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Interpreting the Beveridge curve. An agent-based approach. *

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and

Eric Guerci[‡]

Abstract

We construct an agent based computational model of the labour market with heterogeneous workers and firms to study the behaviour of the Beveridge curve along the business cycle. In this framework, search frictions arise because filling a vacancy is a costly activity that takes time, whereas productivity mismatch comes from firms' imperfect information about the value of the workers before the job interview takes place. The model offers an interpretation for the outward movement exhibited by the U.S. Beveridge curve since the last months of 2009. Sectoral misallocation plays a role. Moreover, when the speed of recovery from a recession is not uniform across sectors, unemployed workers are less selective in their application strategy and firms must spend more time in choosing the best match. Unemployment remains high in spite of an increase in the number of vacancies.

Keywords: Beveridge curve; mismatch; unemployment; agent-based modelling.

JEL codes:

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1 Introduction

The Beveridge curve illustrates the empirical relationship between unemployment and vacancies. It usually has a negative slope, meaning that high (respectively, low) unemployment is associated with exiguous (resp. many) job opportunities. This makes economic sense. In recessions, firms post less vacancies and the stock of unemployed people increases. We would expect the vacancy-unemployment coordinates to be in the bottom right area of the positive quadrant. As the economic conditions improve, there should be a shift of these variables towards the top left corner.

It is more difficult to understand why for some quarters (or even years) the Beveridge curve shifts outward¹. Recent data on the U.S. labour market witness quite starkly the occurrence of this phenomenon. Figure 1 illustrates the U.S. Beveridge curve at monthly level. From June 2009 to mid June 2011, in the wake of Great Recession, the unemployment rate has remained quite stable in spite of a substantial increase in job openings. Since then, the position of curve has shifted to the right, implying that for a given vacancy rate the U.S. economy now exhibits a higher unemployment rate².

The main objective of this paper is to offer an explanation for the behaviour of vacancies and unemployment along the business cycle. To do so, we construct a model that merges the search and matching approach to labour markets with the agent-based computational method to represent the functioning of complex decentralized systems. Differently from standard equilibrium search models (see Pissarides, 2000), labour market frictions do not originate from an exogenous matching function. Rather, they are the result of each single agent's decision in a world where (i) employers do not know the value of the unemployed worker before the job interview takes place and (ii) searching for a suitable job / worker is an activity that takes time and mismatch may arise.

¹More rare are the cases of an upward sloping schedule. Pissarides (2010) cites the early 80s in U.K. as an example of simultaneous increase in vacancy and unemployment. Barlevy (2011) has shown that a similar pattern can be found during two periods of high unemployment in the U.S., namely 1973-1975 and 1981-1982.

²Some works (Elsby *et al.*, 2010, Daly *et al.*, 2012, and Hall, 2010) find that at the 2011 level of vacancies, the pre-recession unemployment-vacancies relationship predicts an unemployment rate between 1 and 2 percentage points lower than its 2011 value.

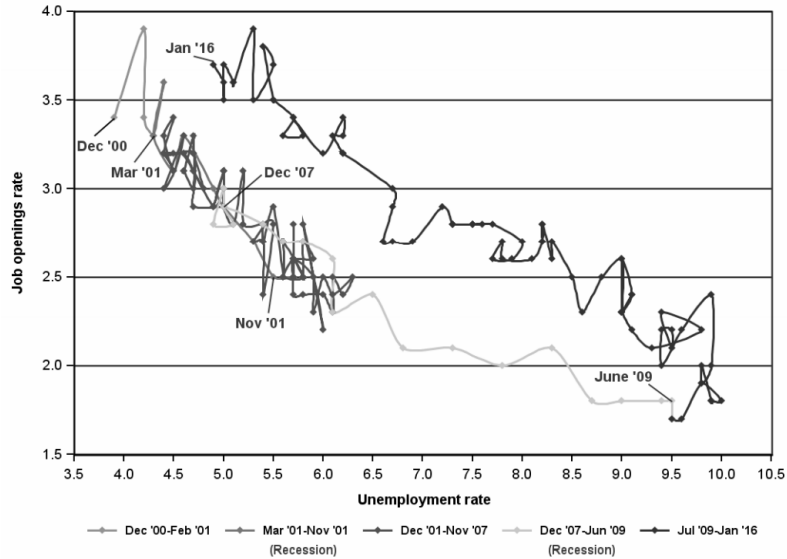


Figure 1: The U.S. Beveridge curve. The Job Openings and Labor Turnover Survey (JOLTS) collects vacancy data from December 2000. Source: B.L.S. (2016b).

More precisely, we consider a framework in which firms and workers are heterogeneous in terms of productivity. Workers are better informed than firms about their qualifications for a certain job and select the job openings accordingly, taking into account both the productivity of the resulting match and the labour market conditions of each sector. Once an employer interviews an applicant, the value of the match becomes common knowledge. So he has to decide whether to hire him or to wait for a better match. Clearly, the value of the delay option is higher the longer the queue of applicants for that job.

In this setup, we simulate the effects of the changes in value added experienced by the 15 U.S. macro-sectors during the 2008–2011 period. We choose this interval because, according to official statistics, the Great Recession began in December 2007, it reached the bottom in the second quarter of 2009, and real valued added returned to its pre-crisis value in mid-2011. As many papers have documented (see for instance Barnichon *et al.*, 2012), both the recession and the recovery periods

were quite asymmetric, as some industries have been more severely affected by the crisis than others³.

In our model, such an uneven variation in the sectoral growth rates has a twofold effect on the Beveridge curve. The first one is "pure" misallocation: unemployed workers in sectors worst affected by the crisis are not suitable for the job opportunities created in more robust industries. For a given level of aggregate vacancy creation, the unemployment rate is higher than would be in case of a perfectly symmetric recovery. The result stems from congestion effects, so that it takes more time to fill one more vacancy posted in a sector on the road to recovery than in a sector still in recession, where there is scarcity of employers willing to hire and abundance of workers eager to accept a job position. This process is similar to the one applied by Şahin *et al.* (2014).

The second effect that our study aims to capture concerns the recruitment process. At the onset of the recovery unemployed workers are eager to find a job and apply even for positions for which they are not particularly well suited. Firms receive a larger and more heterogeneous list of applicants and this lengthens the recruitment process. The hiring mechanism becomes more inefficient and, for a given vacancy rate, the unemployment level is now higher. This explanation on why the screening process takes longer time in recessions has been first advanced by Hall (2005)⁴.

Notice that in case of perfectly segmented labour markets, where each worker just searches for jobs opened in his own sector, only the first, misallocation effect would be present, and at its largest magnitude. Conversely, if workers are allowed to send applications by looking at the number of vacancies available in the other sectors of the economy, the recruitment congestion effect gains importance. Therefore, by varying the parameter that determines the importance of productivity over sectoral labour market conditions on workers' applications decisions, we are able to gauge the weight of both forces in the shift of the Beveridge curve.

³At the beginning of 2010, for instance, the utilities industry had regained all the jobs lost in the previous two years, while in the construction sector the employment level was 20 % lower than in December 2007. See the Bureau of Labour Statistics, 2016. Tables 7 and 8 present the different growth rates in the main macro sectors of the U.S. economy.

⁴More recently, Ravenna and Walsh (2012) find that a greater heterogeneity in the pool of job applicants may explain the relative volatility of unemployment to output, but in their model job searchers differ in terms of TFP and not in terms of skills.

By running a first numerical exercise with perfectly segmented labour markets, we get that the misallocation effect can explain at most 40% of the 5 percentage points increase in unemployment from the beginning of 2008 to the end of 2009. This is in line with the results of Şahin *et al.* (2014), in which the sectoral misallocation makes up for about 30% of the 5.4 percentage points increase in the unemployment rate from 2006 to 2009⁵. Moreover, the same exercise produces a U-shaped Beveridge curve, qualitatively similar to what observed in data in the same period. However, this first simulation is unable to replicate the observed magnitude of the unemployment and vacancy fluctuations over the period taken in exam. In particular, the model response of unemployment to the observed variations in productivity is too small, and the labour market variables show no persistence over the period. These are drawbacks that our set-up shares with canonical search and matching models, as first documented by Shimer (2005) and Fujita and Ramey (2007).

Introducing the hiring congestion mechanism into the model considerably improves our exercise. For some reasonable parameter values, the simulated Beveridge curve is able to reach the peak in the unemployment rate observed in data (10% at the end of 2011). Since then, our numerical exercises show that the increase in the vacancy rate (from 1.8% to 2.4%) is not accompanied by a reduction in the unemployment rate, that hovers around 9 – 10%, in line with the empirical evidence.

The paper is organized as follows. Section 2 surveys the literature on the subject. Section 3 illustrates the theoretical model. In section 4 we illustrate our numerical procedure, while in section 5 we show the quantitative results. Section 6 concludes.

