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One Dynamic Game for Two Veblenian Ideas. Income Redistribution is Pareto-Improving in the Presence of Social Concerns

Frédéric Gavrel*

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Abstract

In a status game, homogenous individuals first decide on their income (and on the effort necessary to that end) with the aim of *Getting ahead of the Smiths* (GAS). Next, they make use of a pure positional good to make incomes visible. Although the GAS hypothesis is ordinal, the signaling costs induce cardinal social concerns. The GAS hypothesis, translated into the *pure pride* concern, generates an equilibrium in which identical agents have unequal income levels. Because individuals decide on their income without taking into account its effect on the signaling costs of higher-ranked participants, this equilibrium is inefficient. Introducing a Pigovian tax to reduce conspicuous consumption generates a rat-race effect in the income-setting stage which neutralizes the effect of this tax on utilities. But a redistributive income tax, if coupled with an appropriate Pigovian tax on conspicuous consumption, increases all utilities.

Key words : Status game, Social concerns, Income inequalities, Conspicuous consumption, Income redistribution, Well-being, Efficiency.

JEL Classification numbers: D3, D6, D8, I3, Z1.

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"So far as concerns the present question, the end sought by accumulation is to rank high in comparison with the rest of the community in point of pecuniary strength." - Veblen (1899)

"[...] and the means of showing pecuniary strength, and so of gaining or regaining a good name, are leisure and conspicuous consumption of goods." - Veblen (1899)

1 Introduction

This paper builds a simple two-stage game to account for the consequences of two behavioral assumptions made in the spirit of Veblen. In the first stage, homogenous individuals decide on their income (hence, on their effort to obtain that income) with the aim of reaching a high rank *in comparison with the rest of the community*. They are driven by what we refer to as Getting Ahead of the Smiths (GAS). This is the social competition game which explains financial accumulation in Veblen's *Theory of the Leisure Class* (1899). At the second stage participants decide on their conspicuous consumption in order to signal their rank to society. As in Veblen (idem), conspicuous consumption is *the means of showing pecuniary strength*. The signaling game in which individuals engage provides a rationale for rich people consuming apparently useless goods.

Concerning equilibrium, the main insights of our contribution are as follows. First, the "indirect" utilities, deduced from the equilibrium of the signaling game, have a cardinal form. More precisely, according to these reduced utilities, derived from the ordinal principle, social concerns reflect the *pure pride* feeling, a special case of the generalized KUJ (*Keeping Up with the Joneses*) motivation (Friedman and Ostrov, 2008). Individuals behave as if they were preoccupied by the gap between their own income and the average income in the population of lower ranked participants. The intuition behind this finding is that signaling costs grow with the incomes of lower ranked individuals. This result is welcome, since the cardinal version of relative concerns sounds "ad hoc", whereas assuming that people worry about their income rank,

the so-called GAS behavior, is quite plausible and indirectly confirmed by empirical papers in the spirit of Easterlin (1974, 1995). Second, the equilibrium distribution of income is non-degenerate. This result means that the desire for (advantageous) inequality is sufficient to explain income inequalities across *ex ante* identical individuals. Third, since individuals do not internalize the effect of low-ranked incomes on the signaling costs associated with high-ranked incomes, the non-cooperative equilibrium is inefficient. Incomes are too high.

If public policy is possible, then, as with Hopkins and Kornienko (2004), we first examine the effects of a Pigovian tax on conspicuous consumption. For an exogenous income distribution this tax policy clearly improves welfare by lowering signaling expenses. But if incomes are set according to the GAS principle, it turns out that implementing a Pigovian tax does not affect utilities. The decrease in conspicuous consumption generates a rat-race effect which lowers utilities by increasing incomes. Conversely, a redistributive income tax is Pareto-improving if coupled with an appropriate Pigovian system.

To position our contribution in the literature, we can clearly distinguish between two behavioral issues. The first issue is: what motivates wealth accumulation? As clearly expressed in the introductory quote, *the end sought by accumulation is to rank high in comparison with the rest of the community*. This is what we refer to as the social competition argument. The second issue is: why do we observe such large expenditures on apparently useless items? An answer is that *the means of showing pecuniary strength, and so of gaining or regaining a good name, are leisure and conspicuous consumption of goods*. We refer to this answer as the signaling argument. Assuming that incomes are private information, Hopkins and Kornienko (2004) develop a signaling game of this kind and show that implementing an appropriate tax on conspicuous consumption makes market equilibrium coincide with the cooperative situation. See also Frank (1985), Robson (1992), Ireland (2001), Hopkins and Kornienko (2009), Bilancini and Boncinelli (2012). As in the present paper, individuals are affected by their rank in the income hierarchy. In our terminology, they are driven by the GAS attitude. These papers thus belong to the signaling branch in which agents must be *ex ante* heterogeneous.¹ Relative to this strand of literature, our

¹Relative to this strand of literature, our specification of preferences is more restrictive. The

contribution shows how income distribution can be derived from social competition between homogenous individuals. In other words, the usual assumption of continuously dispersed incomes is compatible with the GAS hypothesis. To some extent, this can be seen as a consistency requirement. In addition, as mentioned above, we show that by itself a Pigovian tax on conspicuous consumption loses all effect on utilities when the feedback reaction of incomes is taken into account.

Many papers follow the "cardinal" tradition.² Similar to Duesenberry (1949), individuals' utilities are affected by the gap between their own "conspicuous" consumption and the average in the population. The cardinal version of social concerns is also referred to as the KUJ (Keeping Up with the Joneses) attitude. In general information is perfect.³ Consequently these papers belong to the social competition branch, in which agents can be homogenous. Friedman and Ostrov (2008) study the dynamics of conspicuous consumption. They also show that social competition may generate *ex post* heterogeneities among *ex ante* homogenous agents. When individuals only care about people whose conspicuous consumption is lower - the *pure pride* hypothesis - the underlying static model possesses a stable dispersed equilibrium. Relative to this strand of literature, our contribution provides a rationale for cardinal relative concerns as well as a general version of the pure pride hypothesis. In general, KUJ applies to the consumption of a positional good, but KUJ can also apply to income (models have the same structure) or to real assets, as in Cole et al. (1992), Corneo and Jeanne (1997, 1999), Cooper et al. (2001), or Cahuc and Postel-Vinay (2005) who develop behavioral growth models in which social concerns affect savings.⁴ Following Boskin and Sheshinski (1978), different papers deal with optimal taxation in a KUJ environment. See for instance Kanbur and Tuomala (2013) or Slack and Ulph (2017) for static analysis, and Aronsson and Johansson-Stenman (2010) for OLG models. Contrary to the latter papers, in our contribution income redistribution is Pareto-improving.

motivation for this specification is that it allows us to solve the signaling game, hence to deduce a general version of the pure pride concern.

²Regarding the distinction cardinal/ordinal, Bilancini and Boncinelli (2008) provide a thoughtful discussion. See also Hopkins (2008).

³An exception is Bilancini and Boncinelli (2012) who study a signaling game in which social concerns are either ordinal or cardinal.

⁴Hopkins and Kornienko (2006) develop a growth model in which social concerns are ordinal

Our paper can also be related to the literature on premarital investments which resemble a status game (to some extent) without signaling as in Peters and Siow (2002) or more recently Bhaskar and Hopkins (2016), or with signaling as in Bilancini and Boncinelli (2014) who show that when potential mates attributes are private knowledge, instrumental concerns for social status become cardinal.

To summarize, to the best of our knowledge earlier papers either develop the social competition argument⁵ or the signaling argument. We here develop a synthetic analysis which both accounts for the signaling argument (in the second stage of the status game) and for the social competition argument (in the first stage). In so doing, we show that social competition with cardinal concerns can be seen as the result of a signaling game with ordinal concerns, and we derive a generalization of pure pride preferences which is driven by the specification of the GAS concern. Overall, we develop an analysis of efficiency and taxation which delivers new insights about public policy in a Veblenian environment.

Fundamentally, the present paper seeks to unify this approach to status games, developing and bringing together two complementary ideas of Veblen⁶(whose empirical relevance is confirmed by happiness economics) within the same analytical framework. Our investigation is organized as follows. Section 2 describes our setting and solves the signaling game. Section 3 shows how the pure pride version of social preferences can be seen as an outcome of GAS and derives the equilibrium of the (reduced) social competition game. Section 4 studies the effects of taxation measures. Section 5 concludes.

⁵See Gavrel and Rebière (2018) for social competition with ordinal concerns.

⁶Historians could object that these ideas were already present in A. Smith (1776) *Wealth of Nations*. In this respect, see Tribe (2016). According to Smith, emulation (i.e. social competition) is one of the main cause of the wealth of nations, together with the increase of labor efficiency in the production of "necessities". However, Th. Veblen is much clearer on the signaling role of luxury goods, "conveniences" in Smith's terminology. For example, he notices that the rich would reduce their conspicuous consumption if the poor knew that they are rich, as in a small community where individuals' income ranks are common knowledge. Veblen can be seen as a forerunner of information theory. In fact, Smith's conception of social emulation is closer to the usual KUJ argument, whose attribution to Veblen is questionable.

2 Environment and signaling equilibrium

In this economy, there is a "very large number" (a continuum whose measure is normalized to one) of identical individuals whose peculiarity is that they are sensitive to their rank in the income hierarchy. Their utility grows with an increase in the share of participants whose incomes are lower than their own.

In the first stage, referred to as the social competition game, individuals decide on their income, y . The higher the income y , the higher the required effort $E(y)$. The effort function, $E(\cdot)$, satisfies the assumption below.

Assumption 1. $E(\cdot)$ is a strictly convex increasing function, such that $E(0) = 0$, and whose continuous derivative, $E'(y)$, satisfies $E'(0) = 0$ as well as $E'(+\infty) = +\infty$.

