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Collusion, antitrust policy, leniency programs, multimarket contact, organizational form

JEL codes:
K42, L22, L41
Slowdown antitrust investigations by decentralization

Emilie DARGAUD* Armel JACQUES†

May 19, 2020

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1 Introduction

The fight against cartels is the priority of antitrust authority and leniency program is widely used to facilitate their detection.\(^1\) Despite a consensus regarding its utility, the design and effects of this program are yet a matter of many debates.\(^2\)

This article analyzes the interactions between leniency program and internal organization of multiproduct collusive firms. Theoretical works have often been produced on the premise that inter-firm contact may facilitate collusion.\(^3\) Antitrust authorities have incorporated this into their procedures and investigate all anticompetitive conducts inside a firm when they detect a collusive agreement. In 1999 the US adopted amnesty plus programs which consist of reduced fines for a convicted firm if it reports another, undetected, cartel.\(^4\)

Multiproduct firms may adopt appropriate collusive strategies. In Choi and Gerlach (2013), firms collude in a first market and subsequently in another once the first cartel is dissolved, if the substitutability of products is low. Dargaud and Jacques (2015) [DJ (2015)] show that the firm’s choice of internal structure affects the detection probability of a cartel inside a firm. Particularly the probability that the antitrust authority uncovers inculpatory evidence of several infringements when investigating only a single market decreases if firms compartmentalize collusive agreements. Dargaud and Jacques (2020) [DJ (2020)] address the role of leniency programs in the previous model and describe how they can defeat the compartmentalization strategy. The antitrust authority seeks to substitute for its own investigations by inducing CEOs to launch internal investigations so as to collect evidence and apply for leniency. However, by defeating the compartmentalization strategy, leniency may promote centralization with higher collusive prices.

Assumptions of this article are in a large part those used in DJ (2015, 2020) but we study another effect of the organizational structure. In DJ (2015, 2020) firms may compartmentalize collusive agreements to avoid the contagion of antitrust authority investigations. In this paper we eliminate this effect by assuming that the contagion probability is the same for each organization but we focus on different durations for the investigations. Decentralized firms, by compartmentalizing activities, make investigations slower. In this case these firms are much more likely to seek leniency for the second cartel (before its detection) than centralized firms.

When a cartel is either denounced by whistle-blowers or suspected by antitrust authorities, firms are not immediately informed. Authorities can wiretap firm’s line\(^5\) or raid the head office in order to obtain hard evidence. If a second cartel is detected during the first wave of investigations then firms lack the time to apply

\(^{1}\text{See Miller (2009) and Brenner (2009).}\)
\(^{2}\text{See Spagnolo (2008) and Marvão and Spagnolo (2018) for a literature review on leniency.}\)
\(^{3}\text{See Bernheim and Whinston (1990), Spagnolo (1999), Matsushima (2001).}\)
\(^{5}\text{Following a press article in 2005, Canadian authorities wiretap gasoline stations’ line and several gasoline price-fixing cases have been detected, see Clarke and Houde (2013). Another example is the lysine cartel: at the end of June 1995, after wiretapping, FBI agents raided the head office of a ADM firm. The incident attracted widespread press coverage and has been adapted into a film by Steven Soderbergh (The informant!). Other offences have been revealed during the investigations.}\)
for leniency. Otherwise they may do so before a deeper investigation. The probability that the second cartel is discovered during one-market investigations is higher for centralized firms since both cartels are managed by the same people.

This article analyzes the impact of leniency on the choice of compartmentalization. In the main part, we assume that the authority always finds hard evidence of a second cartel when it investigates the other market and both cartels are dissolved. This reduced the number of collusive strategies and firms, where possible, always apply for leniency for the second cartel since its conviction is inescapable. Firms face a trade-off: competition between divisions of decentralized firms results in low collusive prices (firms do not internalize the effects of the price on the other product demand) but expected fine is lower since they are much more likely to seek leniency for the second collusive agreement. Without leniency, firms never select decentralized structure. Leniency can affect this result if goods are weak substitutes and may cause collusive prices to fall. We show that leniency’s overall level of generosity may lead to pro-collusive as well as pro-competitive effects.

Then we check the robustness of the results in a richer model and assume that the detection of the second cartel by serendipity is lower than one, irrespective of the organizational structure. Then firms can sometimes continue to collude on the second market after the condemnation of the first cartel. Equilibria depend on the serendipitous probability and the design of leniency programs. If the contagion probability is high, it is more difficult to pursue collusion in the second market and the major results obtained in the main part of the paper are robust. If leniency is sufficiently generous, then firms seek leniency for the second cartel and prefer a decentralized organization if product substitutability is low. If the contagion probability is low then results depend on the amnesty plus program. Without it, firms should collectively commit not to apply for leniency and adopt a centralized organization. But pursuing collusion in the second market once the first cartel is detected is destabilized if we introduce amnesty plus. Then, with amnesty plus, firms wish to apply for leniency for this second cartel and firms may select decentralized organization to seek leniency early and to reduce the risk of being fined. Thus we obtain similar results as in the main part of the paper if we consider high contagion probability or a lower value together with amnesty plus program.

The paper is organized as follows. In section 2, we describe our model. Section 3 characterizes collusive strategies without leniency programs and puts in evidence that firms never select a decentralized organization. Section 4 analyzes the role of leniency in facilitating decentralization adoption. In section 5 we study the effects of different levels of fine reduction and show that leniency may lead to pro-competitive and pro-collusive effects. In section 6 we check the robustness of our results. The conclusion follows in section 7.

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6In 1995 SC Johnson blew the whistle on a cartel. Then antitrust authority raided a meeting of the managers and the head offices of the firms involved. Among these was Colgate-Palmolive, which earlier had applied twice for leniency. The first application concerned the cartel under investigations, but the second an undetected cartel. If different people manage cartels then the risk of a fast detection by contagion is lower.

7A counter-argument is that a centralized firm can quicker collect hard evidences for both cartel and apply for leniency. But it seems reasonable to expect that this counter-argument is not prevailing.
2 The model

This model largely takes over the assumptions used in DJ (2015, 2020). Two identical firms, 1 and 2, each produce two differentiated products, A and B. Marginal costs are assumed to be identical and equal to zero. The firms determine their organizational form and then they play an infinitely repeated game of price competition. Firms can decide to implement a collusive agreement on one or two products but we introduce a third player, antitrust authority, which seeks to detect and fight collusion.

Organizational design: Before competing in price, firms determine their organizational form cooperatively. Each firm can choose between a centralized (unitary) organizational structure (U-form) and a decentralized (multidivisional) structure (M-form). In the first case each CEO chooses prices in order to maximize overall firm profit, whereas in the second option two divisional managers each determine the price of a single product so as to maximize the profit of their business unit. In this case firms do not internalize the effects of the price on the other product demand.

Price competition: Once organizational structures have been decided, firms play an infinitely repeated game of price competition. In each period, they can decide to implement a collusive agreement. Collusive outcomes are modelled on the basis of grim trigger strategies (Friedman, 1971): as soon as one firm deviates from the agreement, the other plays non-cooperatively forever. Both firms face the same discount factor $\delta$.

The demand function for product $i$ ($i = \{A, B\}, j = \{A, B\}$ and $i \neq j$) is\(^8\) ($d$ is a positive parameter reflecting product substitutability):

$$Q^i(p^i,p^j) = \max\left\{0, \min\left\{a - p^i + d(p^j - p^i), \frac{1+2d}{1+d}(a - p^i)\right\}\right\}$$

Probabilities of detection and fines: Collusion among firms or their divisions generates hard evidence which can be found by the authority with probability $\rho$. The authority can still find collusion during a deviation period, but past offences cannot be detected once firms have reverted to competitive pricing. If a cartel is detected, the authority launches an investigation which always leads to successful prosecution, and a fine $F$ is imposed on each cartel member. Cartels are dissolved once they have been convicted and firms cannot collude again on the market in which they have been condemned. During such a one-market investigation, the authority may find, which probability $\mu$, actionable evidence of a collusive agreement in the other market which leads firms to pay a second fine $F$. In DJ (2015, 2020) the probability of this event depends on the organizational structure. In this article we remove this assumption since we focus on the investigations slowdown allowed by the decentralized structure. To simplify presentation of the major results we assume that $\mu = 1$ until Section 6.

