



HAL
open science

Neurofunctional plasticity in fraction learning: An fMRI training study

Silke Wortha, Johannes Bloechle, Manuel Ninaus, Kristian Kiili, Antero Lindstedt, Julia Bahnmüller, Korbinian Moeller, Elise Klein

► To cite this version:

Silke Wortha, Johannes Bloechle, Manuel Ninaus, Kristian Kiili, Antero Lindstedt, et al.. Neurofunctional plasticity in fraction learning: An fMRI training study. *Trends in Neuroscience and Education*, 2020, 21, pp.100141. 10.1016/j.tine.2020.100141 . halshs-02947077

HAL Id: halshs-02947077

<https://shs.hal.science/halshs-02947077>

Submitted on 25 Nov 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 Neurofunctional plasticity in fraction learning: an fMRI training study

2
3 Silke M. Wortha^{1,2}, Johannes Bloechle³, Manuel Ninaus^{1,4}, Kristian Kiili⁵, Antero Lindstedt⁶, Julia
4 Bahnmueller^{1,7}, Korbinian Moeller^{1,4,7,8} & Elise Klein^{4,9}

5 ¹ LEAD Graduate School & Research Network, University of Tuebingen, Tuebingen, Germany

6 ² Department of Neurology, Universitätsmedizin Greifswald, Greifswald, Germany

7 ³ Department of Psychiatry and Psychotherapy, University of Tuebingen, Tuebingen, Germany

8 ⁴ Leibniz-Institut für Wissensmedien, Tuebingen, Germany

9 ⁵ Faculty of Education and Culture, Tampere University, Tampere, Finland

10 ⁶ Faculty of Information Technology and Communication Sciences, Tampere University, Pori, Finland

11 ⁷ Centre for Mathematical Cognition, School of Science, Loughborough University, United Kingdom

12 ⁸ Individual Development and Adaptive Education Center, Frankfurt am Main, Germany

13
14 ⁹ Université de Paris, LaPsyDÉ, CNRS, Sorbonne Paris Cité, Paris, France
15
16
17
18
19

20 Corresponding author:

21 Silke M. Wortha
22 Department of Neurology,
23 Universitätsmedizin Greifswald,
24 Ferdinand-Sauerbruch-Straße,
25 17475 Greifswald
26 Germany

27
28 silkemaria.wortha@med.uni-greifswald.de
29
30
31
32
33
34
35
36
37

Abstract

38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57

Background: Fractions are known to be difficult for children and adults. Behavioral studies suggest that magnitude processing of fractions can be improved via number line estimation (NLE) trainings, but little is known about the neural correlates of fraction learning.

Method: To examine the neuro-cognitive foundations of fraction learning, behavioral performance and neural correlates were measured before and after a five-day NLE training.

Results: In all evaluation tasks behavioral performance increased after training. We observed a fronto-parietal network associated with number magnitude processing to be recruited in all tasks as indicated by a numerical distance effect. For symbolic fractions, the distance effect on intraparietal activation was only observed after training.

Conclusion: The absence of a distance effect of symbolic fractions before the training could indicate an initially less automatic access to their overall magnitude. NLE training facilitates processing of overall fraction magnitude as indicated by the distance effect in neural activation.

Keywords: fraction processing, number line estimation training, flow experience, numerical distance effect, fMRI

Word count: 150

Introduction

58
59 Over the last decade, research has shown repeatedly that understanding fractions is a
60 crucial predictor of future achievement in higher mathematics [1–3]. However, despite intense
61 research efforts in this area, children’s poor performance when it comes to handling and
62 understanding fractions hardly changed [4–7]. For example, in 2015 Lortie-Forgues and
63 colleagues found that only 27% of 8th graders in the United States were able to choose correctly
64 the number closest to the result of a fraction addition problem out of four given solution probes
65 [8]. A similar result was already reported in 1978 by the National Assessment of Educational
66 Progress [9], when only 24% of the 8th graders chose the correct answer to the same question.
67 Importantly, these difficulties in understanding fractions may be persisting regardless of
68 educational efforts because - unlike natural numbers - fraction magnitude processing seems to
69 be more difficult due to its bipartite structure reflecting the relative relation of numerator and
70 denominator [10]. According to the integrated theory of numerical development, magnitude
71 information is the crucial basis for understanding numbers. Moreover, the understanding that
72 all real numbers (e.g., natural numbers, integers, rational numbers) can be represented on a
73 number line is a key assumption for numerical learning. Therefore, promoting fraction
74 magnitude understanding seems crucial for fostering fraction understanding more generally
75 [11–13]. Thus, interventions with the aim to improve fraction understanding and therefore
76 conceptual knowledge of fractions should focus on fostering mastery in processing and
77 representing fraction magnitude. In the context of (fraction) magnitude understanding number
78 lines have been shown to be a beneficial instructional tool [14].

79 Against this background, we aimed at understanding the neuro-cognitive foundations
80 underlying successful fraction learning and their plasticity. For this purpose, participants had to
81 complete a number line estimation training and a flow questionnaire on five consecutive days.
82 In the following we will highlight the most important research results from the research areas
83 which are relevant for our study. First, we will give a brief overview about the relevance of

84 number line estimation training for fraction magnitude understanding. Second, we will
85 introduce flow as a state which is beneficial for learning and especially for fraction learning.
86 Third, we will summarize present knowledge about the neural correlates of fraction processing
87 and highlight the importance of our study in this context. Finally, we will introduce the aim of
88 the current study including our specific hypothesis.

89 *Number line estimation as predictor for fraction magnitude learning*

90 The mental number line is a metaphor for the nature of the number magnitude
91 representation whereby numbers are represented spatially with their magnitude increasing from
92 left to right (at least in Western cultures [15]). In numerical cognition research, number line
93 estimation (henceforth NLE) is used repeatedly to assess number magnitude understanding –
94 especially in children ([16–18], but see [19] for additional aspects). In the NLE task,
95 participants have to indicate the spatial position of a target number on a given number line for
96 which only start- and endpoint are specified [18].

97 As magnitude is the semantic core for any type of number, the task can not only be used
98 to assess, but also to train fraction magnitude understanding [12]. For instance, Hamdan &
99 Gunderson [20] conducted a training study with three conditions for fraction learning (i.e.,
100 number line estimation training, area model training, and a non-numerical control). They
101 observed that although children in both the NLE training and the area model training improved
102 in the respective tasks, only children completing the NLE training showed transfer effects to a
103 not trained magnitude comparison task with fractions.

104 Moreover, Barbieri and colleagues [21] used a number line-based intervention to
105 improve fraction understanding in children with poor conceptual knowledge of fractions and
106 compared the number line intervention group to a standard mathematics curriculum group. The
107 number line intervention group showed significantly larger learning gains than the control
108 group. Finally, computerized and game-based versions of the NLE task were used successfully

109 to assess and improve children's fraction understanding [22–24]. Taken together this
110 substantiates that number lines are a powerful instructional tool and the NLE task can be applied
111 successfully to foster fraction (magnitude) understanding. However, successful fraction
112 magnitude learning might not only depend on improving conceptual knowledge of fractions,
113 but also on a more fundamental ability to be able to reach a beneficial cognitive state for
114 learning which is known as flow experience.

115 *Flow experience as an indicator for optimal learning*

116 Learning is not a pure cognitive process but is affected by motivation and emotions [25].
117 A beneficial factor for learning that is considered specifically in computerized approaches on
118 learning is the *flow experience* of the learner [26]. Flow was first coined by Csikszentmihalyi
119 [27] and can be described as a positive emotional state [28,29] and as a holistic approach to
120 motivation [30]. In particular, flow is characterized by a combination of factors such as
121 increased concentration, reduced self-consciousness, sense of control, that are experienced as
122 intrinsically rewarding [29]. Flow is usually reached when task demands meet personal skills
123 or resources in a balanced way. Thus, when the skills of the learner are too low for the demands
124 of a given task – for instance at the beginning of a training – flow experience is rather low. The
125 same is true when the skills of the learner are too high for a given task which leads to boredom
126 and reduced flow experience. Therefore, flow experience seems to be an optimal state for
127 intrinsically motivated learning, which helps focus on the given task and can lead to improved
128 performance [31]. This is further supported by studies on the neural correlates of flow
129 experience. These studies could show that flow experience is associated with increased
130 activation in areas of the multiple demand system such as inferior frontal gyrus, putamen and
131 anterior insula and decreased activation in areas typically associated with the default mode
132 network such as amygdala, medial prefrontal cortex and posterior cingulate cortex [32,33].

133 Flow experience was specifically, but not solely considered in different computer-based
134 learning settings. For instance, in game-based learning (for a review see [34]), collaborative
135 learning of problem-solving in virtual environments [35], hypermedia learning [36], e-learning
136 [37], but also in creative processes like music learning [38,39]. As such, flow experience should
137 also be beneficial for fostering fraction magnitude understanding using a computerized NLE
138 training. However, successful fraction magnitude learning might not only depend on improving
139 conceptual knowledge of fractions and on the learners' flow experience, but also on the
140 successful interplay of certain neural correlates underlying the neuro-cognitive foundations of
141 fraction learning. Therefore, in the following the current state of research on neural correlates
142 of fractional learning is briefly outlined.

143

144 Neural correlates of fraction and proportion magnitude processing

145 Despite above described established relevance of fraction knowledge and longstanding
146 research in educational sciences and psychology, little is still known about the neural
147 mechanisms underlying the processing of proportions and fractions in general and the neural
148 correlates of fraction learning in particular. To date, there are only few neuroimaging studies
149 investigating the neural correlates of processing proportions [40–42] and fractions [43–46] in
150 adults.

151 One important aspect across most studies is that the numerical distance effect was used
152 as an indicator of automatic processing of overall fraction magnitude. The numerical distance
153 effect [47] describes the finding that two numbers are compared faster and more accurately the
154 larger the numerical distance between them (i.e., the farther apart they are on the mental number
155 line, e.g. 3 and 7 is easier to compare than 3 and 4). For instance, for fraction magnitude
156 comparison, Ischebeck et al. [45] observed that neural activation within the right IPS was
157 modulated by the overall numerical distance between the to-be-compared fractions (e.g.,

158 numerical distance between $\frac{2}{4}$ and $\frac{3}{7}$), but not by the numerical distance between numerators
159 or denominators (i.e., numerical distance between 2 and 3 for numerators and numerical
160 distance between 4 and 7 for denominators when comparing $\frac{2}{4}$ and $\frac{2}{7}$). Moreover, Mock and
161 colleagues [41] observed a joint neural correlate of specific occipito-parietal activation
162 including the right intraparietal sulcus (IPS) for the processing of different notations of
163 proportions including not only fractions, but also pie charts, dot patterns and decimals.

164 Finally, Klabunde et al. [48] conducted a first fMRI training study on proportions in
165 participants with fragile X syndrome and a control group with intellectual disabilities.
166 Participants were trained for two days in 10 min sessions until they were able to have over 80%
167 accuracy on matching fractions to pie charts and pie charts to decimals. Neurofunctional
168 changes from before to after the training indicated significantly increased brain activation in
169 the left inferior parietal lobule, left postcentral gyrus, and left insula for both groups. However,
170 the mechanism of interest in this study was not the distance effect but to investigate neural
171 correlates of stimulus equivalence relations.

172 In summary, previous studies indicate that the distance effect for overall magnitude of
173 the to-be-compared proportions/fractions seems a good measure reflecting automatic
174 processing of overall fraction magnitude. As such, the numerical distance effect as a hallmark
175 effect for magnitude processing indicated that the (right) IPS seems to play a crucial role in the
176 processing of proportion and fraction magnitude independent of the actual task, which is in line
177 with its involvement in the processing of natural number magnitude [49]. As such, one might
178 argue that the presence of a numerical distance effect seems to indicate automatic processing
179 of overall fraction magnitude in proficient fraction processing (see [50] for a similar argument
180 on the relation between distance effect and arithmetic performance), while absence of a
181 numerical distance effect might indicate that the fractions' magnitude is not automatically
182 accessed.

