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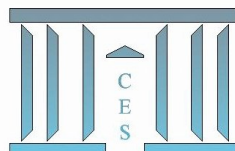
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Abstract

The relation between price elasticities computed at different levels of aggregation is a prominent question in the evaluation of price effects in the empirical literature on households' consumption, production or international trade. We propose to model it under a strong separability assumption in order to estimate price elasticities at a disaggregated level when only price aggregates are available in the dataset. The estimation of price elasticities for aggregate consumptions uses an original domestic production model combining monetary expenditures and time uses for aggregate activities in order to calculate full prices of these aggregates. The method is applied for food expenditures on surveys of two different countries, France and Burkina Faso.

JEL classification: C33, D1, D13, J22

Keywords: domestic production, full price, price elasticity, Pigou law.

1. Introduction

Micro-simulation exercises often oblige to calibrate income and price effects at a precise level of the expenditures than what is available in the dataset, for instance for alcoholic beverages when a specific tax is applied to them. Suppose this specific item i is included in the broad consumption activity (named semi-aggregate commodity) I (food). The total expenditure for the semi-aggregate commodity I writes: $P_I X_I = \sum_{i \in I} p_i x_i$ with P_I and X_I the price and quantity aggregates for I . *Income elasticities* could be estimated by means of the estimation of demand equations for disaggregate items of a semi-aggregate expenditure (using for all demand functions the price index of the broad category to which this item pertains). For such a demand function, biases may occur in the estimation of the income coefficient only in the unlikely case where the price of the broad category is correlated to income, while the specific price for the disaggregate item is not.

As far as price effects are concerned, *own price elasticities* for individual items within the semi-aggregate consumption I may be estimated using the price index of the broad activity P_I . But this would suppose that all the variations in the individual price p_i of the specific expenditure i are reflected in the variations of P_I (which, for instance, is not the case for

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changes in special taxations on goods or services such as tobacco or restaurants). Moreover, the substitutability or complementarity between disaggregate consumptions inside the semi-aggregate I can only be estimated by means of individual prices for these disaggregate items. We propose in this article to examine the relation between the own-price elasticity of the aggregate (which depends on all own *and* cross-price of the items composing the aggregate) and those of the items within this aggregate. Even when data are available on individual prices, differences between the price elasticity of the aggregate and the average of the price elasticities of these items may arise through the substitution and complementarities between these items, which are difficult to estimate. That problem has been well documented as concerns for instance elasticities of imports and exports. Our method is based on the assumption that the structure of price-elasticities for sub-categories calculated under strong separability indicates the true structure of these elasticities, which is supported by the estimation on French data. The estimation of the own price-elasticities for the aggregate serves as an average values over which these disaggregated elasticities are calibrated.

The own-price elasticities of semi-aggregate consumptions such as Food, Transportation or Leisure activities are estimated over full prices which are calculated with an opportunity cost of time estimated using households' monetary expenditures and Time Uses. In one country, the Burkina Faso, both informations are known in the survey over rural households. In France, a Family Expenditure survey, containing monetary expenditures, is matched with a Time Use survey in order to recover time use for each household of the Family Expenditure survey. Both dataset furnish significant and negative own-price elasticities of 3 to 8 semi-aggregate consumptions.

Section 1 defines the price index for the semi-aggregate expenditure I. Section 2 analyses the relation between price elasticities for the semi-aggregate and the individual items of expenditures. Section 3 presents a method to estimate price elasticities for broad consumption activities using a domestic production framework and section 4 applies the method to a French dataset and a Burkina Faso survey.

Section 1. Relation between aggregate and disaggregate price elasticities

First, in order to calculate price elasticities for disaggregate items, we define the price index P_I for the broad consumption activity I as a true cost-of-living index proportional to the cost function $C(p,u)^2$ for consumption I: $P_I = kC(p,u)$. This cost corresponds to the expenditure for prices p at the constant utility level u : $C(p,u) = \sum_{i \in I} p_i q_i$ for the Hicksian demand functions $x_i = h_i(p,u)$. Using the Shephard lemma to calculate the hicksian demand we obtain the elasticity of the aggregate price P_I over the individual price p_i as:

$$E_{P_I/p_i} = \frac{\partial \ln P_I}{\partial \ln p_i} = \frac{k \partial C}{\partial p_i} \frac{p_i}{kC} = \frac{p_i x_i}{P_I x_i} = w_{i/I} \quad (1)$$

² We make here the assumption of a strong separability between expenditures in different semi-aggregate commodities, so that the utility level in this cost function depends only on consumptions in the group I. Another way would be to consider that this utility is conditional to all other expenditures in other broad groups.

where $w_{i/I}$ is the budget share of item i in its semi-aggregate group I , so that the logarithmic price index can be written as an Aftalion-Stone index: $\ln P_I = a_I + \sum_{i \in I} w_{i/I} \ln p_i$.