\(^8\)The last terms correspond to the demands for each product when the other is priced above its choke price. We include these terms for completeness, but such prices do not arise in equilibrium.
Timing of leniency programs: They are three moments at which firms can apply for leniency. The first is before collusive agreements are detected. When deviating from a cartel, firms simultaneously apply for leniency as the authority can still find collusion during this deviation period. Second, firms could apply for leniency in the interval between the prosecution of the first cartel and the serendipitous detection of the second. This event depends on the duration of authority investigations. Firms can seek leniency for the second cartel with probability $q_M$ or $q_U$ (the subscripts M and U refer respectively to decentralized and centralized firms) and we assume $q_U \leq q_M$ since decentralization slowdowns investigations. Finally, it is possible for the firms to apply for leniency after completion of the investigation if the cartel has not been detected.

Reduced fines with leniency policy: If only one firm applies for leniency before the cartel is detected, then it receives total immunity from fines. If both firms apply simultaneously, the expected fine is $0.5F$. Offering total immunity to the first firm applying for leniency is optimal since it weakens collusive sustainability by maximizing the deviation profit of the firm.

Firm applying for leniency for the second cartel, during the first market investigation or shortly after this investigation is closed, obtains a reduced fine for the second cartel: $\tau F$, with $\tau \in [-1, 1]$. Once again only the first firm benefits from the reduced fine. $\tau = 0$ is the total immunity and $\tau \in [0; 1]$ the partial immunity case. In 1999 the US adopted amnesty plus programs which consist of reduced fines for a convicted firm if it reports another, undetected, cartel. This option is introduced in our model for $\tau \in [-1; 0]$.

Timing of each period: The timing of each period is: (1) CEO or managers choose their competitive or collusive prices. Collusion generates hard evidence and firms may stay on collusive path or apply for leniency if they deviate from the cartel agreement. (2) The authority suspects an active cartel with probability $\rho$ and launches investigations. (3) During the first-market investigation, firms can apply for leniency for the second cartel with probability $q_U$ or $q_M$ (depending on the organizational choice). (4) If firms do not apply for leniency then the authority detects the second cartel with probability $\mu$. (5) Firms can apply for leniency if this second cartel has not been detected by the authority.

Who applies for leniency? CEOs of centralized structures have clear evidence concerning both cartels and are free to request leniency. Under decentralization collusive agreements are organized by managers who hold proof of their own cartel. DJ (2020) study CEO and manager divergence concerning the revelation to the cartel to the authority. This potential conflict does not appear for $\mu = 1$. In section 6, the case $\mu < 1$ is

---

9Firms can never apply for leniency for the first cartel after its detection. A major reason of the leniency application (see Motta and Polo (2003)) is then rejected.

10DJ (2020) study the following cases: $q_U = q_M = 0$ or $q_U = q_M = 1$. In this case the organizational structure does not influence the ability to apply for leniency for the second cartel during the first market investigation.

11Sauvagnat (2014) suggests that total immunity from fines be implemented if only one firm applies for leniency. If there is more than one applicant (apply simultaneously) the fine reduction is negligible. Our paper follows the traditional approach.

12See Spagnolo (2004) and Harrington (2008). We do not introduce the possibility for the antitrust authority to reward informed employees reporting evidence as was the case in Aubert et al. (2006).
already rather complex and we assume that when CEOs launch internal audits in order to gather evidence then audit cost is negligible and CEOs always obtain information about collusive agreements.

3 No leniency programs

In this section we compute the expected profits and sustainability conditions obtained without any leniency programs. Until section 6, we assume that $\mu = 1$.

Firms can select one of the following four strategies. They can choose not to collude. In this case they obtain zero profit since we assume Bertrand competition with homogeneous products. Firms can collude simultaneously on the two markets. Since $\mu = 1$ the detection of one collusive agreement automatically leads to the second cartel detection and both cartels are dissolved. We denote by $Us$ this strategy for centralized firms or $Ms$ in the opposite case. Firms finally can choose to cartelize in only one market and, if the cartel is discovered, they start collusion in the second market: this is the $Seq$ strategy. If the firms select this strategy we assume that they choose a centralized structure.\(^{13}\)

3.1 Comparison of collusive strategies

Without any leniency programs the $Ms$ strategy is dominated by the $Us$ one.\(^{14}\) Indeed detection probabilities and fines are the same but collusive prices and profits are higher if firms play the $Us$ strategy thanks to a coordination of the two prices. We now compute the expected profits and sustainability conditions considering only the $Us$ and $Seq$ strategies.

3.1.1 $Us$ strategy

Solving the individual maximization program for each firm, we obtain the following equilibrium values ($i = 1, 2; X = A, B$):

<table>
<thead>
<tr>
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<tr>
<td>Price</td>
<td>$p_i^A = \frac{a}{2}$</td>
<td>$p_i^{AX} = \frac{a}{2} - \varepsilon$</td>
<td>$p_i^{AX} = c = 0$</td>
</tr>
<tr>
<td>Quantity</td>
<td>$q_i^X = \frac{a}{4}$</td>
<td>$q_i^{AX} = \frac{a}{2}$</td>
<td>$q_i^{AX} = \frac{a}{2}$</td>
</tr>
<tr>
<td>Profit</td>
<td>$\pi_i^{cU} = \frac{a^2}{4}$</td>
<td>$\pi_i^{dU} = \frac{a^2}{2}$</td>
<td>$\pi_i^{dU} = 0$</td>
</tr>
</tbody>
</table>

The present discounted value of a firm from colluding is given by:

$$\Pi_i^{Us} = \pi_i^{cU} - \left[ 1 - (1 - \rho)^2 \right] 2F + \delta (1 - \rho)^2 \Pi_i^{Us} \Leftrightarrow \Pi_i^{Us} = \frac{\pi_i^{cU} - 2\rho (2 - \rho) F}{1 - \delta (1 - \rho)^2}$$

Deviations are punished with Nash reversion, so the current gain from deviation is:

$$\Pi_i^{dUs} = 2\pi_i^{cU} - 2\rho (2 - \rho) F$$

\(^{13}\)Centralized firms can easily coordinate themselves on the time to put in place the second agreement.

\(^{14}\)Except if we consider independent products ($d = 0$). In this case firms are indifferent between the two organizational structures.
The $U_s$ strategy is sustainable if and only if:

$$\Pi_i^{Us} \geq \Pi_i^{dUs} \iff F \leq F_{Us} = \frac{\delta (1 - \rho)^2 - \frac{1}{2} \pi_i^{cU}}{\delta (1 - \rho)^2 \rho (2 - \rho)}$$

### 3.1.2 Seq strategy

Firms cartelize in only one of the two markets (named market B), maintaining price competition in the second market. If the cartel is discovered, firms start collusion in the market A. We successively determine equilibrium values in the second market and in the first cartels.