2 Related Literature

Many papers have attempted to explain the outward movement of the Beveridge curve in the wake of the Great Recession. In their thorough survey, Elsby *et al.* (2015) have presented and put under empirical scrutiny the different answers that have been advanced.

⁵Şahin *et al.* also calculate the contribution of geographical mismatch to unemployment and find it plays no role in explaining the unemployment persistence

A first, perhaps intuitive, hypothesis points to the behavior of the job separation rate. In a standard search and matching model (see Pissarides, 2000 chapter 1) the anecdotal curve may move to the right if jobs exhibit a shorter duration. However, Elsby *et al.* show that the unemployment inflow rate recovered quickly in the wake of the Great Recession and since then it seems to have continued on its downward pre-recession trend.

A more interesting explanation is offered by Davis, Faberman, and Haltiwanger (2013). In this influential work, they show that firms' recruiting intensity (i.e. all the tools and policies adopted by the employers to speed up the job filling rate) is not insensitive to aggregate fluctuations, as the standard matching model predicts, but it is pro-cyclical. Moreover, it has fallen significantly in the Great Recession. The bottom line is that if job openings had been accompanied by the same amount of "recruiting intensity" as before the crisis, a lower amount of vacancies would have been needed to support the observed level of unemployment. However, as Elsby *et al.* point out, these pro-cyclical movements of the recruitment intensity are not persistent enough to explain why the unemployment/vacancy loci are not back at their pre-recession position.

More in general, the outward movement of the Beveridge curve can be interpreted as a decrease in the efficiency of the matching process, for since 2009 in the U.S. labour market more vacancies are needed for a given level of unemployment. In this sense, mismatch, i.e. any form of geographical or sectoral misallocation of workers across jobs, seems a quite natural culprit. The work of Barnichon *et al.* (2012) belongs to this vein. They find that the Beveridge curve gap (defined as the difference between the current level of unemployment and the value that would be implied given the number of vacancies opened) is almost fully attributable to a shortfall in the number of hires per vacancy and they point to mismatch as one of the potential explanations. In another paper, Barnichon and Figura (2011) study the impact of two distinctive factors on the matching efficiency in the U.S. labour market over the period 1976-2010: the composition of the unemployment pool and the misallocation of jobs and workers across different labour markets (both in a geographic sense and in terms of skill requirements). They conclude that the latter - that is a measure of mismatch - is the main responsible for the decline in matching efficiency over the period 2006-2010. The previously cited paper of Şahin *et al.* (2014) (and their companion work of 2011) also corroborates such

findings. Moreover, they find that taking into account the hypothesis of a decrease in search intensity of the unemployed during the Great Recession does not decrease the contribution of sectoral misallocation to the U.S. unemployment rise.

Many theoretical works have studied the role of mismatch in understanding labour market fluctuations. Similarly to our work, Birchenall (2011) builds a framework in which mismatch arises due to informational problems, although he puts more emphasis on the medium run fluctuations in the aggregate labour market than on the shifts of the short run Beveridge curve.

Carrillo-Tudela and Visschers (2014) also construct a similar model of mismatch to study the contributions of different sources of unemployment (i.e. rest, search, reallocation, and job separation) in explaining its cyclical features. Similar to our paper is the idea that productivity changes modify workers' application decisions, that in turn have an impact on the matching process. More specifically, in their model workers are more willing to change sector in upturns, as returns to a good match are higher. Knowing that, in booms firms will post even more vacancies, thereby amplifying the negative relation between unemployment and vacancies.

The logic behind the model is similar to the reallocation theory of unemployment fluctuations described by Lilien (1982), in which high unemployment is caused by the contemporaneous growth of some sectors and shrinkage of others. More recently, Hall (2005) has developed a similar mechanism to provide a rationale for the high persistence of unemployment following a recession. In his model, high unemployment makes job-seekers less selective, entailing a higher screening cost for employers and raising labour market frictions. Differently from our paper, the focus is more on the amplification of unemployment fluctuations than on the resulting effects on the Beveridge curve.

As concerns the methodology used in the present work, standard economic analyses often adopt a comparative statics approach, i.e. make a comparison of equilibria in response to the change in the parameters of the system. However, in labour markets there is a need for accounting for out-of equilibrium dynamics in the short term (Petrongolo and Pissarides, 2001 and Pissarides, 1985 and 2009). In this sense, search and matching models are particularly well suited to be investigated as an agent-based computational method (see for instance Bergmann, 1990, Freeman, 1998 and Tesfatsion and Judd, 2006). Agent-based models allow researchers to observe and analyse the dynamics properties of the matching process in great

details. This aspect is particularly relevant in our study, where the adjustment process following a shock, either a economy-wide recession or a sectoral recovery, must be considered. In addition, the agent-based computational approach has been successfully adopted to model the flows in and out in the labour market (endogenous hirings, quits, and firings), the employer-employee meeting process (Fagiolo *et al.*, 2004, Martin and Neugart, 2008, Tassier and Menczer, 2008 and Tesfatsion, 2001), spatial labour market frictions (Dawid *et al.*, 2009), and the dynamic impact of matching on economic mobility (Boudreau, 2010). In this vein, Neugart (2004) is probably the paper that most resembles our work. He studies the behaviour of unemployment and vacancies in an agent-based computational model in which the matching process is endogenous. Specifically, a strategy is assigned to each firm and worker, respectively consisting on a number of vacancies to post and applications to send. Those that do not earn positive payoffs are eliminated from the market and substituted by new born workers and firms using one of the strategies of the surviving agents. Although Neugart is able to obtain a downward sloping Beveridge, he does not investigate its short run shifts, the focus of the paper being more on the differences between the exogenous matching function and its endogenous mechanism. Neugart (2008) adopts a similar framework to analyze the impact of subsidies on workers' training costs in an economy hit by sector specific shocks.

3 The Model

Time is discrete and β is the discount rate. The economy is composed by two types of risk neutral agents: firms and workers. Let L denote the total discrete number of workers. The number of firms in the economy is determined by a free-entry condition. In line with the search and matching literature (see Pissarides, 2000, chapter 1), we make the one firm - one job assumption. So, at each period of time, each firm has either one employee or an open vacancy and workers can be either employed or unemployed.

At time 0, workers and firms are distributed in N different sectors of the economy. Once a match between a worker in $s \in \{1, \dots, N\}$ and a firm in $n \in \{1, \dots, N\}$

is formed, a certain amount $y(s, n)$ of the unique consumption good is produced:

$$y(s, n) = z_n \cdot \alpha(s, n), \quad (1)$$

This quantity is the product of an aggregate part z_n - common to all jobs in sector n - and an idiosyncratic one, $\alpha(s, n)$, that depends on the kind of worker hired by the firm in n . In general, we assume that there exists a metric that measures the distance between sector s and sector n , and that the lowest the distance is, the highest the productivity of the resulting match. So, if $s = n$, workers and firms belong to the same sector and $\alpha(s, n)$ reaches its highest value. Conversely, when $s \neq n$, the worker is not perfectly suited for that job and $\alpha(s, n)$ is lower⁶.

3.1 Workers' application decision

Applying for a job is a costless activity. Unemployed workers send one application per unit of time t . The probability for a worker located in point $s \in \{1, 2, \dots, N\}$ of applying for a generic vacancy located in point $n \in \{1, 2, \dots, N\}$ is determined by the following softmax function⁷:

$$p_{s,n} = a \cdot \frac{V_n \cdot e^{\lambda \cdot y(s,n)}}{\sum_{n=1}^N V_n \cdot e^{\lambda \cdot y(s,n)}}, \quad (2)$$

in which $a \in (0, 1)$ is an efficiency parameter, V_n is the number of vacancies opened at point n and $\lambda \in [0, +\infty)$. To ease the notation, we omit the subscript t . Under this rule, workers send their application by considering both the number of vacancies posted in each sector and the resulting productivity of the matches. The higher V_n and $y(s, n)$ are, the more likely the probability for a worker in s to apply for a position in sector n . Because of the limit properties of the softmax function, the value of parameter λ determines the relative importance of labour market conditions over productivity in the application choice. In particular, if $\lambda \rightarrow 0$, then $p_{s,n} = V_n / \sum_{n=1}^N V_n$ and workers apply for a vacancy just looking at the number of open job positions in each industry. Conversely, as $\lambda \rightarrow +\infty$, the

⁶Due to the lack of data, in the quantitative analysis we will consider a simplified version of equation (1): $y_n(s, n) = z_n$ when $s = n$ and $y_n(s, n) = \alpha \cdot z_n$ when $s \neq n$ and $0 < \alpha < 1$.

⁷This kind of functions is quite common in the field of reinforcement learning. See Sutton and Barto (1998).

probability to apply for the sector that ensures the highest productivity tends to 1.

Notice also that $\sum_{n=1}^N p_{s,n} = a$. So there is a probability $1 - a$ that the worker, for some unexplained frictions, does not send an application to the right places.

