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**Treating future people impartially implies avoiding
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Geir B. ASHEIM, Kohei KAMAGA, Stéphane ZUBER

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Treating future people impartially implies avoiding future lives with low well-being*

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4 August 2020

Abstract

It has been claimed that climate policies can be evaluated by the Pareto principle. However, climate policies lead to different identities and different numbers of future people. Even if one assumes that the number of future people is countably infinite independently of policy choice, the problem is that there exists no *natural* one-to-one correspondence between the components of the compared alternatives. This non-existence means that the components of streams are indexed by natural numbers that do not correspond to particular people, making a case for impartiality in the sense of Strong anonymity. Strong anonymity is incompatible with Strong Pareto. The paper re-examines this incompatibility and investigates how far sensitivity for the well-being at any one component can be extended without contradicting Strong anonymity. We show that Strong anonymity combined with four rather innocent axioms has two consequences: (i) There can be sensitivity for the well-being at a particular component of the stream if and only if a cofinite set of people have well-beings that are more than an ε (> 0) higher, and (ii) adding people to the population cannot have positive social value.

Keywords: Infinite streams, Intergenerational equity, Population ethics.

JEL Classification numbers: D63, D71.

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1 Introduction

Among others, Foley (2009) and then Broome in various papers, including Broome (2018), have pointed to the importance of the Pareto principle in the context of climate policies. Climate change is an externality, since those who emit greenhouse gases do not pay the long-term negative consequences of their emissions. Stern (2007) has even characterized climate change as “the greatest market failure the world has ever seen.” Correcting for a market failure by internalizing externalities usually improves efficiency and provides an opportunity for realizing a Pareto improvement whereby some parties gain without other parties losing. A Pareto improvement is in principle possible in the climate change setting by reducing emissions and investments now, improving the situation both for the present and the future. One can argue that Nordhaus (2008) seeks to find the potential for efficiency gains when using observed behavior to motivate discounting of poor and future people through Negishi weights and time-discounting in the DICE and RICE models.

However, climate policies will influence fertility, leading to a different number of people in each future generation. In particular, population measures might serve as a means to reducing greenhouse gas emissions. Moreover, reproductive choices will be affected, implying that different people will exist in the future; this is what philosophers call the non-identity effect (see Parfit, 1984). The concept of Pareto improvement is only applicable in same people choices, where there is a natural one-to-one correspondence between people in the two alternatives, comparing the vector of well-beings in two alternatives person by person. In contrast, as a result of fertility choices and population measures, we are faced with different people–different number choices in the case of climate policies, so that no such natural one-to-one correspondence exists (see Broome, 2018).

While the ratio of people that have ever lived to the ones living now is low (estimated to 15:1 by Kaneda and Haub, 2018), there might potentially be many people living in future.¹ This observation might justify the modeling choice often made in the literature on sustainability and intergenerational equity, namely that the future will consist of infinitely many people, independently of the policy choices that are made.

In this case, the number of people is countably infinite independently of policy choice, and in a formal sense there exist one-to-one correspondences between people in any two alternatives. For example, people might be counted in the order of the time of birth and compared according to this sequence. However, with different fertility choices in different alternatives, the bearer of well-being at component i in one alternative will not coincide with the bearer of well-being at component i in another alternative. Therefore, comparison of well-beings according to the one-to-one correspondence that follows the temporal sequence of birth does not represent a proper

¹With 500 million years left of the earth as acceptable habitat for humans, population stable at 1.1 billion with an average length of life equal to 71 years, the ratio of people who will potentially live in the future to people living now is about 1 million to 1.

application of the Pareto principle. The underlying problem is that there is no *natural* one-to-one correspondence between people in two alternatives since the identities of people in the two alternatives will differ.

One possibility is to deem one alternative better than another if there exists a one-to-one correspondence between the components of the two alternatives such that one alternative strictly dominates the other, in the sense that, for all natural numbers j , the former alternative yields a well-being level at least as high as the latter, and for some natural number i , the former alternative yields a strictly higher well-being level. However, the stream

$$\mathbf{x} = (1, 0, 1, 0, 1, 0, \dots, 1, 0, \dots)$$

can be transformed to the streams $\mathbf{x}_\pi = (1, 1, 1, 0, 1, 0, \dots, 1, 0, \dots)$ and $\mathbf{x}_{\pi'} = (0, 0, 1, 0, 1, 0, \dots, 1, 0, \dots)$ by permuting an infinite number of components, leading by the above dominance criterion to the nonsensical conclusion that the stream \mathbf{x} is both better and worse than itself.

This observation is an illustration of the conflict between sensitivity and impartiality in the evaluation of infinite well-being streams which has been analyzed in many contributions, including Diamond (1965), Svensson (1980), Basu and Mitra (2003), Zame (2007) and Lauwers (2010), as well as Asheim (2010) for an overview. In particular, as shown by Van Liedekerke and Lauwers (1997, p. 163) the axioms of *Strong Pareto*, in the sense of being sensitive to an increase of any one component of the stream if no other component is reduced, is incompatible with the axiom of *Strong anonymity*, in the sense of invariance to any permutation of a stream. Indeed, $\mathbf{x}_\pi \succ \mathbf{x} \succ \mathbf{x}_{\pi'}$ if the binary relation \succsim satisfies Strong Pareto, while $\mathbf{x}_\pi \sim \mathbf{x} \sim \mathbf{x}_{\pi'}$ if \succsim satisfies Strong anonymity, as \mathbf{x}_π and $\mathbf{x}_{\pi'}$ are infinite permutations of \mathbf{x} .

There are two main routes out of the dilemma that this impossibility poses:

- (1) Stick with Strong Pareto and weaken Strong anonymity.
- (2) Stick with Strong anonymity and weaken Strong Pareto.

Moreover, one can consider weakening both Strong Pareto and Strong anonymity.

Route (1) has been extensively explored. Strong Pareto *is* compatible with the axiom of *Finite anonymity*, in the sense of invariance to any *finite* permutation of a stream (Svensson, 1980). In fact, there is a literature on how to extend impartiality beyond Finite anonymity. Specifically, a number of papers examine whether versions of the anonymity axiom that require invariance for specific set of permutations are compatible with Strong Pareto; see, for example, Fleurbaey and Michel (2003), Lauwers (1997, 1998), and Sakai (2010). As showed by Lauwers (1997), *Fixed-step anonymity*, which requires invariance to so-called fixed-step permutations rearranging components of a stream within each fixed range of consecutive coordinates, is compatible with Strong Pareto. Mitra and Basu (2007) present a general analysis of anonymity axioms that are compatible with Strong Pareto from the view point of the requisite algebraic structure of a set of permutations. They showed that an anonymity axiom that is defined by a group of cyclic permutations is

compatible with Strong Pareto (see also Adachi, Cato and Kamaga, 2014). However, Lauwers (2012) proved that a maximal group of cyclic permutation is a non-constructible object because its existence relies on the use of non-constructive mathematics like the Axiom of Choice. Therefore, it is impossible to give an explicit definition of a maximal anonymity axiom that is compatible with Strong Pareto. Moreover, when satisfied by a binary relation \succsim , all versions of anonymity but Strong anonymity require the existence of some underlying natural one-to-one correspondence between the components of the alternatives being compared.

In this paper we take route (2), not yet entirely explored, for avoiding the conflict between sensitivity and impartiality. The non-existence of a natural one-to-one correspondence between the components of the compared alternatives means that the components of streams are indexed by natural numbers that do not necessarily correspond to particular people. Thus, this non-existence makes the case for Strong anonymity as an axiom of impartiality, and we ask how far sensitivity for the well-being at any one component can be extended without contradicting Strong anonymity.² Under Strong anonymity the word ‘stream’ is actually a misnomer, as we will be concerned with countably infinite vectors of well-beings where there might not be any natural order, like time, and where the bearers of well-being at particular components with the same index i might differ between vectors. However, we assume that each vector is presented by a particular one-to-one correspondence with the set of natural numbers. The framework that we have in mind is formally introduced in Appendix A.

Strong anonymity is clearly compatible with some sensitivity for the well-being at a single component of the stream, as illustrated by the *Maximin* order. Maximin is represented by the inferior of well-being taken over all components. It is invariant to any permutation of the stream, thus satisfying Strong anonymity. If there is a sole component indexed by the natural number i with a smaller well-being than all other components, then the goodness of a stream is determined by component i , making this component a positional dictator. This also means that an increase in the well-being at component i , keeping the well-being of all other components constant, makes the stream better. This is an example of *positional dominance*, as the sensitivity depends on the position in the ranking of the component whose well-being is increased: Maximin is sensitive to an increase in the well-being at a single component if and only if the well-being of this component is strictly less than the well-being at all other components. We will refer to this specific form of positional dominance as *Inf-restricted dominance*.

The *Leximin* order constitutes a way of extending the sensitivity of Maximin. Leximin restores Strong Pareto in the setting of finite well-being streams, while extensions of Leximin to the infinite-stream setting that insist on satisfy Strong Pareto must necessarily contradict Strong anonymity. For example, in the infinite-stream versions of Leximin characterized by Asheim and Tungodden

²Also Sakai (2016) explores the consequences of imposing Strong anonymity. However, he combines Strong anonymity and supnorm continuity with his Weak non-substitution axiom, which rules out sensitivity for an increase in one person’s well-being.

(2004) and Bossert, Sprumont and Suzumura (2007) Strong Pareto is maintained at the expense of weakening Strong anonymity to Finite anonymity. Hence, to define a version of Leximin in the infinite-stream setting that satisfies Strong anonymity, Strong Pareto must be weakened. Asheim and Zuber (2013) do so by defining and characterizing an infinite-stream version of Leximin that has sensitivity for an increase in, not only the well-being of the worst-off, but the well-being at any component that is finitely ranked, in the sense that there is a cofinite set of people that have well-beings that are more than an ε (> 0) higher. This stronger form of positional dominance, which we will refer to as *Liminf-restricted dominance*, is also satisfied by the Extended rank-discounted utilitarian order defined and characterized by Zuber and Asheim (2012).

A central question posed in this paper is whether positional dominance can be extended beyond Liminf-restricted dominance while insisting on Strong anonymity. The answer is that Liminf-restricted dominance is as far as we can go under four additional, seemingly innocent, axioms: *Monotonicity*, *Continuity*, *Very weak Pigou-Dalton transfer* and *Critical-level consistency*. We are thereby able to characterize the extent of a Paretian principle under Strong anonymity, provided that these four axioms are imposed: *Liminf-restricted dominance is the maximal sensitivity axiom*.

