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Abstract

This paper aims to present a new explanation for environmental traps, by the presence of endogenous hazard rate. We show that adaptation and mitigation policies have different effects on the occurrence of environmental traps: the former could cause an environmental trap, whereas the latter could help society avoid such a trap, since it decreases the harmful event probability. As a result, we present a new trade-off between adaptation and mitigation policies other than the usual dynamic trade-off highlighted in many studies, which is crucial for developing countries. Contrary to the literature, when the economy is in a trap, the economy at the high environmental quality equilibrium tends to be more conservative for resource exploitation than the low environmental quality equilibrium economy, which implies a heterogeneous reaction against the endogenous hazard rate.

Keywords : Environmental damage, Harmful event, Occurrence hazard, Tipping points, Multiple equilibria, Environmental traps, Adaptation, Mitigation.

JEL Classification : O13, D81, Q2, Q54,

1 Introduction

A social planner should consider ways of avoiding damage as a result of hazardous events that might entail negative consequences. A direct response requires action to reduce the probability of a harmful event taking place. In many cases, mitigation activities are able to reduce the risk of a harmful event by improving the environmental quality¹ although they can not eliminate it completely. In such situations, a possible action could be alleviating the negative consequences of a damage due to a harmful event. The measures taken to reduce the loss due to the harmful event can be considered as adaptation. The management of adaptation and mitigation activities raises an interesting dynamic trade-off that can be described as "adaptation and mitigation dilemma" in the environmental economics literature (Zemel (2015), Tsur and Zemel (2015), Crépin et al. (2012)).

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¹Since we focus on environmental quality in the paper, mitigation activities are aimed at increasing environmental quality. A possible definition of mitigation activity can be reforestation, which enhances carbon sinks. (IPCC (2007))

To elaborate more on these concepts, we consider concrete examples. Improvements in energy efficiency, activities such as carbon capture and storage and reforestation address the root causes, by decreasing greenhouse gas emissions and reducing the risk of a harmful event and therefore can be referred to as mitigation. Mitigation activity can also be seen as a tool to avert a harmful event ([Martin and Pindyck \(2015\)](#)). Whereas, installing flood defenses and developing irrigation systems aim to reduce the damage inflicted by a possible harmful event and hence can be classified as adaptation. In this context, adaptation plays a *proactive* role, which means that it has no concrete effect prior to the harmful event ([Smit et al. \(2000\)](#) ; [Shalizi and Lecocq \(2009\)](#)). In this example, the problem is to decide an optimal combination of risk-reducing and damage-reducing measures within a given budget.

In this paper, we study the optimal management of natural resources and its link with adaptation and mitigation policies in a simple growth model under endogenous hazard rate depending on the environmental quality level². For example, the arrival rate of harmful events such as droughts, crop failures and floods³ is linked to the exploitation of natural capital. Our model uses a general definition for natural capital which encompasses all natural amenities such as the stock of clean water, soil and air quality, forests, biomass and so on.

The contribution of this paper is threefold: The first contribution of the paper is to analyze the implications of the endogenous hazard rate on the occurrence of the multiplicity of equilibria (i.e the environmental trap). We show that one of the reasons behind environmental traps is the endogenous hazard rate. The reason why harmful events may cause an environmental trap is as follows: when an economy faces an endogenous hazard rate, a second trade-off arises between consumption and the endogenous hazard rate other than the usual intertemporal trade-off between present and future consumption. An economy with serious environmental quality problems is supposed to be more impatient due to the endogenous hazard rate. Therefore, agents tend to increase their consumption at earlier dates since they face a higher event probability, which again stresses the environmental quality over time. This trade-off between consumption and harmful events results in a vicious cycle of "low level of environmental quality and consumption" in the long run that can be defined as an environmental trap.

The possibility of multiple stationary equilibria in growth models with endogenous hazard⁴ is already mentioned by [Tsur and Zemel \(2016\)](#). However, one cannot understand the economic intuition behind the multiplicity of equilibria mentioned in [Tsur and Zemel \(2016\)](#). In this paper, not only do we offer an economic explanation for environmental traps, but we also prove the existence of the multiplicity of equilibria. The mathematical proof of the existence of environmental traps allows us to know within which range of damage due to a harmful event, the economy finds itself in a multiple equilibria economy.

Our study relates also to a substantial literature on resource exploitation under uncertainty. The consideration of uncertain events for optimal management starts with [Cropper \(1976\)](#), who finds that the depletion of resources is either faster or slower if the available resource stock is uncertain. [Clarke and Reed \(1994\)](#) offer a framework where the stochastic dynamic problem for optimal management is transformed into a deterministic problem that can be solved with the Pontryagin maximum principle. The authors find that the endogenous hazard rate either increases or decreases the pollution stock when there is a single occurrence

²We use interchangeably the terms environmental quality and natural resource stock.

³The depletion of forests in a region increases the probability of floods, since the soil loses its ability to absorb the rainfall.

⁴There is also a large literature on growth models with endogenous discounting pointing out the possibility of multiple equilibria ([Das \(2003\)](#), [Drugeon \(1996\)](#), [Schumacher \(2009\)](#)). However, these papers do not include neither the uncertainty component or the environmental aspects.

event that reduces indefinitely the utility to constant level. A more recent work by [Polasky et al. \(2011\)](#) presents a growth model with exogenous/endogenous collapse risk and conclude that the endogenous risk either increases or decreases the natural resource exploitation when there is an irreversible damage in the economy after the harmful event. So, the effect of endogenous hazard rate on resource exploitation is ambiguous. A paper by [de Zeeuw and Zemel \(2012\)](#) follows a similar modeling approach to [Polasky et al. \(2011\)](#) and tries to figure out the conditions under which the economy tends to conserve the natural resources with the hazard rate. The conservation of natural resources with respect to the hazard rate can be considered as a *precautious behavior*. They show that the ambiguous effect is due to the characteristics of the post-event value. If there is a doomsday event and economic activity stops, the endogenous hazard rate enters as an additional factor to the discount rate, which makes the management policy ambiguous with respect to harmful event risk. [Ren and Polasky \(2014\)](#) also study optimal management under the risk of potential regime shift and show that it is also possible that the economy adopts an aggressive management policy under a low endogenous hazard rate, different from many studies which show that the economy becomes either precautionous or the overall effect is ambiguous. Apart from the literature on resource exploitation under uncertainty, there are some recent contributions regarding the implications of harmful events on the long run behavior of the economy. A paper by [van der Ploeg \(2014\)](#) focuses on how a first-best optimal carbon tax should be adjusted along time when the economy faces a harmful event probability. [van der Ploeg and de Zeeuw \(2017\)](#) extend the framework offered by [van der Ploeg \(2014\)](#) and show that a positive saving response can help the economy to dampen the discrete change in consumption at the time of the harmful event. A recent study by [Akao and Sakamoto \(2018\)](#) offers a general framework to justify the empirical evidence that shows the positive correlation between disasters and long term economic performances.

The existing literature tries to figure out the reaction of the economy to uncertain events. One of the common arguments in the above-cited papers is that the uncertainty pushes the economy to be more precautionous. However, these studies overlook the possibility of multiple equilibria which can imply a heterogeneous reaction to uncertain events. What exactly is the heterogeneity against the uncertainty? An economy can find itself at an equilibrium point with low or high environmental quality. Therefore, an economy at high environmental quality is more precautionous than one at low environmental quality level. In other words, an economy at the low environmental quality equilibrium adopts an "aggressive" exploitation policy relative to an economy at the high environmental quality equilibrium.

In this sense, differently from the literature, our paper shows that, given uncertainty, an economy adopts precautionary behaviour, although the level of precaution is not the same when an economy converges to a low or a high environmental quality level.

The second and the main contribution of the paper is to present a new trade-off between adaptation and mitigation other than the usual dynamic trade-off highlighted by numerous studies ([Bréchet et al. \(2012\)](#), [Le Kama and Pommeret \(2016\)](#), [Millner and Dietz \(2011\)](#)). A recent work by [Zemel \(2015\)](#) studies the time profile of the optimal mix of adaptation and mitigation in a simple growth model with uncertainty. [Tsur and Zemel \(2015\)](#) extend [Zemel \(2015\)](#) by relaxing the linearity assumption on adaptation investments and find optimal interior solutions. However, these studies did not take into account the multiplicity of equilibria and its implications regarding the adaptation and mitigation policies. Indeed, our contribution is to show that adaptation policy increases the possibility of environmental traps. On the other hand, mitigation policy makes more likely that the economy admits a unique equilibrium. Hence, concentrating on environmental traps in an economy with endogenous hazard rate allows us to present a new trade-off between adaptation

and mitigation regarding the environmental traps.

What is the role of adaptation and mitigation on the environmental traps mentioned above? Our contribution is to show that adaptation policy can cause multiple equilibria while mitigation can avoid it. This depends largely on the occurrence probability. This is because adaptation capital is shown to decrease the optimal steady state level of environmental quality, since agents worry less about the consequences of a harmful event when there is an increasing adaptation capacity. Then, since the endogenous hazard rate increases, the trade-off between present consumption and the endogenous hazard rate becomes more important, which is likely to raise multiple equilibria. Contrary to this mechanism, mitigation activity improves the environmental quality and the trade-off between present consumption and harmful event probability turns to be weaker. To better understand this result, assume for a moment that mitigation activity can eliminate the endogenous hazard rate⁵. Then, the trade-off between present consumption and hazardous event disappears, since the endogenous hazard rate disappears. Consequently, the environmental trap is not a possible outcome.

The third contribution is to analyze how the trade-off between adaptation and mitigation affects the occurrence of environmental traps. The recent papers by [Tsur and Zemel \(2015\)](#) and [Zemel \(2015\)](#) focus on the dynamic trade-off between adaptation and mitigation but overlook its qualitative implications regarding the multiplicity of equilibria. We show that when an economy undertakes more mitigation than adaptation, the economy avoids an environmental trap.

The remainder of the paper is organized as follows: Section 2 presents the benchmark model. Section 3 describes the model with adaptation and mitigation policies and explains in a greater-depth the implications of adaptation and mitigation on the occurrence of environmental traps. Section 4 provides numerical illustrations and Section 5 concludes the paper.

2 Model

Let $S(t)$ represent environmental stock available or environmental quality at time t , e.g, the stock of clean water, soil quality, air quality, forests, biomass. We refer to a broad definition of environmental quality which encompasses all environmental amenities and existing natural capital that have an economic value⁶. Obviously, disamenities such as waste and pollution stemming from consumption decrease environmental quality stock. The stock $S(t)$ evolves in time according to

$$\dot{S}(t) = R(S(t)) - c(t) \quad (1)$$

where the control variable $c(t)$ stands for consumption at time t . With a given initial state $S(0)$, an exploitation policy of environmental stock $c(t)$ generates the state process $S(t)$ according to the equation (1) and provides the utility $u(c(t), S(t))$. Similar to [Le Kama and Schubert \(2007\)](#), we use a framework where consumption comes directly from environmental services and causes environmental damages.

We make use of the following assumptions.

A.1 The regeneration of environmental quality is characterised by $R(.) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $R(S) > 0$, $R_{SS}(S) < 0$, $R_S(S) \leq 0$ and $R(\bar{S}) = 0$ where \bar{S} is the carrying capacity of the natural resource stock.

⁵In our model, an endogenous hazard rate exists always whatever the environmental quality level is. This is also the justification for a proactive adaptation policy.

⁶We exclude mining and oil industry from our definition of natural capital.

A.2 The utility function $u(.) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice continuously differentiable with the following properties ; $u_c(c) > 0$, $u_{cc}(c) < 0$, $\forall c$ and $\lim_{c \rightarrow 0}, u_c(c) = \infty$.

An important issue highlighted by [Schumacher \(2011\)](#) is related to the use of a utility function with a positive domain in growth models with endogenous discount. [Schumacher \(2011\)](#) shows rigorously that the models with endogenous discounting should use a utility function with a positive domain since the utility function shows up in the optimal path of consumption. Indeed, since our model focuses on recurrent events such as droughts and floods, the utility function does not show up in the Keynes-Ramsey rule. Therefore, the use of a utility function with a negative or positive domain does not lead to qualitatively different solutions.

In addition to the fact that the environmental stock $S(t)$ represents a source of consumption for the economy, it also affects the endogenous hazard rate. To elaborate this in greater depth, forests, for example influence considerably the environmental conditions in a given area. ([Dasgupta and Mäler \(1997\)](#), chapter 1) and helps to decrease the endogenous hazard rate (see [Jie-Sheng et al. \(2014\)](#), [Bradshaw et al. \(2007\)](#))⁷. When harmful events occur, they inflict environmental damages. The consequences of these recurrent harmful events are defined by the post-event value $\varphi(S)$ that we will discuss later.

Let T be the event occurrence time and let $F(t) = Pr\{T \leq t\}$ and $f(t) = F'(t)$ denote the corresponding probability distribution and density functions, respectively. The endogenous hazard rate $h(S)$ is related to $F(t)$ and $f(t)$ with respect to

$$h(S(t)) \Delta = Pr\{T \in (t, t + \Delta) \mid T > t\} = \frac{f(t) \Delta}{1 - F(t)} \quad (2)$$

where Δ is an infinitesimal time interval. We have $h(S(t)) = -\frac{\ln(1-F(t))}{dt}$. The term $h(S(t)) \Delta$ specifies the conditional probability that a harmful event will occur between $[t, t + \Delta]$, given that the event has not occurred by time t . A formal specification for probability distribution and density functions gives

$$F(t) = 1 - \exp\left(-\int_0^t h(S(\tau)) d\tau\right) \text{ and } f(t) = h(S(t)) [1 - F(t)] \quad (3)$$

We assume that the hazard function converges to a constant \bar{h} when the natural resource stock tends to infinity.

