frac{1-\alpha}{1-\alpha(1-\theta)(1-\beta\theta)} \theta \kappa \varphi \sigma_{1b}^{-2} < \sigma_{0b}^{-2} + \sigma_{0b}^{-2}$.
- (ii) $(1 - \alpha) \kappa \varphi \sigma_{1b}^{-2} < \sigma_b^{-2} + \sigma_{0b}^{-2}$.
- (iii) $(1 - \alpha) \kappa \varphi \sigma_{1b}^{-2} < \tau(\sigma_b^{-2} + \sigma_{0b}^{-2})$.

¹⁶See the Appendix for details

We can show that a demand-noise shock is inflationary under Condition 2 (i). This condition boils down to Condition 1 when $\alpha = 0$, and it is less restrictive when $\alpha > 0$.

The key difference is that now aggregate demand y_t depends also on firms' expectations on households' demand $\bar{E}_t^f(c_t)$. We can show that under Condition 2 (ii), demand-noise shocks have a positive effect on firms' demand for intermediate input. Indeed, under this condition, firms believe that monetary policy is not going to offset what they believe is a demand shock. It is more likely to be satisfied if inflation reacts less to the expected marginal cost of labor, that is, if α is large, triggering a milder adverse response of the policy rate. This condition is similar to Condition 2 (i), under which a demand-noise shock is inflationary, except that Condition 2 (i) also depends on nominal rigidities θ .

Suppose now that this condition is satisfied. The overall effect of demand noise shocks on aggregate demand is ambiguous, because the household component reacts negatively while the firms' component reacts positively. We establish in the Appendix that under Condition 2 (iii), aggregate demand responds positively. This condition is more restrictive: the share of intermediate input in aggregate demand τ must also be large enough.

If this condition is satisfied, then demand-noise shocks are both inflationary and expansionary. How can we then distinguish them, empirically, from fundamental demand shocks and from supply-noise shocks? In fact, we show that the expectation errors still follow Proposition 2. Namely, when a demand-noise shock occurs, output and inflation increase, but less than anticipated. This is in contrast to actual demand shocks, which generates more output and inflation than anticipated. In the case of supply-noise shocks, output expectations are also over-optimistic, but not inflation expectations.

Adding other demand shocks We introduce aggregate monetary policy shocks and government spending shocks. We prove that, in the benchmark case with no persistence and exogenous information, these shocks correspond to aggregate demand shocks, as the corresponding fundamental and noise shocks generate a response of output, inflation and expectation errors that is qualitatively the same as preference shocks. Our empirical procedure therefore identifies a large set of demand shocks, and not only preference shocks.

More specifically, the Taylor rule (6) is modified as follows:

$$i_t = i + \varphi E_t^g(\pi_t) - u_t^v, \quad (26)$$

where u_t^v is a monetary policy shifter. This shifter can be viewed as a change in velocity or in the term premium. Alternatively, we could add additional noisy signals received by the central bank on the fundamentals of the economy.

We introduce a government. The government finances spending G_t by raising debt B_t or through taxes: $G_t + B_t = T_t + R_t B_{t+1}$, where $G_t = Y_t \bar{g} e^{u_t^g}$. This means that government

spendings are on average proportional to GDP but G_t/Y_t is subject to a shifter u_t^g . We assume that the government purchases equal amounts of goods in the different islands. Each household i pays tax T_{it} , so that $\int_0^1 T_{it} di = T_t$, which modifies their budget constraint. The resource constraint (15) then becomes:

$$y_{it} = c_{l^c(i,t)t} + \bar{\chi} u_t^g. \quad (27)$$

with $\bar{\chi} = \bar{g}/(1 - \bar{g})$.

We assume that u_t^n , $n = v, g$ follow autoregressive processes:

$$u_t^n = \rho_n u_{t-1}^n + \epsilon_t^n. \quad (28)$$

where ϵ_t^n are gaussian i.i.d. shocks with mean zero and a standard deviation equal to σ_n , $n = v, g$.

Regarding information, all agents (households, firms, surveyors) in the economy observe public signals s_t^n on the fundamental shock ϵ_t^n , $n = v, g$, of the form described in (8), where e_t^n , is a gaussian i.i.d. noise shock with mean zero and a standard deviation equal to σ_{0n} . Besides, on island i , the household, firms and the surveyor receive private signals x_{it}^n on ϵ_t^n , $n = v, g$, of the form described in (9), where λ_{it}^n , is a gaussian i.i.d. shock with mean zero and a standard deviation equal to σ_{1n} and that satisfies $\int_0^1 \lambda_{it}^n di = 0$, for $n = v, g$.

Apart from these new signals, the information structure stays unchanged. The exogenous and endogenous information assumptions (Assumptions 1 and 2) are only slightly modified. Now I_t includes s_t^v and s_t^g and I_{it} includes x_{it}^v and x_{it}^g . Note that, since households observe the interest rate, they can identify the monetary policy shock, so they have an informational advantage over firms regarding this shock, just like the preference shock. However, neither firms nor households have any informational advantage regarding the government spending shock.

If information is exogenous (Assumption 1 holds), then the key equations (16) and (17) write as follows:

$$y_t = \bar{E}_t^c \{y_{t+1} + \pi_{t+1}\} - \varphi E_t^g \{\pi_t\} + u_t^b + u_t^v + \bar{\chi} (u_t^g - \bar{E}_t^c \{u_{t+1}^g\}). \quad (29)$$

$$\begin{aligned} \pi_t = & \theta \left[\kappa \left(\bar{E}_t^f \{y_t\} - u_t^a - \chi \bar{E}_t^f \{u_t^g\} \right) + \beta \bar{E}_t^f \{\pi_{t+1}\} \right] \\ & + (1 - \theta) \left[\bar{E}_t \{\pi_t\} - \pi_t \right] \\ & + \theta(1 - \theta) \beta \bar{E}_t \{p_{it+1}^* - p_{t+1}^*\}. \end{aligned} \quad (30)$$

with $\chi = \bar{\chi}/(1 + \zeta)$. The monetary policy shifter u_t^v plays exactly the same role as the preference shifter u_t^b , by shifting aggregate demand (29). The government spending shifter however plays a dual role. It shifts aggregate demand as well, but only to the extent that the current government spending exceeds the future amount expected by the households. It also plays the role of a supply shifter in (30). Indeed, government spending plays a positive role on labor supply. Therefore, firms take into account the expected government spending when setting prices. Yet,

we show in the Appendix that the predictions applying to the preference shock described in Propositions 1 and 2 apply to the monetary shock as well as government spending shock.

Adding persistence The full model, with all 4 fundamental shocks and 4 noise shocks and exogenous information is simulated, and we allow for some persistence in demand shocks (preference, monetary, government spending). Consistently with the benchmark case and Propositions 1 and 2, the only cases where our empirical predictions do not hold are those where we assume an unrealistically large level of precision for the private signals. In those cases, inflation reacts negatively to demand noise. Otherwise, our predictions remain very robust (see Appendix).

Endogenous information Moreover, we assume that the households use prices as a source of information, so we replace Assumption 1 with Assumption 2. In equilibrium, the observed price is equal to the aggregate price up to some idiosyncratic shocks τ_{it}

$$p_{l^c(i,t)t} = p_t + \tau_{it}$$

where τ_{it} is a function of the idiosyncratic shocks in island $l^c(i, t)$, which is orthogonal to the information of island i 's household. The price $p_{l^c(i,t)t}$ therefore does not perfectly reveal the aggregate fundamental and noise shocks to household i . The model with endogenous information is thus similar to the model with exogenous information, except that it includes additional private information. This case however is not easily tractable. Indeed, aggregation is more complex, and the information structure is endogenous. We therefore solve it numerically. Numerical simulations show that all our results carry through as well (see Appendix).

Better informed central bank In our benchmark model, we assume that the central bank is less informed than the private agents. This assumption is supported by the fact that central banks typically communicate about their assessment of the state of the economy when setting their policy rate, so their information is public. However, it might still be that the central bank keeps part of its information private. Besides, in practice, the central bank has better means than the agents to evaluate aggregate inflation. We therefore explore the case where the central bank is perfectly informed about inflation, and hence sets $i_t = \varphi\pi_t$. Since households can use the interest rate as a source of information, the households would be able to infer the fundamental supply shock, so the supply noise shock would not have any effect. We therefore consider an alternative structure where the aggregate interest rate i_t is observed with an idiosyncratic noise. This case can be rationalized through local credit markets affected by local credit shocks.

We examine numerically the consequences of this assumption on our predictions, and find that it does not change our qualitative results (see Appendix). One difference though is that the interest rate now responds positively to a supply-noise shock (and not negatively), thus

stabilizing output fluctuations due to this shock. The response of the interest rate to the demand noise and hence its negative effect on output are unchanged.¹⁷ Regarding the information structure, this case is very close to a situation where we allowed households to observe another noisy price signal.

Temporary technology shocks In the model, we have considered only permanent technology shocks. We consider in this extension the possibility of temporary technology shocks. Indeed, because in our empirical exercise we rely on long-run restrictions to identify supply shocks, which excludes temporary shocks, we have to rule out the possibility that temporary supply shocks can be confused with other temporary shocks, especially demand-noise shocks. We therefore suppose that the level of technology u_t^a can be written as a function of a permanent component x_t and a temporary one μ_t :

$$u_t^a = x_t + \mu_t \tag{31}$$

where x_t follows a random walk $x_t = x_{t-1} + \epsilon_t^a$, and μ_t is an iid shock with variance σ_{a2}^2 . We assume now that while total productivity u_t^a is publicly observed, its permanent component x_t is known only by firms. Agents receive a public signal on the permanent innovation ϵ_t^a , $s_t^a = \epsilon_t^a + e_t^a$ and a private island-specific signal, $x_{it}^a = \epsilon_t^a + \lambda_{it}^a$. Past values of x are known. The rest of the model is identical to the baseline. The results are shown in the Appendix.

We establish that the fundamental permanent supply shock ϵ_t^a and the supply-noise shock e_t^a have the same effect on output and inflation as in the baseline model. The temporary supply shock μ_t has qualitatively the same effect on output and inflation as a permanent shock. It has a negative effect on inflation because it decreases the current marginal cost. It has a positive effect on output because it increases the households' expectations about the permanent component of technology. As a result, a temporary supply shock cannot be confused with a supply-noise shock or a demand shock, which drive a positive response of both inflation and output. However, because the response of output to a demand-noise shock cannot be constrained, we need additional restrictions to distinguish a demand-noise shock from a temporary supply shock.¹⁸ Importantly, a temporary supply shock makes surveyors over-optimistic about output. As a result, the demand-noise shock cannot be confused with a temporary supply shock based on inflation and the error on output. Indeed, they both drive a positive response of expectation errors on output but an opposite response of inflation (positive in the case of demand noise, negative in the case of temporary supply).

