Challenging pollution and the balance problem from rare earth extraction: How recycling and environmental taxation matter

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Abstract

Rare earth elements extraction induces pollution and the balance problem. In this article, we investigate how far recycling and environmental taxation challenge both questions. In a two-period framework, we assume a monopoly extractor in the first period that is in competition with one recycler in the second period. Our results depend on whether the recycling activity is bounded or not by extracted quantities. When recycling is not constrained, it does not change extraction in period 1 but has pro-competitive effects in period 2. The balance problem favors recycling in period 2 and reduces environmental damages in both periods. If recycling is limited, the extractor adopts a foreclosure strategy in the first period. The balance problem reduces extraction in both periods but also recycling. A second-best environmental taxation enables to reach the first-best outcome except in the second period of the bounded case. Environmental taxes have to be amended in order to take into account the recycling effect. They are never equal to the marginal damage.

Keywords

Rare earth elements, Pollution, Balance problem, Recycling, Taxation, Cournot competition.

JEL Codes

L13, L72, Q53, Q58.

1 Introduction

Rare earth elements (REEs) play an essential role in modern technology such as energy generation and storage, energy effi cient lights, electric cars and auto catalysts as well as military and aerospace applications (Golev et al. 2014).[1] They, therefore, play a crucial role in the transition towards a low-carbon economy and they have gained a strategic status. According to the US Geological Service (USGS) Mineral Commodity Summaries 2016, REEs reserves worldwide are estimated to be 130 million tons. China and Brazil hold the largest shares of such reserves with respectively 16.9 % and 42.3%, followed by Australia (2.5%), India (2.4%) and the United States (2%). Regarding mine extraction, out of the 124,000 metric tons estimated to have been produced in 2015, China contributed with 87.5%, followed by Australia 8.3% and the United States 3.4% (Fernandez, 2017).

If the USA long dominated the rare earth industry from the mid-1960s to the mid-1980s, China has become the main producer and now holds a quasi-monopoly. This leading status is attributed to lower labor costs and willingness to accept lower environmental standards (Campbell, 2014; Muller et al. 2016). Nowadays, China controls approximately 97% of the world REEs exports (Hurst, 2010), importing countries are highly impacted by China's market power. For instance, China has set export quotas, which have drastically increased the vulnerability of key industries, especially in the USA (Muller et al. 2016).

The REEs processing is water and energy intensive and requires chemicals use (EPA, 2012). The mining and processing of REEs usually results in significant environmental impacts. Many deposits are associated with high concentrations of radioactive elements such as uranium and thorium (Massari and Ruberti, 2013) and acidic substances (Elshkaki and Graedel, 2014) that are released into the environment without being treated (Folger, 2011). The Asian Rare Earth company located in Malaysia (1982–1992) has often been reported as an example of radioactive pollution associated to the processing of monazite rare earths (Ichihara and Harding, 1995). Ionic clay REEs are the most accessible REEs in China. Their mining induces severe damages due to severe erosion, air, water and soil pollution, biodiversity loss, and health problems (China Development, 2011; Packey and Kingsnorth, 2016). We can also quote fish health and human health issues (Chinafolio, 2014): inhabitants of Baotou in China are affected by cancers, respiratory diseases, and dental loss (Schüler et al., 2011). Moreover, radioactive sludges prohibit agricultural activities around this city.

Note that, despite their name, REEs are not all rare (Falconnet, 1985; Wübbeke, 2013). The balance between the demand and their natural abundance in REE ores is called the «balance problem» (Elshkaki and Graedel, 2014). It is a major concern for extractors in that they bear storage costs for REEs which are in excess. The balance problem is a more important issue than the availability of REEs for manufacturers (Binnemans, 2014). For instance, some REEs such as neodymium, dysprosium, terbium and lanthanum are not abundant and are high in demand (Binnemans, 2014), whereas others - such as cerium - are abundant and low in demand (Golev et al., 2014). For illustration, the supplies of neodymium and dysprosium in 2015 were respectively 35, 000 tons and 2,000 tons, while the demands they faced amounted respectively to 40,000 tons and 3,000 tons. The supply for cerium was 80,000 tons, whereas the demand it faced was 70,000 tons in 2015 (IMCOA cited in Kingsnorth, 2015).

[1] The REEs constitute a group of 17 chemically similar metallic elements, composed of 15 lanthanide elements (lanthanum, cerium, praseodymium, neodymium, promethium, samarium, europium, gadolinium, terbium, dysprosium, holmium, erbium, thulium, ytterbium, and lutetium) and two other elements (scandium and yttrium).

The balance problem has been the subject of several studies. Elshkaki and Graedel (2014) investigate the effect of increasing the production of dysprosium and neodymium - that are non-abundant REEs - on the supply and the demand of host metals, and then on the balance problem. Several ideas have been explored in order to mitigate the balance problem (Binnemans, 2014; Binnemans et al. 2013; Binnemans and Jones, 2015). The different options could be the diversification of REEs, their substitution, the reduced use of critical REEs and recycling. To the best of our knowledge, these ideas have not been theoretically investigated so far.

Several authors emphasize that by combining different strategies can help mitigate the balance problem. We argue that attention must also be paid to a thorough analysis of a peculiar strategy. In this paper, we focus on REEs recycling. REEs recycling deserves attention for two reasons. First, recycling can contribute to enhance environmental quality by postponing extraction. For example, it is expected that the supply of neodymium and dysprosium coming from recycling of these elements will cover about 5% of the demand in 2050 (Elshkaki and Graedel, 2013). Second, recycling may mitigate the extractor's market power if it increases the available stock of rare earths and therefore reduces the vulnerability of industries to supply shortages and to sharp price increases.

Several countries have already started to recycle REEs. These include China, which can recover REEs up to a maximum level of 95% (Yang et al., 2002) or Japan which recycles a third of REEs used in the production of magnets (Hetzel and Bataille, 2014). For instance, the Solvay Group has recently developed the process for recovering REEs from lamp phosphors, batteries, magnets and tailings in France (Saint Fons Lyon, La Rochelle), and also in Belgium (Binnemans et al., 2013). Hitachi Ltd has developed technologies to recycle rare earths magnets from hard disk drives and has successfully extracted rare earths from rare earths magnets (Hitachi, 2010 cited in Binnemans et al., 2013). Osram has developed a process to recover REEs from used phosphors (Binnemans and Jones, 2014). There are also other ongoing activities into reclaiming REEs from the scrap generated in the various end-uses sectors (Schüler et al., 2011). It is worth stressing that mining companies such as Molycorp have also implemented recycling schemes for magnets in order to reduce overproduction of some abundant REEs (Binnemans et al., 2013). However, those efforts made by several companies are insufficient and it seems that recycling still remains near zero (OECD 2015). One can therefore wonder whether environmental policies can have an effect on the intensity of recycling.

Recycling has been studied in the economics literature. A first group of papers investigates the relationship between recycling and natural resources exhaustion. André and Cerdà (2006), Weikard and Seyhan (2009) and Seyhan et al. (2012) show that recycling delays the depletion of these resources. A second group explores the impact of recycling on an exhaustible resource extractor' smarket power. There is no consensus on the result. Some papers show that the presence of recycling does not substantially affect the extractor's long-run market power (see Gaskins, 1974; Swan, 1980; Martin, 1982; Suslow, 1986; Hollander and Lasserre, 1988 and Grant, 1999) whereas others show that the presence of recycling increases the extractor' s market power (see Gaudet and Van Long, 2003; Baksi and Long, 2009).

The main purpose of this paper is to investigate how far recycling and environmental taxes can alter the balance problem and pollution. To the best of our knowledge, this paper is the first one that takes into account the balance problem. It is also the first one that designs Pigouvian taxes in presence of recycling. To do that, we consider a model in which a monopolist extracts two types of REEs - abundant and non-abundant REEs - over two consecutive periods. In the second period of the game, it engages in competition with one firm that recycles one part of the non-abundant REEs consumed in the first period.

We first analyze the impact of the recycling on the extracted quantities and so on environmental damages as well as far as the balance problem is concerned. Our results depend on whether the recycling activity is bounded or not by the available quantity of extracted REEs in the first period. Without constraint, recycling does not change extraction in period 1 but has a pro-competitive effect in period 2. The balance problem favors recycling in period 2 and reduces environmental damages in both periods. If recycling is limited, the extractor adopts a foreclosure strategy in the first period. The balance problem reduces extraction in both periods but also recycling. Both situations are not optimal, because of the presence of pollution and market power in each period. We, therefore, propose to implement a Pigouvian tax on extracted quantities.

A second-best environmental taxation in each period delivers distinct results depending on the existence of a constraint on recycling. Without constraint the second-best environmental taxation enables to reach the first-best outcome in the first period. This is however not the case in the second period. When the constraints binds, the first-best can be reached in both periods. The levels of the environmental taxes depend on the marginal damage, on the market power but also on the recycling effect. Due to the presence of recycling, the regulator has to amend the tax rates. If the Pigouvian tax is always lower than the marginal damage, it may be higher in the first period in the bounded case. Eventually the implementation of a tax scheme enables to indirectly strengthen the role of recycling in mitigating the balance problem and reducing pollution.

