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A Radical Proposition on the Origins of the Received Mathematical Classic *The Gnomon of Zhou* (Zhoubi 周髀)

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Abstract In 656, *The Gnomon of Zhou* (Zhoubi 周髀) was canonised as one of *The Ten Mathematical Classics* (*Suanjing shishu* 算經十書) that would serve as the basis of contemporary curricula and testing and, later, the study of the history of mathematics in ancient China. More specifically, *The Gnomon of Zhou* was canonised as the *first* of said classics, and scholars ever since have treated it as the earliest of its kind, most today following Qian Baocong (1968) in positing that it was redacted from earlier materials *circa* 100 BCE. In this paper, I will present linguistic evidence that supports Christopher Cullen (1996) and early Qian Baocong's (1924; 1932) position that the received *Gnomon of Zhou* came together much later, in the first or second century CE, at around the same time or later than *The Nine Chapters on Mathematical Procedures* (*Jiuzhang suanshu* 九章算術). In addition, I will present evidence connecting its early reception and transmission history to the far north / north-west that might explain features of this strange text and its strange position discussions of cosmology in the Middle Period Chinese capitals further south.

Key words Chinese Mathematics, *The Gnomon of Zhou* (Zhoubi 周髀), text dating

1 Introduction

The *Zhoubi* 周髀—*The Gnomon of Zhou*—is one of the earliest mathematical texts to have been passed down through the Chinese written tradition. For those unfamiliar with the text, scholars typically follow the earliest commentary in dividing the text into three discrete parts. Part 1 is a dialogue between the semi-legendary eleventh-century BCE figures of Shang Gao 商高 and the Duke of Zhou 周公 (r. 1042–1036 BCE) on the origin of numbers and the *ju* 矩 ‘trysquare’. Part 2 is a dialogue between the presumably later figures of Chenzi 陳子 and Rong Fang 榮方 on how to calculate the dimensions of the cosmos using gnomon shadows, the base and height (*gougu* 勾股) of right triangles and the rule that one *cun* 寸 (≈ 2.31 cm) at the observer corresponds to a difference of 1000 *li* 里 (≈ 415.8 km) at the cosmic scale. Lastly, part 3—the body—goes on to describe the *gaitian* 蓋天 ‘umbrella heaven’ world model for which the text is famous as well as the daily travel of the mean moon, a shadow table, and the lunisolar resonance periods typical of a ‘quarter-remainder’ astronomical system, or *sifen li* 四分曆.¹

* My thanks to Tang Quan 唐泉 and Qu Anjing 曲安京 for graciously inviting me to this event, and to Alexis Lycas and Eric Trombert for helping with my historical geography. A PDF version of this paper is available for download at <https://halshs.archives-ouvertes.fr/halshs-01900269>.

¹ This traditional division of the text into three chronological layers is as per Qu Anjing (2002: 7–8); cf. Cullen (1996: 138–156).

As to what this ‘umbrella heaven’ looks like, *The Gnomon of Zhou* is somewhat vague, which, combined with other, conflicting versions of ‘umbrella heaven’ in early sources, has led to considerable disagreement. In a nutshell, what *The Gnomon of Zhou* has to say is this: Heaven and Earth are like inverted pans of the same shape and size;² Heaven is 80 000 *li* above the Earth, resting (apparently) on a grand pivot called the *xuanji* 璇璣;³ and as to account for seasonal differences in altitude, and the hour and direction of its apparent rising and setting, the sun, being affixed to Heaven, shifts seasonally through seven circular tracks (the *qi heng* 七衡) of different diameters concentric with the celestial pole. As to what this configuration is meant to explain, the text focuses on geographical conditions at the edges of the Chinese world:

故日運行處……極西，西方日中，東方夜半。

Therefore when the sun’s rotation has brought it to a position[...] west of the pole, it is noon in the western region and midnight in the eastern region. [...] ⁴

北極左右，夏有不釋之冰……中衡去周七萬五千五百里。中衡左右冬有不死之草。

Around the north pole, there is unmelting ice in summer. [...] The middle *heng* is 75 500 *li* (\approx 31 393 km) from Zhou, and near the middle track (the equator) there are plants that do not die in winter. ⁵

To help visualise what this might look like, I offer the Christopher Cullen’s reconstruction in Fig. 1 as one plausible interpretation. ⁶

In 656 CE, *The Gnomon of Zhou* was canonised as one of *The Ten Mathematical Classics* (*Suanjing shishu* 算經十書) that would go on to serve as the basis of the contemporary mathematical curriculum and later historiography. Of the ten, *The Gnomon of Zhou* was placed *first*, prior to the famous *Nine Chapters on Mathematical Procedures* (*Jiuzhang suanshu* 九章算術). This is despite the fact that the two are both *pseudepigraphal*, the tradition surrounding both texts equally claiming the Duke of Zhou as its author. ⁷ No one still believes this. ⁸ Rather, it is probably safe to say that most everyone now believes that the two were compiled from earlier sources

² More specifically, ‘Heaven resembles a covering rain-hat, while Earth is patterned on an inverted pan’ 天象蓋笠，地法覆槃, *Zhoubi suanjing* 周髀算經 (*Sibu congkan* 四部叢刊 edn.), 2.2b; tr. Cullen (1996: 189).

³ This is a reference to the famous *xuanji yuheng* 璇璣玉衡 ‘rotating mechanism and jade traverse’ mentioned in the *Book of Documents*, on which see Cullen and Farrer (1983).

⁴ *Zhoubi suanjing*, 2.1b–2a; tr. Cullen (1996: 189).

⁵ *Zhoubi suanjing*, 2.8b–9a; tr. modified from Cullen (1996: 192).

⁶ For a survey of alternative interpretations, see Qu Anjing (2002: 91–143).

⁷ In addition to featuring a dialogue with the Duke of Zhou in its introduction, in *Zhoubi suanjing*, 1.38a, we read: ‘Rong Fang asked “What is meant by [the term] *zhou bi*?” Chen Zi replied “In ancient times the Son of Heaven ruled from Zhou. This meant that quantities were observed at Zhou, hence the term *zhou bi*. *Bi* means *biao* 表, gnomon.”’ 榮方曰：「周髀者，何？」陳子曰：「古時天子治周，此數望之從周，故曰「周髀。」 (tr. Cullen 1996: 179); cf. Liu Hui’s 劉徽 263 CE preface to *The Nine Chapters*: ‘The Duke of Zhou fixed the rites and we had “the nine numbers” [mentioned in the *Zhouli* 周禮], and the school of the Nine Numbers was none other than *The Nine Chapters*’ 周公制禮而有「九數」，「九數」之流，則「九章」是矣 (tr. modified from Chemla and Guo 2004: 127).