183

184 The present study

185 Against this background, we aimed at investigating neuro-functional correlates and their
186 plasticity associated with an NLE training of fraction magnitude understanding. In particular,
187 we evaluated changes in fraction magnitude processing on the neural level as reflected by the
188 numerical distance effect for overall fraction magnitude. To the best of our knowledge, this is
189 the first study investigating the neural correlates of fraction learning through a NLE training in
190 healthy adult participants. We assessed neural activation associated with fraction magnitude
191 processing using fMRI before and after a five-day consecutive NLE training on fractions.
192 Additionally, we assessed participants' flow experience in each training session.

193 Similar to previous studies applying NLE training to children, we expected the training
194 to improve participants' conceptual knowledge of fraction magnitude on a behavioral level
195 [20,21]. In particular, we expected significant improvements in NLE performance over the five-
196 day training sessions. Additionally, we expected participants flow experience to be associated
197 with NLE training improvements over the five-day training. Finally, on the neural level, we
198 expected significant changes of IPS activation associated with the numerical distance effect
199 from pre- to post fMRI session indicating more automatic processing of fractions' overall
200 magnitude after the NLE training. This should become evident by a more pronounced numerical
201 distance effect in behavioral measures but also neural activation in IPS after the training.

202

203

Methods

204 **Participants**

205 48 right-handed adult participants ($M = 23.73$, $SD = 3.65$, female = 32) took part in this
206 fMRI study. All participants were German native speakers with normal or corrected to normal
207 vision and reported no history of psychiatric or neurological disorders or drug abuse. The study
208 was approved by the local Ethics Committee of the Medical Faculty of the University of
209 Tübingen. Participants gave written informed consent and received monetary compensation for
210 their participation.

211 **Study Design**

212 The study was designed as a pre-post fMRI training study with five consecutive days of
213 training between the fMRI measurements. On the first day before the training and on the last
214 day after the training, participants had to complete two different magnitude comparison tasks
215 (i.e., comparison of symbolic fractions and comparison of line proportions, respectively) and a
216 fraction-line proportion matching task (i.e., indicating whether the magnitude reflected by a
217 fraction matched that of a line proportion or not) while their brain activation was measured
218 using fMRI (see Figure 1). In addition to these computerized tasks, participants also completed
219 a paper-pencil-based NLE task prior to entering the scanner for pre- and post-test measurement
220 (for more details see below). Due to technical problems, behavioral data of fMRI measurements
221 could only be obtained from 24 right-handed adult participants ($M = 22.50$, $SD = 3.76$, female
222 = 16). Imaging results did not differ substantially between the participant group with and
223 without behavioral data (i.e., no suprathreshold clusters at an uncorrected $p < .001$ and cluster
224 size of 10). Therefore, imaging results as well as all behavioral data obtained outside the scanner
225 (i.e., training data, flow experience and paper-pencil-based number line estimation task) will be
226 reported for all 48 participants, while behavioral fMRI results are reported for the respective 24
227 participants only.

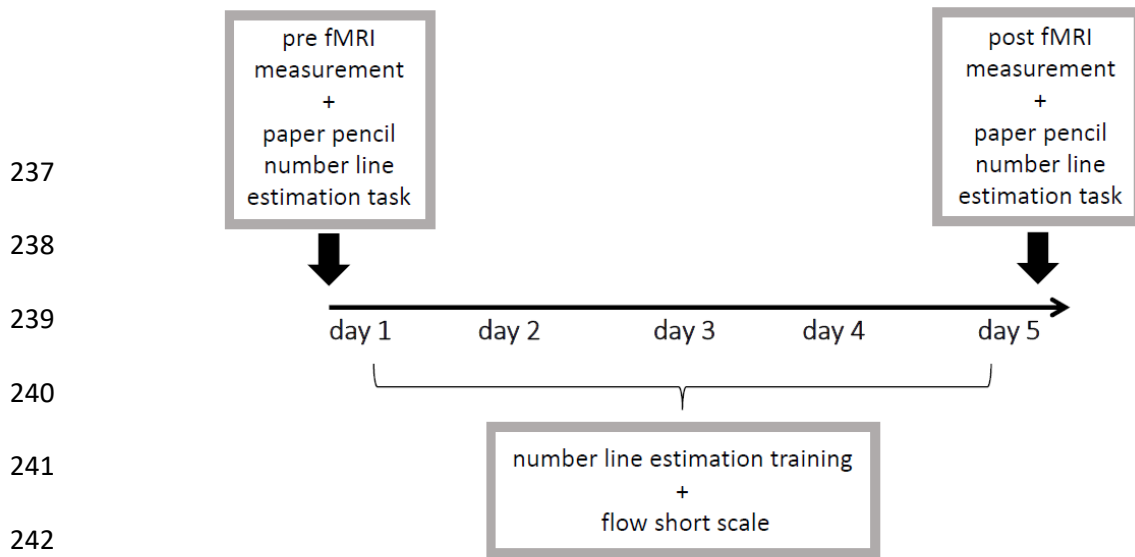
228 The training consisted of a fraction number line estimation task. It took place outside
229 the scanner and each training session lasted around 15-20 minutes depending on participants
230 individual performance. After each training session participants completed a brief questionnaire
231 on mental flow to evaluate possible changes in flow experience over the training period (for
232 more details on the flow questionnaire see below).

233

234

235

236



243
 244 **Fig. 1: Study design.** Before and after a five-day number line estimation training fMRI measurements were
 245 conducted to evaluate neurofunctional changes in brain activation through the training. After each training session
 246 flow was assessed using the flow short scale [51]. Prior to entering the scanner for the pre and post measurement
 247 a paper-pencil number line estimation task was administered.
 248

249 **Item sets and stimuli**

250 In this study, two different item sets (one to be trained and one not to be trained) were
 251 used. The order of sets was counterbalanced across participants so that whichever set a
 252 participant did not train on served as the untrained set at the pre- and post-test. However, each
 253 participant was tested during the fMRI sessions on both item sets. Each item set consisted of
 254 48 stimuli with items always presented in randomized order. To ensure comparable difficulty
 255 of item sets, they were matched on overall problem size and numerical distance between
 256 fraction pairs used. All fractions used in the stimuli sets were proper fractions with nominators
 257 and denominators ranging from 1 to 30. To evaluate neurofunctional changes in fraction
 258 magnitude processing, pre-post-test evaluation tasks performed in the fMRI scanner included
 259 both trained and untrained items. Items of the two item sets can be found in the Appendix.

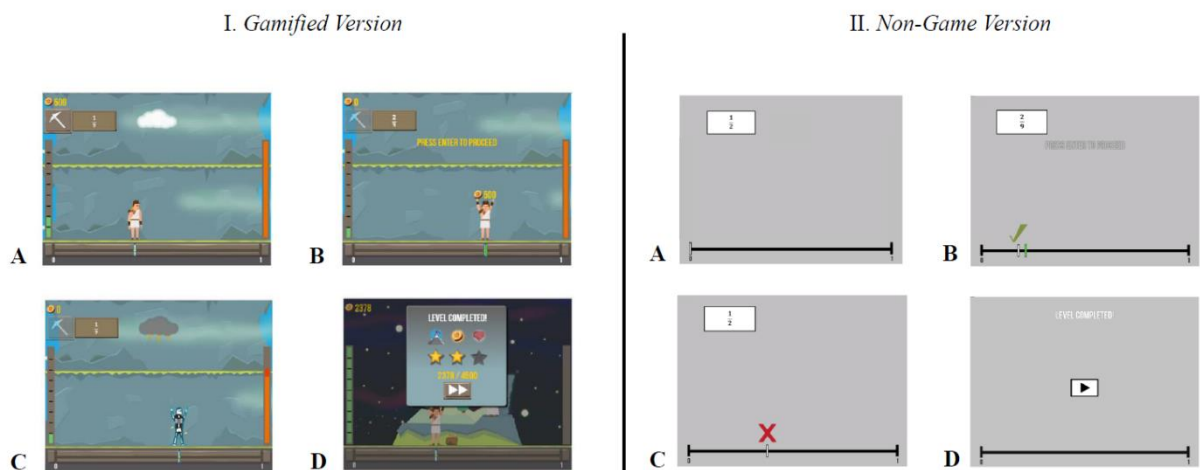
260 **Training Procedure**

261 For the training, we used two variations of a fraction number line estimation (NLE)
 262 training with feedback, which differed in appearance and framing, but not in numerical content
 263 and task. In particular, half of the participants were trained using an NLE task set within a
 264 *gamified* environment (see Figure 2), while the other half were trained in the same NLE task in
 265 a *non-game* environment (see Figure 2). For the gamified NLE task we used the *Semideus*

266 research engine, which was already applied successfully in previous studies for assessing and
 267 training fraction knowledge [23,24,52–54].

268 In both versions of the NLE task, participants had to indicate the correct position of a
 269 given fraction on a number line ranging from 0 to 1 by maneuvering an avatar in the gamified
 270 version and moving a white slider along the number line using the left and right arrow keys of
 271 a computer keyboard in the non-game version. After reaching the estimated correct position,
 272 participants had to press the space bar to select that position. Participants were instructed to
 273 indicate the right position as quickly and accurately as possible within a time limit of 10
 274 seconds. They received positive feedback (i.e., cheering avatar plus coins awarded in the
 275 gamified vs. green checkmark in the non-game version) when their answer fell within a range
 276 $\pm 5\%$ around the correct position. In case they failed to answer or did not answer accurately
 277 enough, negative feedback was given (i.e., avatar struck by lightning plus loss of virtual energy
 278 in the gamified vs. red cross shown in the non-game version) and participants had to repeat the
 279 item until it was correctly solved within the $\pm 5\%$ range. At each training session, participants
 280 worked through 96 items in 12 runs containing 8 items each. Each item from the trained
 281 stimulus set was encountered twice within a training session. Items were presented in
 282 randomized order and were identical in both versions of the NLE training.

283



284

285 **Fig. 2: Examples of different stages of the gamified (I.) and non-game version (II.) of the NLE training.**
286 I. *Gamified version*: (A) Beginning of a new trial. A fraction is shown in the left upper corner and the avatar
287 Semideus has to be moved on a number line between 0 and 1 towards the anticipated position on the number line
288 of the shown fraction. (B) The result of a successful trial. If the position of the fraction is estimated correctly
289 (tolerated range: $\pm 5\%$) Semideus is rewarded with coins. (C) The result of a failed trial. In case the position of the
290 fraction is not estimated correctly Semideus is struck by lightning and the participant needs to try again. (D)
291 Completed level with feedback. Mountain: For completing the level, they earned one star; Coin: for collecting at
292 least 2000 points they earned a second star; Heart: for maintaining at least 80% of the life points, they earned a
293 third star. II. *Non-game version*: (A) Beginning of a new trial. A fraction is shown in the left upper corner and the
294 participant has to move the white slider on a number line between 0 and 1 towards the anticipated position on the
295 number line of the shown fraction. (B) The result of a successful trial. In case the position of the fraction is
296 estimated correctly (tolerated range: $\pm 5\%$) a green check mark appears. (C) The result of a failed trial. If the
297 position of the fraction is not estimated correctly a red cross appears and the participant must try again. (D)
298 Completed level.
299

300 **Flow Short Scale**

301 Flow was assessed using the German version of the flow short scale [51]. This
302 questionnaire consists of 16 items. Thirteen items are associated with the flow scale (7-point
303 Likert-scale ranging from 1 = totally disagree to 7 = totally agree), which has a three-
304 dimensional structure: The first dimension assessed by the scale is *fluency of performance* (6
305 items, i.e., “My thoughts or activities run fluently and smoothly”, $\alpha = .92$). The second
306 dimension is *absorption by activity* (4 items, i.e., “I’m completely focused on what I’m doing
307 right now”, $\alpha = .80$). Finally, the third dimension is *perceived importance* or concern (3 items,
308 i.e., “I’m worried about failure”, $\alpha = .90$).

309 Additionally, the questionnaire includes 3 items of a demand scale (10-point Likert-
310 scale ranging from 1 = easy to 10 = difficult), which aims at assessing how demanding the
311 current task was for the participant (i.e., “For me personally, the current requirements are....”).
312 To monitor flow experience over the course of the training, participants had to complete the
313 flow short scale following each completed training session resulting in five completed flow
314 short scales per participant.

