

Lifa 101: Reading Chinese Astronomical Procedure Texts

A workshop organized by: Daniel P. Morgan, Dror Weil, and Michelle McCoy

Max-Planck-Institut für Wissenschaftsgeschichte
Boltzmannstrasse 22, 14195 Berlin
Room R265
12–13 April 2018

Led by: Daniel Patrick Morgan, Centre national de la recherche scientifique, Laboratoire SPHere

A two-day crash-course reading group in *li* 曆 mathematical astronomy for those at the MPIWG who are already trained in classical Chinese and are interested in learning to read classical Chinese mathematics. The goal is to supplement current scholars' training in other facets of the Chinese astral sciences and to lay a foundation for future collaboration. Roughly 200 pages of background reading will be assigned in advance. The event will be open to roughly 5 (but no more than 10) MPIWG scholars.

Tentative Schedule:

Day 1, 12 April, 09:30 – 17:30: Introduction to the real-world celestial phenomena treated in *li* procedure texts; to software helpful for learning, demonstrating, and retrodicting these phenomena; and to the way that primary sources put these phenomena into words, numbers, and action.

Day 2, 13 April, 09:30 – 17:30: Close reading of a *li* procedure text—the Han *Sifen li* 四分曆 of 85 CE). The reading will be at once philological, mathematical, and digital, so to speak, as we will be working from a version of the text prepared in spread sheet format so that as we translate Chinese to English we translate also from instructions into computations – and, thus, from ancient Chinese into computer code (Excel) – with the goal of reproducing worked examples. This interactive, live-action, multi-level approach to text reading is not only an effective method for learning this rather difficult genre of text, it leaves the reader with a working software tool to revisit, replicate, complete, expand, and to do with later as he/she desires.

Beer, to be arranged: Doldenmadel Braugasthaus, Mehringdamm 80, 10965 Berlin

Access to all course documents may be had here:

<https://drive.google.com/open?id=1qyQrWTxnNIggQQar73M9nMMITCsFj0cu>

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D.P. Morgan

Bibliography

Free software

Stellarium, <http://stellarium.org/>

Beautiful, intuitive, open source, multi-platform software for visualising the heavens including a rather accurate and complete set of Chinese constellations under ‘Korean’.

Nasa Eclipse Website, <https://eclipse.gsfc.nasa.gov/eclipse.html>

The Espenak’s *Five Millennium Catalog of Solar Eclipses: –1999 to +3000* and *Five Millennium Catalog of lunar Eclipses: –1999 to +3000* are to my knowledge the most reliable ready-made sources for ancient eclipse data available. The Javascript Explorer allows you to adjust catalogue data to the latitude, longitude, and altitude of your choosing.

PSLV, http://www.alcyone.de/planetary_lunar_and_stellar_visibility.html

Handy tool for calculating the rising, setting, and visibility of the luminaries according to latitude, longitude, and altitude of a place. Note that visibility is at best a helpful approximation.

Background reading for course (in order of importance)

Kuhn, Thomas S. 1957. *The Copernican Revolution: Planetary Astronomy in the Development of Western Thought*. Cambridge, MA: Harvard University Press. (pp. 8–77)

A simple, concise overview of *apparent* astronomical phenomena and the ancient Greek approach to them, well-written and replete with diagrams and metaphors.

Cullen, Christopher. 1996. *Astronomy and Mathematics in Ancient China: The Zhou Bi Suan Jing*. Cambridge: Cambridge University Press. (pp. 7–35)

A clear introductory account of the sort of phenomena covered in Kuhn as experienced and modelled in Han-era *li*, the 四分曆 in particular.

Morgan, Daniel Patrick. 2017a. ‘Remarks on the Mathematics and Philosophy of Space-time in Early Imperial China’, forthcoming chapter in Garret Olberding, ed., *Designing Space: the Exercise of the Spatial Imagination in Pre-modern China* (?).

My recent sundry thoughts on contradictory truisms sinology teaches us to repeat about 曆, on the origins and (in)appropriateness of the translation ‘calendars/ics’, on the mathematics and philosophy through which the ancients conceived of their astronomical calculations.

***Morgan, Daniel Patrick. 2017b.** *Astral Sciences in Early Imperial China: Observation, Sagehood and the Individual*. Cambridge: Cambridge University Press. (pp. 9–49)

If you want: Brief critical overview of the sources, legend, terminology, history, participants, institutions, values, and sociology of *li* in the Han.

And if you would like some day to read more, allow me to suggest....

Translations – historical monographs

Chavannes, Édouard. 1898. *Les Mémoires historiques de Se-ma Ts'ien, tome troisième.* reprint 1967. Paris: Librairie d'Amérique et d'Orient. [Chineancienne.fr | Gallica]

Includes translation of the 史記 • 曆書 for the sake of completion. The translation is not bad, if memory serves, but it's by no means *expert*.

Cullen, Christopher. 2017. *The Foundations of Celestial Reckoning: Three Ancient Chinese Astronomical Systems.* New York: Routledge.

Having divided up and rearranged their contents, this book gives you what is an essentially complete translation of the *li* half of the 漢書 • 律曆志 and 後漢書 • 律曆志. Its translations of the historical contents surrounding the procedure texts is reliable, but the apparatus is bare-bones, being something of an appendix to this and his other 2017 book. Read my review [here](#).

Morgan, Daniel Patrick and Howard L. Goodman. (in preparation). *The Sui shu 'Lü-li zhi' : Li Chunfeng's history of Metrological Standards and Mathematical Astronomy over the Period of North-South Division, 502-618 CE* (provisional title). Routledge.

One day, this will be the first complete translation of a 'Lü-li zhi' by experts in said fields.

Translations – procedure texts

Liu Hongtao 劉洪濤. 2003. *Gudai lifa jisuanfa 古代曆法計算法.* Tianjin: Nankai daxue chubanshe.

A lucid explanation of the basics of early imperial *li* and the specifics of every system manual from Han-Tang, *line-by-line*, with everything described in both prose and symbolic algebra. Not fun reading, but arguably the most important **resource** available on the topic. Qu Anjing and his team are planning to supersede this with a *complete series* in the same vein, Han to Qing, under the title 中國曆法通史研究.

Cullen, Christopher. 2017. *The Foundations of Celestial Reckoning: Three Ancient Chinese Astronomical Systems.* New York: Routledge.

In the same exact vein as Liu Hongtao, but in English, slightly more sensitive to the original, and covering only three Han-era procedure texts. For my review, see [here](#).

Sivin, Nathan. 2009. *Granting the Seasons: the Chinese Astronomical Reform of 1280, with a Study of Its Many Dimensions and a Translation of its Records.* New York: Springer.

An expansive work translating the epoch-making 授時曆 plus descriptions of the instruments and the data analysis that went with it in 1280 and covering every possible dimension of the reform in the prolegomena. The major detraction is that Sivin completed this with Yabuuti and Nakayama in 1969 or so – you can look up the 40-years-prior version in the NRI archives if you're ever there – and it remains theoretically very much where Sivin was as many decades ago. Also, there are no sample problems, and no one I know in the field has ever gotten his maths to work. See Cullen's review [here](#).

Yabuuti Kiyosi 藪内清 and Nakayama Shigeru 中山茂. 2006. *Jujireki: yakuchū to kenkyū 授時曆: 譯注と研究.* Kawasaki: I. K. Corporation.

The Japanese-language edition of the 1960s Yabuuti-Nakayama-Sivin translation. See Cullen's review [here](#).

Guan Yuzhen 關瑜楨. 2015 (unpublished). 'The Treatment of Eclipses in Early Chinese Astral Sciences', Ph.D. diss., Brown University.

Purported to include a complete translation of the 景初曆 of 237 CE, but I have not seen it.

Morgan, Daniel Patrick. 2014 (unpublished). ‘Luminous Inception system Draft Translation with Apparatus’, <[halshs-01341876](#)>

My own complete translation of the 景初曆, covering everything up to the planets, which I abandoned when I heard that Guan Yuzhen was doing the same work.

Morgan, Daniel Patrick and Howard L. Goodman. (in preparation). *The Sui shu ‘Lü-li zhi’ : Li Chunfeng’s history of Metrological Standards and Mathematical Astronomy over the Period of North-South Division, 502-618 CE* (provisional title). Routledge.

One day...

Major secondary sources (2009–2018)

Cullen, Christopher. 2017. *Heavenly Numbers: Astronomy and Authority in Early Imperial China*. Oxford: Oxford University Press.

Solid, accessible history of mathematical astronomy and related *tianwen* for the Han, arguably the best introduction to the topic to date. Read my eventual review [here](#).

Martzloff, Jean-Claude. 2016. *Astronomy and Calendars the Other Chinese Mathematics: 104 BC - AD 1644*. Berlin: Springer.

English translation of his 2009 French-language monograph. The book focuses only on the *calendar* portion of *li*, how to calculate the civil calendar, and how to understand the calculations behind extant calendars preserved in the archaeological record, etc. The prolegomena is interesting, but the book is even harder to read than Qu Anjing’s, in a way, as it invents its own symbolic algebra by which to render Chinese text. See Cullen’s review [here](#).

Pan Nai 潘鼐. 2009. *Zhongguo hengxing guance shi 中國恆星觀測史*. 2d ed. Shanghai: Xuelin chubanshe.

Everything you ever wanted to know about star catalogues 星經, with a focus on data analysis and arguments on dating, errors, etc. Comprehensive, but not particularly up to date (1st edn. 1989). Note that the argumentation concerning star-identification is circular and its purported value can be taken with a grain of salt.

Qu Anjing 曲安京. 2008, *Zhongguo shuli tianwenxue 中國數理天文學*, Shuxue yu kexue shi congshu 4. Beijing: Kexue chubanshe.

A mathematical genius lays bare every question ever thought of (and numerous ones never thought of) in the entire history of mathematical astronomy in China. Comprehensive, organised, thematically for easy reference, but made difficult to understand due to the extent to which Chinese text is abstracted into modern symbolic algebra. See Cullen’s review [here](#). (Note that this is being translated by someone).

中國天文學史大系 (2008)¹

Zhang Peiyu 張培瑜, Chen Meidong 陳美東, Bo Shuren 薄樹人, and Hu Tiezhu 胡鐵珠. 2008. *Zhongguo gudai lifa 中國古代曆法*. *Zhongguo tianwenxueshi daxi 中國天文學史大系*. Beijing: Zhongguo kexue jishu chubanshe.

A very unevenly organised and realised book on the technical advances from one historical *li* to another running from Han to Yuan. It’s not bad, but I generally turn to this after the studies of Chen Meidong.

¹ Note that the following works exist in multiple editions in different series, some published in 2007. WTF?

Chen Meidong 陳美東. 2008. *Zhongguo gudai tianwenxue sixiang* 中國古代天文學思想. *Zhongguo tianwenxueshi daxi* 中國天文學史大系. Beijing: Zhongguo kexue jishu chubanshe.

Probably the single best history of Chinese astronomy every written in any language – come here first, come here last, and come here whenever you’re in need of inspiration.

Lu Yang 盧央. 2008. *Zhongguo gudai xingzhanxue* 中國古代星占學. Beijing: Zhongguo kexue jishu chubanshe.

A poorly organised mess of copy-and-pasted primary sources with an insipid thesis.

Chen Jiujin 陳久金, ed. 2008. *Zhongguo gudai tianwenxuejia* 中國古代天文學家. *Zhongguo tianwenxueshi daxi* 中國天文學史大系. Beijing: Zhongguo kexue jishu chubanshe.

A collection of biographies in what is essentially a modern version of 疇人傳 with more details and a couple of interesting arguments here and there.

Wu Shouxian 吳守賢, and Quan Hejun 全和鈞. 2008. *Zhongguo gudai tianti celiangxue ji tianwen yiqi* 中國古代天體測量學及天文儀器. *Zhongguo tianwenxueshi daxi*. Beijing: Zhongguo kexue jishu chubanshe.

The first history of its kind to my knowledge. It’s complete, but it’s pretty bland, and it makes little to no attempt to engage outside scholarship.