¹⁷Note that the Taylor rule still plays a critical role in generating fluctuations driven by the demand noise. However, in the absence of a *separate* assessment of the shocks, the Taylor rule could still be the optimal policy.

¹⁸In fact, in the benchmark case, as output responds negatively to a demand-noise shock, it looks like a negative temporary supply shock.

To understand, note that output is driven by the households' average expectations about the current permanent component x_t . Since x_{t-1} is known, the households need only to infer ϵ_t^a . Obviously, a temporary supply shock, which moves the observed total productivity innovation $\epsilon_t^a + \mu_t$, has a positive effect on households' expectations about the permanent component and will thus affect positively aggregate demand. From the point of view of households, the temporary supply shock plays the same role as a supply-noise shock. From the point of view of surveyors, it will play the same role as well: a positive temporary shock will lead surveyors to overestimate the permanent component of technology, and hence aggregate demand, as aggregate demand is positively affected by the permanent component.

While the demand-noise shock cannot be confused with the temporary supply shock, it can now be confused with the supply-noise shock. Indeed, a supply-noise shock now drives a positive response of the surveyors' error on inflation. Indeed, the deflationary effect of total productivity is now common knowledge, so it does not generate an error on behalf of surveyors. But surveyors will still infer imperfectly the positive response of aggregate demand to the permanent component, and hence the associated positive response of inflation. Therefore, they will overestimate inflation both following a positive demand-noise shock or a positive temporary supply shock, because these shocks lead surveyors to overestimate the permanent shock.¹⁹ A supply-noise shock can then look like a demand-noise shock, as it generates a positive response of inflation and of the expectation errors on output and inflation. In our estimation strategy, this could lead us to identify supply-noise shocks as demand-noise shocks. This would bias our results towards a positive response of output to the demand-noise shock. However, since we identify a negative response, this possible confusion can only lead to a downward bias in the magnitude of the response, and would make us underestimate the contribution of the demand-noise shock to fluctuations. At worst, our estimated contribution is thus a lower bound to the actual one.

4 Assessing Noise and Fundamental Shocks

We now exploit the theoretical predictions established in the previous section to gauge the contribution of demand and supply shocks to the business cycle. To this aim, we estimate a SVAR model from which structural shocks are identified through sign restrictions derived from the theoretical framework. Nowcast errors provide information regarding the misperception of agents on economic activity. Consistently with our setup, we use this piece of information to

¹⁹Note that this implies that the permanent supply shock leads to a negative response of the error on inflation, which is at odds with Table 1. However, we do not use sign restrictions to identify fundamental supply shocks in our baseline estimation.

disentangle the transmission channels of fundamental and noise shocks. We first describe the estimation strategy before turning to the results.

4.1 Estimation Strategy

We first describe the estimation strategy and the identification restrictions on the structural shocks which lies on a mixture of sign and zero restrictions. We estimate the canonical VAR(p) model

$$Y_t = \tilde{\Phi}(L) Y_t + \nu_t, \quad (32)$$

where $Y_t = (Y_{1,t}, \dots, Y_{n,t})'$ is an $(n \times 1)$ vector of endogenous variables, $\tilde{\Phi}$ is the $(n \times 1)$ matrix of estimated parameters, ν_t is an $(n \times 1)$ vector of reduced-form residuals such that $\nu_t \sim \text{iid}(0, \Sigma)$, with Σ , a symmetric positive definite matrix. Canonical innovations, ν_t , are related to structural innovations, ξ_t , by the following linear combination $\nu_t = \Gamma \xi_t$, where structural shocks are by assumption orthogonalized, such that $\xi_t \sim \text{iid}(0, \mathbf{I}_{n \times n})$ and Γ is a $(n \times n)$ non singular matrix. Relation (??) can be re-written as $\Sigma = \tilde{\Gamma} Q Q' \tilde{\Gamma}'$, where $\tilde{\Gamma}$ is a Choleski decomposition of Σ and Q is an orthonormal matrix (i.e. $Q Q' = \mathbf{I}_{n \times n}$). The QR decomposition is used to find an orthonormal (or rotation) matrix Q . There is an infinite number of possible combinations in Q and therefore the structural shocks are identified by drawing randomly Q and imposing identifying restriction on the impulse response functions (IRFs) of selected variables to shocks.

Our identification strategy requires to express the MA(∞) representation of the VAR(p) model $Y_t = \sum_{i=0}^{\infty} r_i \xi_{t-i}$, where $r_i = \partial Y_{t+i} / \partial \xi_t'$ is interpreted as IRF of the system, Y_{t+i} , to a variation of ξ_t , $\forall i \geq 0$. The estimated IRFs are asymptotically normal (Lütkepohl, 2005).

Following the Monte Carlo strategy suggested by Hamilton (1995), we randomly generate a set of coefficients $\hat{\Phi}(L)$ drawn from the normal distribution of the estimated reduced-form parameters and a matrix $\hat{\Sigma}$ drawn from the asymptotic distribution of the variance-covariance matrix of the reduced-form residuals associated to the canonical VAR (32). We repeat this K times. For each of these K draws, we follow Arias et al. (2016) and draw a rotation matrix Q , then apply a transformation to this matrix in order to satisfy the zero long-run restrictions, and finally build the corresponding IRFs. We select only the set of IRFs among the K draws for which the sign restrictions are satisfied.²⁰ The identification restrictions are detailed below. Therefore, our methodology takes into account both the uncertainty inherent to sign restrictions and the uncertainty of the estimated parameters in (32).²¹

²⁰The methodology is detailed in the online appendix.

²¹In a similar spirit, Arias et al. (2016) build an algorithm where (i) they draw IRFs from the unrestricted posterior distribution of the BVAR parameters, (ii) draw a rotation matrix, (iii) build the corresponding IRFs and (iv) select the IRFs that satisfy the sign restrictions. They argue that this methodology provides an agnostic shocks' identification since the distribution of the IRFs conditional to sign restrictions is used.

The baseline VAR model (32) includes the set of observable

$$Y_t = [\log(h_t), \Delta(y_t - h_t), \pi_t, E_t\{\Delta y_t\} - \Delta\tilde{y}_t, E_t\{\pi_t\} - \tilde{\pi}_t], \quad (33)$$

where $\Delta(y_t - h_t)$ denotes the annualized growth rate of labor productivity, measured as the real output per hour and h_t denotes hours worked.^{22,23} The response of real GDP, y_t , to the shocks are recovered from these two series. Let $(E_t\{\Delta y_t\} - \Delta\tilde{y}_t)$ and $(E_t\{\pi_t\} - \tilde{\pi}_t)$ denote the nowcast error of real GDP growth and GDP deflator inflation, respectively. They are measured as the difference between the nowcast prediction of the variable and the corresponding first-release observation. The nowcast predictions are produced by the SPF of the Fed of Philadelphia.²⁴ The series covers the sample 1968q4-2014q2. However, we restrict ourselves to the period 1983q1-2014q2, because of the changes in the nature of business cycles that happened after the early 1980's.²⁵

We now turn to the identification strategy of the shocks. The structural innovations are

$$\xi_t = [\epsilon_t^a, e_t^a, \epsilon_t^b, e_t^b, \varepsilon_t], \quad (34)$$

where the first four shocks – as defined in Table 1 – are supply (fundamental and noise) shocks and demand (fundamental and noise) shocks, respectively. Notice that ε_t is a remaining unconstrained structural shock which captures unidentified expansionary fluctuations.

As argued earlier, the set of restrictions described in Table 1 is enough to identify our four shocks. Fundamental demand shocks as well as supply noise and demand-noise shocks are then identified by sign restrictions on the IRFs on impact, while we impose all shocks except

²²Both variables are for the business sector and they are taken from the Bureau of Labor Statistics.

²³The effects of TFP shocks on hours series have been extensively debated in the literature since the hours transformation (in level, in first-difference or filtered) might alter the results (see Galí 1999; Francis and Ramey, 2009; Canova and Paustian, 2011, among others). Since this question strays from the scope of our paper, we estimate the SVAR model in level and following Canova and Paustian (2011) we use hours per capita in order to avoid any potential non-stationarity issues. We are not interested in the response of hours to TFP per se, the restrictions impose on hours and productivity are simply used to recover the sign restriction targeted for output. Nevertheless, the online appendix provides the estimation results on hours in the benchmark estimation and when hours are linearly quadratic detrended.

²⁴A nowcast prediction coincide with the median forecast of the variable within the quarter (laid out in the online appendix). The questionnaire is sent at the end of the first month of the quarter and the deadline to submit it is in the middle of the second month of the quarter. At the time of the forecast, the information set of the forecasters consists of data until the previous quarter (included). The realized values of real GDP growth and inflation, $\Delta\tilde{y}_t$ and $\tilde{\pi}_t$, are the first release provided by the Real-Time Data Set of the Federal Reserve Bank of Philadelphia. In the SPF, output is initially measured by GNP, later by GDP. The measures of realized output is adapted accordingly. All variables are expressed as the annualized percentage change with respect to the previous quarter.

²⁵These changes are often attributed to changes in the conduct of monetary policy, see for example Clarida et al. (2000).

Table 2: Baseline Identification Strategy

	Short-run Restriction (impact)				Long-run Restriction
	y_t	π_t	$E_t\{\Delta y_t\} - \Delta \tilde{y}_t$	$E_t(\pi_t) - \tilde{\pi}_t$	$y_t - h_t$
Supply (ϵ_t^a)					+
Supply noise (e_t^a)	+	+	+	-	0
Demand (ϵ_t^b)	+	+	-	-	0
Demand noise (e_t^b)		+	+	+	0
Rest (ε_t)					0

Note: Unfilled elements correspond to unconstrained responses. Labor productivity in the long run is obtained by taking the cumulated response of labor productivity. The response of GDP is constructed by taking the sum of the cumulated response of labor productivity and the response of hours.

the supply shock to have no long-run effect on labor productivity, as presented in Table 2. Consistently with our set-up, we assume that the fundamental supply shock is the only shock that has a permanent effect on labor productivity.²⁶ We relax the restriction of a negative response of GDP to the demand-noise shock, as this effect is ambiguous in theory, see Section 3.2. It can nevertheless be distinguished from other short-term shocks through expectation errors. Since we aim at assessing the contribution of each shock to economic volatility, we have chosen to use all the other robust sign restrictions suggested by our theoretical framework to improve identification.²⁷ As shown by Chari et al. (2008), SVAR models with long-run restrictions suffers from a lag-truncation bias which leads us to select 8 lags to circumvent this issue.

