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Keywords:

beauty-contest, information acquisition, overreaction, central bank communication

JEL codes:

D82, E52, E58

Double overreaction in beauty-contests with information acquisition: theory and experiment*

Romain Baeriswyl[§] Kene Boun My[#] Camille Cornand[†]

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1 Introduction

The role of central bank communication in the conduct of monetary policy has increased considerably in recent decades. While communication is essential to establish the democratic legitimacy of an independent central bank, it has increasingly been used as a monetary policy instrument for managing market participants' expectations. As pointed out by Woodford (2005, p.3), "for [monetary policy to be most effective] not only do expectations about policy matter, but, at least under current conditions, very little else matters." Forward guidance is perhaps the best example of an instrument for managing expectations. By disclosing information on the policy it intends to pursue, the central bank aims to influence market expectations of future policy rates and, thereby, long-term interest rates and inflation expectations.

The coordinating role of public information has received considerable attention in the ongoing debate on how a central bank should communicate. In environments characterised by strategic complementarities, such as financial markets or price settings under monopolistic competition, economic agents seek to coordinate their actions, while remaining close to economic fundamentals. When it is common knowledge, public information enhances coordination among economic agents and, thereby, mitigates strategic uncertainty. However, despite its prominence in theory, common knowledge is difficult to achieve in reality. This difficulty stems from the intertwined relationship between the sender and the receivers of public information: common knowledge not only requires that the sender discloses public information, but also that the receivers pay attention to public information and that they know that all other receivers pay attention to the same public information as well, up to an infinite level of iteration. Thus, understanding how agents pay attention to various sources of information is essential for designing an optimal communication policy.

This paper contributes to understanding how people pay attention to various sources of information when they face strategic complementarities. First, it provides a theory of *double overreaction* to the most common and least private signal in a beauty-contest with information acquisition. On the one hand, strategic complementarities increase the equilibrium weight assigned to the most common and least private signal because it helps to coordinate with other agents. This is the overreaction mechanism described by Morris and Shin (2002). On the other hand, strategic complementarities also increase the equilibrium attention given to the most common and least private signal, which further increases the equilibrium weight assigned to it in the beauty-contest.

Second, in a laboratory experiment, we investigate the double overreaction mechanism with three treatments. Treatment A implements the beauty-contest with exogenous information to control for the overreaction to public information. Treatment B implements the beauty-contest with information acquisition where it is optimal to pay full attention to the most common and least private signal. Treatment C implements a fundamental-estimation game with information acquisition where it is optimal to pay no attention to the most common and least private signal. In response to strategic complementarities, participants increase both the attention and the weight assigned to the most common and

least private signal, confirming the double overreaction mechanism predicted by theory. Moreover, also in accordance with theory, participants tend to assign a higher weight to the most common and least private signal, the more attention they pay to it. The overreaction, however, is weaker than theoretically predicted. Deviations from equilibrium are explained by a suboptimally weak effect of strategic complementarities on both the allocation of attention and the expectations of other participants' action.

The experiment highlights the role of information acquisition on the reaction of participants to public disclosures. This finding provides some insight into the so-called forward-guidance puzzle documented by Del Negro et al. (2015) and which refers to the surprisingly weak reaction of market participants to central banks' disclosures about their intended future policy, compared to the theoretical predictions of the standard New Keynesian models. Shaping market expectations seems more challenging in practice than in theory. Angeletos and Lian (2018) highlight the lack of common knowledge and Garcia-Schmidt and Woodford (2018) the limited level of reasoning as possible cause of this puzzle. Both hypotheses are supported by our experiment: the lack of attention to disclosures prevents the emergence of common knowledge among participants, who, given their information, operate only a limited level of reasoning.

Section 2 discusses the related literature, section 3 presents the model, section 4 describes the experiment, section 5 gives the results, and section 6 concludes.

2 Related literature

Our paper relates to the literature on information acquisition in coordination environments under heterogeneous information, with a continuum of actions and exhibiting a quadratic payoff structure (mimicking the business cycle type of environment). Different approaches have been proposed to account for the fact that information processing depends not only on the issuer but also on the recipients. Focusing, as we do, on a beauty-contest payoff structure and allowing for the endogenous acquisition of multiple information sources, Myatt and Wallace (2012) show that: “if actions are more complementary, the information endogenously acquired in equilibrium is more public in nature.” In this approach, different signals may be acquired by the players at some cost (or ignored): the publicity of each information source is an equilibrium phenomenon. Myatt and Wallace (2014) provide an application to a Lucas-Phelps island economy. Our theoretical contribution is in the spirit of Myatt and Wallace. Nevertheless, rather than introducing some information costs, our model requires agents to allocate their limited information-processing resources between different pieces of information. Pavan (2016) – distinguishing between attention and recall – adopts the same information structure, but proposes a more general payoff structure allowing to address normative questions, as in Angeletos and Pavan (2007) and Colombo et al. (2014). Among the other alternative approaches to information acquisition in coordination games, Hellwig and Veldkamp (2009) show that strategic complementarities in actions imply that information acquisition is characterised by strategic

complementarities as well. Assuming an exogenous publicity of each information source and a binary allocation of attention to each source results in equilibrium indeterminacy. Focusing on the optimal central bank disclosure, Chahrour (2014) investigates agents' miscoordination depending on the source of costly information they pay attention to. Llosa and Venkateswaran (2013) instead compare the equilibrium vs. efficient acquisition of information in different specifications of the business cycle.

Our paper also belongs to the experimental literature on the beauty-contest game under heterogeneous information, formalised by Morris and Shin (2002). Considering the original beauty-contest with an exogenous information structure, Cornand and Heinemann (2014) experimentally study agents' overreaction to public information. Baeriswyl and Cornand (2014) show that partial publicity (consisting in providing a public signal to a subset of agents only) and partial transparency (consisting in providing a public signal with an idiosyncratic noise) may effectively reduce overreaction to public disclosures. To our knowledge, the present paper is the first to study information acquisition in an experimental beauty-contest.

While the literature mentioned above considers economies with a continuum of actions and continuous payoffs, information acquisition in games of regime change (with discontinuous payoffs and binary actions) is studied experimentally by Szkup and Trevino (2017). They show that the endogenous determination of the information structure by subjects (i.e. agents choose at a cost the precision of their private signal) affects the standard results in terms of information precision comparative statics and improves actions' efficiency. In contrast to the latter, the quadratic payoff structure of our study avoids the selection issue of equilibrium action and presents the advantage to find many applications in the business cycle literature.

Finally, the present paper is related to the literature on rational inattention (see e.g. Sims (2003) for a pioneer work and Mackowiak and Wiederholt (2015) for a business-cycle application). Recurring to information theory and focusing on the receiver (rather than the sender) noise, this literature shows that agents may rationally neglect information when its acquisition and processing is costly and the expected returns are small. This literature also offers an experimental counterpart. For example, in an experimental study, Cheremukhin et al. (2015) estimate and compare different models of rational inattention and show that subjects have heterogeneous costs of information processing. Caplin and Dean (2015) provide evidence that subjects collect more information, use more time and put more effort to process information if the rewards for doing so are higher. Caplin and Dean (2013) emphasise however that subjects respond less to changes in payoff incentives than the Shannon-entropy theory predicts.

3 The theoretical model

The problem of information acquisition is derived from a quadratic payoff beauty-contest game in the spirit of Myatt and Wallace (2012). For the purpose of the laboratory exper-

iment, our model shares the two following features. First, we limit the model to the case where there are only two sources of information rather than an indefinite number. Second, rather than introducing observation costs as Myatt and Wallace do, we postulate that an agent allocates his limited information-processing resources between the two sources of information. An agent decides how to spend his available time between the acquisition of information from both sources. An agent's strategy consists then in choosing two weights (between 0 and 1), one determining the allocation of his information-processing resources between the two sources of information, the other determining the weight of the two signals in his beauty-contest action.

3.1 A beauty-contest game

The economy is populated by a *continuum* of agents indexed by the unit interval $i \in [0, 1]$. The utility of agent i decreases in both the distortion of his action a_i from the unknown fundamental $\theta \in \mathbb{R}$ and the dispersion of his action from the average action of other agents \bar{a}_{-i} :

$$u_i(a, \theta) = \bar{u} - (1 - \gamma)(a_i - \theta)^2 - \gamma(a_i - \bar{a}_{-i})^2 \quad (1)$$

The parameter γ is the weight assigned to the strategic component which drives the strength of the coordination motive in the utility function and decision rule. If $\gamma = 0$, agents' utility is independent of the dispersion of their action and agents take their action to be as close as possible to the fundamental θ . If $0 < \gamma < 1$, agents' actions are strategic complements: agents tend to align their action with those of others.

3.2 Information sources

Agents have access to two information sources, x_1 and x_2 , on the unknown fundamental. Each of these information sources contains an error term that is common to all agents and is normally distributed with a zero mean and some variance:

$$\begin{aligned} x_1 &= \theta + \eta_1 \quad \text{where } \eta_1 \sim N(0, \kappa_1^2) \\ x_2 &= \theta + \eta_2 \quad \text{where } \eta_2 \sim N(0, \kappa_2^2) \end{aligned}$$

However, agents do not directly observe the information sources x_1 and x_2 . Instead, they observe these information sources with some idiosyncratic noise, which depends on the attention given to each source. Each agent disposes of a total attention of 1 that he allocates between the two information sources. The total attention can be interpreted as the time available to an agent for processing information. The information-acquisition problem of agent i consists in choosing the share of his total attention $z_i \in [0, 1]$ devoted to observing the information source x_1 . The residual attention of agent i , $1 - z_i$, is devoted to observing the information source x_2 . Assuming that attention reduces additively the variance of the idiosyncratic noise of observing the information sources, the two signals

received by agent i are characterised by the following noise terms:

$$x_{1,i} = x_1 + \epsilon_{1,i} \quad \text{where } \epsilon_{1,i} \sim N(0, \xi_1^2 + 1 - z_i) \quad (2)$$

$$x_{2,i} = x_2 + \epsilon_{2,i} \quad \text{where } \epsilon_{2,i} \sim N(0, \xi_2^2 + z_i) \quad (3)$$

and where all noise terms are independently distributed.

The variance of signals has a component common to all agents, κ_1^2 and κ_2^2 , that can be interpreted as the accuracy of the source or as the *sender* noise, and a component specific to each agent, $\xi_1^2 + 1 - z_i$ and $\xi_2^2 + z_i$, that can be interpreted as the transparency (clarity) of the source or as the *receiver* noises. The receiver noise terms are specific to each agent (i) because they are drawn independently for each agent from a normal distribution and (ii) because each agent shapes the variance of the normal distribution from which they are drawn by choosing z_i . A signal with a low sender noise is said to be *precise*, while a signal with a low receiver noise is said to be *clear*.

3.3 Equilibrium

An equilibrium strategy for agent i is a set $\{z_i, \omega_i\}$ that specifies his attention and action decision as a function of model parameters. While z_i determines the allocation of attention between information sources, ω_i determines the relative weight of each signal in the agent's action. As Myatt and Wallace (2012) show, the equilibrium action a_i is a weighted average of the two signals $x_{1,i}$ and $x_{2,i}$. Defining ω_i as the weight assigned to the signal $x_{1,i}$, the equilibrium action is a linear combination of both signals:

$$a_i = \omega_i \cdot x_{1,i} + (1 - \omega_i)x_{2,i} \quad (4)$$

With $\bar{\omega}$ being defined as the average weight $\int_i \omega_i di$ over all agents, the conditional expected utility yields

$$\begin{aligned} \mathbb{E}_i(u) &= \bar{u} - (1 - \gamma)\mathbb{E}_i(\omega_i \cdot x_{1,i} + (1 - \omega_i)x_{2,i} - \theta)^2 \\ &\quad - \gamma\mathbb{E}_i(\omega_i \cdot x_{1,i} + (1 - \omega_i)x_{2,i} - \bar{\omega} \cdot \bar{x}_1 - (1 - \bar{\omega})\bar{x}_2)^2 \\ &= \bar{u} - \omega_i^2 [(1 - \gamma)\kappa_1^2 + \xi_1^2 + 1 - z_i] - (1 - \omega_i)^2 [(1 - \gamma)\kappa_2^2 + \xi_2^2 + z_i] \\ &\quad - \gamma(\omega_i - \bar{\omega})^2(\kappa_1^2 + \kappa_2^2) \\ &= \bar{u} - \omega_i^2 [\nu_1 + 1 - z_i] - (1 - \omega_i)^2 [\nu_2 + z_i] - \gamma(\omega_i - \bar{\omega})^2(\kappa_1^2 + \kappa_2^2) \end{aligned} \quad (5)$$

where $\nu_1 = (1 - \gamma)\kappa_1^2 + \xi_1^2$ and $\nu_2 = (1 - \gamma)\kappa_2^2 + \xi_2^2$.

Depending on the parameter values, two equilibria are possible: an optimal one and a suboptimal one.

3.3.1 Optimal equilibrium

Since agents are all identical, $\omega_i = \bar{\omega} = \omega$ and $z_i = z \quad \forall i$, the conditional expected utility becomes

$$\mathbb{E}_i(u) = \bar{u} - \omega^2 [\nu_1 + 1 - z] - (1 - \omega)^2 [\nu_2 + z] \quad (6)$$

Differentiating (6) with respect to z and ω yields the following first and second order conditions:

$$\frac{\partial \mathbb{E}_i(u)}{\partial z} = 2\omega - 1 \stackrel{\leq}{\geq} 0 \quad \Leftrightarrow \quad \omega \stackrel{\leq}{\geq} \frac{1}{2} \quad (7)$$

$$\begin{aligned} \frac{\partial \mathbb{E}_i(u)}{\partial \omega} &= -2\omega(\nu_1 + 1 - z) + 2(1 - \omega)(\nu_2 + z) \\ &= 0 \quad \Leftrightarrow \quad \omega = \frac{\nu_2 + z}{\nu_1 + \nu_2 + 1} = \frac{(1 - \gamma)\kappa_2^2 + \xi_2^2 + z}{(1 - \gamma)(\kappa_1^2 + \kappa_2^2) + \xi_1^2 + \xi_2^2 + 1} \end{aligned} \quad (8)$$

$$\frac{\partial^2 \mathbb{E}_i(u)}{\partial \omega^2} = -2(\nu_1 + \nu_2 + 1) < 0 \quad (9)$$

$$\frac{\partial^2 \mathbb{E}_i(u)}{\partial \omega \partial z} = 2 \quad (10)$$

$$\frac{\partial \omega}{\partial z} = \frac{1}{\nu_1 + \nu_2 + 1} > 0 \quad (11)$$

The partial derivative (7) says that increasing the attention given to the signal $x_{1,i}$ improves utility when the weight assigned to it in the decision a_i , ω , is greater than the weight assigned to $x_{2,i}$, $1 - \omega$. Because $z \in [0, 1]$, the optimal attention is either 0 or 1, and it is never optimal to choose an intermediate level of attention, unless $\omega = 1/2$, in which case the optimal attention is indeterminate. It is thus optimal to pay full attention to the signal that is most weighted in the action.

The optimal weight ω assigned to $x_{1,i}$ is given by equation (8). The degree of strategic complementarities increases the weight assigned to the signal with the highest common noise or lowest private noise, giving rise to an overreaction to the most public signal. The negative second derivative (9) ensures that the utility function is concave in ω and that the critical point is a maximum.

The positive cross partial derivative with respect to z and ω (10) indicates that attention z and weight ω are complements. Finally, (11) shows that optimal weight ω always increases with attention z .