In the signaling subgame, the income cumulative distribution $F(y)$, treated as exogenous at this second stage, is common knowledge⁷. But, as incomes are private information, participants are led to dedicate part of their income to "conspicuous consumption", c , in order to make their rank visible. The rest ($y - c$) is dedicated to "ordinary consumption" which, contrary to conspicuous consumption, has an intrinsic value. Consider an individual whose social rank is r in the interval $[0, 1]$. Her utility, $u(y, r, c)$ is assumed to be

$$u(y, r, c) = -E(y) + (\alpha + \beta S(r))(y - c) \quad (1)$$

In this expression for utility⁸, parameters α and β are two positive scalars. The positive function $S(\cdot)$, referred to as the "GAS multiplier", satisfies the following assumption:

Assumption 2. The GAS multiplier, $S(\cdot) > 0$, is an increasing function ($S'(r) > 0$). With no loss in generality, the top, $S(1)$, is set to one while the bottom, $S(0)$, is some positive scalar strictly lower than one.⁹

⁷At the signaling stage effort, which is seen as investment (in human capital, for example), is treated as exogenous, implying that we search for a subgame perfect equilibrium.

⁸The motivation for this simple specification is that it allows us to solve the signaling game, hence to deduce a general version of the pure pride concern.

⁹In the literature, following Frank (1985), the GAS multiplier often coincides with the rank: $S(r) = r$. As the second derivative $S''(r)$ can be zero, Assumption 2 is more general.

The component $\alpha(y - c)$ ($\alpha > 0$) reflects the fact that ordinary consumption also increases utility independently of the rank in the income scale.

Let $F(y^-)$ denote the share of strictly lower income than y . The social rank, $r = r(y)$, of an individual with income y is defined as

$$r(y) = F(y) - \rho(F(y) - F(y^-)) \quad (2)$$

with ρ being a positive scalar in the interval $]0, 1]$.

For $\rho = 1$, the status reflects a pure-pride ordinal motivation: individuals only care about strictly lower incomes, that is $F(y^-)$. The value of a tie is zero. By contrast, for $\rho = 0$, the status translates a pure-envy ordinal motivation. The value of a tie is equal to one. In what follows, the case of ordinal pure envy is ruled out, implying that the (pride) parameter, ρ , is strictly positive.

For the sake of simplicity, the cumulative distribution function $F(\cdot)$ is assumed to be differentiable and strictly increasing in some interval $[m, M]$, ($M > m \geq 0$). In other words, we apply a “guess” method, meaning that we search for an asymmetric equilibrium (with pure strategies) of that type. Since $F(\cdot)$ is continuous, the rank of an individual with income y coincides with $F(y)$.¹⁰

The equilibrium of the signaling game is a relation, $c = c(y)$, between income and conspicuous consumption. From the second stage (signaling) equilibrium, one can deduce individuals’ indirect (i.e. reduced) utility which only depends on their own income and the income distribution, $F(\cdot)$. With these indirect utility functions, the first stage game becomes static. This static-reduced game determines the (global) equilibrium distribution, $F^*(\cdot)$, hence the bounds m^* and M^* of its support.

Let us solve this signaling game.¹¹ We show that any income distribution, $F(\cdot)$, is associated with a single conspicuous consumption function, $c(\cdot)$, which makes incomes visible. This means that an individual whose conspicuous consumption is c has the income $y = c^{-1}(c)$. Let $G(c(z))$ denote the share of individuals whose positional

¹⁰See below for the treatment of a discontinuous distribution.

¹¹The derivations below are similar to Hopkins and Kornienko (2004). The main difference is that, under our assumptions on utility, the signaling game has an explicit solution.

consumption is lower than $c(z)$. Suppose that for each observed consumption, $c = c(y)$, society will "deduce" that the individual has an income of y .

Assuming that $c(\cdot)$ is strictly increasing¹² the rank of an individual who decides on conspicuous consumption $c(z)$ coincides with the share of participants who have an income lower than z . Since $F(\cdot)$ is common knowledge, it follows that $G(c(z)) = F(z)$.

As incomes are given at this stage, the utility of a y -individual who chooses positional consumption $c(z)$ only depends on

$$V(z, y, c(\cdot)) = [\alpha + \beta S(G(c(z)))] (y - c(z)) \quad (3)$$

Each unit of income spent on ordinary consumption, $(y - c)$, provides a utility of $\alpha + \beta S(F(z))$. For any given z , the higher the consumption $c(z)$ of the positional good, the less the individual will benefit from consuming the ordinary good. Thus the term $[\alpha + \beta S(F(z))] c(z)$ is a utility loss which represents the cost of mimicking the behavior of z -participants.

Consistency requires that y -individuals are truthful, in the sense that their conspicuous consumption should be equal to $c(y)$. In other words, a (deviant) y -individual should not be prompted to mimic the behavior of some z -individual, whether z is higher or lower than y . Therefore, the derivative of $V(\cdot)$ with respect to z should be zero at $z = y$ for all y in the support of $F(\cdot)$:

$$\frac{\partial V(z, y, c(\cdot))}{\partial z} \Big|_{z=y} = \beta S'(G(c(y))) G'(c(y)) c'(y) (y - c(y)) - [\alpha + \beta S(G(c(y)))] c'(y) = 0 \quad (4)$$

Since $G(c(y))$ should coincide with $F(y)$ for all y in $[m, M]$, it follows that

$$\beta S'(F(y)) F'(y) (y - c(y)) - [\alpha + \beta S(F(y))] c'(y) = 0 \quad (5)$$

The previous equation is referred to as the "marginal" truth-telling condition. Knowing that mimicking a higher ranked z -individual means consuming more of the positional good, the marginal utility loss, $[\alpha + \beta S(F(y))] c'(y)$, from increasing the

¹²*A priori*, to be a perfect signal $c(\cdot)$ only needs to be reversible. It could then be decreasing, but this case is easy to rule out.

positional consumption must equal the marginal benefit from getting ahead of the Smiths, namely $\beta S'(F(y))F'(y)(y - c(y))$, also referred to as the rat-race term.

From this truth-telling condition, we can deduce the following Lemma:

Lemma 1. *For any income distribution, $F(\cdot)$, there exists a unique equilibrium of the signaling game such that $c(y) = c(m) = 0$ for $y \leq m$, $c(y) = c(M)$ for $y \geq M$, and that*

$$c(y) = \beta \frac{\int_m^y S'(F(z))F'(z)zdz}{\alpha + \beta S(F(y))}, \forall y \in [m, M] \quad (6)$$

Proof. The proof is similar to Hopkins and Kornienko (2004). See Appendix A.

An interesting and intuitive point is that the cost of making income y observable depends on the incomes of lower-ranked agents. This is because the incentive of a low-ranked individual mimicking highly-ranked individuals is all the stronger the higher her income is. Consequently, her conspicuous consumption should grow with her income. But, the positional consumption of highly-ranked individuals should be greater than that of low-ranked ones. This explains why the conspicuous consumption of an individual grows with the increase of income for lower-ranked participants.

3 Global equilibrium

3.1 Social competition equilibrium

We now turn to the first stage of this game of status.¹³

Anticipating her second-stage rational decision (and so using (6)), each individual derives an indirect (or reduced) utility function.

$$U(y, F(\cdot)) = -E(y) + V(y, \cdot) = -E(y) + \alpha y + \beta S(F(y))y - \beta \int_m^y S'(F(z))F'(z)zdz \quad (7)$$

¹³What follows can be seen as the derivation of the equilibrium of a generalized pure pride static game (according to the terminology of Friedman and Ostrov, 2008). See below, par. 3.2.

In the social competition stage participants decide on their income by maximizing $U(y, \cdot)$ for a given distribution $F(\cdot)$. A global equilibrium - i.e. an equilibrium of the static reduced game - can be defined as below.

Definition 1. *A global equilibrium is a distribution of incomes, $F^*(\cdot)$, such that utility $U(y, F^*(\cdot))$ is maximized with respect to y for all incomes in its support.*

One implication of this definition is that all individuals have the same utility in an equilibrium. This equilibrium would not otherwise be stable, in the sense that some of the players would find it profitable to deviate. In other words, $U(y, F(\cdot))$ should be a constant in the interval $[m, M]$ associated with $F(\cdot)$, and lower than this constant outside this interval.

This equal-utility condition implies that the derivative of $U(y, F(\cdot))$ with respect to y is zero in the support $[m, M]$. This condition can be written as follows:

$$\frac{\partial U(y, F(\cdot))}{\partial y} = -E'(y) + \alpha + \beta S(F(y)) + \beta S'(F(y))F'(y)[y - c(y)] - [\alpha + \beta S(F(y))]c'(y) = 0.$$

From the marginal truth-telling equation (5), we obtain that the equal-utility condition can be rewritten as

$$\frac{\partial U(y, F(\cdot))}{\partial y} = -E'(y) + \alpha + \beta S(F(y)) = 0.$$

As a consequence of the determination of positional consumptions in stage 2, it emerges that utility $U(y, r)$ is maximized with respect to income y for a given rank, $r = F(y)$, in the social hierarchy. Due to the truth-telling condition, the upgrading effect of an income increase (the rat-race term $S'(F(y))F'(y)(y - c)$) is neutralized by the increase in conspicuous consumption which is needed to make this income increase visible (term $(\alpha + \beta S(F(y)))c'(y)$). This is an interesting point. The GAS ambition gives rise to a kind of rat race in the signaling subgame: the conspicuous consumption of an individual grows with an increase in the income of lower-ranked participants. But because of this effect in stage 2, the static-reduced game is not a rat race.

It follows that in the range $[m, M]$, the equilibrium distribution satisfies

$$S(F(y)) = \frac{E'(y) - \alpha}{\beta} \quad (8)$$

We can state the proposition below.