In the online appendix, we follow Forni et al. (2013) who propose a strategy to test the non-fundamentalness of the VAR model. Non-fundamentalness means that the variables used by the econometrician do not contain enough information relative to the set of information that the agent has. In that case, the structural moving average representation of the VAR model is not invertible and it cannot be used to recover the structural shocks. In our approach, survey expectations can help retrieve the fundamental and noise shocks only if they share the

²⁶Besides, we do not need to assume that inflation reacts negatively to a supply shock, as the effect of TFP on inflation is not fully consensual in the literature. Galí (1999), who identifies TFP through long-run restrictions and Basu et al. (2006), who identify it through an adjusted Solow residual, find a clear deflationary effect. However, Dedola and Neri (2007), who identify TFP shocks using sign restrictions based on DSGE models, show that TFP shocks do not conclusively lead to a deflation.

²⁷In the robustness analysis, we relax several restrictions and show that our results still hold.

same public information as the agents of the economy. We show in the online appendix that non-fundamentalness is always rejected (or close to be rejected with a small R^2 as suggested by Beaudry et al. (2016)), involving that the set of information used in the SVAR estimation is sufficient to recover the structural expectation-driven shocks.

4.2 Noise Shocks as Sources of Fluctuations?

We argue that noise shocks, originating from noisy information on supply or demand, matter for the business cycle. To do so, we examine the effects of the fundamental and noise shocks and their contribution to GDP fluctuations. A bunch of extensions to the empirical framework confirms this result.

4.2.1 Benchmark Estimation

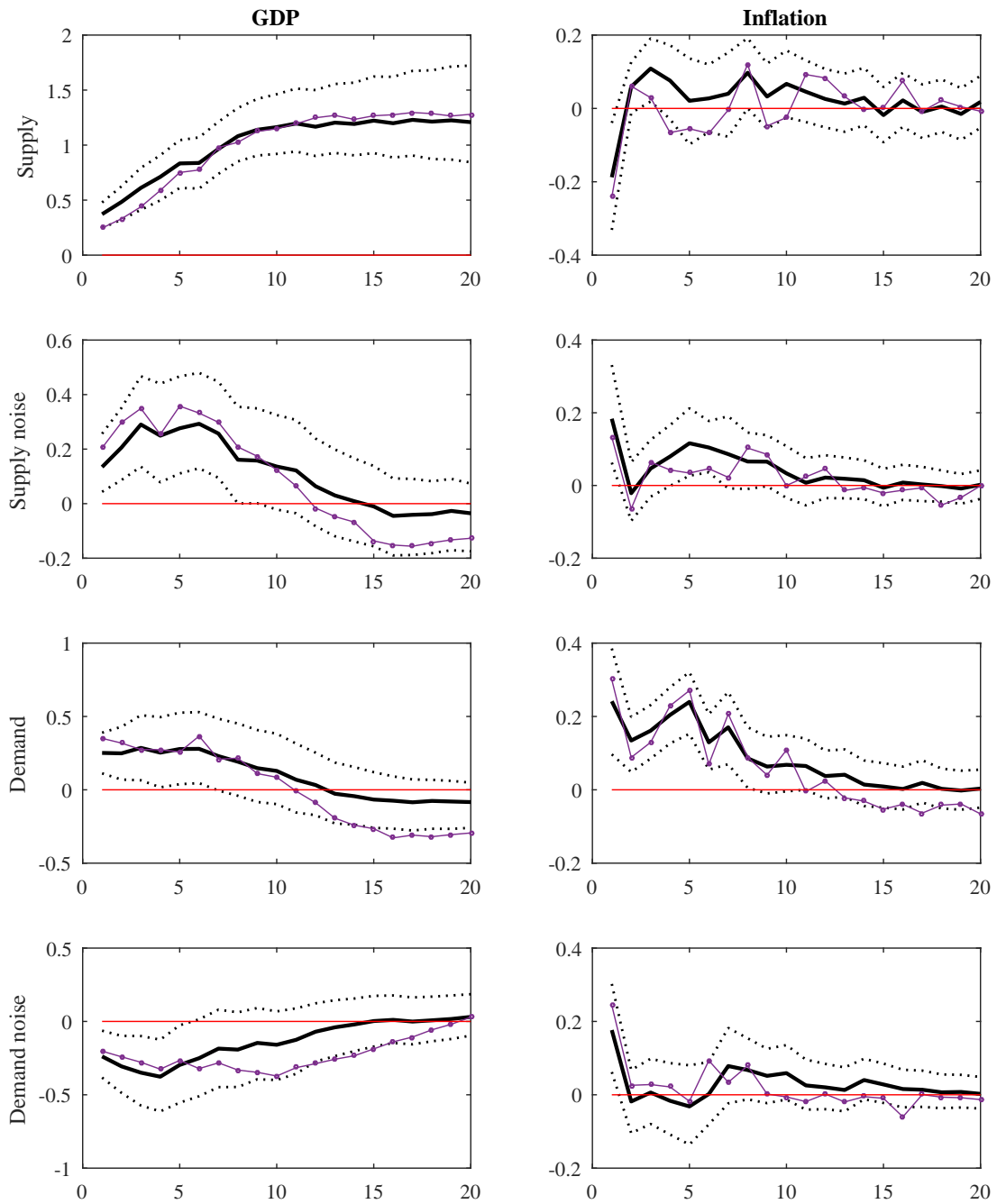
In Figure 3, we focus on the median responses of real GDP and the inflation rate to the fundamental and expectation shocks along with the 16% and 84% quantiles, as suggested by Uhlig (2005). We can interpret these bands as confidence intervals since we resorted to Monte Carlo procedure to take into account the uncertainty of estimated parameters in the SVAR model.²⁸ The use of the median response has been criticized by Fry and Pagan (2011) since it summarizes conditional moments over different draws of Q . They suggest to compute the “median-target” which corresponds to the responses associated to a particular draw which are as close as possible to the medians.²⁹ Looking at inflation, we find that the median-target response (lines with circles) is most of the time close to the pointwise median response (solid lines). Differences are bigger for output responses – at least on the medium-run. In the following, we focus our discussion on the pointwise median responses.

Figure 3 deserves several further comments. First, a supply shock has a permanent effect on output as we restrict it to do so. Second, supply-noise and fundamental demand shocks generate responses of a similar shape on output and inflation, as suggested by Lorenzoni (2009). They are differentiated through their response of the output nowcast error which is of opposite sign (see online appendix). Third, a demand-noise shock generates a rise in inflation in the very short-run, just like the other short-run shocks. However, this shock can be distinguished from a fundamental demand shock because it makes agents over-optimistic about output, and from a supply-noise because it makes agents over-optimistic about inflation.

²⁸The responses of the nowcast errors of these variables are provided in the online appendix.

²⁹In practice, the impulse response functions are standardized in order to be unit-free: for each draw, we subtract the IRF of a variable to a shock from its median and the difference is divided by the standard deviation (computed across draws). We select the draw that minimizes the unconditional distance between all the IRFs and the associated median value.

Figure 3: IRFs - Benchmark Estimation

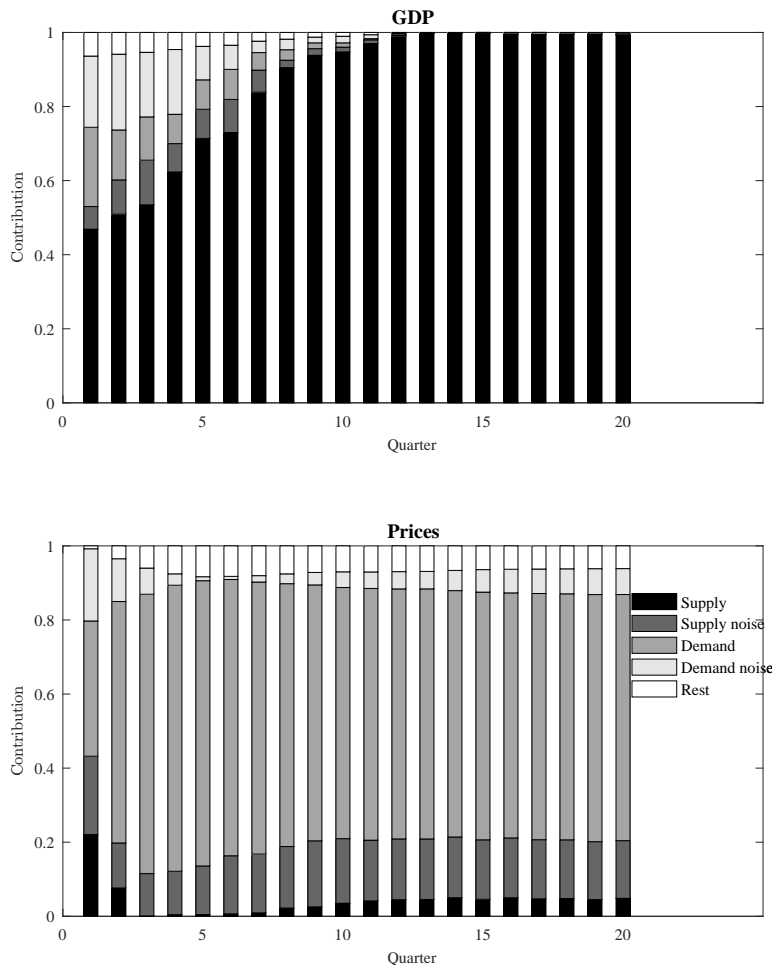


Note: The solid lines depict the median impulse response, The lower (upper) dotted lines indicate the 16th and 84th percentile region. The lines with circles are the median-target responses. Identification restrictions follow Table 2.

Note that these results only reflect our identifying restrictions summarized in Table 2. Consider now the unrestricted responses. Notably, we find that fundamental supply shocks generate a negative response of inflation, confirming the theoretical predictions of our baseline model summarized in Table 1.

The contribution of the shocks to business cycles is also left for the data to decide. The variance decomposition of output and inflation is a useful tool to address the question whether expectation shocks matter for fluctuations. Figure 4 displays the forecast error variance decomposition of GDP and prices to fundamental and noise shocks, based on the median IRF. The figures for GDP are also reported in panel (a) of Table 4.

