Cheap Talk with Coarse Understanding

Jeanne Hagenbach, Frédéric Koessler

Cheap Talk with Coarse Understanding*

Jeanne HAGENBACH† Frédéric KOESSLER‡

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Abstract

We use the analogy-based expectation equilibrium (Jehiel, 2005) to study cheap talk from a sender who does not perfectly understand all the messages available to him. The sender is endowed with a privately known language competence corresponding to the set of messages that he understands. For the messages that he does not understand, the sender has correct but only coarse expectations about the equilibrium response of the receiver. An analogy-based expectation equilibrium is always a Bayesian solution but usually differs from a standard communication equilibrium and from an equilibrium with language barriers (Blume and Board, 2013). We characterize conditions under which an outcome remains an equilibrium outcome when the sender's competence decreases. Partial language competence rationalizes information transmission and lies in pure persuasion problems, and can facilitate information transmission from a moderately biased sender.

KEYWORDS: Analogy-based expectations; bounded rationality; cheap talk; language; pure persuasion; strategic information transmission.

JEL CLASSIFICATION: C72; D82

1 Introduction

In cheap talk games, the meaning of messages is not literal but comes from equilibrium strategies. In particular, in equilibrium, a privately informed sender is supposed to perfectly understand the receiver's reaction to his messaging strategy. However, strategic situations in which

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†Sciences Po Paris — CNRS. *E-mail:* `jeanne.hagenbach@sciencespo.fr`. Jeanne Hagenbach thanks the European Research Council (grant 850996 – MOREV) for financial support.

‡Paris School of Economics — CNRS. *E-mail:* `frederic.koessler@psemail.eu`. Frederic Koessler acknowledges the support of the ANR (Investissements d'Avenir ANR-17-EURE-001 and ANR grant StratCom ANR-19-CE26-0010-01).

messages are wrongly interpreted and/or induce unexpected responses are prevalent: the choice of a casual outfit can be perceived as an expression of indifference even if it was made to convey confidence; arriving right on time to a social event can be viewed as impolite, even when it is supposed to show enthusiasm; the opening of a gift just received can be viewed as indelicate, even though the action is meant to express gratitude; a foreign word can be used without a full understanding of its usage and exact meaning to natives. In all these examples, the informed sender can send a message despite having an imperfect expectation of the receiver's reaction to it. This paper proposes a way in which to incorporate such a sender into cheap talk games and to study how his presence affects information transmission.

Blume and Board (2013) already propose a way in which to incorporate the fact that a privately informed sender may not understand all the messages available to him. In their sender-receiver framework, every player is endowed with a privately known *language competence*, which is a set of messages that he understands. A key assumption is that a sender cannot emit messages out of his language competence. That is, language competence translates into a restriction on the sender's set of strategies. While we also endow the sender with a privately known set of messages that he understands, we assume that the sender can practically send every available message. However, since it can be particularly complex to learn how the receiver reacts facing any possible message, we consider a sender who has a simplified view of the receiver's strategy. In our model, the sender's language competence relates to his ability to finely predict the impact of his messages on the receiver.

More precisely, we incorporate the sender's language competence into the solution concept by adopting the notion of analogy-based expectation equilibrium (ABEE), developed by Jehiel (2005), Jehiel and Koessler (2008), and Ettinger and Jehiel (2010). This concept assumes that bounded rational players do not perceive the strategy of other players as finely as do rational players. In our communication context, the sender has bounded cognitive rationality: he does not fully understand how the receiver's strategy maps each message received into actions. Instead, the sender bundles all messages he does not understand into the same analogy class and perceives only coarsely, but correctly on average, the response of the receiver to such messages. The bundling of messages by the sender can be interpreted in different ways. Depending on his knowledge, motivation, or cultural and social environment, a sender may only have access to limited data and feedback about the actual response of a decision maker to different messages. Alternatively, the sender may receive full feedback, but the language may be too complex or unfamiliar to him. In this case, the sender would simplify the language by bundling some messages into the same broader category, or he would only identify the most salient correlations between messages and actions.

In an ABEE of our cheap talk game, the sender expects the receiver to play according to the same mixed distribution of actions after each message he does not understand, and this mixed distribution is assumed to coincide with the aggregate distribution of play for all such messages. As is the case for a self-confirming (or conjectural) equilibrium (Battigalli, 1987; Fudenberg and Levine, 1993), we can view the sender's perception of the receiver's strategy as the steady state of a learning process: senders and receivers from a large population are randomly matched and play a best response to the feedback they receive about past behavior in the population, with the property that the feedback of a sender is aggregated whenever a message outside of his language competence has been used.[1] In a self-confirming equilibrium, players can have any subjective theory that is consistent with their feedback. An ABEE is a refinement of self-confirming equilibrium in which players' subjective theory is the simplest one that is compatible with the observed feedback. This subjective theory selection also corresponds to the principle of the maximum-entropy extrapolation in the theory of Bayesian networks (see, e.g., Spiegler, 2016, 2020), which has been used recently in other classes of signaling and communication contexts (see, e.g., Bilancini and Boncinelli, 2018; Eliaz, Spiegler, and Thysen, 2019a,b).[2]

We describe how we incorporate the sender's language competence into sender-receiver cheap talk games in Section 2. By doing so, we distinguish what we call a sender's *payoff type*, which directly affects his payoff, and a sender's *language type*, which is the set of messages he understands, and allow for these types to be correlated. We define an analogy-based expectation equilibrium (ABEE) for such games in Section 3. When it is commonly known that the sender understands all messages available to him, then the definition of an ABEE coincides with perfect Bayesian equilibrium (PBE). In Section 4, we derive general properties of ABEE outcomes and examine how they change with the sender's competence. We also compare our approach to that of Blume and Board (2013) and show that ABEE outcomes usually differ from PBE outcomes with restricted strategies.

As in standard cheap talk games, there always exists an equilibrium with no information transmission; that is, the babbling (nonrevealing) outcome is always an ABEE outcome. Hence, the set of ABEE outcomes is always nonempty. In addition, regardless of the language competence of the sender, an ABEE outcome is always a Bayesian solution (Forges, 1993, 2006) of the basic game without communication. A Bayesian solution corresponds to an outcome that can be achieved with some information system, consistent with the prior probability distribution over states. For some language competences, the set of ABEE outcomes is equal to the set of Bayesian solutions. In particular, the two sets coincide if the sender is fully incompetent and if the message space is large enough. In that case, the set of PBE outcomes is always included in the set of ABEE outcomes. It is then natural to ask when an ABEE outcome is robust to

[1]See, e.g., Ettinger and Jehiel (2010) or Esponda and Pouzo (2016) for more details.

[2]For a recent survey on ABEE, with further interpretations and comparisons with alternative solution concepts, see Jehiel (2020).

some decrease in the sender's language competence.

When language competence decreases (in the sense that the new language types are included in the initial ones), the sender's perception of the receiver's reaction to some messages becomes coarser. This can modify the sender's informational incentive constraints in various ways: given a receiver's strategy, some messages are more attractive to the sender when not understood than when understood, and vice versa. We first provide some conditions on the decrease in the sender's competence under which an ABEE strategy profile is still an ABEE profile under such decreased competence (Proposition 2). Intuitively, the lower sender's competence should not change the perception of messages sent with positive probability according to this profile. Second, we characterize two conditions under which an outcome (but not necessarily a strategy profile) of a pure strategy PBE is an ABEE outcome with a partially competent sender. The first condition is to have a structure on the language types that allows us to change the PBE strategy to a strategy that achieves the same outcome but stays incentive-compatible despite the lower sender's competence (Proposition 3). The second condition is simply that the game involves only two actions (Proposition 4).

In Section 5, we apply our approach to situations in which the language competence of the sender is perfectly correlated with his payoff type and increasing with this type: a sender with a higher payoff type understands strictly more messages than a sender with a lower payoff type (a particular case of the structure considered in Proposition 3). This setting could correspond, for example, to a job interview situation in which the candidate (sender) is characterized by his ability (payoff type) to perform a certain task, and his understanding of technical jargon (message) depends on this ability. To convince the recruiter (receiver) to give him a job, the sender must decide whether he speaks only about what he correctly understands or whether he tries to persuade the recruiter by using technical jargon beyond his own competence. Considering such a language setting, we first study pure persuasion problems in which the sender's preference is state-independent: he only cares about the receiver's estimate of the state and wants to maximize it. Second, we consider a moderately biased sender, as in Crawford and Sobel (1982): he wants the receiver's estimate to match the state plus a bias.

In the pure persuasion context, cheap talk involving a fully competent sender cannot be influential. In contrast, we show that there exists a threshold ABEE in which (i) sender types above the threshold reveal that threshold (which is the highest message they commonly understand); (ii) sender types below the threshold lie about their type by randomly sending messages that they do not understand. This outcome constitutes an ABEE for any threshold that corresponds to a payoff type of the sender. If the sender's utility can only take two values (for example, the sender wants to convince the receiver to choose an action that goes against the status quo), then there exists a threshold equilibrium that implements the best Bayesian solution for the sender (i.e., the one that maximizes the probability that the alternative action is chosen). When the sender is moderately biased, we show that there exists a unique threshold equilibrium such that (i) sender types below the threshold reveal their type truthfully, and (ii) sender types above the threshold pool. This ABEE Pareto dominates and is more informative than every PBE.

Related Literature The most closely related papers in the literature are those of Blume and Board (2013) and Giovannoni and Xiong (2019). They introduce players' language competence, which restricts the set of possible strategies by directly modifying the extensive form of the cheap talk game (the set of available messages for the sender and the information sets for the receiver), and use the standard PBE as a solution concept. Hence, in their framework, a sender can only use a message that he perfectly understands, so our approaches do not usually lead to the same equilibrium outcomes. This difference in modeling language competence has a significant impact on our applications to pure persuasion and cheap talk with a biased sender. In the approach of the above authors, if a higher payoff type of sender understands messages that are not understood by lower types, then the games in these applications are equivalent to disclosure games with hard information (as in Grossman, 1981; Milgrom, 1981 and Seidmann and Winter, 1997), and complete information disclosure would always be an equilibrium.