Using the result that optimal attention z is either 0 or 1, equilibrium is obtained by comparing expected utility with zero vs. full attention on $x_{1,i}$. Unconditional expected utility with zero attention on $x_{1,i}$, $\mathbb{E}_i(u)|_{z=0}$, is higher than expected utility with full

attention on $x_{1,i}$, $\mathbb{E}_i(u)|_{z=1}$, when the following condition holds true:

$$\begin{aligned}
\mathbb{E}_i(u)|_{z=0} > \mathbb{E}_i(u)|_{z=1} &\Leftrightarrow \nu_1 > \nu_2 & (12) \\
z^* &= 0 \\
\omega^* &= \frac{\nu_2}{\nu_1 + \nu_2 + 1} \\
\mathbb{E}_i(u) &= -\frac{(\nu_1 + 1)\nu_2}{\nu_1 + \nu_2 + 1}
\end{aligned}$$

In the opposite case, unconditional expected utility with full attention on $x_{1,i}$, $\mathbb{E}_i(u)|_{z=1}$ is higher than expected utility with zero attention on $x_{1,i}$, $\mathbb{E}_i(u)|_{z=0}$, when

$$\begin{aligned}
\mathbb{E}_i(u)|_{z=0} < \mathbb{E}_i(u)|_{z=1} &\Leftrightarrow \nu_1 < \nu_2 & (13) \\
z^* &= 1 \\
\omega^* &= \frac{\nu_2 + 1}{\nu_1 + \nu_2 + 1} \\
\mathbb{E}_i(u) &= -\frac{\nu_1(\nu_2 + 1)}{\nu_1 + \nu_2 + 1}
\end{aligned}$$

Conditions (12) and (13) show that it is optimal to pay full attention to the signal with the lowest adjusted variance of error terms $(1 - \gamma)\kappa^2 + \xi^2$. The variance of the error terms common to all agents, κ_1^2 or κ_2^2 , is adjusted downward with the degree of strategic complementarities γ . Without strategic complementarities, i.e. when $\gamma = 0$, full attention should be given to the signal with the lowest variance of error terms $\kappa^2 + \xi^2$. With increasing strategic complementarities, the variance of common error terms becomes relatively less detrimental to utility than the variance of private error terms because it enhances coordination. Attention is thus to be given to the most common and least private signal.

3.3.2 Suboptimal equilibrium

The optimal equilibrium is not necessarily unique. If other agents' average behaviour deviates sufficiently from optimum equilibrium, it is optimal for agent i to deviate as well. We compare utility (5) when agent i plays the optimal equilibrium strategy $\{z_i^*, \omega_i^*\}$ given by (12) or (13) while other agents play the suboptimal strategy \tilde{z}_{-i} and $\omega(\tilde{z}_{-i})$, $\mathbb{E}_i(u)|_{z_i^*, \tilde{z}_{-i}}$, with utility when agent i plays the same suboptimal strategy \tilde{z}_{-i} and $\omega(\tilde{z}_{-i})$ as other agents, $\mathbb{E}_i(u)|_{z_i=\tilde{z}_{-i}}$:

$$\begin{aligned}
\mathbb{E}_i(u)|_{z_i^*, \tilde{z}_{-i}} &= \bar{u} - \omega_i^{*2} [\nu_1 + 1 - z_i^*] - (1 - \omega_i^*)^2 [\nu_2 + z_i^*] - \gamma (\omega_i^* - \omega(\tilde{z}_{-i}))^2 (\kappa_1^2 + \kappa_2^2) \\
\mathbb{E}_i(u)|_{z_i=\tilde{z}_{-i}} &= \bar{u} - \omega(\tilde{z}_{-i})^2 [\nu_1 + 1 - \tilde{z}_{-i}] - (1 - \omega(\tilde{z}_{-i}))^2 [\nu_2 + \tilde{z}_{-i}]
\end{aligned}$$

When $\nu_1 > \nu_2$, the optimal strategy (12) with $z^* = 0$ yields a lower utility to agent i

than the suboptimal strategy \tilde{z}_{-i} when

$$\mathbb{E}_i(u)|_{z_i^*=0, \tilde{z}_{-i}} < \mathbb{E}_i(u)|_{z_i=\tilde{z}_{-i}} \Leftrightarrow \tilde{z}_{-i} > \frac{(\nu_1 + 1)^2 - \nu_2^2}{\kappa_1^2 + \kappa_2^2 + \xi_1^2 + \xi_2^2 + 1} \quad (14)$$

Provided that other agents play the optimal ω given their suboptimal \tilde{z}_{-i} , it is no longer optimal for agent i to play the optimal strategy (12) as soon as the average attention of other players \tilde{z}_{-i} is larger than the threshold value (14). This means that $\{\tilde{z}_{-i}, \omega(\tilde{z}_{-i})\}$ is a suboptimal equilibrium.

Symmetrically, when $\nu_1 < \nu_2$, the optimal strategy (13) with $z^* = 1$ yields a lower utility to agent i than the suboptimal strategy \tilde{z}_{-i} when

$$\mathbb{E}_i(u)|_{z_i^*=1, \tilde{z}_{-i}} < \mathbb{E}_i(u)|_{z_i=\tilde{z}_{-i}} \Leftrightarrow \tilde{z}_{-i} < \frac{\nu_1(\nu_1 + 1) - \nu_2(\nu_2 + 1) + \gamma(\kappa_1^2 + \kappa_2^2)}{\kappa_1^2 + \kappa_2^2 + \xi_1^2 + \xi_2^2 + 1} \quad (15)$$

It is no longer optimal for agent i to play the optimal strategy (13) as soon as the average attention of other players \tilde{z}_{-i} is smaller than the threshold value (15). This means that $\{\tilde{z}_{-i}, \omega(\tilde{z}_{-i})\}$ is a suboptimal equilibrium.

3.4 Double overreaction at the optimal equilibrium

In the beauty-contest game with information acquisition, strategic complementarities give rise at the optimal equilibrium to a double overreaction to the signal with the highest common noise and lowest private noise. On the one hand, strategic complementarities increase the weight assigned to the most common and least private signal because it better helps to coordinate with other agents. This is the overreaction mechanism described in Morris and Shin (2002). On the other hand, strategic complementarities increase the attention given to the most common and least private signal, which in turn increases the weight assigned to it in the beauty-contest game. Through their effect on information acquisition, strategic complementarities reinforce the overreaction to the most common and least private signal.

The overreaction due to the direct effect of strategic complementarities on the equilibrium weight in the action is obtained from varying γ in (8) when attention z is held constant. Differentiating ω with respect to γ shows that increasing strategic complementarities increases overreaction under the following conditions:

$$\frac{\partial \omega}{\partial \gamma} \begin{matrix} \leq \\ > \end{matrix} 0 \Leftrightarrow \frac{\kappa_1^2}{\xi_1^2 + 1 - z} \begin{matrix} \leq \\ > \end{matrix} \frac{\kappa_2^2}{\xi_2^2 + z}$$

An increase in strategic complementarities increases the weight assigned to the signal with the largest ratio of common to private variances. Because common variance is less detrimental to utility when strategic complementarities are stronger, the most common and least private signal becomes more weighted following an increase in strategic complementarities.

The second overreaction due to the indirect effect of strategic complementarities on the equilibrium action through its effect on equilibrium attention results from variations of z in (8). The conditions for an increase in strategic complementarities to bring about a change in equilibrium attention z are given in (12) and (13). An increase in z from 0 to 1 yields an overreaction in ω of $1/((1 - \gamma)(\kappa_1^2 + \kappa_2^2) + \xi_1^2 + \xi_2^2 + 1)$.

Double overreaction is illustrated in Figure 1. It plots attention z (dotted line) and weight ω (solid line) assigned to signal $x_{1,i}$ when $\kappa_1^2 = 2$, $\kappa_2^2 = 0.5$ and $\xi_1^2 = 0.5$ as a function of ξ_2^2 for two degrees of strategic complementarities, $\gamma = 0$ (black) and 0.85 (blue).

When $\xi_2^2 = 0.375$, $\nu_1 > \nu_2$ and optimal attention is $z^* = 0$. Both signals have the same ratio of common to private variances $\kappa_1^2/(\xi_1^2 + 1) = \kappa_2^2/\xi_2^2$, which means that strategic complementarities do not lead to overreaction to any of the two signals: both with $\gamma = 0$ and 0.85, it is optimal to give zero attention and a weight of 0.2 to $x_{1,i}$ in the game.

When $\xi_2^2 < 0.375$, attention $z^* = 0$ remains optimal for both γ , meaning that strategic complementarities do not affect agents' information acquisition. However, signal $x_{2,i}$ is more common and less private than signal $x_{1,i}$, $\kappa_1^2/(\xi_1^2 + 1) < \kappa_2^2/\xi_2^2$, so that strategic complementarities entail an underreaction to $x_{1,i}$ (i.e. an overreaction to $x_{2,i}$), illustrated by the surface A. This overreaction is due to the direct effect of strategic complementarities on optimal action, not on optimal attention.

Similarly, surfaces B and C illustrate parameter configurations where strategic complementarities do not affect optimal attention and where overreaction (now to $x_{1,i}$) results solely from the direct effect of strategic complementarities on optimal action. When $0.375 < \xi_2^2 < 0.725$, $z = 0$ is optimal for both values of γ , when $\xi_2^2 > 2$, $z = 1$ is optimal.

Double overreaction arises when strategic complementarities also affect optimal attention. This occurs when $0.725 < \xi_2^2 < 2$. Overreaction can be split between the effect of strategic complementarities on optimal action and their effect on optimal attention.

Surfaces D and E show the overreaction due to the effect of strategic complementarities on action when ignoring their effect on attention. Surface D shows the overreaction due to the effect of strategic complementarities on action when optimal attention for $\gamma = 0$, i.e. $z = 0$, also applies to the game with strategic complementarities $\gamma = 0.85$. Surface E shows the overreaction due to the effect of strategic complementarities on action when optimal attention for $\gamma = 0.85$, i.e. $z = 1$, also applies to the game without strategic complementarities $\gamma = 0$. Finally, surface F shows the overreaction due to the effect of strategic complementarities on optimal attention.

3.5 Theoretical predictions

Theoretical predictions focus on the optimal equilibrium because, as will become clear in Sections 4 and 5, this is the only relevant equilibrium given the parameter values in the experiment.¹ The beauty-contest game with information acquisition derived above

¹As explained in Section 4, given the parameter values in the experiment, the suboptimal equilibrium is very unlikely as it requires that all other participants in the experiment choose an attention z of less

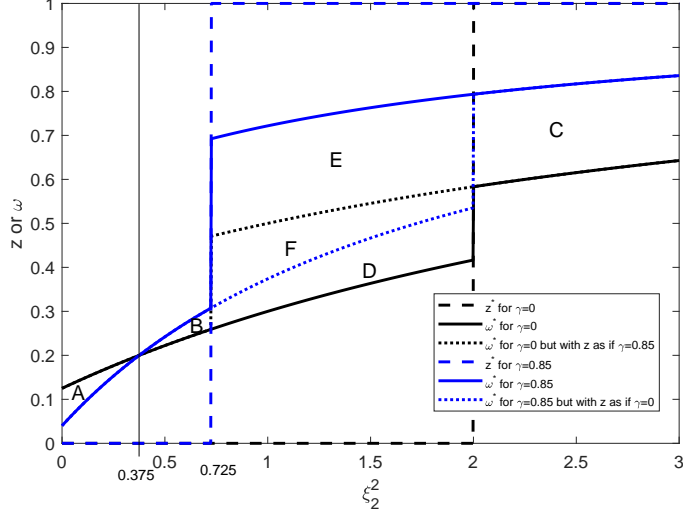


Figure 1: Decomposition of double overreaction for $\kappa_1^2 = 2$, $\kappa_2^2 = 0.5$ and $\xi_1^2 = 0.5$ as a function of ξ_2^2

provides several results that can be tested in the laboratory experiment. These results are summarised as follows:

Proposition 1 (attention) *Full attention must be given to the signal with the lowest adjusted variance of error terms $(1 - \gamma)\kappa^2 + \xi^2$. An increase in strategic complementarities γ tends to shift attention to the signal with the highest common noise and the lowest private noise.*

Proposition 2 (double overreaction) *Strategic complementarities give rise to a double overreaction to the signal with the highest common noise and the lowest private noise (i) through an increase in its weight in equilibrium action over its weight in the first-order expectation of the fundamental, and (ii) through an increase in equilibrium attention and in its weight in the first-order expectation.*

Proposition 3 (attention-action relation) *The optimal weight assigned to a signal in the beauty-contest increases with the attention given to it. Full attention is given to the signal with the greatest weight in the action.*

3.6 Exogenous information

In the experiment, to assess the role of information acquisition, the beauty-contest game with information acquisition is compared to the beauty-contest game with exogenous information. We thus present the theoretical counterpart under exogenous information. The variance of the common (*sender*) noise of signals $x_{1,i}$ and $x_{2,i}$ under exogenous information

than 2.25% on average for $z = 0$ to be optimal from an individual participant's standpoint. In Section 5, we indeed check that such a situation never occurred in the experiment.

is identical to that under information acquisition. By contrast, the private (*receiver*) noise of signals $x_{1,i}$ and $x_{2,i}$ is given by $\epsilon_{1,i} \sim N(0, \sigma_{\epsilon,1}^2)$ and $\epsilon_{2,i} \sim N(0, \sigma_{\epsilon,2}^2)$, and independently distributed across agents. The variances of the private noises $\sigma_{\epsilon,1}^2$ and $\sigma_{\epsilon,2}^2$ are exogenous, that is independent of the attention given to the signals.

The equilibrium action and expected utility are equivalent to those under endogenous information after substituting the exogenous variance of idiosyncratic noises for the endogenous variance. Equilibrium action is determined by the weight assigned to the signal $x_{1,i}$

$$\omega^* = \frac{(1-\gamma)\kappa_2^2 + \sigma_{\epsilon,2}^2}{(1-\gamma)(\kappa_1^2 + \kappa_2^2) + \sigma_{\epsilon,1}^2 + \sigma_{\epsilon,2}^2} \quad (16)$$

and unconditional expected utility is

$$\mathbb{E}_i(u) = -\frac{((1-\gamma)\kappa_1^2 + \sigma_{\epsilon,1}^2)((1-\gamma)\kappa_2^2 + \sigma_{\epsilon,2}^2)}{(1-\gamma)(\kappa_1^2 + \kappa_2^2) + \sigma_{\epsilon,1}^2 + \sigma_{\epsilon,2}^2} \quad (17)$$

3.7 Expectation elicitation

The equilibrium weight ω given in (8) can also be expressed as a function of the expectation of the fundamental θ and the expectation of others' action \bar{a}_{-i} . This is useful for decomposing the causes behind possible deviations of the realised ω in the experiment from the optimal equilibrium. Participants might behave suboptimally because they form biased expectations of the fundamental and of others' action.

