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Quantitative Easing, Excess Reserves, Transmission Channels, Securitization Crisis

JEL codes:

E32, E44, E52, E58, G01

Quantitative Easing and Excess Reserves*

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February 28, 2019

Abstract

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1 Introduction

A lot remains unknown about the effects of Quantitative Easing (QE), from the size of the effects that it had, to the transmission channels through which programs worked. At the onset of the Great Recession (GR thereafter), the FED lowered dramatically the effective fund rate until it hit the zero lower bound. While engaging in large rescue programs providing exceptional liquidity facilities in the discount window and in the interbank market¹, the FED also announced the first of its QE asset purchase programs.² QE appeared as a potentially good solution for several reasons. To start with, QE could be qualitative and remove toxic assets from the capital markets. Further, QE could reduce long-term nominal yields and then flatten the yield curve. Finally, QE could offer an additional degree of freedom for banks to reallocate portfolios, roll over or bail-out debts and resort to recapitalization. Nevertheless, QE and the liquidity rescue programs had a dramatic effect on the size of the FED's balance sheet (see Figures 8 and 9 in Appendix A). In particular, the expansion of the liabilities has been covered by the issuance of deposits; that is, interest-bearing excess reserves (see Figures 10 and 11 in Appendix A).³

Hence, by what avenues excess reserves transmit to the real economy? In other words: To what extents such maturity transformation (swaps of excess reserves for assets) affects macroeconomic variables?

So far in the theoretical literature, the absence of excess reserves limits/overlooks the identification of the transmission channels of QE, as will be clear later. We address this issue by using a general equilibrium model that incorporates the two most popular types of financial frictions in the theoretical literature, namely banking frictions and collateral constraints. Our model is equipped to quantify the relative importance of three theoretical transmissions channels of QE programs, the portfolio balance, bank lending (credit) and asset prices channels. Our contributions are twofold.

First, we provide a more realistic way of modeling the implementation of QE programs. In their seminal papers, Gertler & Karadi (2011, 2013) (GK) design QE as a *public intermediation*

¹Discount window programs: Term Auction Facility (TAF), Primary Dealer Credit Facility (PDCF), and Term Securities Lending Facility (TSLF). Market liquidity facilities: Commercial Paper Funding Facility (CPFF), Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF), Money Market Investor Funding Facility (MMIFF), and the Term Asset-Backed Securities Loan Facility (TALF). See more information in <https://www.federalreserve.gov/monetarypolicy/bst.htm>

²Forward guidance was also a great part of the unconventional monetary instruments. Often defined as *Delphic* and *Odyssean* forward guidance, these explicit commitments to the future stance of monetary policy consistently paced the monetary announcements during the ZLB period.

³The interest rate on required reserves (IORR) and the interest rate on excess reserves (IOER) were settled by the Emergency Economic Stabilization Act of 2008

in the supply of credit: the central bank elastically issues short-term liabilities subject to negligible efficiency costs to supply private capital at a lower interest rate. We offer an alternative, and include reserve requirements in the balance sheet of bankers. In addition, we impose limits to arbitrage to cause an excess return in reserves. In Curdia & Woodford (2011), the nominal interest rate is always greater or equal to the interest rate on reserves. Here, we assume the opposite, in line with the practice of the FED. Then, we design QE programs as the exogenous supply of excess reserves. This way of modeling QE programs is much closer to the actual QE programs than any alternative in the literature.

Second, we disentangle the different transmission channels of QE, not only with the introduction of excess reserves, but also by having various financial frictions. The model includes banking frictions in the shadow banking system, banking frictions on commercial banks à la GK, and a collateral constraint in the housing market in the spirit of Guerrieri & Iacoviello (2017). The latter basically sets an upper borrowing limit for impatient households. In their framework, impatient agents borrow directly from patient households. In our model, we impose a private financial intermediation in the funding process. Doing so not only directly constrains the housing consumption of impatient agents by the market value of their collateral, but also indirectly constrains it through the moral hazard problem that bankers face.

For the sake of reality towards the episode of GR, we take into account a shadow banking system à la Meeks et al. (2017) that provides ABS after the securitization of loan bundles. Shadow banks support the supply of credit in the economy by providing high-quality collateral to bankers, and the mechanism is in line with the observed countercyclicality of the size of the shadow banking system (see Figure 12 in Appendix A). In many contributions that include financial frictions, the GR is modeled as a negative capital quality shock. We adopt a more realistic mechanism and consider a fall in the quality of ABS. In this greater risk environment, bankers proceed to portfolio reallocations that lead them to reduce the supply of loans, and cut the demand for ABS. As a consequence, shadow banks engage in fire sales of securitized assets.

The latter is important in terms of results. Indeed, we find that our model successfully replicates key features of the GR when driven by a negative shock on the quality of ABS. In this greater risk environment, we are able to capture an endogenous mechanism within the financial system that causes ABS to be procyclical, reconciling theory with the data. We demonstrate that in such environment, a public intermediation à la GK limits the understanding of the transmission channels of QE programs because it bypasses the endogenous portfolio reallocations, and neglects the endogenous response of the financial system to the program. In addition, it greatly overstates – roughly by a factor of 1 to 10 – the macroeconomic effects

of QE programs. The main reason is that QE à la GK overlooks the relative changes in the balance sheets. Finally, we find that most macroeconomic effects generated by QE programs are channeled through the wealth effect – asset prices. In this paper, we find that the portfolio balance channel of QE programs is always present, working as the first stage of a multi-stage rocket. But on top of the portfolio balance channel, the model allows to gauge the relative contribution of the credit channel and the asset prices channel. The introduction of excess reserves is crucial because it allows to trigger changes in the relative supply of assets; an essential transmission mechanism of QE within the financial system.

While the model performs reasonably well in replicating the GR, it is not its main focus. Indeed, we abstract from important New-Keynesian features such as wage rigidities, important non-linearities like the ZLB or the occasionally binding constraints. However, our results do not qualitatively suffer from these lacks, since the addition of such features would rather amplify the magnitude of the effects of QE programs, making our evaluation here a lower-bound estimate.

In the literature, the transmission channels of QE programs are still being discussed. Indeed, the economic literature mostly focused on the quantitative effects of QE programs empirically (see Krishnamurthy & Vissing-Jorgensen (2011) or Weale & Wieladek (2016) among others).⁴ In most cases, empirical contributions find that QE programs have very large effects on financial conditions and the real economy. However, these results are in stark contrast with the relative neutrality of QE suggested by the theoretical literature. The neutrality assertion is embraced by the monetarist’s views of Curdia & Woodford (2011) and Williamson (2012, 2016), unless financial markets are segmented or if there are financial frictions. For instance, under the preferred-habitat assumption (Modigliani & Sutch (1966), Vayanos & Vila (2009)) and/or according to Tobin (1969)’s portfolio balance theory of the term structure of interest rates, QE should be effective/relevant. In the former, markets are segmented by the preferences of investors over asset maturities and yields. For investors to be willing to invest in other segments, the incurred risk must at least be offset by the expected return in the asset. In this context, QE fosters demand for imperfect substitutes because of the change in the supply of the targeted asset, and QE programs reduce the yields of close substitutes. In the latter, a financial crisis raises private and public spreads over risk-less assets because of financial frictions, and QE programs help alleviate those. QE was also shown to have large effects in models of financial frictions: in GK a premium on risky assets emerges as the result of bankers diverting a fraction of the total assets intermediated. The friction limits the arbitrage in financial markets among assets, and any exogenous drop in asset prices tightens the balance sheet constraints. This

⁴For further empirical literature on QE see Borio & Zabai (2016).

drop produces an endogenous financial accelerator mechanism, that in turn freezes the supply of credit by private intermediaries.

The paper is organized as follows. Section 2 presents the baseline model. Section 3 investigates the effects of securitization in the transmission of a GK negative capital quality shock, and investigates the effects of a fall in the quality of ABS. It also quantifies the effects of QE programs, modeled either as in GK (public credit intermediation), or as a rise in excess reserves. Section 4 concludes.

2 The Baseline Model

Our baseline model is a generalization of the seminal paper of GK. Extensions capture key features of the housing market and the shadow banking system to allow for a better understanding of the origins of the GR – lining-up with usual narratives – and a better understanding of the effects of QE programs.

In the model, patient households hold financial assets and acquire housing. They fund impatient households and firms in their investment projects. However, patient workers are unable to perfectly enforce payments from borrowers, and resort to commercial banks. Commercial banks are financial intermediaries specialized in lending activities. Due to financial frictions, bankers are limited in their actions. Bankers must hold sufficient amounts of high-quality collateral to meet their balance sheet constraints. Shadow banks trade loan bundles for high-quality ABS. This section offers details about the respective behavior of commercial and shadow banks, households, producers, and the central bank.

2.1 Commercial banks

Commercial banks offer multiple services to three types of agents. First, they grant loans to intermediate good producers. Second, they facilitate government expenditures by holding long-term government bonds. Last, they provide mortgage loans to impatient households. Each bank is owned and managed by a fraction of patient households, namely, bankers. Their assets are backed by the issuance of deposits and net worth (equity). However, financial frictions limits the amount of funds patient households are willing to deposit. Accordingly, a rational banker would circumvent frictions by entirely relying on equity. We then bound the expected lifetime of bankers with an i.i.d probability σ to create a rotation within the workforce. The financial frictions arise from a typical moral hazard problem à la GK in which bankers can

be shady and divert a fixed fraction of their assets. Depositors demand that bankers pledge a fraction of each asset, and the composition of the bankers balance sheet shapes the relationship of trust between both parties. Bankers hold a variety of assets, that may differ in their degrees of pledgeability. In particular, primary loans induce strong suspicion in the eyes of patient households. Bankers then must hold high-quality collateral such as ABS.

The asset side of the individual balance sheet of a banker (superscript c) is made of the remainder of primary loans s_t^c , ABS a_t^c , long-term government bonds b_t^c and mortgage loans to impatient households m_t^c . Liabilities consist of deposits d_t^c and net worth n_t^c . The balance sheet identity writes:

$$Q_t s_t^c + q_t^a a_t^c + q_t^b b_t^c + q_t^h m_t^c = d_t^c + n_t^c \quad (1)$$

in which lower case symbols represent individual quantities. Let be Q_t , q_t^a , q_t^b and q_t^h the respective market prices of assets. The net worth evolves as:

$$\begin{aligned} n_t^c &= R_t^s Q_{t-1} s_{t-1}^c + R_t^a q_{t-1}^a a_{t-1}^c + R_t^b q_{t-1}^b b_{t-1}^c + R_t q_{t-1}^h m_{t-1}^c - R_t d_{t-1}^c \\ &= (R_t^s - R_t) Q_{t-1} s_{t-1}^c + (R_t^a - R_t) q_{t-1}^a a_{t-1}^c + (R_t^b - R_t) q_{t-1}^b b_{t-1}^c + R_t n_{t-1}^c \end{aligned} \quad (2)$$

where R_t^s is the real stochastic return on primary loans, R_t^a on ABS and R_t^b on long-term government bonds. The return on deposits is the real interest rate R_t . In the interest of parsimony we do not design financial frictions in the housing market. In other words, mortgage loans are riskless, hence their cost in the real interest rate.

Going deeper into details, the timeline of a banker's decisions proceeds as follows: In the dawn of t , a banker issues deposits, grants loans to firms and impatient households, funds the government and receives ABS. She then makes a choice between two options: Be honest or shady. An honest banker accumulates net worth until all stakeholders receive their payoffs in $t + 1$. Conversely, a dubious banker sells a fraction of her assets to outside investors. For the sake of discretion, the amount of fraud cannot exceed an upper bound θ^c . Her home household enjoys the booty in period $t + 1$. Unfortunately, this larceny leads the bank to a fragile position. Indeed, rational depositors stop funding the banks in this context. As a consequence, bankers are in danger of bankruptcy. Both parties benefit from reaching an agreement, that takes the form of an incentive compatibility constraint, that states that the discounted franchise value of the bank must be at least as large as a fraction of diverted assets:

$$V_t^c \geq \theta^c (Q_t s_t^c + \Xi q_t^a a_t^c + \Delta q_t^b b_t^c) \quad (3)$$

in which Ξ and Δ are weights related to the assets' pledgeability. By fixing $\theta^c > \theta^c \Delta > \theta^c \Xi$

$\in [0,1]$, we assume that ABS have the highest collateral quality. In line with Meeks et al. (2017), it is quite natural to think that securitization mitigates risk through diversification. In addition, ABS increase the liquidity of the underlying assets. It is also reasonable to define a higher collateral value for long-term government bonds than private loans, public issuers being less likely to default than private issuers. The franchise value of a bank V_t^c depends on the sum of accumulated net worth conditional on the death and survival probabilities:

$$V_{t-1}^c = \mathbb{E}_{t-1} \Lambda_{t-1,t} ((1 - \sigma)n_t^c + \sigma V_t^c) \quad (4)$$

which is a standard Bellman equation. Variable $\Lambda_{t-1,t}$ is the stochastic discount factor derived from the patient households program. The banker's objective is then to maximize the franchise value (4) subject to the incentive constraint (3), the balance sheet identity (1) and the law of motion of net worth (2). Using the method of undetermined coefficients, we guess and later verify that the bank's franchise value V_t^c is linear in the time-varying coefficients $\mu_t^c, \mu_t^a, \mu_t^b, \eta_t^c$. Accordingly, we conjecture that the value function is in period t :

$$V_t^c = \mu_t^c Q_t s_t^c + \mu_t^a q_t^a a_t^c + \mu_t^b q_t^b b_t^c + \eta_t^c n_t^c \quad (5)$$

Hereafter, insert the particular solution (5) into the recurrence relation (4) and differentiate using the method of Lagrange multipliers. Let λ_t^c be the associated Lagrangian multiplier of the incentive constraint (3). The differentiations yield the following first-order necessary conditions, respectively for $s_t^c, a_t^c, b_t^c, \lambda_t^c$:

$$(1 + \lambda_t^c)\mu_t^c = \lambda_t^c \theta^c \quad (6)$$

$$\mu_t^a = \mu_t^c \Xi \quad (7)$$

$$\mu_t^b = \mu_t^c \Delta \quad (8)$$

$$Q_t s_t^c + \Xi q_t^a a_t^c + \Delta q_t^b b_t^c = \phi_t^c n_t^c \quad (9)$$

where,

$$\phi_t^c = \frac{\eta_t^c}{\theta^c - \mu_t^c} \quad (10)$$

The LHS of equation (6) shows the marginal benefit from expanding what we call primary loans. The RHS indicates the marginal cost of fraud. Due to the fact $1 > \Delta > \Xi$, the holdings of ABS and government bonds are marginally less attractive than loans. In this respect, as long as $\mu_t^c < \theta^c$, it is always profitable for bankers to divert assets. The latter is also a necessary condition for the incentive constraint to bind, otherwise, bankers would indefinitely supply loans to offset frictions. In other words, the incentive constraint imposes that the holding of

assets is curbed by a maximum equity multiplier ϕ_t^c (see equation (10)). Hence, as long as the incentive constraint binds, the size of a banker balance sheet is endogenously constrained by fluctuations in its net worth. The latter also induces an equality between the gains from larceny $\theta^c \phi_t^c$ and the costs of losing franchise value $\mu_t^c \phi_t^c + \eta_t^c$.

At the end of period $t + 1$, a shady banker entirely consumes her net worth with probability $(1 - \sigma)$. In this case the marginal gain of net worth is then one. Conversely, an honest banker ensures the continuity of her services. Hence, either the marginal gain from larceny or the marginal cost of losing franchise value apply. Thereby, the weighted shadow value of a banker net worth is defined by:

$$\Omega_{t+1}^c = 1 + \sigma(\theta^c \phi_{t+1}^c - 1) \quad (11)$$

Finally, the time-varying coefficients linear in $s_t^c, a_t^c, b_t^c, n_t^c$ satisfy the Bellman equation with:⁵

$$\mu_t^c = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c (R_{t+1}^s - R_{t+1}) \quad (12)$$

$$\mu_t^a = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c (R_{t+1}^a - R_{t+1}) \quad (13)$$

$$\mu_t^b = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c (R_{t+1}^b - R_{t+1}) \quad (14)$$

$$\eta_t^c = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c R_{t+1} \quad (15)$$

which implies that financial frictions induce discounted excess returns. To give the reader a preview of the model mechanics, imagine an exogenous financial crash that lowers the price of capital. The market value of securities is reduced, hence the bankers' net worth falls. Assets tighten in proportion to the equity multiplier. In addition, equation (6) indicates that a tightening in the constraint substantially increases the marginal gain from expanding primary loans; that is, the marginal cost of fraud. The latter then raises the excess return in loans and the cost of capital. In a nutshell, an exogenous crash in asset prices is in turn amplified by the endogenous tightening in the size of the balance sheet. This feedback effect further reduces the lending activities of the commercial banks.