Proposition 1. *The first stage game has a unique continuous¹⁴ equilibrium $F^*(.)$ such that $F^*(y) = S^{-1}([E'(y) - \alpha]/\beta)$ in the support $[m^*, M^*]$ with m^* being the solution to $E'(y) = \alpha + \beta S(0)$ and M^* the solution to $E'(y) = \alpha + \beta$.*

Proof. We first check that $F^*(.)$ is a c.d.f. Since $(E'(y) - \alpha)/\beta$ is equal to $S(0)$ for $y = m^*$, increasing in y in the interval $[m^*, M^*]$ under Assumption 1, and equal to $S(1) = 1$ for $y = M^*$, the reciprocal $S^{-1}(.)$ is correctly defined in this interval. Since $S(.)$ and $E'(.)$ are strictly increasing, $F^*(.)$ is a c.d.f., and it is unique. We can now see that $F^*(.)$ is an equilibrium. Indeed, $F^*(.)$ has been derived in such a way that $U(y, F^*(.)) = U^* \equiv -E(m^*) + [\alpha + \beta S(0)]m^*$, for all y in its support, $[m^*, M^*]$. From the strict convexity of $E(.)$, we deduce that the derivative of $U(y, F^*(.))$ is positive in $[0, m^*[$, and negative in $]M^*, +\infty[$. Consequently, the distribution $F^*(.)$ is such that $U(y, F^*(.)) < U^*$, for all y outside its support. This implies that $U(y, F^*(.))$ is maximized for all y in the support of $F^*(.)$, meaning that $F^*(.)$ is an equilibrium according to Definition 1.

Note that this equilibrium is easy to derive. The bounds m and M are determined by using the fact that $F(m) = 0$ and $F(M) = 1$ respectively. Between the two extremes $F^*(y)$ is computed so as to make utility constant.¹⁵ An important property of this equilibrium is its endogenous asymmetry. Indeed it means that the GAS motivation is sufficient to generate income inequality across identical agents.

We have seen that an increase in the income of low ranked individuals increases the signaling costs of higher ranked participants. This suggests that equilibrium incomes are too high according to the Pareto criterion. The formal proof consists

¹⁴Note that the dispersion of incomes does not come from mixed strategies. Strategies are pure. It comes from an endogenous asymmetry.

¹⁵This derives from the complementarity of ordinary consumption with status.

of studying the effect on utilities of a small variation $\Delta(r) = r\varepsilon$ in all incomes¹⁶ in the neighborhood of equilibrium $F^*(.)$. Since the income level associated with rank r becomes $\hat{y}(r) \equiv y^*(r) + \Delta(r)$, with $y^*(r)$ being the reciprocal of $F^*(.)$, utilities $U(r, \hat{y}(.))$ can be written as

$$U(r, \hat{y}(.)) = -E(\hat{y}(r)) + [\alpha + \beta S(r)]\hat{y}(r) - \beta \int_0^r S'(t)\hat{y}(t)dt.$$

Differentiating $U(r, \hat{y}(.))$ with respect to ε at $\varepsilon = 0$ yields:

$$\frac{\partial U(r, \hat{y}(.))}{\partial \varepsilon} = -E'(y^*(r))r + [\alpha + \beta S(r)]r - \beta \int_0^r S'(t)t dt.$$

Since the global equilibrium satisfies (see Proposition 1)

$$-E'(y^*(r)) + \alpha + \beta S(r) = 0,$$

for all r in $[0, 1]$, we deduce

$$\frac{\partial U(r, \hat{y}(.))}{\partial \varepsilon} = -\beta \int_0^r S'(t)t dt.$$

It turns out that the previous derivative is strictly negative for $0 < r \leq 1$. Consequently, a small decrease in all incomes (ε being negative and close to zero) raises all utilities $U(r, .)$ such that $0 < r \leq 1$, and leaves utility $U(0, .)$ unchanged. In other words, we can state the following proposition:

Proposition 2. *According to a Paretian planner, laissez-faire equilibrium is inefficient.*

The origin of inefficiency is very intuitive: individuals do not internalize that an increase in their income raises the signaling costs of higher ranked participants. Consequently, incomes are too high, meaning that if all agents could commit to lower their income, this would raise all utilities (for $F(y) > 0$) by reducing conspicuous consumptions.

¹⁶Notice that $\Delta(0)$ should be equal to zero since $y^*(0)$ maximizes $U(0, y^*(.))$.

Are there other equilibria? Appendix B shows that equilibria with atoms and (or) holes are ruled out as soon as the value of a tie is strictly lower than one.¹⁷

On the other hand, it is worth noting that any asymmetric equilibrium with identical agents creates a specific coordination problem. Contrary to Friedman and Ostrov (2008) we do not here deal with this coordination problem; the “dynamics of conspicuous consumption” in their terminology. Note however that the extension to heterogenous individuals (see Appendix E), where equilibrium incomes depend on the skill levels, does not suffer from this limitation.

3.2 Cardinal pure pride case as an outcome of GAS. Relation to Friedman and Ostrov (2008)

To conclude this section, we show that the **cardinal** *pure pride* case of Friedman and Ostrov (2008) can be derived from the signaling game where participants are driven by the GAS attitude.

Let us consider the benchmark case where $S(F(\cdot)) = F(\cdot)$. Under this assumption, agents’ payoffs (equation (7)) are given by

$$U(y, F(\cdot)) = -E(y) + \alpha y + \beta F(y) \left[y - \int_m^y \frac{F'(z)}{F(y)} z dz \right] \quad (9)$$

It is worth comparing the previous expression for utility with the generalized cardinal version as proposed by Friedman and Ostrov (2008) (FO, henceforth). In this paper, the authors retain the following generalized expression for cardinal concerns

$$FO \equiv \kappa F(x) \left[x - \int_0^x \frac{F'(z)}{F(x)} z dz \right] + (1 - \kappa) (1 - F(x)) \left[x - \int_x^\infty \frac{F'(z)}{1 - F(x)} z dz \right]$$

with x being conspicuous consumption and $F(\cdot)$, the corresponding c.d.f. The positive parameter κ is less than or equal to one.

Suppose $\kappa = 1/2$ and let \bar{x} denote the average consumption of the population. In this particular case, the FO criterion coincides with the usual KUJ version of

¹⁷This condition amounts to reject the extreme case of ordinal pure envy, where status only depends on the share of higher incomes (negatively).

relative concerns: $FO = KUJ = 1/2(x - \bar{x})$. In general, one can distinguish between two terms in the FO criterion. The first term, which is positive, reflects a feeling of “pride”, while the second translates a feeling of “envy”. One interesting outcome is that, assuming that the GAS multiplier, $S(\cdot)$, coincides with $F(\cdot)$, the cardinal pure-pride motivation ($\kappa = 1$) (in terms of incomes) is a particular case of the first stage (indirect) utilities in our setting (see equation (9)). In other words, assuming that people are preoccupied with their rank in the income hierarchy, and that their conspicuous consumption make their rank visible, this gives rise to a static game in which individuals behave as if they were driven by the pure pride cardinal motivation in deciding on their income (i.e. on their effort).

This outcome is close in spirit to Bilancini and Boncinelli (2014). In a marriage setup, these authors show that *allowing for asymmetric information leads to reduced preferences for social status that induce concern not only for one’s own rank in the distribution of attributes, but also for how much higher or lower one’s attribute is*. This is what we also observe in the present game of status. Reduced utilities reflect a generalized version of the cardinal pure pride case.

4 Information, welfare and policy

In this section¹⁸ we first provide a comparison of our results with earlier relevant papers. In the literature on ordinal social concerns (GAS), we can distinguish two kinds of papers. On the one hand, there are papers like Gavrel and Rebiere (2018) where, information being perfect, the game of status has only one stage, that of social competition. On the other hand, there are papers like Hopkins and Kornienko (2004) where, the income distribution being exogenous, imperfect information leads to a signaling game. The comparison with Gavrel and Rebiere (idem) shows that individuals reach the same utilities, whether information is perfect or imperfect. Relative to the second strand of literature, we find interesting results. We first show that, as in Hopkins and Kornienko (idem), a Pigovian tax schedule applied to conspicuous consumption (associated with subsidies to income, or to ordinary consumption) reduces the sig-

¹⁸We do not here provide existence and uniqueness proofs. These proofs are simple extensions based on Gavrel and Rebiere (2018) and on section 3 of the present paper. Note that we retain the same a priori assumptions regarding income distribution.

nalizing costs for a given income distribution. Thus, for a given income distribution, this tax/subsidy policy clearly increases all utilities, meaning that it raises the utility associated with any positive rank in the income hierarchy.¹⁹ The question we ask is the following: what happens when, at the income-setting stage (stage 1), individuals take into account the effects of this Pigovian tax? Surprisingly, the answer is: nothing. In other words, as regards utilities, the equilibrium of the static-reduced game is left unchanged, implying that the introduction of a Pigovian tax is ineffective. This unexpected result, which resembles the Lucas argument against public policy, clearly shows why the study of the present two-stage Veblenian game is fruitful. On the other hand, we show that, if associated with an appropriate Pigovian tax, a redistributive income tax is Pareto-improving.

4.1 Equilibrium utilities do not depend on information

As already mentioned, earlier work usually focussed on the determination and on the efficiency of the signaling game. In particular, Hopkins and Kornienko (2004) - henceforth referred to as (HK) - show that an appropriate tax on positional consumption is capable of making *laissez-faire* coincide with the cooperative case which corresponds to perfect information. When incomes are exogenous, perfect information obviously improves welfare by reducing signaling costs to zero. Does this hold true when incomes are endogenous?

Let us compare the imperfect information equilibrium (IIE) with the perfect information equilibrium (PIE). When all incomes are observable, the status game has a single stage in which players maximize their utility, $\hat{U}(y, F(\cdot)) = -E(y) + (\alpha + \beta S(F(y)))y$, for a given distribution, $F(\cdot)$. An equilibrium is a distribution $\hat{F}(\cdot)$ such that $\hat{U}(y, \hat{F}(y))$ is maximized for any income y in its support. Let $y(r)$ denote the reciprocal of the c.d.f, referred to as the income function. Following the same line of reasoning as in the proof of Proposition 1, the condition for equal utilities implies that, for all r , the associated (PIE) income function satisfies²⁰

¹⁹Bear in mind that the conspicuous consumption of 0-ranked people is zero.

²⁰See Gavrel and Rebiere (2018) for existence and uniqueness.

$$E'(y(r)) = \alpha + \beta S(r) + \beta S'(r)F'(y(r))y(r) \quad (10)$$

Note that with perfect information, individuals take into account the (positive) effect of their income on their rank through the rat-race term $\beta S'(r)F'(y(r))y(r)$. They all are trying to get ahead of the Smiths including the Smiths themselves.

