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**International Trade under Monopolistic  
Competition beyond the CES**

Badis TABARKI

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# International Trade under Monopolistic Competition beyond the CES

**Badis Tabarki\***

*Université Paris1 Panthéon-Sorbonne and PSE*

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## **Abstract**

This paper considers a general yet tractable demand system encompassing directly- and indirectly-separable preferences, with homothetic CES as a common ground. An added flexibility of this demand system is that it allows for two alternative curvatures of demand. Beyond the CES, demand may be either "sub-convex": less convex than the CES, or "super-convex": more convex than the CES. Embedded in a general equilibrium trade model featuring standard assumptions on the supply side, this flexible demand system yields new comparative statics results and a wide range of predictions for the gains from trade, while illustrating existing ones in a simple and compact way. The main finding of this paper is that while demand curvature governs comparative statics results and plays a crucial role in determining the structure and the magnitude of welfare gains from trade, the type of preferences has only a second-order importance from a welfare standpoint.

*JEL Classification:* **F12, D11, L11**

*keywords:* gains from trade, heterogeneous firms, non-CES preferences, demand curvature.

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# Introduction

Recent empirical research in international trade has revealed that the price elasticity of demand varies significantly across firms. This central observation stands in stark contradiction with CES demand which imposes a constant demand elasticity under monopolistic competition. This suggests that it is crucial to depart from the homothetic CES to examine gains from trade under more realistic patterns of price sensitivity. In theoretical trade literature, transition to non-CES preferences took place gradually. While in earlier departures from the CES, a large body of work has focused on specific types of preferences, only few recent papers have proposed more general demand systems encompassing prominent alternatives to the CES case.

However, increased generality raises tractability issues, which in turn requires some concessions. Accordingly, a large body of work always impose a restriction on the curvature of demand: they generally assume that demand is “sub-convex”. They also abstract from fixed costs of accessing markets. Instead, they resort to assuming the existence of a choke price to ensure self-selection of firms into markets, see e.g. (Melitz and Ottaviano, 2008; Bertolotti et al., 2018; Arkolakis et al., 2018; Feenstra, 2018; Fally, 2019). Alternatively, other papers squarely focus on a specific type of preferences, while keeping demand curvature unrestricted and taking fixed costs in due account (Zhelobodko et al., 2012; Mrázová and Neary, 2017). Both existing modeling approaches exhibit the same limitations for welfare analysis. First, they imply that at least one channel of welfare gains from trade is ruled out. Second, they preclude the welfare implications of the curvature of demand and those of the type of preferences to be studied in a common framework.

In contrast, this is what the current chapter aims for. Towards this goal, I propose a theoretical framework combining standard assumptions on the supply side with a flexible and restriction-free demand system. In particular, the supply side is identical to (Melitz, 2003; Chaney, 2008; Arkolakis et al., 2012, 2018; Fally, 2019), and incorporates monopolistic competition, firm heterogeneity and Pareto distribution of firm productivity. However, here the novel aspect is that I consider a flexible demand system which encompasses two commonly used families of preferences (directly- and indirectly-separable), and nests two alternative curvatures of demand (beyond the CES, demand can be either sub-convex or super-convex). The modeling approach proposed in this paper offers then a theoretically clean way to examine the welfare implications of these alternative assumptions on the curvature of demand and the nature of preferences.

The goal of this paper is to examine three major questions in trade theory with heterogeneous firms under more realistic consumer behavior than allowed by CES preferences. First, does demand curvature play a role in determining the toughness of firm selection and the degree of their partitioning by export status? Second, under which demand conditions, net variety gains and gains from selection coexist in general equilibrium? Third, to which extent the curvature of demand and the type of preferences determine the magnitude of the gains from trade?

The main finding of this paper is that demand curvature plays a crucial role in driving comparative statics results, shaping the structure of the gains from trade as well as determining the magnitude of these gains, whereas the type of preferences affects only marginally the results. In particular, taking the CES as a boundary case, I show that when demand is sub-convex, selection into markets is more relaxed, the partitioning of firms by export status is more pronounced, net variety gains and gains from selection coexist, and gains from trade are smaller than those obtained under CES demand. I also emphasize that the type of preferences plays only a second-order role. For instance, under sub-convex demands, directly-separable preferences provide an upper bound for the gains from trade, while indirectly-separable preferences provide a lower bound. All these patterns are reversed when demand is super-convex.

The remainder of this paper is organized as follows. Section I describes in detail the general demand system considered in the current paper. Section II offers a simple characterization of demand curvature. Section III illustrates novel comparative statics results. Section IV examines the gains from trade and highlights novel welfare implications of demand curvature. The last section concludes. Appendix A provides the proofs for the main results as well as a detailed explanation of the "EEM" method.

# I. A Flexible Demand System

This section describes the generalized Gorman-Pollak demand system considered by Fally (2019) and recalls sufficient conditions under which such demand system is integrable, following Fally (2018). It also proposes a simple and useful parameterization that allows for a subtle nesting of directly- and indirectly-separable preferences.

## A. Generalized Gorman-Pollak Demand

Consider a representative consumer whose income  $w$  is entirely spent on a set of varieties, denoted by  $\Omega$ . For each variety  $\omega \in \Omega$ , suppose that demand is determined by its price  $p_\omega$ , consumer income  $w$  and an aggregator  $\Lambda$  :

$$x_\omega = Q(\Lambda) D_\omega(V(\Lambda) \frac{p_\omega}{w}), \quad (1)$$

where  $\Lambda = \Lambda(p, w)$  is itself a scalar function of all prices and income, homogeneous of degree zero in  $(p, w)$ .  $\Lambda$  is implicitly determined by the budget constraint, i.e. it is the implicit solution of :

$$\int_{\omega \in \Omega} p_\omega Q(\Lambda) D_\omega(V(\Lambda) \frac{p_\omega}{w}) d\omega = w \quad (2)$$

## B. Conditions for Integrability

Integrability conditions can be defined as regularity restrictions that are sufficient to ensure that a demand system can be derived from a rational utility maximizing consumption behavior (Fally, 2018).<sup>1</sup> Following Fally (2018), the generalized Gorman-Pollak demand system is integrable under the following conditions:

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<sup>1</sup>See Fally (2018) for further details on integrability conditions.

1.  $D_\omega$  is differentiable and sufficiently downward sloping and elastic, i.e.  $\sigma_\omega = -\frac{d \log D_\omega}{d \log p_\omega} > 1$ .
2.  $Q$  and  $V$  are differentiable and  $[\varepsilon_V \sigma_\omega - \varepsilon_Q]$  has the same sign for all  $\Lambda$  and  $\frac{p_\omega}{w}$ .
3. For any set of normalized prices  $\frac{p_\omega}{w}$ , equation (1) admits a solution in  $\Lambda$ ,

where  $\varepsilon_Q$  and  $\varepsilon_V$  denote the elasticity of  $Q$  and  $V$  with respect to  $\Lambda$ .

### C. A Useful Parameterization

In order to nest indirectly- and directly-separable preferences in a simple way, I propose the following parameterization:  $Q(\Lambda) = \Lambda^{-\beta}$  and  $V(\Lambda) = \Lambda^\alpha$ . It is without loss of generality to assume that  $\alpha$  and  $\beta$  are both dummies (whose values can be either 0 or 1), such as the case ( $\alpha=0$  and  $\beta=1$ ) corresponds to indirectly-separable preferences, while directly-separable preferences correspond to ( $\alpha=1$  and  $\beta=0$ ). This implies that the difference  $[\varepsilon_V \sigma_\omega - \varepsilon_Q]$  is positive under both cases, which is sufficient to ensure integrability.

## II. Characterization of Demand Curvature

At this stage, the flexibility of the demand system described above is reflected in how departing from homothetic CES preferences allows the demand elasticity to vary either with normalized prices when preferences are indirectly-separable, or with consumption levels when preferences are directly-separable. However, such flexibility raises the following question: under which conditions the demand elasticity increases, decreases, or ceases to vary with normalized prices or individual consumption?

This section aims at addressing this question, and by doing so it completes the characterization of the demand side of the model. Towards this goal, the current section draws heavily on [Mrázová and Neary \(2017\)](#). For instance, it adopts their approach that they call "a firm's eye view of demand".

## A. A simple Measure of Demand Curvature

Following [Mrázová and Neary \(2017\)](#), the starting point is the fact that a monopolistically competitive firm takes the demand function it perceives as given. As this approach is partial-equilibrium by definition, it is more convenient to express the Gorman-Pollak demand in equation (1) solely as a function of the price:  $x(\Lambda, \frac{p}{w}) \equiv x(D(p)) \equiv x(p)$ .<sup>2</sup>

Let us recall that the price elasticity of demand is given by:

$$\sigma = -\frac{d \log x(p)}{d \log p} = -\frac{d \log D(p)}{d \log p} > 0$$

As in [Mrázová and Neary \(2017\)](#), I measure the convexity of demand using the elasticity of the slope of direct demand :

$$\zeta = -\frac{d \log D'(p)}{d \log p}$$

Now in order to measure demand curvature in a simple and unit-free way, I work with the "superelasticity" of [Kimball \(1995\)](#), defined as the elasticity with respect to price of the elasticity of demand:

$$S = \frac{d \log \sigma}{d \log p} = (1 + \sigma) - \zeta$$

Clearly, the sign of the "superelasticity"  $S$  is pinned down by the relationship between the elasticity  $\sigma$  and the convexity  $\zeta$  of the direct demand function, the so-called "demand manifold" by [Mrázová and Neary \(2017\)](#). Interestingly, the "superelasticity", due to [Kimball \(1995\)](#), can be considered as a sufficient statistic for demand curvature. That is, its sign clearly indicates whether demand is CES, sub-convex, or super-convex :

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<sup>2</sup>I drop the variety subscript for expositional simplicity. Since consumer income and the aggregator are taken as given, they are dropped from this simplified demand function, so as to concentrate on the relationship between  $x$  and  $p$ .

$$\text{demand is } \begin{cases} \text{sub-convex} & \text{if } S > 0 \\ CES & \text{if } S = 0 \\ \text{super-convex} & \text{if } S < 0 \end{cases}$$

A peculiar property of CES demand is that it exhibits an exogenous demand elasticity, which is reflected by zero "superelasticity". It is then convenient to take the CES case as a benchmark to characterize both alternative curvatures in a simple way. Following [Mrázová and Neary \(2017\)](#), a demand function is locally sub-convex if for the same level of demand elasticity  $\sigma$ , it exhibits a lower degree of convexity  $\zeta$  as compared to the CES case. Similarly, a demand function is super-convex if it is more convex than the CES at a given level of demand elasticity.

It is now possible to graphically illustrate these three possible cases of demand curvature in a simple and compact way in the  $(\zeta, \sigma)$  space. Before proceeding, I resort to the first- and second-order conditions of profit maximization to impose restrictions on the values of  $\sigma$  and  $\zeta$ , as in [Mrázová and Neary \(2017\)](#).

As indicated in the beginning of this section, I consider a monopolistically competitive firm that takes the direct demand function it perceives as given and maximizes its profit accordingly. From the first-order condition, a positive price-cost margin implies that the price elasticity of demand must be greater than one:

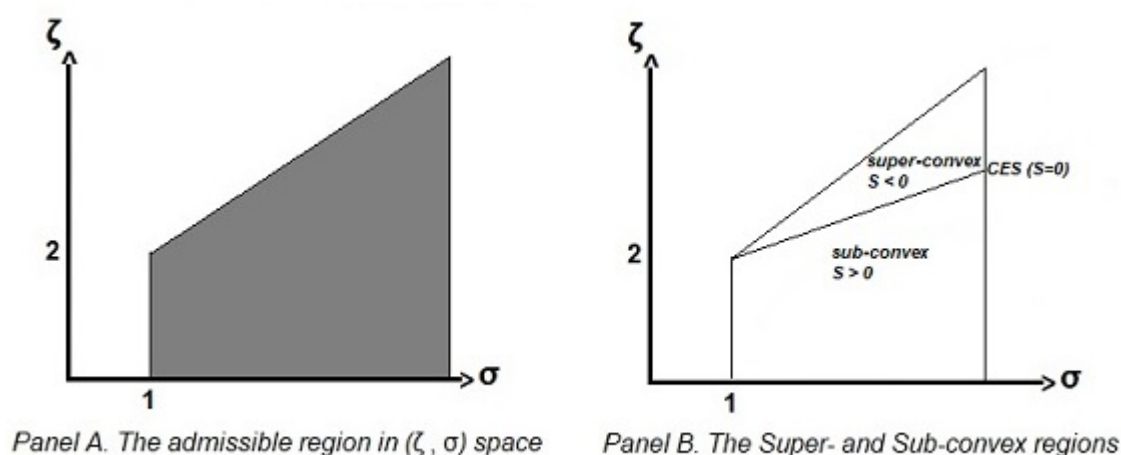
$$(p - \varphi^{-1})x' + x = 0 \Rightarrow \sigma > 1 \quad (3)$$

where  $\varphi^{-1}$  is the marginal cost of a  $\varphi$ -productivity firm.

From the second-order condition, decreasing marginal revenue requires that the degree of demand convexity  $\zeta$  must be smaller than twice the demand elasticity:

$$2x' + (p - \varphi^{-1})x'' < 0 \Rightarrow \zeta < 2\sigma \quad (4)$$

Figure 1: Localising Demand Curvature in the space of Elasticity and Convexity.



As in Mrázová and Neary (2017), the above restrictions imply an admissible region in  $(\zeta, \sigma)$  space, as shown by the shaded region in Figure 1, panel A.<sup>3</sup>

As illustrated in Figure 1, panel B, the CES line (whose equation is given by  $\zeta = 1 + \sigma$ ) divides the admissible region in two. While points located above the CES line correspond to super-convex demands, points below the CES boundary correspond to the case of sub-convex demands. Within the sub-convex region, the "superelasticity" is positive ( $S > 0$ ), which implies that the demand elasticity increases in price (or, equivalently, decreases with consumption) if and only if demand is sub-convex.

In contrast, the super-convex region is characterized with a negative "superelasticity" ( $S < 0$ ). This implies that the demand elasticity decreases in price (or, equivalently increases with consumption) when demand is super-convex. Clearly, CES demand is a boundary case under which the demand elasticity does not vary with price or consumption levels, as reflected by the zero "superelasticity" ( $S=0$ ). Hence, moving along the CES line only changes the value of the elasticity of demand while preserving its exogenous nature.

<sup>3</sup>Ideally, one would borrow an upper bound estimate of the demand elasticity from the empirical literature, and thus concentrate on a more realistic part of the admissible region.

### III. Illustrating Comparative Statics Results

#### A. Variable markups and Relative pass-through

Again, following [Mrázová and Neary \(2017\)](#), the starting point of the analysis is the fact that a monopolistically competitive firm producing a variety  $\omega$  at a  $\varphi^{-1}$  marginal cost takes the price aggregator  $\Lambda$  as given. Whether it perceives the partial equilibrium demand function in its direct or inverse form, the first-order condition of profit maximization in equation (3) yields a unique optimal pricing rule:

$$p(\varphi) = \varphi^{-1} m(\varphi), \quad (5)$$

where  $m(\varphi) = \frac{\sigma(\varphi)}{\sigma(\varphi)-1}$  is the markup set by the  $\varphi$ -productivity firm.

Let us now denote by  $\eta(\varphi) = -\frac{d \log p(\varphi)}{d \log \varphi}$  the absolute value of the elasticity of price with respect to firm productivity :

$$\eta(\varphi) \equiv 1 + \frac{d \log m(\varphi)}{d \log \sigma(\varphi)} S$$

The above expression clearly shows how demand curvature governs the degree of completeness of the relative pass-through.<sup>4</sup> Under the CES case, demand elasticity is exogenous ( $\forall \varphi, \sigma(\varphi) = \sigma$ ), the "superelasticity" of demand  $S$  collapses to zero, and so  $\eta$  is fixed to unity :  $\eta = 1$ . Departing from the CES benchmark allows then for a positive or negative deviation of  $\eta(\varphi)$  from unity that is pinned down by the sign of the "superelasticity"  $S$  :

$$\eta(\varphi) \begin{cases} < 1 & \text{if } S > 0 \\ = 1 & \text{if } S = 0 \\ > 1 & \text{if } S < 0 \end{cases}$$

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<sup>4</sup>Notice that the final expression of  $\eta(\varphi)$  has been simplified using the fact that an increase in productivity, other things equal, must lower a firm's price (and equivalently, increases individual consumption of the variety it supplies).

Therefore, when demand is sub-convex, a higher productivity, which other things equal implies lower price (or equivalently, higher individual consumption), is associated with a lower demand elasticity and so, a higher markup, implying less than 100 percent pass-through. By contrast, when demand is super-convex, a higher productivity is associated with a higher demand elasticity and so, a lower markup, implying more than 100 percent pass-through.

