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► **To cite this version:**

Maarten Bullynck. The engineer's " peculiar sort of frustration " with Shannon's Boolean algebra.: Tensions between theory and practice in designing switching circuits during the 1940s. 2016. halshs-01541609

HAL Id: halshs-01541609

<https://shs.hal.science/halshs-01541609>

Preprint submitted on 19 Jun 2017

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The engineer's "peculiar sort of frustration" with
Shannon's Boolean algebra
Tensions between theory and practice in
designing switching circuits during the 1940s.

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December 22, 2016

It belongs to the lore of computer science that Claude Shannon's master's thesis from 1937, published 1938 in the *Transactions of the American Institute for Electrical Engineers*, revolutionized the design of (relay) switching circuit design. However, as often is the case when taking a closer look at the historical records, things were slightly more complex. Neither was Shannon's paper an isolated result in switching theory, nor was it immediately absorbed into the engineers' daily practice. As a matter of fact, Shannon's Boolean algebra proved to be only usable in a small number of situations and had to be used in conjunction with other techniques and the engineer's know-how. Boolean algebra would only become more important and more generally useful once standard situations in engineering networks of relays and, later, of vacuum tubes and transistors, would be created.

It is often asserted that Shannon's paper has transformed the art of circuit design into a science, but in fact, it rather is a chapter in a longer process that had started in the 1920s where (U.S.) electrical engineering had been moving on from a practical art to a science aided by mathematics. People like G.A. Campbell had been promoting the use of advanced mathematical techniques like Fourier analysis for electrical engineering since the early 20th century. Other techniques such as matrices, quadratic forms and Heaviside's operational calculus also slowly found their way in the curricula during the 1930s and 1940s. While classically most of these mathematical tools were borrowed from analysis, the synthesis of large switching circuits using electromechanical relays called for techniques from other mathematical domains. Because of the discrete behaviour of relays in circuits, elements of algebra, graph theory or algebraic logic were mostly deployed. As large relay networks became more common in the 1920s, a number of researchers in the U.S. (Shannon, Aiken), in Japan (Nakasima and Hanzawa), in Germany and Austria (Piesch, Plechl, Zuse) and in Russia (Sheshtakoff) had proposed formalisms and techniques usable in the synthesis of relay

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circuits. Shannon’s formalism for relay circuits was thus but one among many other mathematical formats proposed.

Shannon’s theory was not directly or generally picked up by circuit designers.¹ It first had to be translated to a practical context, also, engineers had to be made familiar with Boolean algebra² and other algebraic techniques. It apparently took more than a decade before Shannon’s method became better-known. The delay in absorbing Shannon’s results is not the result of bad communication, Shannon’s article was published in a well-known and respected journal and similar results had been found and published in Japan, Germany-Austria and Russia.³ Rather, one has to look at the concrete problems the circuit engineers were facing. Only in the late 1940s it was successfully applied in practice, and, as a result, became interesting for a larger group of engineers.

1 Ingenuity – bringing theory to practice (1938-1951)

One main practical problem was that Shannon’s algebra could only be applied to a restricted set of switching elements, essentially relays without memory. Even relay circuits with memory (sequential circuits) as well as a lot of other circuit elements, such as flip-flops, rotary switches etc., could not be formalized using Shannon’s symbolism, let alone continuous circuitry such as resistors-capacitor networks, wave filters, non-linear oscillators, etc. Another problem was that Boolean algebra was useful to minimize the number of contacts once a network realising a function had been found, but that it could not be used when one wanted to find a realisation of a given function, nor could it be practically used when the network was too large. “Switching algebra could be used directly for the simplification of contact networks”, but “the situation was different with respect to the synthesis of a network” (Caldwell 1953). Indeed, “design tools for the synthesis of the building blocks [are] outside the province of Boolean algebra.” (Washburn 1953), and when “when problems of any magnitude were attempted, the method broke down both because of the difficulty of writing word statements and because of the difficulty of converting bulky word statements into algebraic expressions.” (Caldwell 1953) Hence, when “one attempted to use the method, there arose a peculiar sort of frustration.”

It was only through the experience gathered during the production of large (relay-based) translation circuits in a telephone system and during the development of digital calculating machines at Harvard and at Bell that Shannon’s ideas became integrated into a more systematical framework that connected

¹Note that this paper focuses on U.S. reception, the evolution might be different in other countries.

²J.L. Martin who wrote a PhD on circuit logic in 1948 writes that “Symbolic logic is a subject that is unfamiliar to most persons in industry.

³For a rich documentation of early publications in algebraic switching theory, see the second part of Stankovic, Radomir S., Astola, Jaakko (eds.), *From Boolean Logic to Switching Circuits and Automata. Towards Modern Information Technology*. Springer, 2011

theory with practice. In these contexts a number of practical tools were developed that could be used in conjunction with Boolean algebra to dispell some of its complexities. In Aiken's team the minimization chart was developed, later perfected by Veitch and adapted in Bell Labs by Karnaugh to the form that is now known as the Karnaugh map (1953). This general development of moving theory into practice is also witnessed by two textbooks that would become classic references. The first is a textbook by Keister et al. (1951) based on a course given by Bell engineers for MIT students in 1949-1950, the second is written by H. Aiken et al. (1951), summarizing their own courses at Harvard from 1947-1948.

