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Jean-Bernard Chatelain, Kirsten Ralf. Ramsey Optimal Policy versus Multiple Equilibria with Fiscal and Monetary Interactions. 2019. halshs-02278791v1

HAL Id: halshs-02278791

<https://shs.hal.science/halshs-02278791v1>

Preprint submitted on 4 Sep 2019 (v1), last revised 12 Feb 2020 (v2)

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WORKING PAPER N° 2019 – 48

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JEL Codes: E63, C61, C62, E43, E44, E47, E52, E58

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Ramsey optimal policy, Interest Rate Rule, Fiscal Rule**

Ramsey Optimal Policy versus Multiple Equilibria with Fiscal and Monetary Interactions

Jean-Bernard Chatelain* and Kirsten Ralf†

August 29, 2019

Abstract

The reference model of frictionless endowment economies includes a Fisher relation for the real interest rate and government intertemporal budget constraint. *For this model, Ramsey optimal policy mix is a unique equilibrium with an interest rate peg and a "passive" fiscal rule with a negative-feedback value of its parameter stabilizing public debt.* This is a third equilibrium with respect to the two usual equilibria with ad hoc policy rules. The first one has passive fiscal policy and an active monetary policy rule parameter destabilizing inflation. The second one has an active fiscal policy rule parameter destabilizing public debt and a passive monetary policy which includes the case of an interest rate peg.

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"One would feel more comfortable... with rational expectations equilibria if these equilibria were accompanied by some form of 'stability theory' which illuminated the forces which move an economy toward equilibrium". Lucas (1978, p. 1429).

1 Introduction

In the last decade, several OECD countries faced a new monetary and fiscal interaction regime. Although interest rate were pegged at at zero or negative lower bound, there was no deflationary spiral nor trend inflation. In the US, inflation persistence, estimated by auto-correlation coefficients over rolling windows, remained low by historical standards and even slightly decreased. Because of large increase of the stocks of public debt over GDP following the financial crisis, shifting from negative real rate to positive real rate increases the snowball effect of interest expenses boosting future public debt much more than before the crisis. This is a concern for policy makers.

For describing this new regime, a neo-Fisherian model of frictionless endowment economies is put forward by Cochrane (2019). This model includes the intertemporal

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budget constraint of the government and a Fisher relation with a constant real interest rate equal to the representative household's discount factor (Leeper (1991), Cochrane (2019), chapter 2). For monetary policy, interest rate responds in proportion to inflation. For fiscal policy, lump-sum tax responds in proportion to the stock of real public debt.

Assuming inflation, interest rate and lump-sum tax are forward-looking variables for seeking equilibria with Blanchard and Kahn (1980) determinacy condition, Leeper (1991) obtains two equilibria for monetary and fiscal interactions. These two equilibria are defined by ranges of values of the parameters of ad hoc policy rules.

In the first equilibrium labeled "new-Keynesian" by Cochrane (2019), the interest rate rule parameter destabilizes inflation ("active monetary policy" according to Leeper (1991) with trend) and the fiscal rule parameter stabilizes public debt ("passive fiscal policy" according to Leeper (1991)).

Cochrane (2019, p.345) has a concern related to the local instability of inflation for this monetary and fiscal regime: *"To produce those equilibria, the central bank commits that if inflation gets going, the bank will increase interest rates, and by doing so it will increase subsequent inflation, without bound. Likewise, should inflation be less than the central bank wishes, it will drive the economy down to the liquidity trap... No central bank on this planet describes its inflation-control efforts this way. They uniformly explain the opposite. Should inflation get going, the bank will increase interest rates in order to reduce subsequent inflation. It will induce stability into an unstable economy, not the other way around. I have not seen selecting among multiple equilibria on any central bank's descriptions of what it does... That the central bank will react to inflation by pushing the economy to hyperinflation seems an even more tenuous statement about people's beliefs, today and in any sample period we might study, than it is about actual central bank behavior."*

In the second equilibrium related to the fiscal theory of the price level or FTPL (Cochrane (2019)), the fiscal rule parameter destabilizes public debt ("active policy" according to Leeper (1991)) and the interest rate rule parameter stabilizes inflation ("passive monetary policy" according to Leeper (1991)). A peg of interest rate with lack of response of interest rate to inflation is a particular case of "passive monetary policy" equilibrium.