The optimal weight assigned to $x_{1,i}$ in the expectation of the fundamental θ , f , is given by

$$\mathbb{E}_i(\theta) = \underbrace{\frac{\kappa_2^2 + \xi_2^2 + z_i}{\kappa_1^2 + \kappa_2^2 + \xi_1^2 + \xi_2^2 + 1}}_f x_{1,i} + \underbrace{\frac{\kappa_1^2 + \xi_1^2 + 1 - z_i}{\kappa_1^2 + \kappa_2^2 + \xi_1^2 + \xi_2^2 + 1}}_{1-f} x_{2,i}$$

and because we have

$$\begin{aligned} \mathbb{E}_i(x_1) &= \underbrace{\frac{\kappa_1^2 + \kappa_2^2 + \xi_2^2 + z_i}{\kappa_1^2 + \kappa_2^2 + \xi_1^2 + \xi_2^2 + 1}}_{\Omega_1} x_{1,i} + \underbrace{\frac{\xi_1^2 + 1 - z_i}{\kappa_1^2 + \kappa_2^2 + \xi_1^2 + \xi_2^2 + 1}}_{1-\Omega_1} x_{2,i} \\ \mathbb{E}_i(x_2) &= \underbrace{\frac{\xi_2^2 + z_i}{\kappa_1^2 + \kappa_2^2 + \xi_1^2 + \xi_2^2 + 1}}_{\Omega_2} x_{1,i} + \underbrace{\frac{\kappa_1^2 + \kappa_2^2 + \xi_1^2 + 1 - z_i}{\kappa_1^2 + \kappa_2^2 + \xi_1^2 + \xi_2^2 + 1}}_{1-\Omega_2} x_{2,i}, \end{aligned}$$

the optimal weight assigned to $x_{1,i}$ in the expectation of others' action \bar{a}_{-i} , o , is

$$\mathbb{E}_i(\bar{a}_{-i}) = \underbrace{(\omega \cdot \Omega_1 + (1-\omega)\Omega_2)}_o x_{1,i} + \underbrace{(\omega(1-\Omega_1) + (1-\omega)(1-\Omega_2))}_{1-o} x_{2,i}.$$

Given the expectations of the fundamental and of others' action, the equilibrium action

can be expressed as

$$\begin{aligned} a_i &= (1 - \gamma)\mathbb{E}_i(\theta) + \gamma\mathbb{E}_i(\bar{a}_{-i}) \\ &= \underbrace{((1 - \gamma)f + \gamma \cdot o)}_{\omega} x_{1,i} + \underbrace{((1 - \gamma)(1 - f) + \gamma(1 - o))}_{1-\omega} x_{2,i} \end{aligned}$$

The expectations elicited in the experiment can be compared with the theoretical expectations (that are consistent with the optimal equilibrium action) in order to disentangle the various causes of deviations from equilibrium.

4 The experiment

One may question whether the theoretical predictions above hold in practice, when *homines sapientes* are involved in the beauty-contest instead of *homines oeconomici*. A natural way to test this issue is to run a laboratory experiment which implements a beauty-contest with information acquisition, as real data may be difficult to collect. The theoretical model in Section 3 is perfectly suited to run an experiment.² This section presents the experimental treatments and parameter values, the theoretical optimal behaviour, and the general procedure of the experiment.

4.1 Treatments and parameters

We run three experimental treatments. Treatment A consists of the beauty-contest game with exogenous information. Treatment B consists of the beauty-contest game with information acquisition. Treatment C consists of a fundamental-estimation game with information acquisition without strategic complementarities. In every treatment, the fundamental state θ is drawn randomly from a uniform distribution on the interval $[50, 950]$.³ The parameter choice and equilibrium values are summarised in Table 1.

Treatment A Treatment A implements as a benchmark the beauty-contest game with exogenous information as described in section 3.6.⁴ Each participant i takes an action a_i to maximise the payoff function in ECU (experimental currency units) given by the formula

$$50 - 1.5(a_i - \theta)^2 - 8.5(a_i - \bar{a}_{-i})^2$$

where \bar{a}_{-i} is the average action of other participants. Under exogenous information, each participant receives two signals on the fundamental θ . The signal x_1 is common to all participants and contains an error term normally distributed with zero mean and

²The experiment deals with a finite number of participants (instead of a continuum); however, we will consider in the experiment that the average action is that of others, implying that there is no theoretical difference between a model with a finite or an infinite number of agents.

³Note that participants were not told about the support of the distribution, avoiding the skewness of the posterior distribution.

⁴It thus replicates the beauty-contest experiment with exogenous information by Baeriswyl and Cornand (2014) and Cornand and Heinemann (2014).

variance $\kappa_1^2 = 2$. The signal $x_{2,i}$ is private to each participant and contains an idiosyncratic error term normally and independently distributed across participants with zero mean and variance $\sigma_{\epsilon,2}^2 = 2$.

In addition to choosing their action a_i , participants are also required to reveal their expectations of the fundamental θ and of the decision of the other participants \bar{a}_{-i} . The elicitation of expectations is rewarded by 10 ECU when the difference between the expected payoff based on these expectations and the realised payoff is not larger than 4.

Treatment B Treatment B implements the beauty-contest game with information acquisition. The parameters are chosen to replicate treatment A at the optimal equilibrium. As in treatment A, each participant i takes an action a_i to maximise the payoff function in ECU given by the formula

$$50 - 1.5(a_i - \theta)^2 - 8.5(a_i - \bar{a}_{-i})^2$$

Under endogenous information, each participant allocates his attention of 1 between the two information sources x_1 and x_2 . The signal $x_{1,i}$ contains an error term common to all participants normally distributed with zero mean and variance $\kappa_1^2 = 2$, and an idiosyncratic error term private to each participant normally and independently distributed across participants with zero mean and variance $1 - z_i$ (i.e. $\xi_1^2 = 0$). The signal $x_{2,i}$ contains only an idiosyncratic error term private to each participant normally and independently distributed across participants with zero mean and variance $1 + z_i$ (i.e. $\xi_2^2 = 1$).

In addition to choosing their attention z_i and their action a_i , participants are also required to reveal their expectations of the fundamental θ and of the decision of the other participants \bar{a}_{-i} . The elicitation of expectations is rewarded by 10 ECU when the difference between the expected payoff based on these expectations and the realised payoff is not larger than 4.

Treatment C Treatment C implements a fundamental-estimation game with information acquisition. Treatment C differs from treatment B only because the payoff function shows no strategic complementarities. The structure of information sources is the same as in treatment B. Each participant i takes an action a_i to maximise the payoff function in ECU given by the formula

$$50 - 10(a_i - \theta)^2$$

Although the expected fundamental should coincide with the action taken, participants are nevertheless required to reveal their expectations of the fundamental θ , for the sake of symmetry with treatments A and B. The elicitation of expectations is rewarded as above.

4.2 Testable implications

The treatment and parameter choices allow to test in the experiment the following theoretical predictions:

Treatment	γ	κ_1^2	κ_2^2	ξ_1^2	ξ_2^2	$\sigma_{\epsilon,1}^2$	$\sigma_{\epsilon,2}^2$	z^*	ω^*	u^*	f	o
A	0.85	2	0	—	—	0	2	—	0.870	-0.261	0.5	0.935
B	0.85	2	0	0	1	—	—	1	0.870	-0.261	0.5	0.935
C	0	2	0	0	1	—	—	0	0.250	-0.750	0.25	—

Table 1: Experiment parameters and equilibrium

1. Full attention in treatment B: full attention $z^* = 1$ to $x_{1,i}$ is the optimal equilibrium; zero attention $z = 0$ is a suboptimal equilibrium, which yields a higher payoff to agent i than choosing $z = 1$ if the average attention of other agents is $\tilde{z}_{-i} < 2.25\%$, as expressed in (15).
2. Zero attention in treatment C: zero attention $z^* = 0$ to $x_{1,i}$ is the optimal equilibrium; there is no suboptimal equilibrium, as expressed in (14).
3. Overreaction in treatments A and B: optimal weight assigned to $x_{1,i}$, $\omega^* = 0.87$, is identical in both treatments and is larger than the weight without strategic complementarities ($\omega|_{\gamma=0} = 0.5$ in A and $\omega|_{\gamma=0} = 0.25$ in B).
4. No overreaction in treatment C: optimal weight assigned to $x_{1,i}$, $\omega^* = 0.25$, is equal to the weight in the first-order expectation of the fundamental.
5. Relationship between z and ω in treatments B and C: according to (11), attention and weight are strategic complements.

The upper graph in Figure 2 shows optimal weight ω as a function of attention z in treatment B (solid blue line, $\gamma = 0.85$) and in treatment C (dashed black line, $\gamma = 0$). Optimal weight ω increases linearly with attention z . The lower graph plots the corresponding utility. It also illustrates the fact that while multiple equilibria can occur with strategic complementarities (treatment B), the suboptimal equilibrium is very unlikely given the parameter chosen for the experiment. The blue solid line plots the expected utility of each agent when they all play z and $\omega(z)$. The blue dotted line plots the expected utility of playing the optimal equilibrium $z^* = 1$ and $\omega^* = 0.870$, while all others play z and $\omega(z)$. Playing the optimal equilibrium yields a superior payoff, unless the average attention of all the others is below 2.25%, which never occurs in the experiment. It is therefore always preferable to play $z = 1$ independently of the others.

Without strategic complementarities, the same exercise comparing the black line and the black dotted line shows that the optimal equilibrium is always unique.

4.3 Procedure

Each experimental session consists of two games and each game corresponds to a treatment. Each game is repeated 15 periods (hence a total of 30 periods per session). Participants played within the same group of participants during the whole length of the experiment

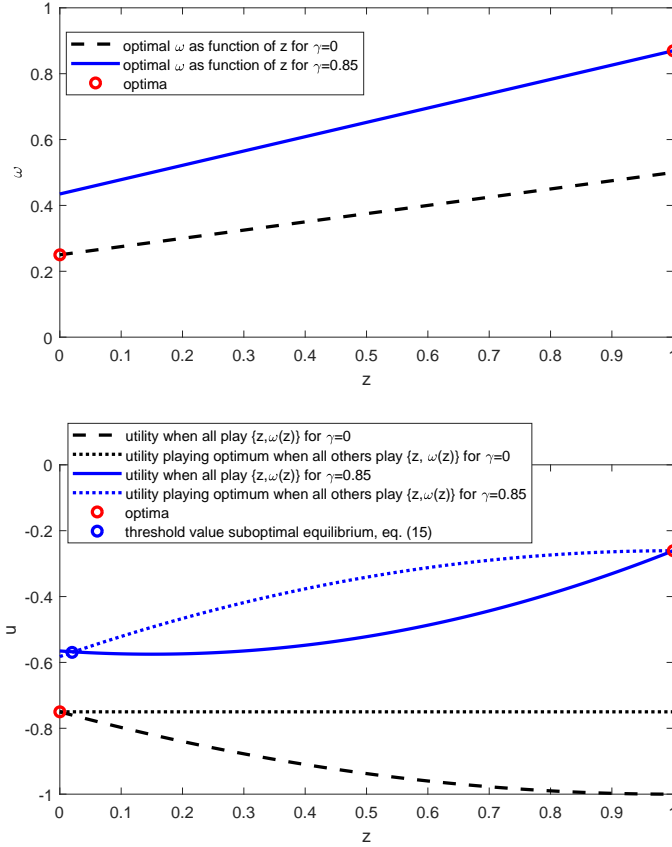


Figure 2: Optimal ω and utility as function of z for $\kappa_1^2 = 2$, $\kappa_2^2 = 0$, $\xi_1^2 = 0$ and $\xi_2^2 = 1$

Session	Date	Groups	Periods 1-15	Periods 16-30
1	19 December 2018	1-3	A	B
2	14 January 2019	4-6	A	B
3	22 January 2019	7-9	B	A
4	23 January 2019	10-12	B	A
5	10 January 2019	13-15	C	B
6	15 January 2019	16-18	C	B
7	22 January 2019	19-21	B	C
8	23 January 2019	22-24	B	C

Table 2: Experimental sessions and treatments

and did not know the identity of the other participants of their group. In each session, the 18 participants were divided into three independent groups of 6 participants (in order to get 3 observations per session).

Sessions were run in December 2018 and January 2019 at the LEES (Laboratoire d'Economie Expérimentale de Strasbourg).⁵ Participants were mainly students from Strasbourg University (most were students in economics, mathematics, biology and psychology). Participants were seated in random order at PCs. Instructions were then read aloud and

⁵The project has been approved by the GATE-Lab local IRB.

questions answered in private. Throughout the sessions, students were not allowed to communicate with one another and could not see each others screens. Each participant could only participate in one session. Before starting the experiment, participants were required to answer a few questions to ascertain their understanding of the instructions. Examples of instructions and screens are reported in the Appendix. After each period, participants were informed about the true fundamental, their partners' decision and their payoff. Information about past periods from the same game (including signals and their own decisions) was displayed during the decision phase on the lower part of the screen.

At the end of each session, 2 periods per game were selected to calculate payoffs. 100 ECU were converted into 12 euros. Payoffs ranged from 14 to 29 euros. The average payoff was about 24 euros. Sessions lasted for around 90 minutes. In every period, it was not possible to earn negative payoffs.

5 Experimental results

The results of the laboratory experiment are presented according to the following structure. In section 5.1, we analyse the average behaviour across groups. In section 5.2, we decompose overreaction, overweight and underweight according to various possible sources of error. Section 5.3 compares participants' individual behaviour between treatments.

Statistical tests are based on Wilcoxon rank sum tests when comparing observed data to theoretical predictions and on Wilcoxon matched-pairs signed rank tests for between treatment tests.

5.1 Overall outcome and treatment comparison at the group level

Table 3 reports the average attention z and the average weight ω realised in each group over all periods. Figure 3 plots the average attention z and the average weight ω realised in every period across all groups in treatments A, B and C.⁶ Figure 4 plots the distribution of attention z and weight ω realised in each period by each participant.

5.1.1 Measuring attention

In accordance with theory, the attention given to signal $x_{1,i}$ is higher in B than in C ($p = 0.001$). Strategic complementarities have the expected effect on attention. However, attention is not as strong as the theory predicts (i.e. $z < 1$) in B and not as weak as the theory predicts (i.e. $z > 0$) in C ($p = 0.000$, $p = 0.000$).

The attention given to signal $x_{1,i}$ depends on the order of play of treatments. The triangles and squares in the upper panel of Figure 3 display the realised attention depending on whether the treatment in question comes first or second in order of play. The blue (black) triangles display the realised attention when the treatment B (C) is played first.

⁶The incentives to converge were not very strong around equilibrium. The sensitivity of payoff to deviations from equilibrium attention z and equilibrium weight ω is shown in Appendix A. This explains why we do not observe much learning. Figure 17 reported in Appendix B analyses convergence.

The blue (black) squares display the realised attention when the treatment B (C) is played second. The attention chosen in the first treatment tends to affect the attention chosen in the second treatment. Playing the low-attention treatment C first induces participants to choose a lower attention in B than when B is played first. Conversely, playing the high-attention treatment B first induces participants to choose a higher attention in C than when C is played first. This order effect does not however challenge the effect of strategic complementarities on attention: attention is always higher in B than in C, independently of the order of play.