Second, in order to establish a relationship between the own-price elasticity of the semi-aggregate expenditure and elasticities of individual expenditures, we calculate the derivative of the average aggregate expenditure over its average aggregate price³. As the price index P_I is a geometric mean of individual prices in group I , the average quantity X_I must also be a geometric mean of individual consumptions x_i so that the value of the semi-aggregate expenditure writes also as a geometric mean of individual expenditures. The average semi-aggregated quantity thus writes $\bar{X}_I = \prod_{i \in I} x_i^{w_{i/I}}$ and the value of the semi-aggregated expenditure I is: $P_I X_I = (P_I \bar{X}_I)^{N_I} = \prod_{i \in I} (p_i x_i)^{N_I w_{i/I}}$ with N_I the number of consumption items in the semi-aggregate group I and $N_I w_{i/I}$ the total weight of item i in the semi-aggregate I such as $N_I = \sum_{i \in I} N_I w_{i/I}$. The price elasticity of this average expenditure is:

$$\frac{\partial \ln P_I \bar{X}_I}{\partial \ln P_I} = \frac{1}{N_I} \sum_{i \in I} w_{i/I} \frac{\partial \ln p_i x_i}{\partial \ln P_I} = \frac{1}{N_I} \sum_{i,j \in I} w_{i/I} \frac{\partial \ln p_i x_i}{\partial \ln p_j} \frac{\partial \ln p_j}{\partial \ln P_I} = \frac{1}{N_I} \sum_{i,j \in I} \frac{w_{i/I}}{w_{j/I}} E_{ij} \quad (2)$$

where E_{ij} is the cross-price elasticity of the individual consumption for item i on price p_j and w_i and w_j the budget shares of consumptions i and j over total expenditure.

Third, we assume that individual expenditures are strongly separable within the semi-aggregates and that these semi-aggregates are also separable between them (which amounts to suppose that all individual expenditures are separable). This hypothesis allows computing direct and cross-price elasticities by means of the Frisch formula:

$$E_{ij} = \Phi E_i \delta_{ij} - E_i w_j (1 + \Phi E_j) \quad (3)$$

Here Φ is the Frisch income flexibility and E_i the income-elasticity for good i . Finally, we normalize these disaggregated price elasticities by multiplying them by the ratio of the aggregate elasticity on the left term of equation (2) over the sum of individual elasticities on the right, in order to define price elasticities verifying this additivity constraint.

Re-writing equation (2) for these disaggregated elasticities gives:

$$\begin{aligned} E_{P_I X_I / P_I} &= \frac{1}{N_I} \sum_{i,j} \frac{w_{i/I}}{w_{j/I}} E_{ij} = \frac{\Phi}{N_I} \sum_i E_i - \frac{1}{N_I} \sum_j \sum_{j \in I} w_{i/I} E_i - \frac{\Phi}{N_I} \sum_i \left[\sum_{j \in I} w_{j/I} E_j \right] E_i \\ &= \frac{\Phi}{N_I} \{1 - w_I E_{I/Y}\} \sum_{i \in I} E_i - \frac{1}{N_I} w_I E_{I/Y} = A \quad (4) \end{aligned}$$

³ We restrict the analysis to the case where no substitution or complementarity exist between item pertaining to different semi-aggregate groups.

with $E_{ij} = E_{p_i x_i / p_j}$, $w_{i/I} = \frac{p_i x_i}{\sum_{i \in I} p_i x_i}$, $w_I = \frac{\sum_{i \in I} p_i x_i}{\sum_i p_i x_i}$ and $E_{I/Y}$ is the income elasticity of the semi-aggregate expenditure I.

