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Abstract

I build a New-Keynesian dynamic stochastic general-equilibrium model with a dual labor market. Firms and workers meet through a matching technology à-la Diamond-Mortensen-Pissarides and face a trade-off between productivity and flexibility at the hiring stage. All else equal, open-ended contracts are more productive than fixed-term contracts, but they embed a firing cost. The share of fixed-term contracts in job creation fluctuates endogenously, which enables to assess the resort to temporary contracts along the cycle and its response to different shocks. I estimate the model using a first-order perturbation method and classic Bayesian procedures with macroeconomic data from the Euro area. I find that the share of fixed-term contracts in job creation is counter-cyclical. The agents react to shocks essentially through the job creation margin and the contractual composition of the hires. Moreover, a general-equilibrium effect arises ; the substitution between fixed-term and open-ended contracts at the hiring stage influences the job seekers' stock, which in turn impacts job creation. Using my previous estimates and solving the model with a third-order perturbation method, I find that fixed-term employment reacts to negative aggregate demand shocks and uncertainty shocks oppositely. This result suggests that fixed-term employment could be used to identify uncertainty shocks in future research. As for inflation, changes in firing costs do not alter its dynamics as long as open-ended and fixed-term matches do not differ much in productivity all else equal.

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1 Introduction

Over the last decades, fixed-term contracts have become a staple of European labor markets. In France, for example, fixed-term contracts cover 12% of employed workers and represent 80% of job creations. Academics have pointed at labor market institutions as responsible for the expanding divide between rigid open-ended contracts and flexible fixed-term contracts; the impact of firing costs on long-run employment and dualism has been discussed extensively[1]. Surprisingly, the literature seldom inspects the short-run behavior of dual labor markets while fixed-term contracts share the same time scale as business cycles and conventional monetary policy.

What are the main cyclical features of a dual labor market ? How does uncertainty impact dualism ? How does dualism impact inflation dynamics ? To address these questions, I estimate a New-Keynesian dynamic stochastic general-equilibrium model with a dual labor market using Euro area data. The New-Keynesian block of the economy remains classic. As for the labor market, I extend the traditional Diamond-Mortensen-Pissarides framework to embed fixed-term and open-ended contracts. Firms post vacancies, unemployed workers search for a job and firm-worker pairs arise following a matching function. New firm-worker pairs differ in quality and optimally choose between going back to search, or carrying on either with an open-ended or a fixed-term contract. While laying off an unprofitable open-ended contract entails the payment of a firing cost, fixed-term contracts split at zero cost and are less productive all else equal. Thus, a trade-off between productivity and flexibility drives the choice between a fixed-term and an open-ended hire. When the quality of the match is high enough, the productivity gains overcome the future payment of firing costs in the doldrums ; the best way to make the most out of the match is hiring through an open-ended contract. Otherwise, the fear of firing costs prevails and a fixed-term contract is preferred.

I estimate the log-linearized model on quarterly data from the Euro area. Overall, the model is able to fit the cyclical features of a dual labor market despite being estimated on classic general macroeconomic time series — Output, nominal interest rates, inflation and employment. One main contribution is the model ability to account for the fluctuations in contractual composition of hires. It is not the case of the few references studying business cycles and dualism. Sala and Silva (2009), Costain et al. (2010) and Sala et al. (2012) either assume that job creation only occurs through fixed-term contracts, or that the share of fixed-term contracts in job creation is exogenous. The model fits well the counter-cyclicality of the share of fixed-term contracts in job creation. I also contribute to the literature dealing with macroeconomic risk. Solving the estimated model with a third-order perturbation method, I find that fixed-term employment responds oppositely to a negative aggregate demand shock and to a volatility shock on aggregate productivity. This result suggests that fixed-term employment could be used to tell apart uncertainty shocks and negative aggregate demand shocks. Measuring uncertainty and identifying uncertainty shocks is a well-known issue in the literature (Jurado et al., 2015; Leduc and Liu, 2016). My last main contribution involves inflation dynamics. I find that dualism does not impact inflation volatility as long as the *ex ante* productivity wedge between open-ended and fixed-term contracts remains small.

The assumption that fixed-term workers produce less than open-ended workers *ceteris paribus* is fundamental to obtain a dual labor market as equilibrium. If fixed-term and open-ended workers with the same idiosyncratic quality produced the same, hires would involve either fixed-term workers or open-workers only. While fundamental, my assumption substantiates with empirical work. Fixed-term workers are likely to undergo successions of short employment periods and long unemployment spans (Fontaine and Malherbet, 2016). Pissarides (1992) shows that the latter reduce concerned workers' skills. Moreover, fixed-term workers benefit less from on-the-job training (Arulampalam and Booth, 1998; Arulampalam et al., 2004; Albert et al., 2005; Cutuli and Guetto, 2012). The productivity-flexibility trade-off is highlighted in several papers related to labor market dualism. Cao et al. (2010) shows

[1] See Cahuc and Postel-Vinay (2002), Bentolila et al. (2012), Cahuc et al. (2016), Créchet (2018) and Rion (2019) among many others

that highly productive workers are offered open-ended contracts, because fixed-term workers have an incentive to search on the job, which depletes their productivity. Caggese and Cuñat (2008) introduces fixed-term contracts, which are less productive than open-ended contracts by assumption, in firms with a decreasing-return-to-scale technology. Firms hire open-ended contracts until the productivity gains are offset by the expected losses from costly separations, and hire fixed-term contracts beyond that point.

Explaining the business cycles in a dual labor market requires relevant data. As far as I know, it does not exist on a quarterly basis concerning the Euro area. Thus, I assess the fit of the model using French data from the *Dares*. Beyond the counter-cyclical share of fixed-term hires mentioned above, open-ended job creation and destruction flows are more volatile than their fixed-term counterparts. While open-ended job destruction is counter-cyclical, fixed-term job destruction is pro-cyclical. The short duration of fixed-term contracts explains this counter-intuitive result ; as most fixed-term contracts last less than a quarter, demand boosts fuel fixed-term job creation that expire within the same quarter. My model also replicates the Beveridge curve. Real business cycles with frictional labor markets a-la Mortensen Pissarides mimic the negative correlation between unemployment and vacancies with difficulty (Shimer, 2005). Talking about dynamics, a substitution effect and general-equilibrium effect shape the labor market. Shocks change the contractual composition of job creation — substitution effect —, which in turn impacts the size of the job seekers' pool and influences job creation — general-equilibrium effect.

Firms resort to fixed-term contracts to cope with uncertainty and demand fluctuations (Dares, 2017). On the one hand, Saint-Paul (1997) explains that fixed-term employment acts as a securing buffer to open-ended workers. In the doldrums, firms lay off fixed-term workers and leave open-ended employment as unchanged as possible to avoid firing costs. Firms restore fixed-term employment during booms. Thus, a negative aggregate demand shock leads to a decrease in fixed-term employment. On the other hand, bolstered uncertainty encourages firms to postpone open-ended hires and opt for fixed-term hires instead[2]. Fixed-term employment should increase following an increase in uncertainty. My model is consistent with both views ; fixed-term employment decreases after an aggregate demand shock and increases after an uncertainty shock. It suggests that fixed-term employment could be used in future research as a way to identify uncertainty shocks.

A leafy literature has considered the effect of labor market frictions on inflation dynamics[3]. As far as I know, my paper is the first one to study the impact of labor market dualism on inflation dynamics. For the sake of comparability, I stick to Thomas and Zanetti (2009) when it comes to the New-Keynesian block. As for the labor market, Thomas and Zanetti (2009) use a Mortensen-Pissarides model with open-ended contracts subject to firing costs. They find that changes in firing costs do not alter the volatility of inflation. My findings corroborate this result. While a 5-percent cut in firing costs significantly alters the cyclical properties of labor market variables, the dynamics of inflation remain the same. This result rests on the small value of the *ex ante* contractual productivity wedge the calibration delivers.

The paper is organized as follows. Section 2 presents the model. Section 3 exposes the calibration and estimation procedure. Section 4 displays the main experiments. Section 5 concludes.

2 Model

The model follows a discrete timing and embeds 4 types of agents; the households, the firms, the fiscal authority and the central bank. Households can be unemployed or employed through a fixed-term or an open-ended contract. Three types of firms coexist. Perfectly competitive firms produce the final good valued by households for consumption and investment. Final good producers aggregate the differentiated goods produced by the retailers. The latter retailers are in monopolistic competition and transform the homogeneous intermediate good into a differentiated retail good. Intermediate firms produce the

[2] Bloom (2014) highlights that firm are more reluctant to go for investments requiring fixed costs in uncertain times.

[3] See Gertler et al. (2008), Trigari (2009), Thomas and Zanetti (2009), Blanchard and Galí (2010) and Christiano et al. (2016) among others

associated intermediate good and experience perfect competition. These intermediate firms use labor as their only input. I now describe the behavior of the different types of agents in more detail.

2.1 Households

Households are identical and constitute a continuum represented by the interval $(0, 1)$. They can be employed under a fixed-term or an open-ended contract, or unemployed. They earn wages or unemployment benefits accordingly. Households also hold firms, consume the homogeneous good produced by final good firms, save using one-period nominal bonds, earn interests on their savings and pay lump-sum taxes. Hence, I assume that taxes do not distort the choice of households over consumption and investment. I do not consider the interplay between payroll taxes and labor market dualism.

If households are identical *ex ante*, their different employment histories make them heterogeneous *ex post*. How labor market dualism impacts the consumption and saving behavior of households is beyond the scope of this paper. As Merz (1995) and Andolfatto (1996) first did, I assume that households pool revenues and that capital markets are perfect. Thereby, I rule out the complication heterogeneity brings on. Households share equal consumption and investments. This assumption is not innocuous: unemployed, open-ended and fixed-term workers have different borrowing constraints and Lise (2013) shows that assets shape the search behavior. I leave these issues for future research.

The household's program boils down to

$$\max_{\{c_t, B_{t+1}\}_{t=0}^{+\infty}} \mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t u(c_t)$$
$$\text{s.t} \quad c_t + \frac{B_{t+1}}{P_t} = R_{t-1}\frac{B_t}{P_t} + \overline{w_t^p} n_t^p + \overline{w_t^f} n_t^f + \rho^b \overline{w} u_t + \Pi_t - \tau_t$$

C_t marks down consumption. B_t is the amount of nominal bond holdings at the beginning of period t, with the associated nominal interest rate R_t between t and $t+1$. $\overline{w_t^p}$, $\overline{w_t^f}$ denote the average real wages for open-ended jobs and temporary jobs. $\overline{w}$ is the steady-state average wage. ρ^b denotes the replacement rate of unemployment benefits over the steady-state average wage. n_t^p is the aggregate open-ended employment and n_t^f denotes its fixed-term counterpart. u_t marks down the measure of unemployed households. Firms transfer their profits to households through Π_t, while the government taxes τ_t to finance the payment of unemployment benefits.

The first order conditions with respect to c_t and B_{t+1} lead to the following Euler equation

$$u'(c_t) = \beta \mathbb{E}_t \left[R_t \frac{P_t}{P_{t+1}} u'(c_{t+1}) \right] \tag{2.1.1}$$

As households own firms, firms discount profits of date s from date t with the factor $\beta_{t,s} = \beta^{s-t} u'(c_s)/u'(c_t)$.

2.2 Final good firms

Final good firms are identical and compete to produce the good consumed by households. They put together retail goods through a Dixit-Stiglitz technology and produce Y_t.

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon_t}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{2.2.1}$$

where ϵ is the elasticity of substitution between retail goods.

The firm takes as given the price of the retail goods $P_{i,t}$ and the price of the final good P_t and maximizes its profits with respect to the components of its input $\{Y_{i,t}\}_{i\in(0,1)}$ under the constraint (2.2.1).

The program of the final good firm boils down to

$$\max_{\{Y_{i,t}\}_{i\in[0,1]}} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$
$$\text{subject to (2.2.1)}$$

The subsequent first order condition provides an expression for the demand of the retail good i.

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t \tag{2.2.2}$$

2.3 Retailers

Retailers buy goods from intermediate firms and sell the obtained production to final good producer. They are in monopolistic competition and lie on the interval $(0, 1)$. Retailers accomplish the one-to-one transformation of the intermediate good into a retail good. Denoting $X_{i,t}$ retailer i's input in the intermediate good, the production technology writes

$$Y_{i,t} = X_{i,t}$$

As a result, retailers face a real marginal cost that equals the relative price of the intermediate good ϕ_t. I assume that retailers adjust prices as Calvo (1983) describe.

$$P_{i,t} = \begin{cases} P_{i,t-1} & \text{with probability } \psi \\ P^*_{i,t} & \text{with probability } 1-\psi \end{cases}$$

A fraction ψ of retailers is able to adjust its prices to the optimal value $P^*_{i,t}$, whereas the remaining retailers stick to their former prices. There is no indexation of non-adjusted princes on inflation in this model. The price-setting retailer i at period t has the following program

$$\max_{P^*_{i,t}} \mathbb{E}_t \sum_{T=t}^{+\infty} \beta_{t,T} \psi^{T-t} \left(\frac{P^*_{i,t}}{P_T} - \phi_T\right) Y_{i,T}$$
$$\text{subject to } Y_{i,T} = \left(\frac{P^*_{i,t}}{P_T}\right)^{-\epsilon} Y_T$$

It leads to the following first order condition

$$\mathbb{E}_t \sum_{T=t}^{+\infty} \beta_{t,T} \psi^{T-t} P_T^{\epsilon} Y_T \left(\frac{P^*_{i,t}}{P_T} - \mu\phi_T\right) = 0 \tag{2.3.1}$$

where $\mu = \epsilon/(\epsilon-1)$ is the relaters' steady-state mark-up.

2.4 Intermediate good firms and the labor market

Intermediate-good firms use labor as sole input. They can employ one worker or maintain one vacancy. Workers can be unemployed or employed under a fixed-term or an open-ended contract. They are identical. There is no on-the-job search, which implies that only unemployed workers search for a job. When a firm and a worker meet, the idiosyncratic productivity of the match z is revealed. I assume that idiosyncratic productivity is i.i.d across time and drawn from a distribution with cumulative distribution function G. I reluctantly make this assumption for the sake of simplicity. With persistent idiosyncratic productivity, the computation of the equilibrium requires keeping track of the productivity distribution of matches as a state variable. Since the literature considering cycles and dual labor market is in early stages, I prefer to leave the distributional issues for future research. Matches also face an aggregate productivity shock A_t with process $\log(A_t) = \rho_A \log(A_{t-1}) + \epsilon_t^A$, where

The number of firm-worker contacts per period is $m(e, v)$, where e is the number of job-seekers and v is the number of vacancies. A classic measure of the matching activity is the labor market tightness $\theta = v/e$. The matching function m has constant returns to scale, which enables the definition of the firm-worker meeting probability $p(\theta)$ on the job seeker's side and its counterpart $q(\theta)$ on the firm's side.

$$q = \frac{m(e, v)}{v} = m\left(\theta^{-1}, 1\right)$$
$$p = \frac{m(e, v)}{e} = m(1, \theta) = \theta q(\theta)$$

p is increasing in labor market tightness, whereas q is decreasing in labor market tightness. Note that the meeting probabilities are not the classic job-finding and vacancy-filling probabilities. A firm-worker meeting does not lead to a production if the idiosyncratic productivity is too low.

The timing in the economy is summed up in Figure 2.4. At the beginning of the period, agents learn the value of shocks and firms manage their workforce accordingly. They lay off poorly productive workers and post vacancies. Next, new matches are revealed. Workers fired in the current period are able to participate to the present meeting round. Hence, I avoid understating labor market flows as most fixed-term contracts last less than a quarter: fixed-term jobs last 1.5 months on average in France (Dares, 2018). Finally, production is carried out, firms pay for wages and firing costs, households consume, and the period ends.

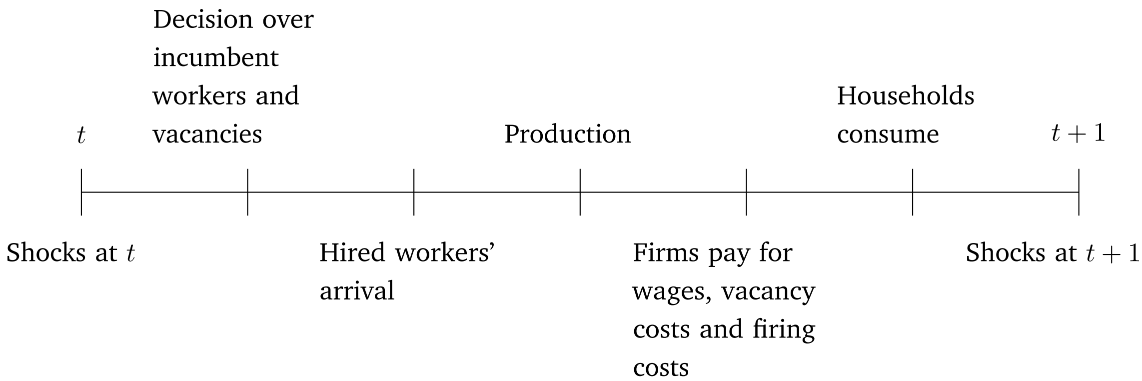

Figure 2.1: The timing of the economy

Vacancies I depart from the literature when it comes to job creation. Sala and Silva (2009) assumes that job creation occurs through fixed-term contracts only; open-ended jobs all come from converted fixed-term jobs, which is counterfactual. As Cahuc and Postel-Vinay (2002) first did, Sala et al. (2012) assume that new matches face hiring restrictions. Firms are allowed to hire either through an open-ended or a fixed-term contract according to an exogenous probability. Otherwise, all hires would take place through fixed-term contracts. As roughly 20% of hires are open-ended in France, different reasons than legal constraints on fixed-term hires explain open-ended hires.