Blume and Board (2013) show that efficiency may not be attained, even if it would be attained if players' language competences were commonly known. Then, they consider a specific class of common-interest sender-receiver games. When only the sender has partial language competence, they show that in the best equilibrium, the sender uses all his messages and that messages also transmit information about language types. When only the receiver has partial language competence, they show that in the best equilibrium, the receiver's response to a message depends on his language type and is therefore stochastic from the point of view of the sender. The role of higher-order uncertainty about language types is investigated further in Blume (2018). Giovannoni and Xiong (2019) show that standard equilibria can be replicated in a game with language barriers (in the sense of Blume and Board, 2013) whenever the set of messages M is extended to multidimensional strings of messages in M^N, for N large enough. Blume and Board (2010) and Giovannoni and Xiong (2019) also show that there exist language barriers that achieve a welfare which is at least as good as (and sometimes strictly better than) the one achievable with mediated or noisy communication.

Another related work is that of Eliaz et al. (2019a,b). They consider sender-receiver games in a pure persuasion context with binary states and actions. In their approach, it is the receiver (instead of the sender) who partially understands the equilibrium messages. Precisely, Eliaz et al. (2019a,b) assume that the receiver obtains partial statistical data about the equilibrium mapping from states to messages, whereas we assume that the sender obtains partial statistical data about the equilibrium mapping from messages to actions. In addition, the above authors allow the sender to choose the analogy partition of the receiver through the choice of a "dictionary" for the receiver. In this framework, they characterize the optimal messaging strategy for the sender and identify conditions under which the sender is able to achieve his first best.

2 Model

Basic Game G. There are two players: the sender (S) and the receiver (R). The set of *payoff types* of the sender is T. The sender is privately informed about his payoff type. Let $p^0 \in \Delta(T)$ be the prior probability distribution over payoff types. The set of actions of the receiver is A. Unless stated otherwise, we assume for expositional simplicity that the sets T and A are finite. In Section 5, we consider simple classes of games in which the setting extends straightforwardly when the type and action sets are intervals of the real lines.

A mixed action of the receiver is denoted by $y \in \Delta(A)$. The utility of the sender (receiver) is given by $u(a;t)$ ($v(a;t)$) when the payoff type of the sender is $t \in T$ and the action of the receiver is $a \in A$. With some abuse of notation, the utilities are extended as follows for every $y \in \Delta(A)$ and $p \in \Delta(T)$:

$$u(y;t) = \sum_{a\in A} y(a)u(a;t) \text{ and } v(y;t) = \sum_{a\in A} y(a)v(a;t),$$

$$u(y;p) = \sum_{t\in T} p(t)u(y;t) \text{ and } v(y;p) = \sum_{t\in T} p(t)v(y;t).$$

Messages and Languages. The set of available messages is M, with $|M| \geq 2$. A *language type* is a subset $\lambda \subseteq M$ of messages, which is interpreted as the set of messages that the sender understands. The sender is privately informed about his language type. The set of all possible language types is a nonempty set $\Lambda \subseteq 2^M$. We allow for the language type to be correlated with the payoff type. Let $\pi : T \to \Delta(\Lambda)$ be the conditional probability distribution over the language types. For every payoff type $t \in T$ and language type $\lambda \in \Lambda$, $\pi(\lambda \mid t)$ denotes the probability that the sender's language type is λ when his payoff type is t. $L = (M, \Lambda, \pi)$ is called a *language system*. We say that the sender is *fully competent* if $\lambda = M$ and *incompetent* if $\lambda = \emptyset$. The language type of the sender is *commonly known* if $\Lambda = \{\lambda\}$. When the language type is perfectly correlated with the payoff type, we denote by $\lambda(t)$ the language type of the sender of payoff type t (i.e., $\pi(\lambda(t) \mid t) = 1$ for every t). In this case, the language system is denoted by $L = (M, (\lambda(t))_{t\in T})$, and the language type of the sender is commonly known if $\lambda(t) = \lambda(t')$ for every t and t'.

Cheap Talk Extension. We are interested in the following cheap talk game (G, L):

1. Nature selects the sender's payoff type, $t \in T$, according to the probability distribution $p^0 \in \Delta(T)$, and the language type $\lambda \in \Lambda$, according to the probability distribution $\pi(\cdot \mid t) \in \Delta(\Lambda)$;
2. The sender is privately informed about $t \in T$ and $\lambda \in \Lambda$;
3. The sender sends a message $m \in M$ to the receiver;
4. The receiver observes the message m and chooses an action $a \in A$.

Note that if Λ has only one element, then the cheap talk game (G, L) is a standard cheap talk extension of the basic game G. A strategy of the sender is a mapping $\sigma_S : T \times \Lambda \to \Delta(M)$. A strategy of the receiver is a mapping $\sigma_R : M \to \Delta(A)$. An outcome (distribution of actions induced by players' strategies in each state) of the cheap talk game is denoted by $\mu : T \to \Delta(A)$. That is, for every $a \in A$ and $t \in T$:

$$\mu(a \mid t) = \sum_{\lambda \in \Lambda} \pi(\lambda \mid t) \sum_{m \in M} \sigma_S(m \mid t, \lambda)\sigma_R(a \mid m).$$

3 Analogy-Based Expectation Equilibrium

Given a strategy σ_S of the sender, denote by

$$\Pr(m) = \sum_{t \in T} p^0(t) \sum_{\lambda \in \Lambda} \pi(\lambda \mid t)\sigma_S(m \mid t, \lambda),$$

the total probability that the sender sends message $m \in M$. The corresponding random variable is denoted by $\mathbf{m}$ when there is possible confusion regarding the realization of a message.

The next definition describes how an actual strategy profile (σ_S, σ_R) translates into a *strategy of the receiver perceived by the sender* of a given language type. For each message that the sender understands, he correctly perceives the receiver's response. For messages that the sender does not understand (if such messages are sent with strictly positive probability according to σ_S), he perceives the actual average reaction of the receiver to all such messages. In the terminology of Jehiel (2005), this perceived strategy corresponds to the analogy-based expectation of a sender who bundles into the same analogy class the receiver's decision nodes that follow a message that the sender does not understand. As discussed in the introduction, the strategy of the receiver perceived by the sender should be interpreted as stemming from a population belief-based learning process, in which each sender of language type λ, who plays the game once, receives precise feedback about the receiver's behavior after messages inside λ and coarse feedback about the receiver's behavior after messages outside λ.

Definition 1 Given (σ_S, σ_R), a *strategy of the receiver perceived by the sender* of language type $\lambda \in \Lambda$ is a strategy $\tilde{\sigma}_R^\lambda : M \to \Delta(A)$ such that for every $a \in A$ and $m \in M$:

$$\tilde{\sigma}_R^\lambda(a \mid m) = \begin{cases} \sigma_R(a \mid m) & \text{if } m \in \lambda \\ \sum_{m' \notin \lambda} \Pr(m' \mid \mathbf{m} \notin \lambda)\sigma_R(a \mid m') & \text{if } m \notin \lambda \text{ and } \Pr(\mathbf{m} \notin \lambda) > 0, \end{cases}$$

where for $m' \notin \lambda$,

$$\Pr(m' \mid \mathbf{m} \notin \lambda) = \frac{\Pr(m')}{\Pr(\mathbf{m} \notin \lambda)} = \frac{\sum_{t \in T} p^0(t) \sum_{\lambda' \in \Lambda} \pi(\lambda' \mid t)\sigma_S(m' \mid t, \lambda')}{\sum_{m'' \notin \lambda} \sum_{t \in T} p^0(t) \sum_{\lambda' \in \Lambda} \pi(\lambda' \mid t)\sigma_S(m'' \mid t, \lambda')}.$$

Note that as in Ettinger and Jehiel (2010) and in the weakly consistent analogy-based expectations of Jehiel (2005, Definition 2), we put no condition on the strategy of the receiver perceived by the sender when $m \notin \lambda$ and $\Pr(\mathbf{m} \notin \lambda) = 0$. With the previous interpretation, the aggregate feedback received by the sender for such a message is not defined because there are no data generating such feedback when strategy σ_S is used (every message in $M \backslash \lambda$ is sent with zero probability).[3]

The following example illustrates the previous definition.

Example 1 Let $T = \{t_1, t_2, t_3\}$, with a uniform prior probability distribution. Let $M = \{m_1, m_2, m_3\}$ and $\Lambda = \{\{m_1\}\}$. The strategy of the receiver perceived by the sender is given by

$$\tilde{\sigma}_R^\lambda(m_1) = \sigma_R(m_1),$$

$$\tilde{\sigma}_R^\lambda(m_2) = \tilde{\sigma}_R^\lambda(m_3) = \Pr(m_2 \mid \{m_2, m_3\})\sigma_R(m_2) + \Pr(m_3 \mid \{m_2, m_3\})\sigma_R(m_3).$$

For example, if the receiver's action matches the message (i.e., $\sigma_R(m_i) = a_i$) and the equilibrium probabilities of messages m_2 and m_3 are given by $\Pr(m_2) = q_2$ and $\Pr(m_3) = q_3$, with $q_2 + q_3 > 0$, then the strategy of the receiver perceived by the sender is uniquely defined: for every message $m \in \{m_2, m_3\}$ that the sender does not understand, he believes that the receiver plays actions a_2 and a_3 with probabilities $\tilde{\sigma}_R^\lambda(a_2 \mid m) = \frac{q_2}{q_2+q_3}$ and $\tilde{\sigma}_R^\lambda(a_3 \mid m) = \frac{q_3}{q_2+q_3}$. $\diamond$

Now that we have defined the sender's coarse perceptions, we are ready to define the equilibrium concept.

[3] It is possible to add conditions on perceived strategies off the equilibrium path by considering perturbed strategies as in the definition of a trembling-hand equilibrium (see Jehiel, 2005, Definition 3 and Jehiel, 2020).

Definition 2 A strategy profile (σ_S, σ_R) is an *analogy-based expectation equilibrium* (ABEE) of the cheap talk extension of G with language system L if there exists a profile of perceived strategies $(\tilde{\sigma}_R^\lambda)_{\lambda \in \Lambda}$ and a belief $\nu : M \to \Delta(T)$ for the receiver such that the following conditions are satisfied:

Belief consistency. The belief ν is obtained from p^0, π and σ_S by Bayes rule whenever possible: for every $t \in T$ and $m \in M$ satisfying $\Pr(m) > 0$, $\nu(t \mid m) = \Pr(t \mid m)$.

