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Efficiency of road pricing schemes with endogenous workplace locations in a polycentric city*

Romain Gaté ^{†‡}

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Abstract

This paper aims to measure the efficiency of different road pricing schemes (*Pigouvian tax*, *flat tax* and *cordon toll*) to address congestion externalities when the locations of jobs and dwellings within a city are endogenous. The model captures the fact that commuters face a trade-off between taking advantage of the wage premium in the Central Business District (CBD) and being stuck in traffic. I find that the *Pigouvian tax* strategy is not a social optimum due to the presence of two market failures in the urban economy: congestion and misallocation of jobs within the city. A *Pigouvian tax* on commuters cannot solve two different problems simultaneously, namely, reducing the congestion level given the locations of jobs and reaching the optimal spatial allocation of firms. Without regulation, the number of jobs in the CBD is too high (and the congestion cost is excessive), while the *Pigouvian tax* generates a CBD that is too small. In addition, a *flat tax* is not necessarily worse than a *Pigouvian tax*, in contrast to the *cordon toll*.

Keywords: Polycentric city; Second-best policies; Congestion; Welfare; Urban land use.

JEL Classification: R13; R14; R21; R41; R48

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[†]Université Paris-Dauphine, Université PSL, LEDA, CNRS, IRD, CGEMP, 75016 PARIS, FRANCE

[‡]*Corresponding author:* Romain GATE, Place du Maréchal de Lattre de Tassigny, 75016 Paris Cedex 16 (France). Email: romain.gate@dauphine.psl.eu

1 Introduction

Urban growth leads to serious traffic congestion in cities worldwide. Emerging economies need to set up some efficient urban policies for congestion management. In the United States, Los Angeles was the most congested city in 2016, and commuters lost 104 hours due to their home-to-work travel (Cookson and Pishue, 2017). In Thailand, commuters lost 61 hours on average in traffic due to congestion, regardless of the city. Long travel time delays are also measured in Colombia, Russia and Indonesia according to a recent study by Cookson and Pishue (2017). To improve travel time reliability, road pricing policies, such as the *cordon toll*, have been implemented in cities around the world (Small and Gómez-Ibáñez, 1997). An urban toll aims to regulate traffic demand within some urban area or over a portion of a road during certain periods. Since 1975, Singapore has charged commuters entering the city center a *cordon toll* to improve traffic flow. Indeed, traffic demand dropped by 45% between 1975 and 1991 (Santos, 2005).

Including a congestion externality in an urban model may reduce the benefits of agglomeration economies. Firms have incentives to be close to each other, hence generating increasing returns and agglomeration economies (external and internal). However, gathering firms in either a Central Business District (CBD) or Secondary Business Districts (SBDs) leads to high land rents at these locations and high commuting costs for workers (Fujita and Thisse, 2013). Over the long run, high home-to-work costs may induce adjustments of job and household locations, which requires close scrutiny. The literature has studied the impacts of congestion on urban density and land use using standard monocentric city models since the seminal work of Solow (1972). He is one of the few authors that incorporated both land use for road infrastructure and congestion into an urban economic model to determine the optimal allocation of land for road infrastructure¹. Several studies have used the standard but empirically questionable monocentric model for convenience (Anas and Kim, 1996). Land in the CBD can be exclusively allotted to firms (Wheaton, 1998; Brueckner, 2007; Larson and Yezer, 2015) or to mixed use (Anas and Kim, 1996; Fujita and Ogawa, 1982). Arnott (1979) developed a theoretical model with congestion without internalizing this externality, unlike Solow (1973, 1972). He extended Solow's work on the relationship between private land value and social land value in both residential and road use. The decision to internalize congestion is at issue, as noted by Arnott's works (1979, 2007). As workers respond to a pricing scheme over the long run, complete internalization of the negative externality imposed on other urban dwellers may not be efficient when a second market failure exists (Tikoudis *et al.*, 2015).

Our objective is to understand the evolution of urban congestion and welfare by relocating jobs in SBDs. This paper assesses which urban land use regulations and road pricing schemes improve the welfare of the whole city. Three different taxes on commuters are evaluated: a *Pigouvian tax*, a *flat tax* and a *cordon toll*. The urban model is a polycentric city with two externalities: (i) positive agglomeration economies yielding a wage premium in the CBD compared to the SBDs and (ii) a negative congestion externality due to home-to-work commutes. Work and residential places are interdependent in household location choices. Several authors have examined road pricing schemes effects using standard monocentric models (Wheaton, 1998; Brueckner, 2007; Tikoudis *et al.*, 2015), whereas only a few have investigated the same effects in polycentric cities (Zhang and Kockelman, 2016a). This approach enables us to reflect the tendency of developed cities (e.g Los Angeles, Paris, Boston) to evolve toward decentralized and non-monocentric forms. Therefore, we can

¹Strotz (1965) was the first to study the optimal provision of road facilities using a monocentric model with congestion.

discuss the efficiency effects and other impacts of urban policies (e.g., taxes and redistribution) in relation to previous work on the monocentric case. In addition, the decentralization of jobs within a city may reduce the average commuting distance and thus reduce traffic congestion for each commuter compared to a monocentric city. This study has similarities with works by Zhang and Kockelman (2016a, 2016b). They evaluate different urban policies and measure their impacts on job decentralization, population density and firm distribution. However, they do not focus on the optimal allocation of jobs within a polycentric city in relation to road pricing schemes. We provide analytical solutions regarding the no-toll equilibrium, as well as the optimal and second-best sizes of SBDs. In addition, our results are not based on simulations.

I find that the *Pigouvian tax* on commuters is not optimal. This can be explained by the presence of two market failures in the urban economy: congestion and job misallocation within the city. This tax cannot kill two birds with one stone, namely, reducing the congestion externality for given workplace locations and yielding the optimal spatial allocation of firms. Firms do not take the social costs of congestion into account when they decentralize jobs in the outskirts yielding too large SBDs. In a no-toll equilibrium, the CBD (the residential area where workers live) is larger than optimal. A large proportion of workers is eager to commute to the city center due to the CBD's wage premium compared to the SBDs. The private marginal gain of being closer to the CBD is greater than the marginal cost incurred by those who already live there.

For a given city size, the *Pigouvian tax* on commuters makes the CBD too small in a polycentric city. In other words, the SBD expands and increases the congestion externality of each road user around the workplace. This second-best policy heavily corrects the congestion externality because firms do not take the effects of jobs decentralization on congestion delay into account. This is due to (i) the free location decisions of workers and (ii) endogenous workplaces. A *flat tax* and a *cordon toll* do not achieve an optimal location for the SBD. This article recaptures the effects of road pricing schemes that have been demonstrated in the literature on monocentric city. Tikoudis *et al.* (2015) use numerical simulations in a different context and include a labor tax in their model with road pricing schemes to study the tax interaction effects. Their results clarify that a road toll is necessary and welfare improving when no distortions exist in the labor market. Tikoudis *et al.* (2015) and Verhoef (2005) agree that a *flat kilometer tax* is more efficient than a *cordon toll* in a monocentric structure. However, when a polycentric structure emerges, this result does not hold, as the efficiency of the *cordon toll* or the *flat tax* depends mainly on the wage gap between the business centers before the implementation of the road pricing scheme. Based on the initial location of the SBD (a SBD that is too small or too large), a *flat tax* may be less (resp., more) harmful than a *cordon toll*.

Related literature . Urban forms and traffic volume in cities with resulting congestion externalities are treated separately. Adding these features affects firm and household location choices within a city. Two bodies of literature address both urban and transport issues. First, the standard urban model (i.e., the monocentric city) has been used since Solow (1972) to address road land use and traffic congestion. One or more urban policies were evaluated in the case of one (Arnott, 1979; Wheaton, 1998) or two (Wheaton, 2004; Tikoudis *et al.*, 2015) externalities in a monocentric city. Wheaton (1998) focused on urban form evolution when the congestion externality is correctly internalized, examining the impacts on resident density and transport capacity in a monocentric circular city. Wheaton (2004) and Arnott (2007) were interested in the interplay between congestion and agglomeration externalities, and Tikoudis *et al.* (2015) questioned the

impacts of various second-best road tolls on the labor market and welfare in the long run in order to observe households' decisions to adjust their labor supply and commuting distances.

Second, congestion management has been considered more recently for the polycentric city. Anas and Kim (1996) studied the impacts of congestion on urban structure in a model of a linear, "narrow" city with a link-node road network and households, firms and shopping centers with endogenous locations. In more recent studies, Anas (2013) and Zhang and Kockelman (2016a) analyzed the effects of road pricing schemes on workplace and residential locations as well as on wages, rents, housing prices and land development. Anas (2013) provided some insights for the city of Chicago, whereas Zhang and Kockelman (2016b) considered a general equilibrium model including both agglomeration and congestion externalities with labor market and land use patterns. Extension of the analysis of the impacts of congestion pricing policies on land use, rents and firm locations to a polycentric structure is a recent development in urban economics, as illustrated by Zhang and Kockelman (2016a). However, the existing literature has not fully addressed the impacts of road pricing policies on the size of the CBD when two externalities interplay. This study provides new insights and thus helps enrich knowledge in this stream of literature.

The paper is organized as follows. In section 2, we describe our model's assumptions. Then, we briefly focus on road pricing scheme effects in a monocentric model in which households face congestion costs. In section 4, we present the configuration in which the polycentric city hosts homogeneous households in the city with a wage gap between the business centers in order to find the equilibrium and optimal size of SBDs. In section 5, we evaluate the efficiency of the three road pricing schemes in terms of congestion management and the allocation of jobs. We then perform one simulation with a set of fixed parameter values to confirm or reverse our analytical results and discuss our results by extending the model to include two transport modes in particular. Section 8 concludes.

2 The model

The urban economy designed here builds on the basic model from Denant-Boèmont *et al.* (2018) and Cavailhès *et al.* (2007). Only one closed-form linear city is built with a fixed population L . While Gaigné *et al.* (2012) analyzed an urban system and the carbon footprints of both firms and households in different city structures, this model copes with congestion and agglomeration externalities within a single city, especially a polycentric structure. The monocentric city is used to compare our findings with those reported in the previous literature (Tikoudis *et al.* 2015; Wheaton, 1998; Arnott, 2007). Locations of SBDs are determined endogenously in the polycentric city.

2.1 The city

Consider a city endowed with $L > 0$ homogeneous workers who are free to choose their residential location and workplace. The city is described by m one-dimensional half-lines sharing the same initial point $x = 0$. Firms are located either in the CBD or in an SBD. Firms do not use land so that households and firms do not compete for land. Both the CBD and the SBDs are surrounded by residential areas occupied by workers and transport infrastructure. Distances and locations to the CBD are expressed by the same variable x measured from 0. Individuals travel only for commuting purposes. They use a single one-way road that ends at the location of their respective workplace. No wasteful commuting occurs in equilibrium. Each worker decides to locate as close to her workplace as possible because spatial mismatches would not maximize her utility. The CBD border \hat{z} and the location of the SBD z_S along each spoke are determined endogenously (the city is assumed to be symmetric around the CBD). The supply of housing floor space δ is constant per unit of distance from the CBD and normalized to 1. At each location x , a and $(1 - a)$ are the exogenous fractions of land devoted to residential purposes and road infrastructure, respectively. The job allocation within the city in relation to transport-related congestion stemming from the flow of commuters is the primary consideration. Therefore, the parameter a is exogenously given, leading to a particular land use pattern (Solow and Vickrey, 1971)². Accordingly, the total housing space available is equal to may , with city size y representing the radius limit from the CBD.

2.2 Households

As a matter of fact, only one class of worker is considered in our model. They earn income ω_i with $i = C$ when they work in the CBD or $i = S$ in a SBD. These households have the same preferences and the same utility function, which depends on two consumption goods: land, which is used as a proxy for housing, and the numéraire given by:

$$U(q, h), \tag{1}$$

where q is the consumption of the numéraire, and h is the consumption of housing floor space. Housing demand is assumed to be constant and normalized to 1; hence, $U(q, h) = q$. As a consequence, the residential density is constant per unit of distance and does not replicate the widely demonstrated fact that population density is decreasing with the distance from the CBD

²The optimal allocation of road facilities has already been discussed in the literature (Strotz, 1965; Solow, 1973, 1972). I do not focus on the optimal land use allocation of residential and road capacities in the model, enabling me to disentangle the different effects at work.

(Brueckner, 1987). Thus, the fixed city size y depends only on exogenous components because our main interest in this study is to observe the rise and fall of the relative share of jobs in the CBD. In addition, the traditional trade-off between low/high land rents and long/short journey to work is respected, as in numerous urban economic models with fixed lot sizes (Lucas and Rossi-Hansberg, 2002; Gaigné *et al.*, 2016)³. Each household reaches a common utility level \bar{U} within the city in equilibrium. The time constraint of a worker located at x is given by:

$$1 = T_L + T(x), \quad (2)$$

where T_L is the amount of labor time, and $T(x)$ is the commuting time from her residential location x to the business district's location z_i , with $z_i = 0$ (resp., $z_i = z_S$) if her job is located in the CBD (resp., SBD). Hence, $T(x) > 0$ is the endogenous time spent commuting per unit of distance. It depends on the congestion imposed by other travelers and the transport mode⁴. The budget constraint of a worker located at x can be written as follows:

$$\omega_i(1 - T(x)) + \bar{G} + \bar{R} = q + \frac{R(x)}{a}h(x) + t_0(x) + \tau_k(x), \quad (3)$$

where $k = \text{Pigouvian tax, flat tax or cordon toll}$, t_0 represents the total pecuniary costs of transportation between the workplace and the residence, and $R(x)$ is the land rent at x . $R(x)/a$ is the price paid by a consumer to reside at x . The transport costs contain a fixed component t_0 reflecting fuel, insurance and average maintenance costs for using a car. Thus, the pecuniary costs of using a car differ only with the distances traveled by workers. In the absence of road pricing schemes, no transfer is received by workers (i.e., it is a competitive market). When a road pricing scheme is implemented, tax revenues and aggregated land rents ALR_j ($j = M$ for a monocentric city and $j = S$ for a polycentric one) are returned via a lump sum with \bar{G} and \bar{R} :

$$\bar{G} = \frac{G_k}{L}, \quad (4)$$

and:

$$G_k = ma \int_0^y \tau_k(x) dx,$$

where G_k is the total amount of tax collected. They receive the second amount given by:

$$\bar{R} = \frac{ALR_j}{L},$$

³Including an endogenous housing demand would increase mechanically the city size while the share of land use devoted to housing is fixed. When a polycentric structure emerges, the average land rent decreases as there are new subcenters. Indeed, these subcenters relax the pressure on land rent near the CBD location. For a given housing size, city limit remains fixed. But, when a monocentric city shifts to a polycentric city, housing demand adjusts. Each household live further out her workplace because the average land rent has lowered. To solve the population constraint, the city has to expand. Therefore they live in larger housing size in a larger city. All things being equal, the size of the CBD increases as well. As a consequence, the congestion delay would rise for each commuter within the city but this negative externality slows down the urban sprawl simultaneously. As long as the size of the CBD does not equal y , each worker will face a lower extra travel time against a monocentric structure. This is also true when the city size is fixed. Our main objective is to determine the effects of three road pricing schemes disregarding the long-run changes of the size of the city.

⁴In this model, time allocated to sleep and leisure is not taken into account. A model with fixed working times and endogenous leisure time would not qualitatively change our results. A case study with two transport modes is discussed at the end.

where:

$$ALR_j = m \int_0^y R(x) dx \quad (5)$$

is the aggregate land rent at the whole city. This approach is in line with previous work by Zhang and Kockelman (2016a) and Tikoudis *et al.* (2015). We implicitly assume that a benevolent planner levies a tax on off-farm land rents and redistributes it to residents. Otherwise, households incur a welfare loss due to a change in aggregate land rents when policies are implemented in this closed-city model (Solow, 1973; Parry and Bento, 2001).

