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# Mismatch and Wage Posting

Frederic Gavrel\*

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## **Abstract**

This paper provides a wage posting model of the labor market in which firms' strategies are pure. To that end, the persistency of vacant jobs results from a mismatch problem, not a pure coordination problem. Since firms cannot commit to an output cutoff lower than the announced wage, laissez-faire is inefficient. Under a binding condition however, public policy can restore market efficiency by associating a minimum wage with a layoff tax.

Key words : Posted wages, Mismatch, Efficiency, Minimum wage, Layoff tax.

JEL Classification numbers: J6. D8.

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# 1 Introduction

In the real world, firms obviously use the wage they post as a means to attract workers. To some extent, as in the household market, the price of labor is likely to be renegotiated ex post, but the assumption of no renegotiation seems to be the right starting point for an acceptable theory of wages. Thus it is not surprising that labor economists have been dissatisfied with the usual assumption of wage bargaining. In the literature on wage posting, also referred to as “directed search” (see Wright et al. 2017) for a synthetic survey of posted wages), we can read two stories: the submarket (or “island”) story, and the mixed strategy story. The submarket story is an interpretation of Moen (1997), which can be found in the textbook by Cahuc and Zylberberg (2004) for example. Surprisingly, although any firm can pick any one of its applicant to fill its vacancy, it is assumed that the probability of this firm filling its vacancy is a continuous increasing function of the number of its applicants. There is something surrealistic in this interpretation of competitive search. Indeed, either the firm has at least one applicant, and its vacancy will be filled with certainty; or the firm has not no applicants at all, and the job will remain vacant. The mixed strategy story is very well exposed in Albrecht et al.(2006), who use urn-ball matching. Each worker decides on the probability with which she will apply to each vacancy<sup>1</sup>. Consequently, if a firm increases its posted wage, all workers will raise their probability of applying to it. Mixed strategies permit the derivation of a continuous (increasing) relationship between the posted wage and the probability of the firm filling its vacancy. But the price to pay in terms of consistency and empirical relevance is high. Although they are more and more familiar to economists - lending a sense of relevance - mixed strategies remain at best a mathematical tool for solving a model which has no solution. But in that respect one can see that this wage posting model actually has a pure strategy equilibrium which is the usual competitive equilibrium. If there are fewer firms than workers the wage will coincide with the utility of leisure; whereas if there are fewer workers than firms, the wage will be set by the zero profit (free-entry) condition. This means that the coordination problem entirely stems from mixed strategies, whereas mixed strategies are not really necessary.

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<sup>1</sup>Since there is a continuum of vacant jobs this probability is a probability “intensity”. See Albrecht et al.

In the present paper we try to tell (what we consider to be) a more convincing story. To that end, we come back to the idea that the vacancy of some jobs results from the mismatch between those jobs and the workers who apply to them, thus not from a pure coordination problem. Like Marimon and Zilibotti (1999), workers and jobs are horizontally differentiated according to Salop’s (1979) circular model. Firms post wages and workers then decide on the (single) firm they will join - not on the probability of going to each firm, as usually encountered in the literature - depending on the wages and on the size of firms’ applicant pools. Ex ante, workers do not know the quality of a match with the firm they choose. Next, firms rank their applicants, select the best, and hire her if her (maximum) productivity is higher than their posted wage. If not, the job remains vacant. Thus, vacancies result from a mismatch problem, not from a pure coordination problem. As usual with wage posting, wages are set by maximizing the expected profits subject to the indifference constraint: in equilibrium, all applicant pools should generate the same expected income for workers. Thus when deciding on its posted wage, a firm faces a tradeoff between its labor costs and its expected output, which increases with the number of applicants, hence with a wage increase. The model is closed by the assumption of free-entry. As regards free-entry, it is worth noting that in this context applicant ranking plays a crucial role, since it creates a decreasing relationship between job creation and expected profits. An important feature of our model is that firms post a single wage (not as many wage rates as possible productivity levels) and that the output cutoff (i.e. the productivity level below which the best candidate is rejected) coincides with the wage. We believe that these assumptions are quite reasonable since the output of a match is not verifiable by a third party, meaning that firms can neither commit to a wage function (of productivity) nor to an output trigger lower than the wage. See for instance Blanchard and Tirole (2008) or Gavrel (2018) for similar assumptions in a model where workers are risk-averse.