2.2 Shadow banks

The shadow banking sector is composed by competitive shadow banks in the exclusive charge of the securitization of primary loans. A shadow bank purchases loan bundles in a secondary market to create high-quality ABS. Each shadow bank is owned and managed by a fraction of patient households, namely, brokers. The existence of shadow banks is motivated by the

⁵Appendix B provide details for commercial and shadow banks problems

obligation of banks to hold high-quality collateral. Hence, the entire stock of ABS is sold to bankers. In this way, the flow of ABS is confined within the financial system. However, financial frictions limit trade between bankers and brokers. A broker survives with probability σ and exits her occupation with probability $(1 - \sigma)$. Some brokers are tempted to divert a fraction of their assets. Therefore, bankers demand that brokers produce a sufficient amount of ABS.

The individual balance sheet of a broker (superscript b) comprises securitized assets s_t^b , ABS a_t^b and net worth (equity) n_t^b with:

$$Q_t s_t^b = q_t^a a_t^b + n_t^b \quad (16)$$

in which the accumulated net worth follows:

$$\begin{aligned} n_t^b &= R_t^s Q_{t-1} s_{t-1}^b - R_t^a q_{t-1}^a a_{t-1}^b \\ &= (R_t^s - R_t^a) Q_{t-1} s_{t-1}^b + R_t^a n_{t-1}^b \end{aligned} \quad (17)$$

For the sake of parsimony, the real returns and market prices in the same class of assets are equivalent across markets.⁶

The timeline of a broker's lifetime is as follows. At the beginning of t , a broker purchases loan bundles and transforms those into ABS. She then chooses to be honest or shady. An honest broker pursues securitization until the final payoffs in $t + 1$. A dubious broker diverts a fraction of her securitized assets. She fixes her greediness to an upper bound θ^b to avoid drawing attention. The related booty goes to her home household at the end of $t + 1$. Unfortunately, the future of the shadow bank is at risk due to this embezzlement. Indeed, bankers are perfectly familiar with this ploy being faced with such a dilemma themselves. In this situation, bankers decide to cut offering loan bundles. However, bankers are in need for high-quality collateral to obtain funds from depositors. Thus, they agree to the condition that the discounted franchise value of the shadow bank must be at least as large as the amount of fraud. The resulting incentive constraint writes:

$$V_t^b \geq \theta^b Q_t s_t^b \quad (18)$$

in which the franchise value V_t^b is the sum of accumulated net worth conditional on the exiting and survival probabilities:

$$V_{t-1}^b = \mathbb{E}_{t-1} \Lambda_{t-1,t} ((1 - \sigma) n_t^b + \sigma V_t^b) \quad (19)$$

⁶It is conceivable to relax this assumption by adding an intermediation between bankers and brokers such as a special purpose vehicle or impose transaction costs.

The broker's objective is to maximize the franchise value (19) subject to the incentive constraint (18), the balance sheet identity (16) and the law of motion of net worth (17). Using the method of undetermined coefficients, the particular solution is linear in the time-varying coefficients ν_t^b , η_t^b . Accordingly, we conjecture that the value function satisfies:

$$V_t^b = \nu_t^b Q_t s_t^b + \eta_t^b n_t^b \quad (20)$$

Then substitute the particular solution (20) into the Bellman equation (19). Let λ_t^b be the associated Lagrangian multiplier of the incentive constraint (18). By differentiating, the first-order necessary conditions for, s_t^b and λ_t^b are:

$$(1 + \lambda_t^b) \nu_t^b = \lambda_t^b \theta^b \quad (21)$$

$$Q_t s_t^b = \phi_t^b n_t^b \quad (22)$$

where,

$$\phi_t^b = \frac{\eta_t^b}{\theta^b - \nu_t^b} \quad (23)$$

The LHS of equation (21) is the marginal gain from increasing the securitization activity. The RHS indicates the marginal cost of embezzlement. As long as $\nu_t^b < \theta^b$, it is always profitable for brokers to divert assets. This is a necessary condition for the incentive constraint to bind, otherwise, brokers would extend securitization to infinite amounts to offset frictions. Hence, an isomorphic equity multiplier ϕ_t^b applies to the brokers balance sheets (see equation (23)). The binding constraint induces an equality between the gains from diverting securitized assets $\theta^b \phi_t^b$ and the costs of losing the franchise value $\mu_t^b \phi_t^b + \eta_t^b$.

As for bankers, a shady broker entirely consumes her net worth at the end of period $t + 1$ with probability $(1 - \sigma)$. In this situation, the marginal gain of net worth is one. An honest broker builds on equity with probability σ until $t + 1$. In this case, either the marginal gain from embezzlement or the marginal cost of squeezing the franchise value apply. The weighted shadow value of a broker net worth is thus:

$$\Omega_{t+1}^b = 1 + \sigma(\theta^b \phi_t^b - 1) \quad (24)$$

The time-varying coefficients in s_t^b and n_t^b satisfy the Bellman equation according to:

$$\mu_t^b = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^b (R_{t+1}^s - R_{t+1}^a) \quad (25)$$

$$\eta_t^b = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^b R_{t+1}^a \quad (26)$$

The above system of equations provides a general description of this financial system. In the

preceding example of the exogenous crash in the price of capital, we highlighted the feedback effect in the absence of shadow banks. Henceforth, both net worth of bankers and brokers are seriously impaired by the fall in the price of capital. Indeed, primary securitized loans are similarly market priced. The incentive constraints tighten and feed the downturn.

2.3 Aggregation

By definition, the components of the equity multipliers ϕ_t^c and ϕ_t^b are identical across commercial and shadow banks. Thus, the sum of the individual balance sheet constraints yield the following aggregate identities:

$$Q_t S_t^c + \Xi q_t^a A_t^c + \Delta q_t^b B_t^c = \phi_t^c N_t^c \quad (27)$$

$$Q_t S_t^b = \phi_t^b N_t^b \quad (28)$$

The same principle applies to the net worth of bankers and brokers. The aggregate net worth consist of the retained earnings of survivors and the initial provision for newcomers:⁷

$$N_t^c = \sigma \left((R_t^s - R_t) Q_{t-1} S_{t-1}^c + (R_t^a - R_t) q_{t-1}^a A_{t-1}^c + (R_t^b - R_t) q_{t-1}^b B_{t-1}^c + R_t N_{t-1}^c \right) + \chi^c \quad (29)$$

$$N_t^b = \sigma \left((R_t^s - R_t^a) Q_{t-1} S_{t-1}^b + R_t^a N_{t-1}^b \right) + \chi^b$$

Let Z_t be the gross profits per unit of capital as will be clear later. The parameter δ represents the depreciation rate of capital. Accordingly, the real return on primary loans is:

$$R_{t+1}^c = \frac{Z_{t+1} + (1 - \delta) Q_{t+1} \xi_{t+1}}{Q_t} \quad (30)$$

where ξ_{t+1} is an exogenous capital quality shock à la GK. Accordingly, the excessive payoff of capital corresponds to the sum of gross profits and the market value of capital stock net depreciation. The ABS are derivative products of primary loans, hence their real return is:

$$R_{t+1}^a = \frac{Z_{t+1} + (1 - \delta) q_{t+1}^a \xi_{t+1}}{q_t^a} \quad (31)$$

Finally, the long-term government bonds return a periodic unit of currency. Let P_t be the aggregate price level of the economy. Thus, the real return on long-term government bonds is:

$$R_{t+1}^b = \frac{\frac{1}{P_t} + q_{t+1}^b}{q_t^b} \quad (32)$$

⁷ $\chi^c = \omega^c (\bar{Q}\bar{S} + \bar{q}^a \bar{A}^c + \bar{q}^b \bar{B}^c)$ and $\chi^b = \omega^b (\bar{Q}\bar{S}^b)$, in which the bars indicate the steady-state values.

2.4 Households and Production

There is a unit mass of households in two different groups, namely, patient and impatient households. In the group of patient households, people switch occupations each period t . A fixed proportion f is a worker while the remaining part is either a banker or a broker. Conversely, impatient households can not work for financial intermediaries. The utility flows for each type of household are then given by:

$$u_t = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left(\Gamma_c \log(C_{t+i} - \varepsilon^c C_{t+i-1}) + \bar{j} \Gamma_h \log(H_{t+i} - \varepsilon^h H_{t+i-1}) - \frac{1}{1+\eta} L_t^{1+\eta} \right) \quad (33)$$

$$u'_t = \mathbb{E}_t \sum_{i=0}^{\infty} (\beta')^i \left(\Gamma'_c \log(C'_{t+i} - \varepsilon^c C'_{t+i-1}) + \bar{j} \Gamma'_h \log(H'_{t+i} - \varepsilon^h H'_{t+i-1}) - \frac{1}{1+\eta} L_t'^{1+\eta} \right) \quad (34)$$

in which the prime symbols denote the impatient households' variables. Variable C_t stands for the consumption of nondurable goods, H_t accounts for housing consumption and L_t for hours worked. Utility flows are discounted by the usual $\beta > \beta'$ factors. Parameter j reflects (constant) housing preferences and $\varepsilon^{c,h}$ captures habit formation.⁸ Patient households face a budget constraint of the form:

$$\begin{aligned} C_t + D_t^h + q_t^h (H_t - H_{t-1}) + Q_t \left(S_t^h + \frac{1}{2} \kappa (S_t^h - \bar{S}^h)^2 \right) + q_t^b \left(B_t^h + \frac{1}{2} \kappa (B_t^h - \bar{B}^h)^2 \right) \\ = W_t L_t + T_t + R_t D_{t-1}^h + R_t^s Q_{t-1} S_{t-1}^h + R_t^b q_{t-1}^b B_{t-1}^h + \Pi_t \end{aligned} \quad (35)$$

where W_t stands for the real wage, T_t denotes lump-sum taxes and Π_t defines dividends from the ownership of commercial banks, shadow banks and capital producing firms. Patient households accumulate deposits D_t^c subject to the moral hazard problem exposed earlier. They also purchase corporate debt and long-term government bond, subject to potential transaction costs. They operate frictionless arbitrage when the convex transaction cost κ is zero. Conversely, they hold assets to their steady-state levels (\bar{S}^h, \bar{B}^h) when the cost tends towards infinity. The impatient households budget constraint is:

$$C'_t + q_t^h (H'_t - H'_{t-1}) + R_t M_{t-1}^h = W'_t L'_t + M_t^h \quad (36)$$

meaning that impatient agents must borrow amounts M_t^h from commercial banks to secure housing consumption H'_t . Following Guerrieri & Iacoviello (2017), we introduce a collateral

⁸The factors $\Gamma_c, \Gamma'_c, \Gamma_h, \Gamma'_h$ scale consumption and housing so that steady-state values of consumption and housing do not depend on habit factors. Hence, $\Gamma_c = (1 - \varepsilon^c)/(1 - \beta\varepsilon^c)$, $\Gamma'_c = (1 - \varepsilon^c)/(1 - \beta'\varepsilon^c)$, $\Gamma_h = (1 - \varepsilon^h)/(1 - \beta\varepsilon^h)$, $\Gamma'_h = (1 - \varepsilon^h)/(1 - \beta'\varepsilon^h)$

constraint on housing demand:

$$M_t^h \leq \gamma_m M_{t-1}^h + (1 - \gamma_m) \chi q_t^h H_t' \quad (37)$$

where $\gamma_m > 0$ governs the inertia of the borrowing upper bound χ . This constraint states that some $(1 - \gamma_m)$ impatient agents can roll over debts.⁹ For simplicity, housing is in a fixed supply. In this context, housing is a durable good that serves as a collateral for consumption. Hence, in trouble times, the fall in housing prices is further amplified by the constraint and largely contributes to the drop in consumption.¹⁰

The production sector comprises different entities that produce the final output of the economy. Competitive intermediate good producers are one of those. Intermediate producers sell goods to retailers by using raw capital and labor in the two groups of households. A standard Cobb-Douglas function captures the production of intermediate goods:

$$Y_t^m = A_t (\xi_t K_t)^\alpha L_t^{(1-\alpha)(1-\varphi)} L_t'^{(1-\alpha)\varphi} \quad (38)$$

where φ is the relative wage share of impatient households.¹¹ Variable A_t is an exogenous productivity measure. After production, firms need to invest in new capital for the next period. Hence, they borrow by issuing state-contingent claims in the capital market. Securities are backed by primary and securitized loans or are in the form of corporate debts in the bond market. Accordingly, the equilibrium condition on the capital market writes:

$$Q_t K_{t+1} = Q_t (S_t^c + S_t^b + S_t^h) \quad (39)$$

Under perfect competition, firms earn zero profits state by state, hence the marginal gain of capital is:

$$Z_t = P_t^m \alpha \frac{Y_t^m}{K_t} \quad (40)$$

in which P_t^m is the relative intermediate output price. The accumulation process of capital consists of the leftover capital stock net depreciation and the investment in new units of capital, as follows:

$$K_{t+1} = \xi_{t+1} (1 - \delta) K_t + I_t \quad (41)$$

⁹This specification is based on findings in Guerrieri & Iacoviello (2017), and implies that housing prices lead changes in the level of debt.

¹⁰In our framework, the collateral constraint is always binding for practical reasons. The latter means that we do not capture the asymmetric effects of housing booms and busts à la Guerrieri & Iacoviello (2017). In the absence of slackness of the constraint, housing booms will then overfeed consumption. However, Guerrieri & Iacoviello (2017) find that exogenous shocks that are not directly related to housing prices and/or consumption cause limited asymmetries. Accordingly, our qualitative results are relatively immune.

¹¹When $\varphi = 0$, the weight of impatient households in production goes to zero.

Note that firms are not directly exposed to financial frictions. However, they indirectly face borrowing constraint due to the relationship between commercial banks and patient households. Therefore, the level of the cost of capital is higher than in a standard new-Keynesian model.

Capital producers are in charge of the creation of the new unit of capital. After production, they sell it to the intermediate good producers at the market price Q_t . The production of capital is subject to adjustment costs. Thus, the profit maximization program is given by:

$$\max \mathbb{E}_t \sum_{i=1}^{\infty} \Lambda_{t,t+i} \left\{ Q_i I_i - \left[1 + f \left(\frac{I_i}{I_{i-1}} \right) \right] I_i \right\} \quad (42)$$

which by optimization yields to a standard Tobin's Q ratio between the assets' market value and the replacement cost:

$$Q_t = 1 + f \left(\frac{I_t}{I_{t-1}} \right) + \left(\frac{I_t}{I_{t-1}} \right) f' \left(\frac{I_t}{I_{t-1}} \right) - \mathbb{E}_t \Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 f' \left(\frac{I_{t+1}}{I_t} \right) \quad (43)$$

A continuum of j different retailers of a unit mass closes the production sector. The following CES aggregator function yields to the final output:

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (44)$$

Following sticky prices principle, we introduce nominal rigidities à la Calvo (1983). Accordingly, retailers change the price of the final output with probability $(1 - \gamma)$. Otherwise, retailers index prices to the inflation rate π_t . The optimal reset price P_t^* is then conditional on these probabilities with:

$$\max \mathbb{E}_t \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^i (1 + \pi_{t+k-1})^{\gamma^p} - P_{t+i}^m \right] Y_{jt+i} \quad (45)$$

The first-order necessary condition writes:

$$\sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^i (1 + \pi_{t+k-1})^{\gamma^p} - \frac{1}{1 - 1/\varepsilon} P_{t+i}^m \right] Y_{jt+i} = 0 \quad (46)$$

for which we derive the aggregate price level dynamic by using the law of large numbers:

$$P_t = \left[(1 - \gamma) (P_t^*)^{1-\varepsilon} + \gamma \left(\pi_{t-1}^{\gamma^p} P_{t-1} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (47)$$

2.5 Market clearing and Equilibrium conditions

The market clearing conditions for primary loans, ABS, long-term government bonds, housing, mortgage loans and deposits are:

$$\begin{aligned} K_{t+1} &= S_t^c + S_t^b + S_t^h & A_t^c &= A_t^b & \bar{B} &= B_t^c + B_t^h \\ H_t + H_t' &= 1 & M_t^h &= M_t^c & D_t^h &= D_t^c \end{aligned} \quad (48)$$

The aggregate resource constraint is:

$$Y_t = C_t + C_t' + \left[1 + f \left(\frac{I_t}{I_{t-1}} \right) \right] I_t + \bar{G} \quad (49)$$

The government budget constraint writes:

$$\bar{G} + R_t^b \bar{B} = T_t + \bar{B} \quad (50)$$

Last, the central bank conducts standard monetary policy following a Taylor rule of the form:

$$i_t = \bar{i} + \kappa_\pi \pi_t + \kappa_y (\log Y_t - \log Y_t^*) + \epsilon_t \quad (51)$$

where \bar{i} is the steady-state level of the nominal interest rate, Y_t^* is the flexible-price output and ϵ_t is an exogenous monetary shock.