2.3 Congestion costs and transport infrastructure

A stationary-state congestion model is implemented in the urban economic model (see Small *et al.*, 2007). All workers take a single road to commute. They face no costs to enter the road, which has no distinctive features such as traffic signals or stops. All homogeneous users are assumed to drive a car with the same characteristics. They face a travel time cost that depends on the number of users on the road at any point and the fraction of land devoted to roads. The road's length corresponds to the city size y . Travel time is increasing with the number of commuters on the road. Households commute to the edge of the CBD and/or of the SBDs according to their residential location. Here, $f(x)$ denotes the cumulative number of travelers using the single road who live beyond the distance x . The travel time per unit of distance at the portion x has the following form⁵:

$$\tau_0 + \tau_1 f(x)^\beta \quad \text{with} \quad \beta = 1 \quad (6)$$

$$f(x) = \begin{cases} \int_x^{\hat{z}} \frac{l}{(1-a)} dz, & \text{if } x < \hat{z} \\ \int_x^y \frac{l}{(1-a)} dz, & \text{if } x > z_S \\ 0, & \text{if } x = y. \end{cases} \quad \text{and} \quad l = \frac{a}{h} \quad (7)$$

The free-flow travel time is equal to τ_0 , and the second term includes the time delay at x induced by the cumulative number of road users living beyond x . This means that the average speed decreases when traffic density increases; namely, there is pure flow congestion. τ_1 is a sensitivity parameter multiplied by the aggregate traffic flow $f(x)$ arriving at a location x along the road. The magnitude of β is widely discussed in the literature (Small, 1992; Arnott *et al.*, 2005) but no consensus has emerged. Arnott (2007) notes that empirical estimates are close to 1.0 when long roads are considered. This travel time function yields a specific traffic congestion at the segment x . The travel demand of drivers living before x is not included in this function. Nevertheless, we must consider the traffic slowdown when they take the road. Furthermore, each commuter living in x imposes a travel delay on other road users living before and beyond x . In turn, these road users cause congestion externalities incurred by each commuter living in x . Thus, we integrate this travel time from the workplace location (the destination) to the trip origin (the residential place) to consider an aggregate congestion externality. We obtain the total commuting time of an

⁵see Small *et al.* (2007), Arnott (2007) and Tikoudis *et al.* (2015)

individual living at distance x , which is expressed as follows:

$$T(x) = \begin{cases} \int_0^x [\tau_0 + \tau_1 \int_x^{\hat{z}} \frac{a}{h(1-a)} dz] dx, & \text{when } 0 < x < \hat{z} \\ \int_{z_S}^x [\tau_0 + \tau_1 \int_x^y \frac{a}{h(1-a)} dz] dx, & \text{when } z_S < x < y \\ - \int_{z_S}^x -[\tau_0 + \tau_1 \int_x^{\hat{z}} \frac{a}{h(1-a)} dz] dx, & \text{when } \hat{z} < x < z_S. \end{cases} \quad (8)$$

It allows us to obtain the travel time per unit of distance near the resident's location. Then, we can quantify the total travel time to her workplace. Incurred congestion is measured at x , which leads to a travel delay along a worker's journey to her business district. As there exists spatial symmetry around the SBD, the second term of the equation can be used to measure congestion along the road section from \hat{z} to z_S . Then, we insert the total commuting time (8) and the time constraint (2) into the budget constraint (3) to obtain the indirect utility of a worker:

$$V_C(x) = \omega_C - \omega_C \left[x(\tau_0 + t_0) + \frac{\tau_1 a}{h(1-a)} \left[\hat{z}x - \frac{x^2}{2} \right] - \frac{R(x)h}{a} + \tau_k(x) + \bar{G} + \bar{R}. \right] \quad (9)$$

when she lives in and commutes to the CBD and:

$$V_S(x) = \omega_S - \omega_S \left[(x - z_S)(\tau_0 + t_0) + \frac{\tau_1 a}{h(1-a)} \left[y(x - z_S) + \frac{z_S^2 - x^2}{2} \right] - \frac{R(x)h}{a} + \tau_k(x) + \bar{G} + \bar{R}. \right] \quad (10)$$

when she lives in and commutes to the SBD. Accordingly, in a competitive market framework, the household that places the highest bid obtains housing at x , which is in line with Alonso (1964) and urban economic models. Households spend their income, net of the opportunity cost of commuting and the monetary costs of transport, on housing and a composite good. This allows us to capture a basic trade-off between land rents and commuting costs.

2.4 Urban toll

We examine three sets of policies in order to tackle the congestion externality caused by travel demand: a *Pigouvian tax*, a *flat kilometer tax* and a *cordón toll*. As congestion delays increase the travel time per unit of distance, this externality may be priced in our model and enter the budget constraint of each household. In the budget constraint, the component $\tau_k(x)$ reflects the level of tax (τ_k) each commuter pays during her home-to-work journey. First, we implement a *Pigouvian tax* ($\tau_k(x) = \tau_P(x)$) that prices the marginal external cost of congestion. A commuter at x imposes travel time delays on other commuters living before and after her location x . It concerns commuters using the same road segment towards a common destination. Accordingly, we price this externality by multiplying the additional travel time per unit of distance by the opportunity cost of working time $\left[\frac{\omega_i}{T_L + T(x)|x - z_i|} \right]$. The labor supply is inelastic within the city, and each worker can place a higher bid to move closer to her workplace with this gross value of time to avoid a travel time delay. The wage rate is divided by the total time endowment normalized to

1 uniformly. Hence, the toll is expressed as follows⁶:

$$\tau_P(x) = \begin{cases} \int_0^{\widehat{z}} \left[\frac{\tau_1 a}{h(1-a)} \int_x^{\widehat{z}} \omega_C \, dz \right] dx, & \text{when } 0 < x < \widehat{z} \\ \int_{\widehat{z}}^x \left[\frac{\tau_1 a}{h(1-a)} \int_x^y \omega_C \, dz \right] dx, & \text{when } \widehat{z} < x < y. \end{cases} \quad (11)$$

This is a location-constrained first-best policy, as workers are free to choose their residential location according to their bid rent. In addition, this tax does not correct its capitalization in aggregate land rents. Second, we analyze another second-best policy that aims to reduce congestion externalities. A *flat kilometer tax* is levied with no differentiation among commuters. This tax does not vary over space or time. It is only proportional to the traveled distance. Each worker commutes to her workplace within the city:

$$\tau_F(x) = t_F x \quad \text{with } t_F > 0 \quad (12)$$

A *cordon toll* is then the other road pricing scheme we consider. Commuters living beyond the location $\alpha\widehat{z}$ ($0 < \alpha < 1$) of the toll are charged, while those living at $x < \alpha\widehat{z}$ face no toll⁷. The charging function is given by:

$$\tau_C(x) = \begin{cases} c & \text{if } \alpha\widehat{z} \leq x \leq \widehat{z} \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

All roads charge tolls to prevent traffic from being reallocated to other roads.

2.5 Wages

In line with the models of Fujita and Ogawa (1982) and Lucas and Rossi-Hansberg (2002), firms produce a composite good. Their only production factor is labor. Each firm makes a profit denoted Π_i whether it produces in the CBD ($i = C$) or in a SBD ($i = S$):

$$\Pi_i = pq_i - \omega_i T_{Li} \quad (14)$$

where p is the output price, q_i is the output size with $q_i = \mathbf{A}_i T_{Li}^\gamma$, $\mathbf{A}_i \geq 1$ and $\gamma \leq 1$, and T_{Li} is the total labor time units⁸. \mathbf{A}_i is a positive agglomeration externality depending on the business centers (CBD and SBDs). This term affects productivity as a positive multiplier. Firms clustered in the CBD take advantage of a more efficient environment that takes concrete form as a productivity drop $\mathbf{A}_C > \mathbf{A}_S = 1$ ⁹. \mathbf{A}_C is increasing with positive agglomeration economies in the

⁶The details of calculations are reported in Appendix B.

⁷ α is purely exogenous in this case and does not represent a location of the cordon, which would maximize the indirect utility as in Verhoef's (2005) numerical simulations. We have left aside the debate about the optimal location of the cordon discussed in the literature (Mun *et al.*, 2005; Verhoef, 2005). Our main interest is analyzing the pecuniary and job relocation effects due to this pricing scheme. However, we indirectly find an optimal location of the cordon ($\alpha\widehat{z}^O$) through the endogenously determined optimal size of the CBD (\widehat{z}^O).

⁸We suppose that the marginal productivity of labor is non-increasing in line with Lucas and Rossi-Hansberg (2002).

⁹Agglomeration economies exist in CBDs, but thanks to new information and communication technologies, decentralization of jobs occurs within the city (Baum-Snow and Pavan, 2012).

CBD. We assume that wages are fixed by firms. The maximization of (14) with respect to labor time T_{Li} implies the following labor demand:

$$p\mathbf{A}_i\gamma T_{Li}^{\gamma-1} = \omega_i \quad (15)$$

so that $\Pi_i = (1 - \gamma)p\mathbf{A}_i T_{Li}^\gamma = (1 - \gamma)[p\mathbf{A}_i]^{\frac{1}{1-\gamma}}[\gamma/\omega_C]^{\frac{\gamma}{1-\gamma}}$, where we have inserted (15) into (14). In equilibrium, $\Pi_C = \Pi_S$ leads to:

$$\omega_C = \mathbf{A}_C^{\frac{1}{\gamma}}\omega_S$$

Hence, $\mu = \omega_S/\omega_C = \mathbf{A}_C^{-1/\gamma}$. As a consequence, the wage gap between business centers depends on the magnitude of agglomeration economies \mathbf{A}_C and diseconomies of scale γ .

2.6 Welfare

Finally, we can derive the Welfare that represents the aggregate indirect utility within the closed city¹⁰. The sum of the indirect utilities of tenant workers, the land rent incomes of absentee landlords are an integral part of this welfare. Under a no-toll equilibrium, welfare is expressed as follows:

$$\begin{aligned} W^* &= \frac{ma}{h} \left[\int_0^y \omega_i (1 - T(x)) dx - \int_0^y t_0 x dx \right] - ALR_j \\ &+ \left[ALR_j - m \int_0^y R_A dx \right] + m \int_0^y R_A dx \end{aligned}$$

with:

$$ALR_j = m \int_0^y R(x) dx$$

Land rent incomes are the aggregate land rents paid by tenant workers minus agricultural land rents R_A redistributed to landlords living outside the city. Finally, agricultural landowners benefit from agricultural land rent incomes. The purpose of this article is to analyze the impacts of the road pricing schemes on welfare and on the city structure. Furthermore, one of the main goals is to determine the conditions under which aggregate welfare is maximized within the closed city.

¹⁰The first part of the aggregate welfare is the difference between aggregate incomes and aggregate congestion, commuting and housing costs in a city (Wheaton, 2004). The second part is the distribution of rents to absentee landlords.

3 The monocentric city

A standard monocentric spatial configuration is used to determine how our policy interventions (i) are capitalized in land rents and (ii) affect welfare. The welfare of the closed city is evaluated under an unpriced congestion framework and three different tax implementations (*Pigouvian tax*, *flat tax* and *cordon toll*). Finally, we compare the policies' impacts.

One single business district (the CBD) gathers all of the city's jobs. Each worker lives in the residential areas within the city. The city border y solves the total population constraint given by:

$$L = m \int_0^y l dx \quad (16)$$

in which l is the population density at each residential location x with $l = a/h$. Solving (16) by using $l = a/h$ yields the city radius size in equilibrium:

$$y_M = \frac{Lh}{ma} \quad (17)$$

when all jobs are located in the CBD (see Appendix A). We observe that the city border decreases with respect to both the number m of built-up areas and the proportion a of residential areas at each unit of distance. The city fringe increases with the total population. Note also that a decrease in housing floor space ($\delta < 1$) per unit of land leads to urban sprawl. In this closed city, the travel time function $\tau(x) = \tau_0 + \tau_1 f_M(x)$ depends on the cumulative flow of users at x given by:

$$f_M(x) = \begin{cases} \int_x^{y_M} \frac{a}{h(1-a)} dz \\ 0 & \text{if } x = y_M \end{cases}$$

The total commuting time of an individual living at distance x from the CBD is expressed as follows:

$$T_M(x) = \int_0^x \left[\tau_0 + \tau_1 \int_x^{y_M} \frac{a}{h(1-a)} dz \right] dx$$

leading to:

$$T_M(x) = x\tau_0 + \tau_1 \frac{a}{h(1-a)} \left[y_M x - \frac{x^2}{2} \right] \geq x\tau_0 \quad (18)$$

The free-flow travel time $x\tau_0$ and the congestion delay $\tau_1 \frac{a}{h(1-a)} \left[y_M x - \frac{x^2}{2} \right]$ are monotonic curves ($x \leq y_M$) and can be represented as follows:

Note that our congestion delay cost function is increasing at a decreasing rate. The marginal travel time cost is higher towards the CBD than outwards from the CBD. Accordingly, the opportunity cost of the commuting time is higher near the city center than in the outskirts. An individual living near the CBD will have a lower commuting time due to an infinitesimal move ($x - dx$) than that of an individual living near the city border. Living near the border implies

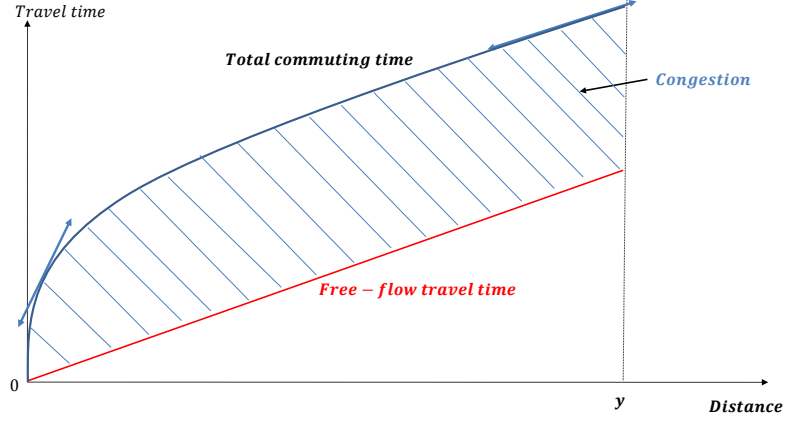


Figure 1: Evolution of the components of the total commuting time function with respect to the distance to the CBD in a monocentric city configuration.

high transportation costs. Congestion delay costs increase with x , but land rents are cheaper in the outskirts. The effect of a on the congestion externality is unclear. Indeed, when a increases, the city limits shrink, which implies a decrease in traveled distances. However, the road's capacity $(1 - a)$ falls, leading to an ambiguous result for a given population density.

Homogeneous workers choose their living places with respect to their budget constraints. Housing rents and commuting costs vary with the distance from the CBD. They maximize their indirect utility. R_A is the agricultural value of land at the city limits. In equilibrium, the bid rent at each location is given by $R(x) = \max \{\Psi_M(x), R_A\}$. The bid rent $\Psi(x)$ must solve $\partial V(x)/\partial x = 0$. All households share the same indirect utility. In equilibrium, the distribution of urban dwellers is such that $V(x) = V(y)$ regardless of x . Hence, there are no incentives to relocate in a no-toll spatial equilibrium. Then, we plug (18) into the indirect utility using (17); hence, we obtain the individual welfare in equilibrium:

$$V_M = \omega_C - \frac{Lh}{ma} \left[\omega_C \tau_0 + t_0 + \frac{\omega_C \tau_1 L}{2m(1-a)} \right] - \frac{R_A h}{a} \quad (19)$$

Using (18) and indirect utility (19), we obtain the bid rent function in equilibrium¹¹:

$$\Psi_M(x) = \frac{a [2h(1-a)(\omega_C \tau_0 + t_0)(y_M - x) + \omega_C \tau_1 a (y_M - x)^2]}{2h^2(1-a)} + R_A \quad (20)$$

At each unit of distance, the land rent capitalizes the opportunity cost of commuting time, the monetary transportation costs and the external costs imposed on all commuters. The land rent decreases with respect to the distance x from the CBD. The bid rent equals the agricultural land rent R_A when an individual lives at the city border ($x = y_M$). When a increases marginally, the effects are unclear, as demonstrated for the congestion externality. On the one hand, land rents are

¹¹The rent gradient (i.e., the partial derivative of (20) with respect to x) is negative, which is in line with other monocentric models (Tikoudis *et al.*, 2015; Verhoef, 2005). In equilibrium, there are no incentives to relocate within the city.

lower due to the greater housing floor space per unit of distance, but on the other hand, the road's capacity decreases, leading mechanically to more congestion. Hence, longer congestion delays raise land rents due to the lack of road capacity.