Sequential rationality of the sender. For every $t \in T$, $\lambda \in \Lambda$ and $m^* \in M$ satisfying $\sigma_S(m^* \mid t, \lambda) > 0$,

$$m^* \in \arg\max_{m \in M} u(\tilde{\sigma}_R^\lambda(m); t).$$

Sequential rationality of the receiver. For every $m \in M$ and $a^* \in A$ satisfying $\sigma_R(a^* \mid m) > 0$,

$$a^* \in \arg\max_{a \in A} \sum_{t \in T} \nu(t \mid m) v(a; t).$$

It should be clear that when it is commonly known that the sender is fully competent ($\Lambda = \{M\}$), the definition of ABEE coincides with the definition of PBE of the cheap talk game. In the rest of the paper, the term PBE refers to an ABEE under full language competence. For every language system L, the set of ABEE outcomes is nonempty because it always includes the set of babbling outcomes. Indeed, it suffices to consider a constant strategy for the receiver, who plays the same mixed action in $\arg\max_{y \in \Delta(A)} v(y; p^0)$ after every message, and a constant strategy for the sender, who sends the same message whatever his type.

In simple cheap talk games, such as those considered in this paper, the set of Nash equilibrium outcomes coincides with the set of PBE outcomes because, given any Nash equilibrium, we can construct an outcome-equivalent PBE in which all messages are sent with strictly positive probability. Alternatively, starting from a Nash equilibrium, we can construct a belief off the equilibrium path which is the same as that for an arbitrary on-path message. Hence, in the standard framework, sequential rationality of the receiver can be imposed only for messages sent with strictly positive probability without affecting the set of equilibrium outcomes. This argument does not apply to ABEE because messages are perceived differently depending on the sender's language type. Example 4 in Appendix A.1 illustrates that the set of ABEE outcomes could be strictly larger if we were to remove the sequential rationality condition of the receiver for messages off the equilibrium path (i.e., for messages not in the support of σ_S).

4 General Results and Relationship with Other Concepts

In this section, we present general properties of the ABEE of our cheap talk game involving a sender of various language competences and compare ABEE with other solution concepts in communication games. We first show that an ABEE is always a Bayesian solution. When it is commonly known that the sender is incompetent, the converse is true as well. It follows that ABEE can lead to different outcomes than can perfect Bayesian or communication equilibria. Next, we provide three sets of conditions under which an ABEE outcome remains an equilibrium outcome when the sender's language competence decreases. Finally, we compare ABEE to PBE with restricted strategies for the sender, such as in Blume and Board (2013).

4.1 Bayesian Solutions

An incompetent sender expects the same reaction of the receiver to any of his messages. It follows that deviating from any given strategy is never strictly beneficial for such a sender. Hence, if it is commonly known that the sender is incompetent, i.e., $\Lambda = \{\emptyset\}$, and if the message space M is large enough, then every Bayesian solution (Forges, 1993, 2006) can be obtained as an ABEE outcome. A Bayesian solution is an outcome that is reached when the receiver plays optimally given some information about the sender's payoff type. Formally, an outcome $\mu : T \to \Delta(A)$ is a Bayesian solution iff it satisfies the following incentive constraints for the receiver:[4]

$$\sum_{t \in T} \mu(a \mid t) p^0(t) v(a; t) \geq \sum_{t \in T} \mu(a \mid t) p^0(t) v(a'; t), \quad \text{for every } a \in \text{supp}[\mu] \text{ and } a' \in A. \qquad \text{(RIC)}$$

The solution of the Bayesian persuasion problem (Kamenica and Gentzkow, 2011) is a Bayesian solution (the one that maximizes the sender's ex ante expected payoff). A fully revealing outcome satisfies $\mu(t) \in \arg\max_{y \in \Delta(A)} v(y; t)$ for every $t \in T$ and is also a Bayesian solution (which gives the first best to the receiver). We deduce that when the sender is commonly known to be incompetent and there are enough messages, the game always has a fully revealing ABEE. As an example, we can think of an incompetent sender who may incidentally reveal his private information (involvement, motivation, etc.) through his outfit, while he expects the receiver's reaction to be independent of his clothes. Example 5 in Appendix A.2 illustrates that there exists a fully revealing ABEE, even when there is no fully informative

[4] The definition already uses the revelation principle, which simplifies the information of the receiver to a recommendation of action, and the optimal strategy of the receiver to an obedient strategy.

perfect Bayesian or communication equilibrium (Forges, 1986; Myerson, 1982, 1986).[5]

In the less extreme cases, in which the sender has some language competence, not all Bayesian solutions can be achieved as ABEE outcomes. However, the converse is true. The next proposition summarizes the link between ABEE and Bayesian solutions.

Proposition 1 *For every language system $L = (M, \Lambda, \pi)$, an ABEE outcome is a Bayesian solution. If $|M| \geq |A|$ and $\Lambda = \{\emptyset\}$, then a Bayesian solution is an ABEE outcome.*

Proof. We first show that for every language system L, an ABEE outcome is a Bayesian solution. Consider an ABEE (σ_S, σ_R) under L. By definition of an ABEE, we know that the receiver's strategy satisfies the following:

$$\sum_t \Pr(t \mid m) v(a;t) \geq \sum_t \Pr(t \mid m) v(a';t), \tag{1}$$

for every $m \in M$, a such that $\sigma_R(a \mid m) > 0$, and $a' \in A$. Multiplying (1) by $\Pr(m \mid a)$ for every $m \in M$, taking the sum over M and simplifying, we obtain (RIC).

Next, to see that a Bayesian solution μ is an ABEE outcome when it is commonly known that the sender is incompetent and when the message space is large enough, let $|M| \geq |A|$ and associate to each action $a \in A$ a different message $m_a \in M$. Consider the sender's strategy σ_S such that $\sigma_S(m_a \mid t) = \mu(a \mid t)$ for every a and t, and an obedient receiver's strategy satisfying $\sigma_R(m_a) = \delta_a$ for every a. By construction, since (RIC) is satisfied, σ_R is a best response to σ_S for the receiver. In addition, since $\lambda = \emptyset$, the strategy of the receiver perceived by the sender is simply the unconditional probability of a under μ, which is independent of m. Hence, the sender has no incentive to deviate from σ_S and μ is an ABEE outcome. ■

4.2 Robustness of Equilibria to Competence Decrease

When the sender is incompetent, every Bayesian solution and, in particular, every PBE outcome is an ABEE outcome. In this section, we provide conditions under which an outcome remains an ABEE outcome when the sender's language competence decreases. First, we show in the next example that even in common-interest games with commonly known language competences, a PBE outcome is not necessarily an ABEE outcome with a partially competent sender.

Example 2 Let $T = \{t_1, t_2, t_3\}$, with uniform priors. Let $M = \{m_1, m_2, m_3\}$. Consider the following basic game with common interest:

[5] In contrast to the definition of Bayesian solution, the notion of communication equilibrium additionally requires the following incentive constraint for the sender: $\sum_{a \in A} \mu(a \mid t) u(a;t) \geq \sum_{a \in A} \mu(a \mid t') u(a;t)$ for every $t, t' \in T$.

	a_1	a_2	a_3
t_1	1	0	−2
t_2	−2	1	0
t_3	0	−2	1

When it is common knowledge that the sender is competent, i.e., L is such that $\Lambda = \{M\}$, there is a fully revealing ABEE (PBE) (σ_S, σ_R) such that $\sigma_S(t_i) = m_i$ and $\sigma_R(m_i) = a_i$ for every $i = 1, 2, 3$. If the commonly known language competence decreases to L' with $\Lambda' = \{\{m_1\}\}$, there is no fully revealing ABEE anymore. First, observe that (σ_S, σ_R) is not an ABEE under L': t_3 deviates from m_3 to m_1 because he expects action a_1 after m_1 and a uniform mixture of actions a_2 and a_3 after m_2 and m_3. More generally, whatever the fully revealing strategy considered, there always exists one sender type with a profitable deviation: if the fully revealing strategy is such that $\sigma_S(t_1) = m_1$, then t_3 deviates from what this strategy prescribes to m_1; if the fully revealing strategy is such that $\sigma_S(t_2) = m_1$, then t_1 deviates from what this strategy prescribes to m_1; and if the fully revealing strategy is such that $\sigma_S(t_3) = m_1$, then t_2 deviates from what this strategy prescribes to m_1. $\diamond$

The example demonstrates that it is not true in general that a strategy profile that is an ABEE when the sender is commonly known to be of type λ is still an ABEE when the sender is commonly known to be of type $\lambda' \subseteq \lambda$. In the following proposition, we consider the more general case of the sender's language type perfectly correlated to his payoff type. We establish that an ABEE under L in which the sender t is of language type $\lambda(t)$ is an ABEE under L' in which the sender t is of language type $\lambda'(t) \subseteq \lambda(t)$ if one additional condition on $\lambda(t)$ and $\lambda'(t)$ is satisfied. The condition is that every message in $\lambda(t)$ that is sent with positive probability in the ABEE under L also belongs to $\lambda'(t)$. If this condition holds, then given a receiver's strategy, a sender of type t has the same perception of the receiver's reaction to any message out of $\lambda(t)$ and out of $\lambda'(t)$. This enables us to deduce that the absence of deviation from an equilibrium strategy profile under L implies the absence of deviation from that same profile under L'. Examples 6 and 7 in Appendix A.3 further show that we cannot relax any of the two conditions linking $\lambda(t)$ and $\lambda'(t)$ for every t in the proposition.

Proposition 2 *Assume that the language type of the sender is perfectly correlated with his payoff type, and let (σ_S, σ_R) be an ABEE under the language system $L = (M, (\lambda(t))_{t\in T})$. If $L' = (M, (\lambda'(t))_{t\in T})$ is another language system such that $\lambda'(t) \subseteq \lambda(t)$ and $\lambda(t) \cap \mathrm{supp}[\sigma_S] \subseteq \lambda'(t)$ for every t, then (σ_S, σ_R) is also an ABEE under L'.*

Proof. Let (σ_S, σ_R) be an ABEE under language system L, with associated belief ν and with the associated strategy of the receiver perceived by the sender $(\tilde{\sigma}_R^{\lambda(t)})_{t\in T}$. Clearly, the equilibrium conditions for the receiver and the belief consistency conditions are the same regardless of the language system. To show that (σ_S, σ_R) is an ABEE under the language system L', it then suffices to show that the equilibrium conditions of the sender are satisfied.

