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# THE SPATIAL SORTING OF INFORMAL DWELLERS IN CITIES IN DEVELOPING COUNTRIES: THEORY AND EVIDENCE\*

HARRIS SELOD<sup>†</sup> AND LARA TOBIN<sup>‡</sup>

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## Abstract

We propose a theory of urban land use with endogenous property rights that applies to cities in developing countries. Households compete for where to live in the city and choose the property rights they purchase from a land administration which collects fees in inequitable ways. The model generates predictions regarding the levels and spatial patterns of residential informality in the city. Simulations show that land policies that reduce the size of the informal sector may adversely impact households in the formal sector through induced land price increases. Empirical evidence from a sub-Saharan African city supports the model's assumptions and outcomes.

**JEL classification:** R14, R52 and P48

**Keywords:** Land markets; Property rights; Tenure security; Multiple sales

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# 1 Introduction

The main forces that shape the spatial structure of cities have long been identified by urban economic theory. In the standard land use model of urban economics (Alonso, 1964; Muth, 1969; Mills, 1972), heterogeneous households compete for where to live in the city while trading-off accessibility for land consumption under a set of endogenously determined prices. This spatial equilibrium representation of cities accounts for universal regularities such as decreasing population densities, a negative land price gradient moving away from city centers, and residential stratification by income (Fujita, 1989; Anas et al., 1998). The general validity of this simple model, however, has been challenged in the more complex setting that characterizes cities in the developing world (Marx et al., 2013). The main critique is that the canonical urban economics model rests on the implicit assumption that all city dwellers hold land with complete tenure security: once buyers have purchased a plot, they own it for good and do not face a risk of dispossession. This is a reasonable assumption in the case of developed countries where the overwhelming majority of owners have clear formal property rights established by a deed or by registration (Arruñada, 2012) and where renters have property rights enforced by legal contracts. In developing countries, however, the assumption does not hold. In many cities in Asia, Latin America, the Middle East, and especially sub-Saharan Africa, land is often—if not mostly—held informally and tenure is insecure (UN-Habitat, 2010).

The objective of our paper is to fill this gap and build an urban land use theory that accounts for tenure insecurity. Although there is an important focus in the economic literature regarding the impact of tenure security on investment decisions and economic outcomes (Besley, 1995; Besley and Ghatak, 2010), the question of *who* has access to secure tenure has somewhat been overlooked—something we also endeavor to explain with our theory. The main source of tenure insecurity in developing countries is land tenure informality although legal and regulatory frameworks, political will, commitments of national and local governments, and the capacity of administrations to deal with the demand for secure tenure also come into play (Durand-Lasserve and Selod, 2009). In this respect, it is important

to acknowledge that there is a wide range of informal tenure situations in the cities of developing countries, including situations where land is squatted upon, where it has been illegally developed or transferred, or where it is held with administrative documents that do not grant legally recognized rights. Similarly, residential formality encompasses various forms of use or ownership rights. Taken together, formal and informal situations constitute a “continuum” of options for holding land for housing (UN-Habitat, 2012) under various levels of tenure security: “More” formal rights are usually more secure, with ownership titles often providing the highest level of tenure security. The existence of this “continuum” also imply that different tenure situations often coexist within a same city, an issue which has largely remained unexplored. Formal and informal land market segments continue to be studied in isolation of each other, which prevents systemic analyses of why informal land markets exist, and of what makes formal tenure unaffordable to a large fraction of urban residents—a major challenge for policy makers (Buckley and Kalarickal, 2006).

A realistic theory of land use in developing countries needs to account for three major characteristics. Firstly, it should allow for the diversity of tenure situations and not just focus on the two extremities of the continuum. Secondly, it needs to specify the exact mechanisms whereby the lack of property rights or lack of enforcement of property rights results in tenure insecurity. So far, the literature has exclusively focused on the risk of eviction of squatters who occupy someone else’s land (Jimenez, 1984; Brueckner and Selod, 2009). Although squatting is an important topic, it only concerns a particular form of tenure insecurity. In many cities, the main source of tenure insecurity that affects the poor as well as a large fraction of the middle class is of a different nature as it stems from the multiplicity of claims (either because of multiple sales to different buyers, or because multiple claimants each managed to obtain a property right for the same plot) (Durand-Lasserve et al., 2013). We retain the second mechanism as the source of insecurity in our model. Lastly, the theory should account for the costly processes faced by agents to obtain property rights from the land administration. We incorporate these three elements in a generalized version of the standard monocentric land use model where households compete for land, trading-off

proximity to the city center and land consumption under cheaper prices further away. In our framework, households who have purchased plots must also choose the type of property right they buy from the land administration along a menu of tenure options. This introduces a second trade-off as weaker property rights can be obtained at a lower cost but entail a higher risk of multiple sale and thus of conflict and property loss. This is an important feature to consider as excessive land administration fees are often blamed for driving households into informality (Udry, 2011; Collier and Venables, 2013). We also allow the land administration to be clientelistic or corrupt, possibly collecting fees in inequitable ways that favor some households at the detriment of others.

Our setting is the first to account for diverse property rights in a unified land market equilibrium that applies to cities in developing countries. It makes it possible to understand the non-trivial interactions between land administration practices, household tenure choices, exposure to multiple sales and conflicts in a context where land prices and city structure are endogenously determined, and thus to understand the general equilibrium effects of urban land policies. Simulation of the model illustrates how policies targeted at improving access to secure tenure for some households will have an impact on the whole system and may actually harm other households through adverse effects on land rents.

The paper is structured as follows: Section 2 presents relevant insights from the literature on the topic. Section 3 presents the model and evidence from West Africa. Section 4 discusses land administration policies, and Section 5 concludes.

## 2 Insights from the literature

The literature on informal land markets is rather small. Apart from empirical papers on the measurement of the tenure security and tradability premia (Jimenez, 1982, 1984; Friedman et al., 1998; Lanjouw and Levy, 2002; Kim, 2004) or on the impacts of land tenure formalization in an urban context (Field, 2007; Di Tella et al., 2007; Galiani and Schargrodsky, 2010), very few papers have analyzed the causes of residential informality (Hidalgo et al., 2010) and, to our knowledge,

none have studied how informality affects the functioning of urban land markets.

The literature on the coexistence of formal and informal types of land use was initiated by [Jimenez \(1985\)](#) and [Hoy and Jimenez \(1991\)](#) who analyzed squatting in a partial equilibrium perspective. These papers present a framework of tenure choice under uncertainty, where in the case of an eviction from a squat, households lose their investments in housing and have to relocate in the formal market. In these “eviction models”, the probability of eviction—a direct measure of tenure insecurity—is endogenous but the price of housing in the formal market is exogenous and not affected by the informal sector. This is a strong assumption, especially considering that a large fraction of the population usually occupies land informally.

Subsequent papers have built on the approach initiated by Jimenez. Relaxing the assumption of a fixed formal land price, [Turnbull \(2008\)](#) considers a stochastic (but still exogenous) formal land price on which the probability of eviction depends. [Brueckner and Selod \(2009\)](#) are the first to adopt a general equilibrium perspective by modeling the effect of squatter settlements on formal land prices in a context of fixed land supply. In their setting, inflation of the squatter settlement “squeezes” the formal market, leading to an increase in the price of land in the formal sector up to the point where landowners would find it profitable to evict squatters. Subsequent extensions of that “no-eviction” model consider the case of squatting on public land instead of private land ([Shah, 2013](#)) and introduce competition among squatter organizers ([Brueckner, 2013](#)). [Da Mata \(2013\)](#) proposes another general equilibrium setting in which heterogeneous households choose between formal housing where they incur a property tax and informal housing where they forgo income and face a disutility from inadequate housing.

The above contributions focus only on the particular case of squatting, and none adopt a spatial framework. Our model fills these two gaps as it determines an equilibrium land use with an endogenous choice of tenure among a continuum of options. It is also the first model to link up tenure insecurity with multiple sales.

## 3 The model

### 3.1 The setup

The economy has a city and a rural area and an overall population of mass  $N$ . The city is linear and is represented by a segment with a Central Business District located at one origin. The rural area encompasses all non-urban locations and does not have an explicit spatial representation. The city is open, meaning that the equilibrium number of households in the city is endogenously determined by migration attempts to the urban area. We assume that there is one unit of land in each point and that each urban household consumes a fixed quantity of land, normalized to one. Within a distance  $x$  from the CBD, there are thus exactly  $x$  units of land and  $x$  households. The model also has absentee landlords who extract rents from households who are competing on the land market.<sup>1</sup>

A household located at a distance  $x$  from the CBD (i.e. in location  $x$ ) pays an endogenous rent  $R(x)$  to an absentee landlord. It also has one member who commutes to the CBD to work at a cost  $xt$ , where  $t$  is the unit transport cost. All workers in the economy earn the same exogenous urban wage  $y_u$  when residing in the city. We assume that the urban wage is significantly higher than the income  $y_r$  of households residing in the rural area who can only engage in low-productivity agricultural activities. When residing in the rural area, households incur a cost  $R_a$  to access land but do not pay any commuting cost since they derive all their income from on-farm activities and do not need to travel to the city in order to work.

Our model departs from standard urban land use models as it allows a same plot of land to be sold to more than one household, which generates conflicts and tenure insecurity. Buyers can reduce tenure insecurity by purchasing a property right from a land administration which offers a menu of options that are more or less efficient in deterring multiple sales. For instance, if a buyer obtains a registered property right, the nature and number of checks that are made

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<sup>1</sup>Landlords are defined in a broad sense and refer to all individuals, groups or institutions that make land available for housing. They can be the primary owners of the land, private developers, or public authorities, and, in practice, are often a combination of these different stakeholders.



before the right can be delivered makes it unlikely—although not impossible—that multiple sales will happen. Formally, we assume that there exists a continuum of tenure situations, each characterized by a level of tenure security measured by the probability  $\pi$  that a right holder can successfully discourage multiple sales.  $\pi$  can take any value between  $\pi_{min} > 0$  and  $\pi_{max} < 1$ . When  $\pi = \pi_{min}$ , the probability of keeping the purchased plot is lowest, corresponding to a situation in which the household has no recognized right and the probability of occurrence of multiple sales is highest.  $\pi = \pi_{max}$  corresponds to the most secure form of tenure for which multiple sales are scarcest. Intermediate values of  $\pi$  correspond to property rights with moderate levels of tenure security.

Different households face different costs to obtain property rights. This is because social status and personal acquaintances are crucial in determining the level of services households can obtain from the land administration and the level of bribes they may have to disburse to obtain land documentation.<sup>2</sup> We model this by having each household characterized by its ability to interact with the land administration, as measured by its “social distance”  $e$  from the administration, with  $e$  uniformly distributed over  $[0, 1]$  across the population. To purchase a property right providing a level  $\pi$  of tenure security, a type- $e$  household incurs a cost  $C(\pi, e)$ . This “tenure cost function” is increasing in  $\pi$  as more secure property rights come at higher fees, and in  $e$  as more socially distant households face higher fees to acquire a given level of tenure security.<sup>3</sup> We further assume that the marginal cost of tenure security is increasing with distance to the administration, implying  $\frac{\partial^2 C}{\partial \pi \partial e} > 0$ , and that the cost of tenure security is convex in  $\pi$ . The tenure cost function is represented on Figure 1 below for two different agents.<sup>4</sup>

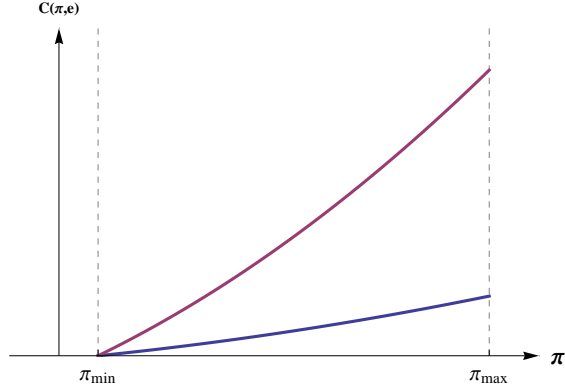
We can now present the timing of the model. In a first stage, households decide whether to purchase land in the city and the type of property rights (if any at

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<sup>2</sup>See [Van der Molen and Tuladhar \(2007\)](#) for a description of corruption in land administrations in a selection of Asian, African, and Central and Eastern European countries.

<sup>3</sup> $C(\pi, e)$  could be regarded as a tenure security premium captured by the land administration. This is consistent with bureaucrats designing complex systems to induce agents to transfer some of the rents to them ([Antwi and Adams, 2003](#)).

<sup>4</sup>Figure 1 and all the other figures in this section are drawn for the specifications and parameter values used in our base case simulation (see Section 4 below).



**Figure 1:** Tenure cost functions of well-connected ( $e=.1$ ) and poorly-connected ( $e=.9$ ) households

all) to purchase from the land administration. Because of multiple sales, conflicts emerge over land. In a second stage, each plot of land is adjudicated to one buyer only. Households who bought land in the first stage remain in the city if they are able to enforce their property right and stay in the rural area otherwise.

A type- $e$  household who bought a plot in location  $x$  and established tenure security  $\pi$  during the first stage has a probability  $\pi$  to retain its plot during the second stage. If so, the household then faces the second-stage budget constraint given by:

$$z_u + xt + R(x) + C(\pi, e) \leq y_u \quad (1)$$

where  $z_u$  is the consumption of a composite good taken as the numeraire.

In the event occurring with probability  $1 - \pi$  that the household loses the plot it purchased, it will have to remain in the rural area and will face the second-stage budget constraint:

$$z_r + R_a + R(x) + C(\pi, e) \leq y_r \quad (2)$$

where  $z_r$  is the consumption of the composite good in this state of the nature. Note that the amount initially paid to the seller to buy the plot as well as the fee paid to the administration for the property right are both lost (sunk costs).