As illustrated by Hilbert's paradox of the Grand Hotel (Hilbert, 2013, p. 730), one can augment an infinite population with a single person, or infinitely many people, without increasing the population's total size—which is already infinite. There is a relationship between, on the one hand, adding new people and moving the existing people to make room for the new ones and, on the other hand, increasing or decreasing the well-being of the people in the particular hotel rooms, indexed by the natural numbers, but where the identities of people in the particular hotel rooms might change when going from one stream to the other. The sensitivity of increasing well-being below the limit inferior entailed by Liminf-restricted dominance means that adding people with well-being below this level has negative value as it can alternatively be interpreted as lowering the well-being of people in some of the hotel rooms. Moreover, the insensitivity of increasing well-being at or above the limit inferior corresponds to zero value of adding people with well-being at or above this level. In both cases, adding people to a population that already is infinite has non-positive value. We show that this is a consequence of Strong anonymity combined with the four additional axioms. So even though Hilbert's Grand Hotel can always accommodate new guests, they are invariably unwelcome in social evaluation.

We thus obtain the conclusion that the *Mere addition principle*, meaning that adding lives worth living cannot have negative social value, need not hold when population is countably infinite.³ Also, zero well-being, interpreted as the well-being level just sufficient to make a life worth living, plays no role in our analysis.⁴

Our contribution differs from Golosov, Jones and Tertilt (2007), who consider Pareto-domi-

³This results can be compared to the arguments by Hamkins and Montero (2000), who argue in favour of the Mere addition principle in that context.

⁴This is consistent with arguments by Broome (2004).

nance in a setting with a finite number of potential identities, and where, in any allocation, well-being is distributed to an observable subset of alive people. Instead, we consider allocations that distribute well-being to a countably infinite number of alive people, and where (as we model formally in Appendix A) these alive people constitute an *unobservable* subsequence of the countably infinite number of potential future identities. This implies that Golosov, Jones and Tertilt’s (2007) concepts of \mathcal{P} -efficiency and \mathcal{A} -efficiency cannot be applied in our context.

We start out in Section 2 by defining four versions of positional dominance and four different complete, reflexive and transitive binary relations that differ with respect to the positional dominance axioms they satisfy. We introduce Strong anonymity and the four additional axioms in Section 3. We then explore possibilities and impossibilities of positional dominance in Section 4. We state and prove the population-ethics result—that adding people cannot have positive social value—in Section 5. We finally discuss the merits of the Strong anonymity axiom in light of these results and point out the consequences for the evaluation of climate policies in the concluding Section 6. In particular, we point out a central implication of the analysis: that one should seek to avoid future lives with low well-being. We present a formal framework where future identities are modeled in Appendix A and show how reflexivity of the binary relation over infinite well-being streams justifies the Strong anonymity axiom if future identities are unobservable. We strengthen our results in Appendix B by weakening the Continuity axiom.

2 Positional dominance

2.1 Preliminaries

Let \mathbb{R} (resp. $\mathbb{R}_+/\mathbb{R}_{++}$) be the set of all (resp. non-negative/positive) real numbers. Let \mathbb{N} (resp. \mathbb{N}_0) denote the set of all positive (resp. non-negative) integers. An infinite stream of well-beings for infinitely many people is generically denoted by $\mathbf{x} = (x_1, x_2, \dots, x_i, \dots) \in \mathbb{R}^{\mathbb{N}}$, where $x_i \in \mathbb{R}$ is the well-being of component $i \in \mathbb{N}$. Throughout the paper, we restrict our attention to the set \mathbf{X} of all bounded streams, which is defined by

$$\mathbf{X} = \left\{ \mathbf{x} = (x_1, x_2, \dots, x_i, \dots) \in \mathbb{R}^{\mathbb{N}} : \sup_{i \in \mathbb{N}} |x_i| < +\infty \right\}.$$

Our notation for vector dominance is as follows. For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x} \geq \mathbf{y}$ whenever $x_i \geq y_i$ for all $i \in \mathbb{N}$, and $\mathbf{x} > \mathbf{y}$ if $\mathbf{x} \geq \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$.

For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and any $N \in \mathbb{N}_0$, $(\mathbf{y}_N, \mathbf{x}) \in \mathbf{X}$ is defined by

$$(\mathbf{y}_N, \mathbf{x}) = \begin{cases} \mathbf{x} & \text{if } N = 0, \\ (y_1, y_2, \dots, y_N, \mathbf{x}) & \text{if } 0 < N < +\infty. \end{cases}$$

Furthermore, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and any $N = +\infty$, we write $\mathbf{z} = (\mathbf{y}, \mathbf{x}) = (\mathbf{y}_N, \mathbf{x})$ to mean $\mathbf{z} \in \mathbf{X}$

defined by

$$z_i = \begin{cases} y_n & \text{if } i = 2n - 1 \text{ and } n \in \mathbb{N}, \\ x_n & \text{if } i = 2n \text{ and } n \in \mathbb{N}, \end{cases}$$

that is, if $N = +\infty$, then $(\mathbf{y}_N, \mathbf{x}) = (\mathbf{y}, \mathbf{x}) = (y_1, x_1, y_2, x_2, \dots)$. For any $\ell \in \mathbb{R}$, let $(\ell)_{\text{con}} = (\ell, \ell, \dots) \in \mathbf{X}$. Thus, $((\ell)_{\text{con}}, \mathbf{x}) = (\ell, x_1, \ell, x_2, \dots)$. Further, for any $\mathbf{x} \in \mathbf{X}$ and any $\ell \in \mathbb{R}$, we write $(\ell, \mathbf{x}) = (\ell, x_1, x_2, \dots) \in \mathbf{X}$.

A *permutation* π of \mathbb{N} is a bijection on \mathbb{N} . Let Π denote the set of all permutations of \mathbb{N} . For any $\mathbf{x} \in \mathbf{X}$ and any $\pi \in \Pi$, we write $\mathbf{x}_\pi = (x_{\pi(1)}, x_{\pi(2)}, \dots) \in \mathbf{X}$.

A binary relation \succsim on \mathbf{X} is a subset of $\mathbf{X} \times \mathbf{X}$. For simplicity, we write $\mathbf{x} \succsim \mathbf{y}$ instead of $(\mathbf{x}, \mathbf{y}) \in \succsim$. The asymmetric and symmetric parts of \succsim are denoted by \succ and \sim , respectively. For any $k \in \mathbb{N} \setminus \{1, 2\}$ and for any $\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^k \in \mathbf{X}$, if a binary relation \succsim on \mathbf{X} is transitive, we write, for simplicity, $\mathbf{x}^1 \succsim \mathbf{x}^2 \succsim \dots \succsim \mathbf{x}^k$ to mean that $\mathbf{x}^\ell \succsim \mathbf{x}^{\ell+1}$ for all $\ell \in \{1, 2, \dots, k-1\}$, so that $\mathbf{x}^\ell \succsim \mathbf{x}^{\ell'}$ holds for all $\ell, \ell' \in \{1, 2, \dots, k\}$ with $\ell < \ell'$.

2.2 Four positional dominance axioms

Consider the following four positional dominance axioms:

Inf-restricted dominance. For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$, $\mathbf{x} \succ \mathbf{y}$ whenever $\inf_{j \in \mathbb{N}} x_j > y_i$.

Liminf-restricted dominance. For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$, $\mathbf{x} \succ \mathbf{y}$ whenever $\liminf_{j \in \mathbb{N}} x_j > y_i$.

Sup-restricted dominance. For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$, $\mathbf{x} \succ \mathbf{y}$ whenever $x_i > \sup_{j \in \mathbb{N}} y_j$.

Limsup-restricted dominance. For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$, $\mathbf{x} \succ \mathbf{y}$ whenever $x_i > \limsup_{j \in \mathbb{N}} y_j$.

Since $\liminf_{j \in \mathbb{N}} x_j \geq \inf_{j \in \mathbb{N}} x_j$, it follows that Liminf-restricted dominance implies Inf-restricted dominance, as it applies to at least as many pairs of \mathbf{x} and \mathbf{y} . For the same reason, as $\sup_{j \in \mathbb{N}} x_j \geq \limsup_{j \in \mathbb{N}} x_j$, we have that Limsup-restricted dominance implies Sup-restricted dominance. Furthermore, note that for any non-decreasing streams \mathbf{x} and \mathbf{y} , $\liminf_{j \in \mathbb{N}} x_j > y_i$ holds if $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$. Thus, Liminf-restricted dominance implies dominance defined for non-decreasing streams. Similarly, Limsup-restricted dominance implies dominance defined for non-increasing streams.

Consider the following four complete, reflexive and transitive binary relations on $\mathbf{X} \times \mathbf{X}$.

Maximin: \succsim_M^+ represented by $W_M^+(\mathbf{x}) = \inf_{i \in \mathbb{N}} x_i = \inf_{\pi \in \Pi} x_{\pi(1)}$.

Progressive rank-discounted (generalized) utilitarianism: \succsim_R^+ represented by

$$W_R^+(\mathbf{x}) = \inf_{\pi \in \Pi} \sum_{i=1}^{\infty} \beta^{i-1} g(x_{\pi(i)}),$$

where $0 < \beta < 1$ and g is a continuous and increasing function.

Maximax: \succsim_M^- represented by $W_M^-(\mathbf{x}) = \sup_{i \in \mathbb{N}} x_i = \sup_{\pi \in \Pi} x_{\pi(1)}$.

Regressive rank-discounted (generalized) utilitarianism: \succsim_R^- represented by

$$W_R^-(\mathbf{x}) = \sup_{\pi \in \Pi} \sum_{i=1}^{\infty} \beta^{i-1} g(x_{\pi(i)}),$$

where $0 < \beta < 1$ and g is a continuous and increasing function.

We observe that Maximin satisfies Inf-restricted dominance, but not the other positional dominance axioms, and that Progressive rank-discounted utilitarianism satisfies Liminf-restricted dominance—and thus also Inf-restricted dominance—but not the two other positional dominance axioms. Likewise, Maximax satisfies Sup-restricted dominance, but not the other positional dominance axioms, while Regressive rank-discounted utilitarianism satisfies Limsup-restricted dominance—and thus also Sup-restricted dominance—but not the two other positional dominance axioms. We note that Progressive rank-discounted utilitarianism is identical to the Extended rank-discounted utilitarian order, as defined (somewhat differently) in Zuber and Asheim (2012, Definition 2). We also note that Progressive and Regressive rank-discounted utilitarianism can be combined to become \succsim_R^α represented by $W_R^\alpha(\mathbf{x}) = \alpha W_R^+(\mathbf{x}) + (1 - \alpha)W_R^-(\mathbf{x})$, where $0 < \alpha < 1$. This combined rank-discounted utilitarianism is related to Sider's (1991) principle GV, but with the important difference that the limit inferior and the limit superior take the place of zero well-being in Sider's setting, being the well-being level just sufficient to make a life worth living.