Thereby, I assume that no constraints bind job creation. When paired with a worker, firms hire through a fixed-term contract or through an open-ended contract. They can also return searching for a worker and get the chance to be matched with another one on the next period. The present discounted value of a vacancy V_t bears witness of these different options:

$$V_t = -\gamma + q(\theta_t) \int max \left[J_t^{0,p}(z), J_t^f(z), \mathbb{E}_t \beta_{t,t+1} V_{t+1} \right] dG(z) \tag{2.4.1}$$

where γ is the cost of a vacancy, $J_t^{0,p}$ is the firm's surplus with a new open-ended match. I denote J_t^p its counterpart for continuing open-ended matches. J_t^f is the firm's surplus with a fixed-term match. The workers' surpluses are denoted W, while the total surpluses are denoted S.

Surplus sharing I assume that wages are set each period by Nash bargaining. It is not realistic considering the evidence supporting rigidity in wages. In addition, wage flexibility seems inconsistent with the stickiness of retailers' prices. Still, I leave rigid wages for future research. Tractability motivates the use of Nash bargaining. It makes hiring and firing decisions jointly efficient and only dependent on the total surplus of a match. Denoting η the worker's share of the match surplus, the sharing rules write

$$W_t^p(z_t) = \eta S_t^p(z_t) \tag{2.4.2}$$
$$W_t^{0,p}(z_t) = \eta S_t^{0,p}(z_t) \tag{2.4.3}$$
$$W_t^f(z_t) = \eta S_t^f(z_t) \tag{2.4.4}$$

Total surpluses verify

$$S_t^p(z_t) = J_t^p(z_t) - (V_t - F_t) + W_t^p(z_t) - U_t \tag{2.4.5}$$
$$S_t^{0,p}(z_t) = J_t^{0,p}(z_t) - V_t + W_t^{0,p}(z_t) - U_t \tag{2.4.6}$$
$$S_t^f(z_t) = J_t^f(z_t) - V_t + W_t^f(z_t) - U_t \tag{2.4.7}$$

where U_t is the value of unemployment at the beginning of period t. The outside option of workers includes taking part in current period's market trades and earning the unemployment benefit if no beneficial meeting occurred.

$$U_t = p(\theta_t) \int \max \left(W_t^{0,p}(z), W_t^f(z), U_t^0 \right) dG(z) + (1 - p(\theta_t)) U_t^0 \tag{2.4.8}$$
$$U_t^0 = b + \mathbb{E}_t \beta_{t,t+1} U_{t+1} \tag{2.4.9}$$

where $b = \rho^b \overline{w} + h$ is the unemployed's flow of utility. It includes unemployment benefits as well as home production h.

In (2.4.5)-(2.4.7), note that the surplus of incumbent open-ended matches includes firing costs, while the one of new open-ended workers does not. In case of disagreement during wage bargaining, a newly paired worker goes back to the pool of job seekers and the firm does not pay firing costs. That way, I do not overstate the role of firing costs.

Now, I describe the open-ended and the fixed-term contracts as well as the associated surpluses.

Open-ended contracts A continuing open-ended contract delivers the wage w_t^p and stipulates a firing tax F. An open-ended match may separate with the exogenous probability s. In this case, the separation bears no cost. Otherwise, the firm chooses whether it keeps or lays off the worker regarding the idiosyncratic productivity of the match. An endogenous separation entails the payment of the firing cost and the opening of a new vacancy. Hence, a firm with an incumbent open-ended worker has the following surplus

$$J_t^p(z_t) = \phi_t A_t z_t - w_t^p(z_t) + \mathbb{E}_t\beta_{t,t+1}\left\{(1-s)\int \max\left[J_{t+1}^p(z), V_{t+1} - F\right] dG(z) + sV_{t+1}\right\} \tag{2.4.10}$$

An incumbent open-ended worker earns a wage and may leave with probability s. If he does not, he faces a productivity shock and the match may split if the latter shock is adverse enough. Laid-off or quitting workers go back to the job seekers' pool immediately and are therefore eligible to participate to the current's period firm-worker meetings. Otherwise, the match goes on.

$$W_t^p(z_t) = w_t^p(z_t) + \mathbb{E}_t\beta_{t,t+1}\left\{(1-s)\int \max\left[W_{t+1}^p(z), U_{t+1}\right] dG(z) + sU_{t+1}\right\} \tag{2.4.11}$$

New open-ended workers have a different wage function $w_t^{0,p}$: their outside option does not include the payment of the firing cost in case of disagreement during the wage bargaining and the possibility to search for a new job in case of disagreement. After one-period, if there is no separation, a wage renegotiation occurs because idiosyncratic productivity changed. The firm pays firing costs if an endogenous split occurs. Otherwise, the surplus of incumbent open-ended contracts weighs in.

$$J_t^{0,p}(z_t) = \phi_t A_t z_t - w_t^{0,p}(z_t) + \mathbb{E}_t\beta_{t,t+1}(1-s)\left\{\int_{z_{t+1}^p}^{+\infty} J_{t+1}^p(z)\, dG(z) - G\left(z_{t+1}^p\right)F + sV_{t+1}\right\} \tag{2.4.12}$$

$$W_t^{0,p}(z_t) = w_t^{0,p}(z_t) + \mathbb{E}_t\beta_{t,t+1}\left\{(1-s)\int \max\left(W_{t+1}^p(z), U_{t+1}\right) dG(z) + sU_{t+1}\right\} \tag{2.4.13}$$

Fixed-term contracts A fixed-term contract stipulates the wage function w_t^f. Fixed-term matches split with the exogenous probability δ. I assume that fixed-term contracts yield a lower productivity than open-ended matches with a factor $\rho < 1$. I discuss this assumption in detail below. Firms value the immediate production net of the wage. As for the continuation value, it simply embeds the possibility of an exogenous separation shock.

$$J_t^f(z_t) = \rho A_t z_t \phi_t - w_t^f(z_t) + \mathbb{E}_t\beta_{t,t+1}\left\{(1-\delta)\int J_{t+1}^f(z)\, dG(z) + \delta V_{t+1}\right\} \tag{2.4.14}$$

Fixed-term workers immediately value their wage. In case of separation, they can immediately indulge in labor market trades.

$$W_t^f(z_t) = w_t^f(z_t) + \mathbb{E}_t\beta_{t,t+1}\left\{(1-\delta)\int W_{t+1}^f(z)\, dG(z) + \delta U_{t+1}\right\} \tag{2.4.15}$$

Using the different definitions of the firms' and the workers' surpluses, the total surpluses write[4]

$$\begin{aligned} S_t^p(z_t) = {} & A_t z_t \phi_t - b - \frac{\eta\gamma\theta_t}{1-\eta} + \left(1 - \mathbb{E}_t\beta_{t,t+1}(1-s)\right)F \\ & + \mathbb{E}_t\beta_{t,t+1}(1-s)\int \max\left(S_{t+1}^p(z), 0\right) dG(z) \end{aligned} \tag{2.4.16}$$

$$S_t^{0,p}(z_t) = S_t^p(z_t) - F \tag{2.4.17}$$

$$S_t^f(z_t) = \rho A_t z_t \phi_t - b - \frac{\eta\gamma\theta_t}{1-\eta} + \rho\mathbb{E}_t\beta_{t,t+1}(1-\delta)\int S_{t+1}^f(z) dG(z) \tag{2.4.18}$$

[4]Detailed calculations are available in Appendix B.1

As intended, the surplus of continuing open-ended workers includes the firing cost in case of endogenous separation. It bolsters the continuing open-ended worker's threat point in Nash bargaining and pushes up his wage. The new open-ended workers does not benefit from this effect since a failure in the bargaining process does not entail the payment of F. The total surplus of fixed-term workers shows the productivity gap ρ in the flows and the exogenous termination of the contract in the continuation value. Fixed-term contracts struck by a job destruction shock split regardless the productivity of the match.

Using the firms' surpluses, joint surpluses and the surplus sharing rules, wages verify

$$w_t^p\left(z_t\right) = \eta\left(A_t z_t \phi_t + \left(1 - E_t \beta_{t,t+1}(1-s)\right) F + \gamma\theta_t\right) + (1-\eta)b \tag{2.4.19}$$

$$w_t^{0,p}\left(z_t\right) = \eta\left(A_t z_t \phi_t - E_t \beta_{t,t+1}(1-s) F + \gamma\theta_t\right) + (1-\eta)b \tag{2.4.20}$$

$$w_t^f\left(z_t\right) = \eta\left(\rho A_t z_t \phi_t + \gamma\theta_t\right) + (1-\eta)b \tag{2.4.21}$$

$$\tag{2.4.22}$$

The new open-ended worker is penalized with higher firing costs to compensate the future wage gains in case of continuation. Moreover, labor market tightness increases the outside option of workers through the higher probability of finding a job, which increases wages.

2.5 Job creation and job destruction

I assume there is free entry on vacancy posting.

$$V_t = 0 \tag{2.5.1}$$

The job creation condition stems from the free-entry condition (2.5.1) and the Bellman equation defining the present discounted value of an unfilled vacancy (2.4.1). Using the Nash-sharing rules (2.4.3)-(2.4.4), the job creation condition becomes

$$\frac{\gamma}{(1-\eta)q\left(\theta_t\right)} = \int max\left[S_t^{0,p}\left(z\right), S_t^f\left(z\right), 0\right] dG(z) \tag{2.5.2}$$

The choice between fixed-term and open-ended contracts lies in the comparison of joint surpluses across contracts. Notice that $\partial S_t^{0,p}/\partial z_t = A_t\phi_t > \rho A_t \phi_t = \partial S_t^f/\partial z_t$. Thus, there exists a productivity threshold z_t^* above which open-ended contracts are preferable to fixed-term ones.

$$S_t^{0,p}\left(z_t^*\right) = S_t^f\left(z_t^*\right) \tag{2.5.3}$$

I also define the threshold z_t^p below which an incumbent open-ended match splits. Similarly, fixed-term contracts become profitable when productivity exceeds z_t^f. I denote z_t^c the analogous threshold for new open-ended contracts.

$$S_t^p\left(z_t^p\right) = 0 \tag{2.5.4}$$

$$S_t^f\left(z_t^f\right) = 0 \tag{2.5.5}$$

$$S_t^{0,p}\left(z_t^c\right) = 0 \tag{2.5.6}$$

Joint surpluses are linear in z_t. I can rewrite them using (2.5.4)-(2.5.5).

$$S_t^p(z) = A_t\phi_t\left(z - z_t^p\right) \tag{2.5.7}$$
$$S_t^f(z) = \rho A_t\phi_t\left(z - z_t^f\right) \tag{2.5.8}$$

As a result, a new definition of z_t^* stems from (2.5.3), (2.5.7) and (2.5.8).

$$(1-\rho)z_t^* = z_t^c - \rho z_t^f \tag{2.5.9}$$

Using the expressions of total surpluses (2.4.16)-(2.4.18), the definition of thresholds (2.5.4)-(2.5.6) and integrations by part, productivity thresholds verify

$$\begin{aligned} & A_t z_t^p\phi_t + \left(1 - \mathbb{E}_t\beta_{t,t+1}(1-s)\right)F + \mathbb{E}_t\beta_{t,t+1}(1-s)A_{t+1}\phi_{t+1}\int_{z_{t+1}^p}^{+\infty}\left(1-G(z)\right)dG(z) \\ & = b + \frac{\eta\gamma\theta_t}{1-\eta} \end{aligned} \tag{2.5.10}$$
$$A_t\phi_t z_t^c = A_t\phi_t z_t^p + F \tag{2.5.11}$$
$$\rho A_t z_t^f\phi_t + \rho\mathbb{E}_t\beta_{t,t+1}(1-\delta)A_{t+1}\phi_{t+1}\left(\mathbb{E}z - z_{t+1}^f\right) = b + \frac{\eta\gamma\theta_t}{1-\eta} \tag{2.5.12}$$

Using these thresholds and integrations by part, I rewrite the job creation condition (2.5.2).

$$\frac{\gamma}{(1-\eta)q\left(\theta_t\right)A_t\phi_t} = \int_{\max[z_t^c, z_t^*]}^{+\infty}\left(1-G(z)\right)dz + \rho\int_{z_t^f}^{\max[z_t^c, z_t^*]}\left(1-G(z)\right)dz \tag{2.5.13}$$

The following proposition sums up job creation.

Proposition 1. *Given* $\left(z_t^p, z_t^c, z_t^f, z_t^*\right)$,

- *if* $z_t^* \leq z_t^f \leq z_t^c$*, job creation is carried out through open-ended contracts only. It occurs when* $z \geq z_t^c$ *as figure 2.2 shows.*

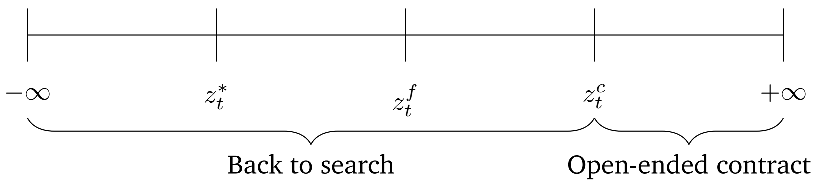

Figure 2.2: Open-ended hires only

- *Otherwise, necessarily,* $z_t^c \leq z_t^f \leq z_t^*$*: job creation is carried out through both open-ended contracts and fixed-term contracts. Fixed-term contracts are hired when* $z \in (z^f, z^*)$ *and open-ended contracts are hired when* $z \in (z^*, +\infty)$*. Figure 2.3 sums it up.*

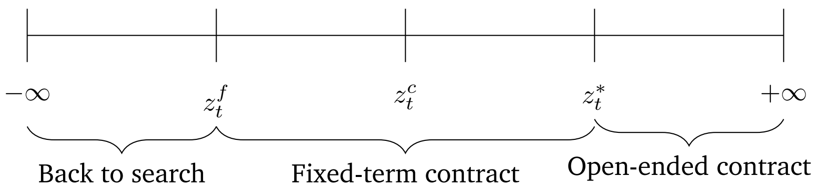

Figure 2.3: Dual job creation

Proof. See Appendix A □

Demonstrating proposition 1 follows the same steps as in Rion (2019). However, the choice between open-ended contract and a fixed-term contract does not stem from the same mechanisms. Here, the trade-off opposes flexibility and productivity. An open-ended contract delivers a full productivity but may lead to the payment of firing costs if an adverse productivity shock hits. Thus, hiring an open-ended worker is worth it if productivity is high enough to overcome the expected firing costs. A fixed-term contract delivers a lower productivity, but it is shorter and separation costs zero. The option of hiring a fixed-term contract makes agents short-sighted: it pushes up the minimum productivity to hire an open-ended contract and tightens the hiring window for open-ended contracts.

Rion (2019) is similar to the present model with two notable exceptions. Firstly, open-ended and fixed-term workers have the same productivity. Secondly, matches face i.i.d productivity shocks, whose occurrence follows a Poisson law. A new match with a high productivity chooses a contract to last as long as possible: it would be a pity to split before any productivity shock hits and to lose the advantage of a high productivity draw. Thus, on average, an open-ended contract lasts longer than a fixed-term contract and enables to lock up highly productive matches. A new match with a lower initial productivity may not find it optimal to face the risk of paying firing costs in the doldrums to secure current gains. In worst cases, going back to search is the best option. In the middle ground, though, fixed-term contracts are relevant; they enable both production and a quick return to search for a better match.

In this paper, productivity shocks no longer hit at random according to a memory-less Poisson law, which cuts out the incentive to secure the most productive matches through open-ended contracts. When meeting, a firm and a worker know that the current productivity draw will last one period. They do not hope a high productivity draw to last forever. Thereby, fixed-term contracts lose their role of median solution between producing and searching for more productive matches. One contract would be systematically preferred to the other without the contractual productivity gap. Still, the assumption that fixed-term contracts are *per se* less productive than open-ended ones is not far-fetched. Fixed-term positions are mainly filled by low-skilled or inexperienced workers (Fontaine and Malherbet, 2016) and benefit less from on-the-job training (Arulampalam and Booth, 1998; Arulampalam et al., 2004; Albert et al., 2005; Cutuli and Guetto, 2012). Aguirregabiria and Alonso-Borrego (2014) estimate that fixed-term workers are 20% less productive than open-ended workers.

The main departure with respect to Sala and Silva (2009) and Sala et al. (2012) is the appearance of the threshold z_t^*. The movements in thresholds z_t^p, z_t^c, z_t^f and z_t^* shape the behavior of the labor market and its fluctuations. If job creation involves both fixed-term and open-ended contracts — as will be the case in my calibration — then Figure 2.4 sums up the hiring and firing policies and the position of the thresholds if idiosyncratic productivity shocks are drawn from a log-Normal distribution.

