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Forgery, Banditry, and Advanced Mathematics: The Curious Academic Lineage of Kong Yingda (574–648) in the Sui and Northern Dynasties

Daniel Patrick Morgan (墨子涵)[*]

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Abstract Lead editor of the *Orthodox Meaning of the Five Classics* (*Wujing zhengyi* 五經正義) and Chancellor of the Tang Directorate of Education, it is hard to think of a Confucian scholar more influential or 'orthodox' than Kong Yingda 孔穎達 (574–648). The same cannot be said of his schoolmasters, Liu Zhuo 劉焯 (544–610) and Liu Xuan 劉炫 (*c*.546–*c*.613). In this talk, I will tell *their story*. Liu Zhuo and Liu Xuan were no less influential in Kong Yingda's day than Kong Yingda is in ours. Known as 'the Two Lius', theirs is a touching story of two best friends from childhood who returned home from the capital in their forties to establish their own place of learning in the provinces – a private school that would attract such elite pupils as Kong Yingda and Zhang Shiheng 張士衡 (d. 645), teacher of Jia Gongyan 賈公彥 (fl. 637), and from which they would write the classical subcommentaries that Kong and Jia would synthesize and, ultimately, supplant. Their school was also something of a madhouse. Liu Zhuo was a visionary in *li* 曆 mathematical astronomy, who, constantly outmanoeuvred at court, took out his frustration by writing classical commentary that, in Kong's words, 'invariably falls back on numbers, invoking the Classics as all but complementary reading, … which leaves the teacher vexed and full of doubts, and the student toiling with little reward'. And the two did not so much *decide* to return home as they were reduced to commoners and ejected from the capital after Liu Xuan was caught forging 'lost works' to collect money from the court's library-restoration campaign. Equally worth mentioning is that, shortly after Kong Yingda's departure and Liu Zhuo's death, the school's student body joined a gang of local bandits, whom they convinced to extract Liu Xuan from behind the walls of the commandery seat to make him their bandit leader. This may sound like a world apart from that of Kong Yingda and Jia Gongyan, in the early Tang (618–907), but it is that of their youth, and, as I hope to show, it is one that deeply informed their later careers.

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I probably need not remind anyone here of who Kong Yingda 孔穎達 (574–648) and Jia Gongyan 賈公彥 (fl. 637) were, what they did, or why they matter. In short, Kong Yingda was a major Confucian scholar, academician (*boshi* 博士), and, eventually, the chancellor of the National University (*Guozi jijiu* 國子祭酒) under the early Tang (618–907), and together with Jia Gongyan, among others, he would author the *Correct Meaning of the Five Classics* (*Wujing zhengyi* 五經正義).[1] Finished in 653, and later grouped with Jia Gongyan's independent work, their subcommentated editions would go on to become the most authoritative versions of no less than seven of the Thirteen Classics (*Shisan jing* 十三經) as they have been transmitted to our day.[2] As educators, and as canonisers, there are few as important as these two men in Chinese history, so we might ask who educated *them*.

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[*] This talk was written very quickly, my apologies for whatever mistakes and missing sources you will inevitably find.

[1] On Kong Yingda, Jia Gongyan, and *The Correct Meaning of the Five Classics* (*Wujing zhengyi* 五經正義) project, see McMullen (1988, 71–112).

[2] Notably, in Ruan Yuan's 阮元 (1764–1849) *Shisan jing zhushu* 十三經注疏 of 1815.

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Jia Gongyan's official biography is rather sparse.[3] However, that of Zhang Shiheng 張士衡 (d. 645), a major scholar of the Rites, identifies Jia Gongyan as his most accomplished student, and it identifies Zhang Shiheng, in turn, as a student of one Xiong Ansheng 熊安生 (489/499–578) and Liu Zhuo 劉焯 (544–610).[4]

Turning to Kong Yingda's biography, one finds the self-same Liu Zhuo named as Kong Yingda's teacher:

> 孔穎達字沖遠，冀州衡水人也……穎達八歲就學，日誦千餘言。及長，尤明『左氏傳』、鄭氏『尚書』、王氏『易』、『毛詩』、『禮記』，兼善算曆，解屬文。
> Kong Yingda, styled Chongyuan 沖遠, was a man of Hengshui, Jizhou Commandery. … Yingda went to school at eight years of age, making a recitation of over a thousand words every day. By the time he was grown, he had become particularly conversant in the *Zuo Tradition*, Mr Zheng [Xuan]'s *Book of Documents*, Mr Wang [Bi]'s *Changes*, *Mao's Odes* and the *Record of Rites*; he was equally good at both **mathematics and mathematical astronomy**, explaining and composing text.
>
> 同郡劉焯名重海內，穎達造其門，焯初不之禮，穎達請質疑滯，多出其意表，焯改容敬之。穎達固辭歸，焯固留，不可。還家，以教授為務。
> **Liu Zhuo**, of his same commandery, had a name that carried weight within the seas, and Yingda delivered himself to his gates (as a 'below-the-gates' disciple). Zhuo did not at first treat him with due ritual propriety, but Yingda asked good and stubborn questions, greatly surpassing his expectations, so Zhuo changed his countenance and [began] to respect him. Yingda firmly bid his leave, and Zhuo firmly insisted that he must stay. [Eventually, Yingda] returned home to take up the profession of teaching.
>
> 隋大業初，舉明經高第，授河內郡博士。
> At the beginning of the Sui Grand Patrimony reign (605–617 CE), [in his early thirties], [Yingda] tested into the upper ranks of the Classics Competency [exam] as was named an academician of Henei Commandery.[5]

Both of their academic lineages thus go back to Liu Zhuo, and as Kong Yingda's subordinate on the *Correct Meaning* project, Jia Gongyan would have received a double-dose of second-hand influence.

Admittedly, it's difficult to know what to make of master–disciple anecdotes like the one we read in Kong Yingda's biography, but there's little doubt about the direct impact that Liu Zhuo had on Kong Yingda's career. In Gai Wenda's 蓋文達 (d. 644) biography, for example, we read about 'the eminent scholar Liu Zhuo' 大儒劉焯 playing the role of 'headmaster' 師首 at a sort of conference panel featuring Kong Yingda and Liu Zhuo's own teacher, Liu Guisi 劉軌思.[6] But more to the point, Kong Yingda describes himself as simply reworking his teacher's subcommentaries in his preface to the *Correct Meaning of Mao's Odes*:

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[3] See *Jiu Tangshu*, 189A.4950; cf. *Xin Tangshu*, 198.5649–5650.

[4] 又從熊安生及劉焯受禮記……士衡既禮學為優，當時受其業擅名於時者，唯賈公彥為最焉, *Jiu Tangshu*, 189A.4949.

[5] *Jiu Tangshu*, 73.2601; cf. *Xin Tangshu*, 198.5643.

[6] *Jiu Tangshu*, 189A.4951, and *Xin Tangshu*, 198.5651.