Chen Jiujin 陳久金. 2013. *Zhongguo shaoshu minzu tianwenxue shi* 中國少數民族天文學史. *Zhongguo tianwenxueshi daxi* 中國天文學史大系. Beijing: Zhongguo kexue jishu chubanshe.

I haven’t bothered. This topic is usually an invitation for nonsense.

Du Shenyu 杜昇云, ed. 2013. *Zhongguo gudai tianwenxue de zhuangui yu jindai tianwenxue* 中國古代天文學的轉軌與近代天文學. *Zhongguo tianwenxueshi daxi* 中國天文學史大系. Beijing: Zhongguo kexue jishu chubanshe.

Not my period.

Xu Zhentao 徐振韜, ed. 2013. *Zhongguo gudai tianwenxue cidian* 中國古代天文學詞典. *Zhongguo tianwenxueshi daxi* 中國天文學史大系. Beijing: Zhongguo kexue jishu chubanshe.

I haven’t really looked.

Zhuang Weifeng 莊威鳳. 2009. *Zhongguo gudai tianxiang jilu de yanjiu yu yingyong* 中國古代天象記錄的研究與應用. *Zhongguo tianwenxueshi daxi*. Beijing: Zhongguo kexue jishu chubanshe.

A really excellent overview of the sort of data analysis modern astronomers are performing on the ancient observational record for modern and historiographic purposes.

Chen Xiaozhong 陳曉中, and Zhang Shuli 張淑莉. 2008. *Zhongguo gudai tianwen jigou yu tianwen jiaoyu* 中國古代天文機構與天文教育. *Zhongguo tianwenxueshi daxi* 中國天文學史大系. Beijing: Zhongguo kexue jishu chubanshe.

Incredibly innovative, solid, and comprehensive study of the institutional and educational framework for the astral sciences. Reads a little more like data analysis than a narrative, so you can read with T.E. Deane.

Major secondary sources (–2008)

Cullen, Christopher. 2007. ‘Actors, Networks, and “Disturbing Spectacles” in Institutional Science: 2nd Century Chinese Debates on Astronomy’. *Antiquorum Philosophia* 1: 237–67.

Really excellent post-Kuhnian, pro-Latourian analysis of astronomical policy debate and epistemology – tosses decades of received wisdom about ‘calendar reform’ out the window with a single well-done case study. See also the relevant section in Chen Meidong 2008 and Niu Weixing 2004.

Chen Zungui 陳遵媯. 2006. *Zhongguo tianwenxue shi* 中國天文學史. 2d ed. Shanghai: Shanghai renmin chubanshe.

A monumental work first written in the 1980s, covering the whole of the history of astronomy in China, *organised topically*, in small, digestible units with lots of figures, tables, etc. *Everything* is in here, if maybe not up to date, self-reflective, or developed to its fullest – an encyclopaedia-like reference.

Henderson, John B. 2006. ‘Premodern Chinese Notions of Astronomical History and Calendrical Time’. In *Notions of Time in Chinese Historical Thinking*, edited by Chun-chieh Huang and John B. Henderson, 97–113. Hong Kong: Chinese University Press.

Argues that ‘astronomical triumphalism’ and ‘astronomical fatalism’ were two serious intellectual positions in Qing historiography of astronomy and thus, contrary to Sivin 1986, that the Chinese had a notion of historical progress. This historiographical argument is picked up in Morgan 2017.

Niu Weixing 鈕衛星. 2004. ‘Han Tang zhi ji lifa gaige zhong de zuoyong yinsu zhi fenxi’ 漢唐之際曆法改革中各作用因素之分析. *Shanghai jiaotong daxue xuebao (zhexue shehui kexue ban)* 上海交通大學學報（哲學社會科學版） 12.5: 33–38, 54.

An excellent critical analysis of the different sorts of motives, objectives, types of ‘calendar reform’ in early times – developed upon by Cullen 2007 and Chen Meidong 2008.

Huang Yi-long 黃一農. 2004. *Shehui tianwenxue shi shi jiang* 社會天文學史十講. Shanghai: Fudan daxue chubanshe.

In short: freak out, all early astronomical records are post-facto political fabrications !!! Also, several chapters on calendar divination and Qing-era Jesuit stuff. You can relax, Huang’s examples are outliers – most records are perfectly believable and reliable except for textual corruption.²

Chen Meidong 陳美東. 2003. *Zhongguo kexue jishu shi: tianwenxue juan* 中國科學技術史：天文學卷. Beijing: Kexue chubanshe.

A very solid complete history of Chinese astronomy, organised chronologically, by person. Includes ‘scientific’ and ‘non-scientific’ info, personal, political, and technical details. It’s not as conceptually interesting as Chen Meidong 2008, above, but it’s an excellent reference work.

Ho Peng Yoke 何丙郁. 2003. *Chinese Mathematical Astrology: Reaching out to the Stars*. Needham Research Institute Series. London: RoutledgeCurzon.

Warning: despite the title, this has nothing to do with mathematics or astronomy/astrology. It is about 式-board calendar divination, and its main argument is one of paranormal nationalism via essentialist auto-orientalising cultural relativism: ‘Chinese science’ is none other than the ‘primitive magic’ described by 19th century anthropologists, *but that’s a good thing*, because Western science is too restricted to the material realm and could stand to learn a thing or two

² See my forthcoming chapter ‘Heavenly Patterns’ [here](#) and the forthcoming version of my talk ‘Remarks on the Source, Selection, Reliability, and Function of the Non-terrestrial Events Recorded in the *Hou Han shu* 後漢書 “Benji” 本紀 Imperial Annals’ [here](#).

from holistic, spiritual alternative ‘sciences’ racially embedded in a homogenous nation like China, for which the 華嶠 author happens to be the official spokesperson.

Jiang Xiaoyuan 江曉原, and Niu Weixing 鈕衛星. 2001. *Ouzhou tianwenxue dongjian fawei* 歐洲天文學東漸發微. Shanghai: Shanghai shudian chubanshe.

Essay collection. Has some really interesting articles on Chinese developments and Chinese–Indian interactions in the Six Dynasties and Tang. I cannot speak for the section on Ming–Qing astronomy.

Jiang Xiaoyuan 江曉原. 2001. *Jiang Xiaoyuan zixuan ji* 江曉原自選集. Guilin: Guangxi shifan daxue chubanshe.

Jiang Xiaoyuan recycles his articles into countless overlapping essay collections. Most of what you find here is also in Jiang and Niu 2001. There are three chapters on sex.

Sun Xiaochun 孫小淳, and Jacob Kistemaker. 1997. *The Chinese sky during the Han: constellating stars and society.* Leiden: Brill.

Undoubtedly the best western-language work on star catalogues and star-identification. Concise, clearly written, filled with interesting notes about lore, contains extremely useful diagrams, lists, etc. – an excellent reference work. Note that the argumentation concerning star-identification is circular and its purported value can be taken with a grain of salt.

Cullen, Christopher. 1996. *Astronomy and Mathematics in Ancient China: The Zhou Bi Suan Jing.* Cambridge: Cambridge University Press. (pp. 7–35)

A translation of the 周髀, an odd, genre-defying text which is usually filed under 算 mathematics or 天文 cosmology. The text is of negligible importance for the history of *li* mathematical astronomy, but Cullen’s clear translations and excellent prolegomena made it the best western-language introduction to *li* in its day [now superseded by *Heavenly Numbers*].

Chen Meidong 陳美東. 1995. *Gu li xin tan* 古曆新探. Shenyang: Liaoning jiaoyu chubanshe. Superseded by the work of Qu Anjing, this was the best technically-focused monograph on *li* for a good twenty years, and remains extremely helpful. The book is divided into topics like ‘solstice reckoning’, each topic is treated chronologically across various *li*, and the focus tends to be questions of plotting the accuracy of a particular type of prediction over a 2000-year period to pinpoint the revolutions of particular models and the refinement of particular data in the meantime.

Jiang Xiaoyuan 江曉原. 1991. *Tianxue zhen yuan* 天學真原. Shenyang: Liaoning jiaoyu chubanshe.

A completely revolutionary book given its context: a sociologically-oriented work, the first in Chinese to insist on putting astronomy in a social, political, mythical, and cultural context in which ‘science’ and ‘non-science’ intermingle. This book is the inspiration, acknowledged or otherwise, of most of what comes after it. The problem is that Jiang often goes too far with his out-of-the-box ideas using weak argumentation and landing him back on untenable colonialist nonsense: that the Chinese are an incurious, practically-minded race incapable *a priori* of the sort of ‘real science’ done by the Greeks; looking for western origins of everything nice in Chinese history...

Deane, Thatcher Elliott. 1989. ‘The Chinese imperial astronomical bureau: form and function of the Ming Dynasty *qintianjian* from 1365 to 1627’. Ph.D. diss., Seattle: University of Washington.

A very solid narrative history of the institution, should be combined with the analytics of Chen & Zhang 2008 above.

Yabuuti Kiyosi 戴内清. 1989. *Zōtei Zui-Tō rekihōshi no kenkyū* 增訂隋唐曆法史の研究. Kyōto: Rinsen shoten.

New Edition of the 1944 classic. There is now a Chinese translation somewhere that has apparently screwed up the diagrams.

Sivin, Nathan. 1986. ‘On the Limits of Empirical Knowledge in the Traditional Chinese Sciences’. In *Time, Science, and Society in China and the West*, edited by Julius Thomas Fraser, Nathaniel Morris Lawrence, and Francis C. Haber, 151–69. Amherst: University of Massachusetts Press.

Argues that Chinese astronomers felt they were waging a losing war against intangible anomalies, and thus they had no sense of historical progress. Focus: WJNBC to the Song. Surpassed by Henderson 2006 and Morgan 2017.

Teboul, Michel. 1983. *Les premières théories planétaires chinoises*. Paris: Collège de France. A study of the planetary models in the 三統曆 (c.5 CE) and all those sundry models in *tianwen* sources preceding it: 史記、淮南子、五星占、三家星經. His treatment of these models is anachronistically geometric, and while this is done better in Qu Anjing, Teboul’s diagrams can be helpful in visualising the phenomena. Note also that Teboul is lumping together what are distinct genres of planetary models serving different ends.³

Yabuuti Kiyosi 藪内清. 1969. *Chūgoku no temmon rekihō 中國の天文曆法*. Tōkyō: Heibonsha.

The gold standard of histories of Chinese astronomy – respected, but largely surpassed by Chen Meidong and Cullen.

Sivin, Nathan. 1969. ‘Cosmos and Computation in Early Chinese Mathematical Astronomy’. *T’oung Pao* 2d ser., ser.55, no. 1/3: 1–73.

A sort of complete history of Han-era *li* but told through the lens of a speculative (but probably correct) theory about eclipse-model tweaking. Painfully long, famously unreadable, and well surpassed by Cullen 2017a and 2017b.

Needham, Joseph. 1959. *Science and Civilisation in China*, vol.3: Mathematics and the Sciences of the Heavens and the Earth. Cambridge: Cambridge University Press.

Focusing nearly all of his attention on cosmology and astronomical instruments (building off of Maspero), Needham prefaces his 19-page section on *li* by essentially saying that it’s a waste of everybody’s time.

...

Maspero, Henri. 1939. ‘Les instruments astronomiques des Chinois au temps des Han’. *Mélanges chinois et bouddhiques* 6: 183–370.

Maspero is timeless. This remains one of the best studies of early astronomical instruments to our day – particularly good for the Han and Wei-Jin Nanbei chao.

³ Daniel Patrick Morgan, ‘The Planetary Visibility Tables in the Second-Century BC Manuscript *Wu Xing Zhan* 五星占’, *East Asian Science, Technology, and Medicine* 43 (2016): 17–60; Daniel Patrick Morgan, ‘Mercury and the Case for Plural Planetary Traditions in Early Imperial China’, in *The Circulation of Astronomical Knowledge in the Ancient World*, ed. John M. Steele (Leiden: Brill, 2016), 416–50.

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12–13 April 2018

Daniel Patrick Morgan

This hand-out is designed to be read in as the Chinese primary-source companion to the introduction to apparent astronomical phenomena in Kuhn (1957, 8–77), as recommended in the background readings.