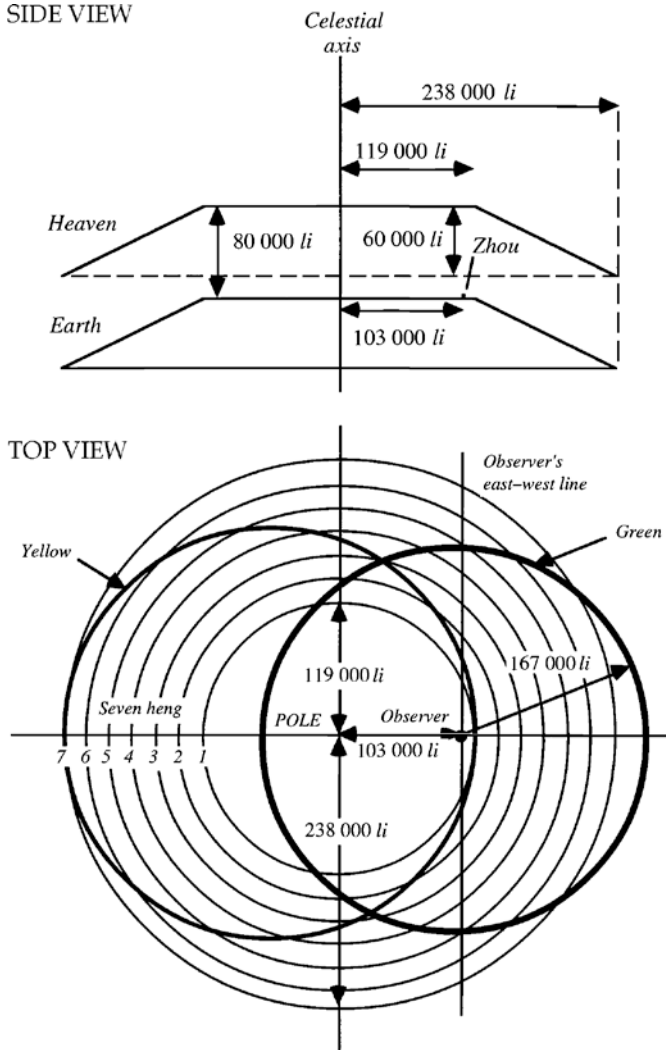


Fig. 1 *Gnomon of Zhou* ‘umbrella heaven’ (*gaitian* 蓋天) world model as reconstructed in Cullen (1996, 136, fig. 13, 222, fig. 31).

⁸ Indeed, one sees scepticism about *The Gnomon of Zhou*’s antiquity as early as He Cheng-tian 何承天 (370–447), Xu Yuan 徐爰 (395–475) and Shen Yue’s 沈約 (441–513) histories of the field in *Song shu*, 13.679, as studied in Morgan (2017: 188–196). Note that some scholars such as Wu Wenjun (1998–2004: vol. 1, 386–390) and Qu Anjing (2002: 4–8) nonetheless maintain that the earliest layer – the Shang Gao dialogue – originates in oral tradition *about the Duke of Zhou* passed down from the eleventh century BCE. Here it is important to note that the language of said dialogue resembles nothing of that of Zhou-era bronze inscriptions or the early classics of *The Book of Documents* or *The Book of Odes*; compare for example with the sources studied in Shaughnessy (1991).

under the Han 漢 (206 BCE–220 CE); that *The Nine Chapters* was in circulation around 90 CE; and that, as per Qian Baocong's 錢寶琮 1963 critical edition of *The Ten Mathematical Classics*, *The Gnomon of Zhou* was redacted two centuries earlier, around 100 BCE.⁹ In this talk I will attempt to flip this traditional order on its head.

2 Prior Attempts to Date *The Gnomon of Zhou*

The one thing that *no one* contests is the *termini ante quem*, the absolute limits before which each text must have been written. We know that *The Nine Chapters* was in circulation by the end of the first century CE as it occurs in Ma Xu's 馬續 (fl. 111–141 CE) childhood curriculum, and given the known dates of his younger brother Ma Rong 馬融 (79–166 CE), we can place this in 93 CE at the very latest:

七歲能通『論語』，十三明『尚書』，十六治『詩』，博觀羣籍，善『九章算術』。

At seven *sui* of age [Ma Xu] was able to master *The Analects*, at thirteen he understood *The Documents*, and at sixteen he had perfected *The Odes*; he was broadly read in a host of books and adept at *The Nine Chapters on Mathematical Procedures*.¹⁰

By contrast, the first mention of *The Gnomon of Zhou* comes a century later in Cai Yong's 蔡邕 (133–192 CE) memorial from Shuofang 朔方, Ordos, in 178 CE:

論天體者三家，宣夜之學，絕無師法。『周髀』術數具存，考驗天狀，多所違失。惟渾天僅得其情……

The discourse on heaven's form is comprised of three expert-lineages, but the study of expansive night has died out and has no master method. The procedures and numbers of *The Gnomon of Zhou* all survive, but when examined and verified against the case of heaven, there is much that misses the mark. It is only sphere heaven which completely grasps the true circumstances. [...] ¹¹

Unless Cai Yong was secretly the author, this means that *The Gnomon of Zhou* was in circulation prior to 178 CE, so the question is how long it took for people like Cai Yong to start talking about it.

Following Qian Baocong (1963 [1998]), most would say about three hundred years. However, Qian Baocong's 1963 argument for dating the text to 100 BCE builds on the earlier, more developed arguments presented in his 'Zhoubi suanjing kao' 周髀算經攷 of 1924 and *Zhongguo suanxue shi* 中國算學史 of 1932. In 1924, Qian Baocong concludes that:

⁹ See for example Qian Baocong (1963 [1998]: 4), Jiang Xiaoyuan (1996: 43), Jiang Xiaoyuan (1997: 207), Wu Wenjun (1998–2004: vol. 1, 386–390), Qu Anjing (2002: 8) and Chemla (2013a: 177; 2014).

¹⁰ *Hou Han shu* 後漢書 (Zhonghua shuju 中華書局 edn.), 24.862. One is born at one *sui* of age and turns two at New Years, thus 16 *sui* = 14 years of age, and given that he was born before Ma Rong, that places the sixteen-*sui*-old Ma Xu sometime prior to 79 + 14 = 93 CE.

¹¹ *Song shu* 宋書 (Zhonghua shuju edn.), 23.673; *Jin shu* 晉書 (Zhonghua shuju edn.), 11.278; *Sui shu* 隋書 (Zhonghua shuju edn.), 19.505; tr. modified from Ho (1966: 49). For a translation of the full memorial, preserved only in the *Song shu*, see Morgan (forthcoming).

The mathematics of *The Gnomon of Zhou* is already rather complete as concerns the multiplication and division of fractions and square root extraction, and compared to the procedures in the chapters 'Rectangular Field' and 'Reduced Width' in *The Nine Chapters on Mathematical Procedures* it is difficult to say which is better or worse. They might well be contemporary works (Qian Baocong 1924 [1983]: 134).¹²

Likewise, in 1932 he writes:

I thus consider that *The Gnomon of Zhou* was written somewhere roughly between Liu Xiang's 劉向 *Seven Summaries* (*Qi lüe* 七略) [of 6/1 BCE] and Wang Chong's 王充 *Balanced Discourse* (*Lunheng* 論衡) [of 80 CE], which is to say around the turn of the [Common] Era (Qian Baocong 1932 [1998]: 199).¹³

So why does Qian Baocong later move this date back a full century to 100 BCE? His simplified argument of 1963 cites four points:

1. That the *huntian* 渾天 'sphere heaven' (Fig. 2), which appears in the first century BCE, had to have come after [and immediately replaced] 'umbrella heaven' [because it is better]
2. That the quarter-remainder style astronomy in *The Gnomon of Zhou* reflects that abandoned by the government in 104 BCE [but continuously studied and re-adopted in 85 CE]
3. That the received text uses the old name, Qizhe 啟蟄 'Awakened Insects', for the third of the twenty-four *qi* rather than the new legal name, Jingzhe 驚蟄 'Startled Insects', relating to a posthumous taboo on Emperor Liu Qi's 劉啟 (d. 141 BCE) given name
4. That the text uses the classical position of the winter solstice sun, 'at the beginning of Ox' 牛初 (β Cap), rather than the new one, 'five *du* before Ox' 牛前五度, [adopted by government experts only in 85 CE]

In conclusion, he offers that:

From these materials and other period-specific language, we have determined that *The Gnomon of Zhou* was written *circa* 100 BCE (Qian Baocong 1963 [1998]: 4).¹⁴