The indirect utility includes income net of pecuniary transport costs (free-flow travel) and land rents ($\omega_C - y_M(\omega_C\tau_0 + t_0) - \frac{R_A h}{a}$). This latter term is called *private welfare*, as it represents the consumption of private goods. The second one refers to the congestion externality related to the number of commuters on the road and the road's width $\left[\frac{\omega_C \tau_1 a y_M^2}{2h(1-a)} \right]$. Standard calculations reveal that the indirect utility is increasing at a decreasing rate with the number of roads m connected to the CBD, as well as with the total capacity of the transportation network $m(1-a)$. Note that a smaller city (i.e., y_M diminishes) reduces the total commuting cost as long as the population density does not adjust. It is clear that a marginal increase in commuting costs and land rents diminishes utility regardless of whether the population density or the number of roads m is given. A growing population has a negative effect on welfare, as it raises the number of commuters and, therefore, the opportunity cost of commuting time through a rise in congestion delay costs. Urban sprawl (i.e., the marginal expansion of y_M) diminishes individual welfare as well, since the maximum traveled distance to the CBD increases.

Welfare and road pricing schemes In a monocentric city, different urban policies do not affect the total congestion level as we consider individuals that have inelastic demand for transport. They do not affect welfare because taxes are redistributed, but they affect each economic agent as a transfer of income¹². The no-toll equilibrium is used as a reference point to discuss our results. Aggregate welfare W_M is calculated under each configuration: (i) the no-toll equilibrium with congestion, (ii) the *Pigouvian tax* and (iii) the outcome with a *flat tax* and a *cordon toll*¹³. A tax increases welfare within the city, as the aggregate land rent and toll revenues are returned as a lump sum and remain in the urban economy. The *Pigouvian tax*, a *flat kilometer tax* and a *cordon toll* yield a same welfare level as a no-toll equilibrium. For given city limits and housing size, the three road pricing schemes differ only in their revenue distribution. In the absence of a lump-sum redistribution of land rents, tenant workers are worse off than in an unpriced congestion context. Indeed, land rents capitalize both congestion externalities and tax levels, leading bids rent to rise at all locations as long as the city limits do not change. Landlords would benefit more from a pricing scheme that yields the highest land value. The second-best policies yield higher bids rent than those in the no-toll equilibrium at all locations as long as housing size and city limits remain fixed (Appendix C). With the *flat tax*, workers who live at the outskirts pay higher prices due to a longer travel distance than those living close to the CBD. Accordingly, workers living close to the CBD benefit more from the revenue return scheme than do those living near the city limits. Thus, they can bid more to live closer to the CBD, explaining why second-best bids are higher than in equilibrium. Regarding the *cordon toll* implementation, households living in the cordon zone face lower prices given the congestion externality they impose on other road users, while those living outside the central zone face higher prices. A *cordon toll* leads bids rent to increase even in presence of a revenue return scheme. To avoid the toll, households may locate closer to their

¹²The calculations are reported in Appendix C.

¹³A *cordon toll* is set at location αy_M within the monocentric city before entering the CBD. Only workers living beyond αy_M incur this fixed charge to commute to their workplace.

workplaces when housing demand adjusts. In conclusion, we can say that congestion externality pricing does not reduce travel time due to the inelastic demand for commutes. However, the road pricing schemes differ according to the distribution of the tax burden and the overall beneficiaries. In the next section, we will analyze how the location of jobs affects congestion and demand for commuting via a wage differential between CBD and SBDs within the polycentric city.

4 Decentralization of jobs and welfare

Traffic congestion and road pricing schemes are now analyzed under a polycentric setting. Jobs are able to relocate in an SBD within the city and wages are fixed. Our main objective is to understand the evolution of urban congestion and welfare by relocating jobs and introducing different taxes.

4.1 The polycentric city

A worker deciding to locate close to the CBD earns a gross wage rate ω_C above the wage ω_S offered by firms in the SBDs. Each business district is surrounded by road infrastructure and residential areas. No worker has an incentive to move from her workplace or residence in the spatial equilibrium. All households live at each location x such that $dV(x)/dx = 0$. In this spatial organization, \hat{z} is the right limit of the area formed by workers commuting to the CBD (i.e., the left limit of the built-up area constituted by individuals working in the SBD). The location of SBDs is determined endogenously through the location of \hat{z} . Individuals locate around each SBD symmetrically. Hence, the endogenous location of the SBD z_S is the midpoint of the area between \hat{z} and the city limit y . Therefore:

$$z_S = \frac{y + \hat{z}}{2} \quad (21)$$

Notice that the bids rent at y and \hat{z} are equal to the opportunity costs of land R_A . The equilibrium border when the city is polycentric reaches¹⁴:

$$y = \frac{Lh}{am} \quad (22)$$

The city limits are identical to the monocentric structure when jobs relocate to the SBDs ($y = y_M$). As housing demand remains fixed, the structural density is the same, and y is unchanged. When jobs relocate to an SBD, the size of the CBD decreases from y_M to \hat{z} ; hence, workers incur lower transport costs (in terms of both time and money).

Congestion delay and traffic flow. The total commuting time of an individual living at x between the CBD and the limit \hat{z} is derived in the same way as in the previous urban structure. Here, the CBD limit substitutes y_M for \hat{z} . Regarding traffic flows toward the SBDs, the commuting time between the SBD z_S and the border y is expressed as follows:

$$T(x) = \int_{z_S}^x [\tau_0 + \tau_1 \int_x^y \frac{a}{h(1-a)} dz] dx \quad \text{when } x > z_S$$

leading to:

$$T(x) = (x - z_S)\tau_0 + \frac{\tau_1 a}{h(1-a)} [y(x - z_S) + \frac{z_S^2 - x^2}{2}] \geq (x - z_S)\tau_0 \quad (23)$$

Notice that whether z_S rises, the total commuting time to this location decreases because the CBD limit expands leading to shrinkage of the SBD area. As there is symmetry around an SBD, a

¹⁴The details of these calculations are reported in Appendix D1.

worker living at location x between \hat{z} and z_S or between z_S and y incurs the same travel distance and congestion delay (see equation 23). Therefore, the subsequent analytical properties focus on workers living at the right side of the SBD. Commuters use one single mode of transport to travel, and they obtain the same level of utility within the city. A worker living in the area between the CBD and \hat{z} has no economic incentives to work in a SBD. No wasteful commuting occurs in equilibrium. Each traffic flow towards the SBD starts from \hat{z} and y . This means that congestion costs at these points may be lower than in the monocentric configuration. Indeed, a commuter incurs a maximum time delay $\frac{\tau_1 a y_M^2}{2h(1-a)}$ at y_M when one single business center exists. Each worker enters a road where the maximum time delay falls to $\frac{\tau_1 a \hat{z}^2}{2h(1-a)}$ ($\hat{z} < y_M$). Trivial calculations show that the congestion externality decreases for each commuter (see Appendix E). The maximum traveled distance diminishes, and the traffic flow is split between the business districts. Therefore, for a given density, each worker is better off when firms relocate. *Ceteris paribus*, congestion costs will rise for inhabitants working in the CBD regardless of whether \hat{z} or y increases. The following proposition summarizes.

Proposition 1. *When a share of jobs relocates to a SBD at the edge of a city, the congestion externality level decreases for each commuter.*

SBD Size. As there are two business districts, a wage gap exists between them in accordance with empirical evidence (see White, 1999; Timothy and Wheaton, 2001). Hence, we define $\omega_S = \mu\omega_C$ ¹⁵ with $0 < \mu < 1$ corresponding to the spread between the two business districts. Furthermore, a worker commuting to the SBD may incur a lower transport cost than one commuting to the CBD. This gives the worker an incentive to work in that subcenter of the city. The size of the CBD \hat{z} is determined by an indifference condition. For a worker living at \hat{z} , the traveled distance to the workplace is equivalent: she is indifferent between commuting to the CBD and commuting to the SBD, yielding the following expression:

$$\omega_C \left[1 - (\hat{z}\tau_0 + \frac{\tau_1 a \hat{z}^2}{2h(1-a)}) \right] - t_0 \hat{z} = \omega_S \left[1 - (z_S - \hat{z})\tau_0 - \frac{\tau_1 a (z_S - \hat{z})^2}{2h(1-a)} \right] - t_0 (z_S - \hat{z}) \quad (24)$$

The size of the CBD \hat{z} is smaller than the city limits y in this model. Individuals accept work from firms in SBDs when the gross wage ω_S offered is strictly above the CBD's wage net of total transport costs (i.e., the opportunity cost of commuting, congestion delays and pecuniary costs) in a monocentric city. Employers benefit from this relocation of jobs within a city because they pay lower wages. Workers face shorter commutes and pay lower land rents. Whatever the gross wage ω_C offered by firms in the CBD, the decentralization of jobs holds for a certain level of ω_S . As a consequence, the free market equilibrium yields a non-linear expression of the CBD border $\hat{z}(\mu)$. For greater clarity, τ_0 is assumed to be equal to zero¹⁶. Hence:

$$\hat{z}^*(\mu) = \frac{-3t_0(1-a) - \frac{\mu\omega_C\tau_1 L}{2m} + 2(1-a)\sqrt{\Delta}}{\omega_C(2 - \mu/2)\tau_1 a} \quad (25)$$

with:

$$\Delta \equiv \left[\frac{3t_0}{2} + \frac{\mu\omega_C\tau_1 L}{4m(1-a)} \right]^2 + \frac{\omega_C(4-\mu)\tau_1 L}{2m(1-a)} \left[\frac{\omega_C(1-\mu)ma}{L} + \frac{t_0}{2} + \frac{\mu\omega_C\tau_1 L}{8m(1-a)} \right] > 0$$

¹⁵ $\mu = A_C^{-\frac{1}{\gamma}} = \frac{\omega_S}{\omega_C}$

¹⁶Calculations for $\hat{z}(\mu)$ without this assumption are reported in Appendix D2.

in which (21) and (22) have been inserted. \hat{z} is implicitly defined by the indifference condition (24). The influence area of the SBD (the number of individuals working in the SBD) rises with μ and reaches a maximum when $\mu = 1$ ($\frac{y-\hat{z}^*(\mu)}{2} = \frac{2y}{3}$). The attractiveness of a peripheral business district depends on the average gross wage observed within this SBD. Each worker considers her wage net of total transport and residential costs. For a given city size, a polycentric structure with a prominent business center enables shorter home-to-work distances and reduces pressure on land rents at and close to the CBD. Therefore, transport and rent costs decrease for each commuter. Urban traffic is divided into three flows, and no cross-border commuting between the business districts occurs in equilibrium. As a consequence, all commuters in the city face fewer congestion delays than they would in a monocentric one, but they do not benefit from the most efficient travel time.

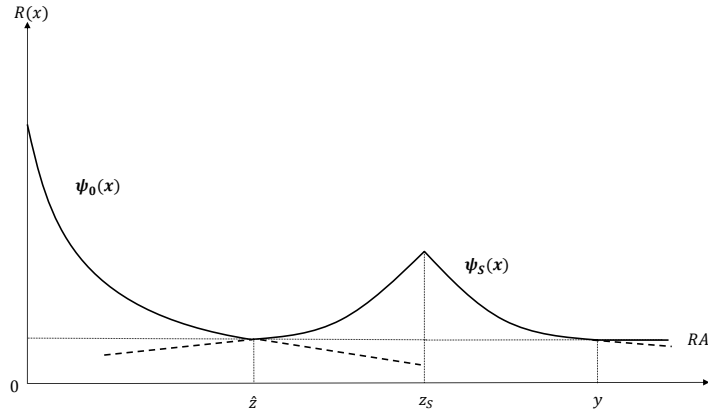


Figure 2: Bids rent in equilibrium near each business district and the locations of the CBD ($x = 0$) and the SBD ($x = z_S$) within a polycentric city configuration.

The equilibrium land rent at each location is given by: $R(x) = \max\{\Psi_0(x), \Psi_P(x), R_A\}$. $\Psi_0(x)$ (resp., $\Psi_P(x)$) is the bid rent of individuals working in the CBD (resp., SBD). $\partial V(x)/\partial x = 0$ implies:

$$\Psi_0(x) = \frac{a [2h(1-a)(\omega_C \tau_0 + t_0)(\hat{z}^*(\mu) - x) + a\omega_C \tau_1(\hat{z}^*(\mu) - x)^2]}{2h^2(1-a)} + R_A$$

and:

$$\Psi_P(x) = \frac{a \left[2h(1-a)(\omega_C \tau_0 + t_0) \left(\frac{y-\hat{z}^*(\mu)}{2} - |z_S - x| \right) + a\omega_C \tau_1 \left(\frac{y-\hat{z}^*(\mu)}{2} - |z_S - x| \right)^2 \right]}{2h^2(1-a)} + R_A$$

Bids rent decrease with the distance to the business districts (see Figure 2). Furthermore, the bids rent remain identical for all $x \in [z_S, y]$ when an SBD is created. Consequently, the average land rent decreases, but the city limits are unchanged when jobs relocate to the edge of the city.

4.2 Equilibrium allocation and optimal location of jobs

The no-toll equilibrium is used as a reference to compare optimal and equilibrium locations of the SBD. In equilibrium, the welfare (i.e., the aggregate indirect utilities) of the polycentric city is

expressed as follows:

$$W_S(\widehat{z}^*) = \frac{ma}{h} \left[\omega_C \widehat{z}^* (1 - \mu) + \mu \omega_C y - \frac{(\mu \omega_C \tau_0 + t_0)(y^2 - \widehat{z}^{*2})}{2} - \frac{(\omega_C \tau_0 + t_0) \widehat{z}^{*2}}{2} - \frac{\omega_C \tau_1 a [4\widehat{z}^{*3} + \mu(y - \widehat{z}^*)^3]}{12h(1-a)} \right]$$

Hence, inserting equation (25) gives us the welfare in equilibrium (Appendix D3). Notice that when $\widehat{z}^*(\underline{\mu}) = y$, the urban economy recaptures the monocentric setting, as $y = y_M$ and $W_S(\underline{\mu}) = W_M$. Because the size of the SBD depends on the wage gap between the business districts, it is crucial to analyze the effects of jobs decentralization on welfare according to the level of (μ) . Indeed, we have:

$$W_S(\widehat{z}^*) - W_M = \frac{ma}{h} \left[\int_0^{\widehat{z}^*} V_C \, dx + 2 \int_{z_S}^y V_S \, dx - \int_0^{y_M} V_M \, dx \right] + ALR_P - ALR_M$$

Job relocation to a SBD has an ambiguous effect on welfare. The effect depends strongly on the magnitude of the wage differential between business districts. Note that in equilibrium without taxation, the welfare of inhabitants working in the CBD improves when the urban economy shifts from a monocentric to a polycentric configuration. The relationship is more complex for inhabitants living near the SBD. On the one hand, they face both shorter travel distances and less congestion. Thus, their opportunity costs of commuting decrease because their income ω_S is smaller than ω_C . On the other hand, they pay the same rent R_A , and the difference in individual utility depends strongly on the wage gap. Indeed, the wage gap is positive for polycentric city's inhabitants. However, the economic shift has no incentive if the wage discrepancy offsets the opportunity costs of commuting and congestion delays. To understand how the wage gap and commuting time costs affect aggregate welfare, the analysis considers the case where the wage rates observed in the CBD and the SBDs are equal. Then, we consider a more realistic case where workers earn a higher hourly wage in the CBD than in the SBDs. For each case, a free market equilibrium is solved, and aggregate welfare is maximized by a benevolent planner who sets the optimal size of the SBD.

