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# A simple model of liquidity

Emanuele Franceschi\*

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## Abstract

We introduce liquidity motives in an otherwise standard monetary model. The Central Bank's policy rule is adapted to target the interest rate on liquid bonds. These deviations are sufficient to relax the requirement for active monetary policy and warrant determinacy in both passive and active policy regimes. We compare this model of liquidity with workhorse models and find that it can substantially replicate usual dynamics. By means of stochastic simulations, we also study how monetary policy stance affect inflation dynamics and find evidence of increased persistence for passive monetary policy.

## 1 Introduction

Liquidity and finance have risen to the central stage of macroeconomic discussion in the wake of the 2008 Global Financial Crisis. As financial and interbank markets froze and interest rates fell to zero, major Central Banks deployed credit and liquidity facilities to unlock the flow of assets and revive their trade, de facto injecting unprecedented amounts of liquidity in the markets. Fed's Chairman Ben Bernanke famously commented on the ex ante effectiveness of Quantitative Easing policies saying that "The problem with QE is [that] it works in practice, but it doesn't work in theory."

This paper contributes to the study of liquidity on monetary policy in general, and how it affects the restrictions on interest rate rules in particular. We introduce liquid bonds and total liquidity in an otherwise standard business cycle model from the New Keynesian tradition. We equip the monetary authority with a simple rule that fixes total liquidity and sets the interest rate on liquid assets looking at expected inflation. This framework yields a stable equilibrium as long as the Central Bank responds positively to expected inflation, regardless of the magnitude of the interest rate adjustment.

This model thus provides a unified system to analyse how monetary policy rules affect the response of the economy under a supposedly indeterminacy regime. We compare this liquidity model to the basic standard model that is at the core of the New Keynesian Dynamic Stochastic General Equilibrium class, with nominal frictions, technological shocks, and rational expectations. The inclusion of liquidity and a simpler monetary policy rule are

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sufficient to broadly match and reproduce the behaviour of the NKDSGE baseline model. This effort is twofold. First, it allows us to move on known territory, presenting the results in a transparent way and keeping contact with the enormous literature flourished around this class of models. Second, it can be easily implemented in existing theoretical structures with negligible adjustments, making possible a direct test against other models.

We further analyse by means of stochastic simulations how the two policy stances affect directly inflation dynamics. We find that an accommodative monetary policy induces more persistence in the inflation process, making it harder to steer toward any target.

Lastly, in close connection to the 2008 crisis, we experiment with a large drop in liquidity and analyse how different policy regimes affect economic responses.

**Related literature** This paper studies the role and consequences of liquidity in monetary policy and formulates a setup encompassing both aggressive and passive monetary policy interest setting rules. This theoretical model builds on Calvo (2016) and relates in spirit with Michaillat and Saez (2015, 2018), B. Diba and Loisel (2017), and M. B. Canzoneri and B. T. Diba (2005) and M. Canzoneri et al. (2008a,b). We further detail Calvo (2016) model by fully specifying the supply side of the economy and studying its behaviour in discrete time. Throughout the exposition, our aim is to propose a small deviation from the standard NKDSGE framework that constitutes the core of modern fluctuations theory in macroeconomics (Gali, 2015; Walsh, 2003; Woodford, 2003).

This paper is also related to studies on conditions for determinacy in several vintages of New Keynesian models. For example, Benhabib, Schmitt-Grohe, and Uribe (2001) include money in the production function and studies under which fiscal and monetary policy regimes the model displays indeterminacy. Similarly, Leeper (1991) studies how fiscal and monetary policy interact and establishes parametrical regions for determinacy. Along the same lines of fiscal interaction, Cochrane further develops a price level theory grounded on debt and taxes, working around the limitations and shortcomings of the monetary NKDSGEs (Cochrane, 2020). In this model, we abstract from fiscal considerations and focus exclusively on the liquidity factor.

The remainder of the paper is structured as follows: Section 2 introduces liquidity in the NKDSGE framework; Section 3 compares the liquidity model with a baseline one; Section 4 studies the effects of a severe liquidity shock; Section 5 analyses how monetary rules influence inflation dynamics; Section 6 concludes.

## 2 A model of liquidity

We introduce liquid bonds in an otherwise standard, small scale NKDSGE model. This extension is motivated by the utility they provide in being liquid assets, much in line with money holdings, which provide utility services. The idea is to model agents' liquidity portfolio allocation between plain money and liquid assets, interchangeably bonds or bank deposits.

Accordingly, we modify the monetary policy behaviour. We postulate a skeletal rule that feedbacks expected inflation to the interest rate paid on liquid assets.

## 2.1 Consumer

We postulate an economy with an infinitely-lived representative agent, who works, consumes, and holds money alongside with two types of assets. Hence, total wealth is divided between a liquid bond  $B$ , cash  $M$ , and an illiquid bond  $X$ . This latter serves the only purpose of smoothing consumption over time and allocate wealth intertemporally. All assets mature after one period. We assume that the agent is willing to hold  $B$  and  $M$  because they provide transaction services, and therefore utility. In addition,  $B$  bonds pay  $s$  nominal interest rate,  $X$  ones pay nominal interest rate  $i$ , while money pays no interest and is carried on to next period. While money balances in the utility function is not new, bond holdings are more exotic. In this respect, our model is close to Michailat and Saez (2018), although the resulting Euler equation at the steady state is not modified by bonds but rather money holdings. In fact, it is completely homomorphic to introduce a cash-in-advance (or rather, liquidity-in-advance) constraint, as in Calvo and Vegh (1996).

Under these assumptions, the utility maximization program of the consumer is structured as follows.

$$\begin{aligned} & \max_{\{C_s, M_s, B_s, N_s\}_{s=0}^{\infty}} E_t \left[ \sum_{s=0}^{\infty} \beta^s (u(C_{t+s}) + h(B_{t+s}) + v(M_{t+s}) - g(N_{t+s})) \right] \\ \text{s.t. } & C_t + M_t + X_t + B_t = W_t N_t + (1 + s_{t-1}) B_{t-1} + (1 + i_{t-1}) X_{t-1} + M_{t-1} \end{aligned} \quad (1)$$

Where we assume additively separable (dis)utilities for consumption, cash, liquid bonds, and hours worked  $N$ . The intertemporal budget constraint summarises expenditures, allocations and income sources: interests promised in  $t - 1$ , carried-on money, and labour income. Moreover,  $c$  is the result of aggregating a measure one of differentiated goods via a Dixit-Stiglitz aggregator with constant elasticity of substitution  $\theta$ . This also implies that  $P$  is the price index of the underlying goods.

It is useful to reformulate the budget constraint in terms of total wealth and real quantities before deriving the system of first order conditions. Therefore, let  $D = X + M + B$  be the total wealth held by the consumer. Replacing  $X = D - M - B$  in the budget constraint and appropriately dividing through  $P$  gives the following real budget constraint.

$$c_t + d_t = w_t N_t + \frac{s_{t-1} - i_{t-1}}{1 + \pi_t} b_{t-1} + (1 + r_{t-1}) d_{t-1} - \frac{i_{t-1}}{1 + \pi_t} m_{t-1} \quad (2)$$

Where  $\pi = \frac{P_t}{P_{t-1}} - 1$  is the inflation rate and lower-case indicates real quantities.

With this reformulation, the FOCs system implies the equilibrium conditions in (3).

$$\begin{aligned} u'(c_t) &= E_t [\beta (1 + r_t) u'(c_{t+1})] & h'(b_t) &= E_t \left[ \beta \lambda_{t+1} \frac{i_t - s_t}{1 + \pi_{t+1}} \right] \\ \frac{g'(N_t)}{u'(c_t)} &= w_t & v'(m_t) &= E_t \left[ \beta \lambda_{t+1} \frac{i_t}{1 + \pi_{t+1}} \right] \end{aligned} \quad (3)$$

where  $\lambda_t$  is the Lagrangian multiplier, the first equation is the usual Euler equation for intertemporal consumption, the second equates marginal cost and benefits of work, and the last two equations govern the allocation decision between liquid bonds  $b$  and real money

balances  $m$ . In particular, last equation discounts money holdings by the real interest rate, that is the loss from inflation; the equilibrium condition on bonds, instead, regulates liquid asset holdings on the spread between nominal interest rate  $i$  and yield of such bonds, both in real terms.

## 2.2 Firms

The production side of this economy is straightforward and assumes a measure one of infinitesimal firms, indexed by  $j \in [0, 1]$ . Each firm is embedded with a technology that employs only labour, so that the production function is

$$Y_{jt} = A_t N_{jt}^a. \quad (4)$$

The term  $A$  captures the stochastic productivity of the economy and follows a simple AR(1) process,  $a \in (0, 1)$  represents the decreasing returns to scale, and  $N_{jt}$  the individual employment of each firm. As the consumption good results from the CES aggregator, every firm  $j$  faces a demand schedule (5), relative to aggregate production, with  $\theta$  being the elasticity of substitution and  $P_{jt}$  the firm's price.

$$Y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\theta} Y_t \quad (5)$$

We assume nominal rigidity à la Calvo (1983):<sup>1</sup> every period  $(1 - \alpha)$  % of firms are given the chance to update their price, while the remaining share will stick to previous prices. This entices a forward-looking behaviour in firms when they optimise their expected discounted profits, as they take into account the duration of their price. Firm  $j$ 's marginal cost is  $\mathcal{MC}_{jt} = \frac{W_t/P_t}{aY_{jt}/N_{jt}}$ , whence the expected discounted profits in eq.(6).

$$\begin{aligned} \max_{P_j^*} \quad & E_t \left[ \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s} \left( P_j^* Y_{jt+s} - \mathcal{MC}_{jt+s} Y_{jt+s} \right) \right] \\ \text{s.t.} \quad & Y_{jt+s} = \left( \frac{P_{jt+s}}{P_{t+s}} \right)^{-\theta} Y_{t+s} \end{aligned} \quad (6)$$

Where  $\alpha$  is the Calvo pricing parameter,  $Q_{t,t+s}$  is the stochastic discount factor between periods  $t$  and  $t + s$  the consumer uses to weight future profits, and  $P_j^*$  is the optimal price chosen by the firm. Factoring in the constraint and solving the program with a symmetry argument gives two results. First, firms price with a constant markup over the marginal cost, and second that the optimal price is specified as a function of expected future marginal costs, price index levels and economic activity, as shown in eq.(7). This same equation also presents the inflation dynamic as a AR(1) process, depending on past prevailing prices and

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<sup>1</sup>The precise modelling of the nominal rigidity is inconsequential, as the main novelties are in the consumers' side of the model. Hence, quadratic adjustment costs as in Rotemberg (1982) might be added without loss of fundamental insights.

current updated prices.

$$\frac{P^*}{P_t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta\alpha)^s Y_{t+s} \left(\frac{P_{t+s}}{P_t}\right)^\theta \mathcal{M}C_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta\alpha)^s Y_{t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\theta-1}} \quad (7)$$

$$P_t^{1-\theta} = (1 - \alpha) P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta}$$