## B. Demand Curvature, Firm Selection, and Partitioning of Firms

In order to examine the role that demand curvature plays in determining the toughness of firm selection and the partitioning of firms by export status,<sup>5</sup> I proceed in two steps.

First, I show how demand curvature determines the nature of the elasticity of a firm's operating profit with respect to its productivity; whether it is constant, increasing or decreasing with firm productivity. Then, I graphically illustrate the results in a compact way and infer new implications of demand curvature for firm selection and the partitioning of firms by export status.

### 1. Constant vs Variable productivity elasticity of operating profits

Again, as in [Mrázová and Neary \(2017\)](#), the starting point is the fact that a  $\varphi$  productivity firm, which engages in monopolistic competition, takes the demand function as given and maximizes its profit accordingly. Its operating profit can be written in an approximate way:<sup>6</sup>

$$\pi^o(\varphi) = \frac{p(\varphi) x(\varphi) L}{\sigma(\varphi)} \equiv \frac{p(\varphi)^{1-\sigma(\varphi)}}{\sigma(\varphi)} L \equiv \varphi^{e(\varphi)} L, \quad (6)$$

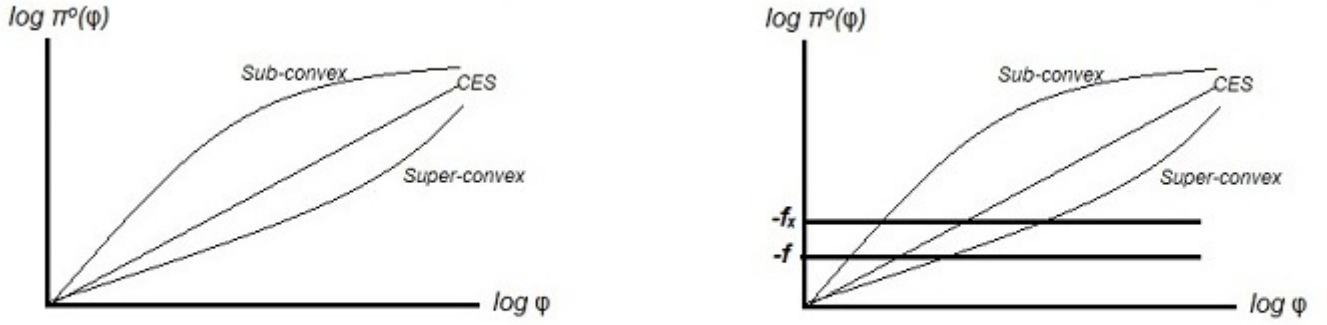
where  $e(\varphi) = \frac{d \log \pi^o(\varphi)}{d \log \varphi} = \eta(\varphi) [\sigma(\varphi) - 1 + S] > 0$ .

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<sup>5</sup>This concept initially introduced by [Melitz \(2003\)](#) refers to the fact that exporting is more selective than serving the domestic market, and so only more productive firms export. This implies then that exporters are on average more productive than non-exporters.

<sup>6</sup>Notice that the price aggregator  $\Lambda$  and individual income  $w$  are absent in this simplified version of the Gorman-Pollak demand function described in equation (1). In deed, as these latter are assumed to be taken as given, and the focus is squarely on the relationship between operating profits and firm productivity, I abstract from both of them in the above expression for expositional simplicity.

Figure 2: Demand Curvature, Firm Selection, and Partitioning of Firms by Export Status.



Panel A. Variable vs Constant Elasticity of Operating Profits

Panel B. Demand Curvature, Selection and Firm Partitioning

It is readily verified that the operating profit is always monotonically increasing in firm productivity regardless of the curvature of demand. However, this latter plays a critical role in determining whether the pace at which the logarithm of operating profits increases with this of firm productivity is constant, or variable.

Clearly, the CES case is very special: as the demand elasticity is exogenous ( $\forall \varphi, \sigma(\varphi) = \sigma$ ), the "superelasticity" collapses to zero ( $S=0$ ), and the relative pass-through is complete ( $\forall \varphi, \eta(\varphi) = 1$ ). This yields a constant elasticity of operating profits with respect to firm productivity (both in logarithms):  $\log \pi^o(\varphi)$  always increases with  $\log \varphi$  at a constant pace regardless of the firm's productivity level ( $\forall \varphi, e(\varphi) = \sigma - 1$ ). This is illustrated by the upward sloping "CES" line in Figure 2, panel A.

Therefore, departing from CES demand allows for a variable elasticity of operating profits:  $e(\varphi)$  may then increase or decrease with firm productivity  $\varphi$ . This implies that the pace at which  $\log \pi^o(\varphi)$  increases with  $\log \varphi$  may be faster or slower, as compared to the CES benchmark, depending on demand curvature.

To show this in a simple way, let us define  $E$  as the productivity elasticity of the elasticity of operating profits:

$$E = \frac{d \log e(\varphi)}{d \log \varphi} = \frac{d \log \eta(\varphi)}{d \log \varphi} + \frac{d \log [\sigma(\varphi) - 1 + S]}{d \log \sigma(\varphi)} \frac{d \log \sigma(\varphi)}{d \log p(\varphi)} \frac{d \log p(\varphi)}{d \log \varphi} \quad (7)$$

After simplification, the final expression of E boils down to:

$$E \equiv -\eta S \begin{cases} < 0 & \text{if demand is sub-convex} \\ = 0 & \text{if demand is CES} \\ > 0 & \text{if demand is super-convex} \end{cases} \quad (8)$$

This reveals that  $\log \pi^o(\varphi)$  is convex/concave in  $\log \varphi$  when demand is super-convex/sub-convex, as shown in Figure 2, panel A. Here, the CES case arises again as a boundary for this comparative statics result. Visibly, under super-convex demands, a movement from left to right along the horizontal axis implies a relatively faster movement (as compared with the CES case) along the  $\log \pi^o(\varphi)$  curve. By contrast, the same movement along the horizontal axis generates a relatively slower movement along the  $\log \pi^o(\varphi)$  curve when demand is sub-convex.

The underlying economics are simple: when demand is super-convex, the demand elasticity increases in firm productivity, and this has three implications for firm profits. First, more productive firms set lower markups, and so a higher productivity induces a more than proportional reduction in price ( $\forall \varphi, \eta(\varphi) > 1$ ). This reveals that under super-convex demands, a firm's initial level of price competitiveness (given by its productivity level) is magnified by lower markups. Second, consumers are more reactive to price variations of varieties supplied by more productive firms. Third, the markup rate ( $\frac{1}{\sigma(\varphi)}$ ) is lower for more productive firms. Combination of the first two effects clearly shows that the super-convex aspect of demand magnifies the sensitivity of firm revenues to firm productivity. As illustrated in Figure 2, panel A, this generates a relatively faster response of operating profits to firm productivity,<sup>7</sup> despite the fact that higher productivity implies lower markup rate.

These patterns are reversed when demand is sub-convex: more productive firms face lower demand elasticity, and so they set higher markups and enjoy higher markup rates ( $\frac{1}{\sigma(\varphi)}$ ). Nevertheless, facing less elastic demand implies that consumers are less sensitive to price variations of the varieties they supply. On top of that, setting higher markups dampens the initial level of price competitiveness of these firms.

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<sup>7</sup>Both in logarithmic terms.

Combination of these last two effects reveals that the sub-convex aspect of demand dampens the sensitivity of firm revenues to firm productivity. Given the dominance of the revenues effect, this immediately implies a relatively slower response of operating profits to firm productivity,<sup>8</sup> as shown in panel A of Figure 2.

Finally, the linearity of the profile of (the logarithm of) operating profits across firms under CES demand clearly shows that this latter is a boundary case. For instance, the CES is a special case where the demand elasticity does not vary with firm productivity. This peculiar property of CES demand has three implications: (i) a firm's level of price competitiveness is solely pinned down by its productivity level; (ii) the elasticity of firm revenues to firm productivity is constant ( $\sigma - 1$ ); and (iii) the markup rate is identical across firms ( $\forall \varphi, \frac{1}{\sigma(\varphi)} = \frac{1}{\sigma}$ ). Such rigidities immediately ensure that the CES delivers an intermediate outcome.

## 2. Novel Implications of Demand Curvature for Firm Selection and Firm Partitioning

### 2.1 Demand Curvature and Firm Selection into the Domestic Market

As illustrated in Figure 2, panel B, for any given level of fixed cost of accessing the domestic market  $f$ , super-convex demands provide an upper bound for the domestic productivity cutoff  $\varphi_d^*$ ,<sup>9</sup> whereas sub-convex demands provide a lower bound. Within these bounds, CES demand delivers an intermediate outcome:  $\varphi_d^*(sub - convex) < \varphi_d^*(CES) < \varphi_d^*(super - convex)$ . This reveals that firm selection is the toughest when demand is super-convex, whereas it is the easiest when demand is sub-convex. Within these two polar cases, the CES yields an intermediate degree of firm selection.

The economic force behind this (partial-equilibrium) result can be explained as follows. When demand is super-convex, the initial level of price competitiveness of more productive firms (implied by their initially high productivity levels) is magnified by lower markups. In addition to that, consumers are more sensitive to price variations of varieties supplied by this category of firms. Hence, as compared with the CES benchmark, the super-convex aspect of demand reinforces the allocation of larger market shares to more productive firms. This induces then a

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<sup>8</sup>Both in logarithmic terms.

<sup>9</sup>According to Melitz (2003), the domestic productivity cutoff corresponds to the productivity level required to make at least zero profits and successfully enter the domestic market.

relatively tougher competitive environment for low productivity firms. As compared to the CES case, setting higher markups makes these less productive firms even less price competitive. This implies then additional difficulty in capturing enough market shares to successfully enter the market, which is reflected by higher domestic cutoff under super-convex demands. All these patterns are reversed when demand is sub-convex.

## 2.2 Demand Curvature and Partitioning of Firms by Export Status

Now let us consider a simple case where the World is comprised of many symmetric countries, and accessing a foreign market via exporting involves only a fixed cost  $f_x$ . In the absence of variable trade costs,<sup>10</sup> I must assume that  $f_x > f$ , to ensure that firms are partitioned by export status as in Melitz (2003). That is, among successful entrants (firms with productivity  $\varphi \geq \varphi_d^*$ ), only more productive firms export (a subset of firms with productivity  $\varphi \geq \varphi_x^* > \varphi_d^*$ ).

Such partitioning of firms is quite standard in heterogeneous firms models. However, the novel idea I explore here is how demand curvature determines the degree of this partitioning of firms by export status. As illustrated in Figure 2, panel B, while the distance between the export and the domestic cutoffs  $[\varphi_x^* - \varphi_d^*]$  is the smallest when demand is super-convex; it is the largest when demand is sub-convex. CES demand, again delivers an intermediate result:

$$[\varphi_x^* - \varphi_d^*] (\text{super-convex}) < [\varphi_x^* - \varphi_d^*] (\text{CES}) < [\varphi_x^* - \varphi_d^*] (\text{sub-convex})$$

The underlying economics are simple: when demand is super-convex, selection is relatively tougher (as compared to the CES benchmark) and only (relatively)<sup>11</sup> more productive firms successfully enter the domestic market. Hence, a relatively large subset of these very productive firms can export. Put differently, these firms are enough productive to successfully enter the domestic market despite tougher competitive conditions implied by super-convex demands. It follows then that a large fringe of these firms is enough price competitive to penetrate the export market. This reveals then that, as compared with the CES benchmark, the partitioning of firms by export status is less pronounced when demand is super-convex. This result is reversed when demand is sub-convex.

<sup>10</sup>Here, I abstract from variable trade costs for expositional simplicity. However, they will be taken in due account in the general equilibrium trade model that I spell out in the next section.

<sup>11</sup>This refers to an immediate comparison with the CES benchmark.

Finally, it is worth noting that these two novel comparative statics results are partial-equilibrium by definition.<sup>12</sup> Yet, the intuitive explanation for both results provides the basis for understanding the general equilibrium behavior. For instance, in Section IV.D, I will show that both results hold in general equilibrium, and that they have crucial implications for the gains from trade.

## IV. Monopolistic Competition with Heterogeneous Firms under Generalized Demands

To illustrate the usefulness of the theoretical approach I have developed in previous sections, I turn next to apply it to a canonical model of international trade, a one-sector, one-factor, multi-country, general equilibrium model of monopolistic competition, where firms are heterogeneous in productivity levels, and countries are symmetric and separated by symmetric trade barriers, as in [Melitz \(2003\)](#).

More specifically, I assume throughout the model that the World economy is comprised of  $N$  symmetric countries which share the same level of labor endowment  $L$  and the same wage  $w$ . This latter is normalized to one ( $w=1$ ) by choice of labor as numéraire. Each economy involves one sector supplying a continuum of horizontally differentiated varieties using labor as a unique production factor. Labor is immobile across countries, and serving foreign markets is only possible via exporting, and this involves both variable and fixed trade costs. Following [Melitz \(2003\)](#), I consider three possible scenarios of higher exposure to trade: (i) a (small) decrease in the variable trade cost; (ii) a (small) decrease in the fixed trade cost; and (iii) a (small) increase in the number of trading countries in the World economy.

I begin with a brief exposition of the supply-side of the model in **Section IV.A**. This latter draws heavily on [Melitz \(2003\)](#), with only one additional assumption: firm productivity is Pareto distributed. In **Section IV.B**, I first embed the general demand system (described in **Section I**) in the model. Then, I derive firm-level variables and spell out the equilibrium conditions. Finally, I solve for the general equilibrium using the "EEM" method. In **Section IV.C**, I expose in detail active sources of welfare gains from trade in closely related literature and in the current paper.

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<sup>12</sup>Thus far, I have worked with [Mrázová and Neary \(2017\)](#)'s "firm's eye view of demand" which is a partial equilibrium approach, as stated by the authors.

I also propose a simple measure for the magnitude of the gains from trade, as well as a sufficient statistic for coexistence of these welfare channels in general equilibrium. In **Section IV.D**, I show that the new comparative statics results (discussed in **Section III.B**) hold in general equilibrium. In particular, I emphasize that demand curvature, by governing these comparative statics at the industry level, plays a critical role in determining the magnitude and the structure of the gains from higher exposure to trade. However, the type of preferences has only a second-order importance from a welfare standpoint. Finally, **Section IV.E** proposes a more granular analysis of the gains from trade. It examines three different scenarios of trade liberalization, and provides a firm-level explanation for the main result of this paper.

## A. Supply

Each country has an endogenous mass  $M_e$  of monopolistically competitive firms that incur a sunk fixed cost  $f_e$  to enter the market. Firms then endogenously enter up to the point at which aggregate profits net of the fixed entry cost,  $f_e$ , are zero. As in [Melitz \(2003\)](#), upon entry, firms draw their initial productivity level  $\varphi$  from a common distribution  $g(\varphi)$ . This latter has positive support over  $[1, +\infty]$  and has a continuous cumulative distribution  $G(\varphi)$ . I assume that  $G(\varphi)$  is Pareto with the same shape parameter  $\theta > 0$  around the World:

**Assumption A1 [Unbounded Pareto]**  $\forall \varphi \in [1, +\infty], G(\varphi) = 1 - \varphi^{-\theta}$ , with  $\theta > 0$ .

Thus far, the above assumption is the unique restriction that I have imposed in the current model.<sup>13</sup> Specifically, I concentrate on the case where the Pareto distribution is unbounded above. Far from being a minor technical detail, this specific feature of the productivity distribution has three main benefits which are worth emphasizing.

First, this unique restriction on the supply side is sufficient to greatly simplify the analysis, while keeping the demand system very flexible and unrestricted. In particular, unbounded Pareto is the central assumption on which rests the simple method that I propose to obtain tractable solutions in general equilibrium under general demands. Second, as is well known, unbounded Pareto is a common distributional assumption in models of monopolistic competition featuring

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<sup>13</sup>Pareto distribution of firm productivity is obviously the most common assumption in models of monopolistic competition incorporating firm-level heterogeneity in productivity levels ([Chaney, 2008](#); [Melitz and Ottaviano, 2008](#); [Feenstra, 2010, 2018](#); [Arkolakis et al., 2012, 2018](#); [Fally, 2019](#)).

firm-level heterogeneity and CES preferences. Hence, imposing Assumption A1 ensures that the novel results highlighted in the current paper are solely attributable to alternative assumptions about the curvature of demand and the type of preferences.