Because (σ_S, σ_R) is an ABEE under language system L, we have $m^* \in \arg\max_{m'} u(\tilde{\sigma}_R^{\lambda}(m'); t)$ for every $t \in T$ and $m^* \in \text{supp}[\sigma_S(t)]$. In particular, for every $t \in T$ and $m^* \in \text{supp}[\sigma_S(t)]$, the following conditions are satisfied:

1. If $m^* \in \lambda(t)$ and $m' \in \lambda(t)$, then $u(\sigma_R(m^*); t) \geq u(\sigma_R(m'); t)$;
2. If $m^* \in \lambda(t)$ and $m' \notin \lambda(t)$, then $u(\sigma_R(m^*); t) \geq u(\tilde{\sigma}_R^{\lambda(t)}(m'); t)$;
3. If $m^* \notin \lambda(t)$ and $m' \in \lambda(t)$, then $u(\tilde{\sigma}_R^{\lambda(t)}(m^*); t) \geq u(\sigma_R(m'); t)$.

From the assumption that $\lambda'(t) \subseteq \lambda(t)$ and $\lambda(t) \cap \text{supp}[\sigma_S] \subseteq \lambda'(t)$ for every t, we have the following property: for every $m \in \text{supp}[\sigma_S]$ and $t \in T$, $\quad m \in \lambda(t) \Leftrightarrow m \in \lambda'(t)$. This statement implies that for every t such that $\Pr(\mathbf{m} \notin \lambda(t)) \neq 0$ and for every $m \in M$, we have $\Pr(m|\mathbf{m} \notin \lambda'(t)) = \Pr(m|\mathbf{m} \notin \lambda(t))$. This equality allows us to directly compare the perception of a message $m \in M$ by sender t of language type $\lambda'(t)$ and by sender t of language type $\lambda(t)$:

(i) For every $m \in \lambda'(t)$, $\tilde{\sigma}_R^{\lambda'(t)}(m) = \tilde{\sigma}_R^{\lambda(t)}(m) = \sigma_R(m)$;

(ii) For every $m \notin \lambda(t)$, $\tilde{\sigma}_R^{\lambda'(t)}(m) = \tilde{\sigma}_R^{\lambda(t)}(m)$.

(iii) For every $m \in \lambda(t) \backslash \lambda'(t)$, $\tilde{\sigma}_R^{\lambda'(t)}(m) = \tilde{\sigma}_R^{\lambda(t)}(m'')$ for some $m'' \notin \lambda(t)$ if $\Pr(\mathbf{m} \notin \lambda(t)) \neq 0$. If $\Pr(\mathbf{m} \notin \lambda(t)) = 0$, then $\tilde{\sigma}_R^{\lambda'(t)}(m)$ is arbitrary and can be set to $\tilde{\sigma}_R^{\lambda'(t)}(m) = \sigma_R(m'')$ for some $m'' \in \lambda(t)$.

The equilibrium conditions of the sender under the language system L' are as follows: for every $t \in T$, $m^* \in \text{supp}[\sigma_S(t)]$ and $m' \in M$:

$$u(\tilde{\sigma}_R^{\lambda'(t)}(m^*); t) \geq u(\tilde{\sigma}_R^{\lambda'(t)}(m'); t). \tag{2}$$

We check that these conditions are satisfied in the different possible cases for m^* and m' using the property $m^* \in \lambda(t) \Leftrightarrow m^* \in \lambda'(t)$.

(a) $m^* \in \lambda(t)$ and $m' \in \lambda'(t)$. From point (i), Equation (2) is equivalent to $u(\sigma_R(m^*); t) \geq u(\sigma_R(m'); t)$, which is satisfied by 1.

(b) $m^* \in \lambda(t)$ and $m' \notin \lambda(t)$. From points (i) and (ii), Equation (2) is equivalent to $u(\sigma_R(m^*); t) \geq u(\tilde{\sigma}_R^{\lambda(t)}(m'); t)$, which is satisfied by 2.

(c) $m^* \in \lambda(t)$ and $m' \in \lambda(t)\backslash\lambda'(t)$. From points (i) and (iii), if $\Pr(\mathbf{m} \notin \lambda(t)) \neq 0$, then Equation (2) is equivalent to $u(\sigma_R(m^*); t) \geq u(\tilde{\sigma}_R^{\lambda(t)}(m''); t)$ for some $m'' \notin \lambda(t)$, which is satisfied by 2. If $\Pr(\mathbf{m} \notin \lambda(t)) = 0$, then Equation (2) is equivalent to $u(\sigma_R(m^*); t) \geq u(\sigma_R(m''); t)$ for some $m'' \in \lambda(t)$, which is satisfied by 1.

(d) $m^* \notin \lambda(t)$ and $m' \in \lambda'(t)$. From points (i) and (ii), Equation (2) is equivalent to $u(\tilde{\sigma}_R^{\lambda(t)}(m^*); t) \geq u(\sigma_R(m'); t)$, which is satisfied by 3.

(e) $m^* \notin \lambda(t)$ and $m' \notin \lambda'(t)$. Equation (2) is satisfied because $\tilde{\sigma}_R^{\lambda'(t)}(m^*) = \tilde{\sigma}_R^{\lambda'(t)}(m')$.

■

In Example 2, not only is the fully revealing PBE not an ABEE, but there does not exist any outcome-equivalent ABEE. In the next proposition, we identify a condition on the language system such that any outcome of a PBE in which the sender plays in pure strategy is an ABEE outcome. The condition is that there exists an order on $M = T$ such that every t understands at least all messages that are weakly below t. Under this condition, it is possible to change any PBE strategy σ_S into a new strategy σ'_S, which induces the same beliefs as σ_S for the receiver and which is such that every sender t only sends a message that he understands.

Proposition 3 *Assume that $M = T \subseteq \mathbb{R}$ and that the language type of the sender is perfectly correlated with his payoff type. Let $L = (M, (\lambda(t))_{t \in T})$ be a language system such that*

$$\{s \in T : s \leq t\} \subseteq \lambda(t), \quad \textit{for every } t \in T.$$

Then, every outcome of a PBE in which this sender plays in pure strategy is an ABEE outcome under the language system L.

Proof. Let (σ_S, σ_R) be a PBE in which the sender plays in pure strategy, and let ν be the associated belief. For every t, let $m(t) \in M$ be the message sent with probability one by type t, i.e., $\sigma_S(m(t) \mid t) = 1$.

We first construct a new PBE (σ_S^*, σ_R^*) and belief ν^* that implements the same outcome as (σ_S, σ_R). For every $t \in T$, let $m^*(t)$ be the message sent with probability one by type t under σ_S^*, i.e., $\sigma_S^*(m^*(t) \mid t) = 1$, with

$$m^*(t) = \min\{s \in T : m(s) = m(t)\}.$$

That is, under σ_S^*, every type t sends the smallest type that sends the same message as type

t under σ_S. Let $\tilde{m} \notin \text{supp}[\sigma_S]$ be any off-path message under σ_S. For every $m \in T$, let

$$\sigma_R^*(m) = \begin{cases} \sigma_R(m(t)) & \text{if } m = m^*(t) \text{ for some } t, \\ \sigma_R(\tilde{m}) & \text{otherwise.} \end{cases}$$

Hence, $\sigma_R^*(m^*(t)) = \sigma_R(m(t))$ for every t, so (σ_S^*, σ_R^*) induces that same outcome as (σ_S, σ_R).

For every $m \in T$, let

$$\nu^*(m) = \begin{cases} \nu(m(t)) & \text{if } m = m^*(t) \text{ for some } t, \\ \nu(\tilde{m}) & \text{otherwise.} \end{cases}$$

The belief ν^* is consistent with σ_S^* because $\Pr_{\sigma_S^*}(s \mid m^*(t)) = \Pr_{\sigma_S}(s \mid m(t)),$ for every s, $t \in T$. In addition, σ_R^* is sequentially rational: $\sigma_R^*(m^*(t))$ is sequentially rational given $\nu^*(m^*(t))$ because $\sigma_R(m(t))$ is sequentially rational given $\nu(m(t))$, and $\sigma_R^*(m)$ for $m \notin \text{supp}[\sigma_S^*]$ is sequentially rational given $\nu^*(m)$ because $\sigma_R(\tilde{m})$ is sequentially rational given $\nu(\tilde{m})$. It is also clear that by construction, $\sigma_S^*(t)$ is a best response to σ_R^*, i.e.,

$$u(\sigma_R^*(m^*(t)); t) \geq u(\sigma_R^*(m'); t), \text{ for every } m' \in M. \tag{3}$$

Therefore, (σ_S^*, σ_R^*) with belief ν^* constitutes a PBE.

Now, to show that (σ_S^*, σ_R^*) with belief ν^* is an ABEE under the language system $(M, (\lambda(t))_{t\in T})$, it suffices to verify the equilibrium conditions of the sender for every $t \in T$:

$$u(\tilde{\sigma}_R^{*\lambda(t)}(m^*(t)); t) \geq u(\tilde{\sigma}_R^{*\lambda(t)}(m'); t), \text{ for every } m' \in M. \tag{4}$$

Because $m^*(t) \in \lambda(t)$, the LHS of Equation (4) is the same as the LHS of Equation (3): along the equilibrium path, the perceived expected utility of the sender is correct, so it is the same as the expected utility of the sender in the PBE. Similarly, for $m' \in \lambda(t)$, the RHS of Equation (4) is the same as the RHS of Equation (3), so the deviation to m' is not profitable. Finally, consider a deviation to a message that the sender does not understand, $m' \notin \lambda(t)$. If $\Pr(\mathbf{m} \notin \lambda(t)) > 0$, then $\tilde{\sigma}_R^{*\lambda(t)}(m')$ is a convex combination of some strategies $\sigma_R^*(m'')$ for $m'' \notin \lambda(t)$, so Equation (3) implies Equation (4) for $m' \notin \lambda(t)$. If $\Pr(\mathbf{m} \notin \lambda(t)) = 0$, then $\tilde{\sigma}_R^{*\lambda(t)}(m')$ is arbitrary and could be set to $\tilde{\sigma}_R^{*\lambda(t)}(m') = \sigma_R^*(m'')$ for some $m'' \in M$. ■

As an illustration of the above proposition, consider an alternative language system $\tilde{L}$ in Example 2: $\tilde{\lambda}(t_1) = M$, $\tilde{\lambda}(t_2) = \{m_1, m_2\}$, $\tilde{\lambda}(t_3) = \{m_1\}$. Under this system, the sender's strategy $\sigma_S(t_1) = m_3, \sigma_S(t_2) = m_2$ and $\sigma_S(t_3) = m_1$ enables the construction of a fully revealing ABEE. By relabeling message m_1 as t_3, message m_2 as t_2 and message m_3 as t_1, the language system $\tilde{L}$ satisfies the conditions of Proposition 3, with $t_3 < t_2 < t_1$. This language system is in fact one in which $\lambda(t)$ is exactly equal to $\{s \in T : s \leq t\}$ for every t (it is the system with the lowest competence that is covered by Proposition 3). In Section 5 on applications, we focus on such language systems and illustrate that the set of ABEE outcomes can be strictly larger than the set of PBE outcomes.