The remainder of the paper is structured as follows. Section 2 describes the model. Recycling is introduced in Section 3. In Section 4 we describe the first-best outcome of this economy and we introduce second-best taxation in Section 5. Section 6 concludes the paper and technical proofs are relegated in an appendix.

2 The model

In this section, we present the assumptions of the model and the equilibrium of the economy without recycling.

2.1 Assumptions

We consider a two-period model where one firm extracts REEs from one mine. The extracted ore contains abundant and non-abundant REEs. Let x_t denote the supply of non-abundant REEs and $\bar{x}_t$ the supply of abundant REEs, where $t = 1, 2$ is an index over time periods. Since both types of REEs are extracted from the same ore, the extraction of one type induces mechanically the extraction of the other type such that $\bar{x}_t = \alpha x_t$, where α is a positive parameter. The extraction cost is denoted by $C_t(x_t)$ that has usual properties ($C' > 0$ and $C'' > 0$). For purposes of simplification, we assume that the discount factor is normalized to one.

We assume that the market of non-abundant REEs is cleared while the supply of abundant REEs exceeds the demand such as $\bar{x}_t > {x_t}^d \; \forall \bar{P}_t$, where ${\bar{x}_t}^d$ is the demand of abundant REEs and $\bar{P}_t$ their price. The balance problem for abundant REEs incurs a storage cost borne by the extractor given by $c_s \sum_{t=1}^{2} (\bar{x}_t - x_t^d)$.

In the first period, the extractor is a monopolist whereas it faces one recycler in the second period who recycles a quantity $r \leq kx_1$. The parameter k is either the recycling technology efficiency or the changes in that technology with $k \in]0; 1]$. If $r < kx_1$, this condition depicts depreciation that occurs during recycling. Note that the non-abundant REEs are recycled at a cost $C_r(r)$ that is an increasing and convex function.

The extracted quantity of non-abundant REEs and the recycled quantity are perfectly substitutable. P_t is the price set by the market. The inverse demand is $P_t = P(Q_t)$ with $P' < 0$ and $P'' \leqslant 0$.[2] We note Q_t the global quantity offered of non-abundant REEs such as $Q_1 = x_1$ and $Q_2 = x_2 + r$.

Lastly we assume that extracting x_t units creates an environmental damage $D(x_t)$, that is an increasing and convex function. We assume that pollution is only caused by REEs extraction.

2.2 Equilibrium without recycling

Without recycling, the extractor acts as a monopolist in both periods. The profit of the extractor is the sum of inter temporal revenue earned from selling both types of REEs minus extraction costs and storage costs:

$$\pi^e(x_1, x_2) = P_1(x_1)x_1 - C_1(x_1) - c_s[\bar{x}_1 - x_1^d] + x_1^d \bar{P}_1 + P_2(x_2)x_2 - C_2(x_2) + x_2^d \bar{P}_2 - c_s[\bar{x}_1 - x_1^d + \bar{x}_2 - x_2^d]$$

First-order conditions take into account the relationship between both types of REEs:

$$P_1(x_1^{wr}) + P_1'(x_1^{wr})x_1^{wr} - C_1'(x_1^{wr}) - 2\alpha c_s = 0 \tag{1}$$

$$P_2(x_2^{wr}) + P_2'(x_2^{wr})x_2^{wr} - C_2'(x_2^{wr}) - \alpha c_s = 0 \tag{2}$$

where the superscript wr means without recycling. Equations (1) and (2) indicate that in each period the price of the non-abundant REEs is equal to the sum of the marginal costs of extraction and storage, adjusted for the monopoly market power. The balance problem, by introducing storage costs, leads to reduce extraction in each period, but mostly in period 1. In period 2, the storage marginal cost is reduced. Eventually, the extracted quantities in period 1 are lower than the quantities extracted in period 2:

$$x_1^{wr} < x_2^{wr}$$

Proposition 1 *Due to the balance problem, extracted quantities without recycling in period 2 are higher than extracted quantities in period 1.*

3 The recycling activity

In this section, recycling is taken into account. As highlighted above, it occurs only in the second period. Using backward induction, we first find the equilibrium quantities in period 2 and we then solve the quantity produced by the extractor in period 1.

[2] In what we will call the binding case, we will set $P'' = 0$ and $C''' = 0$.

3.1 The second stage: the equilibrium quantities in period 2

We define the subgame-perfect Nash equilibrium in period 2. The extractor's profit maximization program in the second period is the following:

$$\pi^e(x_2, r) = P_2(x_2 + r)x_2 - C_2(x_2) + y_2^d \bar{P}_2 - c_s[\alpha x_1 - x_1^d + \alpha x_2 - x_2^d]$$

The FOC gives:

$$P_2(x_2 + r) + P_2'(x_2 + r)x_2 - C_2'(x_2) - \alpha c_s = 0$$

The recycler maximizes its profit, subject to the constraint on the available resource:

$$\begin{aligned} \pi^r(r, x_2) &= P_2(x_2 + r)r - C_r(r) \\ r &\leq kx_1 \end{aligned}$$

We find:

$$\begin{aligned} P_2(x_2 + r) + P_2'(x_2 + r)r - C_r'(r) &= 0 \text{ if } r < kx_1 \\ r &= kx_1 \text{ otherwise} \end{aligned} \tag{3}$$

Thus the best response functions are the following:

$$\begin{aligned} r &= -\frac{P_2(x_2 + r) - C_r'(r)}{P_2'(x_2 + r)} \text{ if } r < kx_1 \\ r &= kx_1 \text{ otherwise} \end{aligned} \tag{4}$$

We have two cases, depending on whether the recycling constraint is bounded (denoted by the superscript c) or not (denoted by the superscript nc).

- If the available quantity of scrap is higher than the unconstrained profit-maximizing quantity, the extractor and the recycler produce the quantities that satisfy the following FOCs:

$$\begin{cases} P_2(x_2^{nc} + r^{nc}) + P_2'(x_2^{nc} + r^{nc})x_2^{nc} - C_2'(x_2^{nc}) - \alpha c_s = 0 \\ P_2(x_2^{nc} + r^{nc}) + P_2'(x_2^{nc} + r^{nc})r^{nc} - C_r'(r^{nc}) = 0 \end{cases} \tag{5}$$

  The Implicit Function Theorem on FOCs given by (5) shows that reaction functions are decreasing. The recycled scrap and the extracted output are strategic substitutes. Recycling reduces the quantity of non-abundant REEs which is extracted by the monopolist in the second period. This behavior was coined "business-stealing" effect after Mankiw and Whinston (1986). The strategic response of existing firms to new entry results in reducing their production when a new entrant "steals business" from incumbent firms. Solving the system given by (5) gives the non-constrained subgame-perfect Nash equilibrium (x_2^{nc}, r^{nc}). The extracted quantities and the level of recycled scrap in period 2 do not depend on quantity extracted in period 1.

- If the available quantity of scrap is lower than unconstrained profit-maximizing quantity, the equilibrium in period 2 is given by:

$$\begin{cases} P_2(x_2^c + r^c) + P_2'(x_2^c + r^c)x_2^c - C_2'(x_2^c) - \alpha c_s = 0 \\ r = kx_1^c \end{cases} \tag{6}$$

  Solving this system gives the constrained subgame-perfect Nash equilibrium (x_2^c, r^c). We find $x_2 = f(x_1)$, with $\frac{dx_2}{dx_1} < 0$. Hence, the extraction level in period 1 will affect the level of recycling as well the extracted quantity in period 2.

Proposition 2 *Whatever the level of recycled scrap, the recycling activity reduces extraction in the second period.*

3.2 The first stage: the equilibrium quantities in period 1

In order to obtain the equilibrium quantity in period 1, we replace equilibrium quantities in period 2 in the extractor's profit function. The quantity in the first stage depends on the subgame-perfect Nash equilibrium obtained in the second stage. Again, we consider two cases.

The no binding case: If the available quantity of scrap does not constraint the production of the recycler, we find:

$$\begin{aligned}\pi^e(x_1, x_2^{nc}, r^{nc}) &= P_1(x_1)x_1 - C_1(x_1) + x_1^d\bar{P}_1 - c_s[\alpha x_1 - x_1^d] \\ &+ P_2(x_2^{nc} + r^{nc})x_2^{nc} - C_2(x_2^{nc}) + x_2^d\bar{P}_2 - c_s[\alpha x_1 - x_1^d + \alpha x_2^{nc} - x_2^d]\end{aligned}$$

The FOC is:

$$P_1(x_1^{nc}) + P_1'(x_1^{nc})x_1^{nc} - C_1'(x_1^{nc}) - 2\alpha c_s = 0 \tag{7}$$

Recycling does not affect the quantity of REEs extracted by the monopolist in the first period (Equation 7 is similar to Equation 1). As expressed above, recycling slowdowns extraction in the second period. Thus, recycling helps to mitigate the balance problem by reducing the stock of abundant REEs and contributes also to reduce pollution in the second period. As far as global quantities exchanged in period 2 , we find

$$Q_2^{nc} \gtrless Q_1^{nc} = Q_1^{wr}$$

On the one hand, without recycling, the storage cost induces more extraction in the second period than in the first period. On the other hand, recycling reduces extraction in the second period. Thus, depending on the storage cost, the global quantities in period 2 can be higher or lower than in period 1. We also find:

$$\frac{\partial x_1^{nc}}{\partial c_s} < 0; \frac{\partial x_2^{nc}}{\partial c_s} < 0; \frac{\partial r^{nc}}{\partial c_s} > 0$$

If the storage costs reduces extracted quantities in each period, they boost the recycled quantity in period 2.