Third, in more recent trade models with heterogeneous firms incorporating non-CES preferences, some authors work with bounded Pareto distribution, which has important implications for the gains from trade. As demonstrated by [Feenstra \(2018\)](#), bounded Pareto is a sufficient condition for gains from (i) selection, (ii) variety, and (iii) reduction in domestic markups to coexist in general equilibrium. Hence, assuming instead that the Pareto distribution is unbounded above rules out the supply side effect in [Feenstra \(2018\)](#), and opens the door for a purely demand-driven condition for the coexistence of these gains in general equilibrium.

Accordingly, I will be able to properly address the following questions: under such standard assumptions on the supply side, what are the novel implications of the flexible demand system considered in this chapter for the gains from trade? What matters more from a welfare standpoint: the curvature of demand or the type of preferences? Clearly, this is a theoretically clean way to highlight the novel welfare predictions that can be derived by solely departing from the homothetic CES benchmark.

## B. Trade Equilibrium

In this section, I characterize the trade equilibrium for arbitrary values of trade costs. I proceed in three steps. I first show how the general demand system, introduced in **Section I**, shapes firm-level variables. Using these latter, I write then the equilibrium conditions more explicitly. Finally, I introduce a new and simple method that I call the "Exponent Elasticity Method" (EEM, hereafter), and I show how it delivers tractable solutions in general equilibrium under general demands.

### 1. Firm-level Variables

Following [Melitz \(2003\)](#), I assume that each firm must incur an overhead production cost  $f$  (in labor units) to start producing for the domestic market. Serving foreign markets is only possible via exporting, and is more costly than operating on the domestic market. For instance, exporting involves two types of costs: a fixed cost of accessing foreign markets  $f_x$ , and a variable trade cost

$\tau$  modeled in the standard iceberg formulation, whereby  $\tau > 1$  units of a good must be shipped for 1 unit to arrive at destination.

Accordingly, for a firm with productivity  $\varphi$ , the constant marginal cost of serving the domestic, and export markets are respectively given by  $\varphi^{-1}$  and  $\tau\varphi^{-1}$ . The first-order condition of profit maximization from equation (3) implies that a firm's pricing rule on these respective markets is given by:

$$\begin{cases} p_d(\varphi) = \varphi^{-1} \frac{\sigma(\varphi)}{\sigma(\varphi)-1} \\ p_x(\varphi) = \tau\varphi^{-1} \frac{\sigma(\varphi)}{\sigma(\varphi)-1} \end{cases} \quad (9)$$

The above expressions of firm-level markups  $\frac{\sigma(\varphi)}{\sigma(\varphi)-1}$  stem from a combination of firm-level heterogeneity in productivity levels on the supply side and flexibility in preferences on the demand side.

As stressed in **Section II**, in the current setting, the demand elasticity always varies with firm productivity  $\sigma(\varphi)$  except under the CES case which imposes a constant demand elasticity.<sup>14</sup>

As to whether the demand elasticity increases or decreases with firm productivity, this hinges on the curvature of demand. As shown in **Section II**, when demand is sub-convex, the price elasticity of demand is decreasing in firm productivity, and thus more productive firms charge higher markups as they face less elastic demand. In contrast, when demand is super-convex, the price elasticity of demand is increasing in firm productivity, and thus more productive firms charge lower markups since they face more elastic demand.

Now by rewriting the generalized Gorman-Pollak demand function in equation (1) more explicitly using the parameterization (described in **Section I.C**), along with invoking the symmetry assumption at the country level and rearranging, the revenues earned from domestic sales and export sales to a given country can be, respectively, written as:<sup>15</sup>

<sup>14</sup>As previously mentioned in **Section II**, beyond the CES case, the demand elasticity may vary with individual consumption (under directly-separable preferences) or with price levels (under indirectly-separable preferences). Since both variables are pinned down by firm productivity  $\varphi$  at equilibrium, it is then both useful and meaningful to write the demand elasticity as a function of firm productivity.

<sup>15</sup>See Appendix A.1 for more details.

$$\begin{cases} r_d(\varphi) = p_d(\varphi)^{1-\sigma(\varphi)} P^{a(\varphi)} L \\ r_x(\varphi) = p_x(\varphi)^{1-\sigma(\varphi)} P^{a(\varphi)} L \end{cases} \quad (10)$$

where  $L$  and  $P$  denote the aggregate expenditure and the partial equilibrium price index in every country, and  $a(\varphi) = \frac{\beta + \alpha \sigma(\varphi)}{\beta + \alpha \bar{\sigma}}$ . Then, with the aid of the Lerner index,<sup>16</sup> I can simply express operating profits as revenues divided by the price elasticity of demand, as is standard in the literature:

$$\begin{cases} \pi_d^o(\varphi) = \frac{r_d(\varphi)}{\sigma(\varphi)} \\ \pi_x^o(\varphi) = \frac{r_x(\varphi)}{\sigma(\varphi)} \end{cases} \quad (11)$$

Finally, taking into account the presence of fixed costs, domestic and export profits can be, respectively, written as:

$$\begin{cases} \pi_d(\varphi) = \pi_d^o(\varphi) - f \\ \pi_x(\varphi) = \pi_x^o(\varphi) - f_x \end{cases} \quad (12)$$

Two key implications of the symmetry assumption are worth emphasizing. First, by ensuring that all markets are identical both in terms of size  $L$  and intensity of competition (captured by the price index  $P$ ), it implies that the difference between domestic and export profits is solely driven by the presence of trade frictions. Second, the symmetry assumption also ensures that all countries share the same average demand elasticity  $\bar{\sigma}$ . I can thus concentrate on the firm-specific aspect of the demand elasticity and examine its implications in general equilibrium.

## 2. Equilibrium Conditions

As in [Melitz \(2003\)](#), since operating profits are monotonically increasing in productivity, the presence of fixed costs of accessing domestic and foreign markets,  $f$  and  $f_x$ , implies the existence

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<sup>16</sup>As is well known, the Lerner index stems from the first-order condition of profit maximization, and implies that the markup rate is inversely related to the demand elasticity:  $\frac{p(\varphi) - \varphi^{-1}}{p(\varphi)} = \sigma(\varphi)^{-1}$ .

of two productivity cutoffs. The first is the domestic productivity cutoff, denoted by  $\varphi_d^*$ , and defined as the minimum productivity level required to make non-negative profits on the domestic market. The second is the export productivity cutoff  $\varphi_x^*$  such that among successful entrants in any country, only those with a productivity level of at least  $\varphi_x^*$  find it profitable to export.

By their definition, the domestic cutoff must then satisfy the zero profit condition on the domestic market (ZPCD):  $\pi_d(\varphi_d^*) = 0$ . Similarly, the export cutoff must satisfy the zero profit condition on the export market (ZPCX):  $\pi_x(\varphi_x^*) = 0$ . Using these two equilibrium conditions, the cutoff levels can be identified implicitly by:

$$\begin{cases} \text{(ZPCD)} \quad \varphi_d^* : \sigma_d^*(\varphi_d^*)^{-1} p_d^*(\varphi)^{1-\sigma_d^*(\varphi_d^*)} P^{a_d^*(\varphi_d^*)} L = f \\ \text{(ZPCX)} \quad \varphi_x^* : \sigma_x^*(\varphi_x^*)^{-1} p_x^*(\varphi)^{1-\sigma_x^*(\varphi_x^*)} P^{a_x^*(\varphi_x^*)} L = f_x \end{cases} \quad (13)$$

where  $P$  is the price index in any country and is given by:

$$P = M_e^{-1} \left[ \int_{\varphi_d^*}^{+\infty} p_d(\varphi)^{1-\sigma(\varphi)} g(\varphi) d\varphi + (N-1) \int_{\varphi_x^*}^{+\infty} p_x(\varphi)^{1-\sigma(\varphi)} g(\varphi) d\varphi \right]^{-1} \quad (14)$$

As previously mentioned in **Section II.B**, for any arbitrary values of the per-unit trade cost  $\tau$ , assuming that the fixed cost of exporting is larger than the fixed cost of accessing the domestic market,  $f_x > f$ , ensures that exporting is always a more selective activity:  $\forall \tau \geq 1, \varphi_x^* > \varphi_d^*$ . This partitioning implies then that exporters are on average more productive than firms serving only the domestic market.

Upon sinking the fixed entry cost  $f_e$ , all entering firms expect positive profits. Yet, as mentioned above, only successful entrants on a given market (either domestic, or export) earn positive profits. This, in turn, implies that the average profit at the industry level is positive. Following [Melitz \(2003\)](#), the average profit, net of the sunk entry cost, must be set to zero to ensure a bounded mass of entrants at equilibrium. This requires imposing the Free Entry condition (FE). This equilibrium condition equalizes the average expected profit conditional on successful entry to the sunk entry cost  $f_e$ :

$$[1 - G(\varphi_d^*)] \int_{\varphi_d^*}^{+\infty} \pi_d(\varphi) \mu_d(\varphi) d\varphi + (N - 1)[1 - G(\varphi_x^*)] \int_{\varphi_x^*}^{+\infty} \pi_x(\varphi) \mu_x(\varphi) d\varphi = f_e \quad (15)$$

where  $\mu_d(\varphi)$  and  $\mu_x(\varphi)$  correspond to the productivity distribution conditionally on successful entry, respectively, on the domestic, and the export market:

$$\begin{cases} \mu_d(\varphi) = \frac{g(\varphi)}{[1-G(\varphi_d^*)]} & \forall \varphi \geq \varphi_d^* \\ \mu_x(\varphi) = \frac{g(\varphi)}{[1-G(\varphi_x^*)]} & \forall \varphi \geq \varphi_x^* \end{cases}$$

Finally, the last equilibrium condition is the Labor Market Clearing condition (LMC). This latter ensures that in any country, total labor demand equates total labor supply:

$$M_e [f_e + [1 - G(\varphi_d^*)] \int_{\varphi_d^*}^{+\infty} l_d(\varphi) \mu_d(\varphi) d\varphi + (N - 1)[1 - G(\varphi_x^*)] \int_{\varphi_x^*}^{+\infty} l_x(\varphi) \mu_x(\varphi) d\varphi] = L \quad (16)$$

where  $l_d(\varphi)$  and  $l_x(\varphi)$  correspond, respectively, to the amount of labor used by a  $\varphi$ -productivity firm to serve the domestic, and the export market:

$$\begin{cases} l_d(\varphi) = \frac{\sigma(\varphi)-1}{\sigma(\varphi)} r_d(\varphi) + f & \forall \varphi \geq \varphi_d^* \\ l_x(\varphi) = \frac{\sigma(\varphi)-1}{\sigma(\varphi)} r_x(\varphi) + f_x & \forall \varphi \geq \varphi_x^* \end{cases}$$

To summarize, there are 4 equilibrium conditions: the Labor Market Clearing condition (LMC), the Free Entry condition (FE), and two Zero Cutoff Profit conditions: (ZPCD) and (ZPCX). By imposing the symmetry assumption at the country level, along with normalizing wage to unity (by choice of labor as numéraire), the set of unknown equilibrium variables is reduced to 4: the mass of entrants  $M_e$ , the price index  $P$ , and the two productivity cutoffs  $\varphi_d^*$  and  $\varphi_x^*$ .

### 3. Solving for the General Equilibrium

I start with solving for the equilibrium mass of entrants  $M_e$  using the Free Entry (FE) and the Labor Market Clearing (LMC) conditions:

$$M_e = \frac{L}{\bar{\sigma} [f_e + [1 - G(\varphi_d^*)] f + (N - 1)[1 - G(\varphi_x^*)] f_x]} \quad (17)$$

where  $\bar{\sigma} = \frac{1}{N} \int_{\varphi_d^*}^{+\infty} \sigma(\varphi) g(\varphi) d\varphi + \frac{N-1}{N} \int_{\varphi_x^*}^{+\infty} \sigma(\varphi) g(\varphi) d\varphi$  is the weighted average demand elasticity to be faced by a successful entrant while serving the World market.

Since increased generality raises tractability issues, I resort to a new method that I call "the Exponent Elasticity Method" (EEM, hereafter). The objective of this simple method is to deliver a tractable solution for the general equilibrium price index despite added flexibility on the demand side. The starting point is the partial equilibrium price index in equation (14):

$$P = M_e^{-1} \left[ \underbrace{\int_{\varphi_d^*}^{+\infty} p_d(\varphi)^{1-\sigma(\varphi)} g(\varphi) d\varphi}_{I_d} + (N-1) \underbrace{\int_{\varphi_x^*}^{+\infty} p_x(\varphi)^{1-\sigma(\varphi)} g(\varphi) d\varphi}_{I_x} \right]^{-1}$$

Clearly, the mathematical challenge consists in solving for both integrals ( $I_d, I_x$ ) without assuming that the demand elasticity is identical across firms ( $\forall \varphi, \sigma(\varphi) = \sigma$ ) and then using this latter as a constant for integrating. Such simplicity is only possible under CES demand, which is the unique case where it is possible to solve for these integrals. As the general demand system considered in this paper encompasses the CES and more flexible alternatives allowing the demand elasticity to vary across firms, it is then impossible to solve for these integrals under general demands.

Given the impossibility to solve for these integrals in the current setting, the key idea that the EEM method proposes is to locally approximate both integrals ( $I_d, I_x$ ) around the equilibrium with a multiplicative equivalent which has a finite number of determinants, such as the exponent of each determinant embodies the elasticity of the average price with respect to it.<sup>17</sup> This requires a five-step procedure that I expose in detail in Appendix A.2. By implementing this simple method, I obtain a tractable solution for the general equilibrium price index:

<sup>17</sup>To be precise, this corresponds to the average price to the power of  $(1 - \bar{\sigma})$ :  $\bar{p}_d^{1-\bar{\sigma}_d}$  and  $\bar{p}_x^{1-\bar{\sigma}_x}$ .

$$P \equiv c_E^{[1 + \varepsilon_{I_x}(P)]^{-1}} L^{-[\frac{1 + \varepsilon_{I_x}(L)}{1 + \varepsilon_{I_x}(P)}]} T(\tau, f_x, N)^{-[1 + \varepsilon_{I_x}(P)]^{-1}} \quad (18)$$

where  $c_E = \bar{\sigma} [f_e + [1 - G(\varphi_d^*)] f + (N - 1)[1 - G(\varphi_x^*)] f_x]$ , and  $T(\tau, f_x, N) = N \tau^{-\theta} f_x^{-\frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1)]}{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]}}$  is an index of exposure to trade. That is, an increase in an economy's exposure to trade occurring through a decrease in variable or fixed trade costs, or an increase in the number of trading countries in the World economy, implies a higher level of  $T(\tau, f_x, N)$ . As shown in Appendix A.2, both elasticities  $\varepsilon_{I_x}(L)$  and  $\varepsilon_{I_x}(P)$  take a simple form, and are respectively given by:

$$\begin{cases} \varepsilon_{I_x}(L) = \frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1)]}{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]} > 0 \\ \varepsilon_{I_x}(P) = a_x^*(\varphi_x^*) \frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1)]}{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]} > 0 \end{cases} \quad (19)$$

Given the positive sign of  $\varepsilon_{I_x}(P)$ , it is then readily verified that higher exposure to trade induces tougher competition on the domestic market of any economy by lowering its price index. Now by plugging the above general equilibrium expression of the price index in the zero profit conditions in equation (13) and rearranging, I obtain the following expressions of the domestic and the export productivity cutoffs, in general equilibrium :

$$\varphi_d^* \equiv f^{\varepsilon_{\varphi_d^*}(f)} c_E^{\gamma_d^*} L^{\varepsilon_{\varphi_d^*}(L)} T(\tau, f_x, N)^{\varepsilon_{\varphi_d^*}(T)} \quad (20)$$

where the above exponents are respectively given by:

$$\begin{cases} \varepsilon_{\varphi_d^*}(f) = \frac{1}{\eta_d^*[\sigma_d^*(\varphi_d^*) - 1 + S]} \\ \gamma_d^* = -\frac{a_d^*(\varphi_d^*)}{\eta_d^*[\sigma_d^*(\varphi_d^*) - 1 + S]} [1 + \varepsilon_{I_x}(P)]^{-1} \\ \varepsilon_{\varphi_d^*}(L) = \frac{1}{\eta_d^*[\sigma_d^*(\varphi_d^*) - 1 + S]} [a_d^*(\varphi_d^*) \frac{1 + \varepsilon_{I_x}(L)}{1 + \varepsilon_{I_x}(P)} - 1] \\ \varepsilon_{\varphi_d^*}(T) = \frac{a_d^*(\varphi_d^*)}{\eta_d^*[\sigma_d^*(\varphi_d^*) - 1 + S]} [1 + \varepsilon_{I_x}(P)]^{-1} \end{cases}$$

