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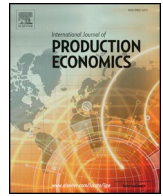
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## Coordinating contracts for VMI systems under manufacturer-CSR and retailer-marketing efforts

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### ABSTRACT

This paper studies the coordination of a two-echelon consignment channel in vendor managed inventory systems. The market demand is affected by retailer's marketing effort, retail price, manufacturer's CSR (Corporate Social Responsibility) effort and the changes in economic and business conditions. We propose four contracts that combine revenue and cost sharing to effectively coordinate the channel members which are referred to as "revenue and production cost sharing"; "revenue, production cost and marketing cost sharing"; "revenue, production cost and CSR cost sharing" and "revenue, production cost, marketing cost and CSR cost sharing". Each contract is represented by a fraction of sharing ( $\alpha$ ). In the deterministic demand, our analysis show that these sharing contracts lead to Pareto improvements in comparison with the wholesale price contract for some ranges of  $\alpha$  values. Furthermore, we found that the first three contracts cannot coordinate the channel while the last contract leads to a perfect coordination of the channel. In the stochastic demand, numerical examples show that the sharing contracts where retailer shares the production cost of all consigned stocks always lead to Pareto improvements and the channel can be perfectly coordinated if the channel members share all of the costs. In contrast, the Pareto improvements may not always be achieved with sharing contracts where retailer shares the production cost of sold stocks and none of them can coordinate the channel. From managerial insights, our research could help channel managers to improve the CSR implementation as well as the channel performance in the short and long term.

### 1. Introduction

In current business practices, most manufacturers increasingly address their concern on CSR issues due to the social and environmental impacts of industrial activities in channel (Hsueh, 2015) and the influence of their customers, who increase their socially responsible consumption practices (Gonzalez et al., 2009). Therefore, the manufacturer (M) can invest in CSR activities to meet the expectations of consumers and support their business. An increase in CSR performance requires higher CSR investment cost but leads to greater market demand (Eltantawy et al., 2009; Bhattacharya and Sen, 2004). In addition to M's CSR efforts, the retailer (R) can exploit the sales channel to promote the market demand and boost sales (Wang and Hu, 2011). R's sales channel includes different types of promotional activities such as advertising, on-site shopping assistance, rebates and post-sales service. However, these activities may constitute a significant portion of a firm's operating expenses (Xiao et al., 2005). Hereafter, as did Ma et al.

(2013), we use the term "marketing efforts" to denote R's sales promotional activities.

It is also well known that when the channel member's decisions on efforts are made separately and each party pays the associated costs of efforts to maximize their own profit, these strategies lead to local optimum solutions which may lower total profit of the whole channel.

In the past decades, the issues of designing coordinating contract have received a great deal of research attention since it improves the profit of both the channel and the individual channel member. Coordinating contracts provide incentives to induce channel members to behave in ways that are best for the whole channel while maximizing their own profit. This situation leads to a coordination of the channel. However, some coordinating contracts only reach the cooperation state (Pareto improvement) where the channel members are better off with the coordinating contract than any other different contracts (Chakraborty et al., 2015). In practice, to coordinate the channel, many coordination mechanisms such as consignment contract, Vendor

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Managed Inventory (VMI), revenue-sharing contract and cost-sharing contract have been introduced and implemented in industry.

The consignment contract and VMI are two common channel practices that can be used separately (Gümüş et al., 2008). In a consignment contract, M as a consignor has the ownership of the inventory at R (Bichescu and Fry, 2009). By contrast, M as a vendor manages inventory for the R and decides when and how much to replenish in the VMI system (Lee and Cho, 2014). However, the consignment process is more complicated on a large scale, often involving VMI (Sarker, 2014) and the consignment contract is assumed to be a part of the VMI system (Bichescu and Fry, 2009). Under the VMI system complemented by a consignment contract (VMI-CC), M monitors the R's inventory levels and makes periodic replenishment decisions in terms of quantity and frequency (Wong et al., 2009) while retaining ownership of the inventory (Chen et al., 2010). VMI-CC has been adopted by many industries such as personal computer and automobile. Readers may refer to Chen et al. (2010) for more examples of the VMI-CC. When the VMI-CC was put into practice, an important issue arises is how to share the revenue and costs between channel members to improve the channel's profit while ensuring that each partner in the channel can benefit from the VMI-CC. The issues of channel coordination under a VMI-CC with a revenue sharing agreement have been widely studied in the literature (Bernstein et al., 2006; Li et al., 2009). Besides, the cost sharing contract has recently been used in coordinating a VMI system (Lee and Cho, 2014; Lee et al., 2016). However, none has addressed the channel coordination issues in a VMI-CC with the presence of both R's marketing effort and M's CSR effort using a revenue and cost sharing contract.

Therefore, in this paper, we investigate the effectiveness of a revenue and cost sharing contract embedded in VMI-CC for coordinating the channel integrating M's CSR and R's marketing efforts. We use the sharing contracts according to which M supplies the products to be sold to R's store while retains the ownership of these products, and the two firms share their revenue and costs according to a sharing parameter negotiated. Specifically, we propose four kinds of sharing contracts to coordinate the channel including revenue and production cost sharing (RP); "revenue, production cost and marketing cost sharing" (RPM); "revenue, production cost and CSR cost sharing" (RPC) and "revenue, production cost, marketing cost and CSR cost sharing" (RPMC). We study the efficiency of each sharing contract in a two-echelon channel wherein the market demand is affected by R's marketing effort, retail price, M's CSR effort and the changes in economic and business conditions. These settings represent realistic business practices for firms dealing with channel coordination issues for the different decisions impacting on the channel performance including not only the operational choices (quantity, price) and marketing decisions of the firms but also the sustainable channel management. Then, in order to evaluate the two important aspects of channel coordinating contracts including the coordination and Pareto improvement, we model the decision-making of the two firms in the decentralized channel as the M-Stackelberg game and carry out equilibrium analysis with consideration of wholesale price contract (WP) and four kinds of sharing contracts in two situations: deterministic demand (DD) and stochastic demand (SD). We use the results of decentralized channel under WP as a benchmark for the evaluation of channel cooperation with the sharing contracts. We also develop a corresponding model for centralized channel and use the optimal results to investigate channel coordination.

Our contributions are two folds. First, we extend existing literature to address the channel coordination issues in a channel with stochastic demand which depends on retail price, R's marketing and M's CSR effort. Moreover, we show the effect of demand uncertainty on the coordination of such a channel. Second, we construct the new sharing contracts combining the VMI-CC with revenue sharing and cost sharing contract in order to coordinate the chain. We further found that the sharing contract proposed lead to a win-win-win situation, namely, R and M earn more profit while the CSR performance of channel is improved.

This paper is organized as follows: after this introductory section, the literature review is presented in Section 2. We provide the problem description with notations and assumptions in Section 3. Section 4 focuses on analyzing a centralized model and a decentralized model with DD. Section 5 considers these two models under SD. In Section 6, we conduct numerical studies to validate the proposed models. A summary of the findings, the managerial insights and suggested directions for future research are described in the last section.

## 2. Literature review

Integrating CSR into the channel coordination under a VMI-CC is one of the distinctive features of the present research. In addition, we aim the demonstration of the proposed sharing contracts in coordinating such a channel. Therefore, we focus most of our attention on the literature relating to the use of a revenue sharing and/or cost sharing contract for coordinating a channel with the presence of M's CSR and R's marketing effort, and the literature exploring the benefits of a revenue and/or cost sharing contract within the framework of a VMI-CC.

Studies on how to integrate CSR into channel coordination issues have been receiving considerable attention in the academic community. Some researchers have developed coordination mechanisms for coordinating channel members considering the effects of CSR. Modak et al. (2014) analytically discussed channel coordination using quantity discounts along with an agreement on franchise fee and surplus profit division while Hsueh (2015) considered a bi-level programming model in order to analyze a CSR collaboration problem in a three-echelon channel to maximize the profit of whole channel. In contrast, many researchers used a revenue sharing and/or cost sharing contract to coordinate the channel. Ni et al. (2010) developed a two-echelon channel where the CSR cost is only incurred by the upstream firm and is shared by a downstream firm through a WP. Hsueh (2014) proposed a revenue sharing contract under which M invests in CSR and charges R a wholesale price to coordinate CSR effort in a two-stage channel. Panda et al. (2016) used revenue sharing and quantity discount contracts to coordinate the channel where one of the firms, either M or R, is socially responsible. In another study, Panda et al. (2017) proposed a revenue sharing contract to resolve conflicts in a channel with product recycling by using of Nash bargaining for dividing surplus profit. Recently, Raj et al. (2018) used revenue and greening-cost sharing contracts to coordinate a channel where M is responsible for greening and R is accountable for social responsibility.

Notably, the terms of the revenue and/or cost sharing contracts in the aforementioned research are bound by a WP. On the contrary, our proposed sharing contracts are implemented in a channel undertaking a VMI-CC. Under a VMI-CC, M sets the stocking quantity and retail price but does not charge a wholesale price to R who sells the product for M without suffering from the inventory risk (Wang et al., 2004; Li et al., 2009; Chen et al., 2011). However, a few researchers have investigated the suitability of a revenue sharing contract in coordinating VMI-CC. Bernstein et al. (2006) identify the conditions according to which a VMI-CC with a sharing mechanism works well for the entire channel. Li et al. (2009) investigate the suitability of a VMI-CC with revenue sharing by using a game-theoretical approach and find that the VMI-CC can be perfectly coordinated but only under a very mild restriction on the demand distribution. Ru and Wang (2010) indicate that it is beneficial both to M and R when delegating the inventory decision to M rather than to R in the VMI-CC with revenue sharing rules. Chen et al. (2011) deal with the problem of coordinating a VMI-CC when price-dependent revenue-sharing and show that the contract with a price-decreasing revenue share performs worse than the one with fixed or price-increasing revenue share. Chen (2013) extends their earlier work to consider the dynamic joint effects of price and time on demand for vertically decentralized two-echelon channel coordination and shows that a VMI-CC with a revenue-sharing agreement tends to achieve lower

retail prices, larger stock quantity, improved channel efficiency, and increases in the profit of channel member. Cai et al. (2017) establish the dynamic game relationship under a revenue sharing contract to coordinate a VMI-CC facing service-sensitive customers.

Besides, cost sharing contract has recently embedded in the VMI-CC to improve the channel performance. Lee and Cho (2014) propose a VMI-CC that specifies fixed and proportional penalties charged to M when stockouts occur at R. Lee et al. (2016) extend their study by incorporating limited storage capacity and fixed transfer payments, they show that VMI-CC along with fixed transfer payments as well as stockout-cost sharing can lead to the coordination regardless of M's reservation cost. Considering the effects of marketing effort, the study of De Giovanni et al. (2018) is the only paper that evaluates the benefits of a cooperative advertising program (i.e., a cost sharing contract on R's advertising) within the framework of a VMI-CC. It is also worth mentioning that our paper differs from theirs in two distinct ways. Firstly, De Giovanni et al. (2018) don't take the effects of the CSR effort into consideration whereas we discuss channel coordination issues by simultaneously considering the effects of R's marketing and M's CSR efforts. Secondly, instead of considering a VMI-CC where M sets the stocking quantity while R decides both the retail price and the marketing effort (i.e., advertising), we address a VMI-CC where M decides both on the stocking quantity and retail price while R only decides on the marketing effort.

According to the literature we reviewed, our research is the first which attempts to explore the benefits of revenue and cost sharing contract in coordinating a channel undertaking a VMI-CC integrating M's CSR and R's marketing efforts.

### 3. Problem description, notations and assumptions

#### 3.1. Problem description

We consider a two-echelon VMI-CC consisting of one M and one R, in which M produces the product and sells it through R who then sells the products to the final consumers. M can invest in CSR activities to increase demand, while R can influence the demand by exerting marketing efforts. The trade between M and R can be either a WP or a sharing contract. We define a sharing contract as being the combination of the revenue sharing and cost sharing between M and R, and embedded in VMI-CC. Under such a contract, M retains the ownership of the consignment stock, decides on the retail price and directly manages the inventory at R's store (i.e., decides on stocking quantity). Besides, M and R enter into a long-term commitment to share their costs and revenues. Therefore, the sharing contract also specifies the sharing parameters to allocate the channel's costs and revenue. For simplicity, we assume that the same sharing terms for revenue are used to share the costs meaning that if one kind of cost is shared, the fraction of cost sharing is equal to that of revenue sharing and we call it the sharing fraction for short. Further, we extend our model using different sharing parameters for efforts costs. Under the sharing contracts, the decision on the level of sharing fraction has to be made before deciding on the level of efforts. Based on which, R is free to determine the marketing effort level and M is free to determine the CSR effort level to maximize their own profit. Specifically, we propose four kinds of sharing contract to coordinate a VMI-CC. All sharing contracts are constructed from the revenue sharing perspective and the main difference is the cost sharing perspective.

Contract RP: The Revenue and Production cost sharing.

Contract RPM: The Revenue, Production cost and Marketing cost sharing.

Contract RPC: The Revenue, Production cost and CSR cost sharing.

Contract RPMC: The Revenue, Production cost, Marketing cost and CSR cost sharing.

In each kind of sharing contracts, we distinguish between two types of contracts depending on how the production cost of the consignment

stock is shared. The first one is called "OS contract" if the production cost of *only sold* stock is shared. The second one is called "AS contract" if the production cost of *all consignment* stock (sold and unsold) is shared.

In the RP OS contract, the inventory at R is owned by M, R does not pay M upon receipt of the stock but shares the sales revenue on units sold. For each unit of any sold stock, R keeps a fraction  $\alpha \in (0, 1)$  of the revenue for herself and returns the rest  $1 - \alpha$  to M and R incurs a fraction  $\alpha$  of production cost for each unit of stock sold.

In the RPM OS contract, R keeps a fraction  $\alpha \in (0, 1)$  of the revenue per unit sold, incurs a fraction  $\alpha$  of production cost for sold stock and M is willing to absorb a fraction  $1 - \alpha$  of R's marketing cost.

In the RPC OS contract, R keeps a fraction  $\alpha \in (0, 1)$  of the revenue per unit sold, incurs a fraction  $\alpha$  of production cost for sold stock and R is willing to absorb a fraction  $\alpha$  of M's CSR cost.

In the RPMC OS contract, R keeps a fraction  $\alpha \in (0, 1)$  of the revenue per unit sold, incurs a fraction  $\alpha$  of production cost for sold stock and R and M share their costs of marketing and CSR with each other according to a fraction  $\alpha$ , i.e., R absorbs a fraction  $\alpha$  of M's CSR cost while M absorbs a fraction  $1 - \alpha$  of R's marketing cost.

The RP AS, RPM AS, RPC AS and RPMC AS contracts are similar to the RP OS, RPM OS, RPC OS and RPMC OS contracts, respectively, but R incurs a fraction  $\alpha$  of production cost for all consignment stock.

#### 3.2. Notations

The following notations are used to formulate the channel model discussed in this paper.

p: Unit retail price	$(p_j^D, e_j^D, \theta_j^D)$ : The optimal decisions for decentralized channel under contract j in DD
z: Stocking factor of inventory	$(p_j^S, e_j^S, \theta_j^S)$ : The optimal decisions for decentralized channel under contract j in SD
w: Unit wholesale price that M charges to R	$\Pi_m^{WP}, \Pi_r^{WP}, \Pi_c^{WP}$ : M's, R's and channel's profit <sup>1</sup> under WP in DD
$\theta$ : M's CSR effort level	$\Pi_m^j, \Pi_r^j, \Pi_c^j$ : M's, R's and channel's profit under sharing contract j in DD
e: R's marketing effort level	$E[\Pi_m^{WP}], E[\Pi_r^{WP}], E[\Pi_c^{WP}]$ : M's, R's and channel's expected profit under WP in SD
c: Unit production cost for M	$E[\Pi_m^j], E[\Pi_r^j], E[\Pi_c^j]$ : M's, R's and channel's expected profit under sharing contract j in SD.
a: Market scale parameter	$\Pi_I$ : The profit of the centralized channel
b: Price elasticity of the demand	$E[\Pi_I]$ : The expected profit of the centralized channel
$\gamma$ : Marginal effect of marketing effort on demand parameter	
$\lambda$ : Marginal effect of CSR effort on demand parameter	
$\eta$ : Marginal marketing effort cost parameter	
$\kappa$ : Marginal CSR effort cost parameter	
i: Indicator of firm, i = m (M), r (R)	
j: Indicator of the sharing contract	
m, r, c: The subscript corresponding to M, R, and the decentralized channel	

#### 3.3. Assumptions

We use the following specific assumptions: (1) The market demand  $\bar{D}$  has the functional form of  $\bar{D} = D + \xi$  where  $D = D(p, e, \theta)$  is the expected demand and  $\xi$  is a random scaling factor, representing randomness of the market demand due to changes in economic and business conditions,  $\xi$  is supported on  $[A, B]$ , having cumulative distribution function  $F(x)$ , and probability density function  $f(x)$ . We also model the expected demand as a multi-variable linear function of marketing

effort, CSR effort and retail price, i.e.,  $D = a - bp + \gamma e + \lambda \theta$  with  $a, b, \gamma$  and  $\lambda$  being positive parameters and  $a - bc > 0$  are assumed for the demand function. The parameter,  $a$ , is the base market size,  $b$  is the price elasticity of demand,  $\gamma$  measures the influence of marketing effort on demand, and  $\lambda$  indicates the demand-enhancing effectiveness of M's CSR effort (per unit of effort). This kind of expected demand function has been widely used to incorporate the price, marketing and CSR effort impacting on the demand (e.g., Ghosh and Shah, 2015; Ma et al., 2017). Here, the demand is decreasing in the retail price, increasing in both the R's Marketing effort and the M's CSR effort level. In practice, the M's CSR effort level can be measured by some social or environmental criteria, such as employment of the disabled, overtime hours, energy consumption, or CO2 emissions (Hsueh, 2015). (3) Quadratic functions are assumed to formulate R's marketing cost and M's CSR cost, i.e., the cost of the marketing efforts at level  $e$  is  $\eta e^2/2$  where  $\eta > 0$ ; the cost of the CSR efforts at level  $\theta$  is  $\kappa \theta^2/2$  where  $\kappa > 0$ . This type of cost functions has been used by several researchers as shown in the literature (e.g., Bhaskaran and Krishnan, 2009; Ma et al., 2017). (4) Our model does not include either holding or stockout costs and the unsold stock's salvage value is zero. (5) Both M and R possess full and symmetric information regarding costs and demand.

#### 4.2. The decentralized channel model

In a decentralized channel, the channel members make their own decisions separately to maximize their own profits, but the decision making results are mutually influential. The sequence of events under a WP is as follows: In the first stage, M decides on the CSR effort level and the wholesale price. In the second stage, for a given wholesale price and CSR effort level chosen by M, R determines the marketing effort level, the retail price and uses a stocking quantity equal to demand to maximize her profit. We model the decision-making problems of the two channel members under WP as a Stackelberg game in which the M acts as the leader and the R as the follower.

