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JEL codes:

D43, D82, L13

The social value of information and the competition motive: Price vs. quantity games*

Camille Cornand[†]
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August 31, 2017

Abstract

We propose a unified framework bridging the gap between team and competition issues, in order to reconsider the social value of private and public information in price and quantity games under imperfect and dispersed information, and to compare the corresponding outcomes in terms of equilibrium and social welfare. The informational distortion associated with the competition motive may lead to a negative social value of private information and reverse the perfect information result in favor of strategic substitutability as the source of higher profit and social welfare.

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1 Introduction

The payoffs of Keynes's beauty contest, as modelled by Morris and Shin (2002), involve three motives (Cornand and Heinemann, 2008): a *fundamental motive*, making agents strive to predict and fit some exogenous fundamental value, a *coordination motive*, giving them an incentive to match the conventional value to be set by the market, and a *competition motive*, making them better off when beating the market.

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Should information be perfect, the fundamental and the coordination motives would be compatible: all agents would simply coordinate on the fundamental value. As information becomes imperfect, blurring the fundamental, and dispersed, obstructing coordination, a conflict emerges between the fundamental and coordination motives. The competition motive introduces a further conflict, even in the absence of informational frictions, since pursuing the coordination and the competition motives means meeting and beating the market at the same time. While the conflict between the fundamental and the coordination motives has been largely explored in the beauty contest literature, less attention has been paid to the influence of the competition motive, which we want to emphasize now as a source of inefficiency under perfect as well as under imperfect and dispersed information.

Our contribution builds upon a unified framework bridging the gap between team and competition issues. On the one hand, this unified framework allows us to reconsider the social value of private and public information in a large category of contexts going beyond the Keynesian beauty contest, since they extend from situations of strategic complementarity generated by the coordination motive to situations of strategic substitutability resulting from an *anti*-coordination motive, when agents want to move away from each other. On the other hand, it enables us to formulate direct comparisons between the performances of price and quantity games when information is imperfect and dispersed.

To obtain the unified framework and capture the specific impact of the competition motive, we model an economy with a simple multidivisional company,¹ where each division supplies a diversified good and tries to maximize a convex combination of its own profit and of the company joint profit. We thus obtain as polar cases full competition (leading to a standard oligopolistic equilibrium) and full cooperation (entailing the collusive solution). The latter outcome, the solution to a team problem, is of course efficient from the company's point of view and may be taken as a benchmark.

The model is studied in the context of both price and quantity games. Introducing the competition motive has two effects. First, independently from informational issues, this motive distorts the equilibrium strategies away from their efficient value, thus generating a (price or quantity) *strategy distortion*. Second, under coexisting public and private information on the stochastic fundamental (the market size), the competition motive distorts the relative weights put on the two kinds of information, generating an *informational distortion*, which counter-weighs the effect of the coordination (or anti-coordination) motive. In both price and quantity games, the informational distortion generated by the competition motive thus translates into an inefficiently low weight on the public signal in the presence of the coordination motive (hence under strategic complementarity), and into an inefficiently high weight on the public signal in the presence of the anti-coordination motive (hence under strategic substitutability).

After examining the well-known case of perfect information where the competition motive is responsible for the sole strategy distortion, we first establish

¹On the multidivisional form of organization, see Mahoney (1992).

some results regarding the social value of information. In the absence of the competition motive, but also when this motive combines with the anti-coordination motive (hence under strategic substitutability), the social value of both public and private information is always positive. By contrast, when it combines with the coordination motive (hence under strategic complementarity), while the social value of public information is still positive, that of private information becomes negative for a high enough intensity of competition. Indeed, increasing the precision of either public or private information directly reduces the information cost, but may also augment the informational distortion. This additional effect of an increase in the precision of private information results under strategic complementarity in an excessive weight put on the private signal. The negative indirect effect eventually dominates the positive direct effect as the intensity of competition becomes larger and larger, reducing the importance of the fundamental motive.

We then compare from a welfare point of view the outcomes of the price and quantity games. Under imperfect and dispersed information, the efficient and socially optimal² uses of information coincide in the absence of the competition motive, ensuring higher performance levels under the influence of anti-coordination rather than coordination (hence under strategic substitutability rather than strategic complementarity). This result obtains under the sole influence of the coordination/anti-coordination motive and only for informational reasons. However, the competition motive may reverse the result, as it generates strategy and informational distortions with opposite effects on social welfare. If the information quality is low (under small precisions of both public and private signals), the effect of the informational distortion dominates the effect of the strategy distortion, so that profit and social welfare are higher with the coordination motive (hence under strategic complementarity). Interestingly, the informational distortion generated by the competition motive thus mitigates the perfect information result in favor of anti-coordination (hence of strategic substitutability).

The paper contributes to two main strands of the literature. First, it extends to a context of imperfect and dispersed information the systematic comparison of price and quantity competition initiated by Singh and Vives (1984) for a differentiated duopoly with linear demand (recent contributions, also under perfect information, are provided by Amir and Jin, 2001, and Amir, Erickson and Jin, 2017).³ Second, it belongs to the growing literature on the social value of information initiated by Morris and Shin (2002), highlighting the conflict between

²Our economy is reduced to a representative consumer with quasilinear utility and to a multidivisional company. This modelling approach rationalizes the standard view of "social" welfare (here identified with consumer's added utility) as the sum of the consumer surplus and distributed profits. Clearly, in a more diversified economy with profits distributed to only part of the consumers, the evaluation of social welfare would be more problematic.

³More recently and in the same vein, we observe the development of a literature on the aggregation of private information in oligopolies with uncertain demand or costs (Raith 1996, Vives 1988, 2011). This literature goes however beyond the scope of the present paper in dealing with firms' strategic decisions concerning information sharing and acquisition, or else with private information revelation at equilibrium.

the fundamental and the coordination motives, and showing that an increase in the precision of public information is harmful in a context of strategic complementarity if private information is already precise enough. Angeletos and Pavan (2007) show that this result must be qualified, in particular by referring to the relation between the equilibrium and the efficient uses of information. Ui and Yoshizawa (2015) provide a necessary and sufficient condition for welfare to increase with public or private information, welfare being represented as a linear combination of the variance of a common term and that of an idiosyncratic term in the equilibrium strategy. Both propose IO (Bertrand and Cournot) applications, without pointing to the source of the discrepancy between the equilibrium and efficient uses of information, namely the competition motive.⁴ The exploitation of the same IO illustrations of beauty contest games, focusing on the conflict between the fundamental and the coordination motives, are provided by Myatt and Wallace (2012, 2015, 2016). By taking as the fundamental targets pursued by the oligopolistic firms the Bertrand and the Cournot equilibrium strategies rather than the collusive ones, Myatt and Wallace merge the fundamental and competition motives. Such a procedure does not allow to consider the effect of the competition motive separately, which is our aim in the present paper.⁵

The remaining of the paper is structured as follows. Section 2 presents the set-up of the economy. Section 3 focuses on the perfect information benchmark, emphasizing the strategy distortion generated by the competition motive, while section 4 examines equilibrium and welfare under dispersed information, emphasizing the informational distortion. Section 5 makes a comparative welfare assessment of price and quantity games. Section 6 concludes.

