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# Description-dependent Choices

Dino Borie\*      Dorian Jullien†

## Abstract

The standard model of choice behavior relies on an implicit assumption that a decision maker is not affected by different descriptions of a given problem (description invariance). However, the behavioral economics and psychology literatures provide well-established evidence that descriptions do in fact influence decision makers. In this paper, we distinguish between descriptions of objects of choice and consequences of objects of choice in order to deduce a decision maker’s preferences over the descriptions from observed choices over the consequences. We provide a choice theoretical foundation for maximizing preference relations subject to the class of framing effects where description invariance is violated.

**JEL Classification:** D89, D90, D91.

**Keywords:** Choice correspondence, framing effects, rational choice, description invariance, description dependence.

## 1 Introduction

A decision maker exhibits a framing effect when she or he makes different choices under different presentations of a given decision problem. There is a variety of framing effects depending on what makes a difference in the presentations. For instance, there can be different alternatives of a given decision problem set as the “default” (i.e., automatically chosen if the agent makes no active choice) or different order of presentation of the alternatives. A few theoretical contributions characterize the axiomatic implications of some framing effects in terms of revealed preferences or choice correspondences (e.g., [Masatlioglu and Ok, 2005](#); [Salant and Rubinstein, 2008](#); [Bernheim and Rangel, 2009](#); [Dietrich and List, 2016](#))<sup>1</sup>. In this paper, we provide such a characterization for a specific type of framing effects that these contributions do not fully address, if at all: different choices under different descriptions of the alternatives of a given decision problem. To illustrate, “25% fat ground beef” and “75% lean ground beef” are two descriptions of the same alternative. If a decision maker is not indifferent between these two descriptions, then she or he exhibits *description-dependent choices*, which violates the implicit but standard

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<sup>1</sup>Throughout the paper, we characterize choice correspondence, i.e., set of possible choices, instead of choice function, i.e., effective choice, because we are interested in a decision maker with a weak order which exhibits description dependency.

behavioral axiom of “description invariance”. Empirically this can correspond to at least two situations. The first situation consists in one choice set containing two pieces of ground beef which are the same except for their descriptions. An example of description-dependent choices here is a decision maker who chooses the piece described as “75% lean ground beef” from {“25% fat ground beef”, “75% lean ground beef”}. The second situation consists in two choice sets containing an opportunity to choose nothing and the same piece of ground beef under different descriptions in each choice set. An example of description-dependent choices here is a decision maker who chooses the ground beef from {“75% lean ground beef”, nothing} and nothing from {“25% fat ground beef”, nothing}. These choices have indeed been observed in a number of experiments with different objects of choice – among which the “25% fat”/“75% lean” ground beef is the classical one (see [Levin et al., 1998, 2002](#); [Janiszewski et al., 2003](#); [Keren, 2007](#)).

Several contributions make the case that description-dependent choices are not necessarily irrational (see, e.g., [Sher and McKenzie, 2006, 2008, 2011](#); [Keren, 2011](#); [Bourgeois-Gironde and Giraud, 2009](#)). The main argument is that there are cases where the fact that an object of choice is described in one way instead of another is a choice-relevant information about the decision situation, which is “leaked” by the one who poses the decision problem and inferred by the decision maker. For instance, [Keren \(2007\)](#) shows how decision makers can be aware that the two pieces of ground beef are of the same quality but infer different information about the butcher (e.g., he or she is more or less honest) from the different descriptions. We capture this possibility by introducing a distinction between the decision modeler – henceforth denoted by  $d_{mo}$  – who poses or observes a decision problem and the decision maker – henceforth denoted by  $d_{ma}$  – who makes the choice. Description invariance implies two requirements. Firstly, the  $d_{ma}$  agrees with the  $d_{mo}$  on which different descriptions are about the same alternative. Secondly, the  $d_{ma}$  is necessarily indifferent between all the descriptions of a given alternative. We propose a framework to make these requirements of description invariance explicit and we weaken them to construct choice correspondences that can represent description-dependent choices<sup>2</sup>.

The paper is structured in three sections. The first one provides the preliminaries of the framework and shows how to construct description-dependent choice correspondences from observed preferences – with the standard description-independent choice correspondences as a special case. The second section presents the axioms that description-dependent choice correspondences need to respect to ensure that we can recover the underlying preferences (i.e., the underlying weak ordering). The third section concludes by a discussion of our framework with respect to other theoretical contributions on choice correspondences for framing effects. All proofs are in the Appendix.