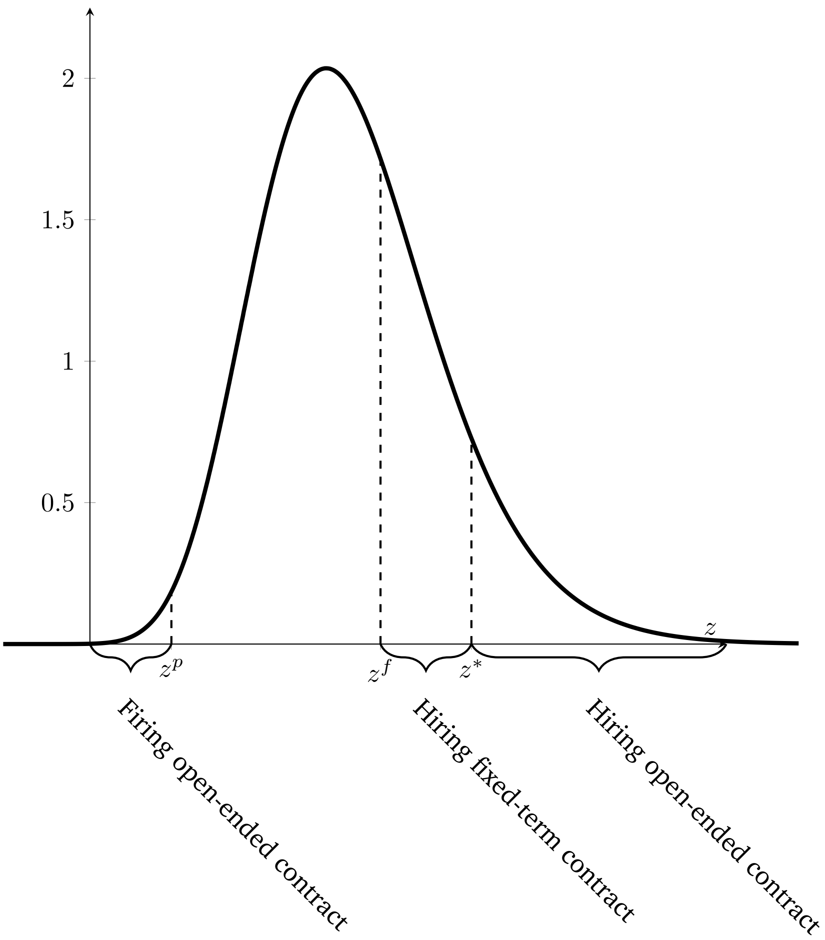

Figure 2.4: The probability distribution function of idiosyncratic shocks and the location of thresholds

The displayed pdf belongs to a log-Normal low with zero mean and standard deviation 0.2.

2.6 Fiscal and monetary policy

The government taxes households in order to provide for the unemployment benefits. The revenues from the firing tax get back to the government. Thus, the budget constraint of the government is

$$\tau_t + (1-s)G\left(z_t^p\right)n_t^p F = \rho^b \overline{w} u_t + g_t \tag{2.6.1}$$

where g_t is the government expenditure and follows the AR(1) log-process $\log\left(g_t\right) = (1-\rho_g)\log\left(\overline{g}\right) + \rho_g log\left(g_{t-1}\right) + \epsilon_t^g$, $\epsilon_t^g \sim \mathcal{N}\left(0, \sigma_g^2\right)$.

The monetary policy is decided in accordance with the interest rate rule

$$\log\left(R_t/R\right) = \rho_R \log\left(R_{t-1}/R\right) + (1-\rho_R)\left[\rho_\pi \mathbb{E}_t \log\left(\frac{P_{t+1}}{P_t}\right) + \rho_y \log\left(\frac{y_t}{y}\right)\right] + \epsilon_t^m \tag{2.6.2}$$

where y is the steady-state output and $\epsilon_t^m \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_m^2\right)$.

Fixed-term contracts are known to behave as buffers in front of workload fluctuations, as Saint-Paul (1997) documented. Thus, the case of indeterminacy in the Taylor rule and the subsequent appearance of sunspot equilibria may prove interesting. For the sake of simplicity, though, I shall restrain the present analysis to the determinate case with $\rho_\pi > 1$ and $\rho_y > 0$ and leave indeterminacy and its consequences on dual labor markets for future research.

2.7 Aggregate values and the equilibrium

This paragraph sums up the different conditions enabling an utter closing of the model. The employment values sum to the measure of households, namely 1.

$$n_t^p + n_t^f + u_t = 1 \tag{2.7.1}$$

The aggregate stock of job-seekers includes the formerly unemployed households and the new entrants in the unemployment pool from the current period.

$$e_t = u_{t-1} + \delta n_{t-1}^f + \xi_t n_{t-1}^p \tag{2.7.2}$$

The employment variables drop by the job destruction flow and augment by the job creation flow.

$$n_t^p = (1-\xi_t)\, n_{t-1}^p + \mu_t^p v_t \tag{2.7.3}$$
$$n_t^f = (1-\delta)\, n_{t-1}^f + \mu_t^f v_t \tag{2.7.4}$$

where $\mu_t^p = q(\theta_t)(1 - G(z_t^*))$ and $\mu_t^f = q(\theta_t)\left(G(z_t^*) - G\left(z_t^f\right)\right)$ are the open-ended and fixed-term job-filling rates. $\xi_t = s + (1-s)G(z_t^p)$ is the the job destruction rate of open-ended contracts.

As for firms, the aggregate demand for final goods is

$$Y_t = c_t + g_t + \gamma v_t \tag{2.7.5}$$

The retailers face only one real marginal cost from the intermediate firms, which entails a unique equilibrium value for the optimal price-setting program, *id est* $P_{i,t}^* = P_t^*$.

The market clearing condition for intermediate goods states that intermediate goods are produced by incumbent workers or new workers through either open-ended or fixed-term contracts.

$$\begin{aligned}\int_0^1 Y_{i,t} di = {} & A_t E_z\left[z \mid z \geq z_t^p\right](1-\xi_t)\, n_{t-1}^p + \mu_t^p v_t A_t E_z\left[z \mid z \geq z_t^*\right] \\ & + \rho A_t \mathbb{E}_z[z](1-\delta)\, n_{t-1}^f + \rho \mu_t^f v_t A_t \mathbb{E}_z\left[z \mid z_t^* \geq z \geq z_t^f\right]\end{aligned}$$

Using the first order condition from the final good firm's program (2.2.2), I get

$$\begin{aligned}Y_t \Delta_t = {} & A_t E_z\left[z \mid z \geq z_t^p\right](1-\xi_t)\, n_{t-1}^p + \mu_t^p v_t A_t E_z\left[z \mid z \geq z_t^*\right] \\ & + \rho A_t E_z[z](1-\delta)\, n_{t-1}^f + \rho \mu_t^f v_t A_t E_z\left[z \mid z_t^* \geq z \geq z_t^f\right]\end{aligned} \tag{2.7.6}$$

with $\Delta_t = \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} di$ measures price dispersion. Yun (1996) demonstrated that the associated law of motion is

$$\Delta_t = (1-\psi)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + \psi\left(\frac{P_t}{P_{t-1}}\right)^{\epsilon} \Delta_{t-1} \tag{2.7.7}$$

while the price level follows

$$P_t = \left[\psi P_{t-1}^{1-\epsilon} + (1-\psi)\left(P_t^*\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{2.7.8}$$

As detailed in Appendix B.3, the log-linearization of the price-level equation (2.7.8) and the price-setting equation (2.3.1) leads to the New-Keynesian Phillips curve.

$$\widehat{\pi}_t = \beta \mathbb{E}_t \widehat{\pi_{t+1}} + \kappa \widehat{\phi}_t + \epsilon_t^\mu$$

where $\kappa = (1-\beta\psi)(1-\psi)/\psi$ and ϵ_t^μ is an i.i.d cost-push cost that follows a normal law $\mathcal{N}\left(0, \sigma_\mu^2\right)$. I depart from Thomas and Zanetti (2009), which introduces a mark-up shock. Mark-up shocks weigh in the Phillips curve in a manner that makes the price rigidity parameter ψ and the parameters defining the shock hard to identify. Cost-push shocks avoid this pitfall.

Given the path of exogenous shocks $\left\{\epsilon_t^A, \epsilon_t^\mu, \epsilon_t^g, \epsilon_t^m\right\}_{t=0}^{+\infty}$, laws of motions of exogenous shocks $\left\{A_t, g_t\right\}_{t=0}^{+\infty}$ and initial values for the state variables $\left\{R_{-1}, n_{-1}^p, n_{-1}^f, \Delta_{-1}, P_{-1}\right\}$, the equilibrium sums up into the set of endogenous variables $\left\{R_t, c_t, Y_t, n_t^p, n_t^f, u_t, \Delta_t, z_t^p, z_t^c, z_t^f, z_t^*, \theta_t, \phi_t, v_t, P_t, P_t^*\right\}_{t=0}^{+\infty}$ pinned down by equations (2.1.1), (2.3.1), (2.5.10)-(2.5.12), (2.5.9) and (2.6.2)-(2.7.8).

3 Calibration and estimation procedure

I bridge the model to the data through two steps. Some of the parameters are firstly calibrated to match moments shaping typical dual European labor markets. Then, parameters driving shock processes and the nominal rigidity are estimated through a Bayesian technique.

3.1 Calibration

The intended calibration exercise needs to meet two requirements to be relevant heuristically speaking. A natural objective is the faithful reproduction of labor market dualism in the Euro area. Comparability with existing research is another one ; labor market dualism has not been combined with a New Keynesian framework already, as far as I know. I shall rely on modeling choices Thomas and Zanetti (2009) make, as it looks at a classic labor market embedding firing costs within the Euro Area.

The central role of the distribution for idiosyncratic productivity shocks in the shaping of hiring and separation decisions falls reluctantly within the need for comparability, in absence of a proper estimation procedure. Consequently, I assume that the standard deviation for these shocks amounts to 0.2. I follow Thomas and Zanetti (2009) in several other dimensions. The discount factor is simarly set to 0.99. The matching function is assumed to be in a Cobb-Douglas form $m(e, v) = me^\sigma v^{1-\sigma}$ with σ set to 0.6, which stands in the middle of the 0.5-0.7 range Burda and Wyplosz (1994) estimated for some European countries. The Hosios condition being still verified, I set the workers' bargaining power to 0.6 as well so that the congestion externalities do not weigh in. The elasticity of demand curves is set to 6. I also assume that the government-spending-to-gdp ratio is 20 % .

β	σ	η	σ_z	ϵ
0.99	0.6	0.6	0.2	6

Table 3.1: Initial parameters

I depart from Thomas and Zanetti (2009) in several dimensions. I assume that the vacancy-worker meeting probability from the firm's point of view is 0.7 instead of 0.9. The latter figure replicates flows on the US labor market, which are known to be bigger than the European ones. One important feature of the labor market is its size, since it influences labor market tightness and job-finding rates. Should we consider ILO-defined unemployment *stricto sensu* or include the inactive population ? Elsby et al. (2015) and Fontaine (2016) demonstrated the importance of the participation margin to explain unemployment fluctuations respectively in the United States and in France. This is all the more true with precarious employment, which involves people at the blurred frontier between unemployment and inactivity. According to data from Eurostat extending between 2002 and 2017, the participation rate in the Euro Area rate is around 67 %. Thus, I set steady-state employment to 0.67. In the same manner, I target a ratio of temporary contracts over total employment of 13 % in accordance with Eurostat estimates from 2006 to 2017.

The contractual composition of creation flows matters much, as it impacts the turnover of the labor force. Targeting the share of fixed-term contracts among new jobs enables to control for the contractual composition of hires. However, an issue arises: the model implicitly assumes that fixed-term and open-ended contracts are substitutes for all tasks in the economy. It is a strong assumption as fixed-term workers could be better suited to some specific tasks, such as one-time tasks. Some industries frequently cope with one-time demand. In France, for example, the audiovisual, catering and medico-social sector represent the bulk of hires through fixed-term contracts lasting one month or less. Thus, targeting the share of fixed-term contracts in job creation should be done with caution, as it mixes fixed-term contracts with diverse degrees of substitutability with respect to open-ended contracts. Ruling out very short fixed-term contracts in the calibration may help to consider fixed-term contracts and open-ended contracts that may substitute. In France, data from *Acoss - Dares* witness that 83 % of job creations occur through fixed-term contracts and 30 % of these new fixed-term contracts last less than one day. Excluding these contracts leads to a share of fixed-term contracts in jobs creation of 77 %. Unfortunately, I do have data about the proportion of very short contracts in other countries. In Spain, Felgueroso et al. (2017) documents that 90 % of job creations occur through fixed-term contracts. At the other end of the spectrum, Addison et al. (2019) report a 45 % share of temporary contracts in German job creation. I target a share of fixed-term contracts in job creation of 70 % to reach a middle ground in the Euro area.

I also set the quarterly separation probability of permanent matches ξ to 3 % consistently with the magnitude of data from Eurostat. This leads to a steady-state separation probability of fixed-term matches of 20 %, which is equivalent to an average duration of 15 months. 15 months lies far above the average durations of fixed-term contracts in the Euro area because I excluded fixed-term contracts that last less than one month from the calibration. Among separations involving permanent contracts, an essential factor is the probability of paying the firing cost. Jolivet et al. (2006) explain that more sclerotic markets lead to a higher share of voluntary quits, whereas lay-offs tend to be more significant in countries with more flexible labor markets. The French case is described as the most representative of the former phenomenon, with 80 % of separations involving permanent matches happening through voluntary channels according to Dares (2018). A reasonable value for the Euro Area is thus inferior. As a result, I target a rate of 60 % for the ratio of exogenous separations among total separations for permanent matches. The resulting value of s is 2.1 %.

The calibration of firing costs constitutes a challenge. The data is scarce, which makes reasonable proposals difficult to spell. It is all the more true since heterogeneity between countries is high, whether it be from a legal or an economic point of view. Moreover, labor court decisions often meddle with firing decisions, bringing their share of uncertainty and hidden costs as Bentolila et al. (2012) documents. The 0-to-5 OECD indicator for employment protection legislation enables a cursory comparison between Euro area countries. While French and German indexes of EPL against collective and individual dismissals of permanent contracts are close to 2.8, the Spanish index is roughly 2.6 and the Italian index is close to 3. Employment protection legislation in the Euro Area seems roughly closer to the French and German case. Fortunately, Kramarz and Michaud (2010) is one of the scarce studies that estimates firing costs and it considers the French case. The latter assess that individual lay-offs marginally cost 4 months of the median wage, while the marginal cost of lay-off within a collective-termination plan represents 12 months of the median wage[5]. The former being the most frequent case, we reckon that total firing costs represent 4 months of the permanent workers' average wage . As in Bentolila et al. (2012) and Cahuc et al. (2016), we assume that red-tape costs actually embodied by F only represent one third of total firing costs for the firm[6]. Thus, we target a ratio of 4/9 for firing costs with respect to the quarterly permanent workers' average wage.

$F/\overline{w^p}$	$\mu^f/\left(\mu^p+\mu^f\right)$	n^f/n	ξ	s/ξ	n	$q(\theta)$
4/9	0.7	0.15	0.03	0.6	0.67	0.7

Table 3.2: Targets for a calibration of the labor market in the Euro area

The last parameters to be pinpointed are F, b, m, ρ and γ. The empirical evidence concerning b is thin. Indeed, the flow value of non-employment is a highly debated issue in labor economics. Hagedorn and Manovskii (2008) advocate a high relative value of non-employment with respect to employment to make the Mortensen-Pissarides model able to replicate faithfully fluctuations in unemployment and vacancies following productivity shocks of a realistic magnitude. The obtained b is coherent with this view. In the same manner, no proper empirical evidence is available to assess the productivity difference between fixed-term contracts and open-ended contracts in European countries, which explains why I prefer leave it as a free parameter in the calibration. I find a 2% productivity deficit among temporary contracts with respect to permanent contracts. The vacancy cost represents 1.5 % of the average wage, which is coherent with the (scarce) available evidence put forward by Kramarz and Michaud (2010).

F	b	s	δ	ρ	m	γ
0.38	0.84	0.02	0.2	0.98	0.44	0.01

Table 3.3: Calibrated parameters

3.2 Estimation

The unknown parameters are related to shock processes and nominal rigidities. I estimate them using a Sequential Monte Carlo procedure[7]. Following Thomas and Zanetti (2009), I choose the same time period extending from 1997-Q1 to 2007-Q4 and a similar set of quarterly time series as observables (Y_t, π_t, R_t, n_t), namely GDP at constant prices, employment, EONIA rates in annual terms and the GDP deflator[8]. All involve the 19-country Euro area and are obtained from the ECB Data Warehouse. The demeaned growth rate of the GDP deflator is plugged to inflation π_t. Since R_t corresponds to the real interest rates, they correspond to the demeaned quarterly equivalent of EONIA rates diminished by estimated inflation. Real GDP and employment are logged and reluctantly linearly detrended. As a matter of fact, the use of any detrending method or filter as well as the comprehension of adequate observable equations to match growth rates in the data is mainly arbitrary and may deliver heterogeneous and mis-specified estimation results. Filippo (2011) and Canova (2014a) roughly propose to simultaneously estimate the parameters of flexible specifications for trends along deep parameters so as to let the model

[5]To be accurate, Kramarz and Michaud (2010) assess that firms with more than 50 employees face a marginal cost of 97,727 FFr (Table 1b), which represents 14 months of the workers' median wage. Consequently, the associated median wage of fired workers is 6980 FFr. Thus, Table 2 shows that individual terminations cost 27,389 FFr, which amounts to 4 months of the fired workers' median wage, while the termination within a collective firing plan marginally costs 81,850 FFr, which equals 12 months of the median wage.

[6]Transfers between separated firms and workers are not taken into account in firing costs because they do no allocational role, in contrast with red-tape firing costs

[7]See Herbst and Schorfheide (2016) for an extensive treatment of this method.

[8]Table C.2 details the series used for estimation

explain the part of the data it is able to account for. However, as Canova and Sala (2009) and Iskrev (2010); Iskrev et al. (2010) testify, identification issues are real in the estimation of DSGE models. The resort to the methods advocated by Filippo (2011) and Canova (2014a) would magnify these problems through the introduction of new parameters. For this reason, I stick to the detrending method employed by Thomas and Zanetti (2009), which remains simple and enables comparisons between my model and their own. In Appendix C.3 Table C.3, I report the estimation results using different filtering methods and observables ranging from 1995Q3 to 2019Q2. Overall, the estimates are pretty robust.

Most chosen priors are chosen to be diffuse, reflecting the void of the DSGE literature with respect to dual labor markets. Thus, the priors for autocorrelations of the shock processes are uniform laws on $[0, 1]$, while standard deviations follow inverse gamma distributions with mean 0.5 and standard deviation 4[9]. As for the Taylor parameters r_π and r_y, the prior needs to be vague and embed the fact that $r_\pi > 1$ and $r_y > 0$. As a result, truncated normal laws with large standard deviations are employed. Druant et al. (2012) assess the average duration of firms' prices in the Euro Area to 10 months, which corresponds to a value of ψ around 0.7. A tight normal law around this value is subsequently chosen.