There are a number of problems with this argument, as indicated by my use of brackets and openly admitted in Qian Baocong's earlier writings. As concerns the shift from Qizhe to Jingzhe in 141 BCE, for example, Qian Baocong (1924 [1983]: 134) acknowledges that the former does continue to see sporadic use into the Common Era, notably in the works of Li Chunfeng 李淳風 (602–670 CE), the lead editor of the *Ten Mathematical Classics*, and that it may well be, I quote, 'that while editing and commentating *The Gnomon of Zhou* he changed *jing* "startled" to

¹² 『周髀』算學于分數乘除及開方算法，已甚完備。與『九章算術』方田、少廣兩章之術，難分軒輊。或是同時代之著作。

¹³ 故余以為『周髀』之撰述時代，大約在劉向『七略』以後，王充『論衡』以前。或為西曆紀元左右之書也。

¹⁴ 我們根據這些資料和其他有時代性的文字，斷定『周髀』是公元前 100 年前後的作品。

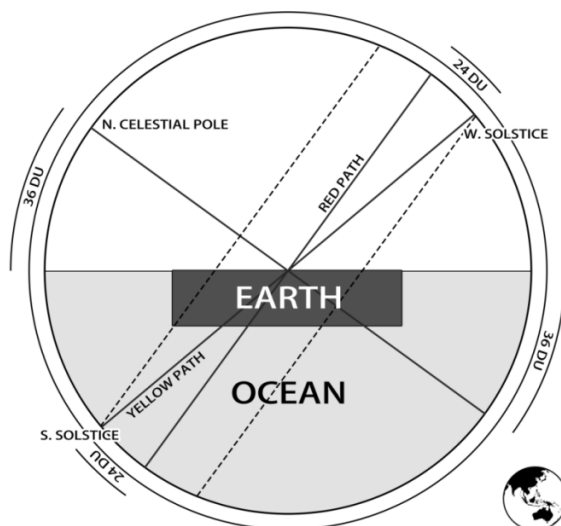


Fig. 2 Zhang Heng's 張衡 (78–139 CE) 'sphere heaven' (*huntian* 渾天) world model. Dotted lines represent the daily course of the sun at the winter and summer solstices; at the equinoxes, the sun travels on the Red Path (*chidao* 赤道). The globe in the bottom right corner is to scale with Zhang Heng's dimensions of heaven. At 1000 *li* (415.8 km) in diameter, the sun is about twice the size of Sri Lanka. Source: CC BY D.P. Morgan.

qi "awakened".¹⁵ The biggest challenge to this argument is however Christopher Cullen's (1996) observation that *The Gnomon of Zhou* incorporates elements of 'sphere heaven' cosmology and instrumentation such as the measurement of an object's polar distance (*quji du* 去極度), or anti-declination, which goes back no earlier than Xianyu Wangren's 鮮于妄人 observation programme of 78–74 BCE.¹⁶ If, as Cullen convincingly argues, the received *Gnomon of Zhou* was written as an anti-sphere heaven polemic pseudepigraphically set in ancient times for purposes of legitimization, then that stands to explain its more ancient technical contents: they are not so much *antique* as they are purposefully *antiquarian*.

Scholars such as Fu Daiwie 傅大為 (1988) have underlined that there are strong parallels in contents between *The Gnomon of Zhou* and the *weishu* 緯書 apocrypha that first appear in the mid-first century CE: the cosmic dimensions, the solar tables, the interest in cosmology and the return to an ancient quarter-remainder astronomy.¹⁷ What is more, I think that it is equally important to recognise the similarity in

¹⁵ 李淳風造『麟德曆術』，主用啟蟄舊名。當其校注『周髀』時，改「驚」為「啟」。One notes that the ancient term *Qizhe* 啟蟄 also appears in Wang Chong's *Lunheng* of 80 CE, Wang Su's 王素 third-century CE *Kongzi jiyu* 孔子家語, Zhang Zhouxuan's 張胄玄 modified *Daye li* 大業曆 of 608 CE and Fu Renjun's 傅仁均 *Wuyin li* 戊寅曆 of 618 CE (*Lunheng* (Sibu congkan edn.), 25.16b; *Kongzi jiyu* (Sibu congkan edn.), 3.8b; *Daye li*, cited in *Sui shu*, 17.454 ff.; *Wuyin li*, cited in *Jiu Tang shu* 舊唐書 (Zhonghua shuju edn.), 32.1153 ff.).

¹⁶ On Xianyu Wangren's observation programme and the measurement of north polar distance by armillary sphere in 78–74 BCE, see Sun and Kistemaker (1997).

¹⁷ On the apocrypha, see Dull (1966). On the astronomical contents of the apocrypha, see also Takeda (1989), Chen Meidong (2003: 167–176; 2007: 475–483) and Wu Jiabi (2007).

presentation: the apocrypha too are revelations written in antiquarian language in later times and attributed to ancient authors. It was these texts that would come to define the political and intellectual culture of the first and second centuries CE, and if it is with these texts that *The Gnomon of Zhou* fits in terms of contents and presentation, we might consider the possibility that it was a product of the same age. Still, we have yet to deal with the question of ‘period-specific language’.

3 Linguistic Markers

To be clear, Qian Baocong (1963 [1998]: 4) does not specify what ‘other period-specific language’ later led him to the date 100 BCE, but this raises the question of whether we might be able to identify linguistic markers of a *terminus post quem*.

When last I was in Xi'an, in 2015, I presented my preliminary findings from a comparison of the language of basic operations in *li* 曆 mathematical astronomy up to 600 CE. The way that these texts prescribe operations such as division is variable and ambiguous, so the idea was to identify *every operation* in *every text*, to individually determine what every instance of a word like *chu* 除 ‘eliminate’ must mean in context for the sums to work out, then, from the data compiled, to try to discern some pattern in usage.¹⁸ This summer, I successfully extended this analysis to all sources for *suan* 算 mathematics, excavated and received, covering the same period.¹⁹ It is in the context of this work I began to question *The Gnomon of Zhou*'s place as the earliest of the classics.

In the earliest sources we now possess, the third- and second-century BCE manuscripts *Numbers* (*Shu* 數), **Writings on Calculation* (*Suan shu* 算書) and *Writings on Calculating Numbers* (*Suanshu shu* 算數書),²⁰ the word *ying* 盈 – ‘fill’ – is used in two senses. The first is ‘greater than or equal to’, particularly as applied to remainders:

如法而一步，不盈步者，以法命之。

As it is up to (\div) the divisor, then 1 *bu*, and take that which does not fill (*ying*) a *bu* and name it by the divisor (*Shu*, slip 15).

¹⁸ Daniel P. Morgan, ‘Reflections upon the presentation of parallel algorithms across the astral and mathematical sciences in first-millennium China’, talk delivered at the International Conference on History of Ancient Mathematics and Astronomy, Northwest University, Xi'an, 23–29 August 2015 (<https://halshs.archives-ouvertes.fr/halshs-01333721>); cf. the earlier version Daniel P. Morgan, ‘By process of elimination: further remarks on the operation *chú* 除 in early imperial mathematical astronomy’, talk delivered at Mathematical Practices in Relation to the Astral Sciences, Université Paris Diderot, 26–31 March 2015 (<https://halshs.archives-ouvertes.fr/halshs-01333725v1>).

¹⁹ The resulting book chapter has been submitted for consideration in the forthcoming volume *Mathematical Practices in Relation to the Astral Sciences*, ed. Matthieu Husson and Agathe Keller (Springer) (submission available online at <https://halshs.archives-ouvertes.fr/halshs-01333725v2>).