The analysis of realised attention shows that the suboptimal equilibrium derived in section 3.3.2 is never relevant in the experiment. As exposed in section 4.2, the suboptimal equilibrium $z = 0$ yields a higher payoff in B than the optimal equilibrium $z^* = 1$ only if the others' average attention realised in B is below the threshold value of 2.25%. Figure 18 in Appendix B shows however that the lowest average attention realised in treatment B within a group in any period was 45.8%. This implies that it has never been optimal for

	Attention z		Weight ω		
	B	C	A	B	C
Group 1	0.777	—	0.776	0.708	—
Group 2	0.550	—	0.681	0.638	—
Group 3	0.776	—	0.617	0.546	—
Group 4	0.744	—	0.749	0.759	—
Group 5	0.646	—	0.588	0.637	—
Group 6	0.665	—	0.578	0.544	—
Group 7	0.918	—	0.708	0.657	—
Group 8	0.849	—	0.744	0.735	—
Group 9	0.766	—	0.840	0.810	—
Group 10	0.706	—	0.529	0.508	—
Group 11	0.773	—	0.650	0.673	—
Group 12	0.830	—	0.645	0.622	—
Group 13	0.702	0.361	—	0.562	0.379
Group 14	0.718	0.244	—	0.447	0.399
Group 15	0.589	0.388	—	0.525	0.311
Group 16	0.867	0.280	—	0.678	0.377
Group 17	0.695	0.329	—	0.612	0.402
Group 18	0.803	0.157	—	0.820	0.231
Group 19	0.672	0.336	—	0.579	0.476
Group 20	0.880	0.344	—	0.612	0.437
Group 21	0.864	0.436	—	0.622	0.416
Group 22	0.775	0.159	—	0.660	0.328
Group 23	0.903	0.442	—	0.666	0.328
Group 24	0.833	0.517	—	0.705	0.326
Average	0.763	0.333	0.675	0.639	0.368
Equilibrium	1	0	0.870	0.870	0.250

Table 3: Realised attention and weight

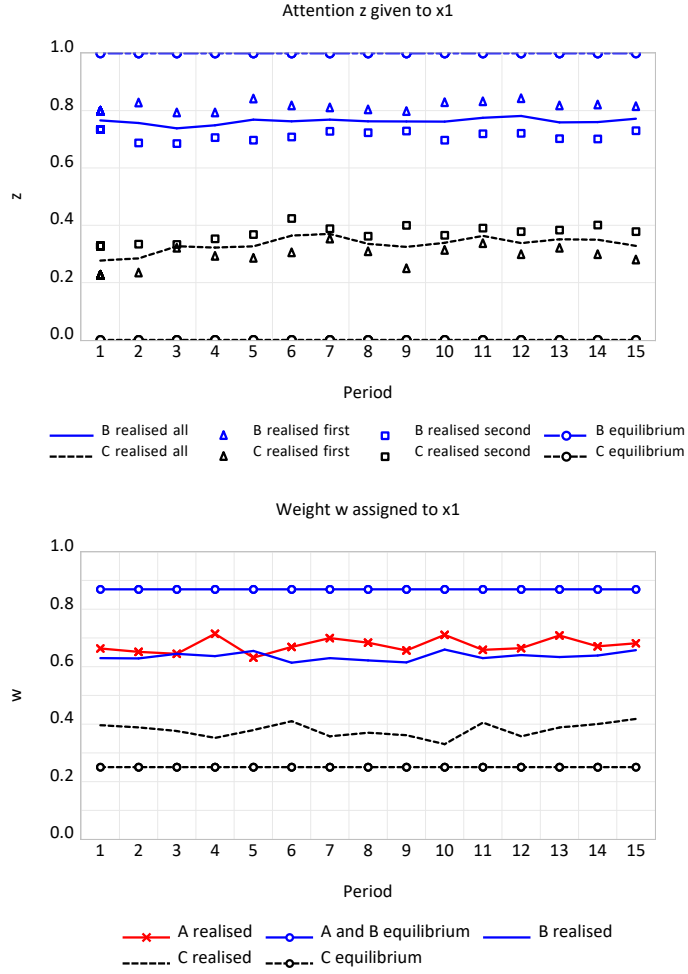


Figure 3: Averaged realised attention z and weight ω given to $x_{1,i}$

any participant in the experiment to choose $z = 0$ at any period of treatment B, making the suboptimal equilibrium irrelevant for our analysis.

Result 1 (a) *Introducing strategic complementarities induces participants to pay more attention to $x_{1,i}$, in accordance with theory. Attention is however lower than the theory predicts under strategic complementarities (treatment B) and higher than the theory predicts without strategic complementarities (treatment C).* (b) *The suboptimal equilibrium under strategic complementarities (treatment B) is irrelevant in the experiment because the lowest average attention in any period is always higher than the threshold value at which the suboptimal-equilibrium attention $z = 0$ dominates the optimal-equilibrium attention $z = 1$.*

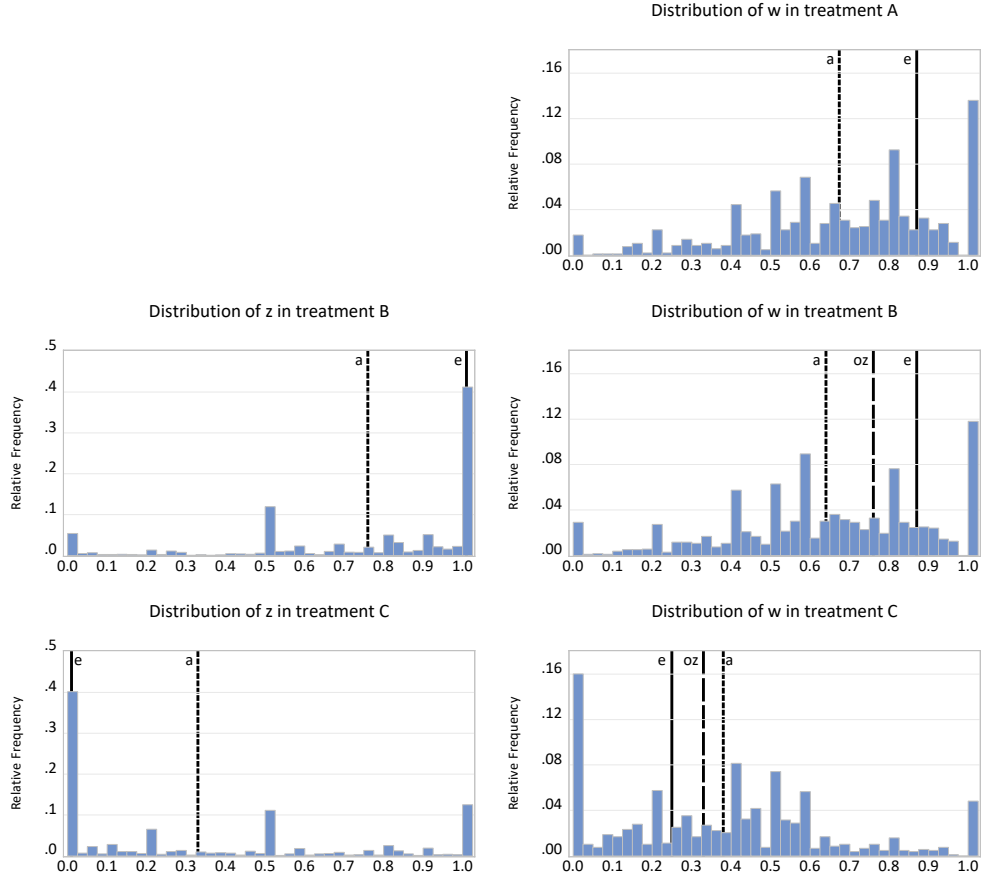


Figure 4: Distribution of attention z and weight ω ; e: equilibrium; a: realised average; oz: average optimum given individually realised z .

5.1.2 Measuring overreaction

In treatment A, participants overreact to the public signal x_1 but not as much as the theory predicts.⁷ Participants overreact to x_1 because the realised weight ω is significantly larger than the weight on x_1 without strategic complementarities, which corresponds to the weight in the first-order expectation of the fundamental, 0.5 ($p = 0.000$). However, participants underweight x_1 because the realised weight ω is significantly smaller than the equilibrium weight in the beauty-contest, 0.87 ($p = 0.000$). The average weight $\omega = 0.66$ is close to the weight of 0.67 realised in a similar experiment by Baeriswyl and Cornand (2014).

In treatment B, participants also overreact to the public signal x_1 but, again, not as much as the theory predicts. The realised weight ω is significantly larger than the weight on x_1 without strategic complementarities, 0.25 ($p = 0.000$). The weight ω is significantly smaller than the equilibrium weight of 0.87 ($p = 0.000$). However, the weight ω is smaller than in A, albeit their equilibrium value is the same. The average weight amounts to 0.62

⁷With regard to terminology, recall that ‘overreaction’ refers to the theoretical effect that strategic complementarities exert on the equilibrium weight ω . By contrast, we use the terms ‘overweight’ and ‘underweight’ to refer to deviations from equilibrium of the weight realised in the experiment.

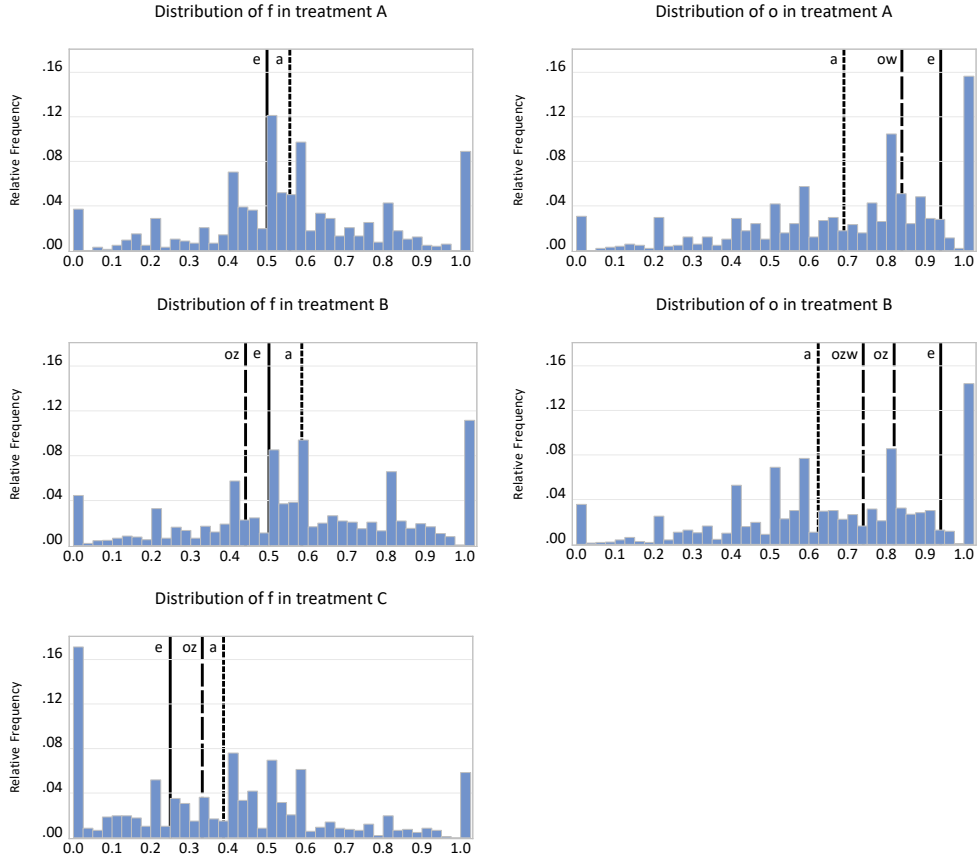


Figure 5: Distribution of weights assigned to $x_{1,i}$ in the expectations of fundamental θ and of others' action \bar{a}_{-i} ; e: equilibrium; a: realised average; oz: average optimum given individually realised z ; ozw: average optimum given individually realised z and ω .

in B, compared to 0.66 in A ($p = 0.084$).

In treatment C, participants overweight signal $x_{1,i}$. The realised weight ω is significantly greater than its equilibrium value of 0.25 ($p = 0.000$). In accordance with theory, it is significantly smaller than in B ($p = 0.001$).

Result 2 (a) Under strategic complementarities (treatments A and B), participants overreact to $x_{1,i}$ as they assign a larger weight to it than without strategic complementarities. (b) Under strategic complementarities (in treatments A and B), participants underweight $x_{1,i}$ as they assign a smaller weight to it than theory predicts. (c) The weight is lower with information acquisition (treatment B) than without information acquisition (treatment A). (d) Without strategic complementarities (treatment C), participants overweight $x_{1,i}$ as they assign a larger weight to it than theory predicts.

5.2 Decomposing overreaction, overweight and underweight

The deviations of the realised weight ω from the optimal equilibrium can be decomposed into four possible sources of error. The realised weight can deviate from equilibrium because

- participants pay a suboptimal attention to $x_{1,i}$,
- participants form a suboptimal expectation of the fundamental given their information,
- participants form a suboptimal expectation of the average action of others given their information, or
- participants chose a suboptimal weight on $x_{1,i}$ in their action given their expectation about the fundamental and others' action.

Figure 5 plots the distribution of realised weight assigned to $x_{1,i}$ in the expectation of the fundamental, f , and in the expectation of others' action, o , for treatments A, B and C; their theoretical value is derived in section 3.7.

5.2.1 Treatment A

Figure 4 (upper right panel) plots the relative frequency of weights ω in treatment A. The distribution exhibits a focal point in $\omega = 1$. Realised weights are mainly concentrated between 0.5 and 1. As already emphasised, the realised average ω in treatment A is significantly lower than its equilibrium value. Figure 6 breaks down the deviation of realised ω from equilibrium by the four types of error.

Error in attention In treatment A, because information is exogenous, there is no attention to allocate, so that suboptimal attention cannot be the cause of a suboptimal equilibrium. Signal $x_{1,i}$ is always common knowledge, while signal $x_{2,i}$ is fully private.

Error in the expectation of the fundamental Although expectations of the fundamental θ are biased upward, they contribute very little to the suboptimal realised weight ω . Figure 5 (upper left panel) plots the relative frequency of f , the weight assigned to $x_{1,i}$ in the expectation of the fundamental θ . Errors in the expectation of the fundamental only contribute to raise the beauty-contest weight by 0.8 pp from optimal equilibrium because the contribution of the quadratic distance to the fundamental is small in the utility function (1.5) in comparison to the quadratic distance to the average action of the others (8.5).

The too large weight in the expectation of the fundamental can be explained by the positive correlation observed between the weight in fundamental expectation and the weight chosen in action. Participants tend to weight $x_{1,i}$ more in the expectation of the fundamental f when they weight it more in the beauty-contest action ω , albeit in theory f is

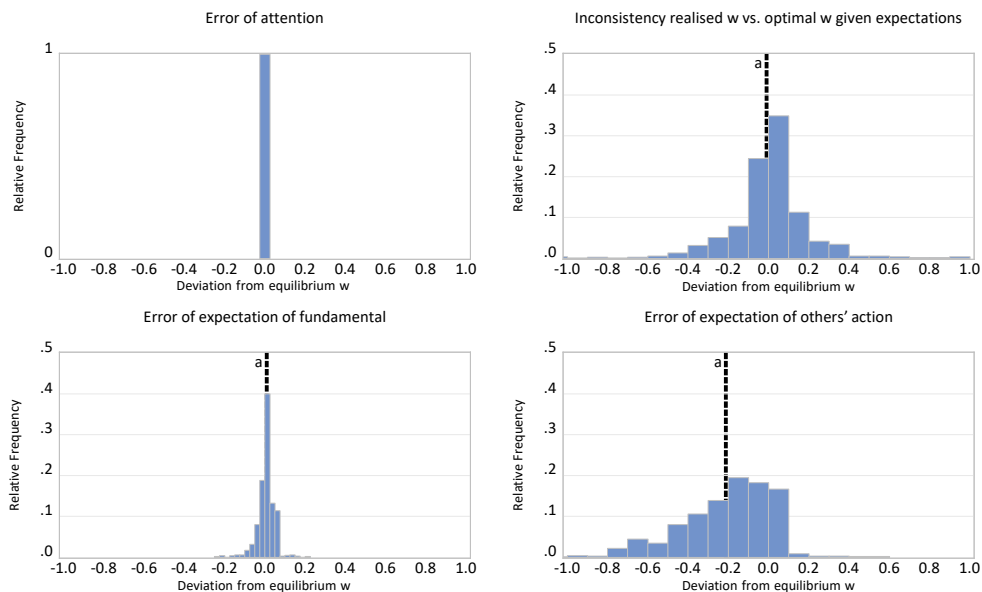


Figure 6: Decomposition of deviation of realised ω from equilibrium by error types in treatment A

independent of ω (see Figure 8 (a) in Appendix B). That $\omega > 0.5$ in average can explain why realised f is larger than optimal.