Proposition 3 *In the no binding case, recycling does not change extraction in period 1. The balance problem favors recycling in period 2 and mitigates environmental damages in both periods.*

The binding case: If the collected scrap constraints the production level of the recycler, the profit of the extractor reads as follows:

$$\begin{aligned}\pi^e(x_1) &= P_1(x_1)x_1 - C_1(x_1) + x_1^d\bar{P}_1 - c_s[\alpha x_1 - x_1^d] + P_2(x_2^c(x_1) + kx_1))x_2^c(x_1) - C_2(x_2^c(x_1)) \\ &+ x_2^d\bar{P}_2 - c_s[\alpha x_1 - x_1^d + \alpha x_2^c(x_1) - x_2^d]\end{aligned}$$

The FOC is the following:

$$P_1(x_1^c) + P_1'(x_1^c)x_1 - C_1'(x_1^c) - 2\alpha c_s + \frac{dx_2^c(x_1^c)}{dx_1^c}[P_2(x_1^c) + P_2'(x_1^c)x_2^c(x_1^c) - C_2'(x_1^c) - \alpha c_s] + kP_2'(x_1^c)x_2^c(x_1^c) = 0 \tag{8}$$

Comparing Equations (7) and (8) gives $x_1^c < x_1^{nc}$. Contrary to the non-binding case, recycling reduces the first period extracted quantity of REEs. By reducing extraction, the extractor, acting as a leader, curtails recycling in the second period. That enables reducing future competition. The extractor adopts a foreclosure strategy in order to keep strong market power in period 2. Hence, recycling strengthens the market power in period 1. At last, one obtains a similar result on global quantities:

$$Q_2^{nc} \gtrless Q_1^c.$$

We also find:

$$\frac{\partial x_1^c}{\partial c_s} < 0; \frac{\partial r^c}{\partial c_s} < 0 \text{ and } \frac{\partial x_2^c}{\partial c_s} < 0$$

If a high storage cost favors recycling in the non-binding case, it reduces recycling if $r = kx_1$. In this case, the balance problem indirectly limits recycling and, hence, competition in the second period.

Proposition 4 *In the binding case, recycling has anti-competitive effects in the first period. The balance problem limits recycling and, hence, accentuates the environmental damages in the second period.*

4 First-best outcome

We define the first-best outcome as a situation where there is no market power, which cancels out strategic interactions and allows taking into account environmental damages. The first-best outcome that maximizes social welfare is computed by considering a benevolent government acting under perfect information. The program of the regulator is the following:

$$\begin{gathered} MaxW(x_1, x_2, r, \lambda) = \int_0^{x_1} P(u)du + x_1^d \bar{P}_1 - C_1(x_1) - c_s[\alpha x_1 - x_1^d] - D(x_1) \\ + \int_0^{x_2+r} P(z)dz + x_2^d \bar{P}_2 - C_2(x_2) - c_s[\alpha x_1 + \alpha x_2 - x_1^d - x_2^d] - C_r(r) - D(x_2) \\ s.t.\ r \leq kx_1 \end{gathered} \tag{9}$$

where λ is a Kuhn and Tucker multiplier. The first-order conditions are:

$$P_1(x_1) - C_1'(x_1) - 2\alpha c_s - D'(x_1) + \lambda k = 0 \tag{10}$$

$$P_2(x_2 + r) - C_2'(x_2) - \alpha c_s - D'(x_2) = 0 \tag{11}$$

$$P_2(x_2 + r) - C_r'(r) - \lambda = 0 \tag{12}$$

$$\lambda[kx_1 - r] = 0 \tag{13}$$

Let us explore the first-best by distinguishing our both cases:

The no binding case: When the recycling constraint is not binding i.e. with $r < kx_1$ and $\lambda = 0$,, we get after several rearrangements of (10), (11) and (12):

$$P_1(x_1^{*nc}) = C_1'(x_1^{*nc}) + 2\alpha c_s + D'(x_1^{*nc}) \tag{14}$$

$$P_2(x_2^{*nc} + r^{*nc}) = C_2'(x_2^{*nc}) + \alpha c_s + D'(x_2^{*nc}) \tag{15}$$

$$P_2(x_2^{*nc} + r^{*nc}) = C_r'(r^{*nc}) \tag{16}$$

At the first-best allocation, the price set over each period is equal to the private marginal costs augmented by the marginal environmental damage induced by extraction. Recycling and extracted quantities in period 2 are such that social marginal costs of production are identical. We find: $\frac{\partial x_1^{*nc}}{\partial c_s} < 0$, $\frac{\partial x_2^{*nc}}{\partial c_s} < 0$, $\frac{\partial r^{*nc}}{\partial c_s} > 0$. The effects of the balance problem are similar to ones under "laissez faire".

Comparing equations (14), (15), (16) with (5) and (7) shows that the market equilibrium does not reach the first-best. The balance problem is taken into account by the extractor within the storage cost but market power and pollution inhibit reaching the first-best outcome.

The binding case: When the recycling constraint is binding i.e. with $r = kx_1$ and $\lambda > 0$, equations (10), (11) and (12) give the following equilibrium conditions:

$$\begin{aligned} P_1(x_1^{*c}) - C_1'(x_1^{*c}) - 2\alpha c_s - D'(x_1^{*c}) + k[P_2(x_2^{*c} + kx_1^{*c}) - C_r'(kx_1^{*c})] &= 0 \\ P_2(x_2^{*c} + kx_1^{*c}) - C_2'(x_2^{*c}) - \alpha c_s - D'(x_2^{*c}) &= 0 \end{aligned} \tag{17}$$

The regulator defines the level of the quantity extracted in period 1 taking into account the marginal profit of the extractor in period 1 and the marginal profit of the recycler in period 2. We find: $\frac{\partial x_1^{*c}}{\partial c_s} < 0$, $\frac{\partial x_2^{*c}}{\partial c_s} < 0$ and $\frac{\partial r^{*c}}{\partial c_s} < 0$. The balance problem reduces extraction in both periods but also the level of recycled quantities. Comparing equations given by (17) with (8) and (6) again shows that the market outcome is not a first-best.

As widely acknowledged in the literature, one way to restore the social optimum is to tax negative externalities. In the sequel, we will analyze what will happen with the implementation of a tax scheme by the benevolent government.

5 Second-best taxation of polluting REEs

In order to internalize the negative externality, i.e. pollution induced by extraction, the regulator sets Pigouvian taxes. The timing of the game between the regulator and the extractor can be described in the following way. In the first stage, the regulator sets the tax levels τ_t, that it levies on each extracted unity in each period. In the second stage, both producers decide the quantities of REEs they sell.

5.1 Decentralized decisions

As in Section 3, we solve the game by backward induction. We first define equilibrium quantities in period 2, then in period 1.

5.1.1 The second stage: the equilibrium quantities in period 2

The extractor's profit maximization program in period 2 is the following:

$$\pi^e(x_1, x_2, r) = P_2(x_2 + r)x_2 - C_2(x_2) + y_2^d \bar{P}_2 - c_s[\alpha x_1 - y_1^d + \alpha x_2 - y_2^d] - \tau_2 x_2$$

The FOC gives:

$$P_2(x_2 + r) + P_2'(x_2 + r)x_2 - C_2'(x_2) - \alpha c_s - \tau_2 = 0$$

The recycler maximizes its profit, subject to the constraint on the available resource:

$$\begin{aligned} \pi^r(r) &= P_2(x_2 + r)r - C_r(r) \\ r &\leq k.x_1 \end{aligned}$$

From the FOCs, we obtain the following best response functions:

$$\begin{aligned} r &= -\frac{P_2(x_2+r) - C_r'(r)}{P_2'(x_2+r)} \text{ if } r < kx_1 \\ r &= kx_1 \text{ otherwise} \end{aligned} \tag{18}$$

Extracted quantities in period 2 depend on the constraint on recycling:

- If the available quantity is higher than the unconstrained profit-maximizing quantity, the extractor and the recycler produce the quantities that satisfy the following FOCs:

$$\left\{ \begin{array}{c} P_2(x_2^{nct}+r^{nct}) + P_2'(x_2^{nct}+r^{nct})x_2^{nct} - C_2'(x_2^{nct}) - \alpha c_s - \tau_2 = 0 \\ P_2(x_2^{nct}+r^{nct}) + P_2'(x_2^{nct}+r^{nct})r^{nct} - C_r'(r^{nct}) = 0 \end{array} \right. \tag{19}$$

  Solving this system gives the unconstrained subgame-perfect Nash equilibrium $(x_2^{nct}(\tau_2), r^{nct}(\tau_2))$, with $\frac{\partial x_2^{nct}}{\partial \tau_2} < 0$ and $\frac{\partial r^{nct}}{\partial \tau_2} > 0$. The equilibrium in period 2 does not depend on the extracted quantities in period 1.

