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# Inference on time-invariant variables using panel data: a pretest estimator\*

Jean-Bernard Chatelain<sup>†</sup> and Kirsten Ralf<sup>‡</sup>

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## Abstract

This paper proposes a new pretest estimator of panel data models including time-invariant variables based on the Mundlak-Krishnakumar estimator and an “unrestricted” Hausman-Taylor estimator. Furthermore, the paper evaluates the biases of currently used estimators: repeated between, ordinary least squares, two-stage restricted between, Oaxaca-Geisler estimator, fixed effect vector decomposition, and generalized least squares. Some of these may lead to erroneous conclusions regarding the statistical significance of the estimated parameter values of time-invariant variables, especially when time-invariant variables are correlated with the individual effects.

**JEL classification numbers:** C01, C22, C23.

**Keywords:** Time-invariant variables, panel data, time-series cross-sections, pretest estimator, Mundlak estimator, Hausman-Taylor estimator.

## 1 Introduction

Panel data generally have a time-series and a cross-section dimension, but we also observe time-invariant explanatory variables. Examples of time-invariant variables are: geographical distance for cross-country data in gravity models of foreign trade and foreign direct investments (Serlenga and Shin (2007)); years of schooling, gender and race when testing Mincer’s wage equation using survey data (Hausman and Taylor (1981)); colonial, legal or political systems, international conflicts, institutional and governance indicators, initial gross domestic product per head when testing the convergence of incomes in growth regressions. These variables are often highly relevant and have a high expected correlation with a cross-section and time-varying variable of interest in the cross-section dimension.

We are interested in estimating and testing these time-invariant variables (observed for  $N$  individuals) and the consequence of including these time-invariant variables for the estimation and significance of the parameter of a time- and individual-varying variable (observed for  $N$  individuals during  $T$  periods). Since a time-invariant variable has no

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variance in the time direction, it can only explain the variance of a time- and individual-varying variable in its individual direction. The reason why it matters for inference is that *“the effect of a random component can only be averaged out if the sample increases in the direction of that random (time or individual) component”* (Kelejian and Stephan (1983), see also Greene (2012), section 11.4.5, pp. 404-411, Hsiao (2014), section 3.6, pp. 58-64). The pooled ordinary least squares (OLS) estimator with time-invariant explanatory variables, however, uses the dimension  $NT$  when drawing inference on time-invariant variables.

This paper makes four contributions:

(1) It proposes an “unrestricted” Hausman-Taylor estimator including the average over time of endogenous time-varying variables, which reduces the root-mean-square error and the bias of parameters of time-invariant variables as compared to the usual “restricted” Hausman-Taylor estimator (1981) when internal instruments are weak.

(2) It proposes a pretest estimator for the selection of internal instruments based upon the Mundlak (1978) and Krishnakumar (2006) estimator and the “unrestricted” Hausman-Taylor estimator, extending the pretest estimator proposed by Baltagi, Bresson and Pirotte (2003).

(3) The paper computes the determinants of the biases of the estimated parameters and the estimated standard errors of several other available estimators. Firstly, an omitted variable bias on the estimated parameter may occur due to omitted average over time of endogenous time-varying variables. Secondly, the estimator of the standard error of the parameters of time-invariant variables may be biased because of an excessive weight on within mean square error and on within degrees of freedom (depending on the time dimension  $T$ ).

(4) It presents the results of a Monte Carlo simulation that investigates small-sample characteristics of the pretest and of the other available estimators.

The paper proceeds as follows. Section 2 defines the pretest estimator and compares it to alternative estimators used in panel data estimations. Section 3 presents the design of the Monte Carlo simulation and section 4 the results of this simulation for nine estimators including the pretest. Section 5 concludes.

## 2 A pretest estimator including time-invariant variables and correlated individual effects

### 2.1 The model

The model of time-series cross-section regression estimates the following equation

$$y = X\beta + Z\gamma + \alpha + \varepsilon, \quad (1)$$

where the  $NT \times 1$  vector  $y = [y_{it}]$  denotes the endogenous variable. Observations are ordered first by individual and then by time. Subscripts indicate variation over individuals ( $i = 1, \dots, N$ ) and time ( $t = 1, \dots, T$ ).  $X$  is a  $NT \times k$  matrix with rows  $X_{it}$  of cross-sections time-series data,  $Z$  is a  $NT \times g$  matrix with rows  $Z_i$  of time-invariant variables,  $\beta$  is a  $k \times 1$  vector and  $\gamma$  is a  $g \times 1$  vector of coefficients associated with time-varying and time-invariant observable variables, respectively. The constant can be included in the matrix of time-invariant variables, in which case  $Z_{1it} = 1$ ; it is also individual invariant. The individual effect  $\alpha$  is assumed to be a time-invariant random variable, distributed

independently across individuals with variance  $\sigma_\alpha^2$  with  $E(\alpha_i | X_{it}, Z_i) \neq 0$ .  $\alpha$  and each column of  $Z$  are  $NT$  vectors having blocks of  $T$  identical entries within each individual  $i = 1, \dots, N$ . The disturbance  $\varepsilon_{it}$  is assumed to be uncorrelated with the columns of  $(X, Z, \alpha)$  and has zero mean and constant variance  $\sigma_\varepsilon^2$ .

This kind of model exhibits a problem of identification since the time-invariant variables and the individual effects are not distinguishable leading to biased estimators for  $\gamma$  and  $\beta$ . Furthermore, problems of endogeneity of different degrees may occur due to correlations of a time-varying or a time-invariant variable with the individual effect. Without additional assumptions these problems cannot be solved.

## 2.2 The pretest estimator based on the Mundlak-Krishnakumar unrestricted GLS estimator

Mundlak (1978) and Krishnakumar (2006) introduce an auxiliary equation that assumes a linear relation between the individual random effect and the explanatory variables:

$$\alpha = \bar{X}.\pi + Z\phi + \alpha^M, \quad (2)$$

where  $\bar{X}$  is a  $NT \times k$  matrix that repeats  $T$  times the average over time of each time-varying variable. The disturbance term  $\alpha^M$  is normally distributed,  $\alpha_i^M \sim (0, \sigma_{\alpha^M}^2)$ ,  $i = 1, \dots, N$ , and  $\pi$  is a  $k$  vector of coefficients associated with the average over time of time-varying variables.  $\phi$  is a  $g$  vector of coefficients associated with the time-invariant observable variables. Combining the auxiliary equation with the initial equation (1) leads to the model:

$$y = X\beta + Z(\gamma + \phi) + \bar{X}.\pi + \alpha^M + \varepsilon. \quad (3)$$

When  $E(\alpha_i | Z_i) = \phi_i \neq 0$ ,  $\gamma_i$  and  $\phi_i$  cannot be identified separately, only the sum  $\gamma_i + \phi_i$  can be estimated. When  $\pi_m \neq 0$ , for all  $m \in \{1, \dots, k\}$  and  $\phi_m = 0$ , for all  $m \in \{1, \dots, g\}$ , the **best linear unbiased estimator** is an unrestricted generalized least-squares estimator (unrestricted random effects) estimator, denoted MK-GLS, see Mundlak (1978) and Krishnakumar (2006):

$$\hat{\beta}_{MK-GLS} = \hat{\beta}_W, \hat{\pi}_{MK-GLS} = \hat{\beta}_B - \hat{\beta}_W \text{ and } \hat{\gamma}_{MK-GLS} = \hat{\gamma}_B,$$

with the variances

$$\begin{aligned} var(\hat{\beta}_{MK-GLS}) &= var(\hat{\beta}_W), var(\hat{\pi}_{MK-GLS}) = var(\hat{\beta}_B) + var(\hat{\beta}_W), \\ \text{and } var(\hat{\gamma}_{MK-GLS}) &= var(\hat{\gamma}_B), \end{aligned} \quad (4)$$

where  $\hat{\beta}_W$  and  $\hat{\beta}_B$  are the within and between estimators for time-varying variables. For the time-invariant variables, the estimator  $\hat{\gamma}_{MK-GLS}$  and the estimator of the standard deviations  $\hat{\sigma}_{\gamma_{GLS}}$  are exactly the ones of the between estimator. The MK-GLS estimator deals with the endogeneity of time-varying variables  $E(\alpha | X) \neq 0$ , but the problem of endogeneity of time-invariant variables  $E(\alpha | Z) \neq 0$  remains to be solved. The usual GLS or random effects estimator of (3) is a "restricted" estimator in the sense of Mundlak (1978), since the vector of parameters of the averages over time of the time-varying variables are constrained to be equal to zero:  $\pi = 0$ . It will be referred to as R-GLS and will be used as a benchmark for the Monte-Carlo simulations.

In a different approach Hausman and Taylor (1981) address the problem of endogene-

ity of time-invariant variables. They assume that the econometrician has prior information, namely which columns of  $X$  and  $Z$  are asymptotically uncorrelated with  $\alpha$  and which are correlated. Then  $[X_{it}] = [X_{1it}, X_{2it}]$  and  $[Z_i] = [Z_{1i}, Z_{2i}]$  are split into two sets of variables such that  $X_1$  is  $NT \times k_1$ ,  $X_2$  is  $NT \times k_2$ ,  $Z_1$  is  $NT \times g_1$ ,  $Z_2$  is  $NT \times g_2$  with  $k_1 + k_2 = k$  and  $g_1 + g_2 = g$  where  $k_1$  is the number of exogenous variables and  $k_2$  the number of endogenous variables.  $X_1$  and  $Z_1$  are exogenous and neither correlated with  $\alpha$  nor with  $\varepsilon$ , while  $X_2$  and  $Z_2$  are endogenous due to their correlation with  $\alpha$ , but are not correlated with  $\varepsilon$ .

The pretest estimator proposed in this paper combines the two approaches when no prior information is available for the time-varying variables. It proceeds in two steps, see figure 1, appendix B: In a first step the internal instruments are selected and in the second step the estimation procedure is chosen depending on the instruments found in the first step. To find the internal instruments we start with a MK-GLS regression (equation 3). Then we identify the group of  $k_1$  exogenous variables and the group of  $k_2$  endogenous variables by testing the exogeneity of each of the  $k$  time-varying variables using the  $k$   $t$ -tests of the null hypothesis:  $H_0 : \pi_m = \beta_{B,m} - \beta_{W,m} = 0$  against  $H_1 : \pi_m \neq 0$ , for  $m \in \{1, \dots, k\}$ . These tests are identical to  $k$  Hausman (1978) tests specific to each time-varying variable.

Then, an upward testing procedure does an Hausman test for a joint null hypothesis, for example,  $H_0 : \pi_m = \pi_r = 0$  if both variables indexed by  $m$  and  $r$  variables did not reject the null hypothesis separately  $H_0 : \pi_m = 0$  and  $H_0 : \pi_r = 0$ . The advantage of an upward testing procedure is that it is more restrictive than a direct pooled test of joint hypothesis  $H_0 : \pi_m = \pi_r = 0$  based on an average statistics, which may not reject a joined null hypothesis for a set of variables which include a subset of endogenous variables and a subset of exogenous variables (Andrews (1999), Chatelain (2007)).