In the next proposition, we establish that if the action set is binary, every pure strategy PBE outcome is an ABEE outcome under any language system. If the PBE outcome is babbling, it is also an ABEE outcome, as explained under Definition 2. If the PBE outcome is not babbling, then it is a first-best outcome for the sender. The intuition of the result then goes as follows: with pure strategies and two actions, any PBE outcome can be obtained as the outcome of a PBE in which the sender uses only two messages, m_1 and m_2. With exactly two messages used on the equilibrium path, as long as one of these messages belongs to the sender's language type, the sender properly understands the receiver's reaction to both messages. No deviation is profitable, as the sender then rightly expects his first best. If neither m_1 nor m_2 belongs to the sender's competence, the sender expects a mixture of the reaction to both of these messages. We construct an ABEE so that every off-path message is followed by that exact mixture; hence, the sender has no profitable deviation from m_1 or m_2.

Proposition 4 *Consider a game with a binary set of actions, $A = \{a_1, a_2\}$. If μ is a pure strategy PBE outcome, then for every language system $L = \{M, \Lambda, \pi\}$, μ is an ABEE outcome under language system L.*

Proof. Let μ be a pure strategy PBE outcome. If in the corresponding equilibrium the receiver chooses the same action for every message, then μ is clearly an ABEE outcome regardless of the language system. Hence, assume in what follows that there exists at least one message leading to action a_1 and at least one message leading to action a_2. For $i = 1, 2$, let $T_i = \{t \in T : \mu(a_i \mid t) = 1\}$ be the set of types inducing action a_i. Note that $\{T_1, T_2\}$ forms a partition of T. The equilibrium condition of the sender implies that he obtains his first-best outcome:

$$u(a_1; t) \geq u(a_2; t) \text{ for every } t \in T_1 \text{ and } u(a_1; t) \leq u(a_2; t) \text{ for every } t \in T_2.$$

Consider any language system $L = \{M, \Lambda, \pi\}$. We construct below an ABEE (σ_S, σ_R) with belief ν inducing the outcome μ. Consider two different messages m_1 and m_2 in M. On the sender side, let $\sigma_S(m_1|t, \lambda) = 1$ if $t \in T_1$ and $\sigma_S(m_2|t, \lambda) = 1$ if $t \in T_2$. On the receiver side, let $\sigma_R(m_1) = \delta_{a_1}$, $\sigma_R(m_2) = \delta_{a_2}$ and for every $m \in M \backslash \{m_1, m_2\}$ let $\sigma'_R(m) = \alpha\delta_{a_1} + (1-\alpha)\delta_{a_2}$ for some $\alpha \in [0, 1]$. The belief ν is obtained by Bayes rule along the equilibrium path. For every $m \in \{m_1, m_2\}$, $\sigma_R(m)$ is sequentially rational for the receiver because he obtains the same outcome as under μ, but the sender's strategy σ_S is less informative than any equilibrium strategy of the sender inducing the outcome μ. Off the equilibrium path, i.e., for $m \in M \backslash \{m_1, m_2\}$, let $\nu(m)$ be a receiver's belief such that he is indifferent between both actions, i.e., $v(a_1, \nu(m)) = v(a_2, \nu(m))$. Such a belief making the receiver indifferent exists because $v(a_1, \nu(m_1)) \geq v(a_2, \nu(m_1))$, $v(a_1, \nu(m_2)) \leq v(a_2, \nu(m_2))$, and the receiver's expected utility v is continuous in his belief. We conclude that for every $\alpha \in [0, 1]$, (σ_S, σ_R) with belief ν satisfies belief consistency and sequential rationality of the receiver. It remains to be shown that there exists $\alpha \in [0, 1]$ such that σ_S is sequentially rational for the sender. To do so, fix $t \in T$ and $\lambda \in \text{supp}[\pi(t)]$, and consider the three following cases regarding how the sets λ and $\{m_1, m_2\}$ overlap.

If $\{m_1, m_2\} \cap \lambda = \{m_1, m_2\}$, then the sender finely perceives the receiver's reaction to messages m_1 and m_2: $\tilde{\sigma}_R^\lambda(m_i) = \sigma_R(m_i) = \delta_{a_i}$ for $i = 1, 2$. It follows that there is no deviation from $\sigma_S(t, \lambda)$, as it gives the sender of type t and language type λ his first best.

If $\{m_1, m_2\} \cap \lambda = \{m_1\}$ (the argument is similar for the symmetric case with $\{m_1, m_2\} \cap \lambda = \{m_2\}$), then the sender finely perceives the reaction to message m_1: $\tilde{\sigma}_R^\lambda(m_1) = \sigma_R(m_1) = \delta_{a_1}$. Regarding the reaction to m_2, the sender perceives $\tilde{\sigma}_R(m_2) = \Pr(m_2 | \mathbf{m} \notin \lambda)\delta_{a_2} = \delta_{a_2}$ because m_2 is the only message in $M \backslash \lambda$ sent with positive probability. It follows that the sender of type t and language type λ correctly perceives the reaction to m_1 and m_2 and does not deviate from σ_S as he reaches his first best.

If $\{m_1, m_2\} \cap \lambda = \emptyset$, then the sender's perception of the reaction to m_1 or m_2 is

$$\tilde{\sigma}_R^\lambda(m_1) = \tilde{\sigma}_R^\lambda(m_2) = \Pr(m_1 | \mathbf{m} \notin \lambda)\delta_{a_1} + \Pr(m_2 | \mathbf{m} \notin \lambda)\delta_{a_2}$$

$$= \frac{\Pr(T_1)}{\Pr(T_1) + \Pr(T_2)}\delta_{a_1} + \frac{\Pr(T_2)}{\Pr(T_1) + \Pr(T_2)}\delta_{a_2} = \Pr(T_1)\delta_{a_1} + \Pr(T_2)\delta_{a_2}.$$

When sending a message $m \in \lambda$, the sender perceives the reaction $\tilde{\sigma}_R(m) = \sigma_R(m) = \alpha\delta_{a_1} + (1 - \alpha)\delta_{a_2}$. Letting $\alpha = \Pr(T_1)$, we obtain a constant perceived strategy, $\tilde{\sigma}_R^\lambda(m) = \Pr(T_1)\delta_{a_1} + \Pr(T_2)\delta_{a_2}$ for every $m \in M$; therefore, the sender does not deviate from σ_S. ■

Example 5 in Appendix A.2 shows that even in games with two actions, an ABEE outcome might not be a PBE outcome.

4.3 Language Barriers and Strategy Restrictions

Blume and Board (2013) and Giovannoni and Xiong (2019) also study privately known language competence in sender-receiver cheap talk games, but with a different approach. They assume that when the sender's language type is $\lambda \subseteq M$, he is only able to send messages in λ (they assume that $\lambda \neq \emptyset$). That is, the set of strategies of the sender is the set of mappings σ_S : $T \times \Lambda \to \Delta(M)$ such that $\sigma_S(m \mid t, \lambda) = 0$ whenever $m \notin \lambda$. An equilibrium is then defined as a standard PBE in the cheap talk game with restricted strategy sets.

As in Blume and Board (2013), Example 8 in Appendix A.4 demonstrates that even in common-interest games, the efficient outcome may not be an ABEE when language competence is privately known even if the language competence is sufficient to do so (that is, efficiency would be reached for every possible language competence considered in the example, provided that this competence was commonly known). More generally, ABEE outcomes differ from PBE outcomes with restricted strategy sets. In particular, while in Blume and Board (2013), the sender never uses a message that he does not understand (by assumption), with our approach, the sender might strictly prefer to use such a message. This can be seen in the following example.

Example 3 Let $T = \{t_1, t_2, t_3\}$, with a uniform prior probability distribution. Consider the following language system: $M = \{m_1, m_2, m_3\}$, $\Lambda = \{\{m_1\}, M\}$, and assume that the language type of the sender is independent of his payoff type. Consider the following payoff matrix:

	a_1	a_2	a_3
t_1	$2, 1$	$0, 0$	$0, 0$
t_2	$2, 0$	$4, 1$	$1, 0$
t_3	$2, 0$	$1, 0$	$4, 1$

Let $\sigma_R(m_i) = a_i$, $i = 1, 2, 3$, be the strategy of the receiver. There is a PBE with strategy restrictions from Blume and Board (2013), in which the sender fully reveals his payoff type when his language type is $\lambda = M$ (i.e., $\sigma_S(t_i, \lambda) = m_i$, $i = 1, 2, 3$) and always sends the message that he understands when his language type is $\lambda = \{m_1\}$ (i.e., $\sigma_S(t_i, \lambda) = m_1$, $i = 1, 2, 3$). However, this is not an ABEE. When $t \in \{t_2, t_3\}$, the sender of language type $\lambda = \{m_1\}$ deviates by sending a message that he does not understand because he expects such a message to induce actions a_2 and a_3 with probability $\frac{1}{2}$, instead of inducing a_1 with message m_1. $\diamond$

In the next section, we will consider an ordered set of payoff types and assume that the set of messages that the sender of payoff type t understands is $\lambda(t) = \{s \in T : s \leq t\}$. That is, the sender's language and payoff types are perfectly correlated, and a sender of higher payoff type understands more messages than a sender of lower type. With the strategy restriction of Blume and Board (2013) and Giovannoni and Xiong (2019), the game is equivalent to a disclosure game with evidence (as in, e.g., Grossman, 1981; Milgrom, 1981 and Seidmann and Winter, 1997). As illustrated in the next section, ABEE outcomes differ from PBE outcomes in disclosure games.