Parameter	Prior distribution			Posterior distribution			
	Distr.	Para (1)	Para(2)	Mean	Std. Dev.	5%	95%
ρ_A	Uniform	0.0	1.0	0.7	0.08	0.56	0.83
ρ_g	Uniform	0.0	1.0	0.89	0.04	0.81	0.94
ρ_R	Uniform	0.0	1.0	0.77	0.05	0.68	0.85
ρ_π	Normal	1.5	0.75	2.17	0.62	1.25	3.28
ρ_y	Normal	0.12	0.15	0.29	0.06	0.19	0.4
ψ	Beta	0.7	0.05	0.83	0.03	0.78	0.87
σ_A	IGamma	0.5	4.0	0.31	0.04	0.26	0.37
σ_μ	IGamma	0.5	4.0	0.3	0.03	0.25	0.36
σ_g	IGamma	0.5	4.0	5.59	1.51	3.86	8.41
σ_m	IGamma	0.5	4.0	0.08	0.01	0.07	0.1

Table 3.4: Prior and posterior distributions of structural parameters.

Para(1) and Para(2) correspond to mean and standard deviation of the prior distribution if the latter is Normal or Inverse Gamma. Para(1) and Para(2) correspond to lower and upper bound of the prior distribution when the latter is uniform

I run the Sequential Monte Carlo procedure with 500 likelihood-tempering steps, which involve a swarm of 30,000 particles and one Random-Walk-Metropolis-Hastings step each. Table 3.4 compound results. I similarly estimate a classic labor market without fixed-term contracts to compare results. All details are available in Appendix D. The results are displayed in Table D.3. Overall, the estimates are pretty similar across both models and estimates are not significantly different. The log marginal density do not significantly differ either, as Table D.2 shows.

Government spending shocks are more volatile and more auto-correlated in the estimated classic model than in its dual counterpart. Government spending is a pure aggregate demand shock that does not impact the production function. Moreover, given the absence of capital, note that the shock on government spending g_t also includes the missing values aggregate demand need to match fluctuations in GDP. Productivity shocks impact production, inflation and employment. Thus, productivity shocks alone cannot account for the whole extent of fluctuations in GDP without producing crazy results in term of interest rate, inflation and employment. Government spending shocks capture the fluctuations in GDP

[9]Standard deviations are expressed in percentage.

that productivity shocks cannot fully account for. Table D.2 reports that both models have comparable logged marginal data densities. Thus, roughly speaking, the more volatile and persistent government spending shocks are, the more mis-specified aggregate demand is to fit the GDP time series. To this extent, the dual model fits observed fluctuations in GDP better than the classic model. The volatility and persistence of government spending shocks remains high in the dual model, though. The model lacks essential features — capital and investment for example — to properly fit fluctuations in GDP. The estimates of the government spending shock in Thomas and Zanetti (2009) exhibit a similar behavior.

The cost-push shock μ_t is the counterpart of the government spending shock for inflation. Its volatility amounts to one standard deviation of the inflation time series. The auto correlation parameter of productivity shocks is 0.7, lower than usual estimates in the literature. The parameters driving the interest rate rule (2.6.2) are similar to Thomas and Zanetti (2009), with the exception of the weight of output. The Calvo pricing parameter amounts to 0.83, a bit lower than the estimate of Thomas and Zanetti (2009) around 0.9.

4 Experiments

4.1 Labor market moments

In this paragraph, we assess the ability of the model with respect to the reproduction of labor market moments. The absence of proper series for the computation of the latter in the Euro area represents a significant obstacle. As a result, I use time series from French data as a proxy to account for the main features of a dual labor market. I detail the sources in Table C.2. Figure 4.1 shows the logged and filtered data along the lines of Hamilton (2018). For this graph only, I do not use linear detrending because the 2008's crisis belongs to the time interval of data; a break of trend occurs for most variables[10]. It is not a problem as the qualitative cyclical features I describe below remain when using linear detrending.

[10] See Gorodnichenko and Ng (2010), Ferroni (2011), Lafourcade and de Wind (2012), Canova (2014b) and Hamilton (2018) for thorough developments about the consequences of filtering data

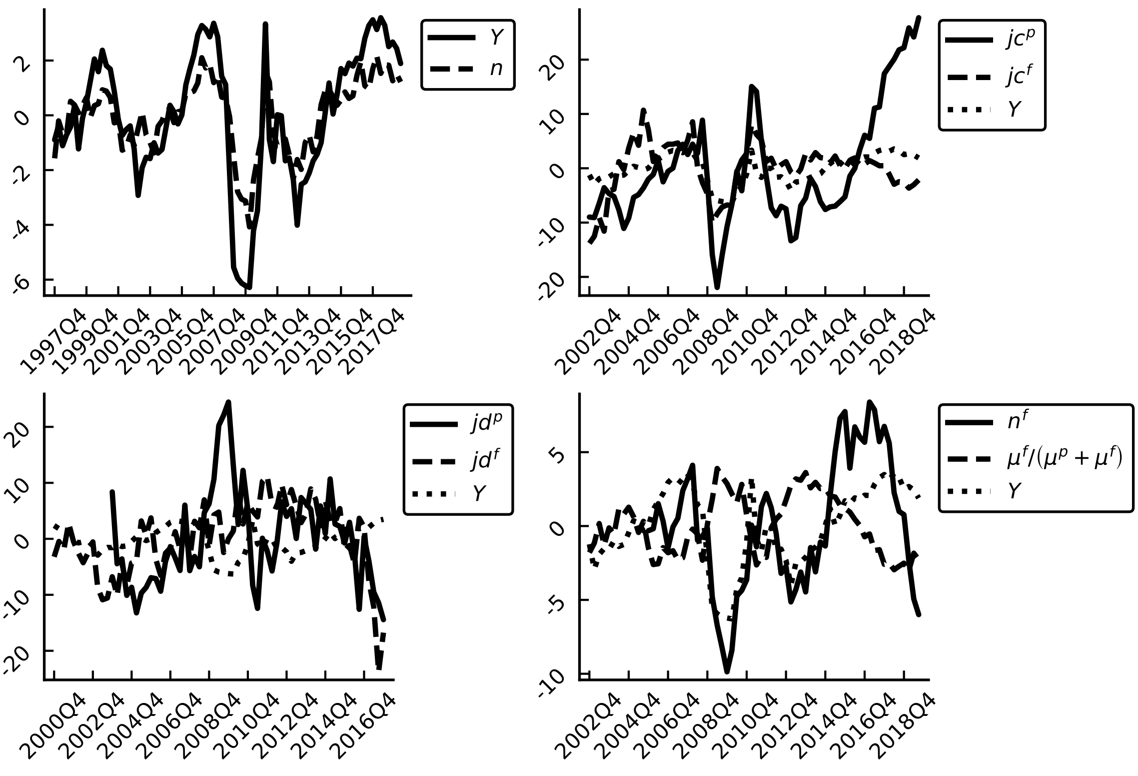

Figure 4.1: Fluctuations in labor market values.

Aggregate employment n is pro-cyclical and less volatile than output, as is well known. Fixed-term employment n^f is also pro-cyclical and displays a volatility larger than aggregate employment. On the flows side, job creation and job destruction of open-ended contracts seems more volatile than their fixed-term counterparts. While open-ended job creation jc^p and fixed-term job creation jc^f are both pro-cyclical as one would expect, job destruction flows show opposite behaviors across contract types. Open-ended job destruction is counter-cyclical whereas fixed-term job destruction is pro-cyclical. In favorable times, as demand increases, firms hire fixed-term workers. Fixed-term contracts are short ; most of them last less than a quarter. Thus, an increase in fixed-term job creation immediately generates an increase in fixed-term job destruction. The share of fixed-term contracts in job creation seems counter-cyclical. Open-ended job creation jc^p and fixed-term job creation jc^f are pro-cyclical. Open-ended job creation is more volatile than fixed-term job creation. The share of fixed-term contracts in job creation $\mu^f / \left(\mu^p + \mu^f\right)$ is counter-cyclical.

Variables	Data		Model	
	Std. Dev.	$Cor\,(Y,.)$	Std. Dev.	$Cor\,(Y,.)$
Y	1.16	1.0	0.79	1.0
π	0.32	0.15	0.31	0.01
R	0.33	0.29	0.18	0.38
n	0.73	0.93	0.87	0.85
jc^p	7.19	0.65	12.16	0.45
jc^f	4.97	0.52	3.89	0.21
$\mu^f / (\mu^f + \mu^p)$	1.29	-0.4	2.89	-0.47
n^f	1.87	0.18	2.0	0.1
jd^p	5.7	-0.43	7.05	-0.54
jd^f	2.84	0.39	2.0	0.24
v	10.32	0.61	7.29	0.41

Table 4.1: Actual and simulated correlations with output of different variables

Is the model able to reproduce cross-correlations and volatility of these labor market time series ? I simulate the model to answer this question. For each of the 30,000 particles of the SMC algorithm, I compute the correlations over 200-period chains of shocks with a 100-period burn-in time. Then, I derive a weighted average of cross covariances using the particle-specific weights generated by the SMC procedure. Then, I compare the simulated moments to those of the time series displayed in Figure 4.1, which are linearly detrended between 1997-Q1 and 2007-Q4. I also consider vacancies v, inflation π and EONIA interest rates R. Table 4.1 compounds the results. Overall, the model reproduces well the standard deviations and correlation with output of the considered time series. The model retrieves the pro-cyclicality of job creation flows. On the job destruction side, the model reproduces the pro-cyclicality of the fixed-term job destruction rate and the counter-cyclicality of the open-ended job destruction. It also mimics the higher volatility of flows on the open-ended side of the labor market with respect to its fixed-term counterparts. Compared to Sala and Silva (2009) and Sala et al. (2012), my main contribution is replicating the volatility and counter-cyclicality of the share of fixed-term workers in job creation $\mu^f / (\mu^p + \mu^f)$. Sala and Silva (2009) and Sala et al. (2012) assume that it is exogenously set by law.

Shimer (2005) showed that replicating the Beveridge curve in real business cycles models with a frictional labor market is challenging. The actual negative correlation between unemployment and vacancies is difficult to replicate without making productivity implausibly volatile. In the original Diamond-Mortensen-Pissarides model, a positive productivity shock increases the surplus of matches and encourages the posting of vacancies. It pushes up labor market tightness and workers benefit from a higher job-finding probability that bolsters their threat point in Nash-bargaining. Wages end up increasing and fuel job destruction. Therefore, the positive effect of the productivity shock on wages significantly weakens the movements of job creation and vacancies, making the Beveridge curve flatter or even positively sloped. Ljungqvist and Sargent (2017) explain that the endeavors to obtain sufficiently strong responses of unemployment to productivity shocks consist in reducing the fundamental surplus of the match. The so-called fundamental surplus is the share of the surplus that can be allocated to job creation. The literature teem with implementations of wage rigidity, high utility of unemployment or firing costs ; all boil down to reducing the fundamental surplus of new matches. What are the dimensions impacting the fundamental surplus in my model ? The utility flow associated with unemployment is calibrated at a high value. Moreover, firing costs discourage open-ended job destruction. Both reduce the fundamental surplus. Fixed-term contracts ambiguously impact the fundamental surplus. On the one hand, it makes production possible for low values of idiosyncratic productivity for which firms and workers would get back to searching if only open-ended contracts were available. On the other hand, for a given idiosyncratic productivity, the new fixed-term contracts produce less. Numerically speaking, the simulated correlation between unemployment and vacancies amounts to -0.4, which fits the data well.

My model suffers from a widespread problem in the DSGE literature: it is unable to generate enough persistence in cross-correlations and covariances. Figures C.2 and C.2 bear witness of this issue. Introducing wage rigidity and persistent idiosyncratic shocks could help. I leave it for future research.

4.2 Impulse Response Functions

The computation of impulse response functions highlights several interesting features of the model. First of all, a substitution effect between fixed-term and open-ended contracts emerges, with long-run consequences on employment whether it be fixed-term or open-ended and a strong influence on impact. Secondly, a general-equilibrium effect appears. Indeed, shocks imply changes in the size of the job seekers' pool, which, in turn, has important implication for job creation flows. Finally, the common source of both effects being the evolution of (z^p, z^f, z^*, θ), the two effects interact through time depending on the persistence of the considered shock.

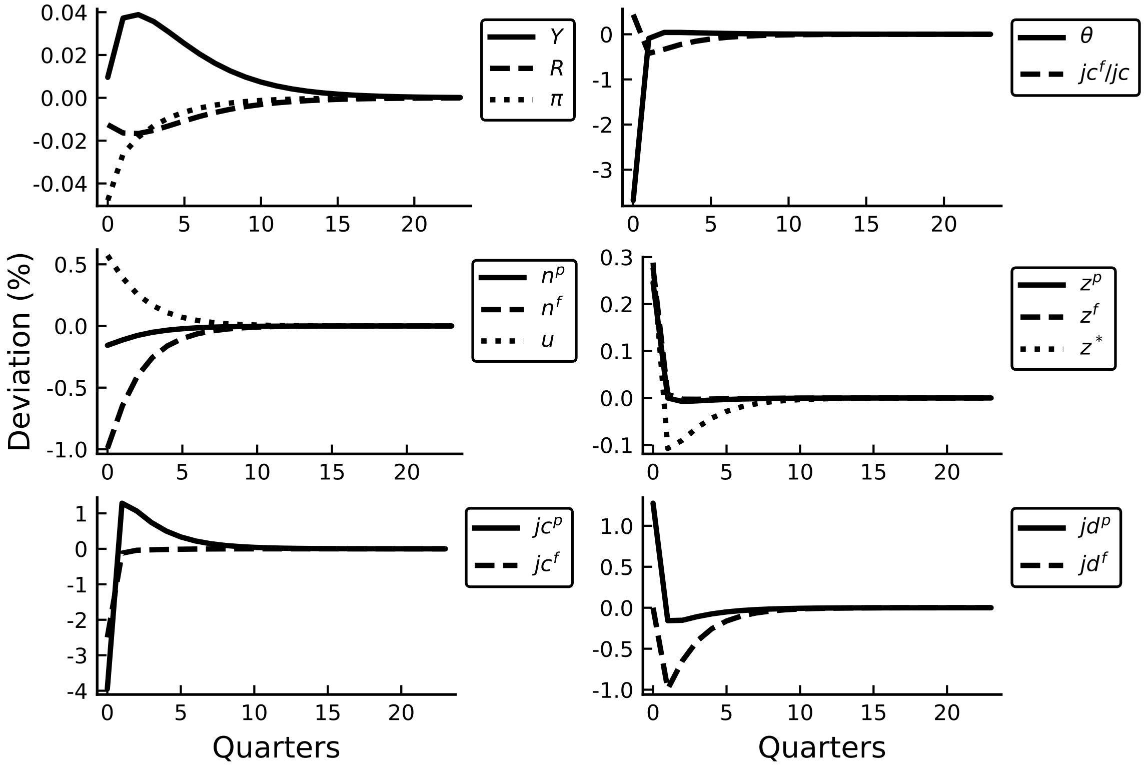

Figure 4.2: IRF of main variables to a one-standard-deviation productivity shock

Productivity shocks Figure 4.2 displays the reaction of main variables to a productivity shock. An increase in labor productivity cuts real marginal costs. Inflation decreases and the central bank reduces nominal interest rates to encourage consumption. The support to demand is insufficient to prevent a decrease in employment. Unemployment increases while vacancies reduce, delineating a downward-sloping Beveridge curve and a decrease in labor market tightness. These results are classic with New Keynesian DSGE models. The downward sloping Beveridge curve is a traditional feature of Mortensen Pissarides models.

The behavior of flows constitute an important feature of labor markets. The enhanced productivity makes agents more demanding to carry on production or to start a new match: z^p and z^f increase. The forces at stake when it comes to the evolution of z^* are more complex. On one hand, the agents seek to benefit from productivity gains in full, which bolsters the attractiveness of open-ended workers. On the other hand, firms are more demanding in terms of productivity when hiring a new open-ended worker. The choice between a fixed-term and a open-ended contract at the hiring stage also reflects the compromise between the fear of future rigidity and the appetite for immediate production gains. The transitory nature of the productivity shock makes productivity gains insufficient to encourage open-ended job creation. Overall, open-ended and fixed-term job creation immediately decrease. The behavior of thresholds, job creation and job destruction reverts after the impact. A general equilibrium effect accounts for it. Just after the shock, aggregate job destruction increases and job creation shrinks, which inflates the job seekers' pool for the next periods. This mechanically increases the subsequent job creation flows. Open-ended employment unambiguously shrinks under the joint diminution in job creation and increase in job destruction. As for fixed-term employment, job creation shrinks and job destruction occurs at a constant exogenous rate. Consequently, fixed-term employment decreases.