A.3 The endogenous hazard rate $h(S) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice continuously differentiable with the following properties ; $\lim_{S \rightarrow \infty} h(S) = \bar{h}$, $h(S) > 0$, $h_S(S) < 0$.

Since $S(t)$ is a beneficial state, the endogenous hazard rate h is a non-increasing function (a higher environmental quality stock entails a lower occurrence probability). We do not put any assumptions on the second derivative of the endogenous hazard rate.

We consider recurrent events which entail an immediate damage $\bar{\psi}$. Recurrent events are repeated events such as droughts and floods that occur frequently in our current society. In addition, the economy is always under the same risk of more events occurring later on, with value function $V(S)$. The post-value function describing the economy with recurrent events after the occurrence of harmful event is defined as

$$\varphi(S) = V(S) - \bar{\psi} \quad (4)$$

⁷ An interesting real world example could be the reforestation project in Samboja Lestari conducted by Borneo Orangutan Survival Foundation. The project helped to increase rainfall by 25% and to avoid droughts by lowering air temperature by 3 degrees Celcius to 5 degrees Celcius. ([Boer \(2010\)](#), [Normile \(2009\)](#))

Given the uncertain arrival time T , the exploitation policy $c(t)$ yields the following payoff

$$\int_0^T u(c(t)) e^{-\rho t} dt + \varphi(S(T)) e^{-\rho T} \quad (5)$$

where ρ is the social discount rate. Taking expectations of the expression (5) according to distribution of T and considering (3) gives the expected payoff

$$V(S) = \max_{c(t)} \int_0^\infty [u(c(t)) + h(S(t)) [V(S(t) - \bar{\psi})]] \exp\left(-\int_0^t [\rho + h(S(\tau))] d\tau\right) dt \quad (6)$$

The maximization problem (6) at hand is deterministic, yielding a value that corresponds to the maximum expected value of the uncertainty problem.

The solution of maximising (6) with respect to evolution of environmental stock (1) leads to the Keynes-Ramsey rule (see Appendix (A) for details),

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} \right] \quad (7)$$

When there are recurrent harmful events, the economy adopts a precautionary behavior due to the presence of the precaution term $\frac{\bar{\psi} h_S(S)}{u_c(c)}$ in equation (7). This result is similar to [de Zeeuw and Zemel \(2012\)](#), where the authors show that recurrent events induce a precautionary behavior.

Proposition 1. (i) *The necessary condition for multiple equilibria in an economy exposed to endogenous hazard rate is the following: If $A = \{S : G(S) = 0\}$, then $\text{card}(A) > 1$. If the hazard rate is exogenous, the multiplicity of equilibria is not a possible outcome.*

(ii) *The sufficient conditions to have an environmental trap for recurrent events are $G(\tilde{S}) > 0$ and $\exists S < \tilde{S}$, $G(S) < 0$ and also*

$$G_S(S) = R_{SS}(S) - \frac{\bar{\psi} h_{SS}(S)}{u_c(R(S))} + \frac{\bar{\psi} h_S(S) u_{cc}(R(S)) R_S(S)}{(u_c(R(S)))^2} > 0 \quad (8)$$

Proof. See Appendix (B)

The first part of the sufficient conditions is for the existence of low steady state. The second part ensures the existence of a high steady state. We denote steady states as $0 < S_{\text{low}} < S_{\text{mid}} (< \tilde{S}) < S_{\text{high}} (< \bar{S})$.

In a standard neoclassical growth model, this condition cannot be satisfied, since all terms with the endogenous hazard rate vanish and the condition reduces to $R_{SS}(S) > 0$, which is not possible according to A.1.

In a Ramsey-Cass-Koopmans model, the usual inter-temporal trade-off is between the present and future consumption. Moreover, an economy exposed to harmful events faces an additional trade-off between present consumption and harmful event risk. Indeed, this is the reason why the economy could find itself in a trapped equilibrium. More technically, it is possible to understand how the trade-off between present consumption and the endogenous hazard rate through the precaution term $\frac{\bar{\psi} h_S(S)}{u_c(c)}$ in equation (7). When the marginal utility is at a high level, the weight of the precaution term decreases. This means that agents care less about the endogenous hazard rate due to a higher opportunity cost of decreasing present consumption.

Note that without the precaution term, the function $G_S(S)$ becomes linear in S and the economy always admits a unique equilibrium. The first term relates to convexity of regeneration function which is a negative term. The second and third term in (8) are positive and negative terms respectively.

The explanation behind the mathematical expressions in (8) are as follows: the second term stands for the convexity of the endogenous hazard rate⁸. The more the endogenous hazard rate is sensitive to marginal changes of environmental quality, the more there are chances that $G_S(S) > 0$. The third term relates to the concavity of the utility function. A more concave utility function (i.e, higher risk aversion) decreases $G_S(S)$ if $R_S(S) > 0$. Overall, the multiple equilibria condition depends on the close relationship between the endogenous hazard rate depending on environmental quality and consumption.

Lemma 1. *The steady-states S_{low} and S_{high} are saddle path stable. However, S_{mid} could have complex dynamics.*

Proof. See Appendix (D)

This means that the economy could converge to either high or low environmental quality equilibrium. Once the economy reaches an equilibrium (low or high), it remains in this state permanently. The economy reaching the low-quality equilibrium is said to be "trapped", where the consumption and environmental quality are lower relative to high-quality equilibrium.

One can understand the occurrence of environmental traps due to endogenous hazard rate by making a phase diagram analysis. Recall that the consumption rule is $\dot{c}/c = \sigma(r - \rho)$ ⁹ without the endogenous hazard rate. Then, the steady-state curve $\dot{c} = 0$ is vertical and implies a unique equilibrium, which is not the case with endogenous hazard rate. Considering equation (7), we can observe that steady state curve $\dot{c} = 0$ is non-linear on a phase plane (S, c) . Therefore, multiple equilibria is a possible outcome with endogenous hazard rate. To illustrate this explanation, we refer to a phase diagram analysis in the following section.

2.1 Phase Diagram Analysis

In the next two subsections, we present two different phase diagram configurations¹⁰ on a plane (S, λ) . The first case is a multiple equilibria with a unique path. This means that, given an initial value of S , there is only one reachable steady-state, then it is optimal to go there. The second case is where there exist, given an initial value of S , multiple growth paths. The reason why we make the phase diagram analysis on a plane (S, λ) and not (S, c) is that differentiating Hamiltonian with respect to a co-state variable (here λ) gives directly the isocline $\dot{S} = 0$ (See Appendix (A) for details). Then, it is easier to find the value of different optimal trajectories and compare them to find the global optimum.

2.1.1 Multiple equilibria: Unique growth path

In this subsection, in order to present clearly which path to choose among different trajectories, we present a phase diagram on a phase plane (S, λ) where λ is the shadow price for natural resources. First, we present the dynamics of co-state and state variable (see Appendix (A))

$$\dot{\lambda} = -\lambda \left(R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{\lambda} \right) \quad (9)$$

$$\dot{S} = R(S) - c(\lambda) \quad (10)$$

⁸Assume that the endogenous hazard rate has a convex form.

⁹where $r = R_S(S)$. With usual growth model, one can maximise $\int_0^\infty \frac{c^{1-\sigma}}{1-\sigma} e^{-\rho t} dt$ with respect to $\dot{S} = R(S) - c$ and find the usual Euler equation.

¹⁰Of course, there can be different kinds of phase diagrams. However, the analysis for other phase diagram configurations is not different than what we present in this section.

Figure (1) shows that S_{mid} acts as a threshold. An economy starting with an initial stock of natural resources $S_0 < S_{\text{mid}}$ always converges to S_{low} , while an economy which starts with $S_0 > S_{\text{mid}}$ converges to S_{high} . There are two convergence groups and the choice between them is made depending on history (initial condition).

The phase diagram with unique path is as follows

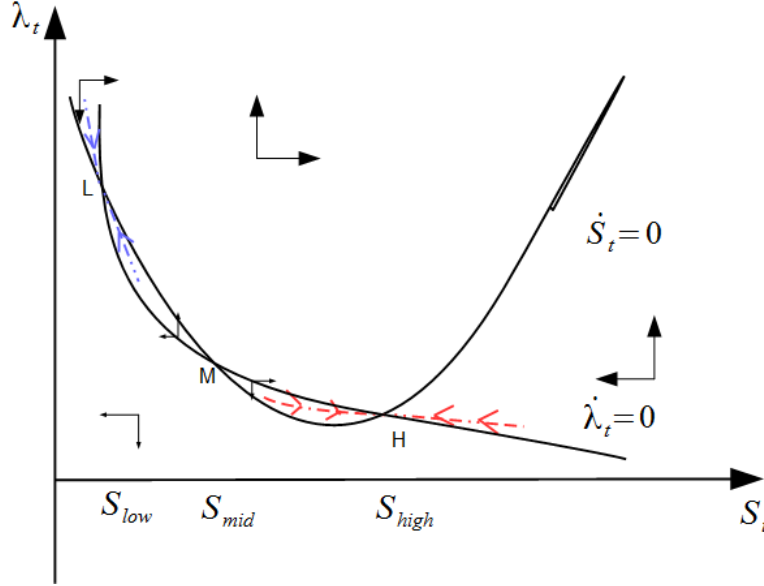


Figure 1: Phase diagram on the plane (S, λ)

The economic explanation behind the figure (1) is as follows: postponing consumption is too costly for survival ($u_c(0) = \infty$) for very poor agents and preferences are directed toward the present. In this case, the weight of precaution term goes to zero and we have the dynamics of a standard Ramsey growth model with a unique equilibrium. Conversely, if agents are very rich, they tend to be very precautionary for the environment and the economy would admit a unique equilibrium. However, when agents are neither very poor nor very rich, the multiplicity of equilibria occurs. [Wirl \(2004\)](#) explains that when it becomes difficult to navigate between the goal of capital accumulation and natural resource protection, the multiplicity of equilibria arises. In our case, when it becomes difficult to decide between consumption and protecting the environment in order to decrease the endogenous hazard rate, the multiplicity of equilibria occurs.

The same kind of reasoning can be made for the penalty rate ψ . On one hand, if the penalty rate is zero, we have a standard Ramsey growth model. On the other hand, if the penalty rate is too high, agents become very precautionary. In the next subsection, we will show analytically this possibility, by giving a sufficient condition on the penalty rate for the multiplicity of equilibria.

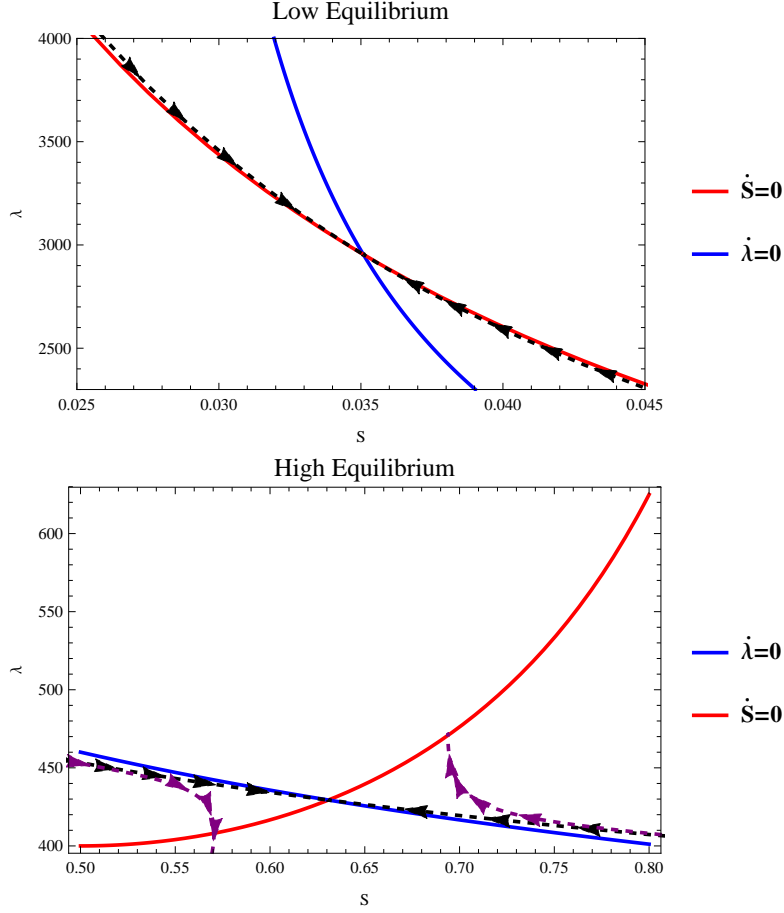


Figure 2: Phase diagram with converging trajectories for low and high equilibrium

For the sake of clarity, we present trajectories for both equilibrium separately and show that there exist different optimal converging paths to both low and high steady states. Figure (2) shows that, depending on the initial conditions, either the economy converges to steady state or diverges. The phase diagram analysis clearly shows that the existence of different convergence groups rules out a possibility of a global stability in the model.

An important issue concerns the stability of the equilibrium and the corresponding trajectories converging to different equilibria. Wagener (2003) analyses the shallow lake dynamics and shows that there could be heteroclinic connections between different saddle points. In the existence of heteroclinic bifurcations, only one of saddle points may be relevant (see Figure 3 in Wagener (2003)). In this study, heteroclinic bifurcations cannot occur, since Lemma 1 rules out the possibility of bifurcations.