After this correction, the disaggregated elasticities write finally:

$$E_{p_i x_i / p_j}^* = E_{p_i x_i / p_j} \frac{E_{p_I x_I / p_I}}{A} \quad (5)$$

with $E_{p_I x_I / p_I}$ the estimate of the own price elasticity of the semi-aggregate I.

Section 2. Full price elasticities

The theoretical foundation of the model used for the empirical estimation of the own price elasticity of the semi-aggregate expenditures is based on Gardes (2014; 2016) model, presenting a household domestic production using a direct utility approach (see Appendix A). The new method consists of computing *full prices* for individual agents based on Becker's model of time allocation. A Cobb-Douglas specification being adopted both for the direct utility function depending on the quantities of domestic activities and for the domestic production functions of these activities (which depend on the monetary expenditure and time use for each activity), the opportunity cost of time is the ratio of the marginal utilities of money and time and can be recovered by means of the first order conditions of the optimization of the direct utility (equation (A4) in Appendix A). Then, full prices are calculated supposing either that the two inputs of the domestic productions (money and time) are complements, as in the case examined by Becker (1965), or substitutes, as supposed by Becker and Michael (1973) and Gronau (1977). Both assumptions give rise to similar estimates of the price effects (see Alpman and Gardes, 2016 and Gardes, 2017).

Becker's full price can be written: $p_{Iht}^f = p_{It} + \omega_{ht} \tau_{Ih}$ with τ_{Ih} the time use necessary to produce one unit of activity I. Suppose that a Leontief technology allows the quantities of the two factors to be proportional to the quantity x_{Iht} of activity I:

$$x_{Iht} = \xi_{Ih} z_{Iht} \text{ and } t_{Iht} = \theta_{Ih} z_{Iht}, \text{ so that: } t_{Iht} = \tau_{Ih} x_{Iht} \text{ with } \tau_{Ih} = \frac{\theta_{Ih}}{\xi_{Ih}}$$

This case corresponds to an assumption of complementarity between the two factors in the domestic technology, which allows calculating a proxy for the full price of activity i by the ratio of full expenditure over its monetary component:

$$\Pi_{Iht} = \frac{(p_{It} + \omega_{ht} \tau_{Iht}) x_{Iht}}{p_{It} x_{Iht}} = \frac{p_{It} + \omega_{ht} \tau_{Iht}}{p_{It}} = 1 + \frac{\omega_{ht} \tau_{Iht}}{p_{It}} = \frac{1}{p_{It}} p_{Iht}^f \quad (6)$$

Note that under the assumption of a common monetary price p_I for all households in a survey made in period t , this ratio contains all the information on the differences of full prices between households deriving from their opportunity cost of time ω_h and the coefficient of production τ_{Ih} . With these definitions, it is possible to measure the full prices, observing only monetary and full expenditures. If the monetary prices change between households or periods, the full price can be computed as the product of this proxy π_{Ih} with p_{Iht} : $p_{Iht}^f = p_{Iht}\pi_{Ih}$.

Another definition of full prices under the hypothesis of a complete substitution between the two factors is discussed in Alpman and Gardes (2017). The logarithm of these two full prices differ only by $\beta_I \log \frac{m_{Iht}}{t_{Ih}}$. We choose to estimate the demand system using the proxies of the full prices under the complementary factors hypothesis because these proxies do not depend on the estimates of parameters β_I of the domestic production model presented in Appendix, and are therefore supposed to be more robust estimates of the full prices.

Finally, using full price elasticities E_{X_I/π_I} , the elasticities over the monetary price E_{X_I/p_I} are easily recovered by derivation:

$$E_{X_I/p_I} = E_{X_I/\pi_I} \frac{p_I}{\pi_I} = E_{X_I/\pi_I} \frac{P_I X_I}{\pi_I X_I} = E_{X_I/\pi_I} \frac{P_I X_I}{P_I X_I + \omega t_I} \quad (7)$$

Note that in the three elements of the last term of this formula (the elasticity and the two terms of the ratio corresponding to average monetary expenditures and time uses in the population) are informed by the estimation and the dataset.

Section 3. Application

As an application, we consider a survey made over rural households in Burkina Faso containing both monetary expenditures and time use for three semi-aggregated consumptions: food, domestic activities (housing, moveable and household durables, leisure and other activities (health, transport, culture, leisure and other). This dataset and the estimation of an Almost Ideal Demand System has been described in Gardes and Thiombiano (2016).