Benassy (2009) has a concern related to the local instability of public debt dynamics for this monetary and fiscal regime: *"The basic idea behind the FTPL is that the government pursues fiscal policies such that, in off-equilibrium paths, it will not satisfy its intertemporal budget constraint, and run an explosive debt policy. This leaves only one feasible equilibrium path. Now although such off-equilibrium paths are not observed in the model's equilibrium, it would be extremely optimistic to assume that in real life situations the economy would follow at every instant the equilibrium path while the government pursues such policies. As a result many people would be reluctant to advise such policies to a real life government."*

Cochrane (2019) and Benassy (2009) concerns suggest that *the local instability of inflation or of public debt dynamics implies the lack of credibility of policy makers' policy and conversely*. Cochrane (2011) also mentions that Ramsey optimal policy is a relevant model of government behaviour: *"In most (Ramsey) analysis of policy choices,..., we think of governments choosing policy configurations while taking first order conditions as constraints; we think of governments acting in markets."* But so far, he does not investigate Ramsey optimal policy in the reference frictionless model of endowment economies in Cochrane (2019).

Because the policy maker has at least some *minimal credibility* which allows him to

anchor optimally initial inflation, the local stability of Ramsey optimal policy equilibrium is obtained. A minimal credibility allows the policy maker to select the unique optimal equilibrium path. Therefore, he rules out alternative stable paths which are sub-optimal off-equilibrium paths. Formally, the co-state of inflation is optimally predetermined at zero. Because of the linear relation between the co-state variable of inflation (or Lagrange multiplier) and the monetary policy instrument (the interest rate), this is equivalent to assume that the interest rate is an optimally predetermined variable.

The paper demonstrates that Ramsey optimal policy is a third equilibrium with respect to the two usual equilibria with ad hoc policy rules (Leeper (1991), Cochrane (2019)). *For the frictionless endowment economy model, this paper shows the Ramsey optimal policy mix under quasi-commitment (Schaumburg and Tambalotti (2006), Debor-toli and Nunes (2014)) is a unique equilibrium with an interest rate peg (a particular case of "passive monetary policy" according to Leeper (1991)) and with a "passive fiscal policy" (which stabilizes public debt dynamics).* This equilibrium is locally stable in the space of inflation and public debt (table 1).

Table 1: Three equilibria (λ_π inflation eigenvalue, λ_b public debt eigenvalue).

Fiscal policy:	Passive monetary policy: $ \lambda_\pi < 1$	Active: $ \lambda_\pi > 1$
Passive: $ \lambda_b < 1$	Ramsey optimal policy: $\lambda_\pi = 0, 0 < \lambda_b < 1$	New-Keynesian
Active: $ \lambda_b > 1$	Fiscal theory of the price level	No equilibrium

For this transmission mechanism, with Ramsey optimal policy, the interest rate is pegged at its long run value because (1) inflation persistence is already equal to zero with a zero response of interest rate to inflation with Fisher relation and (2) there are non-zero cost of inflation volatility and of interest rate volatility. In order to decrease the costly volatility of interest rate and of inflation, a *rational* central banker decides that monetary policy shocks are not auto-regressive and their variance is zero.

For this transmission mechanism, Ramsey optimal policy implies that forward-looking inflation should jump instantaneously to its long run value. To find an optimal initial anchor of inflation on public debt with Ramsey optimal policy, public debt dynamics and inflation dynamics should no longer be decoupled in the policy transmission mechanism. One may assume that the real interest is not constant as in the new-Keynesian model with public debt (Cochrane (2019), chapter 5). One may assume that there is a wealth effect of public debt on consumption (Sims (2016)).

With respect to ad hoc policy rules, Ramsey optimal policy under quasi-commitment adds two relevant assumptions on policy makers behavior. Firstly, it assumes the *rationality of the policy makers* using microeconomic foundations for an optimal choice of policy rule parameters with at least a minimal cost for the volatility of the *interest rate and lump sum tax* (interest rate smoothing and tax smoothing). Secondly, it assumes at least a *minimal credibility of the policy makers*.

Section two presents the policy transmission mechanism, the two equilibria with ad hoc policy rules and the Ramsey optimal policy equilibrium. Section three concludes.

2 Ramsey Optimal Policy in a Frictionless Endowment Economy

2.1 Policy Transmission Mechanism

Leeper and Leith (2016), Bai and Leeper (2017) and Cochrane (2019, chapter 2) omit money in the frictionless endowment economy model with respect to Leeper (1991) seminal paper, while still obtaining Leeper (1991) two regimes for monetary and fiscal inter-actions.