In the next subsection, we analyse a different configuration for the phase diagram where there are multiple growth paths.

2.1.2 Multiple equilibria: Multiple growth paths

When the Hamiltonian is not concave in S , the uniqueness of Pontryagin paths is no longer ensured. Therefore, starting with an initial value of natural resource stock, there may exist several paths leading to different saddle points, i.e, there may exist multiple reachable equilibria (see Palivos (1995)). In this case, the values of different trajectories, starting from a given initial condition of S , should be compared to determine the global optimum. Based on Skiba (1978) and Davidson and Harris (1981), we can determine the optimal path when there are two reachable saddle points.

As drawn in (3) there are three steady states B, D, H . As mentioned before, this is not the only phase diagram configuration that may occur. An analysis based on figure (3) is enough to make clear all the general issues about the dynamics in the model.

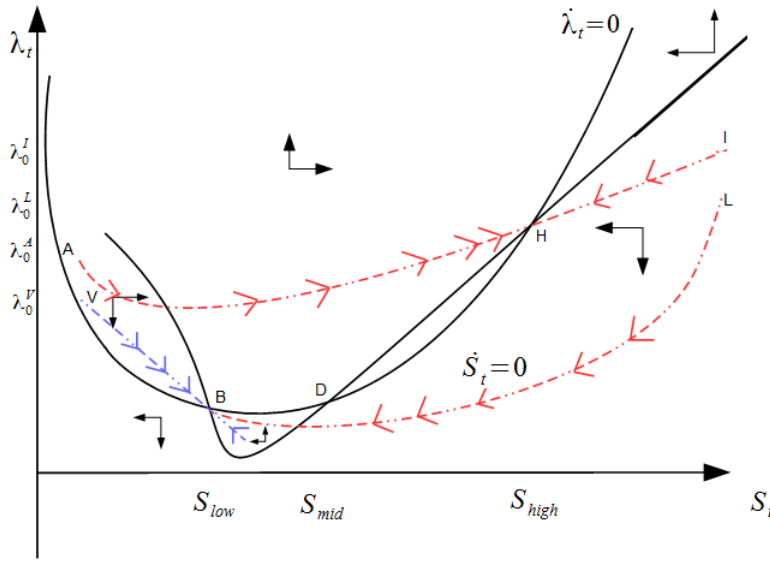


Figure 3: Phase diagram on the plane (S, λ)

Figure (3) shows that an economy can enjoy a higher consumption level and lower natural resource stock or vice versa (see equilibrium B and H). Then, what would be the preference of an economy between these two long run equilibria?

Consider an economy with an initial value of $S_0 < S_{low}$ (see figure (3)). Between two feasible paths AH and VB , which one gives the global maximum?

To answer this question, one should calculate the value of paths V and compare them. Then, the value of the infinite horizon problem, denoted $V(S_0)$ is given by

$$V(S_0) = \frac{\mathcal{H}(S_0, \lambda_0)}{\rho} \quad (11)$$

where \mathcal{H} is the Hamiltonian associated to the infinite horizon program presented in (6) (See Appendix (A) for details). From (11), the choice between AH and VB is immediate. Since $\mathcal{H}_\lambda = \dot{S} = R(S) - c(\lambda) > 0$ for all λ above $\dot{S} = 0$ locus and that $\lambda_0^A < \lambda_0^V$ (i.e, $c_0^A > c_0^V$), the optimal solution is to take the path AH .

Note that the system can not display any limit cycles since it is shown in Lemma 1 that the trace of the Jacobian matrix in two dimensional system is not equal to zero. This implies that it is impossible to be optimal for one country to start at $S_0 < S_{low}$ going to H and for another to start at $S_0 > S_{high}$ going to B .

In the existence of multiple reachable steady states, one would like to know whether there is a threshold at which the economy is indifferent between going to either one of these steady states. This leads us to do an analysis to find a *Skiba point*.

A Skiba threshold is the point where the economy is indifferent between converging to either one of the steady-states.

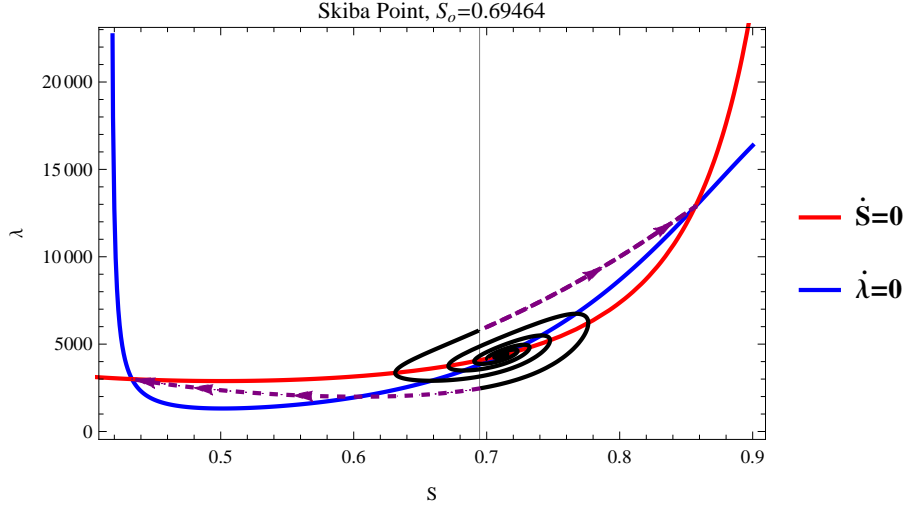


Figure 4: Skiba point with trajectories converging to saddle-point equilibria

Figure (4) shows that there exists a Skiba point where the economy is indifferent to go to either one of the steady-states. Figure (4) is line with (Wagener (2003)) where it is shown that a Skiba point may not be associated with the unstable equilibria. Departing from the initial condition $S_0 = 0.69464$, the value of the optimal path on the left and on the right of Figure (4) (see thick dashed lines) is the same. In fact, the existence of a such indifference point is quite plausible in the present phase diagram configuration. Either the economy prefers having a higher consumption with a lower natural resource stock or vice versa. The existence of a Skiba point shows that an economy may prefer a higher consumption level at the cost of facing a higher endogenous hazard rate.

An important question to which there is no answer in the literature is which kind of behaviour (precautious or aggressive) the economy adopts with respect to the endogenous hazard. We show that the level of precaution for natural resource protection changes with respect to the equilibrium path (convergence to a low or to a equilibrium).

Proposition 2. $S_{high}^* - S_{NR}^* > S_{low}^* - S_{NR}^*$ where S_{NR}^* is the steady state value of natural resources in an economy without risk. This means that a high environmental quality equilibrium economy adopts a more conservative exploitation policy than low environmental quality equilibrium economy.

Proof. See Appendix (E)

According to the precaution term in equation (7), when there are recurrent events, the endogenous hazard rate makes always an economy more precautionary for resource exploitation (see [de Zeeuw and Zemel \(2012\)](#), [Tsur and Zemel \(2016\)](#), [Polasky et al. \(2011\)](#), [Clarke and Reed \(1994\)](#)). The novelty of this proposition with respect to the literature is that the endogenous hazard rate leads to different levels of precaution (different steady-states) against harmful events when there is a multiplicity of equilibria.

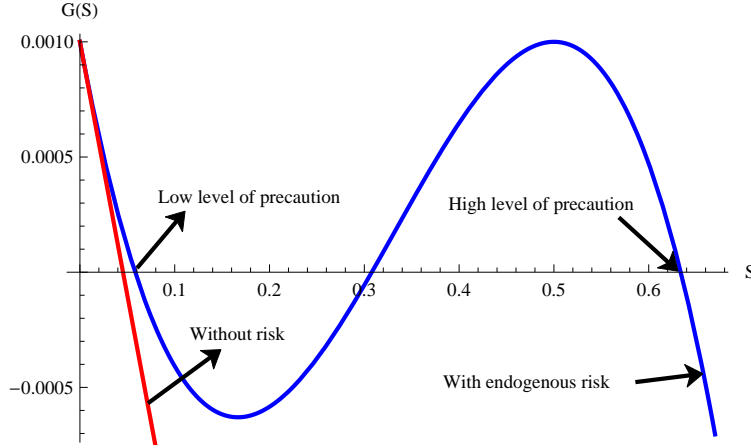


Figure 5: Different reaction against the endogenous hazard rate

As seen in Figure (5), the endogenous hazard rate always pushes the economy to be precautionary with respect to the case without any endogenous hazard rate. However, countries characterized by high equilibrium become more conservative about the environment relative to countries characterized by low equilibrium when there is an endogenous hazard rate (see Figure (5)).

2.2 An example of multiple equilibria

The multiplicity of steady states is an issue related to functional forms and parameter values. We provide explicit conditions ensuring this multiplicity in a case with functional forms that have been extensively used in the literature. The regeneration of the environment, the endogenous hazard rate and the utility function are given respectively as follows:

$$R(S) = g(1 - S)S \quad (12)$$

$$h(S) = (1 - \bar{h}S^2) \quad (13)$$

$$u(c) = \log(c) \quad (14)$$

where g is the intrinsic growth rate of the environmental quality and $0 < \bar{h} < 1$ shows at which extent the endogenous hazard rate depends on the environmental quality level. At the steady-state, the equation (7) can be written as

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi}h_S(S)}{u_c(c)} = 0 \quad (15)$$

By using the functional specifications, we reformulate the equation (15) combined with (1) at the steady

state

$$-2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 - 2gS + (g - \rho) = 0 \quad (16)$$

Proposition 3. *The sufficient condition to have three positive real roots for the third degree polynomial equation (16) is as follows:*

$$\begin{aligned} & \frac{\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2\right) - \sqrt{\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2\right)^2 - 2^9g^3(g - \rho)}}{8(g - \rho)2g\bar{h}} \\ & < \bar{\psi} < \frac{\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2\right) + \sqrt{\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2\right)^2 - 2^9g^3(g - \rho)}}{8(g - \rho)2g\bar{h}} \end{aligned} \quad (17)$$

Proof. See Appendix (F)

This result suggests that the damage rate due to harmful events should not be either too low or too high for the multiplicity of equilibria. The trade-off between present consumption and harmful events can be understood through this condition as well: When the damage rate is too high, this implies that the economy adopts a precautionary behavior since the marginal benefit of protecting the environment is higher. On the other hand, when the damage rate is too low, the economy tends to care less about the environment and exploits natural resources. Therefore, in these two cases, the trade-off is not very binding. However, when the damage lies in between a certain range of values, the trade-off between harmful events and present consumption becomes more binding, since the economy is neither very precautionary nor careless about the environment.

3 Can environmental policy cause/avoid an environmental trap?

In this section, we consider the benchmark model and analyze how adaptation and mitigation policies affect the main results obtained in the model without environmental policy. Since the focus of our study is to figure out the effects of adaptation and mitigation policies on environmental traps, the objective is to understand the effect of these two policies on necessary condition (8) for multiple equilibria. To facilitate understanding of the mechanisms, we prefer to analyze separately the effect of adaptation and mitigation policies on environmental traps.

3.1 An economy with adaptation policy only

As mentioned in the benchmark model, when a harmful event occurs, the economy suffers from environmental damage. However, a social planner could reduce the damage ψ via adaptation capital K_A . This way of modeling adaptation is consistent with Zemel (2015) and Tsur and Zemel (2016) but differs from Bréchet et al. (2012) and Le Kama and Pommeret (2016), where the adaptation capital affects directly the damage function for all time t . In our specification, as mentioned above, adaptation plays a *proactive* role. Hence, tangible benefits of adaptation can be gained only if a harmful event occurs. However, this is not to say that investing in the adaptation decision makes no difference. Its contribution is accounted for by the objective

function of the social planner. Investing at rate A contributes to adaptation capital K_A , which follows the stock dynamics

$$\dot{K}_A(t) = A(t) - \delta K_A(t) \quad (18)$$

where δ represents the capital depreciation rate and damage function $\psi(K_A)$ decreases when adaptation capital K_A increases. We assume that

A.4 The damage function is characterised by $\psi(\cdot): \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $\psi(0) = \bar{\psi}$, $\psi(\infty) = \underline{\psi}$, $\psi(K_A) > 0$, $\psi_{K_A}(K_A) < 0$ and $\psi_{K_A K_A}(K_A) > 0$

When there is no adaptation capital, the inflicted damage will be a constant term. Moreover, it is realistic to assume that reduction in damage has a limit, since we cannot get rid of the negative effects of a harmful event completely by accumulating adaptation capital. For that reason, we assume that damage function is constrained between an upper $\bar{\psi}$ and a lower bound $\underline{\psi}$.

In an economy with adaptation policy, the expected payoff is

$$V(S, K_A) = \max_{\{c(t), A(t)\}} \int_0^\infty [u(c(t)) - Q^1(A(t)) + h(S(t)) [V(S(t), K_A(t)) - \psi(K_A(t))] \exp\left(-\int_0^t [\rho + h(S(\tau))] d\tau\right) dt \quad (19)$$

where $u(c(t))$ is the utility function. $V(S, K_A)$ stands for the value of the maximization problem. $Q^1(A) = \phi_A \frac{A^2}{2}$ with the unit cost of adaptation ϕ is the convex cost function for adaptation that enters in the utility function as a social cost as in [Zemel \(2015\)](#) and [Tsur and Zemel \(2016\)](#). A convex cost function implies that every marginal unit of adaptation investment is more costly in terms of disutility. an example).