On the other hand, we know (from Proposition 1) that, for all r in $[0, 1]$, the (IIE) income function²¹, $y^*(r)$, satisfies $E'(y(r)) = \alpha + \beta S(r)$, hence, in particular, $E'(y(0)) = \alpha + \beta S(0)$, for $r = 0$. In addition, since the conspicuous consumption of lower-ranked individuals is zero, all participants have the same utility level: $-E(m^*) + (\alpha + \beta S(0))m^*$ with $m^* = y^*(0)$ being the lowest income.

We observe that, through the rat-race effect, perfect information tends to increase the derivative $E'(y(r))$. Since the effort function is strictly convex, we obtain the result that perfect information increases the income $y(r)$. In other words, the income function associated with perfect information is above the income function, $y^*(r)$, associated with asymmetric information. The reason for this is that, with imperfect information, the increase in signaling costs neutralizes the GAS effect of an income increase (climbing the social ladder). Consequently, perfect information tends to degrade welfare by introducing a rat-race effect in the social competition game. On the other hand, imperfect information creates signaling costs which are dead-weight losses. One could then conclude that perfect information is not necessarily better for welfare.

This intuition is correct. Unexpectedly, individuals' utilities are exactly the same whether information is perfect or imperfect.²² This is because, in both situations, the utility of lower ranked participants is obtained by maximizing $-E(m) + (\alpha + \beta S(0))m$. With imperfect information, we know that the rat-race effect of an income increase is neutralized by an increase in signaling costs. With perfect information, Gavrel and Rebiere (2018) show that the the rat-race effect is zero for lower ranked individuals, meaning that the density $F'(m)$ is zero. A simple proof is as follows. Suppose that $F'(m) > 0$ in equilibrium. This implies that the derivative $-E'(m) + \alpha + \beta S(0) =$

²¹The reciprocal of $F^*(y)$.

²²This result is unlikely to extend to the case of heterogenous individuals of Appendix E

$-\beta S'(0)F'(m)m < 0$. Then, for any income y lower but close to m , the corresponding utility, $-E(y) + (\alpha + \beta S(0))y$, is strictly higher than for m . This contradicts the definition of an equilibrium (outside the support, utilities should be lower than inside) and sets that $-E'(m) + \alpha + \beta S(0) = F'(m) = 0$. Thus, the lower bound, m^* , is the same in both equilibria. Consequently, due to the equal-utility (equilibrium) condition, individuals receive utility $U^* = -E(m^*) + (\alpha + \beta S(0))m^*$ in both equilibria. Relative to perfect information, imperfect information generates a decrease in incomes which tends to enhance utilities. But imperfect information also induces signaling costs which exactly offset this increase in utilities.

This leads us to examine whether public policy could improve welfare by taxing conspicuous consumption.

4.2 Implementing a Pigovian tax on conspicuous consumption leaves utilities unchanged

We now assume that, unlike all individuals, the social planner has perfect knowledge of individuals' incomes. This makes public policy possible, although the equal-utility equilibrium condition cannot be overcome. Similar to (HK), we first show that, for an exogenous income distribution, introducing an appropriate Pigovian tax on conspicuous consumption (associated with a subsidy to income) reduces conspicuous consumption, leading to an increase in utilities. Next, we examine the effect of this reduction in conspicuous consumption on the setting of incomes in a global equilibrium. It transpires that when incomes are made endogenous (according to the same GAS principle), this Pigovian tax no longer affects utilities. In other words, reducing the signaling costs by taxing positional expenses is ineffective by itself. To be useful, public policy should also act on income-setting decisions, in stage 1 of the status game. Specifically, we will see below (next paragraph) that combining this tax/subsidy system with an appropriate (self-financed) redistributive income tax permits the decentralization of the corresponding social optimum which, in terms of output (not utility), coincides with the non-cooperative equilibrium, $y^*(.)$.

In stage 2, individuals decide on their positional expenses for a given income distribution, $F(.)$. From Lemma 1, we can deduce *laissez-faire* positional consump-

tion, $c(y, F(\cdot))$, for any income $y \geq 0$. We will see that a Pigovian tax can induce conspicuous consumption pattern, $p(y)$, which is lower than in laissez-faire.

Following (HK), it is assumed that the rate of the Pigovian tax, $\tau(y)$, depends on individuals' income, y . This Pigovian tax is associated with a subsidy, $s(y)$, which also depends on income. This tax/subsidy scheme is neutral as regards individuals' income, implying that

$$\tau(y)p(y) = s(y).$$

This system clearly requires perfect knowledge on the part of “tax authorities”.

Consider any continuously differentiable function, $p(y)$, which satisfies: $0 < p(y) < c(y, \cdot)$ for $m < y \leq M$, $p(y) = c(y, \cdot) = 0$ for $0 \leq y \leq m$, and $p(y) = p(M)$ for $y \geq M$. In addition, to be reversible, this function has a strictly positive derivative in y in the interval $[m, M]$.

We show that there exists a neutral tax/subsidy system $(\tau(y), s(y))$ such that individuals decide on conspicuous consumption $p(y)$, meaning that lower conspicuous consumption (for $y > m$) can be decentralized.

It is worth noting that, because tax authorities know individuals' income, a change in the “announced” income, z , does not affect the tax rate, $\tau(y)$, nor the subsidy, $s(y)$. Thus, their utility becomes

$$V(z, y, p(z)) = [\alpha + \beta S(G(p(z)))] [y + s(y) - (1 + \tau(y))p(z)].$$

The derivative of $V(\cdot)$ with respect to (the announced income) z should be equal to zero for $z = y$. It follows that the truth-telling condition is rewritten as (see the proof of Lemma 1)

$$\beta S'(F(y))F'(y)(y - p(y)) - [\alpha + \beta S(F(y))](1 + \tau(y))p'(y) = 0 \quad (11)$$

for all y in $[m, M]$

Note that, in the above equation, the tax/subsidy system leaves the income unchanged: ordinary consumption is $y - p(y)$. Consequently, the basic truth-telling condition (equation (5)) is obtained by setting the income tax rate to zero.

This modified truth-telling equation determines the tax rate, $\tau(y)$, as a function of $F(\cdot)$ and $p(\cdot)$. We obtain:

$$\tau(y) = \frac{\beta S'(F(y))F'(y)(y - p(y))}{[\alpha + \beta S(F(y))]p'(y)} - 1.$$

Knowing the Pigovian tax $\tau(y)$, the subsidy, $s(y)$, can be deduced from the income-neutrality condition $s(y) = \tau(y)p(y)$.

To summarize:

Proposition 3. *For any income distribution $F(\cdot)$, there exists a Pigovian system, $(\tau(y, F(\cdot)), s(y, F(\cdot)))$, that is capable of decentralizing lower positional expenses.*

As a consequence, holding the income distribution as a constant, the introduction of a Pigovian tax on conspicuous consumptions can increase individuals' utilities. We have:

$$[\alpha + \beta S(F(y))][y - p(y)] > [\alpha + \beta S(F(y))][y - c(y, F(\cdot))],$$

for all income, y , in the interval $]m, M]$.

What happens if incomes are made endogenous according to the GAS hypothesis? Does this Pigovian system still improve welfare? The answer is negative. Due to the feedback of incomes, (global) equilibrium utilities are not affected by a reduction of positional consumptions. Appendix C proves the following Proposition:

Proposition 4. *Introducing a Pigovian system to lower positional expenses leaves utilities unchanged by increasing income.*

Proof. Proposition 4 is proved by showing that the lowest agent has the same utility (as in the result about information). See Appendix C.

This insight might look surprising. In fact, it is the logical implication of our previous results. Indeed, if individuals' utilities are the same whether information is perfect or imperfect - hence, the same whether signaling costs are zero or maximum - then it is not so surprising that utilities also are unaffected in any intermediate situation. More precisely, the intuition behind Proposition 4 is that lowering the

signaling costs induces a rat-race dynamic of individuals' incomes which cancels the benefits of the Pigovian tax, as does perfect information with respect to imperfect information. Since, initially, any income $y^*(r)$ maximizes $-E(y(r)) + [\alpha + \beta S(r)]y(r)$ (with respect to $y(r)$ for a given rank r , see Proposition 1), this income increase lowers utilities, which return to their initial level, U^* . Appendix D (Claim 2) shows that this result holds true with an absolute value of conspicuous consumption, as in Hopkins and Kornienko (2004).

At first glance, Proposition 4 calls into question the opportunity of taxing signaling costs. In fact, a Pigovian system is ineffective by itself. But if coupled with a redistributive policy which neutralizes the rat-race effect (that it generates), then an appropriate Pigovian tax is capable of decentralizing a social optimum. This claim is proved in the next paragraph.

4.3 The magic of income redistribution in the Veblenian world

In this policy analysis we establish that taxing the rich to subsidize the poor not only increases the utility of the poor, but also that of the rich.²³

For expositional simplicity, the analysis is restricted to the case where, due to an appropriate Pigovian tax, conspicuous consumption is very close to zero.²⁴ Consequently, we can study the effects of income redistribution as if information was perfect, that is, as if individuals' incomes were public knowledge.

Thus, let us introduce an additive income tax, $\theta(y)$. In the presence of this tax and knowing that conspicuous consumptions are close to zero, utility is rewritten as

$$u(y, r) = -E(y) + (\alpha + \beta S(r))(y - \theta(y)) \quad (12)$$

²³Boskin and Sheshinski (1978) study redistributive taxation in a KUJ environment, in the absence of signaling issues. Conversely, Hopkins and Kornienko (2004) and more recently Bilancini and Boncinelli (2014) study redistributive taxation in a signaling game where the income distribution is exogenous. In this context, income redistribution increases (can decrease) the signaling costs with ordinal (cardinal) concerns.