In contrast to usual wage posting models, market equilibrium is inefficient. The reason for this is that firms cannot commit to a hiring cutoff lower than the posted wage. As a consequence, wages are too high in terms of job rejection, but too low in terms of job creation. To make this point crystal-clear, we consider a hypothetical “unconstrained equilibrium” in which firms actually can commit to the wage as well

as to the hiring cutoff (different from the wage). We find that this unrealistic equilibrium is a social optimum which satisfies two conditions. On the one hand, the hiring cutoff coincides with the utility of leisure and, on the other hand, job creation fulfills the so-called “generalized Hosios condition”, as Julien and Mangin (2017) put it. Firms internalize the usual congestion effect. Overall, they also internalize that additional vacancies reduce the number of applicants per firm, thus lowering the expected productivity of the best applicant: the productivity effect. The social optimality of this hypothetical equilibrium also indicates how public policy could restore efficiency: public intervention should make the output trigger equal to the value of leisure. This can be achieved through a self-financed tax/subsidy system. We show that, under a binding condition, an appropriate layoff tax associated with a minimum wage - equal to the unconstrained-equilibrium wage - can decentralize the social optimum.

Section 2 describes our setup and provides the definition of a (credibility-constrained) equilibrium. Section 3 develops the welfare analysis and shows how public policy can decentralize a social optimum.

## 2 Market structure and equilibrium

### 2.1 Market structure

In this market, there are “very large” numbers of workers and firms who are risk-neutral and heterogeneous. The number of workers is denoted by  $U$ . An important feature of this economy is that workers and jobs are horizontally differentiated according to Salop’s (1979) circular model. See Marimon and Zilibotti (1999) for the application to the labor market. The distance between the firm  $j$  and the worker  $i$  along the skills circle, denoted by  $x$ , measures the mismatch between the offered skills of worker  $i$  and the skill requirements of firm  $j$ . The output of a match  $(i, j)$  is then a decreasing (mismatch) function  $y(x)$  of this distance. Assuming that the workers and the jobs are uniformly distributed on the circle, Gavrel (2012) shows that the circular model is formally equivalent to match specific productivities. Let  $F(y)$  be the cumulative distribution of output according to a model of match specific productivities. It is not difficult to see that for any circular model defined by the mismatch function  $y(x)$  one can associate a model of match specific productivities

whose cumulative distribution satisfies  $F(y) = 1 - 2x(y)$ , with  $x(y)$  being the reciprocal of  $y(x)$ .<sup>2</sup> This isomorphism allows us to use match specific productivities which are simpler and more familiar than the circular model of differentiation; but at this stage we would like to stress that this simple tool of analysis can be derived from first principles. Thus, the productivity of a match is a random variable whose cumulative distribution,  $F(y)$ , is strictly increasing in the interval  $[0, 1]$ , and such that  $F(0) = 0$  and  $F(1) = 1$ . Its density is denoted by  $f(y)$  ( $f(y) > 0$ ).

Another peculiarity concerns the matching process. We retain very simple but understandable assumptions. As usual in wage posting models, workers know where to find the firms as well as the wages which they have posted. But they don't know the quality of a match with these jobs. Depending on the posted wages and on the number of applicants to each firm, a worker chooses which applicant pool to join. We then exclude multiple applications, implying that *ex post* firms' competition for workers is ruled out. The same holds for *ex post* competition between the workers of the same applicant pool.

The timing of this game is the following. In the first stage, firms freely decide whether or not they will enter the market. If they enter the market, they create a single vacancy, paying the cost  $c$  to that end. Next, firms post a wage knowing that an increase in this wage affects the size of their applicant pool. And, knowing those wages, workers decide on the applicant pool they will join (the job they will apply to) by comparing the expected incomes associated with the different jobs. Finally, firms interview all their applicants and select the best. If the output of the best worker is higher than the wage, then they hire this worker. Notice that, as usual with wage posting, firms are able to commit to the wage that they announce. As a consequence, the wage plays two roles here. On the one hand, it is used as a means of competing for workers; and on the other it determines the output cutoff below which firms decide not to hire their best applicant. This assumption sounds reasonable, but it clearly raises an efficiency issue, as emphasized in the welfare and policy analysis.