Under the assumption that the consumption of land is fixed, the only endogenous argument in the household's utility function is its consumption of the composite good  $z$ . We assume without loss of generality that  $u(z) = z$ . The expected utility of a household purchasing land in the city (at location  $x$ ) is therefore:

$$\mathbb{E}(u) = \pi z_u + (1 - \pi) z_r \quad (3)$$

In this setting, a type- $e$  household purchasing land in a given location  $x$ , chooses its level of tenure security  $\pi$  and its anticipated consumption levels  $z_u$  and  $z_r$  in each state of the nature so as to maximize its expected utility function (3) subject to budget constraints (1) and (2). Recognizing that budget constraints must be saturated at the optimum, the household's optimization program conditional on  $x$  and  $R(x)$  simplifies to the choice of  $\pi$  which maximizes the objective function:

$$\mathbb{E}(u)(\pi, x, e) = \pi [y_u - xt - R(x) - C(\pi, e)] + (1 - \pi) [y_r - R_a - R(x) - C(\pi, e)] \quad (4)$$

Solving the model requires identifying, in equilibrium, the set of households who purchase land in the city, the location of each household's land purchase and associated tenure choice, the expected utilities of all households, and the profile of land rents prevailing in the city. Note that our equilibrium definition may have several households choose a same location *ex-ante* due to the possibility of multiple sales but, *ex-post*, when all buyers find out whether they are able to keep the plot they purchased, only a fraction of the initial buyers will effectively reside in the city. The equilibrium is determined *ex-ante* as all decisions are made in the first stage, anticipating outcomes in the second stage.

We solve the model in two steps. In a first step, we parametrize by  $x$ . In other words, we consider a given location  $x$ , and determine, for a given household of type  $e$ , the optimal choice of  $\pi$  as function of  $x$  and  $R(x)$ . In a second step, we account for competition for land in a context of multiple sales and determine the land market equilibrium mapping between household types and land purchase locations. We then derive all the endogenous variables of the model. These two steps are presented sequentially in the two subsections below.

### 3.2 Tenure choice

Let us denote  $\pi^*(x, e)$  the optimal level of tenure security chosen by a type- $e$  household in location  $x$ .  $\pi^*(x, e)$  is a solution to the maximization problem:

$$\max_{\pi \in [\pi_{min}, \pi_{max}]} \mathbb{E}(u)(\pi, x, e) \quad (5)$$

Since the tenure cost function is convex in  $\pi$ , the expected utility function (4) is concave in  $\pi$  and therefore has a unique maximum reached in  $\pi = \pi^*(x, e) \in [\pi_{min}, \pi_{max}]$ .

If  $\pi^*(x, e)$ , is an interior solution to the maximization problem, it must verify the first order condition obtained from differentiating equation (4):

$$y_u - xt - (y_r - R_a) = \frac{\partial C}{\partial \pi}(\pi, e) \quad (6)$$

This means that the optimal level of tenure security must equate the marginal cost of tenure security with the marginal gain from tenure security improvement. Because the cost of tenure security and the land rent are sunk costs paid in both states of nature, the gain associated with a marginal increase in tenure security is simply the difference between the urban wage net of commuting costs and the rural wage net of the agricultural land rent as expressed by the LHS of (6). If the marginal gain and the marginal cost of tenure security improvement do not intersect over  $[\pi_{min}, \pi_{max}]$  then  $\pi^*(x, e)$  is a corner solution. If the marginal gain is greater (respectively smaller) than the marginal cost of tenure security over  $[\pi_{min}, \pi_{max}]$ , then the optimal level of tenure security is  $\pi^*(x, e) = \pi_{max}$  (respectively  $\pi^*(x, e) = \pi_{min}$ ).

Differentiating the right and left hand side of equation (6) with respect to  $x$  and  $e$ , we derive the following proposition:

**Proposition 1.** *The demand for tenure security is non-increasing with physical distance to the CBD and with social distance to the land administration (see*

proof in Appendix B). Where  $\pi^*$  is differentiable, we have:

$$\frac{\partial \pi^*(x, e)}{\partial x} \leq 0 \quad \text{and} \quad \frac{\partial \pi^*(x, e)}{\partial e} \leq 0$$

The intuition for these results is straightforward: Consider first a household of given type  $e$ . Inspection of equation (6) shows that the relative gain from residing in the city as opposed to the rural area,  $y_u - xt - y_r + R_a$ , increases with proximity to the CBD. The closer the location to the CBD, the greater the saving on commuting costs and thus the stronger the incentive to seek secure tenure so as to be able to stay in the city. Now consider a location  $x$ , because the marginal cost of tenure security is increasing in  $e$  ( $\frac{\partial^2 C}{\partial e \partial \pi} > 0$ ), a household which is socially more distant from the land administration will choose to purchase property rights that are less secure than a household which has better connections.

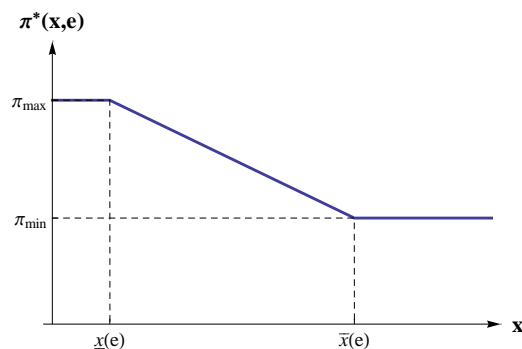
We can now derive the household's demand for tenure security throughout the city. Due to possible corner solutions in the optimization of program (5), the tenure choice function may be characterized by a piecewise function. We have:

**Proposition 2.** *The household tenure choice function  $x \mapsto \pi^*(x, e)$  can be defined by parts over at most three zones in the city (see proof in Appendix B), namely:*

- a central zone defined by  $x \leq \underline{x}(e)$  where a type- $e$  household chooses  $\pi^*(x, e) = \pi_{max}$ ,
- an intermediate zone defined by  $\underline{x}(e) < x < \bar{x}(e)$  where a type- $e$  household chooses  $\pi^*(x, e) \in ]\pi_{min}, \pi_{max}[$  verifying the first order condition (6),
- a peripheral zone defined by  $\bar{x}(e) \leq x$  where a type- $e$  household chooses  $\pi^*(x, e) = \pi_{min}$ .

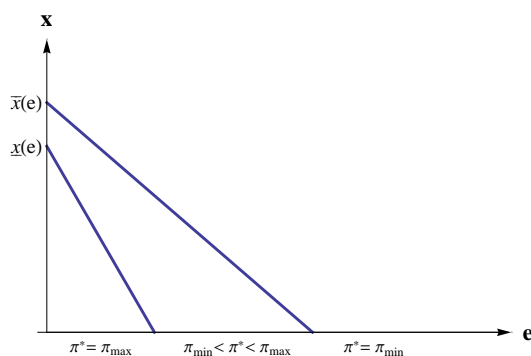
Proposition 2 simply states that there are locations below and beyond which a household's optimal tenure choice is a corner solution. These tenure thresholds depend on the household's type and are thus denoted  $\underline{x}(e)$  and  $\bar{x}(e)$ . In locations

up to  $\underline{x}(e)$ , transport costs are small and the relative gains to residing in the city area high, so that the household chooses the highest level of tenure security. In locations beyond  $\bar{x}(e)$  the household has high transport costs and low relative gains to being in the urban area, therefore choosing the lowest level of tenure security. Between these two thresholds, the household chooses intermediate values of tenure security. The optimal tenure choice  $\pi^*(x, e)$  is represented on Figure 2 below as a function of distance to the CBD .



**Figure 2:** Household demand for tenure security as a function of location (for  $e = 0.2$ )

Appendix B explicits how these functions are derived from the maximization of equation (5) and shows that  $\underline{x}(e)$  and  $\bar{x}(e)$  are decreasing in  $e$ . The three zones may not necessarily exist for all values of  $e$ . For some sufficiently high  $e$  for instance, the central and intermediate zones may not exist if for all values  $x \geq 0$ ,  $\pi^*(x, e) = \pi_{min}$ . Figure 3 graphs tenure threshold locations as functions of household type.



**Figure 3:** Tenure threshold locations  $\underline{x}(e)$  and  $\bar{x}(e)$  as functions of  $e$

### 3.3 The Land Market Equilibrium

Now that we have determined how tenure choice is affected by location and type, we can solve the land market equilibrium. We proceed in four steps. We first (i) determine the payments that each household is willing to make to purchase a plot in all possible locations. Taking into account competition on the land market, we can then derive (ii) the overall city structure (i.e. household locations and tenure zones), (iii) the city fringe beyond which no plots are purchased, and (iv) land prices and household utilities.

#### 3.3.1 Bid Rents

The bid rent of a type- $e$  household is defined as the maximum payment that the household would be willing to make to purchase a plot in location  $x$  in order to be indifferent between all locations and achieve a given level of expected utility  $\nu(e)$ . Inverting the indirect utility function (4), we obtain:

$$\Psi_e(x, \nu(e)) = \pi^*(x, e)[y_u - xt] + [1 - \pi^*(x, e)][y_r - R_a] - C(\pi^*(x, e), e) - \nu(e) \quad (7)$$

where  $\pi^*(x, e)$  is the solution to the household's optimization program determined in the previous subsection. Differentiating the bid-rent function with respect to  $x$ , we have the following Proposition:

**Proposition 3.** *Bid-rent functions are downward sloping and convex, with:*

$$\frac{\partial \Psi_e}{\partial x}(x, \nu(e)) = -\pi^*(x, e)t < 0 \quad (8)$$

Proposition 3 states that a household located in  $x$  is willing to pay more for a location marginally closer to the CBD because of the *expected* marginal saving in commuting costs under the household's optimal level of tenure security. This is a generalized version of the Alonso-Muth-Mills condition which states that, under complete tenure security, land prices should exactly compensate for transport costs (Fujita, 1989). In our setting, introducing tenure insecurity, makes bid-rents flatter,<sup>5</sup> which will have a dampening effect on equilibrium land prices.

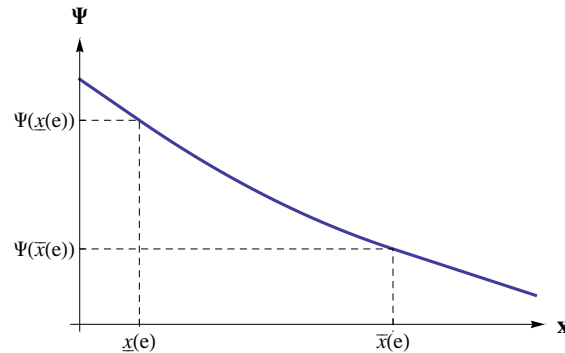
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<sup>5</sup>Constraining  $\pi^*(x, e)$  to be equal to 1 would lead the standard condition that  $\frac{\partial \Psi_e}{\partial x} = -t$

It follows from Proposition 2 that the bid rent  $\Psi_e$  can be defined piecewise over at most three zones in the city:

- In the central zone defined by  $x \leq \underline{x}(e)$  where the type- $e$  household chooses  $\pi^*(x, e) = \pi_{max}$ , the slope of the bid rent is  $-\pi_{max}t$  and the corresponding portion of the bid rent is a linear function of  $x$ .
- In the peripheral zone defined by  $\bar{x}(e) \leq x$  where the household chooses  $\pi^* = \pi_{min}$ , the slope of the bid rent is  $-\pi_{min}t$  and that portion of the bid rent is also a linear function of  $x$ .
- For  $\underline{x}(e) < x < \bar{x}(e)$ , the bid rent is strictly convex.<sup>6</sup>

Over the portion of the city where the household would choose intermediate levels of tenure security, the convexity of the bid-rent function reflects the willingness to pay a higher land price for a plot marginally closer to the CBD, with the increment compensating for the marginal increase in tenure security in addition to the marginal saving in transport costs. This is represented on Figure 4 below.



**Figure 4:** The bid rent function for household  $e = 0.2$

To continue our demonstration, we now establish the following Lemma:

**Lemma 1.** *Whenever they may intersect, the bid rent of a lower type- $e$  household has a steeper or same slope than that of a higher type- $e$  household.*

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<sup>6</sup>Since  $\frac{\partial^2 \Psi_e}{\partial x^2}(x, \nu(e)) = -t \frac{\partial \pi^*}{\partial x} > 0$  with  $\frac{\partial \pi^*}{\partial x} < 0$  following Proposition 1.



$$\frac{\partial^2 \Psi_e}{\partial x \partial e}(x, \nu(e)) = -t \frac{\partial \pi^*}{\partial e}(x, e) \geq 0 \quad (9)$$

The proof is obtained by differentiating (8) with respect to  $e$ , and using the fact that  $\pi^*$  is a non-increasing function of  $e$  (see Proposition 1). The intuition is that lower-type households demand higher levels of tenure security due to their advantage in terms of land administration fees. They thus have an incentive to bid more for a location marginally closer to the CBD.

We can now characterize the land market equilibrium. The requirement in competitive land use models that land gets allocated to the highest bid also holds in the presence of multiple sales.<sup>7</sup> When comparing purchase location choices, we use the standard result that, in equilibrium, agents are ranked by order of bid-rent steepness: agents that have steeper bid rents bid away other agents to more remote locations (Fujita, 1989). Under Lemma 1, this would have households purchase plots in order of increasing type. Although we will show that this is indeed true in our model, the determination of the spatial equilibrium is more complex than in the standard case and requires addressing three specific issues.

First, we are dealing with a continuum of households instead of a discrete number of agents, which implies comparing a continuum of bid rents. Building on an assignment problem first analyzed by Beckmann (1969), several papers in the urban economics literature have developed methods to ensure that the hypothesized mapping between types and space results from competition in the land market (Brueckner et al., 2002; Selod and Zenou, 2003; Brueckner and Selod, 2006; Behrens et al., 2014). We resort to a similar approach.

Second, the model has multiple sales, which means that more land is transacted with households than available in the city (there are more buyers than sellers). In order to define the equilibrium mapping between households and locations, we will need to account for the way the risk of multiple sales is attenuated by the purchase of property rights and how these choices translate into land use.

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<sup>7</sup>In the model, multiples sales occur at identical prices to ensure profit maximization of sellers.

Third, the unambiguous ranking of households according to type is valid only for bid rents that intersect over isolated points. In our model, however, bid rents may intersect over a whole interval. This happens in zones of the city where the optimal tenure choice of households yields a common corner solution. Over such zones, the bid rents will all have the same slope (see equation (8) and Figure 4) irrespective of household type. It is possible to determine which households purchase a plot in that zone but not the exact location purchased by each household within the zone. This indeterminacy means that there can be an infinity of spatial configurations in equilibrium. Fortunately, this will not affect the model in any significant way since these equilibria share the same general spatial structure of the city and all the other endogenous variables of the model will take the exact same value.