²⁰ For pictures and transcriptions, see *Yuelu shuyuan cang Qin jian (er)* 嶽麓書院藏秦簡 (貳), ed. Zhu Hanmin 朱漢民 and Chen Songchang 陳松長 (Shanghai: Shanghai cishu chubanshe, 2011), Han Wei (2013) and *Zhangjiashan Han mu zhujian (ersiqi hao mu)* 張家山漢墓竹簡 (二四七號墓), ed. Zhangjiashan ersiqi hao Han mu zhujian zhengli xiaozu 張家山二四七號漢墓整理小組 (Beijing: Wenwu chubanshe, 2001).

The second is ‘excess’ versus ‘insufficiency’ (*ying buzú* 盈不足) as used in the rule of false position:

贏（盈）不足互乘母為實，子相從為法。

The **excess** (*ying*) and insufficiency mount (×) one another’s numerators to make the dividend, and the denominators go with (+) one another to make the divisor (*Suanshu shu*, slips 133–134).²¹

The next earliest text of whose date we can be sure is Liu Xin’s 劉歆 (c.50 BCE–23 CE) Triple Concordance *li* (*Santong li* 三統曆) of 1 BCE/5 CE,²² which sees two additional applications of the word. One is as an auxiliary particle used in division:

盈章歲得一，名曰積月。

As it **fills** (*ying*) the rule years get 1, and this is named the ‘accumulated months’.²³

The other, by abbreviation, is as the operation of division itself:

見復餘盈其見復數，一以上見在往年

If [the number of times] the appearance and return remainder **fills** (*ying*) the appearance and return number is 1 or higher, then the appearance is in the previous year.²⁴

As illustrated in Table 1, however, starting with the Han Quarter-remainder *li* (*Sifen li* 四分曆) of 85 CE, all sources for *li* and *suan* mathematics are consistent in switching to the synonym *man* 滿 ‘fulfil’ for all usages excepting ‘excess’. Why? One possibility is that it relates to the posthumous taboo on Emperor Liu Ying’s 劉盈 (d. 188 BCE) given name,²⁵ but if so, the dates do not quite work out: the manu-

²¹ Tr. modified from Cullen (2004: 81).

²² We know that the Triple Concordance *li* was written after 1 BCE from *Han shu* 漢書 (Zhonghua shuju edn.), 36.1972: ‘When Emperor Ai fell (in 1 BCE), Wang Mang took the reins of government. [...] The empress dowager retained [Liu] Xin as bureau head of the left and [then?] transferred him to [the posts of] colonel of the capital rampart, Xi-he and governor of the capital, putting him in charge of the Bright Hall and Circular Moat and conferring him the title of Marquis of Hongxiu. He was charged with the scholarly offices of [grand] clerk and diviner, and he studied and fixed tono-metrology and mathematical astronomy, composing the Triple Concordance astronomical system and chronology’ 會哀帝崩，王莽持政……太后留歆為右曹太中大夫，遷中壘校尉，羲和，京兆尹，使治明堂辟雍，封紅休侯。典儒林史卜之官，考定律曆，著『三統』曆、譜。We know that the procedure text preserved in the *Han shu* was written prior to 5 CE based on how Ban Gu 班固 (32–92 CE) identifies his source: ‘In the mid-Epochal Beginning period (1 BCE–5 CE), Wang Mang held control of the government, and wishing to glorify his reputation he summoned more than a hundred experts on bells and pitch-pipes from all under heaven and had Xi-he Liu Xin et al. take charge of memorialising [their findings] item by item. Theirs is the most detailed discussion [of the topic], thus I have expunged their more spurious words, picked out what contents are correct and written them into the present piece’ 至元始中王莽秉政，欲耀名譽，徵天下通知鐘律者百（餘）餘人，使羲和劉歆等典領條奏，言之最詳。故刪其偽辭，取正義，著于篇 (*Han shu*, 21A.955).

²³ *Santong li*, cited in *Han shu*, 27B.1001; cf. Cullen (2017: 91).

²⁴ *Santong li*, cited in *Han shu*, 27B.1002; cf. Cullen (2017: 100).

²⁵ According to Xun Yue 荀悅 (148–209), ‘to avoid the character *ying* one said *man*’ 諱「盈」之字曰「滿」 (*Han shu*, 2.86 comm.); cf. Wang Yankun (1997: 555–556).

Word	Use	S	SSS	STL	JZSJ	ZBSJ	SFL	QXL	JCL	SSSJ	ZBSJZ	YJL	DML	ZQJSJ	ZGL	XHL	ZBCS	WJSS	DYL
滿 <i>man</i> 'fulfil'	≥				10	13	18	15	18	5	10	13	6	1	16	15	1	3	8
	÷						58	44	41	2	36	33			52	50	5	5	39
盈 <i>ying</i> 'fill'	≥	14	8	24															
	÷			25															
	excess		10		27		20	17	6	1	26	16		9	32	33			39

Table 1 Distribution of the synonyms *ying* '盈' and *man* '滿' 'fulfil'. The columns are in approximate chronological order, with works of *li* 曆 astronomy marked in grey and *suann* 算 mathematics in white. Note that texts in which these words do not appear in these capacities are omitted. The row '≥' includes remainders resulting from division ('that which does not [ful]fill' 不盈/滿, etc.), and the row '÷' includes said character as it appears anywhere in an expression that produces a quotient, be it in a primary or supporting role.

Abbreviations in Table 1 and Fig. 3 (below): **Mathematics**: **S** (*Shu* 數, anon. MS, ≤ 212 BCE, Yuelu Academy); **SSS** (*Suan shu shu* 算數書, anon. MS, ≤ 186 BCE, Zhangjiashan tomb 247); **JZSS** (*Jiuzhang suanshu* 九章算術, anon., 5/c.93 CE); **ZBSJ** (*Zhoubi suanjing* 周髀算經, anon., 5/178 CE); **SSJY** (*Shushu jiyi* 數述記遺), Xu Yue 徐岳, c.226 CE); **JZSSZ** (*Jiuzhang suanshu zhu* 九章算術注, Liu Hui 劉徽, 263 CE); **HDSJ** (*Haidao suanjing* 海島算經, Liu Hui, c.263 CE); **SZSJ** (*Sanzhi suanjing* 孫子算經, anon., 280/317 CE); **ZBZ** (*Zhoubi zhu* 周髀注, Zhao Ying 趙嬰, c.314/322 CE); **ZQJSJ** (*Zhang Qiuqian suanjing* 張邱建算經, Zhang Qiuqian 張邱建, 431/484 CE); **SSJYZ** (*Shushu jiyi zhu* 數述記遺注, Zhen Luan 甄鸞, c.535/570 CE); **ZQJSJZ** (*Zhang Qiuqian suanjing zhu* 張邱建算經注, Zhen Luan, c.535/570 CE); **ZBCS** (*Zhoubi chongshu* 周髀重疏, Zhen Luan, c.535/570 CE); **WJSS** (*Wujing suanshu* 五經算術, Zhen Luan, c.535/570 CE); **WCSJ** (*Wucaosuanjing* 五曹算經, Zhen Luan, c.535/570 CE). **Astronomy**: **STL** (*Samtong li* 三統曆, Liu Xin 劉歆, 1 BCE/5 CE); **SFL** (*Sifen li* 四分曆, Bian Xin 編訥 and Li Fan 李梵, 85–86 CE); **QXL** (*Qianxiang li* 乾象曆, Liu Hong 劉洪, fin. 206 CE); **JCL** (*Jingchu li* 乾象曆, Yang Wei 楊偉, 237 CE); **YJL** (*Yuanjia li* 元嘉曆, He Chengtian 何承天, 444 CE); **DML** (*Daming li* 大明曆, Zu Chongzhi 祖沖之, 465 CE); **ZGL** (*Zhengguang li* 正光曆, Li Yexing 李業興 and Zhang Longxiang 張龍祥, 518/520 CE); **XHL** (*Xinghe li* 大興曆, Li Yexing 李業興, 539 CE); **DYL** (*Daye li* 大興曆, Zhang Zhouxuan 張胃玄, 597 CE).

script *Writings on Calculating Numbers* was found in a tomb sealed two years later, in 186 BCE,²⁶ and the Triple Concordance *li* was written a couple of years before the House of Liu was deposed and the taboo lifted in 9 CE.²⁷ Neither does this explanation account for why *ying* was kept in the sense of 'excess' vs 'insufficiency'. Another possibility is that this terminological shift occurred for simple reasons of *disambiguation*, potentially in combination with irregular censorship.