### 2.3 Monetary authority and market clearing

A second exotic ingredient is the Central Bank, its policy, and total liquidity management. We assume the existence of a monetary authority that operates in two ways in the economy. First, it sets the total amount of liquidity in circulation, namely eq.(8). Importantly, the Central Bank does not determine the allocation between cash and liquid bonds, but only the sum of the two, irrespectively of the portfolio composition. For the sake of simplicity, we will also assume a fixed nominal amount of liquidity in the economy, so that

$$Z_t = B_t + M_t \quad (8)$$

$$Z_t = \bar{Z} \quad \forall t \quad (9)$$

A fixed amount of liquidity implies no issuance of liquid bonds, which are instead traded and circulated among agents. The same principle applies to money, which is injected or withdrawn to seamlessly counteract the movements in liquid bonds trade. In this light,  $B$  closely resembles bank deposits, for both can be readily exchanged for cash. One can think of  $Z$  as a fixed money supply (M2), although a full characterisation would have  $B$  issued by government in the form of liquid, risk-free treasury bonds. Along the same lines, the Central Bank would influence the circulation of these bonds to manage total liquidity and, more importantly, to coordinate with fiscal policy.<sup>2</sup>

The Central Bank also sets the policy interest rate  $s_t$  paid on liquid bonds following a constant rule. It is useful to think in this context to the link between the Federal fund rate and the interest rate on the shortest-maturity Treasury Bill. In detail, the rule responds solely to inflation expectations and does ignore any level of economic slack contrary to more usual Taylor feedback rules:

$$\exp(s_t) = E_t \exp(\gamma\pi_{t+1}). \quad (10)$$

This skeletal structure is clearly a simplified Taylor rule, but the values for  $\gamma$  are crucial in showing that an accommodative Central Bank does not necessarily drive the economy down a hyper-inflation spiral, specifically when it takes values smaller than one.<sup>3</sup> It can easily be extended to a full fledged Taylor Rule including real slack without impairing or modifying the final results. For example, this specification of the Taylor Rule relates to that

<sup>2</sup>Notable examples are the CARES Act in the US, or the combined SURE and Next Generation EU in Europe, both in response to Covid-19 crisis. These were preceded by ECB's APP-PSPP and Fed's Open Market Operations as measures for the 2008 crisis.

<sup>3</sup>Although it is worth recalling that a Central Bank complying to the Taylor principle actually *destabilizes* the economy (Benhabib, Schmitt-Grohe, and Uribe, 2001).

of the European Central Bank, which in its mandate contemplates explicitly only prices stability and not employment or economic slack.

The rationale behind this specification is that keeping under control the liquidity in circulation the Central bank assures that inflation follows a specified path. This particular specification of the monetary policy links the inflation rate and the return yield on the liquid bonds. Therefore, the Central Bank in our model levers the liquidity allocation via interest rate setting.

Lastaking eq.(9) and dividing through by the prices level, one can obtain the values for liquidity allocation in real terms, as well as a backward-looking expression for real liquidity depending on current inflation.

$$z_t = m_t + b_t \iff z_t = \frac{z_{t-1}}{1 + \pi_t} \quad (11)$$

This last equation, with the market clearing condition  $C_t = Y_t$ , closes the model.

## 2.4 Linearised model

In this section we briefly present the system of equations resulting from loglinearising eqs. (3), (7), (10), and (11) around a zero-inflation steady state. To this end, we assume precise functional forms for the utility functions, namely

$$\begin{aligned} u(c) &= \frac{c^{1-\sigma}}{1-\sigma} & v(m) &= \frac{m^\psi}{\psi} \\ h(b) &= \frac{b^\phi}{\phi} & g(N) &= \chi \frac{N^{1+\eta}}{1+\eta} \end{aligned} \quad (12)$$

With  $1 \geq \phi > \psi > 0$ , which implies that the consumer is more sensitive to bonds rather than money, in line with everyday financial decisions. The other functional forms assumed are consistent with more traditional exercises and well settled in the NKDSGE literature. This shared ground highlights how consequential liquid assets can be, once modelled as a complement to cash, especially on the policy rule.

Once we log-linearise the model we obtain a system of linear equations whose properties can be easily and extensively studied. We perform a comparison with the simple 3-equation model presented in Galí (2015), for example.<sup>4</sup>

$$\hat{y}_t = \frac{1-\psi}{\sigma} \hat{m}_t + E_t \hat{y}_{t+1} + \frac{1}{\sigma} E_t \pi_{t+1} \quad (13)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (\hat{y}_t - \hat{y}_t^f) \quad (14)$$

$$\left[ \frac{1-\psi}{m^2} + m^{\frac{\phi-\psi}{1-\psi}} (1-\psi) \right] \hat{m}_t + (\beta y^{-\sigma} m^{1-\psi}) s_t - (1-\phi) \left( 1 + m^{\frac{\phi-\psi}{1-\psi}} \right) \hat{z}_t = 0 \quad (15)$$

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<sup>4</sup>A more detailed walk-through for obtaining this system is provided in the Appendix (A.1)

$$\hat{z}_t = \hat{z}_{t-1} - \pi_t \quad (16)$$

$$s_t = \gamma E_t \pi_{t+1} \quad (17)$$

Briefly describing the system above, eq.(13) is the Euler equation augmented with the real money balances, which affect positively the contemporaneous output gap. Eq.(14) is the Phillips curve of this economy, in line with more classical models, eq.(15) is the money demand function (depending on liquid bonds interest rate  $s$ , total real liquidity  $z$ ), eq.(16) captures intertemporal changes in total real liquidity, and finally eq.(17) represents the monetary policy rule. We adopt the convention that  $\hat{x}$  is the percentage deviation of  $x$  from its steady state, while all lower-case, unhatted, and time independent variables are steady state values. Moreover, we use  $y^f$  for flexible prices output. To add more details:

$$\begin{aligned} \kappa &= \left[ \frac{(1-\alpha)(1-\alpha\beta)a}{\alpha(a+\theta(1-a))} \right] \left[ \frac{1+\eta+a(\sigma-1)}{a} \right] & m &= \left( \frac{y^\sigma}{1-\beta} \right)^{\frac{1}{1-a}} \\ \hat{y}_t^f &= \frac{\eta+1}{1+\eta+a(\sigma-1)} \hat{A}_t & y &= \frac{\eta+1}{1+\eta+a(\sigma-1)} \end{aligned} \quad (18)$$

To finalise the model, we introduce two classic shock to perturb the model around its steady state to display convergence dynamics. Both shocks follow a stationary  $AR(1)$  process:

$$\begin{aligned} \hat{A}_t &= (1-\rho_A)\bar{A} + \rho_A\hat{A}_{t-1} + \epsilon_t^A & \epsilon_t^A &\sim N(0, \sigma_A) \\ v_t &= \rho_v v_{t-1} + \epsilon_t^v & \epsilon_t^v &\sim N(0, \sigma_v) \end{aligned}$$

Both shocks are simply added to their respective equations and throughout the whole simulations we calibrate their persistence parameters  $\rho$  to the same value.

As this approximation of the full model can be easily simulated, we exploit it to check for which calibration sets the model generates a unique and stable equilibrium. In addition to the real, technological disturbance, we add a policy shock impulsed by the Central Bank. The former induces a change in the total factor productivity  $A$  from its steady-state value, set to 1. The latter is a shock to the monetary rule detailed in eq.(10). These two shocks allow the comparison with the aforementioned standard models, so to perform a horse race and check consistency of our augmented model. Assessing whether our setup replicates the classic reactions of well-known models is paramount to validate its structure and internal workings.

### 3 Calibration and IRFs

We calibrate the model to the values presented in Table (1), taking the most common values used in the literature. The only restriction involved in the model concerns the exponents of bonds and real balances utility functions. For the "Taylor Principle" parameter of our

policy rule, we explore two values: the first is supposed to violate the Blanchard and Kahn (1980) condition and thus produce an unstable solution, whilst the second is the one most commonly found in both empirical studies and theoretical exercises.

Parameter	Descr.	Value
$a$	ret. to scale	.6
$\beta$	discount rate	.975
$\sigma$	intertemp. el. of subst.	5
$\theta$	intratemp. el. of subst.	3.8
$\alpha$	price duration	.75
$\psi$	bond el.	.02
$\phi$	money el.	.65
$\eta$	Frisch elast.	1
$\chi$	labour disutility	1
$\gamma$	Taylor param.	{.5; 1.8}
$\rho_A$	persistence, TFP shock	.65
$\rho_v$	persistence, MP shock	.65

TABLE 1: CALIBRATION FOR MODEL SIMULATIONS

First, our calibrated model generates a unique, stable equilibrium for both values of  $\gamma$ , meaning that sunspot equilibria are ruled out even if the Central Bank reacts passively the inflation expectations of the economy. Second, our model behaves as expected once compared with the 3-equation NKDSGE counterpart.<sup>5</sup> In fact, under the same calibration the two models show the same behaviour in terms of reactions to shock, as it is possible to see in the next figures. We first compare side-by-side the effects of a technological shock (Fig.(1)), then compare the effects of a monetary policy shock under two regimes for our model (Fig.(2) and Fig.(4)).

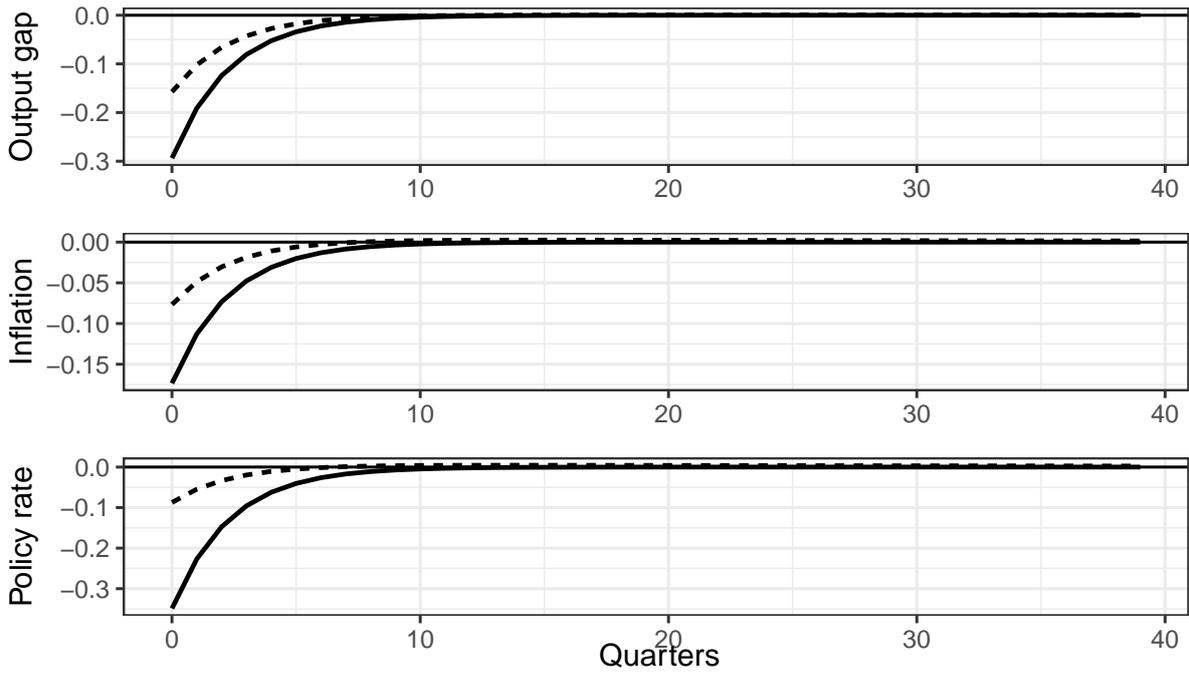
### Technological shock

From Fig.(1), a one-standard-deviation, positive shock to total factor productivity produces the same response in our model and in the standard NKDSGE one. This is due to the very same structure of the production side of the two modelled economies. We focus on three common aggregates. Following a TFP shock, in both cases the output gap turns negative, inflation falls, and the reference interest rate falls in response. While this reaction is common across models, magnitudes mark little differences. In particular, while the output gap and inflation falls more in the NKDSGE model, our proposal implies less movement in these aggregates: for inflation and output gap, the response in our model is broadly half on impact. This milder response in our model might be driven by the stripped-down version of our monetary policy rule, which prevents output gap movements to percolate into the variations in  $s$ . Alternatively, this dampened propagation might result from our assumptions in the consumption and financial blocks of the model, where the TFP shock is dissipated smoothly through two complete financial markets.