Another natural approach to partial language understanding would be to restrict the sender to using all the messages that he does not understand with the same (possibly zero) probability (that is, $\sigma_S(m \mid t, \lambda) = \sigma_S(m' \mid t, \lambda)$ for every $m, m' \notin \lambda$). This is a weaker strategy restriction than in Blume and Board (2013) and Giovannoni and Xiong (2019). In Example 3, the ABEE in which (i) the strategy of the receiver is $\sigma_R(m_i) = a_i$, $i = 1, 2, 3$, (ii) the sender fully reveals when his language type is $\lambda = M$ or his payoff type is t_1 (i.e., $\sigma_S(t_i, M) = m_i$, $i = 1, 2, 3$ and $\sigma_S(t_1, \{m_1\}) = m_1$) and sends the messages that he does not understand (m_2 and m_3) with probability $\frac{1}{2}$ when his language type is $\lambda = \{m_1\}$ and his payoff type is $\{t_2, t_3\}$, is also a PBE of the game with the restricted strategies we just mentioned. However, this is a coincidence coming from the fact that in this equilibrium, the probabilities of messages m_2 and m_3 are equal. If the prior probability distribution of payoff types is no longer uniform, then with the strategy profile described above, the equilibrium probabilities of m_2 and m_3 are no longer equal. The sender of language type $\lambda = \{m_1\}$ and payoff type t_2 would deviate to message m_1 if $\Pr(m_2 \mid \{m_2, m_3\}) < \frac{1}{3}$, and the sender of language type $\lambda = \{m_1\}$ and payoff type t_3 would deviate to message m_1 if $\Pr(m_2 \mid \{m_2, m_3\}) > \frac{2}{3}$.

5 Applications

In this section, we apply the ABEE concept to two standard communication situations: pure persuasion and cheap talk by a biased sender. These two applications differ with respect to the sender's preferences, but in both cases, we assume that the receiver's optimal action is the expected value of the state given his belief. This assumption is common in the economic literature on strategic communication. It can be represented by the set of actions $A = [0, 1]$ and the receiver's utility function $v(a; t) = -(a - t)^2$. Hence, the receiver's strategy is always a pure strategy, which we denote by $\sigma_R : M \to A$.

Regarding the language system, we consider situations in which the set T is ordered and the sender's language types are ordered according to his payoff types: a sender of a given payoff type understands all the messages that lower payoff types understand. Precisely, we assume that $M = T \subseteq [0, 1]$ and that for each payoff type t, there is only one possible language type given by:

$$\lambda(t) = \{s \in T : s \leq t\}.$$

This language system is relevant if one considers, for example, that the sender's payoff type represents his ability to use or provide a service related to a technical product. The ability can then be the knowledge or experience with more or less complex features of a software, a network server or a camera. To communicate this ability to a receiver, the sender can use a technical jargon related to the various features of the product, which he understands better when of greater true ability. The sender's payoff type could also be viewed as his privately known motivation level for, for example, getting a job. At the job interview, a truly more motivated sender has potentially made a better search about the company and its interview practices, and therefore understands better than a less motivated candidate which answers allow to score points.[6]

The language system considered here is a particular case of the one considered in Proposition 3, so we know that the set of PBE outcomes of the cheap talk game in which the sender plays in pure strategy is included in the set of ABEE outcomes. The following two subsections will show how partial language competence of the sender can lead to equilibrium outcomes that are impossible to implement when the sender is fully competent.

5.1 Pure Persuasion

In the pure persuasion case, the sender's utility is independent of the state and can be written as $u(a;t) = u(a)$.[7] We further assume that $u(a)$ is nondecreasing, meaning that the sender always wants the receiver's action to be as high as possible. Hence, every PBE outcome of the cheap talk game is the babbling outcome.

Let $T = \{t_1, \ldots, t_n\}$. The optimal action of the receiver is uniquely defined along the equilibrium path as the expected value of the type given the message and the sender's strategy:

$$\sigma_R(m) = \sum_{t \in T} \Pr(t \mid m)\, t, \text{ for every } m \in \text{supp}[\sigma_S], \tag{5}$$

where $\Pr(t \mid m) = \frac{p^0(t)\sigma_S(m|t)}{\sum_s p^0(s)\sigma_S(m|s)}$. For every $m \notin \text{supp}[\sigma_S]$, we fix $\sigma_R(m) = \tilde{\sigma}_R^{\lambda(t)}(m) = t_1$; this is w.l.o.g. because action $a = t_1$ is the most severe sequentially rational punishment (with belief $\nu(m) = \delta_{t_1}$) for every sender type t.

First observe that, contrary to disclosure games with evidence (Milgrom, 1981), there is no fully revealing ABEE. In particular, there is no ABEE in which the two highest types, t_{n-1} and t_n, obtain different payoffs (and hence send distinct messages when $u(a)$ is strictly increasing in a). By assumption regarding the sender's language competences, there is a unique message that the sender of type t_{n-1} does not understand, namely, t_n, so he correctly perceives the receiver's reaction to this message. It follows that if these two sender's types obtain distinct equilibrium payoffs, then one of them has an interest in mimicking the other one.

[6]These examples suggest that the receiver could also have an active communication role, for example, by asking some clarifications to the sender when he uses more technical messages or when he seems better informed about the company. The analysis of such bilateral communication is beyond the objective of the current paper.

[7]This setting is sometimes called "cheap talk with transparent motives" (Lipnowski and Ravid, 2020)

One-Threshold ABEE. The following proposition establishes that for every threshold type $t^* \in T$, there exists an ABEE in which (i) each sender type t strictly below t^* overreports his type by sending every message $m > t$ with the same probability; (ii) each sender type t above t^* truthfully reports that his type is above t^* by reporting $m = t^*$. In such an equilibrium, low types use all messages that they do not understand, while high types use the highest message they commonly understand. The intuition of this equilibrium is simple: if sender types below the threshold uniformly overreport their types, then the receiver's expectation about the state (and therefore his action) is increasing with the message he receives, except when he receives message t^* corresponding to the threshold, for which his expectation is the highest (see Equation (11) in the proof below). Hence, on the one hand, sender types above the threshold report the threshold because they commonly understand this message and it induce the highest equilibrium action of the receiver. On the other hand, types below the threshold understand that reporting their type or underreporting induces a lower action than overreporting. However, since they do not understand the meaning of the overreporting messages, they are unable to identify the best message (the message that consists of reporting the threshold t^*), and it is a best response for them to overreport uniformly. Note that it is not crucial that types below the threshold precisely use a uniform overreporting strategy. The uniform strategy is simple and natural, and it guarantees that the ordering of the receiver's strategy (11) is satisfied regardless of the prior distribution of types.

Proposition 5 *For every threshold $t^* \in T$, the following strategy for the sender constitutes an ABEE:*

$$\sigma_S(m \mid t_k) = \begin{cases} \frac{1}{n-k} & \text{if } m > t_k \\ 0 & \text{if } m \leq t_k, \end{cases} \quad \text{for every } t_k < t^*, \tag{6}$$

$$\sigma_S(m \mid t_k) = \begin{cases} 1 & \text{if } m = t^* \\ 0 & \text{if } m \neq t^*, \end{cases} \quad \text{for every } t_k \geq t^*. \tag{7}$$

Proof. If $t^* = t_1$, then we have a trivial pooling equilibrium, so let $k^* \in \{2, \ldots, n\}$ be such that $t_{k^*} = t^*$. From the sender strategy σ_S and the priors, we can compute the total probabilities of the different messages sent by the sender and then obtain the conditional probabilities (beliefs of the receiver) for every message sent with positive probability. Simplifying, we obtain the following optimal actions for the receiver as a function of message m that he receives:

- For $t_1 < m = t_{k'} < t_{k^*}$,

$$\sigma_R(m) = \frac{\frac{1}{n-1} p^0(t_1) t_1 + \cdots + \frac{1}{n-(k'-1)} p^0(t_{k'-1}) t_{k'-1}}{\frac{1}{n-1} p^0(t_1) + \cdots + \frac{1}{n-(k'-1)} p^0(t_{k'-1})}. \tag{8}$$

- For $m > t_{k^*}$,

$$\sigma_R(m) = \frac{\frac{1}{n-1}p^0(t_1)t_1 + \cdots + \frac{1}{n-(k^*-1)}p^0(t_{k^*-1})t_{k^*-1}}{\frac{1}{n-1}p^0(t_1) + \cdots + \frac{1}{n-(k^*-1)}p^0(t_{k^*-1})}. \tag{9}$$

- For $m = t_{k^*}$,

$$\sigma_R(m) = \frac{\frac{1}{n-1}p^0(t_1)t_1 + \cdots + \frac{1}{n-(k^*-1)}p^0(t_{k^*-1})t_{k^*-1} + p^0(t_{k^*})t_{k^*} + \cdots + p^0(t_n)t_n}{\frac{1}{n-1}p^0(t_1) + \cdots + \frac{1}{n-(k^*-1)}p^0(t_{k^*-1}) + p^0(t_{k^*}) + \cdots + p^0(t_n)}. \tag{10}$$

Hence, we have:

$$t_1 = \sigma_R(t_1) = \sigma_R(t_2) < \sigma_R(t_3) < \cdots < \sigma_R(t_{k^*-1}) < \sigma_R(t_{k^*+1}) = \sigma_R(t_{k^*+2}) = \cdots = \sigma_R(t_n) < \sigma_R(t_{k^*}) \tag{11}$$

From this ordering, we can observe that the sender has no profitable deviation. Indeed, for every type $t_k < t^*$, the equilibrium distribution of actions that he perceives by sending a message that he does not understand is a distribution over $\{\sigma_R(m) : m > t_k\}$, which is always higher than $\sigma_R(t_{k+1})$; if he deviates to a message that he understands, he obtains at most $u(\sigma_R(t_k)) \leq u(\sigma_R(t_{k+1}))$, so he does not deviate. Any type $t_k \geq t^*$ receives a payoff equal to $u(\sigma_R(t_{k^*}))$, which is the maximum payoff that the sender can receive given the strategy of the receiver. ■

A particular case of the one-threshold ABEE we just described is one in which the threshold is $t^* = t_1$, which is equivalent to pooling. For all other possible thresholds, some information is transmitted. We can note that $\sigma_R(t^*)$ is increasing in t^*, so when t^* increases, sender types $t \geq t^*$ are better off.