3 Experiments

In this section we explore the ability of the model to mimic features of the GR and QE episodes. As stated before, our main goal is not quantitative but qualitative. Indeed, for the sake of parsimony, we ignore standard new-Keynesian features such as wage rigidities, and abstract from any securitization process in the housing market. In addition, we abstract from occasionally binding constraints, that would certainly refine the results. However, we do believe that the addition of these features would strengthen rather than reduce our key findings, and leave these refinements for further research.

The first two experiments consist in highlighting the comovements between primary and securitized loans. In the literature, much of the empirical and theoretical evidences support a countercyclical nature of the shadow banking system (see empirical evidences in Nelson et al. (2015) and Meeks et al. (2017) among others). The Figure 12 in Appendix A depicts the cyclical relationship between the sum of business non-financial corporate and non-corporate business loans by depository institutions and securitized other loans and advances by issuers

of ABS.¹² Securitized loans are mostly countercyclical and highly volatile until 2004. A clear break appears after 2004 (see Figure 13 in Appendix A for further details). This change coincides with the contractionary US monetary policy of the summer 2004 in response to the concerns about housing prices. Usually, securitization helps support the supply of credit by the banking sector. Issuers of ABS are mainly highly leveraged bankruptcy-remote companies that hold pools of assets. Pools cover a large range of assets such as consumer credits, Treasury securities and mortgages, and business loans. Their liabilities essentially consist of the issuance of pass-through securities, namely ABS. In other words, the interests, the principal payments and the amortization pass to the investors through the securities. In the end, these entities are inclined to cover more risk because they are only intermediaries between borrowers and investors. After securitization, the assets are more liquid and more marketable. Nevertheless, ABS are associated to higher risk. Indeed, if interest rates increase, borrowers are more likely to default which induces a great drop in the market value of the ABS. On the contrary, if interest rates decrease, borrowers are more likely to refinance debts which causes a fall in the interests of the ABS. In our view, the former case is a plausible explanation of the procyclical spiral in the credit market for firms between 2005Q1 and 2009Q1. Indeed, the traditional sector has incentives to expand securitization in such a period of asset bubbles and deflating interest rates. Thus, they benefit from the decrease in the cost of capital and the soaring in asset prices by clearing the risk out of their balance sheets. But there is a point where the explosion of the assets bubble and defaults inevitably cause the disruption of this mechanism. The latter is the main interest of our simulated experiments. In this respect, Figure 13 in Appendix A displays the cyclical components in both business and securitized loans in deviation from their levels in 2007Q4. Their joint rise in the beginning of the recession are certainly related to the strong inertia in the credit market for firms. In other words, the downturn in the real economy leads the crash in the market. To convince the reader, we expose the model to an exogenous degradation of the collateral value of ABS and find an intrinsic procyclical spiral.

As exposed earlier, the remainder of the exercises serves to dissociate over the transmission channels of QE. To do so, we refine the standard specification of QE by introducing banking reserve requirements and the supply of excess reserves to the model.

¹²Other loans and advances by issuers of ABS are non-financial business loans securitized by depository institutions and finance companies and syndicated loans to non-financial corporate businesses. Ref: https://www.federalreserve.gov/apps/fof/Guide/z1_tables_description.pdf

3.1 Calibration

The parameters of the baseline model are calibrated along the values of GK, Meeks et al. (2017) and the estimates of Guerrieri & Iacoviello (2017).¹³ These values are adjusted to a quarterly frequency. They mostly reflect economic conditions before the GR in the US.

The estimates of Guerrieri & Iacoviello (2017) are: the discount factors for patient β and impatient households β' , the capital share α , the capital depreciation rate δ , the habit parameters in consumption ε^c and housing ε^h , the weight for housing preferences \bar{j} , the wage share between patient and impatient workers φ , the maximum loan-to-value ratio χ , the labor disutility η and the factor of inertia γ^m . The inverse elasticity of investment η_i , the price rigidity parameters γ and γ^p and the Taylor rule coefficients κ_π , κ_X , ρ , are taken from GK.

The parameters related to the financial system are set to hit the following data targets: the survival probability σ matches an expected lifetime for bankers and brokers of roughly ten years. The parameters θ^c , ξ^c , Δ and Ξ are specific to the commercial banks and are adjusted to target data on the excess returns $(\bar{R}^s - \bar{R})$, $(\bar{R}^a - \bar{R})$, $(\bar{R}^b - \bar{R})$ and the equity multiplier $\bar{\phi}^c$. The excess return on primary loans $(\bar{R}^s - \bar{R})$ corresponds to the average Moody's AAA corporate bond yield relative to yield on 10-year Treasury constant maturity between 1990Q1 and 2007Q4; that is, 100 basis points. According to estimates from Meeks et al. (2017), we fix the excess return on ABS $(\bar{R}^a - \bar{R})$ to 25 basis points.¹⁴ The excess return on long-term government bonds $(\bar{R}^b - \bar{R})$ is of 50 basis points, in line with the average estimates of the term premium on a 10-year zero coupon bond by Adrian et al. (2013) between 2000Q1 and 2007Q1.

The average equity capital to total assets for commercial banks is 9.5 between 2000Q1 and 2007Q1.¹⁵ However, in the model, the moral hazard problem overstates risk within the financial system, hence fluctuations in asset prices. In that respect, GK argue that values between 4 and 6 compensate the absence of the risk-sharing components between borrowers and commercial banks. We follow their lead and fix the steady-state value for the bankers' equity multiplier $\bar{\phi}^c$ to 4. The two parameters θ^b , ξ^b dedicated to the shadow banks are adjusted to match a broker's equity multiplier of 8. For the same above reasons, this value reflects half of the average of investment banks' leverage ratio reported in GK and Gertler and Kiyotaki (2015).

The portfolio adjustment cost κ ensures that steady-state returns are equivalent along the

¹³Readers are reminded that the main contribution of this paper is not quantitative, hence the estimation of parameters is not of high interest. In addition, most of the parameter values are adjusted to match data targets.

¹⁴This value reflects an average pre-crisis ABS spread over swap rates for high-quality securitized assets such as credit card, equipment or mortgages, see Meeks et al. (2017) for further details.

¹⁵This figure is obtained using time series for the Total Equity Capital for Commercial Banks in United States and Total Assets, All Commercial Banks in the F.R.E.D database.

capital markets; in other words, credit and bond markets. In doing so, borrowers will most likely turn to the bond market to bypass financial frictions during bust periods. In addition, the bond market represents a large fraction of the capital market according to the steady-state values \bar{S}^h/\bar{K} and \bar{B}^h/\bar{B} . Those are in line with the calibration of GK as well as the shares of long-term government bonds and public expenditure to output (\bar{B}/\bar{Y} , \bar{G}/\bar{Y}). Parameter values are summarized in Table 1 below.

Table 1: Parameters values

Parameter values	
Capital share in the production function, α	0.300
Discount rate patient households, β	0.995
Discount rate impatient households, β'	0.992
Capital depreciation rate, δ	0.025
Habit in consumption, ε^c	0.684
Habit in housing, ε^h	0.879
Wage share patient/impatient φ	0.501
Labor desutility, η	1.000
Inertia in the borrowing constraint, γ^m	0.694
Maximum loan-to-value ratio, χ	0.900
Inverse elasticity of investment, η_i	1.728
Elasticity of substitution between goods, ε	4.167
Probability of price rigidity, γ	0.779
Measure of price indexation, γ^p	0.241
Taylor Rule inflation coefficient, κ_π	1.500
Taylor Rule markup coefficient, κ_X	-0.125
Survival probability, σ	0.972
Divertibility of primary loans, θ_c	0.382
Divertibility of securitized assets, θ_b	0.271
Relative divertibility of government bonds, Δ	0.500
Relative divertibility of ABS, Ξ	0.250
Transfers to new bankers, ξ_c	0.002
Transfers to new brokers, ξ_b	0.001
Bond market, transaction costs, κ	1.000
Weight on housing preferences, \bar{j}	0.040
Steady-state values	
Proportion of securities held by patient households, \bar{S}^h/\bar{K}	0.500
Proportion of gov. bond held by patient households, \bar{B}^h/\bar{B}	0.750
Government bonds to output, \bar{B}/\bar{Y}	0.450
Government expenditures to output, \bar{G}/\bar{Y}	0.200
Share of securitized assets on capital, \bar{S}^b/\bar{K}	0.150

3.2 Financial crisis experiments

Figures 1 and 14 in Appendix A, report the impulse response functions (IRF) to an exogenous capital quality shock à la GK. This shock either captures a huge depreciation of the price of capital or a fall in the quality of capital. In this context, we compute the IRF of the model with and without shadow banks to quantify the marginal contribution of the securitization. The results indicate that the securitization weakly amplifies the macroeconomic fluctuations. This result can be understood along the following lines. The decline in the market price of primary and securitized loans deteriorates the net worth of bankers and brokers. Brokers' balance sheet constraint explodes due to their high degree of leverage. As a consequence, brokers proceed to fire sales of assets to dampen losses, and cease supplying ABS, which raises their market price. The leverage constraint also leads bankers to experience losses. The marginal gains from diverting primary loans shoots up. Hence, bankers roll over debts by purchasing cheaper primary securities, which further diminish the demand for deposits. The shock causes a sharp reduction in the supply of ABS, which serves as a buffer mechanism inside the financial system. The latter is particularly visible in the response of bankers' net worth. However, this buffer mechanism can not entirely offset the contraction in investment and output.

While this GK exercise is widely used in the literature to mimic the effects of the GR, it can not generate the procyclical spiral in the credit market for firms that actually characterized the GR. In the next experiment, we look at the effects of a direct disruption in the securitization mechanism, that actually captures this salient feature of the GR.

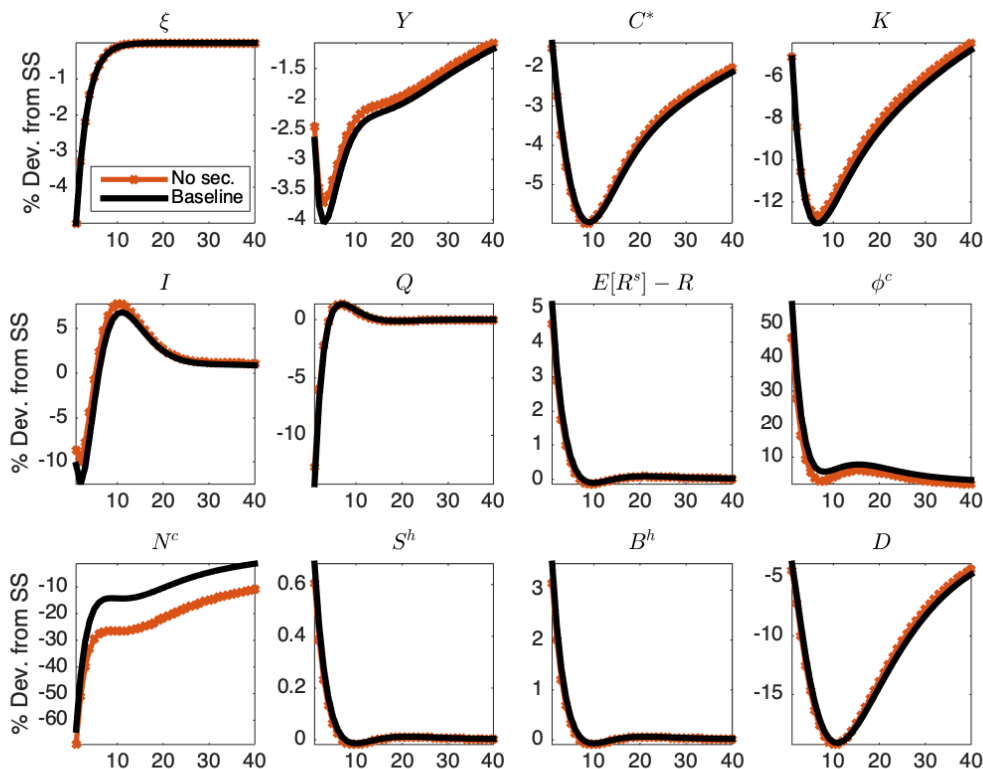
The next experiment lowers the collateral value of ABS by exogenously increasing the weight Ξ in the bankers balance sheet constraints (see Equation (3)). In the interest of gauging the quantitative properties of the baseline model, the shock targets the 2009Q2 detrended drop of real GDP (see Figure 13 in Appendix A).¹⁶ Figure 2 shows the matching of the IRF.¹⁷ The exogenous change in the weight parameter Ξ distorts the optimal condition in Equation (7). The marginal benefit from diverting ABS suddenly jumps to a huge level, and bankers swiftly cut their demand for ABS. In response, the market price of ABS falls and triggers a big contraction in the net worth of bankers. Bankers then sell large amounts of assets in return for short-term liabilities, reflecting a greater discipline. As a consequence, the net worth of brokers falls.

The procyclicality spiral applies when bankers are intrinsically subject to a greater risk

¹⁶The persistence parameter of the shock is 0.96 in order to match a persistent decline in real GDP. We loop over the variance parameter of the shock to minimize the distance between the data target and the deviation in output in the model.

¹⁷Figures 15 and 16 in Appendix A provide other key IRF.

Figure 1: Capital quality shock



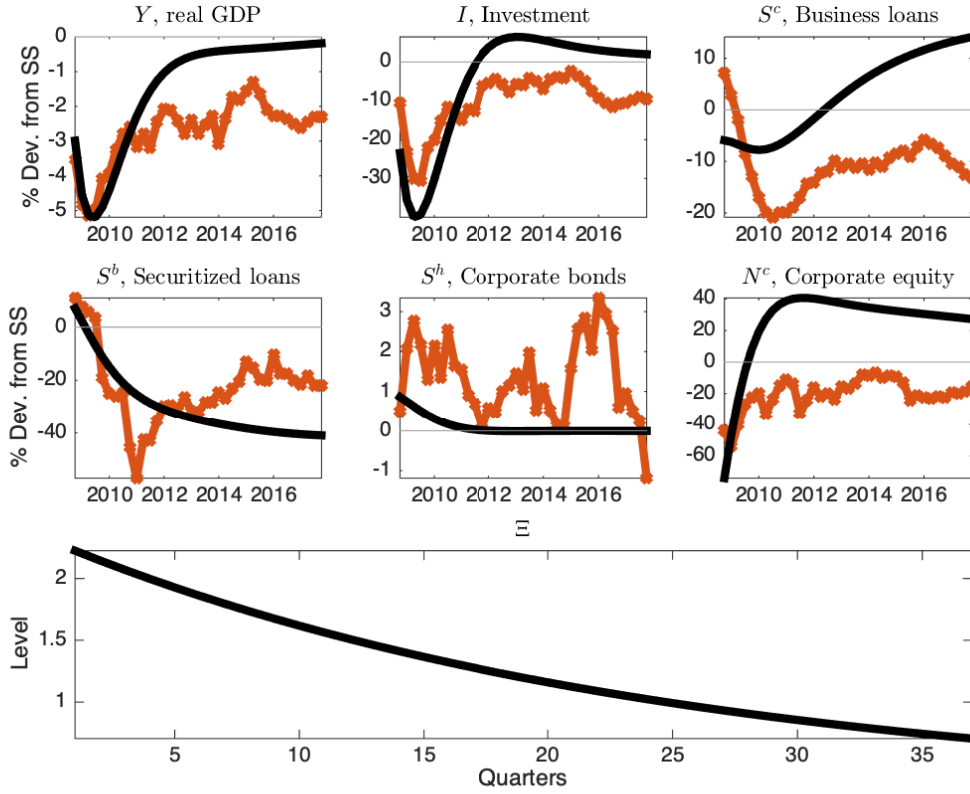
IRF of the baseline model to a 5% unanticipated capital quality shock with and without the securitization mechanism. The black solid lines show the responses in the baseline model with securitization. The red solid lines with crosses are responses related to the model without securitization. C^* is the sum of consumption by patient and impatient households. Each response is expressed in percent deviation from steady-state.

exposure; that is, when the collateral quality of ABS shrinks. Along the lines of Gertler et al. (2012), bankers adopt more discipline in the presence of a riskier environment. Indeed, the Figure 17 in Appendix A displays evidence that macroeconomic fluctuations are more pronounced in a safer environment. In our view, the baseline model generates convincing qualitative results and adequate quantitative properties. Thus, we believe that the model is a suitable environment to study for the theoretical transmission channels of QE programs.