3 Axioms

Throughout this paper we will insist on impartiality in the sense of the Strong anonymity axiom.⁵

Strong anonymity. For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x}_\pi = \mathbf{y}$ for some $\pi \in \Pi$, $\mathbf{x} \sim \mathbf{y}$.

Also, we insist on monotonicity in the weak sense that the stream is weakly improved by increasing the well-being at some component without reducing the well-being at any other component.

Monotonicity. For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} > \mathbf{y}$, $\mathbf{x} \succsim \mathbf{y}$.

We note that the conjunction of Liminf-restricted dominance and Monotonicity implies sensitivity to an increase in the limit inferior, and the conjunction of Limsup-restricted dominance

⁵See Appendix A for justifications of Strong anonymity in a context where future identities are not observable.

and Monotonicity implies sensitivity to an increase in the limit supremum.

Lemma 1. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} \geq \mathbf{y}$. Let \succsim be a reflexive and transitive binary relation satisfying Monotonicity.

(a) If \succsim also satisfies Liminf-restricted dominance, then $\mathbf{x} \succ \mathbf{y}$ whenever $\liminf_j x_j > \liminf_j y_j$,

(b) If \succsim also satisfies Limsup-restricted dominance, then $\mathbf{x} \succ \mathbf{y}$ whenever $\limsup_j x_j > \limsup_j y_j$.

Proof. Part (a). Let $0 < \varepsilon < \liminf_j x_j - \liminf_j y_j$. By the definition of the limit inferior, there are infinitely many integers k such that $y_k < \liminf_j y_j + \varepsilon$ but only finitely many integers l such that $x_l \leq \liminf_j y_j + \varepsilon < \liminf_j x_j$. So for infinitely many integers m , $x_m > \liminf_j y_j + \varepsilon > y_m$. Consider one of these integers, say i . Let \mathbf{z} be derived from \mathbf{x} by replacing x_i by y_i , where by the choice of ε and i , $\min\{x_i, \liminf_j x_j\} > y_i$. By Monotonicity, $\mathbf{z} \succsim \mathbf{y}$, and by Liminf-restricted dominance, $\mathbf{x} \succ \mathbf{z}$. Hence, by transitivity, $\mathbf{x} \succ \mathbf{y}$.

Part (b). The proof is similar. □

In most of this paper, we also impose a continuity requirement using the supnorm topology, based on the distance function $d: \mathbf{X} \times \mathbf{X} \rightarrow \mathbb{R}_+$ given by, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$,

$$d(\mathbf{x}, \mathbf{y}) = \sup_{i \in \mathbb{N}} |x_i - y_i|.$$

Continuity. For any $\mathbf{x} \in \mathbf{X}$, the sets $\{\mathbf{y} \in \mathbf{X} : \mathbf{y} \succsim \mathbf{x}\}$ and $\{\mathbf{y} \in \mathbf{X} : \mathbf{y} \precsim \mathbf{x}\}$ are closed in the supnorm topology.

The supnorm topology is a rather large topology, but it makes it possible to prove our results in a straightforward manner. In Appendix B we show how we can establish our main results also under weaker continuity properties.

All four binary relations considered in Section 2 satisfy Strong anonymity, Monotonicity and Continuity.

For any reflexive and transitive binary relation \succsim , let $\varepsilon^\succsim \in \mathbb{R}_+ \cup \{+\infty\}$ be the supremum of the set \mathcal{E} defined as

$$\mathcal{E} = \{\varepsilon \in \mathbb{R}_{++} : \text{for any } \mathbf{x}, \mathbf{y} \in \mathbf{X} \text{ with } x_i < y_i \leq y_j < x_j \text{ for some } i, j \in \mathbb{N}, \text{ and} \\ x_k = y_k \text{ for all } k \in \mathbb{N} \setminus \{i, j\}, \mathbf{x} \precsim \mathbf{y} \text{ whenever } x_j - y_j = \varepsilon(y_i - x_i)\},$$

if \mathcal{E} is non-empty and equal to 0 otherwise. The number ε^\succsim measures the highest socially acceptable well-being loss for a rich person when a transfer is made from a rich person to a poor person and the poor receives a well-being gain of size 1. Notice that under the axiom of *Hammond equity* (Hammond, 1976) any loss is acceptable, so that $\mathcal{E} = \mathbb{R}_{++}$. This means that Hammond equity is equivalent to $\varepsilon^\succsim = +\infty$. If non-leaky transfers are acceptable, then it must be the case that $1 \in \mathcal{E}$.

Hence, a reflexive and transitive binary relation \succsim satisfies the *Pigou-Dalton transfer principle* in its weak version (Dalton, 1920; Pigou, 1912) only if $\varepsilon^{\succsim} \geq 1$.

Here we will be concerned with a much weaker axiom than both Hammond equity and Pigou-Dalton (but stronger than the minimal equity axiom proposed by Deschamps and Gevers, 1978): we simply ask that transfers from rich to poor people are acceptable if the well-being loss for the rich person is sufficiently small.

Very weak Pigou-Dalton transfer. $\varepsilon^{\succsim} > 0$.

We have that $\varepsilon^{\succsim_M^+} = \infty$ and $\varepsilon^{\succsim_R^+} \geq 1$ (under an assumption given by Zuber and Asheim, 2012, Proposition 6), while $\varepsilon^{\succsim_M^-} = 0$ and $\varepsilon^{\succsim_R^-} = 0$ since both Maximax and Regressive rank-discounted utilitarianism have sensitivity above the supremum, but no sensitivity below the limit supremum (so that well-being gains for people below the limit supremum are worthless and cannot compensate for a well-being loss for a best-off person). Thus, of the four binary relations considered in Section 2, only Maximin and Progressive rank-discounted utilitarianism satisfy Very weak Pigou-Dalton transfer.

Our last axiom has to do with the effect of adding one person (or infinitely many people) to a population. This kind of question has been addressed in the literature on population ethics stemming from Parfit (1984) and discussed at length in Blackorby, Bossert and Donaldson (2005a). In particular, Blackorby and Donaldson (1984) and Blackorby, Bossert and Donaldson (1995) have argued in favor of the existence of a critical level, such that a person's life contributes positively to the social value of a population if only if the well-being of the additional person is above this critical level. In general, the critical level may depend on the distribution of utility in the population.

The next principle imposes some regularity in the level of the critical level for infinite populations. The axioms asserts that, if it is acceptable to add one person at some level of well-being to a population, then it is also acceptable to add infinitely many people at this level.

Critical-level consistency. For any $\mathbf{x} \in \mathbf{X}$ and $z \in \mathbb{R}$, $\mathbf{x} \succsim (z, \mathbf{x})$ (resp. $\mathbf{x} \precsim (z, \mathbf{x})$) if and only if $\mathbf{x} \succsim ((z)_{\text{con}}, \mathbf{x})$ (resp. $\mathbf{x} \precsim ((z)_{\text{con}}, \mathbf{x})$).

All four binary relations of Section 2 satisfy Critical-level consistency. Indeed:

- $\mathbf{x} \succ_M^+ (z, \mathbf{x})$ and $\mathbf{x} \succ_M^+ ((z)_{\text{con}}, \mathbf{x})$ if $z < \inf_{j \in \mathbb{N}} x_j$, while $\mathbf{x} \sim_M^+ (z, \mathbf{x}) \sim_M^+ ((z)_{\text{con}}, \mathbf{x})$ otherwise,
- $\mathbf{x} \succ_R^+ (z, \mathbf{x})$ and $\mathbf{x} \succ_R^+ ((z)_{\text{con}}, \mathbf{x})$ if $z < \liminf_{j \in \mathbb{N}} x_j$, while $\mathbf{x} \sim_M^+ (z, \mathbf{x}) \sim_M^+ ((z)_{\text{con}}, \mathbf{x})$ otherwise,
- $\mathbf{x} \prec_M^- (z, \mathbf{x})$ and $\mathbf{x} \prec_M^- ((z)_{\text{con}}, \mathbf{x})$ if $z > \sup_{j \in \mathbb{N}} x_j$, while $\mathbf{x} \sim_M^- (z, \mathbf{x}) \sim_M^- ((z)_{\text{con}}, \mathbf{x})$ otherwise,

- $\mathbf{x} \prec_R^-(z, \mathbf{x})$ and $\mathbf{x} \prec_R^-((z)_{\text{con}}, \mathbf{x})$ if $z > \limsup_{j \in \mathbb{N}} x_j$, while $\mathbf{x} \sim_R^-(z, \mathbf{x}) \sim_R^-((z)_{\text{con}}, \mathbf{x})$ otherwise.