Under the sharing contracts, the sequence of events is as follows: In the first step, both firms negotiate a sharing fraction  $\alpha$ . Then, in the second step, M decides the retail price and the CSR effort level and chooses a stocking quantity equal to demand to maximize his own profit. In the third step, based on M's decisions, R decides only on the marketing effort level to obtain her own profit maximization. Therefore, after the sharing fraction was chosen, the behavior of M and R under the sharing contracts can be described by using M-Stackelberg setting where M as the leader and R as follower. Then, the Stackelberg game corresponding to each contract can be expressed as follows:

Contract	Stage 1	Stage 2
WP	$\max_{w, \theta} \Pi_m^{WP} = (w - c)D - \kappa \theta^2/2$	$\max_{p, e} \Pi_r^{WP} = (p - w)D - \eta e^2/2$
RP	$\max_{p, \theta} \Pi_m^{RP} = (p - c)(1 - \alpha)D - \kappa \theta^2/2$	$\max_e \Pi_r^{RP} = (p - c)\alpha D - \eta e^2/2$
RPM	$\max_{p, \theta} \Pi_m^{RPM} = (p - c)(1 - \alpha)D - (1 - \alpha)\eta e^2/2 - \kappa \theta^2/2$	$\max_e \Pi_r^{RPM} = (p - c)\alpha D - \alpha \eta e^2/2$
RPC	$\max_{p, \theta} \Pi_m^{RPC} = (p - c)(1 - \alpha)D - (1 - \alpha)\kappa \theta^2/2$	$\max_e \Pi_r^{RPC} = (p - c)\alpha D - \eta e^2/2 - \alpha \kappa \theta^2/2$
RPMC	$\max_{p, \theta} \Pi_m^{RPMC} = (p - c)(1 - \alpha)D - \frac{(1 - \alpha)}{2}\eta e^2 - \frac{(1 - \alpha)}{2}\kappa \theta^2$	$\max_e \Pi_r^{RPMC} = (p - c)\alpha D - \alpha \eta e^2/2 - \alpha \kappa \theta^2/2$

### 4. Modelling with deterministic demand

**Remark 2.** When the demand is deterministic, it is clearly optimal for the decision-maker to use a stocking quantity equal to demand. In addition, there is no risk associated with overstocking. Therefore, we do not distinguish between two types of contracts: the OS contract and the AS contract for the DD case.

#### 4.1. The centralized channel model

In a centralized channel, the channel members are vertically integrated under the central planner for joint profit maximization. The central planner decides the optimal retail price, stocking quantity, marketing level, and CSR level for the entire channel.

The profit of the centralized channel in the DD is

$$\Pi_I = (p - c)D - \eta e^2/2 - \kappa \theta^2/2 \tag{1}$$

As Ma et al. (2013), we impose a restriction of  $-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 > 0$  and  $2b\eta - \gamma^2 > 0$  to ensure positive values of the decision variables in the centralized channel and the Hessian matrix of  $\Pi_I$  is a negative definite. Under this restriction, the profit of the centralized channel is jointly concave in  $p, e$ , and  $\theta$ , therefore, the optimal decisions of retail price  $p_I^D$ , marketing effort level  $e_I^D$ , and CSR effort level  $\theta_I^D$  can be obtained through the first order optimality conditions. The optimal stocking quantity results from substituting  $(p_I^D, e_I^D, \theta_I^D)$  into the demand function. However, we do not present it to save place. Similarly, substituting  $(p_I^D, e_I^D, \theta_I^D)$  into Eq. (1), we obtain the optimal profit of the centralized channel. The results are listed in Table 1.

We solve the games by backward induction. The equilibrium results with DD are listed in Tables 1 and 2. Please see all Proof of tables in Appendix A.

#### 4.3. Analytical results for channel performance in the deterministic demand

##### 4.3.1. The preliminary conditions

We focus on the condition when  $b > \gamma^2/2\eta$  and  $\kappa > \xi$  where  $\xi = \eta\lambda^2/(2b\eta - \gamma^2)$  to ensure that the Hessian matrix of  $\Pi_I$  in the centralized case is negative definite matrices. Moreover, in order for the channel's members to have incentives to participate in the sharing contracts, we need to assure positive profits for each partner in the channel, i.e.,  $\Pi_m^j > 0$  and  $\Pi_r^j > 0$  (hereafter,  $\Pi_i^j > 0$  for short). Therefore, we restrict the range of  $\alpha$  values considered under each sharing contract. We assume that this holds throughout the paper.

##### 4.3.2. The impact of $\alpha$ on the investment effort cost allocation

The capital investment required to generate a unit of demand is  $\eta/2\gamma^2$  for marketing effort while  $\kappa/2\lambda^2$  for CSR effort. Thus, the ratio,  $H$ , is calculated by  $\eta\lambda^2/\gamma^2\kappa$  demonstrates the cost-effectiveness of investing in marketing effort versus CSR effort. As  $H$  is less than one, the marketing effort is more cost-effective than the CSR effort, whereas,  $H$  is equal to one refers that CSR and marketing activities have the same cost-effectiveness. Some algebraic calculations based on the optimal decisions results show that the rate of investment in CSR compared to marketing effort of the channel (i.e., the CSR investment cost divide by the marketing effort cost) is equal to  $H$  in the WP and RPMC contract. By contrast, this rate depends on both  $H$  and the sharing fraction for other contracts. Namely, the value of this rate is  $\frac{(\alpha - 1)^2}{\alpha^2}H$  in RP contract,  $(\alpha - 1)^2H$  in RPM contract and  $\frac{1}{\alpha^2}H$  in RPC contract. In addition, these



**Table 1**  
The optimal decisions for centralized and decentralized channel in DD.

Models/Contract	Retail price	Marketing effort level (e)	CSR effort level (θ)	
Centralized	$c + \frac{(a - bc)\eta\kappa}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$	$\frac{(a - bc)\gamma\kappa}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$	$\frac{(a - bc)\eta\lambda}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$	
Decentralized channel	WP	$c + \frac{(a - bc)(3b\eta - \gamma^2)\kappa}{b(4b\eta\kappa - 2\gamma^2\kappa - \eta\lambda^2)}$	$\frac{(a - bc)\gamma\kappa}{4b\eta\kappa - 2\gamma^2\kappa - \eta\lambda^2}$	$\frac{(a - bc)\eta\lambda}{4b\eta\kappa - 2\gamma^2\kappa - \eta\lambda^2}$
	RP	$c + \frac{(a - bc)\eta\kappa}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$	$\frac{(a - bc)\alpha\gamma\kappa}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$	$\frac{(a - bc)(1 - \alpha)\eta\lambda}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$
	RPM	$c + \frac{(a - bc)\eta\kappa}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$	$\frac{(a - bc)\gamma\kappa}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$	$\frac{(a - bc)(1 - \alpha)\eta\lambda}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$
	RPC	$c + \frac{(a - bc)\eta\kappa}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$	$\frac{(a - bc)\alpha\gamma\kappa}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$	$\frac{(a - bc)\lambda\eta}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$
	RPMC	$c + \frac{(a - bc)\eta\kappa}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$	$\frac{(a - bc)\gamma\kappa}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$	$\frac{(a - bc)\eta\lambda}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$

**Table 2**  
M's, R's and channel's profit under centralized and decentralized channel in DD.

Models/Contract	R's profit	M's profit	Total profit	
Centralized	Omitted	Omitted	$\frac{(a - bc)^2\eta\kappa}{2(-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2)}$	
Decentralized channel	WP	$\frac{(a - bc)^2\eta\kappa}{2(4b\eta\kappa - 2\gamma^2\kappa - \eta\lambda^2)^2}$	$\frac{(a - bc)^2\eta\kappa(6b\eta\kappa - 3\gamma^2\kappa - \eta\lambda^2)}{2(4b\eta\kappa - 2\gamma^2\kappa - \eta\lambda^2)^2}$	
	RP	$\frac{(a - bc)^2\alpha\eta(2b\eta - 3\alpha\gamma^2)\kappa^2}{2(2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2)^2}$	$\frac{(a - bc)^2(1 - \alpha)\eta\kappa}{-4\alpha\gamma^2\kappa + 4b\eta\kappa - 2\eta\lambda^2 + 2\alpha\eta\lambda^2}$	$\frac{(a - bc)^2\eta\kappa((2b\eta\kappa - \eta\lambda^2) - 2\alpha(\gamma^2\kappa - \eta\lambda^2) - \alpha^2(\gamma^2\kappa + \eta\lambda^2))}{2(2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2)^2}$
	RPM	$\frac{(a - bc)^2\alpha\eta(2b\eta - \gamma^2)\kappa^2}{2(\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2)^2}$	$\frac{(a - bc)^2(1 - \alpha)\eta\kappa}{-2\gamma^2\kappa + 4b\eta\kappa - 2\eta\lambda^2 + 2\alpha\eta\lambda^2}$	$\frac{(a - bc)^2\eta\kappa(\gamma^2\kappa + \eta(-2b\kappa + (\alpha - 1)^2\lambda^2))}{-2(\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2)^2}$
	RPC	$\frac{(a - bc)^2\alpha\eta\kappa(2b\eta\kappa - 3\alpha\gamma^2\kappa - \eta\lambda^2)}{2(2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2)^2}$	$\frac{(a - bc)^2(1 - \alpha)\eta\kappa}{-4\alpha\gamma^2\kappa + 4b\eta\kappa - 2\eta\lambda^2}$	$\frac{(a - bc)^2\eta\kappa(-2\alpha\gamma^2\kappa - \alpha^2\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2)}{2(2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2)^2}$
	RPMC	$\frac{\alpha(a - bc)^2\eta\kappa}{2(-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2)}$	$\frac{(1 - \alpha)(a - bc)^2\eta\kappa}{2(-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2)}$	$\frac{(a - bc)^2\eta\kappa}{2(-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2)}$

values are decreasing functions of the sharing fraction for a given  $H$ . Therefore, the channel will increase the allocation of capital to the investment in CSR in two scenarios: when marketing effort is less cost effective than CSR or when M receives more sales of the channel and accordingly, incurs more cost of the chain. This finding implies that in order to increase the investment in CSR activities in a channel where CSR effort is less cost effective, it is necessary that M undertakes more cost of the channel and accordingly, receive more sales.

4.3.3. The impact of  $\alpha$  on the channel member's profit

In this section, we're interested in understanding how the selection of  $\alpha$  impacts on R's and M's profits. With this aim, we evaluate the relative gains/losses realized by the contracting parties associated with the sharing contract versus WP. We let  $\Upsilon_m^j, \Upsilon_r^j$  represent the relative gains/losses of M and R under the sharing contract j versus WP, respectively, i.e.,  $\Upsilon_m^j = (\Pi_m^j - \Pi_m^{WP}) / \Pi_m^{WP}$ ,  $\Upsilon_r^j = (\Pi_r^j - \Pi_r^{WP}) / \Pi_r^{WP}$ .  $\Upsilon_m^j > 0$ , ( $\Upsilon_r^j > 0$ ) indicates that M (R) is better off under the sharing contract j. From the results in Table 2,  $\Upsilon_i^j$  is a function of  $\alpha$ . Moreover, M's profit and R's profit are independent of  $\alpha$  under WP. Thus, the impact of  $\alpha$  on M's profit (R's profit) under the sharing contract j is the same as that of  $\alpha$  on  $\Upsilon_m^j$  ( $\Upsilon_r^j$ ). By examining the sign of the functions  $\phi_i^j = \partial \Upsilon_i^j / \partial \alpha$  with the condition of  $\Pi_i^j > 0$ , we drive the impact of  $\alpha$  on the profit of M and R in each sharing contract. (Please see the details of our analysis in Appendix B1). We summarize the impact of  $\alpha$  on M's profit with the following results: (1) In the RP contract: As  $b > \gamma^2/\eta$ , M's profit always decreases in  $\alpha$  and M prefers the RP contract to the WP if  $\alpha$  is less than 50%. This also implies that if M has more contractual power than R, he will choose a value of  $\alpha$  approaching zero to attain the highest profit. Conversely, when  $b < \gamma^2/\eta$ , M's profit increases with  $\alpha$  for any  $\alpha$  in the range of  $(0, 2b\eta/3\gamma^2)$ . Thus, M chooses a value of  $\alpha$  approaching  $2b\eta/3\gamma^2$  to attain the highest profit. However, M prefers the RP contract to the WP, only if  $\alpha$  is higher than 50% in this situation. (2) In the RPM contract: M's profit always decreases in  $\alpha$  and M prefers the RPM contract to the WP if  $\alpha$  is less than 50%. M achieves the highest profit if

$\alpha$  approaches zero. (3) In the RPC contract: as  $\eta$  is higher than a threshold level, i.e.,  $\eta = 2\gamma^2\kappa/(2b\kappa - \lambda^2)$ , the smaller the selection of  $\alpha$ , the more profits M gets. This implies that M obtains the highest profit if the value of  $\alpha$  approaches zero. Otherwise, M should raise the value of  $\alpha$  approach  $(2b\eta\kappa - \eta\lambda^2)/3\gamma^2\kappa$  to attract the highest profit. (4) In the RPMC contract: M's profit always decreases with  $\alpha$ . Therefore, M attains the highest profit if  $\alpha$  approaches zero.

Similarly, we summarize the impact of  $\alpha$  on R's profit with the following results: (1) In the RP contract: as  $\kappa$  is higher than a threshold level, ie,  $\kappa > (3\gamma^2\lambda^2 - 2b\eta\lambda^2)/(4b\gamma^2 - 2b^2\eta)$ , R's profit increases with  $\alpha \in (0,1)$ . This suggests that if R has more contractual power than M, she increases the value of  $\alpha$  approaching one to attract the highest profit. This also means that R should incur most of the production costs and extract most of the channel sales to maximize her profit. By contrast, R attains the highest profit if  $\alpha$  approaches  $\alpha_r^{RP} = (2b^2\eta\kappa - b\eta\lambda^2)/(4b\gamma^2\kappa - 3\gamma^2\lambda^2 + b\eta\lambda^2)$ . (2) In the RPM contract: as  $\kappa$  is higher than a threshold level, i.e,  $\kappa = 2\kappa$ , R's profit increases with  $\alpha \in (0,1)$ , therefore, R increases the value of  $\alpha$  to approach one to attract the highest profit. Conversely, R rises  $\alpha$  to approach  $\alpha_r^{RPM} = (-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2)/\eta\lambda^2$  to maximize her profit. (3) In the RPC contract: As  $\eta$  is higher than a threshold level, i.e,  $\eta = 4\gamma^2\kappa/(2b\kappa - \lambda^2)$ , R increases the value of  $\alpha$  to approach one to attract more profit. On the contrary, R rises  $\alpha$  to approach  $(2b\eta\kappa - \eta\lambda^2)/4\gamma^2\kappa$  to attain the highest profit. (4) In the RPMC contract: R's profit always increases with the value of  $\alpha$ . Therefore, R attains the highest profit if  $\alpha$  approaches one.

4.3.4. The impact of  $\alpha$  on the channel's profit

We further investigate the impacts of  $\alpha$  on the channel's profit. With this aim, similar to Cachon (2003), we define the efficiency of the decentralized channel with respect to the centralized channel, as the ratio of the channel's profit to the profit of the centralized channel, i.e.,  $E^j = \Pi_c^j / \Pi_r$ . By examining the sign of the functions  $\phi_c^j = \partial E^j(\alpha) / \partial \alpha$  with the conditions of  $\Pi_i^j > 0$ , we summarize the impact of  $\alpha$  on the channel's profit through the following results (the details of our analysis can

be seen in Appendix B2): (1) The decentralized channel with the RP contract generates the highest profit when  $\alpha$  is chosen at  $\alpha_c^{RP} = \frac{2b\gamma^2\eta\kappa - \gamma^2\eta\lambda^2}{2\gamma^4\kappa + 2b\gamma^2\eta\kappa - 4\gamma^2\eta\lambda^2 + 2b\eta^2\lambda^2}$ . Furthermore, the channel's profit in the RP contract is always less than the profit of the centralized channel. (2) The channel efficiency of the RPM contract always decreases in  $\alpha$ , approaches one as  $\alpha$  approaches zero and approaches  $\frac{\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2}{\gamma^2\kappa - 2b\eta\kappa}$  as  $\alpha$  approaches one. (3) The decentralized channel with the RPC contract generates the highest profit when  $\alpha$  is chosen at  $\alpha_c^{RPC} = \frac{2b\eta\kappa - \eta\lambda^2}{2\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$ . However, the channel's profit in the RPC contract is always less than the profit of the centralized channel. (4) The decentralized channel with the RPMC contract generates the same profit as that of the centralized channel and the channel efficiency of the RPMC contract does not depend on the selection of  $\alpha$ .

**Remark 3.** From the above analysis, we observe that the channel efficiency is highest in the RPMC contract and the RPMC contract perfectly coordinates the channel while the RP, RPM and RPC contracts do not coordinate the channel. Note that when  $\alpha = 0$  in the RPM contract, the channel efficiency is equal to one, thus RPM can lead a perfectly coordinated channel. However, M captures all the channel profits while R obtains zero profit in this situation. Therefore, R has no incentive to accept an RPM contract with the sharing fraction equal to zero.

4.3.5. Channel cooperation and CSR-performance under sharing contracts

Let  $\alpha^j \in (\alpha^j, \bar{\alpha}^j)$  represent a Pareto-improving region where  $\Upsilon_m^j > 0$  and  $\Upsilon_r^j > 0$ , then we find that there exists an interval of  $\alpha^j$  values such that both R and M earn greater profits under the sharing contract j than under a WP. Thus, both R and M are willing to accept the sharing contracts. Furthermore, to ensure the sustainability of channel coordination, we also consider whether CSR is improved in the cooperation state. In the propositions 1–4, we provide the upper ( $\bar{\alpha}^j$ ) and lower bounds ( $\alpha^j$ ) of the Pareto-improving region and investigate the CSR performance in this region corresponding to each sharing contract.

**Proposition 1.** In the RP contract:

- (1) When  $b > \gamma^2/\eta$  and  $\kappa > \underline{\kappa}$  then  $\alpha^{RP} = 0.5u_1 - \sqrt{v_1}$ ;  $\bar{\alpha}^{RP} = 0.5$ , where  $u_1 = \frac{\eta(16b^3\eta^2\kappa^2 + 2\gamma^4\eta\lambda^2 - \gamma^2\eta\lambda^4 + (b\eta\lambda^2 - 4b^2\eta\kappa)(2\gamma^2\kappa + 3\eta\lambda^2))}{4\gamma^6\kappa^2 + b\eta^3\lambda^4 + 4\gamma^4\eta\kappa(2\lambda^2 - 5b\kappa) + \gamma^2\eta^2(24b^2\kappa^2 - 16b\kappa\lambda^2 + \lambda^4)}$ , and  $v_1 = \frac{\eta^2(\gamma^2 - b\eta)(2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)^2(3\gamma^2\lambda^4 - 16b^3\eta\kappa^2 + 8b^2\kappa(\gamma^2\kappa + 2\eta\lambda^2) - b(8\gamma^2\kappa\lambda^2 + 5\eta\lambda^4))}{(4\gamma^6\kappa^2 + b\eta^3\lambda^4 + 4\gamma^4\eta\kappa(2\lambda^2 - 5b\kappa) + \gamma^2\eta^2(24b^2\kappa^2 - 16b\kappa\lambda^2 + \lambda^4))^2}$ .

- (2)  $\theta_{RP}^D > \theta_{WP}^D$  when  $\alpha^{RP} < \alpha < \bar{\alpha}^{RP}$  and the conditions of part (1) are satisfied.