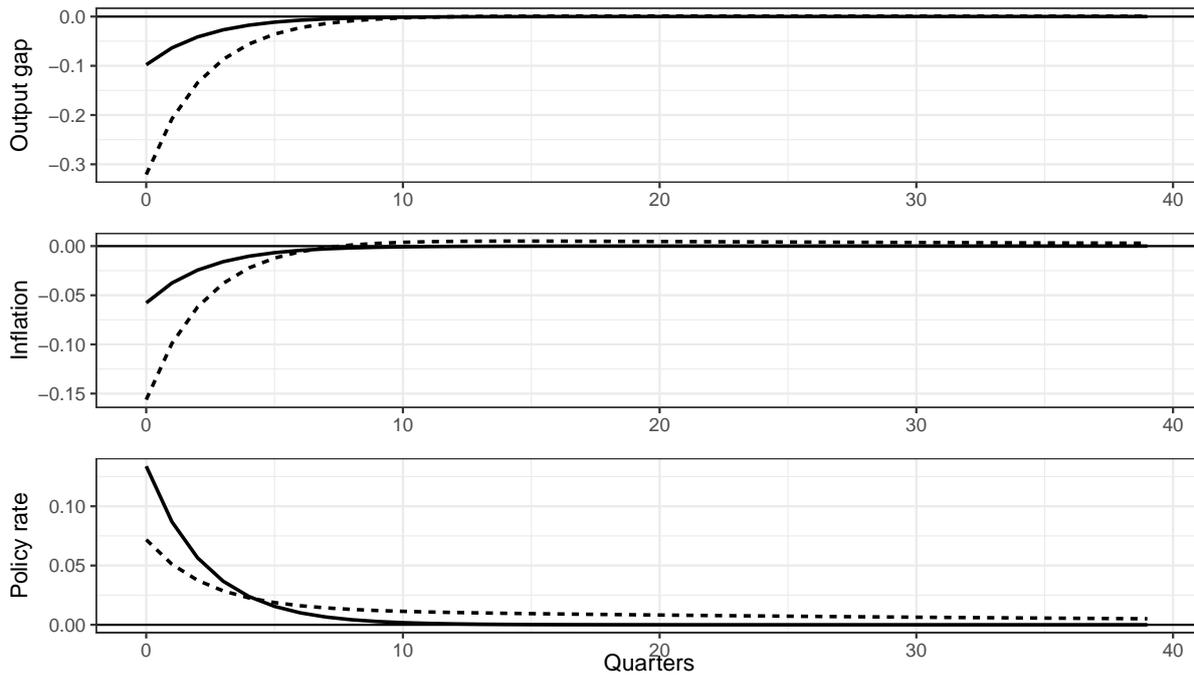
### Monetary policy shock

When the modelled economies are hit by a monetary policy shock and the liquidity

<sup>5</sup>Gali (2015), Chapter III, version with interest rate rule.



**FIGURE 1:** LIQUIDITY MODEL (DASHED,  $\gamma = 1.8$ ) AND GALI (2015) (SOLID): IMPULSE RESPONSE FUNCTIONS FOLLOWING A 1- $\sigma$  TFP SHOCK. VALUES IN PERCENTAGES.



**FIGURE 2:** LIQUIDITY MODEL (DASHED,  $\gamma = 1.8$ ) AND GALI (2015) (SOLID): IMPULSE RESPONSE FUNCTIONS FOLLOWING A 1% KEY RATE HIKE, ANNUALISED. VALUES IN PERCENTAGES.

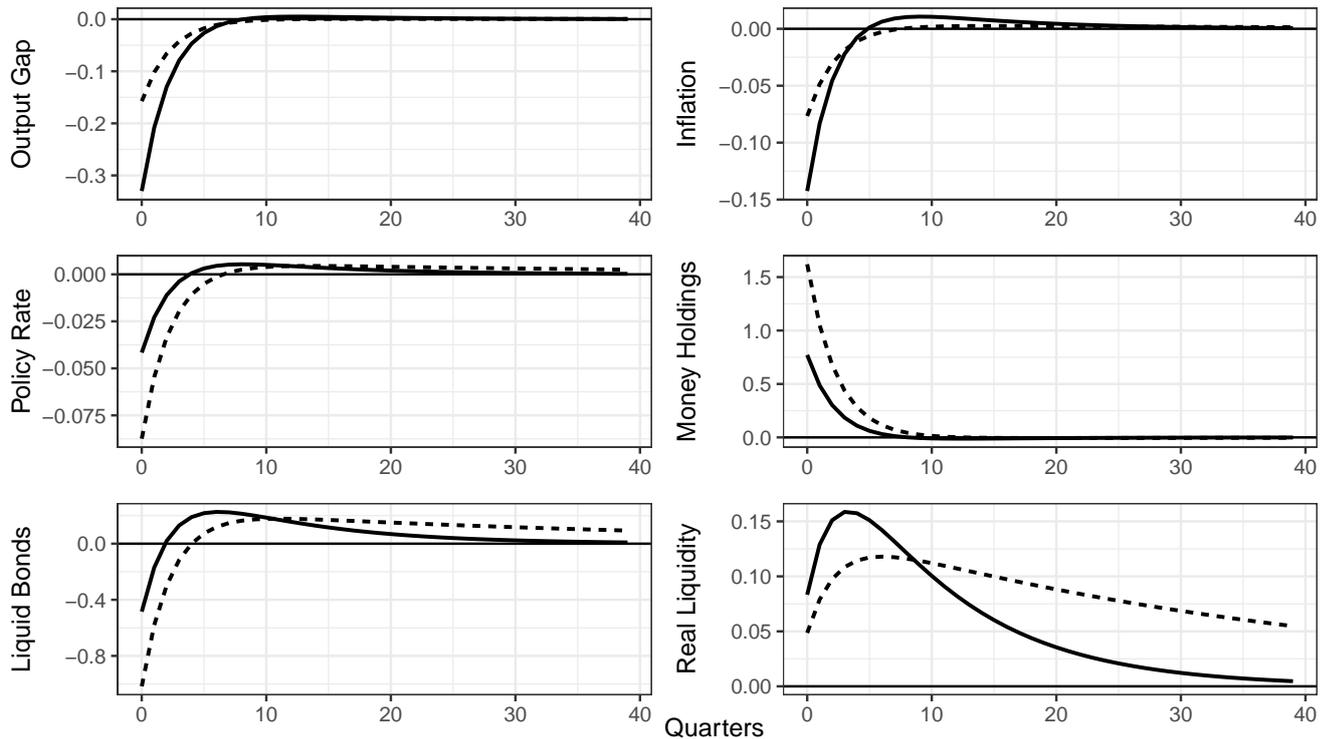
model complies to the Taylor Principle, they generate the IRFs pictured in Fig.(2), again corresponding to common aggregates across models. The IRFs produce the same profiles in both cases, but the liquidity models displays greater impacts on output gap and inflation. In both cases the magnitude is roughly thrice that of the baseline NKDSGE, while

direction and adjustment correspond closely. The skeletal policy rule engrained in the liquidity model abstracts from economic activity level, thus the Central Bank is not facing trade-offs between economic activity and inflation and focuses solely on the latter. This monetary policy rule, moreover, impedes the feedback loops between policy rate, inflation and economic activity, designing a different propagation mechanism: adjustments in the market for money and liquid bonds affect consumption and in turn production.

Focusing on the policy rate, our model responds roughly twice less than the NKDSGE model to the very same shock, pointing again to a propagation mechanisms that is substantially different and occurs through the financial position of the agent. In addition, our monetary shock is well more persistent in the interest rate, converging back to zero only when inflation levels off, too. The absence of economic activity in the rule makes that the policy rate essentially mirrors the inflation profile. In comparison to the baseline model, the liquidity one presents a slower convergence path to the steady state. This model, therefore, entails a higher degree of persistence in its design.

### 3.1 Liquidity with an accommodative Central Bank

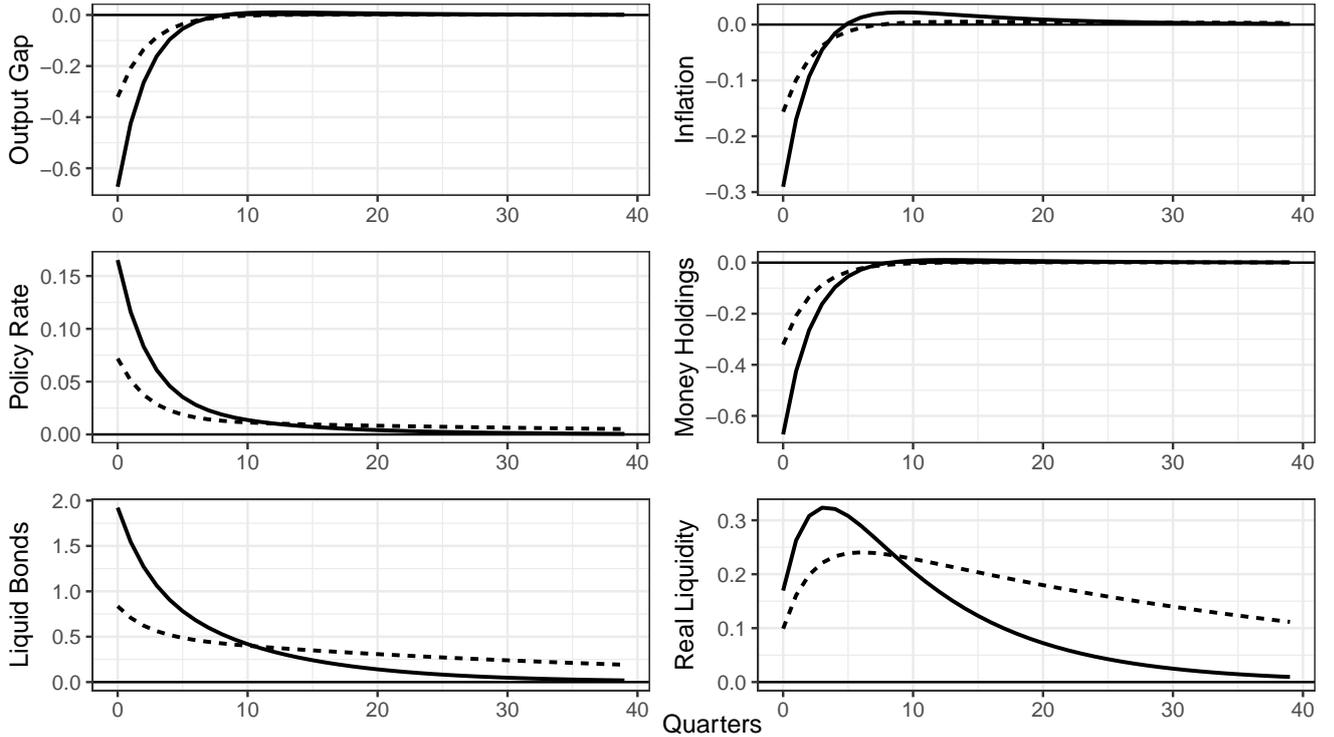
We turn now to showing how the liquidity model reacts to the previous shocks when the Central Bank does not comply to the Taylor principle. In this case, we set  $\gamma = .5$  and expose the behaviour of endogenous aggregates present in the economy, plus the behaviour of the liquid bond holdings, recovered from the FOC for a complete picture.