Insert Table 1 here

The income flexibility $\Phi = \tilde{\omega}^{-1}$ has been calibrated at the value -0.5 proposed by Frisch for the middle income bracket (see a discussion of this calibration in Selvanathan, 1993 and Theil-Meinsner, 1981). Table 1 describes the sensitivity of demand to changes in income and prices. All these variables have a significant effect on demand and they have the expected effects according to economic theory. As expected, the magnitude of the positive effects of income on demand is greater than that of the negative effects of prices and income elasticities are higher for cereals, drinks, compared to condiments that tend to constitute basic diets. Own-price elasticities are much smaller (in absolute values) for food items (average elasticity

equal to -0.35) compared to consumption related to domestic activities or leisure, transport, health and other activities (respectively -0.67 and -0.70). Note also that own-price elasticities amount in average to 31% of income elasticities for food expenditures, while the ratio is much higher (90% and 88%) for the two last semi-aggregates. Thus, what is named the Pigou law (own-price elasticities equal to the half of income elasticities in absolute value) is more exact in this estimation for the estimates concerning food than for other consumptions.

This method is based on the assumption that the structure of price-elasticities for sub-categories calculated under strong separability indicates the true structure of these elasticities. In order to test this assumption, a similar study has been performed for French data. We use a French dataset from INSEE which combines at the individual level the monetary and time expenditures into a common, unique goods and services consumption structure by a statistical match of the information contained in two surveys: the Family Expenditure Survey (FES, INSEE BDF 2001) and the Family Time Budget (FTB, INSEE BDT 1999). We define 8 types of activities or time use types compatible with the available data both from FES and BDT such as *Eating and cooking time* (FTB) and food consumption (FES). Time uses are defined for each household in the FES by means of the time uses of a similar household in the BDT. The estimations of the full price elasticities are presented in Gardes (2017).

Insert Table 2 here

We compare our estimates for disaggregated food expenditures to the own-price elasticities provided by Darmon (1983) for French time-series of households' expenditures. Two calibrations of the income flexibility $\Phi = \tilde{\omega}^{-1}$ are used: the value -0.5 proposed by Frisch and our estimate -1.18 (s.e.=0.0048) obtained by the estimation of a Rotterdam model on pseudo-panel data based on four French Family Expenditures surveys (see Cardoso and Gardes, 1997). We use in this calculus disaggregated income elasticities estimated in Recours et al. (2006). Our estimate of the own price elasticity of the total food expenditure (-0.27) is close to Darmon's time-series estimate (-0.24 in the short term, -0.30 for the weighted sum of Darmon's partial elasticities for the six items, -0.38 in the long term). The cross-section estimates based on separability and on that aggregate price elasticities of the total food expenditure seem plausible, although they differ a lot between the two calibrations of the income flexibility. They indicate that all food items have negative elasticities which are smaller than 0.5 in absolute value, the larger one characterizing Meat and Fish and Vegetables and Fruits, while the smaller applies for Bread, Oil, Egg, Pastes and Potatoes. Notice that Park et al. (1996, Table 8) obtained (by an estimation on quality adjusted unit values using a Linear Expenditures System) the structure of own-price elasticities for the U.S. for semi-aggregates close to ours: -0.45 for Fish and Meat (same estimate as in France), -0.39 for Cheese and Milk (-0.20 in France), -0.49 for Vegetables and Fruits (-0.38 in France), -0.27 for Bread, Fats, Oil and Breakfast cereals (-0.16 in France).

The calibration of the income flexibility is crucial to calculate these price-elasticities as is shown by the comparison between price elasticities under two alternative calibrations in

Table 2⁴. Our preferred set of estimates for France is obtained for the calibration by our estimate (-1.18) rather than the value of -0.5 proposed by Frisch since price elasticities are closer to those of the literature (see appendix B for the estimation of the income flexibility of a pseudo-panel of four French households expenditures surveys). However, the choice of a unique Φ based on Frisch estimates of the income flexibility may also help for the comparison of price elasticities estimated on different datasets.

In progress: estimation for a Polish panel of matched Time Use and monetary expenditures (1997-2000) and U.S. Consumer Expenditures and Time-use surveys, 2003-2011 (in order to re-calculate on these datasets the income flexibility).