Whatever the reason, the shift was absolute, drawing a clear dividing line between all dated sources for *li* and *suan* mathematics before and after the year 5 CE. As to *undated sources*, it should come as no surprise that *The Nine Chapters* systematically uses *man* for 'greater than or equal to', comporting with post-5 CE usages, but so too does *The Gnomon of Zhou*. For example:

實如法得一里，不滿法者，三之，如法得百步。

Get 1 *li* [for each time] the dividend is up to the divisor, and that which does not **fulfil** (*man*) the divisor, triple it and get 100 *bu* [for each time] it is up to the divisor.²⁸

Could it be, as Qian Baocong (1924 [1983]: 134) suggests, that this is because Li Chunfeng et al. harmonised the vocabulary of *The Ten Mathematical Classics* with seventh-century usages? It *could* – though this is not how one normally works with 'classics' (*jing* 經) – but if they did, they hardly left any trace.

For example, as illustrated in Fig. 3, by 85 CE the new term *jian* 減 'diminish' had mostly taken over for *chu* 除 'eliminate' in the operation of subtraction. With the exception of the *Daming li* 大明曆 of 465 CE, *jian* stays in the majority ever after in astronomy, and, in mathematics, it does not dip below 80 per cent of subtractions in any given text with the sole exception of *The Gnomon of Zhou* and its fourth-century commentary. If Li Chunfeng et al. had harmonised the language of these texts, then why don't *The Ten Mathematical Classics* all use the same language of subtraction? Why does that of *The Gnomon of Zhou* and its commentary stand out as so different?

4 Northern Transmission

If we accept the transition between *ying* 盈 and *man* 滿 after the Triple Concordance *li* as a reliable marker of when a mathematical text was composed, then we must conclude that *The Nine Chapters* was compiled sometime between 5 and 93 CE, and *The Gnomon of Zhou* between 5 and 178 CE. This comports with the traditional dating of the former, and it supports Cullen (1996) and early Qian Baocong's (1924 [1983]; 1932 [1998]) dating of the latter, so I dare not say that this is particularly 'radical'. More befitting the title of my talk, I would like to argue that *The Gnomon of Zhou* was probably written in the far north/north-west.

²⁶ One notes that in the example given above *Writings on Calculating Numbers* uses the variant orthography 贏 in place of 盈 for *ying* 'fill' but that the manuscript mixes the two (see slips 42, 56, 58, 64, 82, 160, 165 and 184). That Zhangjiashan 張家山 tomb 247, where *Writings on Calculating Numbers* was found, was sealed in or shortly after 186 BCE is known by the civil calendar running up to that date found also in the tomb (*Zhangjiashan Han mu zhujian* (ersiqi hao mu), 1).

²⁷ See Note 22.

²⁸ *Zhoubi suanjing*, 1.70b; cf. Cullen (1996: 88).

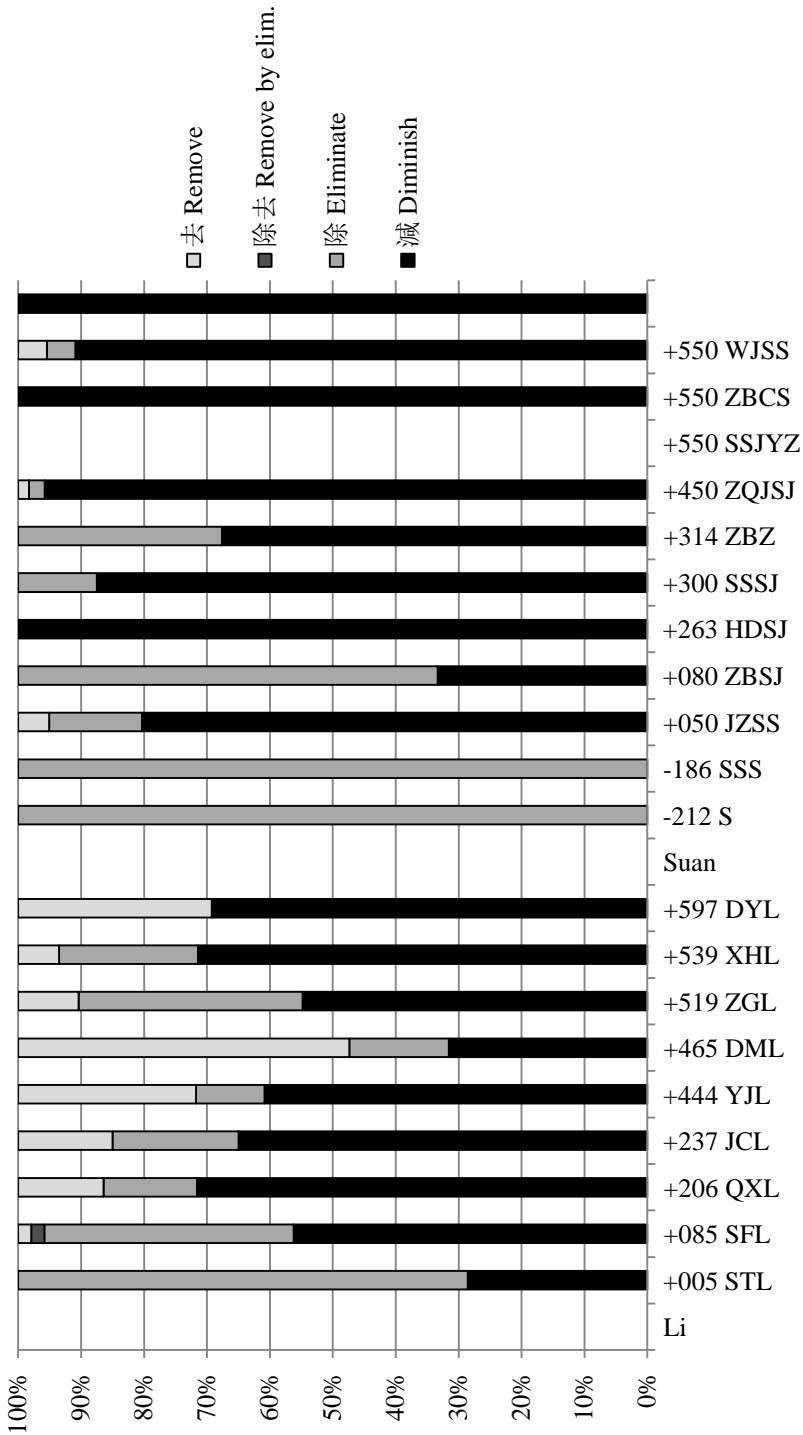


Fig. 3 Distribution of expressions for subtraction in *li* astronomy and *suan* mathematics. Abbreviations given in the caption to Table 1 (above). Note that the dates are in some cases approximate and that I have included instances of sequence subtraction in with normal subtraction.