**Second cartel:** In the market B the cartel has been discovered and equilibrium prices equal marginal cost: $p^B_1 = p^B_2 = c = 0$. In the market A firms act as a monopoly. The best-response function of a firm monopolizing the market A is: $p^A_i = a + dp^B_2(1 + d)$. Setting $p^B = 0$, we obtain:

$$\tilde{p}^A_i = \frac{a}{2(1 + d)} \quad , \quad \tilde{q}^A_i = \frac{a}{4} \quad \text{and} \quad \tilde{\pi}^c_i = \frac{a^2}{8(1 + d)}$$

The present discounted value of a firm from colluding is given by:

$$\tilde{\Pi}^i = \tilde{\pi}_i^c - \rho F + \delta (1 - \rho) \tilde{\Pi}_i \iff \tilde{\Pi}_i = \frac{\tilde{\pi}_i^c - \rho F}{1 - \delta (1 - \rho)}$$

The deviation profit is: $\Pi_i^{d} = 2\tilde{\pi}_i^c - \rho F$. This second cartel is sustainable if and only if:

$$\tilde{\Pi}_i \geq \Pi_i^{d} \iff F \leq F = \frac{2\delta (1 - \rho) - 1}{\delta \rho (1 - \rho)} \tilde{\pi}_i^c$$

**First cartel:** Each firm's expected payoff associated with collusion is defined as:

$$\Pi_i^{Seq} = \tilde{\pi}_i^c - \rho F + \delta (1 - \rho) \Pi_i^{Seq} + \delta \rho \Pi_i \iff \Pi_i^{Seq} = \frac{(1 - \delta + 2\delta \rho)(\tilde{\pi}_i^c - \rho F)}{[1 - \delta (1 - \rho)]^2}$$

The deviation profit is: $\Pi_i^{dSeq} = 2\tilde{\pi}_i^c - \rho F$. Then collusion is sustainable if and only if:

$$\Pi_i^{Seq} \geq \Pi_i^{dSeq} \iff F \leq F_{Seq} = \frac{-1 + 3\delta - 2\delta \rho - 2\delta^2 (1 - \rho)^2}{[1 - \delta (1 - \rho)]^2} \tilde{\pi}_i^c$$

Intuitively this second condition is easier to sustain than: $F \leq F$ since it accounts the expected collusive outcome in the second cartel once the first cartel is detected.

### 3.2 Strategy choice

If they are sustainable, collusive strategies dominate the non-cooperation since they lead to a strictly positive profit. Firms choose the $U_s$ strategy if:

$$\Pi_i^{Us} \geq \Pi_i^{Seq} \iff F \leq F_1 = \frac{[1 - \delta (1 - \rho)]^2 \pi_i^{cU} - [1 - \delta (1 - \rho)^2] (1 - \delta + 2\delta \rho) \tilde{\pi}_i^c}{2[1 - \delta (1 - \rho)]^2 (2 - \rho) - [1 - \delta (1 - \rho)^2] (1 - \delta + 2\delta \rho) \rho}$$
Proposition 1  Firms choose the Us strategy if and only if $F \leq \min(F_{Us}, F_1)$. They choose the Seq strategy if $\min(F_{Us}, F_1) < F \leq \bar{F}$. They do not collude if $F > \max(F_{Us}, \bar{F})$.

The figures 1a and 1b represent these equilibria considering $a = 10$, $\rho = 0.01$ and different values of $\delta$. Changing the value of $\delta$ leaves unchanged the global shape of the graph but the border line $F_1$ moves downward if $\delta$ increases. In the figure 1b we plot the borderlines obtained with $\delta = 0.8$ with dots for comparison.

![Figure 1a: Equilibria obtained without leniency program and $\delta = 0.8$](image-url)
The most influential variable in the organization choice is the substitutability of the products. If the products are strong substitutes (high value of $d$) then collusion is efficient only if collusive agreements are simultaneously effective in the two markets. Firms select the $Us$ strategy if $F$ is low and do not collude at all if $F$ is high. If the products are weak substitutes the $Seq$ strategy is now available. If the firms choose this strategy then the per period profit is lower but the collusion lasts longer and the second fine is paid later. The $Seq$ strategy is selected if $F$ is relatively high. In the other case firms select the $Us$ strategy.\footnote{The value of $F_1$ for $d = 0$ decreases with $\delta$. It is negative if $\delta$ is sufficiently high ($\delta \simeq 0.987$ for $\rho = 0.01$).}

4 Leniency Programs

Since $\mu = 1$ firms never reach the stage (5). Firms can apply for leniency only at stage 1 (before the detection of the second cartel) or stage 3 (between the detection of the first cartel and the detection of the second one).

At stage (1), when deviating from the cartel, firm simultaneously applies for leniency. All the threshold values of the sustainability of the collusion are impacted ($F_{Us}$, $\bar{F}$ and $F_{Seq}$).

The detection of the second cartel once the investigation is launched is inescapable. In this case, firms apply for leniency for the second cartel whenever they can, that is with probability $q_U$ if firms are centralized or $q_M$ under decentralized structure. Since $q_M \geq q_U$ the $Ms$ strategy is not always dominated by the $Us$.

Figure 1b: Equilibria obtained without leniency program and $\delta = 0.95$
4.1 Collusive strategies

Three collusive strategies are now available: the Seq, Us and Ms strategies.

4.1.1 Seq strategy

When a cartel is detected firms cannot apply for leniency for another cartel since firms sequentially collude in the two markets. But they can seek leniency when they deviate from an active cartel which has not been yet detected.

The expected collusive profits are not altered by leniency. Nonetheless, sustainability conditions are modified since deviation profits are increased. The new threshold values are indicated below:

\[
\tilde{F} = \frac{2\delta (1 - \rho) - 1}{\rho} \tilde{\pi}_i^c \quad \text{and} \quad F_{Seq} = \frac{3\delta - 2\delta \rho - 2\delta^2 (1 - \rho)^2 - 1}{(1 - \delta + 2\delta \rho) \rho} \tilde{\pi}_i^c
\]

4.1.2 Us strategy

Leniency programs have a double effect on the Us strategy. First firms can apply for leniency before the detection of the first cartel, increasing the deviation profit and making collusive agreements more difficult to sustain. Second firms can apply for leniency between the detections of the first and the second cartel: the expected profit under the Us strategy is increased (sustainability condition is modified). If \( \mu = 1 \) conviction for the second cartel is inescapable, firms always apply for leniency whenever possible.

Collusive profit is:

\[
\Pi_i^{Us} = \pi_i^{cU} - \rho^2 2F - 2\rho (1 - \rho) \left\{ (1 - q_{U}) 2F + q_{U} \left[ \frac{1}{2} 2F + \frac{1}{2} (F + \tau F) \right] \right\} + \delta (1 - \rho)^2 \Pi_i^{Us}
\]

\[
\Leftrightarrow \Pi_i^{Us} = \frac{\pi_i^{cU} - [2\rho + (1 - \rho) (4 - q_{U} + \tau q_{U})] \rho F}{1 - \delta (1 - \rho)^2}
\]

Deviation profit is: \( \Pi_i^{dUs} = 2\pi_i^{cU} \). Us strategy is sustainable if and only if:

\[
\Pi_i^{Us} \geq \Pi_i^{dUs} \Leftrightarrow F \leq F_{Us} \equiv \frac{2\delta (1 - \rho)^2 - 1}{[2\rho + (1 - \rho) (4 - q_{U} + \tau q_{U})] \rho} \pi_i^{cU}
\]

4.1.3 Ms strategy

The expected per-period payoff of colluding firms playing the Ms strategy is lower than with the Us strategy (because of internal competition between substitutable products) but decentralized firms are much more likely to seek leniency for the second collusive agreement. We obtain the same equilibrium prices under the Ms strategy as in the duopoly case with differentiated products (each firm producing only one product).