**FIGURE 3:** IMPULSE RESPONSE FUNCTIONS FOR OUR MODEL: WITH ACCOMMODATIVE (SOLID) AND AGGRESSIVE (DASHED) REACTION TO EXPECTED INFLATION AFTER A PRODUCTIVITY SHOCK (ONE STANDARD DEVIATION) – ALL ENDOGENOUS VARIABLES, VALUES IN PERCENTAGES.

Fig.(3) collects IRFs generated by a positive TFP shock. In the aggressive regime, output, inflation and the policy rate  $s$  fall. Interestingly,  $s$  and  $\pi$  slightly overshoot before converg-

ing to zero from above from the fifth quarter, roughly. This behaviour is reflected in real liquidity and its components: the technological shock doubles the impact on total real liquidity, the interest rate drop triggers a reallocation away from bonds towards cash  $m$ , until  $s$  overshoots and subsequently balances the overall effect on liquidity reshuffling. All in all, the main discrepancy between the two regimes is in the asset block: the inflation path influences more dramatically and on a shorter period the amount of real liquidity when the Central Bank is passive. Conversely, the path back to the steady state is significantly quicker in this regime.



**FIGURE 4:** IMPULSE RESPONSE FUNCTIONS FOR OUR MODEL: WITH ACCOMMODATIVE (SOLID) AND AGGRESSIVE (DASHED) REACTION TO EXPECTED INFLATION AFTER A MONETARY POLICY SHOCK (ANNUALISED 1% POLICY RATE HIKE) – ALL ENDOGENOUS VARIABLES, VALUES IN PERCENTAGES.

Fig.4 summarises the response to a monetary policy shock that raises the interest rate  $s$  on bonds of 1%. First off, the output gap moves in the same direction and is affected similarly to the  $\gamma > 1$  case. Secondly, inflation responds as expected and after 15 quarters the shock is fully absorbed, without unexpected evolutions of the prices.

Total real liquidity  $z$  and its components,  $m$  and  $b$ , comove in reaction to a monetary policy shock. Two forces are at work in this case, the inflation effect and the reallocation towards the more remunerative asset. On impact, inflation decreases: this path influences real liquidity as future inflation will be higher than today, increasing current liquidity until inflation overshoots its steady state level and turns slightly positive. This happens in the first five quarters, approximately. When inflation turns positive (although extremely close to zero) real liquidity peaks and decreases smoothly.

In the meantime, the agent adjusts its portfolio of assets profiting from the increased return on the liquid asset,  $b$ . In this sense, the IRF for  $m$  mirrors that of  $b$ , with a reallocation

away from money holdings to liquid bonds.

Under the passive monetary policy regime the economy experiences different magnitudes in the aggregates. When the Central Bank is accommodative, output, inflation, and the key interest rate double their impact change, and the latter becomes less persistent, overall. This last outcome falls in line with Primiceri (2005), and sheds light on how monetary policy stances can affect aggregate dynamics in general, and inflation dynamics in particular. Taking this implication to the data would imply that prior to the Great Moderation period inflation was reporting higher degrees of inertia, whilst it would decrease afterwards. We analyse the effects of different monetary policy regimes on inflation dynamics in Section (5).

## 4 The 2008 crisis: severe liquidity shortage

This model lends itself to an interesting experiment: although nominal liquidity is assumed to be constant at  $\bar{Z}$  and real liquidity  $z$  moves with the inflation rate, we hit  $z$  with a negative shock and study the behaviour of our model, as in eq.(19). Although not orthodox, this is a practical short-cut: neglecting where that missing liquidity goes *physically* – and in which proportion money and bonds are affected – lets us focus on the dynamics of convergence to the steady state. One could think of this experiment as a sudden drop in the combined liquidity of bonds and money, closely related to the liquidity dry up triggering the 2008 Global Financial Crisis. In the aftermath of such recession, major Central Bank rapidly hit the zero lower bound on policy interest rates, thus in effect adopting an accommodative monetary policy stance, equivalent to that admitted by the liquidity model. In this respect, analysing how aggregates react to a sudden liquidity dry up sheds light on how different interest rate rules interact and affect the overall dynamics.

We produce IRFs and discuss their economic interpretations under the two regimes of monetary policy.

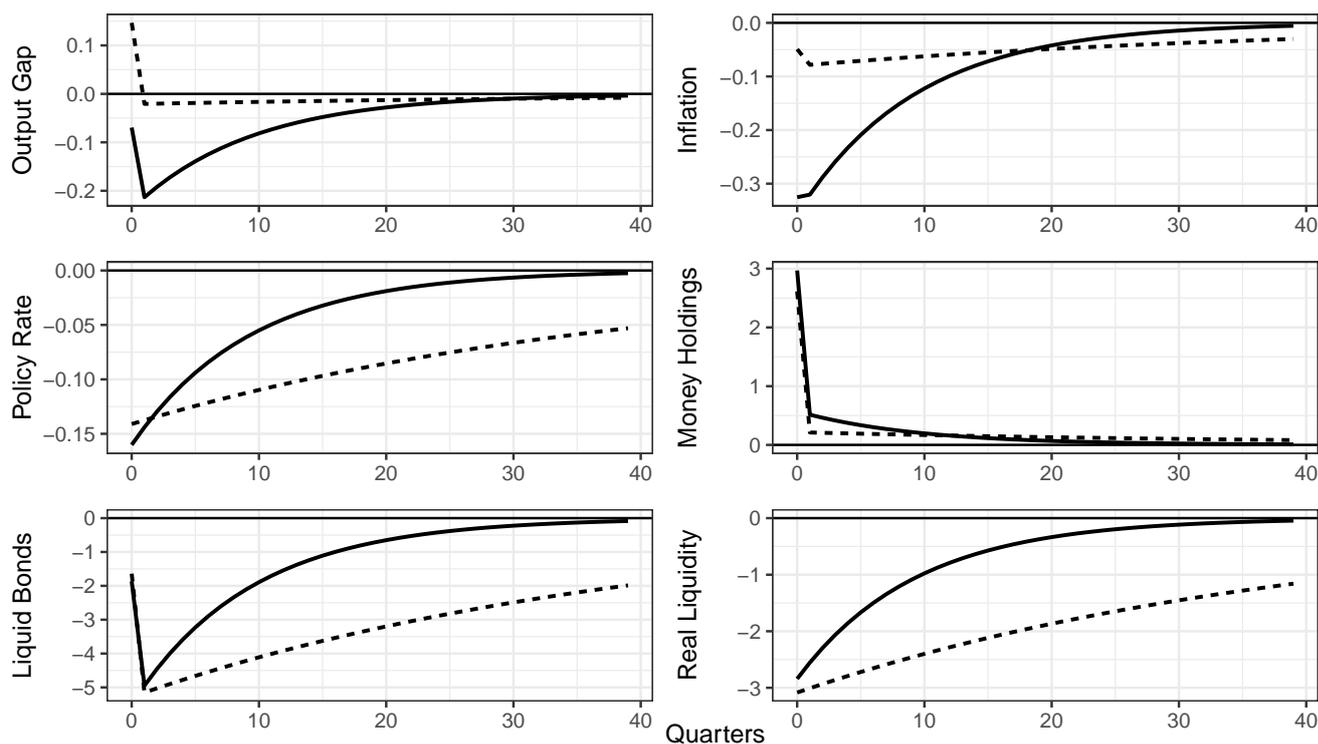
$$\hat{z}_t - \epsilon_t^z = \hat{z}_{t-1} - \pi_t, \text{ with } \epsilon_t^z / z \cong 10 \quad (19)$$

Following an abrupt and violent liquidity dry-up the model shows a general reaction broadly independent from the behaviour of the monetary authority, but substantial differences emerge in magnitude and reversion to the steady state levels.

The shock impacts at first money demand  $\hat{m}$ , for a given policy rate  $s$ , which spikes up. Conversely the representative agent disinvests in the liquid bond, proportionally more than the missing liquidity. This results from the preference for money with respect to bonds that assumed in the calibration.

At this stage, the Phillips curve (14) and the IS equation (13) propagate the shock to the rest of the economy. The money term in the Euler equation (13) transmits the shock to current output that spikes as well on impact. Under a passive monetary policy this translates into a limited effect of the shock on impact, and a degradation in the following quarters. The Phillips curves then squares the expected inflation with current  $\pi$  and widened output gap. Inflation expectations subsequently drive the monetary policy decisions which translate into two distinct paths for realised inflation.

The money term in the IS curve is the telling point between the two regimes, together with the path for liquid bonds,  $b$ .



**FIGURE 5:** REAL LIQUIDITY SHOCK: AGGRESSIVE ( $\gamma = 1.8$ , DASHED) VS ACCOMMODATIVE ( $\gamma = .5$ , SOLID) POLICY RULES, VALUES IN PERCENTAGES.

Under both regimes money converges back to the steady state relatively fast, showing that real liquidity is deeply intertwined with liquid bonds. Most notably, under an accommodative Central Bank real liquidity recovers more rapidly, thanks to a deflation that accelerates the recovery of  $z$  but impedes a quick rebound in output. Interestingly, when the money authority conducts an active policy, the inflation path – contained deflation with slow recovery – turns into persistently low rates, well beyond the case of a passive Central Bank.

The sharp difference in the reaction of the two regimes lies in the severity of the impact and the duration of the recovery. Inflation and output, in particular, show starkly different behaviours: when the Central Bank passively follows inflation expectations a liquidity dry-up triggers a deep recession with a relatively long recovery (more than 30 quarters); the same applies to inflation, too. An active monetary stance against a liquidity shock tames the damage and facilitates the recovery, somewhat controlling the effects within the financial sector of the economy.