Figure 4: Variance decomposition - Output and Inflation



Note: The forecast error variance decomposition of variable i for shock j at horizon h is computed as $\Xi_{ijh} = r_{ijh}^2 / \left[\sum_{j=1}^4 r_{ijh}^2 \right]$, where r_{ijh} is the median IRF of variable i for shock j at horizon h .

Noise shocks explain 25% of GDP fluctuations on impact. This value is greater than Enders

et al. (2015) who show that “optimism” (or noise) shocks contribute to 15% of short-term output volatility.³⁰ Blanchard et al. (2013) also investigate in a fully-fledged DSGE model the contribution of noise shocks: they find that they account for 20% of output volatility. Our approach offers a new perspective to these results since we can disentangle supply-driven and demand-driven noise shocks. We show that a substantial part of our result is attributable to demand-noise shocks. Indeed, they contribute to about 10 – 20% of GDP fluctuations over the short and medium run. On the other hand, supply-noise shocks explain 6 – 8% of GDP variance over the short and medium run. Fundamental supply shocks are important drivers of fluctuations all over the cycle while demand shocks contribute mostly – by construction – to short-run fluctuations (20%). The variance of prices tends to be mostly driven by fundamental shocks, especially fundamental demand shocks (about 30% in the short-run and 75% in the long run), but noise shocks explain 40% of their short-run fluctuations. Note that a very little share of the forecast error variance of both GDP and prices is explained by the “rest” shock.

Interestingly, GDP responds on impact relatively more to the fundamental supply shock than to the supply-noise shock, while it is the opposite for prices. This is consistent with our model. Indeed, if households have a fairly good assessment of the supply shock, aggregate demand responds well to fundamental supply shocks. As a result, firms do not expect their marginal cost to change much as a response to a fundamental supply shock, as the increase in the production volume compensates the productivity gains. Yet, they still have to react to an increase in marginal cost due to noise. Similarly, prices respond on impact relatively more to the fundamental demand shock than to the demand-noise, while it’s the opposite for GDP. On the one hand, the relatively accurate reaction of inflation reflects the relatively good assessment of firms about the demand shock. On the other hand, the relatively large reaction of GDP to the noise shock is driven by a relatively poor assessment of the demand shock by the central bank, which sets an excessively high interest rate as a response to a demand-noise shock.

4.2.2 Robustness

We run several robustness checks. Table 4 gives the contributions to GDP variance in the different specifications and the IRFs of output and inflation are provided in the online appendix.

Identification Restrictions The sign restrictions described by Table 2 and used for identification are based on the theoretical predictions. We can relax some of these restrictions in order to test the predictions of our model and check the robustness of the variance decomposition. We relax all the sign restrictions that are not necessary for identification, as summarized in

³⁰Similarly to our results, noise shocks hardly contribute to long-run output volatility by construction, as supply shocks are identified through long-run restrictions.

Table 3: Identification - Minimum set of sign restrictions

	Short-run Restriction (impact)			Long-run Restriction	
	y_t	π_t	$E_t\{\Delta y_t\} - \Delta \tilde{y}_t$	$E_t(\pi_t) - \tilde{\pi}_t$	$y_t - h_t$
Supply (ϵ_t^a)					+
Supply noise (e_t^a)		+	+	-	0
Demand (ϵ_t^b)		+	-	-	0
Demand noise (e_t^b)		+	+	+	0
Rest (ε_t)					0

Note: Unfilled elements correspond to unconstrained responses. Labor productivity in the long run is obtained by taking the cumulated response of labor productivity. The response of GDP is constructed by taking the sum of the cumulated response of labor productivity and the response of hours.

Table 3.

All shocks are restricted to generate positive inflation. Fundamental demand shocks are differentiated from noise shocks because they induce surveyors to be under-optimistic, while noise shocks lead them to be over-optimistic. Noise shocks on demand are then distinguished from noise shocks on supply through expectation errors on inflation. In the case of noise shocks on demand, surveyors are over-optimistic about inflation, while they wrongly anticipate a deflation in case of noise on supply. Results are robust, the main difference is that the response to supply noise is not significant anymore (see online appendix). Besides, panel (b) of Table 4 shows that the contribution of noise shocks to GDP variance is 22%, which is close to the baseline, although noise is almost exclusively due to demand in this case.

Importantly, as we have seen, the positive inflation response to demand noise is not verified for all values of parameters. We therefore relax this assumption. However, in the absence of any other restriction, it is not possible to distinguish the demand-noise shocks from the fundamental shock. We therefore use also the fact that the inflation response to a demand-noise shock is lower than the inflation response to a fundamental demand shock. Apart from this constraint, we leave all the other restrictions unchanged, as it was in Table 2. As shown in the online appendix, inflation still reacts positively to a demand-noise shock, yet not significantly.³¹ Crucially though, as shown in panel (c) of Table 4, the reaction of output is unchanged, and the variance decomposition of GDP is unaffected.

Lastly, we relax the restriction regarding the long-run effect of fundamental supply shock on

³¹Interestingly, the reaction of the survey nowcasts of inflation react more significantly (result not reported).

TFP that we impose in the benchmark estimation. Instead, we use the theoretical predictions of Table 1 to identify the supply shock. The noise and fundamental shocks are therefore identified only through sign restrictions. Besides, in order to sharpen the identification, we additionally impose that the response of GDP to the demand-noise shock is not greater than its response to the fundamental demand shock, which is consistent with the theory. The reaction of output to the demand-noise shock, the only one whose sign is not restricted, is still negative, although not as significantly as in the benchmark case (see online appendix). The supply shock is the most persistent shock, and explains 64% of long-run fluctuations, as represented in panel (d) of Table 4, despite the fact that no long run restriction has been imposed. Together, noise shocks explain a bit less than 25% of short-term fluctuations, with still a larger contribution of demand-noise shocks in the short-term.

Additional Sensitivity Checks One caveat of SVAR models relying on long-run restrictions lies in the difficulty to recover the matrix of long-run (infinite horizon) responses in truncated sample and finite number of lags. We therefore check the sensitivity of our results in these two dimensions. First, we re-estimate the model on the sample 1968q4-2014q3, ignoring therefore the structural break on inflation observed in the 80's. The IRFs of variables are displayed in the online appendix and panel (e) in Table 4 shows the contribution of each shocks to output volatility. The IRFs and variance decomposition are only marginally affected by adding more observations. Second, we increase the number of lags to 12 in order to check whether our results remains stable with a different lag length. Because the estimated short-run impact of supply shocks is lower, the relative share in the variance of GDP of noise shocks, especially demand-noise shocks, is larger (about 40%), as shown in panel (f) of Table 4.

There is a debate on the literature regarding the effect of technology shocks on hours which might be affected by the transformation of hours series. In our benchmark estimation, we normalize hours by the size of the population in order to have stationary series. As a robustness, we adopt an alternative detrending strategy by removing the linear quadratic trend from hours.³² panel (g) in Table 4 show that noise shocks have a similar quantitative effect on GDP than the benchmark case. However, GDP reacts less to the supply shock in the short run (see online appendix). This is due to the strong and persistent negative response of hours (not shown) to the technology shocks that is usually recovered when hours are detrended. As

³²Note that the question of the removal of low frequency variations in hours is not an issue in our approach. First, noise shocks are our primary object of study. Second, we can think of our identified supply shock as including both labor supply and TFP shocks and adopt a more inclusive definition of supply shocks. Our setup could be extended to generate similar sign restrictions in the case of labor supply fundamental and noise shocks. On the opposite, we favor a minimal data transformation in order to obtain a more faithful variance decomposition of GDP.

a result, noise shocks have an even larger contribution to short-run fluctuations. Namely, they account for about 40% of short-term fluctuations, of which 25% is due to demand noise shocks. However, these results have to be taken with a grain of salt, because some potentially important drivers of fluctuations have been removed from the data.

4.2.3 Discussion

Our benchmark estimation highlights the critical role of demand-noise shocks on fluctuations, as summarized in Panel (a) of Table 4. In order to compare our results with the literature, we assess the contribution of the supply-noise shock when we disregard the demand side of the economy, as is usually done. We argue that omitting demand-noise shocks tends to underestimate the contribution of noise shocks as a whole and over-estimate the contribution of the supply-noise shock. To do so, we focus on Enders et al. (2015), as their methodology is closest to ours. Our specification with detrended hours, in particular, can nest their methodology best. We remove inflation and its nowcast error from the VAR to make it more comparable, and focus our estimation on supply shocks and their noise. Following our methodology, we use long-run restrictions to identify the fundamental supply shock and sign restrictions on GDP and the nowcast error on growth to identify the supply-noise shock. The contribution of the supply-noise shocks to short-term fluctuations, reported in panel (h) of Table 4, is larger than in the case with all shocks, reported in panel (g) (23% against 14%). This value is in line with the finding of Enders et al. (2015) and Blanchard et al. (2013). However, the total contribution of noise shocks is smaller (23% against 39%). This result suggests that noise supply shocks capture some, but not all, of the fluctuations inherent to demand driven shocks when the latter are omitted. This can explain why, in our methodology, noise shocks capture a higher share of short-term fluctuations than in the literature.