Conclusion

The decomposition of the own-price elasticity of a semi-aggregate consumption allows taking care of all substitutions and complementarities between individual items within this aggregate. There remains the challenge to estimate on the same dataset the income flexibility which proves to influence the estimated elasticities at the disaggregated level. Indeed, our estimation on French data shows that price elasticities based on an estimate of the income flexibility are significantly greater than those obtained using Frisch calibration, and more in line with other estimations in the literature.

Cross-price elasticities can be estimated for a couple of semi-aggregates I, J as far as prices are informed for these consumption. It would be possible to infer by similar methods, perhaps under stronger separability assumptions, the cross-price elasticity between items i, j pertaining each to one of these semi-aggregates.

This method could be applied in the literature on international trade, where price-elasticities of exports and imports depend on the level of disaggregation of commodities. Indeed, the literature shows that this difference modifies the conclusions of models dedicated to trade inside sectors of activity (for instance for different type of cars).

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⁴ Note however that the relative structure of price-elasticities for sub-categories remains quite the same under the two calibrations of the income flexibility.

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Table 1
Own-price elasticities for disaggregated expenditures: Burkina Faso

Disaggregated Expenditures	Income elasticities	Own-price elasticity*	Corrected Own-price elasticity**
Cereals	1.2887 (0.02993)	-0.700	-0.419 (0.0077)
Condiments	0.9555 (0.02619)	-0.522	-0.312 (0.0073)
Drinks	1.1014 (0.026599)	-0.595	-0.356 (0.0072)
Protein food	1.0816 (0.0227)	-0.579	-0.347 (0.0062)
Prepared food	1.0271 (0.0381)	-0.532	-0.319 (0.01056)
Housing	0.9804 (0.01133)	-0.633	-0.939 (0.00025)
Movable and household goods(durables)	0.8565 (0.0539)	-0.558	-0.828 (0.0011)
Energy	0.3032 (0.07516)	-0.157	-0.233 (0.0017)
Health	1.0714 (0.04543)	-0.552	-0.911 (0.0074)
Transport	1.0302 (0.0289)	-0.558	-0.920 (0.0042)
Culture	1.0656 (0.04259)	-0.552	-0.911 (0.0073)
Leisure and other	0.8005 (0.02923)	-0.442	-0.729 (0.0052)

Note: * Own-price elasticities are calculated using Frisch's formula under strong separability assumption (equation 3). The inverse of the income flexibility $\Phi = \tilde{\omega}^{-1}$ is set to the average value proposed by Frisch: -0.5 (see a discussion on Gardes, 2014). Income elasticities used in this formula have been estimated as within estimates on a panel dataset of monetary expenditures for years 2004, 2005 and 2006 in order to correct for usual biases on income elasticities estimated in the cross-section dimension (Gardes et al., 2005).

** Correction of own-price elasticities by equation (5). Bootstrapped standard errors.

Table 2
Own-price elasticities for disaggregated food products: France

Inverse of the income flexibility $\Phi = \check{\omega}^{-1}$	Frisch calibration* -0.5	Estimate** -1.18	Park et al. estimations
Fish and meat	-0.322 (0.011)	-0.565 (0.029)	-0.452
Milk and milky products	-0.140 (0.009)	-0.292 (0.020)	-0.387
Vegetables and fruits	-0.270 (0.009)	-0.568 (0.019)	-0.487
Beverages	-0.160 (0.010)	-0.325 (0.020)	-
Bread, oil, eggs, pasts, potatoes	-0.112 (0.011)	-0.220 (0.021)	-0.270
Other food products	-0.146 (0.009)	-0.316 (0.019)	-

Standard errors into parentheses calculated by bootstrap.

The income flexibility $\Phi = \check{\omega}^{-1}$ in equations (3) and (5) is calibrated * first the average value -0.5 proposed by Frisch (1959) and Selvanathan (1993), Theil et al. (1981); ** second at the value estimated on our French dataset by a Rotterdam model ($\Phi = -1.18$). Park et al. estimates (1996) are calculated by aggregating price-elasticities for disaggregate food items (Table 8 of the article).