315 **Paper pencil number line estimation task outside the fMRI scanner**

316 In addition to the computerized in-scanner tasks (see below) participants had to
317 complete a paper pencil version of a number line estimation task prior to both fMRI
318 measurements (pre and post training), respectively. The number line ranged from 0 to 1 with

319 only the endpoints specified and participants had to indicate the spatial position on the number
320 line for all items from both item sets (i.e., the trained and the untrained set, thus 96 items in
321 total prior to and after the training). This task allowed to evaluate potential improvements in
322 spatial localization of fractions on the number line through the training.

323 **Tasks performed inside the fMRI scanner**

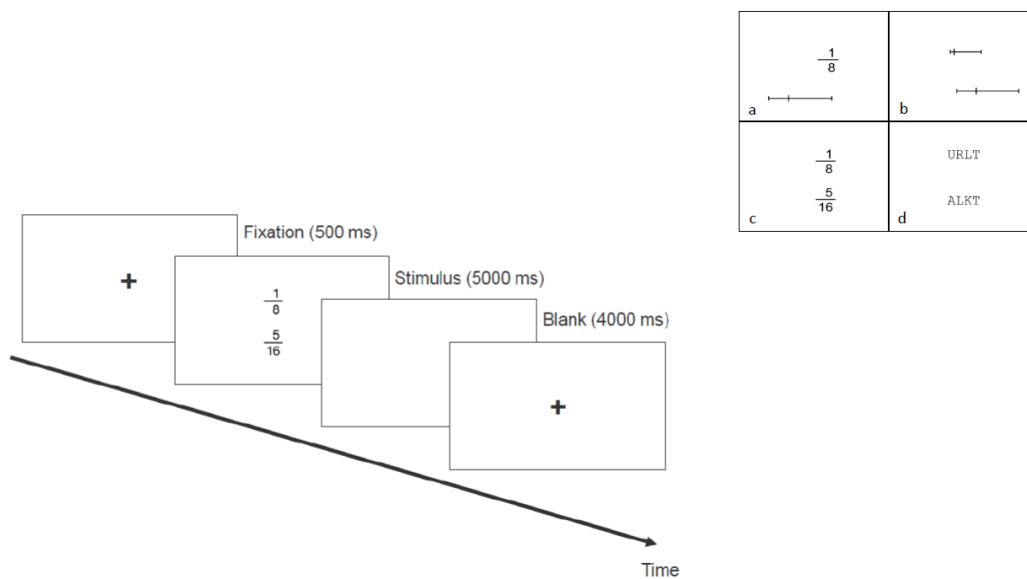
324 For the fMRI paradigm a block design was used, with four separate blocks for the three
325 different tasks (i.e., fraction-line proportion matching task, line proportion comparison task and
326 symbolic fraction magnitude comparison task). For each item in the fraction-line proportion
327 matching task we presented a matching (i.e., magnitude of symbolic fraction and line proportion
328 were identical) and a non-matching version (i.e., magnitude of symbolic fraction and line
329 proportion were not identical) in the fraction-line proportion matching task. Therefore, this task
330 took twice as long than both comparison tasks and was run in two blocks.

331 In the fraction-line proportion matching task, participants were shown a fraction and a
332 line proportion (see Figure 3a). They had to indicate whether the fraction and the line proportion
333 reflected the same magnitude or not (i.e., identical: left button, different: right button). In half
334 of the items, magnitudes of fraction and line proportion matched. In the line proportion
335 comparison task (see Figure 3b), participants were shown two line proportions and had to
336 decide which proportion was the numerically larger one by pressing a corresponding response
337 button (i.e., the right button when the upper and the left button when the lower line proportion
338 was larger). Similar, in the symbolic fraction magnitude comparison task (see Figure 3c),
339 participants were shown two fractions and had to decide which fraction was numerically larger
340 again.

341 Each block consisted of 4 practice followed by 48 critical trials. In both sessions, half
342 of the critical trials consisted of trained stimuli while the other half were untrained stimuli.
343 Stimulus order was random for each participant and each session. Additionally, 22 trials of a

344 scrambled word task were randomly interspersed in each block of each condition to control for
 345 eye-movements and to control that participants would stay focused. During these trials, two
 346 strings of scrambled letters were shown on top of each other and participants had to decide
 347 which of the strings would form a real word (see Figure 3d).

348 To prevent adaptation of the BOLD signal, 6000 ms pauses (white screen, RGB values
 349 = 255 255 255) were randomly interspersed between trials. All stimuli were projected on a
 350 screen outside the scanner and made visible to participants through a mirror mounted on the
 351 head coil of the scanner. Foam pads were used to minimize head movements within the head
 352 coil during fMRI acquisition. Stimuli were presented in black font against a white background
 353 (RGB values = 255 255 255). The experiment was programmed using Presentation® v16.1
 354 software (www.neurobs.com).



355

356 **Fig. 3: Experimental procedure with examples (upper right box) for the different tasks.** Example for a) the
 357 fraction-line proportion matching task, b) the line proportion comparison task, c) the symbolic fraction magnitude
 358 comparison task, and d) the scrambled letters control task. In this example the lower four letters can be unscrambled
 359 to form the word “kalt” (German word for “cold”). The upper four letters cannot be formed to any word used in
 360 German.

361

362 Each trial started with a fixation cross (500 ms), followed by the respective stimulus
363 which appeared for 5000 ms or until participants responded. Subsequently, a blank screen
364 appeared for 4000 ms followed by the beginning of a new trial (see Figure 3). Participants
365 responded by pressing one of two MRI compatible response buttons with either their left or
366 their right thumb. Participants were instructed to answer as fast and as accurately as possible.

367

368 **MRI and fMRI Acquisition**

369 A high-resolution T1-weighted anatomical scan was acquired by a 3T Siemens
370 Magnetom Prisma MRI system (Siemens AG; Erlangen, Germany) equipped with a 64-channel
371 head-neck matrix coil (TR = 2400 s, matrix = 256×256 , 176 slices, voxel size = $1.0 \times 1.0 \times$
372 1.0 mm^3 ; FOV = 256 mm, TE = 2.92 ms; flip angle = 8°). The anatomical scan was always
373 performed at the end of each experimental session.

374 Functional T2*-weighted images were obtained using multiband gradient-echo Echo
375 planar imaging (EPI; TR = 792 ms; TE = 30 ms; flip angle = 58° ; FOV = 192 mm, 64×64
376 matrix; 48 slices, voxel size = $3.0 \times 3.0 \times 3.0 \text{ mm}^3$). Total scanning time was approximately 80
377 minutes. A baseline (rest) condition was accomplished by including about 20% null events in
378 the paradigm.

379 **Analysis**

380 **Preliminary Analysis**

381 Prior to the analysis of the behavioral and imaging data of the present study, possible
382 differences between the two variants of NLE training (*gamified* vs. *non game-based* training)
383 were examined both on the behavioral as well as the neuro-functional level. However, the
384 behavioral analysis after the training showed no significant differences in reaction times or error
385 rates for all three evaluation tasks performed in the scanner (i.e., symbolic fraction magnitude
386 comparison, line proportion comparison, and fraction-line proportion matching task, all *p*-

387 values $> .05$, all $F_s \leq 2.85$). In line with this, the analysis of imaging data revealed no significant
388 difference between the two groups after the training for any of the three evaluation tasks at an
389 uncorrected p -value of $.001$ with a cluster size of $k = 10$ voxels. Because neither behavioral nor
390 neurofunctional differences were observed for the two trainings, we decided to merge both
391 training groups in order to investigate fraction learning across groups with higher statistical
392 power.

393

394 Behavioral analysis

395 *Training and flow data*

396 To evaluate learning outcomes over the five training time points and associations with
397 participants' flow experience during learning we used a latent growth linear mixed-effects
398 model over the five training time points. Regarding dependent variables, we were interested in
399 the mean percentage absolute estimation error (PAE; [55]) and the number of attempts
400 participants needed to estimate a given fraction correctly. Models were fitted in R using 'lmer'
401 from the 'lme4' package [56]. To provide p -values we used the 'summary' function of the
402 "lmerTest" R package [57]. Summary statistics were extracted via the 'analyze' function of
403 'psycho' [58].

404 *Number line estimation task*

405 Mean PAE (cf. [55]) was calculated to reflect performance in the number line estimation
406 task at the two time points. Items without a response were not considered for analyses. To
407 evaluate performance changes in PAE between the pre- vs. post-test, a linear mixed-effects
408 model was fitted using 'lmer' from the "lme4" R package [56]. Again, to provide p -values we
409 used the 'summary' function of the "lmerTest" R package [57]. Additionally, summary
410 statistics were also extracted via the 'analyze' function of "psycho" R package [58].

411

412 *Behavioral fMRI data*

413 For the analysis of the behavioral fMRI data we evaluated reaction times and accuracy
414 as dependent variables. Three separate linear mixed-effects models were run to analyze reaction
415 times for the three different evaluation tasks. Items without a response and items answered
416 incorrectly were not considered for reaction time analyses. To analyze error rates and to include
417 participants' individual performance as random effect, we ran three separate generalized linear
418 mixed-effects models (GLMM) fitted by using the R package 'lme4' [56]. In the GLMMs we
419 assumed a binomial error distribution and used the logit as the link function. For both types of
420 analysis, we provided *p*-values with the 'summary' function of the "lmerTest" R package [57].

421

422 *Imaging analysis*

423 fMRI data analyses were performed using SPM12 (<http://www.fil.ion.ucl.ac.uk/spm>).
424 Images were motion corrected and realigned to each participant's mean image. The mean image
425 was co-registered with the anatomical whole-brain volume. Imaging data was then normalized
426 into standard stereotaxic MNI space (Montreal Neurological Institute, McGill University,
427 Montreal, Canada). Images were resampled every 2.5 mm using 4th degree spline interpolation
428 to obtain isovoxels and then smoothed with a 5 mm full-width at half-maximum (FWHM)
429 Gaussian kernel to accommodate inter-subject variation in brain anatomy and to increase signal-
430 to-noise ratio in the images. Data were high-pass filtered (128 s) to remove low-frequency noise
431 components and corrected for autocorrelation assuming an AR(1) process. Brain activity was
432 convolved over all experimental trials with the canonical hemodynamic response function
433 (HRF) and its first time derivative.

434 For the first level analysis, pre- and post-fMRI training sessions were combined on the
435 subject level in a generalized linear model (GLM). For each participant, we considered the two
436 factors item-set (trained vs. untrained) and session (pre vs post). This resulted in four
437 experimental conditions: trained pre (T1), trained post (T2), untrained pre (UT1) and untrained

438 post (UT2). Additionally, to evaluate the influence of fraction problem difficulty we included
439 the covariate numerical distance between to-be-compared fractions as a parametric regressor in
440 the first level analysis. As a control variable the scrambled word problems (hereafter referred to
441 as “words”) were also included in the first level. Finally, the six movement parameters from
442 preprocessing were entered into the model to capture signal variations due to head motion.

443 These images then entered the second level random-effects group analysis using a
444 flexible factorial design. The SPM Anatomy Toolbox [59], available for all published
445 cytoarchitectonic maps (http://www.fz-juelich.de/ime/spm_anatomy_toolbox), was used for
446 anatomical localization of effects where applicable. For areas not yet implemented, the
447 anatomical automatic labelling tool (AAL) in SPM12 ([http://www.cyceron.fr/web/aal](http://www.cyceron.fr/web/aal_anatomical_automatic_labeling.html)
448 [anatomical_automatic_labeling.html](http://www.cyceron.fr/web/aal_anatomical_automatic_labeling.html)) was used.

449 All contrasts calculated reflect the parametric modulation of the fMRI signal by the
450 numerical distance between two proportions presented (distance effect). Simple contrasts
451 (distance effect in each notation) were family-wise error corrected at $p < .05$ at the whole-brain
452 level with a cluster size of $k = 10$ voxels. Complex contrasts comparing two distance effects
453 (e.g., distance effect in fractions after training vs. distance effect prior to the training) were
454 thresholded at an uncorrected p -value of $< .001$ at the voxel level with a cluster size of $k = 10$
455 voxels and were reported when they remained significant following family-wise error
456 correction (FWE) at the cluster-level with $p_{\text{cluster-corr}} < .05$.