> 其近代為義疏者，有全緩、何胤、舒瑗、劉軌思、劉醜、劉焯、劉炫等，然焯、炫並聰穎特達，文而又儒，擢秀幹於一時，騁絕轡於千里，固諸儒之所揖讓，日下之無雙，於其所作疏內，特為殊絕。
> Among the authors of recent subcommentaries you have Quan Huan 全緩, He Yin 何胤 (446–531), Shu Yuan 舒瑗, Liu Guisi, Liu Chou 劉醜, Liu Zhuo and Liu Xuan 劉炫 (*c*.546–*c*.613), but **[Liu] Zhuo and [Liu] Xuan, together, are the [most] intelligent and outstanding [of the lot]**, matching literary flourish with scholarly [rigour]. [They are] flowers that only bloom in a single season, and [horses] that gallop unbridled for a thousand leagues; [they are] the affirmed object of every scholar's deference, utterly peerless under the sun, and it is in these subcommentaries that they have written that they are particularly transcendent.
>
> 今奉勑刪定，故據以為本。然焯炫等負恃才氣，輕鄙先達，同其所異，異其所同，或應略而反詳，或宜詳而更略……今則削其所煩，增其所簡，唯意存於曲直，非有心於愛增……
> As I have received orders to edit [a new edition of *Mao's Odes*], **we have thus chosen to go by these as our base [texts]**. However, [Liu] Zhuo, [Liu] Xuan and company trust [excessively?] in their talent, looking down on previous accomplishments; they agree where they differ, differ where they agree, and they are go into detail where one should be brief or are brief where one should go into detail. … Here, I have thus cut that which is [overly] prolix and expanded that which is [overly] simplified, my only intention being to [distinguish] wrong [from] right, and not to expand [their texts] for the love of expansion…[7]

The same is true for the *Correct Meaning of the Book of Documents*, but Kong Yingda paints a very different picture there of his teacher's style:

> 然焯乃織綜經文，穿鑿孔穴，詭其新見，異彼前儒，非險而更為險，無義而更生義。竊以古人言誥，惟在達情，雖復時或取象，不必辭皆有意。若其言必託數，經悉對文，斯乃鼓怒浪於平流，震驚飆於靜樹，使教者煩而多惑，學者勞而少功。過猶不及，良為此也。
> However, [Liu] Zhuo interwove the text of the Classic/s, piercing and drilling holes and caverns; he concocted new theories [to] set him apart from previous scholars; and[, in his interpretations,] he opted for danger and created meaning where there was neither. In my humble opinion, the point of the speeches and announcements of the ancients is simply to convey their sentiments, and though they may cycle back in time (?) or resort to symbols, not every word necessarily has [some greater] purpose. For example, **[Liu Zhuo's] words inevitably fall back on numbers to which the text of the Classic serves but as a foil** – this is really beating a level current into a tidal wave, or jolting a peaceful tree into a whirlwind. It leaves the teacher vexed and full of doubts, and the student toiling with little reward, going too far while at the same time falling short. But that is what he was best at.[8]

That should settle the matter of Kong Yingda and Jia Gongyan's academic lineage, but I quote these sources at length to drive home a point that often escapes attention. Namely, Kong Yingda and Jia Gongyan, like many of the great names from Chinese

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[7] *Mao Shi zhushu*, 'Mao Shi zhengyi xu' 毛詩正義序, 3b.

[8] *Shangshu zhushu*, 'Shangshu zhengyi xu' 尚書正義序, 3b–4a. Kong Yingda also mentions working from Liu Xuan's previous subcommentary on the *Zuo Tradition of the Spring and Autumn Annals* in *Chunqiu Zuozhuan zhushu*, 'Chunqiu zhengyi xu' 春秋正義序, 4a–b, and frequently citing him through the course of his own subcommentary.

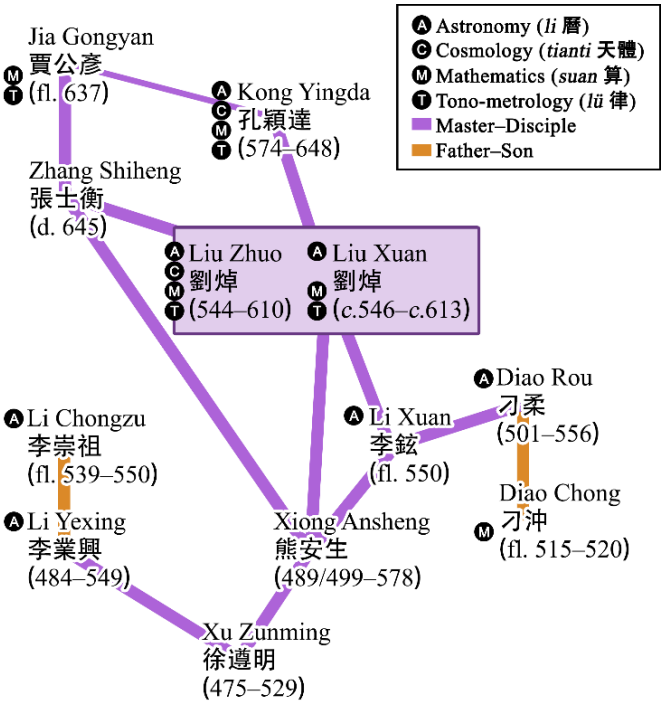

Figure 1 Network of Northern Courts ritual scholars and its links with the astral and mathematical science

history with which you might be familiar, were trained in both the Classics and mathematics,[9] the two of which, particularly as concerns the Rites (*li* 禮), often when hand in hand.

Their subcommentaries, you may not have noticed, are full of mathematical proofs.[10] Kong Yingda, *circa* 633, is recorded as having won a court debate on mathematical astronomy.[11] Their academic lineage of Rites scholars, as illustrated in Figure 1, connects them to some of the most influential astronomers and mathematicians of the northern courts. It should also come as no surprise that their teacher, Liu Zhuo, is mostly known in the history of science as one of the most important astronomers that ever lived.[12] That is all very normal for this period, so I'm going to simply ask you to keep that in mind as I tell you the story of Liu Zhuo and his BFF Liu Xuan – the story recorded in their respective biographies in the *Beishi* 北史 (*j*. 82) and *Suishu* 隋書 (*j*. 75) as collated with the latter's technical treatises ('Tianwen zhi' 天文志, 'Lüli zhi' 律曆志) and other titbits here and there.

[9] For a list of examples from the Han (206 BCE–220 CE), see Cullen (2009).

[10] On the mathematical portions of Kong Yingda and Jia Gongyan's subcommentaries, see Zhu Yiwen (2015; 2016a; 2016b; 2017a; 2017b) and Morgan (forthcoming).

[11] 與諸儒議曆及明堂，皆從穎達之說, *Jiu Tangshu*, 73.2602.

[12] On Liu Zhuo's career in astronomy, see Chen Meidong (2003: 318–331). On his immediate historical reception, see Li Liang (2019).