3.2 Free entry zero-profit condition

Firms must take two decisions. First, they have to decide whether to enter the market by posting a job opening. Second, in case they enter and receive at least one application, they have to choose whether to hire the worker just interviewed or to wait.

Let us focus on the first decision of the firm. Following the standard search and matching literature (see Pissarides, 2000, chapter 1), we assume that the number of vacancies present in the economy is determined by a free entry zero-profit condition. Opening a vacancy involves two kinds of cost for a firm: a fixed cost, k , that must be paid only once when the firm enters the market, and a flow cost h per unit of time, that firms pays as long as the vacancy remains open. The former can be seen as the cost for capital equipment, while the latter can be interpreted as advertising costs⁸. Moreover, in any period of time, there is an exogenous probability δ that the job match is destroyed.

A firm decides to enter the labour market if its future expected revenues from the match are at least equal to the future expected costs. The expected costs are equal to $k + h \cdot \frac{1 - \beta^{q_n}}{1 - \beta}$, in which q_n is the expected duration of vacancy for a firm located at point $n \in \{1, 2, \dots, N\}$. Since firms could get more than one application for each job opening, q_n should be a complex expression that takes into account: (i) the probability of receiving a certain number of job applications I with $I > 0$, (ii) the expected probability of meeting and hiring the i th job applicant with $i \in \{1, \dots, I\}$. For simplicity, we assume that q_n is the expected duration of a vacancy before the first interview takes place. We also assume that in computing its expected revenues firms possess some information at the aggregate level: the total number of unemployed people and their distribution across the sectors. In

⁸We do not model capital markets in this framework. So, more correctly, k can be considered the price - determined in international markets - of one unit of equipment.

this case, the firms' problem belongs to the class of standard urn-ball models⁹ and we have:

$$q_n = \left[1 - e^{-\frac{1}{V_n} \sum_{s=1}^N U_s \cdot p_{s,n}} \right]^{-1}, \quad (3)$$

in which U_s is the number of unemployed people located in sector $s \in \{1, 2, \dots, N\}$. To see this, notice that there is a probability $p_{s,n}/V_n$ that a single unemployed worker located in s send an application to a specific firm with an open vacancy located in n . Then $1 - p_{s,n}/V_n$ is the probability that a firm receives no application from one particular worker located in s and $(1 - p_{s,n}/V_n)^{U_s}$ is the probability that it does not get any application from any worker in s . The probability of receiving at least one application is $1 - \prod_{s=1}^N (1 - p_{s,n}/V_n)^{U_s}$. For large $V_n/p_{s,n}$, $(1 - p_{s,n}/V_n)^{U_s}$ can be approximated to $e^{-\frac{U_s \cdot p_{s,n}}{V_n}}$ and the probability that a firm in n receives at least one application becomes: $1 - e^{-\frac{1}{V_n} \sum_{s=1}^N U_s \cdot p_{s,n}}$. The expected duration of having the first interview is the reciprocal of this number. It can be shown that $\frac{dq_n}{dV_n} > 0$ and $\frac{dq_n}{dU_s} < 0$, $\forall s \in \{1, 2, \dots, N\}$. Intuitively, a larger number of vacancies in sector n enhances their expected duration, whereas an increase in the level of unemployed workers implies job openings are filled more quickly.

The firm's expected discounted revenues $J_e(n)$ for a firm located in n are equal to:

$$J_e(n) = \frac{\beta^{q_n} \cdot (1 - \delta)}{1 - \beta(1 - \delta)} \cdot \sum_{s=1}^N [y(s, n) - w(s, n)] \cdot \mathbb{P}_{s,n} \quad (4)$$

with $\mathbb{P}_{s,n} \equiv \frac{\frac{p_{s,n}}{V_n} \cdot U_s}{\sum_{s=1}^N \frac{p_{s,n}}{V_n} \cdot U_s}$

The term inside the square brackets are the revenues of the match between a firm located at point n and a worker located at s , with $w(s, n)$ denoting the real wage. For simplicity we assume that the wage is simply a fraction of the total productivity value of the match, $w(s, n) = \gamma \cdot y(s, n)$, with $0 < \gamma < 1$.¹⁰ Such revenues are weighted by the factor $\mathbb{P}_{s,n}$, that is the probability of meeting a specific worker located in s , $\frac{p_{s,n}}{V_n}$, conditional on the probability that an application has arrived.

⁹See Neugart (2004) for a survey of the microfoundations of the matching technology.

¹⁰More detailed, micro-founded forms of rent sharing such as the Nash Bargaining rule could be introduced without jeopardizing the results of the model.

For each period of time t , the equilibrium number of vacancies, V_n^* , solves the following conditions:

$$\begin{aligned}
& - \left(k + h \cdot \frac{1 - \beta^{q_n}}{1 - \beta} \right) + J_e(n) \geq 0 \quad \text{with } V_n = V_n^* \\
& - \left(k + h \cdot \frac{1 - \beta^{q_n}}{1 - \beta} \right) + J_e(n) < 0 \quad \text{with } V_n = V_n^* + 1 \quad \forall n \in \{1, 2, \dots, N\}
\end{aligned} \tag{5}$$

It is easy to see that an increase in q_n raises firms' expected costs (the first term at the LHS of 5) and lowers expected revenues, as this means a vacancy remains unfilled for a longer time. Notice also that the number of vacancies does not need to increase from one period to the other. If V_n^* at time t is greater than V_n^* at $t + 1$, no firm enters the market and some of those that posted the vacancy some periods before and did not find any suitable match, exit.

3.3 Firms' recruiting decision

A firm receiving an application at time t , interviews the candidate at time $t + 1$. We assume that no more than one interview per unit of time can be made. So, in case the firm receives more than one application, it randomly decides the order by which any workers is interviewed. For some unspecified delays that may concern both the worker and the entrepreneur, there is a time interval τ between one interview and the other.

Firms realize the productivity value of the match $y(s, n)$ only during the interview process. So, each entrepreneur that is interviewing a worker at time t decides to hire him immediately if the expected discounted value of this match is higher than either the value of waiting and hiring one of the next job applicants or, in case of no applicants, advertising that position again.

For a firm located in n , the expected discounted value of hiring at time t a worker located at s is:

$$J_h(s, n) = \frac{y(s, n) - w(s, n)}{1 - \beta(1 - \delta)} = \frac{(1 - \gamma) \cdot y(s, n)}{1 - \beta(1 - \delta)}$$

Since the productivity of a match depends on the distance between point s and point n , we prefer re-expressing the value of hiring by changing the notation. Let x

denote the distance between the worker's location and the firm' location. Clearly x is equal to 0 when $s = n$, while we denote with X the maximum distance between two sectors. Then the expected discounted value of hiring when is:

$$J_h(x, n) = \frac{(1 - \gamma) \cdot y(x, n)}{1 - \beta(1 - \delta)} \quad (6)$$

with $0 < x < X$. The value of waiting depends on the number of applicants queuing for that job position. Let $J_{d,i}(n)$ denote the expected discounted value for a firm in n of delaying the hiring for a period τ when there are $i \in \{0, 1, \dots, I\}$ remaining applicants to be interviewed. A firm located in n decides to hire a worker that ensures a productivity $y(x, n)$ and forgoes the possibility of interviewing the remaining i job applicants if and only if

$$J_h(x, n) > J_{d,i}(n) \quad (7)$$

with $n \in \{1, 2, \dots, N\}$ and $i \in \{0, 1, \dots, I\}$. We define:

$$x_i^* \text{ is the distance such that } \begin{cases} x \leq x_i^* & J_h(x, n) > J_{d,i}(n) \\ x > x_i^* & J_h(x, n) < J_{d,i}(n) \end{cases}$$

$\forall x \in \{0, \dots, X\}$. Then, we have:

$$J_{d,i}(n) = \frac{(1 - \gamma)(1 - \delta)\beta^\tau}{1 - \beta(1 - \delta)} \cdot \sum_{x=0}^{x_{i-1}^*} y(x, n) \cdot \mathbb{P}_{x,s,n} + \beta^\tau J_{d,i-1}(n) \cdot \sum_{x > x_{i-1}^*}^X \mathbb{P}_{x,s,n} \quad (8)$$

with $n \in \{1, 2, \dots, N\}$ $i \in \{1, 2, \dots, I\}$, and $xx \in \{0, \dots, X\}$. The value of waiting for an employer that has i remaining workers to be interviewed can be split in two parts.

The first term in the RHS of (8) is the expected discounted value of hiring the subsequent worker (and so forgoing the possibility to meet the other $i - 1$ applicants). The employer makes this choice only if $J_h(x, n)$, the value of hiring, is not lower than the value of waiting, $J_{d,i-1}(n)$, that is when the firm - worker distance is equal to x_{i-1}^* at least. This is the reason why, differently from the equation for $J_e(n)$ in (4), the productivity net of the real wage is averaged out for

values of x lower than x_{i-1}^* . The weights are given by $\mathbb{P}_{x,s,n}$ with $0 < x < x_{i-1}^*$. From equation (4), and with a little abuse of notation, they are the probabilities of interviewing a worker located at a distance lower than or equal to x_{i-1}^* , conditional on the fact that an application has arrived. This weighted average is based on the same assumptions made for equation (5), namely that at time t , the firm does not know the productivity value of the job applicant that will be interviewed at time $t + \tau$. However, the firm knows the total number of unemployed people and the distribution of their distance with respect to the entrepreneur's location.