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<sup>2</sup>There is a small number of contributions that tackle the problem in terms of specific preference relations instead of general choice behavior (see [Bacharach 2003](#), [Giraud 2005](#), [Blume et al. 2013](#), [Gold and List 2004](#), [Bourgeois-Gironde and Giraud 2009](#) and [Ahn and Ergin 2010](#)).

## 2 Description-dependent choice correspondences from observed preferences

Let  $\mathcal{X}$  be a non-empty finite set of objects of choice and  $\approx$  a binary relation on  $\mathcal{X}$ . Elements of  $\mathcal{X}$  are interpreted as *described consequences*. For every described consequences  $x, y \in \mathcal{X}$ ,  $x \approx y$  holds if (and only if) from the  $d_{mo}$ 's perspective, they are two descriptions of a same consequence. Observe that  $\approx$ , by definition, is an equivalence relation. If  $x \approx y$ , we say that  $x$  and  $y$  are consequentially equivalent. In other words, the  $d_{mo}$ 's perspective defines equivalence classes of described consequences. In the standard framework, the  $d_{ma}$  and the  $d_{mo}$  share the same perspective on what counts as a consequence. With our notation, the set of standard objects of choice is the set of equivalence classes of  $\mathcal{X}$  under  $\approx$ , denoted by  $X$ . A standard choice problem is a non-empty subset of  $X$ , denoted by  $A, B, \dots$ . By contrast, a *described choice problem* is a non-empty subset of  $\mathcal{X}$ , denoted by  $f, g, \dots$ . We can illustrate these definitions with the following example.

**Example 1.** Consider three consequences. The first one is eating chicken  $[x]_{\approx}$  under whatever description  $x$ . The second one is inspired from Levin and Gaeth (1988): eating ground beef  $[y]_{\approx}$  described either as “25% fat ground beef”  $y_1$  or as “75% lean ground beef”  $y_2$ . The third one is eating beans  $[z]_{\approx}$  under whatever description  $z$ .  $\mathcal{X} = \{x, y_1, y_2, z\}$  and  $X = \{[x]_{\approx}, [y]_{\approx}, [z]_{\approx}\}$ . Choosing between ground beef and beans is a standard choice problem  $A = \{[y]_{\approx}, [z]_{\approx}\}$  that can give rise to three different described choice problems:  $f = \{y_1, z\}$ ,  $g = \{y_2, z\}$  and  $h = \{y_1, y_2, z\}$ .

Our framework allows to relate the set of standard choice problem to the set of described choice problem. Let  $\mathcal{P}^*(\mathcal{X})$  be the set of described choice problem (the set of all non-empty subsets of  $\mathcal{X}$ ) and  $\mathcal{P}^*(X)$  be the set of standard choice problem (the set of all non-empty subsets of  $X$ ). We extend the relation  $\approx$  on  $\mathcal{X}$  to  $\mathcal{P}^*(\mathcal{X})$  and denote this extended relation by  $\approx_{\mathcal{P}}$ . The relation  $\approx_{\mathcal{P}}$  on  $\mathcal{P}^*(\mathcal{X})$  is defined as: for all  $f, g \in \mathcal{P}^*(\mathcal{X})$ ,

$$f \approx_{\mathcal{P}} g \text{ iff } (\forall x \in f, \exists y \in g, x \approx y) \text{ and } (\forall y \in g, \exists x \in f, y \approx x).$$

It is readily seen that the relation  $\approx_{\mathcal{P}}$  on  $\mathcal{P}^*(\mathcal{X})$  is an equivalence relation. More precisely,  $\approx_{\mathcal{P}}$  extends the  $d_{mo}$ 's perspective on consequential equivalence in choice problems. To lighten the notation we remove the subscript so that we use  $\approx$  instead of  $\approx_{\mathcal{P}}$  throughout. To a standard choice problem we can associate all its descriptions, in particular there is a natural correspondence between  $\mathcal{P}^*(\mathcal{X})/\approx$  and  $\mathcal{P}^*(X)$ .<sup>3</sup>

**Example 1 (continued).** From the  $d_{mo}$ 's perspective, there are three descriptions of the standard choice problem  $A$ :  $f, g$  and  $h$ . It is readily seen that  $f \approx g \approx h$  and  $A = [f]_{\approx}$ .