Similarly, the general equilibrium export cutoff can be written as:

$$\varphi_x^* \equiv \tau f_x^{\varepsilon_{\varphi_x^*}(f_x)} c_E^{\gamma_x^*} L^{\varepsilon_{\varphi_x^*}(L)} T(\tau, f_x, N)^{\varepsilon_{\varphi_x^*}(T)} \quad (21)$$

where the above exponents are respectively given by:

$$\begin{cases} \varepsilon_{\varphi_x^*}(f_x) = \frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} \\ \gamma_x^* = -\frac{a_x^*(\varphi_x^*)}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} [1 + \varepsilon_{I_x}(P)]^{-1} \\ \varepsilon_{\varphi_x^*}(L) = \frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} [a_x^*(\varphi_x^*) \frac{1 + \varepsilon_{I_x}(L)}{1 + \varepsilon_{I_x}(P)} - 1] \\ \varepsilon_{\varphi_x^*}(T) = \frac{a_x^*(\varphi_x^*)}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} [1 + \varepsilon_{I_x}(P)]^{-1} \end{cases}$$

Finally, I can solve for the general equilibrium operating profits on the domestic and the export markets by plugging the general equilibrium expression of the price index in their respective partial equilibrium expression provided in equation (11):

$$\begin{cases} \forall \varphi \geq \varphi_d^*, \pi_d^o(\varphi) = c_E^{\gamma(\varphi)} \varphi^{\eta(\varphi)[\sigma(\varphi)-1+S]} L^{\varepsilon_{\pi^o(\varphi)}(L)} T(\tau, f_x, N)^{\varepsilon_{\pi^o(\varphi)}(T)} \\ \forall \varphi \geq \varphi_x^*, \pi_x^o(\varphi) = c_E^{\gamma(\varphi)} \varphi^{\eta(\varphi)[\sigma(\varphi)-1+S]} \tau^{\eta(\varphi)[1-\sigma(\varphi)-S]} L^{\varepsilon_{\pi^o(\varphi)}(L)} T(\tau, f_x, N)^{\varepsilon_{\pi^o(\varphi)}(T)} \end{cases} \quad (22)$$

where  $\gamma(\varphi) = a(\varphi) [1 + \varepsilon_{I_x}(P)]^{-1}$ ,  $\varepsilon_{\pi^o(\varphi)}(L) = 1 - a(\varphi) [\frac{1 + \varepsilon_{I_x}(L)}{1 + \varepsilon_{I_x}(P)}]$ , and finally,  $\varepsilon_{\pi^o(\varphi)}(T)$  is given by  $\varepsilon_{\pi^o(\varphi)}(T) = -a(\varphi) [1 + \varepsilon_{I_x}(P)]^{-1}$ .

### C. Welfare Analysis

In the previous section, I have solved for the main equilibrium variables ( $M_e, P, \varphi_d^*, \varphi_x^*$ ). Now I use their general equilibrium expressions to solve for two key variables for welfare analysis: the total mass of firms competing in a single country,  $M$ , and the weighted average productivity of these firms,  $\bar{\varphi}$ . I proceed as follows.

Let  $M_d$  denote the equilibrium mass of domestic firms. Using the equilibrium mass of entrants  $M_e$ , and the general equilibrium expression of the domestic cutoff  $\varphi_d^*$ ,  $M_d$  can be then written as:

$M_d = M_e [1 - G(\varphi_d^*)]$ . Similarly, the equilibrium mass of exporting firms in any country is given by  $M_x = M_e [1 - G(\varphi_x^*)]$ . The total mass of firms competing in any country (or, alternatively the total mass of available varieties in any country) is then given by  $M = M_d + (N - 1) M_x$ .

As in Melitz (2003), the weighted average productivity of all firms (both domestic and foreign exporters) competing in a single country can be written as:

$$\bar{\varphi} = \frac{M_d}{M_W} \int_{\varphi_d^*}^{+\infty} \varphi \mu_d(\varphi) d\varphi + \frac{(N - 1) M_x}{M_W} \int_{\varphi_x^*}^{+\infty} \varphi \mu_x(\varphi) d\varphi \quad (23)$$

where  $M_W = N M_e$  is the total mass of entrants at the World level, and can be thought of as a proxy for the size of the World market. By solving for the above integrals, and using the general equilibrium expressions of the domestic and the export cutoffs and rearranging,  $\bar{\varphi}$  can be then written as a function of the domestic cutoff:

$$\bar{\varphi} = \varphi_d^* \underbrace{\Psi(\cdot)}_{>1} \quad (24)$$

where  $\Psi(\cdot)$  is larger than one and can be considered as constant since Assumption A1 implies a constant mean-to-min ratio  $\frac{\bar{\varphi}}{\varphi_d^*}$ .<sup>18</sup>

Since Free Entry implies that there are zero net profits at equilibrium, the real wage,  $P^{-1}$ , is then a sufficient measure for welfare per worker  $W$  in this setting:

$$W = P^{-1} = M \bar{p}(\bar{\varphi})^{1-\bar{\sigma}(\bar{\varphi})} \equiv M \bar{\varphi}^{\bar{\eta}[\bar{\sigma}(\bar{\varphi})-1]} \quad (25)$$

Now by plugging the final expression of  $\bar{\varphi}$  from equation (24) in the above expression and simplifying,<sup>19</sup> I can write welfare per capita  $W$  as a function of solely the total number of available

<sup>18</sup>As is well known, this a straightforward implication of unbounded Pareto. For expositional clarity, the explicit expression of  $\Psi(\cdot)$  is relegated to Appendix A.4

<sup>19</sup>Visibly, the simplification consists simply in dropping the constant  $\Psi(\cdot)$  from the final expression of welfare per worker in equation (26).

varieties  $M$ , and the domestic cutoff  $\varphi_d^*$ :

$$W \equiv M \varphi_d^* \bar{\eta}^{[\bar{\sigma}(\bar{\varphi})-1]} \quad (26)$$

It is now worth noting that all the variables that I have solved for in general equilibrium mainly depend on the trade exposure index  $T(\tau, f_x, N)$ . Hence, trade liberalization can be generically modeled as an increase in  $T(\tau, f_x, N)$ , which simply reflects that an economy is more exposed to trade. Following [Melitz \(2003\)](#), I can then go more granular and separately examine three different mechanisms that lead to an increase in the exposure of an economy to trade. As previously mentioned, these scenarios include: (i) a small reduction in the variable trade cost  $\tau$ ; (ii) a small decrease in the fixed cost of exporting  $f_x$ ; and (iii) a small increase in the number of trading countries  $N$ .

### 1. Measuring the Gains From Trade

The final expression of welfare per capita  $W$  in equation (26) clearly shows that consumer welfare is more sensitive to changes in the domestic cutoff  $\varphi_d^*$  than to variations of the total mass of available varieties  $M$ . Moreover, inspection of the general equilibrium expression of the domestic cutoff immediately reveals that an increase in the exposure to trade, higher  $T(\tau, f_x, N)$ , induces an increase in the the productivity cutoff for domestic firms  $\varphi_d^*$ . This ensures then that higher exposure to trade, occurring through any of the aforementioned mechanisms, always generates a welfare gain. In other words, by inducing tougher selection on the domestic market, trade is always welfare improving even if this selection effect may lead to a net decrease in the total mass of available varieties.

Based on this standard result, due to [Melitz \(2003\)](#), I can use the elasticity of the domestic cutoff with respect to the trade exposure index as a sufficient measure of the magnitude of the gains from trade (GFT, hereafter):<sup>20</sup>

$$GFT = \varepsilon_{\varphi_d^*}(T(\tau, f_x, N)) = \frac{d \log \varphi_d^*}{d \log T(\tau, f_x, N)} > 0 \quad (27)$$

<sup>20</sup>Notice that I use the same notation as in **Section I**, whereby  $\varepsilon_Y(x) = \frac{d \log Y}{d \log x}$  is the elasticity of  $Y$  with respect to  $x$ .

Then, when the welfare analysis gets more granular, the magnitude of the gains from higher exposure to trade occurring under each specific scenario is simply given by: <sup>21</sup>

$$\begin{cases} GFT(\tau^-) = -\varepsilon_{\varphi_d^*}(\tau) = \frac{d \log \varphi_d^*}{d \log T(\tau, f_x, N)} \left( -\frac{d \log T(\tau, f_x, N)}{d \log \tau} \right) \\ GFT(f_x^-) = -\varepsilon_{\varphi_d^*}(f_x) = \frac{d \log \varphi_d^*}{d \log T(\tau, f_x, N)} \left( -\frac{d \log T(\tau, f_x, N)}{d \log f_x} \right) \\ GFT(N^+) = \varepsilon_{\varphi_d^*}(N) = \frac{d \log \varphi_d^*}{d \log T(\tau, f_x, N)} \left( \frac{d \log T(\tau, f_x, N)}{d \log N} \right) \end{cases} \quad (28)$$

While the above measures are quite standard, here the novel idea I explore is how alternative assumptions about the curvature of demand and the nature of preferences affect the magnitude of the gains from trade. Does this latter mainly hinge on the curvature of demand or the type of preferences? Departing from the CES benchmark, under which alternative assumptions about preferences and demand, gains from trade are smaller or larger than those obtained under the CES? The first objective of the current paper is to address these two questions. This is what I do in **Sections IV.D and IV.E**.

Before that, I provide first a detailed exposition of the sources of gains from trade that are theoretically possible both in closely related literature and in the current chapter. Then, I recall the second objective of this chapter, which consists in separately examining the novel implications of the curvature of demand and the nature of preferences for the coexistence of these sources of welfare gains from trade in general equilibrium.

## 2. Sources of Welfare Gains From Trade

### 2.1 Potential Sources under Monopolistic Competition with Heterogeneous Firms

Under monopolistic competition, the potential sources of welfare gains from trade are threefold. First, consumers have access to a wider range of products, as newly imported varieties become available on the domestic market. This first source is emphasized by [Krugman \(1980\)](#), and can be referred to as a "gross variety gain". The second gain arises in a setting with heterogeneous

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<sup>21</sup>Notice that  $GFT(\tau^-)$ ,  $GFT(f_x^-)$ , and  $GFT(N^+)$  reflect three different scenarios where higher exposure to trade occurs, respectively, through (i) a small reduction in the variable trade cost ( $\tau^-$ ); (ii) a small reduction in the fixed trade cost ( $f_x^-$ ); and (iii) a small increase in the number of trading countries ( $N^+$ )

firms as in [Melitz \(2003\)](#) and can be explained as follows. As trade reallocates market shares from domestic firms to relatively more productive exporters, the least productive domestic firms are forced to exit the market, which leads to an increase in (weighted) average productivity at the industry level. This welfare channel highlighted by [Melitz \(2003\)](#) can be thought of as a "gain from selection", or equivalently called [Melitz \(2003\)](#)'s "selection effect". The third source consists in a reduction in the markups charged by domestic firms due to import competition. This is so-called "pro-competitive effect of trade" is due to [Krugman \(1979\)](#).

## **2.2 Active Sources in Previous Trade Models with Heterogeneous Firms and CES preferences**

Trade models incorporating firm heterogeneity and CES utility have overwhelmingly substantiated that gains from trade solely stem from the "selection effect", due to [Melitz \(2003\)](#). The absence of the two other welfare channels is caused by the rigidity of CES preferences. First, under monopolistic competition, CES utility imposes constant markups, which precludes then the "pro-competitive effect of trade" to occur. Second, as demonstrated by [Feenstra \(2010\)](#), the consumer's gross gain from newly imported varieties exactly cancels out with the loss of domestic varieties (due to firm exit) when preferences are CES. Thus, trade yields zero net gains from variety.<sup>22</sup>

## **2.3 Active Sources in More Recent Trade Models with Heterogeneous Firms and Non-CES preferences**

In more recent trade models incorporating firm heterogeneity and non-CES preferences, identifying the welfare channel(s) that is (are) operative in general equilibrium is more complex and requires more subtle distinctions between two different modeling approaches.

On the one hand, a large body of work ([Melitz and Ottaviano, 2008](#); [Bertoletti et al., 2018](#); [Feenstra, 2018](#); [Arkolakis et al., 2018](#); [Fally, 2019](#)) abstracted from fixed costs, restricted demand to be sub-convex, and assumed that it exhibits a choke price to ensure selection into both domestic

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<sup>22</sup>[Feenstra \(2010\)](#) derives this result using also unbounded Pareto distribution of firm productivity on the supply side. More recent work by [Melitz and Redding \(2015\)](#) emphasizes that trade yields only gains from selection under CES demand regardless of whether the Pareto distribution is bounded or unbounded above. This ensures then that [Feenstra \(2010\)](#)'s result is solely driven by the rigidity of CES preferences.

and export markets.<sup>23</sup> A key implication of this modeling approach is that the type of preferences determines whether a welfare channel is operative or not, and thus shapes the structure of the gains from trade. [Arkolakis et al. \(2018\)](#) focus on directly-separable as well as homothetic preferences (excluding the CES case). They abstract from variety gains given the absence of fixed costs in their setting and emphasize that trade generates only gains from selection, as in [Melitz \(2003\)](#). In particular, they show that the "pro-competitive effect of trade" does not represent an additional source of gains from trade. In fact, upon trade liberalization, the reduction in domestic markups is either dominated by the increase in foreign markups when preferences are directly-separable, or exactly offset by higher foreign markups when preferences are homothetic. Using indirectly-separable preferences, [Bertoletti et al. \(2018\)](#) find a different result. They show that trade liberalization yields only pure variety gains as in [Krugman \(1980\)](#) despite firm heterogeneity in productivity levels.

On the other hand, only few papers went beyond the CES using a different modeling approach ([Mrázová and Neary, 2017](#); [Zhelobodko et al., 2012](#)). Specifically, while allowing demand to be either sub-convex or super-convex, both papers concentrate on directly-separable preferences.<sup>24</sup> Moreover, both papers assume that trade is frictionless and model globalization as an increase in the size of the World market.<sup>25</sup> The last important detail is that in the absence of a choke price on the demand side, both papers incorporate fixed costs to ensure selection into the domestic market. Given the absence of trade frictions, there is no selection into exporting in both papers, and so all active firms serve the World market. This modeling approach has three implications for the gains from trade that are worth noting.

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<sup>23</sup>Imposing a choke price is a necessary restriction to ensure firm selection in the absence of fixed costs. It also rules out the CES case in papers considering general demand systems, such as [Arkolakis et al. \(2018\)](#) and [Fally \(2019\)](#).

<sup>24</sup>Notice that [Mrázová and Neary \(2017\)](#) and [Zhelobodko et al. \(2012\)](#) characterize the behavior of the demand elasticity in the same way, yet using a different terminology. In deed, [Zhelobodko et al. \(2012\)](#) use the concept of RLV (the Relative Love for Variety) to measure the elasticity of the inverse demand. This latter corresponds to the elasticity of the marginal sub-utility with respect to consumption levels. An increasing RLV implies then that the price elasticity of demand decreases in consumption, which corresponds to the case of "sub-convex demand" in the terminology of [Mrázová and Neary \(2017\)](#). Similarly, a decreasing RLV in [Zhelobodko et al. \(2012\)](#) is equivalent to the case that [Mrázová and Neary \(2017\)](#) call "super-convex demand".

<sup>25</sup>To be precise, while [Zhelobodko et al. \(2012\)](#) study the impact of an enlargement in market size of a given trading country ( $L$ ), [Mrázová and Neary \(2017\)](#) examine the impact of an increase in the number of trading countries at the World level ( $N$ ). Under cross-country symmetry in both papers, this is then equivalent to focusing on the intensive margin of the World market size in the former paper, and to concentrating on the extensive margin in the latter.

First, the variety effect occurs only on "cutoff" domestic varieties. In particular, globalization may induce a decrease or an increase in the number of domestic varieties in any market. Second, Melitz (2003)'s selection effect is operative only when demand is sub-convex (Mrázová and Neary, 2017; Zhelobodko et al., 2012).<sup>26</sup> Third, when the selection effect of trade occurs, it is fully driven by the pro-competitive reduction in markups of less efficient firms (Zhelobodko et al., 2012). This implies that the magnitude of gains from selection is governed by this of the pro-competitive effect of trade. Nevertheless, this decrease in markups of less productive firms (occurring under sub-convex demands) can not be considered as an additional gain from trade because it exactly cancels out with the increase in markups set by more productive firms (Mrázová and Neary, 2017).

In short, all the aforementioned papers convey the same message: regardless of the type of preferences and whether demand is CES or sub-convex, trade liberalization delivers only gains from selection as in Melitz (2003).<sup>27</sup> In this regard, the recent work by Feenstra (2018) where he restores a theoretical role for the pro-competitive effect of trade and gains from variety can be seen as an exception in the literature. In particular, Feenstra (2018) shows that the three welfare channels (that are theoretically possible under monopolistic competition and firm heterogeneity) can be simultaneously operative if and only if firm productivity is drawn from a bounded Pareto distribution.