Specific case with no wage gap between the CBD and the SBDs. First, a simple way to disentangle the different effects at work is to equalize the wage rates of the business centers: $\omega_C = \omega_S(\mu = 1)$. Workers living near the CBD (resp., the SBD) face a maximum distance equal to \widehat{z} (resp., $\frac{y-\widehat{z}}{2}$), and traffic is divided into three flows. Hence, there is lower congestion on the roads when jobs relocate to an SBD within a closed city. In equilibrium, inserting $\mu = 1$ into (25) leads to $\widehat{z}^* = \frac{y}{3}$, so the free market size of the residential zone near the SBD is equal to $(y - \widehat{z}^*) = \frac{2y}{3}$. This result is identical to the optimal size of the left endpoint of the SBD (\widehat{z}^{**}) when welfare is maximized by a benevolent planner¹⁷. As a result, we have:

$$\widehat{z}^* = \widehat{z}^O = \frac{y}{3} \tag{26}$$

The free market solution is identical to the optimal location of the CBD limit when the congestion externality is not internalized, provided that no wage gap exists between the business districts. Furthermore, it is straightforward to check that welfare in a polycentric city ($W_S(\widehat{z}^O)$) with an

¹⁷ $\frac{\partial W_S(\widehat{z})}{\partial \widehat{z}} = 0$ leads to a single maximum for $W_S(\widehat{z})$ when $\widehat{z} \in [0, y]$, which reaches $\widehat{z}^O = \frac{y}{3}$. The details of these calculations are reported in Appendix D4.

equal wage rate is higher than that in a monocentric setting (W_M) (Appendix D4). It follows that each parameter that marginally increases the population density (m, a) yields a higher differential in favor of the polycentric city, all things being equal. The free market equilibrium and optimal solution of \hat{z} yield similar welfare, which is better than that for the monocentric city. Note that welfare in a polycentric city is decreasing when $\frac{y}{3} < \hat{z} \leq y$ ¹⁸. In other words, welfare is increasing as the size of the CBD decreases in favor of the SBD until \hat{z} reaches $\frac{y}{3}$.

Proposition 2. *Assume that there is no wage gap ($A_C = 1$) and that welfare in equilibrium is maximized when $\hat{z}^* = \frac{y}{3}$ when a share of firms relocates to an SBD. In this case, the free market size is equal to the optimal size.*

Proof. *See Appendix D4.*

As a consequence, jobs relocation within a closed city makes workers better off than they would be in a monocentric city. It is straightforward to check that $\Delta_W > 0$:

$$\Delta_W \equiv W_S(\hat{z}) - W_M = \frac{m\omega_C\tau_1^2 a^2 (y - \hat{z})(y + \hat{z})^2}{4h^2(1 - a)} > 0 \quad (27)$$

For all $\hat{z} < y$, the polycentric city is welfare improving, since the maximum distance for home-to-work commutes is shorter than y_M ¹⁹. In addition, workers incur a lower level of congestion for each commuting trip when the city size y is unchanged. Thus, the efficient commuting pattern (the minimum home-to-work distance) is achieved when $\omega_C = \omega_S$ in comparison with the monocentric pattern²⁰.

General case with positive agglomeration externalities. We now return to the case where $\omega_S = \mu\omega_C$ with ($0 < \mu < 1$). The expression of the free market CBD limit \hat{z}^* is highly non-linear and implicitly defined by the indifference condition for a worker living at an equidistant point between the two business centers. \hat{z}^* is expressed as follows:

$$\omega_C \left[1 - \hat{z}\tau_0 - \frac{\tau_1 a \hat{z}^2}{2h(1 - a)} \right] - t_0 \hat{z} = \omega_S \left[1 - (y - z_S)\tau_0 - \frac{\tau_1 a (y - z_S)^2}{2h(1 - a)} \right] - t_0 (y - z_S) \quad (28)$$

The size of the residential zone from which workers commute to the SBD ($y - \hat{z}^*$) depends mainly on the wage gap between the business centers. A wage rate growth in the CBD leads to its expansion at the expense of the size of the SBDs. Standard calculations show that the CBD's size is smaller with free-flow travel time τ_0 , under the three congestion pricing schemes and when the

¹⁸We have $W_S(\hat{z} = 0) > W_S(\hat{z} = y)$ and $\frac{\partial W_S}{\partial \hat{z}}|_{\hat{z}=0} > 0 > \frac{\partial W_S}{\partial \hat{z}}|_{\hat{z}=y}$. In addition, $\frac{\partial W_S(\hat{z})}{\partial \hat{z}}$ has a single extremum when $\hat{z} \in [0, y]$, which is a maximum.

¹⁹Note that when no CBD exists (i.e., $\hat{z} = 0$), the SBD is located in the middle of the total residential area (i.e., $\frac{y}{2}$), as our model is symmetric around the CBD located at $x = 0$. This case is a duocentric city where welfare is higher than in a monocentric city, even when the location of the left endpoint of the SBD does not yield an equilibrium or optimal solution. A particular polycentric pattern leads to the same level of welfare when $\hat{z} = y \frac{\sqrt{5}-1}{2}$ over $[0, y]$. However, traffic is split into two flows, which insufficiently decreases the congestion delay for each commuter. Indeed, only one solution assures that each commuter can minimize her extra travel time (see Proposition 2).

²⁰Note that the city is divided into three equal parts when $\omega_C = \omega_S$ with $\hat{z} = \frac{y}{3}$, and the SBD's size ($y - \hat{z}$) equals $\frac{2y}{3}$.

border y decreases. Workers favor commuting to the SBD when the free-flow travel speed $1/\tau_0$ diminishes. Similarly, the CBD's size decreases when the slope (τ_1) of the congestion curve increases (see Appendix D6 for the details of the calculations). When the available floor space ($\delta > 1$) per land unit grows, the CBD's size \hat{z}^* diminishes, and y decreases with the fraction of residential area a . Conversely, city size grows when the lot size h increases. Note that a marginal increase in a leads to a more compact city but lowers the road's capacity, yielding higher congestion delay costs for a given population density.

The determination of an optimal solution by a benevolent planner considers the maximization of aggregate welfare so that $\frac{\partial W_S(\hat{z})}{\partial \hat{z}} = 0$ yields a non-linear \hat{z}^O (see Appendix D7). In this case, the optimal solution and the free market equilibrium are not similar. When $\mu = 1$, we revert to the previous case where the optimal and free market sizes are equal ($\hat{z}^O = \hat{z}^*$). Then, we determine whether the free market size of the area where individuals live near the CBD is too small or too large. Welfare in the polycentric city can be written as follows²¹:

$$W_S = \frac{ma}{h} \left[\int_0^{\hat{z}} V_C \, dx + 2 \int_{z_S}^y V_S \, dx \right] + ALR_P \quad (29)$$

At the optimum, $\frac{\partial W_S}{\partial \hat{z}} \Big|_{\hat{z}=\hat{z}^O} = 0$ because welfare has reached its maximum. Furthermore, welfare W_S is a concave curve on the interval $[0, y]$ since $\frac{\partial^2 W_S}{\partial \hat{z}^2} < 0$ (see Figure 3 and Appendix D6).

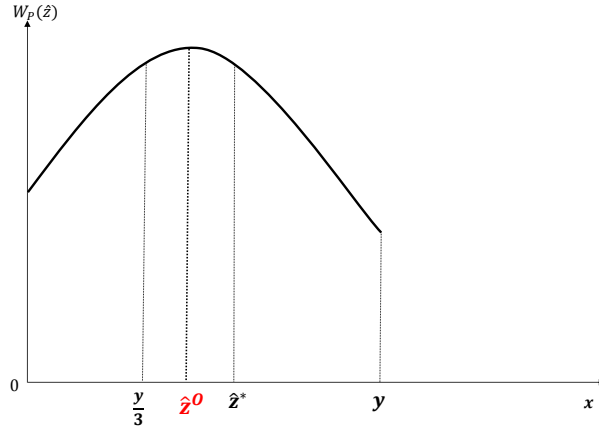


Figure 3: The evolution of welfare with respect to \hat{z} within a polycentric city configuration (from the CBD $x = 0$ to the city limits y).

Hence, to find the location of the CBD limit in equilibrium relative to the optimum, we derive welfare with respect to \hat{z} conditional on $\hat{z} = \hat{z}^*$:

$$\frac{\partial W_S}{\partial \hat{z}} \Big|_{\hat{z}=\hat{z}^*} = \left[\frac{ma}{h} \left((y - \hat{z}^*) \frac{\partial V_S}{\partial \hat{z}} + \hat{z}^* \frac{\partial V_C}{\partial \hat{z}} \right) + \frac{\partial ALR_P}{\partial \hat{z}} \Big|_{\hat{z}=\hat{z}^*} \right] < 0 \quad (30)$$

As a result, the free market size of the CBD is greater than optimal. In other words, the free market SBD area is smaller than the optimum under positive agglomeration and congestion externalities²².

²¹When we derive W_S with respect to \hat{z} , there are no price effect at the aggregate level since all land rents paid by tenant workers are received by absentee landlords.

²²Note that we have $\frac{\partial W_S}{\partial \hat{z}} \Big|_{\hat{z}=\hat{z}^*} = 0$ when there are no market failures in our urban model. The size of the CBD in equilibrium is identical to the optimum.

The intuition is as follows. When the decentralization of jobs occurs, land rents and congestion delays decrease, on average, as a share of workers relocates closer to the new SBD. As a consequence, workers who earned wage rate ω_C have an incentive to remain in the residential zone close to the CBD because their net wage has increased. Then, for a marginal increase of \widehat{z} , the total commuting time for a worker living near the limit of the residential zone is similar when she moves from x to $x + dx$ ²³. Hence, with a higher wage rate offered in the CBD, most inhabitants commute to the CBD instead of the SBD without taking into account the marginal social costs they impose on other commuters. Accordingly, the size of residential zone near the CBD is larger than the optimum. Regarding the optimal size, it is straightforward to check that $\frac{y}{3} < \widehat{z}^O < \widehat{z}^*$ as:

$$\left. \frac{\partial W_S}{\partial \widehat{z}} \right|_{\widehat{z}=\frac{y}{3}} > 0 \quad (31)$$

Briefly, the optimal size of the CBD is larger than $\widehat{z} = \frac{y}{3}$ and smaller than \widehat{z}^* .

Proposition 3. *A free market equilibrium yields a **CBD that is too large** when there is a market failure and a positive agglomeration externality.*

Proof See Appendix D6.

In the next section, we focus on the spatial and economic impacts of internalizing the external costs of congestion according to three different road pricing schemes. We evaluate their efficiency by comparing the tax-induced size of the CBD and the optimal CBD size.

5 Polycentric city and road pricing schemes

The free market equilibrium under a wage gap yields an inefficient outcome for a polycentric city. Here, our objective is to achieve the optimally sized SBD or to bring \widehat{z} closer to that optimal size. Three road pricing schemes are evaluated separately. Each worker pays a tax that affects her income, internalizing the congestion externality imposed on other road users. The tax $\tau_k(x)$ is inserted into the budget constraint of each household, as well as into the revenue return scheme and aggregate land rent redistributions, leading to the following individual welfare:

$$V_S(x) = \omega_i(1 - T(x)) - \tau_k(x) + \bar{G}_S + \frac{ALR_P}{L} - \frac{R(x)h}{a} - t_0x \quad \text{with } i \in [C, S] \quad (32)$$

Land rent is given by $R(x) = \max\{\Psi_0(x), \Psi_P(x), R_A\}$. $\Psi_0(x)$ (resp., $\Psi_P(x)$) is the bid rent of individuals working in the CBD (resp., SBD). $\partial V(x)/\partial x = 0$ implies that:

$$\Psi_0(x) = \frac{a [2h(1-a) [(\omega_C\tau_0 + t_0)(\widehat{z} - x) + \tau_k(\widehat{z}) - \tau_k(x)] + a\omega_C\tau_1(\widehat{z} - x)^2]}{2h^2(1-a)} + R_A,$$

and:

$$\Psi_P(x) = \frac{a \left[2h(1-a) \left[(\omega_S\tau_0 + t_0) \left(\frac{y-\widehat{z}}{2} - |z_S - x| \right) + \tau_k(y) - \tau_k(|z_S - x|) \right] + a\omega_S\tau_1 \left(\frac{y-\widehat{z}}{2} - |z_S - x| \right)^2 \right]}{2h^2(1-a)} + R_A$$

²³In our previous calculations, note that \widehat{z}^* is larger than $(y - \widehat{z}^*)$, and for a longer distance traveled between the CBD and \widehat{z}^* , workers experience a marginal negative effect on their indirect utility, which has a larger magnitude than the positive effect experienced by those working in the small SBD. Hence, $\widehat{z}^* \frac{\partial V_C}{\partial \widehat{z}} > (y - \widehat{z}^*) \frac{\partial W_S}{\partial \widehat{z}}$.

Note that each pricing scheme affects the endogenous CBD size \hat{z} , which is recalculated to fit the model. Welfare in the polycentric city with taxes can now be defined as follows:

$$W_S = \frac{ma}{h} \left[\int_0^{\hat{z}} V_C \, dx + 2 \int_{z_S}^y V_S \, dx \right] + G_k \\ + \left[ALR_P - m \int_0^y R_A dx \right] + m \int_0^y R_A dx$$

Each commuter pays a charge for using her car but benefits from the redistribution of tax revenues and aggregate land rents, since road pricing schemes are capitalized in land rents. Absentee and agricultural landlords continue to receive their revenues. The impacts of these second-best pricing schemes are examined and the results presented.

The Pigouvian tax First, a *Pigouvian tax* is implemented as a location-constrained first-best policy²⁴. Indeed, a benevolent planner decides to fix the location of the SBDs in optimum while we implement this tax when \hat{z} is determined endogenously. This tax must adjust to correctly internalize the behavior of workers who are free to choose their residential locations. A priori, these free decisions would prevent this road pricing scheme from achieving the optimum. Each road user is priced at the exact marginal social cost she imposes on others for a given location and a given workplace. The toll level is expressed as follows:

$$\tau_P(x) = \begin{cases} \int_0^x \frac{\omega_C \tau_1 a}{h(1-a)} (\hat{z} - x), & \text{when } 0 < x \leq \hat{z} \\ \int_{z_S}^x \frac{\omega_S \tau_1 a}{h(1-a)} (y - x), & \text{when } z_S < x \leq y \end{cases} \quad (33)$$

Hence, the tax level for a commuter living in x and commuting to the CBD is $\frac{\omega_C \tau_1 a}{h(1-a)} [\hat{z}x - x^2/2]$. Between \hat{z} and z_S , urban dwellers pay the same tax as those living between z_S and y because jobs located in z_S are in the middle of the residential area. Hence, using the tax level in the indifference condition for the location of the CBD limit \hat{z} yields²⁵:

$$\omega_C \left[1 - \hat{z}\tau_0 - \frac{\tau_1 a \hat{z}^2}{h(1-a)} \right] - t_0 \hat{z} = \omega_S \left[1 - \left(\frac{y - \hat{z}}{2} \right) \tau_0 - \frac{\tau_1 a \left(\frac{y - \hat{z}}{2} \right)^2}{h(1-a)} \right] - t_0 \left(\frac{y - \hat{z}}{2} \right) \quad (34)$$

Then, we have the following welfare level:

$$W_P(\tau_P) = \frac{ma}{h} \left[\int_0^{\hat{z}} \omega_C [1 - T(x)] - t_0(x) dx + 2 \int_{z_S}^y \omega_S [1 - T(x)] - t_0(x - z_S) dx \right]$$

To find the location of the CBD limit \hat{z}_P , we derive (26) with respect to \hat{z} knowing that $\hat{z} = \hat{z}_P$, leading to:

$$\frac{\partial W_S}{\partial \hat{z}} \Big|_{\hat{z}=\hat{z}_P} = \frac{ma}{h} \left[(\omega_S \tau_0 + t_0) \left(\frac{3\hat{z} - y}{2} \right) \right] > 0, \quad (35)$$

²⁴A first-best policy would fix the residential location of each household and the location of the SBD simultaneously.