4 Possibilities and impossibilities of positional dominance

We first introduce and prove two lemmas before turning to the main results. To introduce the first of these lemmas, recall the example of two unbounded streams used by Fleurbaey and Michel (2003, pp. 795–796) to prove that Strong anonymity is incompatible with even the Weak Pareto axiom. A small variation of their example allows us to show the conflict between these two axioms also in our setting of bounded streams: There exists $\pi \in \Pi$ such that

$$\mathbf{z} = \left(\frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2} \dots \right), \text{ and}$$

$$\mathbf{z}_\pi = \left(\frac{3}{4}, \frac{2}{3}, \frac{4}{5}, \frac{1}{3}, \frac{5}{6}, \frac{1}{4}, \dots, \frac{k+2}{k+3}, \frac{1}{k+1} \dots \right),$$

where by Strong anonymity \mathbf{z} is indifferent to \mathbf{z}_π even though $z_i < z_{\pi(i)}$ for all $i \in \mathbb{N}$. Notice that $\mathbf{z} = (\mathbf{z}^+, \mathbf{z}^-)$, where \mathbf{z}^+ is an increasing subsequence

$$\mathbf{z}^+ = \left(\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots, \frac{k+1}{k+2}, \dots \right),$$

and \mathbf{z}^- is an decreasing subsequence

$$\mathbf{z}^- = \left(\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots, \frac{1}{k+2}, \dots \right).$$

Consider now the streams \mathbf{x} and \mathbf{y} , where

$$\mathbf{x} = \left(\frac{2}{3}, \mathbf{z} \right) = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2} \dots \right), \text{ and}$$

$$\mathbf{y} = \left(\frac{1}{3}, \mathbf{z} \right) = \left(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2} \dots \right),$$

Clearly, if \succsim satisfies Monotonicity, then $\mathbf{x} \succsim \mathbf{y}$, as the only difference between the two streams is that $x_1 = 2/3 > 1/3 = y_1$. However, the fact that $\mathbf{z} = (\mathbf{z}^+, \mathbf{z}^-)$ with \mathbf{z}^+ being an increasing sequence with $x_1 = 2/3 = z_1^+$ and \mathbf{z}^- being an decreasing sequence with $y_1 = 1/3 = z_1^-$ implies that there exists $\pi' \in \Pi$ such that $\mathbf{y}_{\pi'} \geq \mathbf{x}$:

$$\mathbf{y}_{\pi'} = \left(\frac{2}{3}, \frac{3}{4}, \frac{1}{3}, \frac{4}{5}, \frac{1}{3}, \frac{5}{6}, \frac{1}{4}, \dots, \frac{k+2}{k+3}, \frac{1}{k+1} \dots \right).$$

So, if \succsim satisfies also Strong anonymity, then $\mathbf{y} \sim \mathbf{y}_{\pi'} \succsim \mathbf{x} \succsim \mathbf{y}$. Hence, by reflexivity and transitivity of \succsim , $\mathbf{x} \sim \mathbf{y}$, showing that \succsim cannot be sensitive to an increase of a component from $1/3$ to $2/3$ when the rest of the stream equals $\mathbf{z} = (\mathbf{z}^+, \mathbf{z}^-)$. The first lemma generalizes this observation.

Lemma 2. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_i > y_i = z_i$ for some $i \in \mathbb{N}$ and $x_j = y_j = z_j$ for all $j \in \mathbb{N} \setminus \{i\}$. If \succsim is reflexive and transitive binary relation satisfying Strong anonymity and Monotonicity, then $\mathbf{x} \sim \mathbf{y}$ whenever $(z_j)_{j \in \mathbb{N} \setminus \{i\}}$ satisfies

- (a) there exists an increasing function $f : \mathbb{N} \rightarrow \mathbb{N} \setminus \{i\}$ such that, for all $k \in \mathbb{N}$, $x_i \leq z_{f(k)} \leq z_{f(k+1)}$, and
- (b) there exists an increasing function $g : \mathbb{N} \rightarrow \mathbb{N} \setminus \{i\}$ such that, for all $k \in \mathbb{N}$, $y_i \geq z_{g(k)} \geq z_{g(k+1)}$.

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j = z_j$ for all $j \in \mathbb{N} \setminus \{i\}$. Assume that \succsim is a reflexive and transitive binary relation satisfying Strong anonymity and Continuity. Let $(z_j)_{j \in \mathbb{N} \setminus \{i\}}$ satisfy (a) and (b).

Consider the permutation function π constructed by

- $\pi(i) = f(1)$ and $\pi(f(k)) = f(k+1)$ for all $k \in \mathbb{N}$,
- $\pi(g(1)) = i$ and $\pi(g(k+1)) = g(k)$ for all $k \in \mathbb{N}$,

and $\pi(j) = j$ otherwise. Then $\mathbf{y}_\pi \geq \mathbf{x} > \mathbf{y}$, so by Strong anonymity and Monotonicity: $\mathbf{y} \sim \mathbf{y}_\pi \succsim \mathbf{x} \succsim \mathbf{y}$. Hence, by reflexivity and transitivity of \succsim , $\mathbf{x} \sim \mathbf{y}$. \square

In fact, Lemma 2 can be used to show that $\mathbf{x}' \sim \mathbf{y}'$, where

$$\begin{aligned} \mathbf{x}' &= (x'_1, \mathbf{z}) = \left(x'_1, \frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2}, \dots \right), \text{ and} \\ \mathbf{y}' &= (y'_1, \mathbf{z}) = \left(y'_1, \frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2}, \dots \right), \end{aligned}$$

whenever $0 = \liminf_{j \in \mathbb{N}} x'_j < y'_1 < x'_1 < \limsup_{j \in \mathbb{N}} y'_j = 1$ since subsequences as specified in (a) and (b) exist. Two such subsequences do not exist if $0 = y'_1 < x'_1 \leq 1$ or $0 \leq y'_1 < x'_1 = 1$, so that Lemma 2 does not apply under these cases. However, the non-sensitivity result of Lemma 2 can be extended to cover also such circumstances if Continuity is imposed. This extension relies on the following lemma, showing that any reflexive and transitive binary relation satisfying Strong anonymity and Continuity is invariant to adding one person (or infinitely many people) with well-being equal to the limit inferior or the limit supremum, or indeed equal to any other cluster point.⁶

Lemma 3. Consider $\mathbf{x} \in \mathbf{X}$ and let z be a cluster point for \mathbf{x} . If \succsim is a reflexive and transitive binary relation satisfying Strong anonymity and Continuity, then $\mathbf{x} \sim (z, \mathbf{x}) \sim ((z)_{con}, \mathbf{x})$.

Proof. Let $\mathbf{x} \in \mathbf{X}$ and let z be a cluster point for \mathbf{x} . Assume that \succsim is a reflexive and transitive binary relation satisfying Strong anonymity and Continuity.

⁶A *cluster point* of an infinite stream is a point such that, for every neighborhood, there are infinitely many components of the stream with well-beings within the neighborhood.

We prove that $\mathbf{x} \sim (z, \mathbf{x})$ and $\mathbf{x} \sim ((z)_{\text{con}}, \mathbf{x})$, which imply $(z, \mathbf{x}) \sim ((z)_{\text{con}}, \mathbf{x})$ by the transitivity of \succsim .

Since z is a cluster point of \mathbf{x} , there exists an increasing function $f: \mathbb{N} \rightarrow \mathbb{N}$ such that $\lim_{i \rightarrow \infty} |x_{f(i)} - z| = 0$. Hence, for every $m \in \mathbb{N}$, there exists $N(m) \in \mathbb{N}$ such that $|x_{f(i)} - z| < 1/(2m)$ for all $i > N(m)$. For each $m \in \mathbb{N}$, define the increasing function $f^m: \mathbb{N} \rightarrow \mathbb{N}$ by $f^m(i) = f(i + N(m))$ for all $i \in \mathbb{N}$. By construction, for every $m \in \mathbb{N}$ and for all $i, j \in \mathbb{N}$,

$$|x_{f^m(i)} - x_{f^m(j)}| \leq |x_{f^m(i)} - z| + |z - x_{f^m(j)}| < \frac{1}{2m} + \frac{1}{2m} = \frac{1}{m}. \quad (1)$$

For all $m \in \mathbb{N}$, let $\tilde{\mathbf{x}}^m = (x_{f^m(k)})_{k \in \mathbb{N}}$ and we define sequences $(\hat{\mathbf{x}}^m)_{m \in \mathbb{N}}$, $(\check{\mathbf{x}}^m)_{m \in \mathbb{N}}$ in \mathbf{X} as follows. For all $m \in \mathbb{N}$,

$$\hat{x}_i^m = \check{x}_i^m = x_i \text{ for all } i \in \mathbb{N} \setminus \{f^m(k) : k \in \mathbb{N}\}$$

and

$$(\hat{x}_{f^m(k)}^m)_{k \in \mathbb{N}} = (z, \tilde{\mathbf{x}}^m) \text{ and } (\check{x}_{f^m(k)}^m)_{k \in \mathbb{N}} = ((z)_{\text{con}}, \tilde{\mathbf{x}}^m).$$

From (1) and the construction of $(\hat{\mathbf{x}}^m)_{m \in \mathbb{N}}$ and $(\check{\mathbf{x}}^m)_{m \in \mathbb{N}}$, it follows that

$$\lim_{m \rightarrow +\infty} \sup_{i \in \mathbb{N}} |x_i - \hat{x}_i^m| = \lim_{m \rightarrow +\infty} \sup_{i \in \mathbb{N}} |\tilde{x}_i^m - \hat{x}_{f^m(i)}^m| = 0, \quad (2a)$$

$$\lim_{m \rightarrow +\infty} \sup_{i \in \mathbb{N}} |x_i - \check{x}_i^m| = \lim_{m \rightarrow +\infty} \sup_{i \in \mathbb{N}} |\tilde{x}_i^m - \check{x}_{f^m(i)}^m| = 0. \quad (2b)$$

Note that for each $m \in \mathbb{N}$, there exist $\pi^m, \rho^m \in \Pi$ such that

$$(z, \mathbf{x}) = \hat{\mathbf{x}}_{\pi^m}^m \text{ and } ((z)_{\text{con}}, \mathbf{x}) = \check{\mathbf{x}}_{\rho^m}^m.$$

By Strong anonymity, we obtain $(z, \mathbf{x}) \sim \hat{\mathbf{x}}^m$ and $((z)_{\text{con}}, \mathbf{x}) \sim \check{\mathbf{x}}^m$ for all $m \in \mathbb{N}$. Since \succsim satisfies Continuity, it follows from (2a) and (2b) that $(z, \mathbf{x}) \sim \mathbf{x}$ and $((z)_{\text{con}}, \mathbf{x}) \sim \mathbf{x}$. \square

Lemma 3 can also be seen as a generalization of the observation that, under Strong anonymity, the streams $(1, 0, 1, 0, 1, 0, \dots)$ and $(0, 1, 0, 1, 0, 1, \dots)$ are equally good, as one stream can be obtained from the other through an infinite permutation. However, the second stream can also be obtained from the first by adding one person with well-being equal to the limit inferior and the first stream can be obtained from the second by adding a person with well-being equal to the limit supremum.

Using our two lemmas, we are able to prove three main results regarding positional dominance. The first proposition shows that any reflexive and transitive binary relation satisfying Strong anonymity, Monotonicity and Continuity is insensitive to increasing the well-being at a particular component of the stream between the limit inferior and the limit supremum.

Proposition 1. *Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$.*

The two following statements are equivalent:

(1) For all reflexive and transitive binary relations \succsim satisfying Strong anonymity, Monotonicity and Continuity, we have that $\mathbf{x} \sim \mathbf{y}$.

(2) $\limsup_{j \in \mathbb{N}} y_j \geq x_i$ and $y_i \geq \liminf_{j \in \mathbb{N}} x_j$.

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$.