### 3.3.2 City structure

In this subsection, we identify the spatial distribution of tenure situations throughout the city as well as the mapping between households and locations of purchased land. Confronting the piecewise bid rents of households described in Proposition 2 and the condition that households must be ranked in decreasing order of their bid rents' steepness, we obtain the following Proposition (See proof in Appendix C):

**Proposition 4.** *In equilibrium, the city can be divided into at most three different “tenure zones”:*

- **A “secure zone” occupying the central section of the city** (between  $x = 0$  and  $x = \underline{x}$ ) where households with the strongest social ties to the land administration ( $e \leq \underline{e}$ ) reside and purchase the most secure type of property rights ( $\pi^* = \pi_{max}$ ).
- **A “precarious zone” in the intermediate section of the city** (for  $x$  between  $\underline{x}$  and  $\bar{x}$ ) where households with intermediate values of  $e$  (belonging to  $]\underline{e}, \bar{e}[$ ) reside. These households purchase property rights with intermediate levels of tenure security  $\pi^*(x, e)$  that depend on their location and their type.

- An **“insecure zone” in the peripheral section of the city** (between  $x = \bar{x}$  and the city fringe) where the households that have the weakest connections with the administration ( $e \geq \bar{e}$ ) reside and do not purchase any property right from the land administration ( $\pi^* = \pi_{min}$ ).

Note that all three zones may not necessarily exist and that a city may exhibit either one, two or three zones depending on the model’s parameters. Although the exact location of households within the secure zone and the insecure zone (if it exists) are undetermined, for mathematical convenience and without loss of generality, we consider only the equilibrium where households are ordered by increasing  $e$ . Taking into account multiple sales and the fact that households are ranked according to type, we have the following proposition:

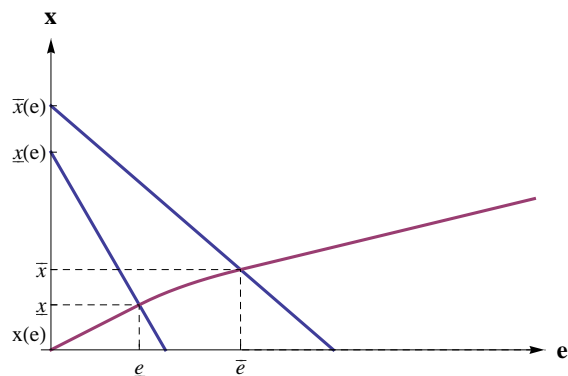
**Proposition 5.** *The equilibrium mapping  $x(e)$  between types and locations must verify the following differential equation and initial condition:*

$$\begin{aligned} \frac{dx}{de} &= \pi^*(x(e), e)N \\ x(0) &= 0 \end{aligned} \tag{10}$$

Proposition 5 maps types and purchase locations considering that a mass of  $Nde$  buyers whose type is comprised between  $e$  and  $e + de$  will purchase land over a portion of the city of size  $dx$  comprised between  $x(e)$  and  $x(e) + dx$ . For an infinitesimal  $de$ , each plot is bought with tenure security  $\pi^*(x(e), e)$ . Equating the quantity of available land with the number of “successful buyers” requires  $dx = \pi^*(x(e), e)Nde$ . Observe that in order to solve the differential equation in proposition 5, an initial condition is needed. Since we consider the case in which all households locate in order of their type, we use  $x(0) = 0$ , which means that CBD plots are always purchased by type-0 households.

We can now determine the thresholds  $\underline{e}$ ,  $\underline{x}$ ,  $\bar{e}$  and  $\bar{x}$  which characterize city structure as described in Proposition 4. The resolution is presented graphically on Figure 5 below which superimposes the mapping  $x(e)$  and the tenure threshold functions

already represented on Figure 3. Consider first that the intersection between  $\underline{x}(e)$  and  $x(e)$  gives  $\underline{e}$  and  $\underline{x}$ . Figure 5 shows that for any type- $e$  household with  $e < \underline{e}$ , we have  $x(e) < \underline{x}(e)$  and  $\pi^*(x, e) = \pi_{max}$ . Therefore,  $x(e) = \pi_{max}Ne$  for  $e < \underline{e}$ . Similarly, note that the intersection of  $x(e)$  with  $\bar{x}(e)$  gives  $\bar{e}$  and  $\bar{x}$ . For any type- $e$  household with  $e > \bar{e}$ , we have  $\bar{x}(e) < x(e)$ , which implies that these households purchase land in the insecure portion of the city and have tenure security  $\pi_{min}$ . For  $e > \bar{e}$ ,  $x(e)$  is a linear function of slope  $N\pi_{min}$ . Finally, for households of type  $e$  with  $\underline{e} < e < \bar{e}$ , we have  $\underline{x}(e) < x(e) < \bar{x}(e)$ , implying that these households reside in the precarious portion of the city and demand intermediate levels of tenure security.



**Figure 5:** Determination of the tenure category zones in the city

### 3.3.3 City composition and urban fringe

Figure 5 represents the type/location mapping as if all households in the economy purchased a plot in the city. There is no reason for this to be the case in equilibrium and we need to determine which households purchase land and the spatial extent of the corresponding market (which will define the true domain of definition for the mapping function). There are two conditions that characterize the city fringe. First, by definition of the land market equilibrium, there is no discontinuity in rural and urban land prices at the city fringe  $x_f$ , implying:

$$R(x_f) = R_a \quad (11)$$

The second condition reflects the open city assumption, where  $e_p$  is the type of the household located at the urban fringe in equilibrium (or the “last” household purchasing land in the city). Equation (12) states that household  $e_p$  is indifferent between purchasing a plot at the urban fringe and not attempting migration.

$$\nu(e_p) = y_r - R_a \quad (12)$$

To show the existence of  $e_p$ , we write the indirect utility of a household  $e$  purchasing a plot at the city fringe, using equation (11) and the mapping function  $x(e)$ :

$$\nu(e) \big|_{R(x(e))=R_a} = \pi^*(x(e), e)[y_u - x(e)t] + [1 - \pi^*(x(e), e)][y_r - R_a] - R_a - C(\pi^*(x(e), e), e)$$

We then differentiate this expression with respect to  $e$ . Using the envelope theorem, we obtain Equation 13 which is always negative given that  $\frac{dx}{de}(e) > 0$ .

$$\frac{d\nu}{de} \big|_{R(x(e))=R_a} = -\pi^*(x(e), e)\frac{dx}{de}(e)t - \frac{\partial C}{\partial e} < 0 \quad (13)$$

There is therefore a unique value  $e_p$  such that all households with a smaller  $e$  purchase land in the urban area, whereas all other households do not attempt to migrate.<sup>8</sup> Depending on the value of  $e_p$  relative to  $\underline{e}$  and  $\bar{e}$  as previously defined, the three sections of the city described in Proposition 4 may or may not exist.  $e_p$  then determines the city fringe  $x_f$  through Equation 14.<sup>9</sup> This is represented on Figure 6 below.

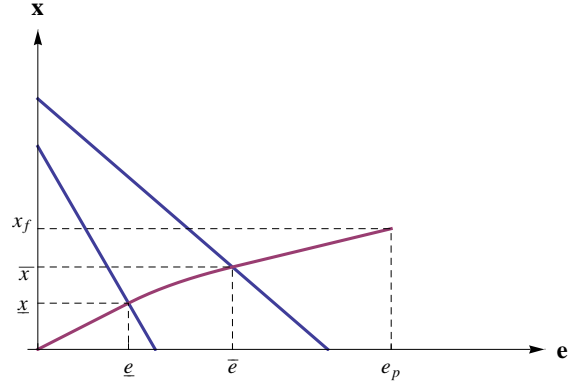
$$x_f = x(e_p) \quad (14)$$

It is not possible to come up with an analytical expression for the city fringe except in the particular case when the cost of buying minimal tenure security is the same for all households ( $C(\pi_{min}, e) = C(\pi_{min})$ ) and the three tenure zones exist ( $\bar{e} < e_p$ ). In this case we can derive an explicit value for  $x_f$  by solving equation (11) considering that  $R(x_f)$  can be written as  $\Psi_e(x, \nu(e))$  (as given by equation (7) with  $x = x_f$ ,  $\pi^* = \pi_{min}$  and  $\nu(e) = y_r - R_a$ ).<sup>10</sup>

<sup>8</sup> $e_p$  is equal to 1 if for all values of  $e$ ,  $\nu(e) \big|_{R(x(e))=R_a} > y_u - R_a$ .

<sup>9</sup>Because of unit land consumption,  $x_f$  is also the number of “successful buyers”.

<sup>10</sup>We obtain  $x_f = \frac{y_u - y_r}{t} - \frac{R_a(1 - \pi_{min}) + C(\pi_{min})}{t\pi_{min}}$ , where the second term is negative. In this particular case, urban extent is thus smaller than in the standard model with complete tenure



**Figure 6:** Determination of the city fringe

### 3.3.4 Land prices and utilities

We now determine the utilities  $\nu(e)$  and the land-rent curve  $R(x)$  which is defined as the upper envelope of bid rents in equilibrium. To determine the utilities of a continuum of households, we use an equilibrium condition that states that the mapping  $x(e)$  is consistent with land being allocated in equilibrium to highest bids:

$$R(x) = \max_e \Psi[x, \nu(e)] \big|_{x=x(e)} \quad (15)$$

This reflects competition for land and trivially holds on the secure and insecure portions of the city since households who purchase land in those zones have the same bid rents. In what follows, we assume all three zones exist<sup>11</sup> ( $\bar{e} < e_p$ ) and consider each zone sequentially, starting with the most remote and using the continuity of land-rent and expected utility in the thresholds identified in Proposition 4.

#### The insecure tenure zone

We start with the insecure zone  $[\bar{x}, x_f]$  where households of type  $e \in [\bar{e}, e_p]$  purchase land. Any household buying land in this zone chooses the lowest level of tenure security  $\pi_{min}$  and has the following bid rent:

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security where it can be shown that the city extends up to  $x_f = \frac{y_r - y_u}{t}$ . This property bears no generality as simulations outside this particular case exhibit situations in which the urban extent is larger in our model.

<sup>11</sup>A similar approach can be applied in the other configurations with less than three zones.

$$\Psi(x, \nu(e)) = \pi_{min}[y_u - xt] + [1 - \pi_{min}][y_r - R_a] - C(\pi_{min}, e) - \nu(e) \quad (16)$$

Since all households have the same bid rent over  $[\bar{x}, x_f]$ , (16) is also the analytical expression for  $R(x)$  on this portion of the city. The border condition (11) equates (16) with  $R_a$  in  $x = x_f$ , such that  $\nu(e) + C(\pi_{min}, e)$  is a constant equal to  $\pi_{min}[y_u - x_f t] + [1 - \pi_{min}][y_r - R_a] - R_a$ . Plugged back into (16), this gives:

$$\begin{aligned} R(x) &= \pi_{min}(x_f - x)t + R_a \\ \nu(e) &= -C(\pi_{min}, e) + K_1 \end{aligned} \quad (17)$$

with  $K_1 = \pi_{min}(y_u - x_f t) + (1 - \pi_{min})(y_r - R_a) - R_a$ .

### The precarious tenure zone

Let us now consider households of type  $e \in [\underline{e}, \bar{e}]$  who purchase land in the precarious section of the city  $[\underline{x}, \bar{x}]$  and choose intermediate values for  $\pi^*$ . Each household's location is given by the mapping  $x(e)$  and must also satisfy the maximization set out in equation (15). We thus derive the first order condition of equation (15) by considering equation (7) for  $x = x(e)$  and differentiating the resulting expression with respect to  $e$ .<sup>12</sup> Using the envelope theorem, we find that the first order condition of equation (15) comes down to the following differential equation:

$$\frac{d\nu}{de}(e) = -\frac{\partial C}{\partial e}[\pi^*(x(e), e), e]$$

Solving this differential equation determines  $\nu(e)$  for all  $e \in [\underline{e}, \bar{e}]$ . This also enables us to determine  $R(x)$ , which is continuous in  $\bar{x}$  due to the continuity of  $\nu$  in  $\bar{e}$ . We have:

$$\forall e \in [\underline{e}, \bar{e}], \nu(e) = -\int_{\bar{e}}^e \frac{\partial C}{\partial e}[\pi^*(x(e), e), e] + K_2 \quad (18)$$

Recognizing the continuity of the function  $\nu(\cdot)$ , we equate expressions (17) and (18) in  $\bar{e}$  and obtain  $K_2 = -C(\pi_{min}, \bar{e}) + K_1$ . Given that the mapping is an invertible

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<sup>12</sup>Differentiating a second time with respect to  $e$ , rearranging the terms, and using (10), we can show that the second order condition always holds.

function over  $[\underline{e}, \bar{e}]$ , we thus have the following expression for the bid-rent over  $[\underline{x}, \bar{x}]$ , where  $e(x)$  is the inverse of the mapping function and  $\nu(\cdot)$  is defined as in equation (18).

$$R(x) = \pi^*(x, e(x))[y_u - xt] + [1 - \pi^*(x, e(x))][y_r - R_a] - C(\pi^*(x, e(x)), e(x)) - \nu(e(x)) \quad (19)$$

### The secure tenure zone

Finally, we consider households of type  $e \in [0, \underline{e}]$  who purchase land over  $[0, \underline{x}]$  and who all choose  $\pi^* = \pi_{max}$ . Their bid rents can be written as:

$$\Psi(x, \nu(e)) = \pi_{max}[y_u - xt] + [1 - \pi_{max}][y_r - R_a] - C(\pi_{max}, e) - \nu(e) \quad (20)$$

Since their bid rents must all be confounded on this portion of the city, we can equate  $\Psi(\underline{x}, \nu(e))$  and  $\Psi(\underline{x}, \nu(\underline{e}))$ . This yields  $\nu(e) + C(\pi_{max}, e) = \nu(\underline{e}) + C(\pi_{max}, \underline{e})$  where  $\nu(\underline{e})$  is provided by the limit when  $e \rightarrow \underline{e}_+$  of the function  $\nu$  over  $[\underline{e}, \bar{e}]$ . Plugging  $\nu(\underline{e}) + C(\pi_{max}, \underline{e})$  back into equation (20) and using equation (18) to express  $\nu(\underline{e})$ , we obtain:

$$\begin{aligned} R(x) &= \pi_{max}(y_u - xt) + (1 - \pi_{max})(y_r - R_a) - K_3 \\ \nu(e) &= -C(\pi_{max}, e) + K_3 \end{aligned} \quad (21)$$

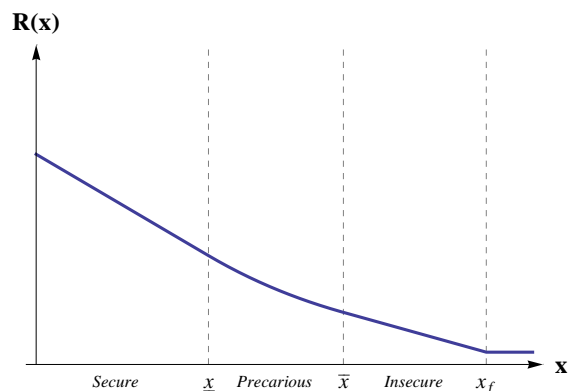
with  $K_3 = C(\pi_{max}, \underline{e}) + \int_{\underline{e}}^{\bar{e}} \frac{\partial C}{\partial e}[\pi^*(x(e), e), e] + K_2$ .