One of the curious features about the early history of ‘umbrella heaven’ is that Wang Chong’s similar description of 80 CE does not mention *The Gnomon of Zhou*, *The Gnomon of Zhou* does not mention Wang Chong, and that Cai Yong, above, mentions only the one. As to why Cai Yong omits Wang Chong’s much earlier work on the topic in writing from Ordos in 178 CE, it turns out that it is because, in 178 CE, he did not yet have the book:

袁山松書曰：「充所作『論衡』，中土未有傳者，蔡邕入吳始得之，恆秘玩以為談助。……」

The *Yuanshan songshu* says: ‘[Wang] Chong’s *Balanced Discourse* had yet to see transmission in the central lands, and when Cai Yong first got a hold of it upon entering Wu 吳 (on the south-eastern seaboard) he was always secretly playing with it as a conversational aid’. [...]

抱朴子曰：「時人嫌蔡邕得異書，或搜求其帳中隱處，果得『論衡』，抱數卷持去。邕丁寧之曰：『唯我與爾共之，勿廣也。』」

The *Baopuzi* says: ‘At the time, people envied Cai Yong’s [ability] to get his hands on rare books. It was in occasionally searching the hidden corners of his home that [Wang Lang 王朗 (d. 228 CE)] indeed found the *Balanced Discourse*, leaving with several rolls in his arms. [Cai] Yong gave him the careful instructions that “Only I and you share this, do not spread it”.’²⁹

According to his biography, Cai Yong was banished to Wang Chong’s hometown of Kuaiji 會稽, in Wu Commandery, only after his release from Ordos in 179 CE.³⁰

Put in the context of Cai Yong’s life, this information is particularly revealing. Cai Yong had spent eight years from 170 to 178 CE working as a textual scholar in the palace archives – the Eastern Observatory (Dongguan 東觀)³¹ – and we know that there were book stores in the capital since at least the day of Wang Chong.³² If it was only in the far south-east at the age of 46+ that this avid collector of rare books could finally get his hands on a copy of *Balanced Discourse* some hundred years after it was written, that tells us that its circulation was indeed extremely regionally limited.

So what do we know about the transmission history of *The Gnomon of Zhou*? The next piece of evidence we have of its reception is the commentary (*zhu* 注) of Zhao Shuang 趙爽 (styled Junqing 君卿) preserved in *The Ten Mathematical Classics*. Nothing is known about Zhao Shuang, and there is even some confusion about his name, which contemporary bibliographies give as Zhao Ying 趙嬰.³³ Most scholars agree that this is an orthographic variant, posit that Zhao was from the Eastern

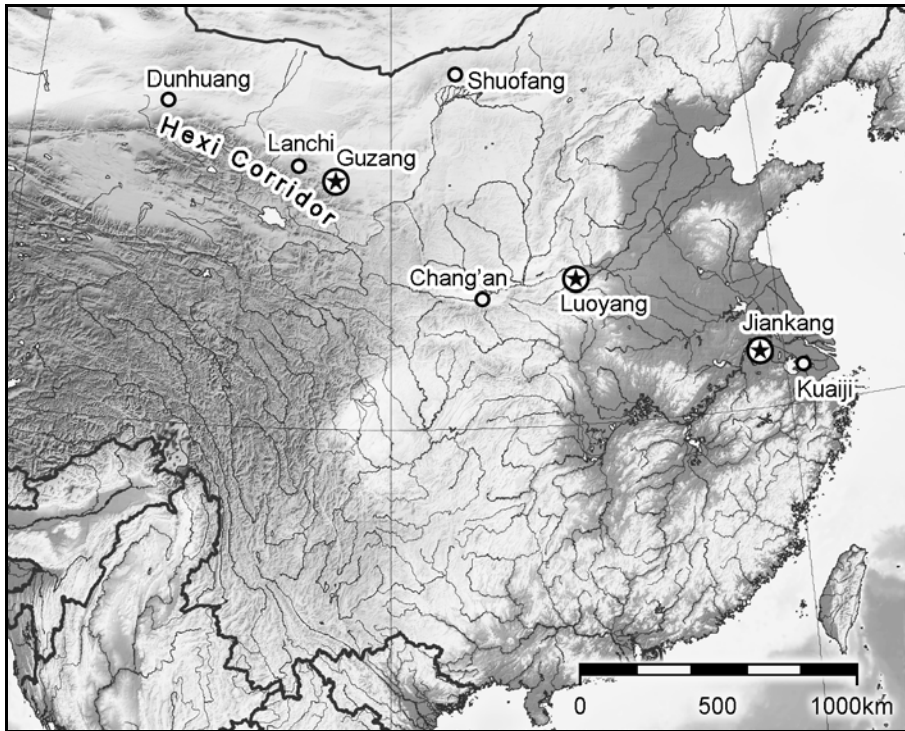
²⁹ *Hou Han shu*, 49.1629 comm. Note that said phrase does not occur in the received *Baopuzi*.

³⁰ *Hou Han shu*, 60B.2003.

³¹ *Hou Han shu*, 60B.1990 ff. On the Eastern Observatory, see Goodman (2005).

³² ‘[Wang Chong] was from a poor family without books, and he would often wander the market stalls of Luoyang reading the books on sale’ 家貧無書，常游洛陽市肆，閱所賣書 (*Hou Han shu*, 49.1629).

³³ Both *Sui shu*, 34.1018, and *Jiu Tang shu*, 47.2036, record a ‘*Gnomon of Zhou* in 1 roll commented by Zhao Ying’ 周髀一卷趙嬰注; *Xin Tang shu* 新唐書 (Zhonghua shuju edn.), 59.1543, records a ‘*Gnomon of Zhou* commented by Zhao Ying in 1 roll’ 趙嬰注周髀一卷; and *Song shi* 宋史 (Zhonghua shuju edn.), 207.5271, records a ‘Zhao Junqing’s *Mathematical Classic of the Gnomon of Zhou* in 2 rolls’ 趙君卿周髀算經二卷.



Map Modified from Topographic Map of East Asia, CC BY [Ksiom](#).

Han 東漢 (25–220 CE) or Sun-Wu 孫吳 (222–280 CE) and go no further.³⁴ If we search the written record, however, we find two men by the same name active in literate circles. The first is an emissary by the name of Zhao Junqing who petitioned the southern court of Chen 陳 (557–589 CE) in 585 CE, as recorded in Guanding's 灌頊 (561–632 CE) *Guoqing bailu* 國清百錄.³⁵ The second is a district administrator who memorialised the Former Liang 前涼 (320–376 CE) court at Guzang 姑臧 in 322 CE, as recorded in Cui Hong's 崔鴻 *Shiliuguo chunqiu* 十六國春秋 of 525/527 CE:

³⁴ See *Chouren zhuan* 疇人傳 (edn. *Chouren zhuan huibian* 疇人傳彙編, Ruan Yuan 阮元 (1764–1849) et al., ed. Peng Weiguo 彭衛國 and Wang Yuanhua 王原華 (Yangzhou: Guangling shushe, 2008)), 4.48–49, Qian Baocong (1924: 134–135), Cullen (1996: 147–148), Wu Wenjun (1998–2004: vol. 3, 23), Qu Anjing (2002: 9), Chemla and Guo (2004: 59).

³⁵ 'In the third year of the Zhide reign (585 CE)... the Young Ruler's (r. 583–589 CE) second decree was responded to by a petition from the emissary Zhao Junqing' 至德三年……少主第二勅。得使人趙君卿啟 (*Guoqing bailu*, in *Taishō shinshū daizō-kyō* 大正新脩大藏經, ed. Takakusu Junjirō 高楠順次郎 and Watanabe Kaigoku 渡邊海旭 (Tōkyō: Taishō is-saikyō kankō-kai, 1924–1934), T. no 1934, 46:799b7).