²⁴For the sake of clarity, suppose for example that $p(y, F(\cdot)) = \alpha c(y, F(\cdot))$, with α being a scalar in $]0, 1]$. Thus, the equilibrium income distribution, denoted by $F^*(y, \alpha)$, depends on parameter α . For $\alpha = 1$, $F^*(y, 1) = F^*(y)$ is determined as in section 3, Proposition 1. On the other hand, as α goes to zero, the equilibrium $F^*(y, \alpha)$ goes to the perfect information equilibrium (PIE). Gavrel and Rebiere (idem) explicit this PIE, but this not required here. Note also that the limit $F^*(\cdot, 0)$ cannot be implemented: the Pigovian tax goes to $+\infty$.

It is worth noting that this tax can be (and will be) negative for low incomes - incomes which are close to the lower bound, m , of the income distribution - , implying that it is positive for high incomes. Indeed, this tax/subsidy system is self-financed, meaning that the budget constraint,

$$\int_m^M \theta(y)F'(y)dy = 0,$$

should be ensured.

Since $F'(y) = 1/y'(r)$, this budget constraint is equivalent to

$$\int_0^1 \theta(y(r))dr = 0.$$

How would a social planner behave in this perfect information situation? Since an appropriate Pigovian tax reduces conspicuous consumptions to (almost) zero, the optimum income function should satisfy

$$E'(y(r)) = \alpha + \beta S(r),$$

for $0 \leq r \leq 1$.

The intuition behind the determination of this social optimum is simple. Knowing that the Pigovian tax reduces signaling costs to almost zero, a social planner chooses the income level which maximizes utility $U(y, r) = -E(y) + (\alpha + \beta S(r))y$ with respect to y , for a given rank, r . In other words, the planner is not engaged in the rat-race. As a consequence, we can observe that the optimum income function coincides with the equilibrium income function, $y^*(.)$ (see Proposition 1). The reason is that, in global equilibrium, the signaling costs outweigh the rat-race effect.

It follows that, to decentralize this optimum, $y^*(.)$, the implemented income tax should reduce the rat-race term to zero in the equal-utility condition, as positional expenses do in the *laissez-faire* equilibrium. In fact, we already know how the income tax should be set. Apart from boundary conditions, the tax $\theta(y)$ should display the same pattern as signaling costs.

Substitution of $\theta(m) = \theta(y(0))$ for $c(m)$ into equation (A.3) of Appendix A yields

$$\tilde{\theta}(r) = \frac{[\alpha + \beta S(0)]\tilde{\theta}(0)}{\alpha + \beta S(r)} + \beta \frac{\int_0^r S'(t)y^*(t)dt}{\alpha + \beta S(r)} \quad (13)$$

with $\tilde{\theta}(r)$ denoting $\theta(y(r))$.

Together with the budget constraint,

$$\int_0^1 \tilde{\theta}(r)dr = 0 \quad (14)$$

equation (13) determines the optimum income tax, $\tilde{\theta}^*(r)$.

Since $\int_0^r S'(t)y^*(t)dt > 0$ for all $1 \geq r > 0$, we deduce that the previous equations generate a negative income tax: $\tilde{\theta}^*(0) < 0$, for lower-ranked participants. It follows that, inside the support of $F^*(\cdot)$, individuals' utilities are all equal to

$$-E(m^*) + [\alpha + \beta S(0)](m^* - \tilde{\theta}^*(0)) = U^* + [\alpha + \beta S(0)](-\tilde{\theta}^*(0)) > U^*,$$

thus, higher than in *laissez-faire*.

To summarize:

Proposition 5. *In the presence of an appropriate Pigovian system that reduces signaling costs to almost zero, income redistribution increases all utilities relative to laissez-faire.*

In other words, taxing the rich (in terms of income) to subsidize the poor (idem) increases the utility of the poor and that of the rich.

Different comments are needed to understand optimum income taxation in the Veblenian world, thus, possibly in the real world, too. It is worth noticing that, in terms of gross income, the implemented tax, $\tilde{\theta}^*(\cdot)$, has been constructed so as to decentralize the optimum outputs, $y^*(\cdot)$. But, in terms of utility, the optimum tax leads to a median situation between two extremes which are:

(a) on the one hand, the non-cooperative equilibrium - without income tax, and with or without a Pigovian tax - in which utilities are all equal to $U^* = -E(m^*) + (\alpha + \beta S(0))m^*$, and, (b), on the other hand, the utilitarian optimum²⁵ - without

²⁵That is the income distribution which maximizes the sum of utilities (in the absence of the equal-utility condition and without signaling costs).

income tax, but with a Pigovian tax such that positional consumption is very close to zero - in which highly ranked individuals would not pay any (positive) income tax, thus enjoying greater utility : $U(r, y^*(.)) = -E(y^*(r)) + (\alpha + \beta S(r))y^*(r)$.

Relative to the first extreme (a), the optimum tax clearly is Pareto-improving. Because lower-ranked people receive a subsidy, all utilities are increased. This is what we see as a main insight of our analysis of tax policy in a Veblenian environment. In this environment, if coupled with a Pigovian tax *à la* Hopkins and Kornienko, income redistribution is Pareto-improving. Paying an income tax improves the utility of the rich. The reason is that the income decrease generates such an increase in utility that the direct effect on disposable income is overcompensated.

Proposition 5 shows that our analysis is a complement to (HK). In (HK) the Pigovian tax is sufficient to decentralize the optimum (cooperative case) for an exogenous income distribution. In our setting where incomes are endogenous, the Pigovian tax is not sufficient. It should be associated with an income tax. The reason is that when the signalling costs are reduced (by the Pigovian tax) individuals are prompted to decide on incomes that are excessive (for any rank r but zero).

This insight is likely to be quite general. First, within the same Veblenian setting, that is, relative to the literature on ordinal social concerns, this result extends to more general specifications of the utility function, as in (HK) where the positional good has an absolute value, meaning that its value not only derives from relative concerns but also, directly, from its intrinsic characteristics.²⁶ Appendix D (Claim 3) provides a direct proof.

One could object that this insight, which seems to be driven by the equal-utility (equilibrium) constraint, is a consequence of the assumption of homogenous agents. Appendix E shows that this result extends to heterogenous agents. In this case, the income distribution replicates the skill distribution. The reason for the robustness of this result is that, fundamentally, our finding stems from the fact that the pattern of the optimum income tax is the same as the pattern of conspicuous consumption in the absence of the Pigovian tax, except for boundary conditions.

Outside the Veblenian framework some kind of Proposition 5 is likely to hold in any game where the payoffs depend on the rank held in some hierarchy, as soon as

²⁶Remember that in (HK), the game reduces to the second subgame.

information asymmetries are resolved expensively through advertising, for example.²⁷

There is something magical in the mechanics of tax policy in this status game. Status concerns profoundly transform the understanding of public policy. As the role of status concerns is supported by empirical evidence, such a change motivates a reexamination of public policies.

5 Conclusion

In the same spirit as Kreps and Scheinkman (1983) in the context of oligopoly theory, this paper brings together two behavioral hypothesis by building a two-stage status game. GAS behavior, the desire for advantageous inequality, gives rise to income inequalities whether income ranks are private information, or common knowledge, as in Gavrel and Rebiere (2018). The GAS attitude generates equilibrium income dispersion in the same way as non-sequential search induces equilibrium price dispersion in Burdett and Judd (1983). Next, the cardinal pure-pride version of social concerns (Friedman and Ostrov, 2008) can be seen as an outcome of the GAS attitude in the signaling subgame. This provides an interpretation for the assumption of cardinal social preferences. The average income in the population of lower-ranked individuals coincides with the cost of making incomes visible. Moreover, regarding welfare, market equilibrium is inefficient because individuals do not internalize the impact of their income upon the signaling costs of higher-ranked participants.

Combining the two Veblenian ideas revealed to be more than a purely theoretical exercise. Assuming that income-setting is also governed by the GAS hypothesis dramatically changes the analysis of public policy. Due to the reaction of incomes, implementing a Pigovian tax on signaling expenses leaves utilities unchanged by creating a rat-race. But, combining this tax/susidy policy with income redistribution improves all utilities by neutralizing the rat-race effect.

Gavrel and Rebiere (*idem*) show that, assuming that anyone's rank in the income scale is known by everyone, GAS equilibrium fairly well accounts for the distribution of observed wages (in France). An interesting line for empirical investigations would

²⁷Bilancini and Boncinelli (2012) also find that redistribution can be Pareto-improving in the presence of status concerns. Contrary to ours, this paper studies the effects of redistribution in a signaling model where the income distribution is exogenous.

be to compare the empirical potential of the GAS hypothesis: whether information is perfect or imperfect.

To conclude, we would like to note that our analysis could be translated to other fields, like industrial organization. For instance, consider firms who first decide on the quality (the rank) of the goods they supply. Suppose then that, in a second stage, they must let customers know this quality. A natural and practically relevant assumption would be that these firms spend (conspicuous) amounts on any item, provided that this expenditure is known to customers. This example shows that ordinal concerns are not incompatible with pure rationality. Ordinal concerns can be instrumental. Notice that, similar to Bilancini and Boncinelli (2014), in the presence of these advertising (signaling) costs, firm's reduced profits (deduced from the signaling subgame) would have a cardinal form. Profits would decrease with an increase in the quality of lower-ranked goods, which raises signaling costs. To some extent, researchers face the same type of situation: producing papers whose value (rank) depends on their effort (among other skills), and then making this value visible. Indeed, if the scholar is not sure that her paper is good, she will not spend as much money in traveling costs and other apparently futile expenses.

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Appendix

A Proof of Lemma 1

Using

$$\{[\alpha + \beta S(F(y))]c(y)\}' = \beta S'(F(y))F'(y)c(y) + [\alpha + \beta S(F(y))]c'(y) \quad (\text{A.1})$$

the first-order condition at $z = y$ is rewritten as:

$$\{[\alpha + \beta S(F(y))]c(y)\}' = \beta S'(F(y))F'(y)y \quad (\text{A.2})$$

From the first order condition of y -participants, we obtain that their utility loss, which reflects the cost of making their income visible, satisfies the condition:

$$[\alpha + \beta S(F(y))]c(y) = (\alpha + \beta S(0))c(m) + \beta \int_m^y S'(F(z))F'(z)zdz \quad (\text{A.3})$$

for all y in the interval $[m, M]$.