Consider an infinitely-lived representative consumer who receives a *constant* endowment of goods each period in the amount y and derives utility only from consumption c_t . The government purchases a *constant* quantity of goods $g > 0$ to the consumer each period. We impose equilibrium in the goods market, so that consumption: $c_t = y - g$.

The consumer makes a consumption-saving decision that produces the Fisher relation where the real rate is here equal to a *constant* discount rate:

$$E_t \frac{1}{\pi_{t+1}} = \frac{1}{\beta} \frac{1}{R_t} \quad (1)$$

where R_t is both the gross one-period nominal interest rate on nominal bonds bought at t and pay off in $t + 1$ and the monetary policy instrument, and $\pi_{t+1} = P_{t+1}/P_t$ is the gross rate of inflation between t and $t + 1$, with P_t the aggregate price level. In the steady state, $P_t = P_{t+1} \Rightarrow \pi^* = 1$ where π^* is the inflation target. Then, $R^* = \pi^*/\beta = 1/\beta$, is the nominal interest rate consistent with the inflation target according to the Fisher relation. The equilibrium real interest rate is constant at $r = (1/\beta) - 1$ where $0 < \beta < 1$ is the consumer's discount factor. The Fisher relation in deviation of steady state values is:

$$E_t \frac{1}{\pi_{t+1}} - \frac{1}{\pi^*} = \frac{1}{\beta} \left(\frac{1}{R_t} - \frac{1}{R^*} \right) \Rightarrow E_t \frac{1}{\pi_{t+1}} - 1 = \frac{1}{\beta} \left(\frac{1}{R_t} - \beta \right). \quad (2)$$

It is linearized around the steady state equilibrium:

$$E_t \pi_{t+1} - \pi^* = \beta (R_t - R^*) \Rightarrow E_t \pi_{t+1} - 1 = \beta \left(R_t - \frac{1}{\beta} \right)$$

Fiscal policy levies lump-sum taxes of τ_t and sets purchases to be constant, $g > 0$ with primary surplus $s_t = \tau_t - g$. Government issues one-period nominal bonds, B_t , that satisfy the flow constraint, where P_t is the aggregate price level and real debt is defined as $b_t = B_t/P_t$.

$$b_t = \frac{B_t}{P_t} = -(\tau_t - g) + R_{t-1} \frac{P_{t-1}}{P_t} \frac{B_{t-1}}{P_{t-1}} = -(\tau_t - g) + \frac{R_{t-1}}{\pi_t} b_{t-1}$$

Using the Fisher relation, we substitute the constant real interest rate in the government intertemporal budget constraint:

$$b_t = -(\tau_t - g) + \frac{1}{\beta} b_{t-1}$$

The dynamics of real debt does not depend on inflation or on the nominal rate R_t . The steady state level of real government debt has an exogenous value: $b_{t+1} = b_t = b^*$.

To be consistent with the government intertemporal budget constraint, the steady state level of tax revenue τ^* is equal to the steady state interest expense:

$$\tau^* - g = \left(\frac{1}{\beta} - 1\right) b^*$$

The government intertemporal budget constraint written in deviation of steady state is

$$b_t - b^* = -(s_t - s^*) + \frac{1}{\beta}(b_{t-1} - b^*)$$

The linearized dynamics are (in deviation from steady state):

$$\begin{pmatrix} E_t \pi_{t+1} - \pi^* \\ E_t b_{t+1} - b^* \end{pmatrix} = \underbrace{\begin{pmatrix} A_\pi = 0 & A_{\pi b} = 0 \\ A_{b\pi} = 0 & A_b = \frac{1}{\beta} \end{pmatrix}}_{=\mathbf{A}} \begin{pmatrix} \pi_t - \pi^* \\ b_t - b^* \end{pmatrix} + \underbrace{\begin{pmatrix} B_{\pi i} = \beta & B_{\pi \tau} = 0 \\ B_{bi} = 0 & B_{\tau \tau} = -1 \end{pmatrix}}_{=\mathbf{B}} \begin{pmatrix} R_t - R^* \\ s_{t+1} - s^* \end{pmatrix}$$

The log-linearized dynamics of inflation is:

$$\frac{E_t \pi_{t+1} - \pi^*}{\pi^*} = \beta \left(\frac{R^*}{\pi^*}\right) \frac{R_t - R^*}{R^*} = \frac{R_t - R^*}{R^*}$$