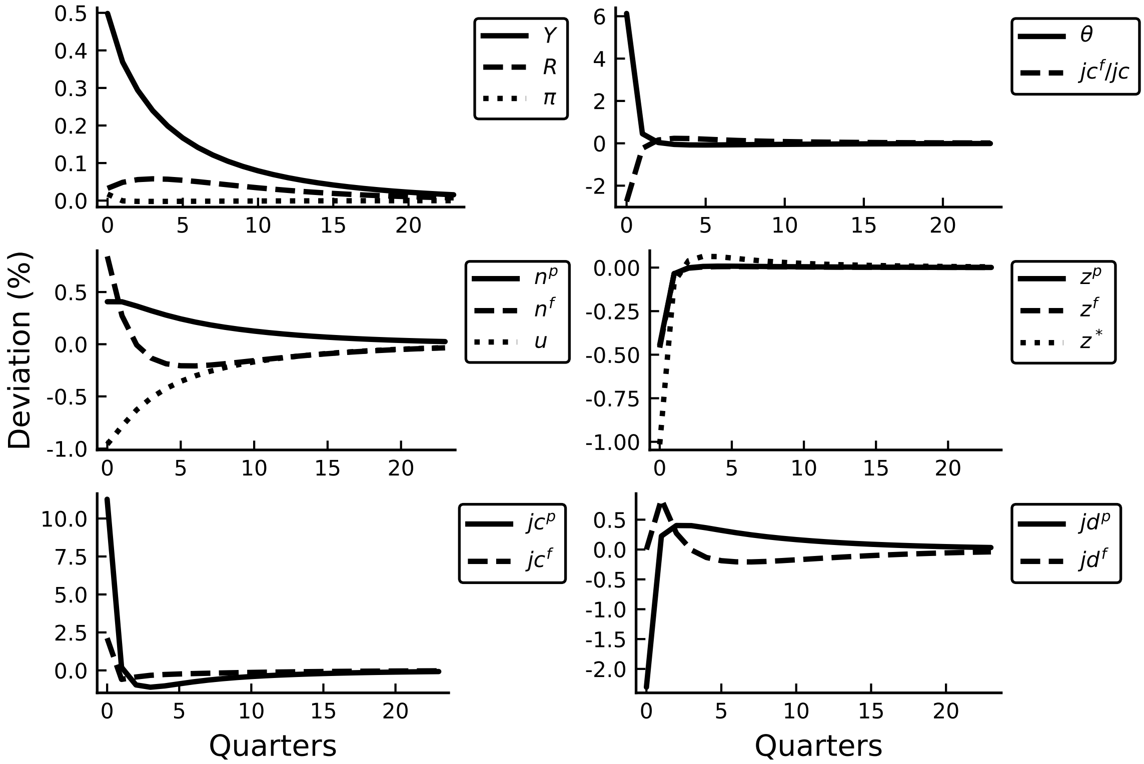

Figure 4.3: IRF of main variables to a one-standard-deviation shock in government spending shock

Government spending shocks Figure 4.3 shows the impulse response functions of main variables after a shock in government spending. As usual in the literature, a sudden increase in government spending enhances output. Real marginal costs increase and the central bank increases the nominal interest rate to prevent the spread of inflation. The labor market tightness increases as firms post more vacancies to cope with the magnified demand of consumption goods. As a result, on impact, job destruction decreases and job creation increases. Interestingly, firms cope with the surplus of demand through an enlarged share of open-ended contracts in job creation. The shock is persistent enough for the future expected losses associated with firing costs to be overcome by lifelong productivity gains. The general-equilibrium effect described in the preceding sections weighs in anew. Indeed, the shrink in the job seekers' stock exerts a downward pressure on job creations, which decrease below their steady-state values. While the transitional path of open-ended employment remains above its steady-state value, fixed-term employment goes below its equilibrium level. As a matter of fact, fixed-term employment experiments a higher turnover, and is therefore highly impacted by fixed-term fluctuations in its associated job creation. This is all the more true since destruction rates of fixed-term jobs are constant.

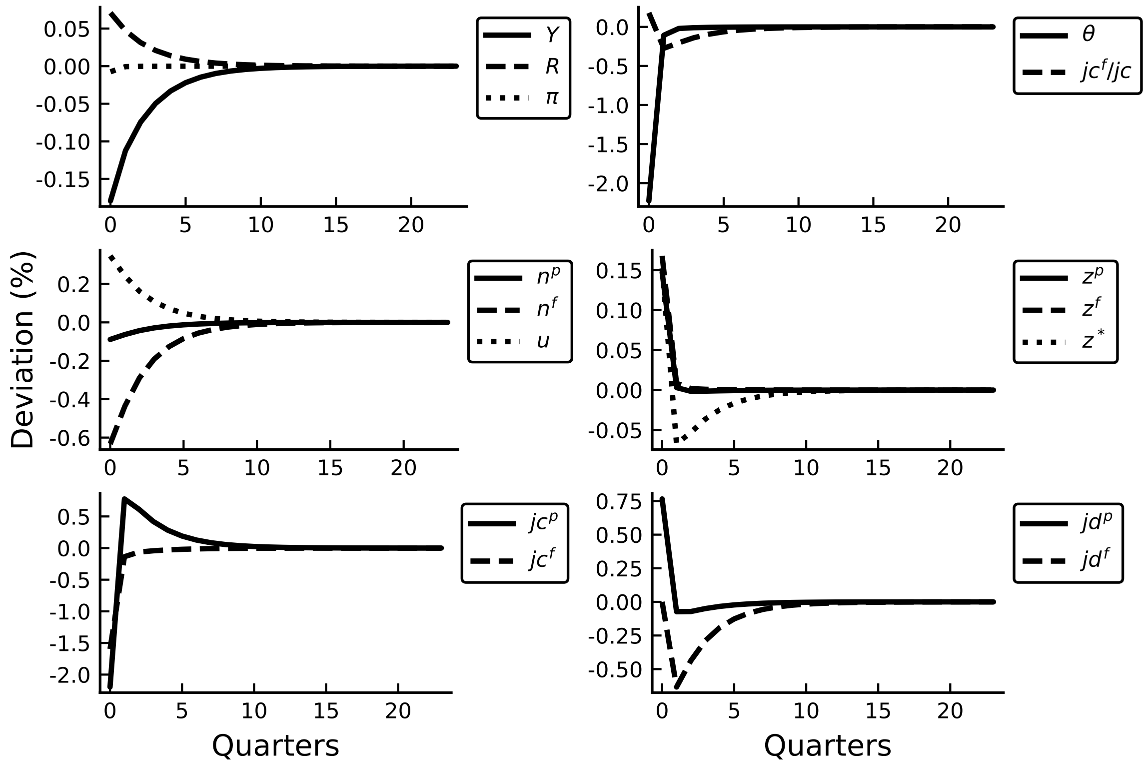

Figure 4.4: IRF of main variables to a one-standard-deviation in the monetary policy shock

Monetary policy shock Figure 4.4 shows the impulse response functions of main variables after a shock in the monetary policy. A monetary policy shock pushes up interest rates, which discourages consumption and subsequently depresses output. Real marginal costs and inflation decrease. The marginal gross revenue from labor decreases and intermediate firms are overall more demanding in productivity terms to compensate the loss in profits ; z^p, z^f and z^* increase. Thus, on impact, job creation shrinks and open-ended job destruction enlarges. In turn, the formerly open-ended employees join the ranks of the job seekers', which enhances job creation. Moreover, firms tend to switch to open-ended contracts on behalf of fixed-term ones at the hiring stage in order to temper the immediate loss in revenue due to prices ; z^* decreases. These two effects combined explain the observed rebound in open-ended job creation.

As a result, viewed from the labor market, the monetary policy shock represents the negative of the government spending shock. The economic schemes at stake are the same. There is however a nuance. The effects of the monetary policy shock on the interest rate vanish much faster and the oscillations between the substitution effect and the general-equilibrium effect develop in a rougher way. This observation is also relevant when one considers the cost-push shock. The IRF of the latter will not be described, as it involves the same economic phenomena as the other shocks.

Volatility shocks As surveys report, uncertainty in demand makes firms willing to resort to fixed-term employment (Dares, 2017). What is the impact of uncertainty in my model ? Solving the model with a first-order perturbation method is convenient to estimate the model within a reasonable time span.

However, it rules out risk as a driving force in policy functions. To catch a glimpse of the impact of uncertainty, I strike a bargain between time feasibility and the introduction of risk. I keep the estimates obtained from the first-order perturbation method, introduce stochastic volatility and solve the model using a third-order perturbation method. The third order is the lowest one where policy functions involve innovations in stochastic volatility. Figure 4.5 shows the impulse response functions of macroeconomic and labor market variables to a one-standard-deviation volatility shock to productivity starting from the stochastic steady state.

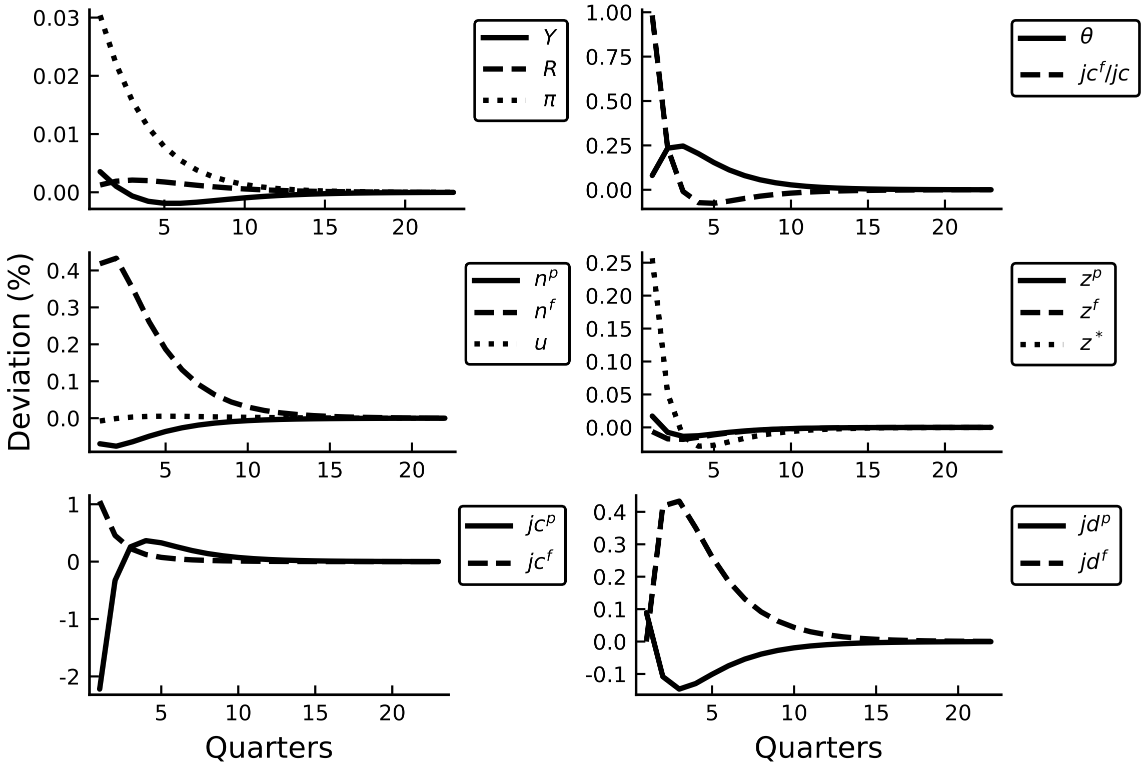

Figure 4.5: IRF of main variables to a one-standard deviation volatility shock to productivity.

The seminal paper Bloom (2009) highlights the impact of uncertainty on investment and hiring. A larger uncertainty motivates a pause in the hiring and investment decisions that involve fixed costs. In my framework, according to this mechanism, increasing uncertainty leads to postponing open-ended job destruction and job creation. It encourages substitution towards fixed-term hires and the share of fixed-term contracts in job creation increases. Following Bloom (2009), I estimate a VAR to corroborate this intuition with CAC40 and its volatility, the share of fixed-term contracts in job creation and total employment. The underpinning assumption is that shocks firstly hit stock markets, then the composition of hires and finally total employment. Including prices and other labor market variables would be better, but the short length of my quarterly time series do not allow me to do so. Figure C.4 shows the impulse response functions to a one standard-deviation volatility shock. I find that the share of fixed-term contracts in hires increases while total employment decreases. An adverse demand shock — embodied by a negative government spending shock — leads to the same response of the share of fixed-term contracts in job creation. However, in contrast with an adverse demand shock, fixed-term employment increases. Saint-Paul (1997) explains that fixed-term employment act as a protective buffer for open-ended workers ; in the doldrums, firms get rid of fixed-term workers and leave open-ended employment as untouched as possible. It is consistent with the response of the model.

As a result, in a dual labor market, an uncertainty shock and an adverse demand shock have different implications. It is a relevant result as the literature over macroeconomic risk argues that uncertainty shocks are difficult to distinguish from negative aggregate demand shocks (Leduc and Liu, 2016). Fluctuations in fixed-term employment may help to identify uncertainty shocks and demand shocks separately. My model is a naive representation of a dual labor market and cannot pretend to do it seriously, but it should be considered in future research.

4.3 Reforms in employment protection legislation

In this paragraph, I examine the macroeconomic effects of a reform on employment protection legislation, which is summed up in firing costs here.

Steady-state analysis The steady-state labor market can be summed up by the thresholds and the labor market tightness $(z^p, z^c, z^f, z^*, \theta)$. Equations (2.5.9) and (2.5.10)-(2.5.13) pinpoint the steady-state equilibrium. Figure 4.6 illustrates it. (2.5.10) defines job destruction for open-ended matches. It defines a positive relationship between the open-ended job destruction threshold z^p and labor market tightness θ ; a higher labor market tightness increases wages and encourages job destruction. I denote JD^p the corresponding locus. For the same reason, a higher labor market tightness reduces the surplus of new matches. As a result, the profitability threshold z^f and substitution threshold z^* increase with labor market tightness. I denote JP and JC^p the corresponding loci. The job creation condition (2.5.13) defines a negative relationship between the open-ended job destruction threshold z^p and the labor market tightness θ. As labor market tightness increases, wages increase and agents are more exacting when it comes to hiring. The expected surplus of a vacancy must increase to compensate firms. The higher labor market tightness pushes up the profitability threshold z^f ; the hiring region tightens and the expected gain from a vacancy decreases. The only manner to enhance the expected benefit from vacancies is to encourage substitution towards open-ended workers that are more productive ; the substitution threshold z^* must decrease, and so does the open-ended profitability threshold z^c and the open-ended job destruction threshold z^p. I denote JC the associated locus.

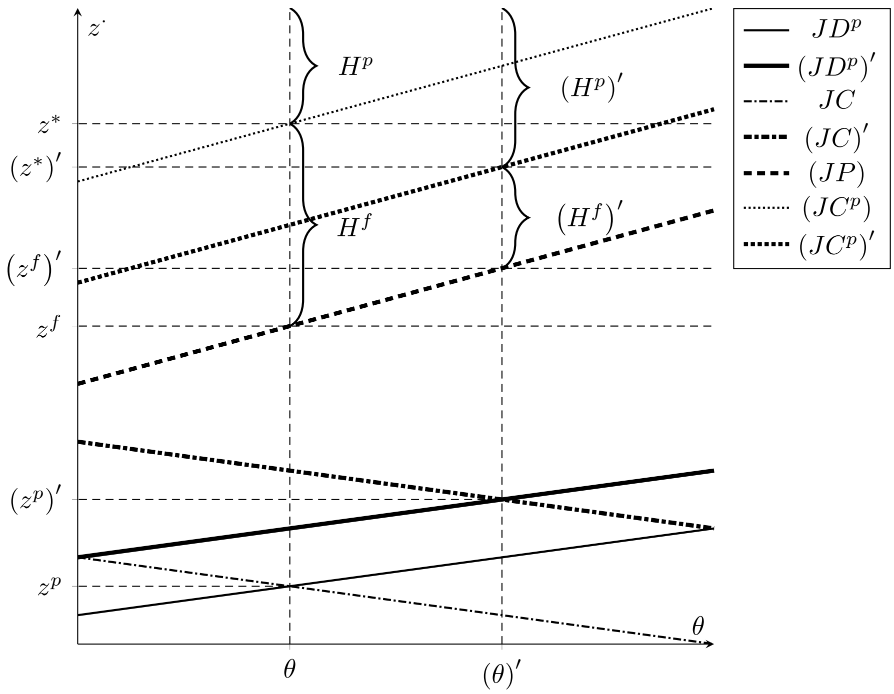

Figure 4.6: A decrease in F and the steady-state equilibrium.
The prime index the equilibrium values after the change in firing costs. H^p denote the hiring region of open-ended contracts.

I consider a steady-state decrease in firing costs. Let me first keep the labor market tightness θ constant as a thought experiment. A cut in firing costs decrease encourages job destruction; the threshold for open-ended job destruction z^p increases and the JD^p curve moves up to $(JD^p)'$. Are open-ended jobs more profitable overall ? On one hand, reduced firing costs directly encourage job creation as separations are cheaper. On the other hand, enhanced job destruction rate pushes up the probability of paying the firing cost in the future, which indirectly depletes open-ended job profitability. The former effect prevails and the profitability threshold z^c decreases. Reduced firing costs leave the profitability of fixed-term contracts unaffected: z_f does not change and the JP locus does not move. As a result, z^* increases and the JC^p curve moves up to $(JC)'$. The hiring region of open-ended contracts H^p widens.

I kept labor market tightness constant in the reasoning above. Now, I loosen up this assumption. The wider hiring region of open-ended contracts increases the expected gain from a vacancy. Open-ended contracts replace highly productive fixed-term contracts in the neighborhood of z^*. As open-ended contracts are more productive than fixed-term ones for a given idiosyncratic quality of the match, the substitution benefits the match. Thus, firms post more vacancies and labor market tightness increases. This indirectly reflects back on the thresholds. The ensuing increase in wages supports the initial increases in job destruction, while it mitigates the higher profitability of open-ended contracts and reduces the profitability of fixed-term contracts ; z^f increases. Proposition 2 mathematically proves the results mentioned above.