The optimal policy is to maximize (19) subject to (J) and (18) (see Appendix G). Optimal dynamics for consumption and adaptation are as follows

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_S(S) - \rho - \frac{\psi(K_A) h_S(S)}{u_c(c)} \right] \quad (20)$$

$$\dot{A} = \frac{Q_A^1(A)}{Q_{AA}^1(A)} \left(\rho + \delta + \frac{h(S) \psi_{K_A}(K_A)}{Q_A^1(A)} \right) \quad (21)$$

When an economy invests in adaptation policy, we have additionally equation (21), which shows the optimal dynamics of adaptation investment. In the next part, we seek to figure out the effect of adaptation investments on the long-run equilibrium.

Proposition 4. *Adaptation policy traps the economy into the multiplicity of equilibria.*

In order to assess the effect of adaptation capital on the environmental trap, we calculate the necessary condition for multiple equilibria (see Appendix (G) and (H) for details).

$$G_S(S) = R_{SS}(S) - \frac{h_{SS}(S) \psi(K_A)}{u_c(R(S))} + \frac{h_S(S) \psi(K_A) u_{cc}(R(S)) R_S(S)}{(u_c(R(S)))^2} - \underbrace{\left[\frac{\psi_{K_A}(K_A) h_S(S)}{u_c(R(S))} \right] \frac{dK_A}{dS}}_{=Z_1} > 0 \quad (22)$$

where the term Z_1 stands for the effect of adaptation capital on necessary condition for the environmental trap.

In order to assess the effect of this term, we should know how environmental quality changes the adaptation capital level at steady state. To do this, we write equation (21) at steady-state

$$Q_A^1(\delta K_A)(\rho + \delta) = -\psi_{K_A}(K_A)h(S) \quad (23)$$

which defines the function $K_A(S)$. The graph of $K_A(\cdot)$ represents a curve in the (K_A, S) plane, denoting the steady-state curve and having the following economic intuition: right-hand side $-\psi_{K_A}(K_A)h(S)$ can be interpreted as the marginal benefit of adaptation capital and left-hand side $Q_A^1(\delta K_A)(\rho + \delta)$ is the marginal cost of adaptation capital. Optimal steady states are located on this curve. We can find the slope of the steady state curve by taking the total derivative of equation (23)

$$\frac{dK_A}{dS} = \frac{h_S(S)\psi_{K_A}(K_A)}{(\rho + \delta)Q_A^1(\delta K_A) + \psi_{K_A K_A}(K_A)h(S)} < 0 \quad (24)$$

Since we have $\frac{dK_A}{dS} < 0$, the term Z_1 is negative and this means that the possibility of multiple equilibria increases with adaptation capital.

The nominator in (24) can be considered as the variation of marginal benefit of adaptation with respect to environmental quality, which is a negative term. This means that the marginal benefit of adaptation is lower when the environmental quality level is higher. This also translates into a decrease of adaptation capital at steady state. The first and second terms in denominator represent the marginal cost of adaptation capital and concavity of penalty function with respect to adaptation capital. According to equation (24), a higher environmental quality requires a lower adaptation capital level at steady state. However, a more concave penalty function with respect to environmental quality means that adaptation capital is able to decrease much more the penalty rate. This results in a lower decrease of adaptation capital with an increase in environmental quality.

The negative slope indicates that when the environmental quality is higher, the economy needs less adaptation capital. This is plausible, since the probability of the harmful event is lower. Correspondingly, one may say that when the economy accumulates more adaptation capital, natural resources start to be overused.¹¹ A higher environmental quality level decreases the endogenous hazard rate. Then, the economy needs less adaptation capital.

We can also explain this result by looking at $h(S)(V(S, K_A) - \psi(K_A))$ in (19). When environmental quality increases, the weight of this component decreases due to a lower endogenous hazard rate, which means that there is less incentive to accumulate adaptation capital.

Another interpretation of this result can be as follows: Since agents expect to face less damage with an adaptation policy and can more easily bear the negative consequences of a harmful event, they tend to care less about the environmental quality.

How can we explain the multiplicity of equilibria when the economy implements an adaptation policy? The mechanism is as follows: when a policymaker starts to invest in adaptation capital, the environmental quality decreases as shown in (24) and the endogenous hazard rate increases. Since the preferences of low-income countries are directed towards the present¹², an increase in endogenous hazard rate due to adaptation

¹¹Zemel (2015) also finds a similar result.

¹²Recall from equation (7) that the multiple equilibria occur when agents are poor and not willing to postpone their consumption. Then, implementing adaptation policy increases the endogenous hazard rate.

capital accumulation amplifies the impatience of low-income countries. Then, these countries become less precautionary in exploiting more natural resources. It follows that the endogenous hazard rate amplifies again, with an increasing need for adaptation capital¹³. This mechanism, yielding a vicious cycle, explains why a developing country with a high level of marginal utility may get trapped into a low steady-state equilibrium by investing only in adaptation capital.

Indeed, the trade-off between present consumption and a harmful event becomes more significant with adaptation policy, since the endogenous hazard rate increases. We prove this claim by an example in the following subsection.

An example with only adaptation policy

To prove the existence of the multiple equilibria in an economy with adaptation policy, we start with an example of an economy without any environmental policy that admits a unique equilibrium and use the following usual functional specifications

$$R(S) = gS(1 - S) \quad (25)$$

$$h(S) = (1 - \bar{h}S) \quad (26)$$

$$u(c) = \log(c) \quad (27)$$

$$\psi(K_A) = \bar{\psi}(1 - aK_A) \quad (28)$$

$$Q(A) = \phi \frac{A^2}{2} \quad (29)$$

For the sake of analytical tractability, note that we use a linear penalty function which is not in line with the definition made in A.3¹⁴.

Differently from the previous example and from the definition given in A.3, we use a linear hazard function and show that a linear endogenous hazard function ensures a unique equilibrium for an economy without environmental policy. Then, we prove that an adaptation policy results in a multiple-equilibria economy.

Proposition 5. *Under the specification of functional forms (25), (26), (27) (i) An economy without an environmental policy admits a unique steady-state.*

(ii) When the economy invests in adaptation activity, multiplicity of equilibria occurs if the following condition on harmful damage $\bar{\psi}$ holds

$$\begin{aligned} & \frac{\left[\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2 \right) - \sqrt{\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2 \right)^2 - 2^9 g^3 (g - \rho)} \right]}{8(g - \rho)g(\bar{h})^2} \\ & < \bar{\psi} < \frac{\left[\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2 \right) + \sqrt{\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2 \right)^2 - 2^9 g^3 (g - \rho)} \right]}{8(g - \rho)g(\bar{h})^2} \end{aligned} \quad (30)$$

Proof. See Appendix (I)

¹³Since the endogenous hazard rate increases, the marginal value of adaptation capital increases.

¹⁴In numerical analysis, we use a penalty function which is line with the definition given in A.4.

This example clearly shows that an economy which is initially not exposed to multiple equilibria faces the possibility of environmental traps under an adaptation policy if the condition (I.6) is ensured.

Indeed, a policy recommendation based on more adaptation capital could cause a multiplicity of steady states and trap an economy at lower equilibrium.

Consider a benchmark case where the economy without environmental policy admits a unique equilibrium. Then, when a social planner takes the benchmark economy and starts to invest in adaptation capital, the multiplicity of steady states occurs if the condition (30) is ensured.

3.2 An economy with mitigation policy only

We now show that an economy implementing mitigation policy could escape an environmental trap. The reason is as follows: since consumption comes from environmental assets, improving environmental quality (mitigation) increases the consumption level in the long run. Therefore, low income countries could have a lower level of marginal utility of consumption, implying that they could make more far-sighted decisions. As the endogenous hazard rate decreases with mitigation policy, agents will be more patient and willing to postpone their consumption to the future.

Investing at rate M for mitigation improves the environmental quality. Then, in the presence of mitigation activity, environmental quality evolves according to

$$\dot{S}(t) = R(S(t)) + \Gamma(M(t)) - c(t) \quad (31)$$

where $\Gamma(M)$ represents the effects of mitigation activities such as reforestation, desalination of water stock, enhancing carbon sinks, etc. Mitigation is defined as "human intervention to reduce the sources or enhance the sinks of greenhouse gases." (IPCC (2014), p.4) In this sense, reforestation can be considered as a means to enhance carbon sinks since forests allow for carbon sequestration.

The specification of the mitigation variable is similar to that in Chimeli and Braden (2005). Alternatively, function $\Gamma(M)$ can be considered as "environmental protection function". The expenditures for environmental protection may be directed not only toward pollution mitigation but also toward the protection of forests and recovery of degraded areas. Equivalently, mitigation activity can be seen as a means of improving environmental quality.

In order to keep the model as simple as possible, we choose to consider mitigation as a flow variable. We use the following assumption

A.4 The mitigation function given by $\Gamma(.) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice continuously differentiable, with the following properties ; $\Gamma_M(M) > 0$, $\Gamma_{MM}(M) < 0$.

The mitigation function is assumed to be an increasing and concave function. Note that mitigation activity can be considered as a complement to the regeneration of environment. In an economy with mitigation activity, the expected payoff is

$$V(S) = \max_{\{c(t), M(t)\}} \int_0^\infty [u(c(t)) - Q^2(M) + h(S(t)) [V(S(t))] - \bar{\psi}] \exp\left(-\int_0^t [\rho + h(S(\tau))] d\tau\right) dt \quad (32)$$

where $Q^2(M) = P_M M$ with the unit price for mitigation P_M is the linear cost function for mitigation that enters in the utility function as a social cost as in Zemel (2015) and Tsur and Zemel (2016). Since

we assume a concave function for mitigation activity $\Gamma(M)$, the use of a linear cost function gives optimal interior solutions and simplifies calculations.

The optimal policy is to maximise (32) subject to (31) (see details in Appendix (L)). Optimal dynamics for consumption and mitigation are as follows

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} \right] \quad (33)$$

$$\dot{M} = -\frac{u_{cc}(c)}{u_c(c)} \frac{\Gamma_M(M)}{\Gamma_{MM}(M)} \left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} \right] \quad (34)$$

In fact, mitigation activity renders the trade-off between present consumption and the endogenous hazard rate less binding. It follows that there are less chances for an economy to be trapped into a multiplicity of steady states.

One can remark the relationship between consumption and mitigation activity. Due to the concavity of the mitigation function, we notice that the sign of \dot{c} and \dot{M} are different. It follows that the optimal level of mitigation activity comes at the cost of a lower consumption. This also explains that the economy harms less the environment when investing in mitigation activity. Consequently, the presence of mitigation activity makes the trade-off between present consumption and the endogenous hazard rate less relevant.

Proposition 6. *Mitigation activity saves the economy from the multiplicity of equilibria.*

In order to understand how mitigation policy can avoid an environmental trap in a formal way, by using equations (33) and (34), we provide the necessary condition for multiple equilibria (Appendix (J) for details)

$$G_S(S) = R_{SS}(S) - \frac{\bar{\psi} h_{SS}(S)}{u_c(c)} + \frac{\bar{\psi} h_S(S) u_{cc} R_S(S)}{(u_c(c))^2} + \underbrace{\frac{\bar{\psi} h_S(S) u_{cc} R_S(S)}{(u_c(c))^2} \frac{dM}{dS}}_{=Z_2} > 0 \quad (35)$$

where the term Z_2 represents the effect of mitigation on the necessary condition for multiple equilibria.

In order to understand the implications of mitigation policy on multiple equilibria condition, we look at how environmental quality level changes the steady state level of mitigation investment. To do that, we write the first order condition for mitigation activity which is (see equations (J.3) and (J.4) in Appendix (J))

$$u_c(R(S) + \Gamma(M)) \Gamma_M(M) = Q_M^2(M) = P_M \quad (36)$$

and take the total derivative of (36) to find

$$\frac{dM}{dS} = -\frac{u_{cc}(c) R_S(S) \Gamma_M(M)}{u_{cc}(c) (\Gamma_M(M))^2 + u_c(c) \Gamma_{MM}(M)} < 0 \quad (37)$$

Since we have $\frac{dM}{dS} < 0$, the term Z_2 is negative and decreases $G_S(S)$. This implies that it is more difficult to ensure the multiple equilibria condition (35).

The economic intuition behind equation (37) is as follows: When environmental quality is higher, the social planner decreases mitigation. In other words, the economy needs less mitigation if environmental quality is higher.

To better understand how mitigation can avoid an environmental trap, suppose for a moment that mitigation activity could reduce the endogenous hazard rate to zero. It follows that the trade-off between present consumption and a harmful event, which causes multiple equilibria, disappears completely. Therefore, the multiplicity of steady state is not a possible outcome. Based on these elements, one can understand that mitigation activity leads to weakening the trade-off between present consumption and a harmful event.

Obviously, mitigation activity not only removes the economy from environmental trap but also increases the steady-state level of environmental quality as expected. Indeed, since consumption comes from natural resource rents, mitigation policy allows the economy to have a higher consumption level. Hence, impatience level of low-income countries decreases and they could postpone their consumption to the future as they can fulfill more easily the basic needs for survival. In a nutshell, a mitigation policy could break the vicious cycle of low consumption and environmental quality that can be triggered by an adaptation policy. Then, one can conclude that social planner should couple an adaptation policy with a mitigation policy in order to avoid a potential environmental trap.