We now check that telling the truth actually is an optimum and that the conspicuous consumption, $c(\cdot)$, actually is an increasing function in y . First, substituting (A.3) into the utility function, we can rewrite the latter as

$$V(z, y, F(\cdot), c(m)) = [\alpha + \beta S(F(z))]y - \beta \int_m^z S'(F(t))F'(t)tdt - [\alpha + \beta S(0)]c(m) \quad (\text{A.4})$$

We see that

$$\frac{\partial V(z, y, \cdot)}{\partial z} = \beta S'(F(z))F'(z)(y - z) \quad (\text{A.5})$$

It emerges that this derivative is strictly positive for $z < y$ and strictly negative for $z > y$. This means that $z = y$ is a global maximum of the function for all y in $]m, M]$. In other words, telling the truth is optimal.

Second, writing $c(y)$ from (A.3) and differentiating, we obtain:

$$c'(y) [\alpha + \beta S(F(y))]^2 = \beta [\alpha + \beta S(F(y))] S'(F(y)) F'(y) [y - c(y)] \quad (\text{A.6})$$

This proves that the consistency requirement, that $c'(y) > 0$, is fulfilled.

To conclude the derivation of the signaling equilibrium, we can see that the positional consumption $c(m)$ of lower-income individuals ($y \leq m$) falls to zero. From equation (A.5), we deduce that those individuals will not decide on higher conspicuous expenditure. Conversely, individuals whose incomes are higher than M , and who cannot improve their rank, decide on conspicuous consumption $c(M)$.²⁸ This proves Lemma 1.

B Allowing for holes and atoms in the income distribution

The extension that we develop here bears on the assumed properties of the income distribution, $F(\cdot)$, where we now allow for holes and mass points. Indeed, what we actually showed in the text is that, assuming that $F(\cdot)$ is continuous with a connected support, then the equilibrium distribution, F^* , does possess such properties. This is not sufficient. There may be an equilibrium with holes and/or mass points. Specifically, we treat the general case where the support of $F(\cdot)$ is the union of n_1 atoms and n_2 intervals of positive length (with $n_1 + n_2 > 0$). We show that the condition for global equilibrium excludes holes as well as mass points in (the support of) income distribution.

Let us first study the signaling game for such a type of income distribution.

Consider an interval $I = [A, B]$ such that $F'(y) > 0$. In such an interval, the same logic as in the proof of Lemma 1 implies that

$$[\alpha + \beta S(F(y))]c(y) = (\alpha + \beta S(F(A)))c(A) + \beta \int_A^y S'(F(z))F'(z)zdz.$$

²⁸In equilibrium, the (measure of the) set of individuals whose income is outside $[m, M]$ reduces to zero. Nonetheless, any income $y \geq 0$ should be associated with a reduced utility $U(y, \cdot)$.

Suppose now that $F(\cdot)$ is constant in $I = [A, B]$. In this case, we note that all incomes are associated with the same rank $F(A) = F(B)$, hence with the same conspicuous consumption: $c(y) = c(A)$ for all y in I .

Consider some mass point, A , of frequency $\phi > 0$. Let $f(x^+)$ ($f(x^-)$) denote the limit of any function $f(x)$ on the right (left) hand side of x . The truth-telling condition imposes that $c(A^+)$ solves the equation

$$(\alpha + \beta S(F(A)))(A - c(A^+)) = (\alpha + \beta S(F(A) - \rho\phi))(A - c(A)).$$

In the previous equation $c(A^+)$ is the (minimum) conspicuous consumption which ensures that an individual with income A will not mimic higher income individuals. We observe that a mass point in the income distribution translates into a discontinuity of conspicuous consumption.

Applying the same line of reasoning as in Lemma 1 shows that the conspicuous consumption associated with the lower bound of $F(\cdot)$ is zero.

One can check that step by step any income distribution can be associated with a unique pattern of conspicuous consumption.

In what follows we use the following Lemma:

Lemma 2. *In any interval I where $F(\cdot)$ is either strictly increasing everywhere or constant everywhere, the derivative of stage-1 utility, $U(y, \cdot)$, with respect to y reduces to*

$$-E'(y) + \alpha + \beta S(F(y)).$$

The proof is straightforward if $F(\cdot)$ is constant, since $c(\cdot)$ is constant in this case. If I belongs to the support of $F(\cdot)$, the lemma is derived from the marginal truth-telling condition, $\beta S'(F(y))F'(y)(y - c(y)) - (\alpha + \beta S(F(y)))c'(y) = 0$.

Excluding atoms

We first show that mass points are excluded in (global) equilibrium.

Suppose by contradiction that $A > 0$ is a mass point of frequency $\phi > 0$. Since $E(\cdot)$ is continuous, we first note that the truth-telling condition,

$$(\alpha + \beta S(F(A) - \rho\phi))(A - c(A)) = (\alpha + \beta S(F(A)))(A - c(A^+)),$$

implies that $U(A^+) = U(A)$.

Let us consider the right hand side of A . We can define an interval $I =]A, B] > 0$ such that $F(\cdot)$ is strictly increasing everywhere in I or that $F(\cdot)$ is constant everywhere in I .

If $F(\cdot)$ is constant, the utility $U(y)$ should be lower than (or equal to) $U(A)$ in I . Since $U(A^+) = U(A)$, we deduce that

$$U'(A^+) = -E'(A) + (\alpha + \beta S(F(A))) \leq 0.$$

If $F(\cdot)$ is increasing, the utility $U(y)$ should be constant (and equal to $U(A)$), implying that $U'(y) = 0$. Using Lemma 2, we obtain that

$$U'(A^+) = -E'(A) + (\alpha + \beta S(F(A))) = 0.$$

Let us now turn to the left hand side of A . The same logic shows that

$$U'(A^-) = -E'(A) + (\alpha + \beta S(F(A) - \rho\phi)) \geq 0,$$

implying that

$$U'(A^+) = -E'(A) + (\alpha + \beta S(F(A))) > 0.$$

This contradiction sets that positive mass points are excluded.

To complete the proof, we show that zero cannot be a mass point. Suppose by contradiction that 0 is a mass point of frequency $\phi > 0$. We have $U(0) = U(0^+) = 0$. In addition, under Assumption 1, the right hand derivative $U'(0^+) = -E'(0) + (\alpha + \beta S(\phi)) = \alpha + \beta S(\phi)$ is strictly positive. It results that $U(y) > U(0)$ in the right hand neighborhood of 0. This contradicts the definition of a global equilibrium and proves that zero cannot be a mass point.

Excluding holes

We can now show that holes are excluded in equilibrium. Suppose by contradiction that $F(\cdot)$ is strictly increasing in the interval $[A, B]$, constant in the hole $I =]B, C[$,

and strictly increasing in the interval $[C, D]$, with $A < B < C < D$. In the interval I , all incomes have the same rank $F(B) = F(C)$. Consequently, the conspicuous consumption, $c(y)$, is constant and equal to $c(B) = c(C)$ in this interval. Since B and C belongs to the support of $F(\cdot)$, we should have:

$$U(B) = -E(B) + (\alpha + \beta S(F(B)))(B - c(B)) = U(C) = -E(C) + (\alpha + \beta S(F(B)))(C - c(B)).$$

From the strict convexity of $E(\cdot)$, we then deduce that, for all y in I , the utility

$$U(y) = -E(y) + (\alpha + \beta S(F(B)))(y - c(B)) > U(B) = U(C).$$

This contradicts the definition of a global equilibrium (in I , utilities should be lower) and proves that holes are excluded.

To summarize, we can state

Proposition 1 includes the case where the support of the income distribution is not necessarily connected and where it possibly contains mass points.

C Proof of Proposition 4

In the social competition stage participants know that the Pigovian tax will make them decide on positional expenses, $p(y)$, without affecting their income. They thus decide on their income by maximizing the utility

$$U(y, F(\cdot)) = -E(y) + [\alpha + \beta S(F(y))][y - p(y)],$$

for a given c.d.f., $F(\cdot)$ and a given pattern of positional expenses, $p(y)$.

In a global equilibrium (Definition 1), we know that utility $U(y, F(\cdot))$ should be constant in the support of $F(\cdot)$, that is, in the interval $[m, M]$, and lower outside this interval.

In the presence of the tax/subsidy system the (equilibrium) equal-utility condition implies

$$E'(y) = [\alpha + \beta S(F(y))][1 - p'(y)] + \beta S'(F(y))F'(y)[y - p(y)] \quad (\text{C.1})$$

for all income y in $[m, M]$. We can observe that lowering the signaling costs generates a (net) rat-race effect in the social competition game (stage 1). This effect is captured through the term

$$\Psi(y) \equiv -[\alpha + \beta S(F(y))]p'(y) + \beta S'(F(y))F'(y)[y - p(y)].$$

Indeed, as mentioned earlier, in the absence of the Pigovian tax, the (gross) rat-race effect was neutralized by *laissez-faire* conspicuous consumption, $c(y, F(\cdot))$.

We will show that the net rat-race terms are strictly positive for positive ranks ($r > 0$). To that end, we first prove that in a global equilibrium (with or without the Pigovian tax), the expression $\Psi(\cdot)$ should be equal to zero for the lowest income, m , i.e., the lower bound of the income-distribution support. That is

$$\Psi(m) = -[\alpha + \beta S(0)]p'(m) + \beta S'(0)F'(m)m = 0.$$

From Lemma 1, we deduce

$$\beta S'(0)F'(m)m = [\alpha + \beta S(0)] \frac{\partial c(m, F(\cdot))}{\partial y}.$$

Since the derivative $p'(m)$ should not be higher than the corresponding derivative of $c(y, F(\cdot))$ (otherwise, positional expenses would not be reduced near m), it turns out that

$$\beta S'(0)F'(m)m \geq [\alpha + \beta S(0)]p'(m),$$

implying that $\Psi(m) \geq 0$. Suppose by contradiction that $\Psi(m) > 0$. Consequently, in the left hand neighborhood of m , the derivative of utility $U(y) = U(y, 0) = -E(y) + [\alpha + \beta S(0)]y$ satisfies

$$U'(y) \simeq -E'(m) + \alpha + \beta S(0) = -\Psi(m) < 0,$$

inducing that $U(y)$ is strictly higher than $U(m) = -E(m) + [\alpha + \beta S(0)]m$. This means that utility is higher for some values of y that are outside the support of the income distribution. This is a contradiction (see Definition 1) which sets that $\Psi(m) = 0$.