Formally, let  $w$  denotes the wage posted by a firm and  $n$  the number of its applicants. The output of the best applicant is the maximum statistic of a sample of  $n$  draws of the distribution  $F(\cdot)$ . Its density is

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<sup>2</sup>Note that, as usual, the circle length is normalized to one.

$$g(y, n) = nF(y)^{(n-1)}f(y),$$

and the expected output of this firm is given by

$$\int_w^1 g(y, n)ydy$$

Consequently, the expected profits of this firm (gross of the entry cost,  $c$ ), denoted by  $\Pi$ , satisfy

$$\Pi = \tilde{\Pi}(w, n) = n \int_w^1 F(y)^{n-1}f(y)(y - w)dy$$

Notice that the probability,  $\tilde{m}(w, n)$ , of the firm filling its vacancy is given by

$$\tilde{m}(w, n) = \int_w^1 g(y, n)dy = 1 - F(w)^n.$$

Indeed, as  $F(w)^n$  is the probability of all candidates having a productivity lower than the wage,  $1 - F(w)^n$  is the probability of at least one applicant having a productivity higher than the wage, hence the probability of the firm hiring a worker.

As all applicants face the same situation, their probability of getting the job is

$$p = \tilde{p}(w, n) = \frac{\tilde{m}(w, n)}{n}.$$

A peculiarity of this model is that the wage, which acts as an output cutoff, directly (negatively) affects the probability of filling a job as well as the probability of finding a job. Regarding the size of the applicant pool, one can see that an increase in the number of applicants raises the probability of the firm hiring a worker. By contrast, the productivity of a candidate getting the job is reduced.

It follows that the expected income of an applicant to this firm, denoted by  $W$ , has the following expression:

$$W = \tilde{W}(w, n) = d + \tilde{p}(w, n)(w - d),$$

with  $d$  being the utility of leisure.

As already mentioned, workers choose an applicant pool according to expected incomes. The stability of the applicant pools then requires that they all generate the same expected income. For given wages, this indifference condition (which is a kind of migration equilibrium) determines the distribution of workers across the applicant pools (i.e. the firms with a vacancy). Since the expected income is a decreasing function of the number of applicants we can deduce that such a distribution of workers across firms is stable.

## 2.2 Market equilibrium

All firms with a vacancy know the indifference condition. They then set their posted wage and the size of their applicant pool by maximizing their expected profit subject to the indifference condition.

Let  $j$  denote a firm with a vacant job, *ex ante*. A market equilibrium can be defined as a number of vacancies,  $V^*$ , a pair of functions  $(w^*(j), n^*(j))$ , and an expected income  $W^*$  such that

- i)  $(w^*(j), n^*(j))$  maximizes  $\tilde{\Pi}(w_j, n_j)$  subject to the indifference condition  $\tilde{W}(w_j, n_j) = W^*$ , for all  $j = 1, \dots, V^*$ ,
- ii)  $\sum_1^{V^*} n^*(j) = U$ ,
- iii)  $-c + \tilde{\Pi}(w_j^*, n_j^*) = 0$ , for all  $j = 1, \dots, V^*$  (free-entry).

In general competitive analysis the agents are price takers. Here, the firms which are “very small” take the expected income  $W$  as given. Thus this model describes a “competitive search” situation.

It is easy to see that, insofar as the maximization problem has a single solution (which is assumed), this equilibrium is symmetric. As a consequence, in this symmetric equilibrium workers are uniformly distributed among firms, implying that all applicant pools have the same size,  $n = U/V = 1/\theta$ , with  $\theta$  denoting market tightness.

In conformity with the notations used, any function,  $z = \tilde{z}(w, n)$ , is now expressed in terms of market tightness,  $\theta = V/U$ , that is  $z = z(w, \theta) = \tilde{z}(w, 1/\theta)$ . We can then define an equilibrium in a (much) simpler way:



**Definition 1.** A market equilibrium is a pair,  $(w^*, \theta^*)$ , which maximizes the expected profit,  $\Pi(w, \theta)$ , subject to the indifference constraint,  $W(w, \theta) = W(w^*, \theta^*) \equiv W^*$ , and which satisfies the free-entry equation,  $-c + \Pi(w, \theta) = 0$ .

As usual in wage posting models, the wage is determined by examining the behavior of a deviant firm who sets the wage  $w$  and (the inverse of) its applicant pool,  $\theta$ , by maximizing its expected gross profit,  $\Pi(w, \theta)$ , subject to the constraint that the expected income of its applicants,  $W(w, \theta) = p(w, \theta)w + (1 - p(w, \theta))d$ , be equal to the expected income of other workers,  $W^*$ .