- If the recycler is limited by the available quantity of scrap, we find the following constrained subgame-Nash perfect equilibrium :

$$\left\{ \begin{array}{c} P_2(x_2^{ct}+r^{ct}) + P_2'(x_2^{ct}+r^{ct})x_2^{ct} - C_2'(x_2^{ct}) - \alpha c_s - \tau_2 = 0 \\ r^{ct} = kx_1 \end{array} \right. \tag{20}$$

  Solving this system gives $x_2^{ct} = f(x_1, \tau_2)$, with $\frac{\partial x_2^{ct}}{\partial x_1} < 0$.

5.1.2 The first stage: the equilibrium quantities in period 1

Quantities in period 1 depend on the subgame-perfect Nash equilibrium.

The no binding case. If $r < kx_1$, quantities in period 2 does not depends on x_1. The extractor maximizes its profit in the first period. We have:

$$\begin{aligned} \pi^e(x_1) &= P_1(x_1)x_1 - C_1(x_1) + x_1^d \bar{P}_1 - c_s[\alpha x_1 - y_1^d] - \tau_1 x_1 + P_2(x_2^{nct}+r^{nct})x_2^{nct} - C_2(x_2^{nct}) \\ &\quad + y_2^d \bar{P}_2 - c_s[\alpha x_1 - y_1^d + \alpha x_2^{nct} - y_2^d] - \tau_2 x_2^{nct} \end{aligned}$$

$$P_1(x_1^{nct}) + P_1'(x_1^{nct})x_1 - C_1'(x_1^{nct}) - 2\alpha c_s - \tau_1 = 0 \tag{21}$$

If we compare Equation (21) with Equation (7), we show that the extractor reduces extracted quantity in period 1under environmental taxation. Solving Equation (21) enables to obtain $x_1^{nct} = f(\tau_1)$, with $\frac{\partial x_1^{nct}}{\partial \tau_1} < 0$. Each per-period extracted quantity decreases with the per-period tax rate. Hence taxes increase the recycled output, since recycling and the second period extracted output are strategic substitutes.

The binding case. We replace $x_2^{ct} = f(x_1, \tau_2)$ and $r^{ct} = kx_1$ in the profit of the extractor. We obtain:

$$\begin{aligned} \pi^e(x_1) &= P_1(x_1)x_1 - C_1(x_1) + x_1^d \bar{P}_1 - c_s[\alpha x_1 - y_1^d] - \tau_1 x_1 + P_2(x_2^{ct}(x_1,\tau_2) + kx_1)x_2^{ct}(x_1,\tau_2) \\ &\quad - C_2(x_2^{ct}(x_1,\tau_2)) + y_2^d \bar{P}_2 - c_s[\alpha x_1 - y_1^d + \alpha x_2^{ct}(x_1,\tau_2) - y_2^d] - \tau_2 x_2^{ct}(x_1,\tau_2) \end{aligned}$$

The first-order condition reads as follows:

$$P_1 + P_1' x_1 - C_1' - 2\alpha c_s - \tau_1 + \frac{\partial x_2^{ct}}{\partial x_1}[P_2' x_2^{ct} + P_2 - C_2' - \alpha c_s - \tau_2] + P_2' k x_2^{ct} = 0 \tag{22}$$

Solving Equation (22) gives $x_1^{ct} = f(\tau_1, \tau_2)$ and hence $x_2^{ct} = f(\tau_1, \tau_2)$ with $\frac{dx_1^{ct}}{d\tau_1} = \frac{dx_2^{ct}}{d\tau_2} < 0$ and $\frac{dx_1^{ct}}{d\tau_2} = \frac{dx_2^{ct}}{d\tau_1} > 0$. The recycling activity increases with the tax in the second period - as in the binding case - but decreases with the tax in the first period. Again, comparing equations (8) and (22) shows that extracted quantities are reduced with environmental taxation.

5.2 The tax levels

The regulator determines the second-best using the following welfare function:

$$\begin{aligned} W(x_1, x_2, r) &= \int_0^{x_1} P(u)du + x_1^d \bar{P}_1 - C_1(x_1) - c_s[\alpha x_1 - x_1^d] - D(x_1) \\ &+ \int_0^{x_2+r} P(z)dz + x_2^d \bar{P}_2 - C_2(x_2) - c_s[\alpha x_1 + \alpha x_2 - x_1^d - x_2^d] - C_r(r) - D(x_2), \end{aligned} \quad (23)$$

replacing quantities depending on the tax levels found in the preceding section. The solution depends on the subgame-perfect Nash equilibrium, i.e. whether the recycler is limited or not by the collected scrap quantity.

5.2.1 The no-binding case

We replace $x_1^{nct} = f(\tau_1)$, $x_2^{nct} = f(\tau_2)$ and $r^{nct} = f(\tau_2)$ in (23) and we maximize the welfare function with respect to τ_1 and τ_2. The first-order conditions are the following:

$$\begin{aligned} \frac{dx_1}{d\tau_1}[P_1(x_1) - C_1'(x_1) - 2\alpha c_s - D'(x_1)] &= 0 \\ \frac{dx_2}{d\tau_2}[P(x_2 + r) - C_2'(x_2) - \alpha c_s - D'(x_2)] + \frac{dr}{d\tau_2}[P_2(x_2 + r) - C_r'(r)] &= 0 \end{aligned} \quad (24)$$

Substituting (19) and (21) into (24) yields the following pair of tax rates:

$$\begin{aligned} \tau_1^{nct} &= \underbrace{D'(x_1) + P'(x_1)}_{\textbf{Usual result}} x_1 \\ \tau_2^{nct} &= \underbrace{D'(x_2) + P'(x_2 + r)x_2}_{\textbf{Usual result}} + \underbrace{\frac{\frac{dr(\tau_2)}{d\tau_2}}{\frac{d\bar{x}_2(\tau_2)}{d\tau_2}} [\overbrace{P_2'(x_2 + r)r}^{\psi}]}_{\textbf{Recycling effect}} \end{aligned} \quad (25)$$

The tax rate in period 1 depends only on distortions of this period. One is the distortion from the negative externality from pollution, the other is the distortion from the extractor' s market power in the market of non-abundant REEs. Since $P_1'(x_1) < 0$, the tax rate is lower than the marginal damage. The benevolent government sets the tax at this level in order to reduce the tendency of the monopolist to underproduce. In this case, the tax could be either positive or negative. Its sign would depend on which distortion outweighs the other. When the distortion from the environmental damage is larger than the distortion from the extractor's market power, i.e. $D'(x_1) > -P_1'(x_1)x_1$, the tax is positive. Otherwise, it is negative and plays the role of a subsidy. It is worth noting that recycling does not influence the first period tax rate because it does not affect the extraction in that period.

According to Equation (25), the second period tax depends also only on distortions in this period. It is also composed of both usual distortions but is adjusted by an additional term emanating from the recycling activity. As $\frac{dr(\tau_2)}{d\tau_2}/\frac{dx_2(\tau_2)}{d\tau_2} < 0$ and $\psi < 0$, the recycling effect is positive. The regulator increases further the second period tax rate in order to foster recycling. If the recycling effect is very strong, the second period tax rate will be higher than the marginal damage. Note that the recycling effect catches capacity of τ_2 to modify the price in the second period. Thus the regulator will increase the tax in the second period in order to favor competition and reduce environmental damage. If we replace τ_1^{nct} and τ_2^{nct} in equations (19) and (21), we find:

$$P(x_1) - C_1^{'}(x_1) - 2\alpha c_s - D'(x_1) = 0 \tag{26}$$

$$P(x_2 + r) - C_2^{'}(x_2) - \alpha c_s - D'(x_2) - \frac{\frac{dr(\tau_2)}{d\tau_2}}{\frac{d\bar{x}_2(\tau_2)}{d\tau_2}} P'(x_2 + r)r = 0 \tag{27}$$

As Equation (26) is similar to Equation (14), taxation in the first period enables to reach the first-best outcome. The tax internalizes both market failures induced by market power and pollution. As Eq. (27) is different than Eq. (15), a tax in period 2 cannot simultaneously cope with distortions enacted by market power and the environmental damage while taking into account the recycled output. Taxation in the second period therefore enables to reach a second-best outcome.

The taxation effect on the balance problem depends on the sign of τ_1^{nct} and τ_2^{nct}. The balance problem is enhanced if tax rates are negative. In contrast, if tax rates are positive, they reduce the extracted quantities, addressing the balance problem.