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Liu Zhuo and Liu Xuan were born in neighboring commanderies near modern-day Tianjin in and around 544. Liu Zhuo's father worked for the commandery personnel office (*jun gongcao* 郡功曹), and among the feats of childhood prodigy typical of an intellectual's biography, we read that 'Zhuo had a rhino's forehead and turtle's back', 'he was weak, and he didn't like to play'.[13]

Politically, they were born under the Eastern Wei (534–550), and they grew up through the Hou Jing 侯景 Rebellion (547–552), the emperor's abdication to the Northern Qi (550–557) in 550, a protracted war with the Northern Zhou (557–581) from 563 on, and the fall of the Northern Qi capital at Ye 鄴 in 577. Luckily, most of the fighting took place to the south and west of where they grew up, and they seem anyways to have spent most of their early years indoors.

It is under the Northern Zhou, in their thirties, that our protagonists first appear in public life: Liu Zhuo, 'famous for his classical studies', as a prefectural academician; Liu Xuan, 'famous for his administrative talents', as a retainer in a local revenue office, then, in 580, in a rites office.[14] That they earned these particular offices and reputations seems to have had everything to do with their schooling, as we read about in Liu Zhuo's biography. In short: the two of them swore an oath of friendship in their youth and went off together to study the *Odes*, the *Zuo Tradition* and the Rites from some of the most important northern scholars of their day; it was from Xiong Ansheng that they learned the Rites and, most likely, their mathematics (see Figure 1 above); they never bothered *finishing* any of their studies; and the most formative period of their education was spent in some guy's private library, where they shut themselves in reading the Classics for ten years.[15]

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A mere five years after they had become subjects of the Northern Zhou and entered into public life, the regent Yang Jian 楊堅 (541–604) coerced the eight-year-old emperor in his charge to yield the throne, and Liu Zhuo and Liu Xuan now found themselves subjects of the Sui 隋 (581–618). More importantly, they found their way to the capital, Daxing 大興 (formerly Chang'an 長安), as the Sui set about establishing its intellectual legitimacy in a flurry of state-sponsored projects.

Liu Zhuo had tested 'Shoot-strips Class A' 射策甲科 in the proto-civil service exam,[16] and, at the capital, our two friends were briefly assigned to a national history project under Editorial Director Wang Shao 王劭, then transferred to the Chancellery (Menxia sheng 門下省) as unpaid consultants. At the Chancellery, they worked with a team of experts on astronomy, astrology and metrology whilst working on the side with the Secretariat (Neishi sheng 內史省) and Palace Library (Bishu sheng 秘書省)

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[13] 焯犀額龜背，望高視遠，聰敏沈深，弱不好弄, *Suishu*, 75.1718.

[14] 遂以儒學知名，為州博士, *Beishi*, 82.2762; *Suishu*, 75.1718 ('Liu Zhuo zhuan'); 周武帝平齊，瀛州刺史宇文。亢引為戶曹從事。後刺史李繪署禮曹從事，以吏幹知名, *Beishi*, 82.2763; *Suishu*, 75.1719 ('Liu Xuan zhuan').

[15] 少與河間劉炫結盟為友，同受『詩』於同郡劉軌思，受『左傳』於廣平郭懋當，問禮於阜城熊安生，皆不卒業而去。武強交津橋劉智海家素多墳籍，焯與炫就之讀書，向經十載，雖衣食不繼，晏如也, *Beishi*, 82.2762; *Suishu*, 75.1718.

[16] On 'shoot strips', see Morgan (forthcoming).

on ‘examining and fixing the various sayings’.[17] Their biographies are not particularly clear about what either project entailed, but it is reasonable to assume that it had to do with consolidating the northern court’s newly acquired human, material and library resources.

As concerns the sciences, the Northern Wei (386–535) had depended on talent from the North China Plain from the mid-fifth century on, and when the empire split in two in the 530s, most of that expert community stayed in the East. Those who made the move to Chang’an carried on as normal, but everything changed in 554: in the South, the Liang (502–557) had fled to Jiangling with what experts, instruments and books they could salvage from rebel-occupied Jiankang, and the Western Wei swooped in to take it all for themselves. What they discovered, to their great delight, was a scientific tradition that had gone its own way for more than a century, and they set about integrating Southern experts and Southern knowledge with their own. This was a delicate process, and when Emperor Wen of Sui 隋文帝 (r. 581–604) began talk of a new ‘calendar’ to inaugurate his reign, it stood divide the capital’s experts along regional lines. Enter Zhang Bin 張賓 (fl. 568–584), a Daoist priest who had served both the Sui and the Zhou before them as propagandist-in-chief as concerned celestial omens. Not only did Zhang Bin have the new emperor’s ear, he used it to bring both parties to work together on the first true North-South synthesis that could outperform the astronomical models it was designed to replace. With public debates and more than seventeen experts involved in its conception, the *Kaihuang li* 開皇曆 took until 584 to complete and enter into public service, and it’s likely that the Two Lius were brought into the Secretariat to consult about this project.[18]

As concerns ‘examining and fixing the various sayings’, we know that the Sui court faced similar challenges in the field of classical scholarship, and it is here where our protagonists’ biographies provide us some details about their involvement. We know, for example, that Niu Hong 牛弘 (545–610) successfully petitioned the throne to ‘offer to buy all the lost books under heaven’ for the Palace Library, and that Liu Xuan worked in acquisitions.[19] Around the same time, Niu Hong and the two Lius are named amongst a list of other experts from various regional traditions called to the National University (Guozi jian 國子監) to ‘debate difficult questions of antiquity and today never understood by previous worthies’.[20] In 586, lastly, the court transported the Stone Classics from Luoyang to Daxing, and it was the two Lius who were charged with deciphering the heavily effaced steles.[21]