**Proposition 2.** In the RPM contract:

- (1) When  $b > \gamma^2/2\eta$  and  $\kappa > \underline{\kappa}$  then  $\alpha^{RPM} = u_2 + \sqrt{v_2}$ ;  $\bar{\alpha}^{RPM} = 0.5$ , where  $u_2 = (4\gamma^4\kappa^2 + 2\gamma^2\eta\kappa(3\lambda^2 - 8b\kappa) + \eta^2(16b^2\kappa^2 - 12b\kappa\lambda^2 + 3\lambda^4))/2\eta^2\lambda^4$  and  $v_2 = ((2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)^2(4\gamma^4\kappa^2 + 8\gamma^2\eta\kappa(\lambda^2 - 2b\kappa) + \eta^2(16b^2\kappa^2 - 16b\kappa\lambda^2 + 5\lambda^4)))/\eta^4\lambda^8$ .
- (2)  $\theta_{RPM}^D > \theta_{WP}^D$  when  $\alpha^{RPM} < \alpha < \bar{\alpha}^{RPM}$  and the conditions of part (1) are satisfied.

**Proposition 3.** In the RPC contract, let

$$u_3 = \frac{\eta^2(2b\kappa - \lambda^2)(16b^2\eta\kappa^2 + 4\gamma^2\eta\lambda^2 + \eta\lambda^4 - 8b\kappa(\gamma^2\kappa + \eta\lambda^2))}{\gamma^2\kappa(8\gamma^4\kappa^2 + 3\eta^2(\lambda^2 - 4b\kappa)^2 + 4\gamma^2\eta\kappa(3\lambda^2 - 10b\kappa))}$$

$$v_3 = \frac{\eta^2(\lambda^2 - 2b\kappa)^2(2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)^2 \left( \gamma^4\kappa^2 + 4\gamma^2\eta\kappa(\lambda^2 - 6b\kappa) + \eta^2(\lambda^2 - 4b\kappa)^2 \right)}{\gamma^4\kappa^2(8\gamma^4\kappa^2 + 3\eta^2(\lambda^2 - 4b\kappa)^2 + 4\gamma^2\eta\kappa(3\lambda^2 - 10b\kappa))^2}$$

- $$\kappa_1 = \frac{1}{4} \sqrt{\frac{\gamma^2\eta^2\lambda^4}{(2b\eta - \gamma^2)(\gamma^2 - b\eta)^2}} - \frac{\eta\lambda^2}{4(\gamma^2 - b\eta)}; \quad \text{and} \quad \kappa_2 = \frac{1}{4} \sqrt{\frac{-\gamma^2\eta^2\lambda^4 - b\eta^3\lambda^4}{(\gamma^2 - b\eta)^3}} - \frac{\eta\lambda^2}{4(\gamma^2 - b\eta)}$$
- we have :
- (1) When  $\gamma^2/\eta < b \leq 5\gamma^2/2\eta$ ,  $\kappa_1 < \kappa \leq \kappa_2$  or when  $b > 5\gamma^2/2\eta$ ,  $\underline{\kappa} < \kappa < \kappa_2$  then  $\alpha^{RPC} = 0.5(u_3 - \sqrt{v_3})$ ;  $\bar{\alpha}^{RPC} = 0.5(u_3 + \sqrt{v_3})$ . Otherwise, when  $b > \gamma^2/\eta$ ,  $\kappa > \kappa_2$  then  $\alpha^{RPC} = 0.5(u_3 - \sqrt{v_3})$ ;  $\bar{\alpha}^{RPC} = (2\gamma^2\kappa - 2b\eta\kappa)/(4\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)$ .
  - (2)  $\theta_{RPC}^D > \theta_{WP}^D$  when  $\alpha^{RPC} < \alpha < \bar{\alpha}^{RPC}$  and the conditions of part (1) are satisfied.

**Proposition 4.** In the RPMC contract:

- (1) When  $b > \gamma^2/2\eta$  and  $\kappa > \underline{\kappa}$  then  $\alpha^{RPMC} = \frac{(\gamma^2 - 2b\eta)\kappa(\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2)}{(2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)^2}$ ,  $\bar{\alpha}^{RPMC} = \frac{(\gamma^2 - 2b\eta)\kappa}{2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2}$ .
- (2)  $\theta_{RPMC}^D > \theta_{WP}^D$  when  $\alpha^{RPMC} < \alpha < \bar{\alpha}^{RPMC}$  and the conditions of part (1) are satisfied.
- (3)  $\theta_{RPMC}^D = \theta_I^D$ ,  $e_{RPMC}^D = e_I^D$ ,  $P_{RPMC}^D = P_I^D$ ,  $\Pi_m^{RPMC} = (1 - \alpha)\Pi_I$  and  $\Pi_r^{RPMC} = \alpha\Pi_I$  for any  $0 < \alpha < 1$ .

**Remark 4.** Part 1 of Propositions 2 and 4 shows that RPM and RPMC always bring the channel to Pareto improvement regardless of the impact of price on market demand, while Part 1 of Propositions 1 and 3 shows that RP and RPC only reach this cooperation state when the effect of price is relatively high i.e.,  $b > \gamma^2/\eta$ . From Part 2 of Proposition 1-4 the sharing contracts can simultaneously achieve the following objectives: (i) improve CSR performance; (ii) improve total channel profits; (iii) ensure that each partner in the channel can benefit from the contract. Moreover, part 3 of Proposition 4 shows that the RPMC contract can maximize the channel profit and arbitrarily allocate the profit between M and R based on each player's negotiation power (i.e., through negotiation to determine  $\alpha$ )

4.4. Bargaining problem

In the previous subsection, we reached the analytical solution to the existence of Pareto-improving region such that both M and R are willing to cooperate under each sharing contract and lead to larger channel's profit. We now use the Nash bargaining model presented by Nash (1950) to determine the optimal sharing fraction. In a Nash bargaining game, two players cooperatively decide to how to split the additional profits that occurs as a result of their interaction. How to split this extra profit depends on the bargaining power of both players and the values of threat points (i.e., the value they are able to obtain when there is no cooperation) (Bhaskaran and Krishnan, 2009; Wu et al., 2009). Assume that M has bargaining power  $\delta$ , while that of R is  $1-\delta$ , where  $\delta \in [0,1]$ . Let  $\Delta\Pi_m^j = \Pi_m^j - \Pi_m^{WP}$ ,  $\Delta\Pi_r^j = \Pi_r^j - \Pi_r^{WP}$  be the extra profit of M, R under each sharing contract versus WP, respectively. Using M's and R's profits in the WP contract reflecting their threat points, we have a bargain problem over  $\alpha^j \in (\alpha^j, \bar{\alpha}^j)$  to the share the joint extra-profit under sharing contract j, i.e.,  $\Delta\Pi_c^j = \Pi_c^j - \Pi_c^{WP} > 0$ . To obtain the Nash's solution, the following optimization needs to be solved:  $Max(\Delta\Pi_m^j)^\delta * (\Delta\Pi_r^j)^{1-\delta}$ , subject to constraints  $\Upsilon_m^j > 0$  and  $\Upsilon_r^j > 0$  (i.e., Pareto-improving region). The analytical solutions of this kind of problems are difficult. Therefore, we leave it to be resolved by a numerical method to get more insights on the effects of bargaining power on the allocation of the joint extra-profit between the channel members.

4.5. The sharing contracts are with different sharing parameters for effort costs

So far, our analysis results above depend heavily on the assumption that the firms use the same sharing rule for all costs. We now extend our analysis using different the sharing parameters for efforts costs. We denote such a contract as  $(\alpha, \phi, \beta)$  where  $\alpha \in (0,1)$  is the fraction of the gross profit (i.e., the total revenue minus the production cost of sold

**Table 3**  
Computational results for the DD case.

A	$\alpha$	RP				RPM				RPC				RPMC			
		$\Upsilon_m$	$\Upsilon_r$	E	$\theta$	$\Upsilon_m$	$\Upsilon_r$	E	$\theta$	$\Upsilon_m$	$\Upsilon_r$	E	$\theta$	$\Upsilon_m$	$\Upsilon_r$	E	$\theta$
A = 0	0.01	0.73	-0.96	0.81	8.19	1.12	-0.95	1.00	10.06	0.73	-0.97	0.81	8.29	1.12	-0.96	1.00	10.17
	0.1	0.61	-0.64	0.84	7.63	0.90	-0.55	1.00	9.03	0.63	-0.67	0.84	8.58	0.93	-0.60	1.00	10.17
	0.2	0.47	-0.27	0.87	6.97	0.67	-0.13	0.99	7.91	0.51	-0.32	0.87	8.93	0.71	-0.20	1.00	10.17
	0.3	0.32	0.09	0.89	6.27	0.44	0.27	0.99	6.83	0.37	0.03	0.90	9.31	0.50	0.21	1.00	10.17
	0.4	0.17	0.45	0.90	5.53	0.22	0.64	0.98	5.77	0.23	0.40	0.92	9.72	0.29	0.61	1.00	10.17
	0.5	0.00	0.80	0.91	4.75	0.00	1.00	0.96	4.75	0.07	0.78	0.94	10.17	0.07	1.01	1.00	10.17
	0.6	-0.18	1.14	0.92	3.91	-0.21	1.34	0.95	3.75	-0.10	1.16	0.96	10.67	-0.14	1.41	1.00	10.17
	0.7	-0.36	1.46	0.91	3.03	-0.42	1.66	0.93	2.77	-0.29	1.55	0.96	11.21	-0.36	1.81	1.00	10.17
	0.8	-0.56	1.77	0.89	2.09	-0.62	1.96	0.92	1.83	-0.50	1.93	0.96	11.82	-0.57	2.21	1.00	10.17
	0.9	-0.77	2.04	0.86	1.08	-0.81	2.24	0.90	0.90	-0.74	2.30	0.94	12.49	-0.79	2.62	1.00	10.17
0.99	-0.98	2.26	0.82	0.11	-0.98	2.49	0.88	0.09	-0.97	2.62	0.91	13.17	-0.98	2.98	1.00	10.17	

stocks) that R receives,  $\varphi \in [0,1]$  is the fraction of R's marketing cost that R undertakes, and  $\beta \in [0,1]$  is the fraction of M's CSR effort cost which R agrees to share with M. Under this setting, the equilibrium results can be derived by following backward induction. However, we obtained complex results that are difficult to analyze following the same procedure used in Section 4.3. Thus, we resort to numerical methods to illustrate the Pareto-improving region for this case in the last subsection of the next section.

4.6. Numerical results for channel performance in the deterministic demand

We assume the parameters as follows:  $a = 101$ ,  $b = 3$ ,  $c = 10$ ,  $\gamma = 0.9$ ,  $\lambda = 1$ ,  $\eta = 0.8$  and  $\kappa = 1.6$ . The parameters of  $a$ ,  $b$ ,  $c$  satisfy that the demand is positive when the retail price is equal to the production cost, i.e.,  $p = c$ , and the parameters of  $\gamma$ ,  $\lambda$ ,  $\eta$  and  $\kappa$  satisfy the preliminary conditions of  $-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 = 5.584 > 0$  and  $2b\eta - \gamma^2 = 3.99 > 0$ . We fix the benchmark parameter values above and focus on analyzing the effects of the sharing parameters on the performance of sharing contracts. We also seek to identify the Pareto-improving region in which the adoption of the sharing contract is feasible.

4.6.1. The sharing contracts use the same sharing rule

By using the parameter values, we first compute the optimal decisions and the profit of the centralized channel. Similarly, we compute the optimal decisions and the profit of M and R under WP (the results are presented in Tables 6 and 7 in Section 6.2). We next compute M's and R's optimal decisions and profit in the decentralized channel under sharing contracts corresponding to the values of  $\alpha$  in the range of [0.01, 0.99]. Based on the computational results (i.e. the profit of M and R), we evaluate the performance of each sharing contract in the two following aspects: (1) the relative gains/losses of M (i.e.,  $\Upsilon_m^j$ ) and R (i.e.,

$\Upsilon_r^j$ ) under sharing contract j versus WP (see Section 4.3.3), (2) the channel efficiency of sharing contract j, i.e.,  $E^j$  (see Section 4.3.4). We present the results of  $\Upsilon_m^j$ ,  $\Upsilon_r^j$  and  $E^j$  for the DD in Table 3. Furthermore, we also present the optimal decision of M on CSR effort level in this table to verify whether the CSR performance is improved in the co-operation state.

4.6.1.1. The effects of the sharing fraction on profits. The results of  $\Upsilon_m^j$  and  $\Upsilon_r^j$  in Table 3 demonstrate that R's profit increases and M's profit decreases with  $\alpha$  in all the sharing contracts. (As we proved in section 4.3.3). We also display the value of  $E^j$  in Table 3 through Fig. 1 to evidence that the channel efficiency is concave in  $\alpha$  under RP and RPC contracts whereas it decreases in  $\alpha$  under RPM contract and remains stable at one under RPMC contract. Following the results in Section 4.3.4, we obtain the value of  $\alpha$  that maximizes the channel efficiency under each sharing contract. Namely,  $E_{Max}^{RP} = 0.916$  at  $\alpha = 0.582$ ,  $E_{Max}^{RPC} = 0.964$  at  $\alpha = 0.726$  and  $E_{Max}^{RPMC}$  approach one when  $\alpha$  approaches to zero. These numerical results demonstrate that the RP, RPM and RPC contracts cannot coordinate the channel while the RPMC contract perfectly coordinates the channel under DD.

4.6.1.2. The Pareto improvements and CSR-performance. As displayed in Table 3, if the beforehand negotiated  $\alpha$  is at 0.3, 0.4 and 0.5 respectively,  $\Upsilon_m^j > 0$  and  $\Upsilon_r^j > 0$ . (For example, when  $\alpha = 0.4$  then  $\Upsilon_m^{RP} = 0.17$ ,  $\Upsilon_r^{RP} = 0.45$ ;  $\Upsilon_m^{RPM} = 0.22$ ,  $\Upsilon_r^{RPM} = 0.64$ ;  $\Upsilon_m^{RPC} = 0.17$ ,  $\Upsilon_r^{RPC} = 0.45$  and  $\Upsilon_m^{RPMC} = 0.29$ ,  $\Upsilon_r^{RPMC} = 0.61$ ). This means that both R and M earn greater profits under the sharing contracts than under WP if the sharing fraction was chosen at 0.3, 0.4 and 0.5 respectively. Thus, the Pareto improvements can be achieved if the sharing fraction was chosen at these values. From Proposition 1-4, we can determine the exact value of  $\alpha$  in the Pareto-improving region which leads to a win-win outcome, namely,  $\alpha^{RP} \in (0.275, 0.5)$ ,  $\alpha^{RPM} \in (0.232, 0.5)$ ,  $\alpha^{RPC} \in (0.291, 0.543)$  and  $\alpha^{RPMC} \in (0.249, 0.533)$ . Moreover, from observing the value of column  $\theta$  in Table 3, we find that the CSR performance of the channel under each sharing contract is higher than those in WP for all  $\alpha$  values within the Pareto-improving region. (For example, when  $\alpha = 0.3$ , M's CSR effort under RP, RPM, RPC and RPMC are 6.27, 6.83, 9.31 and 10.17 respectively. These levels of CSR are higher than that under WP (i.e., 4.746).

4.6.1.3. The effects of the bargaining power on the allocation of the joint extra-profit. We define  $\Delta_m^j = \Delta\Pi_m^j/\Delta\Pi_c^j$  and  $\Delta_r^j = \Delta\Pi_r^j/\Delta\Pi_c^j$  are relative joint extra profits of M and R in the sharing contract j, respectively. The relationship between  $\Delta_m^j$  ( $\Delta_r^j$ ) and the bargaining power parameter,  $\delta$ , is displayed in Fig. 2 (a) (Fig. 2 (b)), respectively. These figures show that the patterns of  $\Delta_r^j$  follow the opposite patterns of  $\Delta_m^j$  in all scenarios. R's share of the joint extra profits has a decreasing behavior with respect to M's bargaining power. Thus, the two parties will divide the joint extra-profit proportionally to their bargaining power. In case that the M (R) is

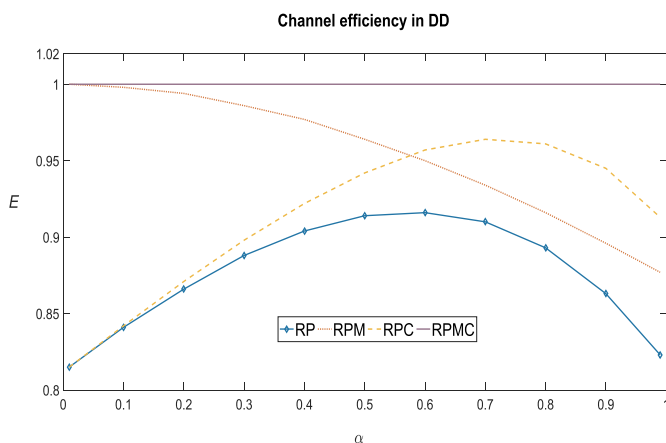


Fig. 1.  $E^j$  changes with  $\alpha$  in the DD.



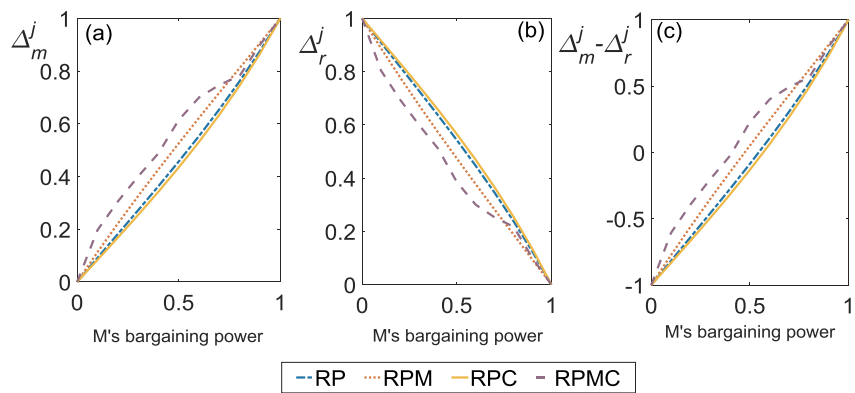


Fig. 2. (a)  $\Delta_m^j$ , (b)  $\Delta_r^j$  and (c)  $\Delta_m^j - \Delta_r^j$ , changes with M's bargaining power.