457

458 **Results**

459 **Behavioral Results**

460 **Training and Flow Experience**

461 *Percentage absolute estimation error*

462 Differences in PAE and possible associations with flow experience over time were
463 analyzed using a latent growth linear mixed effect model, predicting PAE by flow experience
464 and time (i.e., five training time points) as fixed factors while also including a random intercept

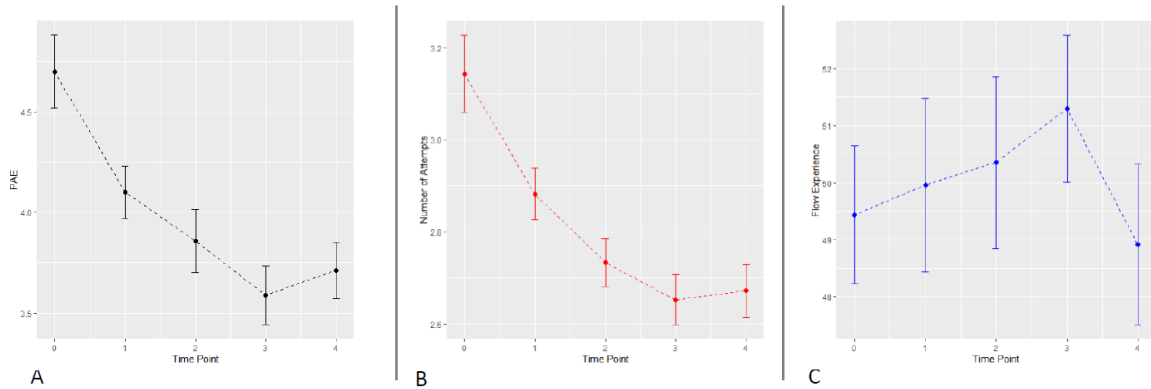
465 to account for participants' individual differences in prior knowledge. The model explained a
466 significant proportion of variance in PAE ($R^2 = 75.22\%$; fixed effects: $R^2 = 12.97\%$) and showed
467 that PAE [$\beta = -0.34$, $SE = 0.15$, $t(193) = -2.29$, $p < .05$] significantly improved over time (see
468 Figure 4A for PAE changes over time). Moreover, the fixed effect of flow [$\beta = -0.02$, $SE =$
469 0.01 , $t(198) = -3.13$, $p < .01$] was significant, indicating that flow experience changed over
470 time (see Figure 4C for general changes in flow experience over time). The interaction between
471 time and flow experience was not significant [$t(193) = 0.62$, $p = .54$].

472

473 *Number of attempts*

474 Identical to the first analysis, differences in the number of attempts participants needed
475 to estimate the given fraction during the NLE training correctly over time and possible
476 associations with flow experience were analyzed using again a latent growth linear mixed effect
477 model. Number of attempts needed was predicted by participants' flow experience and time
478 (i.e., five training time points) as fixed factors. Again, we included a random intercept to
479 account for participants individual differences in prior knowledge. The model explained a
480 significant proportion of variance in the number of attempts needed to estimate the given
481 fraction correctly ($R^2 = 71.24\%$; fixed effects: $R^2 = 16.16\%$) and showed that the number of
482 attempts significantly decreased over time [$\beta = -0.18$, $SE = 0.07$, $t(194) = -2.70$, $p < .01$; see
483 Figure 4B for number of attempts changes over time]. Within this model the fixed effect of flow
484 was significant [$\beta = -0.01$, $SE = 0.06$, $t(201) = -3.40$, $p < .001$], indicating that flow experience
485 changed over time (see Figure 4C for general changes in flow experience over time). Again,
486 the interaction between time and flow experience was not significant [$t(194) = 0.95$, $p = .34$].

487



488
 489 **Fig. 4:** Improvement of PAE (A), number of attempts needed (B), and changes in flow experience (C) over the
 490 training period (i.e., five time points). Error bars indicate standard errors (SEM).
 491
 492

493 **Number line estimation task**

494 To investigate whether PAE for the paper pencil-based NLE tasks differed between pre-
 495 and post-test we ran another linear mixed effect model. We defined the two time points (pre vs.
 496 post) as fixed effect and included a random intercept for subjects to account for individual
 497 variability. The overall model predicting differences in PAE on the two number line estimation
 498 tests (pre vs. post) explained a significant proportion of variance ($R^2 = 63.34\%$, in which the
 499 fixed effects explained $R^2 = 5.56\%$ of the variance). The effect of session was significant [$\beta =$
 500 -0.01 , $SE = 0.00$, $t(47) = -3.80$, $p < .001$] indicating that performance was better on the posttest
 501 than the pretest (reflected by smaller estimation errors).

502
 503 **Behavioral fMRI results**

504 *Fraction-line proportion matching task*

505 Performance changes in reaction time between pre- and post fMRI sessions for the
 506 *fraction-line proportion matching task* were evaluated by a linear mixed effect model. We
 507 defined session (pre vs. post), itemset used (trained vs. untrained), and numerical distance (i.e.,
 508 only for the non-matching items) as fixed effects. Moreover, interactions between session and
 509 item set as well as session and numerical distance were also included as fixed effects to evaluate

510 whether the possible influence of item set or numerical distance on participants reaction time
511 changed from pre- to post-test. Finally, we included a random intercept to account for
512 participants individual differences in prior knowledge.

513 The model explained significant proportions of variance on participants reaction times
514 ($R^2 = 26.18\%$; fixed effects: $R^2 = 9.74\%$) and showed that reaction times significantly improved
515 from pre- to post-test [$\beta = -260.85$, SE = 89.37, $t(1068) = -2.92$, $p < .01$]. Additionally, the
516 fixed effect of numerical distance was significant [$\beta = -1306.84$, SE = 210.59, $t(1069) = -6.21$,
517 $p < .001$], indicating that reaction times improved more strongly for increasing distances.
518 Neither the fixed effect of itemset nor the interactions were significant [all $t \leq 0.48$, all $p_s > .05$].

519 For the evaluation of performance changes in accuracy between pre- and post-test, we
520 ran a generalized linear mixed-effects model, by using logit as the link function and assuming
521 a binomial distribution of the error rates. To avoid overfitting of the model we only included
522 session, itemset used and numerical distance as fixed effects and a random intercept accounting
523 for individual differences in prior knowledge. The model revealed a significant fixed effect of
524 numerical distance [$z = -6.86$, $p < .001$], indicating that increasing distances between fractions
525 and line proportions led to less errors. The fixed effects of session and trained item were not
526 significant [all $z \leq -0.18$, all $p_s > .05$].

527 *Line proportion comparison task*

528 Identical to the analysis of the *fraction-line proportion matching task* we ran the same
529 linear mixed-effects model (including session, itemset used, numerical distance, as well as the
530 interaction session and item set and the interaction session and numerical distance as fixed
531 effects and a random intercept to account for individual differences in prior knowledge) to
532 investigate possible performance changes in reaction time on the *line proportion comparison*
533 *task* between pre- and post fMRI session. The model explained a significant proportion of
534 variance on participants reaction times ($R^2 = 42.71\%$; fixed effects: $R^2 = 8.57\%$) and showed

535 that reaction times significantly improved from pre to post-test [$\beta = -279.33$, $SE = 118.19$,
536 $t(507) = -2.36$, $p < .05$]. Additionally, the fixed effect of numerical distance was significant [β
537 $= -1686.98$, $SE = 295.05$, $t(510) = -5.72$, $p < .001$], indicating that reaction times got
538 significantly faster with increasing distances. Neither the fixed effect of itemset nor the
539 interactions were significant [all $t \leq 0.35$, all $p_s > .05$].

540 Again, to investigate possible performance changes in accuracy for the *line proportion*
541 *comparison task* between pre- and post-test, we ran a generalized linear mixed-effects model,
542 using logit as the link function and assumed a binomial distribution of the error rates. To avoid
543 overfitting of the model we only included session, itemset used and numerical distance as fixed
544 effects and a random intercept accounting for individual differences in prior knowledge.
545 Analyzing error rates for the line proportion comparison task revealed a significant fixed effect
546 of numerical distance [$z = -7.5$, $p < .001$], indicating that increasing distances between line
547 proportions led to less errors. However, there were no significant differences for session and
548 item set [all $z \leq -1.10$, all $p_s > .05$].

549 *Symbolic fraction magnitude comparison task*

550 Finally, possible performance changes in reaction time for the *symbolic fraction*
551 *magnitude comparison task* between pre- and post-test were tested identical to the prior two
552 evaluation tasks. We again ran a linear mixed-effects model with session, itemset used,
553 numerical distance as well as the interaction session and item set and the interaction session and
554 numerical distance as fixed effects and a random intercept accounting for individual differences
555 in prior knowledge. Interestingly, in this model only the fixed effect for numerical distance was
556 significant [$\beta = -1282.76$, $SE = 337.30$, $t(499) = -3.80$, $p < .001$], indicating that reaction times
557 were significantly faster for increasing distances. Neither the fixed effects of session and itemset
558 nor the interactions were significant [all $t \leq 0.43$, all $p_s > .05$].

559 Possible performance changes in accuracy for the *symbolic fraction magnitude*
560 *comparison* task between pre- and post-test, were evaluated again by a generalized linear
561 mixed-effects model, using logit as the link function and assuming a binomial distribution of
562 the error rates. To avoid overfitting of the model we only included session, itemset used and
563 numerical distance as fixed effects and a random intercept accounting for individual differences
564 in prior knowledge. Thus, analyzing error rates for the fraction task revealed a significant fixed
565 effect of session [$z = -2.38, p < .05$], indicating that participants performed better in the post-
566 test compared to the pre-test. Additionally, the fixed effect of numerical distance was significant
567 [$z = -9.17, p < .001$], indicating that increasing distances between fractions led to less errors
568 when comparing two fractions. However, there was no significant difference for item set [$z =$
569 $0.282, p = .079$].

570

571 **Imaging results**

572 Distance effect before the training

573 *Fraction-line proportion matching task*

574 Processing of smaller numerical distances between fractions and line proportions in the
575 fraction-line proportion matching task was associated with stronger magnitude-specific fMRI
576 signal before the training in a bilateral fronto-parietal neural network including areas in the
577 intraparietal cortex (hIP3), the superior parietal cortex (SPL), the inferior frontal gyrus (Areas
578 44 and 45), bilateral inferior temporal gyri as well as bilateral insula. Further activated clusters
579 were found in the bilateral middle frontal gyri as well as right-hemispheric subcortical areas
580 such as thalamus and caudate nucleus as well as the cerebellum (see Table 1, Figure 5A and C
581 depicted in red color).

582 *Line proportion comparison task*

583 Processing of smaller numerical distance between to-be-compared line proportions
584 modulated the fMRI signal before the training in a right-hemispheric fronto-parietal network
585 centered around the right intraparietal sulcus (hIP3). Smaller numerical distance between
586 proportions led to stronger activation in the right IPS and the right anterior IPS reflecting
587 fraction magnitude processing. Additionally, there was a significant cluster of activation in the
588 right inferior frontal gyrus (see Table 1, Figure 5B and C depicted in golden color).

589 *Symbolic fraction magnitude comparison task*

590 Activation in no cluster of voxels was modulated significantly by numerical distance at
591 the given threshold in the symbolic fraction magnitude comparison task before the training.

592

593 Importantly, there were no significant differences in brain activation observed between
594 the two stimulus sets: when comparing the two sets before the training (set to be trained vs.
595 untrained set) in any of the three conditions (i.e., fraction-line proportion matching task, line
596 proportion comparison task, symbolic fraction magnitude comparison task).

597

598

599 **Table 1:** Distance effect for line proportion comparison and fraction-line proportion matching task at a
600 familywise error corrected $p < .05$, cluster size $k = 10$ at the whole brain level.