The first book on the design of switching circuits written by three Bell engineers sums up years of practical experience, especially in designing the No.5 Crossbar Dial Telephone Switching System.⁴ As they write in their preface: "switching art [...] was limited to a few quarters where complex control mechanisms such as telephone switching systems were developed and used", but now (1951) this has changed because of the "appearance of automatic digital computing systems. Their book adequately describes the standard switching elements and the techniques to synthesize them, including Shannon's method. While manipulating and simplifying networks by "inspection"⁵ "may become tedious and time-consuming", switching algebra offers "an extremely useful design tool for setting up, simplifying and combining complex networks". However, "in its present forms it takes no account of time or sequential relationships." A similar opinion is voiced by Staehler, "switching (boolean) algebra in its present state is not to be considered entirely selfsufficient, but, for the most beneficial results, should be applied, when warranted, in conjunction with inspection techniques so that the latter may fill in any limitations in the algebra techniques which have not been completely systematized as yet due to the newness of this field."⁶ In all cases, the application of switching logic should be guided by the engineers "ingenuity"⁷ that is aided but never replaced by design tools.

The second book is the outcome of a 1947-1948 course on large-scale computing machines at Harvard⁸, based upon their years' long experience with building the Mark I, Mark II relay calculators and especially the Mark III and Mark IV vacuum-tube computers. Instead of Shannon's Boolean algebra, Aiken opts for another formalism, an algebraic approach. Essentially, Aiken uses Boolean algebra but works with multivalued Boolean functions which he calls 'switching functions' and correspond to a vacuum-tube operator, a symbolic circuit and

⁴W. Keister, A. Ritchie, and S. Washburn (1951), *The Design of Switching Circuits* (D. Van Nostrand Company, Inc., New York).

⁵Decribed by Staehler as "a background of considerable experience in that the designer must recognize certain contact network arrangements that may allow further rearrangements and thereby he must mentally develop his own rules"

⁶Staehler, "An Application of Boolean Algebra to Switching Circuit Desing", *BSTJ* (1952)

⁷The word "ingenuity" plays a capital role in engineering literature, pointing to the non-mechanisable aspect of the engineer's job. The word has a long history and has literally contributed to shaping the profession of 'engineer', viz. the one who has ingenuity. See H. Vérin, *La gloire des ingénieurs. L'intelligence technique du XVIe au XVIIIe siècle*, 1993.

⁸H. Aiken et al. (1951) *The Synthesis of Electronic Computing and Control Circuits*.

a schematic circuit. In other words, instead of working with relays as basic elements, Aiken develops an algebraic calculus for vacuum tube operators that stand for larger components, viz. triodes, pentodes etc. Just as Keister et al. the book goes on to give usable circuits for a variety of standard units. Of course, whereas Keister et al. focus on telephone-related units, Aiken rather describes the synthesis of computing units.⁹ Aiken's work was quite influential through his students and through the translation of the book into Russian in 1954.¹⁰

2 Getting out of the black box (1951-1960)

While Aiken and his team propose new notations for vacuum tube units that are used repeatedly in a calculator, Keister et al. advocate design techniques of a more general kind that had been used in the Bell System to synthesize large networks or calculators. Instead of attacking general questions, whether a function can be realized or not (which Shannon likened to "somewhat like proving a number transcendental."), a more pragmatic attitude is taken. That is: split a complex problem up in simpler parts, "in general, the more we can decompose a synthesis problem into a combination of simple problems, the simpler the final circuits."¹¹

From a statement of circuit requirements, a functional plan is developed in terms of known or conceptually evident circuit blocks, representing simple circuits similar to single-function circuits [...] as the design proceeds, the functional blocks are coordinated and integrated to the point where a comprehensive block diagram of the proposed circuit exists. [...] The most satisfactory approach to developing a block diagram is to start with a few main subdivisions of the over-all circuit and successively break these down until each block represents a unifunctional circuit. [...] In a surprisingly large number of cases in the planning, familiar functional circuits are found to be applicable. When a new circuit concept is encountered, the designer can usually recognize whether an appropriate circuit can readily be designed. If this is so, the circuit can be designated on the diagram and the design deferred until later. [...] the attempt should be made to obtain the simplest and most efficient arrangement among the various blocks. [...] the designer should from the start make a conscious effort to familiarize himself with different types of basic circuits already in use and to classify them in terms of function.

⁹The team of Stibitz and Williams who developed the Bell relay calculators proceeded in a similar way, developing basic units for their calculators, but they never published on the topic.

¹⁰See *Early Work in Switching Theory and Logic Design in the USSR*, edited by Radomir S. Stankovic, Jaakko T. Astola, Anatoly A. Shalyto, Alexander V. Strukov (2016) for its reception in the Soviet Union.