The log-linearized dynamics of real debt is:

$$\frac{E_t b_{t+1} - b^*}{b^*} = \frac{1}{\beta} \frac{b_t - b^*}{b^*} - \left(\frac{s^*}{b^*}\right) \frac{s_{t+1} - s^*}{s^*} \text{ with } \frac{\tau^* - g}{b^*} = \frac{1}{\beta} - 1$$

Log-linearized dynamics are:

$$\begin{pmatrix} \frac{E_t \pi_{t+1} - \pi^*}{\pi^*} \\ \frac{E_t b_{t+1} - b^*}{b^*} \end{pmatrix} = \underbrace{\begin{pmatrix} A_\pi = 0 & A_{\pi b} = 0 \\ A_{b\pi} = 0 & A_b = \frac{1}{\beta} \end{pmatrix}}_{=\mathbf{A}} \begin{pmatrix} \frac{\pi_t - \pi^*}{\pi^*} \\ \frac{b_t - b^*}{b^*} \end{pmatrix} + \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -\left(\frac{1}{\beta} - 1\right) \end{pmatrix}}_{=\mathbf{B}} \begin{pmatrix} \frac{R_t - R^*}{R^*} \\ \frac{s_{t+1} - s^*}{s^*} \end{pmatrix}$$

This transmission mechanism has the four following properties which drive policy maker's reaction functions:

1. **The system is Kalman (1960) controllable:** ($rank(\mathbf{B}, \mathbf{AB}) = rank(\mathbf{B}) = 2$). If there is no bounds on the values of funds rate and surplus, these two policy instruments responding to policy targets deviations from their set point with proportional feedback rules are able to locate next period public debt and next period inflation at any real value.
2. **The system is decoupled.** Because the real rate of interest is *constant* in the Fisher relation, there is no Granger causality of inflation and nominal interest rate *on* future public debt: $A_{b\pi} = 0$ and $B_{bi} = 0$. This is not the case in the new-Keynesian model with public debt (Cochrane (2019), chapter 5). Because there are no credit constraints on households, there is no Granger causality with a wealth effect of debt and lump-sum taxes on future consumption : $A_{\pi b} = 0$ and $B_{\pi \tau} = 0$. This is not the case in Sims (2016). Hence, the recursive dynamics of inflation and public debt are decoupled (independent): \mathbf{A} and \mathbf{B} are diagonal matrices.

It is possible to analyze the dynamics of inflation or public debt separately with dimension one (scalar) dynamics.

3. **Zero inflation persistence in the open-loop system.** If there is no feedback from policy instruments, the interest rate is pegged to its long run value as well as the tax rate and primary surplus. In this case, the dynamics only driven by the transition matrix \mathbf{A} and are labeled open-loop dynamics. The inflation eigenvalue is not only inside the unit circle, it has already reached the lowest (zero) inflation persistence value: $A_\pi = 0$ without policy feedback. The Fisher equation is a polar case for open-loop zero inflation persistence. By contrast, with accelerationist or new-Keynesian Phillips curves models of inflation dynamics, there is open-loop trend inflation. If monetary policy chooses a peg of interest rate, this would result in unit-root for the estimation of inflation persistence.
4. **Unstable public debt dynamics in the open-loop system.** When the primary surplus is pegged at its optimal value, the public debt eigenvalue is outside the unit circle ($A_b = 1/\beta > 1$): it leads to public debt spirals. Because public debt is controllable by lump-sum tax, a range of negative-feedback values of the parameter of a proportional fiscal rule can stabilize public debt dynamics.

2.2 Ad Hoc Policy Rules

The fiscal authority adjusts lump-sum tax in response to the level of real government debt:

$$s_t - s^* = G_b (b_{t-1} - b^*) + \varepsilon_t^s$$

Monetary policy follows an interest rate rule that responds to inflation and possibly to public debt:

$$R_t - R^* = F_\pi (\pi_t - \pi^*) + \varepsilon_t^i$$

The linearized dynamics are:

$$\begin{pmatrix} E_t \pi_{t+1} - \pi^* \\ E_{t-1} b_{t+1} - b^* \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{\beta} \end{pmatrix} + \begin{pmatrix} \beta & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} F_\pi & 0 \\ 0 & G_b \end{pmatrix}}_{=\mathbf{A}+\mathbf{BF}} \begin{pmatrix} \pi_t - \pi^* \\ b_t - b^* \end{pmatrix} + \underbrace{\begin{pmatrix} \beta & 0 \\ 0 & -1 \end{pmatrix}}_{=\mathbf{B}} \begin{pmatrix} \varepsilon_t^i \\ \varepsilon_t^s \end{pmatrix}$$

$$\begin{pmatrix} E_t \frac{1}{\pi_{t+1}} - \frac{1}{\pi^*} \\ E_{t-1} b_{t+1} - b^* \end{pmatrix} = \begin{pmatrix} \frac{1}{\beta} F_\pi & 0 \\ 0 & \frac{1}{\beta} - G_b \end{pmatrix} \begin{pmatrix} \pi_t - \pi^* \\ b_t - b^* \end{pmatrix} + \begin{pmatrix} \beta & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \varepsilon_t^i \\ \varepsilon_t^s \end{pmatrix} \text{ with } b_0 \text{ given.}$$

Shocks $\varepsilon_t^i, \varepsilon_t^s$ are assumed to be independently and identically distributed, with mean zero and a non-zero variance-covariance matrix.

Leeper (1991) denotes that there is only one predetermined variable (public debt) and three forward-looking variables (inflation, interest rate, surplus). In order to satisfy Blanchard and Kahn (1980) determinacy condition, as the system has two eigenvalues, one should be inside the unit circle and one should be outside the unit circle. But these eigenvalues are not given, they depend on policy rule parameters F_π and G_b , for a given discount factor β . Hence, either the inflation eigenvalue is inside the unit circle and the

public debt eigenvalue is outside the unit circle (first equilibrium) or it is the reverse (second equilibrium).

First equilibrium: $|\beta F_\pi| > 1$ and $\left|\frac{1}{\beta} - G_b\right| < 1$.

Because the parameters are zero: $(\mathbf{A} + \mathbf{BF})_{\pi b} = (\mathbf{A} + \mathbf{BF})_{b\pi} = 0$, it is not possible to anchor inflation on predetermined public debt using Blanchard and Kahn (1980) method with eigenvectors of the stable eigenvalue solution. Therefore, it is usually assume that the monetary shock in the Taylor rule is auto-correlated.

$$\varepsilon_t^i = \rho_{\varepsilon^i} \varepsilon_{t-1}^i + \eta_t^i \text{ with } 0 < \rho_{\varepsilon^i} < 1 \text{ and } \eta_t^i \sim N(0, \sigma_{\varepsilon^i}^2)$$

Then, one finds the anchor for inflation at all periods:

$$\pi_{t+1} - \pi^* = \beta F_\pi (\pi_t - \pi^*) + \beta \varepsilon_t^i = \rho_{\varepsilon^i} (\pi_t - \pi^*) \Rightarrow \pi_t - \pi^* = \frac{\beta}{\rho_{\varepsilon^i} - \beta F_\pi} \varepsilon_t^i$$

The interest rate rule is:

$$R_t - R^* = F_\pi (\pi_t - \pi^*) + \varepsilon_t^i = \left(\frac{\beta F_\pi}{\rho_{\varepsilon^i} - \beta F_\pi} + 1 \right) \varepsilon_t^i = \frac{\rho_{\varepsilon^i}}{\rho_{\varepsilon^i} - \beta F_\pi} \varepsilon_t^i$$

Inflation and nominal interest rate depend the discount factor, the interest rate rule parameter, the auto-correlation of the monetary policy shock and the monetary policy shock.

Public debt is solved backward without needing to assume the auto-correlation of fiscal shocks:

$$b_t - b^* = \left(\frac{1}{\beta} - G_b \right)^t (b_0 - b^*) - \sum_{j=0}^{j=t} \left(\frac{1}{\beta} - G_b \right)^j \varepsilon_{t-j}^s \text{ with } b_0 \text{ given.}$$

Second equilibrium: $|\beta F_\pi| < 1$ and $\left|\frac{1}{\beta} - G_b\right| > 1$.