Proposition 2. *Considering the steady-state equilibrium, the comparative statics with respect to firing costs boil down to:* $\frac{\partial \theta}{\partial F} < 0$, $\frac{\partial z^p}{\partial F} < 0$, $\frac{\partial z^c}{\partial F} > 0$, $\frac{\partial z^f}{\partial F} < 0$, $\frac{\partial z^*}{\partial F} > 0$. *Thus, an increase in firing costs tightens the hiring region of open-ended contracts.*

Proof. See the proof in Appendix A. □

Overall, the higher labor market tightness and the broadened hiring region of open-ended contracts enhance open-ended job creation. The joint increase in job destruction makes the response of open-ended employment unclear. As for fixed-term job creation, the tightened hiring region and the higher labor market tightness lead to an ambiguous response. The change in fixed-term employment is ambiguous. Here, I retrieve the classic result of the inconclusive impact of firing costs on aggregate employment the literature has extensively discussed. Numerically speaking, a 5 % decrease in firing costs significantly increases open-ended employment and decreases fixed-term employment. The lack of persistence in the idiosyncratic productivity of matches explains the high sensitivity of the steady state with respect to firing costs.

Variables	Data		Baseline		Reduced firing costs	
	Std. Dev.	$Cor\,(Y,.)$	Std. Dev.	$Cor\,(Y,.)$	Std. Dev.	$Cor\,(Y,.)$
Y	1.16	1.0	0.79	1.0	0.79	1.0
π	0.32	0.15	0.31	0.01	0.31	0.03
R	0.33	0.29	0.18	0.38	0.18	0.37
n	0.73	0.93	0.87	0.85	0.9	0.85
jc^p	7.19	0.65	12.16	0.45	9.61	0.34
jc^f	4.97	0.52	3.89	0.21	15.64	0.14
$\mu^f / (\mu^f + \mu^p)$	1.29	-0.4	2.89	-0.47	9.95	-0.04
n^f	1.87	0.18	2.0	0.1	8.89	0.03
jd^p	5.7	-0.43	7.05	-0.54	10.18	-0.51
jd^f	2.84	0.39	2.0	0.24	8.89	0.14
v	10.32	0.61	7.29	0.41	12.08	0.35

Table 4.2: Labor market moments and institutions.

Table 4.2 shows that the cyclicality of variables has not changed much. The only exception is the share of fixed-term contracts in job creation, which was counter-cyclical and is now acyclical. Talking about volatility, open-ended job destruction is more volatile as the cut in firing costs hinders its movements less. Note that the volatility of flows on the fixed-term side soars, whereas the volatility of job creation decreased on the open-ended side of the labor market. The tightened hiring region of fixed-term workers strengthens the sensitivity of the related labor market moments. Surprisingly, the change in the composition of flows and employment does not impact inflation and employment volatility and cyclicality.

What happens to inflation dynamics ? Using the job creation condition (2.5.13) to isolate $\widehat{\phi}_t$, the New-Keynesian Phillips curve becomes

$$\widehat{\pi}_t = hc_t + \Pi_t^f + subs_t^p - \frac{\kappa}{1-\beta\rho_A}\widehat{A_t} + \epsilon_t^\mu$$

The hiring cost component hc is bound to the cost of vacancy posting.

$$hc_t = \kappa\sigma\sum_{T=t}^{+\infty}\beta^{T-t}\mathbb{E}_t\widehat{\theta_T}$$

The fixed-term hiring component Π^f represents the profitability of hiring fixed-term contracts only.

$$\Pi_t^f = \frac{(1-\eta)}{\gamma}\frac{\epsilon-1}{\epsilon}\kappa\left(\mu^f+\mu^p\right)\rho z^f\sum_{T=t}^{+\infty}\beta^{T-t}\mathbb{E}_t\widehat{z_T^f}$$

The substitution component $subs^p$ relates to the profitability of substitution fixed-term contracts towards open-ended hiring.

$$subs_t^p = \frac{(1-\eta)}{\gamma}\frac{\epsilon-1}{\epsilon}\kappa\mu^p z^*\sum_{T=t}^{+\infty}\beta^{T-t}\mathbb{E}_t\widehat{z_T^*}$$

The variance of inflation verifies

$$var\left(\widehat{\pi}\right) = var\left(hc\right) + var\left(\Pi^f\right) + var\left(subs^p\right) + \frac{\kappa^2}{\left(1-\beta\rho_A\right)^2}\frac{\sigma_A^2}{1-\rho_A^2} + \sigma_\mu^2 + covs$$

Covariances	Baseline	Reduced firing costs
$var\left(\pi\right)$	1.53	1.55
$var\left(hc\right)$	0.4	0.98
$2cov\left(hc,\Pi^f\right)$	-0.68	-1.64
$var\left(\Pi^f\right)$	0.29	0.69
$var\left(subs^p\right)$	$1.6\cdot10^{-4}$	$4.3\cdot10^{-4}$

Table 4.3: Firing costs and inflation volatility

Table 4.3 reports the variances of inflation and its relevant components in annual terms. As the small value of $var\left(subs^p\right)$ bears witness of, the substitution motive towards open-ended contracts plays a minor role. It is all the more true as the contractual productivity wedge $(1-\rho)$ is tiny. Being able to substitute contracts without facing strong differences in production abilities *ex ante* matters to inflation dynamics. In the present case, the contractual composition of flows and employment can be altered in depth without influencing inflation dynamics. Meanwhile, reduced firing costs push up the profitability threshold z^f to a region where it affects fewer new matches, as Figure 2.4 suggests. Movements in the substitution threshold z^* do not alter much the expected surplus of a vacancy as ρ is close to one. Thus, the same percentage fluctuations in labor market tightness and, thus, hiring costs, require larger adjustments in the profitability threshold z^f. The steady-state value $z^f\left(\mu^p+\mu^f\right)$ does not move much. As a result, reduced firing costs push up the variance of the fixed-term hiring component Π^f. For the same reasons, the same percentage fluctuations in the profitability threshold z^f require larger adjustments in hiring costs ; the volatility of labor market tightness increases and so does the volatility of the hiring cost component hc. The higher volatility of the hiring cost component and the fixed-term profitability component is compensated by the stronger negative co-variance of the labor market tightness and the profitability threshold. When the labor market tightness increases after a transitory shock, the expected cost of a vacancy increases. Consequently, the expected value of a vacancy must increase. A first possible response is to increase the profitability threshold to have more productive new hires, but fewer of them. A second solution is to decrease the profitability threshold to have less productive new hires, but more of them. In my calibration, the second solution prevails, which makes the labor market tightness and the profitability threshold negatively correlated. The enhanced volatility in the hiring cost component and the fixed-term hiring component mechanically increase the magnitude of their co-variance. Overall, in theoretical terms, the dual character of the labor market does not impact inflation dynamics as long as fixed-term contracts and open-ended contracts do not embed wide productivity wedges. Numerically speaking, I tried to check robustness with respect to ρ. As I highlight above, this parameter enable dualism to arise as an equilibrium. Small changes to ρ are enough to lose dualism. Introducing some persistence in the idiosyncratic productivity of matches would help reducing the sensitivity of the model with respect to this parameter.

5 Conclusion

In this paper, I have described and estimated a tractable New-Keynesian model with a dual labor market. The estimated model replicates well the main moments of a typical European dual labor market and shows that fixed-term employment could help identify uncertainty shocks from negative aggregate shocks. Moreover, changes in firing costs do not impact inflation dynamics as long as the *ex ante* contractual productivity wedge remains small.

A few points merit further discussion, though. Tractability comes at a high cost. The assumption that fixed-term produce less than open-ended ones all else equal enable a dual equilibrium to arise, but the resulting model is very sensitive to the parameter embodying this productivity wedge. Assuming that the quality of matches undergoes i.i.d shocks every period explains this lack of robustness. Moreover, ruling out heterogeneity enables to easily delineate the mechanisms at stake, but it leads to an incomplete investigation of inflation dynamics. Inflation dynamics seem unaffected by change in firing costs as long as fixed-term and open-ended workers substitute well. Accounting for firm-worker heterogeneity and rigidity in wages would make the model more realistic. I plan to work out these extensions in future research.

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A Proofs

Proof Proposition 1 The behavior of the thresholds is characterized by the following proposition

Lemma 1. *These assertions are equivalent*

1. $z_t^* > z_t^f$
2. $z_t^* > z_t^c$
3. $z_t^c > z_t^f$

Proof.

- Assume that $z_t^* > z_t^f$. The definition of z_t^* (2.5.9) implies that $z_t^* = (1-\rho)\, z_t^* + \rho z_t^* = z_t^c + \rho \left(z_t^* - z_t^f\right)$. Since $z_t^* - z_t^f > 0$, the latter equality implies $z_t^* > z_t^c$.
- Assume that $z_t^* > z_t^c$. Again, jointly with algebraic manipulations, (2.5.9) implies that $\rho z_t^c = -(1-\rho)\, z_t^c + (1-\rho) z_t^* + \rho z_t^f > -(1-\rho)\, z_t^c + (1-\rho) z_t^c + \rho z_t^f > \rho z_t^f$, which entails that $z_t^c > z_t^f$.
- Assume that $z_t^c > z_t^f$. Algebraic manipulations and (2.5.9) imply that $(1-\rho)\, z_t^* = z_t^c - z_t^f + (1-\rho)\, z_t^f > (1-\rho)\, z_t^f$, which implies $z_t^* > z_t^f$. □

□

Referring to the job creation condition (2.5.13),

- If open-ended workers are the only ones hired, then $max\left[z^f, z^*\right] \leq z^f$, implying that $z^* \leq z^f$. Referring to Lemma 1, the latter inequality entails $z^f \leq z^c$. As a result, $z^* \leq z^f \leq z^c$.
- If job creation is dual, then

$$\begin{cases} 0 < \max\left[z^f, z^*\right] \\ z^f < z^* \end{cases}$$

Using Lemma 1, the latter system of inequalities boils down to $\max\left[0, z^f\right] < z^*$.

For each case, the converse propositions are straightforward using (2.5.13). □

Proof Proposition 2 Differentiating (2.5.10)-(2.5.11), I get.

$$\begin{aligned} (1-\beta(1-\xi))\, \phi \frac{\partial z^p}{\partial F} &= -(1-\beta(1-s)) + \frac{\eta\gamma}{1-\eta} \frac{\partial \theta}{\partial F} \\ \phi \frac{\partial z^c}{\partial F} &= \phi \frac{\partial z^p}{\partial F} + 1 \end{aligned}$$

Combining both equation leads to

$$\phi \frac{\partial z^c}{\partial F} = \frac{\beta(1-s) G\left(z^p\right)}{1-\beta(1-\xi)} + \frac{1}{1-\beta(1-\xi)} \frac{\eta\gamma}{1-\eta} \frac{\partial \theta}{\partial F}$$

Differentiating (2.5.9) and (2.5.12) leads to

$$(1-\rho)\phi \frac{\partial z^*}{\partial F} = \phi \frac{\partial z^c}{\partial F} - \rho\phi \frac{\partial z^f}{\partial F}$$

$$\rho\left(1-\beta(1-\delta)\right)\phi\frac{\partial z^f}{\partial F}=\frac{\eta\gamma}{1-\eta}\frac{\partial\theta}{\partial F}$$

Using these equations in the differentiated (2.5.13), I get

$$\begin{aligned}
-\frac{\gamma q'}{(1-\eta)q^2}\frac{\partial\theta}{\partial F} &= -\left(1-G\left(z^*\right)\right)(1-\rho)\phi\frac{\partial z^*}{\partial F}-\left(1-G\left(z^f\right)\right)\rho\phi\frac{\partial z^f}{\partial F}\\
&= -\left(1-G\left(z^*\right)\right)\phi\frac{\partial z^c}{\partial F}-\rho\left(G\left(z^*\right)-G\left(z^f\right)\right)\phi\frac{\partial z^f}{\partial F}\\
&= -\left(1-G\left(z^*\right)\right)\frac{\beta(1-s)G\left(z^p\right)}{1-\beta(1-\xi)}-\left(\frac{1-G\left(z^*\right)}{1-\beta(1-\xi)}+\frac{G\left(z^*\right)-G\left(z^f\right)}{1-\beta(1-\delta)}\right)\frac{\eta\gamma}{1-\eta}\frac{\partial\theta}{\partial F}
\end{aligned}$$

Isolating $\frac{\partial\theta}{\partial F}$ yields

$$\frac{\partial\theta}{\partial F}=-\frac{\left(1-G\left(z^*\right)\right)\frac{\beta(1-s)G(z^p)}{1-\beta(1-\xi)}}{-\frac{\gamma q'}{(1-\eta)q^2}+\left(\frac{1-G(z^*)}{1-\beta(1-\xi)}+\frac{G(z^*)-G(z^f)}{1-\beta(1-\delta)}\right)\frac{\eta\gamma}{1-\eta}}<0$$

Knowing the sign of $\frac{\partial\theta}{\partial F}$ leads to

$$\begin{aligned}
&\frac{\partial z^p}{\partial F}<0\\
&\frac{\partial z^f}{\partial F}<0\\
&\frac{\partial z^c}{\partial F}\propto-\frac{\gamma q'}{(1-\eta)q^2}+\frac{G\left(z^*\right)-G\left(z^f\right)}{1-\beta(1-\delta)}\frac{\eta\gamma}{1-\eta}>0\\
&\frac{\partial z^*}{\partial F}>0
\end{aligned}$$

B Detailed Calculations

B.1 Nash-bargaining joint surpluses

Using the different definitions of surpluses with the free-entry condition -namely equations (2.4.10), (2.4.11), (2.5.1) and (2.4.5) - I get

$$\begin{aligned}
S_t^p\left(z_t\right)=z_t-U_t+\mathbb{E}_t\beta_{t,t+1}U_{t+1}+\left(1-\mathbb{E}_t\beta_{t,t+1}(1-s)\right)F\\
+\mathbb{E}_t\beta_{t,t+1}(1-s)\int\max\left(S_{t+1}^p(z),0\right)dG(z)
\end{aligned}\tag{B.1}$$

Meanwhile, the definition of U_t (2.4.8) yields

$$U_t=p\left(\theta_t\right)\int\max\left(W_t^{0,p}(z)-U_t^0,W_t^f(z)-U_t^0,0\right)dG(z)+U_t^0\tag{B.2}$$

Nash-sharing rules (2.4.3) imply that

$$U_t=p\left(\theta_t\right)\frac{\eta}{1-\eta}\int\max\left(J_t^{0,p}(z),J_t^f(z),\mathbb{E}_t\beta_{t,t+1}V_{t+1}\right)dG(z)+\left(1-p\left(\theta_t\right)\right)U_t^0\tag{B.3}$$

But the definition of V_t (2.4.1) and the free entry condition (2.5.1) imply that

$$\int \max\left(J_t^{0,p}(z), J_t^f(z), \mathbb{E}_t\beta_{t,t+1}V_{t+1}\right) dG(z) = \frac{\gamma}{q\left(\theta_t\right)} \tag{B.4}$$

Since $p\left(\theta_t\right) = \theta_t q\left(\theta_t\right)$, the definition of U_t boils down to

$$U_t = \frac{\eta\gamma\theta_t}{1-\eta} + U_t^0 \tag{B.5}$$

Consequently, using (2.4.9) the outside value of an incumbent worker is

$$U_t = b + \frac{\eta\gamma\theta_t}{1-\eta} + \mathbb{E}_t\beta_{t,t+1}U_{t+1} \tag{B.6}$$

Reintroducing (B.6) into (2.4.16) leads to the following expression for the surplus of continuing permanent contracts

$$\begin{aligned} S_t^p\left(z_t\right) = A_t z_t \phi_t - b - \frac{\eta\gamma\theta_t}{1-\eta} + \left(1 - \mathbb{E}_t\beta_{t,t+1}(1-s)\right)F \\ + \mathbb{E}_t\beta_{t,t+1}(1-s)\int \max\left(S_{t+1}^p(z), 0\right) dG(z) \end{aligned} \tag{B.7}$$

Following the same steps, I find that

$$S_t^{0,p}\left(z_t\right) = S_t^p\left(z_t\right) - F \tag{B.8}$$

As for temporary contracts, equations (2.4.14), (2.4.15) and (2.5.1) boil down to

$$S_t^f\left(z_t\right) = \rho A_t z_t \phi_t - U_t + \mathbb{E}_t\beta_{t,t+1}\left\{(1-\delta)\int S_{t+1}^f(z)dG(z) + U_{t+1}\right\} \tag{B.9}$$

Using (B.6), the surplus of incumbent fixed-term contracts is

$$S_t^f\left(z_t\right) = \rho A_t z_t \phi_t - b - \frac{\eta\gamma\theta_t}{1-\eta} + \mathbb{E}_t\beta_{t,t+1}(1-\delta)\left\{\int S_{t+1}^f(z)dG(z)\right\} \tag{B.10}$$

B.2 Steady-state equations

Given parameters $\beta, \sigma, \eta, \sigma_z, \epsilon, F, b, \delta, \rho, m, \gamma$, we need to derive the steady-state values of R, c, Y, n^p, n^f, u, Δ, z^p, z^c, z^f, z^*, θ, ϕ, v, π, A, g. Some of them can be directly computed through the following equations

$$\begin{aligned} &R = 1/\beta \\ &\phi = \frac{\epsilon - 1}{\epsilon} \\ &P = P^* = \Delta = 1 \\ &A = 1 \end{aligned}$$

Given (z^p, θ), many variables can be derived in a tractable way.

$$\begin{aligned}
q &= m\theta^{-\sigma} \\
p &= \theta q \\
U &= b + \frac{\eta\gamma\theta}{1-\eta} \\
z^c &= z^p + \frac{F}{\phi} \\
z^f &= \frac{U - \rho\phi(1-\delta)\beta\mathbb{E}z}{\rho\phi\left(1-\beta(1-\delta)\right)} \\
z^* &= \frac{z^c - \rho z^f}{1-\rho} \\
\xi &= s + (1-s)G\left(z^p\right) \\
\mu^p &= p\left(1 - G\left(z^*\right)\right) \\
\mu^f &= p\left(G\left(z^*\right) - G\left(z^f\right)\right)
\end{aligned}$$

Then, one can circumscribe (z^p, θ) by solving the following system numerically.

$$\begin{aligned}
\phi z^p + \left(1 - \beta(1-s)\right)F + (1-s)\beta\phi\int_{z^p}^{+\infty}\left(1 - G(z)\right)dz &= U \\
\frac{\gamma}{(1-\eta)\phi q} = \int_{z^*}^{+\infty}\left(1-G(z)\right)dz + \rho\int_{z^f}^{z_t^*}\left(1-G(z)\right)dz &
\end{aligned}$$

Solving the following linear system yields $\left(n^p, n^f, u, e\right)$.

$$\begin{aligned}
n^p + n^f + u &= 1 \\
e &= u + \delta n^f + \xi n^p \\
n^p &= \left(1-\xi\right)n^p + \mu^p e \\
n^f &= \left(1-\delta\right)n^f + \mu^f e
\end{aligned}$$

The resulting expressions are

$$\begin{aligned}
n^p &= \frac{\delta\mu^p}{\xi(1-\delta)\mu^f + \delta(1-\xi)\mu^p + \xi\delta} \\
n^f &= \frac{\xi\mu^f}{\xi(1-\delta)\mu^f + \delta(1-\xi)\mu^p + \xi\delta} \\
u &= 1 - n^p - n^f \\
e &= u + \xi n^p + \delta n^f
\end{aligned}$$

Now, the steady-state values v, Y, g and c can be obtained.

$$v = \theta e$$

$$
\begin{aligned}
Y &= (1-\xi)\,\mathbb{E}_z\left[z \mid z \geq z^p\right] n^p + \mu^p \mathbb{E}_z\left[z \mid z \geq z^*\right] e \\
&\quad + \rho\,(1-\delta)\,\mathbb{E}_z\left[z\right] n^f + \rho\mu^f \mathbb{E}_z\left[z \mid z^* \geq z \geq z^f\right] e \\
g &= Y\left(\frac{g}{Y}\right) \\
c &= Y - g - \gamma v
\end{aligned}
$$

B.3 Log-linearization

The Euler equation can be log-linearized starting from

$$
\widehat{c_t} = \widehat{E_t c_{t+1}} - \left[-\frac{cu''}{u}\right]^{-1}\left(\widehat{R_t} - \widehat{E_t \pi_{t+1}}\right) \tag{B.1}
$$

where $\pi_t = P_t/P_{t-1}$ denotes inflation.