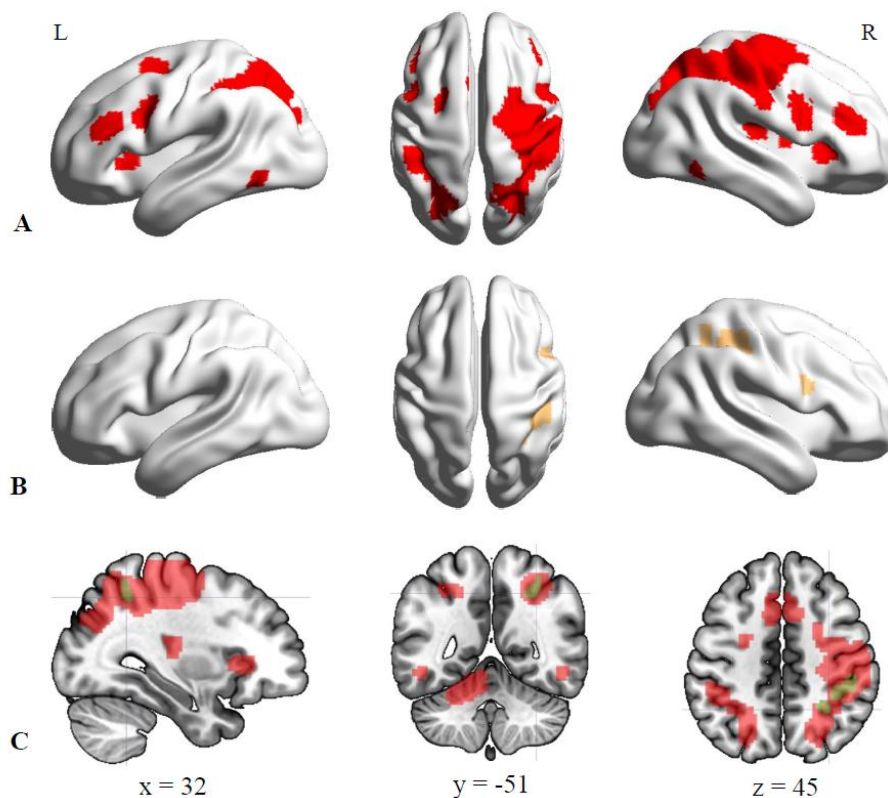
601

Contrast	Brain region	MNI (x, y, z)	Cluster size	t
Distance effect	RH anterior intraparietal sulcus (hIP2)	43 -36 47	61	5.63
<i>Line proportion</i>	RH intraparietal sulcus (hIP3)	33 -52 53	12	5.04
	RH inferior frontal gyrus (44)	51 8 23	15	5.61
Distance effect	RH precentral gyrus	38 -20 55	2195	11.49
<i>Matching task</i>	RH intraparietal sulcus (hIP3)	26 -57 55		7.15
	RH superior parietal lobe (SPL)	18 -60 58		5.46
	LH intraparietal sulcus (hIP3)	-30 -58 42	531	7.29

LH superior parietal lobe (SPL)	-25	-57	53		7.01
LH inferior frontal gyrus (IFG 44)	-42	5	30	167	7.71
RH inferior frontal gyrus (IFG 44)	53	8	28	179	7.38
LH inferior frontal gyrus (IFG 45)	-40	28	23	85	6.77
LH middle frontal gyrus	-20	6	55	58	6.25
RH middle frontal gyrus	41	41	18	85	5.91
LH posterior medial frontal gyrus	-7	8	58	271	7.24
RH inferior temporal gyrus	51	-52	-10	22	5.91
LH inferior temporal gyrus	-50	-57	-10	30	5.79
RH thalamus	18	-22	10	68	7.57
RH caudate nucleus	8	16	3	20	6.24
RH insula	33	-20	15	86	6.76
RH insula	31	28	3	54	6.08
RH insula	41	1	10	13	5.75
LH insula	-35	18	3	17	5.55
LH cerebellum	-17	-52	-23	306	10.56

602 Abbreviations: LH – left hemisphere; RH – right hemisphere; MNI – montreal neurological institute).

603



604

605 **Figure 5:** Effect of overall magnitude processing in the fractions-lines matching task (Panels A and C: red) and
606 the lines proportion comparison task (Panels B and C: gold) as reflected by the distance effect.

607

608 Training and transfer effects

609 *Line proportion comparison / fraction-line proportion matching task*

610 When comparing the distance effect after the training to the distance effect before the
611 training, no suprathreshold clusters of activation were observed for the line proportion
612 comparison or in the fraction-line proportion matching task.

613 *Symbolic fraction magnitude comparison task*

614 Direct comparison of the distance effect after the training to the distance effect before
615 the training in the symbolic fraction magnitude comparison task revealed significant increased
616 activation differences in a bilateral fronto-parietal network centered around the intraparietal
617 sulcus (hIP3; see Table 2, Figure 6). Further clusters of significant increased activation
618 differences were observed in the right superior parietal lobe (SPL) and the left inferior parietal
619 lobe (PFt), the right fusiform gyrus, the bilateral frontal cortex and the left thalamus.

620 When comparing distance effects observed for the trained item set to those observed for
621 untrained item set after the training, no suprathreshold clusters of activation were observed,
622 indicating that the training effect was comparably strong for trained as well as untrained fraction
623 items. This means that for fraction magnitude processing it seemed that the training effect
624 generalized to untrained items after the NLE training.

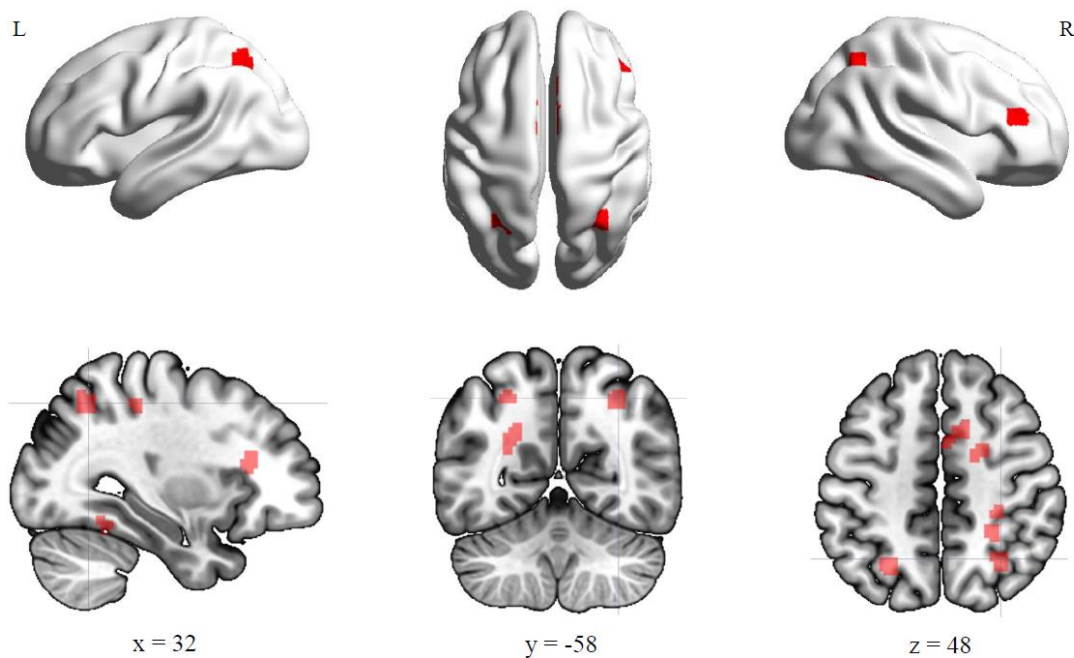
625

626 **Table 2:** Effect of training for processing of overall fraction magnitude as reflected by the distance
627 effect for the symbolic fraction magnitude comparison task.

Contrast	Brain region	MNI (x, y, z)	Cluster size	<i>t</i>
----------	--------------	---------------	--------------	----------

Fractions post vs. pre training	RH superior parietal lobe (SPL)	26	-45	45	59	4.42
	RH intraparietal sulcus (hIP3)	31	-60	48	21	3.70
	LH intraparietal sulcus (hIP3)	-27	-62	50	15	3.53
	LH supramarginal gyrus (PFt)	-34	-28	33	55	4.56
	LH middle frontal gyrus	26	1	40	80	5.36
	RH posterior medial frontal gyrus	13	11	53	133	4.08
	RH inferior frontal gyrus (IFG)	41	36	15	45	4.05
	RH fusiform gyrus	36	-47	-18	15	3.51
	LH thalamus	-17	-5	3	18	5.61
	LH cuneus	-25	-57	28	15	4.07

628 Abbreviations: LH – left hemisphere; RH – right hemisphere; MNI – montreal neurological institute).



629

630 **Figure 6:** Effect of training for processing of overall fraction magnitude as reflected by the distance effect for the
 631 symbolic fraction magnitude comparison task ($p_{\text{cluster-corr}} < .05$, $k = 10$ voxels).

632

633

634

Discussion

635 The aim of the present study was to investigate potential neuro-functional changes of
636 brain activation in adult participants through a five-day consecutive NLE training on fraction
637 magnitude. While there already exist a number of studies indicating the effectiveness of NLE
638 training on the behavioral level [20–22,60] and some few studies investigating the neural
639 correlates of fraction and proportion processing [41,42,44,45], little is still known about the
640 neural correlates of fraction learning.

641 Evaluation tasks included symbolic fraction and non-symbolic line proportion
642 magnitude comparison tasks and a fraction-line proportion matching task. Additionally,
643 learning trajectories over the five time points of the NLE training, participants' corresponding
644 flow experience as well as a paper pencil pre- post NLE task were evaluated. In the following,
645 behavioral as well as neurofunctional results for these measures will be discussed in more detail.

646

647 *The interplay between training performance and flow experience*

648 Flow experience has been described as an optimal state for intrinsically motivated
649 learning, which helps focus on the given task and can lead to improved performance [31].
650 Another explanation for optimal learning was first described by Yerkes and Dodson in 1908
651 and refers to the relationship between arousal and performance (i.e., Yerkes-Dodson-Law;
652 [61]). This law states that learners' optimal performance is achieved on a medium level of
653 arousal reflected by an inverted U-shape relation between the respective parameters.
654 Transferred to flow experience this inverted U-shape suggests that a balance between cognitive
655 demands of the task at hand and individual skill level is the basis for best possible flow
656 experience. Thus, when skills of a learner are too poor for the demands of a given task – for
657 instance at the beginning of a training – flow experience may be rather low. The same is true
658 when skills of a learner are too advanced for a given task. Both non-optimal states can lead to
659 boredom and/or frustration, and reduced flow experience. In turn, this might interfere with
660 learning of the given task [62].

661 In line with these assumptions, our results for training performance and flow experience
662 over the five day NLE training indicated that at the beginning of the training participants' flow
663 experience was significantly lower and PAE as well as number of attempts needed to solve a
664 trial successfully were significantly higher as compared to later training days. This possibly
665 reflects an imbalance between task demands and individual skills. PAE and number of attempts
666 needed on the first training day suggest that participants' ability on the task seemed to be low
667 in the beginning. With each training day passing flow experience increased while at the same
668 time PAE and number of attempts needed decreased. Thus, participants experienced a more and
669 more optimal learning situation in which demands of the task and individual skill level were in
670 balance.

671 Interestingly, on the last training day participants' flow experience dramatically
672 decreased again to values lower than on the first training day. This was accompanied by slight
673 increases of PAE and number of attempts needed. Thus, we assume that participants peak of
674 performance was already achieved at the 4th training day. We can only speculate whether this
675 was caused either by boredom or the fact that fraction magnitudes could not be estimated more
676 accurately by our participants after 4 days of training. Importantly, however, this decrease in
677 flow experience was not associated with a general decrease in training performance. Moreover,
678 we observed that participants significantly improved in the paper pencil NLE task from pre to
679 post session.

680 *Transfer effects in fraction and proportion learning*

681 Behavioral data indicated significant performance improvements for all three tasks.
682 Importantly, these improvements did not differ between trained and untrained items, indicating
683 transfer effects of the training to untrained items. Additionally, neurofunctional data showed
684 similar results: before the training no significant differences in brain activation were observed
685 between the two stimulus sets (trained and untrained set) for all three evaluation tasks. After
686 the training, again, no suprathreshold clusters of activation were observed when comparing

687 trained and untrained items for all three evaluation tasks. Thus, indicating that the training effect
688 was comparably strong for trained as well as untrained items and seemed to generalize to
689 untrained items after the NLE training.