Appendix

The model of domestic production (Gardes, 2014, 2017)

The direct utility U depends on the consumption of final goods in quantities z_i which are produced by the household using the monetary expenditures used to buy the market goods and the time used for the corresponding activity (for instance transportation). Cobb Douglas specifications for the utility and the domestic production functions are chosen in order to allow the calculation of the opportunity cost of time as the ratio of the marginal utilities of monetary expenditures m_i and time use t_i for each activity i . Note that all the parameters of these two functions are estimated locally (i.e. for each household in the dataset). The optimization program is (all variables correspond to a household h which index is omitted in the equations):

$$\max_{m_i, t_i} u(Z) = \prod_i a_i z_i^{\gamma_i} \text{ with } z_i = b_i m_i^{\alpha_i} t_i^{\beta_i} \quad (A1)$$

With m_i and t_i the monetary expenditure and time use for activity i , under the full income constraint:

$$\sum_i (m_i + \omega t_i) = w t_w + \omega (T - t_w) + V \quad (A2)$$

with ω the valuation of time in the domestic production $T - t_w = \sum_i t_i = T_d$, w the wage rate, $w t_w$ the household's wage and V other monetary incomes. Note that the opportunity cost of time ω may differ from the market wage w whenever there exist some imperfection on the labor market or if the disutility of labor is smaller for domestic production.

In order to estimate the opportunity cost of time, the utility function is re-written:

$$u(Z_i) = \prod_i a_i z_i^{\gamma_i} = \prod_i a_i b_i \left[\prod_i m_i^{\frac{\alpha_i \gamma_i}{\sum \alpha_i \gamma_i}} \right]^{\sum \alpha_i \gamma_i} \left[\prod_i t_i^{\frac{\beta_i \gamma_i}{\sum \beta_i \gamma_i}} \right]^{\sum \beta_i \gamma_i} = a m'^{\sum \alpha_i \gamma_i} t'^{\sum \beta_i \gamma_i} \quad (A3)$$

with m' and t' the geometric weighted means of the monetary and time inputs with weights $\frac{\alpha_i \gamma_i}{\sum \alpha_i \gamma_i}$ and $\frac{\beta_i \gamma_i}{\sum \beta_i \gamma_i}$. Deriving the utility over income Y and total leisure and domestic production time T_d gives the opportunity cost of time :

$$\omega = \frac{\frac{\partial u}{\partial T_d}}{\frac{\partial u}{\partial Y}} = \frac{\frac{\partial u}{\partial t'} \frac{\partial t'}{\partial T_d}}{\frac{\partial u}{\partial m'} \frac{\partial m'}{\partial Y}} = \frac{m' \sum \beta_i \gamma_i}{t' \sum \alpha_i \gamma_i} \frac{\frac{\partial t'}{\partial T_d}}{\frac{\partial m'}{\partial Y}} = \frac{\sum \beta_i \gamma_i}{\sum \alpha_i \gamma_i} \frac{T_d El_{t'/T_d}}{Y El_{m'/Y}} \quad (A4)$$

The parameters of the utility (γ_i) and domestic production functions (α_i, β_i) are derived by the substitutions, first between time and money resources for the production of some activity, second between money expenditures (or equivalently time expenditures) concerning two different activities. These substitutions imply the system of equations:

$$m_i \gamma_j = m_j \gamma_i + \omega \gamma_i t_j - \omega \gamma_j t_i \quad (\text{A5})$$

which is estimated under the homogeneity constraint of the utility function: $\sum \gamma_i = 1$. In this system, the opportunity cost of time is over-identified, as well as all γ_j . The resulting estimates of the opportunity cost of time ω and the parameters γ_j of the utility function are then used to calculate α_i and β_i for each household and the opportunity cost of time ω_h for each household in the population by equation (A4). These individual values of ω_h are finally used to value time and calculate the full expenditures and the proxies to full prices.

