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PARIS SCHOOL OF ECONOMICS
ECOLE D'ECONOMIE DE PARIS

WORKING PAPER N° 2020 – 77

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JEL Codes: D31, D43, D82, D86
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Funded by a French government subsidy managed by the ANR under the framework of the Investissements d'avenir programme reference ANR-17-EURE-001

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November 30, 2020

Abstract

This paper reviews the literature in sorting providing with some of the key analytic elements to understand its causes. Among others, sorting has been applied to mechanism design, games and growth theory, allowing for the analysis of strategic behaviors in principal-agent problems. In some applications, an optimal solution can be obtained by introducing a one-dimensional screening device. In practice, screening devices are interest rates, high performance incentives or non-convex technologies that can yield for instance non convex salaries as functions of human capital.
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1 Introduction

Any type of market imperfection allows to generate profits. Firms, governments, or in a general sense, principal agents, will try to sort individuals, i.e., to strategically provide accurate incentives leading agents to reveal their true profile, minimize agent's profits and maximize theirs. In more general and formal terms, sorting is a methodology which ranks individuals and facilitates obtaining a problem's equilibria. Sorting is specially useful in problems with moral hazard, adverse selection or games with asymmetric information. Due to the complexity of these problems, conclussions are usually built on comparative statics, that is, on how outcomes change with a key parameter. After presenting with some detail a first benchmark model of sorting or screening, this survey provides varied applications of sorting in credit rationing in the fields of mechanism design, games with asymmetric information, development and growth theory.

Credit rationing is a well suited problem in which banks need to sort individuals. Roughly speaking, credit rationing arises when risk-averse banks cannot observe the type of potential borrowers. The easiest example of credit rationing is when banks are risk-averse and they cannot observe the type of the potential borrower. Without full disclosure of information, banks will appeal to screening devices to sort individuals. Screening devices have consequences increases, safe-type borrowers drop out from the credit market. This is called adverse selection and it diminishes the number of loan applicants. Second, a higher interest rate may induce borrowers to take riskier projects, which in the end, can cause the problem of debt overhang. For instance, a highly indebted farmer has very little stake of a good harvest. Large repayment implies that the borrower obtains a lower return than late-due returns after the harvest.

This survey is organized as follows. Next section presents the simple strategic models with one-dimensional screening in Freixas and Laffont (1990) and Guesnerie and Laffont's (1984). We also take the opportunity to introduce the single crossing property as a sorting device in higher dimension. Section 3 applies sorting mechanisms to different economic fields, while Section 4 summarizes some results in Hellwig (1994), namely how individuals are sorted and choose either short or long-term investments. Conclussions are presented in Section 5.

2 A Model of Screening

Freixas and Laffont (1990) underline that "a weakness of most contribution is to carry out their analysis with standard debt contracts which have no theoretical justification." Incentive theory has been used since then to enrich (among others) the theoretical framework of debt contracts. Usually, a loan policy providing agent-specific incentives will take into account the distribution of agents or firms and divide them in groups according to their ability to reimburse the loan. Let us illustrate this issue by describing first the strategic behaviour between a screening principal and the agent in Freixas and Laffont (1990). Then, we present the results of Guesnerie and Laffont (1984).

2.1 The strategic behaviour in Freixas and Laffont (1990)

Suppose that agents can be either efficient, with productivity $\bar{\theta}$, or inefficient, characterized by a lower productivity $\underline{\theta}$. All agents would like to obtain a loan from the principal, who ignores agents' types at the time of signing a contract. Regardless of information asymmetry, an interest rate is agreed on the contract.