In this paper, we argue that the effect of demand-noise shocks on GDP is due to the reaction of the policy interest rate. We check the validity of this interpretation by evaluating the effect of the structural shocks we estimated on the Fed Funds rate. We consider the median-target draw from which we first extract the structural shocks series from our benchmark estimation. We then estimate a regression in the spirit of the typical Taylor rule

$$FFR_t = \sum_{j=1}^p \beta_j FFR_{t-j} + RGDP_t + PGDP_t + \gamma shock_t + \varepsilon_{c,t}, \quad (35)$$

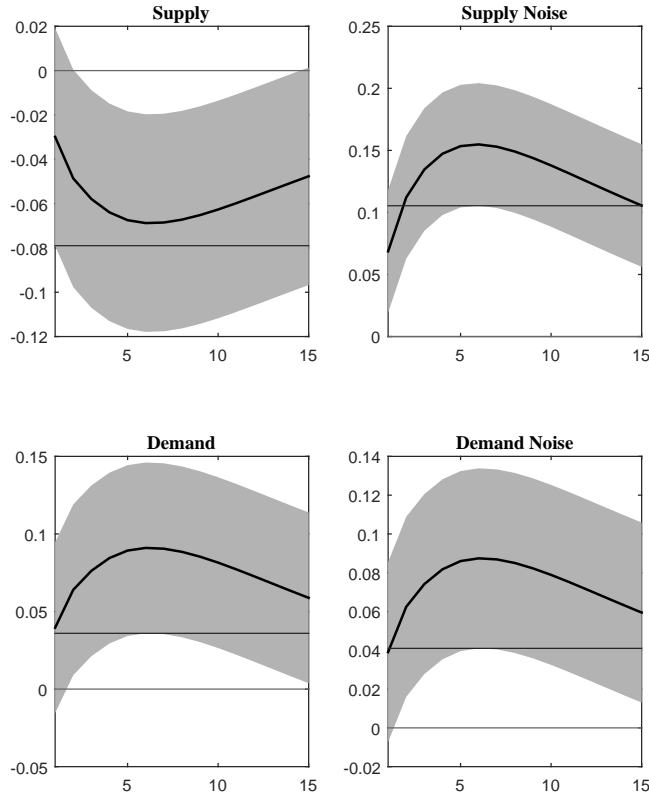
where $shock_t$ is a structural shock and $\varepsilon_{c,t}$ is a vector of residuals. $RGDP_t$ and $PGDP_t$ correspond respectively to the real GDP growth (annualized) obtained from NIPA and the median nowcast prediction of CPI inflation from the SPF database.³³

³³Both measures are included to best approach an actual Taylor rule, our results are robust to different

Table 4: Contribution to GDP variance (in percentage)

		Supply	Supply noise	Demand	Demand noise	Rest
	Horizon					
(a) Benchmark	1	47	6	21	19	6
	5	71	8	8	9	4
	20	99	0	0	0	0
(b) Minimum set of sign restrictions	1	48	1	22	21	8
	5	74	3	8	10	5
	20	99	0	0	0	0
(c) Relax inflation response to demand noise	1	47	6	21	18	8
	5	71	8	8	8	5
	20	99	0	1	0	0
(d) Sign restrictions only	1	24	11	29	15	20
	5	36	18	12	7	26
	20	64	0	13	8	15
(e) Full sample	1	39	4	29	19	8
	5	74	9	9	4	3
	20	99	0	0	1	0
(f) 12 Lags	1	22	9	27	33	10
	5	57	6	11	18	8
	20	99	0	0	0	0
(g) Linear quadratic trend in hours	1	15	14	22	25	24
	5	26	13	2	19	41
	20	89	3	3	5	0
(h) Exclude inflation and nowcast error on inflation	1	73	15	0	0	11
	5	69	16	0	0	15
	20	98	1	0	0	1

Figure 5: Effect of Shocks on the Fed Funds rate.



Note: Estimation of Equation (35). The solid line corresponds to the response of the measure of optimism to noise shocks. The grey area is the 90 percent confidence interval. Both measures of optimism are demeaned. The structural noise shocks are extracted from the median-target draw.

Interestingly, a *recessionary* demand-noise shock generates a significant rise in interest rate. The effect is similar in sign and magnitude to the effect of a positive fundamental demand shock, which suggests that the central bank cannot disentangle fundamental from noise shocks in the case of demand shocks. This is in line with the theory, where the negative effect of a demand-noise shock is driven by policy. Notice that, in contrast, a supply-noise shock has a positive effect on the interest rate, while a fundamental supply shock has a negative effect. This suggests that the central bank has a better assessment of the supply side of the economy than of the demand side, and can better disentangle fundamental and noise shocks in the case of supply shocks. This does not contradict our model, because the response of GDP to the supply noise is mainly driven by the consumers' expectations about TFP, and the increase in interest rate is not enough to counteract the excess demand generated by these expectations. It is also

measures of inflation (current inflation rather than nowcast prediction and GDP deflator rather than CPI inflation).

consistent with the relatively larger estimated contribution of demand-noise shocks to business cycles.

5 Conclusion

This paper assesses the contribution of noise shocks on the business cycle. To the best of our knowledge, this is the first attempt to disentangle supply-related and demand-related noise shocks. Using sign restrictions established through a theoretical model with dispersed information, we show that noise shocks contribute to a large part of output fluctuations, i.e 25% in the short run, and 17% in the medium run (after one year). However, most of this contribution is explained by demand-noise shocks while supply-noise shocks appears to have a negligible impact. Abstracting from demand-driven noise shocks tends to overestimate the role of supply-related noise shocks on the business cycle, while it underestimates the contribution of noise as a whole. Additionally, monetary policy seems to be a key determinant of the effect of demand-noise shocks.

This study opens questions for future research. First, our simple model enables us to derive qualitative restrictions for identification. A richer and more realistic framework could be developed to identify more noise shocks through SVARs or structural estimations of DSGE models, using expectation errors on a larger array of variables. Second, further identifying the information sets of firms, households, government and central banks along these lines, can help design optimal policies that are conditional on both private agents' and policy-makers' imperfect information, thus extending the analysis of Orphanides (2003) and Altavilla and Ciccarelli (2011) in a more structural framework.

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A Proofs

A.1 Firms' price-setting

Firms set prices and supply goods. They observe their individual price and the quantities they supply. They are allowed to reset their price only at random interval with probability $(1 - \theta)$. Let P_{ijt}^* denote the optimal price for firm j on island i that can adjust its price at time t . This firm maximizes over P_{ijt} the following objective

$$E_{it}^f \left\{ \sum_{\tau=0}^{+\infty} \theta^\tau \beta^\tau \lambda_{it+\tau} (P_{ijt+\tau} Y_{ijt+\tau} - W_{it+\tau} N_{kw(i,t)jt+\tau}) \right\},$$

subject to $P_{ijt+\tau} = P_{ijt}$, technology (4) and individual demand $Y_{ijt} = \left(\frac{P_{ijt}}{P_{it}} \right)^{-\gamma} C_{k^c(i,t)t}$. The term between brackets corresponds to the period nominal profits, composed of nominal sales, minus the nominal wage bill. These profits are discounted by the probability θ^τ that price P_{ijt} is still in place and by the stochastic discount factor for nominal profits $\beta^\tau \lambda_{it+\tau}$, where $\lambda_{it+\tau} = P_{it} C_{it} / P_{it+\tau} C_{it+\tau}$ is the multiplier of the budget constraint in $t + \tau$ in household i 's Lagrangian.

Maximizing the objective and linearizing the result yields

$$p_{ijt}^* = p_{it}^* = (1 - \beta\theta) \sum_{\tau=0}^{+\infty} (\beta\theta)^\tau E_{it}^f (w_{it+\tau} - u_{t+\tau}^a)$$

which implies

$$p_{it}^* = (1 - \beta\theta) E_{it}^f (w_{it} - u_t^a) + \beta\theta E_{it}^f (p_{it+1}^*)$$

A.2 Derivation of the New Keynesian model with dispersed information (Equations (16) and (17))

Here we derive the aggregate Euler equation (16) and the aggregate Phillips curve (17), which are obtained under the benchmark case, that is, under exogenous information as described by Assumption 1 and with i.i.d. demand shocks: $\rho_b = 0$.

Consider first the Euler equation (10). We can write $p_{l^c(i,t)t} = p_t + \xi_{it}^1$ where ξ_{it}^1 is a function of the idiosyncratic noise in island $l^c(i, t)$ at date t , and p_t is the average price. Under exogenous information, this noise is orthogonal to the information of household i in stage 2, so $E_{it}^c(p_{l^c(i,t)t}) = E_{it}^c(p_t)$ and $E_{it}^c(p_{l^c(i,t+1)t+1}) = E_{it}^c(p_{t+1})$, hence $E_{it}^c(\pi_{it+1}) = E_{it}^c(\pi_{t+1})$, where π_{t+1} is the average future inflation. Similarly, because of perfect risk-sharing between islands, current idiosyncratic shocks do not affect future consumption, we can write $c_{it+1} = c_{t+1} + \xi_{it+1}^2$ where ξ_{it+1}^2 is a function of the idiosyncratic noise in period $t + 1$. This noise is orthogonal to the information of household i in period t , so $E_{it}^c(c_{it+1}) = E_{it}^c(c_{t+1})$.

Using (6), we then obtain

$$c_{it} = E_{it}^c \{c_{t+1}\} + E_{it}^c \{\pi_{t+1}\} - \varphi E_t^g \{\pi_t\} + u_t^b.$$

Aggregating Equation (15) across islands, we obtain

$$y_t = \int_0^1 y_{it} di = \int_0^1 c_{l^s(i,t)t} di = \int_0^1 c_{it} di = c_t.$$

Then, aggregating the Euler equation and replacing $c_t = y_t$ and $c_{t+1} = y_{t+1}$, we get Equation (16).

Now consider the optimal price (11). The optimal price p_{it}^* depends on the expected nominal marginal cost $w_{it} - u_t^a$. Firms know u_t^a by assumption, but not w_{it} . Plugging Equations (14) and Equation (15) into (12), we can see that the local nominal wage w_{it} is equal to $p_{l^c(k^w(i,t),t)t} + c_{k^w(i,t)t} + \zeta(c_{k^c(i,t)t} - u_t^a)$, so $w_{it} = p_t + c_t + \zeta(c_t - u_t^a) + \xi_t^3$ where c_t is the average consumption and ξ_t^3 is a function of the idiosyncratic noise in islands $l^c(k^w(i, t), t)$, $k^w(i, t)$ and $k^c(i, t)$, which are independent of island i 's firms information at stage 2, so $E_{it}^f(w_{it}) = E_{it}^f[p_t + c_t + \zeta(c_t - u_t^a)]$. Therefore, (11) writes

$$p_{it}^* = (1 - \beta\theta)E_{it}^f [p_t + (1 + \zeta)(c_t - u_t^a)] + \beta\theta E_{it}^f(p_{it+1}^*)$$

Replacing $c_t = y_t$ in the optimal price, and aggregating across islands, we get

$$p_t^* = \int_0^1 p_{it}^* di = (1 - \beta\theta)\bar{E}_t^f [p_t + (1 + \zeta)(y_t - u_t^a)] + \beta\theta\bar{E}_t^f(p_{t+1}^*) \quad (36)$$

Similarly, aggregating prices across islands, we obtain, using (13),

$$p_t = \int_0^1 p_{it} di = \theta \int_0^1 p_{it-1} di + (1 - \theta) \int_0^1 p_{it}^* di = \theta p_{t-1} + (1 - \theta)p_t^* \quad (37)$$