Status Quo vs. Alternative. Now consider a particular case in which the sender cares about whether the receiver's action is above a threshold or not: $u(a) = 0$ if $a < \bar{a}$, and $u(a) = 1$ if $a \geq \bar{a}$, where $\bar{a} \in (0, 1)$. An interpretation is that the receiver has two actions and chooses the *alternative* instead of the *status quo* if his estimate of the expected value of the state is higher than $\bar{a}$. The sender always wants the alternative to be chosen. We assume that $E(t) < \bar{a}$, implying that the alternative is never chosen in the absence of information transmission.

As demonstrated above, there exists a t^*-threshold equilibrium for every $t^* \in T$. We can therefore characterize the best threshold for the sender, that is, the threshold that maximizes the probability that the alternative is chosen. This optimal threshold is the smallest type in the set $\{\tilde{t} \in T : E(t \mid m = \tilde{t}) \geq \bar{a}\}$ if this set is nonempty (otherwise, the alternative is never chosen regardless of the threshold).

Note that in the limit case in which the set of payoff types is $T = [0,1]$, we have $E(t \mid m = t^*) = E(t \mid t \geq t^*)$, and therefore, t^* is the unique solution of $E(t \mid t \geq t^*) = \bar{a}$. For example, if payoff types are uniformly distributed, then $t^* = 2\bar{a} - 1$. It is interesting to observe that this ABEE coincides with the ex ante optimal Bayesian solution of the sender (i.e., the solution of the Bayesian persuasion problem of Kamenica and Gentzkow, 2011). Indeed, in this example, the optimal Bayesian solution for the sender is given by a threshold information structure in which the receiver learns that $t \geq t^{\#}$ and is indifferent between the status quo and the alternative whenever $t \geq t^{\#}$. Hence, $t^{\#}$ solves $E(t \mid t \geq t^{\#}) = \bar{a}$, so $t^{\#} = t^*$.

Equilibrium with Multiple Thresholds. The previous proposition shows that there always exist equilibria with a single threshold. Equilibria with multiple thresholds can exist, but the existence of such equilibria depends on the specific parameters of the game. As an illustration, assume that $T = \{t_1, t_2, t_3, t_4\}$, $p^0(t_1) = p^0(t_2) = p^0(t_3) = 1/3$ and $p^0(t_4) = 0$ (the example is robust to any small perturbation of the prior probability distribution). Consider the following 2-threshold strategy for the sender, where the thresholds are $t^* = t_2$ and $t^{**} = t_3$:

$$\sigma_S(m \mid t_1) = \begin{cases} \frac{1}{3} & \text{if } m > t_1 \\ 0 & \text{if } m = t_1, \end{cases}$$

$\sigma_S(t_2 \mid t_2) = 1$, $\sigma_S(t_3 \mid t_3) = 1$, and $\sigma_S(t_3 \mid t_4) = 1$. Using Bayes rule, the following strategy is a best response for the receiver: $\sigma_R(t_1) = \sigma_R(t_4) = 0$, $\sigma_R(t_2) = \frac{3}{4}t_2$, and $\sigma_R(t_3) = \frac{3}{4}t_3$. Clearly, the sender types t_1, t_3 and t_4 have no incentive to deviate. Consider sender type t_2. By sending message $m = t_2$, his perceived (and correct) expected utility is $\sigma_R(t_2) = \frac{3}{4}t_2$. If he deviates to a message that he does not understand ($m = t_3$ or $m = t_4$), then his perceived expected utility is

$$\frac{\Pr(m = t_3)}{\Pr(m = t_3) + \Pr(m = t_4)}\sigma_R(t_3) + \frac{\Pr(m = t_4)}{\Pr(m = t_3) + \Pr(m = t_4)}\sigma_R(t_4) = \frac{3}{5}t_3.$$

Hence, the 2-threshold strategy above constitutes an ABEE whenever $\frac{3}{4}t_2 \geq \frac{3}{5}t_3$.

5.2 Biased Sender

In this subsection, we assume that the sender's preference over the estimates of the receiver depends on his payoff type and is given by $u(a;t) = -(a - (t+b))^2$ with $b > 0$ a bias parameter. To compare our findings with existing results, we assume that the set of payoff types is $T = [0,1]$, with a uniform prior probability distribution.[8]

[8] The results are similar with a finite grid of types, but the analysis is less tractable due to integer problems. The definition of ABEE in Section 2 extends naturally to a continuum of types with the simple pure strategies considered in what follows.

Partitional ABEE. As shown in Crawford and Sobel (1982), this cheap talk game has informative PBE outcomes if and only if $b < 1/4$. Such equilibria are described by a partition of T into K intervals, $[t^*_{k-1}, t^*_k]$, $k = 1, \ldots, K$, such that the sender reveals to the receiver to which of these intervals his payoff type belongs. There is a unique K-partitional equilibrium outcome for every $K \geq 2$ satisfying $b \leq \frac{1}{2K(K-1)}$. In addition, every K-partitional equilibrium satisfies $t^*_K \leq 1 - 4b$; that is, there exists a pooling interval of size at least $4b$ at the top of the type space. From an ex ante point of view, the best equilibrium is the same for the sender and the receiver and is given by the K^*-partitional equilibrium, where K^* is the highest integer satisfying $b \leq \frac{1}{2K^*(K^*-1)}$.

Using the same idea as in the proof of Proposition 3, it is immediate that these partitional equilibrium outcomes are also ABEE outcomes for the language system assumed in this section. Indeed, an ABEE equilibrium implementing a partitional equilibrium outcome can be constructed as follows: for every $k = 1, \ldots, K$, each type $t \in [t^*_{k-1}, t^*_k]$ sends message t^*_{k-1}, and the receiver responds optimally (i.e., chooses action $\frac{t^*_{k-1}+t^*_k}{2}$ after message t^*_{k-1}) and plays action 0 off the equilibrium path. Consider a sender type $t \in [t^*_{k-1}, t^*_k]$ for some $k = 1, \ldots K$. In a PBE, this type prefers to reveal that his type belongs to the interval $[t^*_{k-1}, t^*_k]$ rather than to any other interval. Hence, a sender with partial language competence does not underreport because he perceives the strategy of the receiver correctly by underreporting, and he does not overreport because his perceived expected utility by overreporting is a convex combination of the utilities that he would obtain by overreporting in a PBE.

One-Threshold ABEE. First, note that the informative threshold ABEE identified in Proposition 5 are never ABEE in the current framework for $b \leq 1/4$ because type $t = 0$ strictly prefers action 0 (what he induces by sending the message $m = 0$) to a distribution of actions whose expectation is $1/2$ (what he perceives to induce by sending a message $m > 0$). Additionally, observe that there is no fully revealing ABEE for $b > 0$: a high enough type $t < 1$ would deviate and overreport because, by over-reporting, he would perceive to induce a distribution of actions slightly above t and hence closer to his ideal action $t + b$.

The next proposition shows that if $b < \frac{1}{4}$, then there exists a unique $t^* \in (0, 1)$ such that the following strategy for the sender constitutes an ABEE: if the sender's type t is strictly below t^*, then he fully reveals the state by sending the highest message that he understands, i.e., by sending message $m = t$; otherwise, if the sender's type is above t^*, then he sends message $m = t^*$.

Proposition 6 *The following pure strategy for the sender constitutes an ABEE if and only if*

$b \leq \frac{1}{4}$ *and* $t^* = 1 - 4b$*:*

$$\sigma_S(t) = \begin{cases} t & \textit{if } t < t^* \\ t^* & \textit{if } t \geq t^*. \end{cases} \tag{12}$$

Proof. Consider a strategy for the sender as stated in the proposition. The best response of the receiver is as follows:

$$\sigma_R(m) = \begin{cases} m & \text{if } m < t^* \\ E(t \mid t > t^*) = \frac{1+t^*}{2} & \text{if } m = t^*. \end{cases}$$

To constitute an ABEE, the sender type t^* must be indifferent between sending message $m = t^*$ and sending message $m = t^* - \varepsilon$ when $\varepsilon \to 0$, i.e.,

$$-(t^* - (t^* + b))^2 = -(\frac{1+t^*}{2} - (t^* + b))^2 \iff b^2 = (\frac{1-t^*}{2} - b)^2 \iff t^* = 1 - 4b.$$

A sender of type $t > t^*$ prefers higher actions than the sender of type t^*, so the former prefers to pool on $m = t^*$ rather than send message $m < t^*$. A sender of type $t < t^*$ does not deviate iff

$$-(t - (t+b))^2 \geq -\frac{1}{1-t}\left(\int_t^{t^*} (m - t - b)^2 dm + (1 - t^*)(\frac{1+t^*}{2} - t - b)^2\right),$$

where the LHS is the utility of type t if he sends message $m = t$, and the RHS is the perceived expected utility of type t if he overreports, i.e., if he sends a message that he does not understand. A sufficient condition for this inequality to be satisfied is

$$b^2 \leq \frac{1}{1-t}\left((1 - t^*)(\frac{1+t^*}{2} - t - b)^2\right).$$

Replacing t^* by $1 - 4b$ and simplifying, we obtain

$$4(1-t)^2 - 25b(1-t) + 36b^2 \geq 0.$$

The LHS is equal to zero at $t = 1 - 4b$ and is decreasing in t for $t \leq 1 - \frac{25}{8}b$. Since $t < t^* = 1 - 4b < 1 - \frac{25}{8}b$, we conclude that the LHS is positive for every $t < t^*$. We conclude that the sender has no profitable deviation; therefore, the strategy profile described above for $t^* = 1-4b$ and $b \leq 1/4$ is an ABEE. ■

In the ABEE of the previous proposition, the strategy of the sender becomes more informative when the conflict of interest b decreases and converges to full revelation when b tends to 0.

The threshold $t^* = 1 - 4b$ is higher than the highest possible threshold t_K^* of a PBE. Hence, this ABEE is more informative and therefore Pareto superior (at the ex ante stage) to *all* perfect Bayesian equilibria of the standard model of Crawford and Sobel (1982).[9]

To conclude, the analysis in Sections 5.1 and 5.2 shows that in two well-known communication settings (pure persuasion and cheap talk with a moderately biased sender), partial language competence can improve communication compared to the case of cheap talk from a fully competent sender. The qualitative properties of the equilibria in the two settings are, however, quite different: in the pure persuasion case, low types lie about their types by systematically overreporting (because they have nothing to lose by doing so), and high types pool by truthfully reporting that they are higher than some threshold. This threshold could be any payoff type on which players coordinate and can be interpreted as a norm used by high type senders to distinguish themselves from lower types. When the sender is only moderately biased, low types truthfully and perfectly reveal their types (because overreporting is too risky), and high types pool. In the latter case, the threshold level above which the high types pool is not arbitrary and decreases with the conflict of interest between the sender and the receiver.