Strategic interaction among players arises because the principal needs to design incentives to reveal the type of each agent. On the one hand, the principal maximizes her expected payoff knowing that the probability of finding a high type is ν. On the other hand, the agent tries to extract the maximum rent of a loan k, up to t, and may agree a risk-free loan at the risk-free interest rate R. Hence in this case, the lender's utility is $V = t - kR$ and the borrower's utility is $U = \theta \cdot f(k) - t$, where $f(k)$ is the net return from the loan, after repayment. θ may be regarded as a productivity shock. It is assumed that a productivity shock has a lesser effect on efficient agents because the high type has more skills to solve problems and pay back loans successfully. Summing up, the principal offers a menu of contracts $(\bar{t}, \bar{k})$ to the high type and $(\underline{t}, \underline{k})$ to the low type, in order to maximize her expected returns

$$\max_{(\bar{t},\bar{k}),(\underline{t},\underline{k})} \nu \cdot \left(\bar{t} - R\bar{k}\right) + (1-\nu) \cdot (\underline{t} - R\underline{k})$$

The information rent of the high-type agent is $\bar{U} = \bar{\theta} \cdot f(\bar{k}) - \bar{t}$ and the rent of the low-type agent is $\underline{U} = \underline{\theta} \cdot f(\underline{k}) - \underline{t}$. If types are revealed truthfully, then the incentive constraints for the high and the low type are respectively

$$\bar{U} = \bar{\theta} \cdot f(\bar{k}) - \bar{t} \geq \bar{\theta} \cdot f(\underline{k}) - \underline{t}$$

and

$$\underline{U} = \underline{\theta} \cdot f(\underline{k}) - \underline{t} \geq \underline{\theta} \cdot f(\bar{k}) - \bar{t}.$$

Agents participate in the market as long as $\bar{U}, \underline{U} \geq 0$. Therefore, the principal problem can be written as

$$\max_{\bar{k},\underline{k}} \nu \cdot \left(\bar{\theta} \cdot f(\bar{k}) - R\bar{k}\right) + (1-\nu) \cdot (\underline{\theta} \cdot f(\underline{k}) - R\underline{k}) - \left(\nu \cdot \bar{U} + (1-\nu) \cdot \underline{U}\right).$$

Since the low type agent has no incentive to mis-report, the principal sets $\underline{U} = 0$, and she does not offer the efficient type more than required so that $\bar{U} = \underline{U} + \Delta\theta \cdot f(\bar{k})$. The first-best solution provides $\bar{\theta} \cdot f'(\bar{k}^*) = R$ and $\underline{\theta} \cdot f'(\underline{k}^*) = R$, so that $\underline{k}^* \leq \bar{k}^*$. That is, at equilibrium, the return on capital equals to the risk-free interest rate:

$$R = \left(\underline{\theta} - \frac{\nu}{1-\nu} \cdot \Delta\theta\right).$$

2.2 One Dimensional Screening *à la* Guesnerie and Laffont (1984)

In the principal-agent model of Guesnerie and Laffont (1984), adverse selection arises since there is a one-dimensional parameter θ, which is known to the agent but unobservable to the principal. There are two types of variables linking the principal and the agent. The first type are multi-dimensional action variables, which both the agent and the principal can observe. The second type of variable is a one-dimensional variable, $t(\theta)$, which is a money transfer from the principal to the agent and which depends on the type θ.

Consider a principal who wants to delegate to an agent the production of x units of a good. The agent can be either efficient or inefficient, measured by a parameter θ in the set Θ, with respective probabilities v and $1-v$. The principal aims at establishing the first-best contract (x, θ) implying a transfer t from the principal to the agent, which fulfills both the utilities of the principal, V, and of each of the agents, U. It implies that higher (efficient) types get more goods.

The optimal necessary conditions describe (1) the marginal rate of substitution between x and θ, (2) the monotonicity of x, and (3) truth-telling incentive compatibility (IC) constraints. The revelation mechanism gives the key constraints under which a contract is *incentive feasible*:

Definition 1. *An action profile $x : \theta \in \Theta \to x(\theta)$ is implementable via a compensatory transfer t, if there exists a function of transfer, $x(\theta)$ such that the revelation mechanism $(x(\theta), t(\theta))$ induces truthful revelation, i.e. $U(x(\theta), t(\theta), \theta) \geq U(x(\theta'), t(\theta'), \theta), \forall \theta, \theta' \in \Theta^2$.*

The following assumption is a necessary condition for implementation.

Assumption 1. *U is strictly increasing in t, continuously differentiable of class C^2.*

Then, Assumption 1 implies that agents' utility is increasing in actions and transfers for almost all types; and the indifference curves for agent θ and another agent $\theta + d\theta$ have constant sign.

