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No-regret Pollution Abatement Options: A Correction of Bréchet and Jouvet (2009)*

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Abstract

In “Why environmental management may yield no-regret pollution abatement options”, Ecological Economics, 2009, Bréchet and Jouvet claim to have theoretically shown that profits maximizing firms can reduce pollution compared to laissez-faire and increase their profits. We correct multiple errors in their paper, with the conclusion that their claim no longer stands.

The empirical existence of no-regret options, also known as “win-win” opportunities, i.e., changes in the production process that decrease pollution and yield higher profits at the same time, is a highly debated topic. It is also a conceptual point of contention as it contradicts the standard assumption of rational behavior

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(profit maximization) at the firm level. If there is no free lunch, how can there be no-regret options?

**BRÉCHET and JOUVET (2009)** purport to explain how it is possible within a standard static microeconomic framework, for a profit maximizing firm. They argue that an environmental management system (EMS), that is a monitoring of the environmental consequences of the production process, transforms the environment into a production factor, which enlarges the production set and may lead to higher profits. They allegedly “show that pollution abatement is always costly but that implementing internal environmental management may lead to increases in factors’ productivity. When comparing situations with and without environmental management, a firm may gain from going green, which is called a no-regret option.” (p.1771). Counter to the conventional wisdom that opposes no-regret options and optimizing firms, Bréchet and Jouvet claim to have reconciled optimizing behaviour of firms and the existence of “win-win” opportunities that save money and reduce pollution.

The alleged possibility of no-regret options is not grounded on an existing gap between the firm’s actual operation possibilities and its production frontier (so-called X-inefficiencies, [Leibenstein (1966)]), due to, e.g., organizational barriers or imperfect information, which is a standard explanation. Bréchet and Jouvet stick to the most simple microeconomic setting, with profit-maximizing firms and no information failure, so that their proof of the existence of no-regret options, if correct, would be an important result.

Unfortunately, their derivation is marred with mathematical ambiguities and errors. It is the sole purpose of this correction to expose them and to show that, within the framework of Bréchet and Jouvet, no-regret options are impossible in
theory, contrary to their claim. We take no stance regarding the empirical existence of no-regret options.

The Stokey (1998) framework and abatement costs

Bréchet and Jouvet introduce pollution and abatement possibilities à la Stokey (1998). In this framework, for a given input vector \( X \), output is \( Y = zF(X) \), with \( F \) a standard production function, and pollution is given by \( P = \psi(z)F(X) \), with \( \psi \) an increasing continuous function of the index of technology \( z \in [0, 1] \). There is a trade-off between environment and production: “Higher values for \( z \) yield more goods but also more pollution.” (Stokey, 1998, p. 4). In this sense, there is no free lunch: the firm cannot have more output and less pollution. In a laissez-faire situation, with no regulation of pollution, a profit-maximizing firm will choose \( z = 1 \) if it has perfect information on the available technologies. A no-regret option, less pollution and more profit, is out-of-sight in this model. Yet this is precisely what Bréchet and Jouvet claim to have found. To see clearly the problem with their claim, let us reframe the question.

The possibility of no-regret options is indeed more easily discussed in the language of abatement and abatement costs. Output is \( Y = F(X)(1 - C(a)) \), that it potential output \( F(X) \) net of abatement cost \( C(a) \) (with \( C(0) = 0 \)). The pollution level is \( P = (1 - a)\sigma F(X) \) with \( a \in [0, 1] \) the abatement, i.e. the fraction of pollution reduced compared to the laissez-faire situation (\( a = 0 \)). Here no-regret options are possible if, at laissez-faire, marginal abatement costs are negative (\( C'(0) < 0 \)). In this case, the firm did not operate on its production frontier, so that reducing pollution yields higher profits. When marginal abatement costs
at laissez-faire are positive, pollution abatement is costly and there is no no-regret option.