Using (36) and (37) and rearranging:

$$\begin{aligned}
p_t - \theta p_{t-1} &= (1 - \theta)p_t^* \\
&= (1 - \theta) \left[(1 - \beta\theta)\bar{E}_t^f[p_t + (1 + \zeta)(y_t - u_t^a)] + \beta\theta\bar{E}_t^f(p_{it+1}^*) \right] \\
&= (1 - \theta) \left[(1 - \beta\theta)[p_t + (1 + \zeta)\bar{E}_t^f(y_t - u_t^a)] + \beta\theta\bar{E}_t^f(p_{it+1}^*) \right] \\
&\quad + (1 - \theta) \left[(1 - \beta\theta)[\bar{E}^f(p_t) - p_t] + \beta\theta\bar{E}_t^f(p_{it+1}^* - p_{t+1}^*) \right] \\
&= (1 - \theta)(1 - \beta\theta)[p_t + (1 + \zeta)\bar{E}_t^f(y_t - u_t^a)] + \beta\theta\bar{E}_t^f(p_{t+1} - \theta p_t) \\
&\quad + (1 - \theta)(1 - \beta\theta)[\bar{E}^f(p_t) - p_t] + \beta\theta(1 - \theta)\bar{E}_t^f(p_{it+1}^* - p_{t+1}^*) \\
&= (1 - \theta)(1 - \beta\theta)[p_t + (1 + \zeta)\bar{E}_t^f(y_t - u_t^a)] + \beta\theta[\bar{E}_t^f(p_{t+1} - p_t) + (1 - \theta)p_t] \\
&\quad + (1 - \theta)[\bar{E}^f(p_t) - p_t] + \beta\theta(1 - \theta)\bar{E}_t^f(p_{it+1}^* - p_{t+1}^*)
\end{aligned}$$

This yields

$$\begin{aligned}
p_t - p_{t-1} &= \frac{(1-\theta)(1-\beta\theta)(1+\zeta)}{\theta}\bar{E}_t^f(y_t - u_t^a)] + \beta\bar{E}_t^f(p_{t+1} - p_t) \\
&\quad + \frac{1-\theta}{\theta}[\bar{E}^f(p_t) - p_t] + \beta(1 - \theta)\bar{E}_t^f(p_{it+1}^* - p_{t+1}^*)
\end{aligned}$$

Then use $\pi_t = p_t - p_{t-1}$ and $\bar{E}^f(p_t) - p_t = \bar{E}^f(\pi_t) - \pi_t$ (as p_{t-1} is common knowledge) to find

$$\begin{aligned}
\pi_t &= \frac{(1-\theta)(1-\beta\theta)(1+\zeta)}{\theta}\bar{E}_t^f(y_t - u_t^a)] + \beta\bar{E}_t^f(\pi_{t+1}) \\
&\quad + \frac{1-\theta}{\theta}[\bar{E}^f(\pi_t) - \pi_t] + \beta(1 - \theta)\bar{E}_t^f(p_{it+1}^* - p_{t+1}^*)
\end{aligned}$$

which yields (17).

A.3 Proof of Lemma 1

We make the following educated guess:

$$\begin{aligned}
\bar{E}_t^c(y_{t+1}) &= a_{t-1} + \bar{E}_t^c(\epsilon_t^a) \\
\bar{E}_t^c(\pi_{t+1}) &= \bar{E}_t^f(\pi_{t+1}) = 0 \\
\bar{E}_t^f(p_{it+1}^*) &= \bar{E}_t^f(p_{t+1}^*)
\end{aligned} \tag{38}$$

with $\bar{E}_t^c(\epsilon_t^a) = \delta_{a0}s_t^a + \delta_{a1}x_{it}^a$. It follows from (17) that

$$\pi_t = \theta\kappa \left(\bar{E}_t^f \{y_t\} - u_t^a \right) + (1 - \theta)\bar{E}_t^f \{\pi_t\} \tag{39}$$

We then make the guess that $\pi_t = \gamma_{a0}s_t^a + \gamma_{a1}x_{it}^a + \gamma_{0b}s_t^b + \gamma_{1b}\epsilon_t^b$ for some $(\gamma_{a0}, \gamma_{a1}, \gamma_{0b}, \gamma_{1b})$. We then replace our guess (38) in the system (16)-(39) to derive y_t and π_t as a function of shocks and signals. We finally check that our guess (38) is satisfied. The first two equations are straightforward. Using the optimal pricing equation (11), along with (12), (14) and (15), we can show that $E_{it}(p_{it+1}^*) = E_{it}[E_{it+1}(p_{t+1})] = E_{it}(p_{t+1}) = E_{it}(p_t)$. Besides, $E_{it}(p_{t+1}^*) = E_{it}[\bar{E}_{t+1}^f(p_{t+1})] = E_{it}[p_t + \bar{E}_{t+1}^f(\pi_{t+1})] = E_{it}(p_t) + E_{it}[\bar{E}_{t+1}^f(\pi_{t+1})] = E_{it}(p_t)$. Therefore, our guess is fully satisfied.

B Extensions

B.1 Quantity decisions by firms

The New Keynesian model with quantity decisions We first explain how Equations (23)-(25) are obtained. We assume Assumption 1 is satisfied and that preference shocks are not persistent: $\rho_b = 0$.

The optimal choice of intermediate input satisfies

$$x_{ijt} = p_{ijt} + E_{it}^f(y_{ijt}) - p_{it} \quad (40)$$

Note that firms on island i share the same information, so p_{it} is common knowledge. Crucially, the demand for intermediate input depends on firms expectation on the demand for their individual good. Taking the island average, and using the fact that $\int_0^1 p_{ijt} dj = p_{it}$, we obtain (22). Combining Equations (22) and (28), we get that $x_{it} = E_{it}^f(c_{k^c(i,t)t})$, so local demand now depends not only on local consumption, but also on firms' expectations on consumption.

The aggregate resource constraint (23) is obtained by aggregating (28) with $x_{it} = E_{it}^f(c_{k^c(i,t)t})$. Under Assumption 1, consumption of household $k^c(i, t)$ is not conditional on the price p_{it} and depends only on the information specific to island $k^c(i, t)$, so the firm can at best forecast c_t : $E_{it}^f(c_{k^c(i,t)t}) = E_{it}^f(c_t)$. This yields (23).

The aggregate Euler equation stays unchanged. So Equation (24) is simply (16), with y_t replaced by c_t .

The aggregate Phillips curve is obtained through the firm's new pricing equation:

$$p_{ijt}^* = p_{it}^* = (1 - \beta\theta)E_{it}^f[\alpha p_{it} + (1 - \alpha)(w_{it} - u_t^a)] + \beta\theta E_{it}^f(p_{it+1}^*) \quad (41)$$

The marginal cost now depends to a lower extent on the expected nominal marginal cost of labor w_{it} , but it now depends also on the intermediate input cost p_{it} .

Consider the expected nominal marginal cost of labor $w_{it} - u_t^a$. Firms know u_t^a by assumption, but not w_{it} . Plugging the aggregated log-linear version of the production equation (20) $y_{it} = \alpha x_{it} + (1 - \alpha)n_{k^w(i,t)t}$ and the resource equation (28) into the labor supply equation (12), we can see that the local nominal wage w_{it} is equal to $p_{it} l^c(k^w(i,t), t) + c_{k^w(i,t)t} + [\zeta/(1 - \alpha)][(1 - \tau)c_{k^c(i,t)t} + (\tau - \alpha)E_{it}^f(c_{k^c(i,t)t}) - (1 - \alpha)u_t^a]$, so $w_{it} = p_t + [1 + \zeta(1 - \tau)/(1 - \alpha)]c_t + [\zeta(\tau - \alpha)/(1 - \alpha)]E_{it}^f(c_t) - \zeta u_t^a + \xi_{it}^4$ where c_t is the average consumption and ξ_{it}^4 is a function of the idiosyncratic noise in islands $l^c(k^w(i, t), t)$, $k^w(i, t)$ and $k^c(i, t)$, which are independent of island i 's firms information at stage 2, so

$$\begin{aligned} E_{it}^f(w_{it}) &= E_{it}^f \left[p_t + \left(1 + \frac{\zeta(1-\tau)}{1-\alpha} \right) c_t + \frac{\zeta(\tau-\alpha)}{1-\alpha} E_{it}^f(c_t) - \zeta u_t^a \right] \\ &= E_{it}^f(p_t) + \left(1 + \frac{\zeta(1-\tau)}{1-\alpha} \right) E_{it}^f(c_t) + \frac{\zeta(\tau-\alpha)}{1-\alpha} E_{it}^f(c_t) - \zeta E_{it}^f(u_t^a) \\ &= E_{it}^f[p_t + (1 + \zeta)c_t - \zeta u_t^a] \end{aligned}$$

Therefore, (41) writes

$$\begin{aligned} p_{ijt}^* = p_{it}^* &= (1 - \beta\theta)E_{it}^f [\alpha p_{it} + (1 - \alpha)[p_t + (1 + \zeta)c_t - \zeta u_t^a - u_t^a]] + \beta\theta E_{it}^f(p_{it+1}^*) \\ &= (1 - \beta\theta) \left\{ \alpha p_{it} + (1 - \alpha)E_{it}^f [p_t + (1 + \zeta)(c_t - u_t^a)] \right\} + \beta\theta E_{it}^f(p_{it+1}^*) \end{aligned}$$

Taking it from there, we follow similar steps as for the baseline model to derive (25).

The effect of fundamental and noise shocks Note that Equations (24) and 25 can be solved independently from (23), to determine c_t and π_t . We can then derive y_t from c_t and $E_t^f(c_t)$.