Theorem 1. *Under Assumption 1, if a piecewise C^1 action profile x is implementable via compensatory transfers t, then necessarily*

$$\left[\frac{\partial}{\partial\theta}\frac{\partial_x U}{\partial_t U}_{(x,t,\theta)}\right]\frac{dx}{d\theta}(\theta) \geq 0 \tag{1}$$

or equivalently:

$$\Sigma_i \left[\frac{\partial}{\partial\theta}\frac{\partial_x U}{\partial_t U}_{(x,t,\theta)}\right]\frac{dx_i}{d\theta}(\theta) \geq 0 \tag{2}$$

for any x, t, θ such that $x = x(\theta)$, $t = t(\theta)$ and x is differentiable at θ.

Theorem 1 shows that if the marginal rate of substitution between x and t increases with the agent's type, then any implementable contract is also necessarily increasing in types. Hence, the higher the type θ, the higher $x(\theta)$.

The following theorem further requires the marginal rate of substitution between x and t to be bounded, meaning that the transfer t cannot tend to infinity as a reaction to an infinitesimal change of x.

Theorem 2. *Assume that the agent's utility function satisfies Assumption 1 and that U is bounded in the action-transfer domain in the following sense $\mid \frac{\partial_x U}{\partial_t U} \mid \leq K \mid t \mid$ for $K > 0$. Then, any piecewise C^1 action profile is implementable via compensatory transfers such that its derivative is non-negative if and only if*

$$\frac{dx}{d\theta} \geq 0. \tag{3}$$

Note that the boundedness assumption $\mid \frac{\partial_x U}{\partial_t U} \mid \leq K \mid t \mid$ for $K > 0$ implies that the transfer t is large enough to fulfill implementability so that $\mid \frac{dt}{d\theta} \mid < K$, with $K > 0$.

Theorem 2 ensures that under certain conditions, there exists at least one transfer function such that $(x(\theta), t(\theta))$ is implementable, and this for any action profile x that optimizes the principal's utility. In this case, both the principal and the agents will all be satisfied, for all $\theta \in \Theta$.

Unfortunately, it's not simple to extend this result to multi-dimensional actions. As Guesnerie and Laffont underline, if $x(\theta)$ has a finite number of increasing portions, then any constant piece in $x(\theta)$ must join two different increasing pieces.

2.3 Multidimensional Sorting with the Single Crossing Property

Sorting conditions simplify allocative distortions. In multi-dimensional problems where the action variable has dimension higher than 1, the *single-crossing condition* of Milgrom and Shannon (1994), or the *Spence-Mirrlees condition* of Diamond and Stiglitz (1974) and Athey (2001) help finding the optimal solution. The Spence-Mirrlees condition requires that the iso-utility curve for agents of different types cross only once. Suppose the action is two dimensional, then broadly speaking, the condition implies that if it is (strictly) preferable to have more of the first component for a particular level for the second component, then it would still be (strictly) preferable to have more of the first component for a greater level for the second component. Let us state this condition formally:

A utility function $U : \mathbb{R}^2 \to v$ which is C^1, $U(x^1, x^2, \theta)$ is said to satisfy the *(strict) Spence Mirrlees condition*

$$\frac{\partial^2 U}{\partial \theta \partial x}(x, \theta) > 0 \tag{4}$$

if $\dfrac{U_{x^2}}{U_{x^1}}$ is (increasing) non-decreasing in θ with $U_{x^1} = 0$ and it has the same sign for every (x^1, x^2, θ). This is equivalent to saying that the ratio $\dfrac{\partial U}{\partial x^1} / \dfrac{\partial U}{\partial x^2}$ is increasing in θ, and $\dfrac{\partial U}{\partial x^2}$ is never 0.

This property is exploited in numerous applications. Despite the importance of the Spence-Mirless condition in multidimensional problems, we cannot develop them in this short survey, choosing instead to overview only one-dimensional problems.

3 Economic Applications of Sorting

This section presents some applications of sorting in the literature, underlining the mechanisms, the intuitions and consequences depending on each context. We start with Stiglitz and Weiss (1981), which uses the interest rate as an incentive mechanism.