Consider an interior private optimum ( $w > d$ ). The indifference constraint implicitly determines the posted wage as a function of market tightness. Differentiating this equation gives the impact of an increase in market tightness on the wage. We obtain:

$$\frac{dw}{d\theta} = -\frac{\frac{\partial p(\cdot)}{\partial \theta}(w-d)}{p(\cdot) + \frac{\partial p(\cdot)}{\partial w}(w-d)} \quad (1)$$

An increase in market tightness (a decrease in the number of applicants) raises expected incomes. Holding the cutoff as a constant, a wage increase raises the expected income as in usual wage posting models (term  $p = \theta m > 0$ ). But here the wage also exerts a negative effect on the probability of an applicant being hired by lowering the lower bound of the output (term  $\partial p(\cdot)/\partial w < 0$ ). As a consequence, a wage increase does not necessarily lead to an increase in the number of applicants (i.e. a decrease in market tightness). If this negative effect is sufficiently strong some applicants will leave the deviant firm, implying that the deviant firm should set a lower wage.

Let  $\varepsilon(m(\cdot), w)$  denote the elasticity of  $m(\cdot)$  to  $w$  in absolute value. Thus, an interior (private) optimum should satisfy the following condition:

**Condition (IC).**  $\varepsilon(m(\cdot), w) < 1$ .

In what follows,  $\bar{y}(\cdot)$  denotes the expected output of a filled job, that is

$$\bar{y}(\cdot) = \frac{\int_w^1 F(y)^{(1-\theta)/\theta} f(y) y dy}{\theta m(w, \theta)}.$$

It can be shown that, in conformity with intuition, the expected maximum output  $\bar{y}$  grows with an increase in the number of applicants, hence with a decrease in market tightness.<sup>3</sup>

Now, differentiating the expected profits with respect to market tightness gives:

$$\frac{d\Pi(\theta)}{d\theta} = \frac{\partial(m(\cdot)\bar{y}(\cdot))}{\partial\theta} - \frac{\partial m(\cdot)}{\partial\theta}w - m(\cdot)\frac{dw}{d\theta}.$$

Consider the term

$$-m(\cdot)\frac{dw}{d\theta} = \frac{m(\cdot)}{m(\cdot) + \frac{\partial m(\cdot)}{\partial w}(w-d)} \left[ \frac{m(\cdot)}{\theta} + \frac{\partial m(\cdot)}{\partial\theta} \right] (w-d).$$

We have:

$$\frac{m(\cdot)}{m(\cdot) + \frac{\partial m(\cdot)}{\partial w}(w-d)} = 1 - \frac{\frac{\partial m(\cdot)}{\partial w}(w-d)}{m(\cdot) + \frac{\partial m(\cdot)}{\partial w}(w-d)}.$$

Let  $\Gamma(\cdot)$  denote the expression

$$\Gamma(\cdot) \equiv \frac{\frac{\partial m(\cdot)}{\partial w}(w-d)}{m(\cdot) + \frac{\partial m(\cdot)}{\partial w}(w-d)} = -\frac{\varepsilon(m(\cdot), w)^{\frac{w-d}{w}}}{1 - \varepsilon(m(\cdot), w)^{\frac{w-d}{w}}} < 0 \quad (2)$$

Substitution into the previous expression for the derivative yields

$$\frac{d\Pi(\theta)}{d\theta} = \frac{\partial(m(\cdot)\bar{y}(\cdot))}{\partial\theta} - \frac{\partial m(\cdot)}{\partial\theta}d + \frac{m(\cdot)}{\theta}(w-d) - \Gamma(\cdot)\frac{m(\cdot)}{\theta}(1 - \eta(\cdot))(w-d).$$

Since

$$\frac{\partial m(\cdot)\bar{y}(\cdot)}{\partial\theta} = -\frac{m(\cdot)}{\theta}[\eta(\cdot)\bar{y}(\cdot) + \alpha(\cdot)(\bar{y}(\cdot) - d)],$$

with  $\eta(\cdot)$  ( $\alpha(\cdot)$ ) being the elasticity of  $m(\cdot)$  ( $\bar{y} - d$ ) with respect to  $\theta$  in absolute value, we obtain that

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<sup>3</sup>The proof follows the same line as Gavrel (2012).

$$\frac{\theta}{m(\cdot)} \frac{d\Pi(\theta)}{d\theta} = w - d - [\eta(\cdot) + \alpha(\cdot)](\bar{y}(\cdot) - d) - \Gamma(\cdot)(1 - \eta(\cdot))(w - d),$$

implying that a private (interior) optimum should satisfy

$$w - d = \frac{[\eta(w, \theta) + \alpha(w, \theta)](\bar{y}(w, \theta) - d)}{1 - \Gamma(w, \theta)(1 - \eta(w, \theta))} \quad (3)$$

It is worth noting that, as the equilibrium value of workers' expected income is deduced from the equilibrium values of the wage and the market tightness, the previous equation only means that, in a private optimum (of firms), firms and workers face the same marginal exchange rate between the wage and the number of applicants. Note also that, under condition (IC), the wage actually is higher than the value of leisure.