Using the same steps as for Lemma 1, we derive the following from Equations (24) and (25):

Lemma 2 *Under Assumption 1 and $\rho_b = 0$, the equilibrium consumption and inflation are*

$$\begin{aligned} c_t &= u_{t-1}^a + \frac{\delta_{a0} + \kappa(1-\alpha)\varphi\delta_{a0}^g(1-\delta_{a1})}{1+\kappa(1-\alpha)\varphi} s_t^a + \delta_{a1}\epsilon_t^a \\ &\quad \frac{-\kappa(1-\alpha)\varphi(\delta_{0b}[1-\alpha(1-\theta)(1-\beta\theta)] + \theta\delta_{1b}\delta_{0b}^g)}{[1+\kappa(1-\alpha)\varphi](1-(1-\theta)[\delta_{1b} + \alpha(1-\beta\theta)(1-\delta_{1b})])} s_t^b + \epsilon_t^b \\ \pi_t &= \kappa(1-\alpha) \left[\frac{\delta_{a0} + \kappa(1-\alpha)\varphi\delta_{a0}^g(1-\delta_{a1})}{1+\kappa(1-\alpha)\varphi} s_t^a - (1-\delta_{a1})\epsilon_t^a \right] \\ &\quad + \frac{\kappa(1-\alpha)}{1-(1-\theta)[\delta_{1b} + \alpha(1-\beta\theta)(1-\delta_{1b})]} \left[\frac{\delta_{0b}[1-\alpha(1-\theta)(1-\beta\theta)] - \theta\kappa(1-\alpha)\varphi\delta_{1b}\delta_{0b}^g}{1+\kappa(1-\alpha)\varphi} s_t^b + \theta\delta_{1b}\epsilon_t^b \right] \end{aligned} \quad (42)$$

with δ_{0j} , δ_{1j} and δ_{0j}^g for $j = a, b$, defined as in Lemma 1.

From Lemma 2, we can derive $E_t^f(c_t)$ and y_t , using (23):

$$\begin{aligned} E_t^f(c_t) &= u_{t-1}^a + \frac{\delta_{a0} + \kappa(1-\alpha)\varphi\delta_{a0}^g(1-\delta_{a1})}{1+\kappa(1-\alpha)\varphi} s_t^a + \delta_{a1}\epsilon_t^a \\ &\quad \frac{\theta[\delta_{0b} - \kappa(1-\alpha)\varphi\delta_{1b}\delta_{0b}^g] + (1-\theta)[1-\alpha(1-\beta\theta)]\delta_{0b}(1-\delta_{1b}[1+\kappa(1-\alpha)\varphi])}{[1+\kappa(1-\alpha)\varphi](1-(1-\theta)[\delta_{1b} + \alpha(1-\beta\theta)(1-\delta_{1b})])} s_t^b + \delta_{1b}\epsilon_t^b \\ y_t &= u_{t-1}^a + \frac{\delta_{a0} + \kappa(1-\alpha)\varphi\delta_{a0}^g(1-\delta_{a1})}{1+\kappa(1-\alpha)\varphi} s_t^a + \delta_{a1}\epsilon_t^a \\ &\quad \frac{\theta[\tau(1+\kappa(1-\alpha)\varphi)]\delta_{0b} - \kappa(1-\alpha)\varphi\delta_{1b}\delta_{0b}^g + (1-\theta)[1-\alpha(1-\beta\theta)]\delta_{0b}(1-[1-\tau(1-\delta_{1b})][1+\kappa(1-\alpha)\varphi])}{[1+\kappa(1-\alpha)\varphi](1-(1-\theta)[\delta_{1b} + \alpha(1-\beta\theta)(1-\delta_{1b})])} s_t^b + [1 - \tau(1 - \delta_{1b})]\epsilon_t^b \end{aligned}$$

Notice that the effect of the supply shock ϵ_t^a and its noise e_t^a has the same effect, qualitatively speaking, on output and inflation.

Consider now the effect of fundamental demand shocks ϵ_t^b . It is straightforward to show that its effect on inflation is unambiguously positive. Some further calculations can also show that its effect on output is also positive.

The effect of the noise shock e_t^b is however ambiguous. Its effect on inflation is of the same sign as $\delta_{0b}[1 - \alpha(1 - \theta)(1 - \beta\theta)] - \theta\kappa(1 - \alpha)\varphi\delta_{1b}\delta_{0b}^g$, which is of the same sign as $[1 - \alpha(1 -$

$\theta)(1 - \beta\theta)](\sigma_{0b}^{-2} + \sigma_{0b}^{-2}) - \theta\kappa(1 - \alpha)\varphi\sigma_{1b}^{-2}$. This implies that demand-noise shock has a positive effect on inflation if and only if Condition 2 (i) is satisfied.

Similarly, the effect of e_t^b on expected consumption $\bar{E}_t^f(c_t)$ is of the same sign as $\theta[\delta_{0b} - \kappa(1 - \alpha)\varphi\delta_{1b}\delta_{0b}^g] + (1 - \theta)[1 - \alpha(1 - \beta\theta)]\delta_{0b}(1 - \delta_{1b}[1 + \kappa(1 - \alpha)\varphi])$ which, we can show, is of the same sign as $\sigma_{0b}^{-2} + \sigma_{0b}^{-2} - \theta\kappa(1 - \alpha)\varphi\sigma_{1b}^{-2}$. This implies that demand-noise shock has a positive effect on expected consumption if and only if Condition 2 (ii) is satisfied. When α , the share of intermediate input in the production function, is sufficiently large, this condition is satisfied.

Finally, the condition for e_t^b to have a positive effect on total output y_t is that $\theta[[\tau(1 + \kappa(1 - \alpha)\varphi)]\delta_{0b} - \kappa(1 - \alpha)\varphi\delta_{1b}\delta_{0b}^g] + (1 - \theta)[1 - \alpha(1 - \beta\theta)]\delta_{0b}(1 - [1 - \tau(1 - \delta_{1b})][1 + \kappa(1 - \alpha)\varphi])$ is positive. This implies that demand-noise shock has a positive effect on output if and only if Condition 2 (iii) is satisfied. Therefore, for demand-noise shocks to have a positive effect on output, we must have not only that the share of intermediate input in production α is large, but also that its share in aggregate demand τ is large.

Regarding the effect of fundamental and noise shocks on expectation errors, we use the fact that s_t^a and s_t^b are common knowledge to derive the following expressions from Lemma 2:

$$\begin{aligned}\bar{E}_t^s y_t - y_t &= \delta_{a1} (\bar{E}_t^s \epsilon_t^a - \epsilon_t^a) + [1 - \tau(1 - \delta_{1b})] (\bar{E}_t^s \epsilon_t^b - \epsilon_t^b) \\ \bar{E}_t^s \pi_t - \pi_t &= \kappa(1 - \alpha) \left[-(1 - \delta_{a1}) (\bar{E}_t^s \epsilon_t^a - \epsilon_t^a) + \frac{\theta\delta_{1b}}{1 - (1 - \theta)[\delta_{1b} + \alpha(1 - \beta\theta)(1 - \delta_{1b})]} (\bar{E}_t^s \epsilon_t^b - \epsilon_t^b) \right]\end{aligned}\tag{43}$$

The surveyors' average expectation errors on output and inflation depend on their average expectation errors on fundamental shocks $\bar{E}_t^s \epsilon_t^j - \epsilon_t^j$, $j = a, b$, as in (19). Therefore, the results of Proposition 2 generalize easily to the case with quantity choices by firms.

B.2 Adding shocks

Here we derive the effect of noise and fundamental shocks affect output, inflation and expectation errors when adding monetary and government spending shocks. We consider the benchmark case, that is, under exogenous information as described by Assumption 1 and with i.i.d. demand shocks: $\rho_n = 0$, $n = b, v, g$.

Using the same steps as for Lemma 1, we derive the following from Equations (29) and (30):

Lemma 3 *Under Assumption 1 and $\rho_b = \rho_v = \rho_g = 0$, the equilibrium output and inflation*

are

$$\begin{aligned}
y_t &= u_{t-1}^a + \frac{\delta_{a0} + \kappa\varphi\delta_{a0}^g(1-\delta_{a1})}{1+\kappa\varphi} s_t^a + \delta_{a1}\epsilon_t^a \\
&\quad \frac{-\kappa\varphi(\delta_{0b} + \theta\delta_{1b}\delta_{0b}^g)}{(1+\kappa\varphi)[1-(1-\theta)\delta_{1b}]} s_t^b + \epsilon_t^b \\
&\quad \frac{-\kappa\varphi(\delta_{0v} + \theta\delta_{1v}\delta_{0v}^g)}{(1+\kappa\varphi)[1-(1-\theta)\delta_{1v}]} s_t^v + \epsilon_t^v \\
&\quad \frac{-\kappa\varphi(\bar{\chi}-\chi)(\delta_{0g} + \theta\delta_{1g}\delta_{0g}^g)}{(1+\kappa\varphi)[1-(1-\theta)\delta_{1g}]} s_t^g + \bar{\chi}\epsilon_t^g \\
\pi_t &= \kappa \left[\frac{\delta_{a0} + \kappa\varphi\delta_{a0}^g(1-\delta_{a1})}{1+\kappa\varphi} s_t^a - (1-\delta_{a1})\epsilon_t^a \right] \\
&\quad + \frac{\kappa}{1-(1-\theta)\delta_{1b}} \left[\frac{\delta_{0b} - \theta\kappa\varphi\delta_{1b}\delta_{0b}^g}{1+\kappa\varphi} s_t^b + \theta\delta_{1b}\epsilon_t^b \right] \\
&\quad + \frac{\kappa}{1-(1-\theta)\delta_{1v}} \left[\frac{\delta_{0v} - \theta\kappa\varphi\delta_{1v}\delta_{0v}^g}{1+\kappa\varphi} s_t^v + \theta\delta_{1v}\epsilon_t^v \right] \\
&\quad + \frac{\kappa(\bar{\chi}-\chi)}{1-(1-\theta)\delta_{1g}} \left[\frac{\delta_{0g} - \theta\kappa\varphi\delta_{1g}\delta_{0g}^g}{1+\kappa\varphi} s_t^g + \theta\delta_{1g}\epsilon_t^g \right]
\end{aligned} \tag{44}$$

with $\bar{\chi}-\chi > 0$, $\delta_{0j} = (\sigma_{j0})^{-2}/[(\sigma_j)^{-2} + (\sigma_{j0})^{-2} + (\sigma_{j1})^{-2}]$, $\delta_{1j} = (\sigma_{j1})^{-2}/[(\sigma_j)^{-2} + (\sigma_{j0})^{-2} + (\sigma_{j1})^{-2}]$ and $\delta_{0j}^g = (\sigma_{j0})^{-2}/[(\sigma_j)^{-2} + (\sigma_{j0})^{-2}]$ for $j = a, b, v, g$.

Notice that the effect of monetary shocks ϵ_t^v is exactly the same as the effect of preference shocks ϵ_t^b . The effect of government spending shocks ϵ_t^g is also the same, up to the coefficients $\bar{\chi}-\chi$ and $\bar{\chi}$. The predictions applying to the preference shock described in Propositions 1 and 2 therefore apply as well to monetary and government spending shocks.