3.1 Credit Rationing *à la* Stiglitz and Weiss (1981)

In Stiglitz and Weiss (1981) the principal is a bank which receives agents' demands for loans. The principal ignores the agents' types, which is here the riskiness of the project that the agent would like to undertake. The agents' type is measured by a parameter $\theta \in \mathbb{R}^+$. Here, the principal uses the interest rate, r, as screening device. In face of some unknown default risk at the individual level, the principal maximizes its expected return. As a result, the interest rate cannot be too low. But, as the interest rate increases, a part of potential borrowers drops out, and the risk of projects undertaken increases, so that the mix of borrowers changes adversely. If an agent borrows an amount B, and the ceiling interest rate is $\hat{r}$, then we say that the agent defaults on her loan if the return R plus the collateral C is insufficient to pay back the promised amount.

Definition 2. *The value of $\hat{\theta}$ for which expected profits are zero satisfies*

$$\Pi(\hat{r}, \hat{\theta}) \equiv \int_0^\infty max\,[R - (\hat{r}+1)B; -C]\, dF(R, \hat{\theta}) = 0 \tag{5}$$

Theorem 3. *For a given interest rate, $\hat{r}$, there is a critial value $\hat{\theta}$ such that an agent borrows from the bank if and only if $\theta > \hat{\theta}$.*

Stiglitz and Weiss also show that there could exist an equilibrium with two interest rates with an excess demand for credit for the lower one. This situation can arise if the bank's expected return has several modes as a function of the interest rate and if the Walrasian interest rate at which credit demand equals credit supply lies between two of these modes.

3.2 High performance incentives of Holmstrom and Milgrom (1994)

Let $x(\theta)$ denote the optimal action of a type θ with incentives t. The objective in higher dimensional screening is to determine how to design types θ of agents (sorting individuals) in a principal-agent relationship. The first step is to characterize how action choices x vary with types θ. Holmstrom and Milgrom (1994) use the following theorem from Topkis (1978) on supermodularity and complementarity:

Theorem 4. *Let $f(x, \theta)$ be a continuous supermodular function and X a compact sublattice of $\mathbb{R}^n$. Let $X(\theta) = argmax\{f(x, \theta) | x \in X\}$ be the set of maximizers with least upper bound $x^*(\theta) = supX(\theta)$ and greatest lower bound $x_* = infX(\theta)$. Then both $x^*(\theta)$ and $x_*(\theta)$ belong to $X(\theta)$ and are nondecreasing functions (from $\mathbb{R}^m$ to $\mathbb{R}^n$).*

When θ is a multidimensional parameter, then sorting depends on the distribution of θ as for instance in the distributional comparative statics in Jensen (2016). Note that in most examples you may think of, not all parameters characterizing types of agents are complementary with all the incentive instruments. For this reason, it is absolutely not straightforward to obtain a general sorting mechanism nor an optimal solution.

In the context of multitask agents Holmstrom and Milgrom (1994) introduce the idea of *associated random variables*. A vector θ of random variables is *associated* if for all real-valued non decreasing functions f and g, $Cov(f(\theta), g(\theta)) \geq 0$, that is, if

$$E\left[f(\theta)g(\theta)\right] \geq E\left[f(\theta)\right]E\left[g(\theta)\right] \tag{6}$$

Theorem 5. *Let $\theta = (\theta_1, ..., \theta_m)$ be a vector of associated random variables. Then:*

1. *any subvector of θ is associated; in particular, any single random variable is associated;*
2. *$Cov(\theta) \geq 0$;*
3. *If γ is a random variable, independent of θ, then (θ, γ) is associated; in particular, any vector of independent random variables is associated;*
4. *if $x_i : \mathbb{R}^m \to \mathbb{R}$, $i = 1, ..., n$, is a collection of nondecreasing functions, then $(x(\theta), \theta)$ is associated; and in particular, $x(\theta)$ is associated.*

Then, if a subvector θ' of the incentive vector θ is associated, then any policy adopted by the principal that increases one of the elements of θ' will increase all the elements of θ'. Note that we can apply Theorem 5 using the law of iterated expectations in a context of low-powered incentives: if $x : \mathbb{R}^n \to \mathbb{R}$ is an arbitrary function and $f : \mathbb{R} \to \mathbb{R}$ a non-decreasing function, then, $f(x(\theta))$ and $x(\theta)$ are associated irrespective of how the vector θ is distributed and whether x is monotone in θ. Using these results, Holmstrom and Milgrom (1994) conclude that in their model of industrial selling, low-powered incentives can be important to inspire cooperation and coordination, and may be enhanced by placing constraints on employees' freedom to act.