<sup>3</sup>Define  $r : \mathcal{P}^*(X) \rightarrow \mathcal{P}^*(\mathcal{X})/\approx_{\mathcal{P}}$  by

$$r(A) := \left[ \bigcup_{[x]_{\approx} \in A} [x]_{\approx} \right]_{\approx_{\mathcal{P}}}.$$

It is straightforward to show that  $r$  is one-to-one and onto.

A standard choice correspondence attaches to every choice problem  $A \subseteq X$  a non-empty subset of  $A$ . There are at least two possible ways of defining description-dependent choice correspondences. The first one attaches to every described choice problem  $f \subseteq \mathcal{X}$  a non-empty subset of  $f$ . This is straightforward in the sense that it would suffice to impose the standard (expansion and contraction) consistency conditions to construct description-dependent choice correspondences. However, this way does not allow to formally represent the key feature that makes description-dependent choices cases of framing effects, namely that some descriptions are about the same consequence. Indeed, in this way the consequential equivalence of the  $d_{mo}$  does not constrain the construction of both description-dependent choice correspondences and (by implication) the underlying preference relations. It follows that all patterns of preferences over described consequences, including classical preference reversals, are rational. Hence the logical principle of extensionality underlying description invariance would disappear from our framework and with it the connection with the standard framework. To bypass this drawback we use another definition: a *description-dependent choice correspondence* attaches to every described choice problem  $f \subseteq \mathcal{X}$  a non-empty subset of  $[f]_{\approx}$ . In this way we can represent the description-dependent choices of a  $d_{ma}$  over described consequences and still track which descriptions are about the same consequence from the consequentialist perspective of the  $d_{mo}$ . Hence we formally identify the key feature of framing effects with choices by the  $d_{ma}$  of one described consequences over alternative described consequences that are equivalent for the  $d_{mo}$ .

Given a weak order  $\succsim$  on  $\mathcal{X}$ , the induced description-dependent choice correspondence  $c_{\succsim}$  is defined by

$$c_{\succsim}(f) = \{[x]_{\approx} \in X \mid \exists x' \in [x]_{\approx} \cap f, x' \succsim y, \forall y \in f\}.$$

If  $\mathcal{X}$  is finite, this set is necessarily non-empty.

**Example 1 (continued).** Suppose the following pattern of preference:  $x \succ y_2 \succ z \succ y_1$  where  $\succ$  is a strict weak ordering. The induced description-dependent choice correspondence is given by  $c(\{x\}) = [x]_{\approx}$ ,  $c(\{y_1\}) = [y]_{\approx}$ ,  $c(\{y_2\}) = [y]_{\approx}$ ,  $c(\{z\}) = [z]_{\approx}$ ,  $c(\{x, y_1\}) = [x]_{\approx}$ ,  $c(\{x, y_2\}) = [x]_{\approx}$ ,  $c(\{x, z\}) = [x]_{\approx}$ ,  $c(\{y_1, y_2\}) = [y]_{\approx}$ ,  $c(\{y_1, z\}) = [z]_{\approx}$ ,  $c(\{y_2, z\}) = [y]_{\approx}$ ,  $c(\{x_1, y_1, y_2\}) = [x]_{\approx}$ ,  $c(\{x, y_1, z\}) = [x]_{\approx}$ ,  $c(\{x, y_2, z\}) = [x]_{\approx}$ ,  $c(\{y_1, y_2, z\}) = [y]_{\approx}$ , and finally  $c(\{x, y_1, y_2, z\}) = [x]_{\approx}$ .

Now that we have seen how to construct description-dependent choice correspondences from observed preferences, we need to characterize the converse.

### 3 Preferences from description-dependent choice correspondences

The following two axioms need to be imposed on description-dependent choice correspondences to ensure that we can recover the underlying weak ordering. The first one is a restatement of a classical consistency condition (usually called property  $\gamma$ ) for described choice problem:

**Set union consistency under one frame** If  $[x]_{\approx} \in c(f)$ ,  $[y]_{\approx} \in c(g)$ , and  $[y]_{\approx} \cap g \subseteq f$ , then for all described choice problem  $h$  such that  $[x]_{\approx} \cap f \subseteq h \subseteq f \cup g$ , it holds that  $[x]_{\approx} \in c(h)$ .