The choice of the estimation procedure in the second step depends on the number of exogenous time-varying variables found in the first step and the number of endogenous time-invariant variables:

- If  $g_2 > k_1$ , the number  $k_1$  of internal instruments is not sufficient to identify all  $g_2$  endogenous time-invariant variables. Only  $k_1$   $t$ -tests reject the null hypothesis:  $H_0 : \pi_m = \beta_{B,m} - \beta_{W,m} = 0$ . Then the pretest estimates for endogenous time-invariant variables are the potentially biased between estimates (MK-GLS) initially found in the first step.

- If  $k > k_1 \geq g_2$ , then the number  $k_1$  of internal instruments is sufficient to identify all  $g_2$  endogenous time-invariant variables, and then the pretest estimates for time-invariant variables are the ones obtained running an unrestricted Hausman-Taylor (denoted U-HT  $k_1$ ) estimation using the  $k_1$  averages over time of exogenous time-varying variables as internal instruments and **including** the averages over time of the  $k_2 = k - k_1$  endogenous time-varying variables in the regression. The usual restricted Hausman-Taylor (denoted R-HT  $k_1$ ) may face a bias because of **omitting**  $k_2$  endogenous variables of the matrix  $\bar{X}$ , as compared to the unrestricted Hausman-Taylor estimator of pretest proposed here.

- If  $k = k_1$ , all time-varying variables are exogenous, the Hausman-Taylor approach is not applicable, and a R-GLS estimation should be done.

The properties of this pretest estimator are investigated in a Monte-Carlo simulation, including the R-GLS estimator, the U-HT estimator, and the MK-GLS estimator since the pretest chooses among these three possibilities. But first, in the next section, the theoretical properties of alternative estimators and their ability to draw inference are discussed.

## 2.3 Alternative estimators of time-invariant variables

### 2.3.1 Unrestricted and restricted OLS estimators (U-OLS and R-OLS)

The first approach to estimate equations (1) and (3) is to use a standard ordinary least-squares estimator. As defined before an estimator is restricted if the parameter values of the averages-over-time of the time-varying variables have to be equal to zero and unrestricted otherwise.

The restricted pooled OLS (R-OLS) estimator faces an omitted variables bias,  $X \cdot (\hat{\beta}_B - \hat{\beta}_W)$ , on the estimated parameters when estimating the Mundlak Krishnakumar model (equation 3). For details see e.g. Oaxaca and Geisler [2003] who evaluate the consistency of the R-OLS estimator of a parameter of a time-invariant explanatory variable.

The unrestricted pooled OLS estimator, denoted U-OLS, however, ignores the random effects  $\alpha_i$ : it is the best linear unbiased estimator when  $\alpha_i = 0$ . It leads to the same parameter estimates as the MK-GLS estimator, but not to the same standard error estimates since it uses the irrelevant  $NT - k - g - 1$  degrees of freedom for the time-invariant variables. The  $RMSE$  is larger because it includes the sum of squares of the error of the within model:

$$\begin{aligned}\hat{\sigma}_{\hat{\gamma}}^{U-OLS} &= \frac{\sqrt{\frac{T \cdot SSE_B + SSE_W}{NT - k - g - 1}}}{\sqrt{T \cdot CSS(z_i)}} \frac{1}{\sqrt{1 - R_A^2(Z_j)}} \\ &= \sqrt{1 + \frac{SSE_W}{T \cdot SSE_B}} \sqrt{\frac{N - k - g - 1}{NT - k - g - 1}} \hat{\sigma}_{\hat{\gamma}}^B,\end{aligned}$$

where subscripts  $W$  and  $B$  refer again to the within and between estimations.  $R_A^2(Z_j)$  is the coefficient of determination of the U-OLS auxiliary regression of the variable  $Z_j$  on all the other regressors: it is the same for the between and U-OLS estimators.

Both estimators are included in the Monte Carlo simulation.

### 2.3.2 Restricted random effects (R-GLS) and restricted Hausman Taylor (R-HT) estimators

The restricted random effect estimator (R-GLS) is one of the most used estimators by applied econometricians when their variables of interest are time-invariant variables in panel data. It uses the same weight  $\hat{\theta}$  for computing quasi demeaned variables as the MK-GLS, when it is based on the Swamy and Arora method (1972):

$$y_{it} - \bar{y}_i + \hat{\theta} \bar{y}_i, \text{ with } \hat{\theta} = \frac{RMSE_W}{\sqrt{T} \cdot RMSE_B}.$$

The between regression includes the averages-over-time of all explanatory variables. Hence, the root mean squared error of the between estimator  $RMSE_B$  is the same in both cases. R-GLS, however, faces an omitted variable bias when the null hypothesis  $\hat{\beta}_B = \hat{\beta}_W$  is rejected for at least one time-varying variable. In this case, the bias of the random effect estimator  $\hat{\gamma}_{R-GLS}$  as compared to the Mundlak-Krishnakumar  $\hat{\gamma}_B$  estimator (which

is equal to the between estimator) is:

$$\hat{\gamma}_{R-GLS} = \hat{\gamma}_B + \sum_{j=1}^{j=k} \left( \hat{\beta}_B - \hat{\beta}_W \right) \hat{\beta}_{\hat{\theta}\bar{x}_j, \hat{\theta}z_j}$$

with  $\beta_{x_{i.}/z_i}$  given by the auxiliary regression using OLS on quasi demeaned variables:

$$\hat{\theta}\bar{x}_{i.} = \beta_{\bar{x}_{i.}/z_i} \hat{\theta}z_i + \beta_{\bar{x}_{i.}/x_{it}} \left( \hat{\theta}\bar{x}_{i.} + x_{it} - \bar{x}_{i.} \right) + \hat{\theta}\beta_0 + \varepsilon_{it}.$$

The restricted Hausman and Taylor estimator (R-HT) uses different weights  $\hat{\theta}_{R-HT}$  in the term  $y_{it} - \bar{y}_{i.} + \hat{\theta}_{R-HT}\bar{y}_{i.}$  because of the choice of internal instrumental variables. A bias similar to the bias of the R-GLS due to the omission of the average-over-time of endogenous time-varying variables may offset the correction of the endogeneity bias of the time-invariant variables using internal instrumental variables.

Both estimators are included in the Monte Carlo simulation.

### 2.3.3 $T$ times repeated-between estimator (T-BE)

Another widely used estimator is the  $T$ -times-between estimator which is not a good choice. We present it because repeating  $T$  times the time-invariant observations turns out to be the major component of the increase of the  $t$ -statistic of time-invariant variables using panel data for several other estimators. Kelejian and Stephan (1983), however, argue that *“the effect of a random component can only be averaged out if the sample increases in the direction of that random (time or individual) component”*. By contrast, Oaxaca and Geisler (2003) assume that the consistency of the estimator of a parameter of a time-invariant explanatory variable *“depends on the time series observations approaching infinity”*. The estimated parameters of time-invariant variables are the same for the repeated-between and the between estimator ( $\hat{\gamma}^B = \hat{\gamma}^{T-BE}$ ), (the subscript for this estimator is  $T-BE$ ). The estimator of the standard error is equal to:

$$\hat{\sigma}_{\hat{\gamma}}^{T-BE} = \frac{\sqrt{\frac{T \cdot SSE_B}{NT-k-g-1}}}{\sqrt{T \cdot CSS(z_j)}} \frac{1}{\sqrt{1 - R_A^2(Z_j)}} = \sqrt{\frac{N - k - g - 1}{NT - k - g - 1}} \hat{\sigma}_{\hat{\gamma}}^B.$$

Inference uses  $NT - k - g - 1$  degrees of freedom instead of  $N - k - g - 1$ . The coefficient of determination of the auxiliary between regression  $R_A^2(Z_j)$  of the variable  $Z_j$  on all the other regressors does not change in the between or repeated-between samples. As the parameter estimator is the same as  $\hat{\gamma}^B$ , the repeated-between  $t^{T-BE}$ -statistic amounts to multiply the between  $t^B$  statistics by the following factor:

$$t^{T-BE} = \frac{\hat{\gamma}^B}{\hat{\sigma}_{\hat{\gamma}}^{T-BE}} = \sqrt{\frac{NT - k - g - 1}{N - k - g - 1}} \frac{\hat{\gamma}^B}{\hat{\sigma}_{\hat{\gamma}}^B} = \sqrt{\frac{NT - k - g - 1}{N - k - g - 1}} t^B$$

When  $N$  is large, the  $t$ -statistic of the repeated-between model is multiplied by around  $\sqrt{T}$  as compared to the between model. Due to these shortcomings the estimator is not included in the Monte-Carlo simulations.



### 2.3.4 Two-stage restricted between (R-BE) and Oaxaca Geisler (2003) estimator

A common practice consists of using a **two-stage restricted between** (denoted R-BE) estimator of time-invariant variables, with the restriction  $\hat{\beta} = \hat{\beta}_W$  for the average-over-time of time-varying variables in a between regression. Let

$$\hat{d} = \bar{y} - \bar{X} \cdot \hat{\beta}_W = \left( B - \bar{X} \cdot (X'WX)^{-1} X'W \right) y$$

be the  $N$  vector of group means estimated from the first stage **within-groups residuals**, with  $B$  and  $W$  the projection matrices on the between subspace of observations and on the within subspace of observations, respectively. Expanding this expression leads to:

$$\hat{d} = Z_1 \gamma_1 + Z_2 (\gamma_2 + \phi_2) + \alpha^M + \left( B - \bar{X} \cdot (X'WX)^{-1} X'W \right) \varepsilon + \bar{X} \cdot \left( \hat{\beta}_B - \hat{\beta}_W \right). \quad (5)$$

Treating the last two terms as an unobservable mean zero disturbance, a two-stage restricted-between estimator considers estimating  $\gamma$  from the above equation using  $N$  observations, without taking into account that the disturbances include  $\bar{X} \cdot \left( \hat{\beta}_B - \hat{\beta}_W \right)$ . But, there is an omitted variables bias,  $X \cdot \left( \hat{\beta}_B - \hat{\beta}_W \right)$ , on the parameters of the time-invariant variable  $Z_i$ :

$$\hat{\gamma}_{R-BE} = \hat{\gamma}_B + \sum_{j=1}^{j=k} \left( \hat{\beta}_B - \hat{\beta}_W \right) \hat{\beta}_{X_{j./Z_i}}$$

with  $\beta_{\bar{X}./z}$  estimated using the following auxiliary regressions in the between dimension:

$$\bar{X} \cdot = \beta_{X./Z} Z + \beta_{0.X./Z} + \bar{\varepsilon} \cdot \bar{X} \cdot / Z.$$

Because of the omitted variable bias, the estimated standard error of the parameter of the time-invariant variables is necessarily smaller than the estimated standard error of the between estimator:

$$\begin{aligned} \hat{\sigma}_{\hat{\gamma}_{R-BE}} &= \sqrt{\frac{SSE_{R-BE}}{N - g - 1} \frac{1}{\sqrt{CSS(Z_i)} \sqrt{1 - R_{A,R-BE}^2(Z_j)}}} \neq \\ \hat{\sigma}_{\hat{\gamma}_B} &= \sqrt{\frac{SSE_B}{N - k - g - 1} \frac{1}{\sqrt{CSS(Z_i)} \sqrt{1 - R_A^2(Z_j)}}} \end{aligned}$$

This stems from the fact that the sum of squares of errors  $SSE_{R-BE}$  is larger than the  $SSE_B$  because the model constrains the parameters of the time-varying variables to their within estimate which may not minimize the between sum of squares of errors. On the other hand, the increase of the estimated standard error may be offset for two reasons due to the fact the averages of  $k$  time-varying variables are on the left hand side of the equation. First, the degrees of freedom increase by  $k$ . This decreases the root mean squared error. Second, the variance inflation factor decreases because  $R_{A,R-BE}^2(Z_j) < R_A^2(Z_j)$ .  $R_A^2(Z_j)$  is the coefficient of determination of an auxiliary regression where the time-invariant variable is correlated with the other  $g - 1$  time-invariant explanatory

variables on the right hand side of the equation. In the between estimator,  $R_A^2(Z_j)$  is the coefficient of determination of an auxiliary regression where the time-invariant variable is correlated with the other  $g - 1$  time-invariant explanatory variables and  $k$  averages of time-varying explanatory variables. As the number of explanatory variables increases by  $k$ , one has  $R_{A,R-BE}^2(Z_j) < R_A^2(Z_j)$ .

