Identifying Choice Correspondences

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A General Method and an Experimental Implementation

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Abstract

We introduce a general method for identifying the *sets* of best alternatives of decision makers in each choice sets, i.e., their *choice correspondences*, experimentally. In contrast, most experiments force the choice of a *single* alternative in each choice set. The method allow decision makers to choose several alternatives, provide a small incentive for each alternative chosen, and then randomly select one for payment. We derive two conditions under which the method may recover the choice correspondence. First, when the incentive to choose several alternative becomes small. Second, we can at least partially identifies the choice correspondence, by obtaining supersets and subsets for each choice set. We illustrate the method with an experiment, in which subjects choose between four paid tasks. In the latter case, we can retrieve the full choice correspondence for 18% of subjects and bind it for another 40%. Using the limit result, we show that 40% of all observed choices can be rationalized by complete, reflexive and transitive preferences in the experiment, i.e., satisfy the Weak Axiom of Revealed Preferences – WARP hereafter. Weakening the classical model, incomplete preferences or just-noticeable difference preferences do not rationalize more choice correspondences. Going beyond, however, we show that complete, reflexive and transitive preferences with menu-dependent choices rationalize 96% of observed choices. Having elicited choice correspondences allows to conclude that indifference is widespread in the experiment.These results pave the way for exploring various behavioral models with a unified method.

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1 Introduction

Samuelson (1938) introduced revealed preferences by positing that an alternative x is revealed preferred to another one y if x is chosen when y is available. We know since Arrow (1959)'s Weak Axiom of Revealed Preferences (WARP) that if there is no alternative x which is strictly revealed preferred to another alternative y while y is revealed preferred to x, then the revealed preference is reflexive, transitive and complete, i.e., is a classical preference, and the decision maker can be modeled *as if* he has maximized his revealed preference to choose.[1] WARP is well known to sometimes fail in practice.[2] In this paper, we explore one possible explanation for the failure of WARP in experiments: forced single choice. In experiments, choices are elicited using a *choice function*: decision makers choose one alternative from the choice set, i.e., the set of available alternatives. In the revealed preference literature, however, choices are commonly modeled with a *choice correspondence*: decision makers choose a non-empty set from the choice set.[3] [4] Choice functions are a special case of choice correspondences, but some theories developed on choice correspondences can only be meaningfully tested with sets being chosen, rather than single alternatives, as shown in Mandler (2005) and Aleskerov, Bouyssou, and Monjardet (2007).[5] This discrepancy has largely not been adressed in practice, as very few experiments have allowed decision makers to choose several alternatives, and all of them differ from the method and experiment we introduce here.[6]

Identifying the choice correspondence of decision makers, rather than a choice function may be key in certain situations, as the following example illustrates.

Example 1.1 (Pizzeria)**.** On a small island, there is a single pizzaiolo who produces three kinds of pizzas, a vegetarian pizza (V), a four-cheese pizza (C) and a ham pizza (H). The preferences of the islanders, which he does not know, are as follows. Half of the islanders are indifferent between the vegetarian and the four-cheese pizza, and prefer both to the ham one, i.e., $V \sim C \succ H$. Another half is indifferent between the ham and the four-cheese pizza, and prefer both to the vegetarian one, i.e., $H \sim C \succ V$. For cost reasons, the pizzaiolo wants to produce only one kind of pizza. The natural choice to make in his situation is to keep the most chosen pizza. Is it the best choice for the welfare of all islanders?

Let us say that from the first half of the population (with preference $V \sim C \succ H$), a proportion p chooses the vegetarian pizza. From the second half of the population (with preference $H \sim$

[1] Completeness is given under the conditions of observing at least all pairwise choices.

[2] See Koo (1963) for an early finding, Choi, Fisman, et al. (2007) and Choi, Kariv, et al. (2014) for more recent illustrations.

[3] One real-life example of the choice of a non-empty set from the choice set is approval voting. Decision makers can vote for all the candidate they deem acceptable.

[4] Aleskerov, Bouyssou, and Monjardet (2007) and Gerasimou (2017) allow the choice of empty sets. The latter interpret the choice of an empty set as deferral of the decision.

[5] To name two: intransitive indifference and menu-dependent choice maximization.

[6] To the best of our knowledge, 5 have done so, in different ways: Danan and Ziegelmeyer (2006) and Costa-Gomes, Cueva, and Gerasimou (2019) have allowed decision makers to postpone their choices, at a cost. Sautua (2017), Cettolin and Riedl (2019) and Ong and Qiu (2018) have allowed decision makers to delegate to a random device the choice. Finally, Agranov and Ortoleva (2017) have made decision makers choose several times in the same choice set, which can be interpreted as eliciting a choice correspondence.

$C \succ V$), a proportion q chooses the ham pizza. The summary of the probability of each pizza being chosen is given in Table 1

Table 1: Fraction of the population choosing each pizza, with $0 \leq p, q \leq 1/2$.

	Vegetarian	Ham	Four-Cheese
Observed Choice	p	q	$1-p-q$
Maximal Alternatives	$\frac{1}{2}$	$\frac{1}{2}$	1

Figure 1 shows the values of the (p, q) for which each kind of pizza is kept by the pizzaiolo. Overall, in two-thirds of the situations, the pizzaiolo keeps the four-cheese pizza. In one-third of the situations, he keeps another pizza, which decreases the welfare of the population. If islanders use a coin toss to decide when they are indifferent, and the number of islanders coming to the pizzeria is large enough, he keeps the four-cheese pizza. If the number of observations is quite small, on the other hand, he may keep the vegetarian or the ham pizzas. Had the pizzaiolo known the preferences of the islander, he would have kept the four-cheese pizza in all cases. It is the pizza that does not decrease welfare for any islander, so it is the best to keep from a collective welfare standpoint, and the best for the pizzaiolo to keep, as he will retain all his customers.

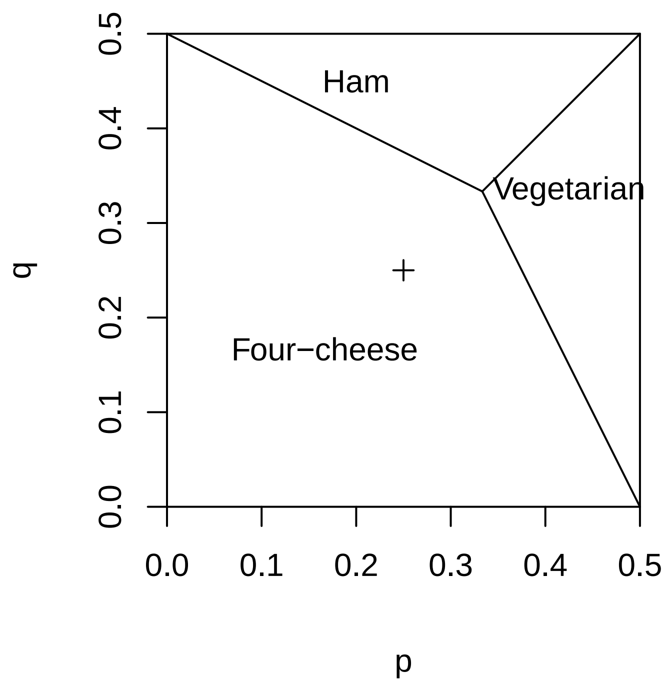

Figure 1: In which probability pair is each pizza the most chosen?

The critical reason for which the pizzaiolo may not keep the right pizza is the indifference of the decision makers. If all islanders had a strict preference, observing only one pizza would perfectly reveal their maximal alternatives. Choice functions may reveal inconsistent choices in the presence of indifference. Eliciting the indifference of decision makers may also have benefit from the a collective welfare standpoint. For instance, Maniquet and Mongin (2015) have shown that it is possible to aggregate preferences when the decision makers have at most two classes of indifferences. More general results on aggregating preferences when indifference is prevalent are lacking however, but we might expect that the more indifference there is, the easier it is to aggregate preferences in general.

We introduce here a method, *pay-for-certainty*, to identify choice correspondences. It incentivizes decision makers to choose *all* their maximal alternatives. For each alternative chosen, the decision maker earns an *bonus payment* $\varepsilon > 0$. In the previous example, the clients are better off by choosing $\{V, C\}$ and getting the vegetarian or the four-cheese pizza *and* 2ε in bonus payment rather than choosing and getting $\{V\}$ or $\{C\}$ and ε. The full characterization of *pay-for-certainty*:

> In each set of alternatives S, the decision maker chooses all the alternatives he wishes, i.e., a subset $c(S)$. He earns a (small) bonus payment of $\$\frac{|c(S)|}{|S|}\varepsilon$ by chosen alternatives. The alternative he gets is picked from her chosen alternatives using a uniform random draw.[7]

The bonus payment of pay-for-certainty implies forgone gains when the chosen set is not the whole set. Choosing only one alternative earns $\frac{1}{|S|}\varepsilon$, whereas choosing two earns $\frac{2}{|S|}\varepsilon$, and so on. We show that under mild monotonicity conditions, the decision maker chooses all maximal alternatives. The choices observed with pay-for-certainty and a ε bonus payment is called the *ε-correspondence* hereafter. This positive bonus payment is essential in order to identify a choice correspondence. Without bonus payment, in the previous example, choosing $\{V, C\}$ or simply $\{V\}$ yield the same satisfaction.

We characterize pay-for-certainty from a theoretical standpoint and provide one partial and two full identification results for the choice correspondence. Strictly speaking, choices with a bonus payment yield a superset of the choice correspondence, whereas choices with no bonus yield a subset. When this is true for all choice sets, the choice correspondence is *partially* identified. When in the same choice set, the choice with no bonus payment and with a positive bonus payment are equal, the set of maximal alternative is exactly identified. When this is true for all choice sets, the choice correspondence is *fully* identified. In theory, it is also possible to fully identify the choice correspondence of the decision makers when ε is small enough. In practice, however, it is difficult to check when this is the case.

Getting menu-choices has benefits wider than studying indifference. It allows the study of models that relax the assumptions behind maximization of a classical preference. We can in particular study intransitive indifference, as first introduced by Luce (1956) and menu-dependent maximization of the choice, while keeping a unique classical preferences, as axiomatized by Aleskerov, Bouyssou, and Monjardet (2007), Frick (2016) and Tyson (2018).

In the experiment, we follow Augenblick, Niederle, and Sprenger (2015) by using the choice between four incentivized tasks, addition, spell-check, copy, and memory. We used tasks because

[7] Other selection mechanisms are possible, but we will not investigate them in this paper. A sketch of some of them is available in Appendix A.1.

we wanted an environment where indifference might not arise easily, and Augenblick, Niederle, and Sprenger (2015) have shown that preferences on tasks tended to be strong. Following the theory of choice correspondence identification, each subject chose at least twice in the same choice set, once with no bonus payment (a 0-correspondence) and one with a positive bonus payment of 1 cent (a 1-correspondence). We use the latter as an approximation of the choice correspondence, following the limit identification results, when full identification fails. The tasks we have implemented in the laboratory are new, to get the counterfactual of forced-single choice, some subjects had to choose one and only one alternative in each choice set. To test the just-noticeable difference and menu-dependent choice, subjects in the experiments chose between all possible subsets of the four tasks, thus performing 11 choices in each bonus payment level. Finally, we used the novelty of the tasks for decision makers to investigate the influence of the quantity of information provided on the indifference. Each subject fell in one of three possible treatments on the information provided: the sentence information treatment, the video information treatment and the training information treatment. In the first one, they received very scarce information on the tasks before choosing, in the second one, they watched a video explaining each task and in the last one, they could train on the tasks before choosing.

214 subjects took part in 13 different sessions of the experiments. We fully identify the choice correspondence for 18% of them, and partially identify it for another 40%. Among subjects whose choice correspondence is not identified, most fail identification by one or two sets, which might be due to errors. 54.32% of all observed choices are sets with strictly more than one alternatives, and only 15% of all subjects always chose single alternatives when they were *not* incentivized to choose more than one alternatives, i.e., without bonus payments. The choice of multiple alternatives when possible is a robust feature of the experiments, as it survives even the training information treatment.

97% of subjects satisfy WARP when their choice correspondence is fully identified. 40% of 1-correspondences satisfy WARP, which is significantly lower from the 57% of choice functions that satisfy WARP.[8]

Additionally, we characterize couples of 0 and 1-correspondences which are compatible with classical rationality. When partial identification is happening, we characterize whether one of the partially identified choice correspondence is compatible with WARP. We find that 92% of subjects with partially identified choice correspondences are compatible with WARP. That is, even if neither their 0 nor 1-correspondence satisfies WARP, it is possible to find at least a choice correspondence in between them that satisfy it.

For 1-correspondences which do not satisfy WARP, we can go beyond it an explore models of intransitive indifference and menu-dependent choice. Just-noticeable preferences rationalize only marginally more than classical preferences do. Menu-dependent choice rationalizes almost all observed data, while being far from void. It allows us to recover a preference for almost

[8]Even though using WARP might not be appropriate with choice functions, as it assumes strict revealed preferences.

all 1-correspondences, and this is significantly higher than what can be achieved with choice functions. With the preferences at hand, we explore the amount of indifference and show that it is significant, which has consequences for aggregation of preferences. We show that the information provided has the expected effect on indifference: the more information is given, the more strict the revealed preferences are. Indifference remains prevalent, however, even in the training information treatment.

The paper is organized as follows. Section 2 defines *pay-for-certainty* and explores the conditions for partial and full identification. Section 3 describes the experiment and the sample of subjects. Section 4 explores classical rationality and WARP. Section 5 goes beyond classical rationality. Section 6 explores the influence of information on indifference and observed choices. Finally, Section 7 discusses the results and concludes.

1.1 Related Literature

1.1.1 Identifying Choice Correspondences

Gerasimou (2017) is the most closely related paper to this one. It studies the relation between choice deferral, i.e., not choosing any of the available alternatives, and incomplete preferences. It models choice deferral with a choice correspondence that might be empty valued, indicating in that case that the decision maker prefers to defer. Using choice correspondences with choice deferral allows to distinguish indifference from indecision. It also expands rationalizability of two phenomena: satisficing, i.e., an alternative is good enough to be chosen, and choice overload, i.e., too many alternatives render the choice impossible. The modeling of the choice correspondence is different from what we adopt here, as deferring is not an option, we do not have empty valued choice and cannot explore the axioms provided by Gerasimou (2017). An experimental study is done by Costa-Gomes, Cueva, and Gerasimou (2019), which we explain in more details later on.[9]

Balakrishnan, Ok, and Ortoleva (2021) use a different approach to identify choice correspondences. It starts from observed stochastic choices, i.e., from several observations from the same choice set. The intuition behind their construction is quite simple, if a decision maker is indifferent between two alternatives, then we are likely to observed switches in the chosen alternatives when choices are repeated. To separate this switches from mistakes, we can consider that only the most chosen alternatives are optimal, whereas the least chosen ones are errors, where most and least are separated by a threshold probability. We can therefore reconstruct a choice correspondence from observed repeated choices. The paper is using a different primitive from ours, and so the methods are not directly comparable.

[9]In this paper, we sometimes refer to Costa-Gomes, Cueva, and Gerasimou (2019) and Costa-Gomes, Cueva, and Gerasimou (2016). The latter is an earlier working paper version of the former, which contains more experimental results related to this paper. We try to cite the version the best related to the point made each time.

1.1.2 Menu Choice in the Literature

Six experiments have tried to elicit menu choice in practice, mainly in attempts to explore incomplete preferences. In the first one, Danan and Ziegelmeyer (2006) looked at the choice between a lottery and certain amounts using a bracketing procedure. Decision makers could postpone the choice, at a cost. This design essentially implements the *flexibility* selection mechanism given in Appendix A.1, in the context of the choice of lotteries. They found a significant proportion of decision makers exhibiting incomplete preferences, that is, preferred to postpone the choices between one lottery and one certainty equivalent.

Costa-Gomes, Cueva, and Gerasimou (2019) use a non-forced-choice procedure in the choice between headphones. Subjects could postpone the choice, at a cost, which them to have a look at the headphones. It provided them with more information about choice objects. They also find a significant proportion of decision makers who postponed, and some subjects where closer to being rationally indecisive and 73% looked like classical decision makers in a non-forced choice setting.

Cettolin and Riedl (2019) and Ong and Qiu (2018) both allowed decision makers to delegate their choices to a random device. Cettolin and Riedl (2019) implement an experiment testing the completeness axiom with ambiguous prospects, looking for Bewley (2002) preferences. They perform three experiments where decision makers had to choose between risky and ambiguous lotteries. The key is, decision makers were allowed to delegate their choice to a random device. A significant proportion of decision makers did so. Ong and Qiu (2018) built an experiment around the ultimatum game. Proposers faced binary choices between the equal allocation and (random) series of unequal allocation. Receivers face binary choices between accepting and rejecting the proposed allocations. Both receivers and proposers can randomize their choices, and had to state a willingness-to-pay for the randomization. Randomizing is only understandable if subjects have incomplete preferences, as is a positive willingness-to-pay. They found that many subjects randomize at some point, a strong indication that subjects value having the possibility to choose several alternatives, and potentially have incomplete preferences.

Agranov and Ortoleva (2017) ran an experiment where decision makers faced several times the same choices between lotteries. Crucially, even though they were aware that the choices were the same decision makers still changed their choices in some specifications. It indicates that they were not so sure about what the best alternative was.

Agranov and Ortoleva (2021) allows decision makers to choose their own probability between two options: a risky lottery and a sure outcome. They elicit a range of certainty equivalents to a given lottery. Their results cannot be reconciled with a traditional maximization of the expected utility. Instead, it can be seen as showing that preferences are kind of vague and that intransitive indifference might happen. For our interest here, it is eliciting a choice correspondence between two outcomes, by allowing subjects to choose two alternatives instead of one.

Except for Agranov and Ortoleva (2017) and Agranov and Ortoleva (2021), it is costly to choose several alternatives in these experiments. It is, therefore, a dominant strategy for subjects indifferent between two or more alternatives to select only one. In Agranov and Ortoleva (2017)'s experiment, it is not a strictly dominant strategy to switch between the repetition of the same choice when indifferent. All these experiments fail or might fail to estimate the extent of the indifference of subjects adequately. Another difference, except for the experiment run by Costa-Gomes, Cueva, and Gerasimou (2019), is that all the experiments used the choice between risk or ambiguous lotteries, and thus differed in a crucial way from the choice of tasks implemented here. The main differences between the experiment of Costa-Gomes, Cueva, and Gerasimou (2019) are the selection mechanism, and the cost of not choosing, which is much higher in their experiment compared to here.

2 Pay-For-Certainty in Theory

This section introduces, describes, and discusses the different feature of the pay-for-certainty method.

2.1 General Setup

Formally, X is a *finite* set of alternatives, $\mathcal{P}(X)$ the set of all non-empty subsets of X, i.e., $\mathcal{P}(X) = 2^X \backslash \emptyset$.[10] S designs a non-empty subset of X, a *choice set*, so that $\mathcal{P}(X)$ is the set of all possible choice sets. Choices of decision makers are modeled with a *choice correspondence*.

Definition 2.1 (Choice Correspondence)**.** A **choice correspondence** on $\mathcal{P}(X)$ associates to non-empty subsets of X a choice $c(S)$, which is a non-empty subset of S (and thus an element of $\mathcal{P}(X)$).

$$\begin{aligned} c : \mathcal{P}(X) &\rightarrow \mathcal{P}(X) \\ S &\rightarrow c(S) \subseteq S \end{aligned}$$

We do not allow the choice correspondence to be empty valued that is, $c(S) \neq \emptyset$. It constrains choices over singletons: $c(\{x\}) = \{x\}$ for all $x \in X$. We will, as a consequence, omit choices over singletons in the paper.[11] *Choice functions* are a special kind of choice correspondences, where the chosen set $c(S)$ contains exactly one alternative.

Alternatives in $c(S)$ are the ones *chosen* in S, whereas alternatives in $S \backslash c(S)$ are *unchosen* in S. For incentive purposes, the difference between chosen and unchosen alternatives is that the decision maker never gets unchosen alternatives, whereas he gets *one* of the chosen alternatives. In this sense, the decision maker wants alternatives from $c(S)$ and does not want alternatives from $S \backslash c(S)$.

Assumption 2.1 (Preferences)**.** *The decision maker has preferences $\succeq$ that are at least a partial order. That is, $\succeq$ is quasi-transitive and reflexive.*

[10] The design does not extend readily to infinite sets. We discuss why in Appendix A.2.

[11] We could expand the definition of choice functions and choice correspondences to the empty set, by assuming that the choice in the empty set is the empty set, i.e., $c(\emptyset) = \emptyset$. It does not provide any additional insight.

Importantly, preferences do not have to be transitive, only the strict part $\succ$ has to. In particular, the indifference part does not have to be transitive, which is essential to explore intransitive indifference.

Saying that chosen alternatives are maximal does not adequately characterize chosen and unchosen sets. Indeed, at least two competing assumptions on the choice correspondence are possible. Call $M(S)$ the set of maximal alternatives.

Definition 2.2 (Set of Maximal Alternatives in S ($M(S)$))**.**

$$M(S) = \{x \in S | \text{there is no } y \in S, y \succ x\}$$

The set of *maximal* alternatives in S is the set of all alternatives which are not strictly worse than any other alternative.

As long as S is non-empty, $M(S)$ is non-empty, as the set S is finite and $\succ$ is transitive. $M(S)$ contains all the alternatives a maximizing decision maker potentially chooses. All else equal, it is sub-optimal for the decision maker to choose an alternative in $S \backslash M(S)$, as he could choose a better alternative. That is, the set of chosen alternatives is a subset of the set of maximal alternatives, i.e., $c(S) \subseteq M(S)$. In that case, we can only assume that the revealed preferences are *weak* (we define formally weak and strict revealed preferences in Section 2.2).