To sum up, and combining these results with previous information, we suggest that Central Banks complying to the Taylor Principle have a firmer control on contagion when a liquidity crisis hits. According to our stylised model, in fact, active monetary policy helps containing and limiting the damage to the sole financial sector of the economy, with reduced impact and consequence on real activity. The stark difference in the set of IRFs, clearly, lies in the fall of the output gap: it widens under passive monetary policy while it marks a mild recessions under active policy, although duration is similar. As we already remarked, recovery speeds are substantially different.

## 5 Dissecting simulated inflation dynamics

One interesting experiment involves the inflation dynamics generated by these two models: does an passive Central Bank produce unexpected inflation dynamics? In this framework, it is straightforward to generate abundant time series and hence conduct some ex-post econometric exploration. To offer a more comprehensive comparison, we simulate and analyse two other DSGE model, namely Ascari and Sbordone (2014) and Smets and Wouters (2007). The former study how trend inflation affects aggregate dynamics and policy in a generalised New Keynesian model, while the latter build a rich environment with nominal frictions and indexation for wages and prices, investment adjustment cost, and a large number of shocks and outperform VARs in short term forecasting.

We generate for each model 500000 observations, or 125000 years of synthetic history: this should assure convergence of the estimators and tight confidence intervals. By the very structure of the models, the Data Generating Processes of these series is a linear system shocked by AR(1) normal innovations: one should not be surprised that the data generated are also Gaussian. Comparability across models is meaningful because of the close parametrisation and the same sequence of shocks used.

For clear reasons, we limit our interest to the global autoregressive properties of the inflation series, which is generated by a single, stable, and well behaved DGP for each case. As we calibrate the persistence of all shocks to  $\rho = .65$ , we expect to find values in this neighbourhood. Any difference between the simulated series is due to the propagation mechanism and, most importantly, the monetary response function. Moreover, when comparing our model in its two policy regimes, we will be able to pick up the different inflation dynamics enticed by the passive monetary policy stance.

We estimate first an  $AR(5)$ : our model complying to the Taylor Principle, the same violating it, and the workhorse NKDSGEs. Secondly, we set an upper bound on the lags to 120, and pick the optimal lag number minimising the Bayesian Information Criterion. Table (2) presents the results for an  $AR(5)$ .

Looking at the coefficients on the first lag, we see our expectations confirmed, as all coefficients are tightly close to the calibrated parameter, no constant is statistically different from zero, and significance decreases after the first lag for classic NKDSGE models. A notable exception is the Smets and Wouters (2007) model, which reports five significant lags with relatively large coefficients. The most remarkable feature of such model, though, is that it displays a quasi unit-root in inflation, as the first lag is statistically close to one.

Interestingly, as we depart from the NKDSGE models to the liquidity model with accommodating Central Bank, the coefficient on the first lag moves away from the calibrated value, downwards. This is interesting because it confirms the consensus that a passive monetary authority has less command over inflation path and fails at taming its dynamics back on target. The flip side of this latter aspect is that, when first lag becomes less relevant, previous ones acquire more weight. Overall, hence, inflation seems to become more persistent when Central Banks do not follow an aggressive Taylor rule.

On the other hand, the magnitude of the significant coefficients of the two parametrisations of the liquidity model are comparatively small. While for an active Central Bank (model (3)) the autoregressive coefficients quickly approach zero, for a passive one these still become smaller but remain roughly ten times bigger than those of the other models, and still significant.

	<i>Dependent variable:</i>				
	Simulated Inflation				
	(1)	(2)	(3)	(4)	(5)
Const.	-.00005 (.0003)	-.0002 (.0005)	-.0001 (.0002)	-.0002 (.0003)	-.0004 (.0005)
1 <sup>st</sup> lag	.648*** (.001)	.597*** (.001)	.640*** (.001)	.652*** (.001)	.966*** (.001)
2 <sup>nd</sup> lag	.001 (.002)	-.017*** (.002)	-.005*** (.002)	-.001 (.002)	-.004** (.002)
3 <sup>rd</sup> lag	.00003 (.002)	-.014*** (.002)	-.003** (.002)	.0004 (.002)	-.004** (.002)
4 <sup>th</sup> lag	-.001 (.002)	-.013*** (.002)	-.003** (.002)	.0001 (.002)	-.005** (.002)
5 <sup>th</sup> lag	.002 (.001)	-.029*** (.001)	-.009*** (.001)	.001 (.001)	-.025*** (.001)
Adjusted R <sup>2</sup>	.421	.338	.402	.424	.875

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

**TABLE 2:** AR (5) ESTIMATES ON SIMULATED DATA FROM FIVE MODELS: (1) GALI (2015), (2) LIQUIDITY MODEL  $\gamma = 1.8$ , (3) LIQUIDITY MODEL  $\gamma = .5$ , (4) ASCARI AND SBORDONE (2014), (5) SMETS AND WOUTERS (2007). ONLY TECHNOLOGICAL AND MONETARY POLICY SHOCKS ARE ALLOWED, EACH MODEL IS SIMULATED FOR 500000 PERIODS, AFTER DISCARDING THE FIRST 100000 ITERATIONS. ALL SHOCKS ARE SET TO HAVE ZERO MEAN, EQUAL VARIANCE, AND ARE IID. SECOND AND THIRD COLUMNS PRESENT ESTIMATES FOR OUR MODEL WITH LIQUIDITY, COMPLYING TO THE TAYLOR PRINCIPLE AND VIOLATING IT, RESPECTIVELY.

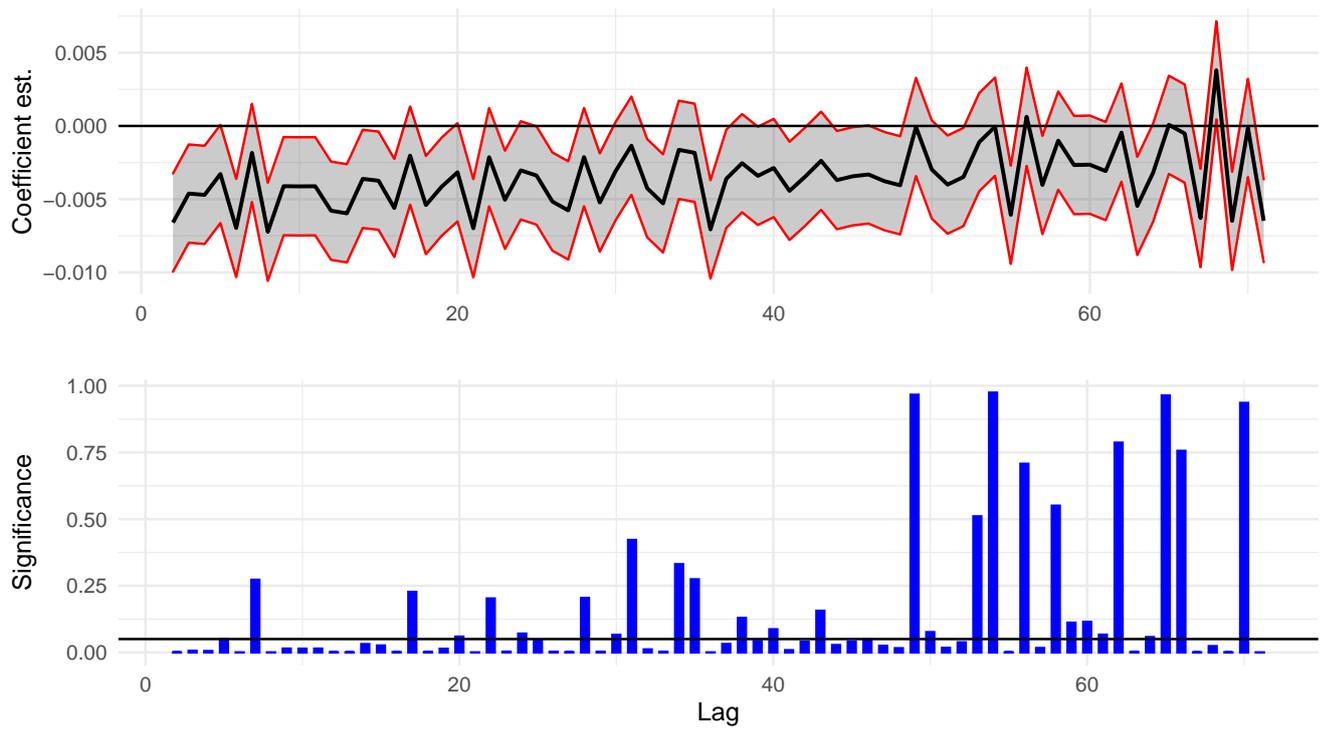
These results point to a role for monetary policy stance in substantially influence the inflation dynamics, along the same lines traced by Cogley, Primiceri, and Sargent (2008), Cogley and Sargent (2002, 2005), and Primiceri (2005).

A more interesting exercise is to compare the optimal lags for an  $AR(k)$  process. This procedure finds that the optimal number of lags for our model with liquidity and Taylor principle (model (3) in Tab.(3)) is around 70, whilst for the version parametrised in accordance to the Taylor Principle (model (2), *ibid*) it is around 50, for the standard NKDSGE it is merely 2. Table (3) offers more details on this result.

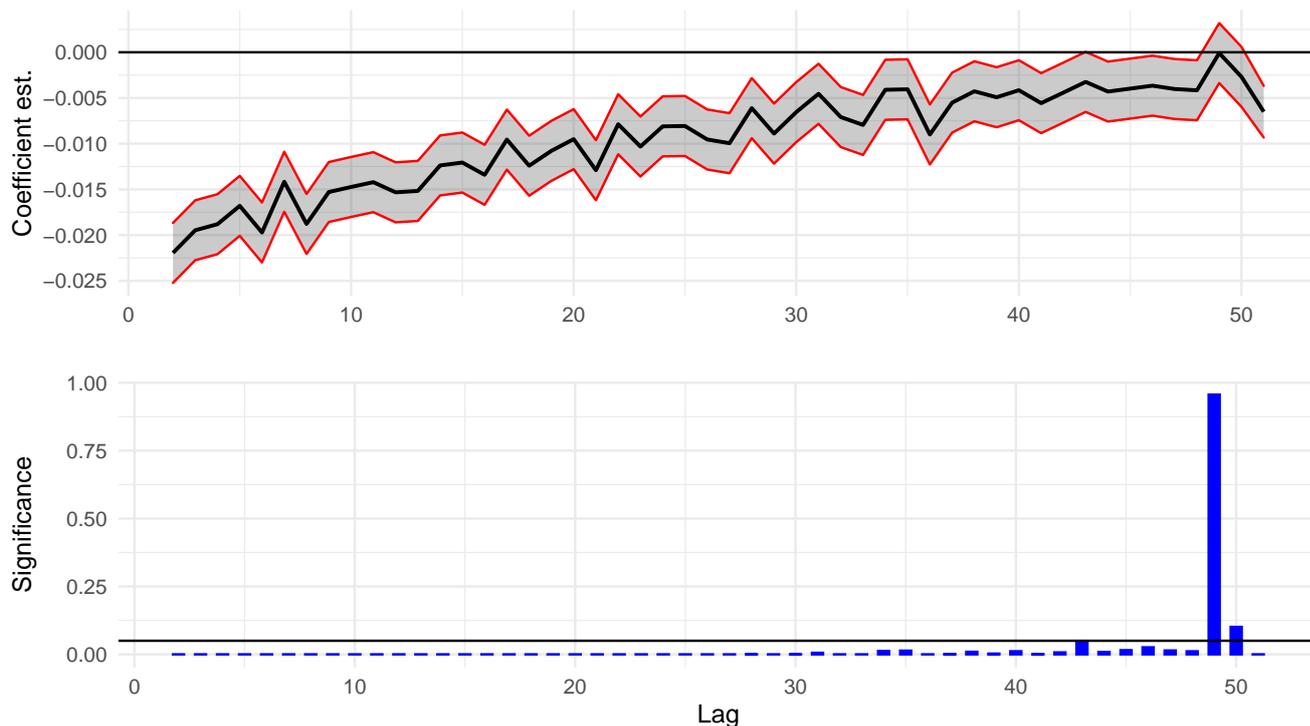
Models	Stats			
	Opt. lags	Sign. lags	adj. R <sup>2</sup>	BIC
(1) Gali (2015)	2	50%	.421	-280676.9
(2) Liq. $\gamma = 1.8$	51	80%	.348	-413028.9
(3) Liq. $\gamma = .5$	71	32%	.401	-1033066
(4) Ascari and Sbordone (2014)	2	50%	.421	-422629.6
(5) Smets and Wouters (2007)	13	38%	.719	-486558.7