Proposition 5 *The second-best tax in period 1 is always lower than the marginal damage. Recycling increases the tax level in period 2 which can be higher than the marginal damage. In the first period, first-best quantities are reached contrary to period 2 where second-best quantities are obtained.*

5.2.2 The binding case

In this case, we replace $x_1^{ct} = f(\tau_1, \tau_2)$, $x_2^{ct} = f(\tau_{1,}\tau_2)$ and $r^{ct} = f(\tau_1, \tau_2)$ in the welfare function given by Equation (23). After rearranging the first-order conditions, we find:

$$\begin{array}{c} \frac{dx_1}{d\tau_1}[P_1(x_1) - C_1'(x_1) - 2\alpha c_s - D'(x_1) + P_2(x_2 + kx_1)k - kC_r'(kx_1)] \\ +\frac{dx_2}{d\tau_1}[P_2(x_2 + kx_1) - C_2'(x_2) - \alpha c_s - D'(x_2)] = 0 \\ \frac{dx_1}{d\tau_2}[P_1(x_1) - C_1'(x_1) - 2\alpha c_s - D'(x_1) + P_2(x_2 + kx_1)k - kC_r'(kx_1)] \\ +\frac{dx_2}{d\tau_2}[P_2(x_2 + kx_1) - C_2'(x_2) - \alpha c_s - D'(x_2)] = 0 \end{array} \tag{28}$$

Substituting (20) and (22) in (28), we find the following taxes:

$$\begin{aligned} \tau_1^{ct} &= \underbrace{P'(x_1)x_1 + D'(x_1)}_{\textbf{Usual result}} \underbrace{-k[P_2(x_2 + kx_1) - C_r'(kx_1) - P_2'(x_2 + kx_1)x_2]}_{\textbf{Recycling effect}} \\ \tau_2^{ct} &= \underbrace{P_2'(x_2 + kx_1)x_2 + D'(x_2)}_{\textbf{Usual result}} \end{aligned} \tag{29}$$

As the quantity extracted in period 1 has an effect on period 2, the design of τ_1^{ct} has to consider effects in both periods. The two first terms catch usual effects in period 1 and other terms take into account effects in period 2. τ_1^{ct} diminishes with the marginal profit of the recycler and with the price variation induced by recycling. Eventually τ_1^{ct} is always inferior to the marginal damage as well τ_2^{ct}. Replacing both taxes in (20) and (22) gives conditions (17). The regulator is able to implement the first-best outcome if the recycling constraint is bounded.

Proposition 6 *Second-best taxation scheme enables to reach first-best quantities in each period. Both second-best taxes are lower than the marginal damage.*

6 Conclusion

Currently, the REEs market is dominated by China. The extraction of REEs raises serious pollution problems and leads to the balance problem. The aim of this article was to explore theoretically these questions. More precisely, this paper explores the effect of recycling and environmental tax scheme on both the balance problem and environmental pollution. It contributes to the theoretical analysis of green policies aiming at promoting reducing resource use and recycling.

We setup a Cournot model where one firm involved in the extraction sector produces two types of REEs - abundant and non-abundant - over two consecutive periods. In the second period, it competes with a recycler of non-abundant RREs which are consumed in the first period. The recycler can be limited in its activity by the extraction level in the first period. Our results crucially depend on whether the recycler can or cannot recycle the whole quantity it wants. In the no-binding case, we show that recycling has always pro-competitive effects in period 2 and no effect in period 1. The balance problem favors recycling in period 2 and reduces environmental damages in both periods. In the binding case, the extractor adopts a foreclosure strategy in the first period. The balance problem also reduces extraction in both periods but recycling as well. Due to the presence of market power and negative externality, both the first-best outcome cannot be reached whenever the recycling constraint binds or not. So we introduce a second-best environmental taxation in each period. It is shown that, except in the second period of the no-binding case, the first-best can be implemented: the regulator is able to fully internalize market imperfections in both periods. The regulator has to amend the tax rates when recycling activities are taking place. The levels of environmental taxes depend on the marginal damage, on the market power but also on the recycling effect. The Pigouvian tax is always lower than the marginal damage, except in the second period in the no-binding case. Eventually the implementation of a tax scheme enables to indirectly strengthen the role of recycling, that mitigates the balance problem and reduces pollution.

This article is, according to our knowledge, the first to theoretically investigate the balance problem and to design environmental taxes with recycling. It can be considered as providing several theoretical insights about the circular economy. It nevertheless have several limitations. It would be interesting to extend this model over several periods. This would enable us to analyze whether our results would be amended under a temporal dynamic setting. We do not consider that the stock of abundant REEs can be polluting. Likewise, we do not take into account that recycling REEs is water and energy intensive. This would lead the regulator to consider another damage and, consequently, to further amend the environmental tax rates. We also neglect the strategic aspect for a country related to the holding of REEs. Further research is needed to investigate these different questions.

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Appendix

APPENDIX 1: The no binding case

A: The recycling activity

Stability of the equilibrium
As we have:
$\pi^e_{\bar{x}_2x_2} = 2P_2' + P_2''x_2 - C_2'' < 0$
$\pi^e_{\bar{x}_2r} = P_2' + P_2''\bar{x}_2 < 0$
$\pi^r_{rr} = 2P_2' + P_2''r - C_r'' < 0$
$\pi^r_{rx_2} = P_2' + P_2''r < 0$
We find: $\pi^e_{\bar{x}_2x_2} < \pi^e_{\bar{x}_2r} < 0$ and $\pi^r_{rr} < \pi^r_{\bar{x}_2r} < 0$, so $\Delta = \pi^e_{\bar{x}_2x_2}\pi^r_{rr} - \pi^r_{\bar{x}_2r}\pi^e_{rx_2} > 0$. Thus the Gale-Nikaido condition is satisfied, meaning global uniqueness of the Cournot equilibrium. As $\pi^e_{\bar{x}_2r} < 0$ and $\pi^r_{rx_2} < 0$, the quantity $\bar{x}_2$ and r are strategic substitutes.

Effect of a change in c_s
We know that $x_1(c_s)$ solves:
$P_1(x_1) + P_1'(x_1)x_1 - C_1'(x_1) - 2\alpha c_s = 0$
We set: $F(x_1, c_s) = \pi_{x_1} = P_1(x_1) + P_1'(x_1)x_1 - C_1'(x_1) - 2\alpha c_s$
We apply the Implicit Function Theorem, and we find:
$\frac{\partial x_1}{\partial c_s} = -\frac{\partial F(x_1,c_s)/\partial c_s}{\partial F(x_1,c_s)/\partial x_1} = -\frac{-2\alpha}{2P_1'+P_1''x_1-C_1''} = -\frac{-2\alpha}{\pi_{x_1x_1}} < 0$

$x_2(c_s)$ and $r(c_s)$ solve:
$$\begin{cases} P_2(x_2(c_s)+r(c_s)) + P_2'(x_2(c_s)+r(c_s))x_2(c_s) - C_2'(x_2(c_s)) = \alpha c_s \\ P_2(x_2(c_s)+r(c_s)) + P_2'(x_2(c_s)+r(c_s))r(c_s) - C_r'(r(c_s)) = 0 \end{cases}$$
If we differentiate this system with respect to c_s, we obtain after simplification:
$$\begin{cases} \frac{dx_2}{dc_s}[2P_2' + P_2''x_2 - C_2''] + \frac{dr}{dc_s}[P_2' + P_2''x_2] = \alpha \\ \frac{dx_2}{dc_s}[P_2' + P_2''r] + \frac{dr}{dc_s}[2P_2' + P_2''r - C_r''] = 0 \end{cases}$$
$$\begin{cases} \frac{dx_2}{dc_s}[\pi^e_{x_2x_2}] + \frac{dr}{dc_s}[\pi^e_{x_2r}] = \alpha \\ \frac{dx_2}{dc_s}[\pi^r_{rx_2}] + \frac{dr}{dc_s}[\pi^r_{rr}] = 0 \end{cases}$$
We can therefore say that:
$$\begin{bmatrix} \frac{dx_2}{dc_s} \\ \frac{dr}{dc_s} \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} \pi^r_{rr} & -\pi^e_{rx_2} \\ -\pi^r_{x_2r} & \pi^e_{x_2x_2} \end{bmatrix}\begin{bmatrix} \alpha \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} \frac{dx_2}{dc_s} \\ \frac{dr}{dc_s} \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} \alpha\pi^r_{rr} \\ -\alpha\pi^r_{x_2r} \end{bmatrix}$$
with the property that $\begin{bmatrix} (<0) \\ (>0) \end{bmatrix}$
with $\Delta = \pi^e_{\bar{x}_2x_2}\pi^r_{rr} - \pi^r_{x_2r}\pi^e_{rx_2} > 0$

B: The first-best

Concavity of the program
From (10), (11) and (12) we find:
$$H(W(x_1,x_2,r)) = \begin{bmatrix} P_1' - C_1'' - D_1'' & 0 & 0 \\ 0 & P_2' - C_2'' - D_2'' & P_2' \\ 0 & P_2' & P_2' - Cr'' \end{bmatrix}$$

$M_1 = P_1' - C_1'' - D_1'' < 0$
$M_2 = [P_1' - C_1'' - D_1''][P_2' - C_2'' - D_2''] > 0$
$M_3 = [P_1' - C_1'' - D_1'']\{[P_2' - C_2'' - D_2''][P_2' - C_r''] - P_2'^2\}$
$M_3 = [P_1' - C_1'' - D_1'']\underbrace{\{[-C_2'' - D_2''][P_2' - C_r''] + P_2'[-C_r'']\}}_{\nabla} < 0$