Most theoretical work (see Schwartz (1976), Sen (1997), Nehring (1997), Aleskerov, Bouyssou, and Monjardet (2007), among others) adopt the following stronger statement. The set of chosen alternatives *is* the set of maximal alternatives, i.e., $c(S) = M(S)$. This stronger assumption implies that any unchosen alternative in S is dominated by another alternative in S, and by transitivity and finiteness of S, by an alternative in $M(S)$:

$$\text{for all } x \in S \backslash M(S), \text{ there exists } y \in M(S), y \succ x$$

It implies that choices in binary sets reveal the *strict* preferences of the decision maker, i.e., $c(S) = M(S)$. If we observe that $c(\{x, y\}) = \{x\}$, we can say for sure that $x \succ y$.

While very convenient, the strict revelation assumption is by no means guaranteed to hold in practice. The remainder of this section studies the conditions under which this assumption on chosen alternatives is legitimate with pay-for-certainty. That is, under which assumptions can we say that $c(S) = M(S)$. To be consistent, we will note from now on $c(S)$ for $M(S)$.

2.2 Revealed Preferences

Before jumping into the subject, it may be useful to offer a reminder on revealed preferences. In all the paper, R, P and I are binary relations on X, i.e., subsets of X^2. We will note xRy for $(x, y) \in X^2$, and similarly, for P and I. We can define *revealed preferences* in these three different cases.

Definition 2.3 (Revealed Preferences)**.** Revealed preferences are a collection of six binary relations, (R^0, P^0, I^0, R, P, I), defined as:

- R^0 is the *directly revealed preference.*
- P^0 is the *strictly directly revealed preference.*
- I^0 is the *directly revealed indifference.*
- R is the *revealed preference*: x is *revealed preferred* to y, noted xRy, if there exists $x_1, x_2, \dots, x_n$ such that $xR^0x_1, x_1R^0x_2, \dots, x_nR^0y$ (potentially with some R^0 being P^0 or I^0).
- P is the *strict revealed preference*: x is *strictly revealed preferred* to y, noted xPy, if there exists $x_1, x_2, \dots, x_n$ such that $xR^0x_1, x_1R^0x_2, \dots, x_nR^0y$ and at least one of them is strict P^0.
- I is the *revealed indifference*: x is *revealed indifferent* to y, noted xIy, if there exists $x_1, x_2, \dots, x_n$ such that $xI^0x_1, x_1I^0x_2, \dots, x_nI^0y$.

The three last relations are the transitive closures of the first three.

In general, P is the *asymmetric* part of the revealed preference relation R, and I is its *symmetric* part. That is, xPy if and only if xRy and not yRx and xIy if and only if xRy and yRx. R^0, P^0, and I^0 are mostly transitory tools. We are mostly interested in R, P, and I, which are supersets of R^0, P^0, and I^0. It means that R provides a summary of all the information in (R^0, P^0, I^0, R, P, I), and is often called the *revealed preference.* We will sometimes denote R as $\succeq$ with $\succ$ being P and $\sim$ being I. We can now turn to the definition of revealed preferences in our setup. We introduce two definitions of revealed preferences on finite data, the *strict* and the *weak* revealed preferences.

Definition 2.4 (Strict Revealed Preferences)**.** With strict revealed preferences, we assume that chosen alternatives are strictly better than unchosen alternatives:

- xR^0y if and only if there exists $i \in \mathcal{N}$, $x \in c(S_i), y \in S_i$.
- xP^0y if and only if there exists $i \in \mathcal{N}$, $x \in c(S_i), y \in S_i \backslash c(S_i)$.
- xI^0y if and only if there exists $i \in \mathcal{N}$, $x \in c(S_i), y \in c(S_i)$.

It is equivalent to assuming that the set of chosen alternatives $c(S_i)$ is the set of *all* the best alternatives in S_i. Arrow (1959) and Richter (1966) have used strict revealed preferences, for instance.

Definition 2.5 (Weak Revealed Preferences)**.** With weak revealed preferences, we only assume that chosen alternatives are not worse than unchosen alternatives.

- xR^0y if and only if there exists S_i, $x \in c(S_i), y \in S_i$.
- xP^0y if and only if xR^0y and not yR^0x.

- xI^0y if and only if xR^0y and yR^0x.

It assumes that some unchosen alternatives might be among the best alternatives in S. Sen (1971) provides the link between strict and weak revealed preferences. Weak revealed preferences are particularly meaningful with choice functions, as the decision makers are forced to choose one alternative and therefore cannot reveal *all* their best alternatives if they have more than one, say if they are indifference.

A simple example illustrates the differences between weak and strict revealed preferences. Take a grand set $X = \{x, y, z\}$, and choices observed in all subsets as $c(\{x, y, z\}) = \{x\}$, $c(\{x, y\}) = \{y\}$, $c(\{y, z\}) = \{y\}$, and $c(\{x, z\}) = \{x\}$. With strict revealed preferences, we deduce from the first choice xP^0y and xP^0z, from the second, yP^0x, from the third, yP^0z and from the fourth, xP^0z, which yield to xPy, yPx, xPz, and yPz. With weak revealed preferences, we deduce from the first choice xR^0y and xR^0z, from the second, yR^0x, from the third, yR^0z and from the fourth, xR^0z, which yield to xIy, xPz, and yPz. Weak revealed preferences yield a consistent revealed preference, whereas strict revealed preferences yield a logical problem if we interpret the choices in terms of welfare, as we have both that x is strictly revealed preferred to y and y is strictly revealed preferred to x. The choice of using weak or strict revealed preferences should depend on the context, and what interpretation of choice is the most sensible.

In this paper, we are looking to use strict revealed preferences, rather than weak revealed preferences, as the former has more explanatory power and is more stringent than the latter. Appendix D.3 quantifies this difference in a particular case, which corresponds to the experiment we run.

2.3 Definition of *Pay-For-Certainty*

The objective of pay-for-certainty is to recover the set of maximal alternatives of a decision maker in a set S. In most experiments, decision makers are forced to choose a single choice. For each choice set, decision makers choose precisely one alternative, which is then given to them. It is perfect for identifying the choice function of the decision maker. Arguably, this is close to the situation in the field, where decision makers generally choose one and only one alternative. There is one key difference, however. In the field, it is generally possible to postpone the choice, which is rarely the case in experiments. Dhar and Simonson (2003) have shown that forcing choice modifies the choice of decision makers. Danan and Ziegelmeyer (2006) and Costa-Gomes, Cueva, and Gerasimou (2019) show experimentally that decision makers value the possibility to postpone their choice. Agranov and Ortoleva (2017) show that sometimes, decision makers are not sure of which alternative is the best in a choice set. These pieces of evidence imply that decision maker like some flexibility in their choices. Choice correspondences introduce this flexibility in experiments by not forcing decision makers to select precisely one alternative.

There are two keys to incentivize decision makers to choose exactly their set of maximal alternatives. First, we must incentivize them to choose all their maximal alternatives. Second, they must not choose *more* than their maximal alternatives. The *selection mechanism* takes this second role. Contrary to choice functions elicitation, there are different ways to elicit a choice correspondence. The simplest was introduced in Example 1.1. Call it the *0-correspondence elicitation*:

> In every choice set S, the decision maker chooses a non-empty subset $c(S)$. A selection mechanism selects the alternative.

The *selection mechanism* is useful in practice for incentive purposes: we must select one alternative for payment in an experiment. It associates to a set of alternatives one alternative from this set. The selection mechanism we use in the paper is the *uniform selection mechanism.*

Selection Mechanism: *Uniform* When the chosen set contains more than one alternative, the decision maker gets the alternative drawn using a uniform random draw over the set of chosen alternatives.

The likelihood of getting a chosen alternative is $\frac{1}{|c(S)|}$ with the uniform selection mechanism. Adding an alternative in the chosen set has two consequences: it is now possible to get this alternative *and* it decreases the chances of getting other chosen alternatives. We describe and discuss alternative selection mechanisms in Appendix A.1.

The 0-correspondence elicitation procedure does not guarantee that decision makers choose the set of best alternatives. This non-maximal problem is very general and arises for a classical decision maker because of indifference. It can only guarantee that $c(S) \subseteq M(S)$. One solution to elicit indifference dates back to Savage (1954) and was formalized by Danan (2008): costly strict preferences. If a decision maker is indifferent between two alternatives x and y, then any small gain (cost) added to one alternative will tip the choice in its direction (the opposite direction).

Adding a small gain for each alternative chosen incentivizes the decision maker to choose larger sets when he is indifferent between alternatives. We build the *pay-for-certainty* method on this intuition.

> From the choice set S, each alternative chosen adds a *bonus payment* of $\frac{1}{|S|}\varepsilon > 0$ per alternative to the gain of the decision maker.[12] The total additional payments are $\frac{|c(S)|}{|S|} \times \varepsilon$. The alternative he gets is selected using a uniform random draw.

The introduction of the payment breaks indifference but comes at a cost. If a decision maker slightly prefers x to y, and his preference is so weak that the difference is hardly perceptible, he will choose $\{x, y\}$ and *we* will think he is indifferent between x and y, which is not the case. If ε is large enough, choosing $\{x, y\}$ and getting ε is better than choosing $\{x\}$ (or $\{y\}$) and getting $\frac{\varepsilon}{2}$. Pay-for-certainty might bundle some strict preferences with indifference. In theory, this problem vanishes when ε tends to zero. In practice, ε cannot be vanishingly small, and the problem might persist. The error made is, by construction, bounded above in monetary terms by ε.

[12] The gain or loss does not have to be monetary. It only has to be perceived as a cost or a gain to be used as payment. Time, for instance, could be used. The payment would then increase or decrease the time spent in the laboratory.

The bonus payment for each alternative chosen depends on the size of the choice set. Another possible incentive mechanism is to use a linear bonus payment, which is more straightforward than a proportional one to explain. We find, however, that the proportional bonus payment has better incentive properties in a simple case, as explained in Appendix A.3.

One benefit of pay-for-certainty for indifference elicitation is that the payment is not directly related to the alternative. Indeed, one way to implement indifference is, following Danan (2008), to observe choices where each alternative has a small bonus payment associated with it, in turns. So in the set $\{x, y\}$, we would need two choices: $\{(x+\varepsilon), y\}$ and $\{x, (y+\varepsilon)\}$. One loss, however, is to lose the direct identification of intransitive indifference (or incompleteness): observing that in the first set, the whole set is chosen and in the second set, the whole set is also chosen cannot be explained with (transitive) indifference. It is, however, quickly hard to implement when X becomes large.

A potential problem with pay-for-certainty with a uniform selection mechanism are individuals with a preference for randomization, as observed in Agranov and Ortoleva (2017) and modeled in Cerreia-Vioglio et al. (2018), for instance. Individuals with a preference for randomization will value the randomization process itself. Most of these models are built with risky or ambiguous lotteries in mind, which cannot be used with pay-for-certainty as presented here (Appendix A.2 explains why). More broadly, these models start with the idea that when outcomes are hard to distinguish, decision makers might value randomization over getting one for certainty, in particular in order to hedge between different risks. If the preference for randomization influences the preferences between outcomes in X, then we identify the order including the preference for randomization, and no problem arises. Now if x and y are hard to distinguish and it has some effects on the choice patterns, this will be identified with models of intransitive indifference or menu-dependence, and eliciting a choice correspondence is precisely the right thing to do here. One potential effect of preference for randomization is that indifferent decision maker will not select by themselves when $\varepsilon = 0$, and will choose the whole set. Again, this is is not a problem for pay-for-certainty. It might solely imply that if the whole sample has a preference for randomization, eliciting the 0-correspondence is enough to identify indifference. It is an empirical question that will be tackled in Section 3.

2.4 Identification of the Choice Correspondence

This section provides the formal conditions needed to identify the choice correspondence.

2.4.1 Assumptions

With pay-for-certainty with a uniform selection mechanism, decision makers choose a couple made of a set of alternatives and a bonus payment associated with the – size of – set. The natural choice space is the Cartesian product of the set of all non-empty subsets of X, and $\mathbb{R}$, call it Ω: $\Omega = \mathcal{P}(X) \times \mathbb{R}$. An element in this set is a couple (S, r), where S is a non-empty subset of X, and r is a real. When r is positive, it is interpreted as a payment *to* the decision maker, when it is negative, as a payment *from* the decision maker. Call *set* preferences and note $\succeq_2$ the preferences of the decision maker on this new choice space.[13] As usual, $\sim_2$ is the symmetric part of $\succeq_2$, and $\succ_2$ is its asymmetric part.

It makes sense to relate set preferences with preferences, as Ω is partly built from $\mathcal{P}(X)$. First, we start with a remark on notation.

Remark (Notation when the real part is null). When the real part is equal to zero, that is, when we consider set preferences on the space $\mathcal{P}(X) \times \{0\}$, we omit the real part:

- For two sets S and S' in $\mathcal{P}(X)$, we note $S \succeq_2 S'$ for $(S, 0) \succeq_2 (S', 0)$;
- For two alternatives x and y in X, we note $x \succeq_2 y$ for $(\{x\}, 0) \succeq_2 (\{y\}, 0)$;
- We use similar abuses of notations for $\succ_2$ and $\sim_2$.

With these notations in mind, we give the first links between preferences and set preferences.

Assumption 2.2 (Link between preferences and set preferences)**.** *We impose some structure on set preferences $\succeq_2$, in relations with properties of preferences $\succeq$. For any two elements $S, S' \in \mathcal{P}(X)$:*

1. *Preferences and set preferences are the same when payments associated with the alternatives are null, and the sets are singletons:*

$$\text{for all } x, y \in X, x \succeq y \Leftrightarrow (\{x\}, 0) \succeq_2 (\{y\}, 0)$$

2. *If for all elements $x \in S$ and for all elements $y \in S'$, $x \succeq y$, then $S \succeq_2 S'$. Additionally, if there exists $x \in S$ and $y \in S' \backslash S$ or $x \in S \backslash S'$ and $y \in S'$ with $x \succ y$, then $S \succ_2 S'$.*
3. *Take two alternatives $x, y \in X$ with $y \in S$, if $x \succ y$, then $S \cup \{x\} \backslash \{y\} \succ_2 S$.*
4. *Take two alternatives $x, y \in X$, if $x \notin S$ and there is no $y \in S$, $y \succ x$, then $S \cup \{x\} \succeq_2 S$.*

Property 1 identifies set preferences with preferences when sets are singletons, and the payments are null. Notice that we avoid as much as possible assumptions on $\sim_2$, as its interpretation in our setup is not very clear. As we do not assume anything on $\sim$, we want to avoid as much as possible assumptions on $\sim_2$ that could be translated back by this property to $\sim$. Property 2 means that if all the alternatives in a set are at least as good as all the alternatives in another set, the first set is at least as good as the second one. For the set preference to be strict, the two sets must be different by at least one alternative, and this alternative must be strictly ordered with one in the other set. Property 3 means that replacing an alternative in a set with an alternative that is strictly better imply that the new set is strictly better than the old one. Note that if x is in the original set, it implies that removing a strictly worse alternative from the original set yield a strictly better set, or the converse: adding a strictly worse alternative to the set yield a worse set. Property 4 means that adding a strictly undominated alternative yield a new set that is at least as good as the original one. These requirements are minor in the case of certain

[13]It is for clarity of the exposition, as elements of X could be sets themselves, for instance, if we think of bundles of goods.

alternatives, and knowing that the decision maker will only get one of them, which is the focus of the pay-for-certainty procedure. They might not be so minor, however, in at least two cases: risky or uncertain lotteries, or if the decision maker gets the set at the end and the alternatives might be complement or substitute (say, a chair and a table).[14]

We impose some structure on set preferences, with three assumptions, *significant differences*, *quasi-linearity*, and *quasi-transitivity*, and link preferences on X and preferences on Ω.

Assumption 2.3 (Monotonicity of set preferences)**.** *If the only difference between two alternatives in Ω is the payment, the decision maker always prefers the highest payment to the lowest one.*

$$\textit{for all } S \in \mathcal{P}(X), \textit{for all } r, r' \in \mathbb{R}, r > r' \Leftrightarrow (S, r) \succ_2 (S, r')$$

Assumption 2.4 (Significant differences on set preferences)**.** *$\succeq_2$ is sensitive to small variations in the real part.*

$$\textit{for all } S, S' \in \mathcal{P}(X) \textit{ with } S \succeq_2 S' \textit{ and } r, r' \in \mathbb{R} \textit{ with } r > r', (S, r) \succ_2 (S', r')$$

Significant differences implies monotonicity and extends it to two sets that are comparable according to $\succeq_2$.[15] It is particularly relevant when two sets are indifferent. In that case, it tells us that a small payment in favor of one set will shift the preference towards this set. It does not impose anything on $\sim$, as the variations are on the real part, not on the set part.

Assumption 2.5 (Quasi-linearity of set preferences)**.** *If one set is preferred to the other at a given level of payment, it is preferred at all levels of payments.*

$$\textit{for all } S, S' \in \mathcal{P}(X), (S, 0) \succeq_2 (S', 0) \textit{ if and only if, for all } r \in \mathbb{R}, (S, r) \succeq_2 (S', r)$$

Quasi-linearity expands the structure imposed in Assumption 2.2 from the real element being null to any value in $\mathbb{R}$, as long as it remains constant.

Assumption 2.6 (Quasi-transitivity of set preferences)**.** *For all S, S', S'' in $\mathcal{P}(X)$ and r, r', r'' in $\mathbb{R}$ such that $(S, r) \succ_2 (S', r')$ and $(S', r') \succ_2 (S'', r'')$, then $(S, r) \succ_2 (S'', r'')$.*

Quasi-transitivity imposes transitivity of strict preferences, but nothing on indifference, i.e., on $\sim_2$. Using identity, quasi-transitivity of set preferences implies quasi-transitivity of preferences, which is Assumption 2.1. The converse is not true in general.

Definition 2.6 (ε-correspondence)**.** The *ε-correspondence* c_ε is the choice correspondence obtained on Ω when the bonus payment for choosing an alternative in S is equal to $\frac{1}{|S|}\varepsilon$.

[14] Take two complementary events R (rain) and NR (not rain), and three (ambiguous) lotteries $x = (1, R; 0, NR), y = (0, R; 1, NR)$ and $z = (1/3, R; 1/3, NR)$, which should be read as: with lottery x, the decision makers gets 1€ if it rains and 0 otherwise. A sufficiently ambiguity averse decision maker will prefer lottery z to lotteries x or y. He will, on the other hand, prefer lottery $(x + y)/2$ to lottery z. So he might prefer the set $\{x, y\}$ over the set $\{x, z\}$, which is a contradiction property 3.

[15] The name will be clear with Assumption 2.8.

The ε-correspondence is the observed choices. $c(S)$ is a theoretical object. We want to link c and c_ε. We use the classical idea that decision makers should choose a maximal alternative. Here, however, an alternative is not an element of X, but an element of Ω, as the alternative they have to choose from are from Ω.

Assumption 2.7 (Maximal Choice)**.** *The decision maker chooses one of the maximal set according to $\succsim_2$.*

$$c_\varepsilon(S) \in \underset{S' \subseteq S}{\operatorname{argmax}} \left\{ \left(S', \frac{|S'|}{|S|}\varepsilon \right) \right\}$$

This argmax is not necessarily unique, as the following example illustrates.

Example 2.1 (Uniqueness of the argmax)**.** Take $S = \{x, y\}$ with preferences $x \sim y$ and no bonus payment ($\varepsilon = 0$). Then, using property 2 of Assumption 2.2, it is easy to see that $\{x\} \sim_2 \{x, y\} \sim_2 \{y\}$: the argmax is not unique.

2.4.2 Identification of the Choice Correspondence

We have talked so far of identifying the choice correspondence, that is, being able to *observe* $c(S)$. We provide now formal contents to this definition.

Definition 2.7 (Full Identification)**.** We fully identify the choice correspondence of the decision maker when $c(S)$ is the only element in the $\operatorname{argmax}_{S' \subseteq S} \left\{ \left(S', \frac{|S'|}{|S|}\varepsilon \right) \right\}$ for a given ε.

This definition has two components. First, we must guarantee the uniqueness of the argmax. Second, this argmax must be the maximal alternatives in each set. We now give the conditions for these two components to be met.

2.4.2.1 Partial Identification of the Choice Correspondence First, we need to introduce a preliminary result on the link between 0-correspondences, the choice correspondence, and the ε-correspondence when $\varepsilon > 0$, i.e., between c_0, c, and c_ε. The *0-correspondence* is the observed choices when there is no incentive to choose several alternatives. The *ε-correspondence* when $\varepsilon > 0$ is the observed choices when there is a strict positive incentive to choose several alternatives. Formally, for all sets $S \in \mathcal{P}(X)$, $c_0(S)$ is one element of $\operatorname{argmax}_{S' \subseteq S} \{(S', 0)\}$ and $c_\varepsilon(S)$ is one element of the $\operatorname{argmax}_{S' \subseteq S} \left\{ \left(S', \frac{|S'|}{|S|}\varepsilon \right) \right\}$. Before looking at the full identification of choice correspondences, let us introduce a preliminary *partial* identification result, which will help us understanding when the full identification happens.

Proposition 2.1 (Partial Identification)**.** *When strong monotonicity, the structure imposed in Assumption 2.2 and maximal choice are satisfied, for all $\varepsilon > 0$, $c_0(S) \subseteq c(S) \subseteq c_\varepsilon(S)$ for all $S \in \mathcal{P}(X)$. It is true for c_0 and any c_ε.*

Proof. The proof is in Appendix B.1. All subsequent proofs are in Appendix B. □

As long as we observe the 0-correspondence and one ε-correspondence with $\varepsilon > 0$, it is easy to check the condition $c_0(S) \subseteq c_\varepsilon(S)$ for all S. Section 3 illustrates exactly this process.

2.4.2.2 Full Identification of the Choice Correspondence

Proposition 2.1 does not guarantee that we can fully identify the choice correspondence of the decision maker. It only guarantees the partial identification of the choice correspondence. There is an obvious corollary, however.