The definition of price level dynamics (2.7.8) can be log-linearized as

$$
\widehat{\pi_t} = (1-\psi)\left(\widehat{P_t^*} - \widehat{P_{t-1}}\right) \tag{B.2}
$$

The retailers' price-setting equation (2.3.1) becomes

$$
\widehat{P_t^*} = (1-\beta\psi)\mathbb{E}_t \sum_{k=0}^{+\infty} (\beta\psi)^k \left(\widehat{P_{t+k}} + \widehat{\phi_{t+k}}\right) \tag{B.3}
$$

Substracting $\widehat{P_{t-1}}$ on each side of this equation, we get

$$
\begin{aligned}
\widehat{P_t^*} - \widehat{P_{t-1}} &= (1-\beta\psi)\mathbb{E}_t \sum_{k=0}^{+\infty} (\beta\psi)^k \left(\widehat{P_{t+k}} - \widehat{P_{t-1}} + \widehat{\phi_{t+k}}\right) && \text{(B.4)} \\
&= \sum_{k=0}^{+\infty} (\beta\psi)^k \mathbb{E}_t \widehat{\pi_{t+k}} + (1-\beta\psi) \sum_{k=0}^{+\infty} (\beta\psi)^k \mathbb{E}_t \widehat{\phi_{t+k}} && \text{(B.5)} \\
&= \beta\psi\left(\mathbb{E}_t \widehat{P_{t+1}^*} - \widehat{P_t}\right) + (1-\beta\psi)\widehat{\phi_t} + \widehat{\pi_t} && \text{(B.6)}
\end{aligned}
$$

Introducing an *i.i.d* cost-push shock ϵ_t^μ and using (B.2), we get

$$
\widehat{\pi_t} = \beta \mathbb{E}_t \widehat{\pi_{t+1}} + \kappa \widehat{\phi_t} + \epsilon_t^\mu \tag{B.7}
$$

where $\kappa = (1-\beta\psi)(1-\psi)/\psi$. I assume that ϵ_t^μ follows a normal law with mean 0 and standard deviation σ_μ.

The log-linearization of exogenous processes yields

$$
\begin{aligned}
\widehat{A_t} &= \rho_A \widehat{A_{t-1}} + \epsilon_t^A \\
\widehat{g_t} &= \rho_g \widehat{g_{t-1}} + \epsilon_t^g
\end{aligned}
$$

The other log-linearizations of equations (2.5.10)-(2.5.12), (2.5.9) and (2.6.2)-(2.7.6) yield

$$A\phi z^p\left(\widehat{A_t}+\widehat{z_t^p}+\widehat{\phi_t}\right)+\beta(1-s)\left(A\phi\int_{z^p}^{+\infty}(1-G(z))\,dz-F\right)\left(c_t-\mathbb{E}_t\widehat{c_{t+1}}\right)$$
$$+\beta(1-s)A\phi\left(\int_{z^p}^{+\infty}(1-G(z))\,dz\left(\mathbb{E}_t\widehat{A_{t+1}}+\mathbb{E}_t\widehat{\phi_{t+1}}\right)-(1-G(z^p))\,z^p\mathbb{E}_t\widehat{z_{t+1}^p}\right)$$
$$-\frac{\eta\gamma\theta}{1-\eta}\widehat{\theta_t}=0$$
$$\rho A\phi z^f\left(\widehat{A_t}+\widehat{z_t^f}+\widehat{\phi_t}\right)+\rho\beta(1-\delta)A\phi\left(\mathbb{E}z-z^f\right)\left(\mathbb{E}_t\widehat{A_{t+1}}+\mathbb{E}_t\widehat{\phi_{t+1}}+\widehat{c_t}-\mathbb{E}_t\widehat{c_{t+1}}\right)$$
$$-\rho\beta(1-\delta)A\phi z^f\mathbb{E}_t\widehat{z_{t+1}^f}-\frac{\eta\gamma\theta}{1-\eta}\widehat{\theta_t}=0$$
$$(1-\rho)z^*\widehat{z_t^*}-z^p\widehat{z_t^p}+\frac{F}{A\phi}\left(\widehat{A_t}+\widehat{\phi_t}\right)+\rho z^f\widehat{z_t^f}=0$$
$$\frac{\gamma}{(1-\eta)\phi q(\theta)}\left(-\widehat{A_t}-\widehat{\phi_t}+\sigma\widehat{\theta_t}\right)+(1-\rho)\left(1-G\left(z^*\right)\right)z^*\widehat{z_t^*}+\rho\left(1-G\left(z^f\right)\right)z^f\widehat{z_t^f}=0$$
$$\widehat{R_t}=\rho_R\widehat{R_{t-1}}+(1-\rho_R)\left[\rho_\pi\mathbb{E}_t\widehat{\pi_{t+1}}+\rho_y\widehat{Y_t}\right]+\epsilon_t^m$$
$$\widehat{v_t}=\widehat{\theta_t}-\frac{(1-\xi^p)\,\theta n^p}{v}\widehat{n_{t-1}^p}-\frac{(1-\delta)\,\theta n^f}{v}\widehat{n_{t-1}^f}+\frac{(1-s)\theta g\left(z^p\right)z^pn^p}{v}\widehat{z_t^p}$$
$$\widehat{n_t^p}=(1-\xi)\,\widehat{n_{t-1}^p}-(1-s)z^pg\left(z^p\right)\widehat{z_t^p}+\frac{\left(1-G\left(z^*\right)\right)q(\theta)v}{n^p}\left(\widehat{v_t}-\sigma\widehat{\theta_t}\right)-\frac{z^*g\left(z^*\right)q(\theta)v}{n^p}\widehat{z_t^*}$$
$$\widehat{n_t^f}=(1-\delta)\,\widehat{n_{t-1}^f}+\frac{\left(G\left(z^*\right)-G\left(z^f\right)\right)q(\theta)v}{n^f}\left(\widehat{v_t}-\sigma\widehat{\theta_t}\right)+\frac{q(\theta)v}{n^f}\left(z^*g\left(z^*\right)\widehat{z_t^*}-z^fg\left(z^f\right)\widehat{z_t^f}\right)$$
$$\widehat{Y_t}=\frac{c}{Y}\widehat{c_t}+\frac{g}{Y}\widehat{g_t}+\frac{\gamma v}{Y}\widehat{v_t}$$
$$\widehat{Y_t}=\widehat{A_t}+\frac{(1-s)\left(\int_{z^p}^{+\infty}zg(z)dz\right)n^p}{Y}\widehat{n_{t-1}^p}+\frac{\rho\,(1-\delta)\,\mathbb{E}zn^f}{Y}\widehat{n_{t-1}^f}-\frac{(1-s)n^p\left(z^p\right)^2g\left(z^p\right)}{Y}\widehat{z_t^p}$$
$$-(1-\rho)\left(z^*\right)^2g\left(z^*\right)\frac{vq(\theta)}{Y}\widehat{z_t^*}-\rho\left(z^f\right)^2g\left(z^f\right)\frac{vq(\theta)}{Y}\widehat{z_t^f}$$
$$+\left(\int_{z^*}^{+\infty}zg(z)dz+\rho\int_{z^f}^{z^*}zg(z)dz\right)\frac{vq(\theta)}{Y}\left(\widehat{v_t}-\sigma\widehat{\theta_t}\right)$$

C Estimation

C.1 Calibration

I carry out the calibration using known parameters and steady-state targets. A first block of steady-state values and parameters is tractable.

$$\phi=\frac{\epsilon-1}{\epsilon}$$
$$u=1-n$$
$$n^f=n\left(\frac{n^f}{n}\right)$$
$$n^p=n-n^f$$
$$s=\xi\left(\frac{s}{\xi}\right)$$

$$\delta = \frac{\left(1-\left(\frac{n^f}{n}\right)\right)\left(\frac{\mu^f}{\mu^p+\mu^f}\right)}{\left(\frac{n^f}{n}\right)\left(1-\left(\frac{\mu^f}{\mu^p+\mu^f}\right)\right)}\xi$$
$$e = u + \delta n^f + \xi n^p$$
$$\mu^p = \frac{\xi n^p}{e}$$
$$\mu^f = \frac{\delta n^f}{e}$$

Knowing ξ, s and σ^z, it is possible to numerically solve for z^p using the fact that $\xi = s + (1-s)G\left(z^p\right)$. Then, the following system in $(\overline{w^p}, U, F)$ — where $U = b + \frac{\eta\gamma\theta}{1-\eta}$ — is tractable.

$$\overline{w^p} = \eta\phi\mathbb{E}\left[z \mid z \geq z^p\right] + \eta(1-\beta(1-s))F + (1-\eta)U$$
$$\phi z^p + (1-\beta(1-s))\,F + (1-s)\beta\phi\int_{z^p}^{+\infty}(1-G(z))\,dz = U$$
$$F = \rho^F\overline{w^p}$$

Combining these three expressions, I get

$$F = \frac{\rho^F\phi}{1-\rho^F(1-\beta(1-s))}\left(\eta\mathbb{E}\left[z \mid z \geq z^p\right] + (1-\eta)\left(z^p + \beta(1-s)\int_{z^p}^{+\infty}(1-G(z))\,dz\right)\right)$$
$$U = \phi z^p + (1-\beta(1-s))\,F + (1-s)\beta\phi\int_{z^p}^{+\infty}(1-G(z))\,dz$$

It is possible to compute z^c.

$$z^c = z^p + \frac{F}{\phi}$$

A non-linear system in $\left(z^f, z^*, \rho\right)$ can then be solved numerically.

$$\rho z^f\phi + \rho\mathbb{E}_t\beta(1-\delta)\phi\left(\mathbb{E}z - z^f\right) = U$$
$$(1-\rho)z^* = z^c - \rho z^f$$
$$\frac{\mu^f}{\mu^p+\mu^f} = \frac{G\left(z^*\right) - G\left(z^f\right)}{1-G\left(z^f\right)}$$

Then, one can derive (θ, γ, b, m).

$$p = \frac{\mu^p}{1-G\left(z^*\right)}$$
$$\theta = \frac{p}{q}$$
$$\gamma = (1-\eta)\phi q\left(\int_{z^*}^{+\infty}(1-G(z))\,dz + \rho\int_{z^f}^{z^*}(1-G(z))\,dz\right)$$

$$
\begin{aligned}
b &= U - \frac{\eta\gamma\theta}{1-\eta} \\
m &= q\theta^{\sigma}
\end{aligned}
$$

The parameters $(F, b, s, \delta, \rho, m, \gamma)$ are all determined at this point

C.2 Data

Observable	Time range	Unique Identifier
GDP at constant prices	1995:Q1 - 2019:Q3	MNA.Q.Y.I8.W2.S1.S1.B.B1GQ._Z._Z._Z.EUR.LR.N
GDP deflator	1995:Q2 - 2019:Q2	MNA.Q.Y.I8.W2.S1.S1.B.B1GQ._Z._Z._Z.IX.D.N
Nominal interest rates	1994:Q1 - 2019:Q3	FM.M.U2.EUR.4F.MM.EONIA.HSTA
Employment	1995:Q1 - 2019:Q3	ENA.Q.Y.I8.W2.S1.S1._Z.EMP._Z._T._Z.PS._Z.N

Table C.1: Times series used for estimation

All these time series are drawn from the ECB Data Warehouse

Observable	Symbol	Time range	Source	Unique Identifier
Open-ended job creations	jc^p	2000:Q1 - 2019:Q3	Acoss	Acoss Stat 296
Fixed-term job creations	jc^f	2000:Q1 - 2019:Q3	Acoss	Acoss Stat 296
Share of fixed-term contracts in job creation	$\mu^f / (\mu^p + \mu^f)$	2000:Q1 - 2019:Q3	Acoss	Acoss Stat 296
Share of fixed-term employment	n^f	2003:Q1 - 2019:Q3	Insee	010605905
Endogenous open-ended job destruction	jd^p	2001:Q1 - 2017:Q4	DARES	DMMO - EMMO
Fixed-term job destruction	jd^f	1998:Q1 - 2017-Q4	DARES	DMMO - EMMO
Vacancies	v	1989:Q1 - 2019:Q2	OECD	LMJVTTNVFRQ647S

Table C.2: Labor market times series

C.3 Additional graphs and tables

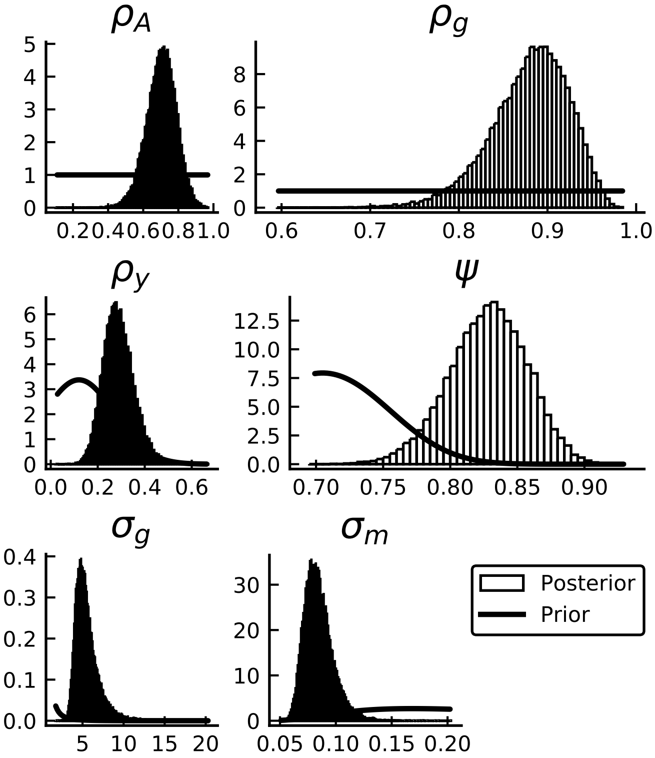

Figure C.1: Prior and posterior distributions

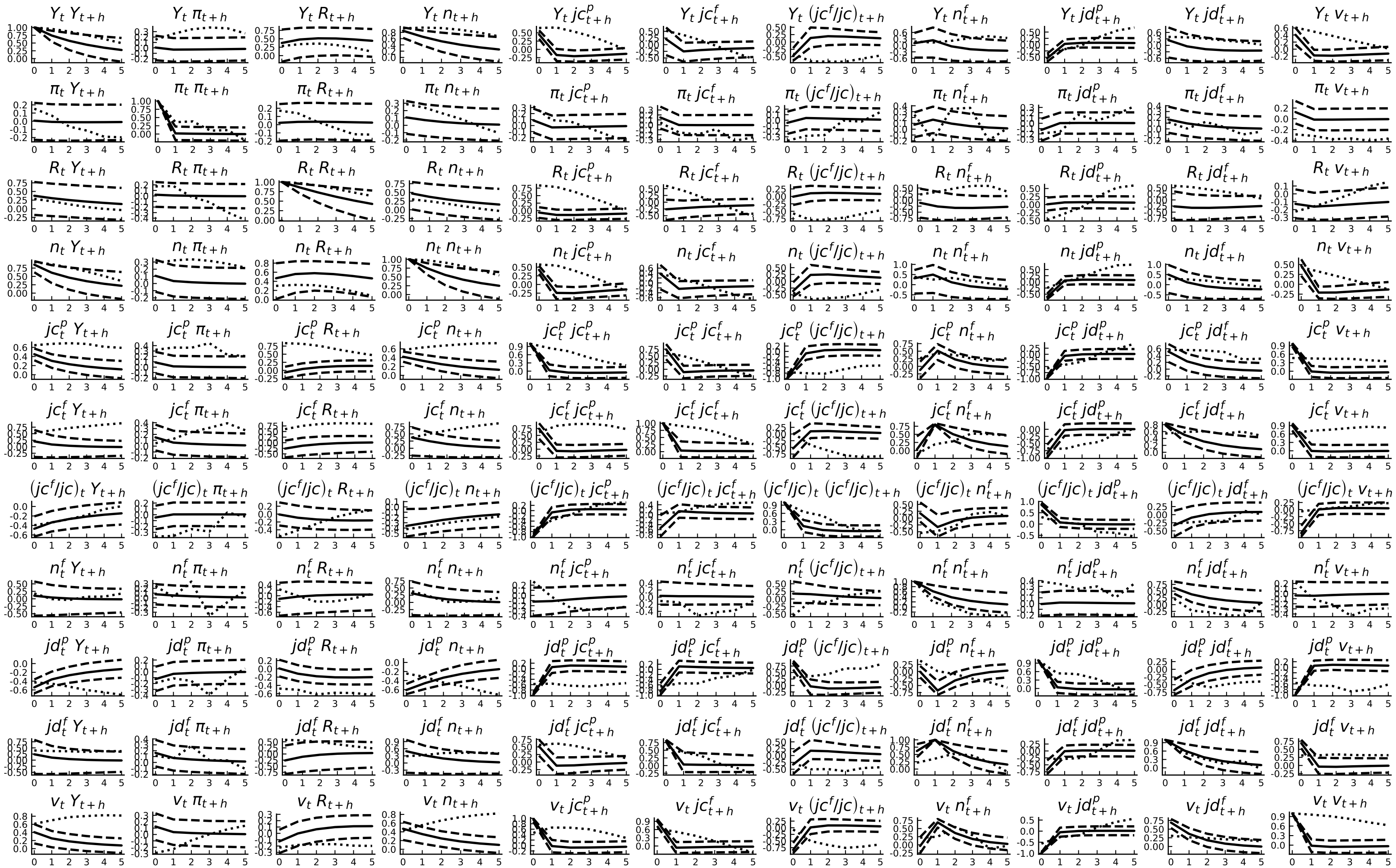

Figure C.2: Simulated and data cross-correlations

The x-axis is the lag h and the y-axis is the correlation between the variable x_t and the variable x_{t+h}. The solid line is the simulated value, the dashed line is the 95 % confidence interval around the latter and the dotted line is the value from the data.