This axiom simply requires that if we observe that chicken is chosen out of {chicken, “75% lean ground beef”} and that “75% lean ground beef” is chosen out of {“75% lean ground beef”, beans}, then we should observe that chicken is chosen when the three alternatives are available, i.e., out of {chicken, “75% lean ground beef”, beans}. (In fact the axiom implies that chicken should be chosen whenever it is available against any combination of the other two options). The crucial part here is that ground beef is presented under the same frame (i.e., the “75% lean” frame in the example) in both described choice problems (i.e., the first two choice sets in the example).

The core of our contribution is the second axiom, which we specifically design for decision problems with framing of descriptions:

**Set union consistency across frames** If  $[x]_{\approx} \in c(f) \cap c(g)$ ,  $[y]_{\approx} \in c(h)$ ,  $[x]_{\approx} \cap g \subseteq h$ , and  $[y]_{\approx} \cap h \subseteq f$ , then for all described choice problem  $k$  such that  $[x]_{\approx} \cap f \subseteq k \subseteq f \cup g$ , it holds that  $[x]_{\approx} \in c(k)$ .

To illustrate the requirements of this axiom we need to use two descriptions of eating chicken, e.g., as “85% lean chicken” and as “15% fat chicken”. This axiom requires that if we observe “85% lean chicken” chosen out of {“85% lean chicken”, “75% lean ground beef”}, “75% lean ground beef” chosen out of {“75% lean ground beef”, “15% fat chicken”} and “15% fat chicken” chosen out of {“15% fat chicken”, “25% fat ground beef”}, then “85% lean chicken” should be chosen when the four options are available, i.e., out of {“85% lean chicken”, “15% fat chicken”, “75% lean ground beef”, “25% fat ground beef”}. (In fact “85% lean chicken” should be chosen whenever it is available against any combination of the other three options). Observe that despite the preference reversals between ground beef and chicken, the  $d_{ma}$  exhibits a maximizing behavior (lean is preferred to fat).

**Proposition 1.** *Let  $\mathcal{X}$  be a finite choice set of described consequences such that  $X$  is not trivial and  $c$  a description-dependent choice correspondence. Then,  $c$  is a description-dependent choice correspondence that satisfies Set union consistency under one frame and Set union consistency across frames if, and only if, there exists a weak order  $\succsim$  on  $\mathcal{X}$  (not necessarily unique) such that  $c = c_{\succsim}$ .*

This proposition has implications for how our framework can or cannot account for the basic examples of description-dependent choices presented in the introduction. Our framework cannot account for the case of a  $d_{ma}$  choosing “75% lean ground beef” out of {“25% fat ground beef”, “75% lean ground beef”}. This is due to the way we construct description-dependent choice correspondences, i.e., with the  $d_{mo}$  who observes only consequences. Here the two objects of choice are identical for the  $d_{mo}$  hence whatever the  $d_{ma}$  chooses, it will give no information to the  $d_{mo}$  about the underlying weak order on described consequences. In other word, the  $d_{ma}$  violates the  $d_{mo}$ ’s consequential equivalence, but the  $d_{mo}$  cannot observe this. By contrast, the first, straightforward, way of constructing description-dependent choice correspondences on the set of described consequences rather than on the set

of consequences, would account for this case. In this way, the  $d_{mo}$  observes described consequences and the violation of his or her consequential equivalence by the  $d_{ma}$  (if the  $d_{mo}$  cares about consequentialism). Furthermore, this example illustrates the difficulty to observe preferences between two descriptions of the same consequence when we focus only on consequences. For instance, in the notation of Example 1, the following two profiles of preferences generate and are generated by the same description-dependent choice correspondence whereas the behaviour is indeed not the same:  $x \succ y_1 \succ y_2 \succ z$  and  $x \succ y_2 \succ y_1 \succ z$ . Here it is possible to define a preference relation over the consequences:  $[x]_{\approx}$  is strictly preferred to  $[y]_{\approx}$  which is strictly preferred to  $[z]_{\approx}$ . But we cannot infer a preference relation between the descriptions of the ground beef (i.e., between  $y_1$  and  $y_2$ ). Formally, this illustrates that the weak order is not necessarily unique in the above proposition.