Corollary 2.1 (Full Identification by Set Inclusion)**.** *When for one $\varepsilon > 0$ and for all S in $\mathcal{P}(X)$, we have that $c_0(S) = c_\varepsilon(S)$, we fully identify the choice correspondence of the decision maker. Moreover, $c(S) = c_0(S) = c_\varepsilon(S)$.*

Corollary 2.1 tells us that if we are lucky enough, it is possible to identify the choice correspondence of the decision makers. One difficulty of this result lies in the 0-correspondence. As pointed out in Example 2.1, the argmax may not be unique, and in particular, it may not be exactly the set of all maximal alternatives. One last result gives us a hint on when full identification is more likely to happen. We have to introduce one last assumption in order to do so, *insignificant difference.*

Assumption 2.8 (Insignificant Difference)**.** *$\succ_2$ is not sensitive to small variations in $\mathbb{R}$. For all $S, S' \in \mathcal{P}(X)$, and for all $r, r' \in \mathbb{R}$ with $(S, r) \succ_2 (S', r')$, there exists a $t > 0$ such that for all t' with $0 \leq t' \leq t$, we have $(S, r) \succ_2 (S', r' + t')$.*

In other words, if a couple (S, r) is strictly preferred to another one (S', r'), a small enough payment will not reverse the preference. It is akin to a continuity assumption of set preferences, but only on $\mathbb{R}$. It is a kind of counterpart to the significant difference assumption introduced earlier, which introduces a dissymmetry between $\succ_2$ and $\sim_2$

Proposition 2.2 (Full Identification)**.** *Under significant and insignificant difference, quasi-linearity, and quasi-transitivity, there exists $\varepsilon > 0$ such that for all ε with $0 < \varepsilon' < \varepsilon$, the* argmax *is unique and equal to c: $c_{\varepsilon'} = c$.*

Proposition 2.2 tells us that we should use the smallest positive ε possible. In practice, however, we are bounded below on the value of ε, and therefore might not be able to reach a sufficiently small one. We might have to rely on partial identification only.

2.4.3 Rationality when Full Identification Fails

It is a problem from a theoretical perspective. If a 0-correspondence and an ε-correspondence yield partial identification, but none satisfy a property, it does not mean that the choice correspondence of the decision maker, which is in between, would not satisfy it. We tackle this question, by looking under which conditions a property could be satisfied in practice when there is partial identification but not full identification.

Definition 2.8 (Compatibility with a Property)**.** A pair (c_0, c_ε) with $\varepsilon > 0$ is said to be *compatible with a property P* if:

1. For all S in $\mathcal{P}(X)$, $c_0(S) \subseteq c_\varepsilon(S)$;

2. There exists a choice correspondence c, for all $S \in \mathcal{P}(X)$, $c_0(S) \subseteq c(S) \subseteq c_\varepsilon(S)$ and c satisfies property P.

Compatibility with a property means that we cannot reject the fact that the decision maker satisfies that property. We may not observe its satisfaction because our tools are too limited to observe it, not because the decision makers do not satisfy it. On the other hand, the decision maker is not compatible with a property implies that the decision maker cannot satisfy it. An obvious sufficient condition for compatibility with property P is when c_0 or c_ε satisfy P. As we will show soon, it is not the only possible cases, however.

Unfortunately, there is no general method to tell if there exists a choice correspondence between a 0-correspondence and an ε-correspondence that satisfies a property when neither satisfies it. One possibility in practice is to check the property for all possible choice correspondences between the 0-correspondence and the ε-correspondence. This brute force method might quickly be computationally heavy, however. In the following, we provide conditions for compatibility with classical preferences, and in Appendix C.3, we explore compatibility of a pair (c_0, c_ε) beyond classical preferences, and in particular, with just-noticeable preferences. First, let us provide an example where a pair (c_0, c_ε) is compatible with classical preferences.

Example 2.2 (Compatibility with Classical Preferences)**.** Take $X = \{x, y, z\}$, and the observed choices in Table 2. The pair (c_0, c_ε) satisfies the first condition of compatibility with classical preferences. What about the second condition? Note that classical preferences rationalize neither c_0 nor c_ε.

Table 2: The observed choices of one individual, with and without the ε bonus payment.

Choice Set	0-correspondence	ε-correspondence	Choice correspondence
$\{x, y\}$	$\{x\}$	$\{x, y\}$	$\{x\}$
$\{x, z\}$	$\{x, z\}$	$\{x, z\}$	$\{x, z\}$
$\{y, z\}$	$\{z\}$	$\{z\}$	$\{z\}$
$\{x, y, z\}$	$\{x\}$	$\{x, y, z\}$	$\{x, z\}$

It is possible however to build a choice correspondence c between c_0 and c_ε that would be rationalized by a classical prefernce, as the last column of Table 2 shows. For every set S, $c_0(S) \subseteq c(S) \subseteq c_\varepsilon(S)$ and c is rationalized by classical preferences. In fact, the revealed classical preference is $x \sim z \succ y$. In this case, it is the unique preference relation which is compatible with both the 0 and ε-correspondnece.

Generally speaking, compatibility with classical preferences is more natural to tackle from a preference standpoint, rather than looking for a choice correspondence that satisfies WARP. The compatibility of a pair (c_0, c_ε) with classical preference imposes some conditions on the strict preferences $\succ$ and the indifference $\sim$.

1. On the strict preferences, it is clear that if an alternative is not chosen with a bonus, then surely it is worse than at least one chosen alternative. For all unchosen alternatives $y \in S \backslash c_\varepsilon(S)$, there must exist an $x \in c_\varepsilon(S)$, $x \succ y$. It does not have to be all alternatives that are not chosen are worse than all alternatives that are chosen, as potentially only some of the chosen alternatives (i.e., in $c_\varepsilon(S)$) will be in $c(S)$.
2. It is also the case if we want $x \succ y$, then y can never be chosen when x is available. Formally, it implies that if there exists S, with x and y in S, and $y \in c_0(S)$, then not $y \succ x$. Moreover, we might suspect that if for all S, with $x, y \in S$, y is never chosen, then $x \succ y$.
3. On the indifference part, it is also clear that if two alternatives are chosen together with no bonus payments, they are indifferent. Formally, for two alternatives x, y, if there exists S such that $x, y \in c_0(S)$, then certainly x and y are indifferent ($x \sim y$). Indeed, $x, y \in c(S)$.

Definition 2.9 (Compatible Classical Preference)**.** We propose a constructive method to build a classical preference $\succeq$ which is compatible with the pair (c_0, c_ε). For all $x, y \in X$:

- $x \succ y$ if and only if both
  1. For all S, $x, y \in S$ and $y \notin c_0(S)$ (which implies that $c_0(\{x, y\}) = \{x\}$)
  2. There is no sequence of integer $i = 1, \dots, n$ such that $y \in c_0(S_1), x \in S_n$ and $S_i \cap c_0(S_{i+1}) \neq \emptyset$ (an acyclicity condition). Note that this condition implies the first one when $n = 1$.
- $x \sim y$ otherwise.

Note that this defines a unique compatible classical preference for each pair (c_0, c_ε). We have not proved yet that this preference is classical, which is the object of the next proposition.

Proposition 2.3. *The compatible classical preference is reflexive, transitive, and complete, i.e., it is a classical preference.*

Definition 2.10 (Associated Choice Correspondence)**.** Take $\succeq$ a preference which is reflexive and quasi-transitive. Build the *choice correspondence associated with* $\succeq$, $c^{\succeq}$, by:

$$\text{For all } S \in \mathcal{P}(X), c^{\succeq}(S) = \{x \in S | \text{there is no } y \in S, y \succ x\}$$

When preferences are classical, we also have:

$$\text{For all } S \in \mathcal{P}(X), c^{\succeq}(S) = \{x \in S | \text{for all } y \in S, x \succeq y\}$$

This is a consequence of the definition of the associated choice correspondence when preferences are transitive and complete. The associated choice correspondence represents the theoretical choice correspondence of the compatible preference. As the compatible preference classical preference is a classical preference, the associated choice correspondence satisfies WARP.

Proposition 2.4 (Compatibility with Classical Preferences)**.** $c_0(S) \subseteq c^{\succeq}(S) \subseteq c_\varepsilon(S)$ *for all* $S \in \mathcal{P}(X)$ *and* $c^{\succeq}$ *satisfies WARP if and only if there exists a choice correspondence* c*,* $c_0(S) \subseteq$

$c(S) \subseteq c_\varepsilon(S)$ *for all* $S \in \mathcal{P}(X)$ *and* c *satisfies WARP.*

Corollary 2.2 (Minimality of the Indifference)**.** *The compatible classical preference is the classical preference compatible with the pair* (c_0, c_ε) *with the least indifference.*

Proof. In the proof of Proposition 2.4. □

In practice, to check the compatibility with classical preferences, it is easier to build the compatible preference using only the first part of the definition of $\succ$, and then check whether it is acyclic and whether $c^{\succsim}(S)$ is included in $c_\varepsilon(S)$ for all S. The algorithm proposed yield the compatible classical preference with the least indifference. This compatible preference is not unique, however, as Example 2.3 shows.

Example 2.3 (c is not unique)**.** Take $X = \{x, y, z, t\}$, and the choices in Table 3

Table 3: A pair (c_0, c_ε) where the compatible preference is not unique.

Choice Set	c_0	c_ε
$\{x, y\}$	$\{x, y\}$	$\{x, y\}$
$\{x, z\}$	$\{x\}$	$\{x, z\}$
$\{y, z\}$	$\{z\}$	$\{y, z\}$
$\{x, y, z\}$	$\{x\}$	$\{x, y, z\}$
$\{x, t\}$	$\{x\}$	$\{x\}$
$\{y, t\}$	$\{y\}$	$\{y\}$
$\{z, t\}$	$\{z\}$	$\{z\}$
$\{x, y, t\}$	$\{x\}$	$\{x, y\}$
$\{x, z, t\}$	$\{x\}$	$\{x, z\}$
$\{y, z, t\}$	$\{y\}$	$\{y, z\}$
$\{x, y, z, t\}$	$\{x\}$	$\{x, y, z, t\}$

The compatible preference is $x \sim y \succ z \succ t$. It is not unique, as another preference is compatible: $x \sim y \sim z \succ t$.

We have not explained yet other properties, such as the ones defining incomplete preferences. We will explain the properties in Section 5.1. The compatibility of these properties are then explored in Appendix C.3.

3 Experiment

In the previous section, we have explored the conditions under which a choice correspondence can be identified. It is now time to go to the practical implementation of the pay-for-certainty procedure.

3.1 Design of the Experiment

Subjects chose between four different incentivized tasks. They chose three times in all possible subsets of alternatives. Each time with a different gain level or forced single choice. For subjects who chose according to the forced single choice procedure, they always performed it first. We feared that because the forced single choice made subjects think of one alternative in each choice set, any multiple choice elicited after would exhibit smaller chosen sets on average. We kept the results obtained with the 0 and 1-correspondence when it was clear that the priming did not happen.

We need choices from all possible subsets of the grand set of alternatives to be able to falsify the different models we will consider. We are restricted to small sets of alternatives, as the cardinal of the powerset grows exponentially with the size of the set of alternatives. For four alternatives, we have to study 11 choices, for five, 26 choices, and six, 57. In practice, X could contain 4 or 5 alternatives. We wanted the subjects to repeat the same choices for at least two different payments, and thus we settled on a set of 4 alternatives.

Finally, we wanted to study indifference in practice. We suspected that the amount of information provided would influence the amount of indifference.[16] Indeed, it is likely that if the information is scarce, it will be hard for subjects to establish the value of the alternatives, and the bonus payment may have a more substantial influence on their choices.

The experiment has been carried out in the Laboratoire d'Economie Expérimentale de Paris (LEEP), using zTree (Fischbacher (2007)). Subjects were recruited using Orsee (Greiner (2015)). All the sessions were in French, and subjects were paid in Euro. The show-up fee was 5€, and the average total gain was 10.28€ The experiment lasted between 40 and 60 minutes, depending on the different treatments and the speed of the subjects.

3.1.1 Tasks

Subjects chose between four different paid tasks (screenshots of the tasks are in Appendix D.1.1):

- An *addition* task, where subjects had to perform as many additions of three two-digit numbers as possible. They earned 30 cents for each correct sum.
- A *spell-check* task, where subjects faced a long text with spelling and grammar mistakes.[17] They earned 10 cents for each mistake corrected and lost 10 cents for each mistake added. Their earnings were floored at 0 so that they could not lose money in this task.
- A *memory* task, where sequences of letters blinked on the screen and stopped after a random number of letters. Subjects had to give the three last letters that appeared on the screen. They earned 30 cents for each correct sequence.

[16]Another driver is the potential for different characteristics of the objects chosen to conflict (i.e., multidimensional choice), for instance, in the choice between different smartphones or different cars.

[17]For the interested French-speaking readers, it was the famous "dictée de Mérimée" with the modernized orthography of 1990: https://fr.wikipedia.org/wiki/Dictée_de_Mérimée The videos shown and the programmed tasks are available by asking the author.

- A *copy* task, where a large number of sequences of 5 letters appeared on the screen. Subjects had to copy the sequences. They earned 10 cents for each sequence.

Tasks involve some effort, which might influence the valuation made by the subjects. For instance, if they thought that the effort is more important in the memory task than in the spell-check task, and they expect the same gains, they might choose the latter over the former. It is not a problem *per se*, as long as it shapes their choices in the same way. Indeed, we chose effort in part because Augenblick, Niederle, and Sprenger (2015) claimed that real effort tasks induce sharper preferences, as subjects might feel strongly about some tasks. It is corroborated by their answers in a non-incentivized questionnaire administrated at the end of the experiment.

The effort might be a problem, in particular, in the high gain treatment. It may be seen in a first approximation as a discounting of the money increment depending on the effort involved, thus lowering the increment of payment perceived for each new success in a task. It might induce some subjects to include a worse task in terms of monetary payments, especially in the high gains, if they perceive the discounted increment in gains as lower than the bonus payment of choosing an additional task. It is one reason why we do not include the high gain treatment in the principal analysis. We believe, however, that the low gains are so low that this effect should not influence them.

The whole choice process consisted of selecting tasks. At the end of the session, subjects had three minutes to earn as much as possible performing one task. We selected the task they performed by drawing one of the 33 choices they made at random. From this chosen set, we uniformly drew one task.

Additions and sequences were randomly generated and thus did not have an end. We told subjects that it was not possible to finish the spell-checking task in less than three minutes – and indeed, none did. Before performing the paid tasks, subjects always could train for at least 30 seconds, in order to get familiar with the interface. The training was, except for one information treatment, always done after they had made all their choices.

3.1.2 Timing

Subjects in the experiment went through five steps, which we explain in the next subsections. First, we read the instructions about the experiment to the subjects. One example is translated in Appendix D.1.2.[18] This part included a description of the tasks they had to choose. The descriptions differed across treatments. Each subject also had a printed version of the instructions in their cubicles. Second, subjects chose three times eleven choices, according to pay-for-certainty, at different payment levels. For some, we replaced the first payment by forced single choices. Third, we measured the subjects' risk-aversion, following Dohmen et al. (2011) method. Fourth, subjects answered a questionnaire on some socio-economic variable and their choices. Finally, subjects performed the task that had been selected and received their payments for the experi-

[18]The instructions are available (in French) here: http://www.bouacida.fr/files/eliciting-choice-correspondences/instructions/.

ment afterward. The payment of subjects was made by drawing one of the 33 choices and one of the lotteries in the risk aversion elicitation. It is mostly in line with what Azrieli, Chambers, and Healy (2018) show to be incentive-compatible. Indeed, the payments for the lottery and the task are entirely independent and are likely to satisfy the no complementarities at the top hypothesis needed for incentive-compatibility.

3.1.3 Information Treatments

To investigate the influence of the information provided on the size of the chosen sets, we varied the explanations of the tasks. We have always given the explanations before the choices. Each subject faced one of the three possible treatments. In the *sentence* treatment, subjects received a vague description of the tasks, close to the description given in Section 3.1.1. In the *video* treatment, subjects first received the sentence treatment, and then watched a video explaining each task. The video showed the interface of the task and explained how to perform it.[19] The forced single choice elicitation followed this treatment. Finally, in the *training* treatment, subjects first went through the video treatment. Then they trained on each task for 1 minute. This training happened *before* choosing. The quantity of information orders treatments: the sentence treatment is strictly less informative than the video treatment, itself strictly less informative than the training treatment.

3.1.4 Choices

We investigate choices with pay-for-certainty at three different payment levels and compare them to choices with a choice function. The difference between each set of 11 choices is the payment for adding an alternative. We studied three different bonus payment levels, no (0 cent), low (1 cent), and high (12 cents) gains.

Following pay-for-certainty, in each set, choosing an alternative implied a gain of $\frac{1}{|S|}\varepsilon$ where $|S|$ is the size of the choice set. For instance, if the choice set is of size two and the subjects faced a high gain, choosing an alternative pays 6 cents. Fractions of cent were paid by randomization: 0.25 cent corresponds to a 25% probability of getting 1 cent and a 75% probability of getting 0 cent. When the subjects chose several alternatives in a set, the computer used the uniform distribution to select the alternative eventually given to the subjects. The 11 choices at a given level of payment were performed in a row to avoid confusion between the different bonus payment levels. The order of the different bonus payment levels was random. The order of the different choice sets was random.

For each choice set, choosing meant saying "yes" or "no" to each task. Subjects had to choose at least one task. The order of alternatives shown on the screen was random. Once the subject chose, a confirmation screen appeared. It displayed the chosen alternatives and the associated gain. We did that to decrease the risk of errors from choosing hastily. Screenshots of a choice screen followed by a confirmation screen are in Figures 2 and 3. Subjects were reminded of the

[19] The text used for spell-checking was different in the explanation and the real task.

selection mechanism on each screen.

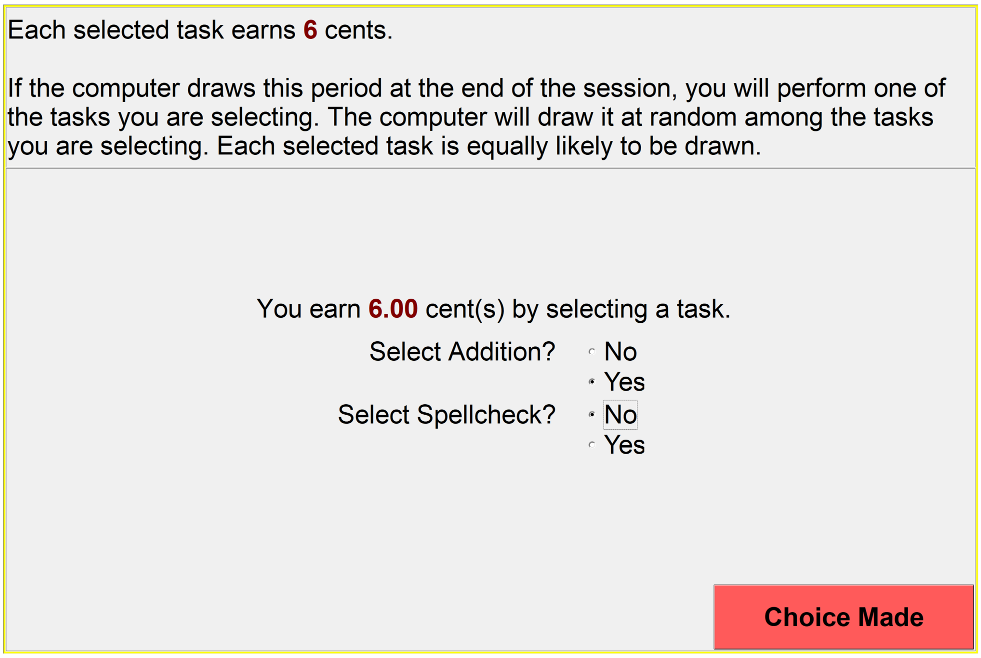

Figure 2: A choice screen.

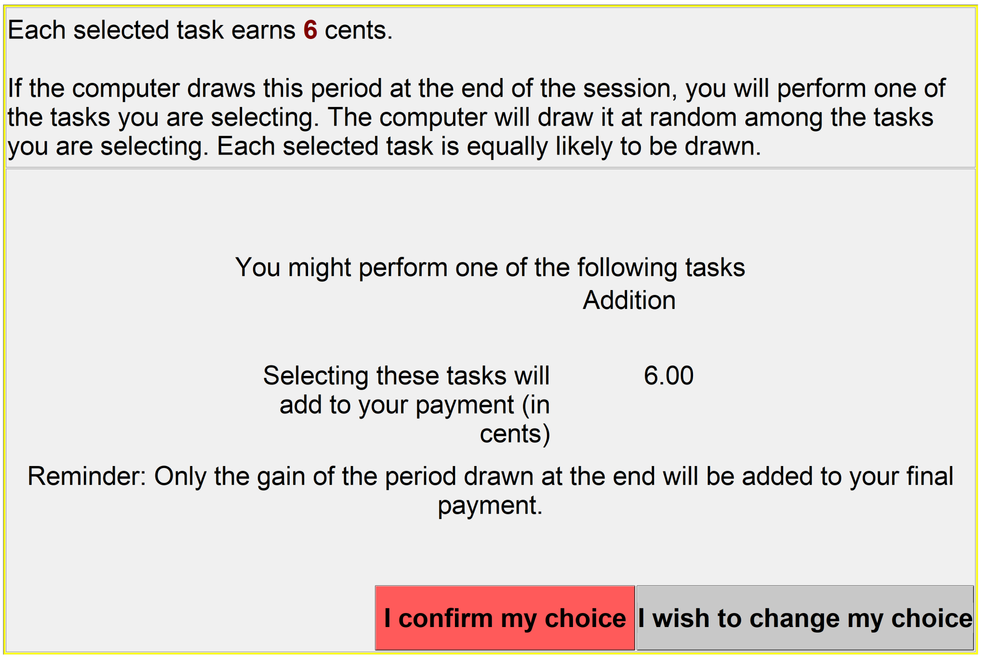

Figure 3: A confirmation screen.