The second term in the RHS of (8) is the expected discounted value of waiting one period more. If $x > x_{i-1}^*$, then $J_h(x, n) < J_{d,i-1}(n)$ and the employer decides not to hire the current applicant and waits yet another τ period. The underlying assumption is that, by delaying its own hiring decision, firms do not incur vacancy costs. This is plausible if we interpret them as advertising expenditures and there is a queue of potential employees.

If the firm has no other potential employee in its list (either because it received only one application or because it has just interviewed the last applicant), waiting implies to start again the search and matching process. This implies incurring advertising costs h per q_n periods of time. On the other hand, the cost of the equipment k must not be paid again. The value of not hiring is then equal to:

$$J_{d,0}(n) = -h \cdot \frac{1 - \beta^{q_n}}{1 - \beta} + J_e(n) \quad (9)$$

Notice that, from the zero profit condition (5), the value of $J_{d,0}(n)$ is approximately equal to k . The system of equations (8) is linear in $J_{d,i}(x, n)$ with $i \in \{1, 2, \dots, I\}$. Using equation (9), it can easily be solved forward.

If a match is destroyed at time t , the firm exits the market while the newly unemployed worker starts sending job applications from period $t + 1$.

4 Numerical analysis

Our quantitative procedure is organized as follows. We assume that the economy is composed by 15 macro sectors¹¹. We calibrate the model on the assumption

¹¹We follow the level of aggregation considered by the Bureau of Labor Statistics of the United States (2016) and (2017) . The sectors are: Education and Health Care, Financial activi-

that, at the beginning of our simulation, the economy is at the steady state and all the sectors are identical, namely that they exhibit the same unemployment and job opening rates¹².

Then we run a first, simplified version of the model presented in section 2. In particular, we impose the assumption that all the sectors are perfectly segmented. This means that unemployed workers are allowed to search for a job only in the sector in which they have lost it. In terms of the decision rule 2, this implies setting $\lambda \rightarrow +\infty$. Indeed, as λ increases, workers assign more importance to the productivity of the resulting match than to the labour market conditions of the sector.

In this first simulation, we also impose that the productivity shocks that hit the economy are the same across sectors. More precisely, we assume that the 15 sectors are affected by the same variation in the value added, as the one experienced on average in the U.S. economy from the first quarter 2008 until the second quarter of 2011. These restrictions (perfect segmentation and identical shocks across sectors) allow us to single out the response on aggregate unemployment and vacancies of a productivity shock when neither mismatch nor congestions in the recruitment process are present in the economy.

In a second exercise, we abandon the assumption that all the 15 sectors are hit by an identical shock but we keep the one about perfect segmentation. The changes in the value added during the same period are sector specific and taken from the Bureau of Labor Statistics (2017) . Tables 7 and 8 present data on value added variations employed in this exercise.

This second simulation identifies the share of the aggregate unemployment and vacancies variation that is due to sector misallocation. Under perfect segmented labour markets, unemployed workers in the sectors worst affected by the crisis cannot search for vacancies in other industries. This tends to raise aggregate unemployment compared to the symmetric simulation run at the first stage. The reason is the following. Like a standard search and matching framework (see

ties, Government, Professional and Business Services, Utilities, Information, Transportation and Warehousing, Wholesale trade, Other services, Leisure and Hospitality, Durable Goods, Mining, Retail Trade, Construction, Nondurable goods.

¹²We make this choice for simplicity and because we lack data on labour market variables for the 15 sectors in exam.

Calibration			
Variables	Values	Interpretation	Source
β	1	discount rate	0% on annual basis.
δ	0.01	separation rate	target initial unemployment rate
q	2.9	exp. duration of a vacancy	target initial vacancy rate
V/U	0.56	labour market tightness	target initial vacancy rate
N	15	number of sectors	macro sectors of the U.S. economy
L	2000	labour force	arbitrary choice
z	1	aggregate part of productivity	normalization
α	0.6	idiosyncratic part of productivity	Abowd <i>et al.</i> (1999) and Postel-Vinay and Robin (2002)
a	0.23	matching efficiency	equation (2)
h	0.05	advertising costs per month	Shimer (2005) and Hagedorn and Manovskii (2008)
k	0.8	fixed cost	Hagedorn and Manovskii (2008)
γ	0.99	workers' rent share	zero profit condition (11)
τ	0	time between an interview and the other	arbitrary choice

Table 1: Calibration procedure. Unit of time: month.

Pissarides, 2000), even our model exhibits decreasing marginal returns in the job filling rate: the larger the number of job positions opened in a sector, the higher the expected duration of a vacancy. So, for any given level of aggregate vacancy rate, in a model with perfectly segmented labour markets jobs creation takes longer time and the unemployment rate results higher if job openings are unevenly spread across all sectors of the economy.

The third exercise replicates the exact model presented in the previous sections. There is no perfect segmentation of labour markets and λ takes positive finite values. This third scenario allows us to gauge the relative importance of congestions in the recruitment process on the unemployment rise and the outward shift of the Beveridge curve, by comparing the difference between the results obtained in the second and first exercise and the ones obtained under this scenario. The hypothesis we aim to test is that the hiring process gets lengthier when firms are inundated by a huge and heterogeneous list of job applicants, thereby raising unemployment for any level of the aggregate vacancy rate.

4.1 Calibration

The model is calibrated on the basis of U.S. data in the period 2006-2007, before the onset of the Great Recession. The numerical values of some variables are taken

from data, others are obtained by evaluating the model at the steady-state¹³. A robustness analysis is performed for some parameters of interest.

Since sector specific labour market data are unavailable, we assume that all the 15 macro-sectors of the economy exhibit the same unemployment and vacancy rates at the beginning of the period. For reasons explained in the previous section, we run our first simulations under the hypothesis perfectly segmented labour markets (i.e. $\lambda \rightarrow +\infty$). So, for simplicity we also make this assumption to calibrate the model¹⁴. Appendix 1 shows how the model becomes under these conditions. We take the month as unit of time. Table 1 summarizes our results and the calibration procedure.

We set β equal to 1. This means a real interest rate of 0. This simplifies the computations, as firms value the future and the present the same. Yet, it is also empirically grounded, as the real U.S. Treasury yield curve in 2007 exhibited a rate close to 0.2 for a 5 years maturity¹⁵. The destruction rate δ is set equal to 0.01, the value obtained by Hobijn and Sahin (2009) for the United States.

We want to target both the unemployment rate and the job opening rate exhibited in December 2007 in the United States (5% and 2.9%, respectively). In Appendix 2, we show that imposing both conditions pin down the average expected duration of a vacancy q , the average vacancy unemployment ratio V/U , and the matching efficiency parameter, a . Despite the simplicity of our search and matching technology (an urn-ball model), we get empirically reasonable values for these variables: $V/U = 0.56$, and $q = 2.9$. These figures imply an expected duration in unemployment close to 5 months¹⁶, a value not so far from the value of 4.25 estimated by the Bureau of Labor Statistics.

At the beginning of the simulation we impose that the aggregate level of pro-

¹³The calibration exercise is computed using Wolfram Mathematica. The files are available on request.

¹⁴If λ takes positive finite values but we keep the assumption that all the sectors exhibit the same unemployment and vacancy rates in steady state, the calibration gets more complicated but the resulting numerical values do not differ in a substantial way. Details are available on request.

¹⁵See the website of the U.S. Department of Treasury <https://www.treasury.gov>.

¹⁶Denote the average job finding rate in the economy by f . Its reciprocal is the expected duration in unemployment. For each vacancy filled a worker is hired, so we have $f \cdot U = V/q$. Then $1/f = q \cdot U/V = 5.16$.

ductivity z_n is equal to 1 for all sectors. From equation (1), the value for the idiosyncratic component $\alpha(s, n)$, with $s, n \in \{1, \dots, N\}$ captures the differences in productivity from workers coming from different sectors s in a job in sector n . To the best of our knowledge, there are no empirical works that estimate the change in productivity for employees working in the same job but coming from different sectors. So, to avoid implausible assumptions, we simplify the equation (1) as follows: $y(s, n) = z_n$ when $s = n$ and $y(s, n) = \alpha \cdot z_n$ when $s \neq n$ and $0 < \alpha < 1$. When firms and workers are not located in the same sector, the resulting productivity is a fraction α of z_n , irrespective of the sector the worker comes from. It remains to find a reasonable value for α . Sunday and Pfuntner (2008) document that in the private sector the wage spread within jobs in the same establishment is about 50%¹⁷. Of course just a fraction of this spread can be attributed to a gap in workers' productivity. According to Abowd *et al.* (1999) the unmeasured individual heterogeneity accounts for more or less fifty percent of the total variance of log wages. On the other hand, Postel-Vinay and Robin (2002) get a much smaller fraction for the person effect (ranging from 40 % for the high skilled to 0% for the low skilled). We remain in the same range of values and choose the values for the idiosyncratic parts $\alpha = 0.6$

As concerns the cost of keeping a vacancy open, we set $h = 0.05$, a value in the middle of the interval of results obtained by Shimer (2005) and Hagedorn and Manovskii (2008)¹⁸. We impose τ equal to 0.