Our framework can however account for the case of the  $d_{dma}$  choosing “75% lean ground beef chosen” out of {“75% lean ground beef”, nothing} and nothing out of {“25% fat ground beef”, nothing}, which is rationalized by the following pattern:  $y_2 \succ \emptyset \succ y_1$ . Note that between two descriptions of the same consequence there is another consequence. This is the only possibility in our framework to compare two descriptions of the same consequence. Moreover, here this guarantees the uniqueness of the preference relation. Hence we can observe a preference reversal over described consequences while focusing only on choices over consequences (by contrast with the straightforward way of defining description-dependent choice correspondences over described consequences).

## 4 Contribution with respect to other approaches

We conclude by highlighting our contributions through a comparison of our framework with two other frameworks that are close to ours: Salant and Rubinstein (2008) and Dietrich and List (2016).

Salant and Rubinstein (2008) do not claim to capture framing effects exhibiting description-dependent choices. Nevertheless, their framework is the most general one in the literature on choice functions for framing effects and understanding how it has difficulties to account for description-dependent choices highlights our contribution to this literature. Salant and Rubinstein extend the traditional approach in terms of choice function on the set of available alternatives  $A$  (i.e., a choice problem) from the set of alternatives  $X$  by constructing choice functions on pairs  $(A, f)$ , where  $f$  is one frame from a set of frames  $F$ . Though they do not deal with frames as descriptions, it is easy to use our notion of frame as description in their framework: their frame  $f$  is our described choice problem  $f$  and their choice problem  $A$  is our standard choice problem  $[f]_{\approx}$ . Hence the pairs  $(A, f)$  in their framework correspond to the pairs  $([f]_{\approx}, f)$  in our framework.

Following Salant and Rubinstein’s approach, an *extended* choice function  $c$  assigns a chosen consequence  $[x]_{\approx}$  to every pair  $([f]_{\approx}, f)$ . Given a weak order  $\succsim$  on  $\mathcal{X}$ , the induced extended choice function  $c_{\succsim}$  is defined by

$$c_{\succsim}([f]_{\approx}, f) = \{[x]_{\approx} \in X \mid \exists x' \in [x]_{\approx} \cap f, x' \succsim y, \forall y \in f\}.$$

If  $\mathcal{X}$  is finite, this set is necessarily nonempty. To see how their approach cannot capture framing effects exhibiting description-dependent choices, we propose the following counter example. From example 1, suppose that the  $d_{ma}$  likes meat very much and derives more utility from what looks healthy, so that she or he prefers the ground beef described as “75% lean” over chicken over beans over the ground beef described as “25% fat”, i.e., he or she exhibits the following profile of preferences:  $y_2 \succ x \succ z \succ y_1$ .

The order on the described consequences violates Salant and Rubinstein’s property ensuring the existence of an equivalence between an extend choice function and a weak order on  $X$ . They call this property  $\gamma^+$ -extended, which, adapted to deal with different descriptions of choice problems, can be stated as follow: if  $c([f]_{\approx}, f) = [x]_{\approx}$ ,  $c([g]_{\approx}, g) = [y]_{\approx}$  and  $[y]_{\approx} \in [f]_{\approx}$ , then there exists a described choice problem  $h$  such that  $[h]_{\approx} = [f]_{\approx} \cup [g]_{\approx}$  and  $c([h]_{\approx}, h) = [x]_{\approx}$ . To illustrate a violation, note that according to the  $d_{ma}$ ’s preferences:  $c([z]_{\approx}, [y]_{\approx}, (z, y_1)) = [z]_{\approx}$ ,  $c([x]_{\approx}, [y]_{\approx}, (x, y_2)) = [x]_{\approx}$  and  $[y]_{\approx} \in ([y]_{\approx}, [z]_{\approx})$ . However,

$$\begin{cases} c([x]_{\approx}, [y]_{\approx}, [z]_{\approx}, (x, y_1, z)) = [x]_{\approx}, \\ c([x]_{\approx}, [y]_{\approx}, [z]_{\approx}, (x, y_2, z)) = [y]_{\approx}, \\ c([x]_{\approx}, [y]_{\approx}, [z]_{\approx}, (x, y_1, y_2, z)) = [y]_{\approx}, \end{cases}$$

hence  $[z]_{\approx} \notin c([f]_{\approx} \cup [g]_{\approx}, h)$  for all description  $h$  of the choice problem  $[f]_{\approx} \cup [g]_{\approx}$ .