For each period we obtain:
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</tr>
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<td>Quantity</td>
<td>( q_i^X = \frac{1 + \delta q}{2 + \delta} )</td>
<td>( q_i^{dX} = \frac{1 + \delta q}{2 + \delta} )</td>
<td>depends on the price in the other market</td>
</tr>
<tr>
<td>Profit of a division</td>
<td>( \pi_i^{cM} = \frac{2(1 + \delta q)q - 1}{2(2 + \delta)\bar{\tau}^2} )</td>
<td>( \pi_i^{dM} = \frac{2(1 + \delta q)q - 1}{2(2 + \delta)\bar{\tau}^2} )</td>
<td>( \pi_i^{pM} = 0 )</td>
</tr>
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A deviating division reduces its price \((p_i^X - \varepsilon)\) in order to absorb the global market. But this price reduction is not computed in order to absorb the consumers buying the other product since the collusive price is the best response to the other market price. The expected collusive profit is:

\[
\Pi_i^{Ms} = \pi_i^{cM} - \rho^2 F - (1 - \rho) F - (1 - \rho) \rho \left[(1 - q_M) F + q_M \left(\frac{1}{2} F + \frac{1}{2} \tau F\right)\right] + \delta (1 - \rho)^2 \Pi_i^{Ms}
\]

\[
\Leftrightarrow \Pi_i^{Ms} = \frac{\pi_i^{cM} - \left[1 + \frac{1}{2} (1 - \rho) (2 - q_M + q_M \tau)\right] \rho F}{1 - \delta (1 - \rho)^2}
\]

The deviation profit is: \(\Pi_i^{dMs} = 2\pi_i^{cM}\). \(Ms\) strategy is sustainable if and only if:

\[
\Pi_i^{Ms} \geq \Pi_i^{dMs} \Leftrightarrow F \leq F_{Ms} = \frac{2\delta (1 - \rho)^2 - 1}{1 + \frac{1}{2} (1 - \rho) (2 - q_M + q_M \tau) \rho} \pi_i^{cM}
\]

### 4.2 Organizational choice

Comparison of the profits is used to endogenize the choice of organizational structure. Firms adopt the \(U_{sim}\) strategy instead of the \(Mc\) strategy if and only if:

\[
\Pi_i^{Us} \geq 2\Pi_i^{Ms} \Leftrightarrow F \leq F_2 = \frac{\pi_i^{cU} - 2\pi_i^{cM}}{(q_M - q_U) (1 - \tau) (1 - \rho) \rho}
\]

Decentralization can appear at equilibrium if \(q_M\) is higher than \(q_U\), if the fine reduction is high (low value of \(\tau\)), if the products are weak substitutes \((2\pi_i^{cM} \text{ close to } \pi_i^{cU})\) and for high values of \(F\).

If the products are independent then \(F_2 = 0\) \((\pi_i^{cU} = 2\pi_i^{cM})\). \(Ms\) strategy dominates \(Us\) whatever the amount of fine.\(^{16}\)

We have to compare \(Seq\) strategy with the others \((\overline{\rho} \equiv 1 - \rho)\):

\[
2\Pi_i^{Ms} \geq \Pi_i^{Seq} \Leftrightarrow F \leq F_3 = \frac{(1 - \delta \overline{\rho})^2 2\pi_i^{cM} - (1 - \delta^2 \overline{\rho}) (1 - \delta + 2\delta \rho) \overline{\pi_i^c}}{(1 - \delta \overline{\rho})^2 (2 + \overline{\rho} (2 - q_M + q_M \tau)) - (1 - \delta^2 \overline{\rho}) (1 - \delta + 2\delta \rho) \rho}
\]

\[
\Pi_i^{Us} \geq \Pi_i^{Seq} \Leftrightarrow F \leq F_1 = \frac{(1 - \delta \overline{\rho})^2 \pi_i^{cU} - (1 - \delta^2 \overline{\rho}) (1 - \delta + 2\delta \rho) \overline{\pi_i^c}}{(1 - \delta \overline{\rho})^2 (2 + \overline{\rho} (4 - q_U + q_M \tau)) - (1 - \delta^2 \overline{\rho}) (1 - \delta + 2\delta \rho) \rho}
\]

Firms choose the sequential collusion if \(F\) is relatively high, \(q_M\) and \(q_U\) are low, \(\tau\) is high, \(\delta\) is high (collusion lasts longer with \(Seq\) strategy) and, most importantly, if the products are weak substitutes.

The comparisons of the threshold values are reported in the Appendix A. The relative positions of the borderlines \(F_{Us}, F_{Ms}\) and \(F_2\) do not depend on parameters values (see Appendix A). However the relative position of \(F_1\) and \(F_3\) with respect to the other thresholds depends heavily on \(\delta\).

\(^{16}\)It is not the case if \(\mu < 1\) (see section 6).
In order to graphically illustrate these results and to compare with the previous graphical representation we consider: $a = 10, \rho = 0.01, q_M = 0.9, q_U = 0.1$ and $\tau = 0$. Then we vary $\delta$ considering always sufficiently high value to make all the collusive strategies sustainable for each figure (the benchmark boundaries without leniency program are plotted with dots for comparison).

![Organizational choice with leniency program (δ = 0.8)](image)

Figure 2a: Organizational choice with leniency program ($\delta = 0.8$)
Figure 2b: Organizational choice with leniency program ($\delta = 0.95$)

Figure 2c: Organizational choice with leniency program ($\delta = 0.99$)

**Proposition 2**  Firms choose the Us strategy if $F \leq \min(F_1, F_{Us}, F_2)$, the Ms strategy if $\min(F_2, F_{Us}) \leq \ldots$
\[ F \leq \min(F_{Ms}, F_3). \text{ They do not collude if } F > \max(\tilde{F}, F_{Ms}, F_{Us}). \text{ In the other cases, firms choose the Seq strategy.} \]

5 Impact of leniency programs

**Major impact on the organizational choice:** The major above mentioned result is that leniency can favor the adoption of decentralized organization. Without leniency firms always select a centralized structure. For some parameters values firms can switch to the Ms strategy when leniency is introduced. Decentralized firms face competition between divisions inside the firms and this impacts the collusive prices. Without leniency, the Ms strategy is always dominated by Us (except for independent products). If we introduce leniency, decentralized firms are much more likely to seek leniency for the second collusive agreement than centralized firms and this reduces the expected fine. If competition between products is not too fierce then the Ms strategy dominates the Us.

**Proposition 3** Leniency programs may favor the adoption of decentralized structure.

**Other effects relating to the organizational choice:** In the previous graphical representation the traditional destabilizing effect of leniency appears. Deviation profits are increased when firms can apply for leniency before the first cartel is detected. The boundary \( F_{Us} \) moves down and \( \tilde{F} \) moves to the left. Leniency makes some collusive agreements more difficult to sustain.

In the graphical representation set for \( \delta = 0.8 \) there is a small area in which firms switch from the Us to the Seq strategy because the former is no longer sustainable.

Moreover there is an area in which firms switch from the Seq strategy to Ms. With this latter strategy leniency programs partially protect firms from a double fine, there are fewer incentives to choose the Seq strategy of which one advantage is to protect firms from this risk.

In the graphical representation set for \( \delta = 0.95 \) there is a collusive area which appears only if leniency programs are introduced. This is due to two reverse effects of leniency on the boundary \( F_{Ms} \). Allowing leniency before the first cartel is detected increases deviation profit and makes the Ms strategy more difficult to sustain. But the decreased expected fine increases the expected profit of the Ms strategy and makes this strategy easier to sustain. If leniency is sufficiently generous (low value of \( \tau \)) the second effect may dominate the first one.

**Impacts on the prices:** In the area in which firms switch from the Us strategy to the Ms strategy, leniency programs decrease collusive prices (except in the extreme case \( d = 0 \)) and do not impact the existence or the duration of the cartel. But when firms switch from the Seq strategy to the Ms strategy collusive price increase (except for \( d = 0 \)) and collusion is shorter-lasting. Finally when firms switch to the Us strategy to the Seq strategy, collusive prices decrease and collusion lasts longer.
Proposition 4 Even when leniency programs do not deter collusive agreements they may impact collusive prices.

Moreover we reach traditional effects of leniency: prices decrease when leniency makes collusive agreements not sustainable and increase if collusive opportunities are strengthened.

Impact of $\tau$: There is a broad consensus regarding the usefulness of leniency programs but their design is still being debated, notably concerning the fine reduction. In our model granting total immunity is optimal if firms apply for leniency before the detection of the first cartel, since the destabilizing effect of leniency is maximized. But we obtain inconsistent results regarding the fine reduction for the second cartel (computed with $\tau$) if leniency is requested during the first cartel investigation.

The following table sums up the impact of a decreased value of $\tau$ on prices, consumer surplus and duration of collusion.