²⁵See Appendix F1 for details of calculations.

as $\hat{z} > \frac{y}{3}$. Hence, the *Pigouvian tax* fails to clear the market due to the free location decisions of workers²⁶. They earn higher wages in the CBD, yielding an equilibrium CBD size that is too large to correctly manage traffic congestion. Including an extra marginal external cost reduces their net wage. They relocate near the SBD, overweighting the marginal loss of remaining near the CBD. As a consequence, the limit of the residential area from which inhabitants commute to the CBD is lower than the optimal location \hat{z}^O . The CBD limit under this kind of tax is lower than both the equilibrium and the optimal location. The hierarchy is as follows:

$$\hat{z}_P < \hat{z}^O < \hat{z}^* \quad (36)$$

As a result, a location-constrained first-best policy excessively internalizes the congestion externality by encouraging SBD overextension within the polycentric city.

Proposition 4. *A Pigouvian tax fails to reach the optimum, yielding a **CBD that is too small** when there is a market failure and a positive agglomeration externality.*

Proof See Appendix F.

The flat tax The following toll is imposed simultaneously on each driver and depends only on the traveled distance:

$$\tau_F(x) = t_F x \quad (37)$$

The CBD limit \hat{z} is now implicitly defined by the following expression including the *flat tax* incurred by each individual:

$$\omega_C \left[1 - \hat{z}\tau_0 - \frac{\tau_1 a \hat{z}^2}{2h(1-a)} \right] - (t_0 + t_F)\hat{z} = \omega_S \left[1 - \left(\frac{y - \hat{z}}{2}\right)\tau_0 - \frac{\tau_1 a \left(\frac{y - \hat{z}}{2}\right)^2}{2h(1-a)} \right] - (t_0 + t_F)\left(\frac{y - \hat{z}}{2}\right)$$

All commuters face the same tax; hence, workers living near the left endpoint of the SBD may be underpriced and those living near the SBD or the CBD may be overpriced. In addition, workers benefit from the revenue return scheme and receive the same lump-sum payment from the collected taxes. At the aggregate level, we have the following welfare:

$$W_F(\tau_F) = \frac{ma}{h} \left[\int_0^{\hat{z}} \omega_C [1 - T(x)] - t_0(x) dx + 2 \int_{z_S}^y \omega_S [1 - T(x)] - t_0(x - z_S) dx \right]$$

Deriving W_S with respect to \hat{z} conditional on $\hat{z} = \hat{z}_F$ to find the location of \hat{z}_F yields:

$$\frac{\partial W_S}{\partial \hat{z}} \Big|_{\hat{z}=\hat{z}_F} \leq 0 \quad (38)$$

Thus, the location of the CBD limit when each commuter pays a *flat tax* could be greater or lower than the optimal location of \hat{z}^O . The location depends mainly on the wage gap between the business centers before the implementation of the *flat tax*. As a consequence, the size of the SBD matters, but the optimum will never be achieved unless the wage rates are equal in both business centers.

²⁶There is no reason why \hat{z} should be lower than or equal to $\frac{y}{3}$. When this *Pigouvian tax* is implemented, there is only one solution yielding $\hat{z} = \frac{y}{3}$, namely, when $A_C = 1$.

The cordon toll A *cordon toll* is set at location $\alpha\widehat{z}$ within the CBD area where some inhabitants work. Each commuter living between $\alpha\widehat{z}$ and \widehat{z} pays a fixed fee to pass the cordon. Workers living inside the cordon are not charged; therefore, they do not pay for the congestion externality they impose on other commuters. The fixed fee to enter the cordon is captured in the rents of those living near the CBD. Hence, there exists a discontinuity in the rent bid at the tollgate location. Moreover, inhabitants living in the SBD area do not pay road taxes and benefit from both tax and aggregate land rent redistribution. The CBD limit is implicitly defined by the following expression:

$$\omega_C \left[1 - \widehat{z}\tau_0 - \frac{\tau_1 a \widehat{z}^2}{2h(1-a)} \right] - t_0 \widehat{z} - c = \omega_S \left[1 - \left(\frac{y - \widehat{z}}{2} \right) \tau_0 - \frac{\tau_1 a \left(\frac{y - \widehat{z}}{2} \right)^2}{2h(1-a)} \right] - t_0 \left(\frac{y - \widehat{z}}{2} \right)$$

A priori, in this case, the implementation of a *cordon toll* induces a decrease in the CBD size due to the fee incurred by a few workers living between $\alpha\widehat{z}_{CT}$ and \widehat{z}_{CT} . Commuters living near the cordon will relocate closer to the SBD until it becomes more expensive than living close to the CBD. The welfare function $W_{CT}(\tau_{CT})$ is expressed as follows²⁷:

$$W_{CT}(\tau_{CT}) = \frac{ma}{h} \left[\int_0^{\widehat{z}} \omega_C [1 - T(x)] - t_0(x) dx + 2 \int_{z_S}^y \omega_S [1 - T(x)] - t_0(x - z_S) dx \right]$$

Deriving W_S with respect to \widehat{z} conditional on $\widehat{z} = \widehat{z}_{CT}$ to find the location of \widehat{z}_{CT} yields:

$$\left. \frac{\partial W_S}{\partial \widehat{z}} \right|_{\widehat{z}=\widehat{z}_{CT}} = \frac{ma}{h} \left[c + (\omega_S \tau_0 + t_0) \left(\frac{3\widehat{z} - y}{2} \right) + \frac{\omega_S \tau_1 a (y - \widehat{z})^2}{8h(1-a)} \right] \leq 0 \quad (39)$$

As a result, the *cordon toll* does not achieve the optimal location of the SBD. The location of the CBD limit may be lower or greater than the optimal location according to the initial wage gap between the subcenters and the transport costs incurred by individuals (see Appendix F3). Since the location of the cordon is exogenous, the fixed fee (c) may be too small or too high as well. Indeed, a fee that overprices commuters yield a small CBD and a fee that underprices them leads to a too large CBD. It is straightforward that an increase in the available land for housing lowers the road capacity leading to a small CBD. Conversely, an increase in the lot size spreads out the CBD for a given road capacity.

Proposition 5. *The flat tax and the cordon toll do not perform better than the Pigouvian tax. They yield a CBD that is either too low or too large depending on initial wage gap between the business districts and the amount of the respective pricing scheme.*

Proof See Appendix F.

Finally, we need to compare the results of the policies with the help of numerical simulations because it is difficult to find convenient analytical solutions.

²⁷The details of the calculations are reported in Appendix F3.

6 Comparisons between road pricing schemes

This section simulates a closed city with a fixed limit $y = 2$, where the total population is fixed at $L = 2$ and the agricultural land rent is $R_A = \frac{1}{4}$ for a given wage gap $\frac{1}{\mu} = 2$. This wage gap is chosen since the polycentric city exists under that condition for all case studies. Table 1 shows the different parameter values used for the scenario²⁸. Parameter calibration is in accordance with the transport costs hierarchy used in other numerical simulations in the literature (Tikoudis *et al.*, 2015; Zhang and Kockelman, 2016a) (*i.e.*, $\tau_0 > \tau_1$).

Table 1: Calibrated parameters

L	R_A	m	h	δ	a	τ_0	τ_1	t_0	ω_S	$\frac{1}{\mu}$
2	$\frac{1}{4}$	2	1	1	0.5	$\frac{1}{5}$	$\frac{1}{6}$	0	2	2

The following parts present the results of simulations conducted for the polycentric city. The numerical settings here guarantee that all individuals benefit from the same amount of toll and rent revenues and share the same indirect utility \bar{V} at all locations under each road pricing scheme.

CBD limit Analytically, this model has demonstrated that a small increase in the wage gap between the CBD and the SBD expands the CBD's area, as measured by \hat{z} , in equilibrium. However, the simulations show that the CBD limit under each road pricing scheme is always lower than in equilibrium without taxation. Therefore, they never reach the \hat{z}^O where welfare reaches an extremum. Land use patterns are affected by second-best policies when each business center offers different wage rates. The endogenous left endpoint of the SBD has changed under a *Pigouvian tax*(PT), a *flat tax*(F) and a *cordon toll*(C). For any $\frac{1}{\mu} > 1$, the polycentric city becomes monocentric, reaching the level $\frac{1}{\mu}$ with second-best pricing policies, except for the *Pigouvian tax*. In this case, the polycentric city always exists even if the CBD is the most attractive workplace. When $\frac{1}{\mu}$ approaches infinity, the CBD's size reaches a limit lower than y in the simulations. With a *flat tax* and a *cordon toll*, a polycentric city prevails provided that $\frac{1}{\mu}$ is lower than approximately 4.31 and 4.03, respectively.

Table 2: Simulated results of CBD size within a polycentric city according to each road pricing scheme. ($1/\mu = 2$).

	No toll	Optimum	τ_P	τ_F	τ_{CT}
CBD's size, \hat{z}	1.577	1.467	1.305	1.489	1.506

In other terms, a subcenter no longer has economic interests, as the gross wage offered ω_S is almost 5 times smaller than ω_C under a *flat tax*. Internalizing the congestion externality at any given location mitigates the desirability of traveling to a unique business center. In a way, these

²⁸The calibration of all parameters is consistent with the analytical model such that indirect utilities and aggregate welfare are strictly positive.

findings are in accordance with the non-monocentric model of Zhang and Kockelman (2016b), although our model does not consider endogenous city size. Congestion costs yield higher land rents near the CBD, as each worker wants to reduce her commuting costs (Solow, 1972; Wheaton, 1998). When jobs relocate endogenously, land rents diminish near the CBD. However, marginal congestion pricing for each commuter strengthens their willingness to pay the higher land rents associated with a wage gap between the two business centers. Finally, the optimal size of the residential area for individuals commuting to the CBD is never reached by these road pricing schemes. For $\frac{1}{\mu} = 2$, a *Pigouvian tax* leads to a CBD limit lower than the optimum, and vice versa for the *cordon toll* and *flat tax*. Note that we consider each worker priced at her exact marginal cost under the *Pigouvian tax*. The *flat tax* and the fixed fee to pass through the cordon are similar ($\tau_F = c = 0.148$). This amount of tax represents the marginal cost that pays a worker living at $\frac{y}{3}$. The CBD limit under *Pigouvian taxation* is always lower than the optimal location, as demonstrated analytically.

6.1 Efficiency of the three road pricing schemes in the polycentric city

This part examines the welfare and land use effects of second-best policies, comparing them to the no-toll equilibrium within a polycentric city. First, the implementation of three different instruments is investigated when there is a wage gap between the CBD and the SBDs. Table 2 illustrates relevant characteristics of the no-toll equilibrium, *Pigouvian tax*, *flat tax* and *cordon toll* with equivalent tax rates, except for the location-constrained first-best policy. A welfare

Table 3: Simulated results of policy instruments within a polycentric city. ($1/\mu = 2$).

	No toll	τ_P	τ_F	τ_{CT}
City limit, y	2.00	2.00	2.00	2.00
Total travel time costs at \hat{z}	2.091	1.611	1.930	1.960
Total travel congestion costs at \hat{z}	0.00	0.567	0.148	0.148
Welfare, $W_P(\hat{z})$	5.267	5.38	5.33	5.32
Percentage change				
against no-toll equilibrium				
City limits (%)		0	0	0
Total travel time costs (%)		-22.9	-7.70	-6.26
Welfare (%)		2.00	1.20	1.00

improvement at an aggregate level is significant under the second-best instruments mainly due to the redistribution of both tax and land rent revenues. Under a *Pigouvian tax*, the welfare level increases from 5.267 to 5.38, that is, 2% higher than the welfare of the no-toll equilibrium. Indeed, the toll and the aggregate land rents are returned as lump sums to each worker who does not own their housing. When the congestion externality is internalized at each given location, the total travel costs (in terms of time and money) falls from 2.091 to 1.611 for a worker living in \hat{z} and working in the CBD. Total welfare covers all inhabitants, landowners, agricultural landlords and makes them better off with second-best instruments compared to the no-toll equilibrium. Indeed,

regarding a *flat tax* and a *cordon toll*, welfare is higher by 1.20% and 1.00%, respectively, for a worker. A *Pigouvian tax* and the optimum lead to an increase in the welfare level of 2%. The CBD area where workers live is too small under the *Pigouvian tax*. That is why households living in \hat{z} benefit from the shortest maximum traveled distance between the CBD and their residence.

Travel demand is unaffected, as each worker must commute every day. The rent bids capitalize the tax effects, increasing strongly near each business center. For a given city limit, aggregate welfare is maximized when the CBD limit $\hat{z} = \hat{z}^O$, and the numerical simulations demonstrate that this size is not achieved with a location-constrained first-best policy where congestion is priced and taxes and land rents are returned as lump sums to workers with a wage gap between the business centers (2). When $\frac{1}{\mu} = 2$, aggregate welfare is compared across road pricing schemes, which yields:

$$\Delta W_{P\tau} = \begin{cases} W_P(\tau_P) > W_P(\tau_F) \\ W_P(\tau_F) > W_P(\tau_{CT}) \\ W_P(\tau_P) > W_P(\tau_{CT}) \end{cases} \quad (40)$$

With these different instruments internalizing the congestion externality, welfare in the polycentric city is always above that in the no-toll equilibrium. Workers are better off under each pricing scheme when they live in a polycentric city until $\frac{1}{\mu}$ reaches $\frac{1}{\underline{\mu}}$ according to the simulations. Hence, an increasing pecuniary cost of transport through each road pricing scheme leads to a decrease in the CBD size compared to the equilibrium location, which is in accordance with the results of Zhang and Kockelman (2016b) for a given wage gap. The derived analytical solutions are non-linear; hence, the numerical simulations help us to confirm that \hat{z}^* is always superior to \widehat{z}_{toll} when $1 < \frac{1}{\mu} < \frac{1}{\underline{\mu}}$ ²⁹. Note that for two business centers offering the same wage rate, the CBD size $\hat{z} = y/3$ is similar under the no-toll equilibrium, the optimum, *Pigouvian taxation*, and *flat tax* schemes. These results are in accordance with the literature, as any taxation leads to an increase in the city's compactness (Zhang and Kockelman, 2016a). Road pricing schemes force workers to relocate closer to the CBD in accordance with the findings of Anas (2013). A *Pigouvian tax* decreases CBD size more than the other pricing schemes. The optimal location of \hat{z} is higher than that of \hat{z}_P and lower than \hat{z}_F and \hat{C} according to our simulations.

²⁹The following values have been selected: $L = 2, \omega_S = 2, \omega_C = \frac{1}{\mu}\omega_S, m = 2, \delta = 1, a = 0.5, \tau_0 = \frac{1}{5}, \tau_1 = \frac{1}{6}, h = 1, t_0 = 0,$.

7 Discussion

This section addresses the implementation of a mass transit service close to the CBD and considers the possible impacts on congestion and land rents. The two spatial configurations are compared according to their welfare, congestion levels and wage rates over the long run. Finally, the implementation of a parking policy instead of a pricing congestion policy is discussed.