3.1.5 Questionnaire

Finally, subjects faced a non-incentivized questionnaire. The questionnaire investigated some socio-economic characteristics: gender, age, level and kind of education, and jobs. The answers are manually encoded using the National Institute of Statistics and Economic Studies of France classification, the Nomenclatures des Spécialités de Formation (NSF) for education, and the Classification of professions and socio-professional categories of 2003 (PCS 2003) for the kind and level of activities.

3.2 Data

The sessions took place between the 14th of November 2017 and the 29th of May 2018. The earliest started around 11.00am and the latest finished around 6.30pm. The time the choice was made during the day varied, but it should not matter too much as we mostly compare within subjects. There is at least some anecdotal evidence that the time of the session influenced choices, however.[20]

51 subjects participated in the forced single choice elicitation, during three separate sessions. These subjects follow the video information treatment. After the forced single choice elicitation, 17 chose according to pay-for-certainty with no and low gains.[21] Results obtained on these 17 subjects are in Appendix D.7. They allow us to compare more directly the forced single choice with pay-for-certainty.

172 subjects participated in the pay-for-certainty elicitation procedure with no, low, and high gains. Among these subjects, 102 followed the video treatment, 33 the sentence treatment, and 37 the training treatment. We drop from the analysis sample subjects who chose everything all the time, as the experimental design had no bite on their behavior. It represents the removal of 9 subjects, 4 in the sentence treatment, 4 in the video treatment, and 1 in the training treatment.

All subjects of the sample have done the measure of risk aversion, the questionnaire, and the tasks. The demographics of the sample shows that it is neither a representative sample of the population nor a typical student pool. In the principal analysis, we use strict revealed preferences, as defined in Definition 2.4.

4 Revealing Classical Rationality

4.1 Comparing Revelations Method

We first compare the possibility to choose multiple alternatives to the classical experimental method. We compare the results obtained in terms of consistency and preference revealed.

4.1.1 The Experimental Benchmark: Forced Single Choice

51 subjects were forced to choose a single alternative in all possible choice sets. Classical preferences rationalize 57% of observed choices. We assume that subjects reveal only strict preferences. As all revealed preferences are strict, each classical subject has six strict binary relations. We weaken this assumption in Appendix D.5 and show that more subjects are rationalized by classical preferences (80% to be precise) and that we can recover some indifference (29% of subjects have exactly one indifference relation, the others have only strict preference relations). We lose a lot of explanatory power, however, in allowing weak revealed preferences, as Appendix D.3 shows.

[20] One subject said that it was just after lunch and she was tired so that she chose an easier task.

[21] All did it, but in two sessions, the results obtained showed that the participants did not understand the instructions. We dropped them.

4.1.2 0-Correspondences

We study one of the simplest ways to allow subjects to choose several alternatives: pay-for-certainty with no bonus payment. The sample has 180 subjects. Classical preferences rationalize 44% of 0-correspondences. The proportion is lower than with choice functions, but the difference is not significant, with a Fisher exact test p-value of 0.14. One benefit of allowing subjects to choose multiple alternatives is to observe indifference directly. The preference is only meaningful, so far, when subjects are rationalized by classical preferences. Figure 4 shows it is significant and heterogeneous. Some subjects are fully indifferent, whereas some have fully strict preferences. On average, 4.35 relations are strict preference relations, and 1.65 are indifference relations.

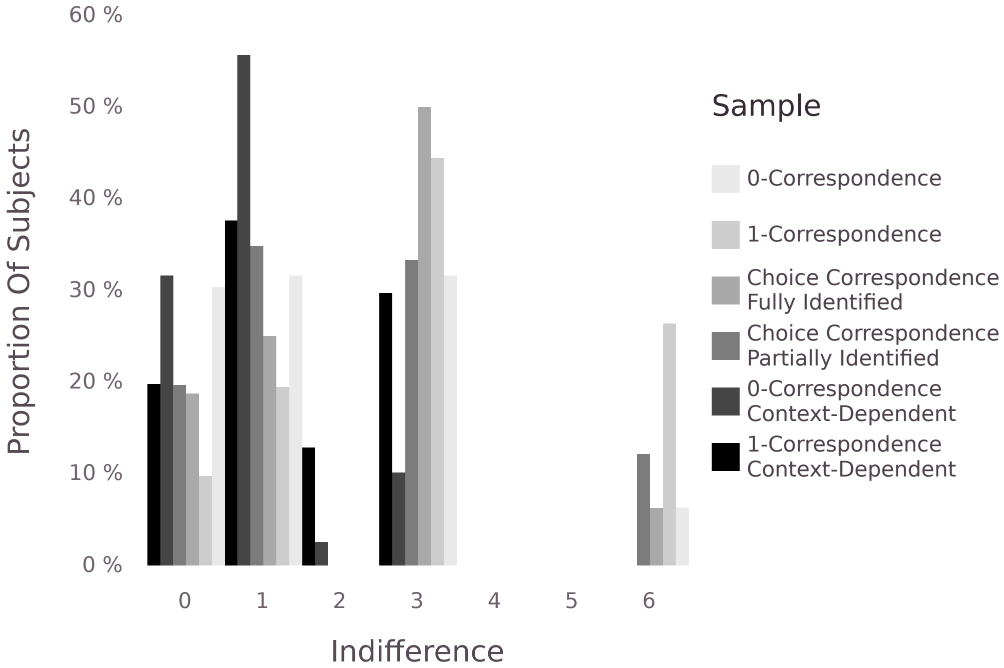

Figure 4: Histogram of the number of indifference relations with different samples.

4.1.3 The Limit Benchmark: 1-Correspondences

We interpret the 1-correspondences as the choice correspondence of the decision makers, thanks to the limit identification result of Proposition 2.2. We assume, therefore, in this subsection that the one cent bonus payment is low enough to warrant the limit interpretation. We will check this assumption in Section 4.1.4.

The sample has 180 subjects, which are the same as with 0-correspondences. Classical preferences rationalize 40% of 1-correspondences, which is significantly lower than with forced single choice. [22] The difference between 0 and 1-correspondences is not significant, however.[23] Figure 4 shows that indifference is significant and heterogeneous, and higher than with 0-correspondences. The difference is significant, as the p-value of the Kolmogorov-Smirnov test of the two samples being

[22]The p-value Fisher exact test is 0.048.

[23]The p-value of the Fisher exact test is 0.52.

Table 4: Average number of strict preference relations and indifference relations for one subject with classical preferences.

	Strict Preference	Indifference	N
Forced Single Choice	6***	0***	29
0-Correspondence	4.35***	1.65***	79
1-Correspondence	3.11	2.89	72
Fully Identified Choice Correspondence	3.88**	2.12**	32
Partially Identified Choice Correspondence	3.92**	2.08**	66

Note:
P-value of two-sided two-sample t-test of equality with respect to the value given by the 1-correspondence. *: p < 0.05, **: p < 0.01, and ***: p < 0.001.

Table 5: Identification of choice correspondences.

	Identification		
	Full	Partial	None
Number of subjects	33	72	75
Proportion of subjects	18%	40%	42%

drawn from the same distribution is lower than 0.001.[24] Table 4 shows that on average, 3.11 relations are strict preference relations, and 2.89 are indifference relations. It is significantly more indifference than with 0-correspondences

4.1.4 The Theoretical Benchmark: Identified Choice Correspondence

We can qualify the results obtained before by using the identification results based on set inclusion, rather than the limit. That is, we partially identify the choice correspondence of a subject when $c_0(S) \subseteq c_1(S)$ for all S, and we fully identify it when all the chosen sets are equal. In that case, we take into account two choices for each choice sets. The total sample is the same as before. We have to introduce some preliminary results, however, regarding the identification of the choice correspondence. Table 5 shows that we partially or fully identify the choice correspondence for a majority of subjects.

We can look at their distance from partial identification. Figure 5 represents the number of sets which violates $c_0(S) \subseteq c_1(S)$. All subjects whose choice correspondence is partially or fully identified never violates the above inclusion, which is why 58% of subjects are at 0. Most subjects whose choice correspondence cannot be identified are not far from identification. The median number of violating sets is 2. Some subjects, however, violate the set inclusion assumption radically. When the number of sets which violates $c_0(S) \subseteq c_1(S)$ is low, we might suspect that it is a mistake. In the remainder of this subsection, we exclude these subjects from the results and concentrate on fully and partially identified choice correspondences.

[24] All Kolmogorov-Smirnov test will be the same, so, from now on, we abbreviate it as the KS-test.

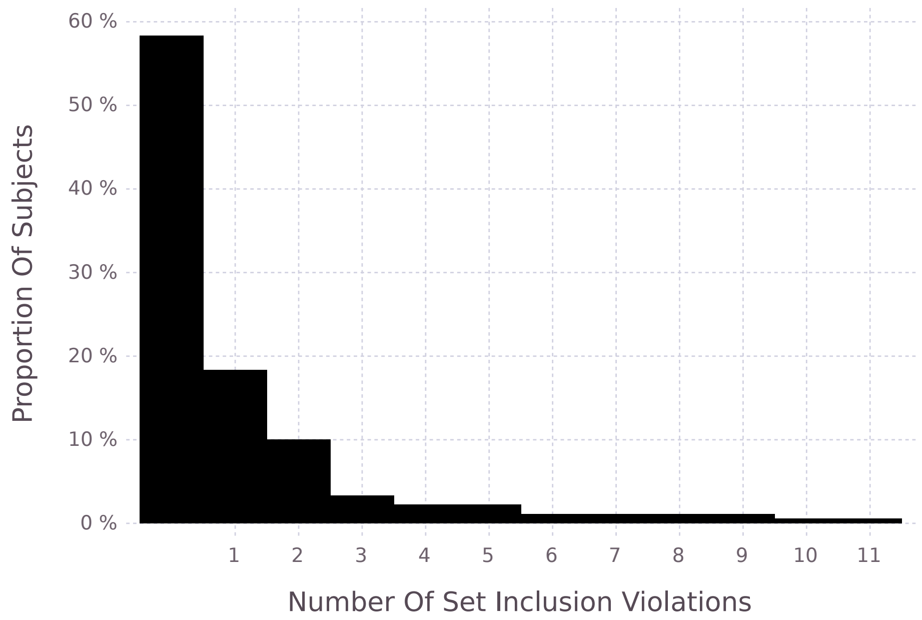

Figure 5: Histogram of the distance from partial identification. It shows the number of sets that violated set inclusion between the 0 and 1-correspondences.

4.1.4.1 Fully Identified Choice Correspondences In this subsection, we restrict the study to the sample of 33 subjects whose choice correspondence is fully identified. Classical preferences rationalize 97% of these subjects. It is significantly higher than with forced single choice and 0 and 1-correspondences, with p-values of Fisher exact test below 0.001.

We quantify the indifference and strict preferences of these subjects. Figure 4 shows a significant heterogeneity again in the kind of preference relations we observe. On average, 2.12 relations are indifference relations, and 3.88 are strict preferences, which is not significantly different from 0-correspondences.[25] The distributions, however, are significantly different, with a p-value of KS-test lower than 0.001. The average number of indifference relations is significantly lower than 1-correspondences, and the distributions are also significantly different. The p-value of the KS-test is 0.04.

4.1.4.2 Partially Identified Choice Correspondences In this subsection, we restrict the study to the sample of 72 subjects whose choice correspondence is partially identified. We can use the partial identification results given in Section 2.4.3. We find that 92% of observed choices are compatible with classical preferences, i.e., may have a choice correspondence that satisfies WARP between their 0-correspondence and their 1-correspondence. This result is lower than with fully identified choice correspondences, but not significantly so, with a p-value of the Fisher exact test of 0.58. It is significantly higher than with forced single choice and 0 and 1-correspondences, however, with p-values of the Fisher exact test below 0.001.

When we use the classical compatible preference, remember that it is the one with the most strict

[25] The p-value of the two-sample two-sided unequal variance t-test of equality of the mean is 0.16.

preferences, as shown in Corollary 2.2, preferences relations exhibit both indifference and strict preferences. We see a significant heterogeneity in the preferences of subjects in Figure 4 again. On average, 2.08 relations are indifference relations, and 3.92 are strict preference relations, which is not significantly different from identified choice correspondences. The distribution of indifference is significantly different; the p-value of the KS-test is lower than 0.001.

4.2 Welfare Exploration

A natural question from an individual welfare analysis is to ask what alternatives are preferred by decision makers. In this experiment, the fact that a subject prefers one task over another is of little relevance to the real world, but if we think about policy recommendation based on observed choices, this is a relevant question. Showing the distribution of the preference of decision makers does not yield a simple conclusion, as they are quite different. We can use a second best, however, which is to show the proportion of the sample with classical preferences that deem each alternative as one of the maximal ones. An alternative is maximal if, according to the revealed preference of the decision maker, no other alternative is strictly better. It allows us to build a crude collective welfare order, by considering that overall, the best alternative collectively is the most approved.

Table 6 shows precisely this. For choice functions, as we reveal only strict preferences, it means that each classical subject has exactly one maximal alternative, the one which is chosen in the grand set of alternatives. For choice correspondences, as we have found many indifference, it is possible, and we observe, that several alternatives are maximal. Indeed, each alternatives is maximal for more than half of the sample once we allow for the choice of multiple alternatives. The difference between the 0-correspondences and forced single choice is significant.[26] 1-correspondences have more indifference than 0-correspondences, sometimes significantly so. It is translated accordingly into the maximal alternatives.

Once we move to partially or fully identified choice correspondences, we observe that the proportion of subjects who deem an alternatives maximal is between the 0 and 1-correspondences, which is inline with the construction of the identified choice correspondences. The figures are significantly higher for partially identified choice correspondences compared with choice functions and not significantly different from 0 or 1-correspondences or fully identified choices correspondences.[27]

One limitation of this assessment is that we have to throw away all subjects who are not classical. Appendix D.8 provides a robustness check by using a non-preference based approach, using Condorcet winners. The results are mostly the same.

[26]The p-value of the Fisher exact test between 0-correspondence and forced single choice are: addition: 0.022; spell-check: 0.022; memory: 0.006; copy: 0.021.

[27]Using a Fisher exact test of equality for each task. For choice functions, the p-values are 0.03 for addition and spell-check and below 0.001 for memory and copy.

Table 6: Proportion of each task being maximal, restricted to subjects who are rationalized by classical preferences.

	Task				
	Addition	Spell-check	Memory	Copy	N[a]
Forced single choice	24%***	24%***	21%***	31%***	29
0-correspondence	51%*	51%*	52%	58%*	79
1-correspondence	72%	72%	65%	78%	72
Fully identified choice correspondence	66%	66%	53%	59%	32
Partially identified choice correspondence	50%*	50%*	59%	71%	66

Note:
An alternative is maximal if, according to the revealed preference of the decision maker, no other alternative is better. P-value of Fisher exact test with respect to the value given by the 1-correspondence. *: $p < 0.05$, **: $p < 0.01$, and ***: $p < 0.001$.
[a] In all the tables, N is the sample size.

4.3 Related Literature and Discussion

Few papers proceed to similar tests of rationalizability by classical preferences on experimental data. The setups can be widely different, but their results are not. They all show significant violations of WARP. Except for Costa-Gomes, Cueva, and Gerasimou (2019), however, they all use choice functions to elicit preferences.

In Bouacida and Martin (2020), 47% of subjects satisfy WARP all the time in the experimental data of Manzini and Mariotti (2010), which is similar to the figure we found on choice functions. Choi, Fisman, et al. (2007) found around 35% of subjects violating WARP for choices over risky assets. In the closely related large-scale field experiment of Choi, Kariv, et al. (2014), around 90% of subjects violate WARP for a similar choice task. The subject pool here is in between their two pools, as shown in Appendix D.2. Costa-Gomes, Cueva, and Gerasimou (2019) have two main treatments in their experiments: one where subjects are forced to choose one alternative and one where they can postpone at a cost. When forced to choose a single alternative, 54% of subjects satisfy WARP, and when not forced, 73% satisfy WARP. The first figure is remarkably close to ours, whereas the second is quite higher.

Comparing the results obtained, assuming the limit identification and the partial and full identification yield several conclusions. Limit identification has the benefit of assessing the consistency of the whole sample. It yields, however, a lower (potential) rationalizability with classical preferences, and a higher indifference, as shown in Table 4. If we believe that the sample selection with full and partial identification is not too severe, these results suggest that fractions of 1 cent were not low enough to use a limit identification result. The rest of the literature inclines us to think that the results obtained with full and partial identification are overly optimistic, however. We believe that the results obtained with 1-correspondences are more representative of what we should expect in terms of consistency in an experiment.

To the best of our knowledge, we are the first to investigate indifference directly in an incentive-compatible manner. We find that indifference is significant in both 0 and 1-correspondences. It might have consequences for collective welfare analysis, as aggregating preferences with some indifference is more accessible than when preferences are fully strict, as shown by Table 6. Assuming 1 cent is a low enough incentive not to bundle together strict preferences and indifference. The significant difference in the number of indifference relations between 0- and 1-correspondences cannot be reconciled with the idea that the whole population has a preference for randomization. Indeed, otherwise, all subjects who value randomization would already choose all the alternatives they are indifference in between with the 0-correspondence, which is not what we observe.

5 Going Further with Correspondences

To simplify the exposition, we use the term correspondences to lump together 0-correspondences, 1-correspondences, partially and fully identified choice correspondences. In addition to qualifying the robustness of the classical experimental method and the consistency of observed choices, the method and the experiment aim at testing models of decision making involving intransitive indifference and menu-dependent choices, which we will discuss now. The models we are going to describe rationalize the same observed choices as classical preferences do on choice functions, as shown in Aleskerov, Bouyssou, and Monjardet (2007). They are therefore indistinguishable on forced single choice and require at least a 0-correspondence elicitation to be explored. We start by exploring menu-independent choice models and then explore menu-dependent choice models.

5.1 Beyond Classical Preferences in Theory

One possibility to explore the failures of revealed preferences is to change the kind of preference it is mapped into. That is, to replace classical preferences with other kinds of preferences. A starting point is to relax the assumptions of classical preferences: transitivity, and completeness.[28]

Transitivity of *strict* preferences has strong normative backing for welfare analysis, as the lack of transitivity means that it is potentially impossible to determine the best alternatives in a set, and thus makes it impossible to think about individual welfare. Violating the transitivity of strict preferences would also imply weird logical conclusions. It is possible, however, to relax transitivity of the indifference, using models of just-noticeable differences introduced by Luce (1956) and Fishburn (1970). Armstrong (1939) provides an early critique of the normative appeal of transitivity of the indifference. Revealed preference conditions for intransitive indifference models have been given by Schwartz (1976) and Aleskerov, Bouyssou, and Monjardet (2007), among others. Luce (1956) provides a famous example of why transitivity of the indifference might not be desirable, involving coffee and sugar. The intuition for intransitive indifference is

[28] Reflexivity does not have implications on revealed preferences.

that decision makers might not perceive the difference between two alternatives under a certain threshold.

Completeness has been criticized quite early on from a normative standpoint too (see Aumann (1962), Bewley (2002)). Various reasons have been put forward in the literature to explain the emergence of incomplete preferences, which might be summarized in two. First, the decision maker might lack information on the alternatives available. Second, he might lack information on his preference over very rare alternatives. Relaxing completeness does not prevent the finding of the best alternatives in a set of alternatives. Eliaz and Ok (2006) and Aleskerov, Bouyssou, and Monjardet (2007) provide revealed preference conditions for incomplete preferences.

From an empirical standpoint and with our modeling of choice correspondences, models of intransitive indifference with no transitivity conditions imposed on the indifference part of the preference are equivalent to models of incomplete preferences which impose transitivity on the strict part of the preference. For this reason, we will mainly talk about intransitive indifference.

It is possible to go even further with choice correspondence, and study models where the choice depends on the choice set, but not the preference. In these models, decision makers only approximately maximize their choice, and this approximation is choice set-dependent. Set-independence maximization has been criticized from a positive standpoint by Sen (1997). He argues that external conditions might influence the choice but not the underlying preference, and in particular moral considerations. Set-dependent models as studied here have strong link with models of intransitive indifference.

The intuition of the set-dependent models we will study here is as follows. Decision makers have difficulties in distinguishing alternatives that are close to each other and thus might choose alternatives that are not the best but close from the best. The threshold to distinguish depends on the set considered in these models, whereas it does not in just-noticeable difference models. It means that the other available alternatives influence the threshold. In Frick (2016), this threshold increases with set inclusion. She called it a *monotone threshold model.*[29] Aleskerov, Bouyssou, and Monjardet (2007) introduced models that are relaxations of the model of Frick (2016). First, the threshold depends on the set considered, but there is no monotonicity condition imposed, it is *menu-dependent threshold model.* Second, the threshold depends on the set and the alternatives considered, is is a *context-dependent threshold model.*[30]

Intransitive indifference models take their roots in the Weber-Fechner law of psychophysics that states that the threshold above which a difference between two stimuli is perceived is proportional to the original stimuli. It captures the idea that the magnitude of the difference between two measurable objects must be large enough to be noticed. Fishburn (1970) is a survey of the theoretical literature on intransitive indifference models. Three models will be tested here, the original semi-order model of Luce (1956), the interval order model of Fishburn (1970) and the

[29]Tyson (2018) has introduced models of set dependent choices that are strengthening of the monotone threshold model.

[30]These models are far from being compatible with every data, as shown in Appendix D.3.

partial order model. The formal definitions of the different intransitive indifference models used in this paper are given in Appendix C.1. Aleskerov, Bouyssou, and Monjardet (2007)'s Chapter 3 provide the corresponding testable conditions for rationalizability. The summary needed here is in Appendix C.2.

In addition to relaxing transitivity of the indifference, we also relax menu-independence. In order to understand the relaxation of menu-independence we use, it is useful to introduce the link between the choice and the utility in these models.