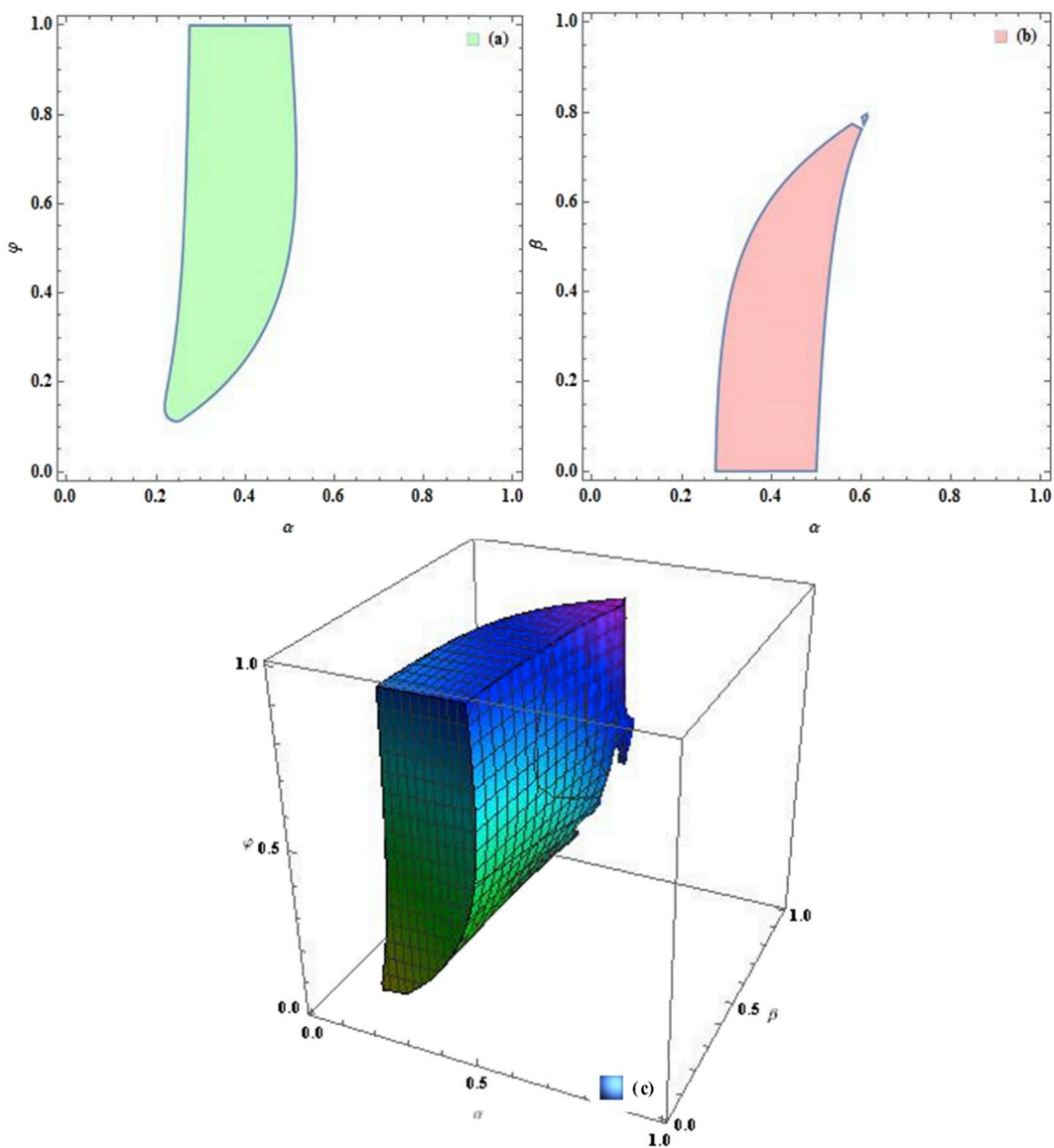


Fig. 3. The Pareto-improving region in the RPM (a), RPC (b) and RPMC contract (c) with different sharing parameters for effort costs.

a dominant player (i.e.,  $\delta = 1$ , res.  $\delta = 0$ ), they are able to obtain the entire joint extra-profit. When M and R have equal power, i.e.,  $\delta = 0.5$ , the common knowledge in the bargaining literature of the Nash's model predicts that the channel members will equally split the joint extra-profits. In our numeric results, M obtains higher joint extra-profit than R under RPM and RPMC contracts but lower under other contracts when the channel member has the same power (see Fig. 2 (c)). The explanation here is that we do not assume that the threat points of both parties are the same. Therefore, our numeric results reflect the different positions of M and R in the negotiation and the effects of marketing and CSR activities on the allocation of the joint extra-profit between the channel members.

4.6.2. The Pareto-improving region with different sharing parameters

The Pareto-improving region is plotted in Fig. 3 (a) with respect to  $\alpha$  and  $\varphi$  for the RPM contract with differentiating between the sharing rule for gross profit,  $\alpha$ , and the sharing rule for the marketing cost,  $\varphi$  (i.e., the sharing contract as  $(\alpha, \varphi, 0)$ ). Every pair  $(\alpha, \varphi)$  in this region presents a feasible solution to the bargaining problem that leads to higher profits of both channel members. Similarly, Fig. 3 (b) illustrates the Pareto-improving region with respect to  $\alpha$  and  $\beta$  for the RPC contract with differentiating between the sharing rule for gross profit,  $\alpha$ , and the sharing rule for the CSR effort cost,  $\beta$  (i.e., the sharing contract as  $(\alpha, 1, \beta)$ ). For a general contract  $(\alpha, \varphi, \beta)$  (i.e., RPMC contract with differentiating the sharing rule for the gross profit,  $\alpha$ , the sharing rule for the marketing cost,  $\varphi$ , and the sharing rule for the CSR cost,  $\beta$ ), we plotted in Fig. 3 (c) all triples of  $(\alpha, \varphi, \beta)$  that identify the feasible solutions for the bargaining problem. Therefore, M and R will get more profit if they decide the sharing parameters  $(\alpha, \varphi, \beta)$  in this region. Compared to the results of original model using the same sharing rules, we find that the use of different sharing coefficients will expand more the feasible region for the effort costs sharing. In addition, when a channel member undertakes more cost of efforts, they will require an increase in the gross profit received accordingly. Interestingly, the negotiable range values of gross profit sharing do not change significantly compared to the original model. This result implies that regardless of efforts cost sharing, the adoption of a cooperative program on efforts is never feasible when most of the gross profits go to either M or R. Further, M is willing to implement a cooperative program on marketing efforts only when R bears marketing cost higher than a threshold (i.e., 13,6%). Whereas, R is willing to implement a cooperative program on CSR efforts only when R incurs CSR cost lower than a threshold, about 80%. This finding is quite intuitive, the increase in cost share borne by the partner leads a decrease in benefits for the party who handles the activities and makes the implementation of a cooperative program on efforts difficult to be feasible.

5. Modelling with stochastic demand

5.1. The centralized channel model

In the centralized channel with the SD, a central decision maker chooses the stocking quantity ( $q$ ), the retail price, the CSR and marketing effort level to maximize the profit of the entire channel. Following Petruzzi and Dada (1999), we define the stocking factor of inventory  $z$  as  $z = q - D$  to cover the randomness of demand. Let  $\Lambda[z] = E[\text{Min}(z, \xi)] = \int_A^z \xi f(\xi) d\xi + \int_z^B z f(\xi) d\xi$ ,  $\Lambda'[z] = \partial\Lambda[z]/\partial z = 1 - F(z)$ ,  $\Lambda''[z] = \partial^2\Lambda[z]/\partial z^2 = -f(z)$ . The expected profit of the centralized channel in the SD, denoted as  $E[\Pi_r^S]$  is  $E[\Pi_r^S] = p(D + \Lambda[z]) - c(D + z) - \eta e^2/2 - \kappa\theta^2/2$  (2)

In Eq. (2), the first term is the expected revenue, the second term is the production cost, the third is the marketing cost and the fourth is the cost of CSR effort.

We apply a sequential procedure (Wang et al., 2004) to find the optimal solutions, denoted by  $(p_r^S, z_r, e_r^S, \theta_r^S)$  that maximize  $E[\Pi_r^S]$  of Eq. (2). That is, we find the optimal decisions  $(p_r^S, e_r^S, \theta_r^S)$  for a given  $z$ , and then maximize  $E[\Pi_r^S]$  over  $z$  to find  $z_r$ . For the optimal solutions, let  $r(\cdot) = f(\cdot)/(1 - F(\cdot))$  represent the hazard rate function of the demand distribution, the following theorem is given.

**Theorem 1.** For any fixed  $z \in [A, B]$ , the optimal retail price ( $p_r^S$ ), marketing effort level ( $e_r^S$ ) and CSR effort level ( $\theta_r^S$ ) in the centralized channel with the SD are given by

$$p_r^S(z) = p_r^D + \frac{\eta\kappa\Lambda[z]}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}; \theta_r^S(z) = \theta_r^D + \frac{\eta\lambda\Lambda[z]}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}; e_r^S(z) = e_r^D + \frac{\gamma\kappa\Lambda[z]}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$$

If  $F(\cdot)$  satisfies  $2r(z)^2 + dr(z)/dz > 0$  and  $\frac{(a - bc + A)\eta\kappa}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2} > 0$ , the optimal  $z_r$  that maximizes  $E[\Pi_r^S]$  is unique in the region  $[A, B]$  that satisfies the first-order optimality conditions  $\frac{1}{\Lambda'[z_r]} = 1 - \frac{\eta\kappa(a - bc + \Lambda[z_r])}{c(\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2)}$ . Otherwise, if  $F(\cdot)$  is an arbitrary distribution, then the entire support must be searched to find  $z_r$ .

**Proof.** Please see Appendix C1.

Since  $-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 > 0$  compared to the optimal decisions in the centralized channel under DD with that under SD, we see that this relationship depends on the sign of  $\Lambda[z_r]$  which may be negative or positive depending on the randomness of demand ( $\xi$ ). For example, if  $\xi$  is modeled by  $E[\xi] = 0$  then  $\Lambda[z]$  is less than or equal to zero for any  $z \in [A, B]$ , thus we have the flowing relationships:  $p_r^S \leq p_r^D$ ,  $e_r^S \leq e_r^D$  and  $\theta_r^S \leq \theta_r^D$ . However, if  $\xi$  is always non-negative then  $\Lambda[z] \geq 0$  for any  $z \in [A, B]$ , thus we find that  $p_r^S \geq p_r^D$ ,  $e_r^S \geq e_r^D$  and  $\theta_r^S \geq \theta_r^D$ .

5.2. Decentralized channel under WP

The sequence of events under a WP in the SD are similar to that in the DD but in the second stage, R decides the stocking quantity (or equivalently, R's stocking factor of inventory) instead of using a stocking quantity equal to demand. Therefore, the behavior of M and R under WP in the SD can be described using M-Stackelberg setting as follows:

Stage 1:  $\text{Max}_{w, \theta} E[\Pi_m^{WP}(w, \theta)] = (w - c)(z + D) - \kappa\theta^2/2$

Stage 2:  $\text{Max}_{p, e, z} E[\Pi_r^{WP}(p, e, z)] = p(D + \Lambda[z]) - w(D + z) - \eta e^2/2$

Through backward induction, the optimal equilibrium solution can be reached for this game. However, it is difficult to find a closed form solution. Therefore, we leave it to be resolved by a numerical method in the next section.

5.3. Decentralized channel under sharing contracts

The sequence of events under the sharing contracts in the SD are similar to that in the DD but in the second step, M decides stocking quantity (or equivalently, the M's stocking factor of inventory) instead of choosing a stocking quantity equal to demand. Let  $\bar{Q} = D + \Lambda[z]$  be the expected sales quantity and  $\bar{L} = z - \Lambda[z] = E[\text{Max}(0, z - \xi)]$  be the expected value of the leftover inventory (or unsold stock). After the sharing fraction was chosen, the behavior of M and R under the sharing contracts can be described using M-Stackelberg setting as follows:

Contract	Stage	Objective	The OS contracts	The AS contracts
RP	Stage 1	$\max_{p,z,\theta} E[\Pi_m^{RP}]$	$(p - c)(1 - \alpha)\bar{Q} - c\bar{L} - \kappa\theta^2/2$	$(p - c)(1 - \alpha)\bar{Q} - (1 - \alpha)c\bar{L} - \kappa\theta^2/2$
	Stage 2	$\max_e E[\Pi_r^{RP}]$	$(p - c)\alpha\bar{Q} - \eta e^2/2$	$(p - c)\alpha\bar{Q} - \alpha c\bar{L} - \eta e^2/2$
RPM	Stage 1	$\max_{p,z,\theta} E[\Pi_m^{RPM}]$	$(p - c)(1 - \alpha)\bar{Q} - c\bar{L} - (1 - \alpha)\eta e^2/2 - \kappa\theta^2/2$	$(p - c)(1 - \alpha)\bar{Q} - (1 - \alpha)c\bar{L} - (1 - \alpha)\eta e^2/2 - \kappa\theta^2/2$
	Stage 2	$\max_e E[\Pi_r^{RPM}]$	$(p - c)\alpha\bar{Q} - \alpha\eta e^2/2$	$(p - c)\alpha\bar{Q} - \alpha c\bar{L} - \alpha\eta e^2/2$
RPC	Stage 1	$\max_{p,z,\theta} E[\Pi_m^{RPC}]$	$(p - c)(1 - \alpha)\bar{Q} - c\bar{L} - (1 - \alpha)\kappa\theta^2/2$	$(p - c)(1 - \alpha)\bar{Q} - (1 - \alpha)c\bar{L} - (1 - \alpha)\kappa\theta^2/2$
	Stage 2	$\max_e E[\Pi_r^{RPC}]$	$(p - c)\alpha\bar{Q} - \eta e^2/2 - \alpha\kappa\theta^2/2$	$(p - c)\alpha\bar{Q} - \alpha c\bar{L} - \eta e^2/2 - \alpha\kappa\theta^2/2$
RPMC	Stage 1	$\max_{p,z,\theta} E[\Pi_m^{RPMC}]$	$(p - c)(1 - \alpha)\bar{Q} - c\bar{L} - (1 - \alpha)\eta e^2/2 - (1 - \alpha)\kappa\theta^2/2$	$(p - c)(1 - \alpha)\bar{Q} - (1 - \alpha)c\bar{L} - (1 - \alpha)\eta e^2/2 - (1 - \alpha)\kappa\theta^2/2$
	Stage 2	$\max_e E[\Pi_r^{RPMC}]$	$(p - c)\alpha\bar{Q} - \alpha\eta e^2/2 - \alpha\kappa\theta^2/2$	$(p - c)\alpha\bar{Q} - \alpha c\bar{L} - \alpha\eta e^2/2 - \alpha\kappa\theta^2/2$

We solve the games by backward induction. In Table 4, we present the first order optimality condition of M's expected profit function with respect to the stocking factor of inventory under each sharing contract, i.e.,  $\partial E[\Pi_m^j]/\partial z = 0$  from which M's optimal stocking factor of inventory  $z^{sj}$  is determined corresponding to each sharing contract. We provide the following theorem with regard to the uniqueness of M's optimal stocking factor.

**Theorem 2.** In a decentralized channel with the sharing contracts in the SD, the optimal stocking factor of inventory  $z^{sj}$  that maximizes M's expected profit  $E[\Pi_m^j]$ , is unique in the region  $[A, B]$  determined by the first-order optimality conditions  $\partial E[\Pi_m^j]/\partial z = 0$  if the following conditions are satisfied:

- (a)  $F(\cdot)$  is a distribution function satisfying the condition  $2r(z)^2 + dr(z)/dz > 0$  for  $z \in [A, B]$
- (b)  $a - bc + A > 0$
- (c) The  $\alpha \in (0, 1)$  chosen assures positive profits for M in the DD, i.e.,  $\Pi_m^j > 0$ .

**Proof.** Please see the Appendix C2.

**Remark 5.** Condition (a) in Theorem 2 guarantees that  $R^j(z) = \partial E[\Pi_m^j]/\partial z$  is either monotone or unimodal which implies that  $R^j(z)$  has at most two roots. Moreover,  $\lim_{z \rightarrow B} R^j(z) = -c < 0$  for the OS contracts and  $\lim_{z \rightarrow B} R^j(z) = -c(1 - \alpha) < 0$  if  $0 < \alpha < 1$  for the AS contracts. Therefore, if  $R^j(z)$  has two roots, the larger of the two represents the maximum and the smaller represents the minimum of  $E[\Pi_m^j]$ . On the other hand, if  $R^j(z)$  has only one root, we require that  $R^j(A) = \frac{2(a-bc+A)\Pi_m^j}{(a-bc)^2} > 0$  (equivalently  $a - bc + A > 0$  and  $\Pi_m^j > 0$ ) then only one root of  $R^j(z)$  corresponds to the maximum of  $E[\Pi_m^j]$ .

Given M's optimal stocking factor of inventory,  $z^{sj}$ , we derive M's optimal decisions on the retail price, CSR effort level and R's optimal marketing effort level in Table 5. It is noteworthy that the optimal decisions on retail price, CSR effort and marketing effort level under OS contracts have the same expressions as those under the AS contracts. The only difference is the selection of M's optimal stocking factor. From the results in Table 5, we observe that the optimal decisions in the RPMC AS contract are the same as those in the centralized channel. Moreover, R's and M's expected profit functions under the RPMC AS contract are linear functions of that of the centralized channel in the SD (i.e.,  $E[\Pi_r^{RPMC}] = \alpha E[\Pi_r^S]$  and  $E[\Pi_m^{RPMC}] = (1 - \alpha)E[\Pi_m^S]$ ). Therefore, the channel members can arbitrarily allocate the maximized joint profit by negotiating the sharing fraction under the RPMC contract. We then arrive at the following conclusion in Proposition 5.

**Proposition 5.** The RPMC AS contract leads to perfect coordination of the channel and arbitrarily allocates coordinated profit among members in the SD.

**Table 4**  
M's optimal stocking factor of inventory under sharing contracts.

Contract	The OS contracts	The AS contracts
RP	$\frac{(1 - \alpha)\eta\kappa(a - bc + A[z])}{c(-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2)}$	$\frac{\eta\kappa(a - bc + A[z])}{c(-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2)}$
RPM	$\frac{(1 - \alpha)\eta\kappa(a - bc + A[z])}{c(-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2)}$	$\frac{\eta\kappa(a - bc + A[z])}{c(-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2)}$
RPC	$\frac{(1 - \alpha)\eta\kappa(a - bc + A[z])}{c(-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2)}$	$\frac{\eta\kappa(a - bc + A[z])}{c(-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2)}$
RPMC	$\frac{(1 - \alpha)\eta\kappa(a - bc + A[z])}{c(-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2)}$	$\frac{\eta\kappa(a - bc + A[z])}{c(-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2)}$

**Remark 6.** The first order optimality condition:  $\partial E[\Pi_m^j]/\partial z = 0$ , (or equivalently,  $1/\Lambda'[z^j] - 1 = "Y"$ , then, the second term in this equation, "Y", is shown in Table 4.

## 6. Numerical examples under uniform distribution

In this Section, we conduct numerical studies to illustrate our results for the SD case. Using the optimal decisions and profits in the equilibrium, we show that for some ranges values of demand variability and sharing fraction, the sharing contracts using the same sharing rules can effectively coordinate the channel and bring the channel to Pareto improvement. Also by changing the demand variability and sharing fraction, we can evaluate the channel performance under sharing contracts.

### 6.1. Experimental parameters

When deriving the equilibrium results for the SD case, we use the benchmark parameter values as in Section 4.6. In addition, we choose to model the randomness of market demand,  $\xi$ , using a uniform random variable defined on the interval  $[-A, A]$ , where  $A > 0$  is the upper bound of the uniform distribution. Therefore,  $f(x) = \frac{1}{2A}$ ,  $F(x) = \frac{x+A}{2A}$ ,  $\Lambda[z] = -\frac{(A-z)^2}{4A}$  and  $\Lambda'[z] = (A - z)/2A$ . The variance of  $\xi$  which is measured by  $A^2/3$  represents demand variability. The higher the value of  $A$ , the greater the demand variability is. We limit our analysis to scenarios where  $A < a-bc$  (equivalent  $A < 71$ ) in order to ensure that demand is positive at  $p = c$  in the SD. We select the results calculated by  $A = 0, 20, 40$  and  $60$  to simulate three levels of demand variability. The case of  $A = 0$  is equivalent to the case of DD ( $\xi = 0$ ).