Oaxaca and Geisler's (2003) two-stage estimator deals with the omitted variable bias of the two-stage restricted between. The second stage estimates the equation (5) with a generalized least-squares estimator using a covariance matrix that takes into account that the disturbances of the restricted between include the omitted term:  $\bar{X} \cdot (\hat{\beta}_B - \hat{\beta}_W)$ . Nevertheless, the estimator exhibits two problems. The first drawback is that the second stage should not be estimated using observations repeated  $T$  times. The second drawback is that there is a straightforward way to deal with the above omitted bias, namely to include the omitted variables  $\bar{X}$  directly in a single stage MK-GLS estimator. For these reasons only the two-stage restricted between estimator, but not the Oaxaca-Geisler estimator is included in the Monte Carlo simulation.

### 2.3.5 Three-stage FEVD restricted estimator

As demonstrated by Greene (2011), the FEVD estimator (Plümper and Troeger (2007)) of time-invariant variables is not valid. Nonetheless, we present this estimator because it turns out to be the extreme case, where the root mean square error of the within regression is used for doing inference on time invariant variables in panel data instead of the root mean square error of the between regression as in the Mundlak-Krishnakumar estimator. Assuming exogeneity ( $E(\alpha | Z) = \phi_2 = \mathbf{0}$  and  $E(\alpha | X) = \pi = \mathbf{0}$ ), a restricted FEVD estimator calculated the time-invariant residuals of stage-two restricted-between, denoted  $\hat{\eta}_{2R-BE}$  :

$$\begin{aligned}\hat{\eta}_{2R-BE} &= \hat{d} - Z\hat{\gamma}_{R-BE} = \bar{y} - \bar{X} \cdot \hat{\beta}_W - Z\hat{\gamma}_{R-BE} \\ &= \alpha^M + \left( B - \bar{X} \cdot (X'WX)^{-1} X'W \right) \varepsilon\end{aligned}$$

The third stage of a restricted FEVD estimator is an OLS regression which includes the time-invariant residuals of the second stage,  $\hat{\eta}_{2R-BE}$ , as an additional regressor with a parameter  $\delta$ :

$$y = X\beta + Z\gamma + \hat{\eta}_{2R-BE} \cdot \delta + \eta_{FEDV}$$

where the third stage disturbances are denoted  $\eta_{FEDV}$ . This is equivalent to running OLS for the following equation including only demeaned and time-invariant variables:

$$\begin{aligned}y - \bar{y} = (X - \bar{X})\beta - \bar{X} \cdot (\beta - \hat{\beta}_W) + Z(\gamma - \hat{\gamma}_{R-BE}) + \\ \hat{\eta}_{2R-BE}(\delta - 1) + \eta_{FEVD}.\end{aligned}$$

Because of the orthogonality of demeaned (within transformed) variables with time-invariant variables ( $cov(y - \bar{y}, z_i) = cov(y - \bar{y}, \bar{x}_i) = cov(y - \bar{y}, \hat{\eta}_{2R-BE}) = 0$ ) and by definition of the within estimator (OLS on demeaned variables), the OLS estimates of third stage FEVD equation are  $\hat{\delta}_{FEVD} = 1$ ,  $\hat{\beta}_{FEVD} = \hat{\beta}_W$ ,  $\hat{\gamma}_{FEVD} = \hat{\gamma}_{RBE}$ . Hence, the restricted third stage FEVD estimator has the same parameter estimates as the two-stage

restricted-between estimator. The only difference is that the third stage residuals  $\hat{\eta}_{3,FEVD}$  are the within-regression residuals:

$$\hat{\eta}_{FEVD} = \hat{\varepsilon}_{it} - \hat{\varepsilon}_i = y - \bar{y}_i - (X - \bar{X}_i) \hat{\beta}_W.$$

Hence, with respect to the two-stage restricted-between estimator, the only change of the third stage restricted FEVD estimator is the estimated standard error of the parameters  $\gamma$  of time-invariant variables  $Z$  which no longer depends on the **mean squared error** of the second-stage time-invariant residuals of the restricted-between estimator  $\hat{\eta}_{2R-BE}$  (observed  $N$  times, with  $MSE_{RBE} = SSE_{RBE} / (N - k - g - 1)$ ), but depends now on the mean square error of the within residuals  $\hat{\eta}_{FEVD}$ , observed  $NT$  times, with  $MSE_W = SSE_W / (NT - N - k)$ .

The estimated parameters  $\hat{\gamma}_{R-BE}$  are related to the projection in the between subspace of observations, but their estimated standard errors are related to residuals in the within subspace of observations, which is orthogonal to the between subspace. This contradicts the Frisch and Waugh (1933) and Lovell (1963) theorem: the same orthogonal projection matrix has to be used in the parameter estimator and in the estimator of the standard error of the estimated parameter.

As observed by Breusch et al. (2011) in Monte Carlo simulations, the FEVD estimated standard error of a time-invariant variable appears "abnormally" small. The restricted FEVD estimator of the standard error of a parameter of a time invariant variable is related to one the best (lowest variance) linear unbiased estimator (BLUE) of the unrestricted Mundlak Krishnakumar model (U-OLS) as follows:

$$\hat{\sigma}_{\hat{\gamma}_{FEVD}} = \frac{\sqrt{\frac{SSE_W}{NT - N - k}}}{\sqrt{T \cdot CSS(z_i)}} \frac{1}{\sqrt{1 - R_A^2(z_j)}} = \sqrt{\frac{N - k - g - 1}{NT - N - k}} \sqrt{\frac{SSE_W}{T \cdot SSE_B}} \hat{\sigma}_{\hat{\gamma}}^{BE}.$$

Therefore,  $\hat{\sigma}_{\hat{\gamma}_{FEVD}}$  is biased downward for two reasons:

- It uses  $NT - N - k$  degrees of freedom (with  $N$  the number of individuals,  $k$  the number of time-varying explanatory variables, with  $T$  the number of periods) instead of  $N - k - g - 1$  degrees of freedom.
- It multiplies the repeated-between estimator of the standard error by a positive factor  $\sqrt{\frac{SSE_W}{T \cdot SSE_B}}$  which can be much smaller than one when  $T$  increases.

For these reasons the FEDV estimator has not been included in the Monte Carlo simulation. Its large number of citations in google scholar database (around 900 citations during 11 years) is correlated with the researchers' demand for statistically significant parameters for time-invariant variable in panel data in order to be published in academic journals (Wasserstein and Lazar (2016)). In order to reach a large number of citations, theoretical econometricians may be tempted to propose new computations of *downward-biased* standard errors of estimated parameters to applied econometricians. By contrast, stating that the correct degrees of freedom that should be used is  $N - k - g - 1 \ll NT - N - k$  for inference of the effect of time-invariant variables is on the opposite side of this demand.

Using  $NT - N - k$  degrees of freedom instead  $N - k - g - 1$  is a "magical solution" to the following issue. If the full population is small (e.g. the cross-section of  $N = 35$  OECD countries), so that the sample size  $N$  is small with a p-value larger than 0.05 and nonetheless the cross-section correlation of the dependent variable is large with a time-invariant variable, then the p-value below 5% may not be a useful criterion with

respect to a loss function of decision makers (Wasserstein and Lazar (2016), Chatelain (2010) section 2).

### 3 A Monte Carlo simulation: The model

#### 3.1 Design

To analyze the finite sample properties of the pretest estimator and compare it to the other methods mentioned in the previous sections, our Monte Carlo simulations extends to an Hausman-Taylor world Guggenberger (2010) design. The reason why we selected Guggenberger (2010) complete design of the correlation matrix between regressors is that Plümper and Troeger (2007) FEVD Monte-Carlo simulations presented some surprising results that could only be theoretically explained if some other correlations between variables were changing at the same time they were changing one parameter in their simulations.

His design controls the correlation matrix of all explanatory variables, including the individual effect, while assuming multi-normality. Additional simulations assuming specific cases of non-normality are available upon request from the authors. The simulated model includes three time-varying exogenous variables ( $X_{1it}, X_{2it}, X_{3it}$ ), two time-invariant variables ( $Z_{1,i}$  and  $Z_{2,i}$ ) and an individual effect  $\alpha_i$  explaining an endogenous time-varying variable  $y_{it}$ :

$$y_{it} = \beta_1 X_{1it} + \beta_2 X_{2it} + \beta_3 X_{3it} + \gamma_1 Z_{1,i} + \gamma_2 Z_{2,i} + \alpha_i + \varepsilon_{it}, \quad t = 1 \dots T, \quad i = 1 \dots N. \quad (6)$$

The disturbance  $\varepsilon_{it}$  is independently normally distributed with mean 0 and standard deviation  $\sigma_\varepsilon > 0$  uncorrelated with the other endogenous variables. The exogenous time invariant variable  $Z_{1,i}$  is a constant,  $Z_{1,i} = 1$ ; it is also individual invariant.

Following Guggenberger (2010), firstly, the time-varying explanatory variables, indexed by  $m = 1, 2, 3$  and denoted  $X_{mit}, t = 1, \dots, T$ , and  $T = 5$ , follow an auto-regressive process of order one, AR(1). Then, in the multi-normal setting, only the correlation matrix has to be defined:

$$\mathbf{R}(X_{m,it}) = \begin{pmatrix} 1 & r & r^2 & r^3 & r^4 \\ r & 1 & r & r^2 & r^3 \\ r^2 & r & 1 & r & r^2 \\ r^3 & r^2 & r & 1 & r \\ r^4 & r^3 & r^2 & r & 1 \end{pmatrix}$$

with  $r$  being the coefficient of autocorrelation between adjacent periods for a time varying variable  $X_{mit}$ . It is the same for the three time varying variables.