<table>
<thead>
<tr>
<th>border line</th>
<th>shift of the border line</th>
<th>potential switch</th>
<th>prices</th>
<th>surplus</th>
<th>duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_2$</td>
<td>↘</td>
<td>Us $\Rightarrow$ Ms</td>
<td>$\nearrow$</td>
<td>$\nearrow$</td>
<td>$=$</td>
</tr>
<tr>
<td>$F_{Ms}$</td>
<td>$\nearrow$</td>
<td>Seq $\Rightarrow$ Ms</td>
<td>$\searrow$</td>
<td>ambiguous</td>
<td>↘</td>
</tr>
<tr>
<td>$F_{Ms}$</td>
<td>$\nearrow$</td>
<td>No collusion $\Rightarrow$ Ms</td>
<td>$\nearrow$</td>
<td>$\nearrow$</td>
<td>-</td>
</tr>
<tr>
<td>$F_3$</td>
<td>$\leftarrow$</td>
<td>Seq $\Rightarrow$ Ms</td>
<td>$\nearrow$</td>
<td>ambiguous</td>
<td>↘</td>
</tr>
<tr>
<td>$F_{Us}$</td>
<td>$\uparrow$</td>
<td>No collusion $\Rightarrow$ Us</td>
<td>$\searrow$</td>
<td>$\searrow$</td>
<td>-</td>
</tr>
<tr>
<td>$F_1$</td>
<td>$\leftarrow$</td>
<td>Seq $\Rightarrow$ Us</td>
<td>$\nearrow$</td>
<td>ambiguous</td>
<td>↘</td>
</tr>
</tbody>
</table>

The above effects of the border lines $F_1$ and $F_3$ appear if the discount factor is very high (in the two graphical representations below, $F_1$ and $F_3$ do not appear since we assume $\delta = 0.8$).

A decreased value of $\tau$ can yields to collusive or competitive effects. We reach the same effects concerning the introduction of amnesty plus ($\tau < 0$).

In the short term, a decreased value of $\tau$ increases the number of active cartels since firms switch from the Seq strategy to Ms strategy (one more cartel) or can decide to start collusion under the Ms or Us strategies (two more cartels). In a longer term, the effect of $\tau$ on the number of active cartels is quite ambiguous since the switch from the Seq strategy to the Ms strategy results in shorter lasting cartel. In any case, leniency applications increase.

The two graphical representations below illustrate these points (for $\delta = 0.8$).

Partial amnesty for the second cartel: Border lines are plotted for $\tau = 0$ (full line) and $\tau = 0.5$ (dotted line).
**Figure 3:** Impacts of $\tau$ in the partial amnesty case

**Amnesty plus:** Border lines are plotted for $\tau = 0$ (full line) and $\tau = -1$ (dotted line).

**Figure 4:** Impacts of $\tau$ in the amnesty plus case
6 Extension: uncertain serendipitous detection ($\mu < 1$)

In this section we check the robustness of the results obtained with $\mu = 1$. We obtain similar results considering $\mu < 1$ if $\mu$ is sufficiently high or if amnesty plus program is active for lower value of $\mu$.

When the antitrust authority investigates the first cartel, then firms, irrespective of the organizational structure, face a risk of detection for the second agreement that is lower than one $\mu \in ]0;1[$.

Another strategy may appear: firms simultaneously collude in the two markets, and when a cartel has been successfully detected they can continue to collude in the other market provided that this second cartel is not detected. This strategy is denoted $Uc$ if firms are centralized and $Mc$ in the other case.

6.1 No leniency programs

Without leniency program, the $Ms$ and $Mc$ strategies are respectively dominated by the $Us$ and $Uc$ strategies.

The expected profit under the $Us$ strategy is:

$\Pi^Us_i = \pi^Us_i - \rho^22F - 2(1 - \rho)\rho(F + \mu F) + \delta(1 - \rho)^2\Pi^Us_i \Leftrightarrow \Pi^Us_i = \frac{\pi^Us_i - 2[1 + (1 - \rho)\mu]\rho F}{1 - \delta(1 - \rho)^2}$

The deviation profit is: $\Pi^dUs_i = 2\pi^Us_i - \rho^22F - 2(1 - \rho)\rho(1 + \mu)F$. This strategy is sustainable if and only if:

$\Pi^Us_i \geq \Pi^dUs_i \Leftrightarrow F \leq F_{Us} = \frac{\delta(1 - \rho)^2 - \frac{1}{2}}{\delta(1 - \rho)^2 - \frac{1}{2} + (1 + (1 - \mu)\rho)} \pi^Us_i$

The expected profit under the $Uc$ strategy is:

$\Pi^Uc_i = \pi^Uc_i - \rho^22F - 2(1 - \rho)\rho(F + \mu F) + 2\delta(1 - \rho)\rho(1 - \mu) \Pi_i + \delta(1 - \rho)^2\Pi^Uc_i$

$\Leftrightarrow \Pi^Uc_i = \frac{\pi^Uc_i - 2[1 + (1 - \rho)\mu]\rho F + 2\delta(1 - \rho)\rho(1 - \mu) \Pi_i}{1 - \delta(1 - \rho)^2}$

The deviation profit is: $\Pi^dUc_i = 2\pi^Uc_i - 2[1 + (1 - \rho)\mu]\rho F$. This strategy is sustainable if and only if:

$F \leq \bar{F}$ and $\Pi^Uc_i \geq \Pi^dUc_i \Leftrightarrow F \leq F_{Uc} = \frac{[1 - \delta(1 - \rho)][\delta(1 - \rho)^2 - \frac{1}{2}]\pi^Uc_i + \delta\rho(1 - \rho)(1 - \mu)\tilde{\pi}_c}{[1 - \delta(1 - \rho)][\delta(1 - \rho)^2 - \frac{1}{2}]\pi^Uc_i + \delta\rho(1 - \rho)(1 - \mu)\tilde{\pi}_c}$

If we consider $F \leq \bar{F}$ then $F_{Uc} \geq F_{Us}$ and $Uc$ dominates $Us$. The only valuable comparison is between $Uc$ and $Seq$ (with $\bar{\rho} = 1 - \rho$):

$\Pi^Uc_i \geq \Pi_i^{Seq} \Leftrightarrow F \leq F_4 = \frac{(1 - \delta \bar{\rho})^2 \pi^Uc_i + \{1 + 3 + 2\rho(1 - \delta \bar{\rho})\} \tilde{\pi}_c}{[1 - \delta \bar{\rho}^2 + 2(1 - \delta \bar{\rho})\bar{\rho}\mu]} \tilde{\pi}_c$

We obtain the following proposition:

**Proposition 5** Firms choose the $Uc$ strategy if $F \leq \min(\bar{F}, F_{Uc}, F_4)$. They choose the $Us$ strategy if $\bar{F} < F < F_{Us}$. They choose the $Seq$ strategy if $\min(F_{Uc}, F_4) < F \leq \bar{F}$. They do not collude if $F > \max(\bar{F}, F_{Us})$. 

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The main difference with the case $\mu = 1$ is that the $Uc$ strategy is now an equilibrium structure. Moreover $F_{Us}$ is increased: some cartels are sustainable only for a lower value of $\mu$.

The figure below illustrates these results ($a = 10$, $\rho = 0.01$, $\delta = 0.8$ et $\mu = 0.6$). We also plot on the graph the border lines obtained with $\mu = 1$ for comparison.

![Equilibria obtained without leniency program ($\mu = 0.6$)](image)

**Figure 5**: Equilibria obtained without leniency program ($\mu = 0.6$)

### 6.2 Leniency programs

To simplify the exposition and highlight the main effects we assume: $q_M = 1$ and $q_U = 0$. The $Uc$ strategy dominates the $Mc$ strategy: firms prefer the centralized structure when they aim to collude as long as possible.\(^{17}\)

#### 6.2.1 Collusive strategies

The $Seq$ strategy does not depend on the value of $\mu$. We compute the expected profits and sustainability conditions for the $Us$, $Uc$ and $Ms$ strategies.