3.3 Quantitative Easing experiments

In order to perform QE experiments, we impose some modification to the baseline model, and first introduce QE shocks à la GK. A more realistic QE experiment is conducted after this first-pass analysis. To this end, a balance sheet of the central bank is introduced to the model. The monetary authority finances its operations by issuing costless short-term debts D_t^q , bought

Figure 2: Securization crisis



The black solid lines represent IRF to an unanticipated securitization shock, characterized by an increase in Ξ . The red solid line with crosses are detrended US data given in percent change from 2007Q4 level. Each response related to the DSGE model is expressed in percent deviation from steady-state, except for Ξ . Details about the data are given in Appendix D.

by patient households. The balance sheet of the central bank then writes:

$$D_t^g = Q_t S_t^g + q_t^a A_t^g + q_t^b B_t^g \quad (52)$$

which is simply zero in the absence of asset purchases. The equilibrium conditions in the financial markets must now read:

$$K_{t+1} = S_t^c + S_t^b + S_t^h + S_t^g \quad (53)$$

$$A_t^c + A_t^g = A_t^b \quad (54)$$

$$\bar{B} = B_t^c + B_t^h + B_t^g \quad (55)$$

The consolidated government budget constraint is now modified:

$$\bar{G} + R_t^b \bar{B} = T_t + \bar{B} + (R_t^s - R_t) Q_{t-1} S_{t-1}^g + (R_t^a - R_t) q_{t-1} A_{t-1}^g + (R_t^b - R_t) q_{t-1}^b B_{t-1}^g \quad (56)$$

In a baseline scenario, we only allow the central bank to purchase assets in the primary market; that is, in the balance sheet of bankers.¹⁸ These asset purchases are driven by exogenous first-order disturbances, as follows:

$$S_t^g = \psi_t^c(S_t^c(+S_t^b + S_t^h)), \quad A_t^g = \psi_t^a(A_t^c(+A_t^b)), \quad B_t^g = \psi_t^b(\bar{B}^c(+B_t^h)), \quad (57)$$

$$\psi_t^i = \rho^i \psi_{t-1}^i + \varepsilon_t^i, \quad i = \{c, a, b\}.$$

in which blue colored variables refer to the interventions of the central bank into the balance sheet of brokers and in the bond market. By relaxing the degrees of interventions, we can easily disentangle how each type of purchase, in each market, is effective in supporting the real economy. In their studies, GK and Meeks et al. (2017) find that the effectiveness of asset purchases is an increasing function of the value of the asset weights in the incentive constraints. Figures 3 and 4 display similar results in our model.¹⁹ Unsurprisingly, a purchase of primary loans is marginally the most effective policy because these assets have, by definition the greatest leverage effect. Hence, QE à la GK is almost transmitted one for one through the rise in credit supply (see the equilibrium relations (53), (54) and (55)), abstracting from the relative changes in the supply of assets. This mechanism artificially overestimates the impacts of QE, as will be clear later.

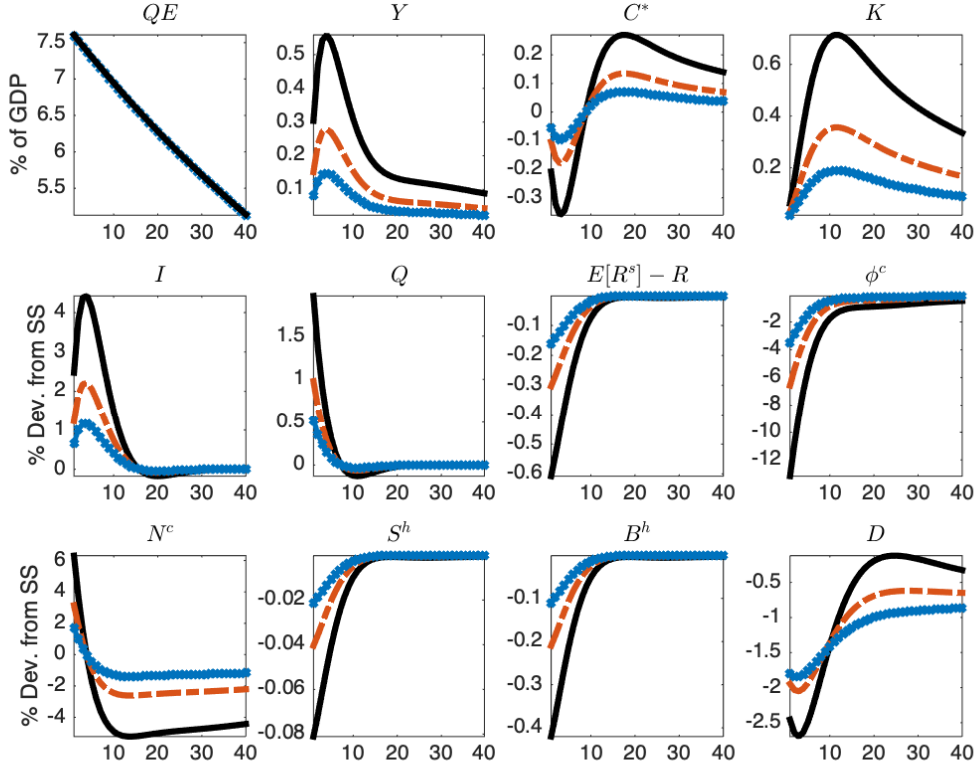
Figure 5 allows for variations in market interventions, and shows that the effects of QE programs depending on the type of asset are similar, regardless of the market on which the intervention takes place. For instance, the impact of primary securities purchases S^c in the primary market are equivalent to the effects of a purchases of households securities S^h in the bond market.²⁰ As such, this way of designing QE intervention clearly shuts down some of the theoretical transmission channels of QE. In particular, it remains silent on the implications of QE programs within the financial system, as it does not trigger changes in the relative supply of assets. These shortcomings are overturned by refining the design of QE intervention, introducing banking reserves in the model and using the supply of excess reserves to implement QE programs. This way of designing QE programs in a DSGE model with financial frictions is both more realistic with respect to the actual implementation of QE programs, and allows for more refined understanding of the different channels through which QE affects the financial system, and *in fine* the real economy.

¹⁸Superscript g indicates central bank related variables. Unlike GK, we ignore efficiency costs per unit of asset purchases, as these costs have negligible implications for the overall dynamics and therefore are of interest only for welfare analysis.

¹⁹In the baseline scenario, we match the increase in the total assets of Federal Reserve banks as a percentage of GDP, *i.e.* 7.59% between 2008Q3 and 2008Q4 to investigate the macroeconomic implications of QE

²⁰This result applies even with changes in the transaction costs κ .

Figure 3: Quantitative Easing: public intermediation



IRF to unanticipated QE shocks with a mechanism à la GK. The shocks are calibrated to match the observed deviation of total assets of Federal Reserve to GDP (7.59% of GDP between 2008Q3 and 2008Q4). The black solid lines are the responses to primary loan purchases. The red dashed line are purchases of long-term government bonds. The blue solid lines with crosses are ABS purchases. Each response is expressed in percent deviation from steady-state, except those of QE, that are given in percentage points of annual GDP.

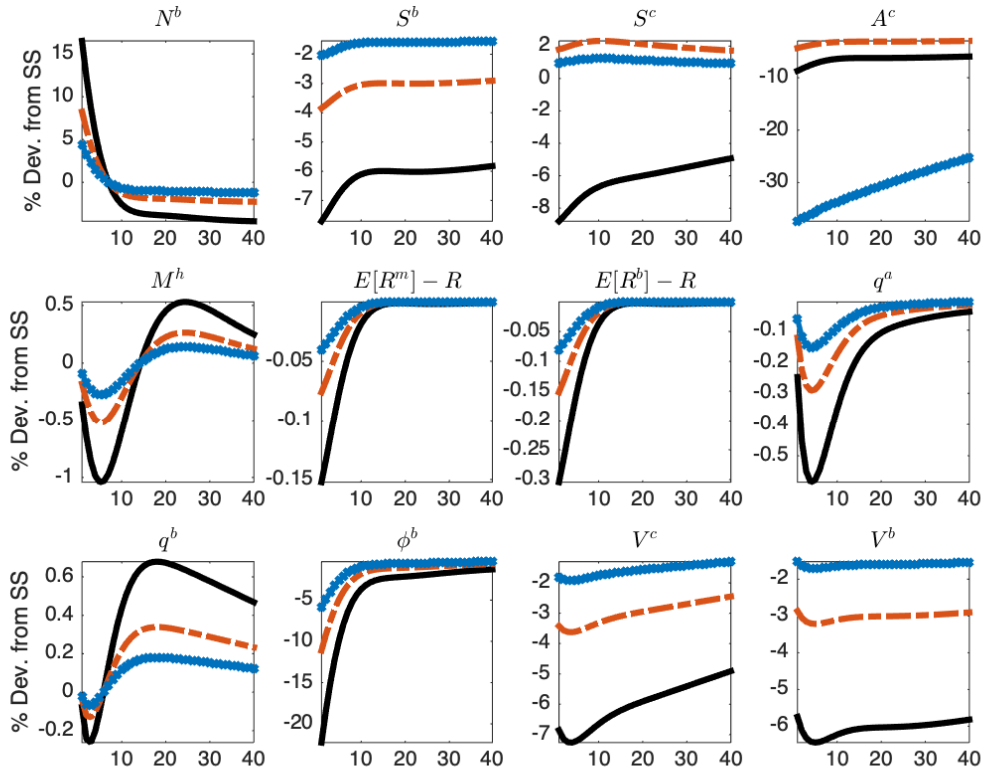
Including banking reserve requirements is more realistic. In addition, the introduction of the supply of excess reserves allows us to monitor the portfolio reallocations within the balance sheets of financial intermediaries. Indeed, it will make it possible to assess the implications of an excess return on both types of reserves, something that the GK mechanism overlooks. The addition of reserve requirements changes the optimization program of a representative banker. The balance sheet identity and the law of motion of net worth become:

$$Q_t s_t^c + q_t^a a_t^c + q_t^b b_t^c + q_t^h m_t^c + q_t^{rr} r r_t^c = d_t^c + n_t^c \quad (58)$$

$$\begin{aligned} n_t^c &= R_t^s Q_{t-1} s_{t-1}^c + R_t^a q_{t-1}^a a_{t-1}^c + R_t^b q_{t-1}^b b_{t-1}^c + R_t q_{t-1}^h m_{t-1}^c + R_t^{rr} q_{t-1}^{rr} r r_{t-1}^c - R_t d_{t-1}^c \\ &= (R_t^s - R_t) Q_{t-1} s_{t-1}^c + (R_t^a - R_t) q_{t-1}^a a_{t-1}^c + (R_t^b - R_t) q_{t-1}^b b_{t-1}^c \\ &\quad + (R_t^{rr} - R_t) q_{t-1}^{rr} r r_{t-1}^c + R_t n_{t-1}^c \end{aligned} \quad (59)$$

where q_t^{rr} is a market price for banking reserves, determined by the general equilibrium. The

Figure 4: Quantitative Easing: public intermediation, cont.



market price for banking reserves will help to dissociate the different transmission channels of QE, as will be clearer later. Realistically, the baseline (first) scenario defines the excess return on both, requirements and excess reserves, as an upper target of the nominal interest rate. In this scenario, the excess return on reserves is given by:

$$\bar{R}_t^{rr} = v \times i_t \quad (60)$$

where v sets the upper limit. In the second scenario (market rate), banking reserve requirements pay a fixed coupon as follows:

$$R_{t+1}^{rr} = \frac{\varsigma + q_{t+1}^{rr}}{q_t^{rr}} \quad (61)$$

Banking reserves are defined as a fixed fraction of households deposits:²¹

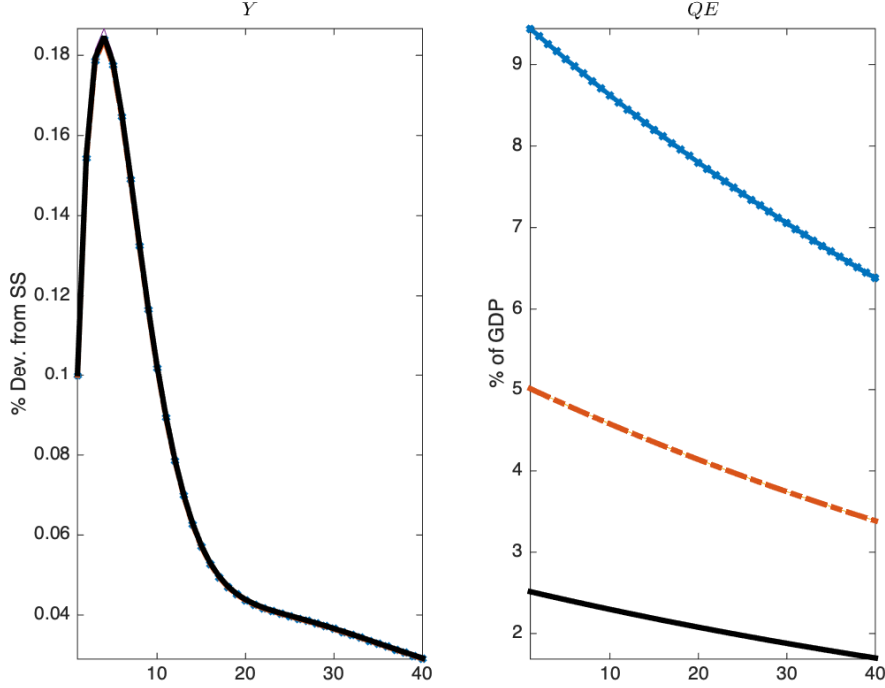
$$rr_t^c = \varphi^{rr} d_t^c \quad (62)$$

where the requirement ratio φ^{rr} is up to 10%.²² We also modify the incentive constraint to allow bankers to divert a fraction of reserve requirements, which is a way for us to model a

²¹ ς is set so that the steady-state level of the market price of reserves is one.

²²This reserve requirement ratio corresponds to the ratio imposed to the highest tranche of depository institutions. Ref: <https://www.federalreserve.gov/monetarypolicy/reservereq.htm>.