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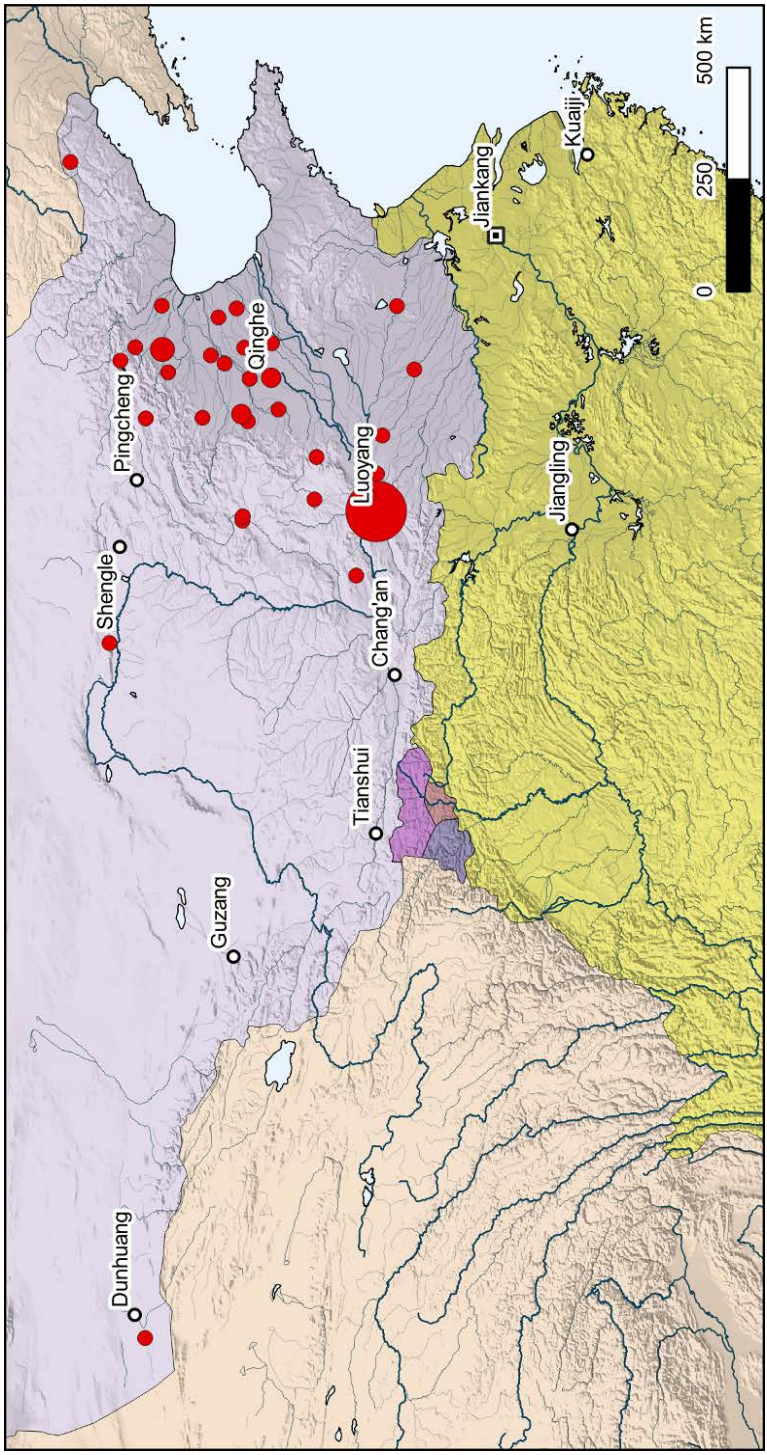
[17] 與著作郎王劭同修國史，兼參議律曆，仍直門下省，以待顧問。俄除員外將軍。後與諸儒於祕書省考定羣言, *Suishu*, 75.1718 (‘Liu Zhuo zhuan’); 奉勑與著作郎王劭同修國史。俄直門下省，以待顧問。又與諸術者修天文律曆，兼於內史省考定羣言, *Suishu*, 75.1719 (‘Liu Xuan zhuan’).

[18] This account of regional politics in the exact sciences is based on work presented in my talk, ‘Mapping Regional Traditions in Chinese Astronomy and Mathematics, 311–618 CE’, at the conference On Cultures of Scientific Practice in Ancient Mathematical Sciences, Université Paris Diderot, 10–13 April 2019 (https://halshs.archivesouvertes.fr/halshs-02021447).

[19] 牛弘奏請購求天下遺逸之書, *Suishu*, 75.1720.

[20] 與左僕射楊素、吏部尚書牛弘……等於國子共論古今滯義，前賢所不通者, *Suishu*, 75.1718.

[21] 六年，運洛陽石經至京師，文字磨滅，莫能知者，奉敕與劉炫等考定, *Suishu*, 75.1718.



Map 1 Known *jiguan* 籍貫 of mathematicians and mathematical astronomers active in the Northern Wei, 439–535 CE (largest dot = 12 people)

Working side by side at the centre of it all on difficult questions and new discoveries, one might say that these childhood friends were living the dream, except, that is, that they were not really being paid.

Liu Zhuo was made a 'supernumerary general' (*yuanwai jiangjun* 員外將軍), which is not a real position and paid only half what collaborators among the court gentlemen were making. Frustrated, he went home, briefly took a position in the district labour section (*gongcao*), then returned. Liu Xuan, by contrast, had nothing at all, and he complained about this to a confidant in the Secretariat who managed to get him a 'generalship' in the Palace Administration. This was apparently not enough for him either, because, taking advantage of the court's book drive,

> 炫遂偽造書百餘卷，題為『連山易』、『魯史記』等，錄上送官，取賞而去。
> [Liu] Xuan forged several hundred rolls of books with such titles as the *Mt Lian Changes* and the *Historical Records of Lu*, which he catalogued and delivered to the [relevant] office, then claimed the reward and left.[22]

Eventually, Liu Xuan was reported, stripped of his office and forced to return home. So too was his friend.

According to Liu Zhuo's biography, the two of them had made such a mockery of other scholars in the debates at the National University that he was sacked in an angry letter-writing campaign sometime after 586.[23] One imagines that this may have had something to do with his questionable choice of associates in classical studies, *who had defrauded them*, but the *Suishu* 'Lüli zhi' suggests that it may also have to do with his equally questionable alliances in the parallel debate unfolding in astronomy.

In astronomy, the *Kaihuang li* was just the sort of landmark success that the new dynasty needed: it was an honest improvement, from a technical perspective, and, politically, it had brought together the Chang'an and Jiangling factions at the Astronomical Bureau (Taishi ling 太史令/Shiguan 史官) in service of the new ruling family. As soon as it was put into service, however, Liu Zhuo joined one Liu Xiaosun 劉孝孫 (d. 594/597) in publishing a scathing criticism of its authors and a score-sheet showing Liu Xiaosun's methods to outperform it at every level.[24]

Liu Xiaosun was a compatriot, from the North China Plain, and a total political outsider in Daxing (Chang'an). To make a long story short, Liu Xiaosun was a student of Zhang Zixin 張子信, who had returned home to the North China Plain after thirty years spent on a sea isle dedicated to observation and experimentation. Zhang's work was revolutionary, but instead of bringing it to court, he took local students, like Liu Xiaosun, who would go on to do so in his place.[25] In 576, Liu Xiaosun and another of Zhang's students were selected to participate in a competitive predictive trial at the Northern Qi's Astronomical Bureau, where they stood to prove to one and all that they had cracked such secrets as the variability of solar and planetary motion.

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[22] *Suishu*, 75.1720.

[23] 後因國子釋奠，與炫二人論義，深挫諸儒，咸懷妬恨，遂為飛章所謗，除名為民, *Suishu*, 75.1718.

[24] See *Suishu*, 17.424–428.

[25] On Zhang Zixin and Liu Xiaosun, see Chen Meidong (2003: 298–304) and the appropriate entries in Martin and Chaussende (forthcoming). For Zhang Zixin's official biography, see *Bei Qishu*, 49.680, and *Beishi*, 89.2941.

Unfortunately, the capital fell to Northern Zhou armies before the results came in;[26] Zhang Zixin died in the fighting,[27] and Liu Xiaosun disappears from the historical record for seven years.