Now, setting $\Psi(m) = 0$ in equation (C.1) determines the lowest income, m :

$$E'(m) = \alpha + \beta S(0).$$

In other words, we have proved that the lower bound does not depend on the Pigovian system. See Proposition 1. It is always equal to m^* (the solution in m to the former equation) whether positional expenses are reduced (by implementing a Pigovian tax) or not. Consequently, due to the (equilibrium) equal-utility constraint, the same holds for individuals' utilities

$$U(y, F(\cdot)) = -E(m^*) + [\alpha + \beta S(0)]m^* = U^*.$$

This means that introducing a Pigovian tax leaves the utilities unchanged.

Furthermore, we can see that the Pigovian tax leads to an increase in incomes for $1 \geq r > 0$. This is because the (net) rat-term, $\Psi(y(r))$, is strictly positive in this interval. Indeed, suppose by contradiction that $\Psi(y(r_1)) \leq 0$ for some rank $r_1 > 0$.

On the one hand, if $\Psi(y(r_1)) = 0$, this income, $y(r_1)$, should be equal to the *laissez-faire* level, $y^*(r_1)$. Since $p(y(r)) < c(y(r), F(\cdot))$ for $1 \geq r > 0$, it follows that the utility associated with rank r_1 ,

$$\begin{aligned} U(y(r_1), r_1) &= -E(y^*(r_1)) + (\alpha + \beta S(r_1))[y^*(r_1) - p(y^*(r_1))] \\ &> -E(y^*(r_1)) + (\alpha + \beta S(r_1))[y^*(r_1) - c(y^*(r_1), r_1)], \end{aligned}$$

would be strictly higher than U^* . From this contradiction we deduce that $\Psi(y(r))$ cannot be equal to zero for $r > 0$. On the other hand, $\Psi(y(r_1)) < 0$ implies $p'(y(r_1)) > c'(y(r_1))$. Since $p(m) = c(m) = 0$ and $p(y) < c(y)$, we obtain that for some rank r_2 , $p'(y(r_2)) < c'(y(r_2))$, hence, $\Psi(y(r_2)) > 0$. Due to the continuity of $\Psi(\cdot)$, it follows that there exists some rank $r_3 > 0$ such that $\Psi(y(r_3)) = 0$. Which is impossible. This contradiction establishes that the net rat-race term is strictly positive, for $r > 0$, implying that the Pigovian tax increases all incomes $y(r)$ for $0 < r \leq 1$ (see equation (C.1)). Now, since any income $y^*(r)$ maximizes $-E(y) + [\alpha + \beta S(r)]y$ with respect to y for a given rank r (see Proposition 1), this income increase lowers utilities, which return to their initial level, U^* .

This proves Proposition 4.

D Allowing for an absolute value of the positional good

Appendix D shows that the results for tax policy obtained in section 4 extend to the more general case where the positional good has an absolute (or intrinsic) value.

The main change is that, in stage 2, individual choices now depend on a utility function $v(c, y - c)$ which is strictly increasing in both its arguments. The function $v(\cdot)$ is assumed to satisfy the usual assumptions in consumer microeconomics. Note that the prices have been normalized to one.

We can first consider the case in which information is perfect, referred to as the cooperative case in (HK). In the absence of information asymmetries, the positional consumption satisfy

$$v_1(c, y - c) = v_2(c, y - c)$$

This optimum condition determines c as a function of y denoted by $c = \pi(y)$. Cooperative conspicuous consumption is assumed to grow with an income increase. Substitution into $v(\cdot)$ yields the indirect utility function

$$\mu(y) \equiv v(\pi(y), y - \pi(y))$$

which is a (strictly) increasing differentiable function in y .

The reason why (HK)²⁹ refer to $\pi(y)$ as the cooperative solution is that if all individuals could commit to $\pi(\cdot)$, they would solve the signaling problem at no cost. This results from the fact that $\pi(y)$, which is strictly increasing, is a perfect signal of individuals' incomes.

We now turn to the determination of conspicuous consumption in the absence of any public intervention. Using the same notation as in the text, individuals maximize

$$V(z, y, F(\cdot)) = [\alpha + \beta S(G(c(z)))]v(c(z), y - c(z))$$

with respect to the announced income, z .

It follows that the marginal truth-telling condition ($z = y$) becomes

²⁹Following Frank (1985).

$$\beta S'(F(y))F'(y)v(c(y), y - c)$$

$$+[\alpha + \beta S(F(y))][v_1(c(y), y - c(y)) - v_2(c(y), y - c(y))]c'(y) = 0.$$

This marginal truth-telling condition together with $c(y, F(\cdot)) = \pi(y)$ for $0 \leq y \leq m$, and $c(y, F(\cdot)) = \text{Max}(c(M, F(\cdot)), \pi(y))$ for $y \geq M$ determines the pattern of conspicuous consumption in the absence of public intervention, $c(\cdot, F(\cdot))$.

It is worth noticing that when the positional good has an absolute value the conspicuous consumption for $y < m$ ³⁰ is not equal to zero (as in the text) any longer. Their consumption of the positional good is $\pi(y)$, whatever the income distribution might be. Which means that their signaling costs are reduced to zero, as in the text. At the other extreme, the positional consumption for $y > M$ is still given by $c(M, F(\cdot))$ as long as it is higher than $\pi(y)$ (because the rank in the income scale cannot be higher than one).

We prove three claims:

Claim 1. Treating the income distribution as exogenous, an optimum Pigovian tax (on conspicuous consumption) is capable of decentralizing the cooperative solution. Claim 1 is proved in (HK).

Claim 2. Treating the income distribution as endogenous - individuals sets their income according to the same GAS hypothesis - , decentralizing the cooperative solution is useless: *Ex ante* utilities which take into account effort costs, are left unchanged.

Claim 3. If coupled with an appropriate income-tax (which is redistributive), the optimum Pigovian tax (which depends on income dispersion) can decentralize the social optimum . This is a Pareto-improving measure relative to the global equilibrium (without any tax).

Proof of Claim 1. The proof follows (HK). Let us introduce a Pigovian tax on conspicuous consumptions. Individuals with income y pay a tax whose rate is $\tau(y)$. Notice that this assumption means that policy makers have perfect knowledge of individuals' incomes. Loosely speaking, we can say that their purpose is to provide people

³⁰Contrary to (HK) where incomes are exogenous, individuals need to know what occurs in the signaling subgame for all $y \geq 0$.

with this information at no cost. The tax collected from the conspicuous consumption of a y -individual is dedicated to the financing of a subsidy, $s(y) = \tau(y)c(y, \cdot)$ to the same individuals. Individuals take these tax/subsidy couples $(s(y), \tau(y))$ as given.

In the presence of a Pigovian tax, utilities are rewritten as

$$V(z, y, F(\cdot), \tau(\cdot)) = [\alpha + \beta S(G(c(z, \cdot)))]v(c(z), y + s(y) - (1 + \tau(y))c(z)).$$

Since the Pigovian tax is set so as to make the cooperative solution a signaling equilibrium, the marginal truth-telling condition now is (with simple notations)

$$\beta S'(F(y))F'(y)\mu(y)$$

$$+[\alpha + \beta S(F(y))][v_1(\pi(y), y - \pi(y)) - (1 + \tau(y))v_2(\pi(y), y - \pi(y))]\pi'(y) = 0.$$

Two comments are needed to understand the latter condition. Firstly, since tax authorities know individuals' income the announced income, z , does not affect the tax on positional consumption. This explains why the derivative, $\tau'(y)$, does not appear in the previous equation. Second, assuming that $y + s(y)$ is strictly increasing, $F(y)$, is the share of individuals whose subsidized income is lower than (or equal to) $y + s(y)$.

Noting that in the cooperative case v_1 is equal to v_2 , this truth-telling condition determines the optimum Pigovian tax as

$$\tau^*(y, F(\cdot)) = \frac{\beta S'(F(y))F'(y)\mu(y)}{[\alpha + \beta S(F(y))]v_2(\pi(y), y - \pi(y))\pi'(y)}.$$

Assuming that $F'(y) > 0$, we can see that the tax is strictly positive. The reason is that, for any income y , the Pigovian tax must make the non-positional good more attractive. If $F'(y) = 0$, then the return to cheating is zero: announcing $z > y$ does not allow individuals to reach a better rank. Consequently, the tax is zero in such case.

Proof of Claim 2. We now study the equilibrium in the income-setting stage depending on whether an optimum Pigovian tax is implemented or not.

Income setting in the presence of the optimum Pigovian tax

Knowing that the Pigovian tax will decentralize the cooperative solution $\pi(y)$ (to the signaling game of stage 2), any y -individual knows that, in stage 1, her utility is

$$U(y, F(\cdot)) = -E(y) + [\alpha + \beta S(F(y))]\mu(y).$$

In the income-setting stage, the income distribution is derived from the (equilibrium) equal-utility constraint:

$$-E'(y) + [\alpha + \beta S(F(y))]\mu'(y) + \beta S'(F(y))F'(y)\mu(y) = 0.$$

The proof of Claim 2 amounts to show that the density $F'(m)$ is zero. To that end, consider the previous equation for the lower bound of the income distribution, $y = m$:

$$-E'(m) + [\alpha + \beta S(0)]\mu'(m) + \beta S'(0)F'(m)\mu(m) = 0.$$

For $y < m$, the utility is

$$-E(y) + [\alpha + \beta S(0)]\mu(y).$$

According to the definition of an equilibrium (Definition 1), for $0 \leq y < m$, the utility should be lower than in m . This equilibrium requirement implies that

$$-E'(m) + [\alpha + \beta S(0)]\mu'(m) \geq 0,$$

hence

$$\beta S'(0)F'(m)\mu(m) \leq 0.$$

Since $\beta S'(0)\mu(m) > 0$, this sets that $F'(m) = 0$.