**Table 5**  
The difference of the optimal decisions under sharing contracts between two cases of SD and DD.

Contract	Retail price	Marketing effort level	CSR effort level
RP	$\frac{\eta\kappa\lambda[z]}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$	$\frac{\alpha\eta\kappa\lambda[z]}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$	$\frac{(1-\alpha)\eta\lambda\lambda[z]}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$
RPM	$\frac{\eta\kappa\lambda[z]}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$	$\frac{\eta\kappa\lambda[z]}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$	$\frac{(1-\alpha)\eta\lambda\lambda[z]}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$
RPC	$\frac{\eta\kappa\lambda[z]}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$	$\frac{\alpha\eta\kappa\lambda[z]}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$	$\frac{\eta\lambda\lambda[z]}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$
RPMC	$\frac{\eta\kappa\lambda[z]}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$	$\frac{\eta\kappa\lambda[z]}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$	$\frac{\eta\lambda\lambda[z]}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$

**Table 6**  
Optimal decisions for centralized channel when  $\xi \sim U(-A, A)$ .

Demand	$p_I$	$z_I$	$q_I$	$e_I$	$\theta_I$	$\Pi_I$
A = 0	26.275		48.825	18.309	10.172	577.765
A = 20	25.574	4.359	54.138	17.520	9.733	454.890
A = 40	24.782	7.719	58.569	16.620	9.238	334.310
A = 60	23.859	9.704	61.819	15.590	8.661	216.510

6.2. Two basic models: the centralized channel and decentralized channel under WP

Table 6 gives the numerical example results for the optimal decisions and the profit of the centralized channel with different demand variances. Similarly, we compute the optimal decisions and the profit of M and R under WP and present them in Table 7. The results of two models serve as benchmarks for the evaluation of the performance of sharing contracts.

From Tables 6 and 7, the stocking quantity, the marketing effort and the CSR effort of the decentralized channel under WP are lower than those of the centralized channel. These results lead to a lower profit for whole channel under WP. As demand variability increases, the profit of the decentralized channel under WP and that of the centralized channel decrease because the channel tends to set a lower price with higher level of demand variability. Moreover, the channel efficiency under WP decreases with increasing in demand variability. In the next section, we will demonstrate how the performance of channel can be altered if our sharing contracts is adopted under the SD case.

6.3. The joint impact of demand variability and sharing fraction on channel performance

Using the same procedure as in Section 4.6.1, we obtain the results of  $\gamma_m^j, \gamma_r^j, E^j$  and the optimal decision of M on CSR effort level for the SD case with two representative values of A (i.e., A = 20, A = 60) in Table 8 (for the OS contracts) and Table 9 (for the AS contracts), the computational results with A = 40 is available from the authors.

**Table 7**  
The optimal decisions under WP when  $\xi \sim U(-A, A)$ .

Demand	w	$\theta$	p	z	q	e	M's profit	R's profit	channel's profit	E
A = 0	22.624	4.746	30.218		22.781	8.543	269.572	143.796	413.368	0.715
A = 20	19.050	4.400	26.679	-8.560	33.088	8.583	206.488	114.010	320.498	0.705
A = 40	16.100	3.900	23.768	-14.180	41.360	8.627	153.632	79.614	233.246	0.698
A = 60	13.950	3.950	21.569	-17.600	47.958	8.572	107.431	40.338	147.770	0.683

6.3.1. The impact on M's profit and R's profit

Based on the computational results, we have the following observations with the SD case: (1) both R's and M's profit decrease with demand variability in all sharing contracts at any value of sharing fraction; (2) R's profit under the OS contract becomes concave in  $\alpha$  (R's profit first increases with  $\alpha$  and then decreases with  $\alpha$  when it passes a certain threshold) while R's profit under the AS contract is always increasing in  $\alpha$  for any level of demand variability. This observation implies that there exists a certain threshold at which R's profit is higher under the AS contract than under the OS contract when the sharing fraction exceeds this threshold. Thus R will have the incentive to share the production cost of unsold stock only if the negotiated  $\alpha$  is greater than this threshold; (3) M's profit always decreases with  $\alpha$  in both the OS contract and the AS contract. However, the difference between the  $\gamma_m^j$  under the AS contract and that in the OS contract (i.e.,  $\gamma_m^{jAS} - \gamma_m^{jOS}$ ) is always positive and becomes concave in  $\alpha$  (first increases in  $\alpha$ , attains the highest when the sharing fraction approaches a certain threshold, approximately 50%, and then decreases in  $\alpha$ ). This implies that M always gets benefit when the retailer undertakes the production cost of unsold products. Specifically, the AS contract is the most attractive for M if the sharing fraction is moderate (neither too high nor too low).

For a more graphical look of these observations above, we provide an example with the RPM contract in Figs. 4 and 5.

6.3.2. The impact on channel efficiency

The value of  $E^j$  in Tables 8 and 9 provides us with the following observations: (1)  $E^j$  decreases as demand variability increases. (2)  $E^j$  is always higher in the AS contract than in the OS contract. The higher the degree of the demand variability (large A) is, the greater the difference of  $E^j$  between the AS contract and the OS contract is. Furthermore, the higher the value of sharing fraction is, the greater the difference of  $E^j$  between the AS contract and the OS contract is. This result suggests that the channel will get more profit with the increase in demand variation if R shares more production costs of unsold stock with M. We provide an example of these observations above in Fig. 6. (3) The channel efficiency under the RPM AS contract is always equal to one (for all values of  $\alpha$  and A). This finding illustrates that RPM AS contract can bring the channel full coordination regardless of the value of demand variability (As we showed in Proposition 5). In contrast, the channel



**Table 8**  
Computational results of the OS contracts for the SD case.

A	$\alpha$	RP				RPM				RPC				RPMC			
		$\Upsilon_m$	$\Upsilon_r$	E	$\theta$	$\Upsilon_m$	$\Upsilon_r$	E	$\theta$	$\Upsilon_m$	$\Upsilon_r$	E	$\theta$	$\Upsilon_m$	$\Upsilon_r$	E	$\theta$
A = 20	0.01	0.71	-0.96	0.79	7.73	1.17	-0.95	1.00	9.62	0.71	-0.96	0.79	7.82	1.18	-0.96	0.99	9.73
	0.1	0.57	-0.59	0.82	7.16	0.92	-0.49	1.00	8.57	0.60	-0.63	0.82	8.06	0.95	-0.57	0.99	9.67
	0.2	0.41	-0.20	0.84	6.50	0.64	-0.03	0.99	7.45	0.46	-0.26	0.85	8.34	0.70	-0.15	0.98	9.59
	0.3	0.25	0.17	0.86	5.79	0.38	0.38	0.97	6.35	0.31	0.12	0.87	8.63	0.45	0.22	0.97	9.50
	0.4	0.08	0.52	0.87	5.05	0.13	0.74	0.95	5.30	0.15	0.49	0.89	8.93	0.21	0.56	0.94	9.38
	0.5	-0.10	0.84	0.87	4.27	-0.10	1.05	0.92	4.27	-0.02	0.84	0.90	9.22	-0.02	0.83	0.90	9.22
	0.6	-0.28	1.10	0.85	3.46	-0.32	1.28	0.88	3.29	-0.21	1.16	0.90	9.51	-0.25	1.01	0.84	9.02
	0.7	-0.47	1.29	0.82	2.60	-0.52	1.43	0.83	2.36	-0.40	1.44	0.89	9.75	-0.46	1.05	0.76	8.75
	0.8	-0.65	1.39	0.76	1.72	-0.70	1.49	0.76	1.49	-0.60	1.60	0.83	9.90	-0.66	0.92	0.63	8.39
	0.9	-0.83	1.35	0.66	0.84	-0.86	1.45	0.68	0.70	-0.80	1.60	0.74	9.87	-0.84	0.50	0.45	7.91
A = 60	0.01	-0.98	1.18	0.55	0.08	-0.99	1.31	0.59	0.06	-0.98	1.42	0.62	9.57	-0.99	0.10	0.28	7.37
	0.01	0.30	-0.92	0.66	6.54	0.97	-0.88	1.00	8.54	0.30	-0.93	0.66	6.62	0.98	-0.90	0.99	8.64
	0.1	0.11	-0.21	0.70	5.93	0.59	0.08	0.99	7.43	0.14	-0.28	0.70	6.70	0.64	-0.02	0.99	8.41
	0.2	-0.09	0.46	0.72	5.22	0.21	0.92	0.96	6.24	-0.04	0.38	0.74	6.76	0.28	0.82	0.97	8.10
	0.3	-0.29	0.97	0.72	4.47	-0.13	1.49	0.89	5.07	-0.22	0.94	0.75	6.76	-0.04	1.46	0.93	7.69
	0.4	-0.47	1.28	0.69	3.68	-0.42	1.72	0.80	3.94	-0.40	1.35	0.73	6.67	-0.33	1.83	0.86	7.15
	0.5	-0.64	1.33	0.61	2.86	-0.64	1.58	0.66	2.86	-0.57	1.52	0.68	6.42	-0.57	1.85	0.74	6.42
	0.6	-0.78	1.05	0.49	2.03	-0.80	1.08	0.49	1.87	-0.73	1.38	0.58	5.93	-0.76	1.46	0.58	5.44
	0.7	-0.88	0.51	0.34	1.25	-0.90	0.38	0.31	1.06	-0.85	0.87	0.42	5.10	-0.88	0.74	0.38	4.23
	0.8	-0.95	-0.09	0.20	0.63	-0.96	-0.19	0.17	0.50	-0.93	0.16	0.25	3.94	-0.95	0.02	0.21	3.03
0.9	-0.98	-0.50	0.10	0.23	-0.99	-0.53	0.10	0.18	-0.98	-0.40	0.12	2.83	-0.98	-0.43	0.11	2.13	
0.99	-1.00	-0.70	0.06	0.02	-1.00	-0.69	0.06	0.01	-1.00	-0.67	0.06	2.11	-1.00	-0.64	0.07	1.62	

**Table 9**  
Computational results of the AS contracts for the SD case.

A	$\alpha$	RP				RPM				RPC				RPMC			
		$\Upsilon_m$	$\Upsilon_r$	E	$\theta$	$\Upsilon_m$	$\Upsilon_r$	E	$\theta$	$\Upsilon_m$	$\Upsilon_r$	E	$\theta$	$\Upsilon_m$	$\Upsilon_r$	E	$\theta$
A = 20	0.01	0.71	-0.96	0.79	7.74	1.18	-0.95	1.00	9.62	0.72	-0.97	0.79	7.83	1.18	-0.96	1.00	9.73
	0.1	0.60	-0.64	0.82	7.22	0.95	-0.55	1.00	8.63	0.63	-0.68	0.82	8.12	0.98	-0.60	1.00	9.73
	0.2	0.47	-0.29	0.85	6.60	0.71	-0.13	0.99	7.56	0.51	-0.35	0.85	8.48	0.76	-0.20	1.00	9.73
	0.3	0.33	0.06	0.87	5.95	0.47	0.27	0.98	6.52	0.39	0.00	0.88	8.86	0.54	0.20	1.00	9.73
	0.4	0.18	0.41	0.89	5.26	0.24	0.64	0.97	5.51	0.25	0.36	0.91	9.28	0.32	0.60	1.00	9.73
	0.5	0.02	0.75	0.90	4.52	0.02	0.98	0.96	4.52	0.10	0.73	0.93	9.73	0.10	0.99	1.00	9.73
	0.6	-0.16	1.08	0.90	3.74	-0.20	1.31	0.94	3.57	-0.07	1.10	0.95	10.23	-0.12	1.39	1.00	9.73
	0.7	-0.34	1.39	0.90	2.90	-0.41	1.61	0.92	2.64	-0.26	1.48	0.96	10.78	-0.34	1.79	1.00	9.73
	0.8	-0.55	1.67	0.88	2.00	-0.61	1.90	0.90	1.74	-0.48	1.85	0.95	11.40	-0.56	2.19	1.00	9.73
	0.9	-0.76	1.92	0.84	1.04	-0.81	2.16	0.88	0.86	-0.72	2.21	0.93	12.08	-0.78	2.59	1.00	9.73
A = 60	0.01	-0.98	1.18	0.55	0.08	-0.98	1.31	0.59	0.06	-0.97	1.42	0.62	9.57	-0.98	0.10	0.28	7.37
	0.01	0.31	-0.96	0.66	6.56	0.99	-0.93	1.00	8.56	0.32	-0.96	0.66	6.64	1.00	-0.95	1.00	8.66
	0.1	0.26	-0.56	0.71	6.16	0.76	-0.35	1.00	7.66	0.29	-0.63	0.71	6.95	0.81	-0.46	1.00	8.66
	0.2	0.18	-0.12	0.75	5.68	0.53	0.24	0.99	6.69	0.24	-0.24	0.76	7.33	0.61	0.07	1.00	8.66
	0.3	0.10	0.32	0.79	5.15	0.30	0.76	0.97	5.75	0.18	0.18	0.81	7.74	0.41	0.61	1.00	8.66
	0.4	-0.01	0.74	0.82	4.58	0.08	1.24	0.95	4.84	0.11	0.62	0.85	8.18	0.21	1.15	1.00	8.66
	0.5	-0.12	1.15	0.84	3.97	-0.12	1.65	0.93	3.97	0.01	1.08	0.89	8.66	0.01	1.68	1.00	8.66
	0.6	-0.25	1.52	0.84	3.30	-0.31	2.03	0.90	3.12	-0.12	1.55	0.91	9.19	-0.19	2.22	1.00	8.66
	0.7	-0.40	1.85	0.83	2.57	-0.50	2.36	0.87	2.30	-0.27	2.02	0.92	9.77	-0.40	2.76	1.00	8.66
	0.8	-0.58	2.12	0.79	1.79	-0.67	2.64	0.84	1.51	-0.47	2.47	0.91	10.41	-0.60	3.29	1.00	8.66
0.9	-0.77	2.30	0.73	0.93	-0.84	2.89	0.80	0.74	-0.71	2.85	0.86	11.12	-0.80	3.83	1.00	8.66	
0.99	-0.98	2.37	0.64	0.10	-0.98	3.09	0.77	0.07	-0.97	3.12	0.78	11.83	-0.98	4.31	1.00	8.66	

efficiency of the RPMC OS contract is less than one for all positive values of A. This finding also implies that the RPMC OS contract does not coordinate the channel under demand uncertainty. The investigation of channel efficiency does not allow the question of whether the sharing contract is attractive for M and R to be answered. Therefore, in the following section, we will show that whether the sharing contracts proposed can bring the channel to Pareto improvement when the demand is uncertain.

6.4. The Pareto improvements and CSR-performance

Because we cannot describe closed form solutions for the Pareto-improving region of  $\alpha$  in the SD case, we highlight the values of  $\Upsilon_m^j, \Upsilon_r^j$  corresponding to the value of  $\alpha$  which guarantees both  $\Upsilon_m^j > 0, \Upsilon_r^j > 0$  in Tables 8 and 9. We find that the Pareto-improving region always exists with the AS contracts at any levels of demand variability. Namely, when  $A = 20$  then  $\Upsilon_m^j > 0, \Upsilon_r^j > 0$  for  $j = RP, RPM, RPC$  and RPMC at  $\alpha = 0.3, 0.4$  and  $0.5$ ; when  $A = 40$  then  $\Upsilon_m^{RP} > 0, \Upsilon_r^{RP} > 0$  at

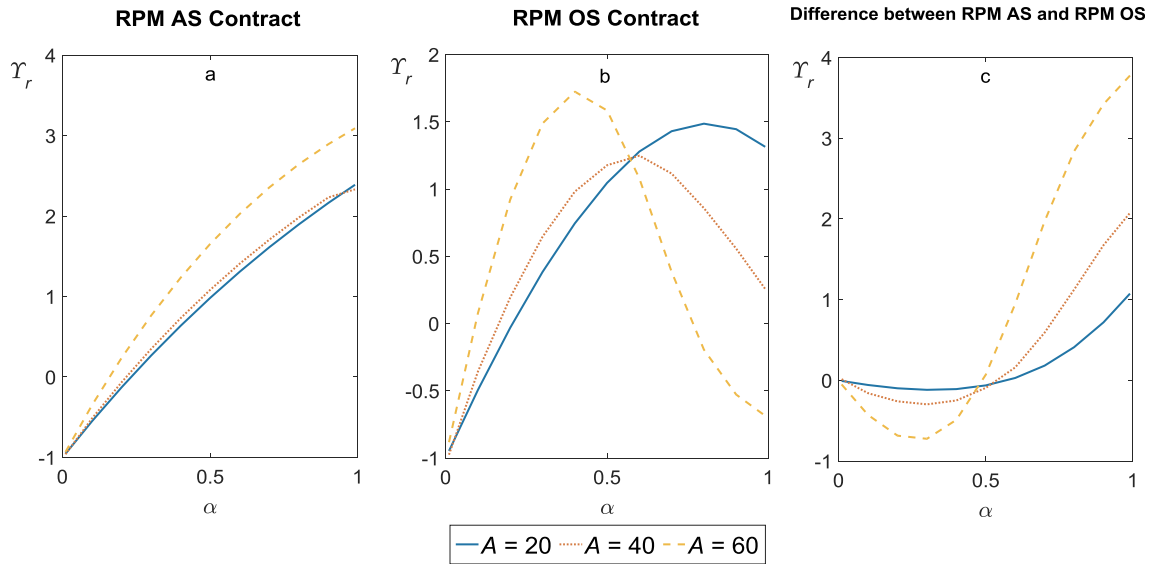


Fig. 4. (a)  $\gamma_r^{RPM AS}$ , (b)  $\gamma_r^{RPM OS}$ , (c)  $\gamma_r^{RPM AS} - \gamma_r^{RPM OS}$  changes with  $A$  and  $\alpha$ .