The apparent motion of the sun (Kuhn 1957, 8–13)

XIAO Yan 蕭衍 (b. 464; r. Empereur Wu of Liang 502–549 CE):

四大海之外，有金剛山，一名鐵圍山，金剛山北，又有黑山，日月循山而轉……

Beyond the four seas there are the Diamond Mountain(s), also called the Iron Enclosure mountains, and north of the Diamond Mountains is/are also the Black Mountain(s), around which the sun and moon revolve. [...]

一晝一夜，圍繞環匝，於南則見，在北則隱，

In one **day** (*zhou* 晝) and one night (*ye* 夜),¹ they turn there-around in circles; to the south [of the Black Mountain(s)] they are visible, and to the north they are invisible.

冬則陽降而下，夏則陽升而高，高則日長，下則日短，寒暑昏明，皆由此作。

In winter, the *yang* descends on low, and in summer the *yang* mounts up high. When it is up high, **the day** (*ri* 日) is long, and when it is on low the day is short. The cold [of winter] and the heat [of summer], the darkness and the light, they are all produced from this.

夏則陽升，故日高，而出山之道遠，冬則陽降，故日下，而出山之道促...二分則合高下之中，故半隱半見，所以晝夜均等，無有長短。

In summer, the *yang* mounts, and thus **the sun** (*ri* 日) is on high, and its path exterior to the mountain(s) is far; in winter, the *yang* descends, and thus is the sun on low, and its path exterior to the mountain(s) is close. [...] At **the two equinoxes** (*er fen* 二分), [the sun] is right in the middle between high and low, and thus is [its path] half hidden and half visible, and so it is that day and night are even and equal, neither the one of them longer or shorter [than the other]

日照於南，故南方之氣燠，日隱在北，故北方之氣寒。南方所以常溫者，冬日近南而下，故雖冬而猶溫，夏日近北而高，故雖夏猶不熱...

The sun shines in/upon the south, and thus is the *qi* of the south so hot; the sun is hidden in the north, and thus is the *qi* of the north so cold. The reason that the south is always warm is that the winter sun approaches the south on low, and thus, though it be winter, is it still warm; [now,] the summer sun approaches the north but *on high*, and thus, though it be summer, it is not *hot*. [...]

故夏日...至寅而見，至戌而隱，春秋分...須至中然後乃見，西方亦複如是，冬則轉夏，所隱亦多，朝至於辰，則出金剛之上，夕至於申，則入金剛之下。

¹ Note that the midnight-to-midnight *ri* 日 *nichthemeron*, usually translated 'day', is divided for the purposes of civil time keeping into *zhou* 晝 'day' / 'daylight' and *ye* 夜 'night'.

Thus, the summer sun [...] appears at *yin*.B03 (04h–06h = ENE) and it disappears at *xu*.B11 (20h–22h = WNW); At the spring and autumn equinoxes [...] it only appears after arriving at the centre (i.e., *mao*.B04 = 06h–08h = E), and such is the case also in the west (i.e., it sets at *you*.B10 = 18h–20h = W); and the winter turns the tables on summer, the hidden part [of the sun’s path] being for its part larger, [the sun] arriving at *chen*.B05 (08h–10h = ESE) in the morning, where/when emerges above the Diamonds, and arriving at *shen*.B09 (16h–18h = WSW) in the evening, where/when it enters below the Diamonds.²

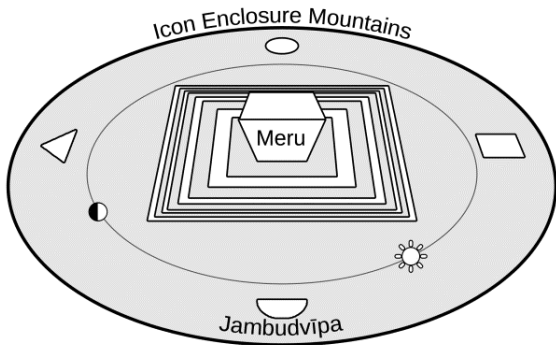


Figure 1 Buddhist cosmology (– Black Mountain(s)).

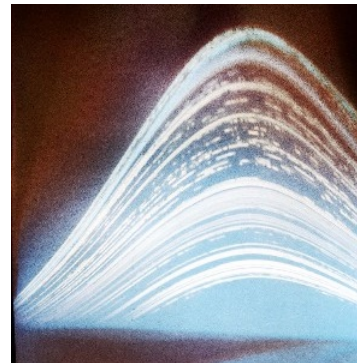


Figure 2 Solargraph showing the daily course of the sun over the course of a year. Source : Observatoire de Meudon

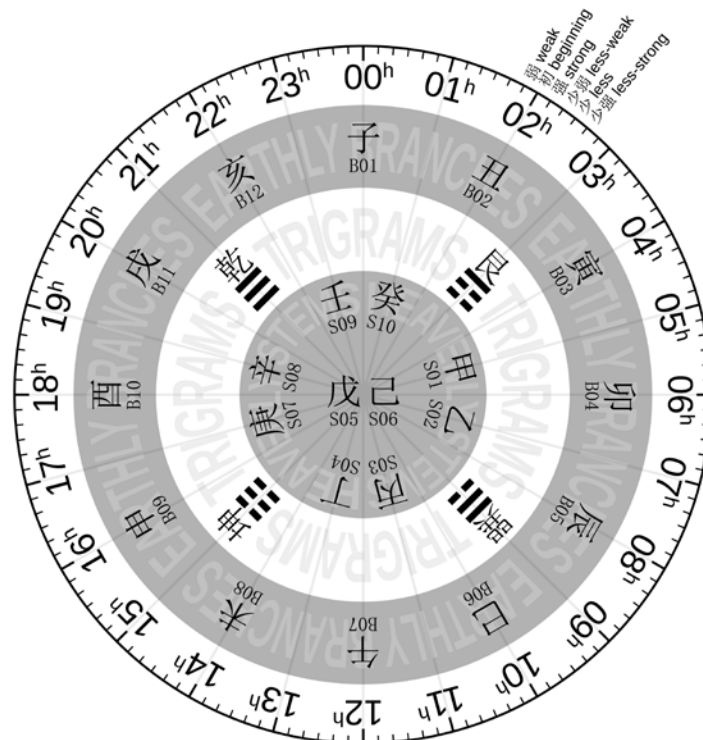


Figure 3 The twenty-four hours and/as horizontal directions

² Cited in *Kaiyuan zhanjing* 開元占經 (Siku quanshu ed.), 1.34a–35b. On Xiao Yan’s cosmology, see Yamada Keiji 山田慶兒, ‘Ryō Bu no gaitenron’ 梁武の蓋天説, *Tōhō gakuho* 東方學報 48 (1975): 99–134; Christopher Cullen, ‘Cosmographical Discussions in China from Early Times up the T’ang Dynasty’ (Ph.D. diss., University of London, 1977), 360–75; Yuan Min 遠敏 and Qu Anjing 曲安京, ‘Liang Wudi de gaitianshuo moxing’ 梁武帝的蓋天說模型, *Kexue jishu yu bianzhengfa* 科學技術與辯證法 25, no. 2 (2008): 85–89, 104..

Preface, *Sifen li* 四分曆 (85 CE):

曆數之生也，乃立儀、表，以校日景。景長則日遠，天度之端也。

The origin of [our] *li* numbers lies in erecting a sight (?) and **gnomon** (*biao* 表) to **compare** (*jiao* 校) **solar shadows** (*ri jing* / *ying* 日景) [taken at noon]. When the shadow is longest the sun is most distant, and this is the starting point for the *du* 度 of heaven.

日發其端，周而為歲，

The sun sets out from that starting point, and, **making a full circuit** (*zhou* 周), that makes a **solar year** (*sui* 歲).

然其景不復，四周千四百六十一日，而景復初，是則日行之終。以周除日，得三百六十五四分度之一，為歲之日數。

However, the *shadow* does not return [in that time]: the shadow returns to its starting point after *four circuits*, 1461 days, and *this* then represents the termination of the sun's motion. Eliminating (*chu* 除, i.e. dividing) the days by the circuits, one gets 365 and $\frac{1}{4}$ *du*, which makes the number of days in a solar year (*sui*).

日日行一度，亦為天度。

The sun (*ri*) travels moves one *du* per day (*ri*), so [this figure] is also the *du* in [a circuit of] the heavens.³

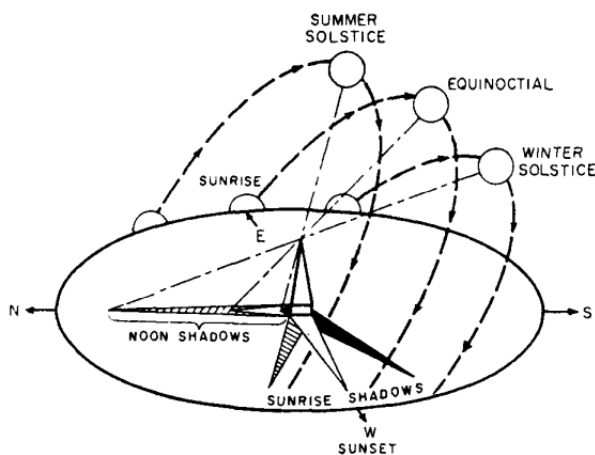


Figure 4 Relation between the position of sunrise, the sun's noon elevation, and the seasonal variation of the gnomon's shadow. Kuhn (1957, p. 10, fig. 2)



Figure 5 Gnomon

³ Cited in *Hou Han shu*, *zhi* 3, 3057; tr. modified from Christopher Cullen, *The Foundations of Celestial Reckoning: Three Ancient Chinese Astronomical Systems*, Scientific writings from the ancient and medieval world (New York: Routledge, 2017), 151.

The stars (Kuhn 1957, 13–20)

Liu Zhiji 劉知幾 (661–721), *Shi tong* 史通, 710 CE:

夫兩曜百星，麗於玄象，非如九州萬國，廢置無恆，故海田可變，而景緯無易。古之天猶今之天也，今之天即古之天也，必欲刊之國史，施於何代不可也？

The two luminaries (*liang yao* 兩曜, i.e. sun and moon) and the hundred stars are fixed in mysterious simulacra (*xuanxiang* 玄象),⁴ not like the nine provinces and myriad states, which are abolished and established without constancy (*heng* 恆). [One says] thus [that] the seas and fields can be transformed but that the light (*jing* 景) and the woof (*wei* 緯) are unchanging (*wuyi* 無易). If the sky of antiquity is like the sky of today it is because the sky of today is the sky of antiquity. If one must insist upon including [a historical monograph (*zhi* 志) on the subject] in a state history, then until what age would it not be appropriate? (I.e., wouldn't a single treatise suffice for the whole of history?)⁵

The Suzhou *Tianwen tu* 天文圖 stele of 1247:

經星：三垣、二十八舍、中外官星，是也。許二百八十三官，一千五百六十五星。其星不動。

Canon stars: These are stars of the Three Enclosures, Twenty-eight Lodges, and the inner and outer offices (*guan* 官). There are approximately 283 offices (*guan*), 1565 stars. The stars do not move.⁶

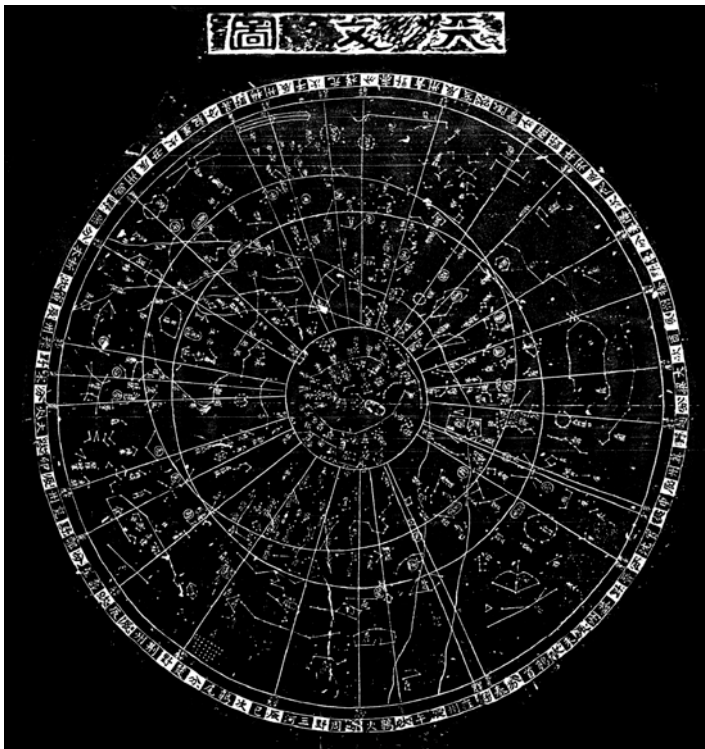


Figure 6 The Suzhou *Tianwen tu* stele of 1247, Suzhou, Jiangsu. Ink rubbing. Source: Rufus and Tien (1945, plate IA).