**TABLE 3:** OPTIMAL LAGS FOR AR PROCESS OF INFLATION FOR STANDARD (1) BASELINE NKDSGE (GALI, 2015), (2) LIQUIDITY MODEL COMPLYING TO THE TAYLOR PRINCIPLE ( $\gamma = 1.8$ ), (3) LIQUIDITY MODEL VIOLATING IT ( $\gamma = .5$ ), A MODEL WITH TIME-VARYING TREND INFLATION (ASCARI AND SBORDONE, 2014), AND (5) A MEDIUM SCALE WORKHORSE DSGE (SMETS AND WOUTERS, 2007). OPTIMAL LAGS ARE THOSE MINIMISING THE BIC. ALL MODELS ARE FED THE SAME SEQUENCE OF SHOCKS OF THE SAME VARIANCE, GENERATING 500000 QUARTERLY OBSERVATIONS.

An optimal lag number does not imply that *all* regressor lags are significant. It only implies that all significant lags are part of the regression, likely a subset of the total lags. With this in mind, our model of liquidity with an accommodative monetary authority shows that today's inflation depends on a long sequence of lags. The number of optimal lags decreases when we let the Central Bank respond aggressively to expected inflation. On the other hand, the baseline NKDSGE model produces a process with extremely short memory, likewise for the model of trend inflation. In line with the result from Tab.(2), the medium scale Smets and Wouters (2007) model displays longer endogenous lags, due to numerous frictions and feedback engrained in this model.



**FIGURE 6:** AUTOREGRESSIVE ESTIMATES ON OPTIMALLY SELECTED LAGS: LIQUIDITY MODEL WITH AGGRESSIVE CENTRAL BANK ( $\gamma = 1.8$ ). TOP PANEL PLOTS THE ESTIMATED COEFFICIENTS IN SOLID, BANDS ARE TWICE THE ESTIMATED STANDARD ERRORS. BOTTOM PANEL PLOTS THE  $p$ -VALUES FOR EVERY COEFFICIENT AND A 5% THRESHOLD. NOTE: FIRST LAG EXCLUDED FROM THE PLOT FOR SCALE READABILITY.



**FIGURE 7:** AUTOREGRESSIVE ESTIMATES ON OPTIMALLY SELECTED LAGS: LIQUIDITY MODEL WITH ACCOMMODATIVE CENTRAL BANK ( $\gamma = .5$ ). TOP PANEL PLOTS THE ESTIMATED COEFFICIENTS IN SOLID, BANDS ARE TWICE THE ESTIMATED STANDARD ERRORS. BOTTOM PANEL PLOTS THE  $p$ -VALUES FOR EVERY COEFFICIENT AND A 5% THRESHOLD. NOTE: FIRST LAG EXCLUDED FROM THE PLOT FOR SCALE READABILITY.

Fig.(6) plots the estimates, confidence bands, and  $p$ -values<sup>6</sup> for the 71 optimal lags of the liquidity model with accommodative policy rule. Of all lags included, only 21 are strongly significant ( $\pm 30\%$ ); interestingly, these are mostly negative in sign and relatively small in magnitude. This last evidence might suggest that some erratic, although not degenerate, dynamic is at play in this policy regime.

Fig.(7) plots the same information for an aggressive policy rule. This model features liquidity and an accommodative Central Bank. What is striking is the length of lags deemed relevant and the share of significant ones ( $\pm 80\%$ ), as opposed to the aggressive policy rule. As remarked in Table (2), coefficients are greater and all significant at 1% up to the 38th one. These features point toward a higher persistence in inflation, something compatible with a Central Bank that, for instance, targets monetary aggregates (prior to Volcker’s chairmanship) or finds itself short of conventional monetary tools (QE at the ZLB), as it has been for long periods recently. Nonetheless, this setting does not imply necessarily sunspot equilibria or spiralling aggregates, the system eventually converges back to steady state even when the monetary policy stance is accommodative.

This result offers a framework to analyse inflation dynamics in light of diverse monetary policy regimes. Higher levels of persistence – as long as it is measured in number of significant lags – are expected in periods like the US hyperinflation of the 70’s, when indeed prices skyrocketed out of control. Conversely, tranquil periods results from aggressive monetary stances, like the Great Moderation. The inclusion of liquidity is extremely

<sup>6</sup>For the sake of readability, we do not plot the first lag. All-inclusive plots are in Appendix A.2.

helpful to systematise the 2008 crisis, while also accounting for the inactive interest rate policy. For this latter case, though, other factors need to be considered, like the level of liquidity and the wide range of unconventional policies put in place.

## 6 Conclusion

Since the Global Financial Crisis, liquidity has gained a central role in the general macroeconomic discussion and in monetary policy in particular. It has been the main concern for major Central Banks in engaging in unconventional policies. We adapt a parsimonious framework with minor departures from the core model of monetary policy and derive relevant results for Central Banks' mandate of price stabilisation. First and foremost, we show that the simple addition of a liquid asset – and the consequent modification of the intertemporal Euler equation – pins down a solution with an accommodative, stripped down policy rate rule. The latter needs only to *positively* correlate the policy rate and expected inflation to rule out degenerate, multiple equilibria. These latter sunspot equilibria would arise in the baseline New Keynesian Dynamic Stochastic General Equilibrium model when the Central Bank does not react to inflation aggressively.

We compare the responses of our liquidity model with those of the baseline NKDSGE and confirm that they broadly match for technological and monetary policy shocks, under identical calibration. We then study how the model respond to such shocks when the Central Bank is allowed to under-react to expected inflation. We find no evidence of degenerate behaviour for the model aggregates, contrary to the predictions of the baseline NKDSGE.

In fact, all aggregates, and inflation especially, broadly react in the same way under the two regimes: we signal though a change in the dynamics, rather than direction. We find that an accommodative Central Banks generates more persistent inflation. We test such hypothesis on simulated data, including two other workhorse models from the field.

We find that the inclusion of liquidity and liquid bonds generates overall more persistence in inflation. Within the sets of calibration parameters, inflation displays more persistence when the monetary policy stance is passive, in line with evidence from the US hyperinflation period.

## References

- Ascari, Guido and Argia M. Sbordone (2014). "The Macroeconomics of Trend Inflation". In: *Journal of Economic Literature* 52.3, pp. 679–739 (cit. on pp. 14–16).
- Benhabib, Jess, Stephanie Schmitt-Grohe, and Martin Uribe (2001). "The perils of Taylor rules". In: *Journal of Economic Theory* (cit. on pp. 2, 5).
- Blanchard, Olivier J. and Charles M. Kahn (1980). "The solution of linear difference models under rational expectations". In: *Econometrica* (cit. on p. 8).
- Calvo, Guillermo A. (1983). "Staggered prices in a utility-maximizing framework". In: *Journal of Monetary Economics* (cit. on p. 4).
- (2016). *Macroeconomics in times of liquidity crises: searching for economic essentials* (cit. on p. 2).
- Calvo, Guillermo A. and Carlos A. Vegh (1996). "Disinflation and interest-bearing money". In: *The Economic Journal* (cit. on p. 3).
- Canzoneri, Matthew B. and Behzad T. Diba (2005). "Interest rate rules and price determinacy: the role of transactions services of bonds". In: *Journal of Monetary Economics* (cit. on p. 2).
- Canzoneri, Matthew et al. (2008a). "Monetary aggregates and liquidity in a neowicksellian framework". In: *NBER WP 14244* (cit. on p. 2).
- (2008b). "Monetary and fiscal policy coordination when bonds provide transactions services". In: *CEPR WP 6814* (cit. on p. 2).
- Cochrane, John H. (2020). *The Fiscal Theory of the Price Level* (cit. on p. 2).
- Cogley, Timothy, Giorgio E. Primiceri, and Thomas J. Sargent (2008). "Inflation-gap persistence in the US". In: *American Economic Journal: Macroeconomics* (cit. on p. 16).
- Cogley, Timothy and Thomas J. Sargent (2002). "Evolving post-World War II US inflation dynamics". In: *NBER Macroeconomics Annual 2001*. Ed. by Mark Gertler and Kenneth Rogoff (cit. on p. 16).
- (2005). "Drift and volatilities: monetary policies and outcomes in the post World War II US". In: *Review of Economic Dynamics* (cit. on p. 16).
- Diba, Behzad and Olivier Loisel (2017). "Pegging the interest rate on bank reserves". In: *WP* (cit. on p. 2).
- Gali, Jordi (2015). *Monetary Policy, Inflation, and the Business Cycle* (cit. on pp. 2, 6, 8, 9, 15, 16, 27, 28).
- Leeper, Eric M. (1991). "Equilibria under 'active' and 'passive' monetary and fiscal policies". In: *Journal of Monetary Economics* 27.1, pp. 129–147 (cit. on p. 2).
- Michaillat, Pascal and Emmanuel Saez (2015). "An economical business-cycle model". In: *NBER WP 19777* (cit. on p. 2).
- (2018). "A New Keynesian model with wealth in the utility function". In: *NBER WP 2491* (cit. on pp. 2, 3).

- Primiceri, Giorgio E. (2005). "Time varying structural vector autoregressions and monetary policy". In: *Review of Economic Studies* (cit. on pp. 12, 16).
- Rotemberg, Julio J. (1982). "Sticky prices in the United States". In: *Journal of Political Economy* (cit. on p. 4).
- Smets, Frank and Rafael Wouters (2007). "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach". In: *American Economic Review* (cit. on pp. 14–16, 26).
- Walsh, Carl E. (2003). *Monetary Theory and Policy* (cit. on p. 2).
- Woodford, Micheal (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy* (cit. on p. 2).