2 The economy

We consider a simple economy composed of a representative consumer and a multidivisional company.

2.1 The representative consumer

We assume that the representative consumer's utility is symmetric⁶ quadratic with respect to the differentiated goods supplied by the company divisions and quasi-linear with respect to a composite good representing the rest of the economy.

⁴Ui (2009) has already built a bridge between the beauty contest literature and the team literature (which goes back to Radner, 1962), focusing on a team problem similar to our collusion benchmark. His result generalizes Cremer (1990), emphasizing that "shared knowledge is better than diversified knowledge if and only if a team exhibits strategic complementarity" (Ui, 2009, p. 273).

⁵The role of the competition motive has already been underlined in Cornand and Dos Santos Ferreira (2017) but its effects have not been isolated.

⁶Symmetry is a convenient but restrictive assumption. Amir and Jin (2001) compare Cournot and Bertrand equilibria, also with linear demand, but by taking asymmetries into account.

The representative consumer, endowed with positive wealth w , buys at prices $p \in \mathbb{R}_{++}^n$ a basket $q \in \mathbb{R}_+^n$ of n differentiated goods to the n divisions of the company, plus a quantity $z \in \mathbb{R}_+$ of a composite numéraire good to a competitive industry, so as to maximize, under the budget constraint $pq + z \leq w$ utility

$$U(q, z) = u(q) + z, \quad (1)$$

with

$$u(q) = 2\theta \sum_{i=1}^n q_i - \frac{1}{2} \sum_{i=1}^n q_i^2 - \frac{\gamma}{2} \left(\sum_{i=1}^n q_i \right)^2. \quad (2)$$

The positive parameter θ is an index of market size and $|\gamma|$ is the degree of interdependence between goods: goods are substitutes if $\gamma > 0$, complements if $-1/n < \gamma < 0$, independent if $\gamma = 0$.

The first order condition for utility maximization leads directly to the *inverse demand function* for each good $i = 1, \dots, n$:

$$p_i = 2\theta - q_i - n\gamma \underbrace{\frac{1}{n} \sum_{j=1}^n q_j}_{\bar{q}} \equiv P_i(q). \quad (3)$$

From this equation system we easily obtain the *demand function* for each good $i = 1, \dots, n$:

$$q_i = \frac{1}{1 + n\gamma} \left(2\theta - (1 + n\gamma)p_i + n\gamma \underbrace{\frac{1}{n} \sum_{j=1}^n p_j}_{\bar{p}} \right) \equiv Q_i(p). \quad (4)$$

2.2 The multidivisional company

The company assigns as objective to each division $i = 1, \dots, n$ a convex combination of its own profit and of the joint profit. The relative weight λ put on the former is a measure of the strength of the *competition motive* within the organization. We thus obtain as polar cases: the fully non-cooperative conduct (oligopolistic competition between the divisions) for $\lambda = 1$, and the fully cooperative conduct (collusion between the divisions) for $\lambda = 0$. More explicitly, the program of each division i is, in the benchmark case of perfect information and assuming zero production costs:

$$\max_{p_i} [\lambda p_i Q_i(p) + (1 - \lambda) pQ(p)] \quad (5)$$

when divisions play in prices, and

$$\max_{q_i} [\lambda q_i P_i(q) + (1 - \lambda) qP(q)] \quad (6)$$

when they play in quantities.

3 The perfect information benchmark: the competition motive and the strategy distortion

We first refer to the benchmark of perfect information, and consider successively equilibria and welfare in the price and quantity games.

3.1 Equilibria

- *The price game*

Instead of referring to the program (5), we can alternatively refer to an equivalent program consisting in minimizing the loss of the objective assigned to division i relative to the collusive profit (obtained for $\lambda = 0$ with all prices equal to the fundamental θ):

$$\theta Q_i(\theta, \dots, \theta) - [\lambda p_i Q_i(p) + (1 - \lambda) p Q(p)] = \frac{1}{1 + n\gamma} L^P(p_i, \bar{p}_{-i}, \theta), \quad (7)$$

with $\bar{p}_{-i} = (\sum_{j \neq i} p_j) / (n - 1)$ and the loss function L^P to be minimized in p_i

$$\begin{aligned} & L^P(p_i, \bar{p}_{-i}, \theta) \\ = & \underbrace{(p_i - \theta)^2}_{\text{fundamental motive}} + (n - 1) \gamma \left(\underbrace{(p_i - \bar{p}_{-i})^2}_{(\text{anti-})\text{coordination motive}} + \lambda \underbrace{\bar{p}_{-i} (p_i - \bar{p}_{-i})}_{\text{competition motive}} \right) \\ & + (n - 1) (1 - \lambda) \underbrace{\left(\frac{1 + n\gamma}{n - 1} \sum_{j \neq i} p_j^2 - n\gamma \bar{p}_{-i}^2 - 2\theta \bar{p}_{-i} \right)}_{\text{externality}}. \end{aligned} \quad (8)$$

This loss function is obviously reminiscent of the loss function introduced by Morris and Shin (2002) in their seminal modelling of the beauty contest, with its three motives:⁷ the fundamental, the coordination and the competition

⁷Morris and Shin (2002) loss function is indeed the sum of a fundamental, a coordination and a competition motives:

$$-(1 - r)(a_i - \theta)^2 - r(a_i - \bar{a})^2 + r \frac{1}{n} \sum_j (a_j - \bar{a})^2,$$

with given weights $-(1 - r)$, $-r$ and r , respectively, and involving individual and mean strategies a_i and \bar{a} . An important difference with respect to our own loss function is that, by assuming a continuum of agents, Morris and Shin make the competition motive inoperative regarding individual decisions. Another important difference concerns aggregate losses: the competition motive exactly balances the coordination motive in the Morris and Shin specification, so that the fundamental motive stands alone as a component of social welfare. By contrast, the competition motive vanishes by aggregation of our loss functions, so that coordination contributes in our case to the agents' welfare.

motives.⁸ We formulate three remarks concerning the present loss function.

First, notice that the fundamental motive stands naturally alone when there is a single division ($n = 1$) or when the goods are independent ($\gamma = 0$). Correspondingly, its relative weight diminishes as the level $(n - 1) |\gamma|$ of interdependence between divisions increases (either through the number $n - 1$ of competitors of each division or through the degree $|\gamma|$ of interdependence between goods).

Second, notice that the coordination motive becomes in fact an *anti*-coordination motive when we switch from the case of substitutable goods ($\gamma > 0$) to the case of complementary goods ($\gamma < 0$). There is however more than a simple change of signs. An important difference between the two cases is that, as the level of interdependence between divisions tends to its maximum, the fundamental motive eventually vanishes relative to the coordination motive ($(n - 1) |\gamma| \rightarrow \infty$), whereas it always dominates the anti-coordination motive ($(n - 1) |\gamma| \rightarrow 1 - 1/n < 1$).