A Appendix: Additional Examples

A.1 ABEE and Sequential Rationality

The following example shows that removing the sequential rationality requirement of the receiver for messages off path may enlarge the set of ABEE outcomes.

Example 4 Let $T = \{t_1^I, t_2^I, t_1^C, t_2^C\}$, with a uniform prior probability distribution. Consider the following language system, where the sender's language type is perfectly correlated with his payoff type: $M = \{m_1^I, m_2^I, m_1^C, m_2^C\}$, $\Lambda = \{\lambda^I, \lambda^C\}$, $\lambda(t_1^I) = \lambda(t_2^I) = \lambda^I = \{m_1^I, m_2^I\}$, and $\lambda(t_1^C) = \lambda(t_2^C) = \lambda^C = M$. Consider the following payoff matrix:

	a_1	a_2	a_3
t_1^I	$-1, 2$	$1, 1$	$1, -2$
t_2^I	$1, -2$	$1, 1$	$-1, 2$
t_1^C	$2, 2$	$0, 1$	$0, -2$
t_2^C	$0, -2$	$0, 1$	$2, 2$

[9] For small biases, this ABEE also ex ante Pareto dominates all (mediated) communication equilibria. This assertion can be verified easily by using the upper bounds of the ex ante expected communication equilibrium payoffs in Goltsman, Hörner, Pavlov, and Squintani (2009, Lemma 2) and checking that when b is small enough, the ex ante expected payoffs at the ABEE constructed above are strictly higher than these upper bounds.

Consider the following strategy profile. The sender reveals whether his type is in $\{t_1^I, t_1^C\}$ or $\{t_2^I, t_2^C\}$ with the strategy $\sigma_S(t_1^I) = \sigma_S(t_1^C) = m_1^C$ and $\sigma_S(t_2^I) = \sigma_S(t_2^C) = m_2^C$. The receiver plays $\sigma_R(a_1 \mid m_1^C) = \sigma_R(a_3 \mid m_2^C) = 1$ and $\sigma_R(a_1 \mid m) = \sigma_R(a_3 \mid m) = \frac{1}{2}$ for $m \in \{m_1^I, m_2^I\}$. Hence, the strategy of the receiver perceived by the sender as a function of his language type is $\tilde{\sigma}_R^{\lambda^C} = \sigma_R$ and $\tilde{\sigma}_R^{\lambda^I}(a_1 \mid m) = \tilde{\sigma}_R^{\lambda^I}(a_3 \mid m) = \frac{1}{2}$ for every $m \in M$. The sender's strategy is a best response to the strategy of the receiver that he perceives, and the receiver's strategy is a best response to the strategy of the sender. However, the strategy of the receiver is not sequentially rational for the off-path messages in $\{m_1^I, m_2^I\}$ because randomizing between a_1 and a_3 is not optimal regardless of his belief. Indeed, if $\nu \in [0, 1]$ is the belief that the receiver assigns to the event $\{t_1^I, t_1^C\}$, then his sequentially rational decision is a_1 if $\nu > \frac{2}{3}$, a_2 if $\nu \in (\frac{1}{3}, \frac{2}{3})$ and a_3 if $\nu < \frac{1}{3}$. It is immediate that more generally, there is no (sequentially rational) ABEE inducing the first-best outcome for the receiver $\mu(a_1 \mid t_1^I) = \mu(a_1 \mid t_1^C) = \mu(a_3 \mid t_2^I) = \mu(a_3 \mid t_2^C) = 1$. $\diamond$

A.2 Fully Revealing ABEE

In the following example, there is no fully informative PBE or communication equilibrium, while there is a fully revealing ABEE when the sender is incompetent.

Example 5 Let $T = \{t_1, t_2\}$, with uniform priors. Let the language system be such that $M = \{m_1, m_2\}$ and $\Lambda = \{\emptyset\}$. The basic game is given by:

	a_1	a_2
t_1	$1, 1$	$0, 0$
t_2	$1, -2$	$0, 0$

Clearly, full revelation cannot be sustained as a PBE outcome because type t_2 would have an incentive to mimic type t_1. For the same reason, full revelation is not a communication equilibrium outcome. Nevertheless, the strategy profile $\sigma_S(t_1) = m_1$, $\sigma_S(t_2) = m_2$, $\sigma_R(m_1) = a_1$, $\sigma_R(m_2) = a_2$ is an ABEE. The sender perceives the same receiver's reaction following m_1 and m_2 and therefore has no incentive to deviate from full revelation. $\diamond$

A.3 Relaxing the Conditions of Proposition 2?

We provide two examples demonstrating that we cannot relax any of the two conditions linking $\lambda(t)$ to $\lambda'(t)$ for every t in Proposition 2. First, we relax the condition $\lambda'(t) \subseteq \lambda(t)$ for t_1 but keep $\{\lambda(t) \cap \text{supp}[\sigma_S]\} \subseteq \lambda'(t)$ for every t. In that case, the sender of type t_1 may deviate from $\sigma_S(t_1)$ because he perceives the reaction to some messages $m \notin \text{supp}[\sigma_S(t_1)]$ more finely when of language type $\lambda'(t_1)$ than when of language type $\lambda(t_1)$.

Example 6 Let $T = \{t_1, t_2, t_3\}$, with a uniform probability distribution, and $M = \{m_1, m_2, m_3\}$. Let the payoff matrix be as follows:

	a_1	a_2	a_3
t_1	1	0	1.5
t_2	0	1	0
t_3	0	0	1

We consider the following language types: $\lambda(t_2) = \lambda'(t_2) = \lambda(t_3) = \lambda'(t_3) = \lambda'(t_1) = M$, while $\lambda(t_1) = \{m_1\}$. Consider the fully revealing strategy profile: $\sigma_R(m_i) = a_i$ and $\sigma_S(t_i) = m_i$ for every $i = \{1, 2, 3\}$. (σ_S, σ_R) is an ABEE under $\lambda(\cdot)$, but it is not an ABEE under $\lambda'(\cdot)$, as t_1 deviates from sending m_1 to sending m_3. $\diamond$

In the second example, we show that it is not sufficient to have $\lambda'(t) \subseteq \lambda(t)$ and $\{\lambda(t) \cap \operatorname{supp}[\sigma_S(t)]\} \subseteq \lambda'(t)$ for every t because, in that case, there may be a message $m(t)$ sent in equilibrium, which is not in $\lambda(t)$ and not in $\lambda'(t)$ but which is such that the perception of this message is different under $\lambda(t)$ and under $\lambda'(t)$ because $\Pr(m''|\mathbf{m} \notin \lambda(t)) \neq \Pr(m''|\mathbf{m} \notin \lambda'(t))$. In the following example, $\lambda'(t) \subseteq \lambda(t)$ and $\{\lambda(t) \cap \operatorname{supp}[\sigma_S(t)]\} \subseteq \lambda'(t)$ for every t, but $\{\lambda(t_3) \cap \operatorname{supp}[\sigma_S]\} \not\subseteq \lambda'(t_3)$.

Example 7 $T = \{t_1, t_2, t_3, t_4\}$, with uniform priors, and $M = \{m_1, m_2, m_3, m_4\}$. Let the payoff matrix be as follows:

	a_1	a_2	a_3	a_4
t_1	1	0	0	0
t_2	0	1	0	0
t_3	1	0	2.5	0
t_4	0	0	0	1

We consider the following language types: $\lambda(t_1) = \lambda'(t_1) = M$; $\lambda(t_2) = \lambda'(t_2) = M$; $\lambda(t_4) = \lambda'(t_4) = M$ and $\lambda(t_3) = \{m_1, m_2\}$,, while $\lambda'(t_3) = \{m_1\}$. Consider the fully revealing strategy profile: $\sigma_R(m_i) = a_i$ and $\sigma_S(t_i) = m_i$ for every $i = \{1, 2, 3, 4\}$. (σ_S, σ_R) is an ABEE under $\lambda(\cdot)$, but it is not an ABEE under $\lambda'(\cdot)$, as t_3 deviates from sending m_3 to m_1. $\diamond$

A.4 Privately Known Language Competence as a Barrier to Communication

In the following common-interest game, private information about the language type prevents players from reaching the efficient outcome in ABEE.

Example 8 Let $T = \{t_1, t_2, t_3\}$, with uniform priors. Let $M = \{m_1, m_2, m_3\}$. Consider the following basic game with common interest:

	a_1	a_2	a_3
t_1	0	-2	-2
t_2	0	1	-2
t_3	0	-2	1

Consider the following sender's language types $\lambda^1 = \{m_1\}$, $\lambda^2 = \{m_2\}$, $\lambda^3 = \{m_3\}$, and assume that they are uniformly distributed. Therefore, the sender has nine equally likely types in $T \times \Lambda$. The efficient (first best) outcome is $\mu(t_i) = a_i$, $i = 1, 2, 3$. This outcome can be implemented only if the sender uses a separating strategy and if the receiver chooses a different action for each message. Consider such a strategy profile. If the receiver chooses a_1 after m_1, then the sender of language type λ^1 and payoff type t_2 or t_3 would like to deviate and send message m_1 since his payoff is 0 by sending m_1, but his perceived payoff is $\frac{1}{2}(1) + \frac{1}{2}(-2) < 0$ if he sends m_2 or m_3. Similarly, if the receiver chooses a_1 after m_2, then the sender of language type λ^2 and payoff type t_2 or t_3 would like to deviate, and if the receiver chooses a_1 after m_3, then the sender of language type λ^3 and payoff type t_2 or t_3 would like to deviate.

However, observe that if the sender's language type is commonly known, then there is an efficient ABEE. For example, if $\Lambda = \{\{m_1\}\}$, then $\sigma_S(t_1) = m_2$, $\sigma_S(t_2) = m_1$, $\sigma_S(t_3) = m_3$ and $\sigma_R(m_1) = a_2$, $\sigma_R(m_2) = a_1$, $\sigma_R(m_3) = a_3$ is an efficient ABEE. A symmetric argument applies when $\Lambda = \{\{m_2\}\}$ or $\Lambda = \{\{m_3\}\}$. $\diamond$

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