# Appendix

## A Model appendix

### A.1 Obtaining the system of equations

We combine the equations presented in the body of the paper so to obtain the system of equations that will be later loglinearised and fed to Dynare for simulations.

The first target is the augmented Euler equation. The version proposed in the paper includes real money balances on top of the usual terms. Take the first intertemporal FOC from the consumer utility maximisation program as the starting point:

$$u'(c_t) = E_t [\beta (1 + r_t) u'(c_{t+1})]$$

recall the Fisher equation  $(1 + r_t) (1 + \pi_{t+1}) = (1 + i_t)$  and replace  $r_t$

$$\begin{aligned} u'(c_t) &= E_t \left[ \beta u'(c_{t+1}) \frac{1 + i_t}{1 + \pi_{t+1}} \right] \\ &= E_t \left[ \frac{\beta u'(c_{t+1})}{1 + \pi_{t+1}} + \frac{\beta u'(c_{t+1}) i_t}{1 + \pi_{t+1}} \right] \end{aligned}$$

Now recall the condition on marginal utility of real money balances,  $v'(m_t)$ , and employ it to define the nominal interest rate,  $i_t$ :

$$\begin{aligned} v'(m_t) &= E_t \left[ \beta \lambda_{t+1} \frac{i_t}{1 + \pi_{t+1}} \right] \\ i_t &= v'(m_t) E_t \left[ \frac{1 + \pi_{t+1}}{\beta \lambda_{t+1}} \right] \end{aligned}$$

moreover  $\lambda_{t+1} = u'(c_{t+1})$

Turning back to the Euler equation, plug in the nominal interest rate relation just recovered:

$$u'(c_t) = E_t \left[ \frac{\beta u'(c_{t+1})}{1 + \pi_{t+1}} + \frac{\beta u'(c_{t+1})}{1 + \pi_{t+1}} v'(m_t) \frac{1 + \pi_{t+1}}{\beta u'(c_{t+1})} \right]$$

which rearranges in

$$= E_t \left[ \beta u'(c_{t+1}) \left( \frac{1}{1 + \pi_{t+1}} + \frac{v'(m_t)}{\beta u'(c_{t+1})} \right) \right].$$

This last expression is then loglinearised to obtain equation (13).

To condense the money equation start with the last two relations in (3):

$$v'(m_t) = E_t \left[ \beta \lambda_{t+1} \frac{i_t}{1 + \pi_{t+1}} \right]$$

$$i_t = v'(m_t) E_t \left[ \frac{1 + \pi_{t+1}}{\beta \lambda_{t+1}} \right]$$

plug this result into  $h'(b_t)$

$$h'(b_t) = E_t \left[ \beta \lambda_{t+1} \frac{i_t - s_t}{1 + \pi_{t+1}} \right]$$

$$\Rightarrow h'(b_t) = E_t \left[ \frac{\beta \lambda_{t+1}}{1 + \pi_{t+1}} v'(m_t) \frac{1 + \pi_{t+1}}{\beta \lambda_{t+1}} - \frac{\beta \lambda_{t+1} s_t}{1 + \pi_{t+1}} \right]$$

exploit the fact that  $\lambda_{t+1} = u'(c_{t+1})$  and  $b_t = z_t - m_t$  to obtain

$$h'(z_t - m_t) - v'(m_t) = E_t \left[ -\frac{\beta s_t u'(c_{t+1})}{1 + \pi_{t+1}} \right]$$

Setting this equation to its steady state and using a first Taylor approximation generates equation (15) in the text.

The backward dependence of real liquidity (11) results as follows:

$$Z_t = \bar{Z} = M_t + B_t$$

$$M_t = P_t m_t \quad B_t = P_t b_t$$

$$\bar{Z} = P_t m_t + P_t b_t$$

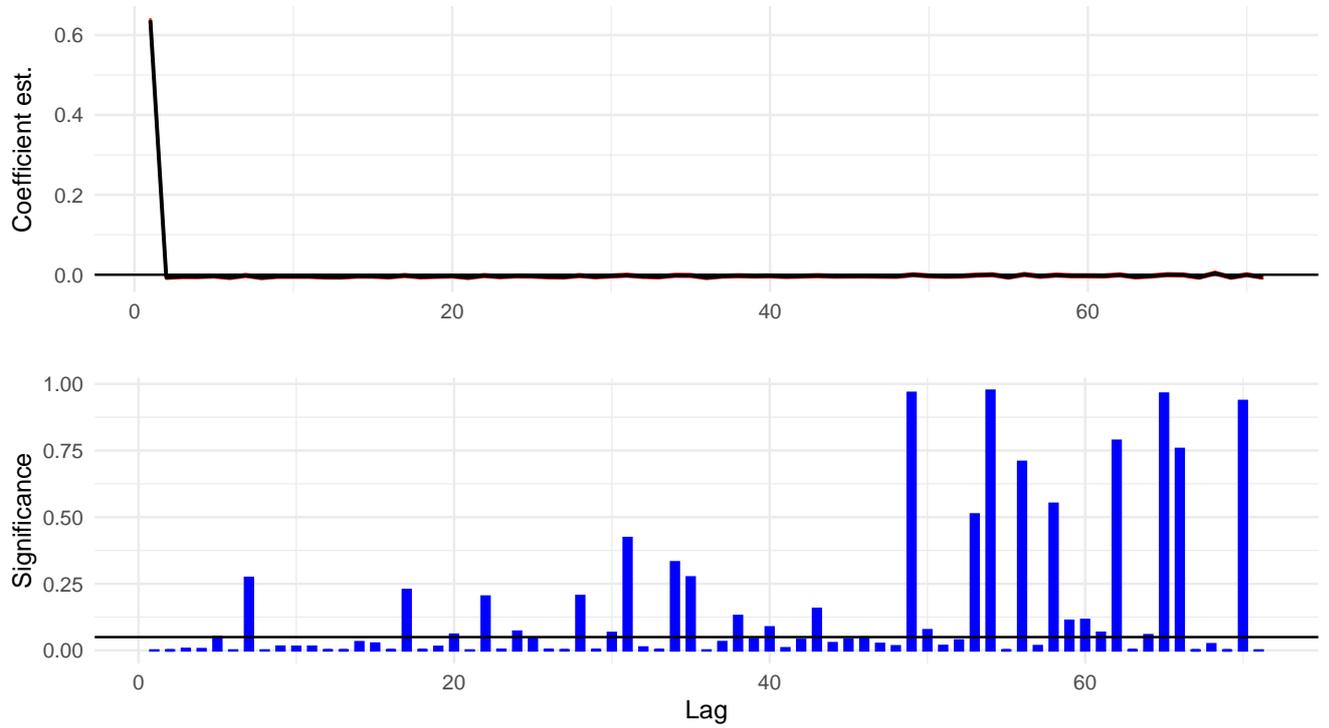
$$\frac{\bar{Z}}{P_{t-1}} = \frac{(m_t + b_t) P_t}{P_{t-1}}$$

$$z_{t-1} = z_t (1 + \pi_t)$$

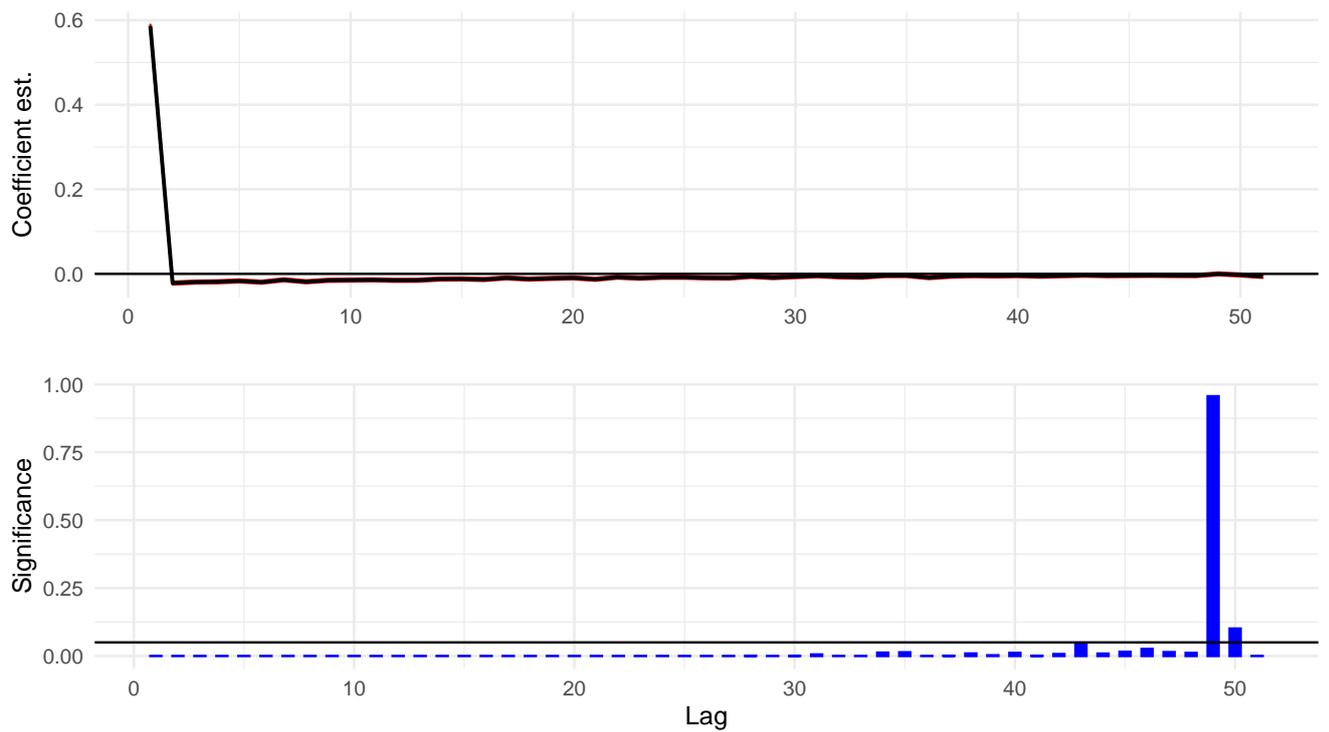
$$\Rightarrow z_t = \frac{z_{t-1}}{1 + \pi_t}$$

Concerning the Phillips curve, it remains unchanged from traditional New Keynesians models and derives from the use of equations (7). The output gap it includes results from the comparison to the flexible prices version of the model. Other relations do not need further manipulation.