#### 2.4 Active Sources of Welfare Gains in the Current Paper

In this paper, I propose a different modeling approach which combines standard assumptions on the supply side with a flexible demand system, while taking variable and fixed trade barriers in due account. The main objective of this approach is to identify a demand-based condition which is sufficient to restore a theoretical role for these three sources of welfare gains from trade. Despite apparent similarity to Feenstra (2018), the main difference is in the nature of the condition under which these three welfare channels are simultaneously operative in general equilibrium. This theoretically possible result can be hereafter referred to as a state of "coexistence of gains from trade". As hinted to in the previous paragraph, in Feenstra (2018), the coexistence of

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<sup>26</sup>As shown by Mrázová and Neary (2017), globalization encourages entry of less efficient firms when demand is super-convex. Similarly, Zhelobodko et al. (2012) find that market size enlargement triggers entry of less productive firms under decreasing RLV.

<sup>27</sup>While retaining Bertoletti et al. (2018) as an exception in this regard.

gains from selection, variety, and domestic markup reduction in general equilibrium hinges on a feature of the supply side: productivity distribution must be Pareto and bounded above. In contrast, in the current model, the distribution is Pareto, but unbounded above, which immediately rules out the supply side effect in [Feenstra \(2018\)](#). Hence, here the novel aspect is that the coexistence of these three sources of gains from trade solely hinges on demand conditions.

### 3. A Sufficient Statistic for Coexistence of Gains From Trade

As stressed in the two previous subsections, [Melitz \(2003\)](#)'s selection effect is always operative in the current model. In fact, the general equilibrium expression of the domestic cutoff clearly shows that for any curvature of demand and type of preferences, higher exposure to trade always constrains less efficient domestic firms to exit the market. While ensuring that trade always delivers welfare gains as in [Melitz \(2003\)](#), this result indicates that trade liberalization has countervailing effects on the total mass of available varieties. On the one hand, higher exposure to trade leads to an increase in the mass of imported varieties. On the other hand, it forces the least productive firms to exit the market, and leads then to a reduction in the mass of domestic varieties. The sign of the net variety effect (NVE, hereafter) will thus depend on a horse race between the initial gross variety gain from newly imported varieties as in [Krugman \(1980\)](#), and the loss from disappearing domestic varieties due to firm exit as in [Melitz \(2003\)](#). Therefore, the NVE can be simply measured as follows:

$$NVE = \varepsilon_{M_m}(T(\tau, f_x, N)) - \varepsilon_{M_d}(T(\tau, f_x, N)) \quad (29)$$

where  $M_m = (N - 1)M_x$  is the total mass of imported varieties in any trading country, and  $\varepsilon_{M_m}(T(\tau, f_x, N))$ ,  $\varepsilon_{M_d}(T(\tau, f_x, N))$  are, respectively, the elasticities of the equilibrium mass of imported varieties, and the equilibrium mass of domestic firms with respect to the trade exposure index.<sup>28</sup> Inspection of the above expression clearly indicates that net variety gains and gains from selection coexist if and only if the net variety effect of trade is strictly positive:  $NVE > 0$ .

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<sup>28</sup>Notice that the general equilibrium export cutoff (embodied in  $M_m$ ) is written as a function of  $\tau$ ,  $f_x$  and  $T(\tau, f_x, N)$ , it is then more convenient to separately examine the net variety effect of trade under each of the three scenarios of trade liberalization considered in this paper.

Departing from the CES benchmark, the flexible demand system considered in this paper allows for the demand elasticity to be firm-specific, which implies the existence of variable markups in the current setting. This raises then the following question: Is it theoretically possible that the three sources of gains from trade coexist in the current model? Put differently, under which condition(s) the coexistence of net variety gains and gains from selection can be accompanied by a pro-competitive reduction in domestic markups?

As I will show in the next two sections (**IV.D** and **IV.E**), the current paper provides a clear and gradual response to this question. Departing from the homothetic CES as a boundary case where trade yields a zero net variety effect ( $NVE=0$ ),<sup>29</sup> I show that the sign of the NVE is solely pinned down by the curvature of demand. That is, whether gains from selection and net variety coexist in general equilibrium ( $NVE > 0$ ) or not ( $NVE < 0$ ) crucially depends on whether demand is sub-convex or super-convex. Then, once the coexistence of these two gains is ensured, I show that whether they are accompanied by the pro-competitive effect on domestic markups or not, depends on whether preferences are directly- or indirectly-separable.

As mentioned earlier in this section, the objective of this paper is twofold. First, I separately examine the role that the curvature of demand and the type of preferences play in determining the magnitude of the gains from trade. Second, I characterize sufficient conditions for the three sources of welfare gains to coexist in general equilibrium. This is what I focus on next.

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<sup>29</sup>As I will show next, under CES demand, I replicate [Feenstra \(2010\)](#)'s result, whereby trade yields zero net gains from variety ( $NVE=0$ ) and generates thus only gains from selection.

## D. Demand Curvature and Gains from Higher Exposure to Trade

This section shows how demand curvature plays a crucial role in determining the magnitude of the gains from higher exposure to trade. It also provides an industry-level explanation for this novel result. To do so, I proceed in three steps.

First, I derive a general formula for the gains from increased exposure to trade occurring through any of the three aforementioned scenarios of trade liberalization. This simple formula is presented and interpreted in **Theorem 1**. Second, I derive two novel results showing how demand curvature determines, not only, the degree of toughness of firm selection on any market, but also, the degree of their partitioning by export status. These two general equilibrium results are respectively presented in **Theorem 2**, and **Theorem 3**. Finally, I connect these three theorems and I provide an industry-level explanation for the new welfare result, whereby demand curvature, by governing the toughness of firm selection and the degree of their partitioning, plays a critical role in determining the gains from trade.

### 1. A Simple Welfare Formula

**Theorem 1.** *Under Generalized Gorman-Pollak demand, unbounded Pareto distribution of firm productivity and country-level symmetry, welfare gains from increased exposure to trade, occurring through any mechanism of trade liberalization, are given by*

$$GFT(T) = \frac{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]}{\eta_d^*[\sigma_d^*(\varphi_d^*) - 1 + S]} \frac{a_d^*(\varphi_d^*)}{a_x^*(\varphi_x^*) \theta + \Delta_x} > 0$$

where  $\theta$  is the shape parameter of the Pareto distribution,  $S$  is the "superelasticity" of [Kimball \(1995\)](#), and  $\Delta_x = [\eta_x^*(\sigma_x^*(\varphi_x^*) - 1) - a_x^*(\varphi_x^*) \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1) + S]$  reflects the curvature of demand.  $a_d^*(\varphi_d^*)$  and  $a_x^*(\varphi_x^*)$ , are respectively given by  $a_d^*(\varphi_d^*) = \frac{\beta + \alpha \sigma_d^*(\varphi_d^*)}{\beta + \alpha \bar{\sigma}}$ ,  $a_x^*(\varphi_x^*) = \frac{\beta + \alpha \sigma_x^*(\varphi_x^*)}{\beta + \alpha \bar{\sigma}}$ .

Finally,  $\alpha$  and  $\beta$  are dummies capturing the type of preferences as described in **Section I**.

Clearly, the above formula offers a parsimonious generalization of previous welfare formulas derived by (Arkolakis et al., 2012, 2018; Fally, 2019) in two respects. First, it encompasses three theoretically possible scenarios of trade liberalization. For instance, it clearly shows that regardless of whether higher exposure to trade occurs through a small reduction in variable or fixed trade costs, or a small increase in the number of trading countries, the welfare gains it delivers always take the same simple form. Second, as compared with Arkolakis et al. (2018), and Fally (2019), here the novel aspect of this formula is that it allows, not only, the type of preferences, but also, the curvature of demand to play a role in determining the magnitude of the gains from trade.

By recalling that under the CES case, the demand elasticity is identical across firms ( $\forall \varphi, \sigma(\varphi) = \sigma$ ), the pass-through is complete ( $\eta=1$ ) and the "superelasticity" is equal to zero ( $S=0$ ), and so  $\Delta_x$  boils down to zero ( $\Delta_x=0$ ), it follows by inspection that Arkolakis et al. (2012)'s result is replicated under the CES case: ( $GFT = \frac{1}{\theta}$ ). Beyond the CES, it is clear that increased generality of the current demand system allows, not only, the curvature of demand, but also, the type of preferences to play a role in determining the magnitude of the gains from trade. Yet, as in the current section the focus is squarely on the role of demand curvature, let us now examine its novel implications for the gains from trade.

When demand is sub-convex ( $S > 0$ ), the demand elasticity decreases in firm productivity. This implies that the firm at the domestic cutoff faces a higher elasticity than the firm at the export cutoff:  $\sigma_d^*(\varphi_d^*) > \sigma_x^*(\varphi_x^*)$ . This latter faces then a higher demand elasticity than the average productivity exporter:  $\sigma_x^*(\varphi_x^*) > \bar{\sigma}_x(\bar{\varphi}_x)$ , and so  $\Delta_x$  is strictly positive ( $\Delta_x > 0$ ). Inspection of the new welfare formula immediately reveals that sub-convex demands yield smaller gains from trade as compared with the CES benchmark (where  $\Delta_x=0$ ). This result is reversed when demand is super-convex ( $S < 0$ ): now the firm at the domestic cutoff faces the lowest demand elasticity, and the cutoff exporter faces a lower demand elasticity than the average productivity exporter:  $\sigma_d^*(\varphi_d^*) < \sigma_x^*(\varphi_x^*) < \bar{\sigma}_x(\bar{\varphi}_x)$ . This yields a strictly negative value of  $\Delta_x$  ( $\Delta_x < 0$ ). This, in turn, immediately implies that super-convex demands deliver larger gains from trade as compared with the CES benchmark.

## 2. General Equilibrium Implications of Demand Curvature for Firm Selection and Partitioning of Firms by Export Status

In **Section III**, I highlighted two novel comparative statics results showing how demand curvature governs the toughness of firm selection as well as the degree of the partitioning of firms by export status. I also provided a partial equilibrium explanation for both results and emphasized that this latter sets the scene for understanding the general equilibrium behavior. Now I show that both results hold in general equilibrium, as clearly stated in the two following theorems:<sup>30</sup>

**Theorem 2.** *Under Generalized Gorman-Pollak demand, unbounded Pareto distribution of firm productivity, country-level symmetry and the presence of fixed costs of accessing markets, super-convex demands provide an upper bound for the degree of toughness of firm selection on any market in general equilibrium, while sub-convex demands provide a lower bound. Within these bounds, CES demand delivers an intermediate outcome.*

$$\begin{cases} \forall f > 0, \varphi_d^* (\text{sub} - \text{convex}) < \varphi_d^* (\text{CES}) < \varphi_d^* (\text{super} - \text{convex}) \\ \forall f > f_x, \varphi_x^* (\text{sub} - \text{convex}) < \varphi_x^* (\text{CES}) < \varphi_x^* (\text{super} - \text{convex}) \end{cases}$$

**Theorem 3.** *Under Generalized Gorman-Pollak demand, unbounded Pareto distribution of firm productivity, country-level symmetry and the presence of fixed costs of accessing markets, super-convex demands provide a lower bound for the degree of partitioning of firms by export status in general equilibrium, while sub-convex demands provide an upper bound. Within these bounds, CES demand delivers an intermediate outcome.*

$$\forall f > f_x, \frac{\varphi_x^*}{\varphi_d^*} (\text{super} - \text{convex}) < \frac{\varphi_x^*}{\varphi_d^*} (\text{CES}) < \frac{\varphi_x^*}{\varphi_d^*} (\text{super} - \text{convex})$$

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<sup>30</sup>Proofs of **Theorem 2**, and **Theorem 3** are respectively provided in sections A.5 and A.6 of the Appendix.

### 3. An Industry-level Explanation for the Welfare Implications of Demand Curvature

Based on the partial equilibrium explanation provided in **Section II.B**, here I emphasize that there is a common economic force driving **Theorem 2** and **Theorem 3**. Then, I show that it is by governing these comparative statics results, that demand curvature determines the magnitude of the gains from higher exposure to trade.

As mentioned in **Section II.B**, departing from the CES benchmark, when demand is super-convex, selection is relatively tougher and only very productive firms successfully enter the domestic market. Hence, a relatively large subset of these very productive firms can export. In other words, these firms are enough productive to successfully enter the domestic market despite tougher competitive conditions implied by super-convex demands. It follows then that a large fringe of these firms is enough price competitive to penetrate the export market. This reveals then that under super-convex demands, the partitioning of firms by export status is less pronounced as compared with the CES case. This result is reversed when demand is sub-convex.

Importantly, these novel comparative statics results have crucial implications for the gains from trade, which can be explained as follows. Departing from the CES case, when demand is super-convex, successful entrants are relatively fewer and more productive since selection is relatively tougher under this curvature of demand. In addition to that, the fact that the partitioning of firms is relatively less pronounced under this case clearly indicates that a relatively large fringe of active firms export. Therefore, when demand is super-convex, the import competition effect is magnified since domestic firms in any trading country face a relatively fiercer competition from a relatively large number of exporters that are on average relatively more productive (as compared to the CES benchmark). This, in turn, induces a relatively stronger selection effect of trade, forcing then a relatively larger fringe of domestic firms to exit the market. Therefore, by magnifying [Melitz \(2003\)](#)'s selection effect, super-convex demands deliver larger gains from trade than those obtained under CES demand. These patterns are reversed when demand is sub-convex.

## E. A More Granular Analysis of the Gains From Trade

As compared with the preceding analysis, this section goes more granular in two respects. First, I separately examine three different scenarios of trade liberalization, including a (small) decrease in either the variable or fixed trade cost, and a (small) increase in the number of trading countries. Second, under each scenario, I provide a more granular explanation where added flexibility in both firm and consumer behaviors plays a critical role.

I will show that increases in the exposure to trade occurring through any of these scenarios will generate very similar results. In all cases, while demand curvature plays a crucial role in determining the magnitude and the structure of welfare gains from trade, the type of preferences has only a second-order importance from a welfare standpoint.

In particular, I show that sub-convex demands provide a lower bound for the gains from trade and super-convex demands provide a higher bound. Within these bounds, CES demand delivers an intermediate outcome. Under sub-convex demands, directly-separable preferences delivers larger gains than those obtained with indirectly-separable preferences. When demand is super-convex, this order is reversed. As for the structure of welfare gains from trade, I will show that net variety gains and gains from selection coexist if and only if demand is sub-convex. Under this case, only when preferences are directly-separable, the pro-competitive effect of trade is restored, and thus the three sources of gains from trade coexist.

### 1. A Small Decrease in the Variable Trade Cost

In this subsection, I examine the gains from higher exposure to trade occurring through a small reduction in the variable trade cost  $\tau$ . I proceed in three steps. First, I show how demand curvature plays a first-order role in determining the magnitude of the gains from trade, while the type of preferences only marginally affects the result. Second, I characterize a sufficient condition for coexistence of gains from selection, net variety gains, and pro-competitive reduction in domestic markups. Third, I provide a finer explanation at the firm-level for these novel results.

## 1.1 Magnitude of the Gains From Trade

For a general demand system which encompasses the CES and two alternative types of preferences (directly- and indirectly-separable) and curvatures of demand (sub-convex and super-convex), I show that gains from a small reduction in the variable trade cost take a simple form:

$$GFT(\tau^-) = -\varepsilon_{\varphi_d^*}(\tau) = \frac{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]}{\eta_d^*[\sigma_d^*(\varphi_d^*) - 1 + S]} \frac{a_d^*(\varphi_d^*) \theta}{a_x^*(\varphi_x^*) \theta + \Delta_x} \quad (30)$$

where  $\theta$  is the shape parameter of the Pareto distribution,  $S$  is the "superelasticity" of [Kimball \(1995\)](#), and  $\Delta_x = [\eta_x^*(\sigma_x^*(\varphi_x^*) - 1) - a_x^*(\varphi_x^*) \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1) + S]$  reflects the curvature of demand.  $a_d^*(\varphi_d^*)$  and  $a_x^*(\varphi_x^*)$ , are respectively given by  $a_d^*(\varphi_d^*) = \frac{\beta + \alpha \sigma_d^*(\varphi_d^*)}{\beta + \alpha \bar{\sigma}}$ ,  $a_x^*(\varphi_x^*) = \frac{\beta + \alpha \sigma_x^*(\varphi_x^*)}{\beta + \alpha \bar{\sigma}}$ , where  $\alpha$  and  $\beta$  are exogenous parameters capturing the type of preferences as described in **Section I**.