⁴ A *xiang* 象 ‘image’, ‘simulacrum’, etc., are the forms of things in the heavens that correspond to things on earth, not just in *form* but in *fate*. See Edward H. Schafer, *Pacing the Void: T’ang Approaches to the Stars* (Berkeley: University of California Press, 1977), esp. 54–56.

⁵ *Shi tong* 史通 (Sibu congkan ed.), 3.4a.

⁶ Will Carl Rufus and Hsing-chih Tien, *The Soochow Astronomical Chart* (Ann Arbor: University of Michigan Press, 1945). On Chinese constellations and star catalogues, see Sun Xiaochun 孫小淳 and Jacob Kistemaker, *The Chinese Sky during the Han: Constellating Stars and Society* (Leiden: Brill, 1997).

Zhang Heng 張衡 (78–139 CE), *Lingxian* 靈憲:

天體於陽，故圓以動；地體於陰，故平以靜……天有兩儀，以儻道中。其可觀，樞星是也。謂之北極。在南者不著，故聖人弗之名焉……陽道左迴，故天運左行……

Heaven is embodied of *yang*, thus is it round and in motion; earth is embodied of *yin*, thus is it flat and still. [...] Heaven has two poles by which to dance in the middle of the *dao* (?). That which can be seen is the pivot star, which we refer to as the North Pole. The one in the south is not visible, thus the Sages did not give it a name. [...] The *dao* of *yang* is to turn to the left, thus does heaven rotate going left-wise (i.e. clockwise, i.e. $E \rightarrow N \rightarrow W$ as observed looking south from the northern hemisphere).⁷



Figure 7 Astrophotography, long-exposure photograph showing the pole and the star-trails made around it in a 24-hour period

The sun as a moving star (Kuhn 1957, 20–25)

The sun moves one *du* per day in the direction opposite the sky's diurnal motion, rising and setting one *du* 'rightwise', counter-clockwise, or eastward vis-à-vis the fixed stars. Its position among the stars – and in the seasons – can thus be indirectly observed.

Liji 禮記, 'Yue ling' 月令:

孟春之月，日在營室，昏參中，旦尾中……是月也，以立春……

First month of spring: the sun is in Hall_{L13} (α Pegasi); at dusk, Triad_{L21} (δ Orionis) is centred (on the meridian); at dawn, Tail_{L06} (μ^1 Scorpii) is centred. [...] This month is the establishment of spring-Q01 (\approx 8 February).

仲春之月，日在奎，昏弧中，旦建星中……是月也，日夜分……

Second month of spring: the sun is in Straddler_{L15} (ζ Andromedae); at dusk, Bow (κ Canis Majoris) is centred; at dawn, the Establishment Star(s)_{L06} (ξ^2 Sagittarii) is centred. [...] This month the day and the night are equal (*fen* 分; i.e., the spring equinox, \approx 26 March).

季春之月，日在胃，昏七星中，旦牽牛中……

Third month of spring: the sun is in Stomach_{L17} (35 Arietis); at dusk, the Seven Stars_{L21} (α Hydrae) are centred; at dawn, Led Ox_{L06} (β Capricorni) is centred. [...]

⁷ Cited in *Kaiyuan zhanjing*, 1.1b–2a, 1.2b.

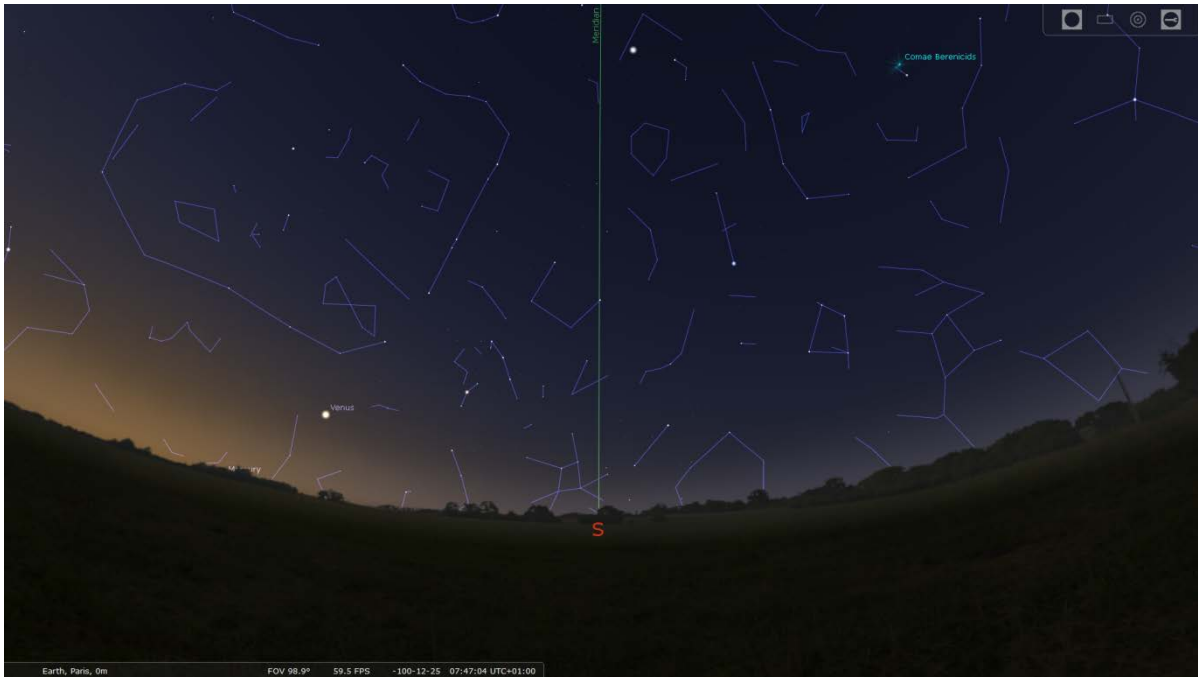


Figure 8 Finding the star ‘centred’ due south on the meridian at dawn with the help of a gnomon (25 December 101 BCE, Stellarium v.0.14.2)

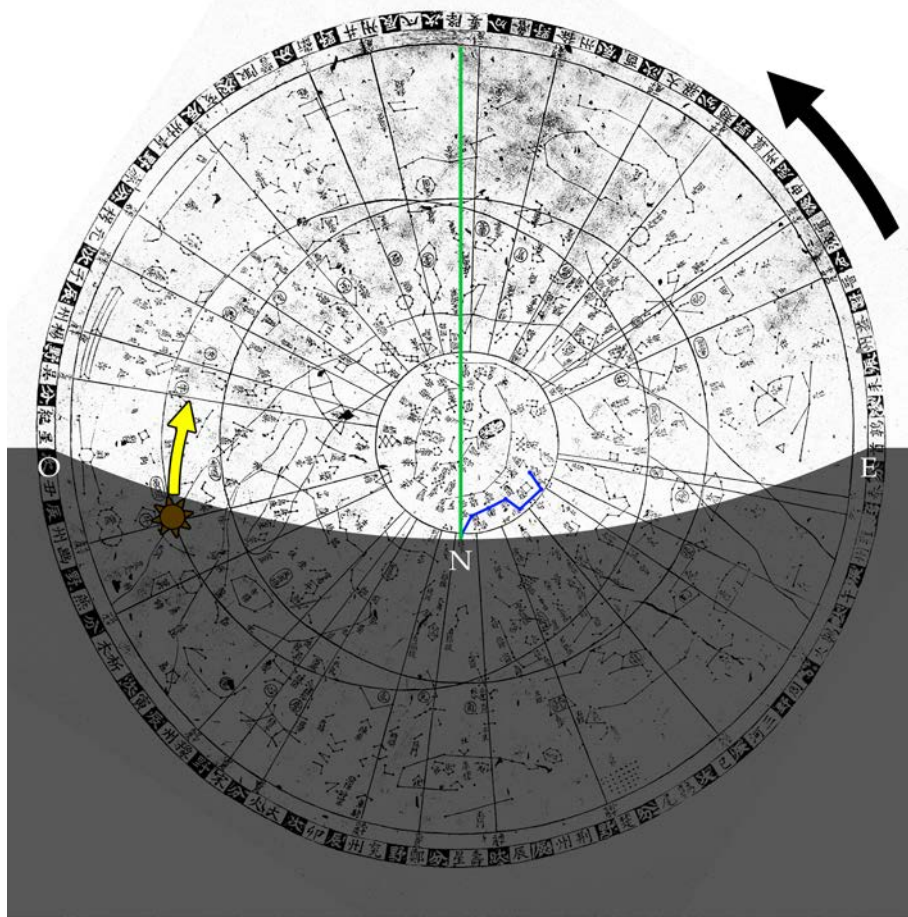
Huainanzi 淮南子, compiled by Liu An 劉安 et al. in 139 BCE:

日行一度，十五日為一節，以生二十四時之變。

[The sun] travels one *du* per day, fifteen days making one node (*jie* 節), producing the transformations of the twenty-four seasons (i.e. *qi*).

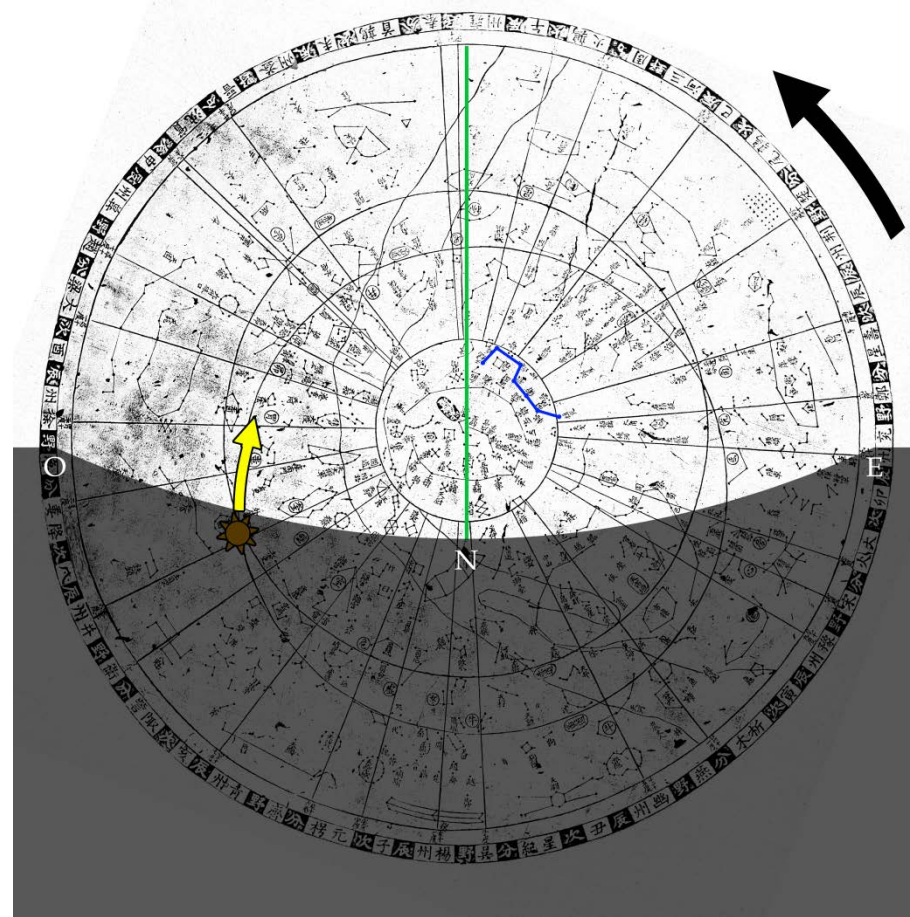
1. 斗指子則冬至...
2. 加十五日指癸，則小寒...
3. 加十五日指丑，則大寒...
4. 加十五日指報德之維，則越陰在地，故曰距日冬至四十六日而立春，陽氣凍解...
5. 加十五日指寅，則雨水.....
6. 加十五日指甲，則雷驚蟄.....
7. 加十五日指卯中繩，故曰春分則雷行.....
8. 加十五日指乙，則清明風至.....
9. 加十日指辰，則穀雨.....
10. 加十五日指常羊之維，則春分盡，故曰有四十六日而立夏，大風濟.....
11. 加十五日指巳，則小滿.....
12. 加十五日指丙，則芒種.....
13. 加十五日指午，則陽氣極，故曰有四十六日而夏至.....
14. 加十五指丁，則小暑.....
15. 加十五日指未，則大暑.....
16. 加十五日指背陽之維，則夏分盡，故曰有四十六日而立秋，涼風至.....
17. 加十五日指申，則處暑.....
18. 加十五日指庚，則白露降.....
19. 加十五日指酉中繩，故曰秋分雷減，蟄蟲北向.....
20. 加十五日指辛，則寒露.....
21. 加十五日指戌，則霜降.....
22. 加十五日指蹄通之維，則秋分盡，故曰有四十六日而立冬，草木畢死.....
23. 加十五日指亥，則小雪.....
24. 加十五日指壬，則大雪.....
25. 加十五日指子。