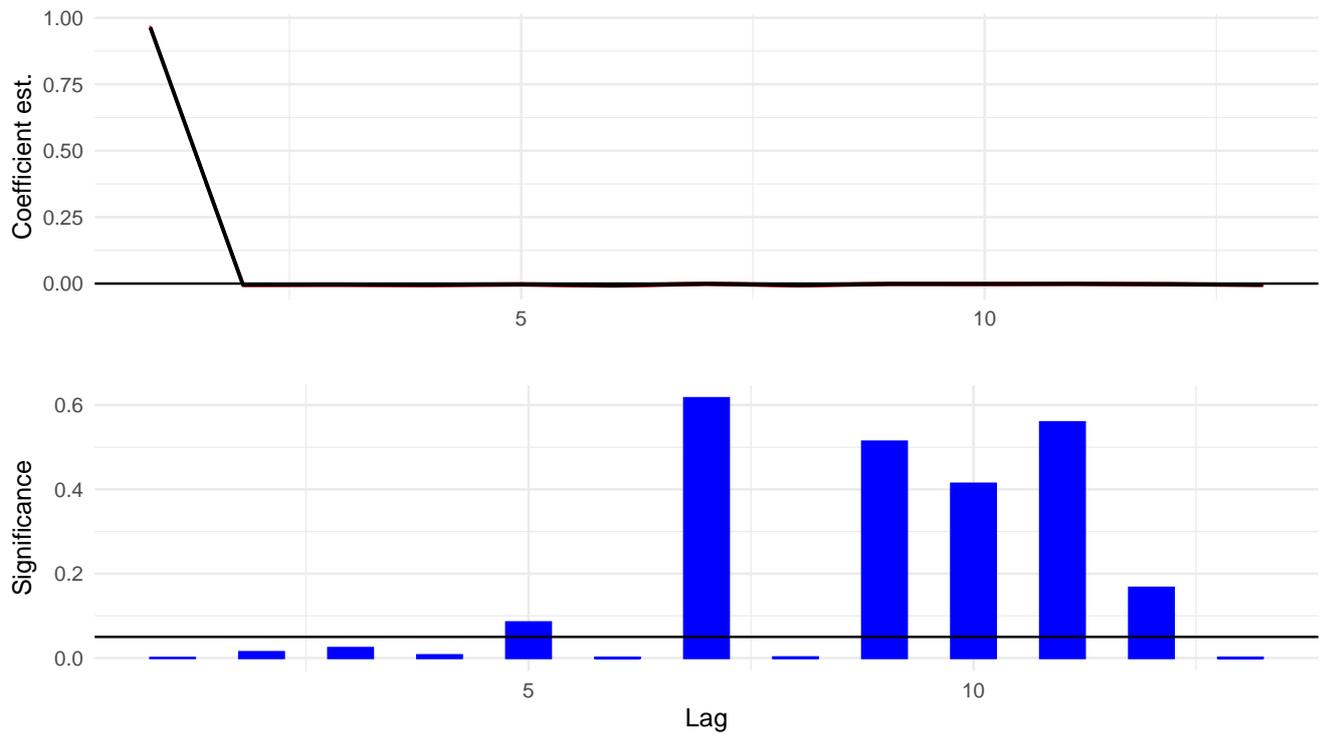
## A.2 Additional graphs



**FIGURE 8:** AUTOREGRESSIVE ON OPTIMALLY SELECTED LAGS: LIQUIDITY MODEL WITH AGGRESSIVE CENTRAL BANK ( $\gamma = 1.8$ ). TOP PANEL PLOTS THE ESTIMATED COEFFICIENTS IN SOLID, BANDS ARE TWICE THE ESTIMATED STANDARD ERRORS. BOTTOM PANEL PLOTS THE  $p$ -VALUES FOR EVERY COEFFICIENT AND A 5% THRESHOLD. FULL LAGS, ONLY THE INTERCEPT IS OMITTED.



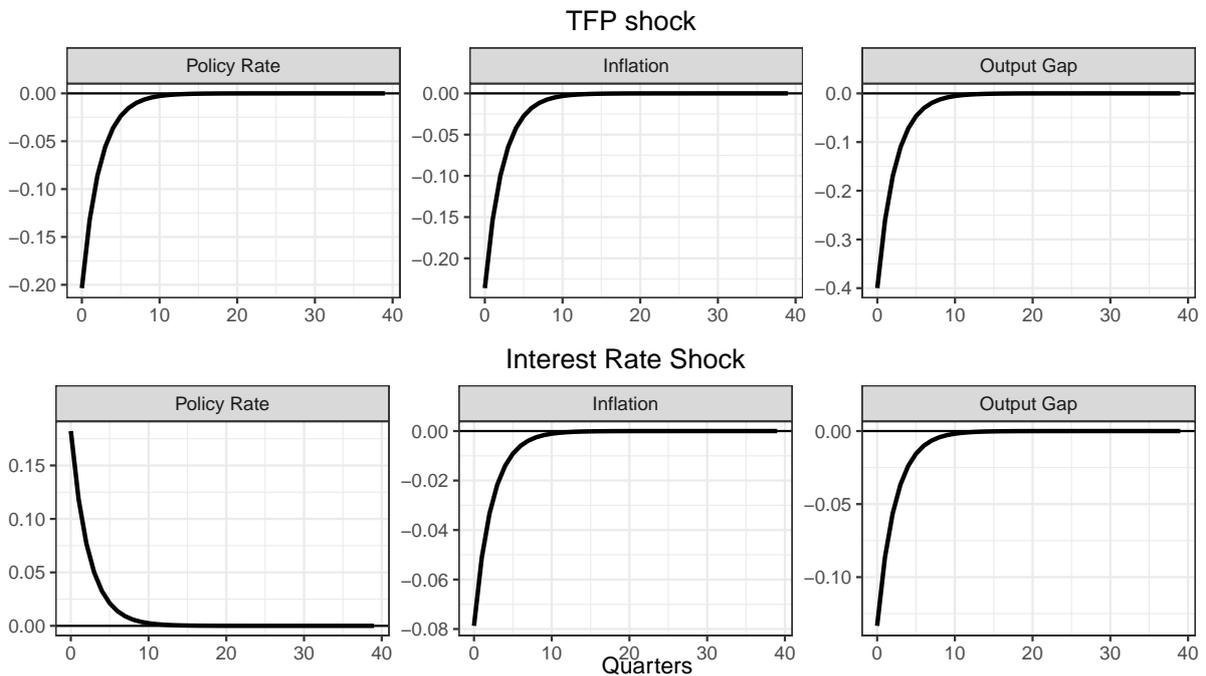
**FIGURE 9:** AUTOREGRESSIVE ON OPTIMALLY SELECTED LAGS: LIQUIDITY MODEL WITH ACCOMMODATIVE CENTRAL BANK ( $\gamma = .5$ ). TOP PANEL PLOTS THE ESTIMATED COEFFICIENTS IN SOLID, BANDS ARE TWICE THE ESTIMATED STANDARD ERRORS. BOTTOM PANEL PLOTS THE  $p$ -VALUES FOR EVERY COEFFICIENT AND A 5% THRESHOLD. FULL LAGS, ONLY THE INTERCEPT IS OMITTED.



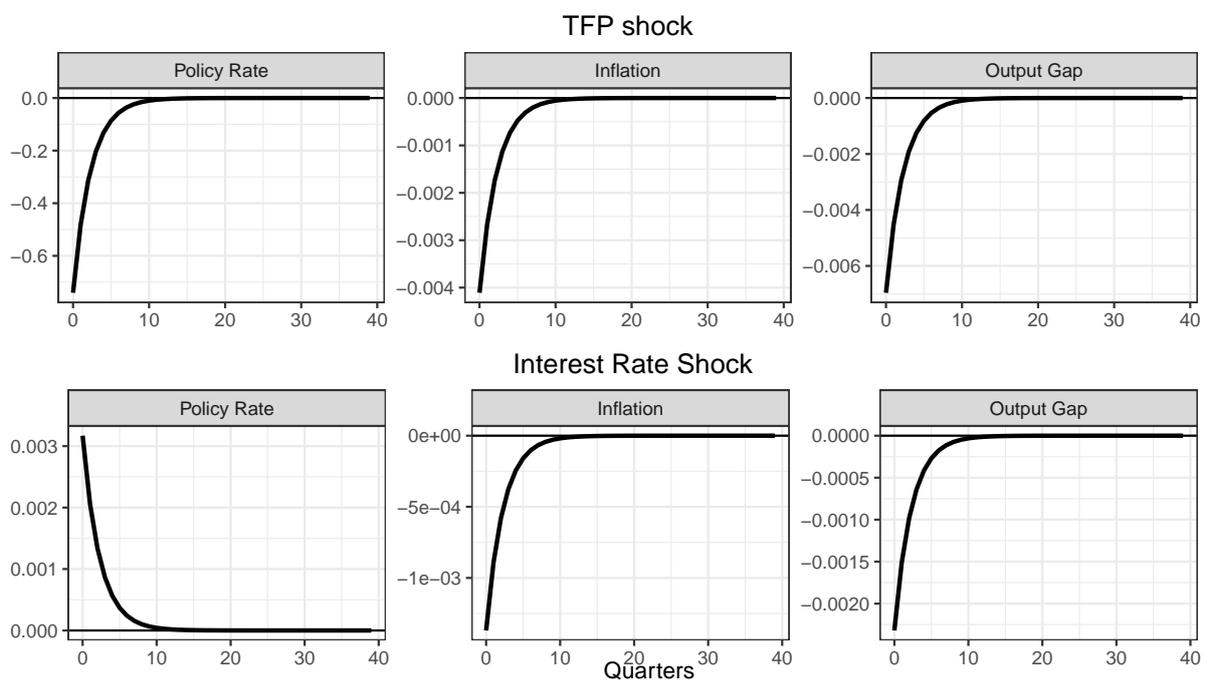
**FIGURE 10:** AUTOREGRESSIVE ON OPTIMALLY SELECTED LAGS: SMETS AND WOUTERS (2007) MODEL. TOP PANEL PLOTS THE ESTIMATED COEFFICIENTS IN SOLID, BANDS ARE TWICE THE ESTIMATED STANDARD ERRORS. BOTTOM PANEL PLOTS THE  $p$ -VALUES FOR EVERY COEFFICIENT AND A 5% THRESHOLD. FULL LAGS, ONLY THE INTERCEPT IS OMITTED.

### A.3 Extreme policies

This section investigates what are the consequences of two extreme calibrations for the textbook NKDSGE models. We let the reaction to expected inflation be negligibly close to its determinacy limit, such that  $\gamma^+ \rightarrow 1$ , and then test for extremely high levels of reaction. The IRFs for these two cases are presented in Figures below. Interestingly, while calibrations close to indeterminacy do not display dramatic changes but only a higher effect on impact, an extremely reactive Central Bank can substantially suppress a sizeable share of volatility in the economy. Indeed, it appear to curb inflation and economic gap in a much more effective way when a fundamental productivity shock hits the economy – at the cost of considerable volatility in the policy rate, though. On the monetary policy shock, the decrease in volatility is even sharper.



**FIGURE 11:** BASELINE NKDSGE (GALI, 2015) WITH VERY LOW REACTION TO EXPECTED INFLATION BY THE CENTRAL BANK:  $\gamma^+ \rightarrow 1$ . VALUES IN PERCENTAGES.



**FIGURE 12:** BASELINE NKDSGE (GALI, 2015) WITH EXTREMELY HIGH REACTION TO EXPECTED INFLATION BY THE CENTRAL BANK. VALUES IN PERCENTAGES.