$\alpha = 0.3$  and  $0.4$ ,  $\gamma_m^{RPM} > 0$ ,  $\gamma_r^{RPM} > 0$  at  $\alpha = 0.3$  and  $0.4$ ,  $\gamma_m^{RPC} > 0$ ,  $\gamma_r^{RPC} > 0$  at  $\alpha = 0.3, 0.4$  and  $0.5$ ; when  $A = 60$  then  $\gamma_m^{RP} > 0$ ,  $\gamma_r^{RP} > 0$  at  $\alpha = 0.3$ ;  $\gamma_m^{RPM} > 0$ ,  $\gamma_r^{RPM} > 0$  at  $\alpha = 0.2, 0.3$  and  $0.4$ ;  $\gamma_m^{RPC} > 0$ ,  $\gamma_r^{RPC} > 0$  at  $\alpha = 0.3, 0.4$  and  $0.5$ ;  $\gamma_m^{RPMC} > 0$ ,  $\gamma_r^{RPMC} > 0$  at  $\alpha = 0.2, 0.3, 0.4$  and  $0.5$ . In contrast, the OS contract is not always a Pareto-improving solution when the demand variability is relatively large. Namely, when  $A = 20$ , all four OS contracts guarantee a Pareto improvement at  $\alpha = 0.2$  and  $0.3$ ; when  $A = 40$  then  $\gamma_m^{RP} > 0$ ,  $\gamma_r^{RP} > 0$  at  $\alpha = 0.3$ ,  $\gamma_m^{RPM} > 0$ ,  $\gamma_r^{RPM} > 0$  at  $\alpha = 0.2$  and  $0.3$ ,  $\gamma_m^{RPC} > 0$ ,  $\gamma_r^{RPC} > 0$  at  $\alpha = 0.3, 0.4$  and  $0.5$ ;  $\gamma_m^{RPMC} > 0$ ,  $\gamma_r^{RPMC} > 0$  at  $\alpha = 0.2, 0.3, 0.4$  and  $0.5$ . However, when the demand is extremely uncertain, ( $A = 60$ ), we can't find the value of  $\alpha$  creating Pareto-improving solutions under the RP OS contract and the RPM OS contract. Regarding the CSR performance, we also observed that when the sharing fraction was chosen to lead to a win-win outcome, the CSR performance under the sharing contract is higher than those under WP. For more insight of the movement of the Pareto-improving region with

the demand variability, we note that the labels of  $\alpha^j$  are intersection points between  $\gamma_r^j$  and horizontal axis (if it has two points, the smaller of the two represents  $\alpha^j$ ) in Fig. 7. Similarly, we note that the labels of  $\bar{\alpha}^j$  are the intersection points between  $\gamma_m^j$  and the horizontal axis in Fig. 8. From the observation of  $\alpha^j$  and  $\bar{\alpha}^j$ , the lower and upper bounds of the Pareto-improving region move in the same directions. That is, the Pareto-improving region under the sharing contract  $j$  ( $\alpha^j, \bar{\alpha}^j$ ) shifts to the left as demand uncertainty increases. However, the Pareto-improving region under the AS contracts shifts to the left and is relatively small and less than that under the OS contracts.

7. Conclusion and perspectives

In this paper, we studied the channel coordination and cooperation issues when the channel members make different decisions to promote the market demand including marketing and CSR efforts. We study a two-echelon channel in the condition that the market demand is

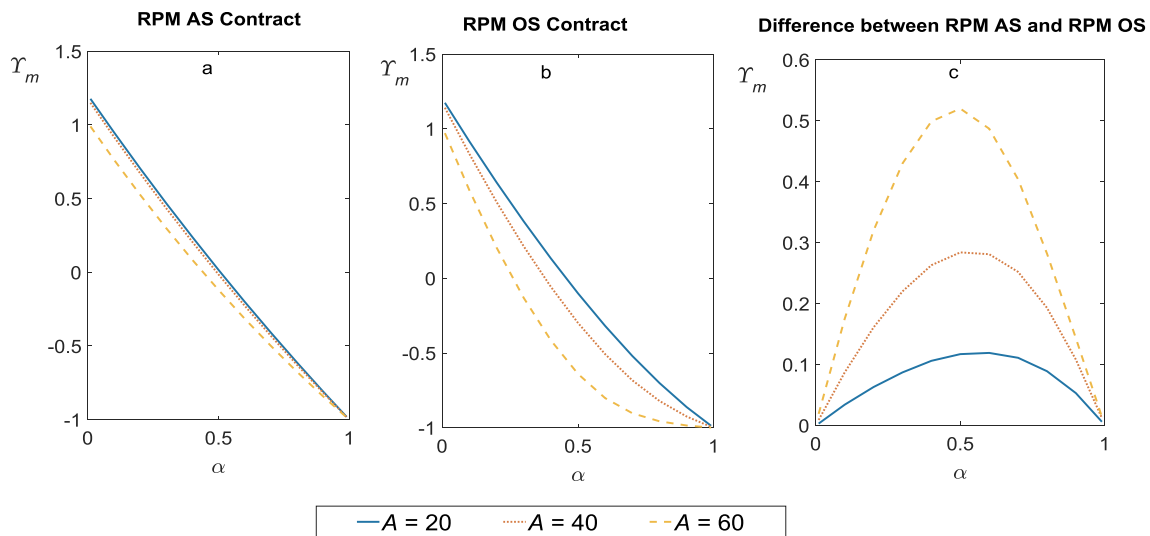


Fig. 5. (a)  $\gamma_m^{RPM AS}$ , (b)  $\gamma_m^{RPM OS}$ , (c)  $\gamma_m^{RPM AS} - \gamma_m^{RPM OS}$  changes with  $A$  and  $\alpha$ .

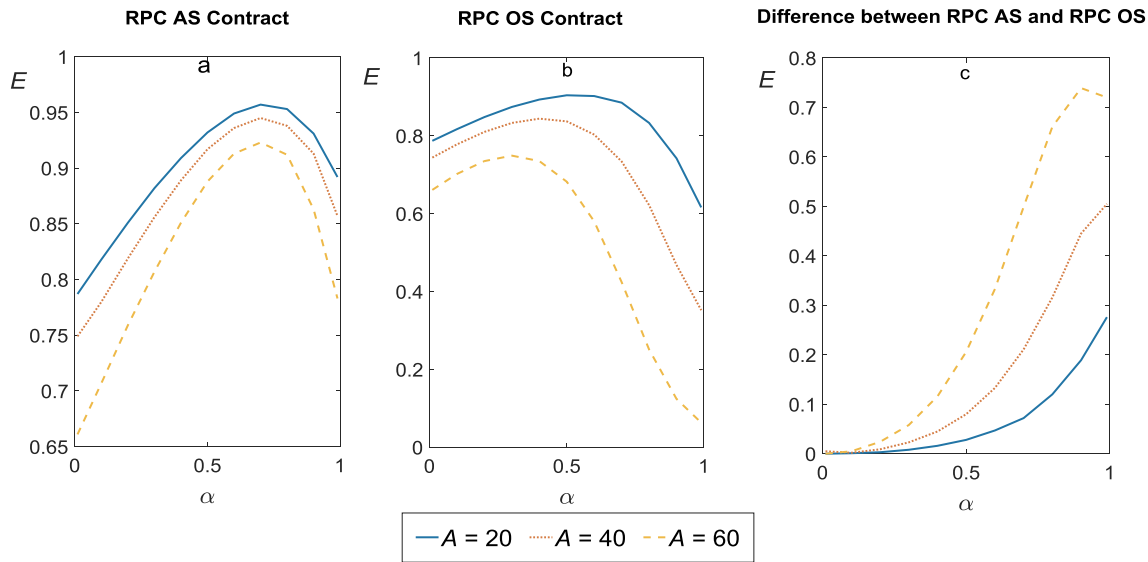


Fig. 6. (a)  $E^{RPC AS}$ , (b)  $E^{RPC OS}$ , (c)  $E^{RPC AS} - E^{RPC OS}$ , changes with  $A$  and  $\alpha$ .

stochastic and affected by R's marketing effort, retail price and M's CSR effort. These complex settings represent a realistic business practices for firms dealing with channel coordination issues for the different decisions impacting on the channel performance including not only the operational choices (quantity, price) and marketing strategy of the firms but also the sustainable channel management. Therefore, we integrate M's CSR and R's marketing efforts into the coordination of VMI-CC and propose the coordination schemes for the chain through the combination of revenue sharing and cost sharing contracts. Under some assumptions on demand and cost function, we prove that our proposed sharing contracts can simultaneously achieve the following objectives: (1) improve CSR performance; (2) increase total channel profits; (3) ensure that each partner in the channel can benefit from the proposed contracts. Therefore, the coordination between M and R in a VMI-CC via sharing contracts may lead to both higher profit and higher CSR performance. Furthermore, we found that the channel can be

coordinated perfectly regardless of demand uncertainty only if R and M share all of the marketing cost, CSR investment cost and production cost of the entire consignment stock.

From managerial insights, our research helps decision-makers to propose the different coordination strategies to improve the channel performance in the short and long term. In terms of the short-term contracts, our findings suggest that the channel's members can avoid profit loss by adopting a VMI-CC with a revenue and cost sharing agreement. In the long term, many sources can lead to instability in product demand such as new products and changes in technological and economic conditions; this uncertainty may have to be taken into consideration when making strategic decisions. Our results show that in order to attract the highest profit under demand uncertainty, the channel's members should share all channel costs including the production costs of unsold stocks. Furthermore, CSR implementation in the channel is indispensable to achieve sustainable development and the

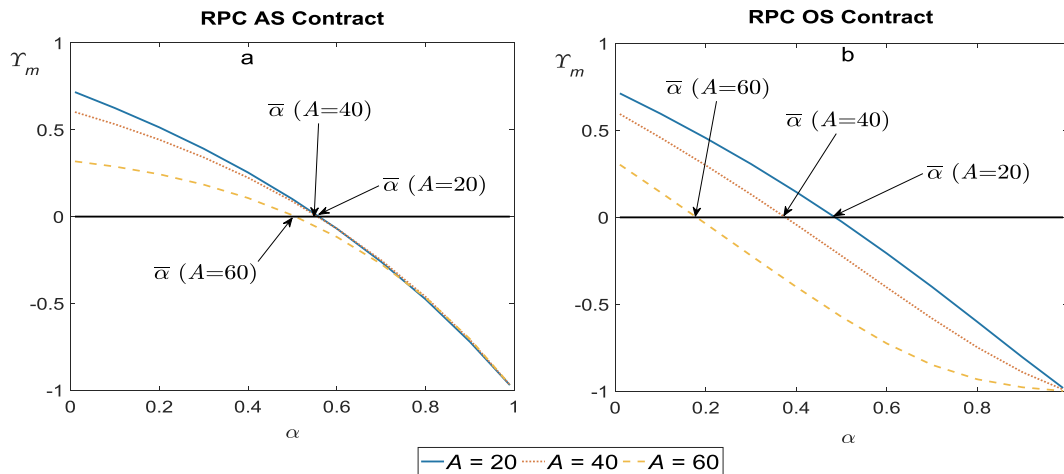


Fig. 7. The upper bound of Pareto-improving region in the RPC AS contract (a) and the RPC OS contract (b).

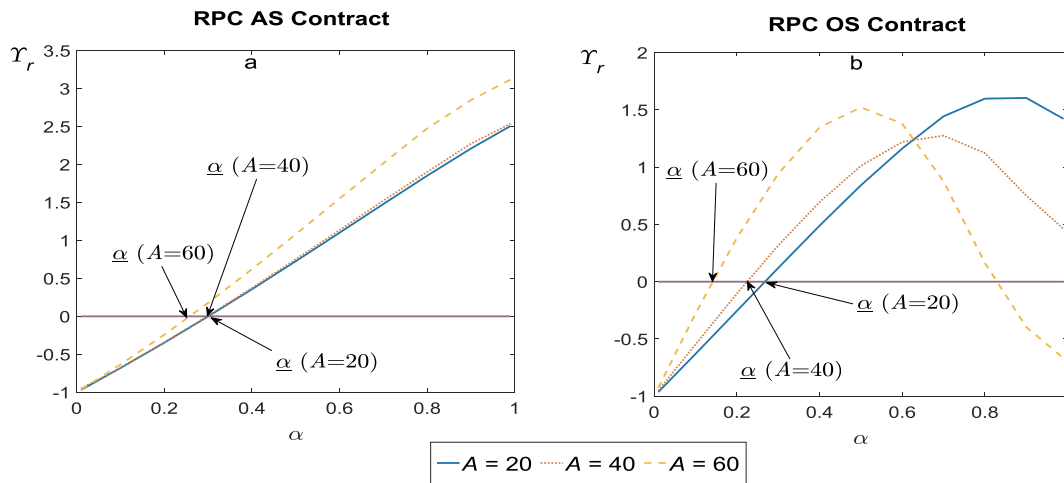


Fig. 8. The lower bound of Pareto-improving region in the RPC AS contract (a) and the RPC OS contract (b).

incentives are important factors in maintaining a long-term relationship of channel CSR collaboration. With our sharing contracts, the channel can increase CSR implementation to meet the expectation of consumers while optimizing their economic performances.

Finally, our current model can potentially be extended in some directions. First, that would be more interesting if the mutual impact of marketing and CSR efforts on the consumers' behavior is taken into consideration. In fact, the potential customer value creation of CSR activities (i.e., self-benefit value and societal benefit value) depends not only on the CSR investment cost but also on type of product and the interactions between firms and customers through marketing strategy. Therefore, future research can examine the case that the marginal effect of CSR effort on demand depends on the marketing strategy (i.e., advertising effort level and advertising appeals) and/or type of product.

Second, since the capital constraints are a great challenge for many firms in the real world, as another opportunity for future investigation to delve into our sharing contracts with a VMI-CC where the capital constraints exist. Last but not least researchers can explore the cases of

asymmetric information. For example, the cost of CSR efforts is the private information of M whereas R can be more knowledgeable about the cost of exerting marketing effort and the demand. These complexities will affect the strategic interaction between the two firms and how to coordinate a VMI-CC with the combination of Revenue - Cost and Information sharing in order to achieve the best performance of the channel will become an important and attractive issue.

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**Appendix A. Proof of Tables**

We only provide Proof for RP contract, the proofs for other contracts are the same logic.

*A1. Proof of Tables 1 and 2*

The results in Table 1 and Table 2 for the cases of centralized channel and wholesale price contract are the same as those in the paper of P. Ma et al. (2013). We provide only proofs for the sharing contract (The proofs for the sharing contracts RPM, RPC, RPMC is conducted according to the same process. We omit the details of proofs). By backward sequential decision-making approach, we first analyze the optimal decision of the retailer, and then derive the equilibrium strategies of the manufacturer. When the demand is deterministic, the retailer's profit under the RP contract can be expressed as  $\Pi_r^{RP} = (p - c)\alpha(a - bp + \gamma e + \lambda\theta) - \frac{\eta e^2}{2}$ . The first order derivatives of  $\Pi_r^{RP}$  with respect to  $e$  and set it to zero yield  $e = \frac{(p - c)\alpha\gamma}{\eta}$ . Substituting  $e$  into the Manufacturer's profit function, i.e.,  $\Pi_m^{RP} = (p - c)(1 - \alpha)(a - bp + \gamma e + \lambda\theta) - \frac{\theta^2\kappa}{2}$ . We have,

$\Pi_m^{RP} = (p - c)(1 - \alpha)\left(a - bp + \frac{(p - c)\alpha\gamma^2}{\eta} + \lambda\theta\right) - \frac{\theta^2\kappa}{2}$ . The first order derivatives of  $\Pi_m^{RP}$  with respect to  $p$  and  $\theta$  as follows:

$$\frac{\partial \Pi_m^{RP}}{\partial p} = (p - c)(1 - \alpha)\left(\frac{\alpha\gamma^2}{\eta} - b\right) + (1 - \alpha)\left(a - bp + \frac{(p - c)\alpha\gamma^2}{\eta} + \lambda\theta\right).$$

$$\frac{\partial \Pi_m^{RP}}{\partial \theta} = (p - c)(1 - \alpha)\lambda - \theta\kappa.$$

The Hessian matrix of  $\Pi_m^{RP}$  is  $\begin{pmatrix} \frac{\partial^2 \Pi_m^{RP}}{\partial p^2} & \frac{\partial^2 \Pi_m^{RP}}{\partial p \partial \theta} \\ \frac{\partial^2 \Pi_m^{RP}}{\partial \theta \partial p} & \frac{\partial^2 \Pi_m^{RP}}{\partial \theta^2} \end{pmatrix} = \begin{pmatrix} 2(1 - \alpha)\left(\frac{\alpha\gamma^2}{\eta} - b\right) & (1 - \alpha)\lambda \\ (1 - \alpha)\lambda & -\kappa \end{pmatrix}$ .

The Hessian matrix of  $\Pi_m^{RP}$  is a negative definite for all values of  $p$  and  $\theta$  if  $\eta > 0$  and  $2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2 < 0$ .

Solving  $\frac{\partial \Pi_m^{RP}}{\partial p} = 0$  and  $\frac{\partial \Pi_m^{RP}}{\partial \theta} = 0$  yields:



$$p = -\frac{-2c\alpha\gamma^2\kappa + a\eta\kappa + bc\eta\kappa - c\eta\lambda^2 + c\alpha\eta\lambda^2}{2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2}, \theta = -\frac{a\eta\lambda - bc\eta\lambda - a\alpha\eta\lambda + bc\alpha\eta\lambda}{2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2}$$

After simplification, we have  $p = c + \frac{(a - bc)\eta\kappa}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$  and  $\theta = \frac{(a - bc)(1 - \alpha)\eta\lambda}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$ .

By substituting  $p$ , the marketing effort level becomes  $e = \frac{(a - bc)\gamma\kappa}{4b\eta\kappa - 2\gamma^2\kappa - \eta\lambda^2}$ .