Error in the expectation of the actions of others The error in the expectation of others' action contributes to the largest part of the deviation of realised ω from optimal equilibrium. The upper right panel of Figure 5 plots the relative frequency of weight assigned to $x_{1,i}$ in the expectation of others' action for treatment A. Because the contribution of the quadratic distance to others' action is large (8.5) in the utility function, errors in expectation of others' action contribute to reduce the beauty-contest weight by 21.2 pp from optimal equilibrium.

The realised average weight o is below the equilibrium value (although $\omega = 1$ is focal) and below the weight conditional on participants' individually realised weight (ow), meaning that the deviation of o from equilibrium cannot only be explained by the weight ω realised individually. As theory predicts, participants tend to weight $x_{1,i}$ more in the expectation of others' action o when they weight it more themselves in action ω . The positive realised relationship between ω and o , and the positive theoretical relationship between both is depicted on Figure 19 (b) in Appendix B.

The realised average weight o can be expressed in terms of limited level of reasoning, which captures the importance agents attribute to the coordinating motive with the others. It is measured by the number of iterations about common information that agents operate when they make their expectation. Appendix C derives the weight assigned to the signal $x_{1,i}$ in an agent's expectation of the others' action for limited levels of reasoning. The average realised weight o is close to the weight a level-2 player would attribute to $x_{1,i}$ in

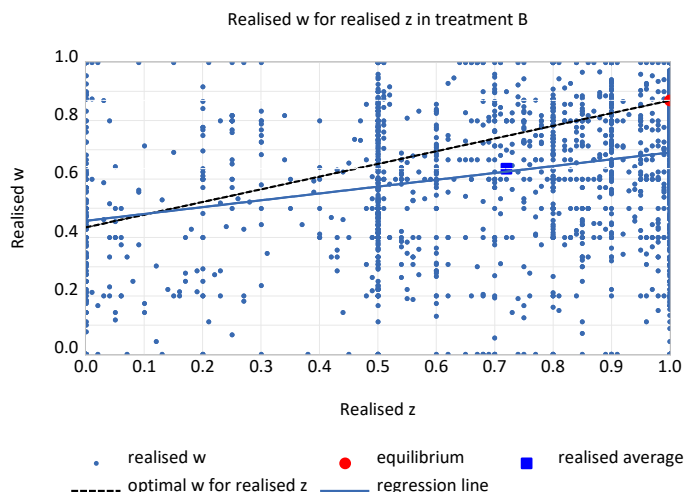


Figure 7: Realised ω for realised z in treatment B

his expectation of others' action.⁸

Inconsistency between expectations and action The weight ω in action is not systematically inconsistent with participants' expectation of the fundamental and of others' action. Figure 6 (upper right panel) shows that the inconsistency between the elicited expectations f and o , and the chosen action ω only accounts for 0.8 pp of the deviation in treatment A.

Result 3 (a) *Without information acquisition (treatment A), the public signal plays a focal role.* (b) *The too low weight realised in the beauty-contest ω can be explained by the too low expectation about the action of others.*

5.2.2 Treatment B

Figure 4 (middle panels) presents the relative frequency of realised attention z and realised weight ω on $x_{1,i}$ in the beauty-contest action in treatment B. As in treatment A, the distribution exhibits a focal point in $\omega = 1$ and the realised average weight is significantly lower than its equilibrium value. Figure 7 illustrates the realised z and ω in B for each of the 1080 individual decisions (12 groups x 6 participants x 15 periods). As theory predicts in (11), participants tend to play a higher weight ω when they have chosen a higher attention z . The realised relationship (solid blue line) is nevertheless weaker than the theoretical one (dashed black line). Figure 8 decomposes the deviation of realised ω from equilibrium by error types.

⁸Accordingly, the average weight ω realised in treatment A is not significantly different from level-2 of reasoning ($p = 0.208$), while levels-1 and 3 can be rejected. This finding is in line with Baeriswyl and Cornand (2014) and Cornand and Heinemann (2014) but slightly below the level found by Nagel (1995) in a pure guessing game.

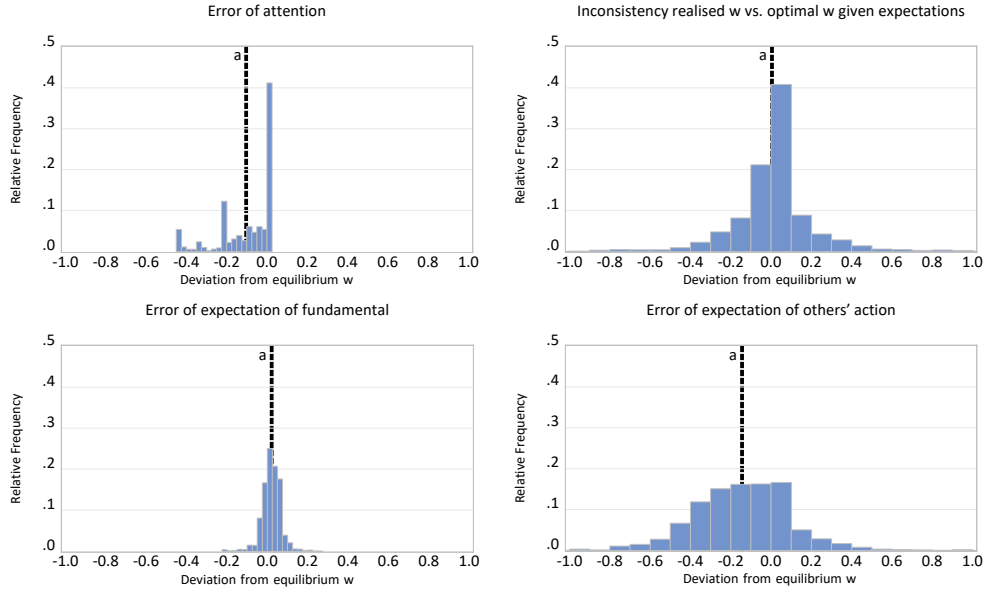


Figure 8: Decomposition of deviation of realised ω from equilibrium by error types in treatment B

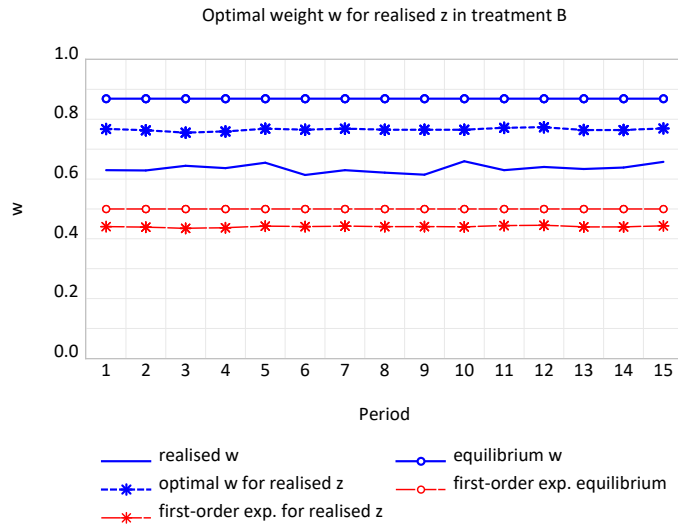


Figure 9: Optimal ω for realised z in treatment B

Error in attention A large part of the negative deviation of realised ω from equilibrium can be explained by a too low realised attention z . The middle left panel in Figure 4 shows that more than 40% of subjects play the equilibrium attention $z = 1$. More than 10% play the mid-point $z = 0.5$ and about 5% attributed no attention $z = 0$ to $x_{1,i}$. Other realised z are mainly distributed between 0.5 and 1. Average realised attention is lower in B than theory predicts, meaning that $x_{1,i}$ contains a private noise. Figure 8 (upper left panel) shows that errors in attention contribute to reduce the beauty-contest weight by 10.2 pp from optimal equilibrium.

Figure 9 provides an alternative illustration of the effect of realised attention z on

the average optimal weight ω per period. Given realised attention z , the optimal weight assigned to $x_{1,i}$ in the beauty-contest (blue stars) as well as in the first-order expectation of fundamental (red stars) are significantly below their equilibrium value (blue and red circles).

Error in the expectation of the fundamental As in treatment A, errors in the expectation of the fundamental do not account for much of the suboptimal weight on $x_{1,i}$ in treatment B. The middle left panel of Figure 5 plots the relative frequency of weight f assigned to $x_{1,i}$ in the expectation of the fundamental θ . The realised average weight $f = 0.58$ is larger than its equilibrium value (0.5). However, as realised attention z is lower than equilibrium attention, it would have been optimal to choose a lower $f = 0.44$. Because the contribution of the quadratic distance from the fundamental is small in the utility function (1.5) in comparison to the quadratic distance to the average action of the others (8.5), errors in the expectation of the fundamental only contribute to raise the beauty-contest weight by 0.5 pp from optimal equilibrium.

The realised weight f increases with the realised attention z as theory predicts, although realised f is systematically higher (see Figure 20 (a) in Appendix B). However, contrary to theoretical predictions, the realised weight f increases also with the part of realised ω that is not due to realised attention, that is with the difference between the realised ω and the optimal ω given the realised z ; this difference captures the variations in realised ω that cannot be explained by realised z (see Figure 20 (b) in Appendix B).

Error in the expectation of the actions of others A large part of the deviation of realised ω from optimal equilibrium can be explained by biased expectations of others' action. The middle right panel of Figure 5 plots the relative frequency of weight assigned to $x_{1,i}$ in the expectation of others' action. The lower right panel of Figure 8 points that errors in forming expectations about other participants' actions contribute to reduce the beauty-contest weight by 14.4 pp from optimal equilibrium.

The realised average weight o is below the equilibrium value (although $\omega = 1$ is focal) and below the weight conditional on participants' individually realised attention (oz) or conditional on both participants' individually realised attention and weight (ozw). Participants tend to weight $x_{1,i}$ in the expectation of others' action more when they attribute more attention to it, as theory predicts, although realised o is systematically lower (see Figure 21 (a) in Appendix B). The realised weight o increases also with the difference between realised ω and optimal ω given realised attention z (see Figure 21 (b) in Appendix B), which indicates that, in line with theoretical predictions, realised o tend to increase with deviations of ω that are not rationalised by the choice of attention z .

As for treatment A, limited levels of reasoning can account for this weight, as it is close to the level-2 weight conditional on realised attention (0.691) (see Appendix C for further details).⁹

⁹Similarly, in treatment B, the average weight ω is not significantly different from level-2 of reasoning

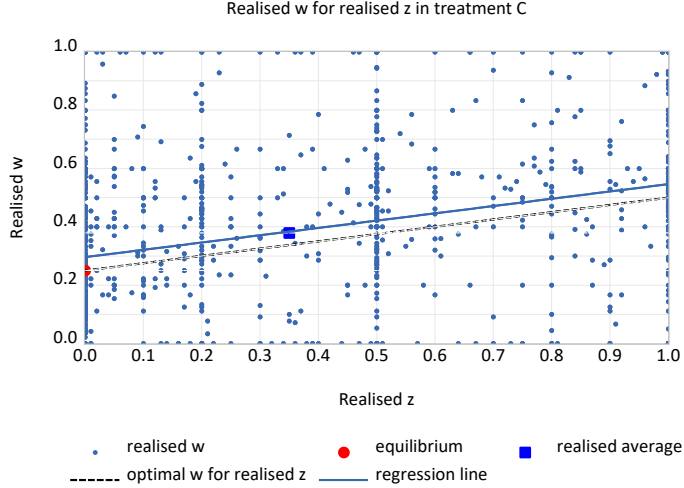


Figure 10: Realised ω for realised z in treatment C

Inconsistency between expectations and action The weight ω in action is not systematically inconsistent with participants' expectation of the fundamental and of others' action. Figure 8 (upper right panel) shows that the inconsistency between the elicited expectations f and o , and the chosen action ω only account for 0.5 pp of the deviation.

Result 4 (a) *Under strategic complementarities and with information acquisition (treatment B), participants increase the weight ω with attention z , but not as much as theory predicts.* (b) *The too low weight realised in the beauty-contest ω can be explained by the realised too low attention z and the too low expectation about others' actions o .*

5.2.3 Treatment C

Figure 4 (lower panels) presents the relative frequency of realised attention z and weight ω on $x_{1,i}$ in the fundamental estimation game in treatment C. The distribution exhibits a focal point in $\omega = 0$ and the realised average weight is significantly higher than its equilibrium value. Figure 10 illustrates the realised z and ω for each of the 540 individual decisions (6 groups x 6 participants x 15 periods). The solid blue line shows that participants tend to increase ω with z , as theory predicts in (11). This line is slightly higher than the dashed black line depicting the optimal weight ω for realised attention z . This means that for their realised z , participants play a weight ω that is slightly larger than what would be optimal. Figure 11 presents the decomposition of deviation from equilibrium weight ω by error types in treatment C.

Error in attention A large part of the positive deviation of realised ω from equilibrium can be explained by a too high realised attention z . The lower left panel of Figure 4 shows

conditional on observed attention z ($p = 0.406$), while unconditional level-2 and conditional levels-1 and 3 can be rejected.

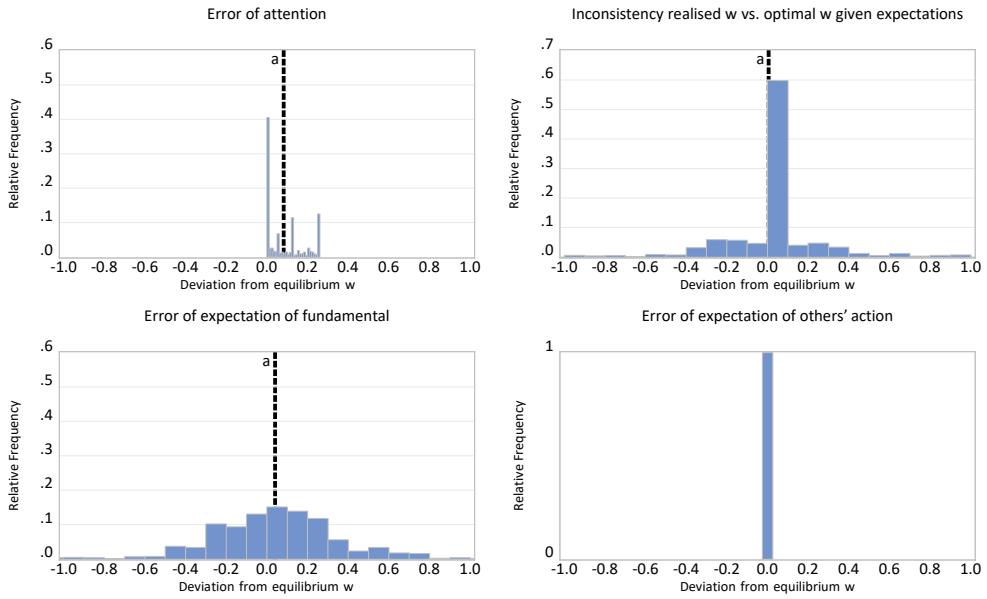


Figure 11: Decomposition of deviation of realised ω from equilibrium by error types in treatment C

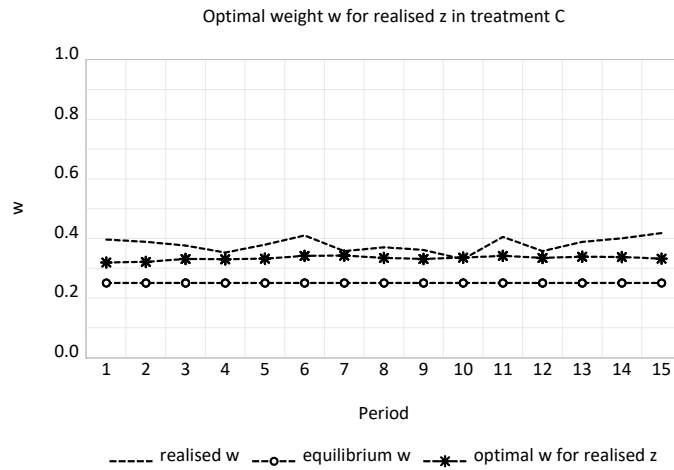


Figure 12: Optimal ω for realised z in treatment C

that while about 40% of participants attribute no attention to signal $x_{1,i}$ in line with theoretical predictions, it seems that $z = 0.5$ and $z = 1$ are focal since more than 20% of subjects play one or the other strategy. Figure 11 (upper left panel) shows that errors in attention contribute to rise the weight ω by 8.3 pp above optimal equilibrium.