Figure 5: Quantitative Easing: interventions on different markets



IRF to unanticipated QE shocks with a mechanism à la GK. The shocks are calibrated to match a 0.10% increase in output in the first period. The black solid lines depict QE purchases of securities in the primary, secondary and bond markets. The red dashed lines are purchases of ABS in the primary and the secondary markets. The blue solid lines with crosses show long-term government bond purchases in the primary and bond markets. For each class of assets, there is only one visible line, because the responses are combined despite changes of market interventions.

limits to arbitrage.²³ We adjust the parameter Θ to match the average interest spread of 10 basis points between the IORR and the effective FFR (see Figure 11 in Appendix A). The banker's incentive constraint and the linear guessed solutions of underdetermined coefficients are modified and become:

$$V_t^c \geq \theta^c(Q_t s_t^c + \Xi q_t^a a_t^c + \Delta q_t^b b_t^c + \Theta q_t^{rr} r r_t^c) \quad (63)$$

$$V_t^c = \mu_t^c Q_t s_t^c + \mu_t^a q_t^a a_t^c + \mu_t^b q_t^b b_t^c + \mu_t^r q_t^r r r_t^c + \eta_t^c n_t^c \quad (64)$$

which yields an additional first-order condition, and a slight change in the incentive constraint. The time-varying coefficient μ_t^r ; that is, the marginal cost of diverting reserves is:

$$\mu_t^r = \mu_t^c \Theta \quad (65)$$

$$Q_t s_t^c + \Xi q_t^a a_t^c + \Delta q_t^b b_t^c + \Theta q_t^{rr} r r_t^c = \phi_t^c n_t^c \quad (66)$$

$$\mu_t^r = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c (R_{t+1}^{rr} - R_{t+1}) \quad (67)$$

²³Some empirical evidences of such frictions are documented in Martin et al. (2015).

where the second equation (66) is the new balance sheet equation.²⁴ Aggregating among bankers, the law of motion of net worth is now:

$$N_t^c = \sigma \left((R_t^s - R_t) Q_{t-1} S_{t-1}^c + (R_t^a - R_t) q_{t-1}^a A_{t-1}^c + (R_t^b - R_t) q_{t-1}^b B_{t-1}^c \right. \quad (68)$$

$$\left. + (R_t^{rr} - R_t) q_{t-1}^{rr} R R_{t-1}^{rr} + R_t N_{t-1}^c \right) + \chi^c \quad (69)$$

We close the model by the QE program, designed as an excess supply of reserves driven by first-order disturbances:²⁵

$$ER_t^g = v_t^c Q_t S_t^g + v_t^a q_t^a A_t^g + v_t^b q_t^b B_t^g \quad (70)$$

$$v_t^i = \rho^i v_{t-1}^i + \varepsilon_t^i, \quad i = \{c, a, b\}. \quad (71)$$

The consolidated government budget constraint and the equilibrium condition for total reserves are:

$$\bar{G} + R_t^b \bar{B} + R_t^{rr} T R_t^g = T_t + (R_t^{rr} - R_t) ER_t^g + \bar{B} + T R_t^g \quad (72)$$

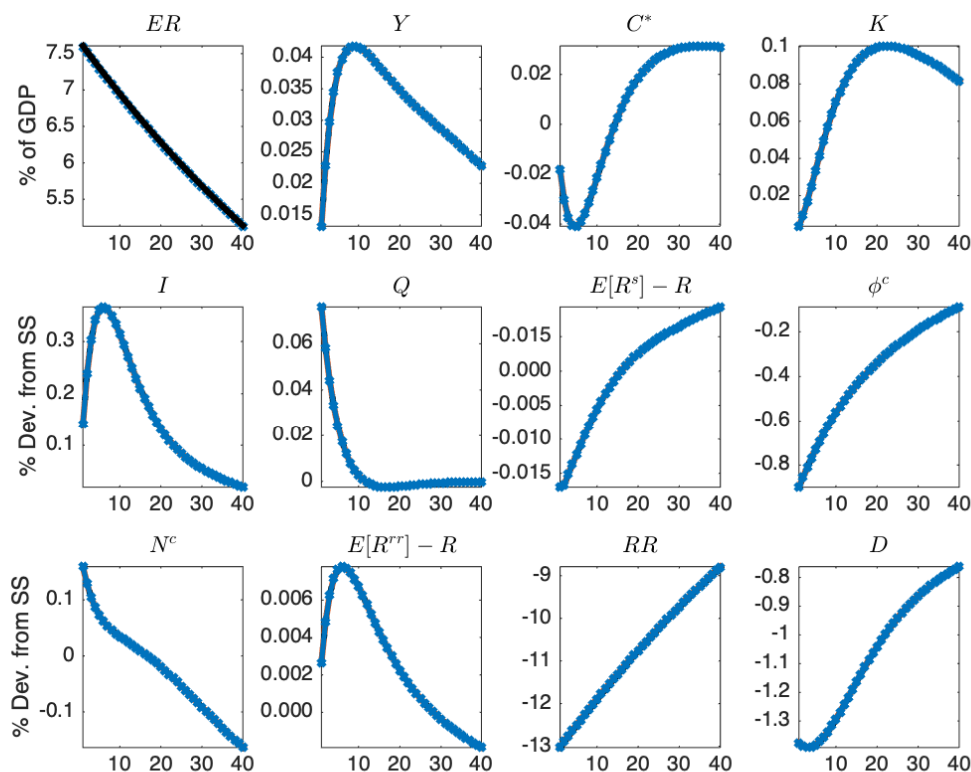
$$T R_t^g = R R_t^c + ER_t^g \quad (73)$$

Before we compare the results obtained with this design of QE program through excess reserves to the GK specification of QE programs, we consider the effects of an excess reserves QE experiment on each type of asset purchases in the baseline framework (first scenario). Figures 6 and 7 reveal that QE programs have similar effects regardless of the type of asset purchases. The mechanism proceeds as follows: The liabilities of the central bank is exogenously expanded with excess reserves. Reserve requirements are mechanically diminished due the equilibrium relation in equation (73). Consequently, in any case, QE here, exerts a relaxing leverage effect proportional to the weight Θ in the balance sheet of bankers (see equation (66)). We believe that this decrease in reserve requirements is analogous to a purchase of assets in the balance sheet of bankers. Here the loosening of constraints is thus indirect. The latter entails a leveraging process. As a result, bankers take more risks and supply more primary loans. These reallocations dampen the demand of bankers for ABS, and brokers suffer from this shortage. However, their net worth is partially protected by the soaring in the market price of capital. From a macroeconomic perspective, the increase in asset prices causes wealth effects for households and firms. In addition, both also benefit from the sharp fall in the cost of capital,

²⁴ $\chi^c = \omega^c \left(\bar{Q} \bar{S}^c + \bar{q}^a \bar{A}^c + \bar{q}^b \bar{B}^c + \bar{q}^{rr} \bar{R} R^{rr} \right)$

²⁵ The persistence parameters $\rho^i, i = \{c, a, b\}$ have similar values as in the GK-like QE experiments; that is 0.96. The market price of reserves q_t^{rr} has to be removed in each of these equations to obtain the first scenario; that is, the upper target of the nominal interest rate.

Figure 6: Quantitative Easing through Excess Reserves



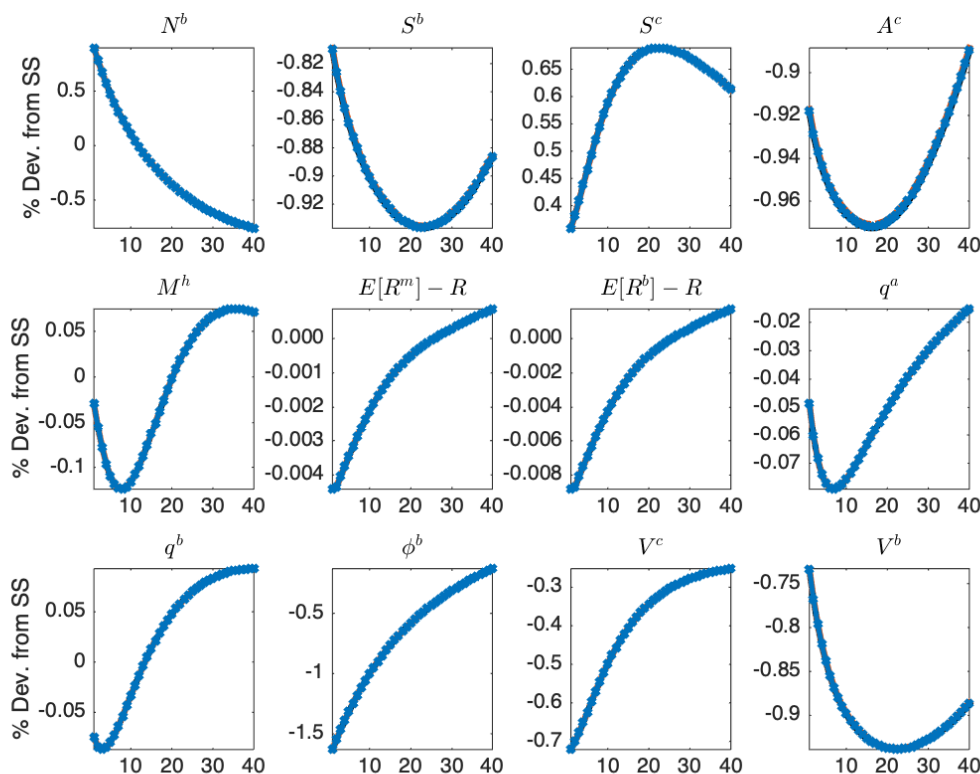
IRF to unanticipated QE shocks through excess reserves. In this scenario, the interest rate in reserves is fixed and proportional to the nominal interest rate. The purchases are similar for any type of asset purchase. For each panel, responses overlap and form a single blue solid line with crosses. Each response is expressed in percent deviation from steady-state, except for excess reserves that are given in percent of GDP.

and from the increase in the collateral value of housing. In brief, QE is clearly effective in fostering financial conditions in the traditional sector. Nevertheless, the macroeconomic effects of QE are extremely small quantitatively in comparison of the huge increase in the supply of excess reserves.

Given the small effects of a QE program modeled as an excess of reserves, we compare our results to an equivalent QE program with a mechanism à la GK. Figures 18 and 19 in Appendix A displays a program of ABS purchases under both specifications.²⁶ We remind the reader that the ABS purchase is the less effective purchase in a GK specification. The results show that the fluctuations are significantly greater with a GK mechanism than when QE is modeled as an excess of reserves. The way QE programs are modeled is thus crucial to determine the overall qualitative and quantitative effects of QE programs, and mechanisms à la GK can seriously overstate their importance. In addition, we compute gradual changes in the weight Θ that as

²⁶We report ABS purchases because responses are closer in magnitude for both specifications. Indeed, other asset purchases with a mechanism à la GK, such as primary loans and long-term government bonds, are by far more effective, as already shown in Figures 3 and 4.

Figure 7: Quantitative Easing through Excess Reserves, cont.



a reminder, symbolizes the steady-state level in the excess return of reserves. Our goal is to find a value for Θ that matches the deviation in output under both specifications. Figure 20 in Appendix A displays the results. A simple back of the envelope calculation indicates that Θ must be roughly 5 for output to get close to the target – the output effect obtained with a GK mechanism. The latter would imply a steady-state excess return on reserves of 500 basis points, very far from empirically realistic values.²⁷ These elements indicate that the macroeconomic effects of QE programs might be significantly overestimated in the literature, that mostly relies on GK mechanisms.²⁸

Let us now focus on a potential drawback of the modeling of QE as an excess of reserves in the first scenario, namely the fact that it is rather tricky to identify the relative importance of the various transmission channels of QE. The portfolio balance channel – changes in the composition of portfolios of financial intermediaries – unquestionably triggers fluctuations within the financial system in the above experiment. In addition, the reduction in the cost of capital and the rise in primary securities can be thought of as a credit channel. Finally, the rise in asset prices generates strong wealth effects. Unfortunately however, the first (baseline) scenario

²⁷Such a large steady-state spread also leads to equilibrium indeterminacy issues.

²⁸see Jouvanceau (2019) for an empirical discussion on this issue.

of QE modeled as an excess of reserves does not help distinguish the different channels at play. Our second scenario addresses this issue.

In the second scenario, we separate the transmission channels of a QE program modeled as an excess of reserves. To do so, we switch to the market rate scenario. The idea is to write a model in which asset prices exhibit more dynamics and affect the financial system more significantly. On the one hand, larger macroeconomic fluctuations would be supportive evidence of the asset prices effects of QE programs (wealth effect). On the other hand, smaller fluctuations would indicate that the credit channel dominates. The results in Figures 21 and 22 in Appendix A support the view that wealth effects are more important. Indeed, the portfolio reallocations are qualitatively similar to the first scenario. However, the fluctuations in asset quantities are dramatically reduced, while deviations in output and investment are of close magnitude. In addition, the baseline and market scenario also exhibit differences in the persistence of the effects. The latter are mostly explained by the sluggish rises in capital, that alters the bank lending transmission channel – this is certainly at play but is dominated in magnitude by the wealth effects. Finally, as in the GK vs. excess reserves comparison exercise, we play with the value of Θ . Figure 23 in Appendix A emphasizes that macroeconomic fluctuations are clearly increasing in the fluctuations in asset prices.

All of these experiments support the view that the relative changes in asset prices clearly alter bankers' decisions. On the one hand, the portfolio rebalancing effects of QE alleviate financial frictions and foster financial conditions in the traditional sector. On the other hand, aggregate demand is slightly stimulated by wealth effects, that in turn boost households' consumption and private investment.

4 Conclusion

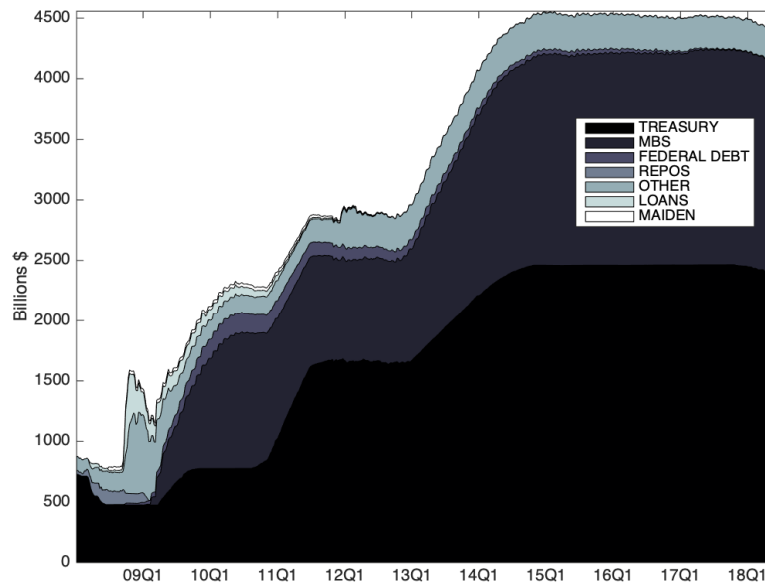
In this paper, we show that financial frictions are useful to understand the GR and the effects of QE programs. We extend the seminal paper of GK and refine the financial sector to include securitization (in the form of ABS and a shadow banking system) and housing, with a collateral constraint that affects households consumption. These elements help explain the procyclical credit market spiral in the wake of the GR. In this context, we demonstrate that a greater exposure to risk triggers discipline in bankers' decisions of portfolio reallocations. Shadow and commercial bankers accumulate losses which further and endogenously magnify the size of the subsequent economic downturn.

We then conduct QE experiments. Our first contribution is to model QE programs as being

implemented through an excess of reserves, closer to the actual implementation schemes. We show that implementation matters for the effects of QE programs: the financial and macroeconomic effects are clearly dampened when QE is implemented through an excess of reserves – despite huge amounts of simulated excess reserves – compared to an simplified implementation scheme à la GK. Our second contribution is to allow for multiple transmission channels of QE programs, and gauge their relative importance. Our assumptions allow for a portfolio balance channel, a credit channel and an asset prices (wealth) channel of QE. Our results suggest that QE programs are mainly effective through wealth effects; that is, through the asset prices channel. Finally, our qualitative results are robust to quantitative refinements.

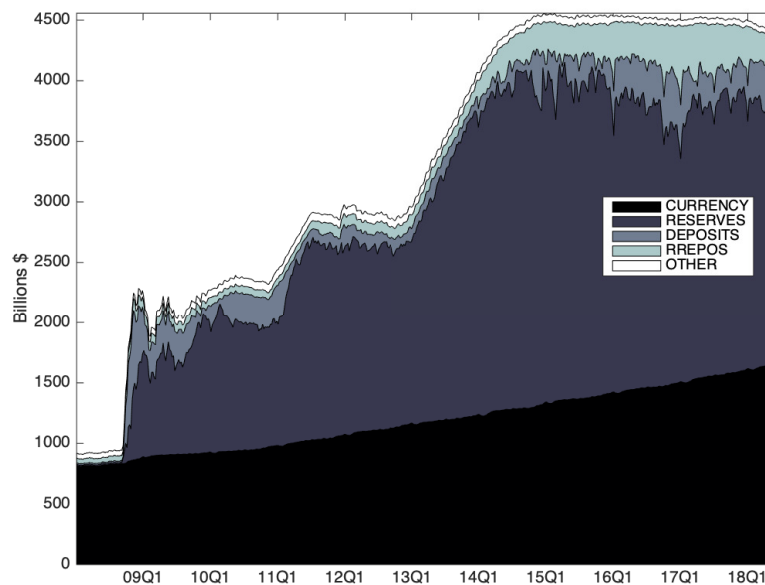
Appendix A - Additional Figures

Figure 8: Assets of the Federal Reserve



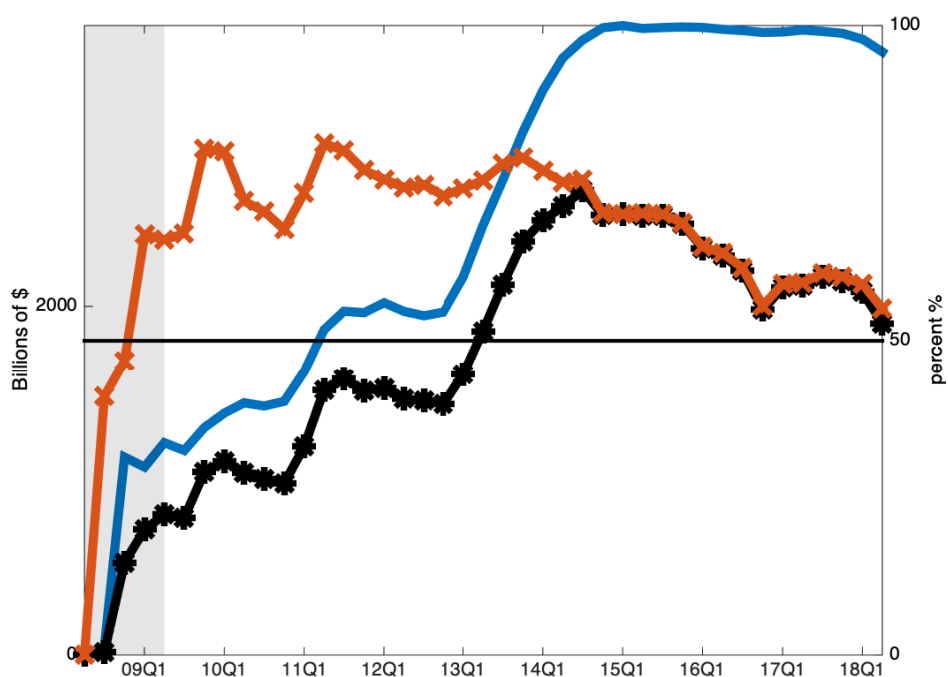
TREASURY: US Treasury securities, MBS: Mortgage-backed securities, FEDERAL DEBT: Federal agency debt securities, REPOS: Repurchase agreements, MAIDEN: Net portfolio holdings of Maiden Lane LLC, OTHER = Float + Central bank liquidity swaps + Other Federal Reserve assets + Foreign currency denominated assets + Gold stock + Special drawing rights certificate account + Treasury currency outstanding + Unamortized premia and discounts on securities held outright.