We begin by showing that (1) implies (2). Suppose that (2) does not hold. Then there are two subcases:

Subcase (i): $x_i > \limsup_{j \in \mathbb{N}} y_j$. Consider \succsim_R^- . We have that \succsim_R^- is a reflexive and transitive binary relation that satisfies Strong anonymity, Monotonicity and Continuity. Furthermore, $\mathbf{x} \succ \mathbf{y}$ since \succsim_R^- satisfies Limsup-restricted dominance.

Subcase (ii): $\liminf_{j \in \mathbb{N}} x_j > y_i$. Consider \succsim_R^+ . We have that \succsim_R^+ is a reflexive and transitive binary relation that satisfies Strong anonymity, Monotonicity and Continuity. Furthermore, $\mathbf{x} \succ \mathbf{y}$ since \succsim_R^+ satisfies Liminf-restricted dominance.

Next, we show that (2) implies (1). Assume that (2) holds, and assume that \succsim is a reflexive and transitive binary relation that satisfies Strong anonymity, Monotonicity and Continuity.

Both $u = \limsup_{j \in \mathbb{N}} x_j = \limsup_{j \in \mathbb{N}} y_j$ and $\ell = \liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ are cluster points for \mathbf{x} and \mathbf{y} , so by repeated use of Lemma 3 we have that

$$\mathbf{x} \sim ((\ell)_{\text{con}}, \mathbf{x}) \sim ((u)_{\text{con}}, ((\ell)_{\text{con}}, \mathbf{x})) \text{ and } \mathbf{y} \sim ((\ell)_{\text{con}}, \mathbf{y}) \sim ((u)_{\text{con}}, ((\ell)_{\text{con}}, \mathbf{y})).$$

Write $\mathbf{x}' = ((u)_{\text{con}}, ((\ell)_{\text{con}}, \mathbf{x}))$ and $\mathbf{y}' = ((u)_{\text{con}}, ((\ell)_{\text{con}}, \mathbf{y}))$. By construction, $x'_{i'} > y'_{i'}$ for some $i' \in \mathbb{N}$ and $x'_j = y'_j = z_j$ for all $j \in \mathbb{N} \setminus \{i'\}$. Furthermore, $(z_j)_{j \in \mathbb{N} \setminus \{i'\}}$ satisfies

(a) there exists an increasing function $f : \mathbb{N} \rightarrow \mathbb{N} \setminus \{i'\}$ such that, for all $k \in \mathbb{N}$, $x'_{i'} \leq z_{f(k)} = u$, and

(b) there exists an increasing function $g : \mathbb{N} \rightarrow \mathbb{N} \setminus \{i'\}$ such that, for all $k \in \mathbb{N}$, $y'_{i'} \geq z_{g(k)} = \ell$.

By Lemma 2, $\mathbf{x} \sim \mathbf{x}' \sim \mathbf{y}' \sim \mathbf{y}$. Thus, $\mathbf{x} \sim \mathbf{y}$ by the transitivity of \succsim . \square

Before stating our two last main results, we prove the following lemma which goes one step further by adding the Very weak Pigou-Dalton transfer axiom. It shows that a binary relation that satisfies also this axiom is invariant to increasing the well-being at a particular component of the stream that is already at the limit inferior or above, in the case where the stream under consideration has a limit inferior different from its limit supremum.

Lemma 4. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$. If \succsim is a reflexive and transitive binary relation \succsim satisfying Strong anonymity, Monotonicity, Continuity and Very weak Pigou-Dalton transfer, then $\mathbf{x} \sim \mathbf{y}$ whenever $\limsup_{j \in \mathbb{N}} y_j > \liminf_{j \in \mathbb{N}} x_j$ and $y_i \geq \liminf_{j \in \mathbb{N}} x_j$.

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$. Assume that \succsim is a reflexive and transitive binary relation that satisfies Strong anonymity, Monotonicity, Continuity and Very weak Pigou-Dalton transfer. Let $\limsup_{j \in \mathbb{N}} y_j > \liminf_{j \in \mathbb{N}} x_j$ and $y_i \geq \liminf_{j \in \mathbb{N}} x_j$.

The result follows from Proposition 1 if $\limsup_{j \in \mathbb{N}} y_j \geq x_i$.

Now, suppose that $x_i > \limsup_{j \in \mathbb{N}} y_j$. By monotonicity, $\mathbf{x} \succsim \mathbf{y}$. We need to establish that $\mathbf{y} \succsim \mathbf{x}$. Write $u = \limsup_{j \in \mathbb{N}} x_j = \limsup_{j \in \mathbb{N}} y_j$, $\ell = \liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ and $\delta = \varepsilon(u - \ell)$, where $\varepsilon > 0$ is chosen such that $\varepsilon \in \mathcal{E}$; this is possible since \succsim satisfies Very weak Pigou-Dalton transfer, implying that $\mathcal{E} \neq \emptyset$.

Assume that there exists $k \in \mathbb{N} \setminus \{i\}$ such that $u \geq x_k = y_k \geq \ell$. Define \mathbf{y}^0 by $y_i^0 = \max\{y_i, u\}$, $y_k^0 = u$ and $y_j^0 = y_j$ for $j \in \mathbb{N} \setminus \{i, k\}$. Define inductively \mathbf{x}^n by $x_i^n = y_i^{n-1} + \delta$ and $x_k^n = \ell$ and $x_j^n = y_j^{n-1}$ for $j \in \mathbb{N} \setminus \{i, k\}$, and \mathbf{y}^n by $y_k^n = u$ and $y_j^n = x_j^n$ for $j \in \mathbb{N} \setminus \{k\}$ for $n = 1, 2, 3, \dots, \bar{n}$ until $\mathbf{y}^{\bar{n}} \geq \mathbf{x}$. Then:

$$\mathbf{y} \sim \mathbf{y}^0 \succsim \mathbf{x}^1 \sim \mathbf{y}^1 \succsim \dots \succsim \mathbf{x}^{\bar{n}} \sim \mathbf{y}^{\bar{n}} \succsim \mathbf{x},$$

where $\mathbf{y} \sim \mathbf{y}^0$ and $\mathbf{x}^{\bar{n}} \sim \mathbf{y}^{\bar{n}}$ follow from Proposition 1, $\mathbf{y}^{n-1} \succsim \mathbf{x}^n$ for $n = 1, 2, 3, \dots, \bar{n}$ follow from Very weak Pigou-Dalton transfer, and $\mathbf{y}^{\bar{n}} \succsim \mathbf{x}$ follows from Monotonicity.

In the case where there does not exist $k \in \mathbb{N} \setminus \{i\}$ such that $u \geq x_k = y_k \geq \ell$, it follows from Lemma 3 that $\mathbf{x} \sim \mathbf{y}$ if and only if $(u, \mathbf{x}) \sim (u, \mathbf{y})$ since $u = \limsup_{j \in \mathbb{N}} x_j = \limsup_{j \in \mathbb{N}} y_j$ is a cluster point for \mathbf{x} and \mathbf{y} . Hence, we can use the argument above on the pair $((u, \mathbf{x}), (u, \mathbf{y}))$ instead of (\mathbf{x}, \mathbf{y}) . \square

Lemma 4 implies that Limsup-restricted dominance is inconsistent with the four axioms of the lemma because such dominance cannot be imposed on streams where the limit inferior is strictly lower than the limit supremum. This almost establishes the result that any reflexive and transitive binary relation satisfying these four axioms cannot exhibit any sensitivity beyond Liminf-restricted dominance. In order to show this result we must remove the restriction made in Lemma 4 that the streams under consideration must have a limit inferior strictly lower than the limit supremum. The next proposition shows that it is actually true in general that if a reflexive and transitive binary relation satisfies Strong anonymity, Monotonicity, Continuity and Very weak Pigou-Dalton transfer, then it cannot exhibit sensitivity beyond Liminf-restricted dominance. Moreover, this result is tight since the converse follows from the fact that Progressive rank-discounted utilitarianism satisfies these axioms as well as Liminf-restricted dominance.

Proposition 2. *Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$. The two following statements are equivalent:*

- (1) *For all reflexive and transitive binary relations \succsim satisfying Strong anonymity, Monotonicity, Continuity and Very weak Pigou-Dalton transfer, we have that $\mathbf{x} \sim \mathbf{y}$.*
- (2) *$y_i \geq \liminf_{j \in \mathbb{N}} x_j$.*

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$.

We begin by showing that (1) implies (2). Suppose that (2) does not hold, implying that $\liminf_{j \in \mathbb{N}} x_j > y_i$. Consider \succsim_R^+ with u being linear. We have that \succsim_R^+ is a reflexive and transitive binary relation that satisfies Strong anonymity, Monotonicity, Continuity and Very weak Pigou-Dalton transfer. Furthermore, $\mathbf{x} \succ \mathbf{y}$ since \succsim_R^+ satisfies Liminf-restricted dominance.

Next, we show that (2) implies (1). Assume that (2) holds, and assume that \succsim is a reflexive and transitive binary relation that satisfies Strong anonymity, Monotonicity, Continuity and Very weak Pigou-Dalton transfer. There are two subcases:

Subcase (i): $\limsup_{j \in \mathbb{N}} y_j > \liminf_{i \in \mathbb{N}} x_i$. The result follows from Lemma 4.

Subcase (ii): $\limsup_{j \in \mathbb{N}} y_j = \liminf_{i \in \mathbb{N}} x_i = \ell$. Since ℓ is a cluster point of \mathbf{x} and \mathbf{y} , it follows from Lemma 3 that $((\ell)_{\text{con}}, \mathbf{x}) \sim \mathbf{x}$ and $((\ell)_{\text{con}}, \mathbf{y}) \sim \mathbf{y}$. Let $(\varepsilon_n)_{n \in \mathbb{N}}$ be a sequence of positive real numbers converging to 0. By Lemma 4, $((\ell + \varepsilon_n)_{\text{con}}, \mathbf{x}) \sim ((\ell + \varepsilon_n)_{\text{con}}, \mathbf{y})$ for each $n \in \mathbb{N}$. Furthermore, $((\ell + \varepsilon_n)_{\text{con}}, \mathbf{x}) > ((\ell)_{\text{con}}, \mathbf{x})$ so that $((\ell + \varepsilon_n)_{\text{con}}, \mathbf{x}) \succ ((\ell)_{\text{con}}, \mathbf{x}) \sim \mathbf{x}$ by Monotonicity. By transitivity, $((\ell + \varepsilon_n)_{\text{con}}, \mathbf{y}) \succ \mathbf{x}$. This is true for each $n \in \mathbb{N}$ so that $((\ell)_{\text{con}}, \mathbf{y}) \succ \mathbf{x}$ by Continuity and $\mathbf{y} \succ \mathbf{x}$ by transitivity. On the other hand, $\mathbf{x} \succ \mathbf{y}$ by Monotonicity. \square

Proposition 2 shows that dominance can occur under Strong anonymity, Monotonicity, Continuity and Very weak Pigou-Dalton transfer, when increasing the well-being at a particular component of the stream, if and only if the well-being at this component is strictly lower than the limit inferior before the increase. Any reflexive and transitive binary relation satisfying these axioms is invariant to an increase in the well-being at a particular component of the stream at or above the limit inferior. Notice that Regressive rank-discounted utilitarianism satisfy Limsup-restricted dominance and all the other axioms except for Very weak Pigou-Dalton transfer. Our minimal equity requirement is essential for obtaining the result. On the other hand, Inf-restricted dominance and Liminf-restricted dominance are clearly compatible with the other axioms: both Maximin and Progressive rank-discounted utilitarianism satisfy Strong anonymity, Monotonicity, Continuity and Very weak Pigou-Dalton transfer.