Secondly, the correlation matrix of all variables corresponds to a Hausman-Taylor world. By construction, the correlations of the regressors  $X_{1it}, X_{2it}, X_{3it}, Z_{2i}$  with the constant  $Z_{1i}$  are equal to zero. The full correlation matrix  $\mathbf{R}$  without the constant variable  $Z_{1i}$  is a  $17 \times 17$  matrix (block matrices are in bold and the matrix is symmetric so that the lower triangular matrix is not reported). The constraint  $\det(\mathbf{R}) \geq 0$  restricts the range of values of simple correlation coefficients.

$$\mathbf{R} = \begin{pmatrix} \mathbf{R}(X_1, r) & \mathbf{0} & \mathbf{0} & \rho_{X_1 Z_2} & \rho_{X_1 \alpha} \\ & \mathbf{R}(X_2, r) & \mathbf{0} & \rho_{X_2 Z_2} & \rho_{X_2 \alpha} \\ & & \mathbf{R}(X_3, r) & \rho_{X_3 Z_2} & \rho_{X_3 \alpha} \\ & & & 1 & \rho_{Z_2 \alpha} \\ & & & & 1 \end{pmatrix}$$

Thirdly, the time- and individual-varying variables  $X_{mit}$ , the time-invariant variable  $Z_{2i}$  and the individual random effects  $\alpha$  are drawn from a standardized *multi-normal* distribution with mean zero and standard deviations for all variables  $\sigma_{X_{m,it}} = \sigma_{Z_{2ii}} = 1$  with the exception of the individual random effects  $\alpha_i$ , for which we assume a variance  $\sigma_\alpha^2$ . In the simulations, we vary the share of the individual effect disturbances  $\sigma_\alpha^2 / (\sigma_\alpha^2 + \sigma_\varepsilon^2)$ . The overall variance of the disturbances is fixed to 3 ( $\sigma_\alpha^2 + \sigma_\varepsilon^2 = 3$ ), as in Im, Ahn, Schmidt and Wooldridge (1999) and Baltagi, Bresson and Pirotte (2003).

Finally, the parameter values of the regression equation are chosen as  $\beta_1 = \beta_2 = \beta_3 = \gamma_1 = \gamma_2 = 1$ . Setting these five parameters is equivalent to setting the simple correlations between the dependent variable and the explanatory variables, once the correlation matrix and the variances of explanatory variables are given.

Guggenberger (2010) multi-normal design has four advantages:

Firstly, it controls all the correlations between the four explanatory variables and the random individual term for all periods.

Secondly, it is checked that the correlation matrix has a strictly positive determinant ( $\det(\mathbf{R}) \geq 0$ ), so that the matrix is positive definite for each simulations.

Thirdly, in the benchmark simulation, the time-invariant variables are *not* drawn from a uniform distribution instead of a normal distribution (the distributions of regressors do not have excess kurtosis in the between space). We nevertheless controlled for non-normality of the individual effect and the random disturbance term in an additional simulation.

Fourthly, and most importantly, this design allows to *separate* the distinct effects of two deviations from the classical model of uncorrelated variables on the size of the Hausman pretest as detailed in Guggenberger (2010). A first deviation is the correlation  $\rho_{Z_2\alpha}$  between the random individual effect and the endogenous time-invariant variable. A second deviation is the relative variance of the individual random term with respect to the variance of the time-varying disturbances  $\sigma_\alpha^2 / (\sigma_\alpha^2 + \sigma_\varepsilon^2)$ . In the design of Im *et al.* (1999) and of Plümper and Troeger (2007), it is not possible to change  $\sigma_\alpha^2 / (\sigma_\alpha^2 + \sigma_\varepsilon^2)$  without changing  $\rho_{Z_2\alpha}$  and  $\rho_{X_3\alpha}$  at the same time. Furthermore, in the multi-normal design, the simulations allows to check the theoretical result that the OLS endogeneity bias is a linear function of  $\sigma_\alpha$  when  $\rho_{Z_2\alpha}$  is fixed.

### 3.2 Parameter values and estimation

In the benchmark simulation, the variance of random effects is  $\sigma_\alpha^2 = 1.5$  so that it represents 50% of the overall variance:  $\sigma_\alpha^2 / (\sigma_\alpha^2 + \sigma_\varepsilon^2) = 1.5/3 = 1/2$ . The correlation coefficients are  $\rho_{X_1Z_2} = \rho_{X_2Z_2} = \rho_{X_3Z_2} = 0.4$ ,  $\rho_{X_1,\alpha} = 0$ ,  $\rho_{X_2,\alpha} = 0$ ,  $\rho_{X_3\alpha} = 0.75$ ,  $\rho_{Z_2\alpha} = 0.52$ . They are close to some of the parameters chosen by Baltagi, Bresson and Pirotte (2003).

We have  $k_1 = 2$  exogenous time-varying variables,  $k_2 = 1$  endogenous time-varying variable,  $g_1 = 1$  exogenous time-invariant variable, and  $g_2 = 1$  endogenous time-invariant variable. In the first step of the pretest procedure, the following Mundlak-Krishnakumar model is estimated using GLS:

$$y_{it} = \beta_1 X_{1it} + \beta_2 X_{2it} + \beta_3 X_{3it} + \pi_1 \bar{X}_{1i} + \pi_2 \bar{X}_{2i} + \pi_3 \bar{X}_{3i} + \gamma_1 Z_{1i} + \gamma_2 Z_{2i} + \alpha_i + \varepsilon_{it}. \quad (7)$$

According to the estimated values of  $\pi_1$ ,  $\pi_2$ , and  $\pi_3$ , the second step chooses one of

the following 8 estimators:

(1) MK-GLS: the outcome of individual tests is that  $\pi_1$ ,  $\pi_2$ , and  $\pi_3$  are statistically different from zero. Then the pretest stops at the first step with the MK-GLS estimator.

(2) U-HT1:  $\pi_1$  is not different from zero and  $\pi_2$ , and  $\pi_3$  are different from zero. Then we use an unrestricted Hausman-Taylor (U-HT1) estimator with  $X_1$  as the exogenous variable and  $X_2$  and  $X_3$  as endogenous variables.

(3) U-HT2:  $\pi_2$  is not different from zero and  $\pi_1$ , and  $\pi_3$  are different from zero. Then we use the U-HT2 estimator, with  $X_2$  as the exogenous variable and  $X_1$  and  $X_3$  as endogenous variables.

(4) U-HT3:  $\pi_3$  is not different from zero and  $\pi_1$ , and  $\pi_2$  are different from zero. Then we use the U-HT3 estimator, with  $X_3$  as the exogenous variable and  $X_1$  and  $X_2$  as endogenous variables.

(5) U-HT12:  $\pi_1$  and  $\pi_2$  are not different from zero and  $\pi_3$  is different from zero. Then we use the U-HT12 estimator with both  $X_1$  and  $X_2$  as exogenous variables and  $X_3$  as the endogenous variable.<sup>1</sup>

(6) U-HT13:  $\pi_1$  and  $\pi_3$  are not different from zero and  $\pi_2$  is different from zero. Then we use the U-HT13 estimator with both  $X_1$  and  $X_3$  as exogenous variables and  $X_2$  as the endogenous variable.

(7) U-HT23:  $\pi_2$  and  $\pi_3$  are not different from zero and  $\pi_1$  is different from zero. Then we use the U-HT23 estimator with both  $X_2$  and  $X_3$  as exogenous variables and  $X_1$  as the endogenous variable.

(8) R-GLS:  $\pi_1$ ,  $\pi_2$ , and  $\pi_3$  are all not different from zero. Then we simply run a R-GLS estimation.

Given the above parameterization of our model in a Hausman-Taylor world, the pretest procedure should choose the U-HT12 estimator and use both  $X_1$  and  $X_2$  as exogenous internal instruments. Alternative U-HT1 and U-HT2 estimators are not that bad since at least one of the variables  $X_1$  and  $X_2$  is chosen as an exogenous variables. The BBP (2003) pretest considers three possible options instead of the 8 cases dealt with the above pretest (1, 5 and 8): (1) fixed effects (FE) for  $\beta$  but no estimator for  $\gamma$  (this paper pretest reports the Between estimator using MK-GLS), (5) with restricted Hausman Taylor (R-HT12) (this paper pretest reports the unrestricted Hausman Taylor (U-HT12)) and (8) R-GLS (same choice for BBP (2003) pretest and for this paper pretest). The simulations report the results of four *restricted* estimators (R-OLS, R-GLS, R-BE, R-HT12), of four *unrestricted* estimators (U-OLS, MK-GLS, BE, U-HT12) and finally of the pretest estimator. Following the advice of Greene (2011) and Breusch et al. (2011), the FEVD estimator is not reported.

The number of individuals  $N$  was chosen to be equal to 100, the number of periods  $T$  equal to 5, and the experiment was repeated 1000 times. For each estimator, we report the bias of the parameter  $\beta_3$  and  $\gamma_2$ , the corresponding simulated root mean square error, and the 5% size, which is the frequency of rejections in 1000 replications of the null hypotheses  $\beta_3 = 1$  and  $\gamma_2 = 1$ , respectively at the 5% significance level. The Monte-Carlo simulation then aims at answering two questions: How does the pretest perform as compared to other methods, i.e. how biased are the results? And does the pretest find the correct alternative, i.e. does it choose both,  $X_1$  and  $X_2$  as endogenous internal

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<sup>1</sup>Additionally, a pooled Hausman test for the contrast of within versus between parameters of  $(X_{1it}, X_{2it})$  could be carried out. If this pooled Hausman test rejects the null joint hypothesis, we revert to the U-HT1 or U-HT2 taking as internal instrument the variable among  $(X_{1it}, X_{2it})$  which has the *lowest p-value* of the test  $\pi_1 = 0$  and  $\pi_2 = 0$  (both p-values being below the 5% threshold).

instruments?

### 3.3 Theoretical results for the bias

Before the simulation results are presented, some theoretical issues are discussed. For the OLS estimator the bias can be calculated as follows:

$$\begin{aligned}\lim \hat{\beta}_{OLS} &= \lim \left( \frac{X'X}{NT} \right)^{-1} \left( \frac{X'Y}{NT} \right) = \lim \left( \frac{X'X}{NT} \right)^{-1} \left( \frac{X'[X\beta + \alpha]}{NT} \right) \\ &= \beta + \lim \left( \frac{X'X}{NT} \right)^{-1} \lim \left( \frac{X'\alpha}{NT} \right)\end{aligned}$$

that is, in our case with  $\beta_i = 1$  and  $\gamma_2 = 1$ :

$$\begin{pmatrix} \hat{\beta}_1 - 1 \\ \hat{\beta}_2 - 1 \\ \hat{\beta}_3 - 1 \\ \hat{\gamma}_2 - 1 \end{pmatrix} = \frac{\sigma_\alpha}{1 - \rho_{X_1Z}^2 - \rho_{X_2Z}^2 - \rho_{X_3Z}^2} \begin{pmatrix} \rho_{X_1Z} (\rho_{ZX_3} \rho_{X_3\alpha} - \rho_{Z\alpha}) \\ \rho_{X_2Z} (\rho_{ZX_3} \rho_{X_3\alpha} - \rho_{Z\alpha}) \\ (1 - \rho_{X_1Z}^2 - \rho_{X_2Z}^2) \rho_{X_3\alpha} - \rho_{X_3Z} \rho_{Z\alpha} \\ \rho_{Z\alpha} - \rho_{ZX_3} \rho_{X_3\alpha} \end{pmatrix}$$

The biases of the OLS estimator due to endogeneity increase linearly with  $\sigma_\alpha$  for all parameters. The bias for  $\gamma$  increases with  $\rho_{Z\alpha}$  and decreases with  $\rho_{X_3\alpha}$  when  $\rho_{ZX_3} > 0$ . The bias for  $\beta_m$  increases with  $\rho_{X_3\alpha}$  and decreases with  $\rho_{Z\alpha}$  when  $\rho_{ZX_m} > 0$ . In what follows, we vary only one parameter with respect to the benchmark case, keeping the other parameters unchanged. We focus on the coefficients of the endogenous regressors  $X_3$  and  $Z$ , i.e.  $\beta_3$  and  $\gamma_2$ , respectively. Results on the other coefficients are available upon request from the authors.