History would ultimately prove them right, but no one in 584 was ready to listen to two no-name former subjects of the Northern Qi who wanted to undo one of the new dynasty's founding accomplishments. As a result, Zhang Bin's protégé Liu Hui 劉暉 was made the director of the Astronomical Bureau, the two worked together to supress their upstart critics, and, 'in the end, Xiaosun, Zhuo and company were dismissed from office on other charges'.[28]

In 594, Zhang Bin passed away. Upon hearing the news, Liu Xiaosun immediately returned to the capital from his district-level post to repetition the throne. After blocking him at court and interrogating him in person, Liu Hui realises that Liu Xiaosun is never going to quit, so he finds him a position in the Astronomical Bureau where he can keep an eye on him. Things quickly get uncomfortable: Liu literally moves in, sleeping at the Observatory, whose resources he uses to advance his own research, and once it's clear that he can't get past the director, he eventually turns to public protest:

> 乃抱其書，弟子輿櫬，來詣闕下，伏而慟哭。執法拘以奏之。
> Holding his book(s) to his chest, [Liu Xiaosun had] his disciples load him into a coffin, which they delivered to the palace gates, prostrating and wailing in sorrow. They were arrested by law enforcement, who memorialised [the matter].[29]

This gets the emperor's attention. He asks around for references, orders a third part to compare his work to the Bureau's, and, finally, grants Liu and his fellow countryman protégé Zhang Zhouxuan 張冑玄 (d. 605/617) the opportunity to prove themselves in a competitive eclipse trial. The results blow everyone away, and the emperor calls them to court, offering them the chance at the sort of reform for which Liu Xiaosun has been campaigning for eighteen years. Unfortunately, Liu and Zhang have a falling out, and Liu agrees, but only under one condition: 'first [You] decapitate [Director] Liu Hui'.[30] Liu Xiaosun is once again removed from office, the opportunity passes to his enemy, and Liu returns home to die a failure.

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Having been ejected from the capital, and whilst their ally was burning their last bridges in astronomy, Liu Zhuo and Liu Xuan 'leisurely wandered from town, indefatigable, devoting themselves to teaching and writing'.[31] They made a living at it for about a decade, *circa* 586–598, in what was the most productive period of their respective careers. As we read in Liu Zhuo's biography:

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[26] *Suishu*, 17.418.

[27] *Bei Qishu*, 49.680; *Beishi*, 89.2941.

[28] 孝孫、焯等，竟以他事斥罷, *Suishu*, 17.428.

[29] *Suishu*, 17.429.

[30] 先斬劉暉, *Suishu*, 17.429.

[31] 於是優遊鄉里，專以教授著述為務，孜孜不倦, *Beishi*, 88.2762 ; *Suishu*, 75.1718.

Title	Author	Rolls
經 CLASSICS		
五經述議 五經正名	劉焯 劉炫	?? 12/15
周易述議* 周易正名*	劉焯 劉炫	?? ??
尚書義疏/述議 尚書述義 尚書正名*	劉焯 劉炫	20 20 ??
毛詩述議 毛詩述義 毛詩譜 毛詩集小序 毛詩正名*	劉焯 劉炫	?? 30/40 2 1 ??
禮記述議* 禮記正名*	劉焯 劉炫	?? ??
春秋左氏傳述議* 春秋左氏傳述義 春秋攻昧 春秋規過 春秋左傳杜預序集解 春秋左傳正名*	劉焯 劉炫	?? 37/40 10/12 3 1 ??
孝經疏？ 古文孝經述義 酒孝經	劉焯 劉炫	?? 5 1
論語章句 論語述義	劉炫	20 10
曆筭 ASTRONOMY AND MATHEMATICS		
稽極 曆書 皇極曆	劉焯	10 10 1
算術	劉炫	1

Table 1 Liu Zhuo and Liu Xuan, recorded works. Note that *yi* 義 and *yi* 議 are often used interchangeably in these titles, that (*) marks an individual work filled in from the title of the canons in the first line.

> 賈、馬、王、鄭所傳章句，多所是非。『九章算術』、『周髀』、『七曜曆書』十餘部，推步日月之經，量度山海之術，莫不覈其根本，窮其祕奧。著『稽極』十卷，『曆書』十卷，『五經述議』，並行於世。
> [Liu Zhuo] had many opinions about what Jia [Kui] 賈逵 (30–101), Ma [Rong] 馬融 (79–166), Zheng [Xuan] 鄭玄 (127–200) and Wang [Su's] 王肅 (195–256) transmitted chapter-and-verse [commentaries] got right and wrong. And as concerned the classics of calculating the sun and moon and the procedures for measuring the mountains and seas in more than ten [mathematical] titles such as the *Jiuzhang suanshu* 九章算術, *Zhoubi* 周髀 and *Qiyao lishu* 七曜曆書, there were none whose fundamentals he did not examine or whose mysteries he did not lay bare. He wrote the *Jiji* 稽極 in ten rolls, a *Lishu* 曆書 in ten rolls and the *Wujing shuyi* 五經述議, all of which saw circulation at the time.
>
> 劉炫聰明博學，名亞於焯，故時人稱二劉焉。
> Liu Xuan was intelligent and broadly learned, [but] his reputation took second place to Zhuo's, thus people of the time called him 'Second Liu'.
>
> 天下名儒後進，質疑受業，不遠千里而至者，不可勝數。論者以為數百年已來，博學通儒，無能出其右者。然懷抱不曠，又嗇於財，不行束脩者，未嘗有所教誨，時人以此少之。
> The world's famous scholars later entered to pose questions and receive training, with more than one could count being ones who 'came not thinking a thousand *li* too far'. The opinion was that they were peerless among the most broadly learned of scholars of the last several hundred years. However, their arms were not particularly wide open, and they were stingy with money, so no one ever got a single lesson without paying some tutorial fee. The people of the time looked down on them for this.[32]

There are four points I think we can extract from this passage and the list of our protagonists' recorded works in Table 1. First, the Two Liu's research and written output had a clear double focus on Classical exegesis and mathematics, as already mentioned. Second, this is the period when the Two Lius apparently composed the subcommentaries that would lay the groundwork for Kong Yingda et al. Third, judging from Kong Yingda's biography, it is also towards the end of this period, *circa* 592–598, that he would have been in residence at Liu Zhuo (and Liu Xuan's) school.[33] Fourth, it is important to note that, even in this new milieu, the theme of their inappropriate moneymaking schemes continues to come up.

This reputation followed them, and it had a negative impact on their reception beyond the walls of their private school. As explained in the *Suishu* bibliographical treatise, for example, the Kong Anguo 孔安國 commentary to the *Xiaojing* 孝經 only comes down to us through Liu Xuan: supposedly, the commentary had been lost in the South, during the civil war that drove the Liang to Jiangling, and it only resurfaced in the North through the Sui Palace Library's book drive. Liu Xuan decided to subcommentate and teach this newly rediscovered edition of the Classic…

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[32] *Beishi*, 82.2762–2763 ; *Suishu*, 75.1718–1719.

[33] Born in 574, 'Yingda delivered himself to [Liu Zhuo's] gates' sometime after 'he was grown' 及長 (592, at 20 *sui*?) and prior his recommendation to the position of academician at 'the beginning of the Sui's Great Patrimony [reign]' 隋大業初 in 605 (*Jiu Tangshu*, 73.2601; cf. *Xin Tangshu*, 198.5643). The Two Liu's return to Daxing by 598, below, provides an amended upper limit.