Our last proposition in this section shows that we can generalize the result to the case where not only one component of the stream, but infinitely many components experience an increase in well-being above the limit inferior. To obtain this result, we need the additional axiom of Critical-level consistency.

Proposition 3. *Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} > \mathbf{y}$. The two following statements are equivalent:*

- (1) *For all reflexive and transitive binary relations \succsim satisfying Strong anonymity, Monotonicity, Continuity, Very weak Pigou-Dalton transfer and Critical-level consistency, we have that $\mathbf{x} \sim \mathbf{y}$.*
- (2) *$\ell = \liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ and $y_i \geq \ell$ for all $i \in \mathbb{N}$ such that $x_i > y_i$.*

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $\mathbf{x} > \mathbf{y}$.

We begin by showing that (1) implies (2). Suppose that (2) does not hold, implying that (i) $\liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ and there exists $i \in \mathbb{N}$ such that $\min\{x_i, \liminf_{j \in \mathbb{N}} x_j\} > y_i$, or (ii) $\liminf_{j \in \mathbb{N}} x_j > \liminf_{j \in \mathbb{N}} y_j$. Consider \succsim_R^+ with u being linear. We have that \succsim_R^+ is a reflexive and transitive binary relation that satisfies Strong anonymity, Monotonicity, Continuity, Very weak Pigou-Dalton transfer and Critical-level consistency. Furthermore, $\mathbf{x} \succ \mathbf{y}$ since \succsim_R^+ satisfies Monotonicity and Liminf-restricted dominance (cf. Lemma 1(a) in case (ii)).

Next, we show that (2) implies (1). Assume that (2) holds, and assume that \succsim is a reflexive and transitive binary relation that satisfies Strong anonymity, Monotonicity Continuity, Very weak Pigou-Dalton transfer and Critical-level consistency. Denote $I_{>} = \{i \in \mathbb{N} : x_i > y_i\}$ the set of coordinates where utility is strictly larger in \mathbf{x} than in \mathbf{y} . There are two subcases:

Subcase (i): $|I_{>}| < +\infty$. In this case, the result follows from repeated applications of Proposition 2 and transitivity.

Subcase (ii): $|I_{>}| = +\infty$. Since ℓ is a cluster point, by Lemma 3 we obtain $((\ell)_{\text{con}}, \mathbf{x}) \sim \mathbf{x}$ and $((\ell)_{\text{con}}, \mathbf{y}) \sim \mathbf{y}$. Let \mathbf{z} and $\hat{\mathbf{z}}$ be two streams such that:

$$z_i = \begin{cases} \ell & \text{if } i = 2n - 1 \text{ and } n \in \mathbb{N}, \\ x_i & \text{if } i = 2n \text{ and } n \in \mathbb{N} \setminus I_{>}, \\ \sup_{j \in \mathbb{N}} x_j & \text{if } i = 2n \text{ and } n \in I_{>}, \end{cases}$$

and

$$\hat{z}_i = \begin{cases} \ell & \text{if } i = 2n - 1 \text{ and } n \in \mathbb{N}, \\ x_i & \text{if } i = 2n \text{ and } n \in \mathbb{N} \setminus I_{>}, \\ \ell & \text{if } i = 2n \text{ and } n \in I_{>}. \end{cases}$$

By definition, $\mathbf{z} \geq ((\ell)_{\text{con}}, \mathbf{x})$ and $((\ell)_{\text{con}}, \mathbf{y}) \geq \hat{\mathbf{z}}$, so that by Monotonicity $\mathbf{z} \succsim ((\ell)_{\text{con}}, \mathbf{x})$ and $((\ell)_{\text{con}}, \mathbf{y}) \succsim \hat{\mathbf{z}}$. Now we want to show that $\mathbf{z} \sim \hat{\mathbf{z}}$ to obtain by transitivity that $((\ell)_{\text{con}}, \mathbf{y}) \succsim ((\ell)_{\text{con}}, \mathbf{x})$ and therefore $\mathbf{y} \succsim \mathbf{x}$.

Let $J = \{i \in \mathbb{N} : \exists k \in I_{>}, i = 2k\}$. In streams $((\ell)_{\text{con}}, \mathbf{x})$, $((\ell)_{\text{con}}, \mathbf{y})$, \mathbf{z} and $\hat{\mathbf{z}}$, coordinates in J are those where the streams differ. Let f be the unique increasing bijection between \mathbb{N} and $\mathbb{N} \setminus J$. Define $\tilde{\mathbf{z}}$ by $\tilde{z}_n = z_{f(n)}$ for all $n \in \mathbb{N}$: stream $\tilde{\mathbf{z}}$ collects all coordinates that are the same in $((\ell)_{\text{con}}, \mathbf{x})$, $((\ell)_{\text{con}}, \mathbf{y})$, \mathbf{z} and $\hat{\mathbf{z}}$. Let $\bar{x} = \sup_{j \in \mathbb{N}} x_j$, by Proposition 2, $(\bar{x}, \tilde{\mathbf{z}}) \sim (\ell, \tilde{\mathbf{z}})$. By Critical-level consistency, $(\bar{x}, \tilde{\mathbf{z}}) \sim ((\bar{x})_{\text{con}}, \tilde{\mathbf{z}})$ and $(\ell, \tilde{\mathbf{z}}) \sim ((\ell)_{\text{con}}, \tilde{\mathbf{z}})$. Clearly, \mathbf{z} can be obtained from $((\bar{x})_{\text{con}}, \tilde{\mathbf{z}})$ and $\hat{\mathbf{z}}$ can be obtained from $((\ell)_{\text{con}}, \tilde{\mathbf{z}})$ through a permutation. Therefore, by Strong anonymity and transitivity, $\mathbf{z} \sim \hat{\mathbf{z}}$.

We thus know that $\mathbf{y} \succsim \mathbf{x}$. But $\mathbf{x} \geq \mathbf{y}$ so that by Monotonicity $\mathbf{x} \succsim \mathbf{y}$. \square

An implication of Proposition 3 is that even an increase in utility at an infinite number of components may not be sufficient to guarantee social dominance. For instance, Proposition 3

implies that the following two streams are equivalent:

$$(1, 0, 1, 0, \dots, 1, 0, \dots) \quad \text{and} \quad (0, 0, 0, 0, \dots).$$

As mentioned in the introduction to this section, Fleurbaey and Michel (2003) proved that Strong anonymity is incompatible with the Weak Pareto axiom using an example. Proposition 3 provides a larger set of cases where the Weak Pareto axiom fails when our other axioms are satisfied: all cases where there is a strict improvement above the limit inferior without changing the limit inferior. For instance, the following two streams are equally good:

$$(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{k}, \dots) \quad \text{and} \quad (0, 0, 0, 0, \dots).$$

Remark 1. Our analysis builds on the assumption that well-being is real-valued. If instead well-being is a binary variable, being equal to 0 or 1, then clearly Continuity and Very weak Pigou-Dalton transfer have no bite. In this case, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} > \mathbf{y}$, the two following statements are equivalent:

- (1) For all reflexive and transitive binary relations \succsim satisfying Strong anonymity, Monotonicity and Critical-level consistency, we have that $\mathbf{x} \sim \mathbf{y}$.
- (2) $\limsup_{j \in \mathbb{N}} y_j = 1$ and $\liminf_{j \in \mathbb{N}} x_j = 0$.

Hence, there is no sensitivity as long as both \mathbf{x} and \mathbf{y} have infinitely many elements at both 0 and 1. If not, \succsim_T^α , the combined rank-discounted utilitarianism defined at the end of Section 2, shows that sensitivity is consistent with Strong anonymity, Monotonicity and Critical-level consistency.

5 Infinite population ethics under strong anonymity

As we have discussed before, the literature on population ethics considers the effect of adding a person, or several people, to a population. Parfit (1984) has introduced the Mere addition principle: the addition of someone with a non-negative well-being should always be acceptable. The problem is that, under mild conditions, this principle may yield the ‘repugnant conclusion’ by which a very large population of people with lives barely worth living may be better than a large but smaller population of people with excellent lives. To avoid this conclusion, Blackorby and Donaldson (1984) and Blackorby, Bossert and Donaldson (1995) have proposed critical-level utilitarianism, according to which adding only sufficiently good lives are socially acceptable.

In any case, most approaches assume that there exist levels of well-being such that adding someone with a well-being at these levels is a strict social improvement. However, this assumption is made in the setting of a finite population. In contrast, the following axiom is concerned with the addition of finitely or infinitely many people to a population that already has infinitely many people. It asserts that the social value of adding finitely or infinitely many lives is non-positive

regardless of their well-being. Note that, in the statement of the axiom, the stream \mathbf{x} can be viewed as being obtained from stream \mathbf{y} by “removing” some people.

Non-positive value of additional lives. For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if there exists an increasing function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $x_i = y_{f(i)}$ for all $i \in \mathbb{N}$, then $\mathbf{x} \succsim \mathbf{y}$.

We obtain the surprising result that this population-ethics axiom is a result of the axioms that we have already imposed.

Proposition 4. *Assume that \succsim is a reflexive and transitive binary relation satisfying Strong anonymity, Monotonicity, Continuity, Very weak Pigou-Dalton transfer and Critical-level consistency. Then \succsim satisfies Non-positive value of additional lives.*

Proof. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ such that there exists an increasing function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $x_i = y_{f(i)}$ for all $i \in \mathbb{N}$. Assume that \succsim is a reflexive and transitive binary relation satisfying Strong anonymity, Monotonicity, Continuity, Very weak Pigou-Dalton, and Critical-level consistency.