Figure 12 provides an alternative illustration of the effect of realised attention z on the average optimal weight ω per period. Given realised attention, the optimal weight assigned to $x_{1,i}$ in the beauty-contest (black stars) is significantly above its equilibrium value (black circles).

Error in the expectation of the fundamental Part of the positive deviation of realised ω from equilibrium can also be explained by errors in fundamental's expectation. The lower panel of Figure 5 plots the relative frequency of weight assigned to $x_{1,i}$ in the expectation of the fundamental. The realised average weight $f = 0.39$ is larger than its equilibrium value (0.25), but in the vicinity of the realised average weight in action $\omega = 0.38$. As realised attention z is larger than equilibrium attention, it would have been optimal to choose on average $f = 0.33$, which is still smaller than realised f but larger than equilibrium f . In contrast to utility functions of treatments A and B, the utility in treatment C only depends on the quadratic distance to fundamentals, implying that errors in forming fundamental expectations matter more than in A and B. Errors in the expectation of the fundamental contribute to raise the weight ω by 4 pp from optimal equilibrium.

The realised weight f increases with the realised attention z as theory predicts (see Figure 22 (a) in Appendix B). However, contrary to theoretical predictions, the realised weight f also increases with the difference between the realised ω and the optimal ω given the realised z , which captures the variations in realised ω that cannot be explained by the choice of z (see Figure 22 (b) in Appendix B).

Error in the expectation of the actions of others The expectation of others' action is not elicited in treatment C because it is irrelevant without strategic complementarities.

Inconsistency between expectations and action The weight ω in action is not systematically inconsistent with participants' expectation of the fundamental. Figure 11 (upper right panel) shows that the inconsistency between the elicited expectations f and the chosen action ω only account for 0.6 pp of the deviation.

Result 5 (a) *Without strategic complementarities (treatment C), participants increase the weight ω with attention z , but slightly more than what theory predicts.* (b) *The too high weight realised in the fundamental-estimation game ω can be explained by the realised too high attention z and the too high expectation about the fundamental f .*

5.2.4 Decomposing double overreaction

As discussed in section 3.4, strategic complementarities give rise to a double overreaction to $x_{1,i}$. Combining the results of sections 5.2.2 and 5.2.3 allows to decompose the double overreaction into the effect of strategic complementarities on the realised attention z and on the realised weight ω , as plotted in Figure 13.

In theory, increasing γ from 0 to 0.85 increases equilibrium attention z from 0 to 1, and the weight on $x_{1,i}$ in the first-order expectation of the fundamental θ from 0.25 to 0.5. Increasing γ from 0 to 0.85 also increases equilibrium weight in the beauty-contest ω to 0.87. Both effects combined, introducing strategic complementarities increases the equilibrium weight assigned to $x_{1,i}$ from 0.25 to 0.87.

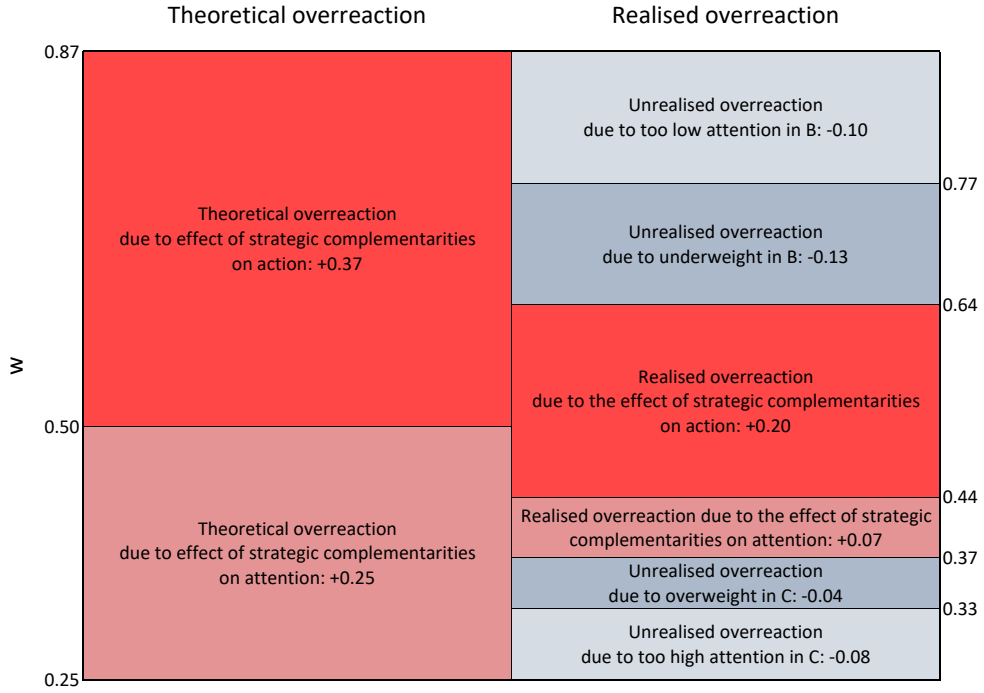


Figure 13: Theoretical and realised decomposition of double overreaction into the effect of strategic complementarities on realised attention z and on realised weight ω

In the experiment, the effect of strategic complementarities is much smaller: the average realised weight of 0.37 in C rises to 0.64 in B. The realised overreaction amounts to about 44% of the theoretical prediction. About half of the unrealised overreaction is caused by the suboptimal allocation of attention (too high in C, too low in B) and the other half by the suboptimal action given realised attention (overweight in C, underweight in B).

Result 6 (a) *Realised double overreaction amounts to about 44% of the theoretical prediction.* (b) *Unrealised double overreaction is caused equally by the suboptimal allocation of attention as by the suboptimal action given the realised attention.*

5.3 Comparison of participants' behaviour between treatments

This section compares participants' behaviour between treatments. It examines whether the behavioural pattern in one treatment predicts the behavioural pattern in another treatment.

The upper panel of Figure 14 plots the average error of each participant in its realised weight ω in B and in A. The average error in the realised weight ω in B is conditional on the realised individual attention in each period. The positive regression line indicates that participants tend to play similarly in B as in A. Those who play a low ω in B also tend to play a low ω in A. This is not really surprising because, apart from the choice of

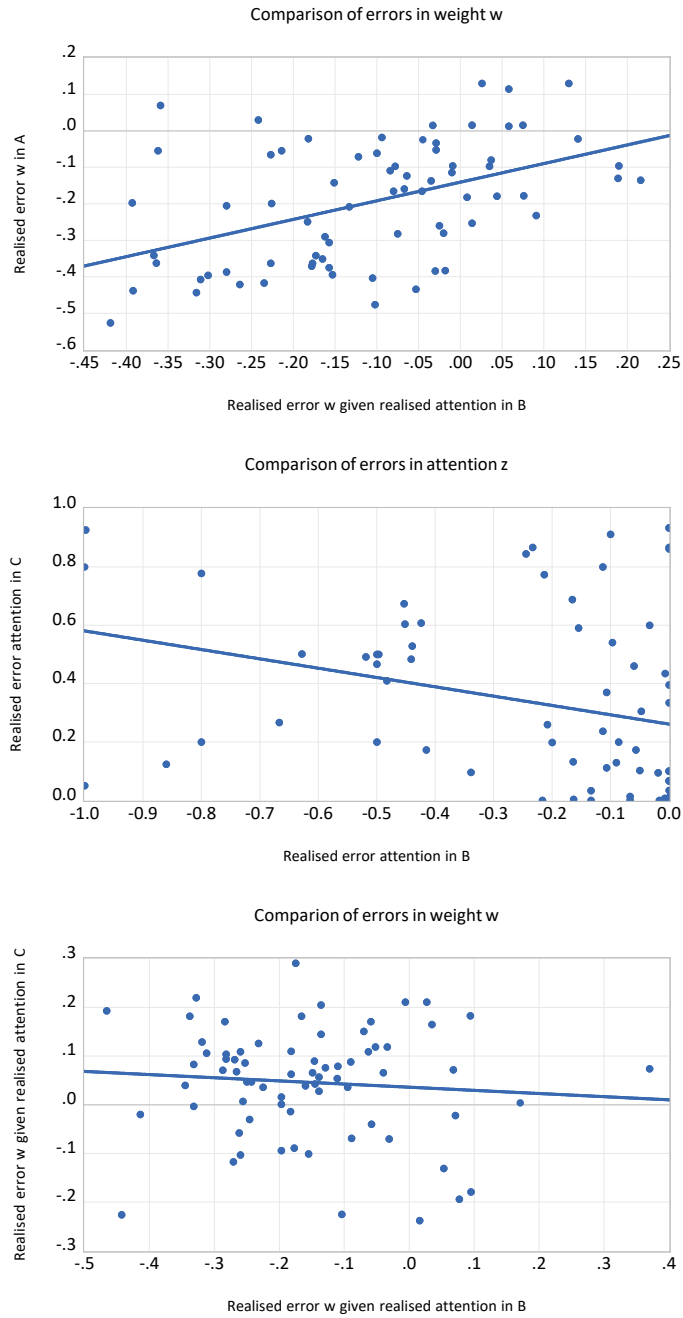


Figure 14: Comparison of realised individual average errors between treatments

attention in B, participants play the same beauty-contest game in A and B.

The middle panel plots the average error of each participant in its realised attention in B and in C. Because optimal attention is 1 in B and 0 in C, errors can only be negative in B and positive in C. The negative regression line indicates that participants who choose a near-optimal attention in B also tend to choose a near-optimal attention in treatment C. Thus, although introducing strategic complementarities shifts optimal attention from 0 to 1, it does not seem to alter the problem of choosing an optimal attention per se.

The lower panel plots the average error of each participant in its realised weight ω in

B and in C. The average error in the realised weight ω in B and in C is conditional on the realised individual attention in each period. The regression line does not indicate a clear relationship between realised errors in B and in C. Participants who perform well in the beauty-contest game do not necessarily perform well in the fundamental-estimation game.

Result 7 (a) *Under strategic complementarities, participants who choose a near-optimal weight ω without information acquisition (treatment A) also tend to choose a near-optimal weight ω with information acquisition (treatment B).* (b) *Participants who choose a near-optimal attention under strategic complementarities (treatment B) also tend to choose a near-optimal attention without strategic complementarities (treatment C).* (c) *Participants who choose a near-optimal weight ω under strategic complementarities (treatment B) do not necessarily choose a near-optimal weight ω without strategic complementarities (treatment C).*

6 Discussion and conclusion

Central bank communication aims to shape public expectations of key macroeconomic factors and to align private sector expectations with the actions of the central bank, enhancing monetary policy effectiveness. The market reaction to disclosures depends as much on how the central bank discloses information as on how much attention market participants give to disclosures. In a beauty-contest model with information acquisition by the private agents, strategic complementarities give rise to a double overreaction mechanism to public disclosures. While strategic complementarities induce agents to give in their action more weight to the most common and least private signal than what would be justified by its face value, they also induce them to give more attention to this signal, further strengthening its weight in action. These theoretical predictions are supported by a laboratory experiment, although the double overreaction is weaker than predicted due to a lower effect of strategic complementarities on both the allocation of attention and the beauty-contest action.

As a weaker attention (than theoretically predicted) explains to a large extent the weaker focal potential of public disclosures (weaker overreaction than theoretically predicted), such limited attention might usefully be accounted for in designing the optimal central bank communication. Though relying on an abstract model, our experiment captures realistic features of the way central banks may communicate in practice. In particular, it offers a rationale for the so-called forward-guidance puzzle, i.e. the fact that macroeconomic responses to forward guidance are much mitigated in comparison to the theoretical predictions of the standard New Keynesian models. Indeed, the experimentally observed weaker attention to common information may rationalise why public disclosures do not fully drive market actions in the direction of announcements. Among the various alternative belief-based interpretations of the forward-guidance puzzle, Angeletos and Lian (2018) attribute it to a lack of common knowledge, Garcia-Schmidt and Woodford (2018)

and Farhi and Werning (2017) to limited level-k of reasoning. If the forward-guidance puzzle is due to a lack of the private sector’s attention, as our experimental results may suggest, central banks should consider how to increase attention. Improving attention operates through model parameters that are not under the control of the central bank in the present model. This would involve, for example, better control of the timing and channels of communication (less frequent to focus attention and vice versa, depending on the context), or improving economic literacy of people to raise their awareness of what monetary policy and forward guidance in particular is all about.

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A Payoff sensitivity

Figures 15 and 16 illustrate the sensitivity of payoff in treatments B and C to deviations from optimal attention and optimal action. The upper graph in figure 15 shows for treatment B the expected utility as a function of z when ω remains at its equilibrium value $\omega = 0.870$ (solid black line) and when ω adjusts to its optimal value relative to z (dotted blue line). The lower graph shows the expected utility as a function of ω when z remains at its equilibrium value $z = 1$ (solid black line) and when z adjusts to its optimal value relative to ω (dotted blue line). The thick lines plot utility when all agents deviate from optimum; the thin lines plot utility when only one agent deviates from optimum.

Figure 16 shows utility as a function of z (upper graph) and ω (lower graph) for treatment C. Without strategic complementarities, deviations from the optimal weight ω by all agents or by only one agent yield the same utility.