Figure 9: Liabilities of the Federal Reserve



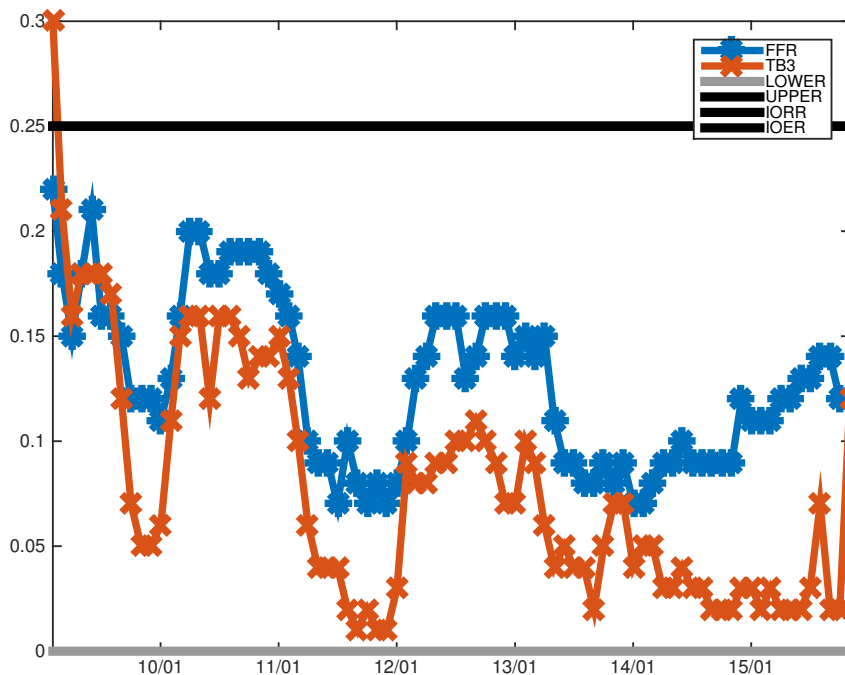
DEPOSITS: Deposits with F.R. Banks, other than reserve balances, RREPOS: Reverse repurchase agreements, OTHER = Treasury cash holdings + Other liabilities and capital

Figure 10: Share of excess reserves in total assets, for all Federal Reserve banks



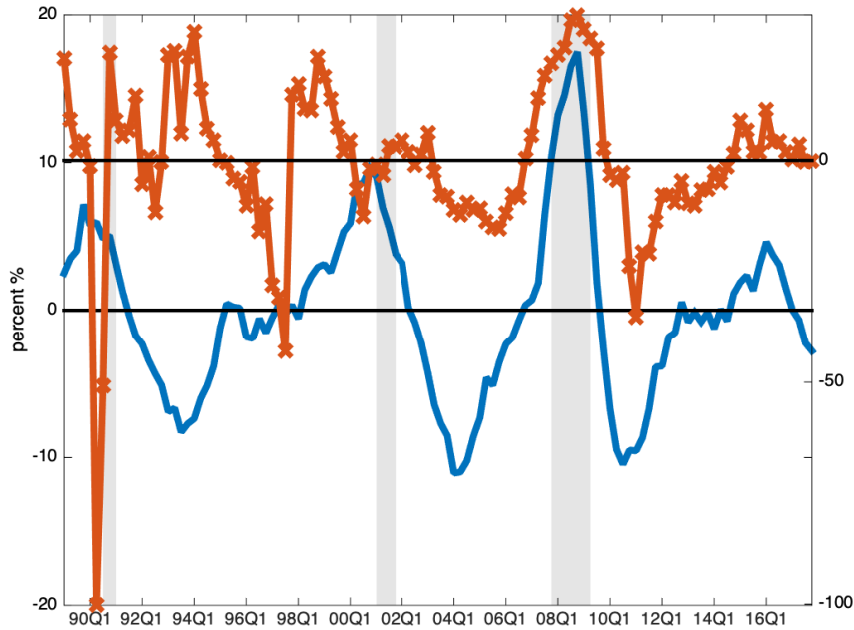
The left-hand scale and blue solid line represents excess reserves of depository institutions. The left-hand scale and black line with stars shows the total assets of all Federal Reserves banks. The right-hand scale in red solid line with crosses is the share of excess reserves in total assets. The gray shaded area is the Great Recession period. Further details on data are given in Appendix D.

Figure 11: Main interest rates for the conduct of conventional monetary policy in the US



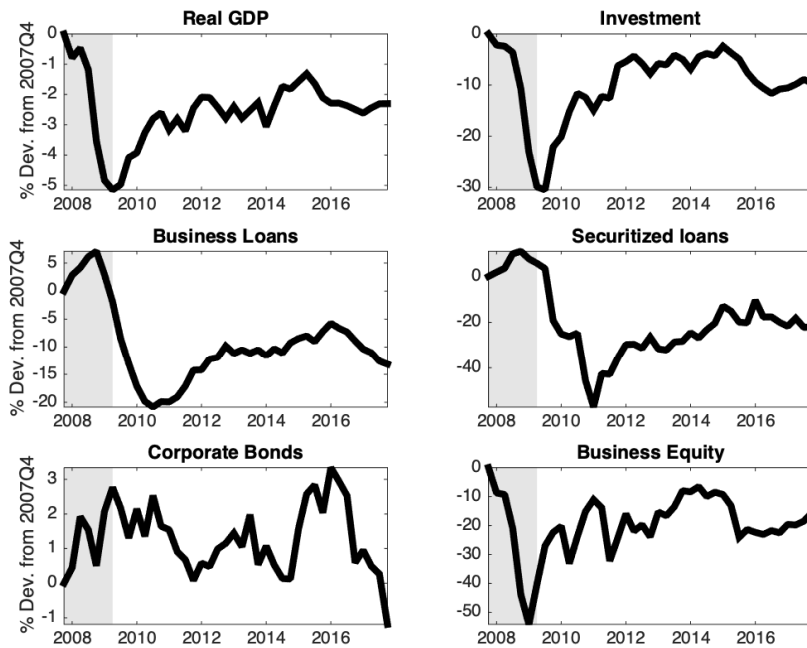
The upper limit of the federal funds target range and interest rates on both required and excess reserves are combined in the black line. The blue line is the effective federal funds rate. The red line shows the 3-month Treasury bill rate.

Figure 12: Cyclical components of business loans and ABS



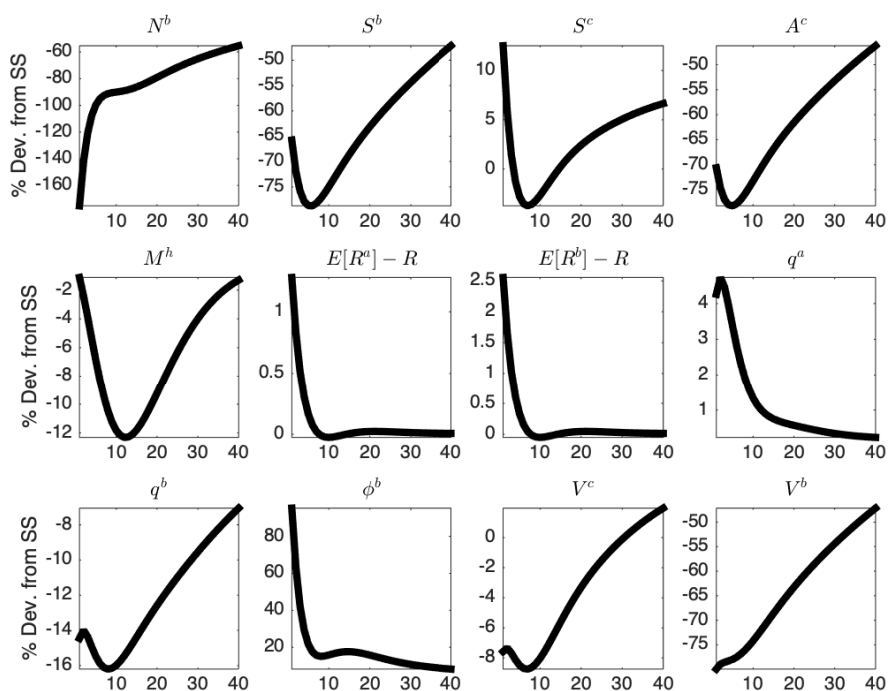
The left-hand scale and blue solid line represent the sum of business non-financial corporate or non-corporate business loans by depository institutions. The right-hand scale and red solid line with crosses shows securitized other loans and advances by issuers of asset-backed securities. Both series are deflated with the implicit price deflator computed by the US Bureau of Economic Analysis. Cyclical components are filtered using a HP-filter with a smoothing parameter of 1600. Further details on the data are given in Appendix D.

Figure 13: US detrended data from 2007Q4 to 2017Q4



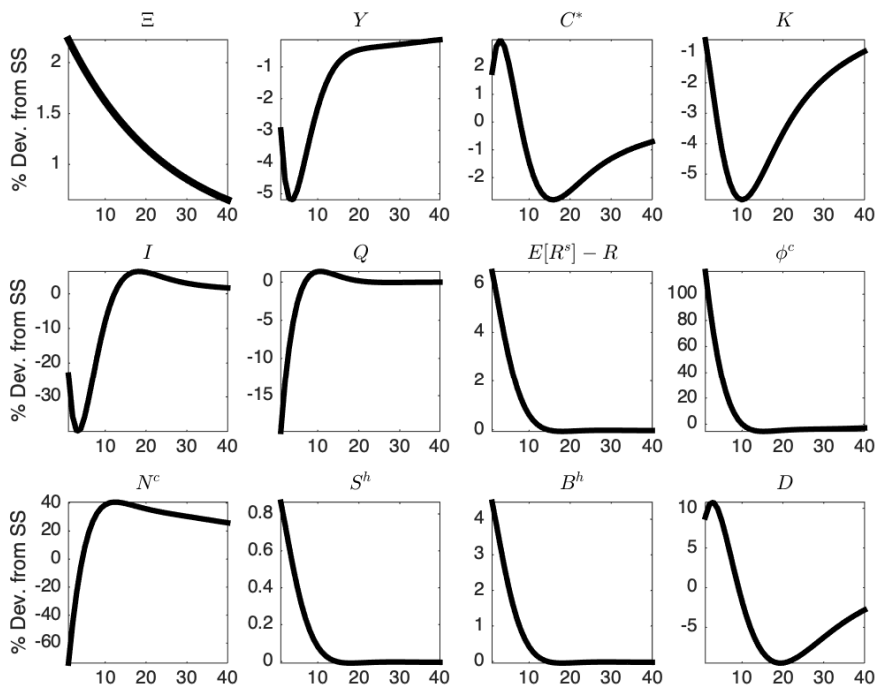
Data are first deflated with the implicit price deflator and then detrended using a HP-filter with a smoothing parameter of 1600. The gray shaded area shows the Great Recession period. All variables are expressed in percent deviation from their 2007Q4 level. Further details on the data are given in Appendix D.

Figure 14: Capital quality shock, cont.



This figure displays additional IRF related to the baseline model with securitization only. Variables are expressed in percentage deviation from the steady-state.

Figure 15: Securitization crisis, cont.



This figure depicts key IRF after an unanticipated securitization crisis. Variables are expressed in percentage deviation from the steady-state.

Figure 16: Securitization crisis, cont.

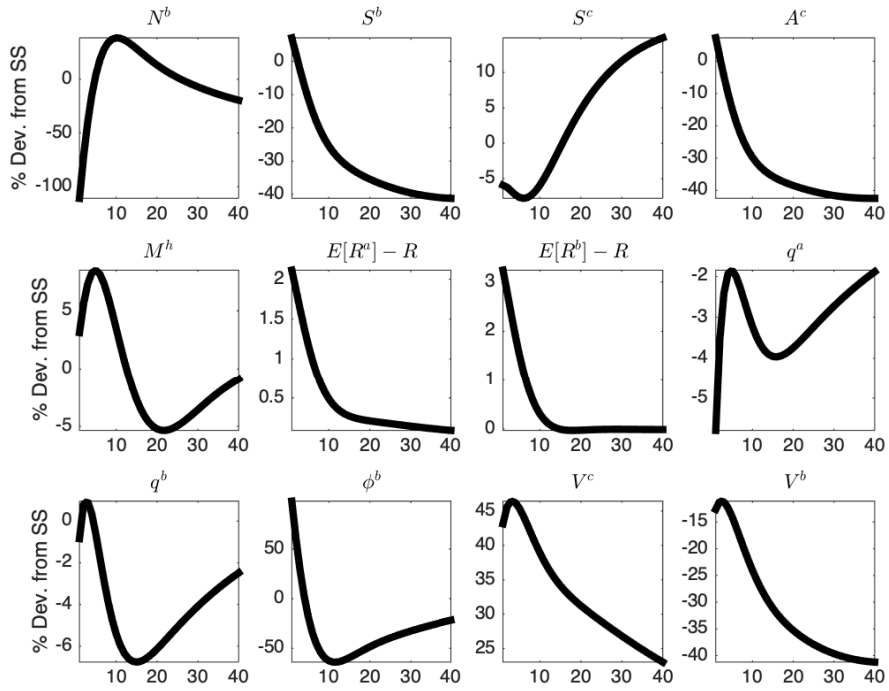
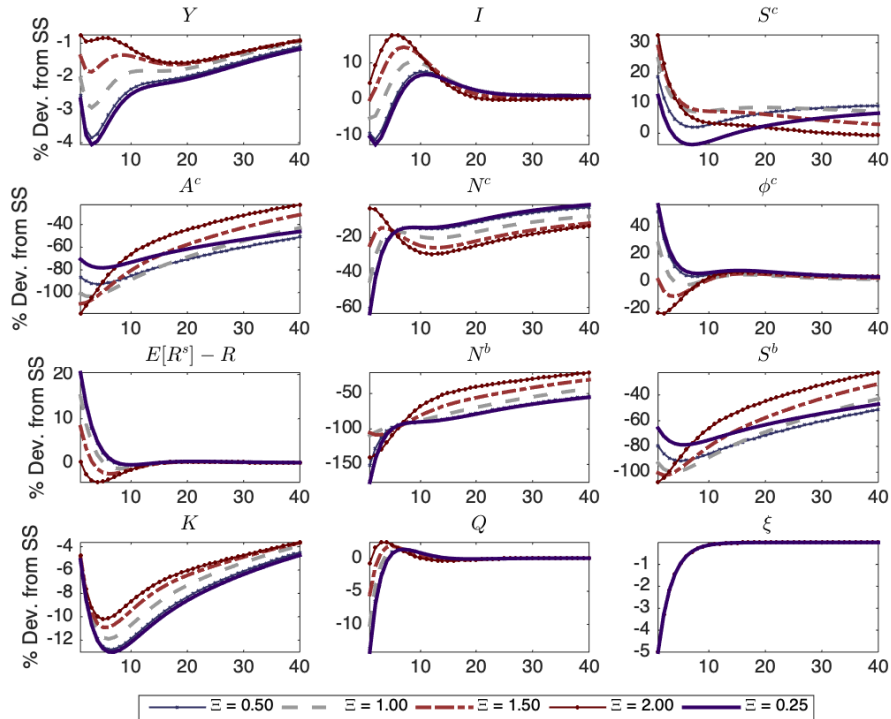
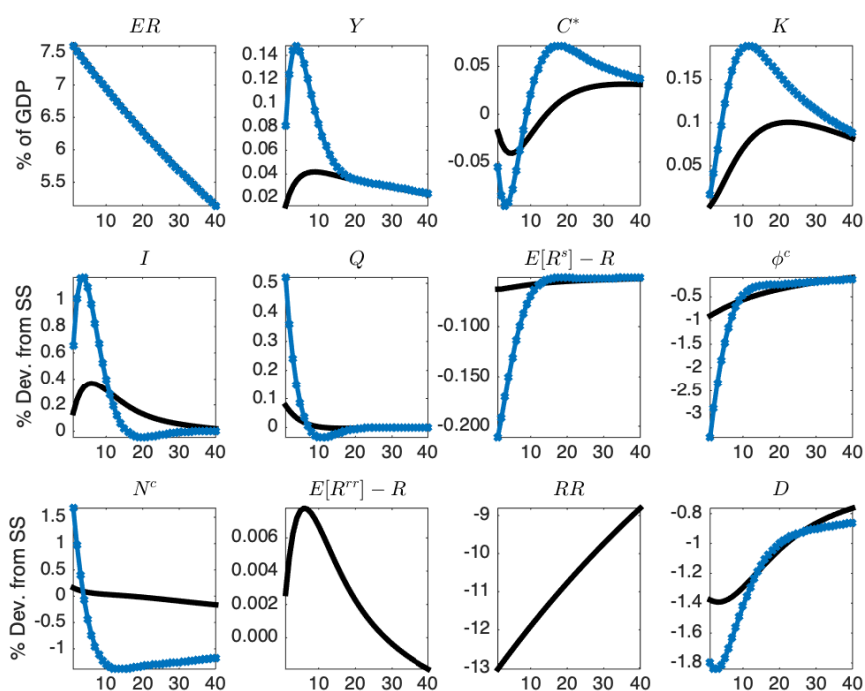