Effect of a change in c_s
$x_1^*(c_s)$ solves:
$P_1(x_1^*(c_s) - C_1'(x_1^*(c_s)) - 2\alpha c_s - D''(x_1^*(c_s)) = 0$
We set: $F(x_1^*(c_s)) = P_1(x_1^*(c_s)) - C_1'(x_1^*(c_s)) - 2\alpha c_s - D'(x_1^*(c_s))$
We apply the Implicit Function Theorem, and we find:
$\frac{\partial x_1^*}{\partial c_s} = -\frac{\partial F(x_1,c_s)/\partial c_s}{\partial F(x_1,c_s)/\partial x_1} = -\frac{-2\alpha}{M_1} < 0$

$x_2^*(c_s)$ and $r^*(c_s)$ solve:
$$\begin{cases} P_2(x_2^*(c_s) + r^*(c_s)) - C_2'(x_2^*(c_s)) - \alpha c_s - D_2'(x_2^*(c_s)) = 0 \\ P_2(x_2^*(c_s) + r^*(c_s)) \quad C_r'(r^*(c_s)) = 0 \end{cases}$$
If we differentiate this system with respect to c_s, we obtain after simplification:
$$\begin{cases} P_2'\frac{dx_2{}^*}{dc_s} + P_2'\frac{dr^*}{dc_s} - C_2''\frac{dx_2{}^*}{dc_s} - D_2''\frac{dx_2^*}{dc_s} = \alpha \\ P_2'\frac{dx_2{}^*}{dc_s} + P_2'\frac{dr^*}{dc_s} - C_r''\frac{dr^*}{dc_s} = 0 \end{cases}$$
$$\begin{cases} \frac{dx_2{}^*}{dc_s}(P_2' - C_2'' - D_2'') + P_2'\frac{dr^*}{dc_s} = \alpha \\ P_2'\frac{dx_2{}^*}{dc_s} + \frac{dr^*}{dc_s}(P_2' - C_r'') = 0 \end{cases}$$
$$\begin{bmatrix} P_2' - C_2'' - D_2'' & P_2' \\ P_2' & P_2' - C_r'' \end{bmatrix}\begin{bmatrix} \frac{dx_2{}^*}{dc_s} \\ \frac{dr^*}{dc_s} \end{bmatrix} = \begin{bmatrix} \alpha \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} \frac{dx_2{}^*}{dc_s} \\ \frac{dr^*}{dc_s} \end{bmatrix} = \begin{bmatrix} P_2' - C_2'' - D_2'' & P_2' \\ P_2' & P_2' - C_r'' \end{bmatrix}^{-1}\begin{bmatrix} \alpha \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} \frac{dx_2{}^*}{dc_s} \\ \frac{dr^*}{dc_s} \end{bmatrix} = \frac{1}{\nabla}\begin{bmatrix} P_2' - C_r'' & -P_2' \\ -P_2' & P_2' - C_2'' - D_2'' \end{bmatrix}\begin{bmatrix} \alpha \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} \frac{dx_2{}^*}{dc_s} \\ \frac{dr^*}{dc_s} \end{bmatrix} = \frac{1}{\nabla}\begin{bmatrix} \alpha(P_2' - C_r'') \\ -\alpha P_2' \end{bmatrix}$$
with the property that $\begin{bmatrix} (<0) \\ (>0) \end{bmatrix}$
with $\nabla = [P_2' - C_2'' - D''][P_2' - C_r''] - [P_2']^2 > 0$

C: The second-best

Effect of a change in τ_1
$x_1^{sb}(c_s, \tau_1)$ solves:
$P_1'(x_1^{sb}(c_s,\tau_1))x_1^{sb}(c_s,\tau_1) + P_1(x_1^{sb}(c_s,\tau_1)) - C_1'(x_1^{sb}(c_s,\tau_1)) - 2\alpha c_s - \tau_1 = 0$
We set: $F(x_1^{sb}(c_s,\tau_1)) = P_1'(x_1^{sb}(c_s,\tau_1))x_1^{sb}(c_s,\tau_1) + P(x_1^{sb}(c_s,\tau_1)) - C_1'(x_1^{sb}(c_s,\tau_1)) - 2\alpha c_s - \tau_1$
We apply the Implicit Function Theorem, and we find:
$\frac{\partial x_1^{sb}}{\partial \tau_1} = -\frac{\partial F(x_1,c_s,\tau_1)/\partial \tau_1}{\partial F(x_1,c_s,\tau_1)/\partial x_1} = -\frac{-1}{P_1''x_1 + 2P_1' - C_1''(x_1)} < 0$

Effect of a change in τ_2
$x_2^{sb}(c_s,\tau_2)$ and $r^{sb}(c_s,\tau_2)$ solve:

$$\begin{cases} P_2'(x_2^{sb}(c_s,\tau_2)+r^{sb}(c_s,\tau_2))x_2^{sb}(c_s,\tau_2)+P_2(x_2^{sb}(c_s,\tau_2)+r^{sb}(c_s,\tau_2))-C_2'(x_2^{sb}(c_s,\tau_2))-\alpha c_s-\tau_2=0 \\ P_2(x_2^{sb}(c_s,\tau_2)+r^{sb}(c_s,\tau_2))+P_2'(x_2^{sb}(c_s,\tau_2)+r^{sb}(c_s,\tau_2)r-C_r'(r^{sb}(c_s,\tau_2))=0 \end{cases}$$

If we differentiate this system with respect to τ_2, we obtain after simplification:

$$\begin{cases} \frac{dx_2^{sb}}{d\tau_2}[2P_2'+P_2''x_2-C_2'']+\frac{dr^{sb}}{d\tau_2}[p_2'+P_2''x_2]=1 \\ \frac{dx_2^{sb}}{d\tau_2}[P_2'+P_2''r]+\frac{dr^{sb}}{d\tau_2}[2P_2'+P_2''r-C_r'']=0 \end{cases}$$

$$\begin{cases} \frac{dx_2^{sb}}{d\tau_2}[\pi^e_{x_2x_2}]+\frac{dr^{sb}}{d\tau_2}[\pi^e_{x_2r}]=1 \\ \frac{dx_2^{sb}}{d\tau_2}[\pi^r_{rx_2}]+\frac{dr^{sb}}{d\tau_2}[\pi^r_{rr}]=0 \end{cases}$$

We can therefore say that:

$$\begin{bmatrix} \frac{dx_2^{sb}}{d\tau_2} \\ \frac{dr^{sb}}{d\tau_2} \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} \pi^r_{rr} & -\pi^e_{rx_2} \\ -\pi^r_{\bar{x}_2r} & \pi^e_{\bar{x}_2x_2} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{dx_2^{sb}}{d\tau_2} \\ \frac{dr^{sb}}{d\tau_2} \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} \pi^r_{rr} \\ -\pi^r_{\bar{x}_2r} \end{bmatrix}$$

with the property that $\begin{bmatrix} (<0) \\ (>0) \end{bmatrix}$

with $\Delta > 0$

APPENDIX 2: The binding case

We assume $P''=0$ and $C'''=0$.

A: The recycling activity

Variation of x_2 with respect to x_1:

From (6), we note $F(x_2,x_1)=P_2(x_2+kx1)+P_2'(x_2+kx1)x_2-C_2'(x_2)-\alpha c_s=0$

$\frac{\partial x_2}{\partial x_1}=-\frac{\partial F(x_2,x_1)/\partial x_1}{\partial F(x_2,x_1)/\partial x_2}=-\frac{kP_2'}{2P_2'-C_2''}<0$

Concavity of the profit in the first-step:

$\Pi^e_{x_1}=P_1(x_1)+P_1'(x_1)x_1-C_1'(x_1)-2\alpha c_s+\frac{dx_2}{dx_1}[P_2(x_2(x_1)+kx1)+P_2'(x_2(x_1)+kx1)x_2(x_1)-C_2'(x_2(x_1))-\alpha c_s]+kP_2'(x_2(x_1)+kx1)x_2(x_1)$

$\Pi^e_{x_1x_1}=2p_1'-C_1''+\frac{dx_2}{dx_1}\left\{\frac{dx_2}{dx_1}[2P_2'-C_2'']+2kp_2'\right\}$

$\Pi^e_{x_1x_1}=2p_1'-C_1''+(-\frac{[kP_2']^2}{2P_2'-c2''})$

$\Pi^e_{x_1x_1}=\frac{1}{[2P_2'-C_2'']}\{[2p_1'-C_1''][2P_2'-C_2'']-[kP_2']^2\}$

$\Pi^e_{x_1x_1}=\frac{1}{[2P_2'-C_2'']}\{\underbrace{[p']^2(4-k)-C_1''[2P_2'-C_2'']-2p_1'C_2''}_{\Delta>0}\}<0$