On the other hand, the free-entry equation can be written as

$$-c + m(w, \theta)(\bar{y}(w, \theta) - w) = 0 \quad (4)$$

To summarize:

**Proposition 1.** *A market equilibrium is a pair,  $(w^*, \theta^*)$ , which jointly satisfies equations (3), and (4).*

### 3 Efficiency and public policy

We now show that, despite wage posting, *laissez-faire* is inefficient. To achieve a precise understanding of this inefficiency result we consider the hypothetical case in which firms can commit to an output cutoff lower than the posted wage. On this basis we show how public policy can restore efficiency.

#### 3.1 Social optimum and market efficiency

As usual, our welfare criterion is the (net) aggregate income, i.e. the social surplus.

Consider the (net) aggregate income of this economy. The aggregate income per head, denoted by  $\Sigma$ , depends on two variables: the output cutoff,  $\hat{y}$ , and the number jobs per worker,  $n = U/V = 1/\theta$ .

This can be written as follows

$$-\theta c + \theta m(\hat{y}, \theta) \bar{y}(\hat{y}, \theta) + [1 - \theta m(\hat{y}, \theta)] d \quad (5)$$

It is easy to see that, whatever market tightness might be, the optimum cutoff coincides with the utility of leisure,  $d$ . Thus, the aggregate income can be rewritten as

$$-\theta c + \theta m(d, \theta) \bar{y}(d, \theta) + [1 - \theta m(d, \theta)] d \quad (6)$$

Differentiating the social surplus per worker with respect to  $\theta$  gives

$$-c + [m(\cdot) + \theta \frac{\partial m(\cdot)}{\partial \theta}] (\bar{y}(\cdot) - d) + \theta m(\cdot) \frac{\partial \bar{y}(\cdot)}{\partial \theta}$$

It follows that the first order condition for the maximization of the aggregate income can be written as

$$-c + m(d, \theta) [1 - \eta(d, \theta) - \alpha(d, \theta)] [\bar{y}(d, \theta) - d] = 0 \quad (7)$$

With  $\alpha(\cdot) = 0$ , we would obtain the usual Hosios condition for the efficiency of job creation. With  $\alpha(\cdot) > 0$ , we face the “generalized Hosios condition” as Mangin and Julien (2017) put it.

This optimality condition takes into account two externalities of an increase in the number of vacancies. The first is the usual congestion effect. The second effect results from the ranking of candidates. An increase in market tightness (a decrease in the number of applicants per firm) lowers the average output. In conformity with intuition, an increase vacancies lowers the number of applicants per firm, implying that the average of the maximum statistic is reduced.<sup>4</sup> This negative productivity effect tends to reduce job creation in an optimum. In other words, if the basic Hosios condition holds, then job creation is excessive.

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<sup>4</sup>As mentioned above, the proof follows the same line as Gavrel (2012).

Let us now compare the laissez-faire equilibrium with the social optimum. Substitution of the wage equation (3) into the free-entry condition (4) yields

$$-c + m(\cdot) \left[ 1 - \frac{\eta(\cdot) + \alpha(\cdot)}{1 - \Gamma(\cdot)(1 - \eta(\cdot))} \right] (\bar{y} - d) = 0 \quad (8)$$

Knowing that the equilibrium wage is higher than the value of leisure, the comparison of equations (7) and (8) show that the laissez-faire equilibrium is inefficient in terms of the output cutoff as well as in terms of market tightness. For obvious reasons, lowering the cutoff for a given labor cost - meaning that only the probabilities  $m(\cdot)$  and  $p(\cdot)$  are (positively) affected - increases the aggregate income. This holds true for any cutoff higher than the value of leisure. Lowering the cutoff means that some viable matches are no longer rejected. It is difficult to locate the equilibrium market tightness relative to the social optimum.<sup>5</sup> However, since  $\Gamma(\cdot) < 0$ , equation (8) implies that the derivative of the aggregate income with respect to market tightness,

$$-c + m(\cdot) [1 - (\eta(\cdot) + \alpha(\cdot)) (\bar{y} - d)],$$

is strictly negative in the neighborhood of market equilibrium. This means that job creation is excessive. On the other hand one can check that, according to the free-entry equation (4), market tightness is a decreasing (implicit) function of the wage. A small increase in the labor cost for a given rejection trigger - meaning that probabilities  $m(\cdot)$  and  $p(\cdot)$  are left unchanged - increases the aggregate income. To summarize, we can state the following

**Proposition 2.** *In market equilibrium, the wage is too high in terms of match rejection but too low in terms of job creation.*

The reason why the wage is too low regarding job creation is that the negative impact on the transition rates ( $m(\cdot)$  and  $p(\cdot)$ ) exerts a wage moderation effect.