Consequently, the lower bound is set so that

$$-E'(m) + [\alpha + \beta S(0)]\mu'(m) = 0.$$

Income-setting in the absence of any Pigovian tax

In the absence of the Pigovian tax, the equilibrium equal-utility condition is written as

$$-E'(y) + \beta S'(F(y))F'(y)v(c(y), y - c(y))$$

$$+[\alpha + \beta S(F(y))]\{[v_1(c(y), y - c(y)) - v_2(c(y), y - c(y))]c'(y) + v_2(c(y), y - c(y))\} = 0.$$

For $y \leq m$, $c(y, F(\cdot))$ coincides with the cooperative solution $\pi(y)$. This implies that for $y = m$, the equal-utility condition leads to

$$-E'(m) + \beta S'(0)F'(m)\mu(m) + [\alpha + \beta S(0)]\mu'(m) = 0.$$

Following the same line of reasoning as above, we deduce that the rat-race term, $\beta S'(0)F'(m)\mu(m)$ is zero in a global equilibrium. Consequently, the lower bound, m , and then the utility $-E(m) + (\alpha + \beta S(0)\mu(m))$ are the same with or without the Pigovian tax. From the equilibrium equal-utility condition, we deduce Claim 2.

Proof of Claim 3.

In this case, the social optimum (in terms of gross incomes), $y^*(r)$, should satisfy

$$-E'(y^*(r))\frac{dy^*(r)}{dr} + (\alpha + \beta S(r))\frac{d\mu(y^*(r))}{dr} = 0 \quad (\text{D.1})$$

In the presence of an appropriate income tax (that decentralizes the optimum), $\tilde{\theta}(r) \equiv \theta(y^*(r))$, the equal-utility equilibrium condition becomes

$$-E'(y^*(r))\frac{dy^*(r)}{dr} + [\alpha + \beta S(r)]\mu'(z(r))z'(r) + \beta S'(r)\mu(z(r)) = 0,$$

with $z(r) = y^*(r) - \tilde{\theta}(r)$ being the disposable income.

Let $\tilde{\mu}(r)$ denote $\mu(z(r))$. Using equation (D.1), the equal-utility equation can be rewritten as

$$\tilde{\mu}'(r) = -\frac{\beta S'(r)}{\alpha + \beta S(r)}\tilde{\mu}(r) + \frac{d\mu(y^*(r))}{dr}.$$

Solving this linear differential equation in $\tilde{\mu}(r)$ gives

$$\tilde{\mu}^*(r) = \exp\left[-\int_0^r \frac{\beta S'(t)}{\alpha + \beta S(t)} dt\right]\tilde{\mu}^*(0) + \int_0^r \exp\left[-\int_t^r \frac{\beta S'(s)}{\alpha + \beta S(s)} ds\right] \frac{d\mu(y^*(t))}{dt} dt,$$

or

$$\tilde{\mu}^*(r) = \frac{\alpha + \beta S(0)}{\alpha + \beta S(r)} \tilde{\mu}^*(0) + \int_0^r \frac{\alpha + \beta S(t)}{\alpha + \beta S(r)} \frac{d\mu(y^*(t))}{dt} dt.$$

Let us show that, under the budget constraint, the disposable income of the lower-ranked individuals is higher than m^* , meaning that the income tax $\tilde{\theta}^*(0)$ is strictly negative. Suppose by contradiction that $\tilde{\mu}^*(0) \leq \mu(m^*) (= \mu(y^*(0)))$. It then follows that, for all $1 \geq r > 0$,

$$\tilde{\mu}^*(r) < \tilde{\mu}^*(0) + \int_0^r \frac{d\mu(y^*(t))}{dt} dt.$$

We obtain

$$\tilde{\mu}^*(r) < \tilde{\mu}^*(0) - \mu(m^*) + \mu(y^*(r)) \leq \mu(y^*(r)).$$

This means that the income tax, $\tilde{\theta}^*(r)$, is strictly positive for all $r > 0$, implying that the budget constraint is not fulfilled. Consequently, we deduce $\tilde{\theta}^*(0) < 0$, and then $\tilde{\mu}^*(0) = \mu(m^* - \tilde{\theta}^*(0)) > \mu(m^*)$. Consequently, due to the equal-utility (equilibrium) condition, utilities are all equal to

$$-E(m^*) + (\alpha + \beta S(0))\mu(m^* - \tilde{\theta}^*(0)) > -E(m^*) + (\alpha + \beta S(0))\mu(m^*),$$

hence strictly higher than in *laissez-faire*.

This proves Claim 3.

E Heterogenous skills

We here briefly prove that Proposition 5 holds true with heterogenous skills. To that end, we assume that, for any income y , the required effort, $(1/\gamma)E(y)$, decreases with an increase in the skill level³¹, denoted by γ . The utility of a γ -individual (individual of skill γ) now is

$$u(y, r, c, \gamma) = -\frac{E(y)}{\gamma} + (\alpha + \beta S(r))(y - c).$$

³¹Results extend to more general specifications for the effort function.

The skill level, γ , is assumed to be continuously distributed in the interval $[\gamma_L, \gamma_H]$ with $0 < \gamma_L < \gamma_H$ according to the c.d.f. $H(\gamma)$.

In line with intuition we search for a non-cooperative equilibrium in which the income distribution, $F(y)$, replicates the skill distribution, $H(\gamma)$. This means that if $y(\gamma)$ is the income of a γ -individual, then $F(y(\gamma)) = H(\gamma)$

The analysis of the signaling stage (Lemma 1) extends to heterogenous skills. This implies that the reduced utility of a γ -individual is

$$U(y, F(\cdot), \gamma) = -\frac{E(y)}{\gamma} + (\alpha + \beta S(F(y)))y - \beta \int_m^y S'(F(z))F'(z)zdz.$$

For any γ , the first order condition for the maximization of utility is

$$E'(y) = \gamma[\alpha + \beta S(H(\gamma))].$$

Since $E(\cdot)$ is strictly convex while $S(\cdot)$ and $H(\cdot)$ are strictly increasing, we deduce that the solution to the previous f.o.c., denoted by $y^*(\gamma)$, is strictly increasing in γ . This implies that $F(y^*(\gamma)) = H(\gamma)$, as assumed in writing this maximization condition.

In the laissez-faire situation, the income hierarchy is described by the income function $y^*(\gamma)$ and the utility of an individual is

$$U(y, F(\cdot), \gamma) = \frac{E(y^*(\gamma))}{\gamma} + (\alpha + \beta H(\gamma))(y^*(\gamma) - c^*(\gamma)),$$

with (as $y'(\gamma)F'(y(\gamma)) = 1$)

$$c^*(\gamma) = \frac{\beta}{\alpha + \beta S(H(\gamma))} \int_{\gamma_L}^{\gamma} S'(H(t))H'(t)y^*(t)dt.$$

We now introduce public policy. First, conspicuous consumption is reduced to almost zero by an appropriate Pigovian system. Second, an income tax, $\theta(y)$, is introduced in the income-setting stage. It turns out that the utility of a γ -individual becomes

$$U(y, F(\cdot), \gamma, \theta(\cdot)) = -\frac{E(y)}{\gamma} + (\alpha + \beta S(F(y)))(y - \theta(y)).$$

The f.o.c. now is

$$\frac{E'(y)}{\gamma} = (\alpha + \beta S(F(y))) - (\alpha + \beta S(F(y)))\theta'(y) + \beta S'(F(y))F'(y)(y - \theta(y)).$$

Suppose that the income tax (which can be negative) is set so as to cancel the net rat-race term, that is

$$-(\alpha + \beta S(F(y)))\theta'(y) + \beta S'(F(y))F'(y)(y - \theta(y)) = 0.$$

Consequently, the income function coincides with $y^*(\gamma)$ and we deduce that the income tax should satisfy the following condition (noting that $F'(y(\gamma)) = 1/y'(\gamma)$)

$$-(\alpha + \beta S(H(\gamma)))\theta'(y^*(\gamma))\frac{dy^*(\gamma)}{d\gamma} + \beta S'(H(\gamma))(y^*(\gamma) - \theta(y^*(\gamma))) = 0.$$

Denoting $\theta(y^*(\gamma))$ by $\tilde{\theta}(\gamma)$, we obtain the following differential equation in $\tilde{\theta}$

$$-(\alpha + \beta S(H(\gamma)))\tilde{\theta}'(\gamma)\frac{dy^*(\gamma)}{d\gamma} + \beta S'(H(\gamma))(y^*(\gamma) - \tilde{\theta}(\gamma)) = 0,$$

whose solution is

$$\tilde{\theta}(\gamma) = \frac{(\alpha + \beta S(0))\tilde{\theta}(\gamma_L)}{\alpha + \beta S(H(\gamma))} + \frac{\beta}{\alpha + \beta S(H(\gamma))} \int_{\gamma_L}^{\gamma} S'(H(t))H'(t)y^*(t)dt,$$

or

$$\tilde{\theta}(\gamma) = \frac{(\alpha + \beta S(0))\tilde{\theta}(\gamma_L)}{\alpha + \beta S(H(\gamma))} + c^*(\gamma).$$

Together with the budget constraint,

$$\int_{\gamma_L}^{\gamma_H} \tilde{\theta}(\gamma)d\gamma = 0,$$

the previous equation determines an income tax $\tilde{\theta}^*(\gamma)$. One can see that $\tilde{\theta}^*(\gamma_L) < 0$, hence that $\tilde{\theta}^*(\gamma) < c^*(\gamma)$ for any γ in $[\gamma_L, \gamma_H]$. Comparing $U(y, F(\cdot), \gamma, \theta(\cdot))$ with

$U(y, F(\cdot), \gamma)$ shows that the income-redistribution schedule, $[\tilde{\theta}^*(\gamma)]_{\gamma_L}^{\gamma_H}$, increases all utilities. This proves that Proposition 5 extends to heterogenous skills.