## 4 Results of the simulation

### 4.1 Changing the variance $\sigma_\alpha^2$ of the individual effect

In a first set of simulations we changed the variance of the individual effect keeping the overall variance constant. The values of  $\sigma_\alpha$  were chosen as in BBP (2003, p.368, table 4), namely  $\sigma_\alpha^2$  varying from 0 to 2.75 which corresponds to a variation of  $\sigma_\alpha^2/(\sigma_\alpha^2 + \sigma_\epsilon^2)$  from 0 to 11/12. Table 1a (all results are reported in appendix C) reports the choice of the pretest estimator for these different values of  $\sigma_\alpha$ . With  $\sigma_\alpha^2 = 0$  the pretest estimator chooses the R-GLS (RE) estimator in 87.4% of all replications and the U-HT12 estimator (unrestricted Hausman Taylor estimator with  $X_1$  and  $X_2$  as internal instruments, the correct choice given the parametrization of the model) in 3.4% of all replications. With  $0.5 \leq \sigma_\alpha^2 \leq 2.75$ , the pretest procedure chooses the U-HT12 estimator in at least 54.4% and at most 86.8% replications.

Table 1b reports the bias, the RMSE, and the 5% size for various estimators of  $\beta_3$  and  $\gamma_2$ . For  $\sigma_\alpha^2$  between 0 and 1, the restricted OLS estimator has the lowest RMSE for  $\beta_3$ . Increasing  $\sigma_\alpha$  reduces the RMSE for the restricted and unrestricted Hausman Taylor estimator, the within estimator (used at the first step of the two-step restricted-between estimator), and the pretest for  $\beta_3$ . For  $\gamma_2$ , R-OLS and R-GLS have the lowest RMSE (0.112), but their value is questionable as they use  $NT$  observations in their degrees for

freedom. By contrast, between (U-OLS) and restricted and unrestricted Hausman and Taylor estimators have the largest RMSE (0.178), but they use  $N$  instead of  $NT$  in their degrees of freedom. When  $\sigma_\alpha$  increases, the restricted Hausman Taylor RMSE for  $\gamma_2$  *increases*, whereas the between RMSE *decreases*, as well as the unrestricted Hausman Taylor (U-HT12), because more variance of the dependent variable is explained when including the average-over-time of endogenous time-varying variables.

The R-OLS, between and R-between biases for  $\hat{\beta}_3$  and  $\hat{\gamma}_2$  are linear increasing functions of  $\sigma_\alpha$ , see figure 1a and figure 1b. The R-GLS (the usual "random effect" model) bias for  $\hat{\beta}_3$  increases for value of  $\sigma_\alpha \leq 2$  and then decreases as in Baltagi, Bresson and Pirotte (2003, p.368). The between estimator has the largest bias for  $\hat{\beta}_3$  and the smallest bias for  $\hat{\gamma}_2$  of those 3 estimators. The endogeneity bias for  $\hat{\beta}_3$  is fully corrected when including  $X_{3i}$  in the three unrestricted estimators (U-OLS, MK-GLS, U-HT12). The unrestricted OLS and GLS provide the between estimate for the parameter  $\hat{\gamma}_2$ . The restricted between is related to a large omitted variable bias for  $\hat{\gamma}_2$ . The biases for  $\hat{\beta}_3$  and for  $\hat{\gamma}_2$  using the restricted Hausman Taylor are negligible. However, they are between 4 to 10 times larger than the one obtained when using the three unrestricted estimators: OLS, GLS and Hausman Taylor. The bias for  $\gamma_2$  of the pretest estimator increases first because it selects the R-GLS (RE) estimator for  $\sigma_\alpha = 0.5$  in 14.5% of replications, then it falls and increases again when  $\sigma_\alpha > 1.41$ , because it increasingly selects the MK-GLS estimator (between estimator for  $\gamma_2$ ), up to 16.8% of replications with  $\sigma_\alpha = 1.66$ .

The 5% size columns in tables 1b report the frequency of rejections in 1000 replications of  $\beta_3 = 1$  and  $\gamma_2 = 1$ , respectively at the 5% significance level. Since the null hypothesis is always true, this represents the empirical size of the test. As expected, R-OLS and R-GLS (RE) estimators perform badly; they reject the (true) null hypothesis frequently, especially when  $\sigma_\alpha$  is large. On the other hand, R-HT12 and U-HT12 perform well for  $\beta_3$  and  $\gamma_2$ , giving the required 5% size, while MK-GLS (FE for  $\beta_3$  and BE for  $\gamma_2$ ) do well for  $\beta_3$  and not so well for  $\gamma_2$ , with 5% size increasing steadily from around 6% for  $\sigma_\alpha = 0.5$  to 59% for  $\sigma_\alpha = 1.66$ . The MK-GLS (BE) for  $\gamma_2$  5% sizes are nonetheless smaller than R-OLS, R-GLS and R-BE with 5% size at 100% when  $\sigma_\alpha > 1$ . The pretest exceeds the 5% size (reaching 11.2%) for small values of  $\sigma_\alpha = 0.5$  (when it selects the R-GLS (RE) in 14.5% of replications) and then for large values of  $\sigma_\alpha > 1.41$  (reaching 8.5% to 19.8%) because it selects the MK-GLS (BE for  $\gamma_2$ ) in 6.2% to 16.8% of replications.

## 4.2 Changing the degree of endogeneity of the time invariant variable $\rho_{Z_2\alpha}$

The correlation between the time-invariant variable and the individual error term indicating the presence of endogeneity has to be contained in the interval  $0.426 < \rho_{Z_2\alpha} < 0.597$  so that  $\det(\mathbf{R}) \geq 0$ . Table 2a shows that in our simulations the pretest never selected the wrong internal instrument  $X_3$ . The pretest chooses the correct alternative U-HT12 in 88.6% of all cases for  $\rho_{Z_2\alpha} = 0.5$ . For large correlations or small correlations also the Mundlak/Krishnakumar estimator is chosen.

The biases, RMSE and 5% size are presented in table 2b and the biases are shown in figures 2a and 2b. The restricted OLS estimator, the between estimator and the restricted GLS estimator for  $\beta_3$  are biased and the bias is decreasing when the correlation increases. The other estimators for  $\beta_3$  are performing well with a RMSE and 5% size approximately the same. The bias for  $\gamma_2$  increases linearly for R-OLS, R-between, between and non-linearly for R-GLS and the pretest. Restricted and unrestricted Hausman Taylor



estimators perform best, even though the unrestricted estimator is slightly better. The less good performance of the pretest comes from the fact that MK-GLS is sometimes chosen. The 5% size of the pretest remains below 19%.

### 4.3 Shifting from weak to strong internal instruments

Results when increasing **simultaneously**  $\rho_{X_1Z_2}$  and  $\rho_{X_2Z_2}$  ( $-0.461 < \rho_{X_1Z_2} = \rho_{X_2Z_2} < 0.461$ , so that  $\det(R) \geq 0$ ) are presented in tables 3a and 3b. Since the problem is symmetric only the results for positive values,  $0 < \rho_{X_1Z_2} = \rho_{X_2Z_2} < 0.461$ , are stated.

The percentage of times the pretest selects the different alternatives when moving from weak to strong internal instruments is shown in table 3a. With the given parametrization the pretest never chooses the wrong instrument. With increasing correlation, however, the pretest chooses also the MK-GLS estimator on takes only one of the averages of the time- and individual varying variables as instrument. This is due to the fact that the overall correlation matrix is nearly non-invertible because of near multi-collinearity. This will be also reflected in the biases.

Biases of the estimators, their RMSE and 5% size are shown in table 3b and figures 3a and 3b. When the internal instruments are weak ( $\rho_{X_1Z_2} = \rho_{X_2Z_2} = 0$ ), the restricted Hausman Taylor bias is identical to the restricted between bias for  $\hat{\gamma}$  (0.62) which is an intermediate step in the restricted Hausman Taylor estimator. By contrast, the unrestricted Hausman bias for  $\hat{\gamma}$  (0.12) is closer to the bias (0.05) of the between estimator. As a consequence, the pretest estimator has a smaller bias than the restricted Hausman Taylor for weak instruments up to the level:  $\rho_{X_1Z_2} = \rho_{X_2Z_2} = 0.25$ . For large levels of these correlation coefficients ( $\rho_{X_1Z_2} = \rho_{X_2Z_2} = 0.45$ ), whose square terms also decrease the denominator of the R-OLS bias, the overall correlation matrix  $\mathbf{R}$  is close to be non invertible because of near-multicollinearity. Then, the bias for  $\gamma_2$  tends to increase faster (non linearly) up to the level of the restricted between (0.62) for R-OLS, R-GLS and between, except for the unbiased R-HT12 and U-HT12 estimators which benefit from strong internal instruments. It is unfortunately also the case for the bias for  $\beta_1$  and  $\beta_2$  in R-OLS, R-GLS and BE, so that the pretest rejects both null hypothesis  $\pi_1 = \beta_{1,BE} - 1$  and  $\pi_2 = \beta_{2,BE} - 1$  and selects the MK-GLS (BE for  $\gamma$ ) estimator in 24.9% of replications. Its bias is then 0.238 with a 5% size equal to 27% in this limit case.

### 4.4 Other simulations

To test the robustness of our results, different levels of significance and different population size were simulated. Furthermore, the assumptions of normality were relaxed, first a uniform distribution for the time-varying error was assumed and secondly a uniform distribution for the individual random disturbance, keeping the overall variance constant.

Results when increasing the size of the sample and the significance threshold are presented in table 4. The number of individuals went from  $N = 100$  to 500 to 1000. When the sample increases, the absolute value of the  $t$ -statistics increases, so that the null hypothesis is more easily rejected, which is also the case when the significance threshold increases from 2% to 5% and to 10%. Then, the pretest selects more often the MK-GLS estimator. In these simulations, the endogenous variable  $X_3$  was never selected as exogenous. We can therefore conclude that the pretest performs well for relatively small samples.

When changing the distribution of the disturbances  $\varepsilon_{it}$  from normal to uniform (with

identical variance but more kurtosis) the results are almost the same. Details are available from the authors upon request. Chatelain and Ralf (2010) used these various estimators on a wage equation evaluating the returns to schooling on the data set used by Greene (2012), section 11.4.5.