圖亥天



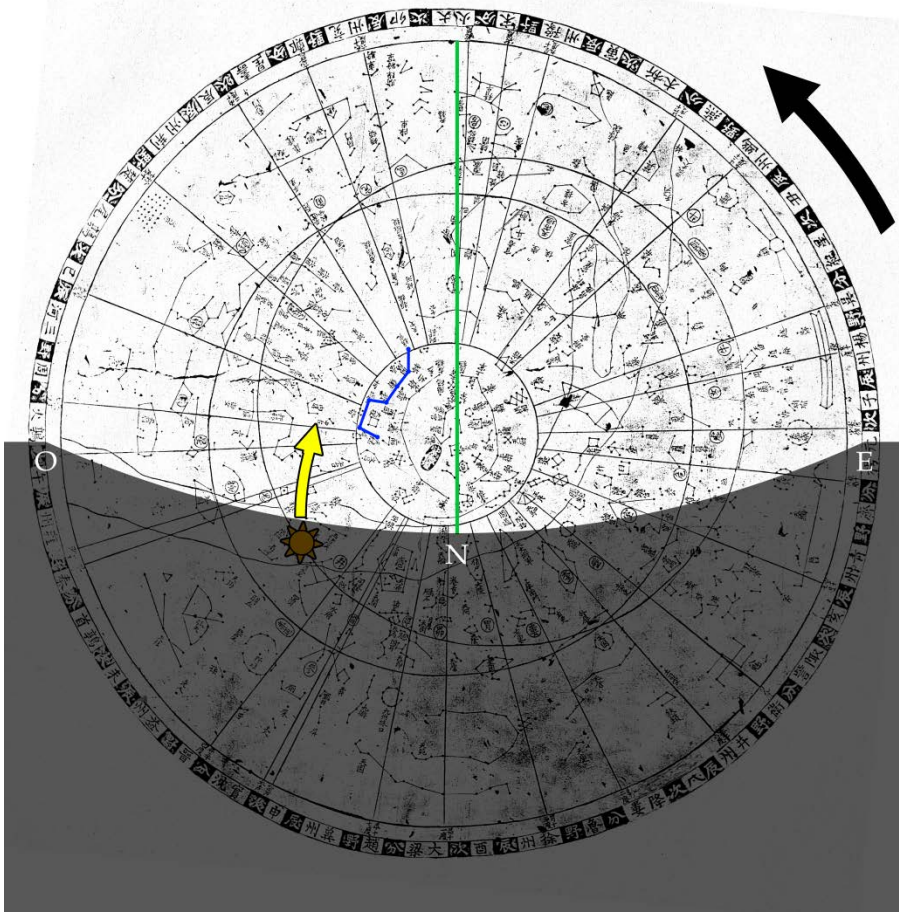
Winter solstice

圖亥天



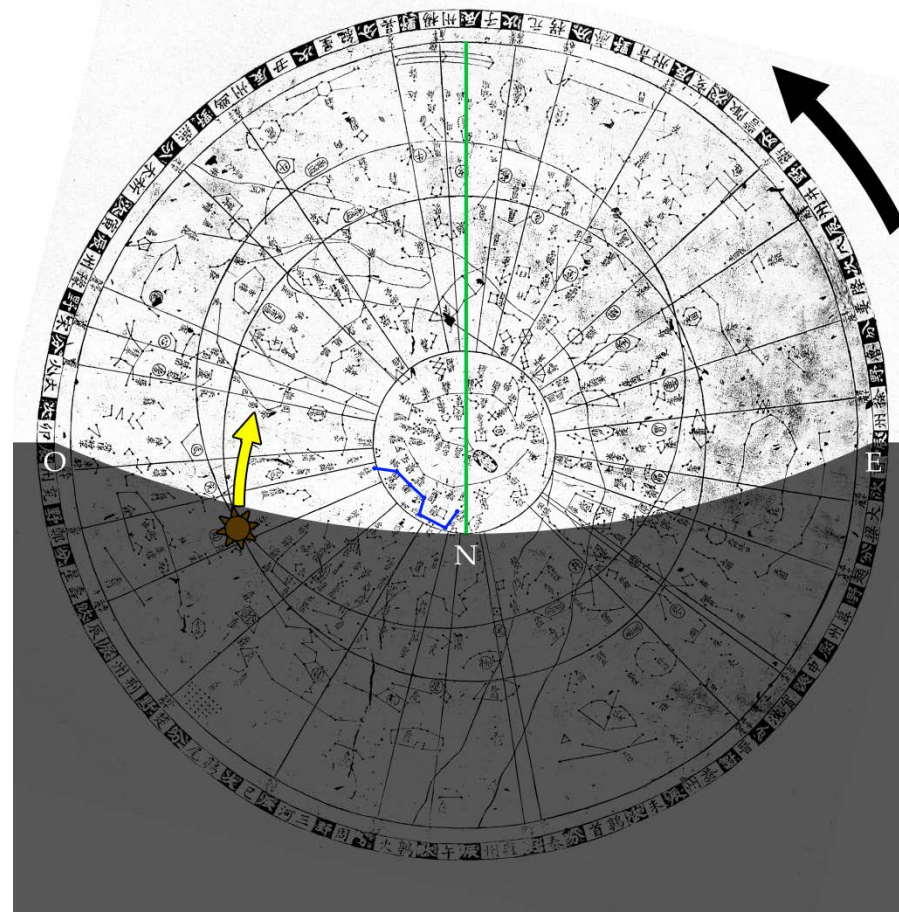
Spring equinox

天 文 圖



Summer solstice

天 文 圖



Autumn equinox

The birth of scientific cosmology (Kuhn 1957, 25–33)

Zhang Heng, *Huntian yi* 渾天儀:

渾天如雞子，天體圓如彈丸，地如雞子中黃，孤居於內。天大而地小，天表裏有水，天之包地，猶殼之裹黃。天地各乘氣而立，載水而浮，

The sphere heaven is like a chicken egg: heaven's form is round like a slingshot pellet, and earth is like the yolk in the middle of the egg, sitting inside all alone. Heaven is big and earth is small, and inside and outside of heaven there is water. Heaven englobes the earth like the shell contains the yolk. Both heaven and earth each stand, mounted on *qi*, and float, conveyed by water.

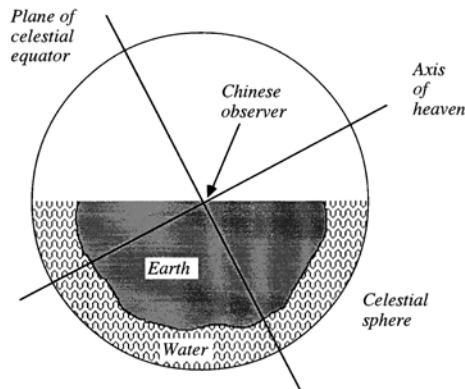


Figure 12 Zhang Heng's *huntian* 渾天 'sphere heaven' as reimaged in Cullen, Christopher. *Astronomy and Mathematics in Ancient China: The Zhou Bi Suan Jing* (Cambridge: Cambridge University Press, 1996), p. 136, fig. 13.

Zhang Heng, *Lingxian* (continued from [above](#)):

八極之維，徑二億三萬二千三百里，南北則短減千里，東西則廣增千里。自地至天半於八極，則地之深亦如之。通而度之，則是渾也……過此而往，未之或知也。未之或知者，宇宙之謂也。宇之表無極，宙之端無窮……

The bounds of the eight limits have a diameter of 232,300 *li*, the north–south of which is shorter, subtracting 1000 *li*, and the east–west of which is broader, adding 1000 *li*. The [distance] from earth to heaven is half [the diameter of] the eight limits, and so too is the depth of the earth. Put together and measured as such, then [heaven] truly is a sphere. [...] We know nothing of what lays beyond this. That of which we know nothing is called spacetime (*yuzhou* 宇宙, i.e. the cosmos). The frontiers of space are limitless, and the bounds of time are inexhaustible. [...]

夫日譬猶火，月譬猶水，火則外光，水則含景。故月光生於日之所照，魄生於日之所蔽，當日則光盈，就日則光盡也。

Now, the sun is, by analogy, like fire, and the moon, by analogy, like water: fire externalises radiance, and water sucks up light/shadow (*jing*). Thus, moonlight is appears where the sun shines [upon it], and moondark appears where the sun is obstructed. When it is facing the sun [in opposition], its light is full, and when it goes to the sun [at new moon] its light is exhausted.

眾星被耀，因水轉光。當日之衝，光常不合者，蔽於地也。是謂闇虛。在星星微，月過則食。

All the stars are illuminated [at night] because the water turns the light [of the sun around the earth]. In the place opposite the sun, [however], its rays never/rarely come together, because [the light] is blocked by the earth. This is called the 'dark void'; when it falls upon a star, the star is diminished, and when the moon passes it [the moon] is eclipsed.⁸

⁸ Cited in *Hou Han shu* 後漢書 (Zhonghua shuju ed.), *zhi* 10, 3215 (comm.).