The land market equilibrium is represented in Figure 7 below. Other aggregates from the model such as the utilities, the land administration's revenue and the total income of landowners can also be calculated. This is presented in Appendix D.

## 3.4 Comparative statics

We discuss here the effects on city patterns of marginal changes in the parameters of the model (see the demonstrations in Appendix E). We consider a growing population ( $dN > 0$ ), decreasing unit transport cost ( $dt < 0$ ) and a rising urban income relative to the rural income ( $d(y_u - y_r) > 0$ ), which effects are presented





**Figure 7:** The equilibrium land rent  $R(x)$  and the threshold values  $\underline{x}$ ,  $\bar{x}$ , and  $x_f$

in Table 1 below. We are able to determine how changes in the values of these parameters affect the size and the composition of the secure zone of the city but the effects on the precarious and insecure zones are in several instances ambiguous.

We find that an increase in the overall population leads to an increase in the size  $\underline{x}$  of the secure section of the city but to a decrease in the share  $\underline{e}$  of households purchasing secure property rights. This is because an increase in  $N$  increases the number of well-connected households purchasing land close to the city center, pushing away other households towards the periphery of the city where they choose lower levels of tenure security. If transport costs are reduced, or the urban rural wage differential is increased, the size of the formal section of the city will also increase but the share of households in the secure section of the city will in this case increase rather than decrease. This is because these variations in transport costs or wages increase the gains a household can expect from purchasing land in the city, providing households with an incentive to purchase higher levels of tenure security for any given location, which inflates both the zone over which households purchase secure rights and the share of households doing so.

In the specific case where the city has three zones, we show that city size  $x_f$  increases with the urban rural wage differential and as the unit transport cost decreases (as the city becomes more attractive). On the other hand, the share  $e_p$

of households purchasing land in the city decreases as  $N$  or  $y_u - y_r$  increase, or when  $t$  decreases, reflecting the fact that households with a smaller  $e$  are taking up more space (either because they are more numerous or because they demand more secure tenure).

	Variation of the parameters		
	$dN > 0$	$dt < 0$	$d(y_u - y_r) > 0$
$d\underline{x}$	+	+	+
$d\underline{e}$	-	+	+
$d\bar{x}$	+	+	+
$d\bar{e}$	-	?	?
$dx_f$	0	+	+
$de_p$	-	-	-

**Table 1:** Comparative statics (general case for  $\underline{x}, \underline{e}, \bar{x}$  and  $\bar{e}$ ; specific case where the city has three zones for  $x_f$  and  $e_p$ )

The other parameter which affects city patterns is  $C(\pi, e)$ , which describes the cost paid by a household  $e$  to establish a property right ensuring security  $\pi$  over a plot of land. This parameter is a function such that it is difficult to say anything about how variations of  $C$  affect the city structure and the land market in the general case. To get some grip on this question, we parameterize  $C$  and resort to simulations. The results of these simulations are presented in Section 4.

### 3.5 Evidence from West Africa

Our model involves general mechanisms at work in cities that have dysfunctional land institutions and exhibit diverse land tenure situations ([UN-Habitat, 2012](#)).

This is the case in different regions of the world and particularly in West African countries for which we present evidence that support the model’s assumptions and results.

### 3.5.1 Multiple sales

Conflicts over land use, especially involving multiple sales and the issuance of multiple property rights over a same plot, have become very common in West Africa. A recent study on land markets in Ghana ranked “double sales of land by traditional owners” as the number one problem in Accra (Omirin and Antwi, 2004). In Benin, the problem is so pervasive that a specific provision has been inserted in the recent land code to punish authors of multiple sales with a heavy fine and a jail sentence of up to 5 years. In the sole district of Bamako, Mali, at least 12,000 cases of double allocations of use rights or of superimposition of property titles have been identified (Coulibaly, 2009). Multiple sales are actually clogging tribunals as an estimated 80% of cases in Mali are related to land (Camara, 2012).

### 3.5.2 The range of property rights and tenure security

Land tenure systems are relatively comparable throughout West Africa (Bruce, 1998; Durand-Lasserve et al., 2013) as land may be held with a property title, with a use right, with and administrative document, or without any such document. Both property titles and use rights are formal rights that provide high levels of tenure security and correspond to the secure tenure situation in the model.<sup>13</sup> As for administrative documents, they are intermediary papers issued by the administration in a process of land allocation or regularization that was never completed.<sup>14</sup> They do not provide any legal right *per se* but are used by households to prove some legitimacy on the land they occupy in case of a conflict. They thus provide intermediate levels of tenure security and correspond to the precarious tenure situations in the model. Finally, land may be held with a private

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<sup>13</sup>Although both are formal rights, property titles may be viewed as more secure to the extent that they involve registration with a cadaster/registry and cannot be challenged once they have been issued.

<sup>14</sup>Such processes are typically long and costly and households often choose or do not have the means to pay for the last stages that would have established a use right.

sales contract only or without any document.<sup>15</sup> The latter type of tenure provides very low security and corresponds to the insecure tenure category in the model. Formal property rights are usually held by a tiny fraction of the population while most people only have administrative documents or no such documents at all (Durand-Lasserre et al., 2013).

The legal and illegal fees necessary to establish secure or precarious tenure are documented in a few descriptive studies (Bertrand, 1998; Djiré, 2007). These studies show that obtaining more secure tenure is more costly as it requires complex processes made of several steps, each requiring the payment of various fees and taxes. In the case of Mali for instance, Djiré (2007) reports an example in a locality at the outskirts of the Bamako district where the average cost of obtaining a property title is 725,000 CFA Francs (about \$1,500). This includes in particular the payment for the topographical survey, the permit fees (valid for five years), the fees for demarcation, registration, notice of public inquiry, the signature of the sub-prefect and the village chief, and the Land Office registration stamp... In comparison, the price of land in that locality is on average 225,000 CFA Francs per hectare (about USD 450) (Djiré, 2007; Bouju, 2009). In addition to these costs, several authors insist on the role of social connections and corruption in the land administration: Membership of the leading political party in the commune, or an influential trade union or association, or links with an NGO working on servicing land in the commune can be determining factors in obtaining a plot, regularizing tenure and speeding up the process of obtaining a property right (Bertrand, 1995, 2006). All these elements are in line with our model's assumptions.

### 3.5.3 Spatial sorting

At the exception of some insights provided by Bertrand (1998), there is very little focus in the literature on the location of the different tenure situations in West African cities. To explore this issue quantitatively, we collected land transaction data for Bamako, Mali, and its surroundings (see Durand-Lasserre et al. (2013) for a detailed presentation of the survey). Our sample comprises

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<sup>15</sup>This is often the case for land obtained from customary owners at the periphery of cities, an increasingly common situation as West African cities expand over rural areas (Becker, 2013).

randomly-selected unbuilt plots that were recently transacted (between 2009 and 2012) and destined to a residential use at the time of transaction.<sup>16</sup> For each transaction in our database, we have information on the characteristics of the plot (surface, infrastructure servicing), its location (GPS coordinates, municipality (*commune*), distance to the closest paved road and to the river), and its tenure status (reported tenure type and documents at the time of transaction and at the time of the survey).

The data allow us to represent the land tenure mix (share of each tenure category) at the time of the survey for different distances from the center of Bamako (see Figure 8 below where we grouped property titles and use rights in the secure tenure category). It can be seen for instance, that among the plots located between 12km and 16km from the city center, 16% of plots have no administrative document regarding their ownership, 57% are held under precarious tenure (an administrative document only), and 27% have either a use right or an ownership title. These bar charts reveal the spatial sorting of plots according to their tenure status: as one moves further away from the city center, the share of secure tenure types diminishes to the benefit of less secure tenure types (precarious and insecure tenures).<sup>17</sup> Although we do not have complete segregation by tenure type as implied by our stylized model, these facts are nevertheless consistent with Proposition 4 of the model defining the tenure zones in the city.

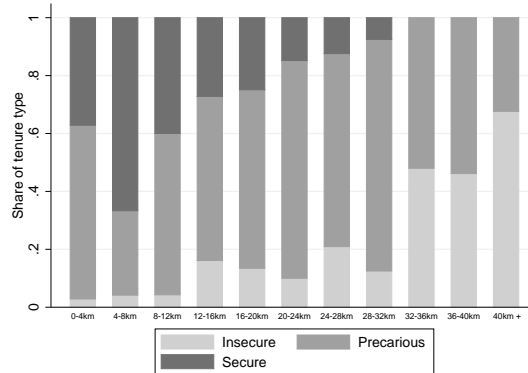
Because we have information on prices and locations, we can also explore the role of the market in organizing the spatial stratification of land tenure as in the model.<sup>18</sup> To do that, we first performed a hedonic regression of the logarithm of sales prices, controlling for the determinants of prices present in the database including distance to the CDB, tenure category and the interaction of distance and tenure category

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<sup>16</sup>Initial sample size including plots for agricultural use is 1,655 and about a thousand observations when restricted to plots for residential use.

<sup>17</sup>A Kolmogorov-Smirnov test shows that the cumulative distribution function for secure tenure is significantly different at the 1% level from the cumulative distribution functions for the precarious and insecure tenure categories.

<sup>18</sup>For this analysis, we removed the top and bottom 1% of the price distribution (expressed in CFA Francs per square meter) as well as all plots located more than 40km away from the CDB. We are left with slightly less than 1,000 observations.

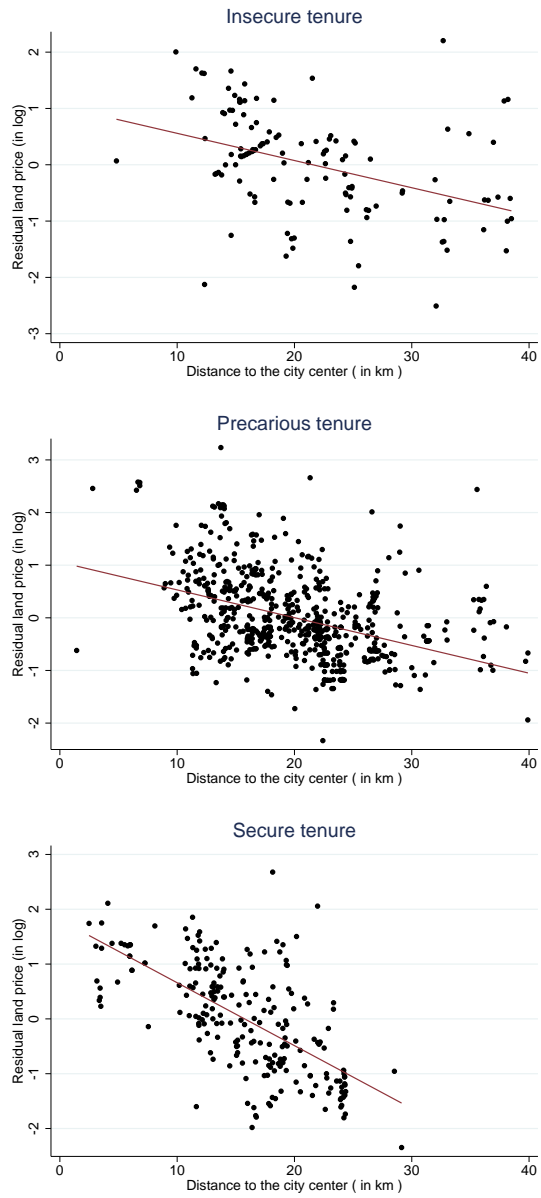


**Figure 8:** Tenure categories by distance to the CBD (in km)

(see Appendix A for a presentation of the methodology and main results). In line with the literature on the tenure premium, the regression is consistent with an overall land market at the scale of the urban area that incorporates different forms of tenure and values formality and tenure security over informality and tenure insecurity. The negative coefficients on the interaction terms between distance and the secure and precarious tenure categories (insecure tenure being the benchmark) are consistent with the model’s prediction of steeper bid rents for more secure forms of tenure. We further explore this point by regressing land prices on all possible determinants excluding distance and tenure category,<sup>19</sup> and then, for each tenure category, by plotting the residuals of this regression over distance to the CBD. This is represented on Figure 9 which shows the land price gradient by tenure category. Consistently with land markets in developed economies (which have property titles only), the gradient of land price to distance is negative. As predicted by our model, the gradient depends on the tenure category, with the most secure forms of tenure having a steeper gradient than less secure forms of tenure.<sup>20</sup> This is suggestive of agents willing to pay more for more secure forms of tenure closer to the city center as found in the model.

<sup>19</sup>Surface of the plot was included in the control variables to capture possible non linear effects of surface on price.

<sup>20</sup>A Fisher test run on the coefficients of the hedonic price analysis show that the land price gradient for the most secure tenure category is significantly different from the land price gradients for the other two tenure categories.



**Figure 9:** Land prices and land gradients by tenure type

## 4 Land administration policies

In this section, we resort to simulations to illustrate the predictions of our model and the likely *general equilibrium effects* of policies.

### 4.1 Specification of the cost function

The key feature of our model is the pricing of property rights by the administration through the tenure cost function  $C(\pi, e)$ . In our simulations, we use the following separable specification

$$C(\pi, e) = c(\pi)\tilde{f}(e)$$

with

$$c(\pi) = K(\pi^2 - \pi_{min}^2)$$

$$\tilde{f}(e) = f(e_m) + \theta[f(e) - f(e_m)]$$

$$f(e) = 1 + \delta e$$

where  $K$ ,  $\theta$ ,  $\delta$  and  $e_m$  are parameters that can be adjusted to account for different practices of the land administration.