690 Moreover, as discussed in more detail below, for the case of the fraction magnitude
691 comparison task we think brain activation associated with numerical distance after the NLE
692 training indicates that overall symbolic fraction magnitude was not automatically processed
693 before training.

694 However, one might argue that our results were elicited by the applied drill-like training
695 approach. In the literature, this is often used to investigate numerical learning in terms of
696 arithmetic fact learning [63,64]. Nevertheless, we are confident that participants did not just
697 learn specific fraction magnitudes by heart for at least two reasons: First, if fraction magnitudes
698 were learned by drill no transfer effect from trained to untrained items should be evident neither
699 on the behavioral nor on the neural level. Moreover, fractions that are learned by heart should
700 not show a numerical distance effect especially for untrained items because their overall
701 magnitude should not be processed. In our study induced automated magnitude activation was
702 also present for untrained items. Thus, it is unlikely that our training supported fact learning but
703 rather improved magnitude representation.

704 Second, previous literature reported different neural correlates for arithmetic fact
705 learning than observed in the current study. In particular, learning arithmetic facts is associated
706 with a shift from bilateral fronto-parietal processing around the IPS to a primarily left
707 hemispheric network in the medial temporary lobe (MTL) involving the hippocampus (cf. [64–
708 66]). However, in the present study, we rather observed a shift towards more activation within
709 the fronto-parietal network of magnitude processing [67–69] – thus, indicating more explicit
710 processing of overall fraction magnitude and not fact retrieval after the training.

711 This supports the notion that the training indeed resulted in a general conceptual
712 improvement and automatization of fraction magnitude processing, in contrast to training fact
713 retrieval of specific fraction magnitudes (cf. [64] for limited evidence of transfer effects in
714 multiplication fact training).

715 *Differential neural activity patterns before training and possible implications*

716 Surprisingly, and not consistent with the previous literature on neural correlates of
717 fraction processing brain activation before the training for the three tasks of interest revealed
718 different activation patterns in the IPS associated with the numerical distance effect. For the
719 non-symbolic line proportion comparison and the fraction-line proportion matching task we
720 found significant neural activation patterns in the typical fronto-parietal network observed
721 previously for proportion and fraction processing (cf. [40]). In particular, the line proportion
722 comparison task led to increased activation in the right intraparietal sulcus, whereas the
723 fraction-line matching task led to increased activation in the bilateral intraparietal sulcus. This
724 is in line with research on brain activation for symbolic and non-symbolic magnitude processing
725 [70].

726 Interestingly, for the symbolic fraction magnitude comparison task activation in no
727 cluster of voxels was modulated significantly by numerical distance before the training. This
728 was surprising as previous studies on the neural correlates of fraction magnitude processing
729 consistently reported IPS activity to be modulated by numerical distance for fraction magnitude
730 processing. Importantly, the presented magnitudes did not differ between the three evaluation
731 tasks. This means, that participants of our study were generally able to process the presented
732 magnitudes. However, access to symbolic fraction magnitudes during the respective fraction
733 comparison task might be reduced probably because of the bipartite nature of fractions [10].
734 Thus, fractions are more difficult to compare than for instance line proportions.

735 Moreover, our fraction items used in this study were more complex because of two
736 reasons: i) Our fraction pairs did not involve fractions with common components. In this case,
737 reasoning about the natural number components alone (i.e., processing numerators and
738 denominators separately) might often not help to find the right solution when comparing two
739 fractions without common components. However, we used fractions without common
740 components as we wanted to specifically investigate and promote overall fraction magnitude
741 processing. For instance, [71] found that mathematic experts showed a distance effect for
742 overall fraction magnitude while comparing fractions without common components but not for
743 fraction pairs with common components. Moreover, comparing fraction pairs with common
744 components is typically susceptible to what has been called the natural number bias [72]. ii)
745 We used fractions with numerators and denominators ranging from 1 to 30. This had two major
746 reasons: a) the difficulty to match two item sets of fraction pairs on both overall numerical
747 distance and problem size including only fractions without common components and b) the fact
748 that we wanted to make sure that fractions were rather unfamiliar to participants to be able to
749 investigate fraction learning on a neural level. Thus, participants may not have had a specific
750 representation of the magnitude of the presented fractions prior to our study.

751 *Training induced distance effect in the intraparietal sulcus for fraction magnitude processing*

752 In line with previous results of training studies, imaging results after the training showed
753 that the processing of overall symbolic fraction magnitude was improved. In particular, distance
754 related neural activation for symbolic fraction processing became significantly stronger from
755 pre- to post-test in the bilateral intraparietal sulcus. This may indicate that our NLE training
756 helped to establish easier access to the representation of overall fraction magnitude. These
757 results are inconsistent with a NLE training study on natural numbers with pre-post-test fMRI
758 comparing children with and without developmental dyscalculia [73]. After the NLE training
759 both groups showed decreased activation of brain areas involved in number magnitude

760 processing (for instance bilateral middle and superior frontal gyrus and left intraparietal sulcus).
761 The authors interpreted these results as reflection of more automatized processing of numerical
762 magnitude after their training.

763 In this context, however, there is inconsistent evidence in the literature about the
764 distance effect as an indicator for better numerical/ mathematical performance (see Moeller et
765 al., 2011). On the one hand, there are studies showing that a larger distance effect was associated
766 with poorer numerical/ mathematical performance [74,75]. On the other hand, there also are
767 studies observing that a more pronounced distance effect was associated with better numerical/
768 mathematical performance [50,76]. To accommodate these inconsistent lines of evidence, [50]
769 suggested that the relation between the size of the distance effect and mathematical performance
770 might not be linear but curvilinear instead. In particular, these authors suggested that the size
771 of the distance effect is depended on two factors: i) automated access to processed magnitudes
772 decreases the distance effect whereas ii) increasing task complexity may increase the distance
773 effect while processing magnitudes.

774 In line with this argument, it needs to be noted that we do not necessarily think that a
775 larger distance effect indicates better number/ fraction magnitude processing. However, in the
776 present study the increase of the distance effect on a neural level in symbolic fraction magnitude
777 comparison might nevertheless indicate more automatic access to overall fraction magnitude as
778 task complexity was very high. Moreover, due to the bipartite nature of fractions [10] overall
779 fraction magnitude may not have been as automatically activated prior to the NLE training.
780 Thus, after the NLE training our participants may have built a more coherent fraction magnitude
781 representation reflected by a larger distance effect. In line with the hypothetical curvilinear
782 model by [50] we think that the distance effect might decrease again after having established
783 the magnitude representation with further training.

784 Moreover, no significant differences between pre- and post-fMRI were found for the
785 line proportion comparison and fraction-line proportion matching task. This may indicate that
786 improvement towards more automated activation on a neural level was not achieved through
787 training as activation related to the numerical distance effect was already there before training.
788 Again, this might reflect that the bipartite nature of fractions might have hindered automated
789 magnitude processing of the fractions before training. In turn, non-symbolic proportions are
790 further reflected by visual-spatial aspects which may have helped process the actual relative
791 magnitude expressed as compared to symbolic fractions for which this relation needs to be built
792 by participants themselves. Thus, the NLE training might not have changed processing of line
793 proportions and fraction-line matching significantly on the neural level as the respective relative
794 magnitudes may have already been processed before the training due to facilitation by visual-
795 spatial aspects of the presentation format.

796 Taken together, these results indicate that even well-educated adults benefitted from a
797 NLE based training of fractions aimed at improving conceptual knowledge of fraction
798 magnitude. Importantly, the training did not only induce significant training effects on the
799 behavioral level but in particular also led to changes in brain activation associated with the
800 processing of symbolic fraction magnitude. This indicates processes of neurofunctional
801 plasticity in fraction learning. In the following, we will discuss implications of these results for
802 education.

803 *Implications for education*

804 The final report of the National Mathematics Advisory Panel states that ‘one key
805 mechanism linking conceptual and procedural knowledge is the ability to represent fractions on
806 a number line’ (p. 28; [77]). However, fraction learning and understanding still is an educational
807 challenge not only in the US but globally. The integrated theory of numerical development,
808 postulates that one core basis of all (rational) numbers is their magnitudes and that these

809 magnitudes can be represented on a mental number line [13]. Students difficulties with fractions
810 often arise from missing conceptual understanding, which among other things but not
811 exclusively involves an understanding of their magnitudes [78]. Therefore, it is a crucial step
812 for students to learn that fractions are numbers with magnitudes that can be represented on a
813 number line as well.

814 Accordingly, the recommendation to use number lines as an instructional tool to foster
815 conceptual understanding of fractions is given in different guidelines for educational practice
816 in fraction teaching (e.g., *Teaching fractions* [78] or *Developing Effective Fraction Instruction*
817 *for Kindergarten through 8th grade* [79]). This recommendation is supported by recent evidence
818 from different intervention studies that used number lines as intervention tools and found
819 significant improvements of children's performance and understanding of fractions [20–23,60].

820 With respect to educational practice, our results support the existing body of literature
821 that processing of proportion and fraction magnitudes can be improved by NLE training.
822 Moreover, to the best of our knowledge this is the first study indicating that such a training
823 improves symbolic fraction processing as reflected by the numerical distance effect on a neural
824 level. In particular, we argue that relative magnitude information of complex fractions may
825 initially not be processed automatically within the IPS as indicated by the missing numerical
826 distance related activation in the IPS before but significant activation associated with numerical
827 distance after the training.

828 *Limitations*

829 When interpreting the results of the current study there are some limiting aspects that
830 need to be considered. First of all, we are well aware that the current study is only a first step
831 towards a better understanding of the underlying neural processes of fraction learning. In
832 particular, this study investigated fraction learning on fractions more complex than those
833 fractions first encounter in school. This was the case for two major reasons: i) Our fraction pairs

834 did not consist of fractions with common components, which limits the number of available
835 fractions when only considering those with numerators and denominators ranging from 1 to 9.
836 Therefore, we used fractions with numerators and denominators ranging from 1 to 30 allowing
837 for proper matching of stimuli sets. ii) We ran our study with adult participants for whom we
838 assumed that they should be more or less proficient with fractions with numerators and
839 denominators up to 9. Thus, to be able to detect training effects we used more complex fractions.

840 As such, to investigate fraction learning more fundamentally, less complex fractions
841 (i.e., with single-digit numerators and denominators or even unit fractions) should be used.
842 Moreover, our study investigated fraction processing in adult participants. To focus more on
843 the fundamentals of fraction learning the developing brain should be investigated. A first
844 attempt, to investigate developmental differences in fraction magnitude processing on a neural
845 level is a study by [80]. In this study, the authors applied 2 mA bilateral tDCS and found that
846 adults and children benefitted differently during fraction processing by tDCS applied to
847 different areas of stimulation (IPS vs. dorsolateral prefrontal cortex (DLPFC)). However,
848 imaging studies on the neural correlates of proportion and fraction magnitude processing in
849 children are still missing.

850 *Conclusion*

851

852 It is well known that fractions are difficult to learn and understand not only for children
853 and students but even adults and teachers [7,81,82]. Therefore, the ability to foster and improve
854 fraction knowledge is of high educational importance. Apart from beneficial effects of NLE
855 training on the behavioral level, our study provides first insights into the neural correlates of
856 fraction learning. In particular, we did not observe numerical magnitude to significantly
857 modulate brain activation before the training for the processing of symbolic fractions. This
858 might indicate that overall fraction magnitude is not yet processed automatically before
859 training. Thus, through the training participants might have built up more automated processing

860 of overall fraction magnitude. As such, our results indicate a specific improvement of overall
861 fraction magnitude processing through NLE training reflected on the neural level. This case of
862 neuronal plasticity in fraction learning indicates neurofunctional changes elicited by training of
863 educationally relevant content. Therefore, our study supports the importance of NLE trainings
864 for fraction learning on a neurophysiological level.