These two modeling of abatement possibilities are somehow equivalent: translating one into another simply involves a change of variable: \( \sigma(1-a) = \psi(z) \), \( 1 - C(a) = z \). The abatements costs in the Stokey framework are thus \( C(a) = 1 - \psi^{[-1]}(\sigma(1-a)) \). With the standard assumption made by Bréchet and Jouvet that \( \psi \) is increasing, its reciprocate also is, so that \( C(a) \) is increasing: marginal abatement costs at laissez-faire are positive. We agree with Bréchet and Jouvet on this point: the Stokey framework is compatible with standard microeconomic theory in the sense that the firm operates on the production frontier. The consequence they failed to notice is that this precisely rules out no-regret options. Let us now expose the errors in their arguments.

**Errors with Proposition 1**

Under laissez-faire, the firm maximizes its profits, taking input prices \( p \) as given (see eq. (5) p.1772): \[
\max_X F(X) - p \cdot X \quad \text{(LF)}
\]

An optimal solution for this program is \( \tilde{X} \) and pollution level is \( \tilde{P} = \psi(1)F(\tilde{X}) \). Bréchet and Jouvet model the implementation of an environmental management system (EMS) as such: the firm sets itself an emission target \( \bar{P} \) below the laissez-faire pollution level \( \bar{P} \leq \tilde{P} \) and maximizes its profits with abatement possibilities modeled à la Stokey. The production set is enlarged along the \( z \) dimension.
Hence, under EMS, the maximization program now reads (see eq. (6) p.1773):

$$\max_{X,z} zF(X) - p \cdot X \quad \text{s.t.} \quad \psi(z)F(X) \leq \bar{P}, \ 0 \leq z \leq 1 \quad \text{(EMS)}$$

Their proposition 1 (p.1773) asserts that, by implementing an EMS, a firm may experience a profit increase, the upper bound of which is

$$\tilde{\Omega} := \psi(1)/\psi'(1)F(\tilde{X}).$$

(1)

It nevertheless contradicts the following trivial result.

**Proposition.** *If a solution to (LF) exists, then the given profit is greater or equal to any profit obtained in (EMS).*

**Proof.** Consider any feasible inputs $X, z$ of the program \((\text{EMS})\). Then:

$$zF(X) - p \cdot X \leq F(X) - p \cdot X \leq F(\tilde{X}) - p \cdot \tilde{X}$$

The first inequality holds because $z \leq 1$ and the second because any feasible input of the constrained program \((\text{EMS})\) is also a feasible input of the unconstrained \textit{laissez-faire} program \((\text{LF})\), so the profits it yields are lower than profits at an optimal solution of \((\text{LF})\).

Trivially and as expected, profits after implementing an EMS are always lower than in the \textit{laissez-faire} situation, contrary to proposition 1 of Bréchet and Jouvet. Let us review their demonstration.

They write the first-order conditions (FOC) of the maximization program\(^1\) (see

\(^1\)The careful reader of \text{BRÉCHET and JOUVET (2009)} would have noticed that, compared to
(8) p.1773) with respect to input maximization, denoting \(\mu\) the Lagrange multiplier for the constraint \(\psi(z) F(X) \leq \bar{P}\):

\[
(z - \mu \psi(z)) F_{X_i}(X) = p_i
\]

(2)

Then, they comment on the FOC and twice call for ‘values of \(\mu\) compatible with’ the pollution level. They seem to imply that the value of \(\mu\) can be arbitrarily chosen in the interval \([0, \hat{\mu}]\), with \(\hat{\mu} = 1/\psi'(1)\) and that for each \(\mu\) there exists a feasible input \(X, z\) that is solution of the FOC. This obscure passage ends with the statement of Proposition 1, where they neither explicate how can the firm gain nor how they compute \(\hat{\Omega}\) of eq. (1).