B.3 Temporary technology shocks

We suppose that the level of technology u_t^a can be written as a function of a permanent component x_t and a temporary one μ_t :

$$u_t^a = x_t + \mu_t \tag{45}$$

where x_t follows a random walk $x_t = x_{t-1} + \epsilon_t^a$, and μ_t is an iid shock with variance σ_{a2}^2 . We assume now that while u_t^a is publicly observed, x_t is known only by firms. Agents receive a public signal on ϵ_t^a , $s_t^a = \epsilon_t^a + e_t^a$ and a private island-specific signals on ϵ_t^a , $x_{it}^a = \epsilon_t^a + \lambda_{it}^a$. Past values of x are known. The rest of the model is identical to the baseline.

Using the same steps as for Lemma 1, we derive the following from Equations (29) and (30):

Lemma 4 *Under Assumption 1, the equilibrium output and inflation are*

$$\begin{aligned}
y_t &= u_{t-1}^a + \frac{\delta_{a0} - \kappa\varphi\delta_{a0}^g\delta_{a1}}{1+\kappa\varphi} s_t^a + \delta_{a1}\epsilon_t^a + \frac{\delta_{a2} + \kappa\varphi(1-\delta_{a2}^g\delta_{a1})}{1+\kappa\varphi} (\epsilon_t^a + \mu_t) \\
\pi_t &= \kappa \left[\frac{\delta_{a0} - \kappa\varphi\delta_{a0}^g\delta_{a1}}{1+\kappa\varphi} s_t^a + \delta_{a1}\epsilon_t^a - \left(1 - \frac{\delta_{a2} + \kappa\varphi(1-\delta_{a2}^g\delta_{a1})}{1+\kappa\varphi} (\epsilon_t^a + \mu_t) \right) \right]
\end{aligned} \tag{46}$$

with $\delta_{a0} = (\sigma_{a0})^{-2}/[(\sigma_a)^{-2} + (\sigma_{a0})^{-2} + (\sigma_{a1})^{-2} + (\sigma_{a2})^{-2}]$, $\delta_{a1} = (\sigma_{a1})^{-2}/[(\sigma_a)^{-2} + (\sigma_{a0})^{-2} + (\sigma_{a1})^{-2} + (\sigma_{a2})^{-2}]$, $\delta_{a2} = (\sigma_{a2})^{-2}/[(\sigma_a)^{-2} + (\sigma_{a0})^{-2} + (\sigma_{a1})^{-2} + (\sigma_{a2})^{-2}]$, $\delta_{a0}^g = (\sigma_{a0})^{-2}/[(\sigma_a)^{-2} + (\sigma_{a0})^{-2} + (\sigma_{a2})^{-2}]$ and $\delta_{a2}^g = (\sigma_{a2})^{-2}/[(\sigma_a)^{-2} + (\sigma_{a0})^{-2} + (\sigma_{a2})^{-2}]$.

Note that since $\epsilon_t^a + \mu_t$ and s_t^a are common knowledge, $\bar{E}_t^s(y_t) - y_t = \delta_{a1}[\bar{E}_t^s(\epsilon_t^a) - \epsilon_t^a]$ and $\bar{E}_t^s(y_t) - y_t = \kappa\delta_{a1}[\bar{E}_t^s(\epsilon_t^a) - \epsilon_t^a]$. Finally, $\bar{E}_t^s(\epsilon_t^a) - \epsilon_t^a = -(1 - \delta_{a0} - \delta_{a1} - \delta_{a2})\epsilon_t^a + \delta_{a0}e_t^a + \delta_{a2}\mu_t$.

We can show that under the condition that $\kappa\varphi\sigma_{a1}^{-2} < \sigma_a^{-2} + \sigma_{a0}^{-2} + \sigma_{a2}^{-2}$, which is close to Condition 1, Proposition 1 applies for the permanent supply shock and for the supply-noise shock. Additionally, the temporary supply shock has the same effect as permanent shock on inflation and output. Finally, a permanent supply shock drives a negative response of the expectation errors on output and inflation while a supply-noise shock and a temporary supply shock drive a positive response.

C Numerical simulation

The numerical simulation method is described in details in the online appendix. Here we discuss the parameter assumptions and perform a sensitivity analysis.

C.1 Parametrization

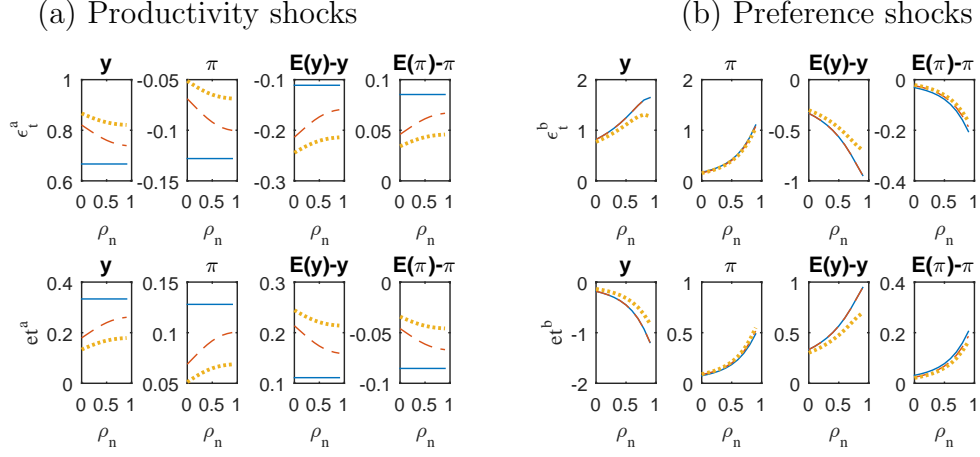
The simulations are run with the parameters described in Table 5. The preference parameters, as well as θ and φ , are standard. The persistence parameters are chosen to generate a high amount of persistence, as is often observed in the data. Finally, \bar{g} is equal to the steady-state ratio of government spending to GDP, so we set it to 0.3, the average government spending share in the US. The variance of the fundamental shocks is normalized to 1. The variance of the aggregate and idiosyncratic noise shocks is set to 1 in the baseline. This implies that the precision of idiosyncratic and public signals is 1.

C.2 Sensitivity analysis

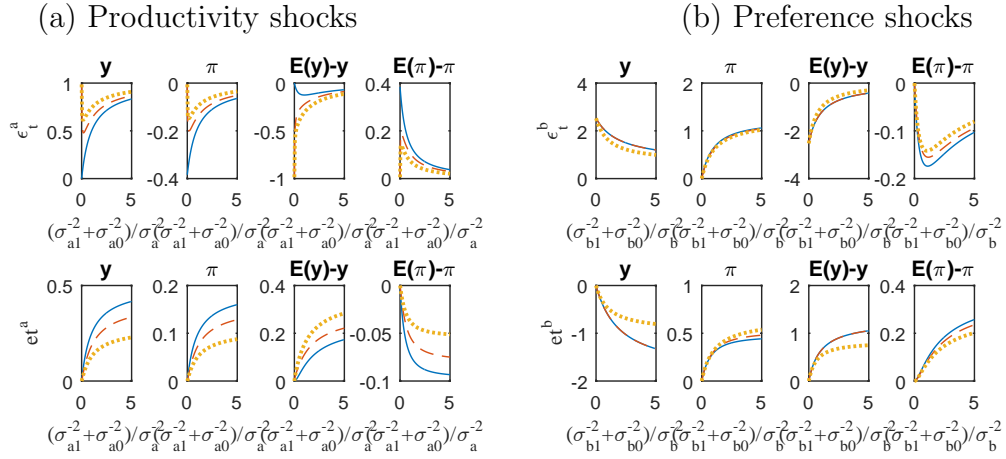
In the sensitivity analysis, we let the persistence parameters ρ_j go from 0 to 0.99. In another exercise, we let the precision of both private and public signals go from 0 to 2. Finally, we let the precision of idiosyncratic signals go from 0 to 5, while maintaining the precision of public signals to 1. The results are represented respectively in panels (a), (b) and (c) of Figure 6. Each panel shows the results under different information sets: exogenous information, endogenous information and full information of the central bank. In this last case, we consider the endogenous information case as well, but where households observe the nominal interest rate with an idiosyncratic noise. The results of this sensitivity analysis is summarized in Section 3.2. Here, because of the lack of space, we represent only the results for productivity and preference shocks. The remaining results are available on request.

Figure 6: Sensitivity analysis

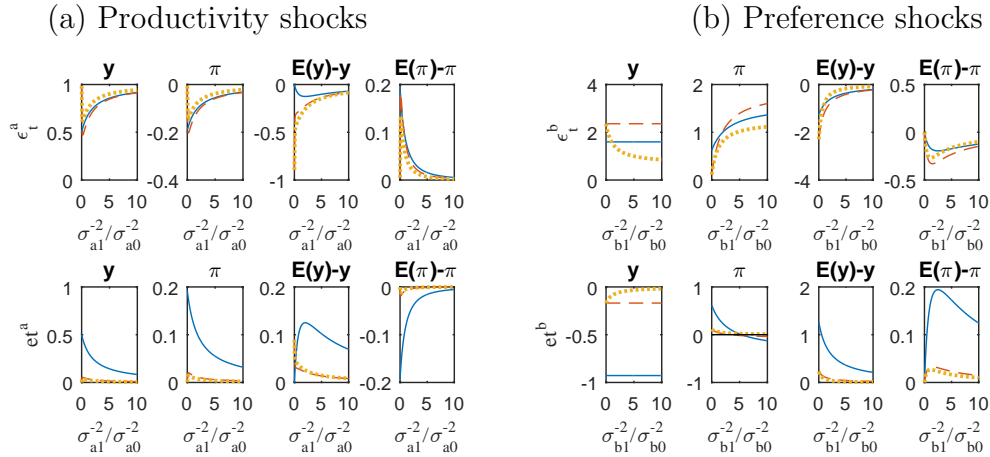
A- Persistence parameter



B- Signal-to-noise ratio



C- Relative precision of private versus public signals



Note: The solid lines represent the case with exogenous information. The dashed lines represent the case with endogenous information. The dotted lines represent the case with full information of the central bank, endogenous information and noisy observation of the nominal interest rate.

Table 5: Baseline parameters

Parameter	Value
β	0.99
θ	0.65
γ	2
ζ	2
\bar{g}	0.3
φ	1.5
$\rho_j, j = b, v, g$	0.8
$\sigma_j^2, j = a, b, v, g$	1
$\sigma_{0j}^2, j = a, b, v, g$	1
$\sigma_{1j}^2, j = a, b, v, g$	1