Third, observe that symmetry in strategy deviations from the targets is broken by the competition motive, which introduces a downward (upward) bias when the coordination (anti-coordination) motive prevails. The weight on the competition motive is the *intensity of competition*, obtained by multiplying the level $(n - 1) |\gamma|$ of interdependence between divisions and the strength λ of the competition motive, thus combining two structural characteristics and a conduct characteristic of competition.

The first order condition for minimization of $L^P(\cdot, \bar{p}_{-i}, \theta)$ gives the best reply function

$$p_i = \frac{\theta + (n - 1) \gamma (1 - \lambda/2) \bar{p}_{-i}}{1 + (n - 1) \gamma}. \quad (9)$$

Naturally, the coordination motive (for $\gamma > 0$) implies strategic complementarity, and the anti-coordination motive (for $\gamma < 0$) strategic substitutability.

By symmetry ($p_i = \bar{p}_{-i}$), we deduce from the best reply function the equilibrium price \bar{p}^P (and the corresponding quantity \bar{q}^P), equal for all i and depending upon the strength λ of the competition motive:

$$\bar{p}^P(\lambda) = \frac{1}{1 + (n - 1) \gamma \lambda/2} \theta \equiv K^P(\lambda) \theta, \quad \bar{q}^P(\lambda) = \frac{1 + (n - 1) \gamma \lambda}{1 + (n - 1) \gamma \lambda/2} \frac{\theta}{1 + n \gamma}. \quad (10)$$

The equilibrium price \bar{p}^P , equal to the fundamental in the absence of the competition motive, is otherwise distorted, taking lower (higher) values in the presence of the coordination (anti-coordination) motive, in other words under strategic

⁸Myatt and Wallace (2012) assume a continuum of competitors supplying differentiated substitutes, and rewrite directly the profit function as a loss function (up to its sign):

$$p_i ((2 - \beta) \theta - p_i + \beta \bar{p}) = - (1 - \beta/2) (p_i - \theta)^2 - (\beta/2) (p_i - \bar{p})^2 + (1 - \beta/2) \theta^2 + (\beta/2) \bar{p}^2,$$

with $\beta \in (0, 1)$. This loss function exhibits the sole fundamental and coordination motives, but the competition motive is actually merged with the fundamental motive, since the fundamental θ is the (monopolistically) competitive price, not the collusive price $\theta (1 - \beta/2) / (1 - \beta)$ as in our framework.

complementarity (substitutability). Of course, we obtain an opposite effect on the equilibrium quantity \bar{q}^P . The distortion – a consequence of the bias introduced by the competition motive – is the larger the larger the intensity of competition $(n - 1) |\gamma| \lambda$.

- *The quantity game*

Proceeding in the same way, we consider the loss of the objective assigned to division i relative to the collusive profit (obtained for $\lambda = 0$ with all quantities equal to $\theta / (1 + n\gamma)$) as

$$\frac{\theta}{1 + n\gamma} P_i \left(\frac{\theta}{1 + n\gamma}, \dots, \frac{\theta}{1 + n\gamma} \right) - [\lambda q_i P_i(q) + (1 - \lambda) q P(q)] = (1 + n\gamma) L^Q(q_i, \bar{q}_{-i}, \theta), \quad (11)$$

with $\bar{q}_{-i} = (\sum_{j \neq i} q_j) / (n - 1)$ and the loss function L^Q to be minimized in q_i :

$$\begin{aligned} & L^Q(q_i, \bar{q}_{-i}, \theta) \quad (12) \\ = & \underbrace{\left(q_i - \frac{\theta}{1 + n\gamma} \right)^2}_{\text{fundamental motive}} - \frac{(n - 1)\gamma}{1 + n\gamma} \left(\underbrace{(q_i - \bar{q}_{-i})^2}_{\text{(anti-)coordination motive}} + \lambda \underbrace{\bar{q}_{-i} (q_i - \bar{q}_{-i})}_{\text{competition motive}} \right) \\ & + \frac{(n - 1)(1 - \lambda)}{1 + n\gamma} \underbrace{\left(\frac{1}{n - 1} \sum_{j \neq i} q_j^2 + n\gamma \bar{q}_{-i}^2 - 2\theta \bar{q}_{-i} \right)}_{\text{externality}}. \end{aligned}$$

Again, this loss function is reminiscent of the one in the beauty contest model of Morris and Shin (2002). The observations formulated for L^P stand for L^Q , but the sign of the weight put on the coordination and competition motives is naturally reversed, an *anti*-coordination motive prevailing now when goods are substitutes. As in the price game, when the intensity of competition tends to its maximum, this weight tends to infinity if applied to the coordination motive but it remains dominated if applied to the anti-coordination motive.

The first order condition for minimizing $L^Q(\cdot, \bar{q}_{-i}, \theta)$ gives the best reply function

$$q_i = \frac{\theta - (n - 1)\gamma(1 - \lambda/2)\bar{q}_{-i}}{1 + \gamma}. \quad (13)$$

As in the case of the price game, coordination generates strategic complementarity, anti-coordination strategic substitutability. By symmetry ($q_i = \bar{q}_{-i}$), we deduce from the best reply function the equilibrium quantity \bar{q}^Q (and the corresponding price \bar{p}^Q), the same for each i and depending upon the strength of the competition motive λ :

$$\bar{q}^Q(\lambda) = \frac{1}{1 + n\gamma - (n - 1)\gamma\lambda/2} \theta \equiv K^Q(\lambda) \theta, \quad \bar{p}^Q(\lambda) = \frac{1 + n\gamma - (n - 1)\gamma\lambda}{1 + n\gamma - (n - 1)\gamma\lambda/2} \theta. \quad (14)$$

Observe that under full cooperation of the company divisions, $\bar{p}^Q(0) = \bar{p}^P(0) = \theta$ and $\bar{q}^Q(0) = \bar{q}^P(0) = \theta / (1 + n\gamma)$. Under perfect information and a vanishing competition motive, it is indifferent to play in prices or in quantities. However, as soon as $\lambda > 0$, the equilibrium price is distorted in the same way as in the price game. Notice further that $\bar{p}^Q(\lambda) > \bar{p}^P(\lambda)$ independently of the nature of the interdependence between goods: the strategy distortion is just larger under strategic complementarity, pushing the price \bar{p}^P downwards when $\gamma > 0$, and the price \bar{p}^Q upwards when $\gamma < 0$.

The following proposition summarizes the consequences of introducing the competition motive in the two games.