**Us strategy**: If $\mu < 1$ firms can reach the stage (5). In this case the antitrust authority can not detect the second collusive agreement since it is no longer active but firms can denounce it. Without amnesty plus

\(^{17}\)For $0 < q_U < 1$ centralized firms could apply for leniency for the second cartel at stage (3) but do not and continue to collude in the second market if they reach the stage (5). A similar strategy could appear for decentralized firms if $0 < q_M < 1$.\)
program ($\tau \geq 0$) two pure strategy equilibria exist: either both firms denounce the cartel or not. We assume that firms can coordinate on the best equilibrium that is the non denunciation. But, if $\tau < 0$, firms denounce the second cartel. We have to distinguish these two cases.

If $\tau \geq 0$, the expected profit is:

$$\Pi_i^{Us} = \pi_i^{cU} - \rho^2 2F - 2\rho (1 - \rho) [\mu 2F + (1 - \mu) F] + \delta (1 - \rho)^2 \Pi_i^{Us} \Leftrightarrow \Pi_i^{Us} = \frac{\pi_i^{cU} - 2 (1 + \mu - \rho \mu) \rho F}{1 - \delta (1 - \rho)^2}$$

The deviation profit is: $\Pi_i^{dUs} = 2\pi_i^{cU}$. This strategy is sustainable if:

$$\Pi_i^{Us} \geq \Pi_i^{dUs} \Leftrightarrow F \leq F_{Us} = \frac{2\delta (1 - \rho)^2 - 1}{2 (1 + \mu - \rho \mu) \rho} \pi_i^{cU}.$$

If $\tau < 0$, the expected profit is:

$$\Pi_i^{Us} = \pi_i^{cU} - \rho^2 2F - 2\rho (1 - \rho) \left\{ \mu 2F + (1 - \mu) \left[ F + \frac{1}{2} F + \frac{1}{2} \tau F \right] \right\} + \delta (1 - \rho)^2 \Pi_i^{Us}$$

$$\Leftrightarrow \Pi_i^{Us} = \frac{\pi_i^{cU} - \{2\rho + (1 - \rho) [3 + \mu + \tau (1 - \mu)]\} \rho F}{1 - \delta (1 - \rho)^2}$$

The deviation profit is: $\Pi_i^{dUs} = 2\pi_i^{cU}$. This strategy is sustainable if:

$$\Pi_i^{Us} \geq \Pi_i^{dUs} \Leftrightarrow F \leq F_{Us} = \frac{2\delta (1 - \rho)^2 - 1}{2 (1 + \mu - \rho \mu) [3 + \mu + \tau (1 - \mu)]} \rho \pi_i^{cU}$$

$Uc$ strategy: This strategy is sustainable if firms do not deviate from the collusive agreement at stage (1) and do not apply for leniency for the second cartel at stage (5). Once the investigation is closed then expected fine is nil for the firms and they do not apply for leniency if $\tau \geq 0$. But if $\tau < 0$ they may apply for leniency if:

$$-\tau F \geq \delta \Pi_i \Leftrightarrow F \geq F_{rep} = \frac{\delta}{\delta \rho - \{1 - \delta (1 - \rho)\} \tau \pi_i^{cU}}$$

If the firms stay on the collusive path then the expected profit under the $Uc$ strategy is the same than without leniency but the deviation profit is altered: $\Pi_i^{dUc} = 2\pi_i^{cU}$. Firms do not deviate from the cartel at stage (1) if:

$$\Pi_i^{Uc} \geq \Pi_i^{dUc} \Leftrightarrow F \leq F_{Uc} = \frac{[1 - \delta (1 - \rho)] \left[ 2\delta (1 - \rho)^2 - 1 \right] \pi_i^{cU} + 2 (1 - \rho) \rho (1 - \mu) \delta \pi_i^{cU}}{2 \left[ 1 - \delta (1 - \rho)^2 \right] (1 - \delta) (1 - \rho) \mu}.$$

$Ms$ strategy: Firms always apply for leniency at stage (3) since the second cartel disappears once the first one has been detected.

The expected profit is:

$$\Pi_i^{Ms} = \pi_i^{cM} - \rho F - (1 - \rho) \rho \left[ \left( \frac{1}{2} + \frac{1}{2} \tau \right) \right] F + \delta (1 - \rho)^2 \Pi_i^{Ms} \Leftrightarrow \Pi_i^{Ms} = \frac{\pi_i^{cM} - [1 + \frac{1}{2} (1 - \rho) (1 + \tau)] \rho F}{1 - \delta (1 - \rho)^2}.$$
The deviation profit of a firm which deviates at stage (1) is: \( \Pi_{i}^{dMs} = 2\pi_{c}^{cM} \). Then this strategy is sustainable if:

\[
\Pi_{i}^{M_s} \geq \Pi_{i}^{dMs} \iff F \leq F_{Ms} = \frac{2\delta(1-\rho)^2 - 1}{[1 + \frac{1}{2}(1-\rho)(1+\tau)]\rho} \pi_{c}^{cM}
\]

### 6.2.2 Strategy choice

Four collusive strategies are available: \( Us, Uc, Ms \) and \( Seq \). If sustainable, the \( Uc \) strategy dominates the \( Us \) strategy. The comparison between \( Uc \) and \( Seq \) remains unchanged. So we have to compare the \( Ms \) strategy with the others.

\[
\Pi_{i}^{Seq} \geq 2\Pi_{i}^{Ms} \iff F \geq F_{3} = \frac{[1 - \delta (1 - \rho)]^2 2\pi_{c}^{cM} - [1 - \delta (1 - \rho)^2] (1 - \delta + 2\delta\rho) \tilde{\pi}_{i}^{c}}{\left\{ [1 - \delta (1 - \rho)]^2 [2 + (1 - \rho)(1 + \tau)] - [1 - \delta (1 - \rho)^2] (1 - \delta + 2\delta\rho) \right\} \rho}
\]

\[
\Pi_{i}^{Uc} \geq 2\Pi_{i}^{Ms} \iff \frac{[1 - \delta (1 - \rho)] (\pi_{i}^{cU} - 2\pi_{c}^{cM}) + 2\delta (1 - \rho) \rho (1 - \mu) \tilde{\pi}_{i}^{c}}{(1 - \rho) \rho} \geq 2\mu (1 - \delta) (1 - \delta) (1 + \tau) + \delta \rho (1 - \tau) F
\]

If \( \mu < \frac{1 + \tau - \delta \rho (1 - \tau)}{2(1 - \delta)} \), the right hand side is negative whereas the left hand side is positive: if sustainable, \( Uc \) dominates \( Ms \).

If \( \mu > \frac{1 + \tau - \delta \rho (1 - \tau)}{2(1 - \delta)} \) then: \( \Pi_{i}^{Uc} \geq 2\Pi_{i}^{Ms} \iff F \leq F_{3} = \frac{[1 - \delta (1 - \rho)] (\pi_{i}^{cU} - 2\pi_{c}^{cM}) + 2\delta (1 - \rho) \rho (1 - \mu) \tilde{\pi}_{i}^{c}}{[2\mu (1 - \delta) (1 - \delta) (1 + \tau) + \delta \rho (1 - \tau) (1 - \rho) F] \rho} \).

**Remark:** For \( d = 0 \), \( \pi_{c}^{cU} - 2\pi_{c}^{cM} = 0 \) and \( F_{5} > 0 \).

To compare \( Ms \) and \( Us \) we have to distinguish two sub-cases depending on the existence of amnesty plus.

**Case 1:** if \( \tau \geq 0 \), then: \( \Pi_{i}^{Us} \geq 2\Pi_{i}^{Ms} \iff \pi_{i}^{cU} - 2\pi_{c}^{cM} \geq 2\mu (1 + \tau) (1 - \rho) F \).

If \( \mu < \frac{1 + \tau}{2} \), the right hand side is negative and the above inequality always applies.