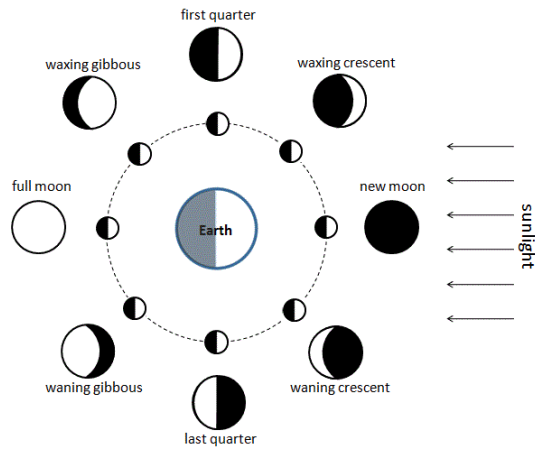


Figure 13 Lunar phases

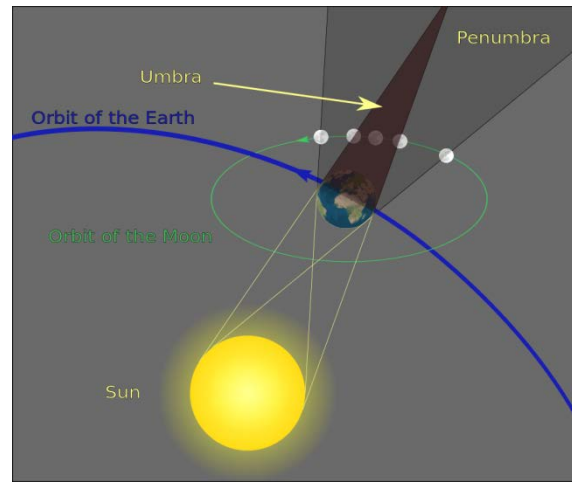


Figure 14 Lunar eclipse

The sun in the [one-]sphere universe (Kuhn 1957, 33–41)

Jia Kui 賈逵 (30–101 CE), petition of 92 CE:

臣前上傳安等用黃道度日月弦望多近。史官一以赤道度之，不與日月同，於今曆弦望至差一日以上，輒奏以為變，至以為日却縮退行。於黃道，自得行度，不為變。

Your servant has previously memorialised about how Fu An et al.'s use of the yellow road to measure the [position of] the sun and moon at quarter and full moon [produced results that were] considerably closer (*jin* 近, i.e. more accurate). The Clerk's Office (*shiguan* 史官, i.e. astronomical bureau) has only ever measured them using the red road, which results in an error of more than one day in comparison with the [predictions] of the current *li*, and this they immediately report as being a portent (*bian* 變), going all the way as to take it that the sun has in fact behind (*suo* 縮) and moving in retrograde (*tuixing* 退行). On the yellow road, [however], its *du* of motion (*xingdu* 行度, i.e. motion model) works out naturally and there are no such 'portents'.⁹

Zhang Heng, *Huntian yi* (continued from above):

周天三百六十五度四分度之一，又中分之，則一百八十二度八分之五覆地上，一百八十二度八分之五繞地下。故二十八宿半見半隱。

The circuit of heaven (*zhoutian*) is 365 *du* and $\frac{1}{4}$ *du*, which, divided in half, [means] 182 *du* and $\frac{5}{8}$ *du* covering the earth above and 182 *du* and $\frac{5}{8}$ *du* wending below the earth. Thus are the twenty-eight lodges half visible and half hidden [at any given moment].

其兩端謂之南北極。北極乃天之中也，在正北，出地上三十六度。然則北極上規徑七十二度，常見不隱。南極，天之中也，在南，入地三十六度。南極下規七十二度，常伏不見。兩極相去一百八十二度半強。

Its two extremities are referred to as the northern and southern poles. The north pole is the centre of heaven, [located] due north, rising 36 *du* above the horizon (*di* 地). As such, the north pole upper ring (i.e., the circumpolar ring) has a diameter of 76 *du*, which is always (*chang* 常) visible and never hidden. The south pole is [also] the centre of heaven, [located] in the south, set 36 *du* below the horizon (*di*). The south pole lower ring [likewise has a radius of] 72 *du*, which is always (*chang*) hidden and never seen. The two poles are 182 *du* and a strong half apart.

⁹ Cited in *Hou Han shu*, *zhi* 2, 3028–29; tr. modified from Cullen, *Foundations of Celestial Reckoning*, 385–86.

天轉如車轂之運也。周旋無端，其形渾渾，故曰渾天也。

Heaven turns [around the poles] like the revolution of a chariot [wheel around its] axle. Spinning in circuits with no beginning [or end] its form is spherey-spherey, and thus is it called 'sphere heaven'.

赤道橫帶天之腹，去南北二極各九十一度十九分度之五。黃道斜帶其腹，出赤道表裏各二十四度。然則黃道斜截赤道者，即春秋分之去極也。

The red road (*chidao* 赤道, i.e. equator) belts the belly of heaven, 91 *du* and $\frac{5}{19}$ *du* from both the southern and northern pole. The yellow road (*huangdao* 黃道, i.e. ecliptic) belts its belly obliquely, protruding 24 *du* inside (north) and outside (south) of the red road [at the respective solstices], whereas where the yellow road obliquely intersects the red road, that – [91 *du* and $\frac{5}{19}$ *du*] – is the north polar distance of the spring and autumn equinoxes.¹⁰

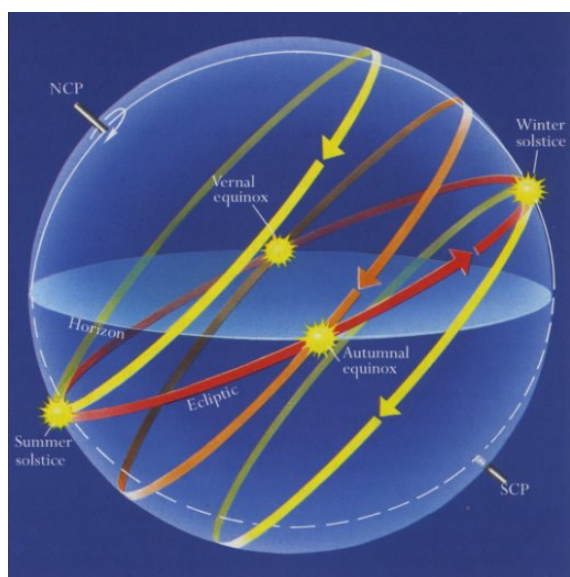


Figure 15 The yellow road (ecliptic) in red, the red road (equator) in orange. The displacement of the sun's position in the sky at culmination (noon) and at the horizon at rising at setting is due to travelling on a 'path' that is 24 *du* oblique to the equator. Measured on the yellow road, the sun's motion is (supposedly) constant, whereas it varies noticeably as measured in red-road coordinates.

Ancient competitors to the [one-]sphere universe (Kuhn 1957, 41–44)

There are a variety of predecessors and competitors of the Chinese flat-earth 'sphere heaven' model, notably various forms of 'umbrella heaven' (*gaitian* 蓋天) and the Buddhist-inspired cosmology espoused by Xiao Yan, [above](#). These haven't much to do with *li* mathematical astronomy, so allow me to skip past them.¹¹

¹⁰ Cited in *Hou Han shu*, *zhi* 10, 3215 (comm.).

¹¹ On *tianti* 天體 'heaven's form' cosmology, see Cullen, 'Cosmographical Discussions'; Chen Meidong 陳美東, *Zhongguo gudai tianwenxue sixiang* 中國古代天文學思想, *Zhongguo tianwenxueshi daxi* 中國天文學史大系 (Beijing: Zhongguo kexue jishu chubanshe, 2007), 128–532. Be wary of any scholarship that declares that 'China has no cosmology' or that its *only* 'cosmology' is *yin-yang*, five-agents correlative thinking, e.g. Nathan Sivin, 'Cosmos and Computation in Early Chinese Mathematical Astronomy', *T'oung Pao* 2d ser., 55, no. 1/3 (1969): 1–73; Benjamin I. Schwartz, *The World of Thought in Ancient China* (Cambridge, Mass.: Harvard University Press, 1985), chap. 9. 'Cosmology' is an observer's category, which means that what we chose to include in it speaks more about contemporary academic politics than ancient knowledge.

Apparent planetary motion (Kuhn 1957, 45–50)

Preface, *Sifen li* (85 CE; continued from [above](#)):

察日月俱發度端，日行十九周，月行二百五十四周，復會于端，是則月行之終也。

If one examines the sun and moon as they together leave the starting point for *du* (i.e., from conjunction at winter solstice, see above), [one finds that] when the sun has travelled 19 circuits, and the moon has travelled 254, they meet (*hui* 會) once again at the starting point – this is the termination of lunar motion.

以日周除月周，得一歲周天之數。

Eliminating (*chu*) the lunar circuits by the solar circuits, one gets the number of [lunar] circuits of heaven in one solar year (*sui*). (i.e., $254 \div 19 = 13 \frac{7}{19}$ lunar circuits per year).

以日一周減之，餘二十九分之七，則月行過周及日行之數也，為一歲之月。

Diminishing (*jian* 減) this by one solar circuit leaves (*yu* 餘) 12 and $\frac{7}{19}$, which is the number of times the moon moves through more than a circuit and catches up with the sun, i.e. the number of months in one solar year (*sui*).

以除一歲日，為一月之數。

Eliminating the [number of] days in one solar year (*sui*) by [that] makes the number [of days] in one month/lunation. ($365\frac{1}{4} \div 12 \frac{7}{19} = \frac{1461}{4} \div \frac{235}{19} = \frac{1461}{4} \times \frac{19}{235} = \frac{27759}{940} = 29 \frac{499}{940}$ days).

月之餘分積滿其法，得一月，月成則其歲大。

When the month/lunation's remainder parts accumulate [to the point where] they fill the divisor, then the [fractional] month is completed and the year is big. (i.e., when the $\frac{7}{19}$ -month gap between twelve lunations and the year add up, $\frac{7}{19} + \frac{7}{19} + \frac{7}{19} = \frac{21}{19}$, we get an extra, 'intercalary' month that year).

月四時推移，故置十二中以定月位。有朔而無中者為閏月。

The months slide around the four seasons (because of the $\frac{7}{19}$ -month gap), thus are the twelve medial [*qi*] (*zhong* 中) set out to fix the positions of the months. If it has a new moon but no medial, it is made an intercalary month (*runyue* 閏月).¹²

Preface, *Sifen li* (85 CE; earlier up):

月有晦朔，星有合見，月有弦望，星有留逆，其歸一也，步術生焉。

The moon has dark moon (*hui* 晦, i.e. the last day of the lunation/month) and new moon (*shuo* 朔, i.e. the first day of the lunation/month), and the stars (i.e. planets) have conjunction (*he* 合) and appearance (*xian* 見); the moon has quarter (*xian* 弦) and full moon (*wang* 望), and the planets have station (*liu* 留) and retrogradation (*ni* 逆) – they all come down to a single [basis], and 'pacing' (*bu* 步, i.e. calculating) procedures (*shu* 術) arose therefrom.

金、水承陽，先後日下，速則先日，遲而後留，留而後逆，逆與日違，違而後速，速與日競，競又先日，遲速順逆，晨夕生焉。

Metal (Mercury) and Water (Venus) ride on *yang*, always right in front of or behind the sun. When they're fast, they precede the sun, and when they are slow they linger behind [it], they linger (i.e., go into station) and after they retrograde; they retrograde and they transgress against [the direction of] the sun, they transgress and after they speed up; they speed up and they go neck and neck with the sun; they go neck and neck and, once again, they get ahead of the sun –

¹² *Hou Han shu*, *zhi* 3, 3057–58; tr. modified from Cullen, *Foundations of Celestial Reckoning*, 151–52.

slowing down and speeding up, prograde (*shun* 順) and retrograde (*ni*) [motion], their dawn (*chen* 晨) and dusk (*xi* 夕) [visibility] is derived therefrom.

日、月、五緯各有終原，而七元生焉。

The sun, moon, and five wefts (i.e., planets) each have their own terminations and/as starting points, and the seven origins are derived therefrom. (i.e., each luminary has its own periodicity and, thus, astronomical zero-point).

見伏有日，留行有度，而率數生焉。

The [planetary periods of] visibility (*xian*) and hiding (*fu* 伏) have their [respective number of] days, their [periods of] station and motion have their [respective] *du*/measures, and their *lü* 率 ratio-element [resonance periods] are derived therefrom.

參差齊之，多少均之，會終生焉。

Those which range and differ one equalises, those which are greater or lesser one evens out, and their coincidence termination (i.e., grand resonance period(s)) is derived therefrom.¹³

The location of the planets (Kuhn 1957, 50–55)

Zhang Heng, *Lingxian* (continued from [above](#)):

夫三光同形，有似珠玉，神守精存，麗其職而宣其明；及其衰，神歇精散，於是乎有隕星。然則奔星之所墜，至地則石矣。

Now, the three lights (Sun, Moon and stars) are identical in shape, having the semblance of jade marbles. Spirit conserved and essence preserved, [this] enhances their function and disseminates their luminance; coming to their decline, spirit is exhausted and essence defeated, and at this there is a falling star. Furthermore, what falls from a speeding star, upon arrival, is a stone.