$K$  is a multiplicative parameter that affects all households equally. If  $K$  is increased then the fees are increased in the same proportion for all households. On the contrary,  $\tilde{f}(e)$  is a type-specific multiplicative term that accounts for the differential treatment of well-connected and poorly-connected households. To understand how this works, first consider the case of  $\theta = 1$  so that  $\tilde{f}(e) = f(e) = 1 + \delta e$ . In this case, a type-0 household pays  $c(\pi)$  to purchase a property right  $\pi$ , whereas a type- $e$  household pays  $c(\pi)(1 + \delta e) > c(\pi)$ . Increasing  $\delta$  will reinforce the advantage of well-connected households (those with a low  $e$ ) relative to poorly-connected ones (those with a high  $e$ ) who will pay a higher fee. Choosing  $\theta < 1$  operates a rotation of the  $f(\cdot)$  function around a pivotal household of type  $e_m \in [0, 1]$ . If  $\theta = 0$  and  $e_m = 0$  then all households are treated as the most-favored household (type  $e = 0$ ). If  $\theta = 0$  and  $e_m = 0.5$  then all households are treated as the median household in the distribution of types.



## 4.2 The base case: a clientelistic land administration

We can now turn to the presentation of the base case which we constructed assuming that the administration serves the interests of a small elite. Examples of such clientelism abound in many different countries (Van der Molen and Tuladhar, 2007; Deininger and Feder, 2009) and it makes sense to make this situation the initial stage or benchmark from which we will explore the desirability and feasibility of policies.

The base case is obtained by maximizing the expected welfare of the best-connected household  $e = 0^{21}$  with respect to  $K$  and  $\theta$ , taking  $\delta$  as fixed. We find that *the well-connected households benefit from a land administration that treats households as unequally as possible*. This is due to a pecuniary externality: the more expensive tenure security is made for higher types, the smaller their demand for tenure security and the more depressed their bid rents (because of the negative tenure insecurity premium which materializes in the form of a flatter bid-rent). In this context, it is thus easier for lower types to bid away higher types to the periphery of the city without having to bid too much for central locations (prices are depressed throughout the city and *in fine* the land rent in  $x = 0$  is low). In a sense, the land administration makes land available under advantageous conditions for the elite while protecting them from the competition of others on the formal segment of the market. As a matter of fact, household  $e = 0$  would always be better off with a higher value of  $\delta$  and with  $\theta = 1$ . Assuming an exogenous value  $\delta = 9$  and considering  $y_u = 111,000$ ,  $y_r = 70,000$ ,  $R_a = 15,000$ ,  $t = 1,000$ ,  $\pi_{max} = 0.95$ ,  $p_{i_{min}} = 0.45$  and  $N = 50$ , we obtain  $K_0 = 8,948$  and  $\theta_0 = 1$ .<sup>22</sup>

## 4.3 More affordable property rights

It is often argued that excessive land administration fees push people into informality. To explore this argument within the framework of our model, we simulate a policy that would make  $K$  smaller (but without making the system

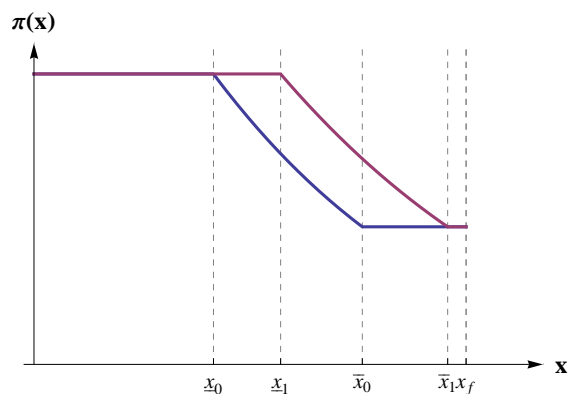
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<sup>21</sup>Alternatively we could consider any other objective function, for instance to satisfy a fraction of the population  $e \in [0, e_c]$  representing the political clientele (where  $e_c$  is the “last” client).

<sup>22</sup>With  $\theta = 1$ , the value of  $e_m$  is irrelevant.

less inequitable, i.e. keeping  $\theta_0 = 1$ ). We simulate a case where  $K$  is decreased by 25 percent ( $K_1 = 0.75K_0 = 6711$ ). There are three interesting impacts we can graphically highlight from those simulations:

First, the effects on the demand for tenure security as a function of distance to the CBD are represented on Figure 10 below. The blue curve is the base case demand for tenure security whereas the red curve is demand under a lower  $K$  (when fees are more affordable to all). The simulation results in *an increase in (and displacement of) the spatial extent of the secure and precarious sections at the expense of the insecure zone.*<sup>23</sup>

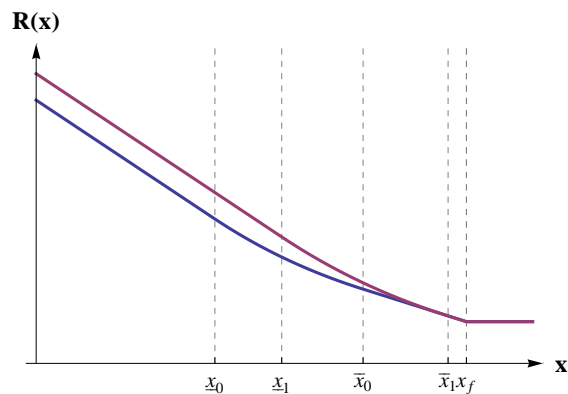


**Figure 10:** Tenure zones when property rights become more affordable (base case in blue; policy in red, with  $K_1 = 0.75K_0$ )

Second, making land administration fees more affordable results in *an increase in land rents associated with tenure security improvement* (see Figure 11 below). This comes from the combination of two mechanisms: (i) the shrinking of the insecure portion of the city, which mechanically pushes all prices up throughout the city, and (ii) the capitalization in land prices of more secure tenures over the precarious

<sup>23</sup>Because in our simulations  $C(e, \pi_{min}) = 0$ ,  $x_f$  will not vary with  $K$  as long as there are three zones, in which case the city size remains the same but the insecure zone is smaller than before. With a more drastic reduction in  $K$ , the insecure zone will completely disappear and the overall city size will increase. Indexing by 0 the base case and 1 the policy simulation, we would have:  $\bar{x}_1 > x_{f1}$  and  $x_{f1} > x_{f0}$  (simulation not shown).

and secure portions of the city. The greater the decrease in  $K$  the higher the equilibrium land prices (simulations not shown).

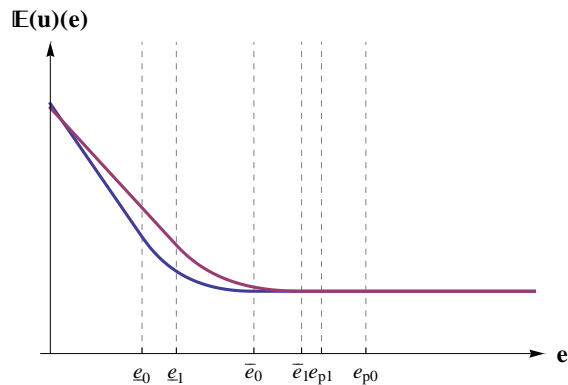


**Figure 11:** Land rents when property rights become more affordable  
(base case in blue; policy in red, with  $K_1 = 0.75K_0$ )

Third, looking at figure 12 below, we see that the impact of lowering land administration fees has *disparate effects on the expected utility*. Lowering  $K$  results in a *decrease* in the expected utility of households in the “social vicinity” of the land administration (this is understandable since the base case is built by choosing the value of  $K$  that maximizes the expected utility of the household  $e = 0$ ). However, lowering  $K$  results in an *increase* in the expected utility of households with relatively higher types who reside in the secure and precarious sections of the city. The reason for this is that, for the lowest type household, the increase in land rent outweighs the decrease in the cost of acquiring secure property rights (since they were paying a low fee anyway), resulting in an expected utility loss, while it is not the case for higher-type households.<sup>24</sup> We find that these effects are magnified for even lower values of  $K$  (not shown). Observe that lowering  $K$  makes no difference for the expected utility of households in the rural sector and in the insecure section of the city (because all households in these

<sup>24</sup>This occurs because land rents are shifted upwards *additively* and irrespective of the exact type of the households who reside in the secure section of the city, whereas fees are reduced *multiplicatively*, favoring households with relatively higher types.

areas have the same expected utility  $y_r - R_a$  by virtue of  $C(\pi_{min}, e) = 0$ .



**Figure 12:** *Ex ante* utilities when property rights become more affordable (base case in blue; policy in red, with  $K_1 = 0.75K_0$ )

We also simulated a policy making the menu of fees more equitable. We obtain an increase of the spatial extent of the precarious section of the city at the detriment of the secure and insecure sections and non-monotonic effects on land prices (see results in Appendix F).

## 5 Conclusion

In this paper, we introduced an endogenous choice of property rights in the standard urban land use model. To our knowledge, this is the first attempt to build a theory of residential informality that allows for a spatial representation of the urban space under endogenously determined land prices. The result is an augmented monocentric land use model which generates the standard urban economics model as a particular case under the assumption of costless and fully secure property rights. Considering the more general case where property rights are costly and provide only limited tenure security, we proposed a suitable framework to analyze urban land markets and land tenure patterns in developing countries. Our approach makes the following four contributions:

First, analyzing land tenure from a systemic perspective shifts the focus from an analysis of informal land markets in isolation from formal land markets to one where the formal and the informal sectors interact: In our model, although households may hold land under a continuum of tenure situations (from very insecure to very secure), they compete for land over the same city. The only previous theoretical paper to have explored such interactions was [Brueckner and Selod \(2009\)](#) who argued, in a context of inelastic land supply, that the greater the informal squatting sector in a city, the greater the prices in the formal sector because of a “squeezing effect” (due to squatters settlements using up scarce land in the city). The assumptions in the present model, however, are different as land supply is not restricted and because informal dwellers are not squatters but actually pay a competitive price to use land under a chosen level of tenure security. Interestingly, holding city size constant, the predictions of our spatial model differ from [Brueckner and Selod \(2009\)](#): Because of competition for land and the lower value put by the market on insecure tenure, informal tenure in peripheral locations contributes to depressing rather than increasing land prices in central locations where formal housing locates. The systemic approach also highlighted that land policies may entail responses from land markets that affect households beyond the targeted populations, with possible trade-offs regarding who benefits from the policy.

A second lesson to be derived from the model is the central role of land administration fees (including side payments and clientelistic practices) in shaping cities. In line with recent research measuring the elasticity of demand for property rights with respect to regularization or registration fees ([Monkkonen, 2012](#); [Ali et al., 2013](#)), our model suggests that land administration fees—which can reach the same order of magnitude as land prices—can significantly alter the tenure choices and location decisions of households within a city. An important implication for policy makers should thus be whether pricing by the land administration is compatible with urban planning objectives. Further studies on the willingness to pay of households for property rights, and more generally on how the menu of land administration fees affect tenure choices and land markets will certainly be needed.

A third lesson is that clientelism in the land administration may determine *who* has access to secure land in the city. In our model, a political elite can benefit from imposing costly fees to others in order to weaken competition for land in the formal sector. Although political scientists and urban economists have long studied how cities serve political clienteles (Ades and Glaeser, 1995), discrimination in the land administration and the way discrimination may affect city patterns or even city composition has been little studied. More generally, empirical studies that identify the losers and winners from the complex processes and barriers to obtain secure land in the cities of developing countries would be very useful. In this respect, Durand-Lasserve et al. (2013) for instance highlight the disproportionate role of expatriates and merchants (the rich) and civil servants (the well-connected) in land purchases in Bamako, Mali.

Finally, linked to the previous comments, our analysis suggests that if governments want to improve access to secure land, improving the land institutions themselves should be a prerequisite. Whereas efforts to improve transparency in land administrations have been successful in some countries, they have failed or faced lack of political will in other countries, especially in sub-Saharan Africa. Further thinking on the conditions that make a transition towards transparent and affordable land administrations possible is needed. More generally, studies on the political economy of the urban land sector are warranted. We leave these important issues for further research.

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## A Hedonic regressions - *For Online Publication*

In theory, land prices are expected to be related to physical characteristics such as plot size, location (physical accessibility to the city center), availability of services (water or electricity) as well as to the tenure status and associated security and embedded rights (e.g. transferability). We resort to hedonic price analysis to investigate how prices are correlated with tenure, while controlling for other characteristics.<sup>25</sup> We are also interested in characterizing the land price gradients for the different tenure statuses.

We run our hedonic price regression on the subsample of plots used for residential purposes and also remove observations for which the price of land per square meter was in the top or bottom 1% of the price distribution. The equation we estimate is of the form :

$$\log(p_i) = \alpha d_i + \beta t_i + \gamma d_i \times t_i + \sum_{j=1}^k \omega_j X_i^j + \varepsilon_i$$

In this equation  $p_i$  is the price per square meter of the plot  $i$  at the date of the transaction,  $t_i$  is a set of dummy variables describing the tenure on the land,  $d_i$  is the euclidean distance to the CDB. The  $X_i^j$  are various covariates present in the database such as the distance to the nearest paved road, the surface of the plot, whether the land is serviced (water and/or electricity) and the bank of the river on which the land is located. We include the surface of the plot in order to capture non linear effects (even though the dependent variable is price per square meter).

We run two different specifications: Model (1) only includes the tenure dummies in levels (insecure tenure being the benchmark). Model (2) adds interaction terms between distance to the CBD and the tenure dummies. The sales are spread over a long period of time and inflation must be accounted for. We follow [Kim \(2004\)](#)

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<sup>25</sup>The reverse causality between the value of land and its tenure affects our hedonic price analysis: households will choose more secure tenure for plots which are more valuable, but plots will also become more valuable as their tenure increases. In the absence of a valid identification strategy, we will have to limit our analysis to a description of correlations.

who introduces a dummy variable for the year of the sale to control for the effect of inflation. The regression results are presented in Table 2 below. As can be seen in Table 2, the land market in Bamako displays expected patterns: prices decrease with distance to the city center and road infrastructure, and increase when serviced. Plots with secure forms of tenure are on average more expensive than plots with less secure forms of tenure. The gradient of land prices to distance depends on the tenure, with more secure forms of tenure having steeper gradients than less secure forms of tenure.