Figure 17: Capital quality shock, changes in Ξ



IRF to unanticipated capital quality shock with gradual changes in the values of Ξ . A low value of Ξ capture an environment with low risk. Each line corresponds to a different value given in the legend. Variables are expressed in percentage deviation from the steady-state.

Figure 18: Public intermediation (GK) vs Excess Reserves



IRF to unanticipated QE shocks with a GK implementation and with excess reserves. The blue solid lines with crosses show ABS purchases with a GK implementation. The gray dashed lines represent purchases of any type of assets in our first (baseline) excess reserves scenario. Variables are expressed in percentage deviation from the steady-state, except for QE that is given in percent of GDP.

Figure 19: Public intermediation (GK) vs Excess Reserves, cont.

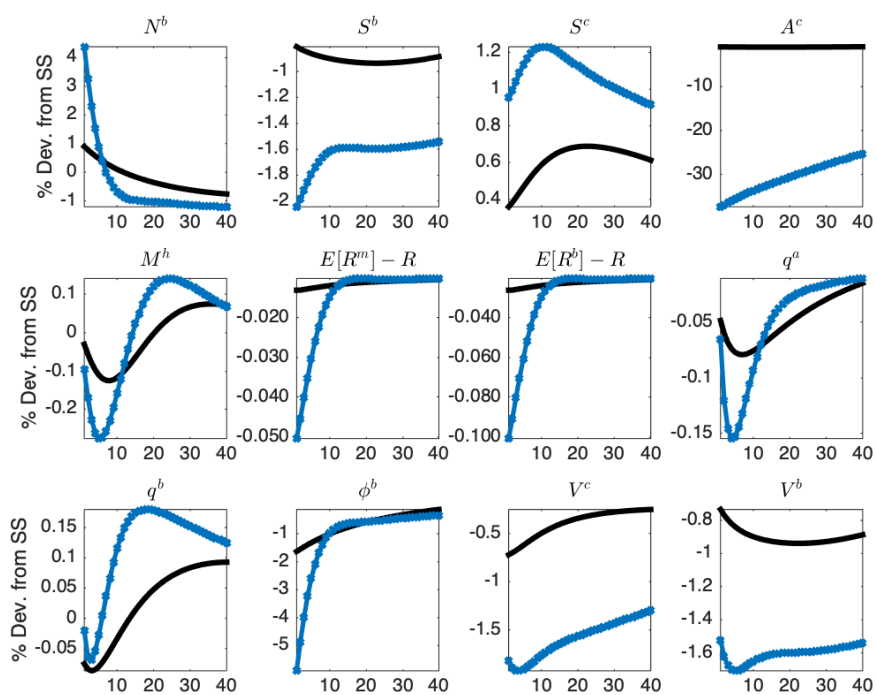
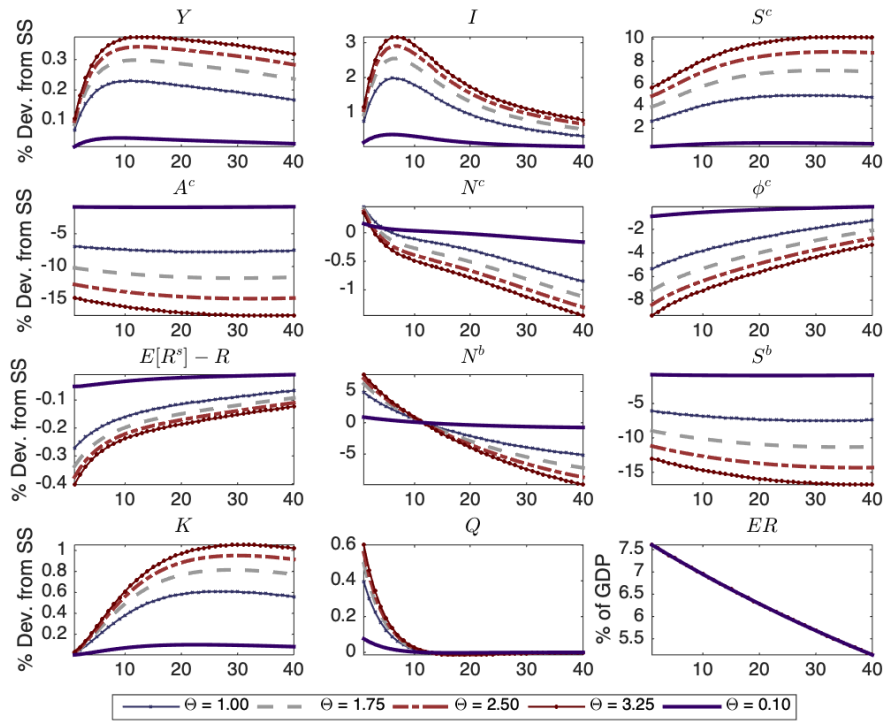
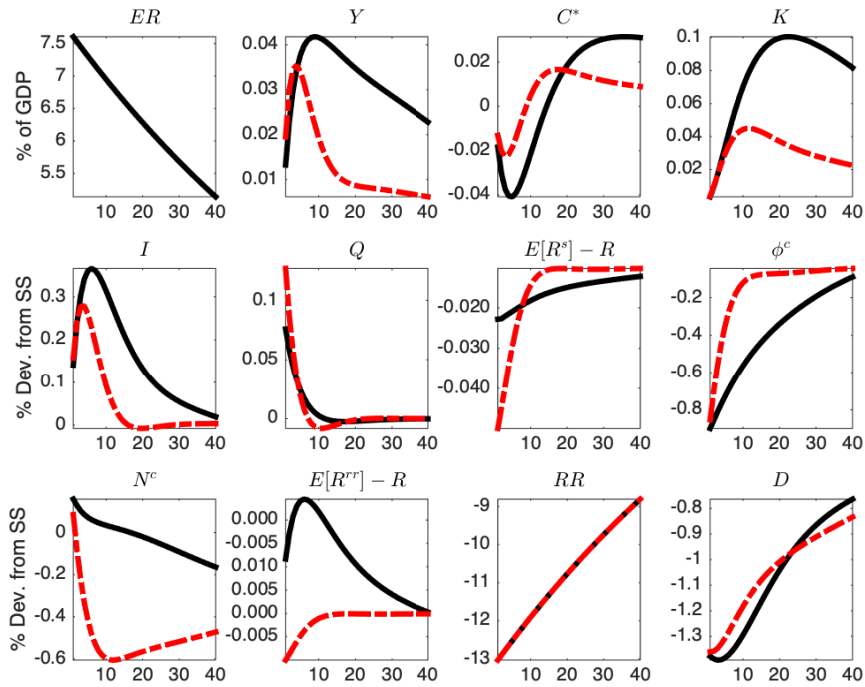


Figure 20: First scenario (fixed rate), changes in the value of Θ



IRF to unanticipated QE shocks implemented with excess reserves in the first (baseline) scenario with multiple values of Θ . Each line corresponds to a different value given in the legend. Variables are expressed in percentage deviation from the steady-state, except for excess reserves that is given in percent of GDP.

Figure 21: Excess Reserves, different scenarios



IRF comparing the two excess reserves scenarios. The black solid lines show responses in the first scenario, *i.e.* with a fixed interest rate. The red solid lines with crosses represent the second scenario, in which the return on reserves is stochastic.

Figure 22: Excess Reserves, different scenarios, cont.

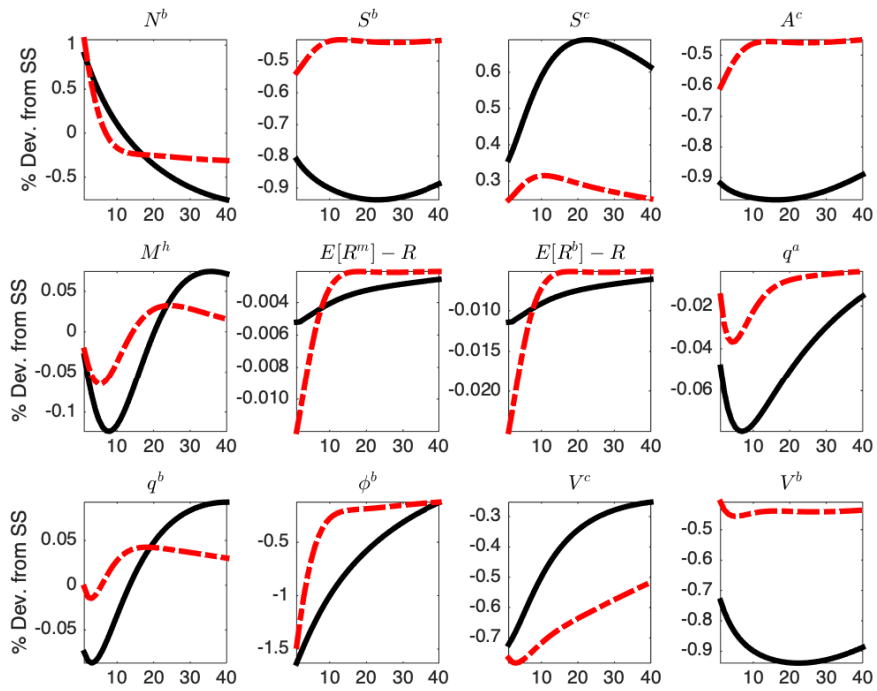
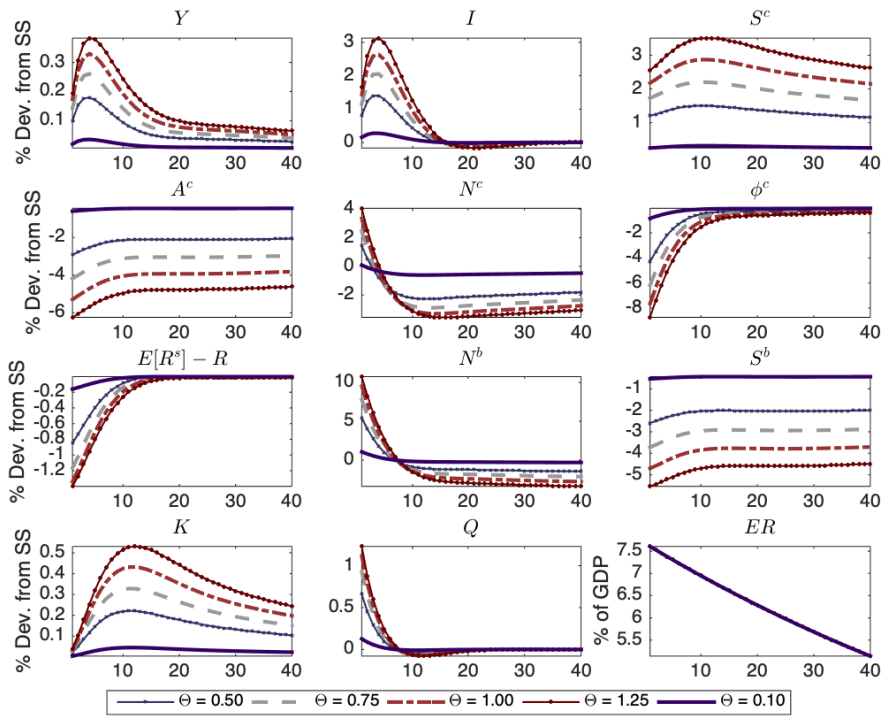


Figure 23: Second scenario (market rate), changes in the value of Θ



The values of Θ are less flexible in this scenario, to satisfy Blanchard-Kahn conditions. However, the macroeconomic effects are much larger. Accordingly, the QE program seems to be effective mostly through the portfolio balance channel and wealth effects for private agents.

Appendix B - Model Equations

The set of equations for the baseline model without excess reserves is:

- Patient households:

$$\begin{aligned} C_t + D_t^h + q_t^h(H_t - H_{t-1}) + Q_t \left(S_t^h + \frac{1}{2}\kappa(S_t^h - \bar{S}^h)^2 \right) + q_t^b \left(B_t^h + \frac{1}{2}\kappa(B_t^h - \bar{B}^h)^2 \right) \\ = W_t L_t + T_t + R_t D_{t-1}^h + R_t^s Q_{t-1} S_{t-1}^h + R_t^b q_{t-1}^b B_{t-1}^h + \Pi_t \end{aligned} \quad (1.B)$$

$$W_t u_{c,t} = u_{L,t} \quad (2.B)$$

$$\mathbb{E}_t \Lambda_{t,t+1} R_{t+1} = 1 \quad (3.B)$$

$$\Lambda_{t,t+1} = \beta \frac{u_{c,t+1}}{u_{c,t}} \quad (4.B)$$

$$S_t^h = \bar{S}^h + \Lambda_{t,t+1} \frac{(R_{t+1}^s - R_{t+1})}{\kappa} \quad (5.B)$$

$$B_t^h = \bar{B}^h + \Lambda_{t,t+1} \frac{(R_{t+1}^b - R_{t+1})}{\kappa} \quad (6.B)$$

$$q_t^h u_{c,t} = u_{h,t} + \beta \mathbb{E}_t q_{t+1}^h u_{c,t+1} \quad (7.B)$$

- Impatient households:

$$C'_t + q_t^h(H'_t - H'_{t-1}) + R_t M_{t-1}^h = W'_t L'_t + M_t^h \quad (8.B)$$

$$M_t^h = \gamma_m M_{t-1}^h + (1 - \gamma_m) \chi q_t^h H'_t \quad (9.B)$$

$$W'_t u_{c',t} = u_{L',t} \quad (10.B)$$

$$q_t^h u_{c',t} = u_{h',t} + \beta' \mathbb{E}_t q_{t+1}^h u_{c',t+1} + u_{c',t} \lambda_t^h (1 - \gamma_m) \chi q_t^h \quad (11.B)$$

$$(1 - \lambda_t^h) u_{c',t} = \beta' \mathbb{E}_t (R_{t+1} - \gamma_m \lambda_{t+1}^h u_{c',t+1}) \quad (12.B)$$

where λ_t^h is the multiplier of the borrowing constraint (37) normalized by the marginal utility of consumption of impatient households $u_{c',t}$.