文曜麗乎天，其動者七，日、月、五星是也。周旋右回。天道者，貴順也。近天則遲，遠天則速，行則屈，屈則留回，留回則逆，逆則遲，迫於天也。行遲者覲于東，覲于東屬陽，行速者覲于西，覲于西屬陰，日與月此配合也。攝提、熒惑、地候見晨，附于日也。太白、辰星見昏，附于月也。二陰三陽，參天兩地，故男女取焉。

Of all the things patterned, scintillating and beautiful in Heaven, those that move are seven: the sun, moon and the five stars (i.e., planets) being these. They revolve in circles winding to the right (i.e., eastward). As for the *dao* of heaven, it considers compliant/prograde motion noble, [so] what gets near to heaven slows and what gets far from heaven hastens. Traveling then arcing, arcing then lingering in return (station), lingering in return then retrograde, retrograde then slow – this is all due to approaching heaven.¹⁴

The theory of homocentric spheres (Kuhn 1957, 55–59)

This is a uniquely Greek conceptual solution to explaining the retrogradation of the planets, which has no counterpart in Chinese, Mesopotamian, Mesoamerican, Egyptian, or pre-contact Indian planetary models.

¹³ *Hou Han shu*, *zhi* 3, 3056; tr. modified from *Ibid.*, 146–47.

¹⁴ Cited in *Hou Han shu*, *zhi* 3, 3057.

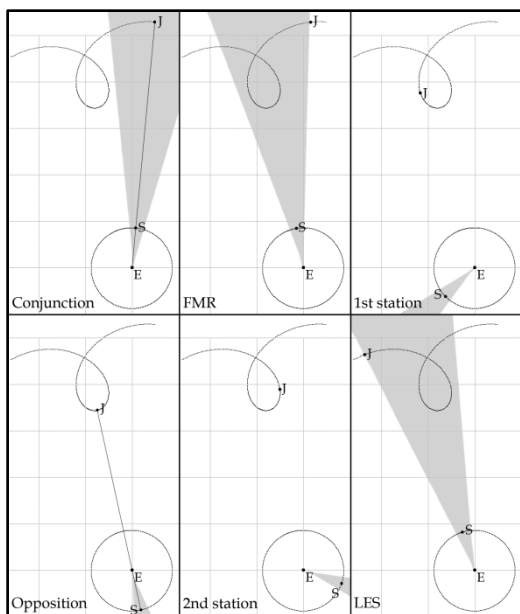


Figure 16 The synodic phenomena of Jupiter as viewed from the earth. Images modified from *Alcyone Ephemeris* v3.2.

Epicycles and deferents (Kuhn 1957, 59–64)

This is a uniquely Greek *geometrical* solution, based on a uniquely conceptual solution, for modelling *planetary retrogradation* and *inequalities of motion*. The Chinese, like the Mesopotamians, were content with arithmetic models and, over time, adding a variety of arithmetic corrections accounting for inequalities and seasonal variations in visibility.¹⁵

The *Sifen li*'s traditional *xingdu* 行度 'du-travelled' or 'motion-degree' Venus model:

金，晨伏，五日，退四度，在日後九度，而見東方。

Metal (Venus), hidden at dawn: [over] 5 days [the planet] retreats 4 *du* before making its [first morning] appearance (*xian*) in the east, 9 *du* behind the sun.

見逆，日行五分度之三，十日，退六度。

Visible in retrograde: [it] travels $\frac{3}{5}$ *du* per day, retreating 6 *du* in 10 days.

留，不行，八日。

Station (*liu*): [it] does not move for 8 days.

旋順，日行四十六分度之三十三，四十六日行三十三度。

Turning into prograde (*shun*): [it] travels $\frac{33}{46}$ *du* per day, travelling [a total of] 33 *du* in 46 days.

而疾，日行一度九十一分度之十五，九十一日行百六度。

Then fast (*ji* 疾): [it] travels 1 *du* and $\frac{15}{91}$ *du* per day, travelling [a total of] 106 *du* in 91 days.

益疾，日行一度二十二分，九十一日行百一十三度，在日後九度，而晨伏東方。

Faster: [it] travels 1 *du* and 22 parts per day, travelling [a total of] 113 *du* in 91 days, hiding in the morning in the east (i.e., last morning rising) 9 *du* behind the sun.

¹⁵ On early planetary models, see Michel Teboul, *Les premières théories planétaires chinoises* (Paris: Collège de France, 1983). Cf. the later models in Nathan Sivin, *Granting the Seasons: The Chinese Astronomical Reform of 1280, with a Study of Its Many Dimensions and a Translation of Its Records* (New York: Springer, 2009).

除伏逆，一見二百四十六日，行二百四十六度。

Eliminating (*chu*, this time subtracting) hiding and retrograde, [this] one visibility [period] is 246 days, [during which it] travels 246 *du*.

伏四十一日二百八十一分，行五十度二百八十一分，而與日合。一合二百九十二日二百八十一分，行星如之。

Hiding: over 41 days and 281 parts [it] travels 50 *du* and 281 parts, going into conjunction (*he*) with the sun. One conjunction is 292 days 281 parts, its travel through the stars being [also] as such (i.e., it is the same number of *du*).

金，夕伏，四十一日二百八十一分，行五十度二百八十一分，在日前九度，而見西方。

Metal (Venus), hidden in the evening: over 41 days and 281 parts [it] travels 50 *du* and 281 parts, making its [first evening] appearance (*xian*) in the west, 9 *du* before the sun.

見順，疾，日行一度九十一分度之二十二，九十一日行百一十三度。

Visible in prograde, fast: [it] travels 1 *du* and $\frac{22}{91}$ *du* per day, travelling [a total of] 113 *du* in 91 days.

微遲，日行一度十五分，九十一日行百六度。

Minutely slower: [it] travels 1 *du* and 15 parts per day, travelling [a total of] 106 *du* in 91 days.

而遲，日行四十六分度之三十三，四十六日行三十三度。

Then slow: [it] travels $\frac{33}{46}$ *du* per day, travelling [a total of] 33 *du* in 46 days.

留，不行，八日。

Station (*liu*): [it] does not move for 8 days.

旋逆，日行五分度之三，十日退六度，在日前九度，而夕伏西方。

Turning into retrograde: [it] travels $\frac{3}{5}$ *du* per day, retreating 6 *du* in 10 days, hiding in the evening in the west (i.e., last evening rising) 9 *du* before the sun.

除伏逆，一見二百四十六日，行二百四十六度，

Eliminating (*chu*, this time subtracting) hiding and retrograde, [this] one visibility [period] is 246 days, [during which it] travels 246 *du*.

伏五日，退四度而復合。

Hiding: over 5 days [it] retreats 4 *du* before returning into conjunction (*he*).

凡再合一終，五百八十四日有五百六十二分，行星如之。通率日行一度。

In total, one termination of its double (i.e., inferior and superior) conjunctions is 584 days 562 parts, its travel through the stars being [also] as such (i.e., it is the same number of *du*). Its overall *li* 率 rate is a daily motion of 1 *du*.¹⁶

¹⁶ Cited in *Hou Han shu*, *zhi* 3, 3071–72; tr. modified from Cullen, *Foundations of Celestial Reckoning*, 212–15.

Jia Kui, on the inequality of lunar motion (continued from [above](#)):

月行當有遲疾，不必在牽牛、東井、婁、角之間，又非所謂朏、側匿。

And lunar motion should [indeed] experience slowing and acceleration (*chi-ji* 遲疾), and not necessarily [just] around Led Ox_{L09} (winter solstice), Eastern Well_{L22} (summer solstice), Pasture_{L16} (spring equinox), and Horn_{L01} (autumn equinox) (as counted in equatorial *du*).¹⁷ Nor is this a matter of supposed *tiao* 朏 or *zeni* 側匿.¹⁸

乃由月所行道有遠近出入所生，率一月移故所疾處三度，九歲九道一復，凡九章，百七十一歲，復十一月合朔旦冬至。

[This] is produced, rather, by the fact that the *dao* it travels varies in distance [from the earth] and takes it above and below [the yellow road qua ecliptic]. The *lü* 率 are,¹⁹ in one lunation/month, the place where it was formerly fastest shifts 3 *du*,²⁰ such that the ‘nine roads’ make one return in in nine solar years (*sui*),²¹ and that in a total of nine ‘rules’ (*zhang* 章), 171 solar years (*sui*), [the nine-roads cycle] returns to [coincide with the coincidence of] syzygy, new moon day, month XI, and winter solstice.²²

合春秋、三統九道終數，可以知合朔、弦、望、月食加時。據官注天度為分率，以其術法上考建武以來月食凡三十八事，差密近，有益，宜課試上。

[The results of this model] coincide with the *Spring and Autumn Annals* [eclipse records] and the ‘nine road termination numbers’ of the Triple Concordance [*li*, circa 5 CE, by Liu Xin 劉歆 (c.50 BCE–23 CE)]. [It] can be [used to] know the added hour (*jiashi* 加時) of lunar syzygy, quarter, opposition, and eclipse.²³ Relying on the heavenly *du* noted by the [Clerk’s] Office (i.e., the astronomical bureau’s observational records) for the parts (fractions) and *lü* (rates) and using the(se) procedure(s) and model(s) to test (*kao* 考) all thirty-eight lunar eclipses from the Jianwu period on (i.e., 25–91 CE), [one finds that that the results’] the difference is tightly close (*mi jin* 密近). [This method] has the advantage, and it would be fitting [to be ordered] ranked (*ke* 課) and tested (*shi* 試) [by] the throne (?).²⁴

¹⁷ On the lodges and the position of the solstices and equinoxes therein, see [Figure 11](#). On the apparent discrepancy of solar velocity experienced around the solstices and equinoxes as a result of postulating a constant velocity around the *red road* (equator) – what he is talking about here – see Jia Kui’s earlier statement [above](#).

¹⁸ *Tiao* and *zeni* are early terms for the appearance of the moon in the west on new moon eve (*hui* 晦) and in the east on new moon day (*shuo* 朔), respectively, which count as ‘anomalies’ (*yi* 異) or, at least, evidence of the *li*’s inaccuracy. Without a model for the inequality of lunar motion, these ‘anomalies’ are hard to avoid.

¹⁹ *Lü* are not so much ‘rates’ as they are the integer components comprising them. On *lü* in mathematics and astronomy, see my paper ‘Remarks on the Mathematics and Philosophy of Space-time in Early China’ [here](#).

²⁰ The ‘place where [the moon] is fastest’ 所疾處 is what we call its *perigee* – the point where it is closest to the earth and, thus, its apparent motion the fastest. This shifts *forward*, in the direction of the moon’s orbit (‘rightward’ or eastward), thus experiencing *precession*, in what we call *apsidal precession*, one type of *lunar precession*.

²¹ After 8.85 years, the perigee returns to its original position, making the nine-year ‘nine roads’ period a reasonable approximation (on the nine roads, see Christopher Cullen, *Heavenly Numbers: Astronomy and Authority in Early Imperial China* [Oxford University Press, 2017]).

²² The nine-year ‘nine-roads’ scheme for lunar anomaly does not fit into the nineteen-year ‘rule’ (19 years = 235 lunations = 254 lunar circuits = 19 × 12 + 7 months), so the coincidence of these periods necessarily takes 9 × 19 = 171 years.

²³ The *jiashi* 加時 ‘added hour’ is the hour ‘added’ to the date, starting from midnight as converted from a fractional number of days, i.e. the hour.

²⁴ Cited in *Hou Han shu*, *zhi* 2, 3030; tr. modified from Cullen, *Foundations of Celestial Reckoning*, 390–91.

Li Chunfeng 李淳風 (602–670) on the discovery of other inequalities:

至後魏末，清河張子信，學藝博通，尤精曆數。因避葛榮亂，隱於海島中，積三十許年，專以渾儀測候日月五星差變之數，以算步之，始悟日月交道，有表裏遲速，五星見伏，有感召向背。

At the end of the [Tuoba] Wei, Zhang Zixin 張子信 (d. 577 CE) of Qinghe, who was broadly conversant in knowledge and skill, and particularly fine at *li* numbers, ensconced himself on a sea isle in order to flee the Ge Rong 葛榮 turmoil (526–528 CE), [and there he] racked up thirty-odd years devoting himself to measuring (*ce* 測) and watching (*hou* 候), via sphere sight (*hunyi* 渾儀, i.e., armillary sphere), the numbers of the change in the sun, moon and five [planets'] discrepancies (from their mean motions). Pacing (*bu*) them through calculation (*suan* 算), [he] first awakened [to the realisation] that the crossed roads of sun and moon have inside and outside (i.e., that the path of the moon is oblique to the ecliptic) and retardation and acceleration (i.e., uneven motion),²⁵ and that the appearance and hiding (*xian fu*, above) of the five stars (i.e., planets) experience stimulus and calling towards and away (i.e., seasonal variation depending on the obliquity of the ecliptic against the horizon at the moment of rising/setting).²⁶

Ptolemaic astronomy (Kuhn 1957, 64–73)

Here again we must diverge from Kuhn, as Ptolemaic astronomy only arrived in China much later, through Islamic sources. Instead, we will focus on a handful of important discoveries and improvements to the Chinese arithmetic tradition.