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<sup>5</sup>To that end, one should retain specific assumptions on the distribution  $F(\cdot)$ .

## 3.2 Unconstrained equilibrium and public policy

### 3.2.1 Relaxing the credibility constraint

To reach a better understanding of the welfare results as well as to clarify the analysis of public policy we here consider the (unrealistic) case in which firms could commit to the wage as well as to the output cutoff. In other words, in this “unconstrained” equilibrium, firms are assumed to be able to disconnect the cutoff,  $\hat{y}$ , from the wage,  $w$ .

Let us first study how the output cutoff is determined in this situation. To that end, we show that for a given applicant pool - meaning that any variation in the cutoff should be associated with an appropriate variation in the wage which keeps workers’ expected income constant - the optimum of expected profits is reached for  $\hat{y} = d$ . From the indifference condition, we deduce workers’ marginal rate of substitution between the wage and the cutoff:

$$\frac{dw}{d\hat{y}} = -\frac{\frac{\partial m(\hat{y}, \theta)}{\partial \hat{y}}}{m(\hat{y}, \theta)}(w - d) \quad (9)$$

For obvious reasons, this MRS is positive. Holding the expected income,  $W$ , constant, an increase in the cutoff leads to a decrease in the probability of getting the job. This is neutralized by a wage increase.

Consequently the total derivative of expected profits with respect to the cutoff, for a given market tightness, is

$$\frac{d\Pi}{d\hat{y}} = -nF(\hat{y})^{n-1}f(\hat{y})(\hat{y} - w) - m(\hat{y}, \theta)\frac{dw}{d\hat{y}} \quad (10)$$

Substituting (9) into (10) yields

$$\frac{d\Pi}{d\hat{y}} = \frac{\partial m(\hat{y}, \theta)}{\partial \hat{y}}(\hat{y} - d) \quad (11)$$

Since  $\partial m(\hat{y}, \theta)/\partial \hat{y} < 0$ , we deduce from the previous derivative that an increase in the cutoff raises (reduces) profits if and only if the cutoff is lower (higher) than  $d$ , whatever the number of applicants is. Unsurprisingly, this means that in an unconstrained equilibrium, the cutoff necessarily coincides with the domestic output.

What implies that firms would be able to commit to a cutoff which can generate a loss.

Let us now turn to wage setting. The wage is derived from the maximization of profits subject to the indifference condition for  $\hat{y} = d$ . In the unconstrained situation, using the indifference constraint, the expected profit can be rewritten as a function of market tightness,  $\theta$ :

$$\Pi(\theta) = m(d, \theta)[\bar{y}(d, \theta) - d] - \frac{W - d}{\theta}.$$

Thus, maximizing the expected profit with respect to  $\theta$  gives

$$\frac{p(\cdot)}{\theta^2}(w - d) = -\frac{\partial m(\cdot)}{\partial \theta}(\bar{y}(\cdot) - d) - m(\cdot)\frac{\partial(\bar{y} - d)}{\partial \theta}.$$

Or

$$w = d + [\eta(d, \theta) + \alpha(d, \theta)](\bar{y}(d, \theta) - d) \tag{12}$$

From the substitution of (12) into the free-entry condition

$$-c + m(d, \theta)(\bar{y}(d, \theta) - w) = 0,$$

we deduce the following

**Proposition 3.** *In the absence of the credibility constraint market equilibrium is efficient.*

In fact, this unconstrained equilibrium is purely theoretical. Nonetheless, this possibility indicates how public policy could deal with the credibility constraint.

### 3.2.2 Public policy

From the analysis above, we can deduce that if firms are given the incentive to decide on a output trigger equal to the value of leisure, then market efficiency will improve. There are different Pigovian tax/subsidy schemes whose introduction in the credibility-constrained equilibrium are capable of creating the right incentive for firms, i.e. “tying their hands”. The simplest one consists of taxing *ex post* vacancies while subsidizing all jobs.