建興三年冬十月，蘭池長趙嬰上言，軍士張冰於青澗水中得一玉璽……

In the third year of the Jianxing reign (322 CE), winter, month X, Lanchi 蘭池 [District] Chief Zhao Ying 趙嬰 memorialised the throne saying that Serviceman Zhang Bing found a jade seal in the waters of Greenbrook. [...] ³⁶

The later *History of the Jin* (*Jin shu* 晉書) of 648 CE records the same event featuring yet another orthographic variation on Zhao's name:

蘭池長趙爽上軍士張冰得璽……

Lanchi [District] Chief Zhao Shi 趙爽 memorialised the throne that Serviceman Zhang Bing found a seal. [...] ³⁷

Of the two, it is the Former Liang district chief that is the most likely to be our commentator. First, given that Zhen Luan 甄鸞 (*fl.* 535–570 CE) wrote a subcommentary (*shu* 疏) to Zhao's work under the Northern Zhou 北周 (557–581 CE), the late-sixth-century southern emissary is a bit too late and too far away for his writings to have seen canonisation at a northern court at this time. Second, the next mention of *The Gnomon of Zhou* in the Chinese written record, in 437 CE, places it at the Northern Liang 北涼 (397–439 CE) capital at Guzang, less than 200 km down the Hexi 河西 Corridor from Lanchi, in the context of an interstate book exchange:

河西人趙敷善歷算。【元嘉】十四年，茂虔奉表獻方物，并獻『周生子』十三卷，『時務論』十二卷，『三國總略』二十卷，『俗問』十一卷，『十三州志』十卷，『文檢』六卷，『四科傳』四卷，『燉煌實錄』十卷，『涼書』十卷，『漢皇德傳』二十五卷，『亡典』七卷，『魏駁』九卷，『謝艾集』八卷，『古今字』二卷，『(乘)(桑)丘先生』三卷，【周髀】一卷，『皇帝王歷三合紀』一卷，『趙敷傳』并『甲寅元歷』一卷，『孔子讚』一卷，合一百五十四卷。茂虔又求晉、趙起居注諸雜書數十件，太祖賜之。

Zhao Fei of Hexi was good at astronomy and mathematics. In the fourteenth year [of the Liu-Song Yuanjia reign (437 CE)], [Emperor Juqu] Maoqian 沮渠茂虔 (AKA Mujian 牧犍) [of the Northern Liang] (r. 433–439 CE) presented a memorial and a tribute of regional items. Also offered as tribute were the *Zhoushengzi* in 13 rolls, *On Seasonal Duties* in 12 rolls, *A Comprehensive Summary of the Three Kingdoms* in 20 rolls, *Questions on Customs* in 11 rolls, *Treatise on the Thirteen Provinces* in 10 rolls, *Literary Examinations* in 6 rolls, *Biography of the Four Classifications [of Writing]* in 4 rolls, *Veritable Records of Dunhuang* in 10 rolls, *The History of Liang* in 10 rolls, *Biography of Han Imperial Virtue* in 25 rolls, *Lost Laws* in 7 rolls, *Wei Arguments* (?) in 9 rolls, *The Collected Works of Xie Ai* in 8 rolls, *Ancient and Modern Characters* in 2 rolls, *The Mulberrymound Sensei* in 3 rolls, ***The Gnomon of Zhou* in 1 roll**, *Annals of the Three Unities of the Sequence of Emperors and Kings* (?) in 1 roll, the *Biography of Zhao Fei* and [his] *Jiayin-origin li* in 1 roll and *Praises of Master Kong* in 1 roll, for a total of 154 rolls. [Emperor Juqu] Maoqian also requested the court diaries of the Jin 晉 (265–420 CE) and Zhao 趙 (318–351 CE) and a various selection of several dozen other books, which Emperor Taizu [of the Liu-Song] 宋太祖 (r. 424–453 CE) granted him. ³⁸

³⁶ *Shiliuguo chunqiu* (Siku quanshu 四庫全書 edn.), 71.1b–2a; cf. *Cefu yuangui* 冊府元龜 (Siku quanshu edn.), 232.3b.

³⁷ *Jin shu*, 86.2227.

³⁸ *Song shu*, 96.2416. As presented from the Northern Liang perspective, see *Shiliuguo chunqiu*, 95.4a–b.

In case the purpose of this book exchange is unclear, one notes that there is a preponderance of north-western authors and subjects among the titles sent to Jiankang 健康 in exchange for the diaries of the major eastern courts: Zhao Fei 趙暉 of Hexi (fl. 412–433 CE) was the astronomer royale of the Northern Liang,³⁹ and like Xie Ai 謝艾 (d. 353 CE), the authors of the *Biography of Han Imperial Virtue*, the *Zhoushengzi*, the *Treatise on the Thirteen Provinces*, the *Veritable Records of Dunhuang* and *The History of Liang* were all natives of Dunhuang 敦煌.⁴⁰ In short, if *The Gnomon of Zhou* is on this list it is probably because the Xiongnu 匈奴 emperor Juqu Maoqian thought it a representative work of the Hexi Corridor that they may not have in the south.

5 Conclusion

Now if I may engage in a little speculation: if the curious ‘survival’ (*cun* 存) of *The Gnomon of Zhou* is only first reported in the Chinese written tradition by a collector of rare books in Ordos in 178 CE, it may well be because he discovered it on the trip north; moreover, limited regional circulation might well explain why *The Gnomon of Zhou* and the *Balanced Discourse* seem to be oblivious of one another’s arguments for umbrella heaven cosmology. As to dating, *The Gnomon of Zhou* would seem to be the product of an intellectual culture defined by the *weishu* apocrypha whose titles only first begin to appear 25 CE, those with which *The Gnomon of Zhou* shares parallel contents first appearing in the Mt Tai Stone Inscription (Taishan keshi wen 泰山刻石文) of 56 CE, the White Tiger Hall debates (*Baihu tongyi* 白虎通義) of 79 CE and court discussions on mathematical astronomy in 85–102 CE.⁴¹ From this, it looks like Qian Baocong (1924 [1983]: 134) is right that *The Gnomon*

³⁹ *Sui shu*, 34.1022.

⁴⁰ Trombert (2005: 17–24). On Zhao Fei, author of the *Jiayin-origin li* (*Jiayin yuan li* 甲寅元歷), see Chen Meidong (2003: 254–255). On Hou Jin’s 侯瑾 authorship of the *Biography of Han Imperial Virtue* (*Han huangde zhuan* 漢皇德傳), see *Sui shu*, 33.961; for his bio, see *Hou Han shu*, 80B.2649. On Kan Yin’s 闕駟 (fl. 401–444 CE) authorship of the *Treatise on the Thirteen Provinces* (*Shisanzhou zhi* 十三州志), see *Sui shu*, 33.985; for his bio, see *Bei shi* 北史 (Zhonghua shuju edn.), 34.1267. For Liu Bing’s 劉昉 (fl. 420–439 CE) bio, including his authorship of the *Veritable Records of Dunhuang* (*Dunhuang shilu* 燉煌實錄) and *History of Liang* (*Liang shu* 涼書), see *Wei shu* 魏書 (Zhonghua shuju edn.), 52.1160–1161. For Xie Ai’s 謝艾 (d. 353 CE) bio, see *Shiliuguo chunqiu*, 75.20b–22b. On Zhousheng Lie’s 周生烈 (3rd cent. CE) authorship of the *Zhoushengzi* 周生子, see *Sui shu*, 34.998; on his hometown and connection to Cai Yong through Wang Lang (above), see *Sanguo zhi* 三國志 (Zhonghua shuju edn.), 13.420. In addition, note that Yao Dan 姚澹 (fl. 317 CE), author of the *Biography of the Four Classifications [of Writing]* (*Sike zhuan* 四科傳), was a Han-Zhao 漢趙 (304–318 CE) military leader who, judging from his position and surname, that was likely from among the Qiang 羌 tribes on the southern edge of the Hexi Corridor (see *Jiu Tang shu*, 46.2003; *Shiliuguo chunqiu*, 2.2b). The two known exceptions in this list are Yang Jun 楊(偉)(俊) of Huojia 獲嘉 (d. 222 CE), author of *On Seasonal Duties* (*Shiwu lun* 時務論) and *The Mulberrymound Sensei* (*Sangqiu xiansheng* 桑丘先生), and Zhang Ji 張揖 of Qinghe 清河 (3rd cent. CE), author of *Ancient and Modern Characters* (*Gujin zi* 古今字). On the authorship of these works, see *Sui shu*, 32.942, 34.1006. For Yang Jun’s bio, see *Sanguo zhi*, 23.663–664. On Zhang Ji’s background, see *Wei shu*, 91.1963.