The last parameters we need to calibrate are γ , workers' bargaining power and k , the capital fixed cost of a vacancy. The latter is set equal to 0.8, a value in line with the findings of Hagedorn and Manovskii (2008). The former is obtained by solving the zero profit condition, as all the other variables in the equation have been set. We get $\gamma = 0.99$ a very large number. The reason lies on the high elasticity of vacancy creation implied by the model. The value of $q = 2.9$ obtained in the calibration (and empirically plausible) implies a relatively low number of job positions opened in each sector: on average it takes just a bit

¹⁷The wage spread is measured as the percentage by which the highest paid worker's wage in the establishment job exceeds that of the lowest paid worker.

¹⁸Shimer obtains a quarterly value of 0.213. This implies a cost of 0.071 per month. For Hagedorn and Manovskii, the labour flow cost of posting a vacancy is about 11 per cent of the average weekly labour productivity. In our model, average monthly labour productivity is 1, weekly productivity is 0.25. So the cost should be equal to 0.0275.

less than three months to fill a vacancy, meaning that few firms compete for the available applicants. Conversely, in our model in which, at this stage, there are no congestions in the hiring process, vacancy creation would be huge. To reduce it (and get $q = 2.9$), firms' expected revenues must be low, and employers' rents share tiny¹⁹.

5 Quantitative Results

5.1 Pseudocode

To summarize the main ingredients of the model, Figure 2 illustrates the steps undertaken to construct our artificial economy. In the following simulations we do not impose a steady state condition in labour market flows, as we did in the calibration procedure. However, the dynamics appear to be very stable, with the economy getting to a new steady state equilibrium shortly after the shock hit the sectors²⁰. In the Figures of this section, we report for each axis the historical Beveridge curve plus the simulated ones on a quarterly basis. Each simulated Beveridge curve corresponds to a different set of values for the parameters of the model and is composed of 15 points/quarters. The first 7 points (plotted with empty markers) are the u/v points during the recession (1st quarter 2008 to 2nd quarter 2009) whereas the 8 final points (filled markers) are the u/v points during the recovery (3rd quarter 2009 to 2nd quarter 2011). We have replicated each simulated Beveridge curve 100 times, thus we report some statistics of the distribution of 100 simulations. For each quarter we have a distribution of 100 X-Y values. We report with an empty or filled marker the average of the X (unemployment rate) and Y values (vacancy rate). Finally, the interval of two standard deviations for both axis are reported as error bars.

¹⁹Although a workers' bargaining power equal to 0.99 is clearly unrealistic, notice that in our model firms are owned by the same workers (the only agents in the economy). So γ does not really stand for capitalist's share of total output compared to the labour share.

²⁰These numerical exercises have been produced using MatLab. The codes and the output files are available on request.

5.2 First Simulation: Segmented Labour Markets and Identical Shock

As explained in section 4, the aim of this first simulation is to look at the behaviour of the Beveridge curve in the simplest scenario, when the economy does not suffer from misallocation effects and congestions in the recruitment process. The 15 labour markets are perfectly segmented and the change in the aggregate component of the match productivity $z_n = z$ is identical across sectors and equal to the average monthly change in value added experienced in the U.S. economy from the January 2008 to June 2011. Table 7 (second line) presents the figures from January 2008 to June 2009, the trough of the Great Recession in the United States (see the Bureau of Labor Statistics, 2017). In Table 8 (second line) we have data from July 2009 until June 2011, as the second quarter of 2011 is the one in which the aggregate value added returned to the pre-crisis level²¹. In absolute terms, z starts from a normalized value of 1 at the beginning of the simulation and bottoms at 0.957 in the second quarter of 2009. Then it recovers, so that in June 2011 $z = 0.999$.

In Figure 3 we present the results and plot the historical Beveridge curve²². This first simplified model is able to mimic the usual downward sloping behaviour of the Beveridge curve. It also exhibits a small outward shift during the recovery. However, the increase in the unemployment rate resulting from this first exercise is tiny compared to those observed in data. In the simulation, unemployment increases by a mere 1 percentage point, going from 5% to 5.5%, whereas the U.S. economy hit a 10% unemployment rate shortly after the trough of the Great Recession. Similarly, the vacancy rate goes from 3% to 2.1 in our exercise while data show it was 1.6% in June 2009.

Another critical drawback of this first exercise is the lack of persistence of the unemployment rate: in our exercise it gets back to the pre-crisis value in June 2011, when data show in that period it was about 9% and returned to a 5% value only in September 2015.

²¹The Bureau of Labor Statistics presents quarterly data. To get the monthly percentage change in value added we simply compute the geometric mean: $(1 + \Delta z_m)^3 = 1 + \Delta z_q$, in which Δz_m and Δz_q are respectively the monthly and the quarterly change in z .

²²Even if the month is the unit of time in our simulations, for the sake of clarity in the Figures we present quarterly data

Such a failure is hardly surprising. Since the seminal work of Shimer (2005), it is well known that a basic search and matching model is unable to replicate the behaviour of unemployment and vacancies at business cycle frequencies in response to productivity shocks of plausible magnitude. In particular, unemployment and vacancies are much more volatile in data than in the model. Although our model is not a canonical search and matching framework with an exogenous matching function, this simulation confirms the poor results documented in the literature²³.

The lack of propagation is another common drawback of search models. Fujita and Ramey (2007) show that in simulations the response of market tightness closely mimics the dynamics of productivity, exhibiting none of the sluggishness observed in the empirical data.

More in general, the poor responsiveness of the model to realistic changes in z can be read at the light of a recent strand of research that has started questioning the role of productivity shocks as the main source of the unemployment variations²⁴.

5.3 Second Simulation: Segmented Labour Markets and Different Shocks across Sectors

In this second exercise, we abandon the assumption of an identical productivity shock across sectors, but we still impose perfectly segmented labour markets. The change in value added in each of the 15 sectors is reported in Tables 7 and 8²⁵. Moreover, to convey the idea that the 15 sectors are not equally important in the economy, the exogenous labour force in each sector is assumed to be different. In particular, the ratio between the labour force in a generic sector n and the total number of workers in the economy is imposed to be equal to the ratio between

²³There is no univocal explanation for such a chasm between theory and data. Shimer (2004) and (2005) identify the lack of wage rigidity as the main culprit. Hagedorn and Manovskii (2008) point to the usual parametrization of the model. Other papers claim the standard setup does not capture some realistic features of moder labour market that would amplify the productivity shocks. See Cardullo (2010) for a detailed survey.

²⁴Hall (2017), for instance, focuses on the discount rate while Farmer (2012) looks at the stock market.

²⁵The construction sector is the one that has suffered most, with a decrease in valued added close to 20%.

the number of employees in that sector and total employment found in data and relative to December 2007.

Figure 4 presents the Beveridge curve resulting from this second scenario and the historical Beveridge curve. Three notable features have to be singled out.

First, almost the Beveridge curve obtained in this second simulation is to the right of the Beveridge curve presented in the first simulation. This means that, for any given level of aggregate job openings, the unemployment rate under asymmetric shocks is always greater than under a symmetric one. Such a gap oscillates between 0.1 and 0.6 percentage points. As we explained before, the matching efficiency is deteriorated because of the combined effect of decreasing marginal returns in the job filing rate and segmented labour markets. Decreasing returns in the hiring rate implies that, for any given level of aggregate vacancies, the employment losses when two sectors reduce job offerings by the same amount are smaller compared to the case one sector's fall is more pronounced than the other one's. With labour freely moving across sectors, this difference would tend to zero as, for instance, unemployed people in the construction sector would apply for jobs in the utility industry. But since we impose perfectly segmented labour markets, the allocation of workers across sectors is not optimal and the second simulation delivers a higher unemployment rate.

The second point worth mentioning is a corollary of the first one. In this second simulation the unemployment rate reached in the 4th quarter of 2009 is much higher than the one obtained in the first exercise. The former is close to 5.5%, whereas the latter is about 7.5%. Since in the 4th quarter of 2009 the U.S. unemployment rate peaked at 10%, this 2 percentage points difference means that misallocation can explain at most 40% of the unemployment rise during the period January 2008 - December 2009. This is in line with the 30% figure found by Sahin *et al.* (2014) in their analysis of mismatch unemployment for the period 2008 – 2009 ²⁶.