## 5 Conclusion

When a researcher does not know which are the exogenous internal instruments in order to correct the endogeneity bias of some time-invariant regressors, a pretest estimator based upon the Mundlak-Krishnakumar and an unrestricted Hausman-Taylor estimator is a viable alternative to other estimators in terms of bias, RMSE and inference. The procedures are easy to program since it is only required to compute the average over time of time-varying variables and merge them with the initial data set, and then to use random effects and Hausman-Taylor procedures available e.g. in STATA. Further work may consider dynamic panel data, where the between estimator is related to the long term value of the parameters, and may as well consider external instruments in between estimations.

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## Appendix A. Unrestricted Hausman and Taylor (U-HT) estimator

If  $k_2 \geq 1$ , to obtain consistent estimators for both  $\beta$  and  $\gamma$  in the second stage, let

$$\hat{d} = \bar{y}_\cdot - \bar{X}_\cdot \hat{\beta}_W = \left( B - \bar{X}_\cdot (X'WX)^{-1} X'W \right) y$$

be the  $NT$  vector of group means estimated from the within-groups residuals. The only difference of the unrestricted Hausman and Taylor with respect to the usual restricted Hausman and Taylor is to keep the average over time of the endogenous time varying variables  $\bar{X}_2$  in each of the steps derived by Hausman and Taylor (1981). Expanding this expression using equation 3 including only  $\bar{X}_2$  leads to:

$$\hat{d} = Z_1 \gamma_1 + Z_2 (\gamma_2 + \phi_2) + \bar{X}_2 \pi_3 + \alpha^M + \left( B - \bar{X}_\cdot (X'WX)^{-1} X'W \right) \varepsilon. \quad (8)$$

Treating the last two terms as an unobservable mean zero disturbance, we estimate  $\gamma$  from the above equation using  $N$  observations. If  $\alpha$  is correlated with the columns of  $Z_2$ ,  $E(\alpha | Z_2) = \phi_2 \neq 0$ , according to prior information, both OLS and GLS will be inconsistent estimators for  $\gamma$ . Consistent estimation is possible, however, if the columns of  $X_1$ , uncorrelated with  $\alpha$  according to the non rejection of the null hypothesis of preliminary tests, provide sufficient instruments for the columns of  $Z$  in equation (8). The two stage least squares (2SLS) estimator for  $\gamma$  in equation (8) is:

$$\hat{\gamma}_{II} = \left( [Z', \bar{X}_2] P_A [Z', \bar{X}_2] \right)^{-1} [Z', \bar{X}_2]' P_A \hat{d} \quad (9)$$

where  $A = [\bar{X}_1, Z_1]$  and  $P_A$  is the orthogonal projection operator onto its column space. The sampling error is given by

$$\hat{\gamma}_{II} - \gamma = \left( [Z', \bar{X}_2] P_A [Z', \bar{X}_2] \right)^{-1} [Z', \bar{X}_2]' P_A \left( \alpha^M + \left( B - \bar{X}_\cdot (X'WX)^{-1} X'W \right) \varepsilon \right)$$

and under the usual assumptions governing  $X$  and  $Z$ , the 2SLS estimator is consistent for  $\gamma$ , since for fixed  $T$ ,  $\text{plim}_{N \rightarrow \infty} \frac{1}{N} A' \alpha = 0$  and  $\text{plim}_{N \rightarrow \infty} \frac{1}{N} X' \varepsilon = 0$ .

Having consistent estimators of  $\beta$  and, under the condition  $k_1 \geq g_2$ ,  $\gamma$ , we can construct consistent estimators for the variance components. A consistent estimator of  $\sigma_\varepsilon^2$  can be derived from the within-group residuals in the first step  $\hat{\sigma}_\varepsilon^2 = MSE_W$ . Whenever we have consistent estimators for both  $\beta$  and  $\gamma$ , a consistent estimator for  $\sigma_\alpha^2$  can be obtained. Let

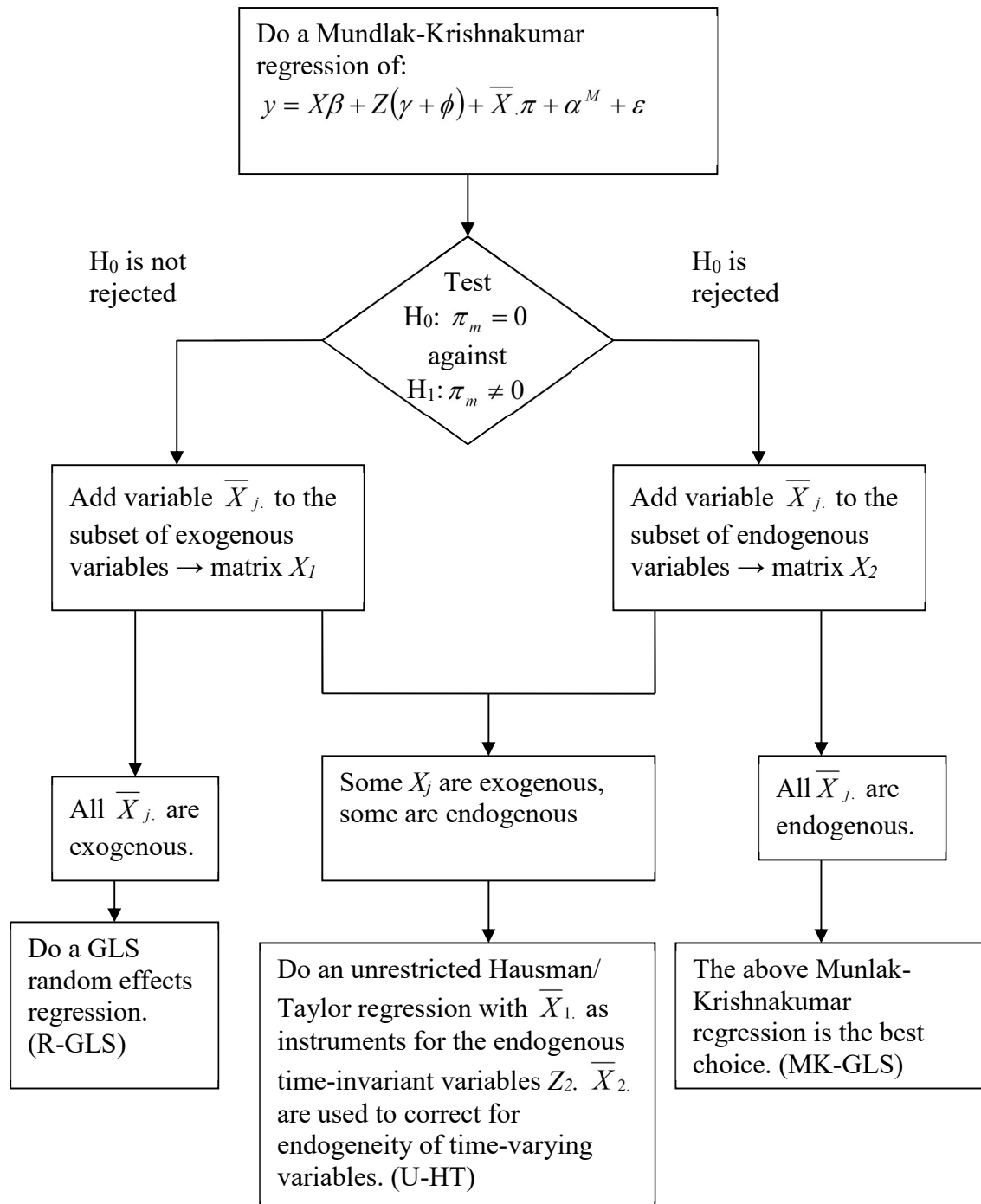
$$s^2 = (1/N) \left( \bar{y}_\cdot - \bar{X}_\cdot \hat{\beta}_W - Z \hat{\gamma}_{II} - \bar{X}_2 \hat{\pi}_{2,II} \right)' \left( \bar{y}_\cdot - \bar{X}_\cdot \hat{\beta}_W - Z \hat{\gamma}_{II} - \bar{X}_2 \hat{\pi}_{2,II} \right)$$

then

$$\text{plim}_{N \rightarrow \infty} s^2 = \text{plim}_{N \rightarrow \infty} \frac{1}{N} (\alpha + \varepsilon)' (\alpha + \varepsilon) = \sigma_\alpha^2 + \frac{1}{T} \sigma_\varepsilon^2$$

so that  $s_a^2 = s^2 - (1/T) s_\varepsilon^2$  is consistent for  $s_\alpha^2$ .

## Appendix B: Figure 1

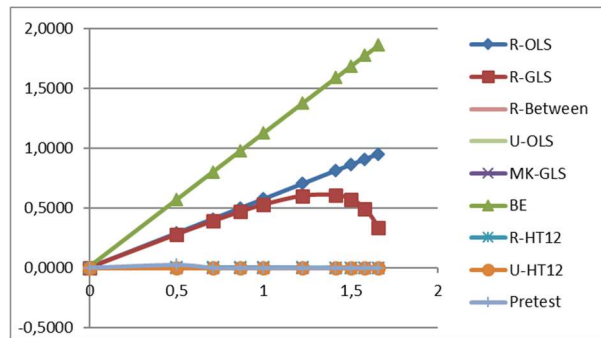


## Appendix C

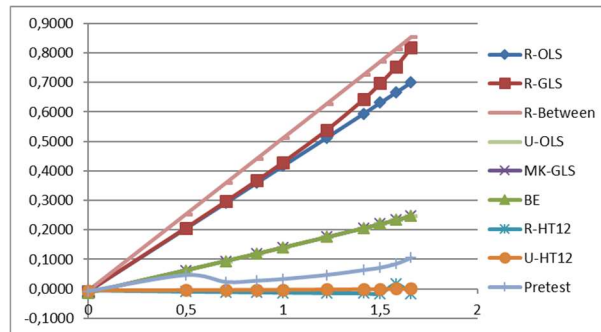
**Table 1a: Percentage of times the pretest selects the internal instruments when changing  $\sigma_u$**

$\sigma_u^2/(\sigma_u^2 + \sigma_\varepsilon^2)$	$\sigma_u$	MK-GLS	U-HT1	U-HT2	U-HT12	R-GLS	U-HT13	U-HT23
0	0	0	0.3	0.2	3.4	<b>87.4</b>	3.2	4.7
1/12	0.5	0.5	3.7	3.3	<b>75.1</b>	<b>14.5</b>	1.2	1.6
1/6	0.71	0	5	5.2	<b>88.1</b>	0.3	0.3	-
1/4	0.87	1	6.1	5.6	<b>86.8</b>	-	-	-
1/3	1	1.5	7.7	6.5	<b>84.1</b>	-	-	-
1/2	1.22	1.7	8.2	8.6	<b>79.8</b>	-	-	-
2/3	1.41	3.4	10.8	10.3	<b>72.7</b>	-	-	-
3/4	1.5	6.2	12.4	11.4	<b>68.2</b>	-	-	-
10/12	1.58	<b>11.2</b>	13.6	12.1	<b>63.1</b>	-	-	-
11/12	1.66	<b>16.8</b>	16.2	12.6	<b>54.4</b>	-	-	-