> 儒者諠諠，皆云炫自作之，非孔舊本，而祕府又先無其書。
> [but] this threw scholars into a hubbub, everyone saying that Xuan had written it himself, that it was not actually the old Kong [Anguo] edition, and that the Archives furthermore had no prior record of this book.[34]

Had it been anyone but Liu Xuan behind it, one imagines that we would be reading this commentary today as self-assuredly as we do any of Zheng Xuan's.

***

In 598, Liu Xuan reappears at the capital, intervening in a debate to deny an official mourning leave for his adopted mother.[35] Hearing of their return, Yang Yong 楊勇 (d. 604), the heir apparent, orders them to enter into the service of his younger brother, King Yang Xiu of Shu 蜀王楊秀 (d. 618). They refuse, the king has them brought to Sichuan in handcuffs, and they are made to serve in his palace guard as punishment. Luckily, the heir is deposed in 600, the king of Shu in 602, and they are immediately recruited into the palace of the new crown prince, Yang Guang 楊廣 (569–618).

Working behind the scenes with Emperor Wen's infamous 'hatchet man', Yang Su 楊素 (d. 606),[36] Yang Guang, the second in line, had manufactured the succession crisis that saw his brothers removed from power. He would need to consolidate his influence to keep the same from happening to him, of course, and upon entering the Eastern Palace in 600 he was granted the perfect opportunity: the emperor entrusted him with the increasingly complicated matter of astronomical policy.[37]

To recap, in 594, after Zhang Bin's death and Liu Xiaosun's dismissal, Zhang Zhouxuan proved himself the only man capable of working with the throne to replace the outdated *Kaihuang li*. The emperor gave him the go-ahead, but he faced an uphill battle against the Chang'an and Jiangling factions at the Bureau, led, respectively, by Liu Hui and Yu Jicai 庾季才 (d. 603).[38] Zhang Zhouxuan, after all, was a no-name Bureau technician from the former Northern Qi with questionable alliances who had just circumvented and embarrassed the Bureau leadership. To make a long story short, in 594–600, Zhang finished his *Daye li* 大業曆, saw its implementation, and managed to oust both Liu Hui and Yu Jicai from the Bureau in favour of his own ally, Yuan Chong 袁充 (544–618), a member of the *fourth wave* of experts captured at Jiankang at the fall of the Chen 陳 (557–589).[39]

This was nothing less than a coup d'État for the Astronomical Bureau, and things under Yuan Chong's directorship began to get messy. First, Yuan Chong inserted himself into the succession crisis, citing celestial omens in support of deposing the

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[34] *Suishu*, 32.935.

[35] 十八年……時永寧令李公孝四歲喪母，九歲外繼，其後父更別娶後妻，至是而亡。河間劉炫以無撫育之恩，議不解任, *Suishu*, 71.1651.

[36] Twitchett (1979: 69).

[37] 開皇二十年，袁充奏日長影短，高祖因以曆事付皇太子, *Suishu*, 18.459.

[38] These and the following events are recounted in *Suishu*, 17.429–435. For Yu Jicai's biography, see *Beishi*, 89.2947–2949, and *Suishu*, 78.1764–1767; cf. Chen Meidong (2003: 310–313), Deane (1989: 115–116) and the relevant entry in Martin and Chaussende (forthcoming).

[39] For Yuan Chong's biography, see *Beishi*, 74.2554–2557, and *Suishu*, 69.1610–1614; cf. Chen Meidong (2003: 307–318, 331–337), Hasebe (1993), Imanishi (2009) and the relevant entry in Martin and Chaussende (forthcoming).

heir apparent.[40] Second, Yuan Chong worked with Zhang Zhouxuan to reform the court's civil timekeeping policy, which, while 'more scientific', resulted in a longer work day, fomenting resentment within the empire's workforce.[41]

This is the mess that was handed to the crown prince in 600, and he conscripted Liu Zhuo and Liu Xuan in 602 to help resolve it. It was the closest that our protagonists had ever come to power, and they intended to make the most of it.

In astronomy, Liu Zhuo led the charge to salvage Liu Xiaosun's reputation and oust his rival, Zhang Zhouxuan. In terms of academic lineage, Zhang Zixin's North China Plain school had come out on top, and this was a fight to the death for its leadership. Having spent a part of his exile improving upon Liu Xiaosun's *Qiyao li* 七曜曆 (above), Liu Zhuo finished and submitted to the throne his own *Huangji li* 皇極曆 under Yang Guang's patronage. In retrospect, the *Huangji li* marks one of the most important technical breakthroughs in the history of Chinese astronomy, and it apparently won him a place in the Academy; at the Bureau, however, Zhang Zhouxuan acted to block Liu Zhuo's work pending further study, placing the crown prince's plans on hold.[42]

Having been brought back to consult upon astronomy, the Two Lius also took the opportunity to re-establish themselves amongst the leading Classical scholars of the capital. Importantly, the two of them managed to get themselves appointed to the team of Classicists charged with 'fixing the Five Rites' and/or 'fixing the Rites and tono-metrological standards' at the basis of the Sui's ritual and economic regime, which earned them an honorary place among the Palace's Commandants of Cavalry (*jiwei* 騎尉).[43]

Everything was going great until August of 604, when Yang Guang had his father killed to put an end to the continuing succession struggle with his elder brother. Having inherited the empire's institutions, Yang Guang, now Emperor Yang of Sui 隋煬帝 (r. 605–617), no longer needed the two outsiders to help him take them over, and, so, he withdrew his unconditional grace.

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In 605, thinking that the new emperor had his back, Liu Zhuo once again set his sights on ousting Zhang Zhouxuan to push through an astronomical reform of his own. This backfires for the simple reason that Emperor Yang listens to both sides, and, instead, Liu Zhuo is removed from office for demanding the same of his rival. In 606, he is brought back onto a project analysing the musical and metrological instruments captured in the South, all of which – the instruments *and* the research – will later sink to the bottom of a river in General Li Yuan's 李淵 (566–635) coming rebellion. In 608, finally, he is called to court to consult upon a failed eclipse prediction, and, seeing

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[40] 時上將廢皇太子，正窮治東宮官屬，充見上雅信符應，因希旨進曰：「比觀玄象，皇太子當廢。」……, *Suishu*, 69.1610; cf. *Beishi*, 74.2554 ('Yuan Chong zhuan');

[41] 此後百工作役，並加程課，以日長故也, *Suishu*, 19.524–525.

[42] 劉焯以太子新立，復增修其書，名曰皇極曆，駁正冑玄之短。太子頗嘉之，未獲考驗, *Suishu*, 18.459. On the technical aspects of the *Daye li* and *Huangji li*, see Chen Meidong (2003: 318–337).