Definition 5.1 (General Threshold Representation)**.** A choice correspondence c on X admits a threshold representation if there exist two functions $u : X \to \mathbb{R}$ and $t : X \times X \times \mathcal{P}(X) \to \mathbb{R}^+$ such that for every S,

$$c(S) = \{x \in S \,|\text{for all } y \in S, u(x) \geq u(y) + t(x, y, S)\}$$

u is the fully rational benchmark, i.e., represents a classical preference, and t the departure threshold of the representation, t is always positive valued. It depends only on combinations of x, y, and S, but one could imagine other dependency structures if more information about the context of choice is available. This dependency structure captures interval orders and semi-orders, as defined in Definition C.4 in Appendix C.1, as well as attraction, decoy or choice overload effects from behavioral economics.[31]

In menu-independent just-noticeable difference models, the threshold between two alternatives may only depend on the alternatives. In menu-dependent models, it also depends on the set. The threshold function can depend on various combinations of x, y, and S, such as $t(x, S)$, for instance. When the threshold also depends on the sets, Aleskerov, Bouyssou, and Monjardet (2007) showed that these threshold representations reduce to menu-dependent ($t(S)$) or context-dependent ($t(x, y, S)$) models.[32] Partial orders are the exception here, as far as we know, it is only representable with a multi-utility representation, as in Ok (2002).

5.2 Intransitive Indifference in Practice

In theory, it is possible to rationalize choice correspondences when classical preferences fail with relaxations of complete and transitive preferences. We relax transitivity of the indifference using the axioms given in Appendix C.2. We can investigate these models because we have observed menu choices and not only a singleton choice. To the best of our knowledge, we are the first

[31] The attraction and decoy effects are the facts that introducing a third dominated alternative in the choice between two alternatives will change the relative probability of each alternative being chosen. See Landry and Webb (2021) for a general model of attraction and decoy effects. The choice overload effect is the fact that adding alternatives to the choice sets might yield worse welfare. See Chernev, Böckenholt, and Goodman (2015) for a review of the origins and effects of choice overload.

[32] More precisely, a threshold model with a threshold of the form $t(y, S)$ can be equivalently represented by a threshold model with a threshold of the form $t(x, y, S)$, as shown in Aleskerov, Bouyssou, and Monjardet (2007)'s theorem 5.1. We have kept the latter representation. A threshold model with a threshold of the form $t(x, S)$ can be equivalently represented by a threshold model with a threshold of the form $t(S)$, as shown in Aleskerov, Bouyssou, and Monjardet (2007)'s theorem 5.2. We have kept the latter representation.

Table 7: Rationalizability by a weakening of classical preferences.

	Correspondence		Identified	
	0	1	Partially	Fully
Classical Preferences	44%	40%	92%	97%
Semi-Order	45%	42%	94%	97%
Interval Order	45%	42%	94%	97%
Partial Order	45%	42%	94%	97%
Occasional Optimality	57%*	57%**	97%	97%
Menu-Dependent	82%***	91%***	100%*	100%
Context-Dependent	88%***	96%***	100%*	100%
N	180	180	72	33

Note:
Significance levels are assessed by using a Fisher exact test. The baseline value are figures obtained with classical preferences. *: $p < 0.05$, **: $p < 0.01$, and ***: $p < 0.001$.

to study the empirical validity of intransitive indifference in a systematic way. It is despite Luce (1956) stating that there is a large body of literature in psycho-physics backing the notion of just-noticeable difference. Sautua (2017) rules out intransitive indifference as explaining his observations, but it is not a test of intransitive indifference models *per se*.

We explore three intransitive indifference models: semi-order, interval order, and partial orders. The first two impose some consistency on the indifference and strict preference relations, whereas the latter only requires the strict preference to be transitive.

Table 7 summarizes the satisfaction of the different models depending on the specification of the choice correspondence considered. Using menu-independent weakening of classical preferences is not very helpful to rationalize correspondences in the experiment. The various weakening rationalizes marginally more correspondences than classical preferences, and the difference is never significant. Interestingly, partial orders, interval orders, and semi-orders rationalize the same correspondences in the experiment. It does not have to be the case in theory, as the difference in explanatory power in Appendix D.3 shows.

5.3 Menu-Dependent Choices in Practice

In addition to intransitive indifference, we can explore models of menu-dependent choice. Specifically, we explore the occasional optimality model of Frick (2016), and the menu- and context-dependent models of Aleskerov, Bouyssou, and Monjardet (2007). Table 7 show the menu- and context-dependent choices rationalize almost all 0- and 1-correspondences. It is significantly more than classical preference. When we consider partially or fully identified choice correspondences, the baseline obtained with classical preferences is much higher, and the results lose much of their significance.

One benefit of the context-dependent choice model is to show that one underlying preference

Table 8: Proportion of each task being maximal, restricted to subjects who are rationalized by context-dependent choice and not classical preferences. In parenthesis are the p-value of the Fisher exact test of equality with classical preferences, task by task and ε-correspondence by ε-correspondence

	Task				
	Addition	Spellcheck	Memory	Copy	N
0-Correspondence	44% (0.524)	35% (0.077)	37% (0.078)	48% (0.264)	79
1-Correspondence	51% (0.009)	39% (<0.001)	50% (0.075)	58% (0.012)	101

Note:
An alternative is maximal if, according to the revealed preference of the decision maker, no other alternative is better. No difference are significant, according to a Fisher exact test.

can explain choices that are menu-dependent. It is quite easy to build this preference with the data of our experiment, as explained in Section C.2.6. We consider the preferences obtained when we can rationalize choices with the context-dependent choice model only. The preferences revealed by the other models are the same than the context-dependent choice model but applied on a smaller sample. On average, subjects who are rationalized by the context-dependent choice model but not rationalized by classical preferences have 0.91 indifference relations on 0-correspondences and 1.52 on 1-correspondences, which is significantly lower than the similar figures obtained with classical preferences.[33] Figure 4 show that the indifference revealed by the context-dependent choice is significantly lower than with classical preferences. The distributions are also significantly different, as the p-values of the KS-tests are below 0.001. Notice that subjects whose choices can be rationalized by classical preferences are excluded from the figures of the context-dependent choice, even though when a subject is classical, his choices can be rationalized by a context-dependent model. We did this in order to avoid double counting subjects who are rationalized by classical preferences.

Finally, the lower number of indifference relations of subjects which satisfy the context-dependent choice but not the classical model has a corollary. Fewer alternatives are maximal, as shown by comparing Tables 8 and 6. The differences with classical preferences on 0-correspondences are mostly insignificant, but they are significant on 1-correspondences.

5.4 Discussion

Using correspondences, we have explored revealed preferences that are not classical. We have shown in Section 4.1 that allowing subjects to choose several alternatives decreased their rationalizability by classical preferences. Intransitive indifference and menu-independent models do yield to significantly higher rationalizability in this experiment. On the flip-side, however, we show in this section that we rationalize almost all correspondences with menu-dependent models. The difference in satisfaction between WARP on forced single choice and FP on 0

[33]The p-values of the two-sided two-sample unequal variance t-test of equality of the means are below 0.001 in both cases.

Table 9: Rationalizability of 0 and 1-correspondences, depending on the information provided.

	0-Correspondence			1-Correspondence		
	Sentence	Video	Training	Sentence	Video	Training
Classical Preferences	55%	40%	47%	52%	39%	33%
Partial Order	59%	41%	47%	52%	42%	33%
Occasional Optimality	66%	57%	53%	69%	52%	61%
Menu-Dependent	83%	79%	92%	100%	88%	94%
Context-Dependent	93%	85%	92%	100%	94%	100%
N	29	115	36	29	115	36

and 1-correspondences is significant.[34] In that sense, we have restored the rationalizability of subjects.

Additionally, the context-dependent models allow us to identify a unique underlying classical preference explaining the choice. We study the preferences obtained when the context-dependent model rationalizes the observed choices of the subjects. They exhibit significantly less indifference, which implies that the samples of subjects satisfying WARP and the sample of subjects which satisfy FP but not WARP are different, not only on their rationalizability but also on their preferences.

6 The Value of Information in the Experiment

In this experiment, we varied the information provided to the subjects. We expect the information provided to influence the choice of decisions makers, as it is likely to influence their valuation of each task. 115 subjects took part in the video treatment, 29 in the sentence treatment, and 36 in the training treatment. The sample sizes are small for the latter two, rendering any exploitation of full and partial identification results hardly meaningful so that we only consider 0 and 1-correspondences.[35] We will not show results on these.

We assess how the information provided influences the consistency of subjects. Table 9 summarizes the satisfaction of the different models depending on the information provided, on 0- and 1-correspondences.[36] Overall, it looks as if the information provided is slightly harmful in terms of consistency, in particular on 1-correspondences. The differences in the different axioms satisfaction between information treatments are not significant, according to Fisher exact tests.

On the other hand, Table 10 shows that the information provided is beneficial in terms of gains in the tasks. The difference between the training and the two other treatments are significant,

[34] The p-values of the Fisher exact test are both below 0.001.

[35] In the sentence treatment, we fully identify the choice correspondence of 4 subjects and partially identify it for 19. In the training treatment, figures are respectively 9 and 12.

[36] Results for semi-order and interval order have been removed from the analysis, as they provide no additional information compared to partial orders.

Table 10: Average gains in the tasks, depending on the information provided and the model rationalizing choices. Context-Dependent choices does not include classical preferences.

	Information		
	Sentence	Video	Training
Classical Preferences	2.63€*	3.08€	3.85€
Context-Dependent	2.52€**	2.84€**	3.78€

Note:
Significance levels are reported with respect to the training treatment, for a two-sided two-sample unequal variance t-test of equality of the means. *: $p < 0.05$, **: $p < 0.01$, and ***: $p < 0.001$.

but the difference between the sentence and the video treatment are not, maybe because of the small sample sizes.

Turning ourselves to the analysis of the preferences revealed by the subjects, we see that indifference is heterogeneous between subjects, and the information treatments indeed influence the amount of indifference. Figure 6 shows that some subjects are entirely indifferent, whereas some have only strict preference relations. Overall, more information leads to less indifference.

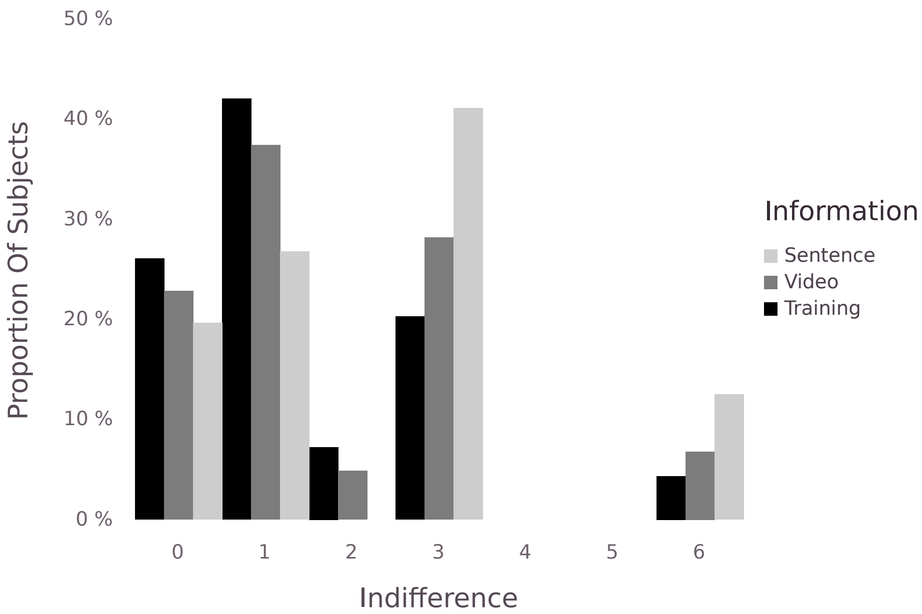

Figure 6: Histogram of the number of indifference relations by information provided. The p-value of the KS-test is equal to 0.01 between the training and the sentence treatment, and below 0.001 between the video treatment and the other two.

We investigate the cross-influence of the information treatment and the bonus payment levels jointly by running an ordinary least-square regression with and without subjects fixed effects. We use interaction effects between the level of bonus payment and the information treatment

Table 11: OLS regressions on the number of indifference relation on the sample of subjects rationalizable by context-dependent choice, controlling for rationalizability by classical preferences.

	Indifference	
	(1)	(2)
Intercept	0.63**	
	(0.215)	
Bonus	1.93***	1.76***
	(0.367)	(0.363)
Video	0.11	
	(0.248)	
Training	0.09	
	(0.328)	
Video & Bonus	-0.99*	-0.97*
	(0.419)	(0.393)
Training & Bonus	-1.47**	-1.31**
	(0.496)	(0.406)
Classical	1.13***	1.13***
	(0.169)	(0.206)
Fixed Effects	No	Yes
N	331	308
Adjusted R^2	0.24	0.79

Note:
Fixed effects are at the subject level. Robust standard deviations are in parenthesis. *: p < 0.05, **: p < 0.01, and ***: p < 0.001.

to capture the idea that the influence of the bonus payment may depend on the information provided.

$$I = \beta_1 \text{bonus} + \beta_2 \text{training} + \beta_3 \text{video} + \beta_4 \text{training} \times \text{bonus} + \beta_5 \text{video} \times \text{bonus} + \beta_6 \text{WARP} + \varepsilon \quad (1)$$

Table 11 shows the coefficient of the regressions given in Equation (1). *Bonus* is a dummy of value one for 1-correspondences and zero for 0-correspondences. *Training* is a dummy of value one if it is the training treatment. *Video* is a dummy of value one if it is the video treatment. *WARP* is a dummy of value one when the decision maker is rationalized with classical preferences, and of value zero otherwise. The baseline treatment considered is, therefore, the sentence treatment with no bonus payment as the information is fixed at the subject levels, no direct effect of information that can be captured in the fixed effect regression.

The regression with fixed effects shows that there is a positive and significant impact of the gain level on the number of indifference relations, as going from 0 to 1 increases on average the number of indifference relations by 1.75. The cross effects of information and the bonus payments are negative and significant and partially cancels out the direct effect of the bonus payments. The more information is provided at a given bonus payment and the lower the number of indifference relation is. The fact that a decision maker can be rationalized with classical preferences also has a positive and significant effect on the number of indifference relations. It indicates that classical and non-classical decision makers are likely to be different. The adjusted R^2 is much higher with fixed effects, indicating that much of the explanatory power is due to subjects heterogeneity. Overall, the small bonus of 1 cent is enough to drive the choice of the subjects, when they lack information about the tasks.

7 Conclusion

In this chapter, we have discussed the difference between the empirical and theoretical literature on revealed preferences. Most of the theoretical literature, and in particular the literature interested in the relaxation of the classical paradigm, start with a choice correspondence, whereas most of the empirical literature identifies a choice function. Decision makers are assumed to choose all the maximal alternatives in a given choice set, whereas, in practice, they are forced to choose a single alternative in a given choice set.

Incentivizing decision makers to choose exactly their maximal alternatives is not easy in practice. We introduced a new method to do so, *pay-for-certainty with a uniform selection mechanism*. We provide two conditions under which the choice correspondence is identified. One is testable in practice, but restrictive, and another that is likely to be more general, but not testable. Full identification might be hard to get in practice, so we also characterize the compatibility with different properties when we have a weaker partial identification.

In this chapter, we have implemented pay-for-certainty with a uniform selection mechanism in a laboratory experiment. The method is working in practice, as a majority of subjects are consistent with partial or full identification with set inclusion, the main requirement for its validity. It is also valuable, as it confirms the idea that restricting the choice of decision makers to one alternative in practice is a real constraint, at least for some applications.

We have shown that when we partially or fully identify the choice correspondence of decision makers, we cannot falsify the fact that they have classical preferences. If we use the limit identification result, on the other hand, classical preferences rationalize less than with forced single choice. The benefit of getting sets that are chosen rather than a single alternative is that we can use the context-dependent model to rationalize most of the 1-correspondences. The difference between classical preferences and context-dependent choice imply that many subjects behave as an approximate maximizer. They do not seem to attribute a precise enough value to each task to maximize their preferences in a classical sense. They behave consistently enough for us to reveal a preference with correspondences, however.

The preferences revealed in this experiment with choice correspondence exhibit widespread indifference. It is significantly higher than what could have been obtained with choice functions, even assuming weak revealed preferences. It does not disappear even when there are no incentives to choose several alternatives, nor when the information provided is quite complete. Even a simple elicitation method for choice correspondence, i.e., 0-correspondence elicitation, which does not identify the choice correspondence alone, is better than choice functions to study indifference. It has important implications for individual welfare analysis, as it implies that we overestimate strict preferences, and thus overestimate the individual loss due to switching to a seemingly Pareto-dominated alternative. It also has collective welfare implication, as the preferences with some indifference may break Arrow's impossibility theorem.

A Extensions of Pay-for-Certainty

A.1 Selection Mechanisms

With choice correspondences, for incentive purposes, precisely one of the selected alternative must be used for the payment. We select this alternative through a *selection mechanism*. It associates to a set of alternatives one alternative from this set. The selection mechanism we use in the core of the paper is the *uniform selection mechanism*.

Selection Mechanism: *Uniform* When the chosen set contains more than one alternative, the decision maker gets the alternative drawn using a uniform random draw over the set of chosen alternatives.

The likelihood of getting a chosen alternative is $\frac{1}{|c(S)|}$ with the uniform selection mechanism. Adding an alternative in the chosen set has two consequences: it is now possible to get this alternative *and* it decreases the chances of getting other chosen alternatives. Other selection mechanisms are possible, and some of them have been used in the literature.

Selection Mechanism: *Own Randomization* When the chosen set contains more than one alternative, the decision maker chooses a distribution on chosen alternatives. The decision maker gets the alternative drawn from her distribution on chosen alternatives.

In this mechanism, the decision maker chooses the distribution. It is more complicated to implement experimentally, as it adds more features to the elicitation procedure, but yields to potentially more fruitful results.

Another possible mechanism is delegation – to someone else.

Selection Mechanism: *Delegation* When the chosen set contains more than one alternative, the decision maker gets an alternative selected by someone else.

In this mechanism, the selection process is now a black box for the decision maker, and potentially the observer. They do not know how the selection is made. Some assumptions are required to ensure correct elicitation. The first one is the absence of preference for delegation when preferences are strict. If subjects prefer to delegate no matter what happens, they will choose the whole set. The second one is the absence of an "experimenter" effect. The beliefs on the selection mechanism might influence the choices made by the decision. The uniform selection mechanism explicitly states the probabilities of getting an alternative. As long as the decision makers trust the experimenter, the beliefs should be correct and commonly known. With delegation, the constraints on the beliefs on the selection mechanism are hard to infer for the observer. For instance, decision makers might believe that the choosers will behave adversely, by choosing, for instance, the least costly alternative to the experimenter. One way to mitigate that might be to delegate anonymously to other subjects in the experiment and not to the experimenter. The point is, beliefs are hard to control with delegation, and the observed choices might reflect both the preference and the beliefs of the decision makers.

The last selection mechanism is flexibility.

Selection Mechanism: *Flexibility* When the chosen set contains more than one alternative, the decision maker will face the chosen set again later and will have to select one alternative.

This mechanism has been used in at least two experiments, one by Danan and Ziegelmeyer (2006) and the other by Costa-Gomes, Cueva, and Gerasimou (2019). It can be used with the pay for certainty choice procedure, under the condition that payments are costs and not gains. Otherwise, the incentive scheme is such that choosing the whole set and then selecting the alternative in the second choice is a strictly dominant strategy. With a cost, the incentive is to reduce as much as possible the chosen set in the first stage. It is not possible to guarantee that decision makers always choose the set of maximal alternatives with flexibility, and thus indifference may be underestimated.

A.2 Extending Pay-For-Certainty to Infinite Sets

We discuss here why extending pay-for-certainty to infinite sets is not straightforward. In particular, why the Andreoni and Sprenger (2012) method for preferences over risky assets cannot be used with pay-for-certainty.

A.2.1 Infinite Choice Sets

Take (X, μ) to be a measurable set. The natural extension of pay-for-certainty in this context is:

- The decision maker can choose any subset of $S \subseteq X$;
- The gain of choosing one subset is $\left(1 - \frac{\mu(c(S))}{\mu(S)}\right)\varepsilon$;
- The alternative given to the decision maker is selected from the chosen set using the uniform selection mechanism.

This procedure is the natural counterpart of the pay-for-certainty with the uniform selection mechanism on finite sets. It reduces to pay-for-certainty on finite sets using the uniform measure.

This naive counterpart does not work as intended, as an example illustrates.

Example: *Infinite choice set* Take $X = [0, 1]$, with the canonical measure on $\mathbb{R}$. The decision maker has to choose the certainty equivalent in $[0, 1]$ of the lottery $(1/2, 0; 1/2, 1)$. Assume that she is imprecise about her exact certainty equivalent but knows it is in the interval $I = [0.4, 0.5]$. The gain from this choice is:

$$\left(1 - \frac{\mu([0.4, 0.5])}{\mu([0, 1])}\right)\varepsilon = \left(1 - \frac{0.1}{1}\right)\varepsilon = 0.9\varepsilon$$

By density of $\mathbb{Q}$ in $\mathbb{R}$, an equivalent choice is to choose the set $I' = \mathbb{Q} \cap I$, a set of measure 0, which yield to a gain of ε.

Avoiding this pitfall requires forcing the choice of convex sets in S, which in turns requires S to be a convex set. It is an additional technical restriction on the choice of the decision maker that has no clear justifications.

A.2.2 Finite Choice Sets on Infinite Sets

One might want to restrict to finite choice sets with an underlying infinite structure. It does not work straightforwardly either. Following the previous example, but now restricting the choice of the certainty equivalent to be one or several of the following values {0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}.

An interpretation problem arises: what happens if his certainty equivalent is 0.41? Assuming that ε is small enough, would he choose 0.4 or {0.4, 0.5}? It is not clear. If she chooses 0.4, what is his certainty equivalent? We certainly cannot guarantee that it is equal to 0.4; it may as well be 0.42 or 0.38.

The pay for certainty procedure can be applied on finite sets with no underlying infinite structure (i.e., not for risky lotteries, not for consumption choices). The extension to a choice set with an underlying infinite structure is not straightforward and requires additional assumptions.