Effect of a change in c_s

At the equilibrium, $x_1^c(c_s)$ and $x_2^c(c_s)$ solve:

$$\begin{cases} P_1(x_1^c)+P_1'(x_1^c)x_1-C_1'(x_1^c)-2\alpha c_s+\frac{dx_2^c}{dx_1^c}[P_2(x_2^c+kx1^c)+P_2'(x_2^c+kx1^c)x_2^c-C_2'(x_2^c)-\alpha c_s] \\ +kP_2'(x_2^c+kx1^c)x_2^c=0 \\ P_2(x_2^c+kx_1^c)+P_2'(x_2^c+kx_1^c)x_2-C_2'(x_2^c)-\alpha c_s=0 \end{cases}$$

If we differentiate this system with respect to c_s, we obtain after simplification:

$$\left\{\begin{array}{c} \frac{dx_1^c}{dc_s}[2p_1' + P_1''x_1 - C_1'' + \frac{dx_2^c(x_1^c)}{dx_1^c}[P_2'k + kP_2''x2 + k^2p_2''\bar{x}_2^b] + \frac{dx_2^c}{dc_s}[kPp_2''x2 + kp_2' + \frac{dx_2^c(x_1^c)}{dx_1^c}(2P_2' - C_2'' + P_2''x_2) \\ = 2\alpha + \frac{dx_2^c(x_1^c)}{dx_1^c}\alpha \\ \frac{dx_1^c}{dc_s}[kP_2' + kp_2''x_2^c] + \frac{dx_2^c}{dc_s}[2P_2' + p_2''x_2^b - C_2''] = \alpha \end{array}\right.$$

$$\begin{bmatrix} \frac{dx_1^c}{dc_s} \\ \frac{dx_2^c}{dc_s} \end{bmatrix} = \begin{bmatrix} 2p_1' - C_1'' + \frac{dx_2^c(x_1^c)}{dx_1^c}[P_2'k] & kp_2' + \frac{dx_2^c(x_1^c)}{dx_1^c}(2P_2' - C_2'') \\ kP_2' & 2P_2' - C_2' \end{bmatrix}^{-1} \begin{bmatrix} \alpha[2 + \frac{dx_2^c(x_1^c)}{dx_1^c}] \\ \alpha \end{bmatrix}$$

$$\begin{bmatrix} \frac{dx_1^c}{dc_s} \\ \frac{dx_2^c}{dc_s} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} 2P_2' - C_2'' & -[kp_2' + \frac{dx_2^c(x_1^c)}{dx_1^c}(2P_2' - C_2'')] \\ -[kP_2'] & 2p_1' - C_1'' + \frac{dx_2^c(x_1^c)}{dx_1^c}[P_2'k] \end{bmatrix} \begin{bmatrix} \alpha[2 + \frac{dx_2^c(x_1^c)}{dx_1^c}] \\ \alpha \end{bmatrix}$$

$$\left\{\begin{array}{c} \frac{dx_1^c}{dc_s} = \frac{1}{\Delta}[2P_2' - C_2'']\alpha[2 + \frac{dx_2^c(x_1^c)}{dx_1^c}] - [kp_2' + \frac{dx_2^c(x_1^c)}{dx_1^c}(2P_2' - C_2'')]\alpha \\ \frac{dx_2^c}{dc_s} = \frac{1}{\Delta}\{-[kP_2']\alpha[2 + \frac{dx_2^c(x_1^c)}{dx_1^c}] + [2p_1' - C_1'' + \frac{dx_2^c(x_1^c)}{dx_1^c}[P_2'k]]\alpha\} \end{array}\right.$$

$$\left\{\begin{array}{c} \frac{dx_1^c}{dc_s} = \frac{1}{\Delta}\{[2P_2' - C_2'']\{\alpha[2 + \frac{dx_2^c(x_1^c)}{dx_1^c}] - \frac{dx_2^c(x_1^c)}{dx_1^c}\alpha]\} - kp_2'\alpha\} \\ \frac{dx_2^c}{dc_s} = \frac{1}{\Delta}\{-[kP_2']\alpha[2 + \frac{dx_2^c(x_1^c)}{dx_1^c}] + [2p_1' - C_1'' + \frac{dx_2^c(x_1^c)}{dx_1^c}[P_2'k]]\alpha\} \end{array}\right.$$

$$\left\{\begin{array}{c} \frac{dx_1^c}{dc_s} = \frac{1}{\Delta}\{[2P_2' - C_2'']\{2\alpha\} - kp_2'\alpha\} \\ \frac{dx_2^c}{dc_s} = \frac{1}{\Delta}[-[kP_2'\alpha][2 + \frac{dx_2^c(x_1^c)}{dx_1^c} - \frac{dx_2^c(x_1^c)}{dx_1^c}] + [2p_1' - C_1'']\alpha] \end{array}\right.$$

$$\left\{\begin{array}{c} \frac{dx_1^c}{dc_s} = \frac{1}{\Delta}[\alpha P_2'[4 - k] - 2\alpha C_2''] \\ \frac{dx_2^c}{dc_s} = \frac{\alpha}{\Delta}[-2kP_2' + [2p_1' - C_1'']] \end{array}\right.$$

$$\left\{\begin{array}{c} \frac{dx_1^c}{dc_s} = \frac{1}{\Delta}[\alpha P_2'[4 - k] - 2\alpha C_2''] < 0 \\ \frac{dx_2^c}{dc_s} = \frac{\alpha}{\Delta}[2P'(1 - k) - C_1''] < 0 \end{array}\right.$$

$$\Delta = [2P_2' - C_2'][2p_1' - C_1'' + \frac{dx_2^c(x_1^c)}{dx_1^c}[P_2'k]] - [kP_2'][kp_2' + \frac{dx_2^c(x_1^c)}{dx_1^c}(2P_2' - C_2'')]$$

$$\Delta = [2P_2' - C_2'][2p_1' - C_1''] + [2P_2' - C_2']\frac{dx_2^c(x_1^c)}{dx_1^c}[P_2'k] - [kP_2'][kp_2' + \frac{dx_2^c(x_1^c)}{dx_1^c}(2P_2' - C_2'')]$$

$$\Delta = [2P_2' - C_2'][2p_1' - C_1''] + P_2'k\{[2P_2' - C_2']\frac{dx_2^c(x_1^c)}{dx_1^c} - kP_2' - \frac{dx_2^c(x_1^c)}{dx_1^c}(2P_2' - C_2'')\}$$

$$\Delta = [2p_1' - C_1''][2P_2' - C_2'] - [kP_2']^2 > 0$$

B: The first-best

Concavity of the program

From (17) we find:

$H(W(x_1, x_2, r)) = \begin{bmatrix} P_1' - C_1'' - D_1'' + k^2P_2' - k^2C_r'' & kP_2' \\ kP_2' & P_2' - C_2'' - D_2'' \end{bmatrix}$

$W_{x_1x_1} = [P_1' - C_1'' - D_1'' + k^2P_2' - k^2C_r''] < 0$

$W_{x_1x_2} = kP_2' = W_{x_2x_1} < 0$

$W_{x_2x_2} = P_2' - C_2'' - D_2'' < 0$

$DetH = \Psi = [P_1' - C_1'' - D_1'' + k^2P_2' - k^2C_r''][P_2' - C_2'' - D_2''] - [kP_2']^2$

$\Psi = [P_1' - C_1'' - D_1'' - k^2C_r''][P_2' - C_2'' - D_2''] + k^2P_2'[P_2' - C_2'' - D_2''] - [kP_2']^2$

$\Psi = [P_1' - C_1'' - D_1'' - k^2C_r''][P_2' - C_2'' - D_2''] + k^2P_2'[-C_2'' - D_2''] > 0$

Effect of a change in $\mathbf{c}_s$

$x_1^{*c}(c_s)$ and $x_2^{*c}(c_s)$ solve:

$$\left\{\begin{array}{c} P_1(x_1^{*c}(c_s)) - C_1'(x_1^{*c}(c_s)) - 2\alpha c_s - D_1'(x_1^{*c}(c_s)) + k[P_2(x_2^{*c}(c_s) + kx_1^{*c}(c_s)) - C_r'(kx_1^{*c}(c_s))] = 0 \\ P_2(x_2^{*c}(c_s) + kx_1^{*c}(c_s)) - C_2'(x_2^{*c}(c_s)) - \alpha c_s - D_2'(x_2^{*c}(c_s)) = 0 \end{array}\right.$$