An example with only mitigation policy

To prove that mitigation can rid the economy of multiple equilibria, we start with an example of a benchmark economy with multiple equilibria and use the following usual functional specifications. In order to have analytical results, we use the following linear functional forms;

$$R(S) = gS(1 - S) \quad (38)$$

$$h(S) = (1 - \bar{h}S^2) \quad (39)$$

$$u(c) = \log(c) \quad (40)$$

$$\Gamma(M) = M \quad (41)$$

$$Q^2(M) = P_M \frac{M^2}{2} \quad (42)$$

Another simplification only used for this example in order to ease the calculations is that the cost function appears in dynamic equation of environmental quality \dot{S} .

$$\dot{S} = R(S) + \Gamma(M) - Q^2(M) - c \quad (43)$$

For the sake of analytical tractability of the example, we use a linear mitigation function and convex cost function for mitigation. We relax these simplifications in the numerical analysis to show that our result is robust, with more realistic functional forms. With the presence of mitigation activity, the function $G(S)$ is as follows (see Appendix (K))

$$G(S) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 + \left(\frac{\bar{\psi}\bar{h}}{P_M} - 2gS\right) + (g - \rho) = 0 \quad (44)$$

Proposition 7. *In an economy with mitigation policy, the multiplicity of steady states can be avoided.*

Proof. See Appendix (K)

When the economy implements mitigation policy, there is a lower number of parameter combination which causes multiple equilibria. As mentioned previously, the economic intuition relies on the fact that mitigation

activity reduces the endogenous hazard rate, which weakens the trade-off between present consumption and endogenous hazard rate.

4 Numerical analysis

The section aims at illustrating the theoretical findings we obtained for the examples with functional forms that we presented in the previous sections. We show the same results regarding the effects of adaptation and mitigation on the occurrence of multiple equilibria by relaxing the linearity assumptions on functional forms that we imposed for the analytical tractability.

For the numerical part, we calibrate the benchmark economy according to New Zealand economy which is one of the rare countries which publicly announces the probability of harmful events such as floods and droughts.

By doing a calibration exercise, we produce a model that matches the EPI index¹⁵ and the endogenous hazard rate that is computed with real data for New Zealand¹⁶.

In this section, we also seek to show numerically the effects of adaptation and mitigation policies that we explained in the text, on the occurrence of the multiplicity of equilibria.

We use the following functional specifications.

Natural Regeneration Function : $R(S) = gS \left(1 - \frac{S}{\bar{S}}\right)$	Utility function : $u(c) = \frac{c^{1-\sigma} - c_{min}^{1-\sigma}}{1-\sigma}$
Source : Ren and Polasky (2014)	Source : Bommier et al. (2015)
\bar{S} Carrying capacity of environment g Intrinsic growth rate of the resource stock	σ Degree of relative risk aversion
harmful Damage : $\psi(K_A) = \bar{\psi} (\omega + (1 - \omega) e^{-\gamma K_A})$	Endogenous Hazard Function : $h(S) = \frac{2\bar{h}}{1 + \exp[\frac{\eta(S - \bar{S})}{\bar{S}}]}$
Source : Br�chet et al. (2012)	Source : Ren and Polasky (2014)
$\bar{\psi}$ Damage rate without adaptation policy ω Lower bound of damage when $\psi(\infty)$ γ Elasticity of adaptation w.r.t to damage rate	\bar{h} Upper bound for hazard rate η Endogeneity level of harmful event \bar{S} Carrying capacity of environment
Mitigation function : $\Gamma(M) = M^\alpha$	Cost of adaptation investment : $Q_1(A) = \phi_A \frac{A^2}{2}$
Source : Le Kama and Pommeret (2016)	
α Elasticity of mitigation activity	ϕ_A Parameter for the change of marginal cost of adaptation

4.1 The economy with mitigation-only policy

We present the multiple equilibria economy without policy and the economy with mitigation policy. In order to understand the implications of mitigation on the economy with multiple equilibria, we first calibrate the benchmark model with multiple equilibria to match the features of New Zealand economy. We calibrate the damage parameter $\bar{\psi}$ and the intrinsic growth rate of the resource stock g such that the high steady-state level of environmental quality S matches the Environmental Performance Index (EPI) score of New Zealand in 2018. We calibrate the important parameters of the endogenous hazard rate (the endogeneity

¹⁵The EPI index gives a score for each country between 0 and 100. Since we use a logistic growth function with carrying capacity set to 1 in our calibration, we have a value of S between 0 and 1. Hence, if a country have a score of 60, we take it as 0.60. The EPI score of New Zealand is 0.75 in 2018.

¹⁶We refer to EMDAT International Disaster Database (2015).

of the hazard η , upper bound for hazard rate \bar{h}) according to the estimates of New Zealand government ¹⁷ which shows that the probability of a flood in any 30 year period is measured as 45%. For the elasticity of substitution σ , we choose the conventional value used in the literature, which is 2 and the pure rate of time preference ρ is set to 0.015 as in Nordhaus (2008). We assume that the elasticity of mitigation and the unit price for mitigation activity are equal to 0.5 and to 250 respectively.

Parameter	Calibrated value
\bar{S}	1.01
g	0.062
$\bar{\psi}$	1315
\bar{h}	0.25
η	7.8
α	0.5
P_M	250

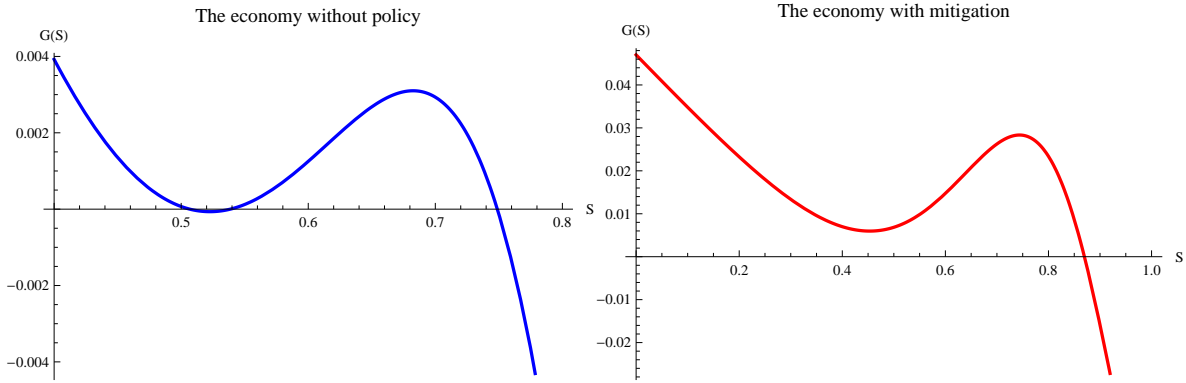


Figure 6: The benchmark economy vs the economy with mitigation

The most important message of Figure (6) is that mitigation activity enables one to avoid the occurrence of multiple equilibria as shown on the graphic on the right. The reason behind this result is that mitigation activity reduces the endogenous hazard rate. Consequently, the trade-off between present consumption and the endogenous hazard rate weakens, which allows an economy avoiding the multiplicity of steady states.

The Figure (6) shows also that mitigation activity not only avoids multiple equilibria but also increases environmental quality. It is easy to remark on the graphic on the right hand side of Figure (6) that the steady state level of environmental quality is higher than all three steady state levels of environmental quality on the graphic on the left.

4.2 The economy with adaptation-only policy

We present the benchmark model without policy and the economy with adaptation policy. For this numerical exercise, we always calibrate the benchmark economy according to New Zealand economy. As in the previous section, we calibrate the damage parameter $\bar{\psi}$ and the intrinsic growth rate of the resource

¹⁷<https://www.ecan.govt.nz/your-region/your-environment/natural-hazards/floods/flood-probabilities/>

stock g such that the steady-state level of environmental quality S equals the Environmental Performance Index (EPI) score of New Zealand in 2018.

Parameter	Calibrated value
\bar{S}	1
g	0.038
$\bar{\psi}$	30000
\bar{h}	0.23
η	8
ω	0.95
γ	0.9
ϕ_A	1

As in the previous numerical exercise for mitigation, we set the elasticity of substitution σ to 2 and the pure rate of time preference ρ to 0.015 as in Nordhaus (2008). Unfortunately, we do not have any available data for the adaptation capital. Therefore, we set parameters ω to 0.95 and γ to 0.9.

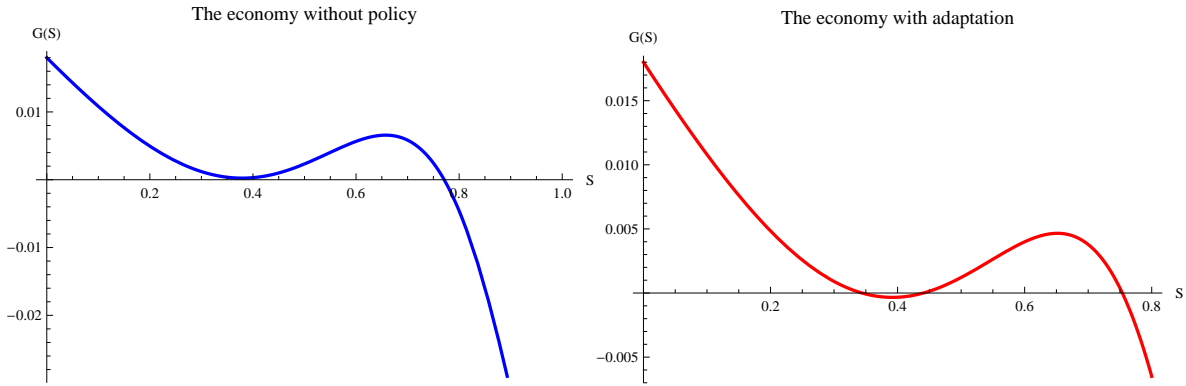


Figure 7: The benchmark economy vs the economy with adaptation

Figure (7) clearly shows that the economy can find itself in a multiple equilibria by only investing in adaptation capital. This result relies on the fact that adaptation capital decreases environmental quality (see equation (24)). As a result, the trade-off between present consumption and the endogenous hazard rate becomes more binding, which is the source of the multiplicity of steady states.

Then, an important policy-based message of this result is that adaptation policy should be coupled with mitigation policy in order to avoid environmental traps.

4.3 Adaptation vs mitigation: what about the multiplicity of equilibria?

An important issue regarding the adaptation and mitigation policies is to know how the trade-off between adaptation and mitigation affects the occurrence of multiple equilibria. The adaptation/mitigation trade-off is intensively discussed in the existing literature. Buob and Stephan (2011) argue that the country's stage of development matters and high-income countries should invest both in adaptation and mitigation while low-income countries should only invest in adaptation. Br chet et al. (2012) study the optimal mix of

adaptation and mitigation and conclude that the substitutability between adaptation and mitigation depends on the country's stage of development. A recent study by [Tsur and Zemel \(2015\)](#) focuses on the optimal mix of adaptation and mitigation under harmful event uncertainty but it remains agnostic about the implications of the trade-off between adaptation and mitigation on the multiplicity of equilibria. We seek to analyze this issue when an economy is subject to an endogenous hazard rate. (see Appendix L for calculations.)

In this subsection, apart from the calibration, the unit cost of adaptation ϕ is a free parameter. In order to assess the role of the trade-off between adaptation and mitigation on the occurrence of multiple equilibria, we present two cases. In the first case, the unit cost of adaptation ϕ is lower with respect to the second case. Consequently, in the first case, the optimal level of adaptation investment is higher than in the second case.

Figure (8) shows an economy with multiple equilibria in an economy with adaptation and mitigation policies. The graphic on the first line shows the existence of multiplicity of equilibria. The two graphics on the second line represent the optimal path of adaptation and mitigation corresponding to the economy with high environmental quality level in the long run. Two graphics on the second line show that both adaptation and mitigation increase from the start date and converge to a stable equilibrium.

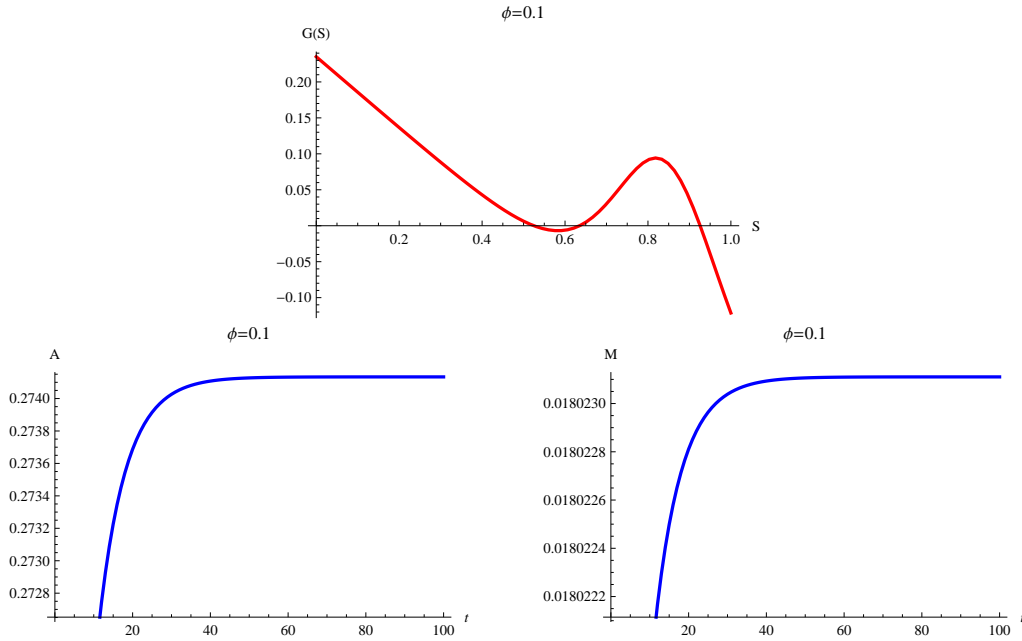


Figure 8: Adaptation vs mitigation trade-off and multiple equilibria

Figure (9) shows that when the unit cost of adaptation ϕ is higher, the economy admits a unique equilibrium. When we look at the two graphics on the second line, as expected, the economy invests in adaptation less than the case where the unit cost of adaptation is lower. However, the economy mitigates much more with respect to the previous case. We can say that a higher cost of adaptation pushes the economy to decrease adaptation and to increase mitigation activity.