A.3 Linear versus Proportional Bonus Payment

The justification behind the choice of a proportional rather than a linear payment scheme is quite simple. Imagine the choice between two alternatives x and y, with a linear payment scheme. That is, for each alternative chosen, the bonus payment is of ε, no matter the size of the choice set. Let us assume, for simplicity, that the decision maker maximizes a quasi-linear expected utility. The selection mechanism is a uniform distribution over the set of chosen alternatives. That is, he values the set $\{x, y\}$, $u(\{x, y\}) = \frac{x+y}{2} + 2\varepsilon$. By a slight abuse of notation, because of quasi-linear expected utility, we note x the utility of x.

Let us also assume that he prefers x to y, that is $x \succ y$. Clearly, the decision maker will never choose y, as $u(\{x\}) > u(\{y\})$. Now, the choice between $\{x\}$ and $\{x, y\}$ depends on ε:

$$\begin{aligned} u(\{x\}) &> u(\{x, y\}) \\ \Leftrightarrow x + \varepsilon &> \tfrac{x+y}{2} + 2\varepsilon \\ \Leftrightarrow x - y &> 2\varepsilon \end{aligned}$$

So $\{x\}$ is chosen when x's utility is higher than y's utility and 2ε.

Introduce now an additional alternative z. Introducing a third alternative z should not change the choice between x and y. That is, if $\{x\}$ is chosen over $\{x, y\}$, then $\{x, z\}$ should be chosen over $\{x, y, z\}$.

$$\begin{aligned} u(\{x, z\}) &> u(\{x, y, z\}) \\ \Leftrightarrow \tfrac{x+z}{2} + 2\varepsilon &> \tfrac{x+y+z}{3} + 3\varepsilon \\ \Leftrightarrow x + z - 2y &> 6\varepsilon \\ \Leftrightarrow \tfrac{x+z}{2} - y &> 3\varepsilon \end{aligned}$$

Now if z is, in fact, a copy of x, then we have in the second that x's utility needs to be higher than y's utility and 3ε, not 2ε. It is quite easy to see that a proportional payment does not have this feature. In general, a proportional payment has the benefit of taking into account the variation in probabilities induced by the uniform selection mechanism. A linear payment scheme cannot capture this externality.

In the next sections, we will study *under which condition the pay for certainty method with the uniform selection mechanism allows one to recover the set of best alternatives.* We need two kinds of assumptions: assumptions on preferences and assumptions on behavior.

B Proofs

B.1 Proof of Proposition 2.1

Take $S \in \mathcal{P}(X)$. We prove the first inclusion and then the second inclusion by contradiction. First inclusion, $c_0(S) \subseteq c(S)$:

- Imagine it is no the case. There exists an alternative x in $c_0(S)$ which is not in $c(S)$.
- $c(S)$ is made of all the undominated alternatives in S so that x must be dominated by another alternative y in S ($y \succ x$).
- Take the set $S' = c_0(S) \cup \{y\} \backslash \{x\}$. x does not belong to $c_0(S)$, we have $S' \succ_2 c_0(S)$ (by property 2 from Assumption 2.2). This is in contradiction with the fact that $c_0(S)$ is in the argmax when $\varepsilon = 0$.
- Therefore, $c_0(S)$ must contain only maximal alternatives in S and $c_0(S) \subseteq c(S)$.
- Example 2.1 above makes clear why it might not be an equality.

Second inclusion, $c(S) \subseteq c_\varepsilon(S)$:

- Imagine it is not the case. There exists an alternative x in $c(S)$ which is in not in $c_\varepsilon(S)$.
- Take $S' = c_\varepsilon(S) \cup \{x\}$. x is a maximal alternative, so by property 4 in Assumption 2.2, we have $S' \succeq_2 c_\varepsilon(S)$.
- x is not in $c_\varepsilon(S)$, so that $|S'| - 1 = |c_\varepsilon(S)|$.
- By strong monotonicity, we therefore have that $\left(S', \frac{|S'|}{|S|}\varepsilon\right) \succ_2 \left(c_\varepsilon(S), \frac{|c_\varepsilon(S)|-1}{|S|}\varepsilon\right)$ Therefore $\left(c_\varepsilon(S), \frac{|c_\varepsilon(S)|}{|S|}\varepsilon\right)$ is strictly dominated when the additional payment is $\frac{1}{|S|}\varepsilon$ which is a contradiction with the fact that it is an element of the argmax.
- It implies that $c(S) \subseteq c_\varepsilon(S)$.

B.2 Proof of Proposition 2.2

Take a set $S \in \mathcal{P}(X)$. The case where $c(S) = S$ is trivially true. The interesting cases are when $c(S) \subset S$.

- First, notice that by the results of the Proposition 2.1, for any $\varepsilon > 0$, the only sets that can be chosen are supersets of $c(S)$.

- Take any strict superset of $c(S)$, call it S'. By definition, $c(S) \succ_2 S'$.
- By insignificant difference, there exists $t_{S'}$ such that $(c(S), 0) \succ_2 (S', t_{S'})$
- Take now any $\varepsilon \geq 0$, by quasi-linearity, we have that $\left(c(S), \frac{|c(S)|}{|S|}\varepsilon\right) \succ_2 \left(S', \frac{|c(S)|}{|S|}\varepsilon + t_{S'}\right)$.
- Take now $0 < \varepsilon' < \min_{\{S' | c(S) \subset S' \subseteq S\}} \frac{|S|}{|S'| - |c(S)|} t_{S'}$. This minimum exists as X, and, therefore, S is finite. It is well defined, as $|S'| - |c(S)| \geq 1$.
- For that ε' and for all S' strict superset of $c(S)$, $\left(S', \frac{|c(S)|}{|S|}\varepsilon' + t_{S'}\right) \succ_2 \left(S', \frac{|S'|}{|S|}\varepsilon'\right)$. It implies, by quasi-transitivity, that $\left(c(S), \frac{|c(S)|}{|S|}\varepsilon'\right) \succ_2 \left(S', \frac{|S'|}{|S|}\varepsilon'\right)$.
- So for all ε' define as above, only $c(S)$ is in the argmax.

B.3 Proof of Proposition 2.3

Reflexivity does not have to be proven.
Completeness is evident from the construction.
Transitivity remains. Take three alternatives x, y, and z such that $x \succeq y \succeq z$. Is it the case that $x \succeq z$? There are four possibilities:

1. $x \succ y \succ z$.
2. $x \sim y \sim z$
3. $x \succ y \sim z$
4. $x \sim y \succ z$

Take the first case. We want to show that $x \succ z$:

1. Imagine it is not the case. There exists a sequence $S_1, \dots, S_n$ of subsets of X, $z \in c_0(S_1), x \in S_n$ and $S_i \cap c_0(S_{i+1}) \neq \emptyset$ (possibly with $n = 1$).
2. It contradicts $x \succ y$. Indeed, $y \succ z$ implies that $y \in c_0(\{y, z\})$. We have that $z \in \{y, z\} \cap c_0(S_1)$, which implies that the intersection is not empty. And finally, $x \in S_n$. We have used the sequence between z and x and added a first set $S_0 = \{y, z\}$ to build a sequence between y and x. However, this is impossible because of $x \succ y$ – so $x \succ z$.

Note that the proof works identically if $y \sim z$, so we have proven cases 1 and 3. It is also similar for case 4.

Let us prove case 2:

1. $x \sim y$ means that there exists a sequence $S_1, \dots, S_n$ of subsets of X, $y \in c_0(S_1), x \in S_n$ and $S_i \cap c_0(S_{i+1}) \neq \emptyset$ (possibly with $n = 1$). $y \sim z$ means that there exists a sequence $S'_1, \dots, S'_m$ of subsets of X, $z \in c_0(S'_1), y \in S'_m$ and $S'_i \cap c_0(S'_{i+1}) \neq \emptyset$ (possibly with $m = 1$).
2. Build a new sequence between x and z by noting that $y \in S'_m \cap c_0(S_1)$, which implies that the intersection is not empty and we can concatenate the sequences.

B.4 Proof of Proposition 2.4

The if direction is obvious, we have to prove the only if part.
Define $\succeq^c$, the revealed preference of c, according to Definition 2.4. Because c satisfies WARP, $\succeq^c$ is transitive and complete. If $x \succ^c y$, then for all S with $x, y \in S$, $y \notin c(S)$, which implies $y \notin c_0(S)$. It implies that $x \succ y$. Indeed, if it were not the case, then the acyclicity condition for c_0 would be violated, which means that it would be violated for c, and $\succeq^c$ could not be transitive. So, if $x \succ^c y$, $x \succ y$. So $\succ \subseteq \succ^c$. So $c^{\succ}(S) \subseteq c(S) \subseteq c_\varepsilon(S)$.

Let us show now that $c_0(S) \subseteq c^{\succeq}(S)$. Imagine it is not the case. Then there exists $x \in c_0(S)$, $x \notin c^{\succ}(S)$. $x \notin c^{\succ}(S)$ means that there exists $y \in S$, $y \succ x$, which implies that $x \notin c_0(S)$, which is a contradiction. So we have proved that $c_0(S) \subseteq c^{\succeq}(S) \subseteq c(S) \subseteq c_\varepsilon(S)$ for all $S \in \mathcal{P}(X)$.

C Beyond Classical Preferences

C.1 Formal Definitions of Intransitive Indifference Models

We introduce some definitions in order to characterize the different intransitive indifference models. In general, they differ on the restrictions they put on the indifference part of the relation. We go from the least structured to the most structured.

Definition C.1 (Quasi-Transitivity)**.** A preference relation $\succeq$ is *quasi-transitive* if the strict part of the preference relation is transitive, that is, $\succ$ is transitive.

Definition C.2 (Strong Intervality (aka Ferrers property))**.** A binary relation $\succeq$ satisfies *strong intervality* if and only if x is strictly better than y and z is strictly better than t, implies that either x is strictly better than t or z is strictly better than y. Formally:

$$\text{for all } x, y, z, t \in X, (x \succ y \text{ and } z \succ t) \Rightarrow x \succ t \text{ or } z \succ y$$

Definition C.3 (Semi-transitivity)**.** A binary relation $\succeq$ satisfies semi-transitivity if and only if x is strictly better than y and y is strictly better than z, implies that either x is strictly better than t or t is strictly better than z. Formally:

$$\text{for all } x, y, z, t \in X, x \succ y \text{ and } y \succ z \Rightarrow x \succ t \text{ or } t \succ z$$

Definition C.4 (Intransitive Indifference)**.** These conditions define three intransitive indifference models:

1. A *partial order* is a binary relation which is reflexive, asymmetric, and transitive. In our settings, it means that the revealed preference is reflexive and quasi-transitive.
2. An *interval order* is a partial order that satisfies the strong intervality condition. In our settings, it means that the revealed preference is reflexive, quasi-transitive, and the strict part P satisfies the strong intervality condition.

3. A *semi-order* is an interval order which satisfies the semi-transitivity condition. In our settings, it means that the revealed preference is reflexive, quasi-transitive, and the strict part P satisfies the strong intervality condition and the semi-transitivity condition.

C.2 Testable Conditions

C.2.1 Partial Order

Eliaz and Ok (2006), following Schwartz (1976) uses a weakening of classical revealed preferences, where now the chosen alternatives are *not worse* than the unchosen alternatives. This assumption on revealed preferences does *not* assume completeness of preferences anymore, or equivalently, does not assume transitivity of the indifference. This yield a consistency requirement called the Weak Axiom of Revealed Non-Inferiority (WARNI hereafter), which is a weakening of WARP.

Axiom C.1 (Weak Axiom of Revealed Non Inferiority (WARNI))**.** For a given alternative y in S, if for all the chosen alternatives in S, there exists a set T where x is in T and y is chosen in T, then y must be chosen in S. This property should be true for all S and y.

$$\text{for all } S \in \mathcal{P}(X), y \in S, \text{ if for all } x \in c(S), \text{there exists a } T \in \mathcal{P}(X), y \in c(T), x \in T \Rightarrow y \in c(S)$$

WARNI states that if an alternative is not worse than all chosen alternatives, it must be chosen too. In other words, an alternative that is not chosen must be worse than at least one chosen alternative. A choice correspondence satisfies WARNI if and only if it is rationalized by a reflexive and quasi-transitive preference relation – i.e., a partial order. This preference $\succeq$ is unique, and we have that $c(S) = \{x \in S | \text{there is no } y \in S, S \succ y\}$. Aleskerov, Bouyssou, and Monjardet (2007) provide an equivalent axiomatization of rationalizability by a partial order.[37] Their axiomatization is linked to Sen (1971)'s decomposition of WARP in axiom α and β, rather than a direct weakening of WARP. Eliaz and Ok (2006) have shown that a choice correspondence that satisfies WARNI also satisfies axiom α.

C.2.2 Interval Order

On a finite set of alternatives, interval orders can be represented according to a general threshold function where the threshold depends on one alternative (which one does not matter). The testable condition for interval order is *functional asymmetry.* More precisely, a choice correspondence is rationalizable by an interval order if and only if it is rationalizable by a partial order – i.e., it satisfies WARNI – and it satisfies the Functional Asymmetry axiom.

Axiom C.2 (Functional Asymmetry (FA))**.** A choice correspondence satisfies *Functional Asymmetry* if some chosen alternatives in S are not chosen in S', it must be that all chosen alternatives in S' that are in S are chosen in S':

[37] We do not formally prove this equivalence. Theorem 2 of Eliaz and Ok (2006) restricted to the partial order shows that if WARNI is satisfied, it is possible to find a partial order that rationalizes the choice correspondence.

$$\text{for all } S, S' \in \mathcal{P}(X), c(S) \cap (S' \backslash c(S')) \neq \emptyset \Rightarrow c(S') \cap (S \backslash c(S)) = \emptyset$$

Interval orders are a strengthening of partial order and weakening of classical preferences. Again, the preference obtained is unique.

C.2.3 Semi-Order

On a finite set of alternatives, semi-orders can be represented according to a general threshold function where the threshold is constant. A choice correspondence is rationalizable by a semi-order if and only if it is rationalizable by an interval order and it satisfies the Jamison-Lau-Fishburn axiom.

Axiom C.3 (Jamison-Lau-Fishburn (JLF))**.** A choice correspondence satisfies the *Jamison-Lau-Fishburn* axiom if S is made of unchosen alternatives in S' and S'' counts some chosen alternatives in S', then chosen alternatives in S'' must be chosen in S (if they belong to it).

$$\text{for all } S, S', S'' \in \mathcal{P}(X), S \subseteq (S' \backslash c(S')), c(S') \cap S'' \neq \emptyset \Rightarrow c(S'') \cap (S \backslash c(S)) = \emptyset$$

Semi-orders are strengthening of partial orders and interval orders and a weakening of classical preferences. Again, the preference obtained is unique.

C.2.4 Monotone Threshold

In the monotone threshold model, the threshold depends only on S and is *non-decreasing* with set inclusion, i.e., $t(S') \leq t(S)$ whenever $S' \subseteq S$. When the size of the choice set is larger, the choice is less precise, and the threshold is larger, which is a simple way to take into account choice overload. Frick (2016) shows that *occasional optimality* characterizes the *monotone threshold* on choice correspondences.

Axiom C.4 (Occasional Optimality)**.** A choice correspondence satisfies occasional optimality if, for all $S \in \mathcal{P}(X)$, there exists $x \in c(S)$ such that for any S' containing x:

1. If $c(S') \cap S \neq \emptyset$, then $x \in c(S')$;
2. If y is in S, then $c(S') \subseteq c(S' \cup \{y\})$.

WARP requires that any alternative the decision makers chooses from S is optimal. Occasional optimality requires that at least some of the decision maker's choices from S be optimal.

C.2.5 Menu-Dependent Threshold

A natural weakening of the monotone threshold model is to allow for a non-monotone threshold. The threshold t depends on S, without constraints. Aleskerov, Bouyssou, and Monjardet (2007) provide the testable condition for a menu-dependent threshold model to rationalize a choice correspondence.

Definition C.5 (Strict Cycle of Observation)**.** A *strict cycle of observation* are n sets $S_1, S_2, \dots, S_n$ in $\mathcal{P}(X)$ such that:

$$
\begin{aligned}
(S_1 \backslash c(S_1)) \cap c(S_2) &\neq \emptyset \\
(S_2 \backslash c(S_2)) \cap c(S_3) &\neq \emptyset \\
&\dots \\
(S_n \backslash c(S_n)) \cap c(S_1) &\neq \emptyset
\end{aligned}
$$

A strict cycle of observations is a cycle of strict revealed preferences.

Axiom C.5 (Functional Acyclicity)**.** A choice correspondence c satisfies *functional acyclicity* if it does not contain strict cycles of observations.

In revealed preferences terminology, functional acyclicity states that there are no cycles of strict revealed preferences. Aleskerov, Bouyssou, and Monjardet (2007) also provide the corresponding revealed preference: it is the *strict revealed preferences.* For subjects which satisfy functional acyclicity, we only directly elicit strict preferences, not indifference. Strict Preferences are acyclic and therefore can be augmented using the transitive closure, but no condition is imposed on indifference. The strict preference is unique, but depending on whether completeness is imposed or not, the indifference part may not be.

C.2.6 Context-Dependent Threshold

In the context-dependent threshold model of Aleskerov, Bouyssou, and Monjardet (2007), the threshold depends on the menu S and on the alternatives x and y that are compared. They give the condition on choice correspondences for context-dependent rationalizability.

Axiom C.6 (Fixed Point)**.** A choice correspondence c satisfies *fixed point* if, for any $S \in \mathcal{P}(X)$, there exists an alternative x in S such that x in S' implies that x in $c(S')$ for any $S' \subseteq S$.

There is a link between fixed point and the α axiom (Sen 1971). α requires that all alternatives chosen in a set are chosen in any subset. Fixed point only requires that one alternative chosen in a set is chosen in any subset.

Aleskerov, Bouyssou, and Monjardet (2007) do not provide the corresponding preference, but it is easy to build it when fixed point and full observability are satisfied. Take the set of fixed points in the whole set X ($FP(X)$). It is made of the most preferred alternatives in X. All alternatives in $FP(X)$ are indifferent, i.e., they are always chosen together when they are both available. Now take the set of fixed points in $X \backslash FP(X)$, it is the most preferred alternatives in $X \backslash FP(X)$, and so on until the set of alternatives that are not fixed point is empty or a singleton. By definition, this procedure will finish, as every nonempty subset of X has a fixed point. It also implies that the constructed preference is unique and complete: it is a classical preference. Compared to strict revealed preferences obtained with functional acyclicity, this revelation of preferences also reveals indifference.

The three models introduced here are ranked, as their threshold representation clearly shows. A choice correspondence that satisfies occasional optimality satisfies functional acyclicity. A choice correspondence that satisfies functional acyclicity satisfies fixed point. Fixed point has a clear advantage: it provides a simple way to reveal the preference, despite a seemingly complicated threshold representation.

C.3 Compatibility

C.3.1 Compatibility with a Partial Order

In Appendix C.1, we have given three intransitive indifference models. Partial orders is the largest class of intransitive indifference preferences, and contain semi-orders and interval orders. This particular intransitive indifference model is observationally equivalent to incomplete preferences, where no relation would replace the indifference part of the relation. For these two reasons, we will focus solely on the study of partial orders with partial identification, and not explore semi-orders and interval orders.

It is again easier to tackle the study of compatibility with partial orders directly with preferences, rather than trying to find a choice correspondence which would be compatible with the pair (c_0, c_ε) and would satisfy WARNI. The conditions imposed on preferences are the same as with classical preferences. The difference between classical preferences and partial orders is on the indifference part: it does not have to be transitive for partial orders. It motivates the definition of a compatible partial order.

Definition C.6 (Compatible Partial Order)**.** We propose a constructive method to build a partial order $\succeq$ which is compatible with the pair (c_0, c_ε). We have three steps:

1. $x \succ y$ if:
    1. There exists $S \in \mathcal{P}(X)$, $x \in c_\varepsilon(S)$ and $y \in S \backslash c_\varepsilon(S)$;
    2. For all $S \in \mathcal{P}(X)$, if $x, y \in S$, $y \notin c_0(S)$.
2. Close $\succ$ transitively;
3. Build $x \sim y$ when not $x \succ y$ and not $y \succ x$.

Because we close transitively $\succ$, it might not be a partial order, as there might be a cycle created. It is not the case, and this is the purpose of the next proposition.

Proposition C.1. *The compatible partial order is a partial order.*

Proof. The compatible partial order is transitive by definition. We have to prove the absence of cycles in the partial order. Assume it would be the case. That is, there is a sequence $x_1, \ldots, x_i, \ldots, x_n$ such that $x_i \succ x_{i+1}$ and $x_n \succ x_1$. The transitive closure cannot generate cycles is it were not originally here. So we can assume that the cycles were cycles generated from the observations. Take the set made of all the alternatives in the cycle: $S={x_1, ..., x_n}. We can do that because we have assumed full observability. $c_0(S)$ is non-empty, implying that one alternative at least is chosen in S. Call x_k this alternative. We cannot have that $x_{k-1} \succ x_k$, because of condition 2 in the definition of a compatible partial order, so we have a contradiction and $\succ$ is acyclic. $\square$

We use the choice correspondence associated with the partial order as defined in Definition 2.10. Compared to the previous subsection, we can only use the first and not the second definition of the associated choice correspondence. As the compatible partial order is a partial order, the associated choice correspondence satisfies WARNI.

Proposition C.2 (Compatibility with a Partial Order)**.** *$c_0(S) \subseteq c^{\succeq}(S) \subseteq c_\varepsilon(S)$ for all $S \in \mathcal{P}(X)$ and $c^{\succeq}$ satisfies WARNI if and only if there exists a choice correspondence c, $c_0(S) \subseteq c(S) \subseteq c_\varepsilon(S)$ for all $S \in \mathcal{P}(X)$ and c satisfies WARNI.*

Proof. The if part is obvious, we have to prove the only if part.
Define $\succ^c$, the revealed preference of c, according to Definition 2.4. Because c satisfies WARNI, $\succ^c$ is transitive. We have two cases.