The budget constraint is an unstable difference equation. Because these parameters are zero: $(\mathbf{A} + \mathbf{BF})_{\pi b} = (\mathbf{A} + \mathbf{BF})_{b\pi} = 0$, it is possible to find the forward-looking solution assuming that the fiscal shock is auto-correlated.

$$\varepsilon_t^s = \rho_{\varepsilon^s} \varepsilon_{t-1}^s + \eta_t^s \text{ with } 0 < \rho_{\varepsilon^s} < 1 \text{ and } \eta_t^s \sim N(0, \sigma_{\varepsilon^s}^2)$$

Then

$$b_{t+1} - b^* = \left(\frac{1}{\beta} - G_b \right) (b_t - b^*) - \varepsilon_t^s = \rho_{\varepsilon^s} (b_t - b^*) \Rightarrow b_t - b^* = \frac{-1}{\rho_{\varepsilon^s} - \left(\frac{1}{\beta} - G_b \right)} \varepsilon_t^s$$

The case of a peg of interest rate with $F_\pi = 0$ is a particular solution for inflation.

By contrast, in the following section, a peg of interest rate is the optimal policy $|\beta F_\pi^*| = 0 < 1$ but with an optimal fiscal rule parameter such that $\left|\frac{1}{\beta} - G_b^*\right| < 1$.

2.3 Ramsey Optimal Policy under Quasi-Commitment

In a monetary policy regime indexed by j , a policy maker may re-optimize on each future period with exogenous probability $1 - q$ strictly below one ("quasi commitment" by

Schaumburg and Tambalotti, 2007 and Debortoli and Nunes, 2014)). Following Schaumburg and Tambalotti (2007), we assume that the mandate to minimize the loss function is delegated to a sequence of policy makers with a commitment of random duration. The degree of credibility is modelled as if it is a change of policy-maker with a given probability of renegeing commitment and re-optimizing optimal plans. The length of their tenure or "regime" depends on a sequence of exogenous i.i.d. Bernoulli signals $\{\eta_t\}_{t \geq 0}$ with $E_t[\eta_t]_{t \geq 0} = 1 - q$, with $0 < q \leq 1$. If $\eta_t = 1$, a new policy maker takes office at the beginning of time t . Otherwise, the incumbent stays on. A higher probability q can be interpreted as a higher credibility. A policy maker with little credibility does not give a large weight on future welfare losses. The policy maker j solves the following problem for regime j , omitting subscript j , before policy maker k starts:

$$V_0^j = -E_0 \sum_{t=0}^{t=+\infty} (\beta q)^t \left[\frac{1}{2} (Q_\pi \pi_t^2 + Q_b b_t^2 + \mu_i i_t^2 + \mu_s s_t^2) + \beta (1 - q) V_t^k \right]$$

$$\text{s.t. } R_t - R^* = \beta q (E_t \pi_{t+1} - \pi^*) + \beta (1 - q) (E_t \pi_{t+1}^k - \pi^*)$$

$$\text{s.t. } b_t - b^* = \beta q (E_t b_{t+1} - b^*) + \beta (1 - q) (E_t b_{t+1}^k - b^*) + \beta q (s_t - s^*) + \beta (1 - q) (s_t^k - s^*)$$

Inflation and public debt texpectations are an average between two terms. The first term, with weight q is the inflation and public debt that would prevail under the current regime upon which there is commitment. The second term with weight $1 - q$ is the inflation and public debt that would be implemented under the alternative regime by policy maker k . A minimal credibility $q = 10^{-7}$ is never the limit of the extreme discretion model where the policy maker continuously re-optimizes for ever (Chatelain and Ralf (2019b)).

This optimal program a discounted linear quadratic regulator (LQR) with a "credibility adjusted" discount factor βq . The log-linear version of dynamic system can also be used instead of the linear version. Preferences of the policy maker are firstly given by positive weights for the two policy targets $Q_\pi \geq 0$, $Q_b \geq 0$ ($\mathbf{Q} = \text{diag}(Q_\pi, Q_b)$). Secondly, there are at least non-zero policy maker's preferences for *interest rate smoothing* and *primary surplus and tax smoothing*, with *strictly* positive weights for these two policy instruments in the loss function: $\mu_i > 0$, $\mu_s > 0$ ($\mathbf{R} = \text{diag}(\mu_i, \mu_s)$). This insures the strict concavity of the LQR program which implies the determinacy (uniqueness) of the solution of this Stackelberg dynamic game, if the system of the transmission mechanism is controllable.

All the results are valid for minimalist limit cases: a minimal interest rate smoothing ($\mu_i \geq 10^{-7}$), a minimal tax smoothing ($\mu_s \geq 10^{-7}$) and a minimal credibility (a non zero probability of not renegeing commitment next period: $10^{-7} \leq q \leq 1$).