Clearly, the above expression offers a parsimonious generalization of [Feenstra \(2010\)](#)'s result where he shows that under CES demand, a small reduction in the variable trade cost leads to a proportional increase in the domestic cutoff. By recalling that under the CES case, the demand elasticity is identical across firms ( $\forall \varphi, \sigma(\varphi) = \sigma$ ), the pass-through is complete ( $\eta=1$ ) and the "superelasticity" is equal to zero ( $S=0$ ), and so  $\Delta_x$  boils down to zero ( $\Delta_x=0$ ), it follows by inspection that [Feenstra \(2010\)](#)'s result is replicated under the CES case. Beyond the CES, it is clear that added flexibility of the current demand system allows, not only, the curvature of demand, but also, the type of preferences to play a role in determining the magnitude of the gains from trade. The welfare implications of these prominent alternatives to the CES case can be studied separately as follows.

### The Role of Demand Curvature

When demand is sub-convex ( $S > 0$ ), the demand elasticity decreases in firm productivity. This implies that the firm at the domestic cutoff faces a higher elasticity than the firm at the export cutoff:  $\sigma_d^*(\varphi_d^*) > \sigma_x^*(\varphi_x^*)$ . This latter faces then a higher demand elasticity than the average productivity exporter:  $\sigma_x^*(\varphi_x^*) > \bar{\sigma}_x(\bar{\varphi}_x)$ . Thus,  $\Delta_x$  is strictly positive ( $\Delta_x > 0$ ), which immediately implies smaller gains from trade as compared with the CES benchmark (where  $\Delta_x=0$ ).

This result is reversed when demand is super-convex ( $S < 0$ ): now the firm at the domestic cutoff faces the lowest demand elasticity, and the cutoff exporter faces a lower demand elasticity than the average productivity exporter:  $\sigma_d^*(\varphi_d^*) < \sigma_x^*(\varphi_x^*) < \bar{\sigma}_x(\bar{\varphi}_x)$ . This yields a strictly negative value of  $\Delta_x$  ( $\Delta_x < 0$ ), and immediately implies larger gains from trade as compared with the CES benchmark.

### The Role of the Type of Preferences

When preferences are indirectly-separable (including the CES case),  $a_d^* = a_x^* = 1$ .<sup>31</sup> However, when preferences are directly-separable and non-CES,  $a_d^*$  and  $a_x^*$  may be higher or lower than 1 depending on demand curvature. In fact,  $a_d^*$  and  $a_x^*$  respectively capture the relative demand elasticity faced by firms at the domestic, and the export cutoffs. Such firms are the least productive among all active firms in their respective markets and so, when demand is sub-convex, they face a relatively higher demand elasticity:  $a_d^* > 1$  and  $a_x^* > 1$ . In contrast, when demand is super-convex, the demand elasticity increases in firm productivity and so, both cutoff firms face a relatively lower elasticity:  $a_d^* < 1$  and  $a_x^* < 1$ .

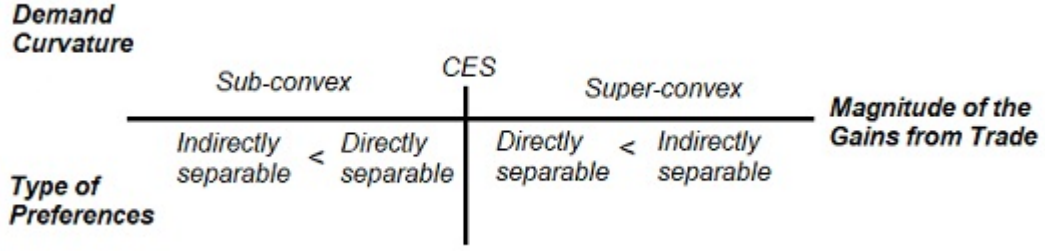
Inspection of the simple expression of the gains from trade in equation (30) reveals that increases less than proportionally with  $a_d^*$ .<sup>32</sup> As mentioned above, this latter positively deviates from unity only when preferences are directly-separable and demand is sub-convex. Hence, under sub-convex demands, directly-separable preferences yield higher gains from trade than indirectly-separable preferences. This order is reversed when demand is super-convex:  $a_d^*$  negatively deviates from unity under directly-separable preferences. This latter delivers then lower gains from trade than indirectly-separable preferences under super-convex demands.

The above results clearly show that while demand curvature plays a critical role in determining the magnitude of the gains from trade, the type of preferences plays only a second-order role. This latter has only a marginal impact on the magnitude of the gains that is initially pinned down by the curvature of demand. This novel finding of the current paper can be graphically illustrated in Figure 3.

<sup>31</sup>As indicated in **Section I**, indirectly-separable preferences correspond to the case where  $\beta = 1$  and  $\alpha = 0$ . This immediately implies that parameter (a) is identical across all firms and fixed to unity under this class of preferences:  $\forall \varphi, a(\varphi) = 1$ .

<sup>32</sup>Since both of  $a_d^*$  and  $a_x^*$  simultaneously deviate from unity and in the same direction, when preferences are directly-separable and non-CES.

Figure 3: Demand Curvature, Type of Preferences, and Magnitude of the Gains from Trade.



## 1.2 Structure of the Gains From Trade

Using the standard measure proposed in equation (29) and the general equilibrium expressions of the domestic and export cutoffs, the net variety effect of a small reduction in the variable trade cost can be simply written as:

$$NVE(\tau^-) \equiv 1 - |\varepsilon_{\varphi_d^*}(\tau)| = 1 - \frac{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]}{\eta_d^*[\sigma_d^*(\varphi_d^*) - 1 + S]} \frac{a_d^*(\varphi_d^*) \theta}{a_x^*(\varphi_x^*) \theta + \Delta_x} \quad (31)$$

where  $\theta$  is the shape parameter of the Pareto distribution,  $S$  is the "superelasticity" of [Kimball \(1995\)](#), and  $\Delta_x = [\eta_x^*(\sigma_x^*(\varphi_x^*) - 1) - a_x^*(\varphi_x^*) \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1) + S]$  reflects the curvature of demand.  $a_d^*(\varphi_d^*)$  and  $a_x^*(\varphi_x^*)$ , are respectively given by  $a_d^*(\varphi_d^*) = \frac{\beta + \alpha \sigma_d^*(\varphi_d^*)}{\beta + \alpha \bar{\sigma}}$ ,  $a_x^*(\varphi_x^*) = \frac{\beta + \alpha \sigma_x^*(\varphi_x^*)}{\beta + \alpha \bar{\sigma}}$ , where  $\alpha$  and  $\beta$  are exogenous parameters capturing the type of preferences as described in **Section I**.

Under the CES case, I can immediately replicate [Feenstra \(2010\)](#)'s result. In the current setting, the CES is a unique exception where the demand elasticity is identical across firms ( $\forall \varphi, \sigma(\varphi) = \sigma$ ), the pass-through is complete ( $\eta=1$ ) and the "superelasticity" is equal to zero ( $S=0$ ). Thus,  $\Delta_x$  boils down to zero ( $\Delta_x=0$ ), and so  $|\varepsilon_{\varphi_d^*}(\tau)|$  equates unity:  $|\varepsilon_{\varphi_d^*}(\tau)| = 1$ . Hence, under CES demand, a small reduction in the variable trade cost yields zero net gains from variety, as in [Feenstra \(2010\)](#). In this sense, the CES can be considered as a boundary case where gross gains from newly imported varieties exactly cancel out with disappearing domestic varieties due to firm exit.

Inspection of the above expression reveals that beyond the CES case, the NVE may be strictly positive or negative depending on demand curvature. When demand is sub-convex, the demand elasticity is decreasing in firm productivity and so, the firm at the domestic cutoff faces the highest demand elasticity:  $\sigma_d^*(\varphi_d^*) > \sigma_x^*(\varphi_x^*) > \bar{\sigma}_x(\bar{\varphi}_x)$ . This implies a strictly positive value of  $\Delta_x$  ( $\Delta_x > 0$ ) and so, a less than proportional increase in the domestic cutoff upon a small reduction in the variable trade cost:  $|\varepsilon_{\varphi_d^*}(\tau)| < 1$ . This, in turn, immediately yields a positive net variety effect (NVE $>0$ ).

This result is reversed when demand is super-convex: the firm at the domestic cutoff faces the lowest demand elasticity:  $\sigma_d^*(\varphi_d^*) < \sigma_x^*(\varphi_x^*) < \bar{\sigma}_x(\bar{\varphi}_x)$ . This implies a strictly negative value of  $\Delta_x$  ( $\Delta_x < 0$ ) and so, a more than proportional increase in the domestic cutoff upon a small reduction in the variable trade cost:  $|\varepsilon_{\varphi_d^*}(\tau)| > 1$ . This, in turn, immediately yields a negative net variety effect (NVE $<0$ ). Therefore, regardless of whether preferences are directly- or indirectly-separable, gains from selection and net variety gains coexist in general equilibrium if and only if demand is sub-convex. As was the case for the magnitude of the gains from trade, the type of preferences has only a marginal effect on the structure of the gains from trade. This what I discuss next.

Now by simply invoking peculiar properties of both families of preferences, I can easily specify the additional condition for the pro-competitive effect of trade to be operative. When preferences are indirectly-separable, the demand elasticity is invariant to changes in the intensity of competition (Bertoletti and Etro, 2017; Bertoletti et al., 2018). This peculiar property precludes any adjustment in domestic markups upon trade liberalization. In contrast, when preferences are directly-separable, the demand elasticity varies with consumption level, and may thus increase or decrease with changes in the intensity of competition depending on the curvature of demand.

As is well known, when preferences are directly-separable and demand is sub-convex, the demand elasticity decreases with the consumption level. As higher exposure to trade lowers demand for domestic varieties, domestic firms face then a higher demand elasticity and are thus forced to reduce their markups. By contrast, under this class of preferences, when demand is super-convex, trade induces an increase in domestic markups (Mrázová and Neary, 2017; Zhelobodko et al., 2012).

Therefore, an increase in the exposure of an economy to trade, occurring through a decrease in the variable trade cost, delivers gains from: (i) selection; (ii) a net increase in product variety; and (iii) a pro-competitive reduction in domestic markups, if and only demand is sub-convex and preferences are directly-separable.

### 1.3 A Finer Explanation at the Firm-level

In order to provide a finer explanation for the results highlighted above, I proceed in three steps. I first study the effect of a small reduction in the variable trade cost on the profile of profits across exporting firms. Then, I show how this initial effect on export profits is transmitted to purely domestic firms through the competition channel. Finally, I connect the last two effects to show how the added flexibility in consumer and firm behaviors gives rise to these novel welfare predictions.

#### The Impact of Trade on the Profile of Operating Profits across Active Exporters

The impact of a small reduction in the variable trade cost  $\tau$  on operating profits of any active exporter ( $\forall \varphi \geq \varphi_x^*$ ) can be derived using the absolute value of the elasticity of its general equilibrium expression with respect to  $\tau$ . Using the general equilibrium expression of operating profits on the export market in equation (22), this elasticity can be simply written as:

$$\forall \varphi \geq \varphi_x^*, \quad \varepsilon_{\pi_x^o(\varphi)}(\tau^-) = \underbrace{\eta(\varphi)[\sigma(\varphi) - 1 + S]}_{I^+} - \underbrace{\frac{a(\varphi)\theta}{[1 + \varepsilon_{I_x}(P)]}}_{C^-} \quad (32)$$

The above expression shows that the net outcome depends on a horse race between two opposite effects. The first is positive, given by ( $I^+$ ), and can be called "the initial positive effect". That is, a reduction in the variable trade cost makes any active exporter more price competitive, and thus raises its revenues and operating profits at the initial level of competition. The second is negative, given by ( $C^-$ ), and can be thought of as a "competition effect". In deed, this reduction in the variable trade cost makes every active exporter more price competitive and may induce

an increase in the number of exporters.<sup>33</sup> This, in turn, leads to an increase in the intensity of competition on the export market.

As shown below in Table 1, the net effect is always heterogeneous across active exporters, except under the CES case where it boils down to zero for all exporters. As to whether this net effect is strictly positive or negative for the least or the most productive exporters, I show that this crucially depends on demand curvature.

Table 1. Heterogeneous vs Identical Impact of Trade on Active Exporters

Demand	Sub-convex	CES	Super-convex
Least productive exporters: $\varphi \rightarrow \varphi_x^*$	$\varepsilon_{\pi_x^o(\varphi)}(\tau^-) > 0$	$\varepsilon_{\pi_x^o(\varphi)}(\tau^-) = 0$	$\varepsilon_{\pi_x^o(\varphi)}(\tau^-) < 0$
Most productive exporters: $\varphi \rightarrow +\infty$	$\varepsilon_{\pi_x^o(\varphi)}(\tau^-) < 0$	$\varepsilon_{\pi_x^o(\varphi)}(\tau^-) = 0$	$\varepsilon_{\pi_x^o(\varphi)}(\tau^-) > 0$

Regardless of the type of preferences, upon a small reduction in the variable trade cost  $\tau$ , operating profits rise for the least productive exporters, whereas they fall for the most productive ones when demand is sub-convex. This is illustrated in Figure 4, panel A, where the solid locus  $\pi_x$  denotes the initial profile of export profits across firms, while the dashed locus  $\pi_x'$  denotes the post-variable trade cost reduction profile when demand is sub-convex. By contrast, when demand is super-convex, this result is reversed. As illustrated in Figure 5, panel A, the outcome exhibits a strong "Matthew Effect": while the most productive exporters (who are initially the most profitable) experience a net increase in their operating profits, the least productive exporters (who are initially the least profitable) experience a net decrease in their operating profits.<sup>34</sup>

<sup>33</sup>Inspection of the general equilibrium expression of the export cutoff clearly indicates that a reduction in the variable trade cost always leads to a proportional increase in the mass of exporters. Yet, this standard result holds only when the instantaneous general equilibrium effect (channeled through the trade exposure index) is ignored.

<sup>34</sup>In the words of Mrázová and Neary (2017), the "Matthew Effect" refers to the case where "to those who have, more shall be given". It is also worth noting that this effect occurs under sub-convex demands in Mrázová and Neary (2017). By contrast, in the current chapter, the "Matthew Effect" occurs when demand is super-convex. I will explain why we obtain the same result under opposite demand curvatures in the next subsection.

Figure 4: Impact of Trade on Active Exporters and Domestic Firms under Sub-convex Demands.