- Intermediate goods producers:

$$Y_t^m = A_t (\xi_t K_t)^\alpha L_t^{(1-\alpha)(1-\varphi)} L_t'^{(1-\alpha)\varphi} \quad (13.B)$$

$$Z_t = P_t^m \alpha \frac{Y_t^m}{K_t} \quad (14.B)$$

$$W_t = P_t^m (1 - \alpha) (1 - \varphi) \frac{Y_t^m}{L_t} \quad (15.B)$$

$$W'_t = P_t^m (1 - \alpha) \varphi \frac{Y_t^m}{L'_t} \quad (16.B)$$

$$R_{t+1}^c = \frac{Z_{t+1} + (1 - \delta)Q_{t+1}}{Q_t} \xi_{t+1} \quad (17.B)$$

$$R_{t+1}^a = \frac{Z_{t+1} + (1 - \delta)q_{t+1}^a}{q_t^a} \xi_{t+1} \quad (18.B)$$

- Capital producers:

$$K_{t+1} = \xi_{t+1}(1 - \delta)K_t + I_t \quad (19.B)$$

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \left(\frac{I_t}{I_{t-1}}\right) f'\left(\frac{I_t}{I_{t-1}}\right) - \mathbb{E}_t \Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right) \quad (20.B)$$

- Final goods producers:

$$Y_t^m = D_t Y_t \quad (21.B)$$

$$D_t = \gamma D_{t-1} \pi_{t-1}^{-\gamma p \varepsilon} \pi_t^\varepsilon + (1 - \gamma) \left(\frac{1 - \gamma \pi_{t-1}^{\gamma p (1-\gamma)} \pi_t^{\gamma-1}}{1 - \gamma} \right)^{-\frac{\varepsilon}{1-\gamma}} \quad (22.B)$$

$$F_t = Y_t P_t^m + \mathbb{E}_t \left[\beta \gamma \Lambda_{t,t+1} \frac{\pi_t^{-\gamma p \varepsilon}}{\pi_{t+1}^{-\varepsilon}} F_{t+1} \right] \quad (23.B)$$

$$J_t = Y_t + \mathbb{E}_t \left[\beta \gamma \Lambda_{t,t+1} \frac{\pi_t^{\gamma p (1-\varepsilon)}}{\pi_{t+1}^{(1-\varepsilon)}} J_{t+1} \right] \quad (24.B)$$

$$\pi_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{F_t}{J_t} \pi_t \quad (25.B)$$

$$\pi_t^{1-\varepsilon} = \gamma \pi_{t-1}^{\gamma p (1-\varepsilon)} + (1 - \gamma) (\pi_t^*)^{1-\varepsilon} \quad (26.B)$$

$$i_t = R_{t+1} \pi_{t+1} \quad (27.B)$$

- Commercial banks:

$$\mu_t^c = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c (R_{t+1}^s - R_{t+1}) \quad (28.B)$$

$$\mu_t^a = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c (R_{t+1}^a - R_{t+1}) \quad (29.B)$$

$$\mu_t^b = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c (R_{t+1}^b - R_{t+1}) \quad (30.B)$$

$$\eta_t^c = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c R_{t+1} \quad (31.B)$$

$$\phi_t^c = \frac{\eta_t^c}{\theta^c - \mu_t^c} \quad (32.B)$$

$$Q_t S_t^c + \Xi q_t^a A_t^c + \Delta q_t^b B_t^c = \phi_t^c N_t^c \quad (33.B)$$

$$N_t^c = \sigma \left((R_t^s - R_t) Q_{t-1} S_{t-1}^c + (R_t^a - R_t) q_{t-1}^a A_{t-1}^c + (R_t^b - R_t) q_{t-1}^b B_{t-1}^c + R_t N_{t-1}^c \right) + \omega^c (\bar{Q} \bar{S} + \bar{q}^a \bar{A}^c + \bar{q}^b \bar{B}^c) \quad (34.B)$$

$$Q_t S_t^c + q_t^a A_t^c + q_t^b B_t^c + q_t^h M_t^c = D_t^c + N_t^c \quad (35.B)$$

$$\Omega_{t+1}^c = (1 - \sigma) + \sigma (\mu_t^c \phi_t^c + \eta_t^c) \quad (36.B)$$

- Shadow banks:

$$\nu_t^b = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^b (R_{t+1}^s - R_{t+1}^a) \quad (37.B)$$

$$\eta_t^b = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^b R_{t+1}^a \quad (38.B)$$

$$\phi_t^b = \frac{\eta_t^b}{\theta^b - \nu_t^b} \quad (39.B)$$

$$Q_t S_t^b = \phi_t^b N_t^b \quad (40.B)$$

$$N_t^b = \sigma ((R_t^s - R_t^a) Q_{t-1} S_{t-1}^b + R_t^a N_{t-1}^b) + \omega^b (\bar{Q} \bar{S}^b) \quad (41.B)$$

$$Q_t S_t^b = q_t^a A_t^b + N_t^b \quad (42.B)$$

$$\Omega_{t+1}^b = (1 - \sigma) + \sigma(\nu_t^b \phi_t^b + \eta_t^b) \quad (43.B)$$

- Market clearing and miscellaneous:

$$K_{t+1} = S_t^c + S_t^b + S_t^h + S_t^g \quad (44.B)$$

$$A_t^c + A_t^g = A_t^b \quad (45.B)$$

$$\bar{B} = B_t^c + B_t^h + B_t^g \quad (46.B)$$

$$H_t + H_t' = 1 \quad (47.B)$$

$$M_t^h = M_t^c \quad (48.B)$$

$$D_t^h = D_t^c \quad (49.B)$$

$$\bar{G} + (R_t^b - 1) \bar{B} = T_t + (R_t^s - R_t) Q_{t-1} S_{t-1}^g + (R_t^a - R_t) q_{t-1}^a A_{t-1}^g + (R_t^b - R_t) q_{t-1}^b B_{t-1}^g \quad (50.B)$$

$$R_{t+1}^b = \frac{\frac{1}{P_t} + q_{t+1}^b}{q_t^b} \quad (51.B)$$

$$Y_t = C_t + C_t' + \left[1 + f \left(\frac{I_t}{I_{t-1}} \right) \right] I_t \quad (52.B)$$

$$i_t = \bar{i} + \kappa_\pi \pi_t + \kappa_y (\log Y_t - \log Y_t^*) + \epsilon_t \quad (53.B)$$

Appendix C, Details to the commercial and shadow banks' problems

- Commercial banks:

Given the balance sheet identity (1) and the evolution of net worth (2), banker's optimization problem comes down to a Bellman equation. Using the method of undetermined coefficients, we guess that the solution is linear in the time-varying coefficients:

$$V_t^c = \mu_t^c Q_t s_t^c + \mu_t^a q_t^a a_t^c + \mu_t^b q_t^b b_t^c + \eta_t^c n_t^c \quad (1.C)$$

Now insert the guessed solution into the Bellman equation then maximize the objective function subject to the incentive constraint (3). Using a Lagrangian multiplier λ_t^c , the problem writes:

$$\mathcal{L} = (1 + \lambda_t^c) (\mu_t^c Q_t s_t^c + \mu_t^a q_t^a a_t^c + \mu_t^b q_t^b b_t^c + \eta_t^c n_t^c) - \lambda_t^c \theta^c (Q_t s_t^c + \Xi q_t^a a_t^c + \Delta q_t^b b_t^c) \quad (2.C)$$

The first-order conditions associated with s_t^c , a_t^c , b_t^c , λ_t^c are:

$$(1 + \lambda_t^c) \mu_t^c = \lambda_t^c \theta^c \quad (3.C)$$

$$\mu_t^a = \mu_t^c \Xi \quad (4.C)$$

$$\mu_t^b = \mu_t^c \Delta \quad (5.C)$$

$$Q_t s_t^c + \Xi q_t^a a_t^c + \Delta q_t^b b_t^c = \phi_t^c n_t^c, \quad \text{if } \lambda_t^c > 0 \quad (6.C)$$

where,

$$\phi_t^c = \frac{\eta_t^c}{\theta^c - \mu_t^c} \quad (7.C)$$

Using the FOCs (4.C), (5.C) and (6.C), step by step, one can rewrite the guessed solution as:

$$\begin{aligned} V_t^c &= \mu_t^c Q_t s_t^c + \mu_t^c \Xi q_t^a a_t^c + \mu_t^c \Delta q_t^b b_t^c + \eta_t^c n_t^c \\ V_t^c &= \mu_t^c (Q_t s_t^c + \Xi q_t^a a_t^c + \Delta q_t^b b_t^c) + \eta_t^c n_t^c \\ V_t^c &= \mu_t^c \phi_t^c n_t^c + \eta_t^c n_t^c \end{aligned} \quad (8.C)$$

which in turn can be plugged into the Bellman equation as follows:

$$\mu_t^c Q_t s_t^c + \mu_t^a q_t^a a_t^c + \mu_t^b q_t^b b_t^c + \eta_t^c n_t^c = \mathbb{E}_t \Lambda_{t,t+1} \left((1 - \sigma) + \sigma (\mu_{t+1}^c \phi_{t+1}^c + \eta_{t+1}^c) \right) n_{t+1}^c \quad (9.C)$$

Define the shadow price of a banker net worth as:

$$\Omega_{t+1}^c = (1 - \sigma) + \sigma (\mu_{t+1}^c \phi_{t+1}^c + \eta_{t+1}^c) \quad (10.C)$$

Then, using the law of motion of a banker net worth (2):

$$\begin{aligned}\mu_t^c Q_t s_t^c + \mu_t^a q_t^a a_t^c + \mu_t^b q_t^b b_t^c + \eta_t^c n_t^c &= \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c n_{t+1}^c \\ \mu_t^c Q_t s_t^c + \mu_t^a q_t^a a_t^c + \mu_t^b q_t^b b_t^c + \eta_t^c n_t^c &= \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c ((R_{t+1}^s - R_{t+1}) Q_t s_t^c + (R_{t+1}^a - R_{t+1}) q_t^a a_t^c \\ &\quad + (R_{t+1}^b - R_{t+1}) q_t^b b_t^c + R_{t+1} n_t^c)\end{aligned}\quad (11.C)$$

which gives the solutions:

$$\mu_t^c = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c (R_{t+1}^s - R_{t+1}) \quad (12.C)$$

$$\mu_t^a = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c (R_{t+1}^a - R_{t+1}) \quad (13.C)$$

$$\mu_t^b = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c (R_{t+1}^b - R_{t+1}) \quad (14.C)$$

$$\eta_t^c = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^c R_{t+1} \quad (15.C)$$

- Shadow banks:

Given the balance sheet identity (16) and the evolution of net worth (17), broker's optimization problem comes down to a Bellman equation. Using the method of undetermined coefficients, we guess that the solution is linear in the time-varying coefficients:

$$V_t^b = \nu_t^b Q_t s_t^b + \eta_t^b n_t^b \quad (16.C)$$

Now insert the guessed solution into the Bellman equation then maximize the objective function subject to the incentive constraint (18). Using a Lagrangian multiplier λ_t^b , the problem writes:

$$\mathcal{L} = (1 + \lambda_t^b) (\nu_t^b Q_t s_t^b + \eta_t^b n_t^b) - \lambda_t^b \theta^b Q_t s_t^b \quad (17.C)$$

The first-order conditions associated with s_t^b, λ_t^b are:

$$(1 + \lambda_t^b) \nu_t^b = \lambda_t^b \theta^b \quad (18.C)$$

$$Q_t s_t^b = \phi_t^b n_t^b, \quad \text{if } \lambda_t^b > 0 \quad (19.C)$$

where,

$$\phi_t^b = \frac{\eta_t^b}{\theta^b - \nu_t^b} \quad (20.C)$$

Using the FOC (18.C), one can rewrite the guessed solution as:

$$V_t^b = \nu_t^b \phi_t^b n_t^b + \eta_t^b n_t^b \quad (21.C)$$

which in turn can be plugged into the Bellman equation as follows:

$$\nu_t^b Q_t s_t^b + \eta_t^b n_t^b = \mathbb{E}_t \Lambda_{t,t+1} \left((1 - \sigma) + \sigma (\nu_{t+1}^b \phi_{t+1}^b n_t^b + \eta_{t+1}^b) \right) n_{t+1}^b \quad (22.C)$$

Define the shadow price of a broker net worth as:

$$\Omega_{t+1}^b = (1 - \sigma) + \sigma (\nu_{t+1}^b \phi_{t+1}^b + \eta_{t+1}^b) \quad (23.C)$$

Then, using the law of motion of a broker net worth (17):

$$\begin{aligned} \nu_t^b Q_t s_t^b + \eta_t^b n_t^b &= \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^b n_{t+1}^b \\ \nu_t^c Q_t s_t^b + \eta_t^b n_t^b &= \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^b \left((R_{t+1}^s - R_{t+1}^a) Q_t s_t^b + R_{t+1}^a n_t^b \right) \end{aligned} \quad (24.C)$$

which gives the solutions:

$$\nu_t^b = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^b (R_{t+1}^s - R_{t+1}^a) \quad (25.C)$$

$$\eta_t^b = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}^b R_{t+1}^a \quad (26.C)$$

Appendix D - Data

Table 2: Data

Description	Units	Sources
Gross Domestic Product	Billions	US B.E.A.
Gross Private Domestic Investment	Billions	US B.E.A.
Gross Domestic Product: Implicit Price Deflator	Index 2012	US B.E.A.
Nonfinancial Corporate Business; Depository Institutions loans; Liability	Millions	US Z.1 F.A.
Nonfinancial Noncorporate Business; Depository Institutions loans; Liability	Millions	US Z.1 F.A.
Nonfinancial Corporate Business; Debt Securities; Liability	Millions	US Z.1 F.A.
Nonfinancial Corporate Business; Corporate Equities; Liability	Millions	US Z.1 F.A.
Other loans and advances by issuers of ABS; asset	Millions	US Z.1 F.A.
All Federal Reserve Banks: Total Assets	Millions	H.4.1 F.A.R.B.
Excess Reserves of Depository Institutions	Millions	H.3 A.R.D.I.M.B.
US Treasury securities	Millions	H.4.1 F.A.R.B.
Federal agency debt securities	Millions	H.4.1 F.A.R.B.
Mortgage-backed securities	Millions	H.4.1 F.A.R.B.
Unamortized premiums on securities held outright	Millions	H.4.1 F.A.R.B.
Unamortized discounts on securities held outright	Millions	H.4.1 F.A.R.B.
Repurchase agreements	Millions	H.4.1 F.A.R.B.
Loans	Millions	H.4.1 F.A.R.B.
Net portfolio holdings of Maiden Lane LLC	Millions	H.4.1 F.A.R.B.
Float	Millions	H.4.1 F.A.R.B.
Central bank liquidity swaps	Millions	H.4.1 F.A.R.B.
Other Federal Reserve assets	Millions	H.4.1 F.A.R.B.
Foreign currency denominated assets	Millions	H.4.1 F.A.R.B.
Gold stock	Millions	H.4.1 F.A.R.B.
Special drawing rights certificate account	Millions	H.4.1 F.A.R.B.
Treasury currency outstanding	Millions	H.4.1 F.A.R.B.
Currency in circulation	Millions	H.4.1 F.A.R.B.
Reverse repurchase agreements	Millions	H.4.1 F.A.R.B.
Treasury cash holdings	Millions	H.4.1 F.A.R.B.
Deposits with F.R. Banks, other than reserve balances	Millions	H.4.1 F.A.R.B.
Effective Federal Funds Rate	Percent	H.15. B.G.F.R.S.
3-Month Treasury Bill: Secondary Market Rate	Percent	H.15. B.G.F.R.S.
Federal Funds Target Range - Upper Limit	Percent	B.G.F.R.S.
Federal Funds Target Range - Lower Limit	Percent	B.G.F.R.S.
Interest Rate on Required Reserves	Percent	B.G.F.R.S.
Interest Rate on Excess Reserves	Percent	B.G.F.R.S.

Initialisms/Acronyms: ABS = Asset-Backed-Securities, B.E.A = Bureau of Economic Analysis, F.A. = Financial Accounts, F.A.R.B. = Factors Affecting Reserve Balances, A.R.D.I.M.B. = Aggregate Reserves of Depository Institutions and the Monetary Base., B.G.F.R.S = Board of Governors of the Federal Reserve System.

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