Proposition 1 *The competition motive generates a strategy distortion in both price and quantity games: as it becomes stronger (through a larger λ), equilibrium prices become lower (higher) and equilibrium quantities higher (lower) when goods are substitutes (complements). This distortion increases with the intensity of competition $(n - 1) |\gamma| \lambda$, and is larger when the coordination motive prevails, hence under strategic complementarity (in the price game when goods are substitutes, and in the quantity game when they are complements).*

3.2 Welfare

Because of the representative consumer assumption, it is straightforward to proceed to welfare analysis. Consumer's utility at equilibrium quantities (be it $q^* = q^P(\lambda)$ or $q^* = q^Q(\lambda)$) can be decomposed as follows:

$$U(q^*, w - q^*P(q^*)) = \underbrace{u(q^*)}_{\text{social welfare}} - \underbrace{q^*P(q^*)}_{\text{profit}} + w. \quad (15)$$

- *The price game*

We start by computing profit, social welfare and the consumer surplus successively. The company profit

$$\begin{aligned} q^P(\lambda) P(q^P(\lambda)) &= \sum_{i=1}^n q_i^P(\lambda) \left(2\theta - q_i^P(\lambda) - \gamma \sum_j q_j^P(\lambda) \right) \\ &= \frac{n\theta^2}{1+n\gamma} 4 \frac{1+(n-1)\gamma\lambda}{(2+(n-1)\gamma\lambda)^2} \end{aligned} \quad (16)$$

is equal to the collusive profit $n\theta^2 / (1 + n\gamma)$ multiplied by a factor which is always smaller than one if $\lambda > 0$, and decreasing in the intensity of competition $(n - 1) |\gamma| \lambda$. As expected, the company profit is maximized at $\lambda = 0$. Social welfare

$$u(q^P(\lambda)) = \frac{n\theta^2}{1+n\gamma} \frac{2(1+(n-1)\gamma\lambda)(3+(n-1)\gamma\lambda)}{(2+(n-1)\gamma\lambda)^2} \quad (17)$$

and the consumer surplus

$$u(q^P(\lambda)) - q^P(\lambda)P(q^P(\lambda)) = \frac{n\theta^2}{1+n\gamma} 2 \left(\frac{1+(n-1)\gamma\lambda}{2+(n-1)\gamma\lambda} \right)^2 \quad (18)$$

are by contrast increasing (decreasing) in the intensity of competition if goods are substitutes (complements). Hence, they are both maximized at $\lambda = 1$ when goods are substitutes, and at $\lambda = 0$ when they are complements. As already shown by Cournot (1838, ch. IX), producers' and consumers' interests converge when goods are complements, whereas they diverge when goods are substitutes.

- *The quantity game*

The company profit

$$\begin{aligned} q^Q(\lambda)P(q^Q(\lambda)) &= \sum_{i=1}^n q_i^Q(\lambda) \left(2\theta - q_i^Q(\lambda) - \gamma \sum_j q_j^Q(\lambda) \right) \\ &= \frac{n\theta^2}{1+n\gamma} 4 \frac{1 - \frac{(n-1)\gamma\lambda}{1+n\gamma}}{\left(2 - \frac{(n-1)\gamma\lambda}{1+n\gamma} \right)^2} \end{aligned} \quad (19)$$

is again equal to the collusive profit $n\theta^2/(1+n\gamma)$ multiplied by a factor which is always smaller than one if $\lambda > 0$, and decreasing in the intensity of competition $(n-1)|\gamma|\lambda$, so that profit is maximized at $\lambda = 0$. Social welfare

$$\begin{aligned} u(q^Q(\lambda)) &= 2\theta \sum_{i=1}^n q_i^Q(\lambda) - \frac{1}{2} \sum_{i=1}^n (q_i^Q(\lambda))^2 - \frac{\gamma}{2} \left(\sum_{i=1}^n q_i^Q(\lambda) \right)^2 \\ &= \frac{n\theta^2}{1+n\gamma} 2 \frac{3 - 2\frac{(n-1)\gamma\lambda}{1+n\gamma}}{\left(2 - \frac{(n-1)\gamma\lambda}{1+n\gamma} \right)^2} \end{aligned} \quad (20)$$

and the consumer surplus

$$u(q^Q(\lambda)) - q^Q(\lambda)P(q^Q(\lambda)) = \frac{n\theta^2}{1+n\gamma} \frac{2}{\left(2 - \frac{(n-1)\gamma\lambda}{1+n\gamma} \right)^2} \quad (21)$$

are, as in the price game, both increasing (decreasing) in the intensity of competition if goods are substitutes (complements). Hence, social welfare and consumer surplus are again maximized at $\lambda = 1$ when goods are substitutes, and at $\lambda = 0$ when they are complements.

We recall in the following proposition the preceding observations.

Proposition 2 *Under perfect information, profit is decreasing in the intensity of competition $(n-1)|\gamma|\lambda$ (as the strategy distortion becomes more severe) in both price and quantity games. Social welfare and the consumer surplus are increasing (decreasing) in the intensity of competition if goods are substitutes (complements): the interests of the company and of the consumer diverge when goods are substitutes, but converge when they are complements.*

4 Dispersed information: the competition motive and the informational distortion

We shall now assume that the company divisions do not know the realization of θ on which they only receive two noisy signals, a public signal $y = \theta + \eta$, with $\eta \sim N(0, \frac{1}{\alpha})$ and a private signal $x_i = \theta + \varepsilon_i$, with $\varepsilon_i \sim N(0, \frac{1}{\beta})$. As in Morris and Shin (2002), the random variables η and $\varepsilon_1, \dots, \varepsilon_n$ are independent.⁹

4.1 Equilibria

- *The price game*

Instead of choosing directly a price strategy p_i , division i is assumed to choose a linear response to the two signals through a non-negative strategy pair (ζ_i^P, ξ_i^P) such that $p_i = \zeta_i^P y + \xi_i^P x_i$. Through this strategy, division i targets the following expected best price response conditional on the two signals:

$$\begin{aligned} \zeta_i^P y + \xi_i^P x_i &= \mathbf{E} \left(\frac{\theta + \gamma(1 - \lambda/2) \sum_{j \neq i} (\zeta_j^P y + \xi_j^P x_j)}{1 + (n-1)\gamma} \middle| (y, x_i) \right) \\ &= \frac{\mathbf{E}(\theta | (y, x_i)) \left(1 + \gamma(1 - \lambda/2) \sum_{j \neq i} \xi_j^P \right) + \gamma(1 - \lambda/2) y \sum_{j \neq i} \zeta_j^P}{1 + (n-1)\gamma}. \end{aligned} \quad (22)$$

As $\mathbf{E}(\theta | (y, x_i)) = (\alpha y + \beta x_i) / (\alpha + \beta)$,

$$\xi_i^P = \frac{\beta}{\alpha + \beta} \frac{1 + \gamma(1 - \lambda/2) \sum_{j \neq i} \xi_j^P}{1 + (n-1)\gamma}, \quad (23)$$

implying at equilibrium

$$\xi_i^P = \xi^P = \frac{1}{1 + \alpha/\beta + (n-1)\gamma(\lambda/2 + \alpha/\beta)}. \quad (24)$$

The next step allows us to determine

$$\zeta_i^P = \zeta^P = \frac{1}{1 + \alpha/\beta + (n-1)\gamma(\lambda/2 + \alpha/\beta)} \frac{1 + (n-1)\gamma}{1 + (n-1)\gamma(\lambda/2)} \frac{\alpha}{\beta}. \quad (25)$$

The strategy sum

$$\xi^P + \zeta^P = \frac{1}{1 + (n-1)\gamma\lambda/2} = K^P(\lambda) \quad (26)$$

does not depend upon the quality of information, as given by the precisions (α, β) . It is in fact equal to the coefficient multiplying the fundamental to

⁹As in Myatt and Wallace (2012), players share an improper prior over θ , which simplifies the statement of the results without loss of generality. This assumption may lead to states where demand is negative. We are implicitly assuming that the parameter α is high enough for the probability of such an event to be small (see Vives, 1984, p. 77).

obtain the equilibrium price under perfect information. We can accordingly apply to it everything we have established in the benchmark game about the strategy distortion and its consequences. In particular, the fact, established by Angeletos and Pavan (2007), that "the Bertrand price reacts too little to θ as compared to the monopoly price" (meaning $K^P(1) < K^P(0)$) is completely dependent upon the strategy distortion and is not modified by the imperfection and dispersion of the information.