Suppose that tax authorities levy a tax  $f$  on (ex post) vacant jobs which is dedicated to the financing of a subsidy for job creation (i.e. a subsidy to all created jobs),  $h$ . In this scenario tax authorities collect the (total) tax amount,  $(1 - m(.))Vf$ , and pay the (total) subsidy amount,  $Vh$ . The balanced budget constraint then implies that  $h = (1 - m(.))f$ .

The Pigovian tax,  $f$ , should ensure that firms decide on the trigger  $\hat{y} = d < w$ . To that end, the *ex post* loss,  $w - d$ , should be equal to the firing (non-hiring) tax,  $f$ . If the maximum productivity (among the applicant pool) is higher (lower) than  $d$ , then the firm hires (rejects) its best applicant.

It is worth noting that this tax/subsidy measure resembles a layoff tax. In general, the motivation for layoff taxes is that firms should perceive that their layoff behavior increase the costs of unemployment. See Blanchard and Tirole (2008) and Gavrel (2017) for instance. These costs result from unemployment insurance. Our analysis shows that introducing a layoff tax is desirable even in the absence of unemployment insurance (workers being risk-neutral).

However, introducing a layoff tax is not sufficient to restore efficiency. Public policy should also act on the wage. As an illustration, we consider the introduction of a minimum wage,  $\sigma$ . From the analysis above (equation (12)), we deduce that the minimum wage should be set to

$$\sigma = \sigma_S \equiv d + (\eta(d, \theta_S) + \alpha(d, \theta_S))(\bar{y}(d, \theta_S) - d),$$

with  $\theta_S$  being the optimum value of market tightness.

We now show that this minimum wage,  $\sigma_S$ , associated with the layoff tax,  $f_S = \sigma_S - d$ , permits the decentralization of the social optimum if and only if the following (binding) condition,

**Condition (BC).**  $\eta(d, \theta_S) < \varepsilon(m(d, \theta_S), w) \frac{\sigma_S - d}{\sigma_S}$ ,

is satisfied.

Due to the budget constraint, we can see that with the pair  $(\sigma_S, f_S)$  the equilibrium equation for job creation,

$$-c + m(d, \theta)(\bar{y}(d, \theta) - \sigma_S),$$



coincides with the (social) optimum equation. But this is not sufficient. The minimum wage,  $\sigma_S$ , should be binding, meaning that firms should not be willing to set a higher wage than this minimum. We must show that, under condition (BC), the total derivative of expected profits with respect to the wage is negative. The calculus of the (total) derivative of the expected profits with respect to  $\theta$  is very close to the proof of Proposition 1. The appendix shows that this derivative has the following expression:

$$\frac{\theta}{m(\cdot)} \frac{d\Pi(\theta)}{d\theta} = w - d - [\eta(\cdot) + \alpha(\cdot)](\bar{y}(\cdot) - d) - [\eta(\cdot)f + \Gamma(\cdot)(1 - \eta(\cdot))(w - d)] \quad (13)$$

For the minimum wage,  $\sigma_S$ , and the layoff tax,  $f_S = \sigma_S - d$ , the derivative reduces to

$$\frac{d\Pi(\theta)}{d\theta} = -\frac{m(\cdot)}{\theta} [\eta(\cdot)f_S + \Gamma(\cdot)(1 - \eta(\cdot))f_S].$$

Thus, the derivative of the expected profits with respect to market tightness,  $d\Pi/d\theta$ , has the same sign as

$$-\eta(d, \theta_S) - \Gamma(d, \theta_S)(1 - \eta(d, \theta_S)),$$

or<sup>6</sup>

$$-\eta(d, \theta_S) + \varepsilon(m(d, \theta_S), w) \frac{\sigma_S - d}{\sigma_S}.$$

The previous expression is positive under condition (BC). Since  $dw/d\theta < 0$ , this proves that the minimum wage,  $\sigma_S$ , is binding under condition (BC).

To summarize:

**Proposition 4.** *Under condition (BC), the minimum wage,  $\sigma_S$ , coupled with the layoff tax,  $f_S = \sigma_S - d$ , permits the decentralization of a social optimum.*

One could wonder why the optimum minimum wage is not necessarily binding. The reason is that the layoff tax creates an incentive to raise the wage. Indeed, a

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<sup>6</sup>Remember that the analysis is restricted to the case in which  $dw/d\theta < 0$ , hence to the case in which  $\varepsilon(m(\cdot), w) < 1$ .

wage increase attracts more applicants to the job. This lowers the probability of firing (not hiring) the best applicant, and so the probability of paying the layoff tax.