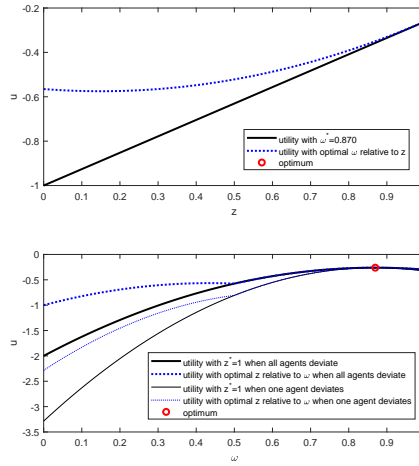


Figure 15: Utility deviation for treatment B

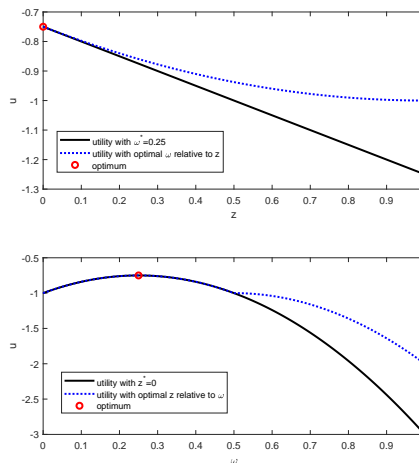


Figure 16: Utility deviation for treatment C

B Additional figures

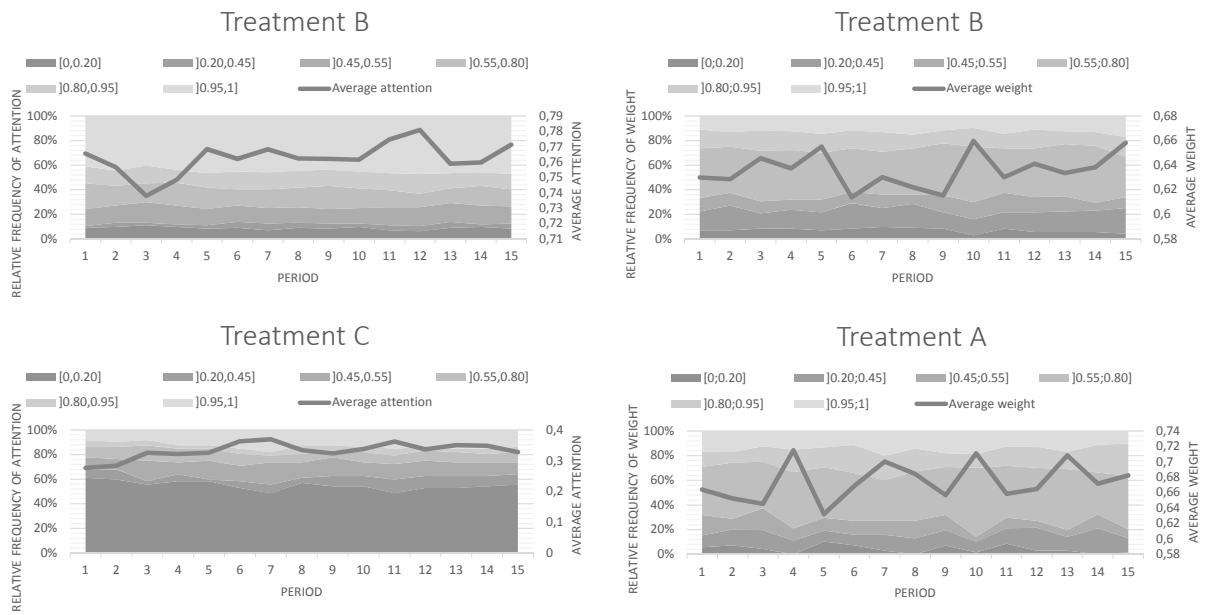


Figure 17: Convergence

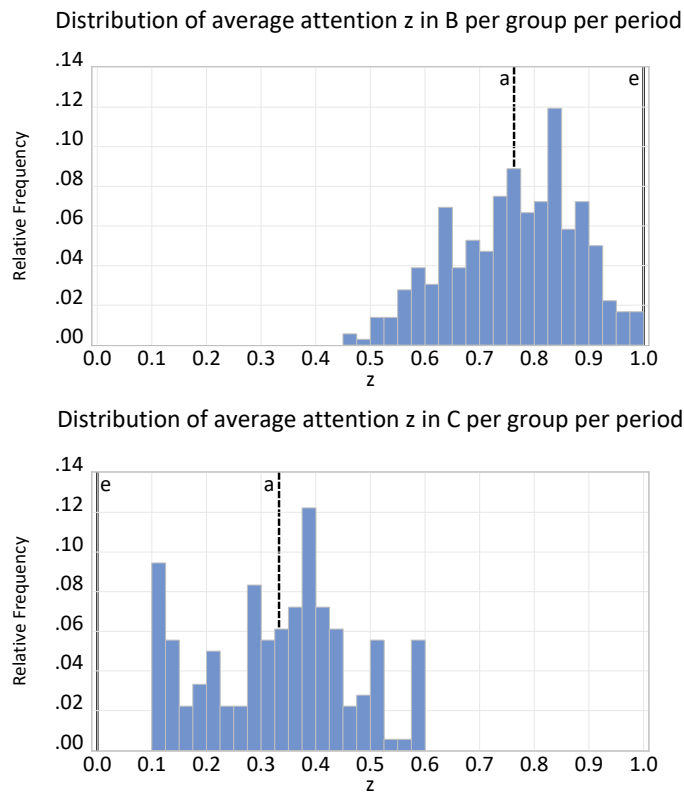
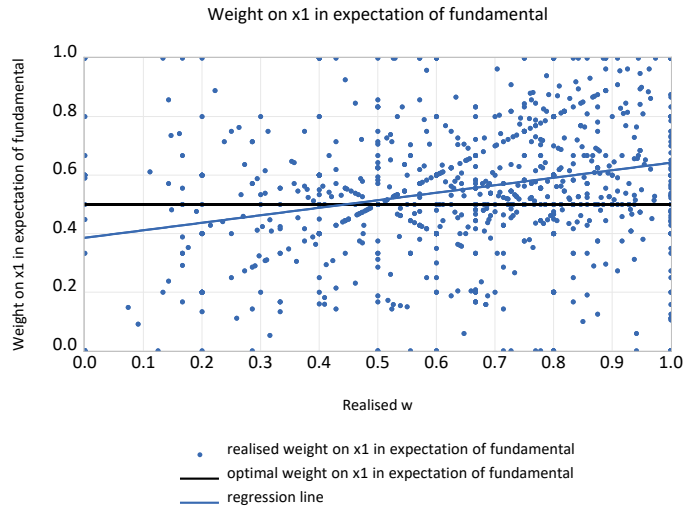


Figure 18: Distribution of average attention given to $x_{1,i}$ within each group in every period

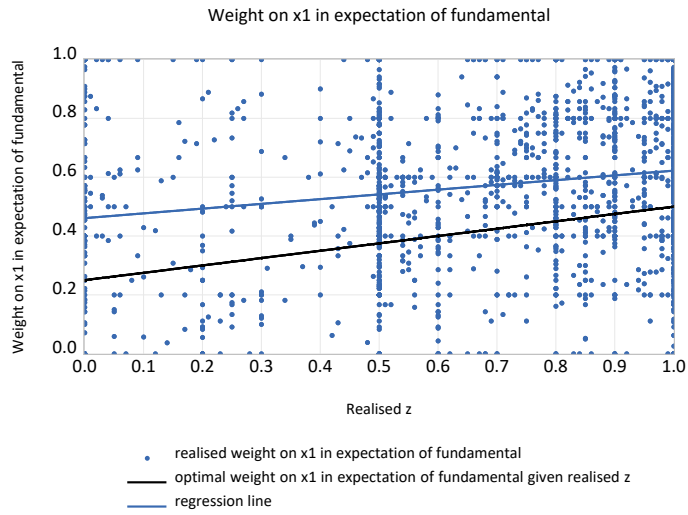


(a) Weight assigned to $x_{1,i}$ in the expectation of the fundamental as a function of ω in treatment A

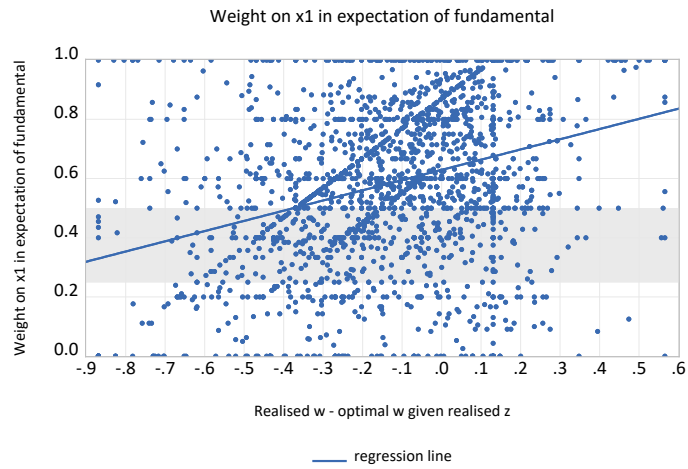


(b) Weight assigned to $x_{1,i}$ in the expectation of others' action as a function of realised ω in treatment A

Figure 19: Effect of realised ω on expectations' formation in treatment A

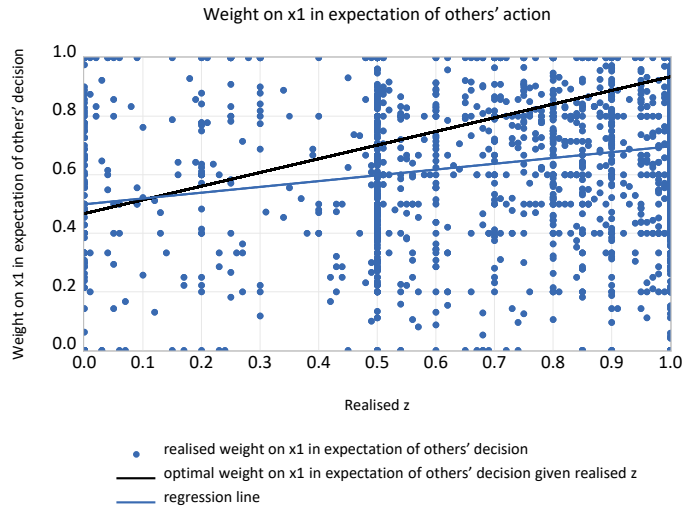


(a) Weight assigned to x_1 in the expectation of the fundamental as a function of realised z in treatment B



(b) Weight assigned to x_1 in the expectation of the fundamental as a function of difference between realised ω and optimal ω given realised z in treatment B

Figure 20: Weight assigned to x_1 in the expectation of the fundamental in treatment B

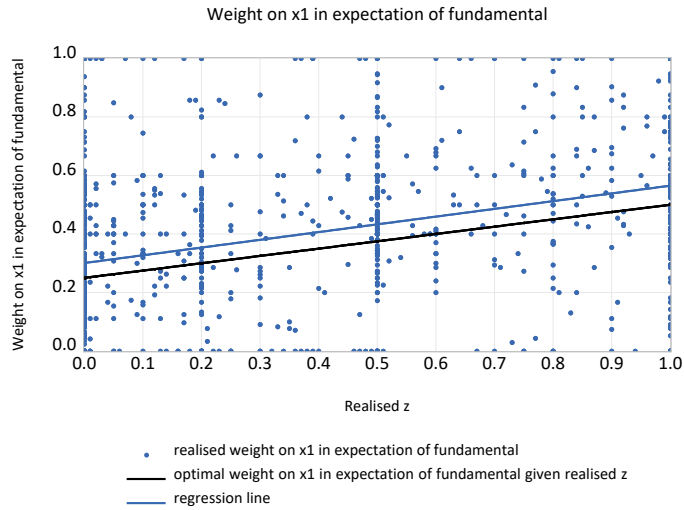


(a) Weight assigned to x_1 in the expectation of others' decision as a function of realised z in treatment B

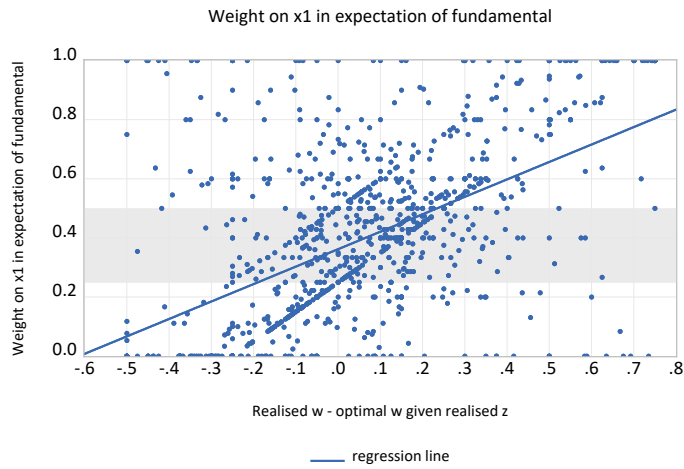


(b) Weight assigned to x_1 in the expectation of others' decision as a function of difference between realised ω and optimal ω given realised z in treatment B

Figure 21: Weight assigned to x_1 in the expectation of others' decision in treatment B



(a) Weight assigned to x_1 in the expectation of the fundamental as a function of realised z in treatment C



(b) Weight assigned to x_1 in the expectation of the fundamental as a function of difference between realised ω and optimal ω given realised z in treatment C

Figure 22: Weight assigned to x_1 in the expectation of the fundamental in treatment C

C Limited level of reasoning

This appendix presents the derivation of weights put on $x_{1,i}$ in expectations about the actions of others under limited levels of reasoning.

Nagel (1995) and Stahl and Wilson (1994) define level-0 types as subjects who choose an action randomly from a uniform distribution over all possible actions. For $k > 0$, a level- k type is playing best response to level- $k-1$. The action for level-1 of reasoning is therefore $a_i^1 = \mathbb{E}_i(\theta)$. Actions for higher levels of reasoning can be calculated as follows. Suppose that the players $-i$ (all players except player i) attach weight ρ^k to their expectation of $x_{1,-i}$. The best response of player i to such behaviour is

$$\begin{aligned}
a_i^{k+1} &= (1 - \gamma)\mathbb{E}_i(\theta) + \gamma\mathbb{E}_i(a_{-i}) \\
&= (1 - \gamma)\mathbb{E}_i(\theta) + \gamma(1 - \rho^k)\mathbb{E}_i(x_{2,-i}) + \gamma\rho^k\mathbb{E}_i(x_{1,-i}) \\
&= (1 - \gamma)\mathbb{E}_i(\theta) + \gamma(1 - \rho^k)\mathbb{E}_i(x_2) + \gamma\rho^k\mathbb{E}_i(x_1) \\
&= (1 - \gamma)(fx_{1,i} + (1 - f)x_{2,i}) + \gamma(1 - \rho^k)(\Omega_2x_{1,i} + (1 - \Omega_2)x_{2,i}) \\
&\quad + \gamma\rho^k(\Omega_1x_{1,i} + (1 - \Omega_1)x_{2,i}) \\
&= x_{1,i} \underbrace{\left[(1 - \gamma)f + \gamma(1 - \rho^k)\Omega_2 + \gamma\rho^k\Omega_1 \right]}_{\rho^{k+1}} \\
&\quad + x_{2,i} \left[(1 - \gamma)(1 - f) + \gamma(1 - \rho^k)(1 - \Omega_2) + \gamma\rho^k(1 - \Omega_1) \right]. \tag{18}
\end{aligned}$$

By identification, we obtain:

$$\begin{aligned}
\rho^{k+1} &= (1 - \gamma)f + \gamma(1 - \rho^k)\Omega_2 + \gamma\rho^k\Omega_1 \\
&= \frac{(1 - \gamma)\kappa_2^2 + \xi_2^2 + z_i + \gamma\rho^k(\kappa_1^2 + \kappa_2^2)}{\kappa_1^2 + \kappa_2^2 + \xi_1^2 + \xi_2^2 + 1}. \tag{19}
\end{aligned}$$

With the parameters of the experiment, we get the following weights for level- k reasoning in treatments A and B: $\rho^1 = 0.500$, $\rho^2 = 0.712$, $\rho^3 = 0.803$, $\rho^4 = 0.841$, $\rho^5 = 0.857$, $\rho^\infty = 0.870$.

The limited level of reasoning approach can not only be expressed in terms of a_i , as done above, but also in terms of the expectation of others' action o derived in section 3.7. Indeed, in Section 5.2, we are interested in the level of reasoning subjects reach in their expectation about others' action (o). Let us now determine the weight O^k a level- k player attaches to $x_{1,-i}$ in her expectation of the others' action.