Write $\ell(\mathbf{x}) = \liminf_{j \in \mathbb{N}} x_j$ and $\ell(\mathbf{y}) = \liminf_{j \in \mathbb{N}} y_j$, and let $z = \sup_{j \in \mathbb{N}} y_j$. By the relationship between \mathbf{x} and \mathbf{y} , $z \geq \ell(\mathbf{x}) \geq \ell(\mathbf{y})$. Write $\tilde{\mathbf{x}} = ((\ell(\mathbf{x}))_{\text{con}}, \mathbf{x})$ and $\tilde{\mathbf{y}} = ((\ell(\mathbf{y}))_{\text{con}}, \mathbf{y})$. By Strong anonymity, adding an additional person with well-being z to $\tilde{\mathbf{x}}$ is equivalent to increasing the well-being of an existing person from $\ell(\mathbf{x})$ to z . So by Proposition 2, $(z, \tilde{\mathbf{x}}) \sim \tilde{\mathbf{x}}$. Thus, by Critical-level consistency, $((z)_{\text{con}}, \tilde{\mathbf{x}}) \sim \tilde{\mathbf{x}}$. By the definition of z , there exists $\pi \in \Pi$ such that $((z)_{\text{con}}, \tilde{\mathbf{x}}) \geq \tilde{\mathbf{y}}_\pi$, so by Monotonicity, $((z)_{\text{con}}, \tilde{\mathbf{x}}) \succsim \tilde{\mathbf{y}}_\pi$. By Strong anonymity, $\tilde{\mathbf{y}}_\pi \sim \tilde{\mathbf{y}}$, while by Lemma 3, $\mathbf{x} \sim \tilde{\mathbf{x}}$ and $\mathbf{y} \sim \tilde{\mathbf{y}}$. Hence,

$$\mathbf{x} \sim \tilde{\mathbf{x}} \sim ((z)_{\text{con}}, \tilde{\mathbf{x}}) \succsim \tilde{\mathbf{y}}_\pi \sim \tilde{\mathbf{y}} \sim \mathbf{y},$$

which by the transitivity of \succsim implies that \succsim satisfies Non-positive value of additional lives. \square

One can even be more specific about cases where adding a person is a matter of social indifference or has negative social value. By Lemma 3 and Proposition 3, adding one person—or finitely or infinitely many people—at the limit inferior or above is a matter of social indifference. Indeed, adding any number of people at the limit inferior is socially indifferent by Lemma 3. Then increasing their well-being is also socially indifferent by Proposition 3.

On the other hand, if a reflexive and transitive binary relation \succsim satisfies Liminf-restricted dominance, then adding at least one person below the limit inferior has negative social value. Indeed, like before, adding any number of people at the limit inferior is socially indifferent. Then decreasing the well-being of one person from the limit inferior to a strictly lower value has negative social value by Liminf-restricted dominance.

6 Concluding remarks

In the view of Van Liedekerke and Lauwers (1997, p. 164) there are good reasons why the route taken in this paper, where we insist on Strong anonymity and weaken Strong Pareto, has been

left unexplored. In their opinion, Strong anonymity is an unreasonable impartiality requirement. This view might be compelling if there exists a natural one-to-one correspondence between the components of different alternatives, like in the case where the identity of people remains the same independently of the well-being they experience. For example, if the number of people in all generations is fixed, then it is not logically impossible to assume that people are the same independently of their well-beings. So if, following an example provided by Van Liedekerke and Lauwers (1997, p. 164), there are 100 people in each generation, with one alternative giving 99 of them a well-being of 1 and the last one a well-being of 0, while in the other alternative 99 gets a well-being of 0 and only one a well-being of 1, it might be hard to argue that the streams are equally good, even though one stream is an infinite permutation of the other. If one agrees with this position, then the fact that Strong anonymity together with four rather innocent axioms implies that adding people to an infinite population never has positive value can be taken as a further indication that Strong anonymity is indeed too strong, giving more weight to the kind of arguments put forward by opponents of this axiom (see e.g., Jonsson, 2019).

However, in comparisons where the number of people in any one generation varies between two alternatives, there exists no such natural one-to-one correspondence. In such settings the arguments against the axiom of Strong anonymity are weakened. We have shown that it is a consequence of Strong anonymity in combination with four other axioms that only people finitely ranked from the bottom matter. The asymmetry between the sensitivity for people finitely ranked from the bottom and the insensitivity for people finitely ranked from the top is a consequence of an innocent equity axiom, Very weak Pigou-Dalton transfer, stating that when making a transfer between a richer and a poorer person, the poorer person's change in well-being must have positive relative weight. Furthermore, we have shown that—when sensitivity is limited in this way—it follows that adding people to an infinite population cannot have positive social value. These conclusions might, at first, appear to be a bitter pill to swallow for those who find Strong anonymity attractive. Further reflection might, however, provide insights into why it is reasonable to require insensitivity at or above the limit inferior and to conclude that an infinite population cannot be improved by adding additional people.

The main implication of the analysis for the evaluation of climate policies is that one should prefer policies that increase the well-being of future people that are finitely ranked and avoid future lives with low well-being. In contrast, increasing the well-being of future people that infinitely ranked or adding future lives with high well-being will not have social value.

Appendix A: Formalizing future identities.

In this appendix we present a framework where future identities are formally modeled. We present an anonymous welfarism theorem, which justifies our analysis of strongly anonymous preorders for infinite well-being streams. The framework we employ for establishing the welfarism theorem assumes that there exists a single and fixed profile of individuals' utility functions. Thus, our welfarism theorem is an infinite and variable population variant of the single-profile welfarism theorems established by Blackorby, Donaldson and Weymark (1990) and Blackorby, Bossert and Donaldson (2006) in a finite and fixed population framework and is an infinite population analogue of the results of Blackorby, Bossert and Donaldson (1999, 2005a) who employed a finite and variable population framework assuming a multi-profile setting.⁷ A distinctive feature of our analysis is that we focus on unobservability of the identities of future people, based on which it is shown that Strong anonymity of the evaluation of infinite well-being streams follows once the evaluation relation is required to be reflexive.

Let A be the *set of social states* in which there exist infinitely many future individuals alive. Each social state $a \in A$ is an alternative to be socially evaluated; it is a complete description which includes information of individuals alive and other relevant aspects of a social state. In all of these alternatives we assume that the identities and well-beings of dead and presently alive individuals are fixed. Hence, we address the question of how to evaluate the effects of policy that changes the identities and well-beings of future individuals without changing the conditions for people that are already dead or have already been born.

Let \mathbb{N} be the *set of all potential future identities*, where \mathbb{N} denotes the set of all natural numbers. Let F^* generically denote an infinite subset of \mathbb{N} ; formally, $F^* \subseteq \mathbb{N}$ and $|F^*| = +\infty$. This will be used to denote a set of the future individuals alive in a framework where their identities are specified. Let \mathcal{N} be the collection of all infinite subsets F^* of \mathbb{N} ; formally, $\mathcal{N} = \{F^* \subseteq \mathbb{N} : |F^*| = +\infty\}$.

Let \mathcal{F} be the collection of all injective functions $f^* : \mathbb{N} \rightarrow \mathbb{N}$. Each $f^* \in \mathcal{F}$ selects the identities of the future individuals that will be alive and orders them in a one-to-one correspondence with the set of natural numbers (it puts them into rooms in Hilbert's Grand Hotel, as it were). Let $f : A \rightarrow \mathcal{F}$ be a function that for any state $a \in A$ determines an injective function $f(a) \in \mathcal{F}$. That is, for social state $a \in A$, $f(a) : \mathbb{N} \rightarrow \mathbb{N}$ selects the identities of the future individuals alive in a and orders them in a one-to-one correspondence with the set of natural numbers between the set of natural numbers; that is, $f(a)(j)$ is the future individual alive in a who is in room j of Hilbert's Grand Hotel. Note that for each $a \in A$, the range $f(a)(\mathbb{N}) \in \mathcal{N}$ is the set of future individuals alive in a , which we denote by $F(a)$ instead of writing $f(a)(\mathbb{N})$.

⁷Other related but different versions of multi-profile welfarism theorem were presented by d'Aspremont and Gevers (1977), Blackorby, Bossert and Donaldson (2005a, 2005b) and Hammond (1979) in a finite and fixed population setting and also by d'Aspremont (2007) and Kamaga (2016) in the context of intergenerational well-being evaluation that relies on the use of the order of generations.

For each potential future identity $i \in \mathbb{N}$, let A_i denote the set of social states in which the individual with identity i is alive, that is, $A_i = \{a \in A : i \in F(a)\}$. For each $i \in \mathbb{N}$, $u_i : A_i \rightarrow \mathbb{R}$ is a utility function of the potential future individual with identity i . Throughout this appendix, we assume that there exists a single fixed profile $u = (u_1, u_2, \dots)$ of utility functions of all potential future identities. Furthermore, we assume that, for each $a \in A$, $\sup_{i \in F(a)} |u_i(a)| < +\infty$. Therefore, a stream of well-beings of future individuals alive is bounded regardless of how we list the individuals alive and what are their identities. Let $\mathbf{x}(a)$ denote the stream of well-beings of the future individuals that will be alive ordered according to the rooms they occupy in Hilbert's Grand Hotel. Formally, for any $a \in A$,

$$\mathbf{x}(a) = (x_1(a), x_2(a), \dots, x_j(a), \dots) \text{ with } x_j(a) = u_{f(a)(j)}(a) \text{ for all } j \in \mathbb{N}.$$

As in the main text, \mathbf{X} denotes the set of all bounded well-being streams.

Throughout this appendix, we make the following assumption on the set A of social states.

Richness. For any $f^* \in \mathcal{F}$ and $\mathbf{x}^* \in \mathbf{X}$, there exists $a \in A$ such that $f(a) = f^*$ and $\mathbf{x}(a) = \mathbf{x}^*$.

Note that the existence of $a \in A$ such that $f(a) = f^*$ for any given $f^* \in \mathcal{F}$ implies that $\{F(a) : a \in A\} = \mathcal{N}$ because, for any $F^* \in \mathcal{N}$, there exists $f^* \in \mathcal{F}$ such that $F^* = f^*(\mathbb{N})$.