7.1 Incidence of modal choice on congestion and urban structure

Over 60 million passenger-kms were covered by mass transit in the Ile-de-France region of France in 2014. This transit activity has grown by more than 30% since 2000 (CGDD, 2016). Therefore, to remain realistic, assume that another mode of transit is introduced in the city, public transport (bus). This mode uses a transportation infrastructure assumed to be provided by a public planner (municipal government) at no cost. The market area of this mass transit mode is between the CBD ($x = 0$) and x_B . As there exists mixed land use for transport modes, workers prefer using mass transit over their car when they live close to the CBD (Limtanakool *et al.*, 2006). Furthermore, car ownership levels increase with the distance from the city center (Dasgupta *et al.*, 1985). Assume that each worker in this area lives close to a bus stop, so they face no costs of access. Hence, between the CBD and the outer border of the public transport area x_B , the road's capacity $(1 - a)$ is equally divided between car and bus lanes on the single road. Beyond x_B to y , each worker uses exclusively her car. A mass transit user faces only a commuting time cost that is higher than the free-flow travel time of car users. This cost depends on the waiting times at bus stops and the travel time to the destination³⁰. In other words, $T_B(x) > T_A(x)$ without a congestion externality, but passengers pay a fare t_B , which is lower than average capital cost t_0 of car ownership. To sum up, transport pecuniary costs are given as follows:

$$t_A(x) = t_0x \quad \text{and} \quad t_B(x) = t_B \quad (41)$$

A share of the population uses now public transport³¹, and each car user incurs congestion linked to the number of commuter on the road between x_B and \hat{z} , as well as reduced road capacity between the CBD's edge and x_B . Each car user incurs a total commuting time expressed as follows:

$$T_A(x) = \begin{cases} \int_0^x [\tau_0 + 3\tau_1 \int_{x_B}^x \int_x^{\hat{z}} \frac{l}{(1-a)} dz] dx, & \text{when } 0 < x_B < x < \hat{z} \\ \int_{z_S}^x [\tau_0 + \tau_1 \int_x^y \frac{l}{(1-a)} dz] dx, & \text{when } z_S < x < y \end{cases} \quad (42)$$

Note that the congestion parameter increases due to the decrease in road capacity near the business center of each worker. The total congestion delay now depends on the size of the residential area where car users live (i.e., $\hat{z} - x_B$).

³⁰Each user faces an exogenous commuting time cost. Waiting times at bus stops depend on the bus fleet size and on the frequency at which the buses run (Small, 2004).

³¹Creutzig (2014) sets up a model including public transport close to the CBD and imposes this mode of travel on the residents living in its market area. We simplify by abstracting from providing public transit and public transit infrastructure at a cost.

The polycentric city: In a polycentric city in equilibrium, the individual welfare of a worker is similar at all locations. An individual living between 0 and x_B has the following indirect utility:

$$V(x)_B = \omega_C(1 - T_B(x)) - \frac{R(x)h}{a} - t_B \quad (43)$$

Between x_B and \hat{z} :

$$V(x)_A = \omega_C(1 - T_A(x)) - \frac{R_A h}{a} - t_0 x \quad (44)$$

The market size for mass transit is specified by the following indifference condition linked to the time and pecuniary costs of each mode:

$$T_B(x_B) + t_B = T(x_B) + t_0 x_B \quad (45)$$

Initially, the residential area where workers live and from which they commute to the CBD is larger than that of the SBD. The introduction of a bus lane on the existing road infrastructure provides a disincentive for residents living close to the CBD to use their car. Workers living near the border \hat{z} will see a rise in their transportation costs (both pecuniary and time costs). Therefore, the equilibrium size of the CBD will decrease until the reduction in car users offsets the higher congestion delay in the zone shared with the bus lane. The size of the SBD will increase; hence, congestion near this business district will increase. From an economic point of view, the introduction of two transport modes may reduce the size of the CBD to achieve the optimal size, which has the same effect as a road pricing scheme³². In the case of a road pricing scheme targeting drivers, ($t_A(x) = t_0 x + \tau_k(x)$), the effect would be similar to our results in section 6, namely, a decrease in the equilibrium size of the CBD and an increase in the number of workers commuting to the SBD. A second transport mode would reinforce the effects of the different taxes as second-best policies. The intuition is as follows: when a second mode of transport is used without improving the road infrastructure for motorists, they face (i) lower transport costs (user effect) and (ii) lower road capacity (time cost effect). When the *user effect* is marginally predominant, the CBD grows, attracting workers from the SBD until there is no incentive to relocate. When the *time cost effect* dominates, CBD size decreases.

Parking policy Urban tolls are designed to restrain travel demand and alleviate congestion within cities. While economists have largely studied the impact of congestion pricing schemes on land use and traffic patterns, urban tolls have been scarcely implemented within cities around the world (Brueckner, 2011). In the model, workers are assured that they can find a parking place at their employment site. Therefore, there is no cruising and no additional congestion. Arnott (2011) points out that parking policy in cities fails to take economic analysis into account. Indeed, Shoup (2005) highlights the huge number of hours lost to cruising for parking in his empirical study. Car parking is a serious issue, as a large urban space is used, each worker incurs extra travel time, and fees do not differentiate between peak and off-peak hours (Albert and Mahalel, 2006). When a fixed cost to park is added to the budget constraint of a worker, her transport costs increase. However, travel demand is inelastic at x and remains unaffected by parking fees in the model. Therefore, implementing different road pricing schemes would not change the nature of our previous results for a given city size and road's capacity.

³²However, from an environmental point of view, the size of the road network used by drivers does not necessarily decrease. This is why CO_2 emissions may remain equivalent for cities of a given size.

8 Conclusion

This chapter develops and explores economic and land use tools in order to tackle the congestion externality in monocentric and polycentric settings. For a given city size, the optimal response is consistent with the development of SBDs, clustering firms that can offer the same wage rate as in the CBD. With unpriced congestion, this urban structure minimizes the negative externality incurred by each road user and maximizes welfare within the city. When the economy in the polycentric city is consistent with empirical findings, the equilibrium size of the CBD is larger than optimum one. The CBD is more attractive due to a higher wage rate and a higher wage net of transport costs in comparison with the monocentric structure for a given city size. Three main results are noteworthy.

First, the implementation of a *Pigouvian tax* on commuters is only a location-constrained first-best policy because the optimal location of the SBDs is not achieved. This tax yields a residential area that is too small and close to a CBD. Welfare is definitely greatest compared with the other road pricing schemes regarding the revenue return schemes. However, at the aggregate level, this tax does not internalize the decentralization of jobs in SBDs, preventing it from achieving the optimum. This *Pigouvian tax* remind us to internalize the congestion externality that road users impose to each other not only at their residential location but also during their daily commute to their workplace. We need also to tax firms that do not internalize effects on congestion when jobs are decentralized.

Second, a *flat tax* and a *cordon toll* are second-best policies that do not yield an optimal location of the SBDs. On the one hand, a *flat tax* is homogeneous across road users; hence, they are not distinguished according to the marginal damage they impose on other commuters. Indeed, workers living near a business center (CBD or SBDs) are underpriced in relation to the congestion delay they impose on other road users and the proportionality of the tax with respect to the traveled distances. On the other hand, a *cordon toll* clearly differentiates among commuters, similar to the *Pigouvian tax*, but the tax burden is asymmetric. The residential area where individuals live and from which they commute to the CBD is larger than that near the SBDs; hence, the *cordon toll* is settled in the former area. Only workers living between the *cordon toll* location and the CBD limit pay a fixed fee to commute to the CBD. Simultaneously, individuals living in other residential areas near the SBDs do not incur higher transport costs because the marginal congestion delays they impose on other users are much lower than those imposed by individuals living close to the CBD. To sum up, the efficiency of these road pricing schemes depends mainly on the wage gap between business centers within the polycentric city before the implementation of this road pricing scheme³³. According to the initial location of the SBDs (too small or too large), a *flat tax* may be less (resp., more) harmful than a *cordon toll*.

Finally, we demonstrate that a polycentric city in an unpriced congestion context yields a suboptimal location of the CBD limit, which is too large compared to the optimal size. According to the magnitude of the transport costs, the CBD may be too small or too large. The location is suboptimal since the indifferent worker living at the CBD limit (the left endpoint of the SBD area) decides to commute to the CBD without taking into account the marginal benefits obtained by individuals living near a SBD. She does not take into account the marginal damage costs incurred by road users commuting to the CBD as well. Conversely, this indifferent worker does not pay

³³Accordingly, the travel time and pecuniary costs incurred by each worker depend on the wage gap, which determines the size of the influence area of each business center.

attention to the marginal damage costs incurred by workers living near a SBD when she decides to drive towards it. In addition, marginal benefits obtained by individuals working in the CBD are not taken into account as well. Her marginal move due to a change in the wage gap will not induce large differences in her transport costs (time and money) compared to her previous location under the suboptimal equilibrium.

Developing subcenters may alleviate the congestion cost for each road user provided that no distortions exist for the labor or housing markets when congestion is unpriced. However, we know that a transport improvement yields direct benefits and indirect costs, as noted by Arnott (1979) in the same context. As long as growth of traffic flows and urban sprawl costs do not substantially decrease the direct benefits from jobs relocations, urban planning remains a key policy for addressing urban transport issues compared to pricing schemes. Indeed, road pricing schemes benefit all landlords regardless of the closed-city structure. Hence, pricing congestion implies political concerns regarding the redistribution of tax revenues when negative externality is internalized. In addition, the desirability of multiple business centers may decrease due to an increase in the average traveled distance; hence, the congestion externality may worsen when housing size adjusts in the long run.

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Appendix

A. The equilibrium border with a monocentric city.

The population constraint is given by:

$$m \int_0^y \frac{a}{h} dx = L$$

so that:

$$y_M = \frac{Lh}{ma}$$

B. The total commuting time of an individual living at distance x from the CBD

Assume that each worker uses a car for home-to-work commutes. The travel time per unit of distance depends on the free-flow travel time and the congestion delay due to commuters living beyond x along the road:

$$\tau(x) = \tau_0 + \tau_1 \left(\int_x^y \frac{a}{h(1-a)} dz \right)^\beta$$

We integrate this travel time over the interval $[0, y]$ to determine the total commuting time of the worker located at the city limits. Hence, we obtain:

$$T_M(y) = \int_0^y \left[\tau_0 + \tau_1 \int_z^y \frac{a}{h(1-a)} dz \right] dx$$

yielding:

$$T_M(y) = y\tau_0 + \tau_1 \frac{a}{h(1-a)} (y^2/2).$$

C. The monocentric city: First- and second-best policies versus no-toll equilibrium

The aggregate welfare under a no-toll equilibrium is expressed as follows:

$$W_M^* = \frac{ma}{h} \left[\int_0^{y_M} \omega_C(1-T(x))dx - \int_0^{y_M} t_0 x dx \right] - ALR_M + \left[ALR_M - m \int_0^{y_M} R_A dx \right] + m \int_0^{y_M} R_A dx$$

with:

$$ALR_M = m \int_0^{y_M} \Psi_M(x) dx.$$

When a road pricing scheme is implemented, the aggregate welfare becomes:

$$W_M^{**} = \frac{ma}{h} \left[\int_0^{y_M} \omega_C(1-T_M(x))dx - \int_0^{y_M} t_0 x dx - \int_0^{y_M} \tau_k(x) dx + G_k \right] - ALR_M + \left[ALR_M - m \int_0^{y_M} R_A dx \right] + m \int_0^{y_M} R_A dx$$

with:

$$G_k = \int_0^{y_M} \tau_k(x) dx$$

Pigouvian tax Each road user is charged a toll at the exact marginal social cost she imposes on others for a given residential location and a given workplace:

$$\tau_P(x) = \int_0^x \frac{\omega_C \tau_1 a}{h(1-a)} (y_M - x) dx$$

Then, the total revenue from the *Pigouvian tax* is written as follows:

$$\begin{aligned} G_P &= \int_0^{y_M} \tau_P(x) dx \\ &= \frac{m\omega_C \tau_1 a^2}{h^2(1-a)} \int_0^{y_M} \left[y_M x - \frac{x^2}{2} \right] dx \\ &= \frac{m\omega_C \tau_1 a^2 y_M^3}{3h^2(1-a)} \end{aligned}$$

The indirect utility V_M includes the *Pigouvian tax* as well as the lump-sum transfer ($\frac{G_k}{L}$) leading to:

$$V_P(y_M) = \omega_C - y_M \left[\omega_C \tau_0 + t_0 - \frac{\omega_C \tau_1 a y_M}{2h(1-a)} \right] - \frac{R_A h}{a} - \tau_k(y_M) + \frac{G_P}{L} + \frac{ALR_P}{L}$$

Note that the *Pigouvian tax* is capitalized in bids rent yielding:

$$\Psi_P(x) = \frac{a [h(\omega_C \tau_0 + t_0)(1-a)(y_M - x) + \omega_C \tau_1 a (y_M - x)^2]}{h^2(1-a)} + R_A,$$

Flat kilometer tax The public authority decides to implement a *flat tax* per mile traveled instead of a *Pigouvian tax*. The uniform tax is the following:

$$\tau_F(x) = t_F x$$

The benefits from this tax are returned as lump sums to the residents and given by:

$$\begin{aligned} G_F &= \frac{ma}{h} \int_0^{y_M} t_F x dx \\ &= \frac{mat_F y_M^2}{2h} \end{aligned}$$

Hence, inserting the *flat tax*, ($\frac{G_F}{L}$) and the aggregate land rent lump sum ($\frac{ALR_M}{L}$) yields:

$$V_F(y_M) = \omega_C - y_M \left[\omega_C \tau_0 + t_0 + t_F + \frac{\omega_C \tau_1 a y_M}{2h(1-a)} \right] - \frac{R_A h}{a} + \frac{ALR_M}{L} + \frac{G_F}{L} \quad (46)$$

The *flat tax* is capitalized in the suboptimal bid rent yielding:

$$\Psi_F(x) = \frac{a [2h(\omega_C \tau_0 + t_0 + t_F)(y_M - x)(1-a) + \omega_C \tau_1 a (y_M - x)^2]}{2h^2(1-a)} + R_A, \quad (47)$$

Cordon toll Instead of a *Pigouvian tax* or a *flat tax*, the public authority decides to set a *cordon toll* at the location αy_M ($0 < \alpha \leq 1$). Traffic flow is assumed to be uninterrupted when passing the cordon. The commuters are charged according to the following toll scheme:

$$\tau_{CT} = \begin{cases} c & \text{if } x \geq \alpha y \\ 0, & \text{otherwise} \end{cases} \quad (48)$$

The revenue return scheme is now given by:

$$G_{CT} = \frac{m\alpha y_M(1-\alpha)^2 c}{2h} \quad (49)$$

Hence, the individual welfare near the CBD is given by:

$$V_{CT}(x > \alpha y_M) = \omega_C \left[1 - x\tau_0 - \frac{\tau_1 a(2y_M x - x^2)}{2h(1-a)} \right] - t_0 x - \frac{\Psi_C h}{a} + \frac{ALR_M}{L} + \frac{G_k}{L} \quad (50)$$

and at the city limits by:

$$V_{CT}(y_M) = \omega_C - y_M \left[\omega_C \tau_0 + t_0 + \frac{\omega_C \tau_1 \alpha y_M}{2h(1-a)} \right] - c - \frac{R_A h}{a} + \frac{ALR_M}{L} + \frac{G_k}{L} \quad (51)$$

Near the CBD, the suboptimal bid rent is expressed as follows:

$$\Psi_0(x < \alpha y_M) = a[\omega_C [T_M(y_M) - T_M(x)] + t_0(y - x)] + \left[R_A + \frac{ca}{h} \right] \quad (52)$$

The product of the lump-sum tax (c) and the population density is now capitalized in the rent between the *cordon toll* and the city limit. The rent between the location αy_M and y_M is given by:

$$\Psi_0(x > \alpha y_M) = a[\omega_C(T_M(y_M) - T_M(x)) + t_0(y_M - \alpha y_M) + R_A] \quad (53)$$

There is a rent discontinuity at the *cordon toll* location.