Thus, it can be argued that our framework is an extension of Salant and Rubinstein’s to deal with description-dependent choices, on two grounds. Firstly, our first axiom (set union consistency under one frame) is a suitable modification (i.e., not a simple translation) of their property  $\gamma^+$ -extended. Secondly, our second axiom (set union consistency across frames) is an addition to their framework which is crucial to account for description-dependent-choices. We would like to acknowledge that Salant and Rubinstein’s framework was an important source of inspiration for the construction of our framework.

The framework of Dietrich and List (2016) is the only one in the choice function literature that claims to account for description-dependent choices. We do not intend to make a formal comparison with their work because the most relevant similarities and differences are at a conceptual level. Their framework is close to ours in the sense that they distinguish two perspectives on a given decision situation: one (“objective”) from the modeler and the other (“subjective”) from the decision maker. The implicit relaxation of description invariance in their work is however different from ours: for them, if the decision maker perceives two descriptions as being about the same object, then he or she is necessarily indifferent between the two descriptions. In our framework there is no such necessity, though it is a possibility. Furthermore, their approach involves more abstract, less observable, primitives (perceptions of objects of choice) than ours (descriptions of objects of choice). This allows them to give a new perspective on a broader range of behavioral phenomena (e.g., attraction and compromise effects, order effects) including framing effects through new notions of context-dependency, while we stick to a more standard approach focused on individual observable behavior. They face the same dilemma concerning the choice set: straightforwardly redefining it as a set of perceptions or enriching it by modeling which perceptions are about the same object of choice



(Dietrich and List, 2016, p.184). They take the second way for two reasons that are immaterial for us (related to the fact that perceptions are never directly observable while descriptions are) and one reason that is essentially the same that we give above (different perspectives on a given object of choice do not imply that there is not one and the same object choice)<sup>4</sup>.

To conclude, Schotter (2008, pp.86-7) notes that frame-sensitive choices are problematic when we just have choice data without information about the frame (e.g., descriptions) under which choices are elicited. Of course, our theoretical contribution by itself does not resolve this problem. However, it offers a framework to formally take such information into account when available for cases of description-dependent choices. Although this may be difficult to obtain in the field (e.g., data about insurance choices are silent about how insurance brokers orally describe their policies during a given deal), it is very easy to obtain in the lab because descriptions are in the instructions.

## Appendix

### Consistency and independence of the axioms

The consistency of the axioms is obvious given that Proposition 1 is true; however, it is of some interest to prove their consistency by providing a very simple model. Let

$$\begin{aligned} \mathcal{X} &= \{x_1, x_2, y_1, y_2\}, \\ \approx &: x_1 \approx x_2 \text{ and } y_1 \approx y_2, \\ \succsim &: x_1 \succ x_2 \succ y_1 \succ y_2, \\ c_{\succsim}(f) &= \{[x]_{\approx} \in X \mid \exists x' \in [x]_{\approx} \cap f, x' \succsim y, \forall y \in f\}. \end{aligned}$$

The reader can check that this system fulfils the axioms. The following examples show the independence of Set union consistency under one frame and Set union consistency across frames. They are itemised according to the property violated. Their verification is left to the reader.

**Set union consistency under one frame.**  $\mathcal{X} = \{x, y, z\}$ ,  $x \not\approx y$ ,  $y \not\approx z$ ,  $x \not\approx z$ ,  $c(\{x\}) = \{[x]_{\approx}\}$ ,  $c(\{y\}) = \{[y]_{\approx}\}$ ,  $c(\{z\}) = \{[z]_{\approx}\}$ ,  $c(\{x, y\}) = \{[x]_{\approx}\}$ ,  $c(\{y, z\}) = \{[y]_{\approx}\}$ ,  $c(\{x, z\}) = \{[z]_{\approx}\}$ , and  $c(X) = \{[x]_{\approx}, [y]_{\approx}, [z]_{\approx}\}$ .

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<sup>4</sup>Comparison with other frameworks proposing choice functions and correspondences for different types of choice reversals (i.e., other than framing effects in general and framing effects triggered by different descriptions in particular) would not be directly enlightening for the basic phenomena we are interested in (for recent references, see Ok et al. 2015 and Furtado et al. 2017. There is nevertheless some similarity in terms of approach with some of them. Generally, these contributions insist on making the standard model a specific case of a more general framework. More specifically, the approach of expliciting an implicit assumption in order to weaken it so that a change of preference can be rationalized without introducing two preference relations is close to Masatlioglu et al. (2012) – who make explicit the implicit assumption of full attention in the standard model and weaken it to represent limited attention.