## 4 Conclusion

This paper provides a wage posting model with heterogeneous agents in which the vacancy of jobs results from a mismatch problem. Under the reasonable assumption that firms cannot commit to a mismatch cutoff which could generate losses, laissez-faire equilibrium is inefficient. But, under a binding condition, associating a minimum wage with a layoff tax permits the decentralization of a social optimum.

From a purely theoretical perspective, it is not surprising that labor theorists first sought to provide a model in which agents are homogenous. We subscribe to this criterion of good theory. But the application of this criterion revealed itself to be quite difficult, implying that one is led to introduce some *ad hoc* hypothesis, rather like the “ether” in pre-Einsteinian physics. We acknowledge that our model is a “lesser evil”, but we really believe that horizontal heterogeneities are a better “ether” (lesser evil) than mixed strategies.<sup>7</sup>

For the sake of brevity we developed a static model, but we can reasonably predict that our results will extend to a dynamic setting.

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<sup>7</sup>Burdett and Mortensen (1998), in which the persistence of vacant jobs stems from on-the-job search is a much lesser evil but it seems to be too far from DMP structure, which is the favorite tool of applied theory since the beginning of the nineties.

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## **Appendix. Derivative of expected profits in the presence of a layoff tax**

In the presence of the layoff tax, the expected profits have the following expression:

$$\Pi(w - f, \theta) = \int_{w-f}^1 g(y, \theta) y dy - m(w - f, \theta) w - (1 - m(w - f, \theta)) f.$$

On the one hand, the derivative of profits with respect to the wage reduces to

$$\frac{\partial \Pi(\cdot)}{\partial w} = -m(\cdot).$$

On the other hand, the derivative of expected profits with respect to market tightness is written as

$$\frac{\partial \Pi(\cdot)}{\partial \theta} = \frac{\partial(m(\cdot)\bar{y}(\cdot))}{\partial \theta} - \frac{\partial m(\cdot)}{\partial \theta}(w - f).$$

Now, from the indifference constraint, we deduce

$$\frac{dw}{d\theta} = -\frac{\frac{\partial p(\cdot)}{\partial \theta}(w - d)}{p(\cdot) + \frac{\partial p(\cdot)}{\partial w}(w - d)}.$$

It follows that the total derivative of expected profits with respect to market tightness has the expression below.

$$\frac{d\Pi(\theta)}{d\theta} = \frac{\partial(m(\cdot)\bar{y}(\cdot))}{\partial \theta} - \frac{\partial m(\cdot)}{\partial \theta}(w - f) - m(\cdot)\frac{dw}{d\theta}.$$

Consider the term

$$-m(\cdot)\frac{dw}{d\theta} = \frac{m(\cdot)}{m(\cdot) + \frac{\partial m(\cdot)}{\partial w}(w - d)} \left[ \frac{m(\cdot)}{\theta} + \frac{\partial m(\cdot)}{\partial \theta} \right] (w - d).$$

We have

$$\frac{m(\cdot)}{m(\cdot) + \frac{\partial m(\cdot)}{\partial w}(w - d)} = 1 - \frac{\frac{\partial m(\cdot)}{\partial w}(w - d)}{m(\cdot) + \frac{\partial m(\cdot)}{\partial w}(w - d)}.$$

Let  $\Gamma(\cdot)$  denote the expression

$$\Gamma(\cdot) \equiv \frac{\frac{\partial m(\cdot)}{\partial w}(w - d)}{m(\cdot) + \frac{\partial m(\cdot)}{\partial w}(w - d)} = -\frac{\varepsilon(m(\cdot), w)^{\frac{w-d}{w}}}{1 - \varepsilon(m(\cdot), w)^{\frac{w-d}{w}}}.$$

Substitution into the previous expression for the derivative yields

$$\frac{d\Pi(\theta)}{d\theta} = \frac{\partial(m(\cdot)\bar{y}(\cdot))}{\partial \theta} + \frac{\partial m(\cdot)}{\partial \theta}(f - d) + \frac{m(\cdot)}{\theta}(w - d) - \Gamma(\cdot)\frac{m(\cdot)}{\theta}(1 - \eta(\cdot))(w - d).$$

Since

$$\frac{\partial m(\cdot)\bar{y}(\cdot)}{\partial \theta} = -\frac{m(\cdot)}{\theta}[\eta(\cdot)\bar{y}(\cdot) + \alpha(\cdot)(\bar{y}(\cdot) - d)],$$

we obtain equation (13) of the text.