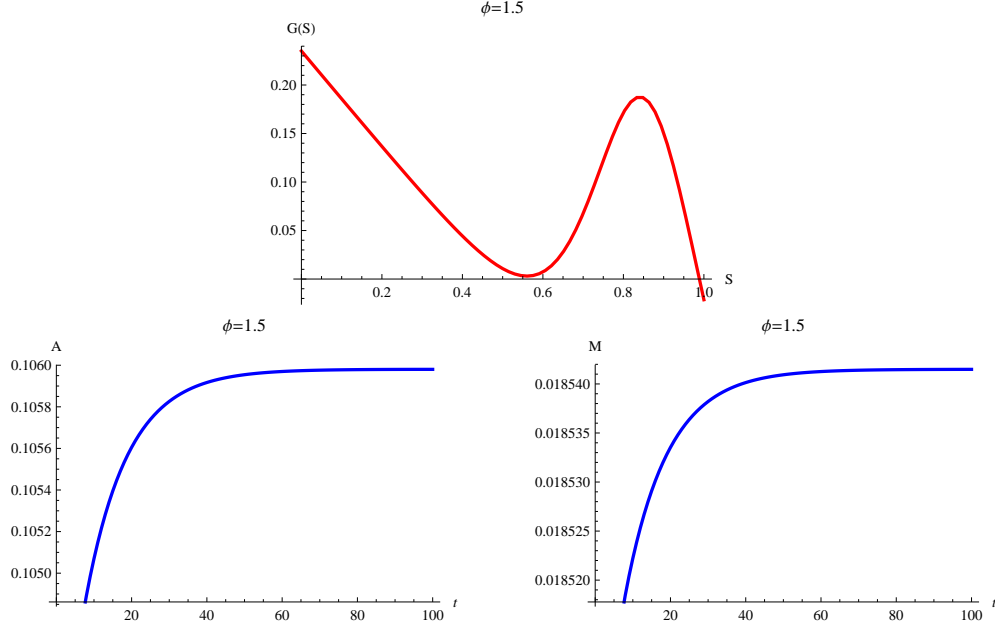


Figure 9: Adaptation vs mitigation trade-off and multiple equilibria

Note that both adaptation and mitigation aim at decreasing the disutility of the presence of the endogenous hazard rate. The former makes it by decreasing the vulnerability against the inflicted damage. The latter makes it thorough the decrease of the hazard rate. Then, the choice between adaptation and mitigation depends on the marginal benefit of these policies. When the cost of adaptation increases, it is evident that its marginal benefit decreases. This is the reason why the economy starts to decrease adaptation and increase mitigation in order to cope with the endogenous hazard rate.

A higher adaptation cost not only affects the dynamic trade-off between adaptation and mitigation but also has qualitative implications such as the occurrence of environmental traps. When the economy mitigates more, the multiplicity of equilibria disappears.

5 Conclusion

In this paper, we analyzed the effect of adaptation and mitigation policies on environmental traps in an economy subject to endogenous hazard rate. The contribution of the study is to offer a new explanation for environmental traps by the endogenous hazard rate and understand how environmental policy plays an important role in causing or avoiding development traps. We believe that this new perspective also provides interesting pointers to policymakers regarding the opposite effects of adaptation and mitigation policies. Our main results show that adaptation policy can lead the economy into an environmental trap, whereas mitigation helps to avoid an environmental trap. We show that a new trade-off appears between adaptation and mitigation with respect to their effect on environmental traps, other than the trade-off between adaptation and mitigation over time mentioned in numerous studies (see [Zemel \(2015\)](#), [Tsur and Zemel \(2015\)](#), [Bréchet et al. \(2012\)](#)). The fact that adaptation policy could cause an environmental trap does not mean that social planner should not invest in adaptation activity. On the contrary, since it is impossible to eliminate completely the endogenous hazard rate, she should invest in adaptation capital but should couple this policy with mitigation activity to avoid the adverse effects of adaptation policy. This

is because mitigation activity weakens the trade-off between present consumption and harmful events, by improving the environmental quality.

Future research could test the model using empirical methods. Currently, this is very challenging since there are no available data on adaptation investments, but it is very desirable and also part of our future research agenda.

Appendix

A Derivation of (7).

To solve the maximization problem, we write the Hamilton-Jacobi-Bellman equation.

$$\rho V^B(S) = \max_c \{u(c) + V_S^B(S)(R(S) - c) - h(S)(V^B(S) - \varphi(S))\} \quad (\text{A.1})$$

where $V^B(S)$ is the value of the maximization program before the event. As also stated in the text, the value of the problem after the event is as follows:

$$\varphi(S) = V^B(S) - \bar{\psi} \quad (\text{A.2})$$

The economy is exposed to an inflicted damage after the event. The first-order condition is given by

$$u_c(c) = V_S^B(S) \quad (\text{A.3})$$

Computing the derivative of (A.1) with respect to the environmental quality stock S yields

$$\rho V_S^B(S) = -\bar{\psi} h_S(S) + V_{SS}^B(S)(R(S) - c) + V_S^B(S) R_S(S) \quad (\text{A.4})$$

Differentiating the equation (A.3) and using equations (A.3) and (A.4) gives

$$\frac{u_{cc}(c)}{u_c(c)} \dot{c} = \frac{V_{SS}^B(S)}{V_S^B(S)} \dot{S} \quad (\text{A.5})$$

$$\rho = -\frac{\bar{\psi} h_S(S)}{V_S^B(S)} + \frac{V_{SS}^B(S)}{V_S^B(S)} \dot{S} + R_S(S) \quad (\text{A.6})$$

Arranging equations (A.5) and (A.6) gives the Keynes-Ramsey rule

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} \right]$$

The associated Hamiltonian of the problem is

$$\mathcal{H} = u(c) + h(S)(V(S) - \bar{\psi}) + \lambda(R(S) - c) - \mu h(S). \quad (\text{A.7})$$

where λ and μ are the shadow price for natural resources and for the endogenous hazard rate respectively. Since $\mu = V(S)$, it is easy to see that

$$V(S) = \max_c \frac{\mathcal{H}(S, \lambda)}{\rho} \quad (\text{A.8})$$

One can easily understand from optimal control and dynamic programming that we have $\lambda = V_S^B(S) = u_c(c)$. Then, by using the equation (A.6), the dynamics of co-state can be derived

$$\dot{\lambda} = -\lambda \left(R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{\lambda} \right) \quad (\text{A.9})$$

B Proof of Proposition 1

The first part of the proof starts with an analysis of the limits of a function (let this function be $G(S)$) that describes the steady state of the economy by a single equation in the long run. Then, in the second part, we focus on the form of $G(S)$ and related necessary conditions for the existence of multiple equilibria.

(a) In a sense, the function $G(S)$ can be considered as the equation $\dot{c} = 0$ in terms of S at the steady state equilibrium. Writing equations $\dot{c} = 0$ and $\dot{S} = 0$;

$$R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} = 0 \quad (\text{B.1})$$

$$R(S) - c = 0 \quad (\text{B.2})$$

Plugging equation (B.2) in (B.1) yields

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(R(S))} \quad (\text{B.3})$$

Note that we limit our analysis between $S \in [0, \bar{S}]$. It is possible to notice that the function $G(S)$ starts with a positive value and has negative values when S approaches \bar{S} . In this framework, \bar{S} is the level of environmental quality level where, the consumption level is equal zero. With these information, it is easy to verify $\lim_{S \rightarrow 0} G(S) = \infty$ and $\lim_{S \rightarrow \bar{S}} G(S) = z < 0$.

(b) In this part, we show the necessary conditions for the existence of multiple equilibria, which also allows to represent the function $G(S)$;

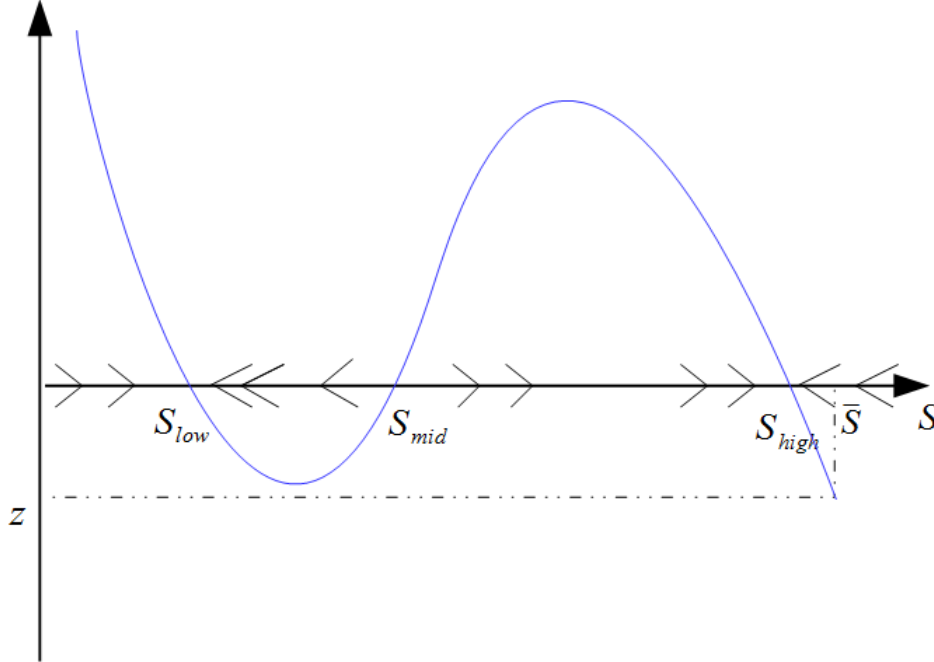


Figure 10: $G(S)$ function with uncertainty

We denote steady states as $0 < S_{low} < S_{mid} (< \tilde{S}) < S_{high} (< \bar{S})$. A sufficient condition for the existence of S_{low} is that $\exists S < \tilde{S}$ ¹⁸ which ensures (i) $G_S(S) > 0$ and (ii) $G(S) < 0$. Unless the condition (i) $G_S(S) > 0$ is satisfied, $G(S)$ starting from an arbitrary positive value may cross x -axis just one more time and converge to z , which results in a unique steady state equilibrium. Additionally, the condition (ii) $G(S) < 0$ is also necessary to ensure that $G(S)$ crosses the x -axis by S_{low} at least once.

The sufficient condition for the existence of S_{high} is that $G(\tilde{S}) > 0$. If this condition does not hold, $G(S)$ does not cross x -axis for the second time and converges to z without changing sign. Then, there exists a unique equilibrium. Once the condition $G(\tilde{S}) > 0$ is satisfied, we notice that the $G(S)$ crosses unambiguously x -axis by S_{mid} and after converges to z when S approaches \bar{S} . With these conditions, we prove the existence of three steady states, one being unstable and two others being stable.

When there is no endogenous occurrence probability, the necessary condition (8) reduces to $R_{SS}(S) > 0$, which makes multiple equilibria an impossible outcome. Therefore, the model reduces to a standard neoclassical growth model. This completes the proof.

C Slope of the steady-state curve

Using equation (B.3) and implicit function theorem, we can find the slope of $\dot{\lambda} = 0$ line.

$$\left. \frac{d\lambda}{dS} \right|_{\dot{\lambda}=0} = \frac{R_{SS}(S) - \frac{\bar{\psi} h_{SS}(S)}{\lambda}}{-\frac{h_S(S)\bar{\psi}}{(\lambda)^2}} \quad (\text{C.1})$$

We can remark that the denominator is always negative. However, the sign of nominator depends on the convexity of the regeneration function and also on the hazard function. This is the reason why we present different phase diagram configurations.