²⁶It must be stressed that we adopt different benchmark models as counterfactual to measure the relative weight of mismatch in the U.S. labour market. For Sahin *et al.*, misallocation explains the difference between the observed unemployment rate and the one that would result in an economy in which workers are all identical and a planner faces no impediment in moving idle labour across sectors, except for the within-market matching frictions. In the present work, we attribute to misallocation the increase in the unemployment rate compared to a model where every sector grows at the same average rate.

The third notable feature of this second simulation is the slope of the Beveridge curve as the economy recovers. As expected, the increase in productivity boosts the vacancy rate. Yet, the unemployment rate continues to increase, reaching an 8% value and hovering around this number in the last quarters considered in our simulation. Given the variety of the forces that affect the mechanism, it is difficult to nail down a single explanation for this behaviour. Our intuition is that such a movement originates from a recovery that has been more unequal than the recession. During the period June 2009 - June 2011, four sectors (Education and Health Care, Financial Activities, Government, and Mining) maintained a real value added above the initial measure registered at the beginning of the recession, showing to be only marginally affected by the crisis. Three sectors (Professional and Business Services, Utilities, and Information) fully recovered. The remaining 8 industries exhibited a value of output below the one observed in December 2007.²⁷ When labour markets are segmented, the surge in vacancies in the best performing sectors may be accompanied with an increase the unemployment rate in industries still scarred by the crisis, since by assumption there is no matching between these two sides of the market.

This second simulation presents some other quantitative inconsistencies. The unemployment rate peaks 8% at the end of 2011, whereas in data the highest value for u was 10% at the end of 2009. Moreover, the decline in vacancy rate is too modest along the recession and its increase is too large along the recovery.

5.4 Third simulation: Congestions in the Recruitment Process

In our third exercise, we abandon the assumption that $\lambda \rightarrow +\infty$. This means that unemployed workers take into account both the number of vacancies available in each sector and the productivity of the resulting match in their job search strategy.

Figure 5 presents the results for different values of λ . When λ takes values above 150, the results are very similar to those obtained in the previous exercise. The Beveridge curve has an U-shaped form and the predicted maximum value for

²⁷For instance, the Wholesale Trade sector experienced a 17 percentage points decline in real value added during the January 2008 - June 2009 period, and in June 2011 it was still 12 percentage points under its pre-crisis level.

the unemployment rate is about 8%, at the end of the recovery (mid 2011) and not at its beginning (end 2009), as observed in data.

With lower values for λ , the Beveridge schedule gets more empirically consistent. Consider for instance the plot for $\lambda = 23$. The peak in the unemployment rate is at 10%, the same figure experienced by the U.S. economy. In data, this value has been registered two quarters after the trough of the Great Recession, while in our exercise it is attained four quarters after. After the unemployment rate has reached its peak, it begins a slow decline, hovering around 9% for the remaining 6 quarters. The same occurs in our simulation. The predicted vacancy rates in the recovery period also take values close to those collected empirically. The former oscillates between 1.6% and 2.4%. The latter are in a 1.8% - 2.4% interval. So our simulation is able to mimic the unusual behaviour of the unemployment - vacancy locus in the aftermath of the Great Recession, when the rate of jobless workers remained quite stable in spite of an increase in the rate of job openings. We think this result confirms that sectoral misallocation alone is unable to explain the movements of the Beveridge curve in the wake of the Great Recession. In terms of our model, with $\lambda \rightarrow +\infty$ misallocation is at its maximum value and yet the simulation falls short of replicating data under several aspects. When the hiring congestions effects gain weight (at the expense of the misallocation effect), as in this third simulation, we have a better fit.

Notice that the best results are obtained for values of λ close to 24. When λ is equal to 30, the predicted Beveridge is much less successful in matching data. According to our computations, with $\lambda = 24$, at the onset of the recovery unemployed 90% of the workers that have lost their job in the sectors worst affected by the crisis (Construction and Wholesale Trade, for instance) continue searching for an occupation in these sectors, while the remaining 10% look for other opportunities in the best performing sectors (Government, Education and Health Services, for instance)²⁸. With these values, the hiring congestion effect succeeds in amplifying the unemployment response to productivity shocks. The recruitment process gets lengthier because firms in the best performing sectors are inundated by applications and the selection mechanism takes more time.

²⁸With $\lambda = 30$, these frequencies are respectively 99% and 1%.

5.5 Sensitivity Analysis

In our model two parameters are key, λ and α . A comparison of the results for different values of λ has been presented in the previous subsection. As respect to α , recall that in the baseline scenario we have assumed that the productivity resulting from the match of a firm in sector n and a worker located in sector s is $y(s, n) = z_n$ when $s = n$ and $y(s, n) = \alpha \cdot z_n$ when $s \neq n$ and $\alpha = 0.6$. Despite this 40% gap being empirically plausible, we perform a robustness check by considering a 35% gap. Figure 6 illustrates the results. It is easy to notice two notable drawbacks that distinguish the Beveridge curves in this exercise from those considered in the baseline model. First, the magnitude of the increase in the unemployment rate is much smaller and, in the best scenario ($\lambda = 23$), it peaks at 8%. Second, during the recovery periods, the model predicts a significant rise in the vacancy rate, that reaches a 3.5% value, when in data it never gets over 2.5%. We do not have a clear explanation for both phenomena. Our intuition starts with the fact that a 35% gap in productivities implies that labour is more free to move across sectors, compared to the baseline scenario. Workers have less to lose if they abandon their own sector to search for job opportunities in other industries. This in turn reduces the misallocation effect during the recession and may explain why the unemployment rate remains well below 10%. The significant increase in vacancies in the recovery quarters may be a direct consequence of the modest increase in unemployment. If the number of job seekers is relatively small, the vacancy rate tends to be larger, as the expected duration to fill a job positions becomes longer.

6 Conclusions

In some recessionary periods the increase in the rate of job openings does not have any impact on the unemployment rate, that remains substantially unchanged. The explanation offered in this paper is based on imperfect information on the true level of productivity that employees may ensure in a given job. A negative economy-wide shock on productivity reduces both unemployment and job openings. But when some sectors start recovering, the increase in the vacancy rate does not lead to a reduction in the number of unemployed people, for two reasons: sectoral mis-

allocation and congestions in the recruitment process. Although both mechanisms are not new, this paper is the first that tries to test their (joint and single) ability to explain the behaviour of the Beveridge curve.

The results show that sectoral misallocation may explain at most 40% of the increase in the unemployment rate during and in the wake of the Great Recession. Moreover, a model with the highest degree of misallocation (i.e. with perfectly segmented labour markets) delivers a U-shaped vacancy unemployment schedule, that is a poor match of the real Beveridge curve.

Conversely, in the model in which the hiring congestion mechanism takes place (and, consequently, sectoral misallocation is less important), the replicated Beveridge curve gets much closer to data.

The paper leaves open several points that need a further analysis. First, our research focuses only on the period 2008 – 2011. It would be interesting to see if the present model is able to replicate the outward shift experienced by the Beveridge curve in the subsequent years. Second, our analysis is based on the well established fact that the Great Recession and the subsequent recovery were quite asymmetric. If some sectors emerged unscarred from the crisis, other sectors produced in 2011 10% less than in 2007. Still, business cycle are hardly periods with even growth among sectors (see for instance Plunkert, 1990 for the 1990s in the United States). Moreover, in the past seventy years the U.S. Beveridge curve has experienced outward and inward shift several times (see Daly *et al.*, 2012). However, a historical analysis on how sectoral changes may have affected the behaviour of the Beveridge curve has not given so far univocal responses. The research agenda in this area is far from being concluded.

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Appendix 1: The steady-state calibrated model

In this Appendix, we show how the model in section 2 simplifies if we assume that (i) labour market conditions are identical across sectors, that is $U_n = U$ and $V_n = V$ for $n \in \{1, 2, \dots, N\}$; (ii) $\lambda \rightarrow +\infty$. We use this simplified framework in the calibration and in the first simulation exercise.

From equation (2), it is easy to see that under these two assumptions, we get $p_{s,n} = 0$ for $s, n \in \{1, 2, \dots, N\}$, $s \neq n$ and $p_{n,n} = a$. The expected duration of a vacancy in equation (3) simplifies to

$$q = \left[1 - \left(1 - \frac{a}{V} \right)^U \right]^{-1}$$

For large values of V , we can also use

$$q = \frac{1}{1 - e^{-\frac{a}{V}}}, \tag{10}$$

with $\theta \equiv V/U$ denoting labour market tightness in each sector. Rearranging the zero profit condition (5) yields:

$$q = \log_{\beta} \left[\frac{[1 - \beta(1 - \delta)] [k(1 - \beta) + h]}{(1 - \beta)(1 - \gamma)(1 - \delta)y + h[1 - \beta(1 - \delta)]} \right]$$

It is easy to show that in the case $\beta = 1$, the zero profit condition (5) becomes:

$$q = \frac{(1 - \gamma)y - \delta \cdot k}{\delta \cdot h}. \quad (11)$$

The exogenous variables at the RHS uniquely determine the value of q and, in turn, θ . In steady state we have for each sector $V/q = (L - U)\delta$, in which L is the labour force in each sector. Let f denote the job finding probability for a generic worker in each sector. Then we have $Uf = (L - U)\delta$. The equilibrium rate of unemployment is then equal to:

$$U = \frac{\delta}{\delta + \frac{\theta}{q}} L.$$

Once U and θ are pinned down, all the other endogenous variables of the model can be easily obtained.