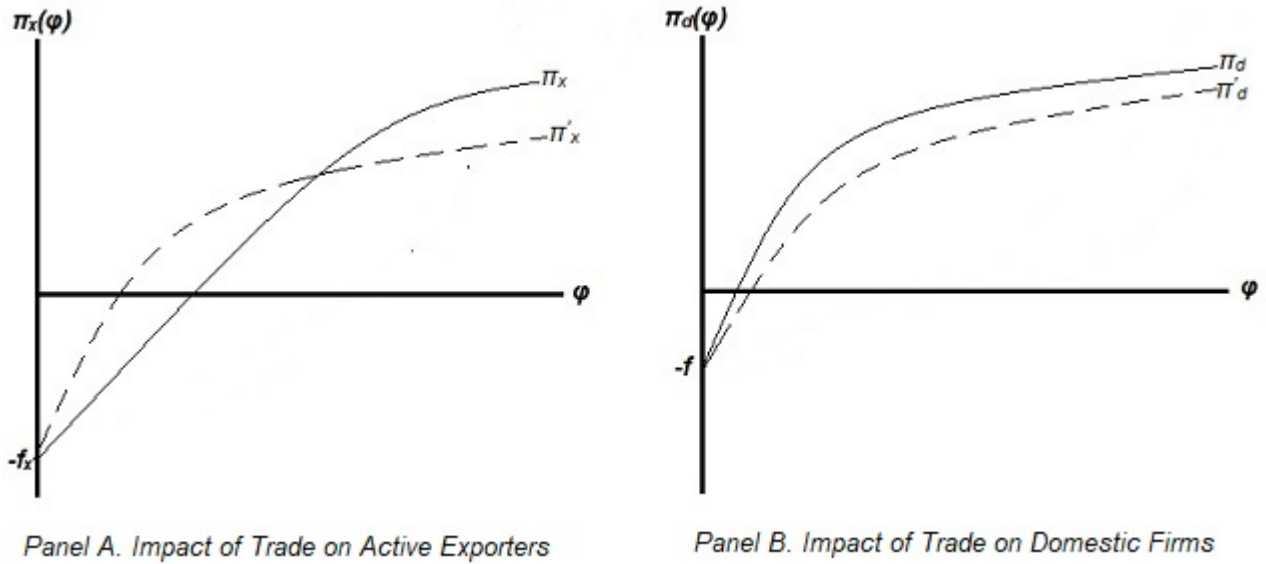
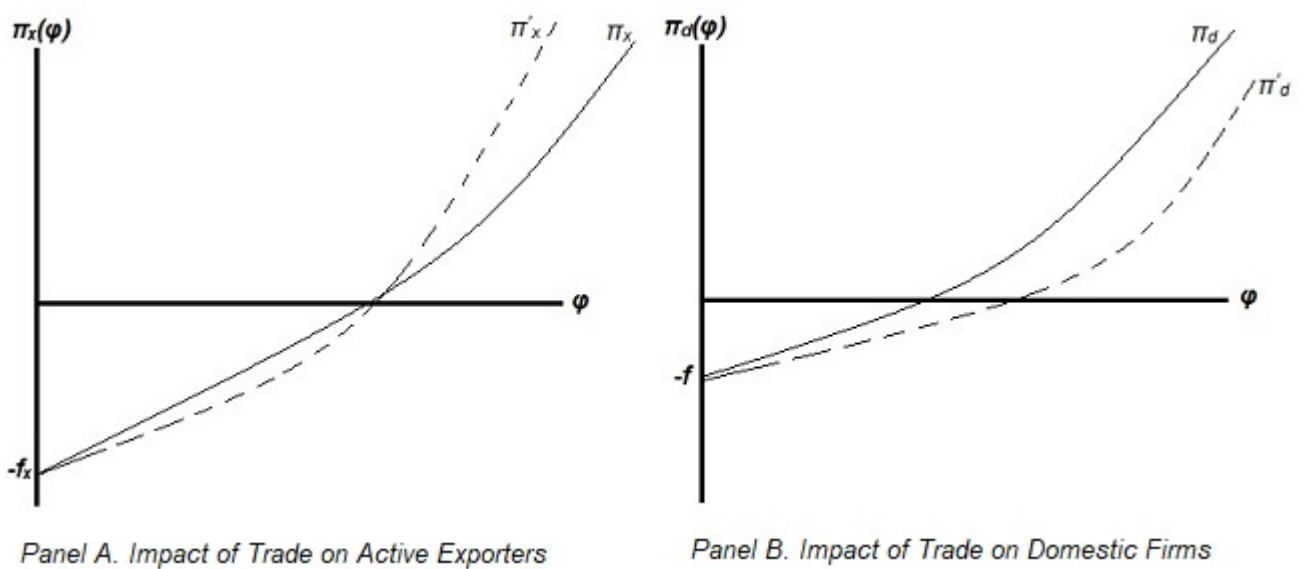


Figure 5: Impact of Trade on Active Exporters and Domestic Firms under Super-convex Demands.



**Proposition 1.** *Whether preferences are directly- or indirectly-separable, the impact of a small reduction in the variable trade cost on the profile of operating profits across active exporters crucially depends on demand curvature. When demand is sub-convex, profits rise for less productive exporters, whereas those of more productive ones fall. By contrast, when demand is super-convex, profits rise for more productive exporters, whereas those of less productive exporters fall. CES demand is a boundary case where the profits of all exporters remain unchanged regardless of their productivity level.*

The economic intuition behind this result can be explained as follows. When demand is sub-convex, the least/most productive exporters face the highest/lowest demand elasticity. This implies that consumers react the most/least to price variations of varieties supplied by the least/most productive exporters. It follows then that upon a small reduction in the variable trade cost, the initial positive effect ( $I^+$ ) is so magnified for less productive exporters that it dominates the competition effect ( $C^-$ ). Hence, profits rise for less productive exporters. Conversely, for more productive exporters, the initial positive effect ( $I^+$ ) is too mild to offset the competition effect ( $C^-$ ), so their operating profits fall.

These results are reversed when demand is super-convex: now consumers react the most to price variations of varieties sold by the most productive exporters, and the least to price variations of those supplied by the least productive exporters. Hence, the initial positive effect ( $I^+$ ) dominates for more productive exporters, so their operating profits rise. In contrast, for less productive exporters, the initial positive effect ( $I^+$ ) is not strong enough to offset the competition effect ( $C^-$ ), so their operating profits fall.

#### Curvature of Demand vs Type of Preferences: A Detailed Discussion

Clearly, the result highlighted above is solely attributable to added flexibility on the demand side in the current model. In deed, incorporating both cases of sub-convex and super-convex demands opens the door for a more realistic modeling of consumer behavior than allowed by the homothetic CES. The added flexibility here is that consumers may exhibit either a weak or strong price sensitivity to the cheapest or the most expensive varieties. This, in turn, allows the general equilibrium effect of trade to be heterogeneous across exporters beyond the CES case.

Here, again the type of preferences has minor implications for this result. Specifically, the nature of preferences solely determines whether the competition effect ( $C^-$ ) is identical across exporters or firm-specific. As previously mentioned, when preferences are indirectly-separable,  $a$  is always equal to one ( $\forall \varphi, a(\varphi) = 1$ ), which clearly indicates that the competition effect is identical across exporters under this class of preferences.

By contrast, when preferences are directly-separable and non-CES,  $a(\varphi)$  may be either strictly higher or lower than one depending on demand curvature and the productivity level of the firm at question. That is, under sub-convex demands,  $a(\varphi)$  is strictly higher than one for the less productive exporters, and strictly lower than one for more productive exporters ( $\forall \varphi \rightarrow \varphi_x^*, a(\varphi) > 1; \forall \varphi \rightarrow +\infty, a(\varphi) < 1$ ).<sup>35</sup> This order is reversed under super-convex demands ( $\forall \varphi \rightarrow \varphi_x^*, a(\varphi) < 1; \forall \varphi \rightarrow +\infty, a(\varphi) > 1$ ). Now I can easily verify that even when the competition effect is firm-specific, the novel result highlighted in Proposition 1 always holds with the aid of the following example:

Under sub-convex demands, and directly-separable preferences, even though the competition effect is magnified for the least productive exporters, these latter experience a net increase in their operating profits, as highlighted in Proposition 1. This immediately reveals that the initial positive effect ( $I^+$ ) always dominates the competition effect ( $C^-$ ). Hence, it is the magnitude of the initial positive effect ( $I^+$ ) that pins down the sign of the net effect of trade on export profits, as reflected by Proposition 1. Finally, as demand curvature governs the magnitude of the initial positive effect ( $I^+$ ), I can then conclude that demand curvature plays a first-order role in driving this result, while the type of preferences has only a second-order importance in this regard.

### Impact of Trade on Purely Domestic Firms

As illustrated in panel B of Figures 4 and 5, trade liberalization always induces an increase in the domestic cutoff, which reflects that increased exposure to trade forces the least productive domestic firms to exit the market. While this is a standard result due to [Melitz \(2003\)](#), the novelty here is twofold.

First, demand curvature determines the nature of the competitive effect of trade. Inspection of Proposition 1 reveals that whether demand is CES or not plays a critical role. When they are,

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<sup>35</sup>Given the definition of demand curvature and the expression of  $a(\varphi)$ , this follows by inspection.

operating profits of all active exporters remain unchanged, and thus additional export market shares are entirely reapt by infra-marginal exporters. Increased labor demand by these new exporters bids up the real wage and forces the the least productive firms to exit, exactly as in [Melitz \(2003\)](#). Hence, as stressed by [Melitz \(2003\)](#), the rigidity of the CES constrains the competitive effect of trade to occur only on the labor market. It precludes then another important and more intuitive channel for the competitive effect of trade, which operates through increases in the intensity of competition on the final good market.<sup>36</sup>

However, beyond CES demand, higher exposure to trade always increases profits of one category of active exporters. Whether this latter corresponds to the most or the least productive exporters, this crucially depends on whether demand is sub-convex or super-convex. This immediately ensures that under both alternatives, the competitive effect of trade operates through an increase in the intensity of competition on the final good market.

Second, demand curvature governs the magnitude of the competitive effect of trade. As hinted to in the previous paragraph, demand curvature determines which category of active exporters reaps additional market shares upon trade liberalization. By doing so, it immediately pins down the magnitude of firm exit. For instance, as highlighted in Proposition 1, when demand is super-convex, additional export market shares are reapt by the most productive exporters. Since this category of exporters has initially large market shares, their increase (upon variable trade cost reduction) leads then to a sharp increase in the intensity of competition. This, in turn, forces a large fringe of domestic firms to exit the market.

In contrast, when demand is sub-convex, additional export market shares are reapt by the least productive exporters. Their market shares are initially small, and so their increase induces a slight increase in the intensity of competition. Hence, only a small fringe of domestic firms is forced to exit the market. The CES is then a special case where additional export market shares are reapt by new exporters. Only under CES demand, these latter face the average demand elasticity ( $\forall \varphi, \sigma(\varphi) = \bar{\sigma} = \sigma$ ). Hence, the magnitude of firm exit under CES demand can be considered as an intermediate outcome.

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<sup>36</sup>[Melitz and Ottaviano \(2008\)](#) restored a theoretical role for this channel using quadratic preferences, which are non-additive

## 2. A small Decrease in the Fixed Trade Cost

Now I study the case where higher exposure to trade occurs through a small reduction in the fixed cost of exporting  $f_x$ . As in the previous case, I separately examine the welfare implications of the curvature of demand and the type of preferences. Using the standards measures proposed in **Section IV.C**, the magnitude of the gains from trade (GFT) and the net variety effect (NVE) which captures their structure are respectively given by:

$$\begin{cases} GFT(f_x^-) = -\varepsilon_{\varphi_d^*}(f_x) = \frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1)]}{\eta_d^*[\sigma_d^*(\varphi_d^*) - 1 + S]} \frac{a_d^*(\varphi_d^*)}{a_x^*(\varphi_x^*) \theta + \Delta_x} \\ NVE(f_x^-) \equiv \frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]} - |\varepsilon_{\varphi_d^*}(f_x)| = \frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]} - \frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1)]}{\eta_d^*[\sigma_d^*(\varphi_d^*) - 1 + S]} \frac{a_d^*(\varphi_d^*)}{a_x^*(\varphi_x^*) \theta + \Delta_x} \end{cases} \quad (33)$$

Inspection of the above expressions clearly shows that a small decrease in the fixed trade cost induces identical results to those described for the small reduction in the variable trade cost.

The only difference here is in the theoretical mechanism driving the result. In deed, under this scenario of trade liberalization, such small reduction in the fixed cost of exporting encourages entry of infra-marginal firms to the export market. As to whether they capture small or large market shares, this crucially depends on demand curvature. When demand is super-convex, these new exporters face the lowest demand elasticity, their relatively low productivity level is thus a mild disadvantage under this case and so, they capture large market shares. This, in turn, magnifies the import competition effect. This induces then a strong selection effect of trade, forcing a large fringe of domestic firms to exit the market. These patterns are reversed when demand is sub-convex. As it imposes the demand elasticity to be identical across firms, CES demand is clearly a boundary case.

Therefore, by magnifying [Melitz \(2003\)](#)'s selection effect, super-convex demands provides an upper bound for the gains from trade. As before, sub-convex demands provide a lower bound, while the CES delivers an intermediate outcome. As for the structure of the gains from trade, it is readily verified that, as before, gains from selection and net variety gains coexist only under sub-convex demands. Additionally, under this curvature of demand, when preferences are directly-separable, the pro-competitive effect of trade on domestic markups is operative, and thus the three sources of welfare gains from trade coexist.

### 3. A small Increase in the Number of Trading Countries $N$

Similarly, I can now investigate the case where higher exposure to trade occurs through a small increase in the number of trading countries at the World level. As in the previous two cases, the objective is to separately examine the welfare implications of the curvature of demand and the type of preferences. As before, using the standards measures proposed in **Section IV.C**, the magnitude of the gains from trade (GFT) and the net variety effect (NVE) which captures their structure are respectively given by:

$$\begin{cases} GFT(N^+) = \varepsilon_{\varphi_d^*}(N) = \frac{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \frac{a_d^*(\varphi_d^*)}{a_x^*(\varphi_x^*) \theta + \Delta_x} \\ NVE(N^+) \equiv 1 - \varepsilon_{\varphi_d^*}(N) = 1 - \frac{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \frac{a_d^*(\varphi_d^*)}{a_x^*(\varphi_x^*) \theta + \Delta_x} \end{cases} \quad (34)$$

Inspection of the above expressions clearly shows that a small increase in the number of trading countries yields identical results to those described for the two previous scenarios.

The only difference here is in the theoretical mechanism underlying the result. In deed, under this scenario of trade liberalization, such small increase in the number of countries implies that domestic firms in any existing country face an additional competition from exporters in these newly trading countries. As to whether this competition effect is strong or mild, this is fully governed by demand curvature. When demand is super-convex, additional export market shares are reapt by the most productive exporters in these newly trading countries. This, in turn, magnifies the import competition effect, which induces then a strong selection effect of trade, forcing a large fringe of domestic firms to exit the market. These patterns are reversed when demand is sub-convex. As it imposes the demand elasticity to be identical across exporters, CES demand is clearly a boundary case.

Hence, as before, by magnifying [Melitz \(2003\)](#)'s selection effect, super-convex demands provides an upper bound for the gains from trade. Sub-convex demands provide a lower bound, while the CES delivers an intermediate outcome. As for the structure of the gains from trade, it is readily verified that, as before, gains from selection and net variety gains coexist only under sub-convex demands. Additionally, under this curvature of demand, when preferences are directly-separable, the pro-competitive effect of trade on domestic markups is restored, and thus these three sources of welfare gains from trade coexist in general equilibrium.

## Conclusion

This paper develops a general yet tractable theoretical framework which combines standard assumptions on the supply side with a flexible demand system, while taking variable and fixed trade barriers in due account. The current model is then well-suited to examine the welfare implications of different scenarios of trade liberalization under general demand conditions. The novelty here is that it is possible to separately examine the implications of the curvature of demand and the type of preferences for the gains from trade. The key finding of this paper is that while demand curvature plays a crucial role in driving comparative statics results and determining the structure and the magnitude of the gains from trade, the type of preferences has only a second-order importance from a welfare standpoint. A key message of the current paper is that rather than assuming a specific type of preferences, more precise estimates of the curvature of demand are necessary to answer comparative statics questions, and to quantify the gains from trade.

# Appendix A

## A.1 Deriving firm-level revenues in partial equilibrium

Let us start with recalling that the price aggregator is the implicit solution of the following equation:

$$\int_{\omega \in \Omega} p_{\omega} Q(\Lambda) D_{\omega}(V(\Lambda) \frac{p_{\omega}}{w}) d\omega = w \quad (35)$$

Now using the parameterization of functions  $Q(\Lambda)$  and  $V(\Lambda)$ :  $Q(\Lambda)=\Lambda^{-\beta}$ ,  $V(\Lambda)=\Lambda^{\alpha}$ , and recalling that the elasticity of each function with respect to its determinant is given by:  $\varepsilon_Q = -\beta$ ,  $\varepsilon_V = \alpha$ , and  $\varepsilon_{D_{\omega}} = -\sigma_{\omega} > 1$ , along with normalizing wage to unity ( $w=1$ ) by choice of labor as numéraire, and rearranging yields:

$$\Lambda = \left[ \int_{\omega \in \Omega} p_{\omega}^{1-\sigma_{\omega}} d\omega \right]^{\frac{1}{(\beta+\alpha\bar{\sigma})}} \quad (36)$$

where  $\bar{\sigma}$  is the average demand elasticity at the industry level,  $\alpha$  and  $\beta$  are both dummies capturing the type of preferences, such as the case ( $\alpha=0$  and  $\beta=1$ ) corresponds to indirectly-separable preferences, while directly-separable preferences correspond to ( $\alpha=1$  and  $\beta=0$ ).

Now let us denote by  $P(\Lambda) = \Lambda^{-(\beta+\alpha\bar{\sigma})} = \left[ \int_{\omega \in \Omega} p_{\omega}^{1-\sigma_{\omega}} d\omega \right]^{-1}$  the conventional price index. Then, with the aid of the parameterization along with the above definition of the conventional price index  $P$ , the Gorman-Pollak demand function described in equation (1) (in the main text) boils down to  $x_{\omega} = p_{\omega}^{-\sigma_{\omega}} P^{a_{\omega}}$ , where  $a_{\omega} = \frac{\beta+\alpha\sigma_{\omega}}{\beta+\alpha\bar{\sigma}}$ . By assuming that any  $\omega$  variety is supplied by a  $\varphi$  productivity firm, I can work throughout with  $\varphi$  as a firm subscript:  $\sigma_{\omega} = \sigma(\varphi)$ ;  $a_{\omega} = a(\varphi)$ . Finally, by recalling that consumers are identical and a firm's market demand is given by  $q(\varphi) = x(\varphi) L$ , firm revenues boil down to:  $r(\varphi) = p(\varphi) q(\varphi) = p(\varphi)^{1-\sigma(\varphi)} P^{a(\varphi)} L$ .