D Proof of Lemma 1

The differential system describing the economy can be written as follows;

$$\begin{bmatrix} \dot{c} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} \frac{dc}{dc} & \frac{dc}{dS} \\ \frac{d\dot{S}}{dc} & \frac{d\dot{S}}{dS} \end{bmatrix}_{\dot{c}=0, \dot{S}=0} \begin{bmatrix} c - c^* \\ S - S^* \end{bmatrix}$$

¹⁸Note that $\tilde{S} > S_{low}$.

$$\frac{d\dot{c}}{dc} = \rho - R_S(S) \quad (\text{D.1})$$

$$\frac{d\dot{c}}{dS} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)} \right] \quad (\text{D.2})$$

$$\frac{d\dot{S}}{dc} = -1 \quad (\text{D.3})$$

$$\frac{d\dot{S}}{dS} = R_S(S) \quad (\text{D.4})$$

We know that for a saddle-stable path system, it is necessary to have one positive and one negative eigenvalue, denoted $\mu_{1,2}$. As the $Tr(J) = \mu_1 + \mu_2$ and $Det(J) = \mu_1\mu_2$. It is sufficient to show that $Tr(J) > 0$ and $Det(J) < 0$. It is easy to see that $Tr(J) = \rho > 0$ and by arranging the terms for the determinant, we can see that determinant reduces to the multiple steady state condition $G(S)$. We conclude that $Det(J)$ is negative if

$$G_S(S) = R_{SS}(S) - \rho - \frac{\bar{\psi}h_{SS}(S)}{u_c(R(S))} + \frac{\bar{\psi}u_{cc}(R(S))}{(u_c(R(S)))^2} < 0 \quad (\text{D.5})$$

$$det(J) = (\rho - R_S(S)) R_S(S) - \frac{u_c(c)}{u_{cc}(c)} \left[R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)} \right] \quad (\text{D.6})$$

Complex dynamics arise if $(Tr(J))^2 - 4Det(J) < 0$.

$$\rho^2 < 4 \left[(\rho - R_S(S)) R_S(S) - \frac{u_c(c)}{u_{cc}(c)} \left[R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)} \right] \right] \quad (\text{D.7})$$

As $Det(J)$ is shown to be negative for low and high steady states, this prevents these two steady states to have complex dynamics. However, for the middle steady state, there is a possibility to have complex dynamics if the condition (D.7) above holds.

E Proof of Proposition 2

The aim of the proof is to show mathematically that all steady state values of environmental quality S take higher values than the unique steady state of the model without uncertainty. This means that the economy becomes always precautionary when exposed to risk relative to an economy without any harmful events. Suppose that the condition (17) holds for the existence of three positive real roots. The function $G(S)$ with risk and the function without risk $G^{WR}(S)$ are as follows:

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi}h_S(S)}{u_c(c)} = 0 \quad (\text{E.1})$$

$$G^{WR}(S) = R_S - \rho \quad (\text{E.2})$$

With functional forms in section (2.2), we have

$$G(S) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 - 2gS + (g - \rho) = 0 \quad (\text{E.3})$$

$$G^{WR}(S) = -2gS + (g - \rho) = 0 \quad (\text{E.4})$$

Since we use a logistic function for natural resource regeneration, we have $S \in [0; 1]$. We know that $G(0) = G^{WR}(0) = (g - \rho)$. This means that two functions start from the same point. It is easy to remark that functions $G(S)$ and $G^{WR}(S)$ never intersect, since $G(S) \neq G^{WR}(S)$ other than the point where S equals 0.

After these information, we can easily show that all three roots of function $G(S)$ take higher values than the unique root of function $G^{WR}(S)$. The additional term $-2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2$ in $G(S)$ is always a positive term when $S \in [0; 1]$. This means that $G(S) > G^{WR}(S)$ for $S \in [0; 1]$. This completes the proof.

F Proof of Proposition 3

The third degree polynomial equation has the following form

$$a_1x^3 + b_1x^2 + c_1x + d_1 = 0 \quad (\text{F.1})$$

With the functional forms given in the text, we remark that terms $b_1 = -a_1$ with $a_1 = -2g\bar{\psi}\bar{h}$, $c_1 = -2g$ and $d_1 = g - \rho$. This simplifies the proof of the existence for three positive real roots. The discriminant of the cubic equation is as follows:

$$\Delta = 18a_1b_1c_1d_1 - 4b_1^3d + b_1^2c_1^2 - 4_1ac_1^3 - 27a_1^2d_1^2 \quad (\text{F.2})$$

- $\Delta > 0$, the equation (F.1) has three distinct real roots.
- $\Delta = 0$, the equation (F.1) has multiple roots and all three roots are real.
- $\Delta < 0$, the equation (F.1) has one real root and two non-real, complex roots.

Since we have $b = -a$, we can reformulate the discriminant (F.2) in the following way

$$\Delta = -b_1 [4d_1b_1^2 - (c_1^2 - 18c_1d_1 - 27d_1^2)b_1 - 4c_1^3] \quad (\text{F.3})$$

Then, we have a second degree equation to be solved for the value of b . The discriminant of this second degree equation is $\Delta_1 = (c^2 - 18cd - 27d^2)^2 - 64dc^3$. The equation (F.3) is written as

$$\Delta = \underbrace{-b_1}_{<0} \left[b_1 - \left(\frac{(c_1^2 - 18c_1d_1 - 27d_1^2) + \sqrt{(c_1^2 - 18c_1d_1 - 27d_1^2)^2 + 64d_1c_1^3}}{8d_1} \right) \right] \left[b_1 - \left(\frac{(c_1^2 - 18c_1d_1 - 27d_1^2) - \sqrt{(c_1^2 - 18c_1d_1 - 27d_1^2)^2 + 64d_1c_1^3}}{8d_1} \right) \right] \quad (\text{F.4})$$

Then, by assuming $\Delta_1 > 0$, the discriminant Δ has a positive value if

$$\begin{aligned}
& \underbrace{\left(\frac{(c_1^2 - 18c_1d_1 - 27d_1^2) - \sqrt{(c_1^2 - 18c_1d_1 - 27d_1^2)^2 + 64d_1c_1^3}}{8d_1} \right)}_{L_E} \\
& < b_1 < \underbrace{\left(\frac{(c_1^2 - 18c_1d_1 - 27d_1^2) + \sqrt{(c_1^2 - 18c_1d_1 - 27d_1^2)^2 + 64d_1c_1^3}}{8d_1} \right)}_{U_E} \quad (F.5)
\end{aligned}$$

where L_E and U_E are lower and upper extremes of the inequality (F.5). By replacing the terms a_1 , b_1 , c_1 , d_1 by their corresponding values, the condition (F.5) becomes

$$\begin{aligned}
& \frac{(4g^2 + 36g(g - \rho) - 27(g - \rho)^2) - \sqrt{(4g^2 + 36g(g - \rho) - 27(g - \rho)^2)^2 - 2^9(g - \rho)}}{8(g - \rho)g\bar{h}} \\
& < \bar{\psi} < \frac{(4g^2 + 36g(g - \rho) - 27(g - \rho)^2) + \sqrt{(4g^2 + 36g(g - \rho) - 27(g - \rho)^2)^2 - 2^9(g - \rho)}}{8(g - \rho)g\bar{h}} \quad (F.6)
\end{aligned}$$

However, the positive discriminant Δ proves only the existence of real roots and does not give an idea if these roots are positive or not. For this purpose, we use the Descartes rule. To see if the equation $G(S) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 - 2gS + (g - \rho) = 0$ has three positive real roots, we write $G(-S)$

$$G(-S) = -2g\bar{\psi}\bar{h}(-S)^3 + 2g\bar{\psi}\bar{h}(-S)^2 - 2g(-S) + (g - \rho) = 0 \quad (F.7)$$

This yields

$$G(-S) = 2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 + 2gS + (g - \rho) = 0 \quad (F.8)$$

By assuming $g - \rho > 0$, we observe that there is no sign change. This proves all roots are positive for $G(S)$. This completes the proof.

G The economy with only adaptation

The Hamilton-Jacobi-Bellman equation of the economy with only adaptation policy is as follows

$$\begin{aligned}
\rho V^B(S, K_A) = \max_{c, A} \{ & u(c) - Q^1(A) + V_S^B(S, K_A)(R(S) - c) \\
& + V_{K_A}^B(S, K_A)(A - \delta K_A) - h(S)(V^B(S, K_A) - \varphi(S, K_A)) \} \quad (G.1)
\end{aligned}$$

First order conditions for an internal optimal solution give

$$u_c = V_S^B \quad (\text{G.2})$$

$$V_{K_A}^B = Q_A^1 V_S^B \quad (\text{G.3})$$

Differentiating (G.1) with respect to S and K_A gives

$$\rho V_S^B = V_{SS}^B (R(S) - c) + V_S^B R_S + V_{K_A S} (A - \delta K_A) - h_S(S) \psi(K_A) \quad (\text{G.4})$$

where $V_{K_A S} = \frac{\partial^2 V}{\partial K_A \partial S}$

$$\rho V_{K_A}^B = V_{SK_A}^B (R(S) - c) + V_S^B R_S + V_{K_A K_A} (A - \delta K_A) - h(S) \psi_{K_A}(K_A) - \delta V_{K_A}^B \quad (\text{G.5})$$

The optimal dynamics of consumption and adaptation investment are

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_S(S) - \rho - \frac{\psi(K_A) h_S(S)}{u_c(c)} \right] \quad (\text{G.6})$$

$$\dot{A} = \frac{Q_A^1(A)}{Q_{AA}^1(A)} \left[(\rho + \delta) + \frac{h(S) \psi_{K_A}(K_A)}{Q_A} \right] \quad (\text{G.7})$$

Using $A = \delta K_A$, at the steady state, we have

$$Q_A^1(\rho + \delta) + h(S) \psi_{K_A}(K_A) = 0 \quad (\text{G.8})$$

$$R_S(S) - \rho - \frac{\psi(K_A) h_S(S)}{u_c(c)} = 0 \quad (\text{G.9})$$

Using the equations (G.6) and $R(S) = c$, we have

$$G(S) = R_S(S) - \rho - \frac{\psi(K_A) h_S(S)}{u_c(R(S))} \quad (\text{G.10})$$

Differentiating the equation (G.10) with respect to S yields

$$G_S(S) = R_{SS}(S) - \frac{h_{SS}(S) \psi(K_A)}{u_c(R(S))} + \frac{h_S(S) \psi(K_A) u_{cc}(R(S)) R_S(S)}{(u_c(R(S)))^2} - \underbrace{\left[\frac{\psi_{K_A}(K_A) h_S(S)}{u_c(R(S))} \right] \frac{dK_A}{dS}}_{=Z_1 > 0} > 0$$

It is also possible to write the equation (G.10) by using the functional forms in (2.2). Using the equations (G.6), (G.7) and functional forms presented in the section (2.2), the function $G(S)$ that describes the steady-state of the economy becomes

$$G(S) = \underbrace{g(1 - 2S) - \rho + \frac{\bar{\psi} \bar{h}}{u_c}}_{\text{Benchmark}} + \underbrace{\frac{a \bar{\psi} (1 - \bar{h} S)}{\phi \delta (\rho + \delta) u_c}}_{\text{Adaptation}} \quad (\text{G.11})$$

The first part for the expression (G.11) is the same as in the benchmark model. The additional last term comes with the adaptation policy. After arranging terms, the equation (G.11) writes

$$G(S) = -\bar{h}gzS^3 + ((g + \bar{h}g)z - \bar{\psi}\bar{h}g)S^2 + (\bar{\psi}\bar{h}g - 2g - zg)S + (g - \rho) = 0 \quad (\text{G.12})$$

H Proof for the multiplicity of equilibria (The economy with adaptation only policy)

Since the dynamical system is four dimensional, the original condition on $G(S)$ is not sufficient to prove the multiplicity of equilibria. Therefore, we give new additional conditions and write the function $G(S) = 0$ given in equation (G.10) in a slightly different form

$$\underbrace{\frac{(R_S(S) - \rho)u_c(R(S))}{h_S(S)}}_{n(S)} = \underbrace{\psi(K_A(S))}_{w(S)} \quad (\text{H.1})$$

By implicit function theorem, we know that $\frac{dK_A}{dS} < 0$. Since the penalty function $\psi(K_A)$ is decreasing in K_A , It is evident that $w(S)$ is increasing in S .

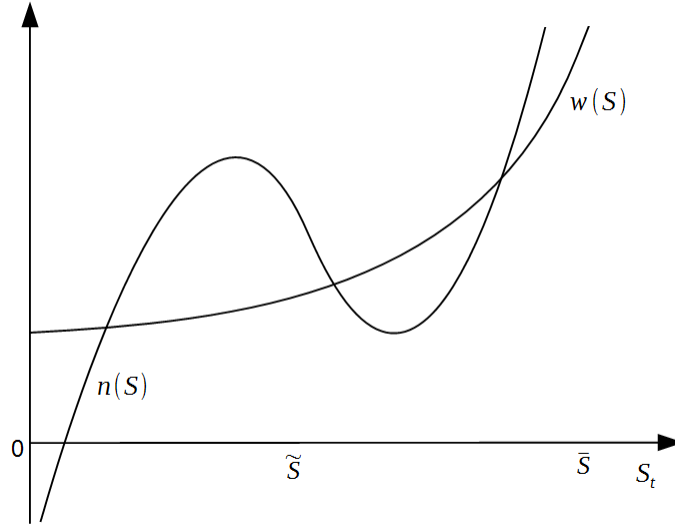


Figure 11: Multiplicity of equilibria with adaptation capital

We remark that $\lim_{S \rightarrow 0} n(S) = -\infty$ and $\lim_{S \rightarrow \bar{S}} n(S) = +\infty$. A sufficient condition for a low steady state is $\exists S < \tilde{S}$ such that $n_S(S) < 0$ with $n(S) > w(S)$. Note that $\tilde{S} < \bar{S}$ is between the low and high steady state. A sufficient condition for high steady state is $\exists S > \tilde{S}$ such that $n(S) < w(S)$.