The novelty introduced by assuming imperfect and dispersed information comes from the relative weights $\zeta/(\xi + \zeta)$ and $\xi/(\xi + \zeta)$ put on the public and private signals, respectively, with

$$\frac{\zeta^P}{\xi^P + \zeta^P} = \frac{(1 + (n-1)\gamma)\alpha/\beta}{(1 + (n-1)\gamma)\alpha/\beta + 1 + (n-1)\gamma\lambda/2} \equiv \kappa^P(\lambda) \quad (27)$$

and $\xi^P/(\xi^P + \zeta^P) = 1 - \kappa^P(\lambda)$. The efficient relative weight on the public signal is obviously $\kappa^P(0)$, which takes into account the fundamental and the coordination (or anti-coordination) motives, without the competition motive. The fundamental motive alone would lead to a relative weight on the public signal given by its relative precision $\alpha/(\alpha + \beta)$. However, for the sake of coordination, and as soon as the goods are substitutes ($\gamma > 0$), the efficient relative weight on the public signal

$$\kappa^P(0) = \frac{(1 + (n-1)\gamma)\alpha/\beta}{(1 + (n-1)\gamma)\alpha/\beta + 1} \quad (28)$$

is increasing under strategic complementarity in the level of interdependence between divisions, as measured by $(n-1)|\gamma|$. Obviously, it is decreasing in the level of interdependence between divisions under the anti-coordination motive ($\gamma < 0$), hence under strategic substitutability.

Introducing the competition motive leads to an *informational distortion*, by decreasing (increasing) $\kappa^P(\lambda)$ under the coordination (anti-coordination) motive, thus by countervailing the effect of this motive. In relative terms, this informational distortion is

$$\frac{|\kappa^P(0) - \kappa^P(\lambda)|}{\kappa^P(0)} = \frac{(n-1)|\gamma|\lambda/2}{(1 + (n-1)\gamma)\alpha/\beta + 1 + (n-1)\gamma\lambda/2}, \quad (29)$$

increasing in the intensity of competition $(n-1)|\gamma|\lambda$ and decreasing in the ratio of precisions α/β .

- *The quantity game*

Following the same line of argument, we now assume that each division i chooses a strategy pair (ζ_i^Q, ξ_i^Q) such that $q_i = \zeta_i^Q y + \xi_i^Q x_i$, and obtain:

$$\zeta^Q y + \xi^Q x_i = \frac{\mathbf{E}(\theta | (y, x_i)) (1 - (n-1)\gamma(1 - \lambda/2)\xi^Q) - (n-1)\gamma(1 - \lambda/2)\zeta^Q y}{1 + \gamma}, \quad (30)$$

so that

$$\xi^Q = \frac{1}{1 + n\gamma - (n-1)\gamma\lambda/2 + (1+\gamma)\alpha/\beta}, \text{ and} \quad (31)$$

$$\zeta^Q = \frac{1}{1 + n\gamma - (n-1)\gamma\lambda/2 + (1+\gamma)\alpha/\beta} \frac{(1+\gamma)\alpha/\beta}{1 + n\gamma - (n-1)\gamma\lambda/2}, \quad (32)$$

with a strategy sum

$$\zeta^Q + \xi^Q = \frac{1}{1 + n\gamma - (n-1)\gamma\lambda/2} = K^Q(\lambda). \quad (33)$$

Again, the strategy sum does not depend upon the quality of information, and is equal to the coefficient multiplying the fundamental in the equilibrium quantity under perfect information, which allows us to apply in the present context, as regards the strategy distortion and its consequences, all that has been established for the perfect information benchmark.

As to the relative weight on the public signal, we have

$$\frac{\zeta^Q}{\zeta^Q + \xi^Q} = \frac{(1+\gamma)\alpha/\beta}{(1+\gamma)\alpha/\beta + 1 + n\gamma - (n-1)\gamma\lambda/2} = \kappa^Q(\lambda). \quad (34)$$

Its efficient value $\kappa^Q(0)$ is again increasing in the level of interdependence $(n-1)|\gamma|$ under the coordination motive (now when $\gamma < 0$), hence under strategic complementarity. It is decreasing under the anti-coordination motive (when $\gamma > 0$), hence under strategic substitutability. The competition motive induces an *informational distortion*, by decreasing (increasing) $\kappa^Q(\lambda)$ under the coordination (anti-coordination) motive, hence by countervailing the effect of this motive. In relative terms,

$$\frac{|\kappa^Q(0) - \kappa^Q(\lambda)|}{\kappa^Q(0)} = \frac{(n-1)|\gamma|\lambda/2}{(1+\gamma)\alpha/\beta + 1 + n\gamma - (n-1)\gamma\lambda/2},$$

increasing in $(n-1)|\gamma|\lambda$ and decreasing in α/β .

To recall,

Proposition 3 *Under imperfect and dispersed information, the competition motive generates, in both price and quantity games, an informational distortion which countervails the effect of the coordination (anti-coordination) motive, making the relative weight put on the public signal inefficiently low (high) under strategic complementarity (substitutability). This distortion is the stronger the higher the intensity of competition $(n-1)|\gamma|\lambda$.*

4.2 Welfare and the social value of information

- *The price game*

The expected profit is

$$\mathbf{E}(\Pi^P | \theta) = \frac{1}{1+n\gamma} \mathbf{E} \left(2\theta \sum_i p_i^* - (1+n\gamma) \sum_i p_i^{*2} + \gamma \left(\sum_i p_i^* \right)^2 \right) \quad (35)$$

with $p_i^* = K^P(\lambda) (\theta + \kappa^P(\lambda) \eta + (1 - \kappa^P(\lambda)) \varepsilon_i)$,

equal to

$$\mathbf{E}(\Pi^P | \theta) = \frac{n (K^P(\lambda))^2}{1+n\gamma} \left(\frac{2}{K^P(\lambda)} - 1 \right) \theta^2 \quad (36)$$

$$- \underbrace{\frac{n (K^P(\lambda))^2}{1+n\gamma} \left(\frac{(\kappa^P(\lambda))^2}{\alpha} + (1+(n-1)\gamma) \frac{(1-\kappa^P(\lambda))^2}{\beta} \right)}_{C^P}.$$

The first term is the profit obtained under perfect information. The second term is an *information cost*, minimized in κ for $\kappa = \kappa^P(0)$:¹⁰

$$\frac{\kappa}{1-\kappa} = (1+(n-1)\gamma) \frac{\alpha}{\beta} = \frac{\kappa^P(0)}{1-\kappa^P(0)}. \quad (37)$$