Each decentralized firm is individually better of applying the leniency at stage (3) but it is not the collectively best action: this is a prisoner dilemma situation. With centralized structure, the stage (3) is removed and firms do not cooperate with the antitrust authority.\(^{18}\)

If \( \mu \geq \frac{1 + \tau}{2} \), we obtain:

\[
\Pi_{i}^{Us} \geq 2\Pi_{i}^{Ms} \iff F \leq F_{2a} = \frac{\pi_{i}^{cU} - 2\pi_{c}^{cM}}{(2\mu - 1 - \tau) (1 - \rho) \rho}
\]

**Case 2:** if \( \tau < 0 \), then: \( \Pi_{i}^{Us} \geq 2\Pi_{i}^{Ms} \iff F \leq F_{2c} = \frac{\pi_{i}^{cU} - 2\pi_{c}^{cM}}{(1 - \rho)(1 - \tau) \rho} \).

Firms tend to prefer decentralization if the product substitutability decreases (\( 2\pi_{c}^{cM} \) closed to \( \pi_{c}^{cU} \)), \( \tau \) decreases, \( \mu \) increases and if \( \rho \) increases \( \text{[assuming} \rho < 1/2\text{]} \).

\(^{18}\text{If} q_{U} > 0 \text{the stage (3) would be partially removed and the trade-off between} Ms \text{and} Us \text{would remain roughly the same.} \)
6.3 Implications of $\mu < 1$ on the previous results

In this section, we do not characterize all the equilibria for each parameter value\textsuperscript{19} but we aim to show that some of the previous results are robust to the case $\mu < 1$. Moreover we obtain new results. Equilibrium choices depend to a large extent on the values of $\mu$ and $\tau$ and on the existence of amnesty plus programs. The results obtained for $\mu = 1$ are robust to the case $\mu < 1$ if $\mu$ is high. For lower value of $\mu$ we obtain similar results only when amnesty plus program is active.

6.3.1 high value of $\mu$

If $\mu > \frac{1+\tau}{2}$ and $\tau \geq 0$, the joint expected profits increase if firms can apply for leniency at stage (3). We reach the same results as obtained for $\mu = 1$. Since decentralization allows firms to apply for leniency for the second cartel once an investigation is opened, firms can select the $Ms$ equilibrium strategy. This benefits consumers in two ways: collusive prices are decreased and collusion is shorter lasting when firms switch from the $Uc$ strategy to the $Ms$ strategy.

Contrary to the case $\mu = 1$ firms can select centralized organization when the products are independent since cartels can be longer sustainable with this type of organization.

Moreover firms may switch from the $Uc$ strategy to the $Us$ strategy since $\tilde{F}$ is reduced with the application of leniency. In such cases, prices are not altered when firms simultaneously collude in the two markets but collusion is shorter-lasting.

The following graphical representation illustrates the previous comments.

We set: $a = 10$, $\rho = 0.01$, $\delta = 0.8$, $\mu = 0.6$ and $\tau = 0$. The boundaries obtained without leniency program are plotted with dots for comparison.\textsuperscript{20}

\textsuperscript{19}It would be too tedious since all the threshold values should be compared and some comparisons depend on the parameters values.

\textsuperscript{20}Similar representation can be obtained for $\delta = 0.95$ ($Ms$ dominates $Seq$ for all the values for which $Ms$ is sustainable and dominates $Uc$).
**Effects of amnesty plus:** If \( \tau \) is slightly negative then firms playing the \( Us \) strategy may apply for leniency for the second cartel in stage (5) with probability \( 1 - \mu \). Then one firm obtains total immunity for the second cartel and a slight reduced fine for the first cartel whereas the other firm must pay a second fine \( F \) for the second cartel. The expected profit under the \( Us \) strategy decreases and \( F_{Us} \) moves downward.

However if \( \tau \) decreases the expected profits are increased and \( F_{Us} \) moves upward. Note that the expression of \( F_{Us} \) is exactly the same for \( \tau = -1 \) and for \( \tau = 0 \).

The upper border of the \( Uc \) area decreases from \( \bar{F} \) to \( F_{rep} \). Amnesty plus program incites firms to reveal the second cartel once the first cartel has been detected and then firms switch from \( Uc \) to \( Us \).

The \( Ms \) area is increased and some firms can switch from the \( Us \) to the \( Ms \) strategy in order to enjoy amnesty plus with certainty.

Moreover there is an area in which firms switch from the \( Uc \) to the \( Ms \) strategy whether because the \( Uc \) strategy is not sustainable or because the expected profit of the \( Ms \) strategy is sufficiently increased compared to the \( Uc \) strategy.

The threshold \( F_{Ms} \) is increased and firms can switch from a no collusion strategy to the \( Ms \) strategy for high values of fine: this is a procollusive effect. Firms can also switch from \( Seq \) to \( Ms \) strategy: collusion is shorter-lasting but collusive prices are increased.

In closing, results obtained with \( \mu < 1 \) are relatively similar than those obtained for \( \mu = 1 \). First, amnesty
plus program favors the decentralization organization. Second it can deter some cartel formations. Finally pro-collusive effects can appear.

The following graphical representation illustrates these points. We set \( a = 10, \rho = 0.01, \delta = 0.8, \mu = 0.6 \) and \( \tau = -0.5 \) (the boundaries obtained for \( \tau = 0 \) are plotted with dots for comparison).

![Equilibria obtained for \( \mu = 0.6 \) and \( \tau = -0.5 \)](image)

### 6.3.2 low value of \( \mu \)

If \( \mu < \frac{1+\tau}{2} - \frac{\delta \rho (1-\tau)}{2(1-\delta)} \) and \( \tau \geq 0 \) then the \( Ms \) strategy is dominated by the \( Uc \) and \( Us \) strategies and is never selected in equilibrium.

Firms playing the \( Ms \) strategy have enough time to apply for leniency. But if \( \mu \) is low and \( \tau \) relatively high, requesting leniency is not the collectively best action. Firms not applying for leniency pay the second fine with a low probability \( \mu \) whereas if one firm applies then the second firm pays the global fine \( F \) and the first firm obtains a reduced fine. The opportunity for seeking leniency in stage (3) decreases the expected profit of firms for low values of \( \mu \). Then firms may prefer centralization since (1) they can not apply for leniency \((q_U = 0)\) and (2) the competition between the two products is removed.\(^{21}\)

In a richer model in which firms should select decentralized form without leniency program\(^{22}\) we could

\(^{21}\)These two explanations support the fact that \( Us \) is sustainable if \( Ms \) is sustainable too. Under the \( Us \) strategy, collusive prices are higher and the expected fine is lower.

\(^{22}\)Organizations theory puts in evidence some factors which drive firms to select decentralization structure. Indeed decentralization structure can increase the provisions of incentives inside the firm (Aghion et Tirole, 1995 ; Maskin, Qian et Xu, 2000),
obtain the opposite effect. If only a slight reduced fine was allowed then firms could select centralized structure in order to decrease their unilateral ability to apply for leniency.

**Effects of amnesty plus:** For low values of $\mu$, the $Uc$ strategy, if sustainable, always dominates the $Ms$ strategy. But amnesty plus program decreases the area for which $Uc$ is sustainable. Firms now compare the $Ms$ structure with $Us$. If $\tau < 0$ $Ms$ dominates the $Us$ strategy if goods are independent. We reach similar comparative static properties as in the case $\mu = 1$, a decreased value of $\tau$ may drive firms to adopt a more decentralized structure. The switch from $Uc$ to $Ms$ induced by amnesty plus reduces the duration of collusion and decreases collusive prices.

Moreover amnesty plus programs may drive firms to switch from $Uc$ to $Us$, $Us$ to $Ms$ and $Us$ to no collusion. For all these switches we obtain procompetitive effects.

But procollusive effect can appear (as in the case $\mu = 1$) since $F_{Ms}$ may shift upward: some cartels are sustainable only if amnesty plus program is introduced. For some parameter values, firms switch from the $Seq$ to the $Ms$ strategy inducing higher collusive prices (but collusion is shorter-lasting).

We illustrate these points with the following graphical representation. We set: $a = 10$, $\rho = 0.01$, $\delta = 0.8$, $\mu = 0.4$ and $\tau = -0.5$ (we also plot with dots the previous graphic obtained with $\tau = 0$).