865

866

867

References

- 868 [1] D.H. Bailey, M.K. Hoard, L. Nugent, D.C. Geary, Competence with fractions predicts
869 gains in mathematics achievement, *J. Exp. Child Psychol.* 113 (2012) 447–455.
- 870 [2] R.S. Siegler, G.J. Duncan, P.E. Davis-Kean, K. Duckworth, A. Claessens, M. Engel,
871 M.I. Susperreguy, M. Chen, Early predictors of high school mathematics achievement,
872 *Psychol. Sci.* 23 (2012) 691–697.
- 873 [3] J. Torbeyns, M. Schneider, Z. Xin, R.S. Siegler, Bridging the gap: Fraction
874 understanding is central to mathematics achievement in students from three different
875 continents, *Learn. Instr.* 37 (2015) 5–13.
- 876 [4] M.J. Behr, I. Wachsmuth, T.R. Post, R. Lesh, Order and equivalence of rational
877 numbers: A clinical teaching experiment, *J. Res. Math. Educ.* (1984) 323–341.
- 878 [5] K. Hart, Fractions, *Math. Sch.* 10 (1981) 13–15.
- 879 [6] R.S. Siegler, A.A. Pyke, Developmental and individual differences in understanding of
880 fractions., *Dev. Psychol.* 49 (2013) 1994.
- 881 [7] X. Vamvakoussi, S. Vosniadou, Understanding the structure of the set of rational
882 numbers: A conceptual change approach, *Learn. Instr.* 14 (2004) 453–467.

- 883 [8] H. Lortie-Forgues, J. Tian, R.S. Siegler, Why is learning fraction and decimal
884 arithmetic so difficult?, *Dev. Rev.* 38 (2015) 201–221.
- 885 [9] T.P. Carpenter, M.K. Corbitt, H.S. Kepner, M.M. Lindquist, R. Reys, Results of the
886 second NAEP mathematics assessment: Secondary school, *Math. Teach.* 73 (1980)
887 329–338.
- 888 [10] M. DeWolf, M.A. Grounds, M. Bassok, K.J. Holyoak, Magnitude comparison with
889 different types of rational numbers., *J. Exp. Psychol. Hum. Percept. Perform.* 40 (2014)
890 71.
- 891 [11] R.S. Siegler, Magnitude knowledge: The common core of numerical development,
892 *Dev. Sci.* 19 (2016) 341–361.
- 893 [12] R.S. Siegler, H. Lortie-Forgues, An integrative theory of numerical development, *Child*
894 *Dev. Perspect.* 8 (2014) 144–150.
- 895 [13] R.S. Siegler, C.A. Thompson, M. Schneider, An integrated theory of whole number
896 and fractions development, *Cogn. Psychol.* 62 (2011) 273–296.
- 897 [14] R. Gersten, R.F. Schumacher, N.C. Jordan, Life on the number line: Routes to
898 understanding fraction magnitude for students with difficulties learning mathematics, *J.*
899 *Learn. Disabil.* 50 (2017) 655–657.
- 900 [15] S.M. Göbel, S. Shaki, M.H. Fischer, The cultural number line: a review of cultural and
901 linguistic influences on the development of number processing, *J. Cross. Cult. Psychol.*
902 42 (2011) 543–565.
- 903 [16] I. Berteletti, D. Lucangeli, M. Piazza, S. Dehaene, M. Zorzi, Numerical estimation in
904 preschoolers., *Dev. Psychol.* 46 (2010) 545.
- 905 [17] D.C. Geary, M.K. Hoard, L. Nugent, J. Byrd-Craven, Development of number line

- 906 representations in children with mathematical learning disability, *Dev. Neuropsychol.*
907 33 (2008) 277–299.
- 908 [18] R.S. Siegler, J.E. Opfer, The development of numerical estimation: Evidence for
909 multiple representations of numerical quantity, *Psychol. Sci.* 14 (2003) 237–250.
- 910 [19] H.C. Barth, A.M. Paladino, The development of numerical estimation: Evidence
911 against a representational shift, *Dev. Sci.* 14 (2011) 125–135.
- 912 [20] N. Hamdan, E.A. Gunderson, The number line is a critical spatial-numerical
913 representation: Evidence from a fraction intervention., *Dev. Psychol.* 53 (2017) 587.
- 914 [21] C.A. Barbieri, J. Rodrigues, N. Dyson, N.C. Jordan, Improving fraction understanding
915 in sixth graders with mathematics difficulties: Effects of a number line approach
916 combined with cognitive learning strategies., *J. Educ. Psychol.* 112 (2020) 628.
- 917 [22] L.K. Fazio, C.A. Kennedy, R.S. Siegler, Improving children’s knowledge of fraction
918 magnitudes, *PLoS One.* 11 (2016) e0165243.
- 919 [23] K. Kiili, K. Moeller, M. Ninaus, Evaluating the effectiveness of a game-based rational
920 number training-In-game metrics as learning indicators, *Comput. Educ.* 120 (2018) 13–
921 28.
- 922 [24] K. Kiili, K. Ojansuu, A. Lindstedt, M. Ninaus, Exploring the educational potential of a
923 game-based math competition, *Int. J. Game-Based Learn.* 8 (2018) 14–28.
- 924 [25] J.L. Plass, U. Kaplan, Emotional design in digital media for learning, in: *Emot.*
925 *Technol. Des. Learn.*, Elsevier, 2016: pp. 131–161.
- 926 [26] K. Kiili, Digital game-based learning: Towards an experiential gaming model, *Internet*
927 *High. Educ.* 8 (2005) 13–24.
- 928 [27] M. Csikszentmihalyi, Beyond boredom and anxiety: Experiencing flow in work and

- 929 play, San Fr. (1975).
- 930 [28] K. Kiili, A. Lindstedt, M. Ninaus, Exploring characteristics of students' emotions, flow
931 and motivation in a math game competition., in: GamiFIN, 2018: pp. 20–29.
- 932 [29] A. Landhäuser, J. Keller, Flow and its affective, cognitive, and performance-related
933 consequences, in: *Adv. Flow Res.*, Springer, 2012: pp. 65–85.
- 934 [30] D.C. Moos, E. Marroquin, Multimedia, hypermedia, and hypertext: Motivation
935 considered and reconsidered, *Comput. Human Behav.* 26 (2010) 265–276.
- 936 [31] M. Csikszentmihalyi, *Flow: The Psychology of Optimal Experience*. Perennial Modern
937 Classics, (1990).
- 938 [32] M. Ulrich, J. Keller, K. Hoenig, C. Waller, G. Grön, Neural correlates of
939 experimentally induced flow experiences, *Neuroimage*. 86 (2014) 194–202.
- 940 [33] M. Ulrich, J. Keller, G. Grön, Neural signatures of experimentally induced flow
941 experiences identified in a typical fMRI block design with BOLD imaging, *Soc. Cogn.*
942 *Affect. Neurosci.* 11 (2016) 496–507.
- 943 [34] A. Perttula, K. Kiili, A. Lindstedt, P. Tuomi, Flow experience in game based learning–
944 a systematic literature review, *Int. J. Serious Games*. 4 (2017) 57–72.
- 945 [35] P. van Schaik, S. Martin, M. Vallance, Measuring flow experience in an immersive
946 virtual environment for collaborative learningjcal_455 350.. 365, (2011).
- 947 [36] U. Konradt, R. Filip, S. Hoffmann, Flow experience and positive affect during
948 hypermedia learning, *Br. J. Educ. Technol.* 34 (2003) 309–327.
- 949 [37] L.-A. Ho, T.-H. Kuo, How can one amplify the effect of e-learning? An examination of
950 high-tech employees' computer attitude and flow experience, *Comput. Human Behav.*
951 26 (2010) 23–31.

- 952 [38] L.A. Custodero, Observable indicators of flow experience: A developmental
953 perspective on musical engagement in young children from infancy to school age,
954 *Music Educ. Res.* 7 (2005) 185–209.
- 955 [39] R. MacDonald, C. Byrne, L. Carlton, Creativity and flow in musical composition: An
956 empirical investigation, *Psychol. Music.* 34 (2006) 292–306.
- 957 [40] S.N. Jacob, A. Nieder, Tuning to non-symbolic proportions in the human frontoparietal
958 cortex, *Eur. J. Neurosci.* 30 (2009) 1432–1442.
- 959 [41] J. Mock, S. Huber, J. Bloechle, J.F. Dietrich, J. Bahnmueller, J. Rennig, E. Klein, K.
960 Moeller, Magnitude processing of symbolic and non-symbolic proportions: an fMRI
961 study, *Behav. Brain Funct.* 14 (2018) 9.
- 962 [42] J. Mock, S. Huber, J. Bloechle, J. Bahnmueller, K. Moeller, E. Klein, Processing
963 symbolic and non-symbolic proportions: domain-specific numerical and domain-
964 general processes in intraparietal cortex, *Brain Res.* 1714 (2019) 133–146.
- 965 [43] J. Cui, L. Li, M. Li, R. Siegler, X. Zhou, Middle temporal cortex is involved in
966 processing fractions, *Neurosci. Lett.* (2020) 134901.
- 967 [44] M. DeWolf, J.N. Chiang, M. Bassok, K.J. Holyoak, M.M. Monti, Neural
968 representations of magnitude for natural and rational numbers, *Neuroimage.* 141 (2016)
969 304–312.
- 970 [45] A. Ischebeck, M. Schocke, M. Delazer, The processing and representation of fractions
971 within the brain: An fMRI investigation, *Neuroimage.* 47 (2009) 403–413.
- 972 [46] S.N. Jacob, A. Nieder, Notation-independent representation of fractions in the human
973 parietal cortex, *J. Neurosci.* 29 (2009) 4652–4657.
- 974 [47] R.S. Moyer, T.K. Landauer, Time required for judgements of numerical inequality,

- 975 Nature. 215 (1967) 1519–1520.
- 976 [48] M. Klabunde, M. Saggarr, K.M. Hustyi, R.G. Kelley, A.L. Reiss, S.S. Hall, Examining
977 the neural correlates of emergent equivalence relations in fragile X syndrome,
978 Psychiatry Res. Neuroimaging. 233 (2015) 373–379.
- 979 [49] M. Piazza, P. Pinel, D. Le Bihan, S. Dehaene, A magnitude code common to
980 numerosities and number symbols in human intraparietal cortex, Neuron. 53 (2007)
981 293–305.
- 982 [50] K. Moeller, S. Pixner, J. Zuber, L. Kaufmann, H.-C. Nuerk, Early place-value
983 understanding as a precursor for later arithmetic performance—A longitudinal study on
984 numerical development, Res. Dev. Disabil. 32 (2011) 1837–1851.
- 985 [51] F. Rheinberg, S. Engeser, R. Vollmeyer, Measuring components of flow: the Flow-
986 Short-Scale, Proc. 1st Int. Posit. Psychol. Summit. (2002).
- 987 [52] M. Ninaus, K. Kiili, J. McMullen, K. Moeller, Assessing fraction knowledge by a
988 digital game, Comput. Human Behav. 70 (2017) 197–206.
- 989 [53] M. Ninaus, K. Moeller, J. McMullen, K. Kiili, Acceptance of Game-Based Learning
990 and Intrinsic Motivation as Predictors for Learning Success and Flow Experience.,
991 (2017).
- 992 [54] & M. Ninaus, Kiili, Wortha, Motivationsprofile bei Verwendung eines Lernspiels zur
993 Messung des Bruchverständnisses in der Schule - Eine latente Profilanalyse., Psychol.
994 Erziehung Und Unterricht. (n.d.).
- 995 [55] J.L. Booth, R.S. Siegler, Developmental and individual differences in pure numerical
996 estimation., Dev. Psychol. 42 (2006) 189.
- 997 [56] D. Bates, M. Mächler, B. Bolker, S. Walker, Fitting linear mixed-effects models using