	Regression of log sales price	
	(1)	(2)
Distance to CBD (km)	-0.076*** (0.005)	-0.056*** (0.011)
Dummy : precarious tenure	-0.020 (0.090)	0.226 (0.296)
Dummy : secure tenure	0.847*** (0.108)	2.069*** (0.307)
Distance x precarious tenure	-	-0.011 (0.013)
Distance x secure tenure	-	-0.070*** (0.015)
Surface (in log)	-0.416*** (0.049)	-0.432*** (0.047)
Dummy : South bank of the river	0.832*** (0.059)	0.792*** (0.061)
Distance to the paved road (km)	-0.078*** (0.009)	-0.078*** (0.009)
Access to water	0.518*** (0.167)	0.557** (0.169)
Access to electricity	0.682** (0.297)	0.766*** (0.279)
Dummy : sold in 2010	0.264*** (0.077)	0.221*** (0.078)
Dummy : sold in 2011	0.261*** (0.077)	0.246** (0.077)
Dummy : sold in 2012	0.434*** (0.114)	0.366*** (0.114)
N	991	991
$R^2$	0.61	0.62

Source: authors' survey. Robust standard errors in parentheses. \*  $p < 0.1$ ,  
\*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

**Table 2:** Hedonic price regressions

## B Optimal tenure choice - *For Online Publication*

To prove Propositions 1 and 2, we first need to consider the optimization program of a type- $e$  household, conditional on location  $x$  and on the land rent function  $R(x)$ . The optimization program is given by Equation (22).

$$\begin{aligned} & \underset{z_u, z_r, \pi}{\text{maximize}} \quad \mathbb{E}(u) = \pi z_u + (1 - \pi) z_r \\ & \text{subject to} \quad z_u + xt + R(x) + C(\pi, e) \leq y_u \\ & \quad \quad \quad z_r + R_a + R(x) + C(\pi, e) \leq y_r \\ & \quad \quad \quad \pi_{min} \leq \pi \leq \pi_{max} \\ & \quad \quad \quad z_u \geq 0, \quad z_r \geq 0 \end{aligned}$$

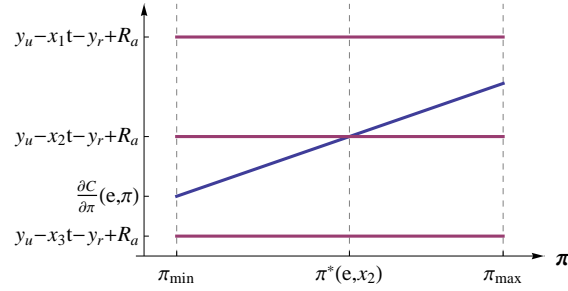
Recognizing that the two budget constraints must be binding at the optimum and considering only parameters of the model which ensure that the consumption of the composite good at equilibrium is positive, the program simplifies to a maximization of expected utility  $\mathbb{E}(u)(\pi, x, e)$  with respect to  $\pi$  as set out in Equation (4).

Under our assumption that the tenure cost function  $C(\pi, e)$  is convex with respect to  $\pi$ , the expected utility function  $\mathbb{E}(u)(\pi, x, e)$  is concave and therefore has a unique maximum over the segment  $[\pi_{min}, \pi_{max}]$ . Observing that  $\frac{\partial \mathbb{E}(u)}{\partial \pi}(\pi, x, e) = y_u - xt - y_r + R_a - \frac{\partial C}{\partial \pi}(\pi, e)$ , there are three cases depending on whether the solution  $\pi^*(x, e)$  is an interior solution or one of two corner solutions:

- If there exists a value  $\pi^* \in ]\pi_{min}, \pi_{max}[$  which satisfies the first order condition  $y_u - xt - y_r + R_a = \frac{\partial C}{\partial \pi}(\pi^*, e)$ , it maximizes the expected utility since the latter is concave. This corresponds to the case in which the expected utility increases over  $[\pi_{min}, \pi^*]$  and decreases over  $[\pi^*, \pi_{max}]$ .
- If  $y_u - xt - y_r + R_a \leq \frac{\partial C}{\partial \pi}(\pi_{min}, e)$ , then  $y_u - xt - y_r + R_a \leq \frac{\partial C}{\partial \pi}(\pi, e)$  for all values of  $\pi \in [\pi_{min}, \pi_{max}]$  since  $\frac{\partial C}{\partial \pi}$  is an increasing function of  $\pi$ . This

means that  $\frac{\partial \mathbb{E}(u)}{\partial \pi}(\pi, x, e) < 0$  and implies that  $\mathbb{E}(u)(\pi, x, e)$  decreases over  $[\pi_{min}, \pi_{max}]$ .  $\pi^* = \pi_{min}$  therefore maximizes the expected utility. .

- If  $y_u - xt - y_r + R_a \geq \frac{\partial C}{\partial \pi}(\pi_{max}, e)$ , then  $y_u - xt - y_r + R_a \geq \frac{\partial C}{\partial \pi}(\pi, e)$  for all values of  $\pi \in [\pi_{min}, \pi_{max}]$  because  $\frac{\partial C}{\partial \pi}$  is an increasing function of  $\pi$ .  $\mathbb{E}(u)(\pi, x, e)$  increases over  $[\pi_{min}, \pi_{max}]$  and  $\pi^* = \pi_{max}$  maximizes the expected utility.



**Figure 13:** Graphical determination of  $\pi^*(x, e)$  for  $e = 0.3$  and three different values of  $x$  :  $x_1 = 0$ ,  $x_2 = \frac{N}{2}$  and  $x_3 = N$ .

We can now present the proofs of Propositions 1 and 2, starting with Proposition 2.

## Proof of Proposition 2

For a given  $x$  and  $e$ , whether we have an interior or a corner solution to the program (5) depends on whether or not  $y_u - xt - y_r + R_a$  and  $\frac{\partial C}{\partial \pi}(\pi, e)$  intersect over  $[\pi_{min}, \pi_{max}]$ . Figure 13 illustrates this for  $e = 0.3$ , three different values of  $x$ , and the specific quadratic tenure cost function used throughout the paper for illustration purposes. The curve  $\pi \rightarrow \frac{\partial C}{\partial \pi}(\pi, e)$  is the upward sloping one. The curve  $\pi \rightarrow y_u - xt - y_r + R_a$  is a horizontal line which shifts downwards with higher values of  $x$ .

- For a small value of  $x$  (case 1 on the graph with  $x = x_1 = 0$ ),  $\pi \rightarrow \frac{\partial C}{\partial \pi}(\pi, e)$  is always below the horizontal line  $\pi \rightarrow y_u - xt - y_r + R_a$  and does not intersect with it. In this case, the solution is  $\pi^* = \pi_{max}$ .
- For an intermediate value of  $x$  (case 2 with  $x = x_2 = \frac{N}{2}$ ) the two curves

intersect and their intersection provides the optimal tenure choice for this household and location.

- For a high value of  $x$  (case 3 with  $x = x_3 = N$ ),  $\pi \rightarrow \frac{\partial C}{\partial \pi}(\pi, e)$  is always above the horizontal line  $\pi \rightarrow y_u - xt - y_r + R_a$  leading to the corner solution  $\pi^* = \pi_{min}$ .

We can thus define two threshold values  $\underline{x}(e)$  and  $\bar{x}(e)$ , such that for  $x \leq \underline{x}(e)$ , the optimal solution is always  $\pi_{max}$ . For  $x \geq \bar{x}(e)$ , the optimal solution is always  $\pi_{min}$ . Between these two threshold values, the optimal solution is an interior solution.

### Proof of Proposition 1

The intuition for Proposition 1 can be understood from inspection of Figure 13. Since the line  $\pi \rightarrow y_u - xt - y_r + R_a$  shifts downwards with greater values of  $x$ ,  $\pi^*(x, e)$  is a decreasing function of  $x$ . We have assumed that  $\frac{\partial^2 C}{\partial \pi \partial e} > 0$ , which means that the curve  $\pi \rightarrow \frac{\partial C}{\partial \pi}(\pi, e)$  shifts upwards with greater values of  $e$ , which implies that  $\pi^*(x, e)$  is also a decreasing function of  $e$ .

Analytically, we can apply the implicit function theorem to the first order condition (6) for  $x \in ]\underline{x}(e), \bar{x}(e)[$ . We obtain:

$$\frac{\partial \pi^*}{\partial x}(x, e) = \frac{-t}{\frac{\partial^2 C}{\partial \pi^2}(\pi^*(x, e), e)} < 0 \quad (22)$$

$$\frac{\partial \pi^*}{\partial e}(x, e) = \frac{-\frac{\partial^2 C}{\partial \pi \partial e}(\pi^*(x, e), e)}{\frac{\partial^2 C}{\partial \pi^2}(\pi^*(x, e), e)} < 0 \quad (23)$$

Both these equations are perfectly defined as we have assumed  $\frac{\partial^2 C}{\partial \pi^2}(\pi^*(x, e), e)$  to be strictly positive. Outside of the interval  $]\underline{x}(e), \bar{x}(e)[$ ,  $x \rightarrow \pi^*(x, e)$  is constant and the derivative is equal to 0. The derivative of  $\pi^*(x, e)$  with respect to  $x$  is not defined in either  $\underline{x}(e)$  or  $\bar{x}(e)$ .



## C Land market equilibrium - *For Online Publication*

### Proof of Proposition 4

In land use models, spatial patterns are determined by the comparison of bid rent functions, with households with the steepest bid rents locating closer to the CBD (Fujita, 1989). In our model however, the bid rent functions of different households cannot systematically be ranked by order of decreasing steepness. This is because for some locations  $x$ , households of different types may have the same demand for tenure security and therefore have bid rents of the same slope (see Equation (8)). Although the equilibrium location of these households cannot be determined precisely, it is nevertheless possible to determine the relative positions of three groups of households defined as follows:

- Group 1 includes all households who choose  $\pi^* = \pi_{max}$  at equilibrium ( $x(e)$  is the equilibrium location of a type- $e$  household)

$$G_1 = \{e | \pi^*(x(e), e) = \pi_{max}\}$$

- Group 2 includes all households who choose  $\pi^* \in ]\pi_{min}, \pi_{max}[$

$$G_2 = \{e | \pi^*(x(e), e) \in ]\pi_{min}, \pi_{max}[ \}$$

- Group 3 includes all households who choose  $\pi^* = \pi_{min}$

$$G_3 = \{e | \pi^*(x(e), e) = \pi_{min}\}$$

In equilibrium  $G_1$ ,  $G_2$  or  $G_3$  could be empty sets.

We will show that all households in group 1 have smaller values of  $e$  than all households in group 2, who in turn all have smaller values of  $e$  than all households

in group 3.

$$\forall e_1 \in G_1, e_2 \in G_2, e_3 \in G_3 : e_1 < e_2 < e_3$$

We will also show that all individuals in group 1 are located at the vicinity of the city center and households in group 3 at the periphery of the city while those in group 2 are located in the middle.

$$\forall e_1 \in G_1, e_2 \in G_2, e_3 \in G_3 : x(e_1) < x(e_2) < x(e_3)$$

Finally, although we will be unable to identify the exact location of households in groups 1 and 3, we will show that the individuals in group 2 locate in order of increasing  $e$  moving outwards from the CBD.

$$\forall \{e_a, e_b\} \in G_2, e_a < e_b \Leftrightarrow x(e_a) < x(e_b)$$

Let us start by considering the different ways bid-rent functions can intersect. We have shown that the bid rent of a type- $e$  household is linear over  $[0, \underline{x}(e)]$ , strictly convex over  $[\underline{x}(e), \bar{x}(e)]$ , and linear over  $[\bar{x}(e), +\infty[$ . From Lemma 1, as also illustrated by Figure 14,<sup>26</sup> we know that the bid rent of the household with the smaller  $e$  dominates (eventually weakly) to the left of the intersection (in the direction of the CBD) whereas the bid rent of the household with the larger  $e$  dominates (eventually weakly) to the right of the intersection (in the direction of the city periphery).

Figures 14a and 14b illustrate the intersection of two bid rents confounded over either of their linear segments:

- On Figure 14a, the bid rent  $\Psi_{e_1}$  dominates  $\Psi_{e_2}$  weakly: over  $[0, \bar{x}(e_1)[$ , we have  $\Psi_{e_2} < \Psi_{e_1}$ , and over  $[\bar{x}(e_1), +\infty[$ , we have  $\Psi_{e_2} = \Psi_{e_1}$ .
- On Figure 14b, the bid rent  $\Psi_{e_2}$  dominates  $\Psi_{e_1}$  weakly: over  $[0, \underline{x}(e_2)[$ , we have  $\Psi_{e_2} = \Psi_{e_1}$ , and over  $[\underline{x}(e_2), +\infty[$ , we have  $\Psi_{e_2} > \Psi_{e_1}$ .

Figure 14c illustrates two bid rents intersecting in a single point  $x_i$ :

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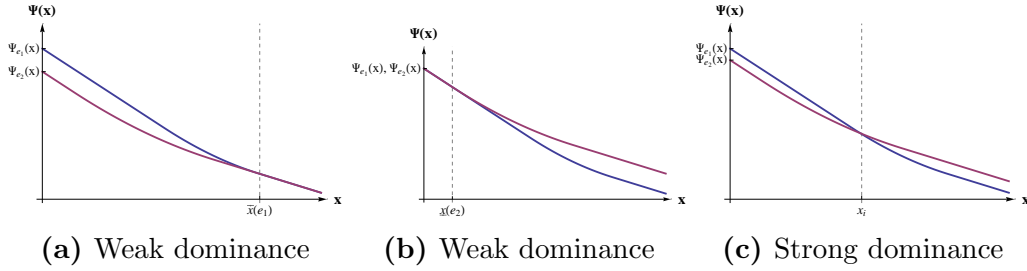
<sup>26</sup>Note that the bid rents in Figure 14 are not represented for the households' respective equilibrium utilities but for arbitrary levels of utility that generate the different cases for intersection.