Substituting  $p$ ,  $\theta$  and  $e$  into the profit function of retailer, manufacturer and channel, we obtain

$$\Pi_r^{RP} = \frac{(a - bc)^2\alpha\eta(2b\eta - 3\alpha\gamma^2)\kappa^2}{2(2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2)^2};$$

$$\Pi_m^{RP} = \frac{(a - bc)^2(1 - \alpha)\eta\kappa}{-4\alpha\gamma^2\kappa + 4b\eta\kappa - 2\eta\lambda^2 + 2\alpha\eta\lambda^2};$$

$$\text{and } \Pi_c^{RP} = \frac{(a - bc)^2\eta\kappa((2b\eta\kappa - \eta\lambda^2) - 2\alpha(\gamma^2\kappa - \eta\lambda^2) - \alpha^2(\gamma^2\kappa + \eta\lambda^2))}{2(2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2)^2}.$$

**A2. Proof of Tables 4 and 5**

When the demand is stochastic, the retailer's expected profit under the RP contract can be expressed as

$$E[\Pi_r^{RP}] = (p - c)\alpha(a - bp + e\gamma + \theta\lambda + \Lambda[z]) - \frac{e^2\eta}{2}.$$

Taking the first derivative of  $E[\Pi_r^{RP}]$  with respect to  $e$  and making them equal to zero, yield

$$e = \frac{(p - c)\alpha\gamma}{\eta}.$$

By plugging  $e$  into M's expected profit, we obtain

$$E[\Pi_m^{RP}] = (p - c)\left(1 - \alpha\right)\left(a - bp + \frac{(p - c)\alpha\gamma^2}{\eta} + \theta\lambda + \Lambda[z]\right) - c(z - \Lambda[z]) - \frac{\theta^2\kappa}{2}.$$

Taking the first derivative of  $E[\Pi_m^{RP}]$  with respect to  $p$  and  $\theta$ :

$$\partial E[\Pi_m^{RP}]/\partial p = (p - c)(1 - \alpha)\left(a - bp + \frac{(p - c)\alpha\gamma^2}{\eta} + \theta\lambda + \Lambda[z]\right) - c(z - \Lambda[z]) - \frac{\theta^2\kappa}{2}$$

$$\text{and } \frac{\partial E[\Pi_m^{RP}]}{\partial \theta} = (p - c)(1 - \alpha)\lambda - \theta\kappa.$$

Solving the first order optimality condition  $\frac{\partial E[\Pi_m^{RP}]}{\partial p} = \frac{\partial E[\Pi_m^{RP}]}{\partial \theta} = 0$ , implies that

$$p_{RP}^S = \frac{-2c\alpha\gamma^2\kappa + a\eta\kappa + bc\eta\kappa - c\eta\lambda^2 + c\alpha\eta\lambda^2 + \eta\kappa\Lambda[z]}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2},$$

$$\theta_{RP}^S = \frac{(-1 + \alpha)\eta\lambda(-a + bc - \Lambda[z])}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}.$$

Substituting the optimal  $p_{RP}^S$  into the optimal marketing effort level, then

$$e_{RP}^S = -\frac{\alpha\gamma\kappa(a - bc + \Lambda[z])}{2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2}.$$

Recall that  $p_{RP}^D = \frac{-2c\alpha\gamma^2\kappa + a\eta\kappa + bc\eta\kappa - c\eta\lambda^2 + c\alpha\eta\lambda^2}{2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2}$ ,  $\theta_{RP}^D = \frac{(a - bc)(1 - \alpha)\eta\lambda}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$  and  $e_{RP}^D = \frac{(a - bc)\alpha\gamma\kappa}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$

After rearranging, we obtain

$$p_{RP}^S - p_{RP}^D = \frac{\eta\kappa\Lambda[z]}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2},$$

$$\theta_{RP}^S - \theta_{RP}^D = \frac{(1 - \alpha)\eta\lambda\Lambda[z]}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2},$$

$$e_{RP}^S - e_{RP}^D = \frac{\alpha\gamma\kappa\Lambda[z]}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2},$$

Substituting the optimal  $p_{RP}^S$ ,  $\theta_{RP}^S$  into the Manufacturer's profit function and taking the first derivative of  $E[\Pi_m^{RP}]$  with respect to  $z$  we have

$$\frac{\partial E[\Pi_m^{RP}]}{\partial z} = \frac{(a(\alpha - 1)\eta\kappa - c(-2\alpha\gamma^2\kappa + b\eta\kappa + bc\alpha\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2) + (\alpha - 1)\eta\kappa\Lambda[z])\Lambda'[z]}{2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2} - c$$

By setting  $\frac{\partial E[\Pi_m^{RP}]}{\partial z} = 0$ , the optimal stocking factor of inventory  $z \in (A, B)$  satisfying

$$\Lambda'[z] = \frac{2c\alpha\gamma^2\kappa - 2bc\eta\kappa + c\eta\lambda^2 - c\alpha\eta\lambda^2}{2\alpha\gamma^2\kappa - a\eta\kappa - bc\eta\kappa + a\alpha\eta\kappa - bc\alpha\eta\kappa + c\eta\lambda^2 - c\alpha\eta\lambda^2 - \eta\kappa\Lambda[z] + \alpha\eta\kappa\Lambda[z]}.$$

After rearranging, we obtain.  $\frac{1}{\Lambda'[z]} - 1 = \frac{(1 - \alpha)\eta\kappa(a - bc + \Lambda[z])}{c(-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2)}$ .

**Appendix B. Analytical results for channel performance in the DD**

*B1. Analyze the impact of  $\alpha$  on the channel member's profit*

From the result in Table 2,  $\gamma_i^j$  is a function of  $\alpha$ . Moreover, the Manufacturer's profit and retailer's profit are independent of  $\alpha$  under wholesale price contract. Thus, the impact of  $\alpha$  on the Manufacturer's profit (the retailer's profit) under the sharing contract  $j$  is the same impact of  $\alpha$  on  $\gamma_m^j$  ( $\gamma_r^j$ ). By examining the sign of the functions  $\phi_i^j = \partial \gamma_i^j / \partial \alpha$  with the conditions of  $\Pi_i^j > 0$ , we summarize the impact of  $\alpha$  on the profit of the manufacturer and the retailer in each sharing contract through properties 1–8 as follows:

**Property 1.** *In the RP contract,*

- (1)  $\gamma_m^{RP} = \frac{2(2\alpha - 1)(\gamma^2 - b\eta)\kappa}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2}$ ;  $\phi_m^{RP} = -\frac{2(\gamma^2 - b\eta)\kappa(2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)}{(2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2)^2}$ ,
- (2) When  $\frac{\gamma^2}{2\eta} < b < \frac{\gamma^2}{\eta}$  then  $\phi_m^{RP} > 0$  if  $0 < \alpha < \frac{2b\eta}{3\gamma^2}$ ,  $\gamma_m^{RP} > 0$  if  $1/2 < \alpha < \frac{2b\eta}{3\gamma^2}$ ,
- (3) When  $b > \frac{\gamma^2}{\eta}$  then  $\phi_m^{RP} < 0$  if  $0 < \alpha < 1$ ,  $\gamma_m^{RP} > 0$  if  $0 < \alpha < 1/2$ .

Property 1 shows that when  $b > \frac{\gamma^2}{\eta}$  the manufacturer's profit decreases in  $\alpha$  for any  $\alpha$  in the range of (0,1) and the manufacturer prefers the RP contract to wholesale price contract if the sharing fraction is less than 50%. This also implies that if the manufacturer has more the contractual power than the retailer, he will choose a sharing fraction approach to zero to attain the highest profit. In this case, the manufacturer's profit increases at most  $\gamma_m^{RP} = \frac{2(b\eta - \gamma^2)\kappa}{2b\eta\kappa - \eta\lambda^2} > 0$  compared with that of the wholesale price. Conversely, when  $b < \frac{\gamma^2}{\eta}$  the manufacturer's profit increases in  $\alpha$  for any  $\alpha$  in the range of  $(0, \frac{2b\eta}{3\gamma^2})$ . Thus, the manufacturer choose  $\alpha$  approach to  $\frac{2b\eta}{3\gamma^2}$  to attain the highest profit. However, the manufacturer prefers the RP contract to wholesale price contract only if the sharing fraction is higher than 50% in this situation.

**Property 2.** *In the RPM contract,*

- (1)  $\gamma_m^{RPM} = \frac{(-1 + 2\alpha)(\gamma^2 - 2b\eta)\kappa}{-\gamma^2\kappa + \eta(2b\kappa + (-1 + \alpha)\lambda^2)}$ ;  $\phi_m^{RPM} = -\frac{(\gamma^2 - 2b\eta)\kappa(2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)}{(\gamma^2\kappa + \eta(-2b\kappa - (-1 + \alpha)\lambda^2))^2}$ .
- (2) When  $b > \frac{\gamma^2}{2\eta}$  and  $\kappa > \bar{\kappa}$  then  $\phi_m^{RPM} < 0$  for any  $0 < \alpha < 1$  and  $\gamma_m^{RPM} > 0$  if  $0 < \alpha < 1/2$ .

Property 2 shows that the profit of the manufacturer always decreases in  $\alpha$  under the RPM contract and the manufacturer prefers the RPM contract to wholesale price contract if the sharing fraction is less than 50%. Furthermore, the Manufacturer's profit in the RPM contract always decreases with  $\alpha$ . Therefore, the manufacturer attains the highest profit if the sharing fraction approach to zero. In this case, the manufacturer's profit increases by at most  $\gamma_m^{RPM} = \frac{(\gamma^2 - 2b\eta)\kappa}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2} > 0$  versus the wholesale price contract.

**Property 3.** *In the RPC contract,*

- (1)  $\gamma_m^{RPC} = \frac{2(2\alpha - 1)\gamma^2\kappa + \eta(b(2 - 4\alpha)\kappa + \alpha\lambda^2)}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$ ;  $\phi_m^{RPC} = \frac{-4\gamma^4\kappa^2 + 4\gamma^2\eta\kappa(3b\kappa - \lambda^2) - \eta^2(8b^2\kappa^2 - 6b\kappa\lambda^2 + \lambda^4)}{(2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2)^2}$ .
- (2) When  $b > \frac{\lambda^2}{2\kappa}$  and  $\frac{\gamma^2\kappa}{2b\kappa - \lambda^2} < \eta < \frac{2\gamma^2\kappa}{2b\kappa - \lambda^2}$  then  $\phi_m^{RPC} > 0$  if  $0 < \alpha < \frac{2b\eta\kappa - \eta\lambda^2}{3\gamma^2\kappa}$ .
- (3) When  $b > \frac{\lambda^2}{2\kappa}$  and  $\frac{2\gamma^2\kappa}{2b\kappa - \lambda^2} < \eta \leq \frac{3\gamma^2\kappa}{2b\kappa - \lambda^2}$  then  $\phi_m^{RPC} < 0$  if  $0 < \alpha < \frac{2b\eta\kappa - \eta\lambda^2}{3\gamma^2\kappa}$ .
- (4) When  $b > \frac{\lambda^2}{2\kappa}$  and  $\eta > \frac{3\gamma^2\kappa}{2b\kappa - \lambda^2}$  then  $\phi_m^{RPC} < 0$  if  $0 < \alpha < 1$ .

From the Property 3, it's seen that the optimal sharing fraction which maximizes the Manufacturer's profit in the RPC contract depends on the value of the coefficient of marketing effort cost. When  $\eta$  is higher than a threshold level, i.e.,  $\eta = \frac{3\gamma^2\kappa}{2b\kappa - \lambda^2}$  the smaller the selection of  $\alpha$ , the more profits the manufacturer gets. This implies that the manufacturer attains the highest profit if the sharing fraction approach to zero. Otherwise, the manufacturer should raise the sharing fraction  $\alpha$  approach to  $\frac{2b\eta\kappa - \eta\lambda^2}{3\gamma^2\kappa}$  to attract the highest profit.

**Property 4.** *In the RPMC contract,*

- (1)  $\gamma_m^{RPMC} = \frac{(2\alpha - 1)\gamma^2\kappa + \eta(b(2 - 4\alpha)\kappa + \alpha\lambda^2)}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$ ;  $\phi_m^{RPMC} = \frac{2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$ .
- (2) When  $b > \frac{\gamma^2}{2\eta}$  and  $\kappa > \bar{\kappa}$  then  $\phi_m^{RPMC} < 0$  if  $0 < \alpha < 1$ .

Property 4 show that the Manufacturer's profit in the RPMC contract always decreases in  $\alpha$ . Therefore, the manufacturer attains the highest profit if the sharing fraction approach to zero.

We next show the impacts of the value of  $\alpha$  on retailer's profit through properties 5-8 as follows

**Property 5.** *In the RP contract, let  $\alpha_r^{RP} = \frac{2b^2\eta\kappa - b\eta\lambda^2}{4b\gamma^2\kappa - 3\gamma^2\lambda^2 + b\eta\lambda^2}$ ,  $\hat{\kappa} = \frac{3\gamma^2\lambda^2 - 2b\eta\lambda^2}{4b\gamma^2 - 2b^2\eta}$ , we have*

- (1)  $\gamma_r^{RP} = \frac{(2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)^2}{2b\eta - \gamma^2} \left( \frac{\alpha(2b\eta - 3\alpha\gamma^2)}{(2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2)^2} + \frac{\gamma^2 - 2b\eta}{(2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)^2} \right)$
- (2)  $\phi_r^{RP} = \frac{2\eta(2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)^2(2b^2\eta\kappa + 3\alpha\gamma^2\lambda^2 - b(4\alpha\gamma^2\kappa + \eta\lambda^2 + \alpha\eta\lambda^2))}{(\gamma^2 - 2b\eta)(2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2)^3}$
- (3) When  $\frac{\gamma^2}{2\eta} < b \leq \frac{\gamma^2}{\eta}$  and  $\kappa > \hat{\kappa}$  then  $\phi_r^{RP} > 0$  if  $0 < \alpha < \alpha_r^{RP}$ ,  $\phi_r^{RP} < 0$  if  $\alpha_r^{RP} < \alpha < 1$ ,

- (4) When  $\frac{\gamma^2}{\eta} < b \leq \frac{2\gamma^2}{\eta}$  and  $\kappa > \hat{\kappa}$  then  $\phi_r^{RP} > 0$  if  $0 < \alpha < \alpha_r^{RP}$ ;  $\phi_r^{RP} < 0$  if  $\alpha_r^{RP} < \alpha < \frac{2b\eta}{3\gamma^2}$ ,
- (5) When  $\frac{2\gamma^2}{\eta} < b < \frac{3\gamma^2}{\eta}$  and  $\underline{\kappa} < \kappa \leq \hat{\kappa}$  then  $\phi_r^{RP} > 0$  if  $0 < \alpha < \alpha_r^{RP}$ ;  $\phi_r^{RP} < 0$  if  $\alpha_r^{RP} < \alpha < 1$ ,
- (6) When  $b > \frac{2\gamma^2}{\eta}$  and  $\kappa > \hat{\kappa}$  then  $\phi_r^{RP} > 0$  if  $0 < \alpha < 1$ ,
- (7) When  $b \geq \frac{3\gamma^2}{\eta}$  and  $\underline{\kappa} < \kappa < \hat{\kappa}$  then  $\phi_r^{RP} > 0$  if  $0 < \alpha < \alpha_r^{RP}$ ;  $\phi_r^{RP} < 0$  if  $\alpha_r^{RP} < \alpha < 1$ .

Property 5 shows that the optimal sharing fraction which maximizes the retailer’s profit in the RP contract depends on the value of the coefficient of CSR effort cost. As the coefficient of CSR effort cost is higher than a threshold level, i.e.  $\kappa > \hat{\kappa}$ , the retailer increases the sharing fraction approach to one to attract the highest profit. This suggest that the retailer should incur most of the production costs and extract most of the channel sales to maximize her profit. On the other hand, the retailer attains the highest profit if the sharing fraction approach to  $\alpha_r^{RP}$ .

**Property 6.** In the RPM contract, let  $\alpha_r^{RPM} = \frac{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}{\eta\lambda^2}$ , we have

- (1)  $\gamma_r^{RPM} = \frac{(4\alpha - 1)\gamma^4\kappa^2 - 2\gamma^2\eta\kappa(2b(4\alpha - 1)\kappa + (1 - 3\alpha)\lambda^2) + \eta^2(4b^2(4\alpha - 1)\kappa^2 + 4b(1 - 3\alpha)\kappa\lambda^2 - (1 - 3\alpha + \alpha^2)\lambda^4)}{(\gamma^2\kappa + \eta(-2b\kappa - (-1 + \alpha)\lambda^2))^2}$ ,
- (2)  $\phi_r^{RPM} = \frac{(2\gamma^2\kappa + \eta(-4b\kappa + \lambda^2))^2(\gamma^2\kappa + \eta(-2b\kappa + (1 + \alpha)\lambda^2))}{(\gamma^2\kappa + \eta(-2b\kappa - (-1 + \alpha)\lambda^2))^3}$ ,
- (3) When  $b > \frac{\gamma^2}{2\eta}$  and  $\underline{\kappa} < \kappa \leq 2\underline{\kappa}$  then  $\phi_r^{RPM} > 0$  if  $0 < \alpha < \alpha_r^{RPM}$ ;  $\phi_r^{RPM} < 0$ , if  $\alpha_r^{RPM} < \alpha < 1$ ,
- (4) When  $b > \frac{\gamma^2}{2\eta}$  and  $\kappa > 2\underline{\kappa}$  then  $\phi_r^{RPM} > 0$  if  $0 < \alpha < 1$ .

Property 6 indicates that the optimal sharing fraction which maximizes the retailer’s profit in the RPM contract depends on the value of the coefficient of CSR effort cost. As the coefficient of CSR effort cost is higher than a threshold level, i.e.  $\kappa = 2\underline{\kappa}$ , the retailer increases the value of  $\alpha$  approach to one attract the highest profit. Conversely, the retailer is raising the fraction of  $\alpha$  approach to  $\alpha_r^{RPM}$  to maximize her profit.

**Property 7.** In the RPC contract,

- (1)  $\gamma_r^{RPC} = -\frac{(2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)^2}{(\gamma^2 - 2b\eta)\kappa} \left( \frac{(\gamma^2 - 2b\eta)\kappa}{(2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)^2} - \frac{\alpha(3\alpha\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))}{(2\alpha\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))^2} \right)$
- (2)  $\phi_r^{RPC} = \frac{\eta(2b\kappa - \lambda^2)(-4\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2)(2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)^2}{(\gamma^2 - 2b\eta)\kappa(2\alpha\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))^3}$
- (3) When  $b > \frac{\lambda^2}{2\kappa}$  and  $\frac{\gamma^2\kappa}{2b\kappa - \lambda^2} < \eta \leq \frac{4\gamma^2\kappa}{2b\kappa - \lambda^2}$  then  $\phi_r^{RPC} > 0$  if  $0 < \alpha < \frac{2b\eta\kappa - \eta\lambda^2}{4\gamma^2\kappa}$
- (4) When  $b > \frac{\lambda^2}{2\kappa}$  and  $\eta > \frac{4\gamma^2\kappa}{2b\kappa - \lambda^2}$  then  $\phi_r^{RPC} > 0$  if  $0 < \alpha < 1$
- (5) When  $b > \frac{\lambda^2}{2\kappa}$  and  $\frac{\gamma^2\kappa}{2b\kappa - \lambda^2} < \eta \leq \frac{3\gamma^2\kappa}{2b\kappa - \lambda^2}$  then  $\phi_r^{RPC} > 0$  if  $\frac{2b\eta\kappa - \eta\lambda^2}{4\gamma^2\kappa} < \alpha < \frac{2b\eta\kappa - \eta\lambda^2}{3\gamma^2\kappa}$
- (6) When  $b > \frac{\lambda^2}{2\kappa}$  and  $\frac{3\gamma^2\kappa}{2b\kappa - \lambda^2} < \eta < \frac{4\gamma^2\kappa}{2b\kappa - \lambda^2}$  then  $\phi_r^{RPC} < 0$  if  $\frac{2b\eta\kappa - \eta\lambda^2}{4\gamma^2\kappa} < \alpha < 1$

The same logic as Property 6, Property 7 shows that the optimal sharing fraction which maximizes the retailer’s profit in the RPC contract depends on the value of the coefficient of marketing effort cost. As the coefficient of marketing effort cost is higher than a threshold level, i.e.  $\eta = \frac{4\gamma^2\kappa}{2b\kappa - \lambda^2}$ , the retailer increases the value of  $\alpha$  approach to one to attract more profit, the retailer’s profit increases at most  $\gamma_r^{RPC} = -\frac{(2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)^2}{(\gamma^2 - 2b\eta)\kappa} \left( \frac{(\gamma^2 - 2b\eta)\kappa}{(2\gamma^2\kappa + \eta(-4b\kappa + \lambda^2))^2} + \frac{-3\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}{(2\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))^2} \right)$ . On the contrary, the retailer is raising the fraction of  $\alpha$  approach to  $\frac{2b\eta\kappa - \eta\lambda^2}{4\gamma^2\kappa}$  to attain the highest profit.

**Property 8.** In the RPMC contract,

- (1)  $\gamma_r^{RPMC} = \frac{(4\alpha - 1)\gamma^4\kappa^2 - (4\alpha - 1)\gamma^2\eta\kappa(4b\kappa - \lambda^2) + \eta^2(4b^2(4\alpha - 1)\kappa^2 + 2b(1 - 4\alpha)\kappa\lambda^2 + \alpha\lambda^4)}{(\gamma^2 - 2b\eta)\kappa(\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2)}$
- (2)  $\phi_r^{RPMC} = \frac{(2\gamma^2\kappa - 4b\eta\kappa + \eta\lambda^2)^2}{(\gamma^2 - 2b\eta)\kappa(\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2)}$
- (3)  $\phi_r^{RPMC} > 0$  for any  $0 < \alpha < 1$

Property 8 show that the retailer’s profit in the RPMC contract always increases in  $\alpha$ . Therefore, the retailer attains the highest profit if the sharing fraction approach to one.