3.3 Sorting, Development and Growth

We present next two papers which evaluate the effect of sorting in inequality and its evolution in time. A common feature is the existence of different types of investments, which end up by sorting individuals. Indeed, an individual's initial wealth determines the type of investment one is entitled to. Although the unicity of investment types and of the associated returns is usually assumed in general equilibrium models, it is debatable whether this assumption is realistic and whether in doing so the modeler does not only lose an opportunity to seize market imperfections but also to better understand access to credit and the ensuing inequality. In

Banerjee and Newman (1993) there are three distinct types of investment which determine individuals' careers, and the focus in put on the evolution of the distribution of wealth. Using a more standard approach, Piketty (1997) analyzes the dynamics of the distribution of wealth in a classic Solow model.

In Banerjee and Newman (1993) there is a continuum of risk neutral individuals who only differ at birth in their bequeathed wealth. There are three ways to invest, which determine the individuals' capacity to accumulate more or less wealth for future generations. Nevertheless, agents are not free to choose their type of investment since they are constrained by their initial wealth.

First, there exist a divisible, safe asset which does not require any labor effort from the agent and which yields a fixed gross return. Second, there exists a risky and indivisible project, which does not require any specific skill but an initial investment, I, and it generates a random return. The third project is a monitoring technology which will allow entrepreneurs to set firms and monitor $\mu > 1$ workers. This project requires an initial investment of $I' > I$ units, and it also yields a random return. The second and the third project have the same expected rate of return, the difference between them being that the second allows an individual to become self-employed and the third generates μ jobs.

The loan market is based on Kehoe and Levine (1993), and through a monetary punishment threat, all loans will be honored. The agent's wealth determines the project they can undertake, since the lender will only accept loan requests with a positive return. In this setup, agents are credit constrained if their initial wealth is not enough as collateral to balance possible losses. At equilibrium, access to credit divides the population in three. The poorer agents are credit constrained, they become workers and can only invest in the safe project. Agents with an intermediate level of wealth become self-employed, and the upper tier of the distribution becomes entrepreneur.

From a dynamic perspective, the limited access to credit can generate a dual society with two types of agents: prosperous entrepreneurs and socially stagnating workers. Note that if all agents have a small wealth, then the whole economy stagnates.

Piketty (1997) analyzes a classic Solow growth model with a continuum of heterogeneous agents. Here, there is moral hazard, uncertainty about physical capital and additionally, interest rate(s) is endogeneously found as the rate which brings equilibrium to the capital market. With respect to the standard Solow model, Piketty shows that under credit constraints, the initial distribution of wealth does have a preeminent role in the long-term distribution of wealth and the economic outcomes.

Since agents can privately choose whether to exert effort or not, lenders need to find contracts that will encourage individuals to exert maximum effort, taking into account that projects can fail. A key founding assumption is that all agents desire to invest an identical amount of capital, the first best optimal, which depends uniquely on the prevailing interest rate. Individual wealth plays a major role here since the poorer an agent, the more she will have to borrow, making risky investments, which are less profitable for the agent. As a result, there exists a critical level of wealth below which the agent will not make any effort. These agents will not receive any loan, they will be credit constrained.