From above, we have

$$a_i^{k+1} = \rho^{k+1}x_{1,i} + (1 - \rho^{k+1})x_{2,i}$$

so that

$$\bar{a}_{-i}^{k+1} = \rho^{k+1}x_1 + (1 - \rho^{k+1})x_2,$$

thus

$$\begin{aligned} \mathbb{E}_i(\bar{a}_{-i}^{k+1}) &= \rho^{k+1}\mathbb{E}_i(x_1) + (1 - \rho^{k+1})\mathbb{E}_i(x_2) \\ &= x_{1,i} \left[\rho^{k+1}\Omega_1 + (1 - \rho^{k+1})\Omega_2 \right] \\ &\quad + x_{2,i} \left[\rho^{k+1}(1 - \Omega_1) + (1 - \rho^{k+1})(1 - \Omega_2) \right]. \end{aligned} \quad (20)$$

By identification, we get

$$O^k = \rho^{k+1} \frac{\kappa_1^2 + \kappa_2^2}{\kappa_1^2 + \kappa_2^2 + \xi_1^2 + \xi_2^2 + 1} + \frac{\xi_2^2 + z_i}{\kappa_1^2 + \kappa_2^2 + \xi_1^2 + \xi_2^2 + 1}. \quad (21)$$

With the parameters of the experiment, we get the following weights in treatments A and B: $O^2 = 0.750$, $O^3 = 0.856$, $O^4 = 0.901$, $O^5 = 0.921$, $O^\infty = 0.935$.

Conditioning on observed attention z in B, we obtain for the weights put on $x_{1,i}$ in the beauty-contest action under limited levels of reasoning: $\rho^1|_z = 0.500$, $\rho^2|_z = 0.653$, $\rho^3|_z = 0.743$, $\rho^4|_z = 0.782$, $\rho^5|_z = 0.798$, $\rho^\infty|_z = 0.810$. And the weights put on $x_{1,i}$ in expectations about the actions of others under limited levels of reasoning conditional on observed attention are given by: $O^2|_z = 0.691$, $O^3|_z = 0.767$, $O^4|_z = 0.812$, $O^5|_z = 0.832$, $O^\infty|_z = 0.846$.

D Translation of instructions

Instructions to participants varied according to the sessions. We present the instructions for sessions 1 and 2 with treatments A and B. For the other sessions, instructions were adapted accordingly and are available upon request.¹⁰

INSTRUCTIONS

Hello and welcome to our laboratory.

During the experiment in which you will participate, you will be asked to make decisions. These instructions explain what these decisions are and how your decisions will earn you a certain amount of money.

The earnings you receive during this experiment will depend partly on your own decisions, partly on the decisions of the other participants, and partly on chance through random draws. All your decisions will be processed anonymously and you will never have to enter your name into the computer. The amount of money earned during the experiment will be paid to you individually at the end of the experiment.

You are 18 people participating in this experiment, and you are divided into three groups of 6 people. These three groups are totally independent and do not interact with each other throughout the experiment. Each participant interacts only with the other participants in his or her group. These instructions describe the rules of the game for a group of 6 participants and are the same for all participants.

At the end of this reading, feel free to ask questions if you wish.

General framework of the experiment

The experiment includes 2 games in total. Each game is repeated for 15 periods so that the experiment has a total of 30 periods. During each of these 30 periods, the decisions you make will determine your earnings. During the experiment your earnings are expressed in ECU. At the end of the experiment, the ECUs you earn will be converted into euros.

The conversion rate is 100 ECU = 12 euros.

Note that there are a total of 15 periods in a game, but only 2 periods will be randomly selected to calculate your payoff. Since all periods are likely to be chosen for the calculation of your payoff, you must give equal weight to each of the 15 periods. At the end of the experiment, a participant will randomly draw the 2 periods of each game that will serve as the basis for your payoff.

Rule determining the payoff in each of the 30 periods

At the beginning of each period, the computer draws a positive integer Z . You will not know the true value of Z . This number Z is different at each period but **identical** for all participants in the same group.

¹⁰What follows is a translation (from French to English) of the instructions given to the participants.

At each period, you are asked to make a decision by choosing a number **D**. Decision **D** will determine the ECU payoff of each participant for each period according to the formula:

$$50 - 1.5(\mathbf{D} - \mathbf{Z})^2 - 8.5(\mathbf{D} - \text{average of decisions } \mathbf{D} \text{ of the other participants})^2$$

This formula indicates that your payoff is higher the closer your decision **D** is to

- on the one hand, the unknown number **Z** and
- on the other hand, the average decision **D** of the other participants in your group.

To maximise your payoff, you must make a decision **D** that is close to both the unknown number **Z** and the average of the decisions **D** of the other participants. Note, however, that it is more important to be close to the average decision of the other participants than to the unknown number **Z**.

No participant knows the true value of **Z** when making his or her decision. However, in order to help you in your choice of **D**, you can observe two hints on the unknown number **Z** as explained below.

Your hints on Z in the first game (periods 1 to 15)

For each period of the first game, you will have two hints (numbers) on the unknown number **Z**. These hints contain unknown errors.

- Common hint **X**

You will receive at each period, as well as all the other members of your group, a common hint **X** on the unknown number **Z**. This common hint is centered on **Z** and contains an error randomly selected from a normal distribution of mean 0 and **variance 2**. The variance of a random variable measures the dispersion of that variable around its mean; it is the inverse of precision, the higher the variance, the lower the precision, and vice versa. This common hint **X** is the **same for all participants**.

- Private hint **Y**

In addition to this common hint **X**, each participant receives at each period a private hint **Y** on the unknown number **Z**. Each private hint is also centered on **Z** and contains an error randomly selected from a normal distribution of mean 0 and **variance 2**. Your private hint and the private hint of each of the other participants are selected independently of each other, so that **each participant will receive a different private hint than the other participants**.

Distinction between common hint X and private hint Y

The common hint **X** and your private hint **Y** have the same accuracy (i.e. the same variance of 2): both hints are on average equally informative on the unknown number **Z**. The only distinction between the two hints is that each participant observes a private hint

Y different from that of the other participants while all participants observe the same common hint X .

How to make your decision?

Since you do not know the errors in your hints, it is natural to choose as decision D a number between the common hint X and your private hint Y if X is less than Y and between Y and X otherwise. To make your decision, we ask you to select a number between the common hint X and your private hint Y using a cursor. You must therefore choose how to combine your two hints in order to maximise your payoff. Note, however, that the payoff formula indicates that it is more important that your decision D is close to the average of the decisions of the other participants than the unknown number Z .

To make it easier for you to choose D , the formula for your payoff will appear on the screen and allow you to estimate your payoff based on your choice of D , your estimate of the unknown number Z as well as your estimate of the average decision of the other participants. If the estimated payoff does not differ by more than 4 ECU from the actual payoff you have made, you will receive a bonus of 10 ECU in addition to the payoff for the period.

Once you have determined your estimate of Z (EZ), your estimate of the decision of others (ED) and your decision D using the sliders, click on the "Validate" button. As soon as all participants have done the same, the period ends and the result of the period is communicated to you. Then, the next period begins.

As soon as the 15 periods of the first game are completed, the second game of the experiment begins.

Note: If the payoff from your decision D at a period is negative, it will be reduced to 0.

Your hints on Z in the second game (periods 16 to 30)

As in the first game, you must make a decision D in order to maximise your payoff given by the formula on page 3 of the instructions. The second game is distinguished from the first by the fact that you do not directly observe any hint on Z . However, you must choose the attention A with which you observe two hints on Z . The attention you give to each of these hints determines the variance of the private error with which you observe these hints as described below.

At each period, you therefore have two choices to make: first, you must choose the attention A that you give to each of the hints M and N ; then, given the hints M and N that you observe, you must choose your decision D in order to maximise your payoff.

Choice of A : allocation of your attention between the two hints M and N

You have a total attention of 100% that you must divide between the two hints M and N . The attention you give to the hint M is denoted by A and is between 0% and 100%. The attention you give to the hint N is equal to $1-A$ and corresponds to the difference between the total attention 100% and the attention given to the hint M .

- Hint **M**

The hint **M** contains an error common to all participants as well as a private error specific to each participant.

- Common error to all participants

The hint **M** is centered on **Z** and contains a common error randomly selected from a normal distribution of mean 0 and **variance 2**. This error term of the hint **M** is common to all participants.

- Private error specific to each participant depending on the choice of **A**

In addition to the common error, you observe the hint **M** with a randomly selected private error from a normal distribution of mean 0 and **variance 1-A**. Thus, the greater the attention **A** you pay to the hint **M**, the more you will observe the hint **M** with a small private error. The private error of each participant is different from that of the other participants because the private errors are derived independently of each other from a normal distribution and because the variance of the normal distribution depends on the attention **A** that each participant has chosen.

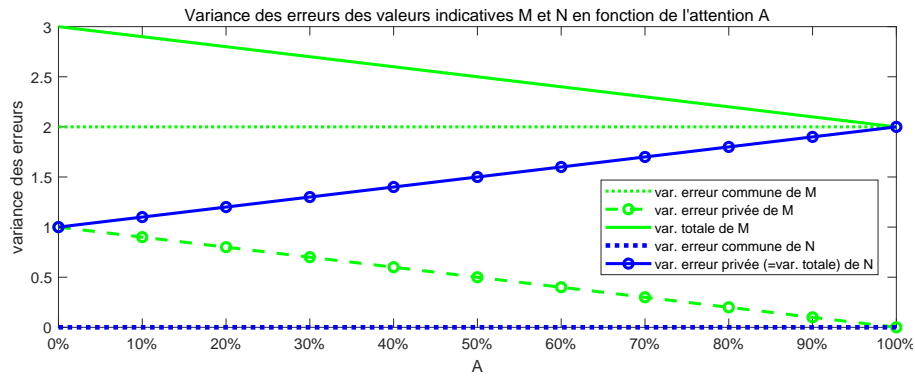
- Hint **N**

The hint **N** contains only one private error specific to each participant, its common error for all participants being equal to 0. The hint **N** is centered on **Z** and contains a private error randomly selected from a normal distribution of mean 0 and **variance 1+A**. Thus, the greater the attention **A** you give to the hint **M**, the lower the attention you give to the hint **N** and the more you will observe the hint **N** with a strong private error. The private error of each participant is different from that of the other participants because the private errors are derived independently of each other from a normal distribution and because the variance of the normal distribution depends on the attention **A** that each participant has chosen.

For example, as you can see on the graph below, if you choose a maximum attention **A** of 100%, you will observe the hint **M** without private error ($1 - 100\% = 0$) and the hint **N** with a private error of variance $1 + 100\% = 2$. If, however, you choose a minimum attention **A** of 0%, you will observe the hint **M** with a private error of variance $1 - 0\% = 1$ and the hint **N** with a private error of variance $1 + 0\% = 1$ also.

In concrete terms, you must select the attention **A** that you give to the hint **M** using a cursor between 0% and 100%. The attention you pay to the hint **N** will automatically correspond to the difference between 100% and the number **A** you have selected using the cursor. The closer your **A** is to 100%, the more attention you will pay to the hint **M** (and less to the hint **N**). Conversely, the closer your **A** is to 0%, the more attention you will pay to the hint **N** (and less to the hint **M**).

Once you have fixed the cursor on the decision **A** of your choice, click on the "Validate" button. As soon as all participants have done the same, the computer will randomly draw



the hints **M** and **N** according to the allocation of attention you have chosen. The hints **M** and **N** will then be displayed on your screen.

Choice of **D**: decision determining your payoffs

Since you do not know the errors in your hints, it is natural to choose as decision **D** a number between your hint **M** and your hint **N** if **M** is less than **N** and between **N** and **M** otherwise. To make your decision, we ask you to select a number between the hint **M** and the hint **N** using a cursor. You must therefore choose how to combine your two hints in order to maximise your payoff.

Note that the formula for payoffs on page 3 of the instructions indicates that it is more important that your decision **D** is close to the average of the decisions of the other participants than to the unknown number **Z**.

Note also that by choosing **A** beforehand, you will have chosen the attention you give to the hint **M** relatively to the hint **N**, which implies that you will have determined the variance of the private error of these hints. To maximise your payoff associated with your decision **D**, you must take into account the choice of **A** you have made beforehand. And conversely, you must choose your **A** by considering its effect on the information you will have to make your decision **D**.


To make it easier for you to choose **D**, the formula for payoffs will appear on the screen and allow you to estimate your payoff based on your choice of **D**, your estimate of the unknown number **Z** as well as your estimate of the average decision of the other participants. If the estimated payoff does not differ by more than 4 ECU from the actual payoff you have made, you will receive a bonus of 10 ECU in addition to the payoff for the period.

Once you have determined your estimate of **Z** (**EZ**), your estimate of the decision of others (**ED**) and your decision **D** using the sliders, click on the "Validate" button. As soon as all participants have done the same, the period ends and the result of the period is communicated to you. Then, the next period begins.

As soon as the 15 periods of the second game are completed, the experiment ends.

Note: If the payoff from your decision **D** at a period is negative, it will be reduced to 0.

E Example of screens

ecun  Bienvenue lees_2 [Sa déconnecter](#)

Vous êtes le joueur 1 Vous êtes dans le jeu 1 Période n° : 1 / 15

Choix de l'allocation de votre attention A

Veillez choisir votre attention A pour la valeur indicative M en déplaçant le curseur ci-dessous.
L'attention pour la valeur N (1 - A) sera calculée automatiquement après votre choix de A.

Vous pouvez également, après avoir déplacé le curseur, entrer directement la valeur de A en utilisant un '.' à la place de la ',' puis appuyez sur la touche Entrée pour qu'elle soit prise en compte.


Valeur indicative M		Valeur indicative N	
Variance commune de M	2	Variance commune de N	0
Variance privée de M	1 - A	Variance privée de N	1 + A
Variance totale de M	3 - A	Variance totale de N	1 + A

Variance des erreurs des valeurs indicatives M et N en fonction de l'attention A

Votre décision A : %

0% 100%

Valider

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Vous êtes le joueur 1 Vous êtes dans le jeu 1 Période n° : 1 / 15

Estimation EZ, ED et choix de D

Veillez choisir vos estimations EZ et ED et votre décision D en déplaçant les curseurs ci-dessous.
Vous pouvez également, après avoir déplacé les curseurs, entrer directement les valeurs de EE, ED et D en utilisant un '.' à la place de la ',' puis appuyez sur la touche Entrée pour que chacune de ces 3 valeurs soit prise en compte.

Votre attention A	69%	Votre attention 1 - A	31%
Valeur indicative M	662,5	Valeur indicative N	659,3
Variance commune de M	2	Variance commune de N	0
Variance privée de M	0,31	Variance privée de N	1,69
Variance totale de M	2,31	Variance totale de N	1,69

Gain espéré = $50 - 1,5 (D - Z)^2 - 8,5 (D - Moy)^2$
 $32,8 = 50 - 1,5 (661,6 - 661)^2 - 8,5 (661,6 - 660,2)^2$

EZ: Estimez la valeur de Z :

ED: Estimez la décision D moyenne des 5 autres :

Votre décision D :

Valider

Jeu 1 - Résultats de la période n° : 1 / 15

Vous êtes le joueur 1

- Nombre inconnu Z : **186**- Votre estimation EZ : **183,9**

Votre attention A	52%	Votre attention 1 - A	48%
Valeur indicative M	183,6	Valeur indicative N	184,2
Variance commune de M	2	Variance commune de N	0
Variance privée de M	0,48	Variance privée de N	1,52
Variance totale M	2,48	Variance totale N	1,52

- Votre décision D : **184,0**- Votre gain provenant de votre décision D : **10**- L'estimation de votre gain : **49,9**- Bonus : **0**- Votre gain de la période : **10**[Suite](#)