**B2. Analyzing the impact of  $\alpha$  on the channel’s profit**

Since the profit of the centralized channel are independent of  $\alpha$  and the channel efficiency of the sharing contract as function of  $\alpha$ , i.e.,  $E^j = E^j(\alpha)$ . Therefore, the impact of  $\alpha$  on the channel efficiency of the sharing contract is the same impact of  $\alpha$  on the profit of channel in the decentralized. By examining the sign of the functions  $\phi_c^j = \partial E^j(\alpha)/\partial \alpha$  with the conditions of  $\Pi_i^j > 0$ , we summarize the impact of  $\alpha$  on the profit of channel in the decentralized system through properties 9-12 as follows:

**Property 9.** In the decentralized channel with RP contract, let  $\alpha_c^{RP} = \frac{2b\gamma^2\eta\kappa - \gamma^2\eta\lambda^2}{2\gamma^4\kappa + 2b\gamma^2\eta\kappa - 4\gamma^2\eta\lambda^2 + 2b\eta^2\lambda^2}$ , we have

- (1)  $E^{RP} = \frac{(\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))(\eta(-2b\kappa + \lambda^2) + 2\alpha(\gamma^2\kappa - \eta\lambda^2) + \alpha^2(\gamma^2\kappa + \eta\lambda^2))}{(2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2)^2}$
- (2)  $\phi_c^{RP} = -\frac{2\kappa(\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))(\gamma^2\eta(-2b\kappa + \lambda^2) + 2\alpha(\gamma^4\kappa + b\eta^2\lambda^2 + \gamma^2\eta(b\kappa - 2\lambda^2)))}{(2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2)^3}$
- (3) When  $b > \frac{\gamma^2}{2\eta}$  and  $\kappa > \underline{\kappa}$  then,  $\phi_c^{RP} > 0$  if  $0 < \alpha < \alpha_c^{RP}$
- (4) When  $\frac{\gamma^2}{2\eta} < b \leq \frac{3\gamma^2}{2\eta}$  and  $\kappa > \underline{\kappa}$  then  $\phi_c^{RP} < 0$  if  $\alpha_c^{RP} < \alpha < \frac{2b\eta}{3\gamma^2}$
- (5) When  $b > \frac{3\gamma^2}{2\eta}$  and  $\kappa > \underline{\kappa}$  then  $\phi_c^{RP} < 0$  if  $\alpha_c^{RP} < \alpha < 1$
- (6)  $E^{RP} = E_{max}^{RP} = -\frac{(\gamma^4\kappa + 2b\eta^2\lambda^2 + \gamma^2\eta(2b\kappa - 3\lambda^2))(\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))}{\eta^2(2b\kappa - \lambda^2)(-3\gamma^2\lambda^2 + 2b(\gamma^2\kappa + \eta\lambda^2))}$  when  $\alpha \rightarrow \alpha_c^{RP}$  and  $E_{max}^{RP} < 1$ .

Property 9 shows that the decentralized channel with RP contract generate the highest profit when the sharing fraction is chosen at  $\alpha_c^{RP}$ . Furthermore, the profit of decentralized channel in the RP contract is always less than the profit of the centralized channel.

**Property 10.** In the decentralized channel with the RPM contract,

- (1)  $E^{RPM} = \frac{(\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))(\gamma^2\kappa + \eta(-2b\kappa + (-1 + \alpha)^2\lambda^2))}{(\gamma^2\kappa + \eta(-2b\kappa - (-1 + \alpha)\lambda^2))^2}$
- (2)  $\phi_c^{RPM} = -\frac{2\alpha\eta(\gamma^2 - 2b\eta)\kappa\lambda^2(-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2)}{(\gamma^2\kappa + \eta(-2b\kappa - (-1 + \alpha)\lambda^2))^3}$
- (3) When  $b > \frac{\gamma^2}{2\eta}$  and  $\kappa > \underline{\kappa}$  then  $\phi_c^{RPM} < 0$  if  $0 < \alpha < 1$
- (4)  $E^{RPM} = E_{max}^{RPM} = 1$  when  $\alpha \rightarrow 0$ ,
- (5)  $E^{RPM} = E_{min}^{RPM} = \frac{\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2}{\gamma^2\kappa - 2b\eta\kappa} > 0$  when  $\alpha \rightarrow 1$

Property 10 shows that the channel efficiency in RPM contract always decreases in  $\alpha$ , approaches to one as  $\alpha$  approaches to zero, approaches to  $E_{min}^{RPM}$  as  $\alpha$  approaches to one. This imply that the channel efficiency of RPM contract is the highest when the manufacturer incurs all of the cost and extracts all of the channel profit. On the other hand, the decentralized channel generate the lowest profit when the retailer incurs all of the production and marketing cost.

**Property 11.** In the decentralized channel with RPC contract, let  $\alpha_c^{RPC} = \frac{2b\eta\kappa - \eta\lambda^2}{2\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}$ , we have

- (1)  $E^{RPC} = \frac{(\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))(2\alpha\gamma^2\kappa + \alpha^2\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))}{(2\alpha\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))^2}$
- (2)  $\phi_c^{RPC} = -\frac{2\gamma^2\kappa(\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))(\eta(-2b\kappa + \lambda^2) + \alpha(2\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2))}{(2\alpha\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))^3}$
- (3) When  $b > \frac{\lambda^2}{2\kappa}$  and  $\eta > \frac{\gamma^2\kappa}{2b\kappa - \lambda^2}$  then  $\phi_c^{RPC} > 0$  if  $0 < \alpha < \alpha_c^{RPC}$
- (4) When  $b > \frac{\lambda^2}{2\kappa}$  and  $\frac{\gamma^2\kappa}{2b\kappa - \lambda^2} < \eta \leq \frac{3\gamma^2\kappa}{2b\kappa - \lambda^2}$  then  $\phi_c^{RPC} < 0$  if  $\alpha_c^{RPC} < \alpha < \frac{2b\eta\kappa - \eta\lambda^2}{3\gamma^2\kappa}$
- (5) When  $b > \frac{\lambda^2}{2\kappa}$  and  $\eta > \frac{3\gamma^2\kappa}{2b\kappa - \lambda^2}$  then  $\phi_c^{RPC} < 0$  if  $\alpha_c^{RPC} < \alpha < 1$
- (6)  $E^{RPC} = E_{Max}^{RPC} = 1 - \frac{\gamma^4\kappa^2}{\eta^2(-2b\kappa + \lambda^2)^2}$  when  $\alpha \rightarrow \alpha_c^{RPC}$  and  $E_{Max}^{RPC} < 1$

Property 11 shows that the decentralized channel with RPC contract generate the highest profit when the sharing fraction is chosen at  $\alpha_c^{RPC}$ . However, the profit of decentralized channel in the RPC contract is always less than the profit of the centralized channel.

**Property 12.** In the RPCM contract,  $E^{RPCM} = 1$  and  $\phi_c^{RPCM} = 0$  for any  $0 < \alpha < 1$ .

Property 12 shows that the decentralized channel with RPCM contract generates the same profit as that in the centralized channel and the channel efficiency of RPCM contract does not depend on the selection of the sharing fraction.

### Appendix C. Proof of Theorem

#### C.1. Proof of Theorem 1

When the demand is stochastic, the expected profit of the centralized channel in Eq. (2) can be expressed as:

$$E[\Pi_f^S] = p(a - bp + \gamma e + \lambda\theta + \Lambda[z]) - c(a - bp + \gamma e + \lambda\theta + z) - \frac{\eta e^2}{2} - \frac{\kappa\theta^2}{2}$$

First, for any fixed z with  $A \leq z \leq B$ , and after taking the first derivative of  $E[\Pi_f^S]$  with respect to p, e and  $\theta$ , we have

$$\frac{\partial E[\Pi_f^S]}{\partial \theta} = (p - c)\lambda - \theta\kappa.$$

$$\frac{\partial E[\Pi_f^S]}{\partial p} = a - bp - b(p - c) + \gamma e + \theta\lambda + \Lambda[z].$$

$$\frac{\partial E[\Pi_f^S]}{\partial e} = (p - c)\gamma - e\eta.$$

Solving the first order optimality condition  $\frac{\partial E[\Pi_f^S]}{\partial \theta} = \frac{\partial E[\Pi_f^S]}{\partial p} = \frac{\partial E[\Pi_f^S]}{\partial e} = 0$  implies that



$$p_I^S(z) = \frac{-c\gamma^2\kappa + a\eta\kappa + bc\eta\kappa - c\eta\lambda^2 + \eta\kappa\Lambda[z]}{\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2} = p_I^D + \frac{\eta\kappa\Lambda[z]}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2},$$

$$\theta_I^S(z) = \frac{\lambda(-a\eta + bc\eta - \eta\Lambda[z])}{\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2} = \theta_I^D + \frac{\eta\lambda\Lambda[z]}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}.$$

$$e_I^S(z) = \frac{\gamma(-a\kappa + bc\kappa - \kappa\Lambda[z])}{\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2} = e_I^D + \frac{\gamma\kappa\Lambda[z]}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2}.$$

Substituting the optimal  $p_I^S, e_I^S, \theta_I^S$  into the Eq. (2), we can reduce the objective function to one variable, z.

$$E[\Pi_I^S] = E[\Pi_I(z)] = \frac{-a^2\eta\kappa + b^2c^2\eta\kappa + 2c(z\gamma^2\kappa - ab\eta\kappa - 2b\eta\kappa + z\eta\lambda^2) + 2(a\eta\kappa - c(\gamma^2\kappa - b\eta\kappa + \eta\lambda^2))\Lambda[z] + \eta\kappa\Lambda[z]^2}{2(\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))}$$

Taking the first derivative of  $E[\Pi_I(z)]$  with respect to z, we have

$$\frac{\partial E[\Pi_I(z)]}{\partial z} = \frac{(-a\eta\kappa + c(\gamma^2\kappa - b\eta\kappa + \eta\lambda^2) - \eta\kappa\Lambda[z])\Lambda'[z]}{\gamma^2\kappa + \eta(-2b\kappa + \lambda^2)} - c$$

By setting  $\frac{\partial E[\Pi_I(z)]}{\partial z} = 0$ , the optimal stocking factor of inventory  $z_I \in (A, B)$  satisfying

$$\Lambda'[z] = \frac{c\gamma^2\kappa - 2bc\eta\kappa + c\eta\lambda^2}{c\gamma^2\kappa - a\eta\kappa - bc\eta\kappa + a\alpha\eta\kappa - bc\alpha\eta\kappa + c\eta\lambda^2 - \eta\kappa\Lambda[z] + \alpha\eta\kappa\Lambda[z]}.$$

After rearranging, we obtain  $\frac{1}{\Lambda'[z]} = 1 - \frac{\eta\kappa(a - bc + \Lambda[z])}{c(\gamma^2\kappa + \eta(-2b\kappa + \lambda^2))}$ .

To identify that the values of z satisfy the first order optimality condition, let  $R(z) = \frac{\partial E[\Pi_I(z)]}{\partial z}$  and consider finding the zero of  $R(z)$

$$dR(z)/dz = \frac{-\eta\kappa\Lambda'[z]^2 + (c\gamma^2\kappa - a\eta\kappa - bc\eta\kappa + c\eta\lambda^2 - \eta\kappa\Lambda[z])\Lambda''[z]}{\gamma^2\kappa + \eta(-2b\kappa + \lambda^2)}.$$

$$\frac{d^2R(z)}{dz^2} = \frac{-3\eta\kappa\Lambda'[z]\Lambda''[z] + (c\gamma^2\kappa - a\eta\kappa - bc\eta\kappa + c\eta\lambda^2 - \eta\kappa\Lambda[z])\Lambda^{(3)}[z]}{\gamma^2\kappa + \eta(-2b\kappa + \lambda^2)}, \text{ where } \Lambda^{(3)}[z] = \frac{\partial^3\Lambda[z]}{\partial z^3}.$$

When  $\frac{dR(z)}{dz} = 0$  then  $\Lambda'[z] = \frac{-\eta\kappa\Lambda'[z]^2}{-c\gamma^2\kappa + a\eta\kappa + bc\eta\kappa - c\eta\lambda^2 + \eta\kappa\Lambda[z]}$ , it follows that

$$\frac{d^2R(z)}{dz^2} \Big|_{\frac{dR(z)}{dz}=0} = \frac{\frac{3\eta^2\kappa^2\Lambda'[z]^3}{-c\gamma^2\kappa + a\eta\kappa + bc\eta\kappa - c\eta\lambda^2 + \eta\kappa\Lambda[z]} + (c\gamma^2\kappa - a\eta\kappa - bc\eta\kappa + c\eta\lambda^2 - \eta\kappa\Lambda[z])\Lambda^{(3)}[z]}{\gamma^2\kappa + \eta(-2b\kappa + \lambda^2)}.$$

and after simplification, we obtain

$$\frac{d^2R(z)}{dz^2} \Big|_{\frac{dR(z)}{dz}=0} = -\frac{\eta\kappa}{\gamma^2\kappa + \eta(-2b\kappa + \lambda^2)} \frac{\Lambda'[z]^3}{\Lambda''[z]} * \left[ \frac{3\Lambda''[z]^2}{\Lambda'[z]^2} - \frac{\Lambda^{(3)}[z]}{\Lambda'[z]} \right].$$

Note that  $\frac{3\Lambda''[z]^2}{\Lambda'[z]^2} - \frac{\Lambda^{(3)}[z]}{\Lambda'[z]} = \frac{dr(z)}{dz} + 2r[z]^2$ . Then,

$$\frac{d^2R(z)}{dz^2} \Big|_{\frac{dR(z)}{dz}=0} = -\frac{\eta\kappa}{\gamma^2\kappa + \eta(-2b\kappa + \lambda^2)} \frac{\Lambda'[z]^3}{\Lambda''[z]} * \left[ \frac{dr(z)}{dz} + 2r[z]^2 \right].$$

Since  $\Lambda'[z] = 1 - F(z) > 0, \Lambda''[z] = -f(z) < 0$  and  $\frac{\eta\kappa}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2} > 0$ . Therefore,

$\frac{d^2R(z)}{dz^2} \Big|_{\frac{dR(z)}{dz}=0} < 0$  if  $\frac{dr(z)}{dz} + 2r[z]^2 > 0$  then it follows that  $R(z)$  is either monotone or unimodal, which implies that  $R(z)$  has at most two roots.

Also,  $\text{Limit}[R(z), z \rightarrow B] = -c < 0$ . Therefore, if  $R(z)$  has only one root, it corresponds to the maximum of  $E[\Pi_I(z)]$ ; if it has two roots, the larger of the two represents the maximum and the smaller represents the minimum of  $E[\Pi_I(z)]$  and we denote it by  $z_I$ . if  $R(z)$  has only one root, we require that  $R(A) = \frac{(a - bc + A)\eta\kappa}{-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2} > 0$  or equivalently  $a - bc + A > 0$  and  $-\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2$ .

### C2. Proof of Theorem 2

We provide Proof for RP contract, the proofs for other contracts are the same logic.

To identify that the values of z satisfy the first order optimality condition,  $\frac{\partial E[\Pi_m^{RP}]}{\partial z} = 0$ , let  $R^{RP}(z) = \frac{\partial E[\Pi_m^{RP}]}{\partial z}$  and consider finding the zero of  $R^{RP}(z)$ .

When  $\frac{dR^{RP}(z)}{dz} = 0$  that is  $\Lambda''[z] = \frac{(\alpha - 1)\eta\kappa\Lambda'[z]^2}{2c\alpha\gamma^2\kappa - a\eta\kappa - bc\eta\kappa + a\alpha\eta\kappa - bc\alpha\eta\kappa + c\eta\lambda^2 - c\alpha\eta\lambda^2 - \eta\kappa\Lambda[z] + \alpha\eta\kappa\Lambda[z]}$ , we have  $\frac{d^2R^{RP}(z)}{dz^2} \Big|_{\frac{dR^{RP}(z)}{dz}=0} =$

$$\frac{(\alpha - 1)\eta\kappa - c(b\eta\kappa - 2\alpha\gamma^2\kappa + bc\alpha\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2) + (\alpha - 1)\eta\kappa f[z]\Lambda^{(3)}[z]}{2c\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2} - \frac{3(\alpha - 1)^2\eta^2\kappa^2\Lambda'[z]^3}{(\alpha - 1)\eta\kappa - c(b\eta\kappa - 2\alpha\gamma^2\kappa + bc\alpha\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2) + (\alpha - 1)\eta\kappa\Lambda[z](2\alpha\gamma^2\kappa - 2b\eta\kappa + \eta\lambda^2 - \alpha\eta\lambda^2)}, \text{ and after simplification, we have}$$

$$\frac{d^2R^{RP}(z)}{dz^2} \Big|_{\frac{dR^{RP}(z)}{dz}=0} = \frac{(1 - \alpha)\eta\kappa}{-2c\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2} * \frac{\Lambda'[z]^3}{\Lambda''[z]} * \left[ \frac{3\Lambda''[z]^2}{\Lambda'[z]^2} - \frac{\Lambda^{(3)}[z]}{\Lambda'[z]} \right].$$

We recall that  $\frac{3\Lambda''[z]^2}{\Lambda'[z]^2} - \frac{\Lambda^{(3)}[z]}{\Lambda'[z]} = \frac{dr(z)}{dz} + 2r[z]^2$ . Then,

$$\frac{d^2 R^{RP}(z)}{dz^2} \Big|_{\frac{dR^{RP}(z)}{dz}=0} = \frac{(1-\alpha)\eta\kappa}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2} * \frac{\Lambda'[z]^3}{\Lambda''[z]} * \left[ \frac{dr(z)}{dz} + 2r[z]^2 \right].$$

Since  $\Lambda'[z] = 1 - F(z) > 0$ ,  $\Lambda''[z] = -f(z) < 0$  and  $-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2 > 0$  to ensure the the Hessian matrix of  $\Pi_m^{RP}$  is a negative definite. Therefore,

$\frac{d^2 R^{RP}(z)}{dz^2} \Big|_{\frac{dR^{RP}(z)}{dz}=0} < 0$  if  $\frac{dr(z)}{dz} + 2r[z]^2 > 0$ , then it follows that  $R^{RP}(z)$  is either monotone or unimodal, which implies that  $R^{RP}(z)$  has at most two roots. Also,  $\lim_{z \rightarrow B} [R^{RP}(z), z \rightarrow B] = -c < 0$ . Therefore, if  $R^{RP}(z)$  has only one root, it corresponds to the maximum of  $E[\Pi_m^{RP}]$ ; if it has two roots, the larger of the two represents the maximum and the smaller represents the minimum of  $E[\Pi_m^{RP}]$ . If  $R^{RP}(z)$  has only one root, we require that  $R^{RP}(A) = \frac{(a+A-bc)(-1+\alpha)\eta\kappa}{-2\alpha\gamma^2\kappa + 2b\eta\kappa - \eta\lambda^2 + \alpha\eta\lambda^2} = \frac{2(a-bc+A)\Pi_m^{RP}}{(a-bc)^2} > 0$  or equivalently  $a - bc + A > 0$  and  $\Pi_m^{RP} > 0$ , then only one root of  $R^{RP}(z)$  corresponds to the maximum of  $E[\Pi_m^{RP}]$ .

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