[43] 王以罪廢，焯又與諸儒修定禮律，除雲騎尉, *Beishi*, 82.2763, *Suishu*, 75.1719 ('Liu Zhuo zhuan'); 及蜀王廢，與諸儒修定五禮，授旅騎尉, *Beishi*, 82.2764, *Suishu*, 75.1720 ('Liu Xuan zhuan').

his chance, he pressures the emperor once again to get rid of Zhang Zhouxuan. This too fails, he is once again removed from office, dying shortly after, in 610, and the court refuses Liu Xuan's request to honour him with a posthumous title.

Meanwhile, according to his biography, Liu Xuan also leaves the capital under peculiar circumstances sometime after 606:

> 歲餘，以品卑去任，還至長平，奉勑追詣行在所。或言其無行，帝遂罷之，歸于河間
> A little over a year later, [Liu Xuan] quit his post and went back to Changping, frustrated with his low rank, and there was an edict ordering that he be followed to find out where he went. [Alternatively,] some say that he did something crooked, that the emperor subsequently fired him, and that he went home to Hejian.[44]

His biography tells us that he arrived to one or the other amid an epidemic of banditry and a hike in grain prices that made it impossible for him to go back to teaching.[45] His biography does not give us an exact date, but one imagines that this was in the context of the economic crisis and rebellions that sprung up around Zhuo Commandery 涿郡 in staging the second and third invasion of the Korean Peninsula in 612–613. Anyways, the conditions force Liu Xuan to move his family several hundred *li* to the commandery seat, and, once there, he meets a fittingly questionable end:

> 時在郡城，糧餉斷絕，其門人多隨盜賊，哀炫窮乏，詣郡城下索炫，郡官乃出炫與之。炫為賊所將，過城下堡。
> At the time, the commandery seat was cut off from provisions, and most of his disciples had joined the bandits. Saddened by [Liu] Xuan's poverty, they showed up below the city wall looking for [him], and the commandery officials thus sent Xuan out [of the gates] to give him over (and, presumably, buy themselves some peace). Xuan was made the bandit leader, and he [turned around and led his merry band] past the city's fortifications.
>
> 未幾，賊為官軍所破，炫飢餓無所依，復投縣城。長吏意炫與賊相知，恐為後變，遂閉門不納。是時夜冰寒，因此凍餒而死，時年六十八。
> Before long, the bandits were crushed by official forces, and Xuan found himself starving with nowhere to go, so he headed back to the district seat. The senior clerk suspected that Xuan and the bandits knew one another, and, afraid of courting trouble, he subsequently closed the gates and refused to take him in. It was freezing cold at night in this season, and thus [Liu Xuan] died of cold and hunger [outside the city gates]. He was sixty-eight years old at the time.
>
> 其後門人謚曰宣德先生。
> His disciples later gave him the posthumous name 'Mister Expounder of Virtue'.[46]

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I have been working on these figures here and there since 2016 in the context of several ongoing projects concerning commentary, mathematical cultures and 'scientific ethics', and while I have considerably more to say about these men's story, I should like to wrap up here in three simple points.

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[44] *Suishu*, 75.1725.

[45] 于時羣盜蜂起，穀食踊貴，經籍道息，教授不行, *Suishu*, 75.1722.

[46] *Beishi*, 82.2767; *Suishu*, 75.1723.

First, Kong Yingda, Jia Gongyan and their teachers are completely unexceptional in the way that their Classical scholarship and political careers are interwoven with mathematics. Such 'broad learning' (*boxue*) was an ideal shared by many of the great 'Confucian scholars' that we study,[47] particularly in the Rites, and, in this case, it plays a crucial role in understanding their story. Unfortunately, we tend not to see this, be it in the history of science or the history of China, as we often focus on the one half of such polymaths that falls into our respective modern specialties. I believe there is something to gain by bridging that divide, and rather than lecture anyone about 'what we should be doing', I simply wanted to offer you an example and solicit feedback on how I might make it more compelling.

Second, inspired by Shapin (1994) and Lloyd and Sivin's (2002: 205) discussion of the importance of gentlemanly conduct in the social construction of truth, I wanted to present you with an example of how, in practice, the disrespect of *li* 禮 might explain the failure of 'good science' to take hold. Of course, 'ritual propriety' is difficult for the modern scholar to judge, which is why, inspired by Béatrice L'Haridon's recent work,[48] it was a serious reading of the horrible things that their biographers, contemporaries and even students have to say about them that brought me to focus on Liu Zhuo and Liu Xuan. These judgements are no more *objective* coming from the seventh century, and they are admittedly contradicted by the picture of the misunderstood genius that Li Chunfeng 李淳風 (602–670) paints of Liu Zhuo in the *Suishu* 'Lüli zhi'.[49] With that said, they are *historical*, and they give us a pretty clear idea of how a number of their contemporaries felt about them: they were arrogant, greedy, deceitful, opportunistic and vengeful; they did not play well with others, not even in their childhood, and when they did make allies it was with all the wrong people. Might not *that* be part of the reason why the Astronomical Bureau wanted nothing to do with their new-fangled algorithms?

Third, as a meditation on the chaos out of which the Tang Empire had so recently emerged by the time that work began on such projects as the *Correct Meaning of the Five Classics*, I wanted to leave you with the following thought: had Kong Yingda been born ten years later, in 584, he very likely would have died a nameless bandit having enrolled in the self-same school.

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[47] See Cullen (2009) and Morgan (forthcoming). On 'broad learning' more broadly, see Goodman (2005; 2010).

[48] Namely, L'Haridon (2019) and a variety of personal communications around Université Paris Diderot in the last two years.

[49] Li Liang (2019). On Li Chunfeng and his authorship of the *Suishu* 'Lüli zhi', see Goodman (2019).