The expected social welfare is

$$\mathbf{E}(u^P | \theta) = \mathbf{E} \left(2\theta \sum_i Q_i(p^*) - \frac{1}{2} \sum_i (Q_i(p^*))^2 - \frac{\gamma}{2} \left(\sum_i Q_i(p^*) \right)^2 \right), \quad (38)$$

with $Q_i(p^*) = \frac{1}{1+n\gamma} \left(\frac{2\theta - K^P(\lambda) (\theta + \kappa^P(\lambda) \eta)}{-K^P(\lambda) (1 - \kappa^P(\lambda)) \left((1+(n-1)\gamma) \varepsilon_i - \gamma \sum_{j \neq i} \varepsilon_j \right)} \right)$,

hence

$$\mathbf{E}(u^P | \theta) = \frac{n\theta^2}{1+n\gamma} \frac{4 - (K^P(\lambda))^2}{2} \quad (39)$$

$$- \frac{n (K^P(\lambda))^2}{2(1+n\gamma)} \left(\frac{(\kappa^P(\lambda))^2}{\alpha} + (1+(n-1)\gamma) \frac{(1-\kappa^P(\lambda))^2}{\beta} \right).$$

The first term is the social welfare obtained under perfect information. The second term, the *information cost*, is half of the information cost beared by the company, and is consequently equal in absolute value to the consumer surplus,

¹⁰We thus verify that the efficient use of information by the company corresponds indeed to the absence of the competition motive ($\lambda = 0$).

but with an opposite sign. Thus, the efficient use of information by the company, prevailing in the absence of the competition motive, is also socially optimal, whereas the consumer surplus would be maximized in κ , among all possible equilibrium outcomes, when $\lambda = 1$ (since the maximization of a strictly convex function has necessarily a corner solution).

Notice that

$$\mathbf{E}(u^P | \theta) = \frac{\mathbf{E}(\Pi^P)}{2} + \frac{n\theta^2}{1+n\gamma} (2 - K^P(\lambda)), \quad (40)$$

with a second term which does not depend upon the information quality. Hence, social welfare and profit are aligned as regards their responses (through the information cost) to changes in the signal precisions α and β . Because of the informational distortion, an increase in these precisions is not necessarily profit and welfare enhancing, as we will now see.

As $(\partial \kappa^P(\lambda) / \partial(\alpha/\beta)) \alpha/\beta = (1 - \kappa^P(\lambda)) \kappa^P(\lambda)$, we have

$$\frac{\partial C^P}{\partial \alpha} = -\frac{(\kappa^P(\lambda))^2}{\alpha^2} \left(1 - \alpha \frac{\partial C^P}{\partial \kappa} \frac{1 - \kappa^P(\lambda)}{\kappa^P(\lambda)} \right) \quad (41)$$

$$\frac{\partial C^P}{\partial \beta} = -\frac{(1 - \kappa^P(\lambda))^2}{\beta^2} \left(1 + \beta \frac{\partial C^P}{\partial \kappa} \frac{\kappa^P(\lambda)}{1 - \kappa^P(\lambda)} \right), \quad (42)$$

so that the information cost unequivocally decreases in response to a higher precision α or β , as long as the competition motive is inactive ($\lambda = 0$), entailing $\partial C^P / \partial \kappa = 0$ by optimality of $\kappa^P(0)$. Under the coordination motive (when $\gamma > 0$), $\kappa^P(\lambda) < \kappa^P(0)$ for $\lambda > 0$, so that $\partial C^P / \partial \kappa < 0$ by strict convexity of C^P relative to κ . The information cost is then decreasing in the precision α of the public signal, but not necessarily in the precision β of the private signal, a result reversing the main conclusion of Morris and Shin (2002), obtained in a situation where social welfare depends upon the sole fundamental motive, given the existence of an exact balance between the coordination and competition motives.¹¹ To check that the information cost may *increase* with more precise private information, consider the term

$$\beta \frac{\partial C^P}{\partial \kappa} \frac{\kappa^P(\lambda)}{1 - \kappa^P(\lambda)} = -\frac{(1 + (n-1)\gamma)(n-1)\gamma\lambda}{1 + (n-1)\gamma\lambda/2} \kappa^P(\lambda), \quad (43)$$

which is smaller than -1 , entailing positivity of $\partial C^P / \partial \beta$, for a high enough intensity of competition. This result contradicts Corollary 11 in Angeletos and Pavan (2007).

¹¹See footnote 7. As the distorted relative weight $\kappa(\lambda)$ on the public signal belongs to the open interval $(\alpha/(\alpha+\beta), \kappa(0))$, the reversal of results is the natural consequence of referring to an optimal $\alpha/(\alpha+\beta)$, as in Morris and Shin (2002) instead of an optimal $\kappa(0)$, as in the present context. See also Angeletos and Pavan (2007, s.6.3) and their comments on such reversal obtained in other contexts.

Under the anti-coordination motive (when $\gamma < 0$), $\kappa^P(\lambda) > \kappa^P(0)$, hence $\partial C^P/\partial\kappa > 0$, so that $\partial C^P/\partial\beta$ is negative. So is $\partial C^P/\partial\alpha$ since

$$\alpha \frac{\partial C^P}{\partial\kappa} \frac{1 - \kappa^P(\lambda)}{\kappa^P(\lambda)} = -(n-1)\gamma\lambda(1 - \kappa^P(\lambda)) < (1 - 1/n)\lambda(1 - \kappa^P(\lambda)) < 1. \quad (44)$$

- *The quantity game*

The expected profit is now

$$\begin{aligned} \mathbf{E}(\Pi^Q|\theta) &= \mathbf{E}\left(2\theta \sum_i q_i^* - \sum_i q_i^{*2} - \gamma \left(\sum_i q_i^*\right)^2\right), \quad (45) \\ \text{with } q_i^* &= K^Q(\lambda)(\theta + \kappa^Q(\lambda)\eta + (1 - \kappa^Q(\lambda))\varepsilon_i), \end{aligned}$$

equal to

$$\begin{aligned} \mathbf{E}(\Pi^Q|\theta) &= n\theta^2 K^Q(\lambda)(2 - (1 + n\gamma)K^Q(\lambda)) \quad (46) \\ &\quad - n(1 + n\gamma)(K^Q(\lambda))^2 \underbrace{\left(\frac{(\kappa^Q(\lambda))^2}{\alpha} + \frac{1 + \gamma}{1 + n\gamma} \frac{(1 - \kappa^Q(\lambda))^2}{\beta}\right)}_{C^Q}. \end{aligned}$$

The first term is again the profit obtained under perfect information. The second term is an *information cost*, minimized in κ when

$$\frac{\kappa}{1 - \kappa} = \frac{1 + \gamma}{1 + n\gamma} \frac{\alpha}{\beta} = \frac{\kappa^Q(0)}{1 - \kappa^Q(0)}, \quad (47)$$

which corresponds to the efficient use of information by the company.