Appendix 2: The calibration procedure

We show that targeting the initial values (December 2007) of the unemployment rate and the job opening rate in the United States pin down the average values in the economy of the expected duration of a vacancy, q , the vacancy unemployment ratio, $\theta \equiv V/U$, and the separation rate, δ . First, recall that in steady state, for each segmented sector, we have:

$$E\delta = V\frac{1}{q}, \quad (12)$$

Using equation (12) and knowing that the labour force $L \equiv E + U$, we have:

$$\frac{V}{V + E} = \frac{q\delta}{q\delta + 1} = 0.029, \quad (13)$$

$$\frac{U}{L} = \frac{q\delta}{q\delta + \frac{V}{U}} = 0.05. \quad (14)$$

If δ is taken from data, eqs. (13) and (14) represent a system of two equations in two unknown, θ and q . Solving for these two variables, we get $\theta = 0.56$ and q equal to 2.98. Using equation 10, we also $a = 0.23$.


```

"matchingFcn" code
{
for s simulations
    for b points of the beveridge curve
        for d number of days
            % job destruction
            for each firm f
                if firm f destroys the job according to probability p
                    firm f is destroyed
                    worker w associated become unemployed
                end
            end
        end
        % vacancy creation
        for each sector n
            find the number of vacancies that satisfy the system of zero profit condition in (5)
        end
        % applying
        for each u unemployed worker
            u send one application to a vacant firm following (2)
        end
        % computing the variable vacancy rate
        % computing the variable unemployment rate
        % interviewing
        for each f firm with vacant position (both new firms and firms already interviewing)
            take the first worker of the list of unemployed having submitted an application to firm f
            if the disc. future profit hiring immediately >= the expected disc. future profit to hire the
                next
                    firm f hires thus
                    firm f has no more vacant position
                    worker u interviewed become employed
                else
                    firm f will interview the next worker of its list at time d+1
                end
            end
        end
    end
end
end
}

```

Figure 2: Pseudocode

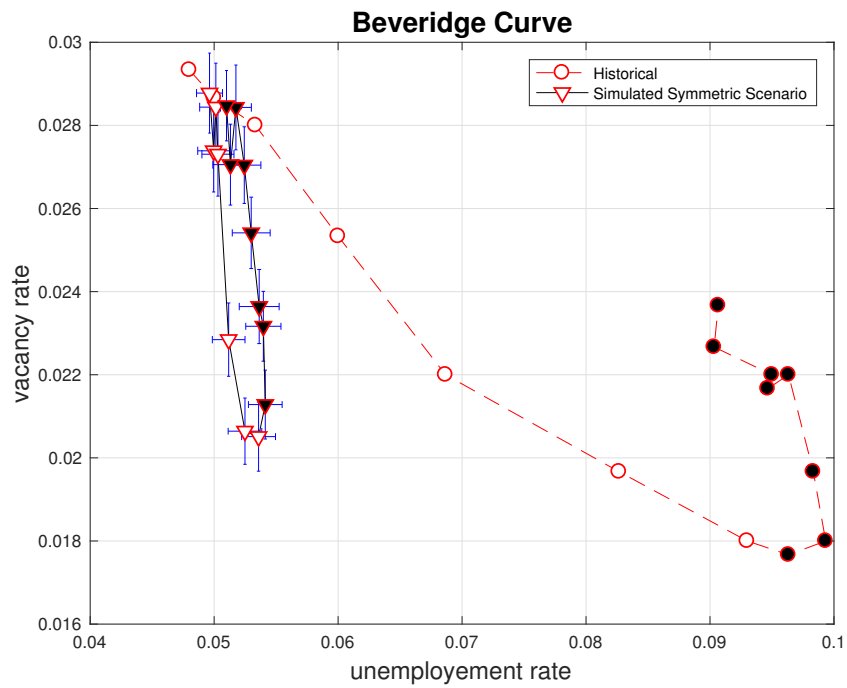


Figure 3: Beveridge curve. First simulation. Empty markers are the u/v points during the recession (1st quarter 2008 to 2nd quarter 2009). Thick markers are the u/v points during the recovery (3rd quarter 2009 to 2nd quarter 2011).

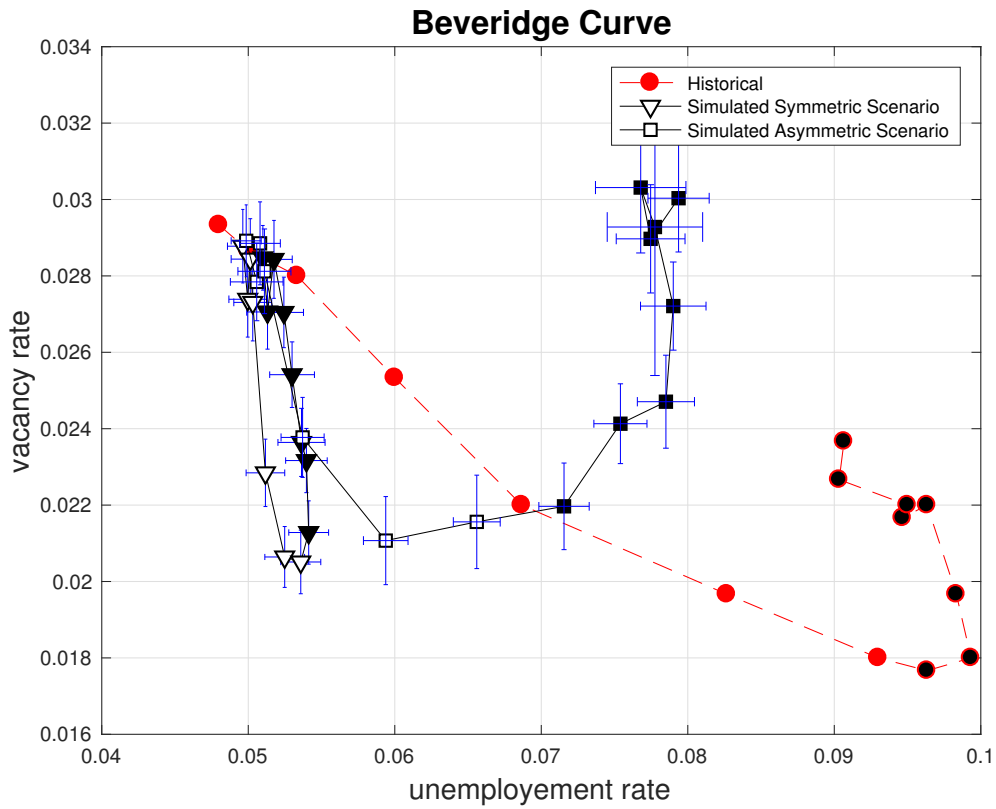


Figure 4: First and Second Simulation. Empty markers are the u/v points during the recession (1st quarter 2008 to 2nd quarter 2009). Thick markers are the u/v points during the recovery (3rd quarter 2009 to 2nd quarter 2011).