**Figure 1a: Bias of  $\hat{\beta}_3$  when changing  $\sigma_u$**



**Figure 1b: Bias of  $\hat{\gamma}_2$  when changing  $\sigma_u$**



**Table 1b: Bias, RMSE and 5% size test for  $\hat{\beta}_3$  and  $\hat{\gamma}_2$  in a Hausman-Taylor world, 1000 replications, N=100 and T=5, changing the variance of the individual effect**

R-OLS					R-OLS			R-GLS			R-GLS			R-Between			R-Between		
$\sigma_a^2$	$\sigma_a$	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size
0	0	0.0020	0.094	6.2%	-0.0089	0.112	4.8%	0.0020	0.093	5.7%	-0.0090	0.112	4.2%	-0.0005	0.124	4.30%	-0.0064	0.120	18.1%
0.25	0.5	0.2889	0.093	91.2%	0.2035	0.111	49%	0.2792	0.094	88%	0.2060	0.111	47%	-0.0005	0.119	4.30%	0.2522	0.122	75%
0.5	0.71	0.4077	0.093	99.1%	0.2917	0.109	79%	0.3894	0.094	99%	0.2965	0.110	79%	-0.0005	0.113	4.30%	0.3594	0.124	91%
0.75	0.87	0.4988	0.092	99.9%	0.3595	0.108	93%	0.4687	0.095	100%	0.3674	0.108	93%	-0.0005	0.108	4.30%	0.4418	0.127	97%
1	1	0.5756	0.092	100.0%	0.4168	0.107	99%	0.5287	0.097	100%	0.4290	0.108	99%	-0.0004	0.101	4.30%	0.5114	0.129	99%
1.5	1.22	0.7044	0.091	100.0%	0.5131	0.104	100%	0.6025	0.100	100%	0.5387	0.107	100%	-0.0004	0.088	4.30%	0.6282	0.134	100%
2	1.41	0.8129	0.090	100.0%	0.5946	0.101	100%	0.6078	0.100	100%	0.6432	0.107	100%	-0.0003	0.072	4.30%	0.7269	0.139	100%
2.25	1.5	0.8620	0.089	100.0%	0.6317	0.099	100%	0.5729	0.098	100%	0.6970	0.109	100%	-0.0003	0.062	4.30%	0.7717	0.141	100%
2.5	1.58	0.9084	0.089	100.0%	0.6669	0.098	100%	0.4948	0.091	100%	0.7543	0.113	100%	-0.0002	0.051	4.30%	0.8142	0.144	100%
2.75	1.66	0.9524	0.089	100.0%	0.7007	0.096	100%	0.3378	0.071	100%	0.8183	0.123	100%	-0.0002	0.036	4.30%	0.8549	0.146	100%

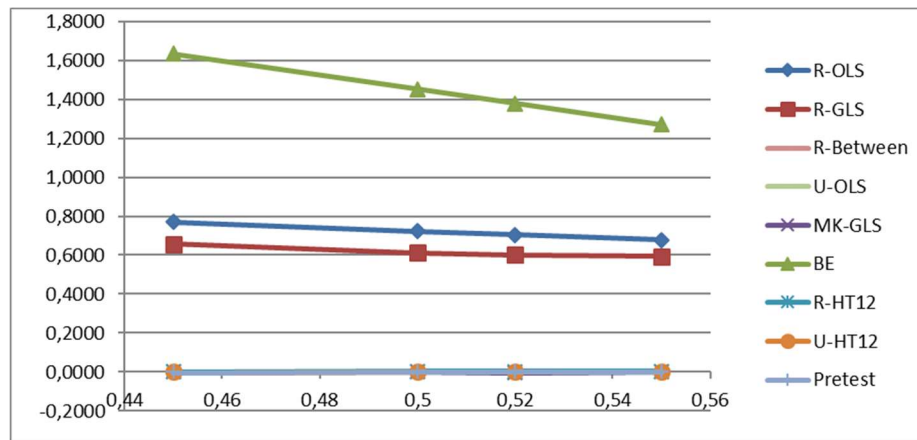
U-OLS					U-OLS			MK-GLS			MK-GLS			Between			Between		
$\sigma_a^2$	$\sigma_a$	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size
0	0	-0.0005	0.124	4.4%	-0.014	0.177	5.9%	-0.0005	0.124	4.60%	-0.0140	0.177	5.5%	0.0070	0.159	6.3%	-0.0140	0.177	6.1%
0.25	0.5	-0.0005	0.119	4.2%	0.0623	0.172	8%	-0.0005	0.119	4.60%	0.0623	0.172	7%	0.5677	0.154	97%	0.0623	0.172	8%
0.5	0.71	-0.0005	0.113	4.1%	0.0943	0.167	11%	-0.0005	0.113	4.60%	0.0943	0.167	10%	0.7998	0.150	100%	0.0943	0.167	10%
0.75	0.87	-0.0005	0.108	3.9%	0.1190	0.162	14%	-0.0005	0.108	4.60%	0.1190	0.162	12%	0.9778	0.145	100%	0.1190	0.162	12%
1	1	-0.0004	0.101	3.8%	0.1400	0.157	18%	-0.0004	0.101	4.50%	0.1400	0.157	15%	1.1277	0.140	100%	0.1400	0.157	15%
1.5	1.22	-0.0004	0.088	3.7%	0.1756	0.146	32%	-0.0005	0.124	4.60%	0.1756	0.15	23%	1.3791	0.130	100.0%	0.1756	0.146	23%
2	1.41	-0.0003	0.072	3.3%	0.2061	0.134	51%	-0.0005	0.119	4.60%	0.2061	0.13	35%	1.5908	0.120	100%	0.2061	0.134	35%
2.25	1.5	-0.0003	0.062	2.9%	0.2202	0.128	62%	-0.0005	0.113	4.60%	0.2202	0.13	42%	1.6865	0.114	100%	0.2202	0.128	42%
2.5	1.58	-0.0002	0.051	2.1%	0.2338	0.122	73%	-0.0005	0.108	4.60%	0.2338	0.12	51%	1.7769	0.108	100%	0.2338	0.122	51%
2.75	1.66	-0.0002	0.036	1.0%	0.2471	0.116	85%	-0.0004	0.101	4.50%	0.2471	0.12	59%	1.8627	0.102	100%	0.2471	0.116	59%

R-HT12					R-HT12			U-HT12			U-HT12			Pretest			Pretest		
$\sigma_a^2$	$\sigma_a$	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size
0	0	0.0001	0.124	4.4%	-0.0053	0.178	3.5%	-0.0003	0.124	4.4%	-0.0056	0.176	4.0%	0.0014	0.100	6.20%	-0.0090	0.118	3.9%
0.25	0.5	0.0030	0.119	4.7%	-0.0095	0.186	4.5%	-0.0003	0.119	4.4%	-0.0047	0.169	3.9%	0.0265	0.152	16.80%	0.0461	0.185	11.2%
0.5	0.71	0.0036	0.114	4.7%	-0.0111	0.193	4.4%	-0.0003	0.114	4.4%	-0.0041	0.164	3.6%	0.0006	0.115	4.70%	0.0224	0.164	3.2%
0.75	0.87	0.0037	0.108	4.7%	-0.0123	0.199	4.6%	-0.0003	0.108	4.4%	-0.0036	0.155	3.8%	-0.0002	0.108	4.50%	0.0268	0.155	3.3%
1	1	0.0035	0.102	4.6%	-0.0132	0.205	4.4%	-0.0003	0.102	4.2%	-0.0032	0.148	3.7%	-0.0003	0.102	4.30%	0.0326	0.149	3.8%
1.5	1.22	0.0029	0.088	4.7%	-0.0146	0.217	4.1%	-0.0003	0.088	4.3%	-0.0023	0.133	3.8%	-0.0003	0.088	4.40%	0.0467	0.148	5.6%
2	1.41	0.0019	0.072	4.7%	-0.0156	0.228	4.2%	-0.0002	0.072	4.4%	-0.0013	0.115	3.8%	-0.0003	0.072	4.50%	0.0633	0.141	8.5%
2.25	1.5	0.0014	0.062	4.5%	-0.0160	0.234	4.3%	-0.0002	0.062	4.4%	-0.0008	0.106	3.9%	-0.0002	0.062	4.50%	0.0716	0.139	10.6%
2.5	1.58	0.0009	0.051	4.6%	0.0162	0.239	4.8%	0.0001	0.051	4.4%	0.0002	0.095	3.8%	0.0002	0.051	4.50%	0.0846	0.143	13.8%
2.75	1.66	0.0004	0.036	4.5%	-0.0162	0.244	5.1%	-0.0001	0.036	4.3%	0.0006	0.083	3.7%	-0.0001	0.036	4.30%	0.1052	0.150	19.8%

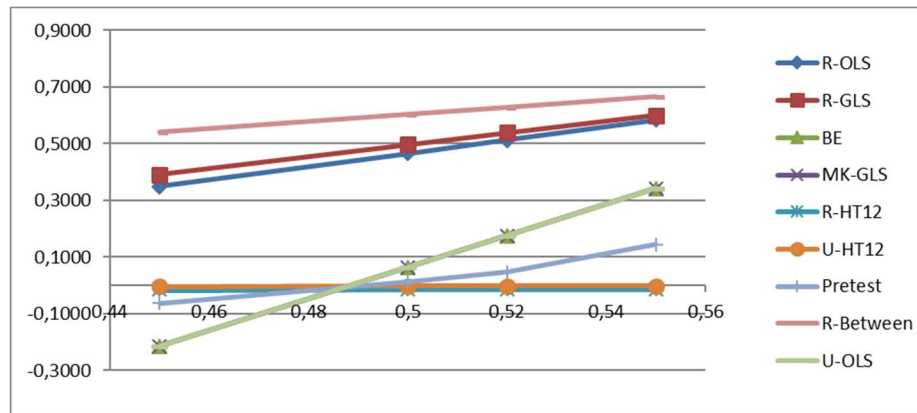
**Table 2a: Percentage of times the pretest selects the internal instruments when changing  $\rho_{z_2\alpha}$**

$\rho_{z_2\alpha}$	MK-GLS	U-HT1	U-HT2	U-HT12	R-GLS	U-HT13	U-HT23
0.45	4.2	10.7	11.4	<b>73.74</b>	-	-	-
0.5	0.7	5.7	5	<b>88.6</b>	-	-	-
0.52	3.4	8.2	8.6	<b>79.8</b>			
0.55	14.2	14.2	16.2	<b>51.1</b>	-	-	-

**Figure 2a: Bias of  $\hat{\beta}_3$  when changing  $\rho_{z_2\alpha}$**



**Figure 2b: Bias of  $\hat{\gamma}_2$  when changing  $\rho_{z_2\alpha}$**





**Table 2b: Bias, RMSE and 5% size test for  $\hat{\beta}_3$  and  $\hat{\gamma}_2$  in a Hausman-Taylor world. 1000 replications, N=100 and T=5, changing the variance of the individual effect**