D. The polycentric city

D1. Equilibrium border

The population constraint is given by:

$$\int_0^{\hat{z}} \frac{a}{h} dx + \int_{\hat{z}}^y \frac{a}{h} dx = \frac{L}{m}$$

so that:

$$\begin{aligned} \int_0^{\hat{z}} \frac{a}{h} dx + \int_{\hat{z}}^y \frac{a}{h} dx &= \frac{L}{m} \\ \int_0^{\hat{z}} \frac{a}{h} dx + 2 \int_{z_p}^y \frac{a}{h} dx &= \frac{L}{m} \\ \frac{a}{h} \hat{z} + 2 \frac{a}{h} \frac{y - \hat{z}}{2} &= \frac{L}{m} \end{aligned}$$

leading to:

$$y = \frac{Lh}{ma}$$

D2. CBD limit in equilibrium with a wage gap

A worker living at \hat{z} is indifferent between traveling to the CBD or the SBD. Therefore, we have:

$$\omega_C \left[1 - \hat{z}\tau_0 - \frac{\tau_1 a \hat{z}^2}{2h(1-a)} \right] - t_0 \hat{z} = \omega_S \left[1 - \tau_0 \left(\frac{y - \hat{z}}{2} \right) - \frac{\tau_1 a (y - \hat{z})^2}{8h(1-a)} \right] - t_0 \left(\frac{y - \hat{z}}{2} \right)$$

Inserting $\mu\omega_C = \omega_S$ due to the wage gap between the business districts yields:

$$\hat{z}^*(\mu) = \frac{-2h(1-a)[\omega_C(2+\mu)\tau_0 + 3t_0] - \mu\omega_C\tau_1 a y + 2\sqrt{\mu\omega_C^2\tau_1^2 a^2 y^2 + \Delta^*}}{\omega_C(4-\mu)\tau_1 a}$$

with:

$$\Delta^* = h^2(-1+a)^2[\omega_C(2+\mu)\tau_0 + 3t_0]^2 + 2h(1-a)[(4+\mu^2)\omega_C^2 - 5\mu\omega_C^2 + \omega_C(2+\mu)\tau_0 y + 3\mu\omega_C^2\tau_0 y] \tau_1 a$$

leading to:

$$\Gamma = \omega_C \left[1 - \mu + \tau_0 \left(\frac{\hat{z}(-2-\mu) + \mu y}{2} \right) + \frac{\tau_1 a [\mu(y - \hat{z})^2 - 4\hat{z}^2]}{8h(1-a)} \right] + t_0 \left(\frac{y - 3\hat{z}}{2} \right),$$

with $\Gamma \equiv 0$. According to the implicit function theorem, we have:

$$\frac{\partial \Gamma}{\partial k} + \frac{\partial \Gamma}{\partial \hat{z}} \frac{\partial \hat{z}}{\partial k} = 0$$

where k is an exogenous variable from the model. Equivalently, we have:

$$\frac{\partial \hat{z}}{\partial k} = - \frac{\partial \Gamma / \partial k}{\partial \Gamma / \partial \hat{z}}$$

Trivial calculations yield:

$$\text{sign} \frac{\partial \hat{z}}{\partial \tau_0} = \text{sign} \frac{\partial \hat{z}}{\partial \mu} < 0, \quad \text{sign} \frac{\partial \hat{z}}{\partial y} > 0$$

$$\text{sign} \frac{\partial \Gamma}{\partial t_0} = \text{sign} \left[\frac{y - 3\hat{z}}{2} \right] < 0$$

where the term in brackets is equal to $\frac{y}{2}$ when $\hat{z} = 0$ and decreases when \hat{z} increases. In addition we have:

$$\text{sign} \frac{\partial \Gamma}{\partial \tau_1} = \text{sign} \frac{\partial \Gamma}{\partial h} = \text{sign} \frac{\partial \Gamma}{\partial a} = \text{sign} [\mu(y - \hat{z})^2 - 4\hat{z}^2] < 0 \quad (0 < \mu < 1)$$

where the term in brackets is equal to μy^2 when $\hat{z} = 0$ and decreases when \hat{z} increases.

D3. Welfare Welfare within the polycentric city is now defined as follows:

$$W_S(\widehat{z}^*(\mu)) = \frac{ma}{h} \left[\int_0^{\widehat{z}} \omega_C [1 - T(x)] - t_0 x dx + 2 \int_{z_p}^y \omega_S [1 - T(x)] - t_0(x - z_S) dx \right] - ALR_P$$

$$+ \left[ALR_P - m \int_0^y R_A dx \right] + m \int_0^y R_A dx$$

$$W_S(\widehat{z}^*(\mu)) = \frac{ma}{h} \left[\omega_C \widehat{z}^*(1 - \mu) + \mu \omega_C y - \frac{(\mu \omega_C \tau_0 + t_0)(y^2 - \widehat{z}^{*2})}{2} - \frac{(\omega_C \tau_0 + t_0) \widehat{z}^{*2}}{2} - \frac{\omega_C \tau_1 a [4\widehat{z}^{*3} + \mu(y - \widehat{z})^3]}{12h(1 - a)} \right]$$

D4. Proof of proposition 2

The aggregate welfare in the no-toll equilibrium can be defined as follows:

$$W_S^* = \frac{ma}{h} \left[\int_0^{\widehat{z}} \omega_C [1 - T(x)] - t_0 x dx + 2 \int_{z_p}^y \omega_C [1 - T(x)] - t_0(x - z_S) dx \right] - ALR_P$$

$$+ \left[ALR_P - m \int_0^y R_A dx \right] + m \int_0^y R_A dx$$

Free-market and optimal sizes of the CBD limit when $\mu = 1$

The indifference function of \widehat{z} is expressed as follows:

$$\omega_C \left[1 - \widehat{z} \tau_0 - \frac{\tau_1 a \widehat{z}^2}{2h(1 - a)} \right] - t_0 \widehat{z} = \omega_C \left[1 - (y - z_S) \tau_0 - \frac{\tau_1 a (y - z_S)^2}{2h(1 - a)} \right] - t_0 (y - z_S)$$

yielding:

$$\widehat{z}^2 \left[\frac{3\omega_C \tau_1 a}{8h(1 - a)} \right] + \widehat{z} \left[\frac{3\omega_C \tau_0}{2} + \frac{3t_0}{2} + \frac{\omega_C \tau_1 a}{4h(1 - a)} \right] - \left[\frac{\omega_C \tau_0 y}{2} + \frac{t_0 y}{2} + \frac{\omega_C \tau_1 a y^2}{8h(1 - a)} \right] = 0$$

leading to:

$$\widehat{z}^* = \frac{y}{3}$$

Then, the size of the CBD in which workers live is determined by a benevolent planner in order to maximize welfare $W_P(\widehat{z})$ subject to:

$$\begin{cases} V_C(\widehat{z}) = \omega_C(1 - x\tau_0 - \frac{\tau_1 a}{h(1-a)}[\widehat{z}x - x^2/2]) - \frac{\Psi_P(x)h}{a} - t_0 x > 0 \\ V_S(\widehat{z}) = \omega_C(1 - (x - z_S)\tau_0 - \frac{\tau_1 a}{h(1-a)}[y(x - z_S) + \frac{z_S^2 - x^2}{2}]) - \frac{\Psi_P(x)h}{a} - t_0(x - z_S) > 0 \end{cases}$$

We have:

$$\frac{\partial W_S(\widehat{z})}{\partial \widehat{z}} = 0$$

Hence:

$$\frac{m\omega_C \tau_1^2 a^2 (y + \widehat{z})(y - 3\widehat{z})}{4h^2(1 - a)} = 0$$

leading to:

$$\widehat{z}^O = \frac{y}{3}$$

Hence, $\widehat{z}^* = \widehat{z}^O$ and their expression has been inserted in the aggregate welfare function (??) yielding:

$$W_S(\widehat{z}^O) = L \left[\omega_C - \frac{(\omega_C \tau_0 + t_0)y}{2} - \frac{\omega_C \tau_1 a y^2}{27h(1-a)} \right]$$

As a result, we have:

$$W_S(\widehat{z}^O) - W_M^* = \frac{m\omega_C \tau_1 a^2 (y - \widehat{z})(y + \widehat{z})^2}{4h^2(1-a)} > 0 \quad (54)$$

D5. Proof of proposition 3

Aggregate welfare, including a wage gap within the city, can be defined as follows:

$$\begin{aligned} W_S &= \frac{ma}{h} \left[\int_0^{\widehat{z}} [1 - T(x)] - t_0 x dx + 2 \int_{\widehat{z}_S}^y \omega_S [1 - T(x)] - t_0(x - z_S) dx \right] - ALR_P \\ &+ [ALR_P - m \int_0^y R_A dx] + m \int_0^y R_A dx \end{aligned}$$

with:

$$ALR_P = m \int_0^{\widehat{z}} \Psi_0(x) dx + 2m \int_{z_S}^y \Psi_P(x) dx$$

yielding:

$$W_P = \omega_C \widehat{z} + \omega_S (y - \widehat{z}) - \omega_C \tau_0 \widehat{z} - \omega_S \tau_0 (y - \widehat{z}) - \frac{\omega_C \tau_1 a \widehat{z}^3}{3h(1-a)} - \frac{2\omega_S \tau_1 a (\frac{y-\widehat{z}}{2})^3}{3h(1-a)} - t_0 \frac{y^2}{2}$$

Then, the left endpoint of the SBD \widehat{z}^O is determined when there exists a wage gap between the business centers. The maximization of aggregate welfare $W_P(\widehat{z})$ is subject to:

$$V_S(\widehat{z}) \begin{cases} \omega_C(1 - x\tau_0 - \frac{\tau_1 a}{h(1-a)}[\widehat{z}x - x^2/2]) - \frac{\Psi_0(x)h}{a} - t_0 x > 0 \\ \omega_P(1 - (x - z_S)\tau_0 - \frac{\tau_1 a}{h(1-a)}[y(x - z_S) + \frac{z_S^2 - x^2}{2}]) - \frac{\Psi_P(x)h}{a} - (x - z_S)t_0 > 0 \\ \widehat{z} < y \end{cases}$$

Then, we obtain:

$$\frac{\partial W_S(\widehat{z})}{\partial \widehat{z}} = \omega_C - \omega_S + \omega_S \tau_0 - \omega_C \tau_0 - \frac{\omega_C \tau_1 a \widehat{z}^2}{h(1-a)} + \frac{\omega_S \tau_1 a (y - \widehat{z})^2}{4h(1-a)}$$

Thus, $\frac{\partial W_S(\widehat{z})}{\partial \widehat{z}} = 0$ leads to a single maximum for $W_S(\widehat{z})$ when $\widehat{z} \in [0, y]$, which reaches:

$$\widehat{z}^O(\mu) = \frac{-2h(1-a)\omega_C(1-\mu)\tau_0 - \mu\omega_C\tau_1 a y + 2\sqrt{\mu\omega_C^2\tau_1^2 a^2 y^2 + \Delta^O}}{\omega_C(4-\mu)\tau_1 a}$$

with:

$$\Delta^O = h^2(-1+a)^2 [\omega_C(1-\mu)]^2 \tau_0^2 + h(1-a) [(4+\mu^2)\omega_C^2 + \mu\omega_C^2\tau_0 y - \mu^2\omega_C^2\tau_0 y] \tau_1 a$$

Furthermore, $W_S(\widehat{z})$ is a concave curve since $\frac{\partial^2 W_S}{\partial^2 \widehat{z}}$ yields the following result:

$$\frac{\partial^2 W_S}{\partial^2 \widehat{z}} = \frac{ma}{h} \left[(\omega_S - \omega_C)\tau_0 - \frac{[24\omega_C\widehat{z} + 6\omega_S(y - \widehat{z})] \tau_1 a}{12h(1-a)} \right] < 0 \quad (55)$$

This optimal CBD size varies positively with the wage rate discrepancy between the business centers within the polycentric setting:

$$\frac{\partial W_S}{\partial \widehat{z}^O} > 0, \quad \text{when} \quad 1 < \frac{1}{\mu} < \frac{1}{\underline{\mu}}$$

E. Monocentric city versus Polycentric city

Congestion level

Note that the differential in the total commuting time cost between the two cities is given by $\Delta T(y)$, with:

$$\begin{aligned} \Delta T(y) &= T_M(y_M) - T(y) \\ &= \tau_0 z_S + \frac{\tau_1 a [y^2 - (\frac{y-\widehat{z}}{2})^2]}{2h(1-a)} \end{aligned}$$

In addition, we have $\Delta T(\widehat{z})$, with:

$$\begin{aligned} \Delta T(\widehat{z}) &= T_M(y_M) - T(\widehat{z}_P) \\ &= \tau_0(y - \widehat{z}) + \frac{\tau_1 a (y^2 - \widehat{z}^2)}{2h(1-a)} \end{aligned}$$

Thus, $\Delta T(y) > 0$ and $\Delta T(\widehat{z}) > 0$, with an average congestion delay that is higher in the monocentric setting.

Welfare comparison. When we assume there is no wage gap within the polycentric city, the welfare is given by:

$$W_S(\widehat{z}^*) = \frac{ma}{h} \left[\omega_C y - \frac{(\omega_C \tau_0 + t_0) y^2}{2} - \frac{\omega_C \tau_1 a (4\widehat{z}^{3*} + (y - \widehat{z}^*)^3)}{12h(1-a)} \right]$$

and in the monocentric city:

$$W_M = L \left[\omega_C - \frac{y_M(\omega_C \tau_0 + t_0)}{2} - \frac{\omega_C \tau_1 a y^2}{3h(1-a)} \right]$$

Therefore, the comparison with the monocentric city yields $\Delta_V = W_M - W_S$, with:

$$\Delta_V = \omega_C(1-\mu)(y - \widehat{z}_{toll}) + \omega_C \tau_0(1-\mu)(\widehat{z}_{toll} - y)$$

$$+ \frac{\omega_C \tau_1 a (\widehat{z}_{\text{toll}}^3 - y^3)}{3h(1-a)} + \frac{\omega_S \tau_1 a (\frac{y - \widehat{z}_{\text{toll}}}{2})^3}{3h(1-a)}$$

Regarding the evolution of the welfare between the monocentric and the polycentric city, we obtain the following expression:

$$\Delta_W(\mu) = \frac{ma(y - \widehat{z}) [h(1-a)(12(\omega_S - \omega_C) + 6\omega_C\tau_0(y + \widehat{z}) - 3\omega_S\tau_0(y - \widehat{z}) + 3t_0(y + 3\widehat{z})) + 4\omega_C\tau_1a(y^2 + y\widehat{z} + \widehat{z}^2)]}{12h^2(1-a)} \quad (56)$$

It is straightforward to check that the aggregate welfare levels are equal when $\widehat{z}(\mu) = y$. In other words, when $\underline{\mu} = 1 - (\tau_0 + \frac{t_0}{\omega_C})y - \frac{\tau_1 a y^2}{2h(1-a)}$. Here, the difference in welfare depends heavily on the magnitude of the wage gap between the business centers within the polycentric city.