⁴¹ Namely, the *Luoshu* 洛書 apocrypha *Zhenyao du* 甄曜度, which is first mentioned in the Mt Tai Stone Inscription (cited in *Hou Han shu*, *zhi* 7, 3165); the *Book of Changes* (*Yijing* 易經) apocrypha *Qianzao du* 乾鑿度, first mentioned in *Baihu tongyi* 白虎通義 (Sibu congkan

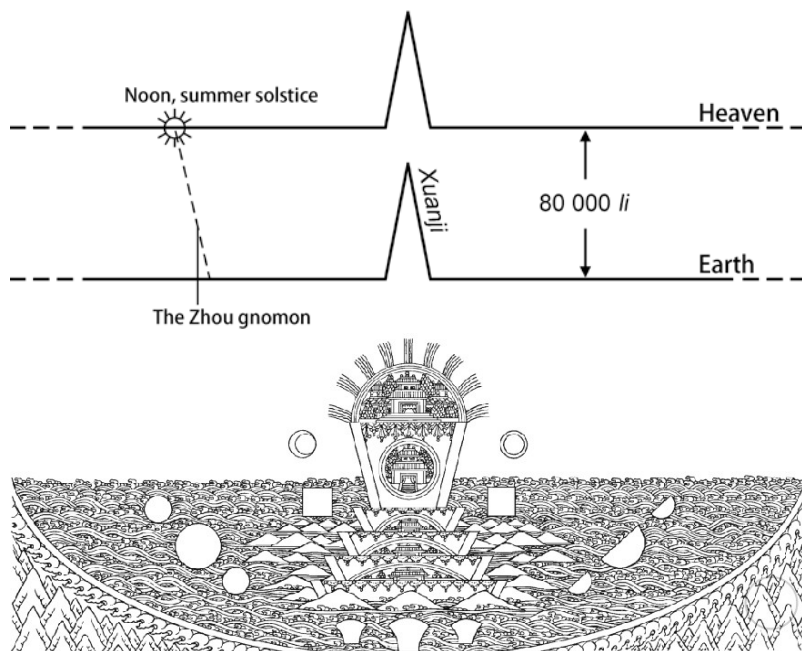


Fig. 4 Above: *Gnomon of Zhou* ‘umbrella heaven’ (*gaitian*) world model, redrawn as per Li Zhichao (1993 [2014], 225, fig. 1). Below: Buddhist cosmology, a random image pulled from the Internet.

of *Zhou* (5/178 CE) and *The Nine Chapters* (5/93 CE) ‘might well be contemporary works’. And while we cannot say for certain which went into circulation *first*, the abundant parallels with the third- and second-century BCE manuscripts⁴² do prove *the tradition of The Nine Chapters* significantly older, justifying Karine Chemla’s (2013b: 63) long-standing assertion that ‘The oldest writing in Chinese devoted to mathematics that has been handed down through the written tradition is the book *The Nine Chapters on mathematical procedures*’.⁴³

Pinning down where and when *The Gnomon of Zhou* was written is crucial for the sort of historical analysis of mathematical language alluded to earlier.⁴⁴ Beyond that, however, it also stands to change our reading of the text. As noted in the open-

edn.), 8.13b (on which see Nielsen 1995); the *Book of Documents* apocrypha *Kaoling yao* 考靈曜, first mentioned in Jia Kui’s 賈逵 court opinion on *li* astronomy of 91 CE (*Hou Han shu*, *zhi* 2, 2027–3028; tr. Cullen 2017: 382); the *Spring and Autumn Annals* apocrypha *Mingli xu* 命曆序, first mentioned in the context of the Quarter-remainder vs Triple Concordance debate of 102 CE (*Hou Han shu*, *zhi* 2, 3033; tr. Cullen 2017: 394). On the apocrypha, see Note 17.

⁴² See Tan Jingnan (2016).

⁴³ Cf. Chemla (2013a: 177).

⁴⁴ See Notes 18 and 19. It has equally important implications for the reconstruction of the historical practices and cultures *behind the language* as studied in Chemla (2013a; 2014; 2016; 2017).

ing, for example, what the text itself uses ‘umbrella heaven’ to explain are the geographical conditions at the edges of the Chinese world – and, well, that makes sense if that is precisely where it was written.⁴⁵ This brings us to Jiang Xiaoyuan’s 江曉原 (1997) famous seven-point argument that *The Gnomon of Zhou*’s peculiar cosmology bears such an uncanny resemblance to that of the Puranas (see Fig. 4) that it must have been a product of Indian influence. Qu Anjing 曲安京 (2002: 99–102) aptly refutes the premise of their resemblance, affirming the fantastical nature of Jiang Xiaoyuan’s claim, but it is hard to ignore the resemblance that *medieval scholars* saw. Himself a Daoist, Li Chunfeng describes Emperor Wu of Liang’s 梁武帝 (r. 502–549 CE) famous Buddhist-inspired world model as ‘completely identical to the text of *The Gnomon of Zhou*’,⁴⁶ while Guanding, the fourth patriarch of Tiantai 天台 Buddhism, describes *The Gnomon of Zhou* in his commentary to the *Mahāparinirvāṇa-sūtra* as ‘the same as the Buddhist model’.⁴⁷ To me, the most difficult part of Jiang Xiaoyuan’s argument to accept has always been *the date*: that a Chinese text written around 100 BCE reflects Buddhist cosmology when the first Buddhist temple in Luoyang 洛陽 was only constructed in 68 CE.⁴⁸ Well, if we accept that *The Gnomon of Zhou* was likely written *after this date* in what was or *would become* an important Buddhist centre on the Silk Road,⁴⁹ that makes the hypothesis of Indian influence considerably more plausible.

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⁴⁵ Compare this to the flat-earth sphere heaven, which only works if the observer is at ‘the centre of the earth’ 地中 in the ancient Zhou capital city of Luoyang; see Yixing’s 一行 (683–727 CE) assessment of the two theories’ strengths and weaknesses as studied in Jin Zumeng (1986).

⁴⁶ 全同周髀之文, *Sui shu*, 19.507. On Liang Wudi’s cosmology, see Yamada (1975), Jiang Xiaoyuan (2001: 212–246), Qu Anjing (2002: 102–114) and Yuan and Qu (2008).

⁴⁷ 同於佛法, *Daban niepan jing shu* 大般涅槃經疏 (in *Taishō shinshū daizō-kyō*), T. no 1767, 12:112a19–20.

⁴⁸ On the early transmission of Buddhism into China, see Zürcher (1959 [2007]).

⁴⁹ See Trombert (2005).

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