**Set union consistency across frames.**  $\mathcal{X} = \{x_1, x_2, y_1, y_2\}$ ,  $x_1 \approx x_2$ ,  $y_1 \approx y_2$ ,  $c(\{x_1\}) = \{[x]_{\approx}\}$ ,  $c(\{x_2\}) = \{[x]_{\approx}\}$ ,  $c(\{y_1\}) = \{[y]_{\approx}\}$ ,  $c(\{y_2\}) = \{[y]_{\approx}\}$ ,  $c(\{x_1, x_2\}) = \{[x]_{\approx}\}$ ,  $c(\{x_1, y_1\}) = \{[x]_{\approx}\}$ ,  $c(\{x_1, y_2\}) = \{[y]_{\approx}\}$ ,  $c(\{x_2, y_1\}) = \{[y]_{\approx}\}$ ,  $c(\{x_2, y_2\}) = \{[x]_{\approx}\}$ ,  $c(\{y_1, y_2\}) = \{[y]_{\approx}\}$ ,  $c(\{x_1, x_2, y_1\}) = \{[x]_{\approx}\}$ ,  $c(\{x_1, x_2, y_2\}) = \{[x]_{\approx}\}$ ,  $c(\{x_1, y_1, y_2\}) = \{[y]_{\approx}\}$ ,  $c(\{x_2, y_1, y_2\}) = \{[y]_{\approx}\}$ , and finally  $c(X) = \{[x]_{\approx}, [y]_{\approx}\}$ .

## Proof of proposition 1

Let  $\mathcal{X}$  be a finite choice set of described consequences such that  $X$  is not trivial and  $c$  a description-dependent choice correspondence. The necessity of the axioms is straightforward. Suppose that there exists a weak order  $\succsim$  on  $\mathcal{X}$  such that  $c = c_{\succsim}$ . To see that  $c$  satisfies set union consistency under one frame, note that if  $[x]_{\approx} \in c(f)$ ,  $[y]_{\approx} \in c(g)$ , and  $[y]_{\approx} \cap g \subseteq f$ , then there exists a  $x' \in f$  such that  $x' \approx x$  and  $x' \succsim z$  for all  $z \in f$  and there exists  $y' \in [y]_{\approx}$  such that  $y' \in f$  and  $y' \succsim z$  for all  $z \in g$ . It follows that  $x' \succsim z$  for all  $z \in f \cup g$ . Let  $h$  such that  $[x]_{\approx} \cap f \subseteq h \subseteq f \cup g$ , hence  $x' \in h$ . Thus  $[x]_{\approx} \in c(h)$ . The proof for set union consistency across frames is handled similarly.

To prove sufficiency, let  $c$  be a description-dependent choice correspondence satisfying Set union consistency under one frame and across frames. Define  $R_c$  on  $\mathcal{X}$  thus:

$$xR_cy \Leftrightarrow \exists f \subseteq \mathcal{X}, [x]_{\approx} \cap f = x, [y]_{\approx} \cap f = y, [x]_{\approx} \in c(f),$$

and  $\succsim_c$  on  $\mathcal{X}$  as follows:

$$\begin{aligned} x \succsim_c y &\Leftrightarrow x = y, xR_cy \\ &\text{or } x \neq y, x \not\approx y, xR_cy \\ &\text{or } x \neq y, x \approx y, \exists t \in \mathcal{X}, t \not\approx x, xR_ct, tR_cy. \end{aligned}$$

Before presenting the main arguments of the proof, we provide an useful fact.

**Claim 1.** *Let  $\mathcal{X}$  be a finite choice set of described consequences such that  $X$  is not trivial and  $c$  a description-dependent choice correspondence satisfying Set union consistency under one frame. Then, for all  $x, y \in \mathcal{X}$  such that  $x = y$  or  $x \not\approx y$ ,*

$$xR_cy \Leftrightarrow [x]_{\approx} \in c(\{x, y\}).$$

By assumption,  $c$  is non-empty; it follows that for all  $x \in \mathcal{X}$ ,  $xR_cx$ , hence  $\succsim_c$  is reflexive. Next, we show that  $\succsim_c$  is transitive. Assume  $x \succsim_c y$  and  $y \succsim_c z$ . (We omit the trivial cases where transitivity follows from  $x = y$  or  $y = z$ .)