The expected social welfare is

$$\begin{aligned} \mathbf{E}(u^Q|\theta) &= \mathbf{E}\left(2\theta \sum_i q_i^* - \frac{1}{2} \sum_i q_i^{*2} - \frac{\gamma}{2} \left(\sum_i q_i^*\right)^2\right) \quad (48) \\ &= \frac{1}{2} \mathbf{E}(\Pi^Q) + \mathbf{E}\left(\theta \sum_i q_i^*\right) = \frac{1}{2} \mathbf{E}(\Pi^Q) + n\theta^2 K^Q(\lambda), \end{aligned}$$

equal to half of the expected profit plus a term which is independent of the signal precisions. Again, social welfare and profit are aligned as regards their responses to changes in these precisions.

How does the information cost respond to an increase in the precisions α and β ? As $(\partial\kappa^Q(\lambda)/\partial(\alpha/\beta))\alpha/\beta = (1 - \kappa^Q(\lambda))\kappa^Q(\lambda)$, we have

$$\frac{\partial C^Q}{\partial\alpha} = -\frac{(\kappa^Q(\lambda))^2}{\alpha^2} \left(1 - \alpha \frac{\partial C^Q}{\partial\kappa} \frac{1 - \kappa^Q(\lambda)}{\kappa^Q(\lambda)}\right) \quad (49)$$

$$\frac{\partial C^Q}{\partial\beta} = -\frac{(1 - \kappa^Q(\lambda))^2}{\beta^2} \left(1 + \beta \frac{\partial C^Q}{\partial\kappa} \frac{\kappa^Q(\lambda)}{1 - \kappa^Q(\lambda)}\right). \quad (50)$$

As in the price game, the information cost unequivocally decreases in response to a higher precision α or β , as long as the competition motive is inactive ($\lambda = 0$), entailing $\partial C^Q/\partial\kappa = 0$ by optimality of $\kappa^Q(0)$. Under the anti-coordination motive (when $\gamma > 0$), $\kappa^Q(\lambda) > \kappa^Q(0)$ for $\lambda > 0$, so that $\partial C^Q/\partial\kappa > 0$ by strict convexity of C^Q relative to κ . The information cost is then always decreasing in the precisions α and β of both signals, since $\partial C^Q/\partial\beta$ is clearly negative, and so is $\partial C^Q/\partial\alpha$ as

$$\alpha \frac{\partial C^Q}{\partial\kappa} \frac{1 - \kappa^Q(\lambda)}{\kappa^Q(\lambda)} = \frac{(n-1)\gamma\lambda}{1+n\gamma} (1 - \kappa^Q(\lambda)) < 1. \quad (51)$$

Notice that this result contradicts Corollary 10 in Angeletos and Pavan (2007). Under the coordination motive however (when $\gamma < 0$), $\partial C^Q/\partial\kappa < 0$, hence $\partial C^Q/\partial\alpha < 0$, but $\partial C^Q/\partial\beta$ may be positive as

$$\beta \frac{\partial C^Q}{\partial\kappa} \frac{\kappa^Q(\lambda)}{1 - \kappa^Q(\lambda)} = \frac{1+\gamma}{1+n\gamma} \frac{(n-1)\gamma\lambda}{1+n\gamma - (n-1)\gamma\lambda/2} \kappa^Q(\lambda) < -1 \quad (52)$$

for a high enough intensity of competition (take $n\gamma \rightarrow -1$).

We may now summarize all the preceding results of this subsection in the following proposition.

Proposition 4 *Under imperfect and dispersed information ($\alpha < \infty$ and $0 < \beta < \infty$), the information cost components of expected profit and social welfare are minimized in κ at $\kappa^P(0)$ and $\kappa^Q(0)$ for the price and quantity games, respectively: efficient and socially optimal uses of information always coincide, prevailing when the competition motive is suppressed. By contrast, the consumer surplus is maximized in κ , under the constraint of equilibrium implementation, when the competition motive reaches its maximum strength.*

Variations of information quality have, in both games, the same qualitative effects on profit and social welfare. In the absence of the competition motive or else under the anti-coordination motive, both precisions are always profit and welfare enhancing: information has always a positive social value. However, under the coordination motive (hence under strategic complementarity), if profit and social welfare always increase with a higher precision α of the public signal, they decrease, for a high enough intensity of competition, when the precision β of the private signal increases.

A few comments on the second part of this proposition are now in order. An increase in the precision of any of the two signals has a favorable direct effect on the information cost, hence on profit and welfare, plus an indirect effect (if $\lambda > 0$) through the relative weight $\kappa(\lambda)$ (see (41), (42) for the price game and (49), (50) for the quantity game). Under the coordination motive (hence under strategic complementarity), $\kappa(\lambda) < \kappa(0)$: the informational distortion leads to an excessive weight put on the private signal. As this distortion is decreasing in the ratio of precisions α/β , an increase in the quality of public – not private – information (see (43) for the price game and (52) for the quantity

game) has a favorable indirect effect on the information cost. So, we are left with two opposite effects of an increase in the precision of the private signal. As the intensity of competition hence the informational distortion increases, the unfavorable indirect effect will eventually dominate.¹² The analysis is not completely symmetric when we switch to the case of the anti-coordination motive, a situation in which the informational distortion leads to an excessive weight put on the public signal. This is because indefinitely increasing the intensity of competition keeps the anti-coordination and competition motives dominated by the fundamental motive. As a consequence, the favorable direct effects always dominate.¹³

5 Price vs. quantity games

In this section we compare price and quantity games from a welfare perspective. Take first, using equations (36) and (46), the difference in profits obtained when playing each kind of strategies.

$$\begin{aligned} & \mathbf{E}(\Pi^Q | \theta) - \mathbf{E}(\Pi^P | \theta) \\ &= \frac{n}{1+n\gamma} \left[\begin{aligned} & \left((1 - K^P(\lambda))^2 - (1 - (1+n\gamma)K^Q(\lambda))^2 \right) \theta^2 \\ & + (K^P(\lambda))^2 C^P - ((1+n\gamma)K^Q(\lambda))^2 C^Q \end{aligned} \right]. \end{aligned} \quad (53)$$

There are two cases in which this expression vanishes. The first case is when $\gamma = 0$, entailing $K^P(\lambda) = K^Q(\lambda) = 1$ and $C^P = C^Q$: it is indifferent to play in quantities or in prices when goods are independent. As $K^P(0) = (1+n\gamma)K^Q(0)$, the sign of the whole expression is that of the difference $C^P - C^Q$ when $\lambda = 0$, so that the second case arises when the competition motive is inactive, if in addition $C^P = C^Q$. This equality obtains if information is perfect ($\alpha = \infty$ or $\beta = \infty$), entailing $C^P = C^Q = 0$, or if it is at least not dispersed ($\beta = 0$), leading to $C^P = C^Q = 1/\alpha$. Thus, when the competition motive is suppressed, it is again indifferent to play in quantities or in prices, provided the absence of information dispersion removes the conflict between the fundamental and the coordination (or anti-coordination) motives.