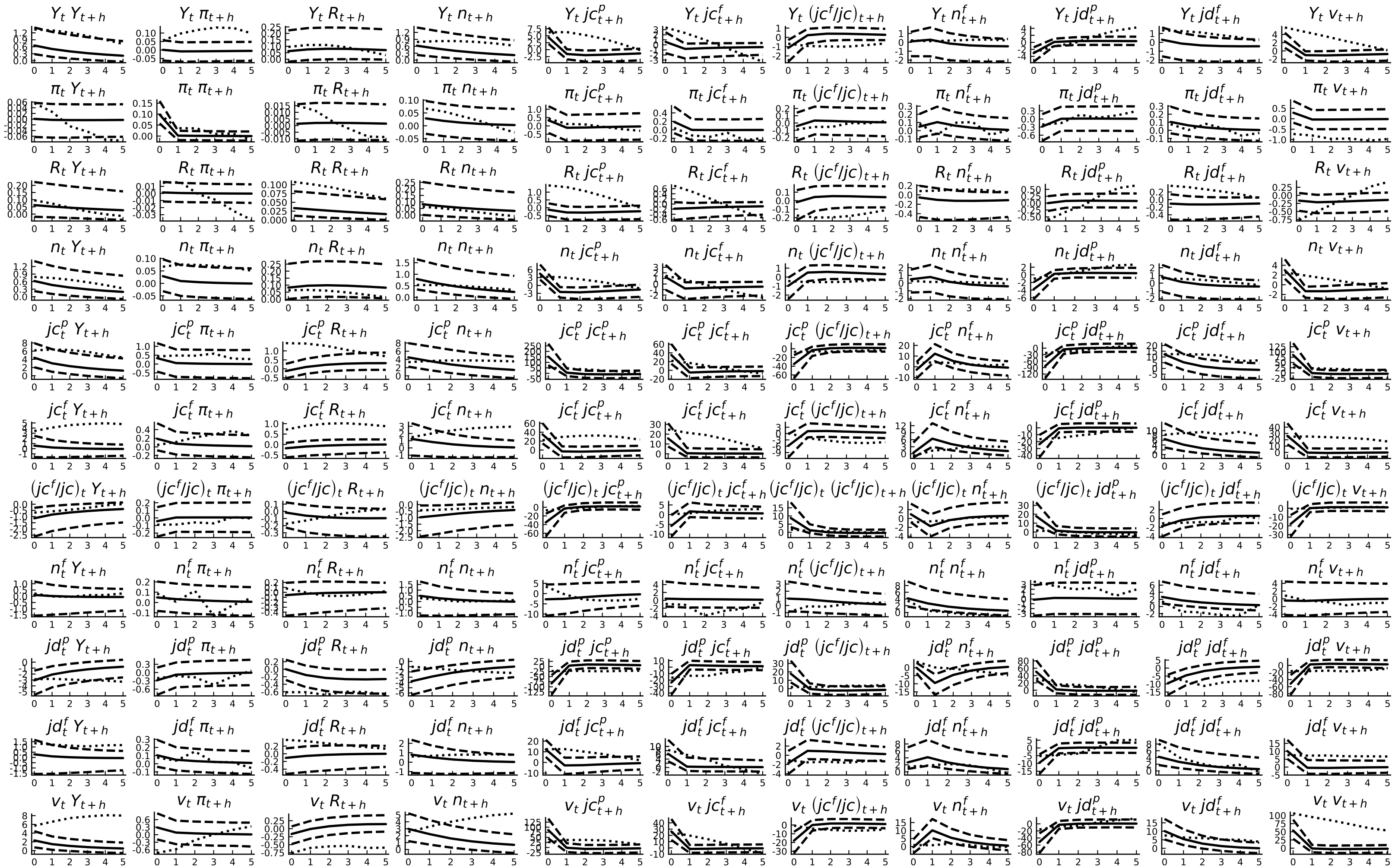

Figure C.3: Simulated and data cross-covariances

The x-axis is the lag h and the y-axis is the covariance between the variable x_t and the variable x_{t+h}. The solid line is the simulated value, the dashed line is the 95 % confidence interval around the latter and the dotted line is the value from the data.

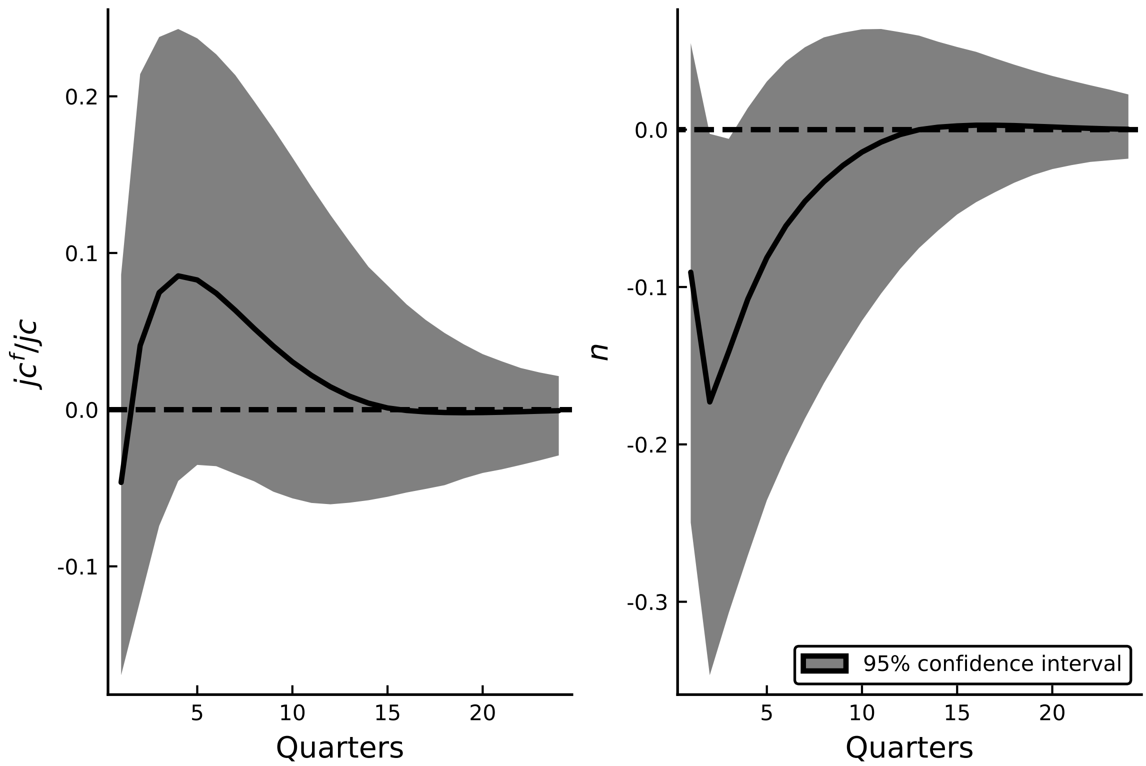

Figure C.4: IRF to a one-standard deviation volatility shock — VAR estimation.

Parameters	Hamilton		Linear trend		First difference		Hodrick-Prescott	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
ρ_A	0.52	0.11	0.69	0.08	0.29	0.27	0.62	0.09
ρ_g	0.86	0.05	0.88	0.04	0.78	0.08	0.86	0.05
ρ_R	0.79	0.04	0.77	0.05	0.79	0.08	0.7	0.05
ρ_π	2.02	0.59	2.06	0.54	1.87	0.74	1.99	0.51
ρ_y	0.25	0.05	0.29	0.06	0.46	0.12	0.43	0.06
ψ	0.84	0.03	0.83	0.03	0.71	0.06	0.8	0.03
σ_A	0.79	0.11	0.31	0.03	0.32	0.04	0.29	0.03
σ_μ	0.23	0.03	0.3	0.03	0.28	0.04	0.3	0.03
σ_g	7.67	1.74	5.61	1.36	3.04	0.87	5.13	1.17
σ_m	0.06	0.01	0.08	0.01	0.09	0.02	0.06	0.01

Table C.3: Estimations with Hamilton, linear-trend, first-difference and Hodrick-Prescott filters

D Classic Model

D.1 Equilibrium equations

$$u'(c_t) = \beta R_t \mathbb{E}_t \left[\frac{P_t}{P_{t+1}} u'(c_{t+1}) \right]$$

$$
\begin{aligned}
&A_t z_t^p \phi_t + \left(1 - \mathbb{E}_t \beta_{t,t+1}(1-s)\right) F + \mathbb{E}_t \beta_{t,t+1}(1-s) A_{t+1}\phi_{t+1} \int_{z_{t+1}^p}^{+\infty} (1 - G(z))\, dG(z) \\
&= b + \frac{\eta\gamma\theta_t}{1-\eta} \\
&z_t^c = z_t^p + \frac{F_t}{A_t\phi_t} \\
&\frac{\gamma}{(1-\eta) A_t \phi_t q(\theta_t)} = \int_{z_t^c}^{+\infty} [1 - G(z)]\, dz \\
&n_t^p = (1-\xi_t)\, n_{t-1}^p + \mu_t^p e_t \\
&v_t = \theta_t \left(1 - (1-\xi_t)\, n_{t-1}^p\right) \\
&Y_t = c_t + g_t + \gamma v_t \\
&Y_t \Delta_t = A_t E_z\left[z \mid z \geq z_t^p\right](1-\xi_t)\, n_{t-1}^p + \left(1 - G(z_t^c)\right) q(\theta_t)\, v_t A_t E_z\left[z \mid z \geq z_t^c\right] \\
&\mathbb{E}_t \sum_{T=t}^{+\infty} \beta_{t,T} \psi^{T-t} P_T^{\epsilon} Y_T \left(\frac{P_{i,t}^*}{P_T} - \mu\phi_T\right) = 0 \\
&\log(R_t/R) = \rho_R \log(R_{t-1}/R) + (1-\rho_R)\left[\rho_\pi \mathbb{E}_t \log\left(\frac{P_{t+1}}{P_t}\right) + \rho_y \log\left(\frac{y_t}{y}\right)\right] + \epsilon_t^m \\
&\log(A_t/A) = \rho^A \log(A_{t-1}/A) + \epsilon_t^A \\
&\log(g_t/g) = \rho^g \log(g_{t-1}/g) + \epsilon_t^g
\end{aligned}
$$

D.2 Log-linearization

$$
\begin{aligned}
&\widehat{c}_t = \widehat{E_t c_{t+1}} - \left[-\frac{c u''}{u}\right]^{-1} \left(\widehat{R_t} - \widehat{E_t \pi_{t+1}}\right) \\
&A\phi z^p \left(\widehat{A_t} + \widehat{z_t^p} + \widehat{\phi_t}\right) + \beta(1-s)\left(A\phi \int_{z^p}^{+\infty} (1-G(z))\, dz - F\right)\left(c_t - \mathbb{E}_t \widehat{c_{t+1}}\right) \\
&+ \beta(1-s) A\phi \left(\int_{z^p}^{+\infty} (1-G(z))\, dz \left(\mathbb{E}_t \widehat{A_{t+1}} + \mathbb{E}_t \widehat{\phi_{t+1}}\right) - (1 - G(z^p))\, z^p \mathbb{E}_t \widehat{z_{t+1}^p}\right) \\
&- \frac{\eta\gamma\theta}{1-\eta} \widehat{\theta_t} = 0 \\
&z^c \widehat{z_t^c} = z^p \widehat{z_t^p} - \frac{F}{A\phi}\left(\widehat{A_t} + \widehat{\phi_t}\right) \\
&\frac{\gamma}{(1-\eta)\phi q(\theta)}\left(-\widehat{A_t} - \widehat{\phi_t} + \sigma\widehat{\theta_t}\right) + \left(1 - G(z^c)\right) z^c \widehat{z_t^c} = 0 \\
&\widehat{n_t^p} = (1-\xi)\, \widehat{n_{t-1}^p} - (1-s) z^p g(z^p)\, \widehat{z_t^p} + \frac{\left(1 - G(z^c)\right) q(\theta) v}{n^p}\left(\widehat{v_t} - \sigma\widehat{\theta_t}\right) - \frac{z^c g(z^c)\, q(\theta) v}{n^p} \widehat{z_t^c} \\
&\widehat{v_t} = \widehat{\theta_t} - \frac{(1-\xi)\,\theta n^p}{v} \widehat{n_{t-1}^p} + \frac{(1-s)\theta g(z^p)\, z^p n^p}{v} \widehat{z_t^p} \\
&\widehat{Y_t} = \frac{c}{Y}\widehat{c_t} + \frac{g}{Y}\widehat{g_t} + \frac{\gamma v}{Y}\widehat{v_t} \\
&\widehat{Y_t} = \widehat{A_t} + \frac{(1-s)\left(\int_{z^p}^{+\infty} z g(z) dz\right) n^p}{Y} \widehat{n_{t-1}^p} - \frac{(1-s) n^p (z^p)^2 g(z^p)}{Y} \widehat{z_t^p} \\
&- (z^c)^2 g(z^c) \frac{v q(\theta)}{Y} \widehat{z_t^c} + \left(\int_{z^c}^{+\infty} z g(z) dz\right) \frac{v q(\theta)}{Y}\left(\widehat{v_t} - \sigma\widehat{\theta_t}\right) \\
&\widehat{\pi_t} = \beta \mathbb{E}_t \widehat{\pi_{t+1}} + \frac{(1-\beta\psi)(1-\psi)}{\psi}\widehat{\phi_t} + \epsilon_t^\mu \\
&\widehat{R_t} = \rho_R \widehat{R_{t-1}} + (1-\rho_R)\left[\rho_\pi \mathbb{E}_t \widehat{\pi_{t+1}} + \rho_y \widehat{Y_t}\right] + \epsilon_t^m
\end{aligned}
$$

$$\widehat{A_t} = \rho_A \widehat{A_{t-1}} + \epsilon_t^A$$
$$\widehat{g_t} = \rho_g \widehat{g_{t-1}} + \epsilon_t^g$$

D.3 Estimation

Parameter	Prior distribution			Posterior distribution			
	Distr.	Para (1)	Para(2)	Mean	Std. Dev.	5%	95%
ρ_A	Uniform	0.0	1.0	0.71	0.09	0.55	0.86
ρ_g	Uniform	0.0	1.0	0.95	0.03	0.9	0.99
ρ_R	Uniform	0.0	1.0	0.7	0.06	0.59	0.8
ρ_π	Normal	1.5	0.75	2.22	0.64	1.25	3.34
ρ_y	Normal	0.12	0.15	0.11	0.07	0.02	0.25
ψ	Beta	0.7	0.05	0.78	0.05	0.7	0.85
σ_A	IGamma	0.5	4.0	0.33	0.04	0.27	0.4
σ_μ	IGamma	0.5	4.0	0.28	0.03	0.23	0.34
σ_g	IGamma	0.5	4.0	9.04	5.73	4.26	18.1
σ_m	IGamma	0.5	4.0	0.08	0.01	0.06	0.11

Table D.1: Prior and posterior distributions of structural parameters.

Para(1) and Para(2) correspond to mean and standard deviation of the prior distribution if the latter is Normal or Inverse Gamma. Para(1) and Para(2) correspond to lower and upper bound of the prior distribution when the latter is uniform

Model	$\log\left(p\left(Y_{1:T}\right)\right)$	Std. Dev.
Dual	-27.4	1.1
Classic	-25.7	1.0

Table D.2: Posterior Odds