**Case 1.** If  $x \not\approx y$ ,  $xR_cy$ ,  $y \not\approx z$  and  $yR_cz$ , then either  $x = z$ ,  $x \neq z$  and  $x \approx z$ , or  $x \not\approx z$ . The two former cases are straightforward. So suppose that  $x \not\approx z$ , by Claim 1,  $[x]_{\approx} \in c(\{x, y\})$ , and  $[y]_{\approx} \in c(\{y, z\})$ . It is readily seen that  $[y]_{\approx} \cap \{y, z\} \subseteq \{x, y\}$ . By Set union consistency under one frame  $[x]_{\approx} \in c(h)$  whenever  $\{x\} \in h \subseteq \{x, y, z\}$ . Then  $[x]_{\approx} \in c(\{x, z\})$  and hence  $x \succsim_c z$ .

**Case 2.** If  $x \not\approx y$ ,  $xR_c y$ ,  $y \approx z$ ,  $y \neq z$ , there exists  $t \in \mathcal{X}$ ,  $t \not\approx y$ ,  $yR_c t$ , and  $tR_c z$ . By hypothesis  $x \not\approx z$  since  $x \not\approx y$  and  $y \approx z$ . Assume that  $x \not\approx t$ , by the preceding argument as in Case 1,  $x \not\approx y$ ,  $xR_c y$ ,  $y \not\approx t$  and  $yR_c t$  give  $[x]_{\approx} \in c(\{x, t\})$ . It follows that  $x \not\approx t$ ,  $xR_c t$ ,  $t \not\approx z$  and  $tR_c z$ . Again, by use of Case 1 and the observation just made above, we have  $[x]_{\approx} \in c(\{x, z\})$ , whence  $x \succ_c z$ . Suppose now that  $x \approx t$ . Setting  $f = \{x, y\}$ ,  $g = \{t, z\}$  and  $h = \{y, t\}$ , we have  $[x]_{\approx} \in c(f) \cap c(g)$ ,  $[y]_{\approx} \in c(h)$ ,  $[x]_{\approx} \cap g = \{t\} \subseteq h$ , and  $[y]_{\approx} \cap h = \{y\} \subseteq f$ , then by Set union consistency across frames, for all  $k \subseteq \mathcal{X}$  such that  $\{x\} \subseteq k \subseteq \{x, y, t, z\}$ , it holds that  $[x]_{\approx} \in c(k)$ . Then  $[x]_{\approx} \in c(\{x, z\})$  and hence  $x \succ_c z$ .

**Case 3.** If  $x \approx y$ ,  $x \neq y$ , there exists  $t \in \mathcal{X}$ ,  $t \not\approx x$ ,  $xR_c t$ ,  $tR_c y$ ,  $y \not\approx z$ ,  $yR_c z$ . The proof is similar to the Case 2.

**Case 4.** If  $x \approx y \approx z$ ,  $x \neq y$ ,  $y \neq z$ , there exist  $s, t \in \mathcal{X}$ ,  $s, t \not\approx x$ ,  $xR_c s$ ,  $sR_c y$ ,  $yR_c t$ , and  $tR_c z$ . If  $x = z$ , there is nothing to prove. Suppose then that  $x \neq z$ . We need to show that there exists  $r \in \mathcal{X}$  such that  $r \not\approx x$ ,  $xR_c r$ , and  $rR_c z$ . We can take either  $s$  or  $t$  and the proofs are similar to the cases 1 or 2.

We have just shown that  $\succ_c$  is transitive. If there exists  $x, y$  incomparable for  $\succ_c$ , then necessarily  $x \approx y$  and there exist  $z_1, z_2$  such that  $z_1, z_2 \not\approx x, y$  and  $z_1 \succ_c x, y \succ_c z_2$  (or just  $z_1$  if  $x, y$  are bottom elements or just  $z_2$  if  $x, y$  are top elements). We can easily extend  $\succ_c$  into a weak order. Let  $\succ_c^*$  be such extension. It is straightforward to show that  $c = c_{\succ_c^*}$ . Q.E.D.

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