Appendix B

Estimation of the Frisch's income flexibility $\Phi = \tilde{\omega}^{-1}$ (Polish panels)

Frisch proposed in his 1959 paper to compute directly price elasticities by means of the income elasticity of each good combined with the inverse of the income elasticity of the marginal utility $\tilde{\omega}$, named by him income flexibility $\Phi = \tilde{\omega}^{-1}$. The method needs only to suppose additive preferences, either for the direct utility (this is the usual hypothesis we will adopt) or the indirect utility of income and prices. Theil has shown in various studies *based on macro time-series* (see Theil, 1980, Theil-Clements, 1987, Selvanathan, 1993) that the income flexibility is quite stable across time and countries, and averages at -0.5 (see Selvanathan, p. 308, for the discussion of 322 estimates over 18 countries and 29 years). The direct additivity hypothesis yields the formula for Frisch direct elasticity (under constant marginal utility of income, see for instance Frisch, 1959, Ayanian, 1959 and a direct proof in Deaton, 1974, p. 339-340) :

$$E_{ii} = -E_i.[w_i - (1 - w_i E_i) \tilde{\omega}^{-1}]$$

with w_i the budget share for good i and E_i its income elasticity. For goods with a small budget share, an approximate value gives $E_{ii} = E_i / \tilde{\omega}$ which corresponds to the Pigou law of proportionality between income and price elasticities (as the income flexibility Φ is supposed to be constant across the population).

We estimate the income flexibility using a Rotterdam specification, as in Theil and Selvanathan studies. The system writes:

$$w_{it} (Dq_{it} - DQ_t) = \beta_i DQ_t + \Phi (w_{it} + \beta_i) Dp'_{it} \quad (\text{see Selvanathan, pp. 30-31 and Chap. 6})$$

with $Dx_t = \ln(x_t/x_{t-1})$, q_{it} real expenditures by unit of consumption for good i in t , $DQ_t = \sum_i m w_{it} . Dq_{it}$, w_{it} the average budget share for periods t and $(t-1)$ and $Dp'_{it} = Dp_{it} - \sum_j (w_{it} + \beta_i) Dp_{jt}$.

We have estimated this system on the *individual data*⁵ of the Polish panel (1987-1990) for four grouped expenditures: 1. food at home and away, 2. alcohol and tobacco, 3. housing and energy, and 4. all other expenditures (financial products excluded).

⁵ We know no other estimation of the income flexibility on individual data.

Table B1. Estimation of a Rotterdam system

	Coefficient	Income elasticity	Own price Frisch elasticity
(a)Income flexibility Φ	-0.4598 (.202)		
θ_1	0.1744 (.0041)	0.421	-0.33
θ_2	0.0327 (.0015)	0.765	-0.37
θ_3	0.3956 (.0063)	2.694	-1.16
θ_4	0.4390 (.0072)	1.370	-0.79
(b1)Inc. Flexibility Φ	-0.3300 (.732)		
θ_2	0.0398 (.0037)	0.960	-0.31
(b1)Inc. Flexibility Φ	-0.8196 (.322)		
θ_2	0.0388	0.943	-0.78
(c1)Inc. Flexibility Φ	-0.3822 (.376)		
θ_2	0.0377 (.0032)	0.815	-0.34
(c2)Inc. Flexibility Φ	-0.3389 (.304)		
θ_2	0.0357 (.0091)	0.786	-0.29
(c3)Inc. Flexibility Φ	-1.2309 (.446)		
θ_2	0.0355 (.0032)	0.789	-0.97

Population : Years 1987, 88, 89 of the Polish panel

(a) Whole population with all positive expenditures (1 to 4), N = 2185

(b1) head aged less than 30; (b2) between 35 and 60

(c1) Income per U.C. (Y) less than average income(y) minus half of the standard error (σ)

(c2) $y - .5\sigma < Y < y + .5\sigma$

(c3) $Y > y + .5\sigma$

Groups of goods : 1. food at home and away, 2. alcohol and tobacco, 3. housing and energy, and 4. all other expenditures (financial products excluded).

Specification : $w_{it}(Dq_{it} - DQ_t) = \theta_i DQ_t + \Phi\theta_i (Dp_{it} - Dp'_t)$ with $Dp'_t = \sum_j \theta_j Dp_{jt}$

θ_i = marginal share = $w_i + \beta_i$

Income elasticity = θ_i / w_i

The different estimates of the income flexibility are very significant and quite stable for the different populations. The figure for the whole population, - 0.46, is quite similar to the results obtained by Theil-Clements and Selvanathan using macro time-series. This estimate can therefore be used with some confidence to calculate price elasticities under want independency.

An estimation on a French pseudo-panel using the same specification gives an income flexibility equal to -1.18.