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**Figure 8:** Equilibria obtained for $\mu = 0.4$ and $\tau = -0.5$

Can allow experiment of creative ideas on small scale (Qian, Roland et Xu, 2006) or avoid heavy workload of CEO (Spiegel, 2009).
7 Conclusion

In the present article, we describe how leniency may induce firms to select decentralized organization and then to compartmentalize hard evidence between divisions inside a firm. Doing so results in slowdown investigations of antitrust authority relating to a cartel once another has been detected. This may facilitate the application for leniency for the undetected cartel. Leniency programs can modify the choice of organizational structure and favor the adoption of a decentralized organization.

DJ (2020) obtain the opposite effect but the two models differ in key respects. This article does not invalidate the results obtained in DJ (2020) but complements them in examining another effect. An interesting topic for future research would be to mix the effects of leniency obtained in this article and in DJ (2020) into a same model but the large potential number of cases could make them difficult to interpret.

The major result of this article is that the introduction of leniency may induce firms to adopt a more decentralized structure with decreased collusive prices.

Despite a consensus regarding the utility of leniency, the designs are yet a matter of many debates. The purpose of this research is, in particular, to further knowledge of the implications of reduced fines when leniency is requested during investigations. We reach procompetitive and procollusive effects depending on the values of some parameters. The determination of optimal fine reduction for the second cartel depends on the distribution of these values and goes beyond the framework of this article but can be studied in future research.

The implications that firms may choose to compartmentalize activities to delay investigations are broader than just the analysis of collusion. Other firm behavior can be analyzed in this context, such as tax evasion or standard (environmental or other) infringement. An interesting topic for future research is to introduce the major assumption of this article into static model of self-reported crimes.

DJ (2015) suggest that firms may prefer decentralized structure in order to decrease the contagion detection probability inside multi-product firms. DJ (2020) show that leniency programs can overcome this strategy. In these two articles, the major assumption relies on the fact that the probability that the antitrust authority uncovers inculpatory evidence of several infringements when investigating only a single market decreases if firms compartmentalize agreements. In this article we eliminate this effect and focus on the investigations slowdown achievable with decentralization.

References


ROUX C and VON UNGERN-STERNBERG T (2007), Leniency programs in a multimarket setting: amnesty plus and penalty plus, *Cahiers de Recherches Economiques du Département d’économie 07.03*, Université de Lausanne, Faculté des HEC, Département d’économie.


8 Appendix A

We have to compare the threshold values obtained in the section 4.2. These values are:

\[
F_2 = \frac{\pi_i^{U} - 2\pi_i^{M}}{(q_M - q_U)(1 - \tau)(1 - \rho) \rho}; \quad F_{Ms} = \frac{2\delta (1 - \rho)^2 - 1}{2\rho + (1 - \rho)(2 - q_M + q_M \tau) \rho^2} \pi_i^{cM}; \quad F_{Us} = \frac{2\delta (1 - \rho)^2 - 1}{2\rho + (1 - \rho)(4 - q_U + \tau q_U) \rho^2} \pi_i^{cU}
\]

\[
\bar{F} = \frac{2\delta (1 - \rho)^2 - 1}{\rho} \pi_i^{cU} ; \quad F_3 = \frac{[1 - \delta (1 - \rho)^2 - 4] [1 - \delta (1 - \rho)^2 (1 - \delta + 2\delta \rho)}{[1 - \delta (1 - \rho)^2(2 + (1 - \rho)(2 - q_M + q_M \tau) - [1 - \delta (1 - \rho)^2(1 - \delta + 2\delta \rho)]}
\]

Note that \( F_{Us} \) does not depend on \( d \).

**Independent products**  If \( d = 0 \) then: \( \pi_i^{cU} = 2\pi_i^{cM} \) and \( \pi_i^{cU} = \pi_i^{cM} \). These conditions imply that:

1) \( F_2 = 0 \) then \( F_{Us} > F_2 \) if \( \delta \) is sufficiently high for \( Us \) to be sustainable.

2) \( F_{Ms} > F_{Us} \Leftrightarrow [2\rho + (1 - \rho)(4 - q_U + \tau q_U)] > [2 + (1 - \rho)(2 - q_M + q_M \tau)] \Leftrightarrow q_M > q_U \)

3) \( \bar{F} > F_{Ms} \Leftrightarrow 4\delta - 2 + q_M (1 - \tau) [1 - 2\delta (1 - \rho)] > 0 \)

And \( 4\delta - 2 + q_M (1 - \tau) [1 - 2\delta (1 - \rho)] > 4\delta - 2 + [1 - 2\delta (1 - \rho)] (1 - 2\delta (1 - \rho) < 0 \) if \( \bar{F} > 0 \)

\( 4\delta - 2 + [1 - 2\delta (1 - \rho)] = 2\delta - 1 + 2\delta \rho > 0 \) if \( \delta > \frac{1}{2} \) (necessary condition for sustainability of collusion).

We conclude that \( \bar{F} > F_{Ms} > F_{Us} > F_2 = 0 \) if \( d = 0 \).

**Substitutable products**  \((d > 0)\)

We have to compare the intersection between the border lines: \( F_{Ms} \) and \( F_{Us} \) and between \( F_{Us} \) and \( F_2 \).

\( \pi_i^{cU} \) does not depend on \( d \) and \( \pi_i^{cM} \) decreases with \( d \) implying that \( F_2 \) increases with \( d \). Moreover \( F_{Ms} \) decreases with \( d \) (\( \pi_i^{cM} \) decreases with \( d \)). We denote by \( d_1 \) and \( d_2 \) the values of product substitutability verifying the following two equalities, respectively:

\[
F_{Us} = F_2 \Leftrightarrow 2\pi_i^{cM} = \left\{ \frac{1 - 2\delta (1 - \rho)^2 - 1}{2\rho + (1 - \rho)(4 - q_M + q_M \tau)} \right\} \pi_i^{cU} \text{ for } d = d_1.
\]

\[
F_{Ms} = F_{Us} \Leftrightarrow \frac{2\delta (1 - \rho)^2 - 1}{2\rho + (1 - \rho)(2 - q_M + q_M \tau) \rho^2} 2\pi_i^{cM} = \frac{2\delta (1 - \rho)^2 - 1}{2\rho + (1 - \rho)(2 - q_M + q_M \tau) \rho^2} \pi_i^{cU} \text{ for } d = d_2
\]

\[
\frac{F_{Ms}}{F_{Us}} = \frac{2\delta (1 - \rho)^2 - 1}{2\rho + (1 - \rho)(2 - q_M + q_M \tau) \rho^2} \pi_i^{cM} = \frac{2\rho + (1 - \rho)(4 - q_U + \tau q_U) [2\pi_i^{cM}]}{2 + (1 - \rho)(2 - q_M + q_M \tau)} \pi_i^{cU} \]

For \( d = d_1 \) then:

\[
\frac{F_{Ms}}{F_{Us}} = \frac{[2\rho + (1 - \rho)(4 - q_U + \tau q_U)] - [2\delta (1 - \rho)^2 - 1]}{2 + (1 - \rho)(2 - q_M + q_M \tau)} \]

We conclude that \( d_1 < d_2 \).
Moreover \( \frac{F_3}{F_{Ms}} = \frac{(1-\delta(1-\rho))^2 \delta \left[ 1-\delta(1-\rho)^2 \right] (1-\delta+2\delta\rho) (2+d)^2}{(1-\delta(1-\rho)^2)[2+(1-\rho)(2-q_M+q_M\tau)] - [1-\delta(1-\rho)^2](1-\delta+2\delta\rho)} \) increases with \( d \) since: \( \frac{(2+d)^2}{(1+d)^2} \) decreases with \( d \). If \( F_3 > F_{Ms} \) for \( d = 0 \) then this inequality applies \( \forall d \geq 0 \). This is the case for intermediate values of \( \delta \), thus the borderline \( F_3 \) does not appear in the graph. However, for high values of \( \delta \) then \( F_3 < F_{Ms} \) if \( d = 0 \). For even higher values, \( F_3 \) goes negative. In this case the \textit{Seq} strategy is selected if \( d = 0 \) whatever the level of the fine.