## A.2 The "Exponent Elasticity Method" (EEM)

As stressed in the main text, increased generality raises tractability issues. Thus, in order to gain in generality without losing in tractability, I resort to a new and simple method that I call the "Exponent Elasticity Method" (EEM, hereafter) which delivers a tractable solution for the general equilibrium price index despite added flexibility in preferences. The starting point is the partial equilibrium expression of the price index provided in equation (14) in the main text:

$$P = M_e^{-1} \left[ \underbrace{\int_{\varphi_d^*}^{+\infty} p_d(\varphi)^{1-\sigma(\varphi)} g(\varphi) d\varphi}_{I_d} + (N-1) \underbrace{\int_{\varphi_x^*}^{+\infty} p_x(\varphi)^{1-\sigma(\varphi)} g(\varphi) d\varphi}_{I_x} \right]^{-1}$$

Clearly, the mathematical challenge consists in solving for both integrals ( $I_d$ ,  $I_x$ ) without assuming that the demand elasticity is identical across firms ( $\forall \varphi, \sigma(\varphi) = \sigma$ ) and then using this latter as a constant for integrating. Such simplicity is only possible under CES demand, which is the unique case where it is possible to solve for these integrals. As the general demand system considered in this paper encompasses the CES and more flexible alternatives allowing the demand elasticity to vary across firms, it is then impossible to solve for these integrals under general demands.

Given the impossibility to solve for these integrals in the current setting, the key idea that the EEM method proposes is to locally approximate both integrals ( $I_d$ ,  $I_x$ ) around the equilibrium with a multiplicative equivalent which has a finite number of determinants, such as the exponent of each determinant embodies the elasticity of the average price with respect to it.<sup>37</sup> This requires a five-step procedure that I explain in detail as follows.

### The "EEM" method: A Five-step Procedure

#### Step 1. Rewrite the integral ( $I_x$ ) using unbounded Pareto:

By invoking this assumption, it is readily verified that  $[1 - G(\varphi_x^*)] = \varphi_x^{*-\theta}$ . Using this, integral  $I_x$  can be rewritten as:

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<sup>37</sup>To be precise, this corresponds to the average price to the power of  $(1 - \bar{\sigma})$ :  $\bar{p}_d^{1-\bar{\sigma}_d}$  and  $\bar{p}_x^{1-\bar{\sigma}_x}$ .

$$I_x = \varphi_x^{*-\theta} \underbrace{\int_{\varphi_x^*}^{+\infty} p_x(\varphi)^{1-\sigma(\varphi)} \mu_x(\varphi) d\varphi}_{I_{x_0}} \quad (37)$$

where  $\mu_x(\varphi) = \frac{g(\varphi)}{[1-G(\varphi_x^*)]}$  is the productivity distribution conditional on successful penetration of the export market. Clearly, the unique difference between the initial integral  $I_x$  and the new integral  $I_{x_0}$  is that this latter is expressed using the conditional productivity distribution  $\mu_x(\varphi)$ .

### Step 2. Approximate integral $I_{x_0}$ with a multiplicative equivalent:

Now by recalling that  $\eta(\varphi) = -\frac{d \log p(\varphi)}{d \log \varphi}$  is our measure of the relative cost-price pass-through and that the variable trade cost  $\tau$  is multiplicative by definition, the integral  $I_{x_0}$  can be locally approximated as follows:

$$I_{x_0} \equiv \tau^{\bar{\eta}_x(1-\bar{\sigma}_x(\bar{\varphi}_x))} \bar{\varphi}_x(\varphi_x^*)^{\bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)-1)} \quad (38)$$

Since operating profits are monotonically increasing in productivity, and exporting involves not only, a variable trade cost  $\tau_{ij}$ , but also a fixed cost  $f_x$ , the equilibrium export cutoff  $\varphi_x^*$  exists and is unique. This, in turn, ensures that this local approximation (around the trade equilibrium) delivers a unique multiplicative equivalent to integral  $I_{x_0}$ .

### Step 3. Obtain a final expression of integral $I_x$ using this of $I_{x_0}$ and unbounded Pareto:

Let us now recall that unbounded Pareto distribution gives rise to constant mean-to-min ratio:  $\bar{\varphi}_x = \frac{\theta}{\theta-1} \varphi_x^*$ . By plugging the multiplicative equivalent of integral  $I_{x_0}$  from equation (38) into the initial expression of integral  $I_x$  in equation (37) and invoking this practical property of unbounded Pareto, I obtain the following multiplicative equivalent for integral  $I_x$ :

$$I_x \equiv \kappa \tau^{\bar{\eta}_x(1-\bar{\sigma}_x(\bar{\varphi}_x))} \varphi_x^{*[\bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)-1)-\theta]} \quad (39)$$

where  $\kappa = \left(\frac{\theta}{\theta-1}\right)\bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)^{-1})$  is a constant.

**Step 4. Approximate the partial equilibrium export cutoff  $\varphi_x^*$  with an explicit multiplicative equivalent:**

Let us first recall that the export cutoff  $\varphi_x^*$  is endogenous and defined as the implicit solution of the zero profit condition on the export market (ZPCX):  $\pi_x(\varphi_x^*)=0$ , described in equation (13) in the main text:

$$(ZPCX) \varphi_x^* : \sigma_x^*(\varphi_x^*)^{-1} p_x^*(\varphi)^{1-\sigma_x^*(\varphi_x^*)} P^{a_x^*(\varphi_x^*)} L = f_x \quad (40)$$

Now by isolating the firm-specific component of the operating profit on the left hand-side, approximating it in a multiplicative way, as in equation (38), and rearranging, I obtain the following explicit equivalent of the partial equilibrium export cutoff:

$$\varphi_x^* \equiv \tau f_x^{\frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*)^{-1+S}]}} L_j^{-\frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*)^{-1+S}]}} P^{-\frac{a_x^*(\varphi_x^*)}{\eta_x^*[\sigma_x^*(\varphi_x^*)^{-1+S}]}} \quad (41)$$

**Step 5. Solving for the general equilibrium price aggregator  $P$  :**

Now by plugging the explicit equivalent of the export cutoff from equation (41) in the expression of integral  $I_x$  in equation (39), this latter can be expressed solely as a function of  $(\tau, f_x, L, P)$ .

Then, by applying the same procedure for integral  $I_d$ , and plugging the final expressions of both integrals in the partial equilibrium price index given by equation (14) in the main text, and rearranging, I obtain a tractable solution for the general equilibrium price index:

$$P \equiv c_E^{[1 + \varepsilon_{I_x}(P)]^{-1}} L^{-\left[\frac{1 + \varepsilon_{I_x}(L)}{1 + \varepsilon_{I_x}(P)}\right]} T(\tau, f_x, N)^{-[1 + \varepsilon_{I_x}(P)]^{-1}} \quad (42)$$

where  $c_E = \bar{\sigma} [f_e + [1 - G(\varphi_d^*)] f + (N - 1)[1 - G(\varphi_x^*)] f_x]$  captures entry conditions in every country,

$T(\tau, f_x, N) = N \tau^{-\theta} f_x^{-\frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1)]}{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]}}$  is an index of exposure to trade, and both elasticities  $\varepsilon_{I_x}(L)$  and  $\varepsilon_{I_x}(P)$  take a simple form, and are respectively given by:

$$\begin{cases} \varepsilon_{I_x}(L) = \frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1)]}{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]} > 0 \\ \varepsilon_{I_x}(P) = a_x^*(\varphi_x^*) \frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1)]}{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]} > 0 \end{cases} \quad (43)$$

### A.3 Solving for the domestic and export cutoffs in general equilibrium

By plugging the general equilibrium price index from equation (42) in the explicit partial equilibrium expression of the export cutoff in equation (41), and applying the same procedure for the domestic cutoff, I can solve for their general equilibrium expressions, respectively, as follows:

$$\varphi_x^* \equiv \tau f_x^{\varepsilon_{\varphi_x^*}(f_x)} c_E^{\gamma_x^*} L^{\varepsilon_{\varphi_x^*}(L)} T(\tau, f_x, N)^{\varepsilon_{\varphi_x^*}(T)} \quad (44)$$

where the above exponents are respectively given by:

$$\begin{cases} \varepsilon_{\varphi_x^*}(f_x) = \frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]} \\ \gamma_x^* = -\frac{a_x^*(\varphi_x^*)}{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]} [1 + \varepsilon_{I_x}(P)]^{-1} \\ \varepsilon_{\varphi_x^*}(L) = \frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]} \left[ a_x^*(\varphi_x^*) \frac{1 + \varepsilon_{I_x}(L)}{1 + \varepsilon_{I_x}(P)} - 1 \right] \\ \varepsilon_{\varphi_x^*}(T) = \frac{a_x^*(\varphi_x^*)}{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]} [1 + \varepsilon_{I_x}(P)]^{-1} \end{cases}$$

Similarly, the general equilibrium domestic cutoff can be written as:

$$\varphi_d^* \equiv f^{\varepsilon_{\varphi_d^*}(f)} c_E^{\gamma_d^*} L^{\varepsilon_{\varphi_d^*}(L)} T(\tau, f_x, N)^{\varepsilon_{\varphi_d^*}(T)} \quad (45)$$

where the above exponents are respectively given by:

$$\begin{cases} \varepsilon_{\varphi_d^*}(f) = \frac{1}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \\ \gamma_d^* = -\frac{a_d^*(\varphi_d^*)}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} [1 + \varepsilon_{I_x}(P)]^{-1} \\ \varepsilon_{\varphi_d^*}(L) = \frac{1}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} [a_d^*(\varphi_d^*) \frac{1 + \varepsilon_{I_x}(L)}{1 + \varepsilon_{I_x}(P)} - 1] \\ \varepsilon_{\varphi_d^*}(T) = \frac{a_d^*(\varphi_d^*)}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} [1 + \varepsilon_{I_x}(P)]^{-1} \end{cases}$$

## A.4 Solving for the weighted average productivity at the industry level in general equilibrium

As mentioned in the main text, the weighted average productivity of all firms (both domestic and foreign exporters) competing in a single country can be written as:

$$\bar{\varphi} = \frac{M_d}{M_W} \int_{\varphi_d^*}^{+\infty} \varphi \mu_d(\varphi) d\varphi + \frac{(N-1)M_x}{M_W} \int_{\varphi_x^*}^{+\infty} \varphi \mu_x(\varphi) d\varphi \quad (46)$$

where  $M_W = NM_e$  is the total mass of entrants at the World level, and can be thought of as a proxy for the size of the World market. By solving for the above integrals,  $\bar{\varphi}$  can be then written as a function of the domestic and the export cutoffs:

$$\bar{\varphi} = \frac{M_d}{M_W} \left( \frac{\theta}{\theta-1} \right) \varphi_d^* + \frac{(N-1)M_x}{M_W} \left( \frac{\theta}{\theta-1} \right) \varphi_x^* \quad (47)$$

Now using the general equilibrium expressions of the export and domestic cutoffs, respectively, from equations (44) and (45), the ratio  $\frac{\varphi_x^*}{\varphi_d^*}$  can be written as:

$$\frac{\varphi_x^*}{\varphi_d^*} = \tau \left( \frac{f_x^{\varepsilon_{\varphi_x^*}(f_x)}}{f_d^{\varepsilon_{\varphi_d^*}(f)}} \right) c_E^{\Delta_{x,d}(c_E)} L^{\Delta_{x,d}(L)} T^{\Delta_{x,d}(T)} \quad (48)$$

where  $\Delta_{x,d}(c_E) = \gamma_x^* - \gamma_d^*$ ,  $\Delta_{x,d}(L) = \varepsilon_{\varphi_x^*}(L) - \varepsilon_{\varphi_d^*}(L)$ , and  $\Delta_{x,d}(T) = \varepsilon_{\varphi_x^*}(T) - \varepsilon_{\varphi_d^*}(T)$ .

Using the above relationship between the export and the domestic cutoff, the weighted average productivity in equation (47) can be rewritten as follows:

$$\bar{\varphi} = \varphi_d^* \Psi(\cdot) \quad (49)$$

where  $\Psi(\cdot) = \left[ \frac{M_d}{M_W} \left( \frac{\theta}{\theta-1} \right) + \frac{(N-1)M_x}{M_W} \left( \frac{\theta}{\theta-1} \right) \tau \left( \frac{f_x^\varepsilon \varphi_x^*(f_x)}{f^\varepsilon \varphi_d^*(f)} \right) c_E^{\Delta_{x,d}(c_E)} L^{\Delta_{x,d}(L)} T^{\Delta_{x,d}(T)} \right] > 1$ .

## A.5 Proof of Theorem 2

**Theorem 2** states that on any market (domestic, or export), super-convex demands provide an upper bound for the productivity cutoff in general equilibrium, while sub-convex demands provide a lower bound. Within these bounds, the CES delivers an intermediate result. This can be shown using the general equilibrium expression of the domestic cutoff in equation (45) (or, equivalently this of the general equilibrium export cutoff) as follows:

Let  $\varphi_d^*$ (super), and  $\varphi_d^*$ (sub) denote the domestic cutoffs, respectively, under super-convex demands, and sub-convex demands. Using equation (45), the ratio  $\frac{\varphi_d^*[\text{super}]}{\varphi_d^*[\text{sub}]}$  can be written as:

$$\frac{\varphi_d^*[\text{super}]}{\varphi_d^*[\text{sub}]} = f^{\Delta_1} c_E^{\Delta_2} L^{\Delta_3} T^{\Delta_4} \quad (50)$$

Using the expressions of the elasticity of the general equilibrium domestic cutoff with respect to each of its determinants provided in equation (45),  $\Delta_1$ ,  $\Delta_2$ ,  $\Delta_3$ , and  $\Delta_4$  are respectively given by:

$$\begin{cases} \Delta_1 = \varepsilon_{\varphi_d^*(f)}[\text{super}] - \varepsilon_{\varphi_d^*(f)}[\text{sub}] \\ \Delta_2 = \gamma_d^*[\text{super}] - \gamma_d^*[\text{sub}] \\ \Delta_3 = \varepsilon_{\varphi_d^*(L)}[\text{super}] - \varepsilon_{\varphi_d^*(L)}[\text{sub}] \\ \Delta_4 = \varepsilon_{\varphi_d^*(T)}[\text{super}] - \varepsilon_{\varphi_d^*(T)}[\text{sub}] \end{cases}$$

Finally, inspecting the expressions of the above elasticities, and recalling that the firm at the cutoff faces the lowest demand elasticity under super-convex demands, whereas it faces the highest demand elasticity under sub-convex demands, immediately reveals that  $\frac{\varphi_d^*[\text{super}]}{\varphi_d^*[\text{sub}]} > 1$ . Since under CES demand, the cutoff productivity firm faces the average demand elasticity, this ensures then that the CES delivers an intermediate result:

$$\varphi_d^*[\text{sub} - \text{convex}] < \varphi_d^*[\text{CES}] < \varphi_d^*[\text{super} - \text{convex}].$$

## A.6 Proof of Theorem 3

**Theorem 3** states that super-convex demands provide a lower bound for the degree of partitioning of firms by export status in general equilibrium, while sub-convex demands provide an upper bound. Within these bounds, the CES delivers an intermediate result. This can be shown using the general equilibrium expression of the ratio  $\frac{\varphi_x^*}{\varphi_d^*}$  in equation (48), as follows:

$$\frac{\varphi_x^*}{\varphi_d^*} = \underbrace{\left( \frac{f_x^{\varepsilon \varphi_x^*}(f_x)}{f^{\varepsilon \varphi_d^*}(f)} \right)}_{(1)} \tau c_E^{\Delta_{x,d}(c_E)} L^{\Delta_{x,d}(L)} T^{\Delta_{x,d}(T)} \quad (51)$$

Since entry conditions  $c_E$ , market size  $L$ , and the degree of exposure to trade  $T$  are identical across countries, the partitioning of firms by export status is mainly driven by the presence of variable and fixed trade barriers. Hence, inspecting the above ratio while restricting our focus on its first component (1), and recalling that under super-convex demands, the firm at the domestic cutoff faces a lower demand elasticity than the firm at the export cutoff and that this order is reversed under sub-convex demands, immediately reveals that:

$$\frac{\varphi_x^*}{\varphi_d^*}[\text{super-convex}] < \frac{\varphi_x^*}{\varphi_d^*}[\text{sub-convex}]$$

Finally, recalling that under CES demands both cutoff productivity firms face the same demand elasticity  $\sigma_d^* = \sigma_x^* = \sigma$  ensures that the CES delivers an intermediate outcome, and thus completes the proof.

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