1. Either there exists a set $S \in \mathcal{P}(X)$, such that there exists $y \in S \backslash c_\varepsilon(S)$.
2. Or for all $S \in \mathcal{P}(X)$, $c_\varepsilon(S) = S$.

Case 2 is trivial because everything is chosen all the time with c_ε, $\succ$ is empty, and $c^{\succ}$ satisfies WARP and therefore WARNI.

Case 1 is the interesting case. It implies that $y \in S \backslash c(S)$. Therefore there exists $x \in c(S), x \succ^c y$ (remember that $\succ^c$ is transitive). Because $x \succ^c y$, for all S such that $x, y \in S$, $y \notin c(S)$. It implies that for all S such that $x, y \in S$, $y \notin c_0(S)$, which implies that $x \succ y$. It implies that for any pair such that $x \succ y$, we must have $x \succ^c y$ (both are transitive). So $\succ \subseteq \succ^c$, which means that for all $S \in \mathcal{P}(X)$, $c(S) \subseteq c^{\succ}(S)$. It proves that $c_0(S) \subseteq c^{\succ}(S)$ for all $S \in \mathcal{P}(X)$.

The inclusion $c^{\succ}(S) \subseteq c_\varepsilon(S)$ comes from the repetition of the reasoning above. As for any $y \in S \backslash c_\varepsilon(S)$, there exist an $x \in c(S), x \succ^c y$, we will have that $x \succ y$, and therefore $y \in S \backslash c^{\succ}(S)$. We have that $S \backslash c^{\succ}(S) \subseteq S \backslash c_\varepsilon(S)$ for all $S \in \mathcal{P}(X)$, which is equivalent to $c^{\succ}(S) \subseteq c_\varepsilon(S)$ for all $S \in \mathcal{P}(X)$. $\square$

Corollary C.1 (Minimality of the Strict Preference)**.** *The compatible partial order is the partial order compatible with (c_0, c_ε) with the least strict preferences.*

Proof. In the proof of Proposition C.2, we have shown that $\succ \subseteq \succ^c$, which means that any partial order compatible with (c_0, c_ε) has at least as many strict preferences as the compatible partial order. $\square$

Proposition C.2 guides us on how to build a partial order compatible with any pair (c_0, c_ε). It also tells us when there is no compatible partial order. In practice, the procedure is to build the compatible order and the associated choice correspondence, and then to check that for all $S \in \mathcal{P}(X)$, we have $c_0(S) \subseteq c^{\succ}(S) \subseteq c(S)$. In that case, the pair (c_0, c_ε) is compatible with a partial order. Otherwise, it is not. Corollary C.1 shows that the compatible partial order is the one with the least strict relation. In other words, it provides the least information possible on the preference of the decision maker. If we want to be conservative from a welfare standpoint, it is the right preference to use. Note that because a choice correspondence which satisfies WARP also satisfies WARNI, compatibility with classical preferences implies compatibility with a partial order. It means that Example 2.3 shows that the compatible partial order is not unique.

C.3.2 Compatibility with Menu-Dependent Threshold

As given in Appendix C.2.5, a necessary and sufficient condition for a choice correspondence to be rationalized by the menu-dependent threshold is to satisfy functional acyclicity. Functional acyclicity is satisfied if and only if there are no cycles of strict preferences. A necessary condition for the compatibility of a pair (c_0, c_ε) with the menu-dependent threshold model mirrors this condition.

Definition C.7 (Strict Cycle of Observations with Partial Identification)**.** A *strict cycle of observation* are n sets $S_1, \dots, S_n$ in $\mathcal{P}(X)$ such that:

$$\begin{aligned}
S_1 \backslash c_\varepsilon(S_1) \cap c_0(S_2) &\neq \emptyset \\
S_2 \backslash c_\varepsilon(S_2) \cap c_0(S_3) &\neq \emptyset \\
\dots \quad \dots &\quad \dots \\
S_n \backslash c_\varepsilon(S_n) \cap c_0(S_1) &\neq \emptyset
\end{aligned}$$

Proposition C.3 (Compatibility with Functional Acyclicity)**.** *A necessary condition for a pair (c_0, c_ε) to be compatible with functional acyclicity is to not have a strict cycle of observation with partial identification.*

Proof. By definition of partial identification, for all S in $\mathcal{P}(X)$, $c_0(S) \subseteq c(S) \subseteq c_\varepsilon(S)$. It means that for all S, $S \backslash c_\varepsilon(S) \subseteq S \backslash c(S)$. If a strict cycle of observations with partial identification exists, it implies that any choice correspondence will have a strict cycle of observations, and thus will violate functional acyclicity. □

C.3.3 Compatibility with Fixed Point

As given in Appendix C.2.6, a necessary and sufficient condition for a choice correspondence to be rationalized by the context-dependent threshold model is to satisfy fixed point (see Axiom C.6). Fixed point requires the existence of *one* alternative that is chosen in a superset, and in any subset, the condition can be directly imposed on c_ε.

Proposition C.4 (Compatibility Fixed Point)**.** *A pair (c_0, c_ε) is compatible with fixed point if and only if c_ε satisfies fixed point.*

In practice, it means that as long as c_ε satisfies fixed point, we can find a c that satisfies it.

D Experiment

D.1 Experimental Design

D.1.1 Tasks

Subjects chose between four different paid tasks. The programs to run the tasks and the videos used in the video and training treatments are available from the author, on request. The tasks were:

- An *addition* task, where subjects had to perform as many additions of three two-digit numbers as possible. They earned 30 cents for each correct sum. A screenshot of one addition is available in Figure 7.
- A *spell-check* task, where subjects faced a long text with spelling and grammar mistakes. They earned 10 cents for each mistake corrected and lost 10 cents for each mistake added. The text they faced (in French), as well as the interface of the task, is in Figure 8.
- A *memory* task, where sequences of blinking letters appeared on the screen and stopped after a random number of letters. Subjects had to give the three last letters that appeared on the screen. They earned 30 cents for each correct sequence. The interface to give the sequence of blinking letters is shown in Figure 9.
- A *copy* task, where a large number of sequences of 5 letters appeared on the screen. Subjects had to copy the sequences. They earned 10 cents for each sequence. The interface of the task is in Figure 10

At the end of the tasks, subjects had feedback on their gains. In the training part, it was on their potential gains. In the training treatment, when subjects performed the tasks before choosing, they also received feedback on their performance.

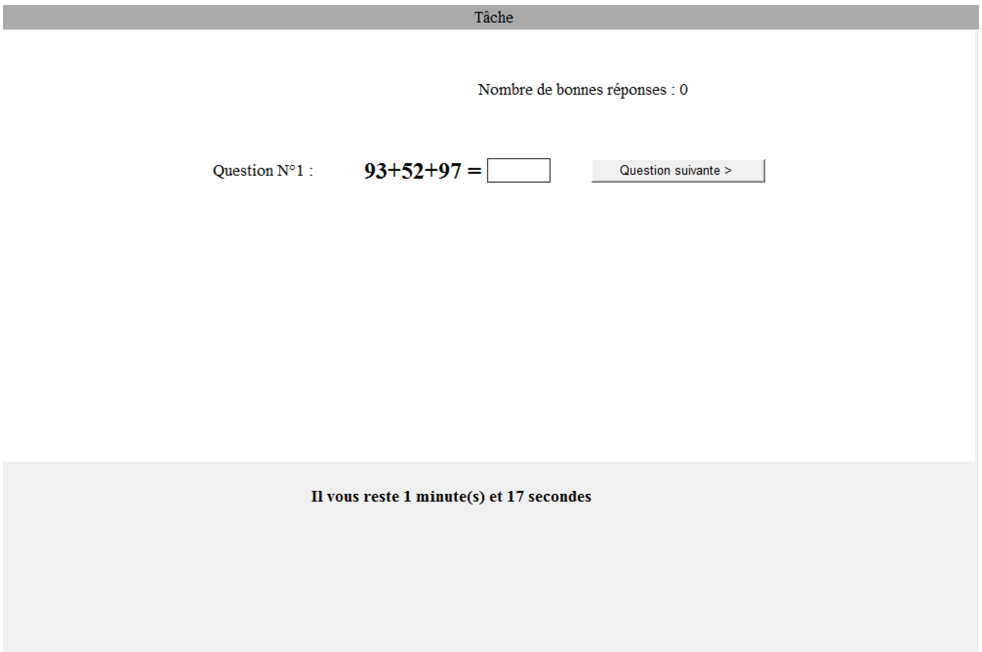

Figure 7: Addition task.

Figure 8: Spell-check task.

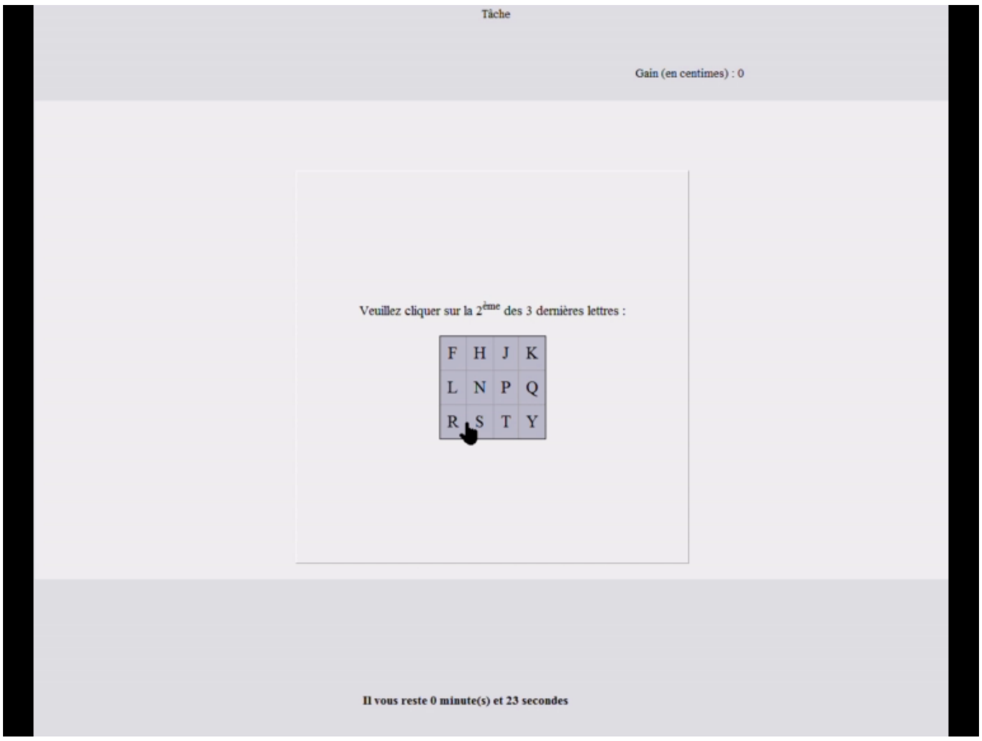

Figure 9: Memory task.

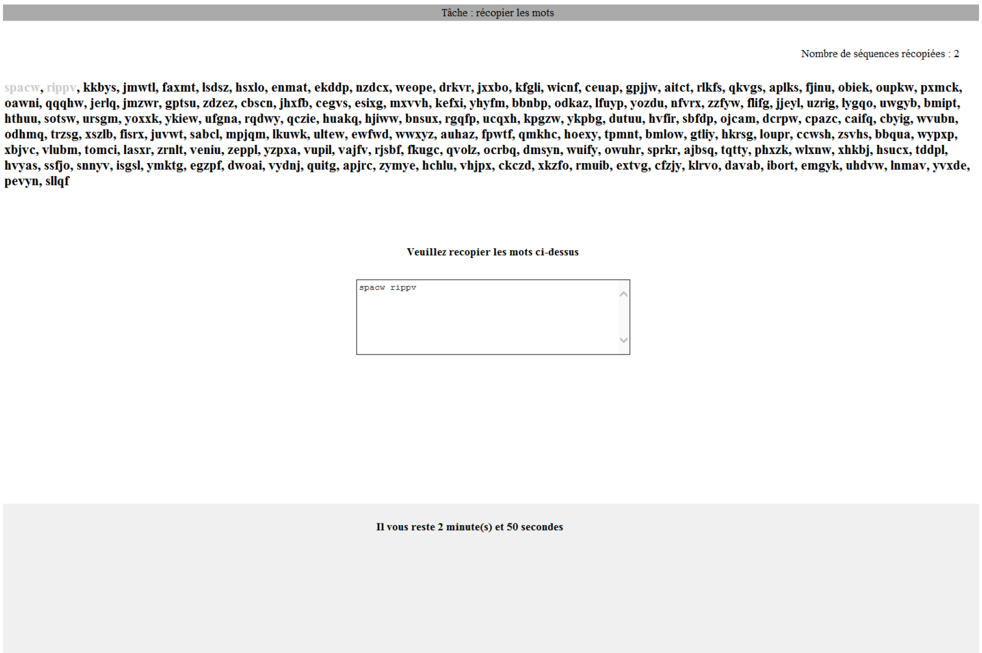

Figure 10: Copy task.

D.1.2 Instructions

These are a rough translation of the instructions associated with the video treatment.

You are participating in an experiment where you will earn money. Your earnings will depend on your decisions and chance. They will not depend on the decisions made by other participants. We are interested in your decisions. There is no right or wrong answer to the questions you will be asked.

During the experiment, we will ask you to answer some questions that will allow us to know you better and understand the decisions you have made.

All this information is strictly anonymous and confidential.

D.1.2.1 Gains in the experiment Your gains in the experiment are:

1. A lump-sum payment of 5€ for your participation in the experiment;
2. Plus: the remuneration corresponding to your performance in the task drawn, usually between 2 and 5€. **Correctly choosing the task you will perform can significantly increase your earnings**;
3. Plus: the gain associated with the selection of the task (between 0 and 12 cents);
4. Plus: the amount you earned in the fourth part (between 0 and 5€).

If in your earnings, hundredths of a cent appear (for example, 2.10 cents), your earnings are computed as follows:

- You win the cent amount with certainty (here 2 cents);
- The decimal amount (here 0.10) corresponds to your odds of earning an extra cent (here 10 chances out of 100).

In this example, you earn 2 cents with 90 chances out of 100 and 3 cents with 10 chances out of 100.

D.1.2.2 Remarks The questions may look similar to you, but they are all different. Please forgive us for the sometimes slow load times. If they are extremely slow, please raise your hand.

D.1.2.3 Instructions For a smooth experiment:

- Please put away your cell phones and put them in silent mode;
- Please do not communicate with other participants during the experiment. The whole experiment takes place on your computer;
- Calculators are forbidden during the experiment;
- If you want to ask a question, please raise your hand, someone will come to answer you.

The experience you participate in is divided into six parts. Before each part, explanations will be displayed on your screen. You will receive your payment at the end of the experiment.

D.1.2.3.1 First part In this part, you will select the task you will perform at the end of the experiment. It is made of 11 periods. Each period is divided into two screens: a choice screen and a confirmation screen. Earnings are associated with each period. The first screen (the choice screen) allows you to select one or more of the tasks displayed (between 2 and 4), by answering "yes" or "no" to each task. You need to keep two things in mind:

- You must select at least one task;
- Each time you select a task, your earnings can increase by a few cents. The exact amount of this gain is specified on the screen;

Once you have made your selection, click on the "choices made" button. The second screen appears, with a summary of your selection in the previous screen: the tasks you have selected and the associated gain. If you are satisfied by your choice, then confirm it by clicking on the button "I confirm my choices". If you want to change it, then click on the button "I want to change my choices", then you will come back to the first screen. You can change your selection as many times as you like. Your selection will only be taken into account once confirmed.

At the end of the experiment, the computer will randomly draw one of the periods. You will then receive the payoff associated with this period and perform one of the tasks you have selected during this period. The task you will perform is determined as follows:

1. If you have selected more than one task, the computer will draw one at random. *For example, assuming that the gain is 10 cents per task selected and you have to choose between fill, close, or stamp envelopes and you select fill and close envelopes, you will have a one in two chance to perform the filling of the envelopes and on in two chances to close the envelopes. The gain related to your selection in this part is 20 cents (since you have selected two tasks) in addition to the remuneration in the task.*

2. If you have selected only one task, you will perform this task. *Using the same example, if you only selected the closing of the envelopes, you will close envelopes, and the gain related to your selection in this part is 10 cents in addition to the remuneration in the task.*

Your selection and the associated gain are taken into account only once confirmed. Only one of the 11 periods in this part will be used to assign you a task and compute the associated gain. It means in particular that you will only earn one of the earnings associated with your selections and will only perform one task.

D.1.2.3.2 Tasks You have 3 minutes to earn a maximum of money and increase your pay. You will have the choice between 4 tasks that I will present to you on the screen in a few moments. The tasks you can perform are:

Addition Additions will appear sequentially on the screen. You must give the result of these additions to increase your remuneration.

Spell-check A text in French will appear on the screen. This text contains spelling and grammar mistakes that you must correct to increase your pay.

Memory On your screen will appear successively letters, each displayed for a split second. When the sequence stops, you will be asked to give the last three letters appeared, in order, to increase your remuneration.

Copy On your screen will appear several sequences of letters separated by commas. You must copy these sequences to increase your pay.

D.2 Demographics Characteristics

The average age of the sample is 35. The youngest subject is 18 and the oldest 76. The majority (56.40%) of the sample is 30 or younger. Figure 11 shows the age distribution of the sample. Figures 12 and 13 show the different kinds of qualifications and studies. The majority of unemployed in the sample are students who do not work, as these two categories are lumped together by the French Statistical Institute.

The sample is almost gendered balanced. There are 53% of female in the sample. The level of education question only led to a response rate of 43%. Among those who answered, 51% have at least started a college education. Overall, the largest population of the sample is a student population.

D.3 Explanatory Power of Different Axioms

With a set of four alternatives, there are 26,254,935 possible choice correspondences and 20,736 possible choice functions. We have tested all the properties on each. Table 12 gathers the results.[38] Most axioms have meager or low pass rates, of the magnitude of the percent or

[38] The procedure we have used to compute the result is to consider that each choice function or choice correspondence has an equal probability of being drawn. It is *as if* the decision maker chose in every choice set a subset at random, using a uniform distribution. It is also *as if* the decision maker chose each alternative in each

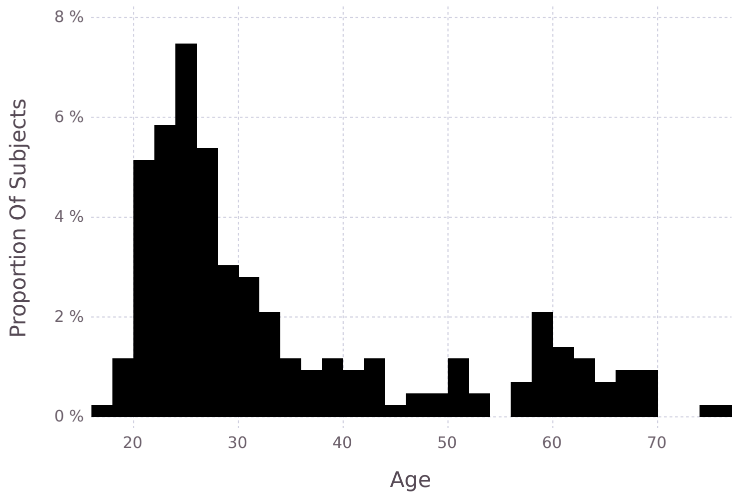

Figure 11: Age distribution of the sample.

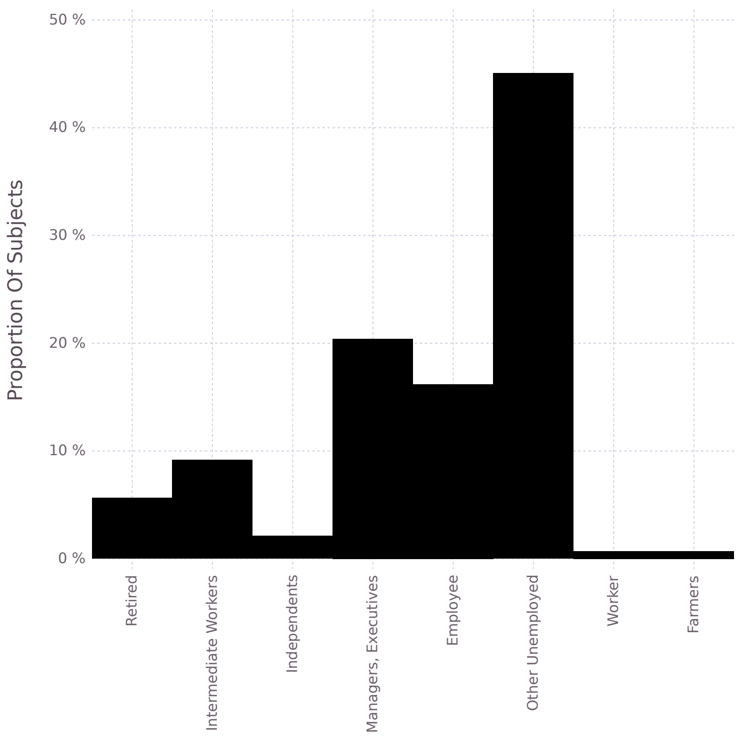

Figure 12: Professional occupation, when given (151 subjects).