¹¹C.E. Shannon (1949), *The synthesis of two-terminal switching circuits* BSTJ.

In this way he develops a constantly growing ‘catalogue’ of circuit building blocks which expedites is planning and design of circuits. (Keister et al.)

Building complex technology out of standard building blocks that could be easily manufactured was an essential part of the Bell System design and development philosophy (as it was of other companies). This explains the importance of Shannon’s switching algebra, because it can be used “to achieve the ultimate in efficiency and economy in that the number of relays used therein approaches the absolute minimum necessary”. Indeed, as another Bell researcher wrote, “better methods for synthesizing any imaginable function whatsoever will be of little help in practice [...] instead one must try to isolate classes of useful switching functions which are easy to build.”¹²

One important feature in the block diagram approach, as described by Keister et al., was the use of ‘black boxes’ for units that were as yet unspecified. This permitted to begin with the overall structure and fill in the gaps later. It was also useful if one had to abstract away from a specific unit (say, an analogue unit), so as to only consider its functioning within a network. The method had its merits for visualizing the general structure, but had its limits when it come to concretize the connections between the boxes. As one engineer remarked: “you cannot calculate with block diagrams.”¹³ A number of mathematically inclined engineers such as Gilbert or Shekel, had been doing trying to get mathematical forms representing particular classes of black boxes, E.F. Moore, who closely collaborated with Shannon in the early 1950s, put the problem in a more general form in his famous paper ‘Gedankenexperiments on sequential machines’ (1954, published 1956)

the machines under consideration are always just what are called ‘black boxes’, described in terms of their inputs and outputs, but no internal construction information can be gained. [...] Suppose an engineer has gone far enough in the design of some machine intended as a part of a digital computer, telephone central office, automatic elevator control etc. to have described hus machine in terms of the list of states and transitions between them [...] he may then wish to perform some gedanken-experiments on his intended machine. If he can find e.g. that there is no experimental way of distinguishing his design from some machine with fewer states, he might as well build the simpler machine.

This leads, in general, to the problem of decomposition of a given circuit (or machine):¹⁴:

¹²E.N. Gilbert (1951), N-Terminal switching circuits BSTJ.

¹³Jeffrey and Reed (1952)

¹⁴This problem was already raised some years before by Shannon: “in general, the more we can decompose a synthesis problem into a combination of simple problems, the simpler the final circuits. The significant point here is that, due to the fact that f satisfies a certain functional relation $f = g(f_1, f_2)$ we can find a simple circuit for it compared to the average

Many problems exist in relation to the question of decomposition [of a machine] into component [machines]. Given a machine with n states, under what conditions can it be represented as a combination of two machines having n_1 and n_2 states such that $n_1 n_2 = n$? [...] One way of describing what engineers do in designing actual automata is to say that they start with an overall description of a machine and break it down successively into smaller and smaller machines, until the individual relays or vacuum tubes are ultimately reached. The efficiency of such a method might be determined by a theoretical investigation on such decompositions. (Moore 1954/1956)

Moore's work, together with Huffman's and Mealy's, would lead to techniques to finally attack the sequential circuits (or automata or machines by that time) using Boolean algebra, a desideratum since the 1940s.¹⁵ "Recent developments in the synthesis of sequential circuits show that the end result of a sequential synthesis is a combinational problem." (Caldwell 1954) Mealy warns, however, "the place of formal methods [...] in the everyday practice of synthesis is smaller than might appear at first glance", in practice, the engineer "must make a judicious selection of design tools and, most likely, must invent methods and diagrammatic devices which fit the particular problem at hand."

3 Instead of a conclusion: Mechanized intelligence

The development of methods for efficient logic-circuit synthesis is presently proceeding along two lines. The first of these involves the construction and/or programming of digital computers for this task. The other involves the improvement of rapid desk-top methods. The former is best for complicated problems, while the latter is to be preferred for problems of little or moderate difficulty. (Karnaugh, 1954)

Even after 15 years of Shannon's paper, its relevance in practice is still conditioned by physics and the engineering day-to-day reality and practice. As Washburn notes, "However, the end result of this process [of using Boolean algebra] in the present case is a circuit composed of more or less idealized elements.

function of the same number of variables. [...] functional separability is often easily detected in the circuit requirements and can always be used to reduce the limits on the numbers of elements required. We will now show that most functions are not functionally separable." (Shannon 1949)

¹⁵The work of Moore, Mealy and Huffman sparked off a new rapprochement, between circuit design and information theory, especially coding theory. This constitutes another chapter in the history of (theoretical) circuit design, where automata theory, information theory and circuit design meet. M. Mahoney's work on the emergence of theoretical computer science provides an angle on that particular chapter, but his studies are based on theoretical publications, not on the actual practices.

Considerable ingenuity may be necessary to replace these elements with their physical equivalents.” (Washburn 1953) Only when the engineering practices and the building blocks for designing an electronic computer became gradually more standardized, did Shannon’s algebra receive its current place. Viz., as part of a partially mechanized and standardized design process, executed by a computer.

References

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