- The bid rent  $\Psi_{e_1}$  dominates  $\Psi_{e_2}$  over  $[0, x_i[$

$$\forall x < x_i, \Psi_{e_2}(x) < \Psi_{e_1}(x)$$

- The bid rent  $\Psi_{e_2}$  dominates  $\Psi_{e_1}$  over  $]x_i, +\infty[$

$$\forall x > x_i, \Psi_{e_2}(x) > \Psi_{e_1}(x)$$



**Figure 14:** Intersections between the bid rents of households  $e_1 = 0$  (blue) and  $e_2 = 0.1$  (red).

With this in mind, let us now consider the case where, in equilibrium, a type- $e_1$  household is in group 1 and a type- $e_2$  household is in group 2 as defined above, and let us show that  $e_2$  is necessarily greater than  $e_1$ . We demonstrate our result *reductio ad absurdum* by assuming that  $e_2$  is smaller than  $e_1$  and showing that it is impossible.

First of all, by definition of group 1, household of type  $e_1$  chooses  $\pi = \pi_{max}$  at an equilibrium location  $x(e_1)$ . Given that  $e_1$  chooses  $\pi_{max}$  in position  $x(e_1)$ , it must be that  $x(e_1) < \underline{x}(e_1) < \underline{x}(e_2)$  because we have assumed that  $e_2 < e_1$  and  $\underline{x}(\cdot)$  is a decreasing function. So in  $x(e_1)$ , both bid rents  $\Psi(e_1)$  and  $\Psi(e_2)$  are linear and of slope  $-t\pi_{max}$ . Furthermore, since  $x(e_1)$  is the equilibrium location of the type- $e_1$  household, the bid rent of the type- $e_2$  household can be either equal or inferior to that of household  $e_1$  in  $x(e_1)$ . Given these assumptions, there are two possibilities (we denote  $\nu(e_1)$  and  $\nu(e_2)$  the equilibrium utilities of the two households):

- If  $\Psi_{e_2}(x(e_1), \nu(e_2)) < \Psi_{e_1}(x(e_1), \nu(e_1))$ , then  $\Psi_{e_2}$  and  $\Psi_{e_1}$  do not intersect at all and  $\Psi_{e_2}$  is completely dominated by  $\Psi_{e_1}$  over all values of  $x$ . This is

due to the our assumption that  $e_2 < e_1$  and therefore  $\Psi_{e_1}$  has a stronger curvature than  $\Psi_{e_2}$ .

- If  $\Psi_{e_2}(x(e_1), \nu(e_2)) = \Psi_{e_1}(x(e_1), \nu(e_1))$ , then  $\Psi_{e_2}$  and  $\Psi_{e_1}$  are confounded over  $[0, \underline{x}(e_1)]$  and then  $\Psi_{e_2}$  is strictly dominated by  $\Psi_{e_1}$  for all values of  $x > \underline{x}(e_1)$ .

Second, observe that the two households' bid rents must intersect in equilibrium because no household can outbid another household over the whole city (otherwise the outbid household would not be able to purchase a plot in equilibrium). This means that for any other tenure choice than  $\pi_{max}$  and location  $x > \underline{x}(e_1)$ , household  $e_2$  is outbid by household  $e_1$  which contradicts the fact that household  $e_2$  is in group 2. Therefore  $e_2$  is larger than  $e_1$ .

Now that we know that  $e_1 < e_2$ , and that the type- $e_1$  household chooses  $\pi^* = \pi_{max}$  in location  $x(e_1)$ , the bid rents  $\Psi_{e_1}$  and  $\Psi_{e_2}$  can only intersect in two ways:

- If  $\Psi_{e_2}(x(e_1), \nu(e_2)) < \Psi_{e_1}(x(e_1), \nu(e_1))$ , which implies that the two bid rents intersect in  $x^* > x(e_1)$ .  $\Psi_{e_2}$  dominates (eventually weakly)  $\Psi_{e_1}$  over  $[x^*, +\infty]$ .  $e_2$  is therefore located further away than  $e_1$  at equilibrium.
- If  $\Psi_{e_2}(x(e_1), \nu(e_2)) = \Psi_{e_1}(x(e_1), \nu(e_1))$ , then in  $x(e_1) < \underline{x}(e_2)$  which means that at equilibrium  $e_2$  locates further out than  $e_1$ .

So in equilibrium,  $e_2$  is located further out than  $e_1$ :  $x(e_1) < x(e_2)$ .

A similar demonstration (not reported) would show that:

$$\forall e_2 \in G_2, e_3 \in G_3 : e_2 < e_3 \text{ and } x(e_2) < x(e_3)$$

Given these properties and the continuity of our problem, we can define four unique threshold values  $\underline{e} < \bar{e}$  and  $\underline{x} < \bar{x}$  which characterize groups 1, 2 and 3 such that:

- Households with  $e$  smaller than  $\underline{e}$  all choose maximum tenure security  $\pi_{max}$ , and are located in the central segment of the city defined by  $x < \underline{x}$ . These are the households of the group 1 we previously defined.

- Households with  $e$  between  $\underline{e}$  and  $\bar{e}$  all choose intermediate levels of tenure security  $\pi \in [\pi_{min}, \pi_{max}]$ , and are located in the middle segment of the city defined by  $\underline{x} < x < \bar{x}$ . These households belong to group 2.
- Households with  $e$  larger than  $\bar{e}$  all choose minimal tenure security  $\pi_{min}$ , and are located in the peripheral segment of the city defined by  $x > \bar{x}$ . These households belong to group 3.

We are now left to show that within the central and peripheral segments, the location of each household cannot be determined precisely, but that in the intermediate area it can.

First of all, all households located in the central or peripheral segments choose levels of tenure security equal to  $\pi_{max}$  or  $\pi_{min}$ . This means that their bid rents have the same slope and are all confounded over the area. It is therefore impossible to determine the exact location of each household since they will all be indifferent between the locations.<sup>27</sup>

We consider two households  $e_a$  and  $e_b$  within group 2 and assume that  $e_a < e_b$ . As previously stated, the two bid rents must intersect so that no household is outbid by another. Moreover, in order for households  $e_a$  and  $e_b$  to be located in the intermediate zone of the city, their equilibrium bid-rent must dominate in at least one location of that zone. This means that in  $x(e_a)$ ,  $\Psi_{e_a}(x(e_a), \nu(e_a)) \geq \Psi_{e_b}(x(e_a), \nu(e_b))$  and in  $x(e_b)$ ,  $\Psi_{e_b}(x(e_b), \nu(e_b)) \geq \Psi_{e_a}(x(e_b), \nu(e_a))$ . Given the typology of possible intersections between bidrents of  $e_a < e_b$ , given that neither household chooses  $\pi_{max}$  or  $\pi_{min}$  in equilibrium, it can only be that the intersection of both bid rents is a point, before which  $\Psi_{e_a}$  dominates strictly and after which  $\Psi_{e_b}$  dominates strictly. Therefore  $x(e_a) < x(e_b)$ , for any  $e_a < e_b$ . QED.

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<sup>27</sup>Different households will have different levels of utility, but each household will be indifferent between all locations within the considered segment.

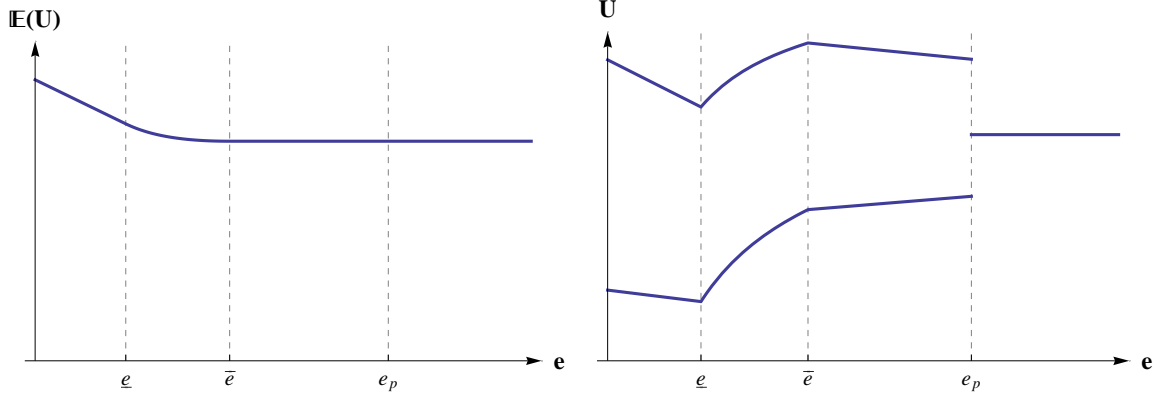
## D Aggregates and Surplus - *For Online Publication*

### Utilities

To illustrate the functioning of the model, it is useful to also discuss the shape of the utility distribution. Figure 15 below represents the *ex ante* utility (left panel) and *ex post* utilities (right panel) as a function of type. On the right panel, over  $[0, e_p]$ , there are two *ex post* utilities which are conditional on the realization of each state of the nature. The higher curve is for households who invested in an urban plot and managed to keep it whereas the lower curve is for those who lost their plot. On  $[e_p, 1]$ , the curve represents the utility of households who did not purchase a plot in the city and remained in the rural area all along. On this segment, there is no distinction between the two states of the nature: both curves are flat and take the same constant value equal to the *expected* utility over the same segment shown on the left panel.

While the *ex ante* utility is decreasing, the *ex post* utilities show an irregular pattern. The decrease in both *ex post* utilities for households in the formal sector ( $e < \underline{e}$ ) reflects the higher fees households with higher types need to pay to the administration. Interestingly, in this simulation, both *ex post* utilities of households with precarious tenure ( $e \in [\underline{e}, \bar{e}]$ ) increase with the type because their purchase of lower tenure security dominates their cost disadvantage (which means that higher types make a smaller rather than a greater payment to the land administration). In the informal section of the city ( $e \in [\bar{e}, e_p]$ ), the *ex post* utility decreases with the type for households who keep their plot but increases with the type for household who lose it. This is due to the fact that on this segment of the land market, variations in utilities are explained by the comparison of rents and transport costs: rents (which are determined in the first stage) compensate for *expected* not *ex post* transport costs, hence a decrease in *ex post* utilities for higher types who manage to keep their plots located further away from the city center.

### Revenues



**Figure 15:** *Ex ante* (left panel) and *ex post* utilities of households (right panel)

The land administration's revenue gathered from collected fees is given by  $I = \int_0^{e_p} C(\pi^*(x(e), e), e)N de$ . The aggregate income of landowners is the sum of all rents paid by buyers (including multiple sales):  $R = \int_0^{e_p} R(x(e))N de$ . The *ex post* welfare of households who bought an urban plot and are able to keep it in the second stage of the model is  $W_{urban,kept} = \int_0^{e_p} \pi^*(x(e), e)[y_u - x(e)t - R(x(e)) - C(\pi^*(x(e), e), e)]N de$ . The *ex post* welfare of households who bought an urban plot but lose it to a multiple sale is  $W_{urban,lost} = \int_0^{e_p} [1 - \pi^*(x(e), e)][y_r - R_a - R(x(e)) - C(\pi^*(x(e), e), e)]N de$ . The welfare of rural households who did not purchase a land plot in the city is  $W_{rural} = \int_{e_p}^1 (y_r - R_a)N de$ .<sup>28</sup> Finally, the *ex ante* welfare of households purchasing a plot in the city is  $W_{urban} = \int_0^{e_p} [[1 - \pi^*(x(e), e)][y_r - R_a - R(x(e)) - C(\pi^*(x(e), e), e)] + \pi^*(x(e), e)[y_u - x(e)t - R(x(e)) - C(\pi^*(x(e), e), e)]]N de$  which is mathematically equivalent to the sum of  $W_{urban,kept}$  and  $W_{urban,lost}$ .

Given the linearity (and *a fortiori* quasi-linearity) of the utility function, we can define a total surplus  $S = I + R + W_{rural} + W_{urban}$  (from *ex ante* definitions). In the absence of any market failure, the surplus is constant but its share among the different agents varies. In this respect, our model is relevant to analyze the

<sup>28</sup>To understand the formulas for  $W_{urban,kept}$  and  $W_{urban,lost}$ , observe that we are summing over  $e$  not  $x$ , hence the multiplicative terms  $\pi^*(x(e), e)$  and  $1 - \pi^*(x(e), e)$ . Also observe that there is no distinction between *ex ante* or *ex post* for  $W_{rural}$ .

redistributional effects of land administration practices.

## E Comparative statics - *For Online Publication*

We are interested in the variation of city structure patterns with respect to the parameters of our model, namely the total population  $N$ , the transport cost  $t$  and the urban rural-wage differential  $y_u - y_r$ . We first show how these parameters affect the curves  $\underline{x}(e)$ ,  $\bar{x}(e)$  and  $x(e)$ , and then their intersections.

The curves  $\underline{x}(e)$  and  $\bar{x}(e)$  are defined by the equations :

$$\begin{aligned}\underline{x}(e) &= \frac{1}{t}[y_u - y_r + R_a - \frac{\partial C}{\partial \pi}(\pi_{max}, e)] \\ \bar{x}(e) &= \frac{1}{t}[y_u - y_r + R_a - \frac{\partial C}{\partial \pi}(\pi_{min}, e)]\end{aligned}$$

The curve  $x(e)$  is determined by the following differential equation :

$$\begin{aligned}x'(e) &= N\pi^*(x(e), e) \\ x(0) &= 0\end{aligned}$$

### A variation in $N$ .

We will now show that for  $N_1 < N_2$ , the corresponding solutions to the differential equation  $x_1(e)$  and  $x_2(e)$  verify  $x_1(e) < x_2(e)$  for  $e > 0$ .

We have  $x_1(0) = x_2(0) = 0$ , and  $x'_1(0) = N_1\pi^*(0, 0) < x'_2(0) = N_2\pi^*(0, 0)$  because  $\pi^*(x, e)$  does not depend on  $N$ . There is therefore a small neighborhood around 0 over which the function  $x_2(e) - x_1(e)$  is strictly positive. We can find  $\varepsilon > 0$  such that  $x_2(e) - x_1(e)$  is strictly positive over  $]0, \varepsilon[$ .



Let us note  $E$  the set of values  $e \geq \varepsilon$  for which the two curves  $x_1$  and  $x_2$  intersect.

$$E = \{e \geq \varepsilon \mid x_1(e) = x_2(e)\}$$

Let us suppose that  $E$  is a non-empty set. It therefore has an infimum  $e_0 \geq \varepsilon$  and because  $x_1(e) - x_2(e)$  is continuous,  $e_0$  belongs to  $E$  and  $x_1(e_0) = x_2(e_0)$ .