- 998 lme4, ArXiv Prepr. ArXiv1406.5823. (2014).
- 999 [57] A. Kuznetsova, P.B. Brockhoff, R.H.B. Christensen, lmerTest Package: Tests in Linear
1000 Mixed Effects Models. R package version 3.0–1, J Stat Softw. (2018).
- 1001 [58] D. Makowski, The psycho package: an efficient and publishing-oriented workflow for
1002 psychological science, J. Open Source Softw. 3 (2018) 470.
- 1003 [59] S.B. Eickhoff, K.E. Stephan, H. Mohlberg, C. Grefkes, G.R. Fink, K. Amunts, K.
1004 Zilles, A new SPM toolbox for combining probabilistic cytoarchitectonic maps and
1005 functional imaging data, Neuroimage. 25 (2005) 1325–1335.
1006 <https://doi.org/10.1016/j.neuroimage.2004.12.034>.
- 1007 [60] E.A. Gunderson, N. Hamdan, L. Hildebrand, V. Bartek, Number line unidimensionality
1008 is a critical feature for promoting fraction magnitude concepts, J. Exp. Child Psychol.
1009 187 (2019) 104657.
- 1010 [61] R.M. Yerkes, J.D. Dodson, The relation of strength of stimulus to rapidity of habit-
1011 formation, Punishm. Issues Exp. (1908) 27–41.
- 1012 [62] S. Greipl, K. Moeller, M. Ninaus, Potential and limits of game-based learning, Int. J.
1013 Technol. Enhanc. Learn. (2020).
- 1014 [63] A. Ischebeck, L. Zamarian, M. Schocke, M. Delazer, Flexible transfer of knowledge in
1015 mental arithmetic—An fMRI study, Neuroimage. 44 (2009) 1103–1112.
- 1016 [64] J. Bloechle, S. Huber, J. Bahnmüller, J. Rennig, K. Willmes, S. Cavdaroglu, K.
1017 Moeller, E. Klein, Fact learning in complex arithmetic—the role of the angular gyrus
1018 revisited, Hum. Brain Mapp. 37 (2016) 3061–3079.
- 1019 [65] V. Menon, Memory and cognitive control circuits in mathematical cognition and
1020 learning, in: Prog. Brain Res., Elsevier, 2016: pp. 159–186.

- 1021 [66] E. Klein, K. Willmes, S.M. Bieck, J. Bloechle, K. Moeller, White matter neuro-
1022 plasticity in mental arithmetic: changes in hippocampal connectivity following
1023 arithmetic drill training, *Cortex*. 114 (2019) 115–123.
- 1024 [67] M. Amalric, S. Dehaene, Origins of the brain networks for advanced mathematics in
1025 expert mathematicians, *Proc. Natl. Acad. Sci.* 113 (2016) 4909–4917.
- 1026 [68] M. Amalric, S. Dehaene, Cortical circuits for mathematical knowledge: evidence for a
1027 major subdivision within the brain’s semantic networks, *Philos. Trans. R. Soc. B Biol.*
1028 *Sci.* 373 (2018) 20160515.
- 1029 [69] M. Amalric, S. Dehaene, A distinct cortical network for mathematical knowledge in the
1030 human brain, *Neuroimage*. 189 (2019) 19–31.
- 1031 [70] H.M. Sokolowski, W. Fias, A. Mousa, D. Ansari, Common and distinct brain regions
1032 in both parietal and frontal cortex support symbolic and nonsymbolic number
1033 processing in humans: A functional neuroimaging meta-analysis, *Neuroimage*. 146
1034 (2017) 376–394.
- 1035 [71] A. Obersteiner, W. Van Dooren, J. Van Hoof, L. Verschaffel, The natural number bias
1036 and magnitude representation in fraction comparison by expert mathematicians, *Learn.*
1037 *Instr.* 28 (2013) 64–72.
- 1038 [72] Y. Ni, Y.-D. Zhou, Teaching and learning fraction and rational numbers: The origins
1039 and implications of whole number bias, *Educ. Psychol.* 40 (2005) 27–52.
- 1040 [73] K. Kucian, U. Grond, S. Rotzer, B. Henzi, C. Schönmann, F. Plangger, M. Gälli, E.
1041 Martin, M. von ASTER, Mental number line training in children with developmental
1042 dyscalculia, *Neuroimage*. 57 (2011) 782–795.
- 1043 [74] B. De Smedt, L. Verschaffel, P. Ghesquière, The predictive value of numerical
1044 magnitude comparison for individual differences in mathematics achievement, *J. Exp.*

- 1045 Child Psychol. 103 (2009) 469–479.
- 1046 [75] I.D. Holloway, D. Ansari, Mapping numerical magnitudes onto symbols: The
1047 numerical distance effect and individual differences in children’s mathematics
1048 achievement, *J. Exp. Child Psychol.* 103 (2009) 17–29.
- 1049 [76] L. Rousselle, M.-P. Noël, Basic numerical skills in children with mathematics learning
1050 disabilities: A comparison of symbolic vs non-symbolic number magnitude processing,
1051 *Cognition.* 102 (2007) 361–395.
- 1052 [77] NMAP, Foundations for success: The final report of the National Mathematics
1053 Advisory Panel, US Department of Education, 2008.
- 1054 [78] L. Fazio, R.S. Siegler, Teaching fractions, International Academy of Education, 2011.
- 1055 [79] R. Siegler, T. Carpenter, F. Fennell, D. Geary, J. Lewis, Y. Okamoto, L. Thompson, J.
1056 Wray, Developing Effective Fractions Instruction for Kindergarten through 8th Grade.
1057 IES Practice Guide. NCEE 2010-4039., What Work. Clear. (2010).
- 1058 [80] S.M. Bieck, M. Splittgerber, K. Keil, R. Salvador, K. Moeller, V. Moliadze, P182
1059 Bilateral tDCS indicates developmental differences in fraction magnitude processing,
1060 *Clin. Neurophysiol.* 131 (2020) e117.
- 1061 [81] J.W. Stigler, K.B. Givvin, B.J. Thompson, What community college developmental
1062 mathematics students understand about mathematics, *MathAMATYC Educ.* 1 (2010)
1063 4–16.
- 1064 [82] L. Ma, Knowing and Teaching Elementary Mathematics: Teacher’s Understanding
1065 OfFundamental Mathematics in China and the United States, Lawrence Erlbaum
1066 Associates, Incorporated, 1999.
- 1067

1068

Appendix

1069 Stimuli and corresponding parameters (i.e, fraction magnitudes, numerical distance, problem size,
 1070 matched distances and problem sizes within and between stimulus sets) for both item sets.
 1071

fraction pairs	magnitude fraction 1	magnitude fraction 2	numerical distance	problem size (PS)	matched distance within and between sets	matched PS within and between sets	Item set
$\frac{1}{8} \frac{5}{16}$	0,125	0,313	0,1875	0,438	0,022	0,078	1
$\frac{2}{3} \frac{8}{11}$	0,667	0,727	0,061	1,394	0,022	0,078	1
$\frac{3}{8} \frac{11}{13}$	0,375	0,846	0,471	1,221	0,027	0,069	1
$\frac{1}{4} \frac{22}{25}$	0,25	0,88	0,63	1,13	0,027	0,069	1
$\frac{5}{12} \frac{10}{17}$	0,417	0,588	0,172	1,005	0,006	0,066	1
$\frac{3}{14} \frac{17}{25}$	0,214	0,68	0,466	0,894	0,006	0,066	1
$\frac{3}{19} \frac{21}{29}$	0,158	0,724	0,566	0,882	0,006	0,066	1
$\frac{1}{24} \frac{16}{25}$	0,042	0,64	0,598	0,682	0,006	0,066	1
$\frac{5}{27} \frac{1}{26}$	0,185	0,039	0,147	0,224	0,008	0,045	1
$\frac{6}{23} \frac{9}{25}$	0,261	0,36	0,099	0,621	0,008	0,045	1

1072

$\frac{23}{27}$	$\frac{13}{22}$	0,852	0,591	0,261	1,443	0,016	0,062	1
$\frac{19}{23}$	$\frac{21}{25}$	0,826	0,84	0,014	1,666	0,016	0,062	1
$\frac{4}{7}$	$\frac{5}{21}$	0,571	0,238	0,333	0,810	0,013	0,087	1
$\frac{3}{7}$	$\frac{9}{22}$	0,429	0,409	0,020	0,838	0,013	0,087	1
$\frac{4}{5}$	$\frac{17}{30}$	0,8	0,567	0,233	1,367	0,022	0,017	1
$\frac{5}{6}$	$\frac{17}{22}$	0,833	0,773	0,061	1,606	0,022	0,017	1
$\frac{9}{14}$	$\frac{15}{26}$	0,643	0,577	0,066	1,220	0,00017897	0,035	1
$\frac{9}{17}$	$\frac{11}{29}$	0,529	0,379	0,150	0,909	0,00017897	0,035	1
$\frac{9}{10}$	$\frac{14}{17}$	0,9	0,824	0,077	1,724	0,00017897	0,035	1
$\frac{9}{13}$	$\frac{11}{17}$	0,692	0,647	0,045	1,339	0,00017897	0,035	1
$\frac{6}{13}$	$\frac{7}{19}$	0,462	0,368	0,093	0,830	0,006	0,033	1
$\frac{6}{11}$	$\frac{9}{26}$	0,545	0,346	0,199	0,892	0,006	0,033	1
$\frac{19}{20}$	$\frac{23}{29}$	0,95	0,793	0,157	1,743	0,001	0,002	1

$\frac{19}{25} \frac{10}{11}$	0,76	0,909	0,149	1,669	0,001	0,002	1
$\frac{1}{9} \frac{6}{25}$	0,111	0,24	0,129	0,351	0,022	0,078	2
$\frac{5}{8} \frac{7}{10}$	0,625	0,7	0,075	1,325	0,022	0,078	2
$\frac{4}{9} \frac{25}{28}$	0,444	0,893	0,448	1,337	0,027	0,069	2
$\frac{2}{9} \frac{13}{14}$	0,222	0,929	0,706	1,151	0,027	0,069	2
$\frac{1}{18} \frac{25}{26}$	0,056	0,962	0,906	1,017	0,006	0,066	2
$\frac{1}{12} \frac{10}{29}$	0,083	0,345	0,262	0,428	0,006	0,066	2
$\frac{5}{19} \frac{17}{20}$	0,263	0,85	0,587	1,113	0,006	0,066	2
$\frac{9}{20} \frac{13}{27}$	0,45	0,482	0,032	0,932	0,006	0,066	2
$\frac{5}{23} \frac{7}{30}$	0,217	0,233	0,016	0,451	0,008	0,045	2
$\frac{7}{27} \frac{1}{22}$	0,259	0,046	0,214	0,305	0,008	0,045	2
$\frac{20}{27} \frac{11}{20}$	0,741	0,55	0,191	1,291	0,016	0,062	2
$\frac{19}{21} \frac{15}{19}$	0,905	0,790	0,115	1,694	0,016	0,062	2

$\frac{2}{5}$	$\frac{3}{29}$	0,4	0,104	0,297	0,504	0,013	0,087	2
$\frac{1}{2}$	$\frac{8}{17}$	0,5	0,471	0,029	0,971	0,013	0,087	2
$\frac{3}{4}$	$\frac{13}{24}$	0,75	0,542	0,208	1,292	0,022	0,017	2
$\frac{8}{9}$	$\frac{22}{29}$	0,889	0,759	0,130	1,648	0,022	0,017	2
$\frac{7}{13}$	$\frac{11}{21}$	0,538	0,524	0,015	1,062	0,00017897	0,035	2
$\frac{9}{11}$	$\frac{21}{26}$	0,818	0,808	0,011	1,626	0,00017897	0,035	2
$\frac{8}{15}$	$\frac{13}{29}$	0,533	0,448	0,085	0,982	0,00017897	0,035	2
$\frac{9}{19}$	$\frac{11}{25}$	0,474	0,44	0,034	0,914	0,00017897	0,035	2
$\frac{8}{13}$	$\frac{9}{23}$	0,615	0,391	0,224	1,007	0,006	0,033	2
$\frac{6}{17}$	$\frac{8}{27}$	0,353	0,296	0,057	0,649	0,006	0,033	2
$\frac{20}{23}$	$\frac{12}{13}$	0,870	0,923	0,054	1,793	0,001	0,002	2
$\frac{15}{22}$	$\frac{14}{15}$	0,682	0,933	0,252	1,615	0,001	0,002	2

1075
1076
1077