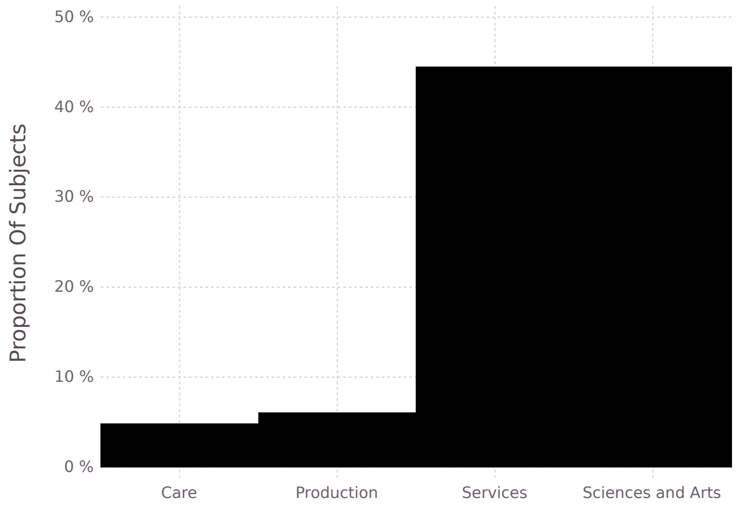

Figure 13: The domain of occupation, when given (172 subjects).

below. A random choice is not very likely to satisfy the properties. The β axiom bites a on choice correspondences and does not bite on choice functions. Lastly, on choice functions, many axioms have the same pass rate and are satisfied by the same choice functions. Choice correspondences are useful because they allow us to separate the different models. The last line shows that using weak revealed preferences and acyclicity instead of strict revealed preferences weakens a lot the explanatory power when the grand set contains four alternatives, in particular on choice functions.

D.4 Size of the Chosen Sets

One of the defining characteristics of pay-for-certainty is to offer subjects the possibility to choose several alternatives. Table 13 shows the size of the chosen sets depending on the size of the choice sets. Overall ε-correspondences, 54.32% of chosen sets are not singletons (when excluding the high bonus payment treatment). In the 0-correspondence treatment, when subjects have no incentive to choose several alternatives, 15% of them always choose singletons. For these subjects, eliciting a choice function is not a restriction. On average, however, subjects took the opportunity to choose several alternatives. Forcing subjects to choose only one alternative is a restriction in these cases.

Table 13 also shows that the size of the chosen sets grows relatively faster than the size of the choice sets. For instance, the ratio of chosen sets of size one over chosen sets of size two decreases when the size of the choice set increases. One explanation for this phenomenon might be that choosing from larger sets is more complicated than choosing from smaller sets, as studied in Section 5.

The incentive to choose more than one alternative introduced by pay-for-certainty is one driver of non-singleton choice, but not the only one. Table 14 shows that indeed, the proportion

choice set with a coin toss.

Table 12: Probabilities that a decision maker choosing with a uniform random satisfies a property.

	Choice	
Property	Correspondence	Function
Classical preferences	2.9e-06	1.2e-03
Semi-order	7.0e-06	1.2e-03
Interval order	7.9e-06	1.2e-03
Partial order	8.3e-06	1.2e-03
Monotone threshold	7.3e-04	1.2e-03
Menu-dependent threshold	1.4e-02	1.2e-03
Context-dependent threshold	7.3e-02	1.2e-03
α	5.7e-04	1.2e-03
β	2.6e-02	1.0e+00
Functional asymmetry	1.5e-02	1.2e-03
Jamison-Lau-Fishburn	1.5e-01	1.0e-02
Acyclicity with weak RP	1.4e-01	7.3e-01

Table 13: Proportion of chosen sets of different sizes for 0-correspondences.

		Chosen set size			
		1	2	3	4
Choice set size	2	68.9%	31.1%		
	3	41.9%	40.7%	17.4%	
	4	28.3%	31.7%	30.0%	10.0%

Table 14: Proportion of singletons chosen at different payments, depending on the size of the choice set.

		Bonus payment		
		No	Low	High
Choice set size	2	68.9%	49.6%	49.1%
	3	41.9%	21.5%	19.6%
	4	28.3%	11.7%	12.3%

Significance
Differences between no gains and positive gains are significant. Differences between low and high gains are not.

Table 15: Proportion of chosen sets that are singletons, depending on the information and the size of the choice set.

	Information		
Set Size	Sentence	Video	Training
2	45.6%	56.8%	62.5%
3	17.5%	30.0%	30.3%
4	9.2%	19.5%	18.5%
N	87	328	108

of singletons chosen drops when the bonus is introduced. The introduction of the additional payment has a non-linear effect on choices: a small gain significantly changes the aggregate proportion of singletons chosen. Small variations inside the gain domains do not have many effects. There is at least two possible explanation for this phenomenon. The first is the salience of even minimal monetary gains in experiments. The second is that very few subjects have a weak strict preference. The jump between no and low gains are due to subjects who, when indifferent between two alternatives and faced with no gains, did not select all their maximal alternatives. When incentivized to choose all the maximal alternatives, they did so. The high gain is then not enough for them to bundle other alternatives with their maximal ones. We cannot disentangle the two explanations in the experiment. This aggregate behavior is compatible with partial identification but is incompatible with the full identification of the choice correspondence for all subjects.

Table 15 shows another driver of non-singleton choice: lack of information. The proportion of singletons chosen grows when the information on the tasks is more precise. The additional information provided by the video has a substantial effect on the choice of singletons. The additional information provided by the training is not as valuable for subjects, judging by the small additional proportion of singletons chosen.

The non-singletons choice is a robust finding in the experiment. It persists even when providing better information on the alternatives or looking only at no gain of choosing more alternatives.

D.5 Weak Revealed Preferences

In Section 4.1, we have assumed that revealed preferences are strict. That is, a chosen alternative is strictly better than an unchosen one. Strictly speaking, this assumption is not warranted with forced single choice and 0-correspondences. With forced single choice, if subjects are indifferent, we force them to select only one alternative. With 0-correspondences, as they have no incentive to choose several alternatives, they can select on their own when they are indifferent. In both cases, it implies that in theory, only saying that a chosen is weakly better than an unchosen one is right. This part is therefore dedicated to exploring weak revealed preferences, first on forced single choice, and then on choice correspondences. We use weak revealed preferences as defined in Definition 2.5. We then test the acyclicity of the preference relation, as given in Definition D.2, as it is the necessary and sufficient condition for weak revealed preferences to be classical.

Definition D.1 (Cycle)**.** A cycle is a pair of alternative $x, y \in X$ such that xRy and yP^0x.

Definition D.2 (Acyclicity of Revealed Preferences)**.** Revealed preferences are acyclic if they contain no cycle of preferences. That is, if, for all $x, y \in X$, xRy implies not yP^0x.

The results should be read keeping in mind the last row of Table 12: weak revealed preferences with acyclicity are more than 600 times more likely to rationalize a choice function than strict revealed preferences with acyclicity in our experiment. With choice correspondences, it is more than 4,000 times.

D.5.1 With Forced Single Choice

80% of the subjects are acyclic with choice functions and thus are rationalized with classical preferences. It is significantly higher than the 57% of subjects which can be rationalized using strict revealed preferences, the p-value of the Fisher exact test is 0.018.

For the 41 subjects whose choices can be rationalized by weak revealed preferences, we can build these preferences and explore indifference. We could not do that in Section 4.1.1. We find that 71% of the subjects have only strict preferences, while the remaining 29% have one indifference relation and five strict preferences relations. The average number of indifference relations is therefore 0.29, and it is significantly different from 0.[39]

Finally, we can compare the maximal alternatives obtained with weak and strict revealed preferences. The former introduces the possibility of indifference. Table 16 shows that copy is the most preferred alternative on average, whereas spellcheck is the least preferred.[40] The proportions are not significantly different between weak and strict revealed preferences, and the social ordering would be similar.

[39]The p-value of the two-sided paired t-test of equality to 0 is lower than 0.001.

[40]The sum of the proportion is higher than 100% because some subjects have several maximal alternatives when they are indifferent.

Table 16: Proportion of each task being maximal with forced single choices, restricted to subjects who are rationalized by classical preferences.

	Task				
	Addition	Spell-check	Memory	Copy	N
Strict RP	24%	24%	21%	31%	29
Weak RP	29%	24%	29%	39%	41
P-value[a]	0.85	1	0.6	0.67	

Note:
An alternative is maximal if, according to the revealed preference of the decision maker, no other alternative is better.
[a] P-values are given for the Fisher exact test, column-wise.

D.5.2 0-Correspondences

85% of the subjects are acyclic with weak revealed preferences on 0-correspondences and thus are rationalized by classical preferences. It is significantly higher than 40% of subjects which can be rationalized using strict revealed preferences.[41] It is not significantly different from the result obtained on choice functions.[42] The acyclicity requirement with weak revealed preferences is much weaker than strict revealed preferences and much higher than with choice functions.

We build the preferences using weak revelation for the 153 subjects which satisfy the acyclicity requirement. We find that on average, each subject has 2.59 binary relations that are indifference. The average is significantly different from both 0-correspondences with strict revealed preferences and forced single choice with weak revealed preferences.[43] The distribution is not significantly different from the 0-correspondence with strict revealed preferences, however, but it is significantly different from the force single choice elicitation.[44] It is, however, quite heterogeneous, as Figure 14 shows. Notably, even with weak revealed preferences, some subjects have only strict preferences.

Finally, we can compare the maximal alternatives obtained with weak and strict revealed preferences. Table 17 shows that copy is the most preferred alternative on average, whereas spell-check is the least preferred. The proportions are mostly not significantly different between weak and strict revealed preferences, and the social ordering would be similar.

Overall, using weak revealed preferences instead of strict revealed preferences increases significantly observed choices that can be rationalized by classical preferences. With forced single choice, it reveals a significant proportion of indifference, but it still underestimates it compared to correspondences.

[41] The p-value of the Fisher exact test is below 0.001.
[42] The p-value of the Fisher exact test is 0.55.
[43] The p-values of two-sided two-sample t-test of equal means are lower than 0.001 in both cases.
[44] The p-value of the first KS-test is 0.06, and the second is lower than 0.001.

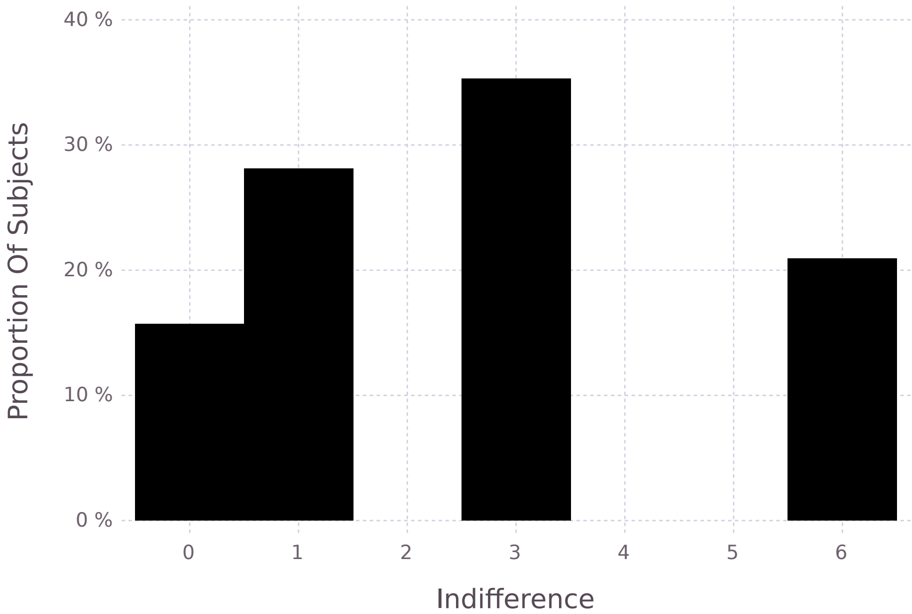

Figure 14: Histogram of the number of indifference relations.

Table 17: Proportion of each task being maximal with 0-correspondences, restricted to subjects who are rationalized by classical preferences. P-values are given for the Fisher exact test, column-wise.

	Task				
	Addition	Spell-check	Memory	Copy	N
Strict RP	51%	51%	52%	58%	79
Weak RP	65%	59%	63%	70%	153
P-value	0.054	0.251	0.122	0.103	

Note:
An alternative is maximal if, according to the revealed preference of the decision maker, no other alternative is better.

D.6 Results with High Payments

In the experiment, 163 subjects faced two positive bonus payment levels: the 1 cent bonus payment level and the 12 cents bonus payment level. In the core of the paper, we reported results only for the low payment levels, as the results obtained with the high payment levels are not very different.

Classical preferences rationalize 47% of the subjects in the high payment treatment (76 subjects), which is not significantly different from the 44% in the low payment treatment.[45] The average number of indifference relation is 3.25, which is not significantly higher than in the low payment.[46] Figure 15 shows the histogram of the number of subjects with their number of indifference relations. They are quite similar, but according to a KS-test, the distributions are different.[47]

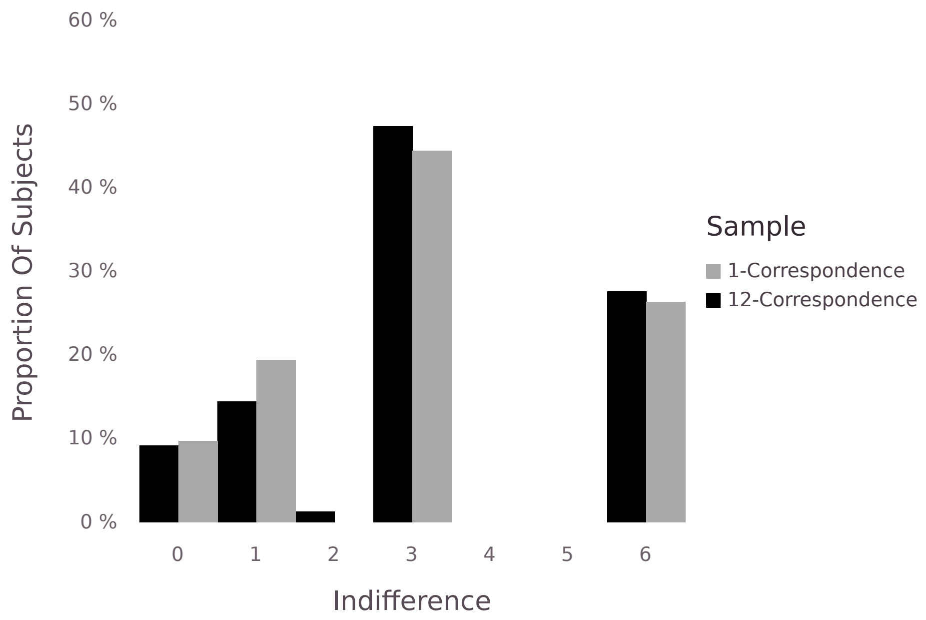

Figure 15: Histogram of the number of indifference relations with 1- and 12-correspondences

If we use 0- and 12-correspondences for identification purposes instead of 0- and 1-correspondences, we fully identify the choice correspondences of 15% of the subjects and partially identify them for another 46%. Both figures are not significantly different than what we obtained in Section 4.1.4, using 0- and 1-correspondences.[48]

Studying 1 and 12-correspondences shows that the difference in results between the two treatments is mostly insignificant. It assuages the concern that 12 cents level was too high compared to the lowest increment of 10 cents for each success in a task. This section also shows that the bonus payments have a non-linear effect, with a jump when the bonus becomes strictly positive, and then small variation when the gains increase.

[45] The p-value of the Fisher exact test is 0.26.
[46] The p-value of the two-sample two-sided t-test of equality is 0.67.
[47] The p-value of the KS-test is lower than 0.001.
[48] The p-values of the Fisher exact tests are respectively 0.55 and 0.31.

Table 18: Preferences deduced for each subject, depending on the elicitation method. Maximal tells us whether the alternatives that are maximal with forced single choice are also maximal according to both 0- and 1-correspondences.

	Observations			WARP			
Subject	FSC	0-C	1-C	FSC	0-C	1-C	Maximal
1201	$C \succ M \succ A \succ S$	$A \sim M \sim C \succ S$	$C \succ M \sim A \succ S$	Yes	No	No	Yes
1202	$C \succ M \succ A \succ S$	$C \succ M \succ A \succ S$	$A \sim S \sim M \sim C$	Yes	Yes	Yes	Yes
1203	$C \succ M \succ S \succ A$		$M \sim C \succ A \sim S$	Yes	No	No	Yes
1204		$M \succ C \succ A \succ S$	$A \sim M \sim C \succ S$	No	Yes	Yes	
1205	$M \succ C \succ A \succ S$	$A \sim S \sim M \sim C$	$A \sim M \sim C \succ S$	Yes	No	No	Yes
1206	$M \sim C \succ S \succ A$	$M \sim C \succ S \succ A$	$M \sim C \succ S \succ A$	No	Yes	No	Yes
1207	$A \sim C \succ S \succ M$	$A \sim S \sim C \succ M$	$A \sim C \succ S \succ M$	No	No	No	Yes
1208	$C \succ A \succ M \succ S$	$A \sim S \sim M \sim C$	$A \sim S \sim M \sim C$	Yes	No	Yes	Yes
1209	$M \succ A \succ C \succ S$	$A \sim M \succ C \succ S$	$A \succ M \succ C \succ S$	Yes	Yes	No	No
1210[a]		$A \sim S \sim C \succ M$	$A \sim S \sim C \succ M$	No	Yes	Yes	No
1211	$M \sim C \succ S \succ A$	$S \sim M \succ C \succ A$	$S \sim M \succ C \succ A$	No	No	No	
1212		$S \sim C \succ M \succ A$	$S \sim M \sim C \succ A$	No	No	Yes	Yes
1213	$A \succ M \sim C \succ S$	$A \sim M \sim C \succ S$	$A \sim M \sim C \succ S$	No	No	No	Yes
1214	$S \succ M \succ C \succ A$	$A \sim S \sim M \sim C$	$S \sim M \sim C \succ A$	Yes	Yes	No	Yes
1215	$S \succ A \succ M \succ C$	$A \sim S \sim M \sim C$	$A \sim S \sim M \succ C$	Yes	Yes	No	Yes
1216	$S \sim C \succ M \succ A$		$C \sim S \succ M \succ A$	No	No	No	Yes
1217[a]	$S \succ A \succ M \succ C$	$S \succ A \succ M \succ C$	$S \succ A \succ M \succ C$	Yes	Yes	Yes	Yes

Legend
A is for Addition, S for Spell-check, M for Memory and C for Copy; FSC for forced single choice, 0-C for 0-correspondence and 1-C for 1-correspondence.
[a] Subjects in bold have fully identified choice correspondences with set inclusion.

D.7 Choice Functions and Choice Correspondences on the Same Subjects

We can look at the 17 subjects for which we have both forced single choice and 0- and 1-correspondences. We use weak revealed preferences with forced single choice, in order to use most of our sample, and because it encompasses strict revealed preferences. Table 18 lists the preferences obtained for each subject, with the different treatments. When the preferences obtained with forced single choice are only strict, the subjects satisfy WARP in addition to acyclicity with weak revealed preferences. For 0- and 1-correspondences, we use preferences obtained with the context-dependent choice model, which is a superset of classical preferences.

We can see on the Table 18 that most of the time, choice functions correctly identify one or two of the maximal alternatives. It also fails to identify the full extent of indifference in general. We identify the choice correspondences of two subjects: 1210 and 1217. For the latter, force single choice yield consistent preferences, but it is not the case for the former.

This small subsample seems to indicate that the use of forced single choice or 0 or 1-

Table 19: Frequency of row chosen over column, in forced single choice. We can deduce the following Condorcet order: Memory $\succ$ Copy $\succ$ Addition $\succ$ Spellcheck.

	Addition	Spell-check	Memory	Copy
Addition		49%	55%	57%
Spell-check	51%		51%	55%
Memory	45%	49%		41%
Copy	43%	45%	59%	

correspondences depends on what we are seeking. If we are interested in finding one of the best alternatives and have no collective welfare purposes in mind, forced single choice looks fine. If on the other hand, we want to study indifference or find a social optimum, 0 or 1-correspondence seem more appropriate.

D.8 An Alternative Determination of Maximal Alternatives

Maximal alternatives are assessed using the preferences of the subjects. It implies that not all the sample is taken into account, as some subjects cannot be rationalized with preferences. In particular, subjects who do not satisfy WARP with forced single choice and subjects who do not satisfy fixed point with choice correspondences are ignored. In order to take them into account, we have to use a non-preference based approach.

One possibility to do so is to use *Condorcet winners* to determine the social ordering, instead of maximal alternatives.

Definition D.3 (winner)**.** The alternative x is a winner in the choice between x and y if it is chosen more often than y in the choice between x and y. In case of equality, both alternatives are winners.

In other words, the frequency of $x \in c(\{x, y\})$ is higher than the frequency of $y \in c(\{x, y\})$. A *Condorcet winner* is an alternative which is a winner in all binary choices with other alternatives. To determine the social order, we then look at the Condorcet winner in all remaining alternatives and so on.

Tables 19, 20, 21, and 22 show that the order we can deduce from Condorcet winner is different between the specifications. It is strikingly different for the fully identified choice correspondences. It is also sometimes different from the order we deduce from the maximal alternatives in Table 6. It is notably different in the case of forced single choice, whereas it is very similar in the case of fully identified choice correspondences and identical in 0-correspondences. This difference is likely to be explained by the difference in the sample considered.

Table 20: Frequency of row chosen from {row, column} with 0-correspondences. We can deduce the following Condorcet order: Copy $\succ$ Memory $\succ$ Addition $\succ$ Spellcheck. In the choice between memory and addition, going further in the decimal shows that memory is more often chosen.

	Addition	Spell-check	Memory	Copy
Addition		61%	67%	69%
Spell-check	70%		63%	74%
Memory	67%	62%		72%
Copy	61%	58%	64%	

Table 21: Frequency of row chosen from {row, column} with 1-correspondences. We can deduce the following Condorcet order: Copy $\succ$ Memory $\succ$ Addition $\succ$ Spellcheck.

	Addition	Spell-check	Memory	Copy
Addition		67%	71%	78%
Spell-check	78%		73%	83%
Memory	79%	72%		81%
Copy	76%	70%	73%	

Table 22: Frequency of row chosen from {row, column} with fully identified choice correspondences. We can deduce the following Condorcet order: Addition $\sim$ Spellcheck $\succ$ Copy $\sim$ Memory.

	Addition	Spell-check	Memory	Copy
Addition		70%	67%	61%
Spell-check	70%		61%	67%
Memory	76%	70%		67%
Copy	79%	73%	67%	

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