Zhou Cong 周琮, petition of 1065 CE:

書舉正南之星以正四方，蓋先王以明時授人，奉天育物。然先儒所述，互有同異。虞喜云：「堯時冬至日短星昴，今二千七百餘年，乃東壁中，則知每歲漸差之所至。」又何承天云：「堯典：『日永星火，以正仲夏；宵中星虛，以正仲秋。』今以中星校之，所差二十七八度，即堯時冬至，日在須女十度。」故祖沖之修大明曆，始立歲差，率四十五年九月却一度。

The *Documents* raises the due-south stars (meridian stars) for rectifying the four directions, [this] being what the former kings used to elucidate the seasons and grant [them] to people, to serve heaven and nurture [the myriad living] things. However, the accounts of former scholars have similarities and differences between them. Yu Xi 虞喜 (fl. 307–346 CE) says: 'In the time of Yao, on the winter solstice_{Q22} – shortest day – the star was Mane_{L18}; now, after 2700-odd years, Eastern Wall_{L14} is centred, which [we] know was arrived at by gradual difference every year' 每歲漸差. Moreover, He Chengtian 何承天 (370–447 CE) says: ... 'By comparison with the centred star of today, the difference is twenty-seven or eight *du*, i.e. on winter solstice_{Q22} in the time of Yao the sun was at Maid_{L10} 10 *du*'. Thus, when Zu Chongzhi 祖沖之 (429–500 CE) made the Grand Enlightenment *li*, he first established the year difference (*suicha* 歲差), [his] *lü* being forty-five years and nine months per *du* of retreat.²⁷

²⁵ Note that the obliquity of the lunar orbit – the 'white road' (*baidao* 白道) – was known from at least Jia Kui's time, *above*, and that it was mathematically modelled by Liu Hong 劉洪 (fl. 167–206), which Li Chunfeng knows. Zhang Zixin's contribution in this regard was, rather, discovering and modelling the inequality of *solar motion*.

²⁶ *Sui shu* 隋書 (Zhonghua shuju edn), 20.561; tr. modified from Daniel Patrick Morgan, *Astral Sciences in Early Imperial China: Observation, Sagehood and the Individual* (Cambridge: Cambridge University Press, 2017), 78.

²⁷ Cited in *Song shi* 宋史 (Zhonghua shuju edn), 74.1689; tr. modified from *Ibid.*, 183.

Zu Chongzhi 祖沖之 (429–500 CE), petition of 462 CE:

其二，以堯典云：「日短星昴，以正仲冬。」以此推之，唐代冬至，日在今宿之左五十許度。漢代之初，即用秦曆，冬至日在牽牛六度。漢武改立太初曆，冬至日在牛初。後漢四分法，冬至日在斗二十一。晉時姜岌以月蝕檢日，知冬至在斗十七。今參以中星，課以蝕望，冬至之日，在斗十一。

[Point] 2, the ‘Canon of Yao’ [chapter of the *Book of Documents*] says, ‘The day being at its shortest, and the asterism being Mao, he thereby determined mid-winter’.²⁸ Extrapolating from this, in [his Tao-]Tang 陶唐 dynasty, the sun at winter solstice was more than 50 *du* to the left (i.e., westward) of today’s lodge.²⁹ At the beginning of the Han dynasty (206 BCE), they used the Qin *li*, [which placed] the winter solstice sun at Led Ox_{L09} 6 *du*.³⁰ Emperor Wu of Han 漢武帝 (r. 141–87 BCE) reformed this [in 104 BCE] and established the Grand Inception *li* (Taichu li 太初曆), [which placed] the winter solstice sun at the beginning of Ox_{L09}. The Later Han Quarter-remainder model (Sifen fa 四分法) [placed] the winter solstice sun at Dipper_{L08} 21.³¹ In the time of the Jin 晉 (265–420 CE), Jiang Ji 姜岌 (fl. 384 CE) [figured out how to] check (*jian* 檢) the [position of the] sun via lunar eclipse, and from this he learned that the winter solstice was at Dipper_{L08} 17.³² Today, [in 462 CE], comparing (*can* 參) via the centred stars (*zhongxing* 中星, *above*) and testing (*ke*) via eclipse opposition, [we know that] the sun on winter solstice is at Dipper_{L08} 11.³³

通而計之，未盈百載，所差二度。

Accounted for together, [one finds that], in just under a hundred 100 years, the difference (*cha* 差) is 2 *du*.

舊法並令冬至日有定處，天數既差，則七曜宿度漸與曆舛。乖謬既著，輒應改制，僅合一時，莫能通遠，遷革不已，又由此條。

The old methods all gave the winter solstice sun a fixed position, [so] once the numbers of heaven differed, then the lodge-*du* [positions] (*xiudu* 宿度)³⁴ of the seven luminaries (*qi yao* 七曜)³⁵ would gradually diverge from *li* [predictions]. Once egregious mistakes became apparent, it would immediately require institutional reform, and since [such *li*] could only accord with a single time (*shi* 時, i.e. ‘age’ or ‘period’), none was able to connect (*tong* 通) with those more distant – all the modifications and reforms without end are due to this point.

今令冬至所在，歲歲微差，卻檢漢注，並皆審密，將來久用，無煩屢改。

Today, [let us] have the position of the winter solstice minutely differ from year to year 歲歲微差, and in going back to investigate (*jian*) Han [observational] notes, [we will find that] all of

²⁸ *Book of Documents*, tr. Bernhard Karlgren, ‘The Book of Documents’, *Bulletin of the Museum of Far Eastern Antiquities* 22 (1950): 3.

²⁹ According to his solar parameters and method for finding meridian stars as preserved in his Great Enlightenment *li* 大明曆, in the *Song shu* 宋書 (Zhonghua shuju edn), 13.291–304, the ‘year difference’ is 1 *du* per 45.92 years, requiring 2295.99 years to produce 50 *du* in precession. This would place the time of Yao in 1834 BCE, and, indeed, according to his solar table, Mao would then be the dusk meridian star at winter solstice.

³⁰ Note that this is an odd claim: most third to first-century BCE sources place the winter solstice at 牛初 or ‘Ox_{L09} 0 *du*’, as with the Grand Inception *li* in the following line.

³¹ Actually, it places it at ‘6 *du* before Ox_{L09}’ or ‘Dipper_{L08} 21¼ *du*’, as we will see when reading the Quarter-remainder *li*.

³² What Jiang Ji figured out is that, because lunar eclipse occurs at *opposition* the position of lunar eclipse allows you to know the exact position of the sun – i.e., opposite the moon – at the moment of eclipse.

³³ Actually Dipper_{L08} 11.71 *du* according to his solar table.

³⁴ E.g., ‘Dipper_{L08} 11 *du*’ 斗[宿]十一度.

³⁵ I.e., the sun, moon, and ‘five stars’ (the five naked-eye planets).

them, together, are carefully tight (*shenmi* 審密) [matches with my retrodictions], and that [my model/li] will [thus] see perpetual use [into the future], without [us needing to go to] the trouble of repeated reform.³⁶

Zu Chongzhi, response to the critique of Dai Faxing 戴法興 about another proposal concerning eclipse prediction (continued from the previous source):

按以日八行譬月九道，此為月行之軌，當循一轍，環帀於天，理無差動也。然則交會之際，當有定所，豈容或斗或牛？同麗一度，去極應等，安得南北無常？

One notes that if we take the eight motions of the sun (?) as an analogy for the nine roads of moon, this would have it that the track (*gui* 軌) of lunar motion should follow a single rut (*zhe* 轍) such that it turns in circles in heaven unerring and unmoving in its principle (*li* 理). Were this so, the juncture(s) of crossing (*jiao* 交)³⁷ and coincidence (*hui* 會) should have a fixed position, [so] how could it abide being sometimes in $O_{X_{L09}}$, sometimes in $Dipper_{L08}$?³⁸ And, stuck to the same *du*, its north polar distance should be equal, so why is there no constancy (*chang*) north and south?³⁹

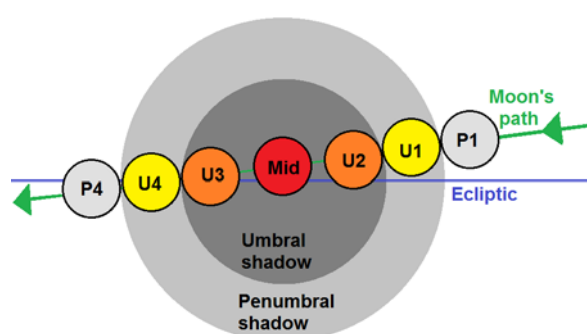


Figure 17 Lunar eclipse – the ‘coincidence’ (*hui*) of full moon and nodal transit, or ‘crossing’ (*jiao*). With a lunar (and solar) disc of diameter $\approx \frac{1}{2}^\circ \approx \frac{1}{2} du$ and an umbra of diameter $\approx 1.5^\circ \approx 1.5 du$, the moon is only eclipsed close to ‘crossing’. $5 du$ above or below the equator, it does not cross the earth’s shadow. Source: Wikipedia.

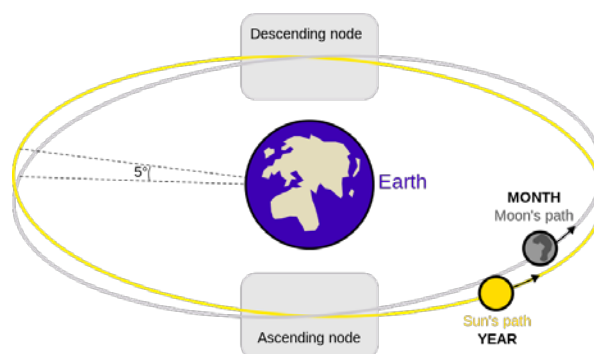


Figure 18 The obliquity of the lunar path and the nodes where it ‘crosses’ (*jiao*) that of the sun. Note that the nodes experience precession ‘left’ (westward), returning to the same position every ≈ 18.6 years. Source: Wikipedia.

³⁶ *Song shu*, 13.289–90. This ‘year difference’ (*suicha* 歲差) – a minute difference between the **tropical** and **sidereal year** – is a solution to **the precession of the equinoxes**, by which the ecliptic moves westward through the stars.

³⁷ ‘Crossing’ (*jiao* 交) is where the path of the moon – the white path (*baidao* 白道) – intersects the path of the sun – the yellow road. In modern terms, ‘crossing(s)’ refer to **the lunar nodes**.

³⁸ ‘Coincidence’ (*hui* 會) is a term that is normally used as concerns eclipses – the point/time where things ‘coincide’ for the sun or moon to be eclipsed. The more specific sense depends on the underlying eclipse model.

³⁹ In short, the moon travels on a path – the white path – approximately $5 du$ oblique to the ecliptic, which ‘crosses’ (*jiao*) it at two points – the lunar nodes, see Note 37. This was theorised and modelled some three centuries earlier by Liu Hong (see Cullen 2017a and 2017b, op. cit.). The nodes experience precession ‘left’ (westward), returning to the same position every ≈ 18.6 years in another form of **lunar precession**. As such, ‘the juncture(s) of crossing (*jiao*) and coincidence (*hui*)’ *do not* ‘have a fixed position’, wandering through ‘ $O_{X_{L09}}$, sometimes in $Dipper_{L08}$ ’, and all the other lodges. As such, ‘stuck to the same *du*’, i.e., at the same position vis-à-vis the stars, ‘its north polar distance’ *is not* ‘equal’ from one year to the next.