F. Road pricing schemes in the polycentric city:

F1. Quasi-first-best vs. equilibrium

The city limits remain identical to the benchmark equilibrium, as housing size is exogenous. Thus, we obtain the bid rent in a suboptimal equilibrium:

$$\Psi_0(x) = \frac{a [h(1-a)(\omega_C\tau_0 + t_0)(\widehat{z}_S - x) + \omega_C\tau_1a(\widehat{z}_S - x)^2]}{h^2(1-a)} + R_A$$

and when $z_S < x < y$:

$$\Psi_P(x) = \frac{a [h(1-a)(\omega_S\tau_0 + t_0)(y - x) + \omega_S\tau_1a(y - x)^2]}{h^2(1-a)} + R_A$$

The "Pigowian" tax is determined as follows:

$$\tau_P(x) = \int_0^x \left[\frac{\tau_1 a}{h(1-a)} \int_x^{\widehat{z}} \omega_C \, dz \right] dx \quad \text{and} \quad \tau_{PT}(x) = \int_{\widehat{z}}^x \left[\frac{\tau_1 a}{h(1-a)} \int_x^y \omega_S \, dz \right] dx,$$

with:

$$\begin{aligned} \tau_P(x) &= \int_0^x \left[\frac{\tau_1 a}{h(1-a)} \int_x^{\widehat{z}} \omega_C \, dz \right] dx \\ &= \frac{\tau_1 a}{h(1-a)} \int_0^x \omega_C (\widehat{z} - x) \, dx \\ &= \frac{\omega_C \tau_1 a (\widehat{z}x - x^2/2)}{h(1-a)} \end{aligned}$$

when $0 < x < \widehat{z}$. Because of symmetry around the SBD, we have:

$$\int_{\widehat{z}}^x \left[\frac{\tau_1 a}{h(1-a)} \int_x^y \omega_S dz \right] dx = \int_{z_S}^x \left[\frac{\tau_1 a}{h(1-a)} \int_x^y \omega_S dz \right] dx$$

$$\begin{aligned}
&= \frac{\tau_1 a}{h(1-a)} \int_{z_S}^x \omega_S(y-x) dx \\
&= \frac{\omega_S \tau_1 a \left[y(x-z_S) - \frac{x^2-z_S^2}{2} \right]}{h(1-a)}
\end{aligned}$$

when $\hat{z} < x < y$. Using the *Pigouvian tax* and the indifference condition of a worker living in \hat{z} yields:

$$\omega_C \left[1 - \hat{z}\tau_0 - \frac{\tau_1 a \hat{z}^2}{h(1-a)} \right] - t_0 \hat{z} = \omega_S \left[1 - \left(\frac{y-\hat{z}}{2} \right) \tau_0 - \frac{\tau_1 a \left(\frac{y-\hat{z}}{2} \right)^2}{h(1-a)} \right] - t_0 \left(\frac{y-\hat{z}}{2} \right)$$

in which $\omega_S = \mu\omega_C$ and (21) have been inserted, leading to:

$$\hat{z}_P(\mu) = \frac{h(-1+a) [\omega_C(2+\mu)\tau_0 + 3t_0] - \omega_S \tau_1 a y + \sqrt{4\mu\omega_C^2 \tau_1^2 a^2 y^2 + \Delta_P}}{\omega_C(4-\mu)\tau_1 a}$$

with:

$$\begin{aligned}
\Delta_P &= h^2(-1+a)^2 [\omega_C(2+\mu)\tau_0 + 3t_0]^2 \\
&+ 4h(1-a) [(4+\mu^2)\omega_C^2 - 5\mu\omega_C^2 + \omega_C(2+\mu)\tau_0 y + 3\mu\omega_C^2 \tau_0 y] \tau_1 a
\end{aligned}$$

Welfare is now given by:

$$\begin{aligned}
W_P &= \frac{ma}{h} \left[\int_0^{\hat{z}} V_0 dx + 2 \int_{z_S}^y V_P \right] + G_P \\
&+ \left[ALR_P - \int_0^y R_A dx \right] + \int_0^y R_A dx
\end{aligned}$$

where aggregate land rents and toll revenues returned as lump sums:

$$ALR_P = \frac{ma}{h} \left[\int_0^{\hat{z}} \frac{\Psi_0 h}{a} dx + 2 \int_{z_S}^y \frac{\Psi_P h}{a} dx \right] \quad \text{and} \quad G_P = \frac{ma}{h} \int_0^y \tau_k(x) dx$$

Thus, we find the following welfare:

$$\begin{aligned}
W_P &= \frac{ma}{h} \left[\omega_C \hat{z} - \frac{(\omega_C \tau_0 + t_0) \hat{z}^2}{2} - \frac{\omega_C \tau_1 a \hat{z}^3}{3h(1-a)} + \omega_S (y - \hat{z}) - \frac{(\omega_S \tau_0 + t_0)(y - \hat{z})^2}{4} - \frac{(\omega_S \tau_1 a (y - \hat{z})^3)}{12h(1-a)} \right] \\
&+ \frac{ma}{h} \left[\frac{(\omega_C \tau_0 + t_0) \hat{z}^2}{2} + \frac{\omega_C \tau_1 a \hat{z}^3}{3h(1-a)} + \frac{(\omega_S \tau_0 + t_0)(y - \hat{z})^2}{4} + \frac{(\omega_S \tau_1 a (y - \hat{z})^3)}{12h(1-a)} \right] + mR_{Ay}
\end{aligned}$$

Then, we derive W_P with respect to \hat{z} leading to:

$$\begin{aligned}
\frac{\partial W_S}{\partial \hat{z}} &= \frac{ma}{h} \left[\omega_C - (\omega_C \tau_0 + t_0) \hat{z} - \frac{\omega_C \tau_1 a \hat{z}^2}{h(1-a)} - \omega_S + \frac{(\omega_S \tau_0 + t_0)(y - \hat{z})}{2} + \frac{(\omega_S \tau_1 a (y - \hat{z})^2)}{4h(1-a)} \right] \\
&+ \frac{ma}{h} \left[(\omega_C \tau_0 + t_0) \hat{z} + \frac{\omega_C \tau_1 a \hat{z}^2}{h(1-a)} - \frac{(\omega_S \tau_0 + t_0)(y - \hat{z})}{2} - \frac{(\omega_S \tau_1 a (y - \hat{z})^2)}{4h(1-a)} \right]
\end{aligned}$$

When $\hat{z} = \hat{z}_P$:

$$\left. \frac{\partial W_P}{\partial \hat{z}} \right|_{\hat{z}=\hat{z}_P} = \frac{ma}{h} [\omega_C - \omega_S] > 0$$

F2. The polycentric city: Second-best vs. equilibrium

The flat tax The bids rent including the *flat tax* are given by:

$$\Psi_0(x) = \frac{a[2h(1-a)(\omega_C\tau_0 + t_0 + \tau_k)(\widehat{z}_F - x) + \omega_C\tau_1a(\widehat{z}_F - x)^2]}{2h^2(1-a)} + R_A$$

and:

$$\Psi_P(x) = \frac{a[2h(1-a)(\omega_S\tau_0 + t_0 + t_F)(y - x) + \omega_S\tau_1a(y - x)^2]}{2h^2(1-a)} + R_A$$

when $z_S < x < y$. Using the indirect utility formula with the *flat tax* and (37) yields the indirect utility given by:

$$V_S = \omega_S \left[1 - \left(\frac{y - \widehat{z}}{2} \right) \tau_0 - \frac{a\tau_1 \left(\frac{y - \widehat{z}}{2} \right)^2}{2h(1-a)} \right] + \bar{G}_T + \frac{ALR}{L} - (t_0 + t_F) \left(\frac{y - \widehat{z}}{2} \right) - \frac{R_A h}{a}$$

The indifference condition of a worker living in \widehat{z} yields:

$$\omega_C \left[1 - \widehat{z}\tau_0 - \frac{\tau_1 a \widehat{z}^2}{2h(1-a)} \right] - (t_0 + t_F)\widehat{z} = \omega_S \left[1 - \left(\frac{y - \widehat{z}}{2} \right) \tau_0 - \frac{\tau_1 a \left(\frac{y - \widehat{z}}{2} \right)^2}{2h(1-a)} \right] - (t_0 + t_F) \left(\frac{y - \widehat{z}}{2} \right)$$

yielding:

$$z_F = \frac{-2h(1-a)[\tau_0(2\omega_C + \omega_S) + 3(t_0 + t_F)] - \omega_S\tau_1 a y + 2\sqrt{\Delta_F}}{(4\omega_C - \omega_S)\tau_1 a}$$

with:

$$\begin{aligned} \Delta_F &= h^2(-1+a)^2[\tau_0(2\omega_C + \omega_S) + 3(t_0 + t_F)]^2 \\ &+ 2h(1-a)\tau_1 a [4\omega_C^2 + \omega_S^2 + 3\omega_C\omega_S\tau_0 y - 5\omega_C\omega_S + (2\omega_C + \omega_S)(t_0 + t_F)y] + \omega_C\omega_S\tau_1^2 a^2 \end{aligned}$$

Welfare is given by:

$$W_F(t_F) = \frac{ma}{h} \left[\int_0^{\widehat{z}} \omega_C [1 - T(x)] - t_0(x) dx + 2 \int_{z_S}^y \omega_S [1 - T(x)] - t_0(x - z_S) dx \right]$$

with:

$$ALR_F = \frac{ma}{h} \left[\int_0^{\widehat{z}} \frac{\Psi_0 h}{a} dx + 2 \int_{z_S}^y \frac{\Psi_P h}{a} dx \right]$$

Hence, the following welfare is:

$$W_F(t_F) = \frac{ma}{h} \left[\omega_C \widehat{z} - \frac{(\omega_C\tau_0 + t_0)\widehat{z}^2}{2} - \frac{\omega_C\tau_1 a \widehat{z}^3}{3h(1-a)} + \omega_S(y - \widehat{z}) - (\omega_S\tau_0 + t_0) \left(\frac{y - \widehat{z}}{2} \right)^2 - \frac{\omega_S\tau_1 a (y - \widehat{z})^3}{12h(1-a)} \right]$$

Then, we derive W_S with respect to \widehat{z} yielding:

$$\frac{\partial W_S}{\partial \widehat{z}} = \frac{ma}{h} \left[\omega_C - (\omega_C\tau_0 + t_0)\widehat{z} - \frac{\omega_C\tau_1 a \widehat{z}^2}{h(1-a)} - \omega_S + (\omega_S\tau_0 + t_0) \left(\frac{y - \widehat{z}}{2} \right) - \frac{\omega_S\tau_1 a (y - \widehat{z})^2}{4h(1-a)} \right]$$

When $\widehat{z} = \widehat{z}_F$:

$$\left. \frac{\partial W_S}{\partial \widehat{z}} \right|_{\widehat{z}=\widehat{z}_F} = \frac{ma}{h} \left[\omega_C - \omega_S - \frac{\omega_C\tau_1 a \widehat{z}_F^2}{2h(1-a)} + \frac{\omega_S\tau_1 a (y - \widehat{z}_F)^2}{8h(1-a)} - t_F \left(\frac{y - 3\widehat{z}_F}{2} \right) \right] \leq 0$$

F3. The polycentric city: Second-best vs. equilibrium

Cordon toll The amount of collected tax is redistributed to each household and expressed as follows:

$$G_{CT} = \int_{\beta\widehat{z}_C}^{\widehat{z}_C} \tau_k(x) dx$$

leading to:

$$\bar{G}_{CT} = \frac{ma(\widehat{z}_C - \beta\widehat{z}_C)^2 c}{2hL}$$

The bid rent including the *cordon toll* is given by:

$$\Psi_0(x) = \frac{a[2h(1-a)(\omega_C\tau_0 + t_0)(\widehat{z}_C - x) + \omega_C\tau_1a(\widehat{z}_C - x)^2]}{2h^2(1-a)} + R_A + \frac{ca}{h}$$

When $\alpha\widehat{z}_C < x < \widehat{z}_C$:

$$\Psi_0(x) = \frac{a[2h(1-a)(\omega_C\tau_0 + t_0)(\widehat{z}_C - x) + a\omega_C\tau_1(\widehat{z}_C - x)^2]}{2h^2(1-a)} + R_A$$

The indirect utility of the inhabitant who lives at \widehat{z}_C and works in the CBD is now given by:

$$V_{CT}(\widehat{z}_C) = \omega_C \left[1 - \widehat{z}_C\tau_0 - \frac{a\tau_1\widehat{z}_C^2}{2h(1-a)} \right] + \bar{G}_{CT} + \frac{ALR_{CT}}{L} - t_0\widehat{z}_C - c - \frac{R_A h}{a}$$

When $z_S < x < y$:

$$V_{CT}(y) = \omega_S \left[1 - \left(\frac{y - \widehat{z}_C}{2}\right)\tau_0 - \frac{a\omega_S\tau_1\left(\frac{y - \widehat{z}_C}{2}\right)^2}{2h(1-a)} \right] + \bar{G}_{CT} + \frac{ALR_{CT}}{L} - t_0\left(\frac{y - \widehat{z}_C}{2}\right) - \frac{RAh}{a}$$

The limit of the CBD \widehat{z} is implicitly defined by:

$$\omega_C \left[1 - \widehat{z}\tau_0 - \frac{\tau_1 a \widehat{z}^2}{2h(1-a)} \right] - t_0\widehat{z} - c = \omega_S \left[1 - \left(\frac{y - \widehat{z}}{2}\right)\tau_0 - \frac{\tau_1 a \left(\frac{y - \widehat{z}}{2}\right)^2}{2h(1-a)} \right] - t_0\left(\frac{y - \widehat{z}}{2}\right)$$

yielding:

$$z_C = \frac{-2h(1-a)[\tau_0(2\omega_C + \omega_S) + 3t_0] - \omega_S\tau_1 a y + 2\sqrt{\Delta_C}}{(4\omega_C - \omega_S)\tau_1 a}$$

with:

$$\begin{aligned} \Delta_C &= h^2(-1+a)^2 [\tau_0(2\omega_C + \omega_S) + 3t_0]^2 \\ &+ 2h(1-a)\tau_1 a [4\omega_C^2 + \omega_S^2 + 3\omega_C\omega_S\tau_0 y - 5\omega_C\omega_S + (2\omega_C + \omega_S)t_0 y - (4\omega_C - \omega_S)c] + \omega_C\omega_S\tau_1^2 a^2 y^2 \end{aligned}$$

Welfare is given by:

$$W_{CT} = \frac{ma}{h} \left[\int_0^{\alpha\widehat{z}} \omega_C [1 - T(x)] - t_0(x) dx + \int_{\alpha\widehat{z}}^{\widehat{z}} \omega_C [1 - T(x)] - t_0(x) dx + 2 \int_{z_S}^y \omega_S [1 - T(x)] - t_0(x - z_S) dx \right]$$

with:

$$ALR_{CT} = \frac{ma}{h} \left[\int_0^{\hat{z}} \frac{\Psi_0 h}{a} dx + 2 \int_{z_S}^y \frac{\Psi_P h}{a} dx \right]$$

Hence, the following welfare is:

$$W_{CT} = \frac{ma}{h} \left[\omega_C \hat{z} - \frac{(\omega_C \tau_0 + t_0) \hat{z}^2}{2} - \frac{\omega_C \tau_1 a \hat{z}^3}{3h(1-a)} + \omega_S (y - \hat{z}) - (\omega_S \tau_0 + t_0) \left(\frac{y - \hat{z}}{2} \right)^2 - \frac{\omega_S \tau_1 a (y - \hat{z})^3}{12h(1-a)} \right]$$

Then, we derive W_{CT} with respect to \hat{z} yielding:

$$\frac{\partial W_{CT}}{\partial \hat{z}} = \frac{ma}{h} \left[\omega_C - (\omega_C \tau_0 + t_0) \hat{z} - \frac{\omega_C \tau_1 a \hat{z}^2}{h(1-a)} - \omega_S + (\omega_S \tau_0 + t_0) \left(\frac{y - \hat{z}}{2} \right) + \frac{\omega_S \tau_1 a (y - \hat{z})^2}{4h(1-a)} \right]$$

When we derive W_S with respect to \hat{z} conditional on $\hat{z} = \hat{z}_C$, it yields:

$$\left. \frac{\partial W_S}{\partial \hat{z}} \right|_{\hat{z}=\hat{z}_C} = \frac{ma}{h} \left[c + \frac{\omega_S \tau_1 a (y - \hat{z})^2}{8h(1-a)} + \frac{(\omega_S \tau_0 + t_0)(3\hat{z} - y)}{2} \right] \leq 0$$