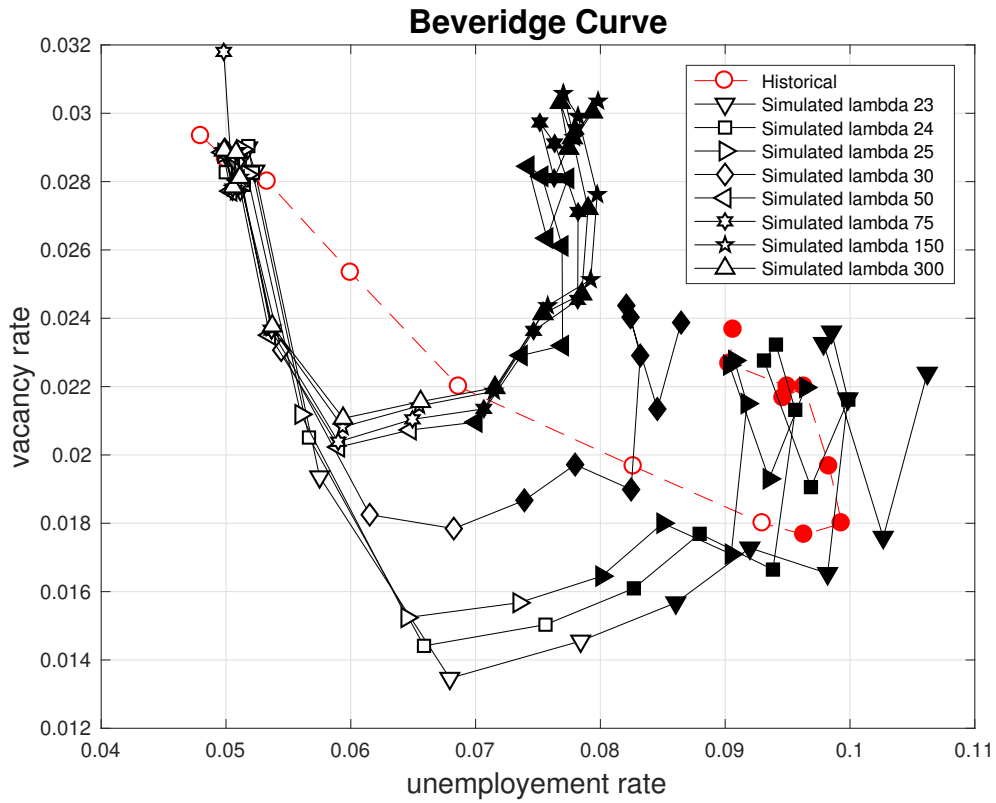


Figure 5: Third Simulation. Empty markers are the u/v points during the recession (1st quarter 2008 to 2nd quarter 2009). Thick markers are the u/v points during the recovery (3rd quarter 2009 to 2nd quarter 2011).

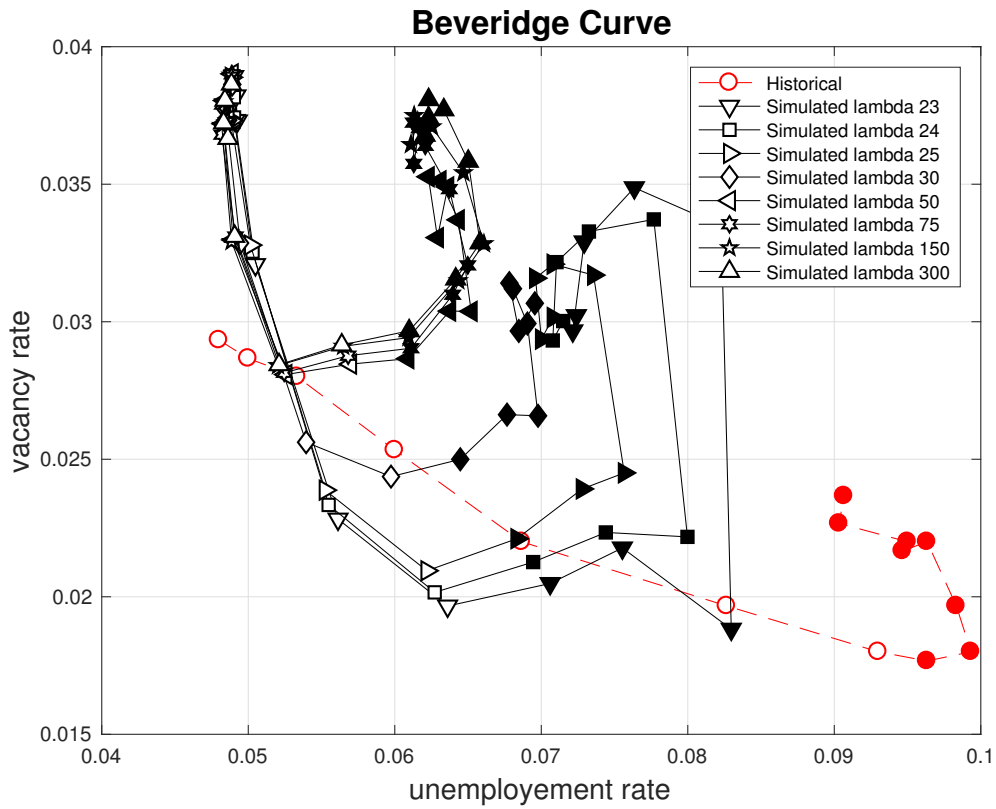


Figure 6: Third Simulation with $\alpha = 0.65$. Empty markers are the u/v points during the recession (1st quarter 2008 to 2nd quarter 2009). Thick markers are the u/v points during the recovery (3rd quarter 2009 to 2nd quarter 2011).

Average Monthly Variation	1st Quarter 2008	2nd Quarter 2008	3rd Quarter 2008	4th Quarter 2008	1st Quarter 2009	2nd Quarter 2009	stdev
AGGREGATE ECONOMY	-0.0024	0.0014	-0.0018	-0.0073	-0.0041	-0.0008	0.0030
MACROSECTORS							
Education and health care	0.0063	0.0043	0.0050	0.0030	0.0009	0.0008	
Financial activities	0.0016	-0.0059	-0.0059	-0.0104	0.0205	0.0062	
Government	0.0021	0.0012	0.0021	-0.0013	-0.0003	0.0020	
Professional and business services	0.0014	0.0058	0.0033	-0.0049	-0.0166	-0.0062	
Utilities	-0.0021	0.0186	-0.0017	-0.0495	-0.0045	0.0077	
Information	-0.0009	0.0084	0.0012	-0.0182	-0.0051	0.0003	
Transportation and warehousing	-0.0024	-0.0042	-0.0003	-0.0217	-0.0203	0.0093	
Wholesale trade	-0.0016	-0.0015	-0.0059	-0.0155	-0.0231	-0.0120	
Other services	-0.0051	-0.0033	-0.0017	-0.0021	-0.0087	-0.0036	
Leisure and hospitality	-0.0044	-0.0007	-0.0038	-0.0060	-0.0124	-0.0037	
Durable goods	-0.0038	0.0011	-0.0031	-0.0208	-0.0313	-0.0103	
Mining	-0.0135	-0.0043	0.0222	0.0740	0.0150	-0.0326	
Retail trade	-0.0075	-0.0034	-0.0038	-0.0098	-0.0003	-0.0007	
Construction	-0.0147	-0.0034	-0.0027	-0.0186	-0.0248	-0.0045	
Nondurable goods	-0.0246	0.0239	-0.0231	-0.0067	-0.0037	0.0138	
st.dev.	0.0078	0.0087	0.0092	0.0258	0.0143	0.0110	

Figure 7: Average monthly change in valued added for each quarter. Source Bureau of Labor Statistics (2017).

	3rd Quarter 2009	4th Quarter 2009	1st Quarter 2010	2nd Quarter 2010	3rd Quarter 2010	4th Quarter 2010	1st Quarter 2011	2nd Quarter 2011	st. dev.
AGGREGATE ECONOMY	0.0008	0.0027	0.0009	0.0029	0.0026	0.0027	-0.0017	0.0023	0.016
MACROSECTORS									
Education and health care	-0.0011	0.0019	-0.0020	0.0022	0.0016	0.0012	0.0008	0.0023	
Financial activities	0.0020	-0.0013	0.0021	0.0024	-0.0017	0.0054	-0.0006	0.0017	
Government	0.0002	0.0008	0.0009	0.0010	-0.0011	-0.0003	-0.0003	-0.0001	
Professional and business services	-0.0009	-0.0004	0.0067	0.0041	0.0054	0.0046	0.0008	0.0052	
Utilities	0.0097	0.0177	0.0175	-0.0118	0.0034	0.0009	-0.0133	0.0205	
Information	0.0013	0.0118	-0.0019	0.0021	0.0106	-0.0031	-0.0057	0.0025	
Transportation and warehousing	0.0028	0.0067	0.0002	0.0023	0.0113	0.0080	-0.0019	0.0034	
Wholesale trade	-0.0015	0.0109	-0.0035	0.0067	0.0089	-0.0005	0.0004	-0.0029	
Other services	-0.0050	-0.0018	-0.0044	0.0022	0.0034	-0.0002	-0.0036	0.0002	
Leisure and hospitality	-0.0024	0.0026	0.0053	0.0052	0.0030	0.0056	0.0017	0.0048	
Durable goods	0.0014	0.0115	0.0096	0.0158	0.0129	0.0086	-0.0007	0.0012	
Mining	-0.0103	-0.0096	-0.0053	0.0042	0.0007	0.0090	-0.0052	0.0080	
Retail trade	-0.0002	0.0066	-0.0013	0.0025	0.0057	-0.0011	-0.0004	-0.0013	
Construction	0.0044	-0.0125	-0.0104	0.0067	-0.0024	-0.0007	-0.0086	0.0071	
Non-durable goods	0.0096	0.0111	-0.0065	-0.0104	-0.0081	-0.0052	-0.0094	0.0017	
st.dev.	0.0050	0.0083	0.0070	0.0065	0.0058	0.0044	0.0045	0.0055	

Figure 8: Average monthly change in valued added for each quarter. Source Bureau of Labor Statistics (2017).