$\rho_{z2a}$	R-OLS			R-OLS			R-GLS			R-GLS			R-Between			R-Between		
	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size
0.45	0.7694	0.095	100%	0.3483	0.108	95%	0.6555	0.107	100%	0.3903	0.111	97%	-0.0004	0.088	4.3%	0.5411	0.137	98%
0.5	0.7230	0.092	100%	0.4661	0.105	100%	0.6126	0.102	100%	0.4978	0.108	100%	-0.0004	0.088	4.3%	0.6034	0.351	100%
0.52	0.7044	0.091	100%	0.5131	0.104	100%	0.6025	0.100	100%	0.5387	0.107	100%	-0.0004	0.088	4.3%	0.6282	0.134	100%
0.55	0.6766	0.089	100%	0.5834	0.102	100%	0.5939	0.097	100%	0.5993	0.104	100%	-0.0004	0.088	4.3%	0.6652	0.133	100%

$\rho_{z2a}$	U-OLS			U-OLS			MK-GLS			MK-GLS			Between			Between		
	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size
0.45	-0.0004	0.088	3.6%	-0.2143	0.147	40%	-0.0004	0.088	4.3%	-0.2143	0.147	34%	1.6329	0.129	100%	-0.2143	0.147	34%
0.5	-0.0004	0.088	3.7%	0.0643	0.147	12%	-0.0004	0.088	4.4%	0.0643	0.147	8%	1.4516	0.131	100%	0.0643	0.147	8%
0.52	-0.0004	0.088	3.7%	0.1756	0.146	32%	-0.0005	0.124	4.6%	0.1756	0.146	23%	1.3791	0.130	100%	0.1756	0.146	23%
0.55	-0.0004	0.088	3.7%	0.3425	0.142	76%	-0.0004	0.088	4.4%	0.3425	0.142	69%	1.2703	0.128	100%	0.3425	0.142	69%

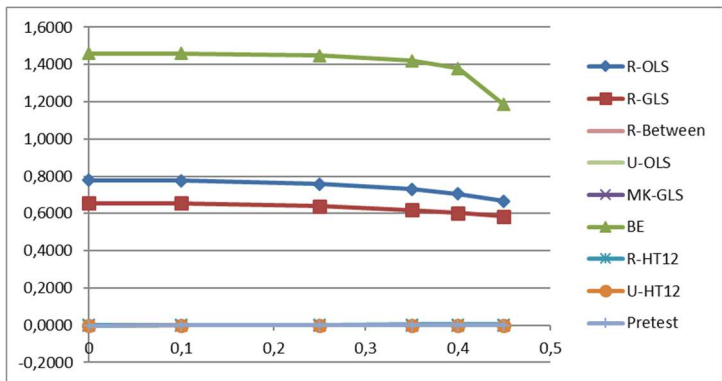
  

$\rho_{z2a}$	R-HT12			R-HT12			U-HT12			U-HT12			Pretest			Pretest		
	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size
0.45	0.0028	0.088	4.3%	-0.0150	0.022	4.4%	-0.0004	0.088	4.2%	-0.0026	0.131	3.5%	-0.0004	0.088	4.4%	-0.0635	0.156	6.7%
0.5	0.0029	0.088	4.7%	-0.0146	0.217	4.0%	-0.0002	0.088	4.2%	-0.0022	0.132	3.7%	-0.0002	0.088	4.4%	0.0138	0.131	3.0%
0.52	0.0029	0.088	4.7%	-0.0146	0.217	4.1%	-0.0003	0.088	4.3%	-0.0023	0.133	3.8%	-0.0003	0.088	4.4%	0.0467	0.148	5.6%
0.55	0.0029	0.088	4.7%	-0.0149	0.217	4.3%	-0.0002	0.088	4.3%	-0.0025	0.133	3.9%	-0.0004	0.088	4.3%	0.1444	0.192	17.8%

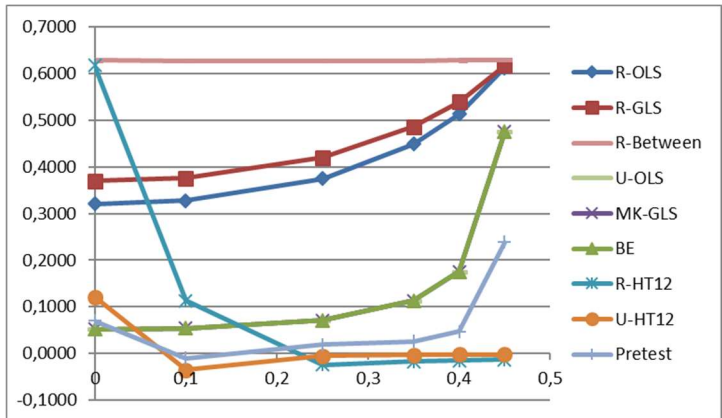
**Table 3a: Percentage of times the pretest selects the internal instruments when changing  $\rho_{x_1 z_2} = \rho_{x_2 z_2}$**

$\rho_{X_1 Z_2} = \rho_{X_2 Z_2}$	MK-GLS	U-HT1	U-HT2	U-HT12	R-GLS	U-HT13	U-HT23
0	0.3	4	5.1	<b>90.6</b>	-	-	-
0.1	0.3	4.1	5.3	<b>90.3</b>	-	-	-
0.25	0.4	4.7	5.7	<b>89.2</b>	-	-	-
0.35	0.8	7	7	<b>85.2</b>	-	-	-
0.4	3.4	8.2	8.6	<b>79.8</b>	-	-	-
0.45	24.9	13.5	13.2	<b>48.4</b>	-	-	-

**Figure 3a: Bias of  $\hat{\beta}_3$  when changing  $\rho_{X_1Z_2} = \rho_{X_2Z_2}$**



**Figure 3b: Bias of  $\hat{\gamma}_2$  when changing  $\rho_{x_1z_2} = \rho_{x_2z_2}$**



**Table 3: Bias. RMSE and 5% size test for  $\hat{\beta}_3$  and  $\hat{\gamma}_2$  in a Hausman Taylor world, 1000 replications, N=100 and T=5: changing from weak to strong internal instruments**

R-OLS				R-OLS			R-GLS			R-GLS			R-Between			R-Between		
$\rho_{x_1z_2}$	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size
0	0.7791	0.091	100.0%	0.3207	0.090	98.3%	0.6561	0.105	100.0%	0.3694	0.097	99.3%	-0.0004	0.088	4.3%	0.6283	0.126	99.8%
0.1	0.7763	0.092	100.0%	0.3280	0.091	98.4%	0.6539	0.105	100.0%	0.3763	0.098	99.1%	-0.0004	0.088	4.3%	0.6275	0.126	99.8%
0.25	0.7582	0.092	100.0%	0.3749	0.096	99.3%	0.6398	0.104	100.0%	0.4200	0.101	99.4%	-0.0004	0.088	4.3%	0.6272	0.129	99.8%
0.35	0.7293	0.091	100.0%	0.4491	0.101	99.8%	0.6189	0.102	100.0%	0.4856	0.105	99.8%	-0.0004	0.088	4.3%	0.6277	0.132	99.7%
0.4	0.7044	0.091	100.0%	0.5131	0.104	99.8%	0.6025	0.100	100.0%	0.5387	0.107	99.9%	-0.0004	0.088	4.3%	0.6282	0.134	99.6%
0.45	0.6655	0.089	100.0%	0.6129	0.107	100.0%	0.5833	0.096	0.0%	0.6169	0.109	100.0%	-0.0004	0.088	4.3%	0.6290	0.081	99.6%

U-OLS				U-OLS			MK-GLS			MK-GLS			Between			Between		
$\rho_{x_1z_2}$	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size
0	-0.0004	0.088	3.7%	0.0519	0.080	15.2%	-0.0004	0.088	4.4%	0.0519	0.080	9.7%	1.4598	0.103	100.0%	0.0519	0.080	9.7%
0.1	-0.0004	0.088	3.7%	0.0542	0.081	15.2%	-0.0004	0.088	4.4%	0.0542	0.081	9.7%	1.4583	0.104	100.0%	0.0542	0.081	9.8%
0.25	-0.0004	0.088	3.7%	0.0716	0.094	17.6%	-0.0004	0.088	4.4%	0.0716	0.094	11.5%	1.4470	0.108	100.0%	0.0716	0.094	11.6%
0.35	-0.0004	0.088	3.7%	0.1127	0.117	23.3%	-0.0004	0.088	4.4%	0.1127	0.117	16.2%	1.4201	0.117	100.0%	0.1127	0.117	16.3%
0.4	-0.0004	0.088	3.7%	0.1756	0.146	32.3%	-0.0004	0.088	4.4%	0.1756	0.146	23.2%	1.3791	0.130	100.0%	0.1756	0.146	23.4%
0.45	-0.0004	0.088	3.7%	0.4746	0.233	62.7%	-0.0004	0.088	4.4%	0.4746	0.233	53.3%	1.1842	0.176	100.0%	0.4746	0.233	53.3%

R-HT12				R-HT12			U-HT12			U-HT12			Pretest			Pretest		
$\rho_{x_1z_2}$	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size	$\hat{\beta}_3$ bias	RMSE	5% size	$\hat{\gamma}_2$ bias	RMSE	5% size
0	0.0015	0.088	44.0%	0.6170	2.498	1.7%	-0.0004	0.088	4.2%	0.1207	1.949	0.1%	-0.0004	0.088	4.3%	0.0702	2.165	0.1%
0.1	0.0025	0.088	4.8%	0.1136	1.515	3.2%	-0.0003	0.088	4.2%	-0.0348	0.801	0.4%	-0.0003	0.088	4.4%	-0.0099	1.024	0.2%
0.25	0.0028	0.088	4.6%	-0.0239	0.364	3.8%	-0.0002	0.088	4.2%	-0.0047	0.218	2.8%	-0.0003	0.088	4.4%	0.0191	0.207	1.6%
0.35	0.0029	0.088	4.6%	-0.0168	0.251	4.2%	-0.0002	0.088	4.2%	-0.0028	0.152	3.5%	-0.0003	0.088	4.4%	0.0262	0.146	2.7%
0.4	0.0029	0.088	4.7%	-0.0146	0.217	4.1%	-0.0002	0.088	4.3%	-0.0023	0.133	3.8%	-0.0003	0.088	4.4%	0.0467	0.142	5.6%
0.45	0.0029	0.088	4.7%	-0.0130	0.192	4.6%	-0.0002	0.088	4.3%	-0.0018	0.118	3.8%	-0.0004	0.088	4.4%	0.2380	0.314	27.4%

**Table 4: Percentage of times the pretest selects the internal instruments when changing the level of significance (in %) and the size of the sample (N)**

	2%	2%	2%	5%	5%	5%	10%	10%	10%
N	100	500	1000	100	500	1000	100	500	1000
MK-GLS	1.1	11.4	<b>34.7</b>	3.4	23.1	<b>52.9</b>	6.9	<b>33.3</b>	<b>68.8</b>
U-HT1	5.2	16	19.5	8.2	17.9	15.2	12.8	19.2	11.1
U-HT2	4.9	17.8	20.4	8.6	20.6	17.6	13.8	21.7	12.5
U-HT12	<b>88.8</b>	<b>54.8</b>	25.4	<b>79.8</b>	<b>38.4</b>	14.3	<b>66.5</b>	25.8	7.6