Proposition 1 *For the transmission mechanism of the Fisher relation with constant real rate and the government intertemporal budget constraint, Ramsey optimal policy has a unique equilibrium. The interest rate is pegged at its long run value $i^* = 0$. A feedback Taylor rule is not optimal and the Taylor principle should not be satisfied: $F^* = 0 < \beta$. This is a "passive monetary policy". The optimal auto-correlation of monetary policy shocks is zero: $\rho_{\varepsilon_i}^* = 0$. The optimal variance of monetary policy shocks is zero $\sigma_{\varepsilon_i}^2 = 0$. Inflation jumps to its steady state value instantaneously following monetary policy shocks $\pi_0^* = 0$, as in a degenerate rational expectations model without predetermined variable. Hence, the price level is constant. Ramsey optimal fiscal rule has a negative feedback parameter ("passive fiscal policy") with insures the local stability of public debt dynamics.*

There are two stable eigenvalues giving the optimal persistence of inflation ($\lambda_\pi^* = 0$) and the optimal persistence of public debt ($0 < \lambda_b^* < 1$).

Proof. Because the system of the policy transmission mechanism is decoupled, the optimal program is identical if we pool the central bank and the treasury as a single policy maker or if we consider that they are distinct policy makers. One can solve Ramsey optimal policy for inflation and public debt separately using Ljungqvist and Sargent ((2015) chapter 5) algorithm (see also Chatelain and Ralf (2019)). For inflation, because the open-loop system is already at the minimal persistence value of inflation: $A_\pi = 0$ and because there is a *non-zero* cost of interest rate volatility in the loss function, it is optimal to set an interest rate peg $i^* = 0$. This implies a Taylor rule parameter equal to zero: $F^* = 0 < \beta$, a zero auto-correlation of monetary policy shocks $\rho_{\varepsilon^i} = 0$, and a zero variance of monetary policy shocks $\sigma_{\varepsilon^i}^2 = 0$. At this stage, any non-zero value of inflation on period t leads to a zero value of expected inflation on period $t + 1$. For the optimal initial anchor of inflation, the optimal value of the loss function is:

$$L^* = \begin{pmatrix} \pi_0 & b_0 \end{pmatrix} \mathbf{P} \begin{pmatrix} \pi_0 & b_0 \end{pmatrix}^T \text{ with } \mathbf{P} = \begin{pmatrix} P_\pi & P_{\pi b} \\ P_{\pi b} & P_b \end{pmatrix}$$

Where \mathbf{P} is a positive symmetric square matrix of dimension two which is the solution of the discrete algebraic Riccati equation (DARE) of the LQR. The marginal values of the loss function with respect to each state variables are equal to the co-state variable or Lagrange multiplier of each state variable. If state variables for the policy maker are jump variables of the private sector (here, inflation), their initial values is found optimizing the loss function at the initial date for each of these variables (initial transversality conditions).

Because the system is decoupled: $A_{\pi b} = A_{b\pi} = 0$ implies $Q_{\pi b} = 0$ in the optimal value of loss function. Because $A_{\pi\pi} = A_{b\pi} = A_\pi = 0$, the solution of the Riccati equation is such that $P_\pi = Q_\pi$, the policy maker's weight in the loss function.

$$P_\pi = Q_\pi + \beta A'_\pi P_\pi A_\pi - \beta' A'_\pi P_\pi B_\pi (\mu_i + \beta B'_{\pi i} P_\pi B_{\pi i})^{-1} \beta B'_{\pi i} P_\pi A_\pi = Q_\pi$$

the optimal anchor of inflation π_0^* on public debt is then zero:

$$\mathbf{P} = \begin{pmatrix} Q_\pi = \frac{Q_\pi}{1-\lambda_\pi^*} & P_{\pi b} = 0 \\ P_{b\pi} = 0 & P_b = \frac{Q_b}{1-\lambda_b^*} \end{pmatrix}$$

Because the system is decoupled: $A_{\pi b} = A_{b\pi} = 0$ implies $Q_{\pi b} = 0$ in the optimal value of loss function. The optimal anchor of inflation π_0^* on public debt is then zero:

$$(\partial L^* / \partial x)_{t=0} = \gamma_0 = P_\pi \pi_0 + P_{\pi b} b_0 = 0 \Rightarrow \pi_0^* = Q_\pi^{-1} P_{\pi b} b_0 = \frac{0}{Q_\pi} b_0 = 0$$

If there is a non-zero cost of inflation in the loss function ($Q_\pi > 0$), because inflation dynamics are decoupled from public debt dynamics, inflation behaves exactly as in a degenerate rational equilibrium model where *there is no predetermined variable*. At any date t , immediately after any monetary policy shock ε_t^s , inflation instantaneously jumps back to equilibrium $\pi_t^* = 0$. There are no transitory dynamics. The volatility of inflation is zero, which minimizes the loss function of the central bank to its lower boundary.