If we differentiate this system with respect to C_s, we obtain after simplification:

$$\begin{cases} P_1'\frac{dx_1^{*c}}{dc_s} - C_1''\frac{dx_1^{*c}}{dc_s} - D_1''\frac{dx_1^{*c}}{dc_s} + k[P_2'[\frac{dx_2^{*c}}{dc_s} + k\frac{dx_1^{*c}}{dc_s}] - k^2C_r''\frac{dx_1^{*c}}{dc_s}] = 2\alpha \\ P_2'[\frac{dx_2^{*c}}{dc_s} + k\frac{dx_1^{*c}}{dc_s}] - C_2''\frac{dx_2^{*c}}{dc_s} - D_2''\frac{dx_2^{*c}}{dc_s} = \alpha \end{cases}$$

$$\begin{cases} \frac{dx_1^{*c}}{dc_s}[P_1' - C_1'' - D_1'' + k^2P_2' - k^2C_r''] + \frac{dx_2^{*c}}{dc_s}kP_2' = 2\alpha \\ \frac{dx_2^{*c}}{dc_s}[P_2' - C_2'' - D_2''] + P_2'k\frac{dx_1^{*c}}{dc_s}] = \alpha \end{cases}$$

$$\begin{bmatrix} \frac{dx_1^{*c}}{dc_s} \\ \frac{dx_2^{*c}}{dc_s} \end{bmatrix} = \begin{bmatrix} P_1' - C_1'' - D_1'' + k^2P_2' - k^2C_r'' & kP_2' \\ kP_2' & P_2' - C_2'' - D_2'' \end{bmatrix}^{-1} \begin{bmatrix} 2\alpha \\ \alpha \end{bmatrix}$$

$$\begin{bmatrix} \frac{dx_1^{*c}}{dc_s} \\ \frac{dx_2^{*c}}{dc_s} \end{bmatrix} = \frac{1}{\Psi} \begin{bmatrix} P_2' - C_2'' - D_2'' & -kP_2' \\ -kP_2' & P_1' - C_1'' - D_1'' + k^2P_2' - k^2C_r'' \end{bmatrix} \begin{bmatrix} 2\alpha \\ \alpha \end{bmatrix}$$

with $\Psi > 0$, we obtain:

$$\begin{cases} \frac{dx_1^{*c}}{dc_s} = \frac{\alpha}{\Psi}\{2[P_2' - C_2'' - D''] - kP_2'\} = \frac{\alpha}{\Psi}\{P_2'(2-k) - 2C_2'' - 2D'']\} \\ \frac{dx_2^{*c}}{dc_s} = \frac{\alpha}{\Psi}\{-2kP_2' + P_1' - C_1'' - D_1'' + k^2P_2' - k^2C_r''\} \end{cases}$$

$$\begin{cases} \frac{dx_1^{*c}}{dc_s} = \frac{\alpha}{\Psi}\{2[P_2' - C_2'' - D''] - kP_2'\} = \frac{\alpha}{\Psi}\{P_2'(2-k) - 2C_2'' - 2D'']\} < 0 \\ \frac{dx_2^{*c}}{dc_s} = \frac{\alpha}{\Psi}\{P'[k^2 - 2k + 1] - C_1'' - D_1'' - k^2C_r''\} < 0 \end{cases}$$

C: The second-best

Effect of a change in τ_1 and τ_2

$x_1^{sbc}(c_s, \tau_1, \tau_2)$ and $x_2^{sbc}(c_s, \tau_1, \tau_2)$ solve:

$$\begin{cases} P_1(x_1^{sbc}(c_s,\tau_1,\tau_2)) + P_1'(x_1^{sbc}(c_s,\tau_1,\tau_2))x_1^{sbc}(c_s,\tau_1,\tau_2) - C_1'(x_1^{sbc}(c_s,\tau_1,\tau_2)) - 2\alpha c_s \\ -\tau_1 + \\ \frac{dx_2^c}{dx_1^c}[P_2(x_2^{sbc}(c_s,\tau_1,\tau_2) + kx_1^{sbc}(c_s,\tau_1,\tau_2)) + P_2'(x_2^{sbc}(c_s,\tau_1,\tau_2) + kx_1^{sbc}(c_s,\tau_1,\tau_2))x_2^{sbc}(c_s,\tau_1,\tau_2) \\ -C_2'(x_2^{sbc}(c_s,\tau_1,\tau_2)) - \alpha c_s - \tau_2] \\ +kP_2'(x_2^{sbc}(c_s,\tau_1,\tau_2) + kx_1^{sbc}(c_s,\tau_1,\tau_2))x_2^{sbc}(c_s,\tau_1,\tau_2) = 0 \\ P_2(x_2^{sbc}(c_s,\tau_1,\tau_2) + kx_1^{sbc}(c_s,\tau_1,\tau_2)) + P_2'(x_2^{sbc}(c_s,\tau_1,\tau_2) + kx_1^{sbc}(c_s,\tau_1,\tau_2))x_2 \\ -C_2'(x_2^{sbc}(c_s,\tau_1,\tau_2)) - \alpha C_s - \tau_2 = 0 \end{cases}$$

If we differentiate this system with respect to τ_1 and τ_2, we obtain after simplification:

$$\begin{cases} 2p_1'\frac{dx_1^{sbc}}{d\tau_1} - C_1''\frac{dx_1^{sbc}}{d\tau_1} + \frac{dx_2^{sbc}}{dx_1}\left\{P_2'[\frac{dx_2^{sbc}}{d\tau_1} + k\frac{dx_1^{sbc}}{d\tau_1}] + P_2'\frac{dx_2^{sbc}}{d\tau_1} - C_2''\frac{dx_2^{sbc}}{d\tau_1}\right\} + kp_2'\frac{dx_2^{sbc}}{d\tau_1} = 1 \\ 2p_1'\frac{dx_1^{sbc}}{d\tau_2} - C_1''\frac{dx_1^{sbc}}{d\tau_2} + \frac{dx_2^{sbc}}{dx_1}\left\{P_2'[\frac{dx_2^{sbc}}{d\tau_2} + k\frac{dx_1^{sbc}}{d\tau_2}] + P_2'\frac{dx_2^{sbc}}{d\tau_2} - C_2''\frac{dx_2^{sbc}}{d\tau_2}\right\} + kp_2'\frac{dx_2^{sbc}}{d\tau_2} = \frac{dx_2^c}{dx_1^c} \\ P_2'[\frac{dx_2^{sbc}}{d\tau_1} + k\frac{dx_1^{sbc}}{d\tau_1}] + P_2'\frac{dx_2^{sbc}}{d\tau_1} - C_2''\frac{dx_2^{sbc}}{d\tau_1} = 0 \\ P_2'[\frac{dx_2^{sbc}}{d\tau_2} + k\frac{dx_1^{sbc}}{d\tau_2}] + P_2'\frac{dx_2^{sbc}}{d\tau_2} - C_2''\frac{dx_2^{sbc}}{d\tau_2} = 1 \end{cases}$$

The system becomes:

$$\begin{bmatrix} \frac{dx_1^{sbc}}{d\tau_1} \\ \frac{dx_2^{sbc}}{d\tau_1} \\ \frac{dx_1^{sbc}}{d\tau_2} \\ \frac{dx_2^{sbc}}{d\tau_2} \end{bmatrix} = \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & 0 & A & 0 \\ C & D & 0 & 0 \\ 0 & 0 & C & D \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \frac{dx_2^c}{dx_1^c} \\ 0 \\ 1 \end{bmatrix}$$

with:

$A = 2p_1' - C_1'' + kP_2'\frac{dx_2^{sbc}}{dx_1} = 2p_1' - C_1'' + kP_2'(-\frac{kP_2'}{2P_2' - C_2''}) = 2p_1' - C_1'' - \frac{(kP_2')^2}{2P_2' - C_2''} < 0$

$B = -\frac{kP_2'}{2P_2' - C_2''}[2P_2' - C_2''] + kp_2' = 0$

$C = P_2'k < 0$

$D = 2P_2' - C_2'' < 0$

$$\begin{bmatrix} \frac{dx_1^{sbc}}{d\tau_1} \\ \frac{dx_2^{sbc}}{d\tau_1} \\ \frac{dx_1^{sbc}}{d\tau_2} \\ \frac{dx_2^{sbc}}{d\tau_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{A} & 0 & 0 & 0 \\ -\frac{C}{AD} & 0 & \frac{1}{D} & 0 \\ 0 & \frac{1}{A} & 0 & 0 \\ 0 & -\frac{C}{AD} & 0 & \frac{1}{D} \end{bmatrix} \begin{bmatrix} 1 \\ \frac{dx_2^c}{dx_1^c} \\ 0 \\ 1 \end{bmatrix}$$

We obtain:

$$\begin{cases} \frac{dx_1^{sbc}}{d\tau_1} = \frac{1}{A} = \frac{1}{2p_1' - C_1'' - \frac{(kP_2')^2}{2P_2' - C_2''}} = \frac{dx_2^{sbc}}{d\tau_2} < 0 \\ \frac{dx_2^{sbc}}{d\tau_1} = -\frac{C}{AD} = -\frac{P_2'k}{2p_1' - C_1'' - \frac{(kP_2')^2}{2P_2' - C_2''}} \frac{1}{[2P_2' - C_2'']} = \frac{dx_1^{sbc}}{d\tau_2} > 0 \\ \frac{dx_1^{sbc}}{d\tau_2} = \frac{1}{A}\frac{dx_2^c}{dx_1^c} = \frac{1}{2p_1' - C_1'' - \frac{(kP_2')^2}{2P_2' - C_2''}}(-\frac{kP_2'}{2P_2' - C_2''}) = \frac{dx_2^{sbc}}{d\tau_1} > 0 \\ \frac{dx_2^{sbc}}{d\tau_2} = -\frac{C}{AD}\frac{dx_2^c}{dx_1^c} + \frac{1}{D} = \frac{1}{2p_1' - C_1'' - \frac{(kP_2')^2}{2P_2' - C_2''}} = \frac{dx_1^{sbc}}{d\tau_1} < 0 \end{cases}$$