We can interpolate their argument. Their discussion suggests that \(\tilde{X}, z = 1\) is a solution of (2) for any \(\mu\) in \([0, \hat{\mu}]\) (“As long as \(0 \leq \mu \leq \hat{\mu}, z = 1\) and all factors’ levels remain the same as under laissez-faire, \(\tilde{X}\),” p. 1773). Then profits at this alleged solution would be:

\[
z F(\tilde{X}) - \sum_i p_i \tilde{X}_i = F(\tilde{X}) - (1 - \mu \psi(1)) \sum_i F_{X_i} \tilde{X}_i = F(\tilde{X}) - (1 - \mu \psi(1)) F(\tilde{X}) = \mu \psi(1) F(\tilde{X}),
\]

(3)

where we have used \(\sum_i F_{X_i} \tilde{X}_i = F(\tilde{X})\), since the production function \(F(X)\) is assumed homogeneous \(^2\) of degree 1 (p.1772). Taking the “highest value of \(\mu\) our (EMS), their maximization program (6) does not have the condition \(z \leq 1\). Yet it is taken into account in FOC (9) p.1773. Note that there is a typo there since the condition for \(z = 1\) should read \(1 - \mu \psi(z) \geq 0\). 

\(^2\)We have not insisted earlier on the homogeneity of degree 1 of \(F\) as it is not essential for discussing the possibility of no-regret options and it only enters the erroneous calculation of \(\hat{\Omega}\). Yet,
compatible”, i.e. $\tilde{\mu}$ according to Bréchet and Jouvet, would yield the result $\tilde{\Omega}$. To be clear, all this is conditional on $\tilde{X}, z = 1$ being a solution of (2) for $\mu = \tilde{\mu}$.

How have Bréchet and Jouvet gone astray? One knows that a profit-maximizing price-taking firm with a production function of degree 1 can only reach a profit which is either $+\infty$ or 0. Under the assumption that a solution to the (LF) program exists, only the second value is appropriate. Thus, the left-hand-side of (3) is at most nil, and this actually shows that a solution to (2) can only exist if $\mu = 0$. Speaking of “values of $\mu$ compatible with”, a strange turn of phrase, misled Bréchet and Jouvet as it suggests that $\mu$ can be freely chosen, which is not the case. As a consequence, the upper bound in profit increase $\tilde{\Omega}$ in Proposition 1 is based on a solution that does not exist.

The remaining theoretical part of Bréchet and Jouvet (2009) attempts to formulate the no-regret option value (Proposition 2, p.1773), that is the difference between the potential benefit raised by the environmental management and the abatement cost. But as it relies on Proposition 1 which is no longer valid, this notion is vacuous. Contrary to their statement, abating pollution (choosing a $z$ strictly below 1) reduces profits for any non-zero output.

Errors with the Cobb-Douglas example

Finally, Bréchet and Jouvet present an example of a firm with a Cobb-Douglas production function $F(K, L) = AK^\alpha L^{1-\alpha}$ in the laissez-faire situation. We follow them to illustrate how their demonstration goes wrong.

It is surprising that Bréchet and Jouvet intend to tackle pollution reduction with such a production function, since the firm can reach its maximum (but nil) profit at any scale of production. It can scale down its output to reach any desired level of pollution, while remaining at its optimal (zero) profit. Closing down the firm is the true no-regret option here.
Bréchet and Jouvet avoid elucidating the optimal profit and the optimal input vector $\tilde{K}, \tilde{L}$ under laissez-faire: the former is indeed null while the latter is under-determined, only satisfying $\frac{r\tilde{K}}{w\tilde{L} + r\tilde{K}} = \alpha$, where $r, w > 0$ are the interest and wage rates. Note that the existence of an optimal non-zero solution to the laissez-faire maximization problem implies a necessary relationship between interest rate and wage rate:

$$F\left(\frac{\alpha}{r}, \frac{1-\alpha}{w}\right) = 1.$$  

With an EMS, in accordance with the Stokey framework, the pollution is a function of the technology index $z$, $P = \psi(z)F(K, L)$, with a specified increasing function $\psi(z) = \varphi z^\beta + 1$ ($\beta > 0$). The production function becomes $zF(K, L)$. However, Bréchet and Jouvet make a normally benign move: they change the decision variable and use the pollution level $P$ instead of the technology index $z$. Since $P = \psi(z)F(K, L)$ or $z = \psi^{-1}\left(\frac{P}{F(K, L)}\right)$, the production function, now denoted $\Phi$, becomes:

$$\Phi(K, L, P) = \psi^{-1}\left(\frac{P}{F(K, L)}\right) F(K, L).$$

Bréchet and Jouvet state that the solution of the maximization problem of the firm under EMS, with pollution level fixed at $\bar{P}$ is

$$K^* = \frac{\alpha \beta}{1 + \beta} \frac{\Phi(K^*, L^*, \bar{P})}{r}, \quad \text{and} \quad L^* = \frac{(1 - \alpha) \beta}{1 + \beta} \frac{\Phi(K^*, L^*, \bar{P})}{w}. \quad (4)$$

These inputs deliver a profit $\Phi(K^*, L^*, \bar{P}) - r.K^* - w.L^* = \frac{1}{1+\beta} \Phi(K^*, L^*, \bar{P})$, which is positive.

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\[3\] Indeed, first-order conditions and homogeneity of degree 1 implies via the Euler theorem: $F(K, L) = r.K + w.L$. For a non-zero solution, this can be re-written as: $F\left(\frac{rK}{r.K + w.L}, \frac{L}{r.K + w.L}\right) = 1$. Since, $\frac{rK}{r.K + w.L} = \alpha$ and $\frac{wL}{r.K + w.L} = 1 - \alpha$, this yields the desired relation.
This is wrong since we know from the general case that profits are at most nil for a homogeneous production function and that this maximum is obtained by $z = 1$. The problem here stems from the fact Bréchet and Jouvet lost tracks of the conditions on the technology index $z$ when they switched from production function $zF$ to $\Phi$ by making pollution level $P$ the decision variable of the firm. When they maximised $\Phi$, they forgot the crucial condition that $z \leq 1$, or equivalently, after the change in decision variable, $\Phi(K, L, P)/F(K, L) \leq 1$.

Indeed, one can see that the technology index $z^*$ implied by their solution is above 1, that is outside the allowed range. Let us compute $z^*$:

$$z^* = \frac{\Phi(K^*, L^*, \tilde{P})}{F(K^*, L^*)} = \frac{\Phi(K^*, L^*, \tilde{P})}{F\left(\frac{\alpha \beta \Phi(K^*, L^*, \tilde{P})}{1 + \beta}, \frac{(1 - \alpha) \beta \Phi(K^*, L^*, \tilde{P})}{1 + \beta}\right)} = 1 + \frac{\beta}{1 + \beta} > 1$$

At the second line we have used (4), at the third line, homogeneity of degree 1 of the function $F$, and at the fourth, the necessary relationship relating interest rate and wage rate.

At their “optimal” solution, the technology index $z$ is outside the allowed range and this is the only reason why profits are positive. Increasing $z$ above 1 keeps costs constant, but increases output, yielding positive profits and more

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Had they maximised the function $G = \min(\Phi, F)$, see p.1772, which is valid over the whole range of input, they would have obtained the correct result.
pollution. Since the firm can operate at any production level, output is scaled so that pollution level is equal to $\bar{P}$. This adapts profits proportionally but keeps them positive.

Recall how the Stokey framework looks like in terms of abatement costs: we have positive marginal abatement costs at the origin. The no-regret option found by Bréchet and Jouvet in their Cobb-Douglas example actually involves choosing negative abatement (i.e. pollution increase) to secure positive profits.

**Conclusion**

Bréchet and Jouvet (2009) allegedly proved that no-regret options are theoretically possible in a framework inspired by Stokey (1998) with maximizing firms and no information failure. We have found and explained errors in their argument, which refute the claim on which their article is based.

**References**