If there is a zero cost of inflation in the loss function ($Q_\pi = 0$), the policy maker does

not care about inflation. There is an *indeterminacy* for the optimal choice of the initial value of inflation π_0^* .

For public debt, the solution is a scalar case of Ramsey optimal policy under quasi-commitment (we use the linear dynamics instead of the log-linear dynamics).

$$E_t b_{t+1} - b^* = \frac{1}{\sqrt{\beta q}} (b_t - b^*) - \sqrt{\beta q} (s_t - s^*) \text{ with } A = \frac{1}{\beta q} \text{ or } \beta q A = 1 \text{ and } B = -1$$

Optimal public debt persistence (or auto-correlation or closed-loop eigenvalue) is the stable root of the characteristic polynomial of the Hamiltonian system:

$$\begin{aligned} 0 &= \lambda^2 - S\lambda + \frac{1}{\beta q} \text{ with } \lambda^* \lambda_2 = \frac{1}{\beta q} \text{ with} \\ S &= A + \frac{1}{A\beta q} + \frac{B^2 Q_b}{A \mu_s} = 1 + \frac{1}{\beta q} + \beta q \frac{Q_b}{\mu_s} \\ \lambda_b^* (\mu_b, \beta q) &= \frac{1}{2} \left(S - \sqrt{S^2 - \frac{4}{\beta q}} \right) \end{aligned}$$

With boundaries given by limits when the cost of changing policy instrument tends to zero or tends to infinity:

$$\lim_{\mu_b \rightarrow 0} \lambda_b^* (\mu_b, \beta q) = \lim_{\mu_b \rightarrow 0} \frac{1}{2} \left(\frac{\beta q}{\mu_b} - \frac{\beta q}{\mu_b} \right) = 0 \text{ and } \lim_{\mu_b \rightarrow +\infty} \lambda_b^* (\mu_b, \beta q) = \frac{1}{\beta q A} = 1$$

The fiscal rule parameter G_b remains in the range of values where where the persistence of public debt is strictly positive:

$$\frac{1}{\beta q} - 1 < G_b^* = \frac{\lambda_b^* - A}{B} = \frac{1}{\beta q} - \lambda_b^* < \frac{1}{\beta q}.$$

The loss function parameter P_b is:

$$P_b = \frac{\frac{1}{A} \frac{Q_b}{q\beta}}{1 - \lambda} = \frac{Q_b}{1 - \lambda} \text{ and } L_0^* = -\frac{1}{2} P_b b_0^2 = -\frac{1}{2} \frac{Q_b}{1 - \lambda} b_0^2.$$

Scilab code is available from the authors to solve Ramsey optimal policy with numerical values. ■

3 Conclusion

Ramsey optimal policy adds a third equilibrium with respect to the fiscal theory of the price level first equilibrium versus the new-Keynesian Taylor rule second equilibrium (Leeper (1991), Cochrane (2019)). There is no longer indeterminacy with multiple equilibrium path converging to long run equilibrium. This property is tied to the minimal credibility of the policy maker ($q > 0$). This occurs because optimal initial condition are derived for the inflation.

Ramsey optimal policy models policy maker's rationality with a minimal credibility, a minimal interest rate smoothing cost and a minimal tax smoothing cost. For the reference model of frictionless endowment economies, *the optimal policy mix is an interest rate peg*

and a "passive" (reactive) fiscal policy. By contrast, the FTPL equilibrium allows an interest peg and a destabilizing fiscal rule parameter labeled active fiscal policy.

Instead of zero inflation persistence in the transmission mechanism with the Fisher relation, if there is trend inflation (unit-root auto-correlation of inflation) in the transmission mechanism for the interest peg as it is the case with the new-Keynesian Phillips curve, the interest rate peg is not optimal (Chatelain and Ralf (2019c)).

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