In  $e_0$  :  $x_2'(e_0) - x_1'(e_0) = (N_2 - N_1)\pi^*(x_1(e_0), e_0) > 0$ . Therefore, there is a neighborhood to the left of  $e_0$  over which  $x_2(e) - x_1(e)$  is strictly negative. We can find  $\eta > 0$  such that  $x_2(e) - x_1(e)$  is strictly negative over  $]e_0 - \eta, e_0[$ .

Therefore, the function  $x_2(e) - x_1(e)$  is strictly positive over  $]0, \varepsilon[$  and strictly negative over  $]e_0 - \eta, e_0[$  which implies that  $\varepsilon < e_0 - \eta$ . By the theorem of intermediate values, there is a value  $e \in [\varepsilon, e_0 - \eta]$  for which  $x_2(e) - x_1(e)$  is equal to zero. This contradicts the definition of  $e_0$ .  $E$  is therefore an empty set and  $x_2(e) > x_1(e)$  for all  $e > 0$ .

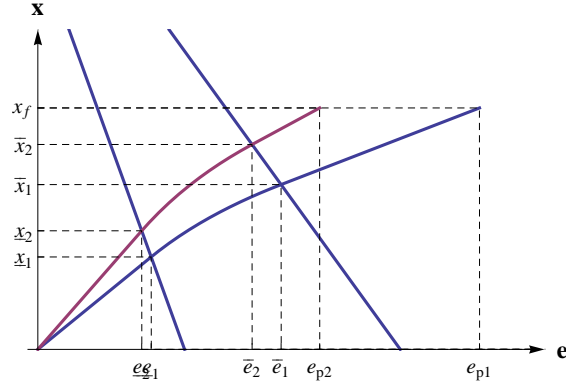
The equations which define  $\underline{x}(e)$  and  $\bar{x}(e)$  do not depend on the parameter  $N$  and are decreasing functions of  $e$ : an increase in  $N$  therefore leads to an increase in  $\underline{x}$  and  $\bar{x}$  and a decline in  $\underline{e}$  and  $\bar{e}$ . Figure 16 illustrates this property: an increase in  $N$  leads to an upward shift in  $x(e)$  (red curve) without changing  $\underline{x}(e)$  and  $\bar{x}(e)$ . Therefore, for  $N_2 > N_1$  :

$$\begin{aligned} \underline{e}_2 &< \underline{e}_1 \quad \text{and} \quad \underline{x}_2 > \underline{x}_1 \\ \bar{e}_2 &< \bar{e}_1 \quad \text{and} \quad \bar{x}_2 > \bar{x}_1 \end{aligned}$$

### A variation in $t$ .

A decrease in transport costs  $t$  or an increase in the urban-rural wage differential  $y_u - y_r$  have the same effect on city structure since they both increase the gains from living in the city.

Let us consider two different unit transport cost  $t_1 > t_2$ , and  $x_1(e)$  and  $x_2(e)$  the solutions to the corresponding differential equations.



**Figure 16:** Variations of  $x(e)$  with  $N$ .

The functions  $\underline{x}(e)$  and  $\bar{x}(e)$  are decreasing functions of  $t$ , such that  $\underline{x}_1(e) < \underline{x}_2(e)$  and  $\bar{x}_1(e) < \bar{x}_2(e)$  for all values of  $e$  for which all functions are not equal to zero. As  $\underline{x}$  and  $\underline{e}$  are defined by the intersection of  $\underline{x}(e)$  and  $N\pi_{max}e$  we have the following inequalities:  $e_1 < e_2$ ,  $x_1 < x_2$  and  $x_1(e)$  and  $x_2(e)$  are confounded over  $[0, e_1]$ . Furthermore, there is a neighborhood of  $e_1$  over which  $x_2(e) > x_1(e) : \forall e \in ]e_1, e_2[$ ,  $x'_2(e) = N\pi_{max} > x'_1(e)$ .

As with  $N$ , let's note  $E$  the set of values  $e \geq e_1 + \varepsilon$  for which the two curves  $x_1$  and  $x_2$  intersect.

$$E = \{e \geq e_1 + \varepsilon \mid x_1(e) = x_2(e)\}$$

Let us suppose that  $E$  is a non-empty set. It therefore has an infimum  $e_0 \geq e_1 + \varepsilon$  and because  $x_1(e) - x_2(e)$  is continuous,  $e_0$  belongs to  $E$  and  $x_1(e_0) = x_2(e_0)$ .

In  $e_0$  either  $x'_2(e_0) > x'_1(e_0)$  or  $x'_2(e_0) = x'_1(e_0) = N\pi_{min}$  because  $t_2 < t_1$  and that  $\pi^*$  is defined by the intersection of  $y_u - y_r + R_a - xt$  and  $\frac{\partial C}{\partial \pi}$  when the intersection exists. Furthermore,  $x'_1(e_0) < \pi_{max}$  as  $e_0 > e_1$ .

- If in  $e_0$ ,  $x'_2(e_0) = x'_1(e_0) = \pi_{min}$ , then  $e_0 = \bar{e}_2$  and  $x_1$  and  $x_2$  are confounded over  $[\bar{e}_2, 1]$ . This leads to  $\bar{e}_1 < \bar{e}_2$ . Over  $]\bar{e}_1, \bar{e}_2[$ ,  $x'_2(e) > x'_1(e)$  which in turn leads to  $x_2(e) - x_1(e) < 0$ . The intermediate value theorem contradicts the definition of  $e_0$ .

- If in  $e_0$ , both derivatives are not equal to  $\pi_{min}$  then it must be that  $x'_2(e_0) > x'_1(e_0)$  as  $y_u - y_r + R_a - x_2(e_0)t_2 > \delta_1 + R_a - x_2(e_0)t_1$ . By a similar argument as previously, this means that  $x_2(e) - x_1(e)$  is strictly negative in a neighborhood around  $e_0$ , which contradicts the definition of  $e_0$  due to the intermediate value theorem.

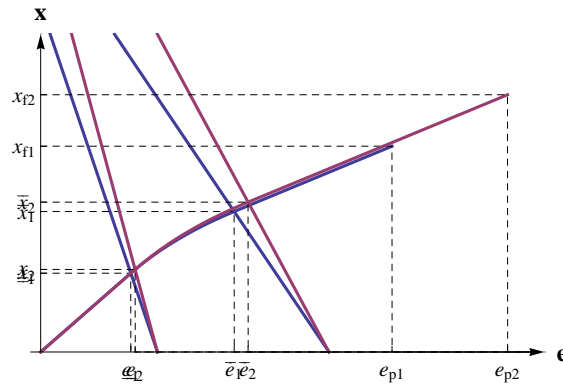
We have therefore shown that for  $t_1 > t_2$ ,  $x_2(e) > x_1(e)$ .

A decrease in transport costs leads to an increase in  $\underline{x}$ ,  $\underline{e}$ ,  $\bar{x}$ , but has an ambiguous effect on  $\bar{e}$ .  $\bar{e}$  is defined by the equation  $\bar{x}(\bar{e}) = x(\bar{e})$ . If we differentiate this equation with respect to  $t$ , we can show that the sign of  $\frac{\partial \bar{e}}{\partial t}$  is not determined:

$$\frac{\partial \bar{e}}{\partial t} \frac{\partial \bar{x}}{\partial e}(\bar{e}) + \frac{\partial \bar{x}}{\partial t}(\bar{e}) = \frac{\partial \bar{e}}{\partial t} \frac{\partial x}{\partial e}(\bar{e}) + \frac{\partial x}{\partial t}(\bar{e})$$

$$\frac{\partial \bar{e}}{\partial t} \left( \frac{\partial \bar{x}}{\partial e}(\bar{e}) - \frac{\partial x}{\partial e}(\bar{e}) \right) = \frac{\partial x}{\partial t}(\bar{e}) - \frac{\partial \bar{x}}{\partial t}(\bar{e})$$

These effects are illustrated on Figure 17.



**Figure 17:** Variations of  $x(e)$  with  $t$ .

### The effect on the urban fringe.

In the most general case, we cannot say what the effect of an increase in  $N$ ,  $y_u - y_r$  or a decrease in  $t$  will have on the size of the city  $x_f$  and the population

who tries to migrate  $e_p$ .

However, in the case in which (i)  $C(\pi_{min}, e) = C(\pi_{min})$ , and (ii) there is an informal area to the city, the urban fringe is defined by the following equations :

$$x_f = \frac{y_u - y_r}{t} - \frac{R_a(1 - \pi_{min}) + C(\pi_{min})}{t\pi_{min}}$$

$$x_f = x(e_p)$$

An increase in  $N$  has no effect on the city size  $x_f$ , but leads to a decrease in  $e_p$  the last migrant to move to the city. This reflects the fact than an increase in  $N$  means that there are more well-connected individuals using up the city space, and that this is pushing the less well-connected to the outskirts where the gains from living in the urban area are smaller.

An increase in  $y_u - y_r$  or a decrease in  $t$  leads to an increase in city size  $x_f$ , and a decrease in  $e_p$ . As the gains from living in the city center increase, well-connected households will be choosing more secure tenure and thereby taking up more of the urban space, hence pushing less well-connected households towards the outskirts.

## F More equitable land administration fees - *For Online Publication*

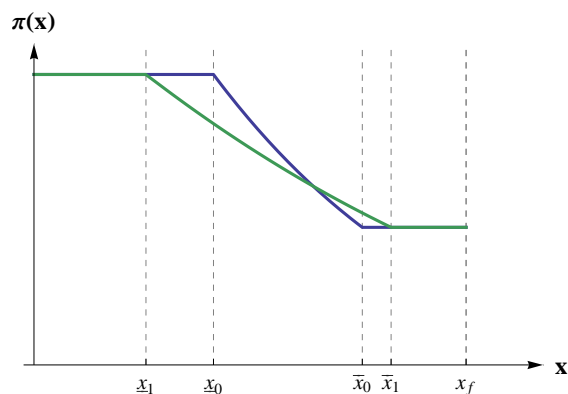
It is often advocated to increase transparency in the land administration in order to avoid bribes and discretionary treatment. Measures include for instance the menu of fees posted on a wall. We simulate such a policy by making the land administration more equitable by changing the parameter  $\theta$ .<sup>29</sup> We explore the case of  $\theta_1 = 0.5$ , which makes acquiring property rights cheaper than under the base case for poorly-connected households of type  $e > e_m$  and more expensive for

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<sup>29</sup>Under  $\theta = 1$ , which we considered up to now, the value of  $e_m$  was irrelevant. The value of  $e_m$  reflects the choice of authorities regarding which household should be the pivot in rotating the fee schedule. We consider  $e_m = \frac{e_0}{2} + \frac{e_0 - e_0}{2} = 0.31$ , where the pivot is the median type of households with semi-formal tenure.

well-connected households with  $e < e_m$ . All curves for this policy are represented in green on the figures below. We have the three following comments:

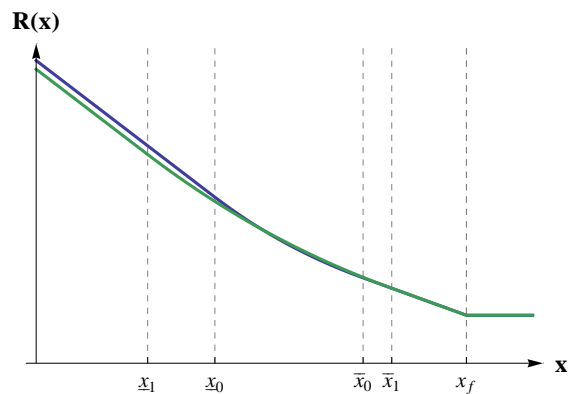
First, *more equitable fees makes the precarious section of the city larger* (at the expense of both the spatial extent of the secure and insecure sections of the city). This is because the highest level of tenure security becomes less affordable for some households who could afford it under the base case and because precarious tenure becomes more affordable for households who would optimally choose insecure tenure under the base case (see Figure 18). The tenure security of poorly-connected households is thus improved at the expense of some well-connected households.



**Figure 18:** Demands for tenure security in each location following a reduction in  $\theta$  ( $\theta_1 = 0.5$ )

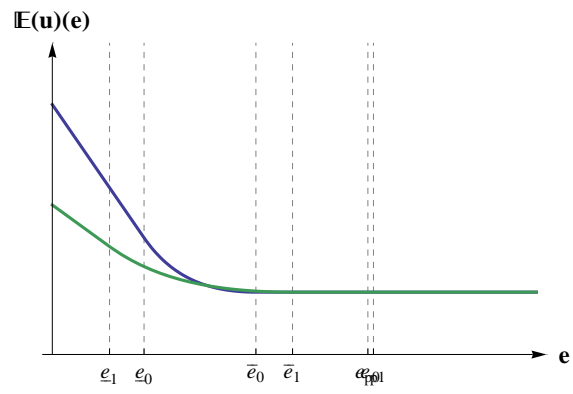
Second, effects on the land rent curve are nuanced, as illustrated by Figure 19 below. At the periphery of the city, some locations held under insecure tenure under the base case are now held under precarious tenure, which raises the price of land for these plots and exerts an upward pressure on land rents in more central locations (due to competition for land). But because of increased fees, households in the precarious section with type  $e < e_m$  and a fraction of households who formerly had secure tenure (who *a fortiori* have a type  $e < e_m$ ) now demand lower tenure security. There is thus a countervailing force towards the decrease in land prices for central locations. In our simulation, the latter effect dominates the

former throughout the secure tenure section and on part of the precarious tenure section of the city, resulting in lower land prices there than under the base case.



**Figure 19:** Land market prices following a reduction in  $\theta$  ( $\theta_1 = 0.5$ )

Third, looking at the expected utility of households (Figure 20), there are nuanced effects as well. Households who have switched from insecure to precarious tenure (in the middle of the distribution) experience an increase in expected utility, whereas households with lower types face a decrease in their expected utility due to an increase in tenure costs (which may result in the choice of a lower level of tenure security) which is not completely compensated by the decrease in land rents in more central locations.



**Figure 20:** *Ex-ante* utility following a reduction in  $\theta$  ( $\theta_1 = 0.5$ )