Piketty analyzes the long-term of the economy which may result in a multiplicity of interest rates depending on the initial distribution of wealth, among other factors. This single interest rate assumption is convenient, among many different issues, to understand business cycles (Jaffee-Modigliani, 1969, Stiglitz-Weiss, 1981); nevertheless, it is inconsistent with Nash equilibria in which the first best may be either no credit rationing (low interest rate) or low investment (high interest rate). The following proposition summarizes the most interesting long-term properties of credit rationing in Piketty (1997):

Proposition 1. *In the long-term, the economy could reach three types of steady state types depending on the probability of failure and the initial wealth distribution. We could observe a steady state in which*

i) there is no credit rationing. The interest rate is sufficiently low because net returns become sufficiently high to give proper incentives to agents with no collateral.

ii) there is some credit rationing. Some agents are credit-rationed and can only make the low investment. Other agents can attain sufficient credit to make the optimum investment.

iii) all agents are rationed, so that everybody chooses the low investment.

4 Short and Long term positions

While the previous section focused on situations in which a multiplicity of projects was available at a given date, we present next Hellwig (1994), where agents may have access to investment goods with different maturities, risks and returns.

The economy goes through 2 periods. All decisions and contracts are undertaken at $t = 0$, and consumption can be made at times 1 or 2. Actually, there is a probability p that an agent will need to consume early, at $t = 1$, and invest short-term accordingly; and a probability $1 - p$ that she will consume late, at $t = 2$. If banks can distinguish among agents who invest either short or longterm, then unregulated (Bertrand) competition will implement the first-best allocation. Otherwise, if banks cannot distinguish them, then competition forces banks to offer contracts that will implement a *second-best allocation.*

Let us denote by k_{01} and k_{02} the allocation of the initial capital to short-term and long-term investment, with returns θ_1 and η, respectively. If agents are not truthful and they do not consume all their investment revenue in the first period, that is, $c_1 - \theta_1 k_{01} > 0$, then they can re-invest it to consume in the second period (with un uncertain return θ_2). Incentive compatibility will require that $c_2 \geq \theta_2 c_1$, that is, consumption in the second period is larger if the agent is truthful than if she cheats, investing short and then re-investing from the first to the second period. If θ_2 is sufficiently low, then in the first best solution $c_1 = \frac{\theta_1 k_{01}}{p}$ and $c_2 = \frac{\eta k_{02}}{1-p}$.

Agents can liquidate prematurely part of their long-term investment, getting a return ϵ. Although there are no premature liquidations in the first best solution, they do arise when the second best is implemented. Then, if the unknown return θ_2 is large enough, premature liquidations can overwhelm the bank, who will not accept all liquidation demands. Forced by competition forces, banks then need to offer the best implementable contracts to the different households so that they will implement a second best allocation, while revealing their type, avoiding a bank-run. First, to avoid bank-runs, banks will suspend payments at date 1 when the fraction of individuals claiming their investment reaches p, the fraction of early consumers. Second, Hellwig shows that if ϵ is sufficiently low, then there exists a value for θ_2, R_2, such that the economy reaches an equilibrium where

$$c_1 = \theta_1 k_{01} + \eta \frac{k_{02}}{R_2},$$

if consumption takes place in the first period, and $c_2 = R_2 \theta_1 k_{01} + \eta k_{02}$ if consumption takes place in the second period. R_2 is actually the equilibrium relative price of date 1 consumption versus date 2 consumption.

We can summarize the results of Hellwig (1994) presented here by saying that there exists a contract which implements a second best allocation in which the bank provides liquidity, and even allows early consumers to liquidate earlier their long term investment. However, in this allocation individuals bear all interest rate risk.

5 Conclusion

After reading this brief review, one may wonder to what extent sorting controls individuals' actions. We have seen that screening devices help achieve expected profits, optimum investment and the second-best welfare. Among all, we have explored various sorting mechanisms like for instance monetary punishment and low powered incentives. These devices sort agents who reveal their type by accepting contracts with monetary transfers (incentives). Hence sorting modifies somehow agents' behavior while helping all parties in a game to reach a second best solution.

A closer examination of all surveyed models reveals similarities among optimal solutions. We believe that the literature would benefit from a nobel comprehensive interpretation of the economy, before and after sorting takes place, that would represent problems in a sufficiently abstract manner so as to exploit the structural similarities among all problems. More precisely, we are thinking about exploiting the most important of these similarities: the fact that the agents' iso-utility curves only cross once.

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