Otherwise, still with $\lambda = 0$ but with $\alpha < \infty$ and $0 < \beta < \infty$, it is straightforward to check that the difference

$$C^P - C^Q = \frac{(2+n\gamma)(n-1)\beta}{[(1+(n-1)\gamma)\alpha + \beta][(1+\gamma)\alpha + (1+n\gamma)\beta]}\gamma \quad (54)$$

has the sign of γ : playing in quantities is better for the company when goods are substitutes, worse when they are complements. In other words, under dispersed information, the company should prefer the game leading to the anti-

¹²The same analysis, although formulated in different terms, underpins Corollary 11 in Angeletos and Pavan (2007), except when stating that expected profits *always* increase with the precision of the private signal.

¹³Again, the same kind of analysis underpins Corollary 10 in Angeletos and Pavan (2007), except when stating that expected profits can *decrease* with the precision of the public signal.

coordination motive (hence to strategic substitutability) if it can implement the suppression of competition between its divisions.

Once the competition motive steps in, the term involving the fundamental in equation (53) ceases to be zero. By equations (10) and (14), if $\lambda > 0$,

$$\begin{aligned} \lim_{\gamma \rightarrow \infty} K^P(\lambda) &= \lim_{\gamma \rightarrow -1/n} (1 + n\gamma) K^Q(\lambda) = 0 \text{ and} \\ \lim_{\gamma \rightarrow -1/n} K^P(\lambda) &= \lim_{\gamma \rightarrow \infty} (1 + n\gamma) K^Q(\lambda) = \frac{1}{1 - (1 - 1/n)\lambda/2} \in (1, 2), \end{aligned} \quad (55)$$

so that the term involving the fundamental tends to a positive (negative) limit as $\gamma \rightarrow \infty$ ($\gamma \rightarrow -1/n$). This reflects the fact, stated in Proposition 1, that the strategy distortion is larger under strategic complementarity. Again (now with an active competition motive but with perfect information), the company should prefer the game leading to the anti-coordination motive (the quantity game when goods are substitutes, the price game when they are complements).

Under imperfect information, we must take into account, in (53), the terms involving the information costs C^P and C^Q . As $\gamma \rightarrow \infty$ ($\gamma \rightarrow -1/n$), the term in C^P (C^Q) vanishes, so that the strategy and information distortions have opposite effects on the company profit. Which of the two effects dominates clearly depends upon the precisions α and β : the effect through the information cost (corresponding to the terms involving C^P and C^Q) will be dominated if this cost is small (if the precision of one of the two signals is high enough), but will dominate if it is large (if the precision of both signals is low enough). Thus, poor information quality and high intensity of competition will result in the company preference for the game leading to the coordination motive (hence to strategic complementarity).

As to the expected social welfare, we know by equations (40) and (48) that

$$\begin{aligned} \mathbf{E}(u^Q|\theta) - \mathbf{E}(u^P|\theta) &= \frac{\mathbf{E}(\Pi^Q|\theta) - \mathbf{E}(\Pi^P|\theta)}{2} \\ &\quad - \frac{n}{1 + n\gamma} [1 - K^P(\lambda) + 1 - (1 + n\gamma) K^Q(\lambda)] \theta^2. \end{aligned} \quad (56)$$

The second term, always negative and independent from the information quality, introduces a bias in favor of the game in prices. As we may infer from Proposition 2, this bias reinforces the effect of the strategy distortion when the goods are complements, but countervails and possibly reverses it when they are substitutes. Indeed, using equation (53), we obtain

$$\mathbf{E}(u^Q|\theta) - \mathbf{E}(u^P|\theta) = \frac{n}{2(1 + n\gamma)} \left[\begin{aligned} &\left((K^P(\lambda))^2 - (2 - (1 + n\gamma) K^Q(\lambda))^2 \right) \theta^2 \\ &+ (K^P(\lambda))^2 C^P - ((1 + n\gamma) K^Q(\lambda))^2 C^Q \end{aligned} \right], \quad (57)$$

where the term involving the fundamental tends to a negative limit as $\gamma \rightarrow 1/n$, but also as $\gamma \rightarrow \infty$.

The term involving the fundamental does not depend upon the information quality, but the combined weight of the two other terms increases with the

information cost, as the precisions of the two signals become lower and lower, so that the effect of the informational distortion may eventually become prevalent, aligning the ranking of the two games obtained for social welfare to that obtained for the expected profit.

We can now summarize the preceding results in the following proposition.

Proposition 5 *It is indifferent to play in prices or in quantities when goods are independent ($\gamma = 0$), or else in the absence of the competition motive ($\lambda = 0$) and of information dispersion ($\alpha = \infty$, $\beta = \infty$ or $\beta = 0$).*

When goods are interdependent, the information dispersed, but still in the absence of the competition motive, the expected equilibrium profit and the social welfare are higher (but the consumer surplus lower) under the anti-coordination motive (under strategic substitutability), hence in the quantity game when goods are substitutes and in the price game when they are complements.

The strategy distortion generated by the competition motive reinforces the advantage of anti-coordination (of strategic substitutability) for the expected equilibrium profit, so that the same result obtains if the precision of one of the two signals is high enough (making the strategy distortion dominant over the informational distortion). However, this advantage stands for social welfare only when goods are complements, turning into a disadvantage when they are substitutes.

As the informational distortion has an opposite effect, if the precision of both signals is low enough (making it dominant over the strategy distortion), the expected equilibrium profit and the social welfare are higher (but the consumer surplus lower) under the coordination motive (hence under strategic complementarity, in the price game when goods are substitutes and in the quantity game when they are complements).

6 Conclusion

We proposed a simple unified framework bridging the gap between team and competition issues under imperfect and dispersed information, and allowing to reconsider the social value of information in price and quantity games and to compare their outcomes in terms of equilibrium and social welfare.

The competition motive creates, even when information is perfect, a well-known strategy distortion and, in addition, when information is imperfect and dispersed, an informational distortion. Both distortions are detrimental to profitability, and the more so the higher the intensity of competition, through either structure (by the degree of interdependence between goods or by the number of competitors) or conduct (by the strength of the competition motive). The informational distortion generated by the competition motive in both price and quantity games is detrimental as it makes the relative weight put on the public signal inefficiently high (low) under the anti-coordination (coordination) motive, hence under strategic substitutability (complementarity). Both distortions interact and the dominance of one over the other depends on the information

precision and naturally lead to different effects in terms of equilibrium and social welfare.

Regarding the social value of information, we show that increasing the precision of public information always improves welfare. By contrast, the social value of private information may be negative under the coordination motive (in the presence of strategic complementarity) for a high enough intensity of competition.

While the competition motive operating under anti-coordination (in the presence of strategic substitutability) yields higher profit and social welfare when information is perfect, imperfect and dispersed information mitigates such a result. Indeed, if information quality is relatively low, making the information distortion dominate the strategy distortion, profit and social welfare are higher under coordination (in the presence of strategic complementarity).

Our results may have some implications on the performance of the competition regimes (price vs. quantity competition). In a context where the company divisions have poorly accurate information on the fundamental (the market size), they may prefer to compete in prices (quantities) if goods are substitutes (complements). A regulator that would increase the precision of public information might reverse such preference by reducing the weight of the informational distortion relative to that of the strategy distortion.

7 References

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