I Proof of Proposition 5 (Adaptation)

The equation (7) combined with (1) at the steady state can be reformulated as

$$G(S) = -\bar{\psi}\bar{h}gS^2 + (\bar{\psi}\bar{h}g - 2g)S + (g - \rho) = 0 \quad (\text{I.1})$$

It is easy to show that this equation has one positive and one negative real root when $g - \rho > 0$:

$$S_{1,2} = \frac{(\bar{\psi}\bar{h}g - 2g) \pm \sqrt{(\bar{\psi}\bar{h}g - 2g)^2 + 4\bar{\psi}\bar{h}g(g - \rho)}}{2\bar{\psi}\bar{h}g} \quad (\text{I.2})$$

We exclude the negative root since it has no economic meaning. Consequently, the economy has a unique equilibrium without any possibility of multiple equilibria. What happens if this economy starts to invest in adaptation capital? For the sake of analytical tractability, we use a linear hazard and adaptation function. In the numerical analysis, we relax the linearity assumption on functional forms and use more general functional forms to show the robustness of our results.

$$\psi(K_A) = \bar{\psi}f(K_A) = \bar{\psi}(1 - aK_A) \quad (\text{I.3})$$

$$Q(A) = \phi \frac{A^2}{2} \quad (\text{I.4})$$

where a shows at which extent the adaptation capital is able to decrease the vulnerability against the inflicted damage $\bar{\psi}$. ϕ stands for a scale parameter for the adaptation cost function. The benchmark economy with an adaptation policy ends up with the following $G(S)$ (see Appendix G)

$$G(S) = -\bar{h}gzS^3 + ((g + \bar{h}g)z - \bar{\psi}\bar{h}g)S^2 + (\bar{\psi}\bar{h}g - 2g - zg)S + (g - \rho) = 0 \quad (\text{I.5})$$

where $z = \frac{\bar{h}(a\bar{\psi})^2}{\phi\delta(\rho+\delta)}$. For analytical tractability, we suppose that $\bar{\psi} = \frac{z}{\bar{h}}$. Then, the equation (I.5) can be reformulated in a simplified form

$$G(S) = -\bar{h}gzS^3 + \bar{h}gzS^2 - 2gS + (g - \rho) = 0$$

This equation is similar to (16) which can be shown to have three positive real roots. Following the same method for the proof of Proposition 3., the condition to have multiple equilibria in the benchmark economy augmented by the adaptation investments is as follows¹⁹

$$\begin{aligned} & \frac{\left[\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2 \right) - \sqrt{\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2 \right)^2 - 2^9g^3(g - \rho)} \right]}{8(g - \rho)g(\bar{h})^2} \\ & < \bar{\psi} < \frac{\left[\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2 \right) + \sqrt{\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2 \right)^2 - 2^9g^3(g - \rho)} \right]}{8(g - \rho)g(\bar{h})^2} \end{aligned} \quad (\text{I.6})$$

J The economy with only mitigation

We write the Hamilton-Jacobi-Bellman equation for the economy with a mitigation activity.

¹⁹The proof is available upon request.

$$\rho V^B(S) = \max_{c,M} \{u(c) - Q^2(M) + V_S^B(S)(R(S) + \Gamma(M) - c) - h(S)(V^B(S) - \varphi(S))\} \quad (\text{J.1})$$

where $V^B(S)$ is the value of the maximisation program before the event. As also stated in the text, the value of the problem after the event is as follows:

$$\varphi(S) = V^B(S) - \bar{\psi} \quad (\text{J.2})$$

The economy is exposed to an inflicted damage after the event. The first-order conditions are given by

$$u_c(c) = V_S^B(S) \quad (\text{J.3})$$

$$Q_M^2 = V_S^B(S) \Gamma_M(M) \quad (\text{J.4})$$

Calculating the derivative of (J.1) with respect to the environmental quality stock S yields

$$\rho V_S^B(S) = -\bar{\psi} h_S(S) + V_{SS}^B(S)(R(S) + \Gamma(M) - c) + V_S^B(S) R_S(S) \quad (\text{J.5})$$

Differentiating the equation (J.3) and using equations (J.3) and (J.5) gives

$$\frac{u_{cc}(c)}{u_c(c)} \dot{c} = \frac{V_{SS}^B(S)}{V_S^B(S)} \dot{S} \quad (\text{J.6})$$

$$\rho = -\frac{\bar{\psi} h_S(S)}{V_S^B(S)} + \frac{V_{SS}^B(S)}{V_S^B(S)} \dot{S} + R_S(S) \quad (\text{J.7})$$

Arranging equations (J.6) and (J.7) gives the Keynes-Ramsey rule

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} \right]$$

At the steady-state, we have

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(R(S) + \Gamma(M))} \quad (\text{J.8})$$

Note that the optimal steady state level of mitigation M depends on S . Differentiating the equation (J.8) with respect to S yields

$$G_S(S) = R_{SS}(S) - \frac{\bar{\psi} h_{SS}(S)}{u_c(c)} + \frac{\bar{\psi} h_S(S) u_{cc}}{(u_c(c))^2} \left(R_S(S) + \Gamma_M(M) \frac{dM}{dS} \right)$$

K Proof of Proposition 7 (Mitigation)

We start the proof by solving the following optimization problem with mitigation policy. Similar to the resolution of the benchmark economy, we write the Hamilton-Jacobi-Bellman equation

$$\rho V^B(S) = \max_{c,M} \{u(c) + V_S^B(S)(R(S) + \Gamma(M) - Q_2(M) - c) - h(S)(V^B(S) - \varphi(S))\} \quad (\text{K.1})$$

where $V^B(S)$ is the value of the maximisation program before the event. The economy is exposed to a constant inflicted damage after the occurrence of the event. As also stated in the text, the value of the problem after the event is as follows

$$\varphi(S) = V^B(S) - \bar{\psi} \quad (\text{K.2})$$

The first-order conditions are given by

$$u_c(c) = V_S^B(S) \quad (\text{K.3})$$

$$\Gamma_M(M) = Q_2^2(M) = P_M \quad (\text{K.4})$$

Using the envelop theorem, we have

$$\rho V_S^B(S) = -\bar{\psi} h_S(S) + V_{SS}^B(S)(R(S) + \Gamma(M) - Q_2(M) - c) + V_S^B(S) R_S(S) \quad (\text{K.5})$$

Differentiating the equation (K.3) and using equations (K.3) and (K.5) gives

$$\frac{u_{cc}(c)}{u_c(c)} \dot{c} = \frac{V_{SS}^B(S)}{V_S^B(S)} \dot{S} \quad (\text{K.6})$$

$$\rho = -\frac{\bar{\psi} h_S(S)}{V_S^B(S)} + \frac{V_{SS}^B(S)}{V_S^B(S)} \dot{S} + R_S(S) \quad (\text{K.7})$$

Arranging equations (K.6) and (K.7) gives the Keynes-Ramsey rule

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} \right]$$

At the steady state, we have

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(R(S) + \Gamma(M) - Q_2(M))} = 0$$

Since we use linear specifications for analytical tractability, we have the optimum level of mitigation investment as $M^* = \frac{1}{P_M}$. Then, we can reformulate the function $G(S)$

$$G(S) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 + \left(\frac{\bar{\psi}\bar{h}}{P_M} - 2gS \right) + (g - \rho) = 0 \quad (\text{K.8})$$

where $a_1 = -2g\bar{\psi}\bar{h}$, $b_1 = 2g\bar{\psi}\bar{h}$, $c_1 = \frac{\bar{\psi}\bar{h}}{P_M} - 2gS$ and $d_1 = g - \rho$. The equation (K.8) is similar to $G(S)$ of the benchmark economy without an environmental policy. The only term that is different in the economy with mitigation policy with respect to the benchmark economy is the term c_1 of the equation (F.1). The term c_1 is higher in the case with mitigation policy with respect to the benchmark economy case.

We look at the effect of a higher value c_1 on the upper and lower extremes of the inequality (F.5). By

assuming $c_1 < 0$ and $d_1 > 0$, the derivative of the upper and lower extremes are as follows

$$\frac{\partial L_E}{\partial c_1} = \frac{1}{2} \frac{\overbrace{(c_1^2 - 18c_1d_1 - 27d_1^2)}^{>0} \underbrace{(2c_1 - 18d_1)}_{<0}}{\left((c_1^2 - 18c_1d_1 - 27d_1^2)^2\right)^{\frac{1}{2}}} - \frac{1}{2} \left(\frac{\overbrace{(c_1^2 - 18c_1d_1 - 27d_1^2)}^{>0} \underbrace{(2c_1 - 18d_1)}_{<0} + 3.64d_1c_1^2}{\left((c_1^2 - 18c_1d_1 - 27d_1^2)^2 + 64d_1c_1^3\right)^{\frac{1}{2}}} \right) > 0$$

$$\frac{\partial U_E}{\partial c_1} = \frac{1}{2} \frac{\overbrace{(c_1^2 - 18c_1d_1 - 27d_1^2)}^{>0} \underbrace{(2c_1 - 18d_1)}_{<0}}{\left((c_1^2 - 18c_1d_1 - 27d_1^2)^2\right)^{\frac{1}{2}}} + \frac{1}{2} \left(\frac{\overbrace{(c_1^2 - 18c_1d_1 - 27d_1^2)}^{>0} \underbrace{(2c_1 - 18d_1)}_{<0} + 3.64d_1c_1^2}{\left((c_1^2 - 18c_1d_1 - 27d_1^2)^2 + 64d_1c_1^3\right)^{\frac{1}{2}}} \right) < 0$$

We observe that the occurrence of multiple equilibria with mitigation policy is less likely, since there is a smaller range of values for damage $\bar{\psi}$ that causes multiple equilibria.

L The economy with adaptation and mitigation

We solve the model with both adaptation and mitigation policies. The social planner maximizes the following program

$$V(S, K_A) = \max_{\{c(t), A(t)\}} \int_0^\infty [u(c(t)) - Q^1(A(t)) - Q^2(M(t)) + h(S(t)) [V(S(t), K_A(t)) - \psi(K_A(t))]] \exp\left(-\int_0^t [\rho + h(S(\tau))] d\tau\right) dt \quad (\text{L.1})$$

subject to

$$\dot{K}_A(t) = A(t) - \delta K_A(t) \quad (\text{L.2})$$

$$\dot{S}(t) = R(S(t)) + \Gamma(M(t)) - c(t) \quad (\text{L.3})$$

The optimal policy is to maximize (L.1) subject to (L.2) and (L.3).

We write the Hamilton-Jacobi-Bellman equation for the economy implementing both adaptation and mitigation policies.

$$\rho V^B(S, K_A) = \max_{c, A} \{u(c) - Q^1(A) - Q^2(M) + V_S^B(S, K_A) (R(S) + \Gamma(M) - c) + V_{K_A}^B(S, K_A) (A - \delta K_A) - h(S) (V^B(S) - \varphi(S, K_A))\} \quad (\text{L.4})$$

where $V^B(S)$ is the value of the maximisation program before the event. As also stated in the text, the value of the problem after the event is as follows:

$$\varphi(S, K_A) = V^B(S, K_A) - \psi(K_A) \quad (\text{L.5})$$

The economy is exposed to an inflicted damage after the event. The first-order conditions are given by

$$u_c(c) = V_S^B(S, K_A) \quad (\text{L.6})$$

$$Q_A^1(A) = V_{K_A}^B(S, K_A) \quad (\text{L.7})$$

$$Q_M^2(M) = V_S^B(S, K_A) \Gamma_M(M) \quad (\text{L.8})$$

Computing the derivative of (L.4) with respect to the environmental quality stock S and adaptation capital K_A yields

$$\begin{aligned} \rho V_S^B(S, K_A) = & -h_S(S) \psi(K_A) + V_{SS}^B(S, K_A) (R(S) + \Gamma(M) - c) \\ & + V_{K_A S}^B(S, K_A) (A - \delta K_A) + V_S^B(S, K_A) R_S(S) \end{aligned} \quad (\text{L.9})$$

$$\begin{aligned} \rho V_{K_A}^B(S, K_A) = & V_{SK_A}^B(S, K_A) (R(S) + \Gamma(M) - c) \\ & + V_{K_A K_A}^B(S, K_A) (A - \delta K_A) - \delta V_{K_A}^B(S, K_A) - h(S) \psi_{K_A}(K_A) \end{aligned} \quad (\text{L.10})$$

Differentiating the equation (L.6) and using equations (L.6) and (L.9) gives

$$\frac{u_{cc}(c)}{u_c(c)} \dot{c} = \frac{V_{SS}^B(S, K_A)}{V_S^B(S, K_A)} \dot{S} \quad (\text{L.11})$$

$$\frac{Q_{AA}^1(A)}{Q_A^1(A)} \dot{A} = \frac{V_{K_A K_A}^B(S, K_A)}{V_{K_A}^B(S, K_A)} \dot{K}_A \quad (\text{L.12})$$

Optimal dynamics for consumption, adaptation and mitigation are as follows

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_S(S) - \rho - \frac{\psi(K_A) h_S(S)}{u_c(c)} \right] \quad (\text{L.13})$$

$$\dot{A} = \frac{Q_A^1(A)}{Q_{AA}^1(A)} \left(\rho + \delta + \frac{h(S) \psi_{K_A}(K_A)}{Q_A^1(A)} \right) \quad (\text{L.14})$$

$$\dot{M} = -\frac{u_{cc}(c)}{u_c(c)} \frac{\Gamma_M(M)}{\Gamma_{MM}(M)} \left[R_S(S) - \rho - \frac{\psi(K_A) h_S(S)}{u_c(c)} \right] \quad (\text{L.15})$$

Using the equations (L.9) and (L.10), at the steady-state, we have

$$R_S(S) - \rho - \frac{\psi(K_A) h_S(S)}{V_S^B(S)} = 0 \quad (\text{L.16})$$

$$\rho + \delta + \frac{h(S) \psi_{K_A}(K_A)}{Q_A^1(\delta K_A)} = 0 \quad (\text{L.17})$$

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