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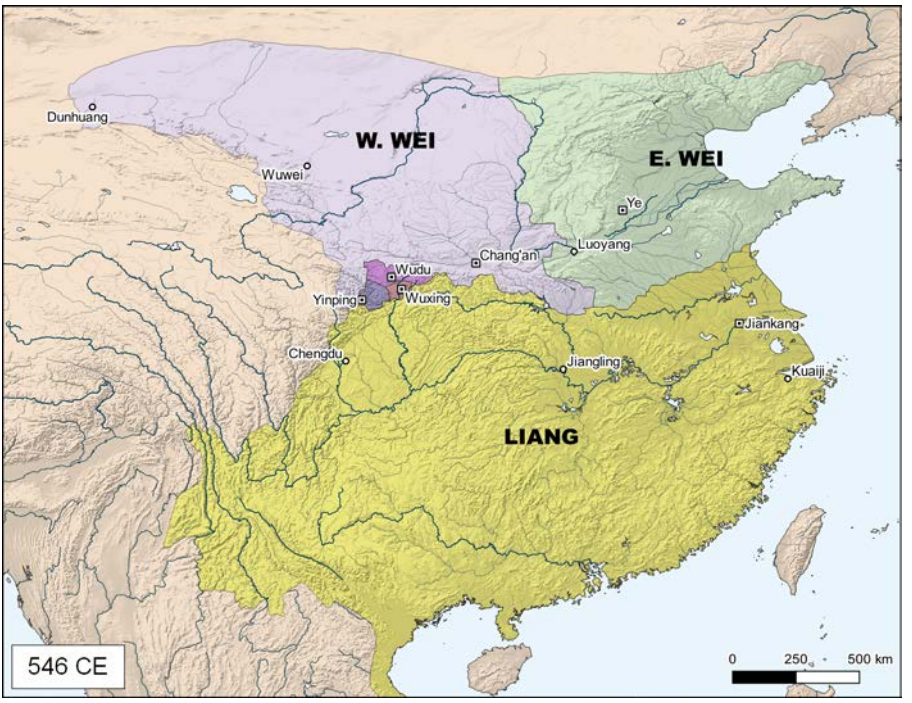
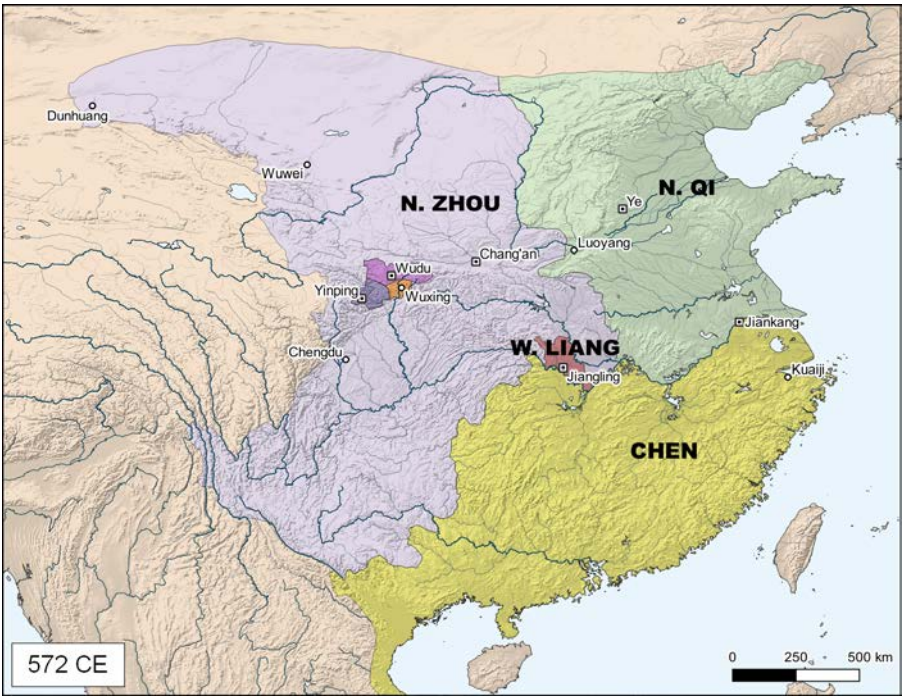
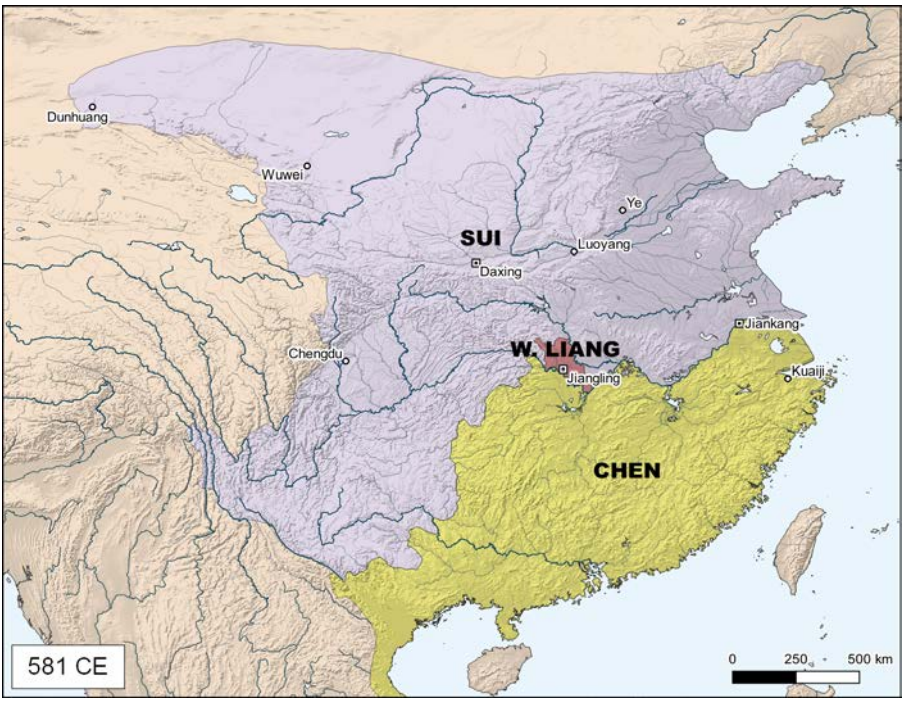
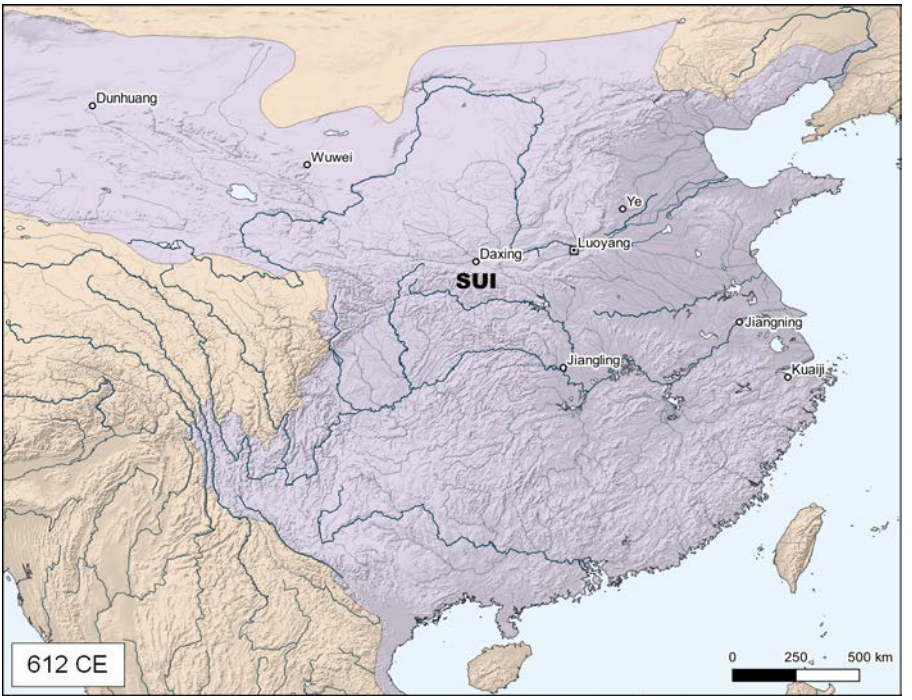




Map 2 Political divisions





Map 2 (continued) Political divisions