We assume that the well-beings of the people in the rooms of Hilbert's Grand Hotel, numbered by the natural numbers, are observable, but not necessarily their identities. Thus, if social states a and b are such that $\mathbf{x}(a) \neq \mathbf{x}(b)$, then a and b are observationally distinguishable even if we cannot observe the identities of the people alive in a and b . On the other hand, if $\mathbf{x}(a) = \mathbf{x}(b)$, whether or not a and b are observationally distinguishable depends on whether the identities of the future people alive in a and b , which are represented by $f(a)$ and $f(b)$, are observable or not. We formalize such an information structure by introducing an information function that generates a partition of set A of social states according to the observational indistinguishability. Let $\{\mathcal{F}(a) : a \in A\}$ be a partition of \mathcal{F} that satisfies that (i) for each $a \in A$, $f(a) \in \mathcal{F}(a)$ and (ii) for any $a, b \in A$, $\mathcal{F}(a) = \mathcal{F}(b)$ or $\mathcal{F}(a) \cap \mathcal{F}(b) = \emptyset$. Note that $\{\mathcal{F}(a) : a \in A\}$ is well-defined as a partition of \mathcal{F} under the Richness assumption. Given a partition $\{\mathcal{F}(a) : a \in A\}$, each element $\mathcal{F}(a)$ is interpreted as the set of functions in \mathcal{F} (for instance, $f(a)$ and $f(b)$) such that the identities of the future people in the underlying social states (for instance, a and b) are not observable. Given a partition $\{\mathcal{F}(a) : a \in A\}$ of \mathcal{F} , we define the *information function* associated with $\{\mathcal{F}(a) : a \in A\}$ by the mapping $\mathcal{I} : A \rightarrow 2^A \setminus \{\emptyset\}$ such that, for each $a \in A$,

$$\mathcal{I}(a) = \{b \in A : f(b) \in \mathcal{F}(a) \text{ and } \mathbf{x}(b) = \mathbf{x}(a)\}.$$

That is, the information function \mathcal{I} associates to each social state $a \in A$ the set $\mathcal{I}(a)$ of social states which are observationally indistinguishable due to the sameness between their observable infinite well-being streams and unobservability of the identities of people alive in the underlying

social states.

Given a partition $\{\mathcal{F}(a) : a \in A\}$ of \mathcal{F} , the associated information function \mathcal{I} induces the partition $\{\mathcal{I}(a) : a \in A\}$ of the set A of social states since it satisfies the following properties: (i) $a \in \mathcal{I}(a)$ for all $a \in A$ and (ii) for all $a, b \in A$, $\mathcal{I}(a) \cap \mathcal{I}(b) \neq \emptyset$ implies $\mathcal{I}(b) = \mathcal{I}(a)$. The first property is straightforward from the definition of \mathcal{I} . The second property is verified as follows. Let $a, b \in A$ and suppose that $\mathcal{I}(a) \cap \mathcal{I}(b) \neq \emptyset$. Let $c \in \mathcal{I}(a) \cap \mathcal{I}(b)$. Then, $\mathcal{F}(a) = \mathcal{F}(b) = \mathcal{F}(c)$ follows and we obtain $\mathcal{I}(a) = \{d \in A : f(d) \in \mathcal{F}(c) \text{ and } \mathbf{x}(d) = \mathbf{x}(c)\} = \mathcal{I}(b)$.

The structure of the induced partition $\{\mathcal{I}(a) : a \in A\}$ of the set A of social states depends on assumptions on observability of future identities that are determined by the structure of a given partition $\{\mathcal{F}(a) : a \in A\}$ of \mathcal{F} . Among all possible assumptions, the two polar cases are worth noting. One is the assumption of perfect observability of future identities defined as follows.

Perfect observability of future identities. $\mathcal{F}(a) = \{f(a)\}$ for all $a \in A$.

The polar opposite assumption is unobservability of future identities defined as follows.

Unobservability of future identities. $\mathcal{F}(a) = \mathcal{F}$ for all $a \in A$; that is, $\{\mathcal{F}(a) : a \in A\} = \{\mathcal{F}\}$.

Note that $\mathcal{I}(a) = \{a\}$ for all $a \in A$ under the assumption of Perfect observability of future identities, while $\mathcal{I}(a) = \{b \in A : \mathbf{x}(b) = \mathbf{x}(a)\}$ for all $a \in A$ under the assumption of Unobservability of future identities.

In the framework of single profile of utility functions, the evaluation of social states in A is determined by a binary relation \succsim^* defined on A . The asymmetric and symmetric parts of \succsim^* are denoted by \succ^* and \sim^* , respectively. We assume that the binary relation \succsim^* is reflexive and transitive (that is, a preorder). Note that for any preorder \succsim^* on A , there exists an induced binary relation for elements of the partition $\{\mathcal{I}(a) : a \in A\}$ determined by an information function \mathcal{I} . Formally, given a partition $\{\mathcal{F}(a) : a \in A\}$ and the associated information function \mathcal{I} , there exists the induced binary relation \succsim on $\{\mathcal{I}(a) : a \in A\}$ defined by, for all $\mathcal{I}(a), \mathcal{I}(b) \in \{\mathcal{I}(a) : a \in A\}$,

$$\mathcal{I}(a) \succsim \mathcal{I}(b) \text{ if and only if } c \succsim^* d \text{ for all } c \in \mathcal{I}(a) \text{ and } d \in \mathcal{I}(b).$$

The induced binary relation \succsim is always transitive, but inherits reflexivity from \succsim^* (without additional assumptions) only if there is Perfect observability of future identities. Under the assumption of Perfect observability of future identities, the induced binary relation \succsim (which is a preorder in this case) coincides with the evaluation of social states determined by \succsim^* since $\mathcal{I}(a) = \{a\}$ for all $a \in A$. On the other hand, under the assumptions of Richness and Unobservability of future identities, the induced binary relation \succsim relates how we should evaluate infinite well-being streams in a consistent manner to the evaluation of underlying social states by \succsim because $\mathcal{I}(a) = \{b \in A : \mathbf{x}(b) = \mathbf{x}(a)\}$ for all $a \in A$, and thus, there exists a bijection between \mathbf{X} and $\{\mathcal{I}(a) : a \in A\}$. In this case, abusing the notation for \succsim , the induced binary relation \succsim can

be equivalently represented as the following binary relation on the set \mathbf{X} of infinite well-being streams. For all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$,

$$\mathbf{x} \succsim \mathbf{y} \text{ if and only if } c \succsim^* d \text{ for all } c \in A \text{ with } \mathbf{x}(c) = \mathbf{x} \text{ and for all } d \in A \text{ with } \mathbf{x}(d) = \mathbf{y}. \quad (3)$$

Note that the induced relation \succsim on \mathbf{X} is always transitive but not necessarily reflexive.

Hereafter, we examine normative properties of the evaluation \succsim^* of social states that are requisite for the reflexivity of the induced relation \succsim for infinite well-being streams under the assumptions of Richness and Unobservability of future identities.

We consider three axioms for a preorder \succsim^* on A . Pareto indifference asserts that the evaluation of social state a depends at most on the set $F(a)$ of identities alive and the well-beings $u_i(a)$ for $i \in F(a)$, not on the sequence that the function $f(a)$ puts these identities into the rooms of Hilbert's Grand Hotel.

Pareto indifference. For any $a, b \in A$, if $F(a) = F(b)$ and $u_i(a) = u_i(b)$ for all $i \in F(a)$, then $a \sim^* b$.

While Pareto indifference is uncontroversial, the next two axioms has the normative content that identities do not matter. As in the main text, Π denotes the set of all permutations of \mathbb{N} .

Extended indifference. For any $a, b \in A$, if $\mathbf{x}(a) = \mathbf{x}(b)$, then $a \sim^* b$.

Strong anonymity for future people. For any $a, b \in A$, if there exists $\pi \in \Pi$ such that $u_{f(a)(\pi(j))}(a) = u_{f(b)(j)}(b)$ for all $j \in \mathbb{N}$, then $a \sim^* b$.

Extended indifference corresponds to what Parfit (1984) calls the *No difference view* in the case with finitely many people. Note that Strong anonymity for future people implies Pareto indifference and Extended indifference. As we will show (in Lemma 5), if a preorder \succsim^* on A satisfies Pareto indifference and Extended indifference, then it satisfies Strong anonymity for future people.

The following is the main result of this appendix.

Proposition 5. *Assume Richness and Unobservability of future identities and suppose that a preorder \succsim^* on A satisfies Pareto indifference. Then the following statements are equivalent, where \succsim denotes the induced binary relation on \mathbf{X} defined in (3).*

- (i) *The preorder \succsim^* satisfies Extended indifference.*
- (ii) *The preorder \succsim^* satisfies Strong anonymity for future people.*
- (iii) *The binary relation \succsim is reflexive.*
- (iv) *The binary relation \succsim is a preorder.*
- (v) *The binary relation \succsim satisfies Strong anonymity.*

(vi) For any $a, b \in A$, if $a \succsim^* b$, then $\mathbf{x}(a) \succsim \mathbf{x}(b)$.

(vii) For any $a, b \in A$, $a \succsim^* b$ if and only if $\mathbf{x}(a) \succsim \mathbf{x}(b)$.

Among the seven statements that are shown to be equivalent to each other in the proposition, the equivalence between (iii), (v) and (vii) presents a novel welfarism result, which means that under the assumptions of Richness and Unobservability of future identities, we can induce the evaluation \succsim of infinite well-being streams from the evaluation \succsim^* of social states, and once we require \succsim^* to be a preorder that satisfies Pareto indifference and \succsim to be reflexive (and thus, to be a preorder since \succsim is always transitive), then the evaluation \succsim^* of social states is equivalently represented by the evaluation \succsim of their well-being streams and \succsim must be strongly anonymous. This result presents a justification for our analysis of a strongly anonymous preorder for infinite well-being streams in terms of reflexivity, a very weak rationality property, of the induced relation under Unobservability of future identities. The equivalence between (ii), (v) and (vii) is seen to be an infinite and variable population variant of the single-profile anonymous welfarism established by Blackorby, Bossert and Donaldson (2006).

To prove Proposition 5, the following lemmas are useful.

Lemma 5. *Assume Richness. Then a preorder \succsim^* on A satisfies Pareto indifference and Extended indifference if and only if \succsim^* satisfies Strong anonymity for future people.*

Proof. It is straightforward that Strong anonymity for future people implies Pareto indifference and Extended anonymity. Thus, we only show that the conjunction of Pareto indifference and Extended indifference implies Strong anonymity for future people. Let $a, b \in A$ and $\pi \in \Pi$. Suppose that $u_{f(a)(\pi(j))}(a) = u_{f(b)(j)}(b)$ for all $j \in \mathbb{N}$. Since A is rich, there exists $c \in A$ such that $f(c) = f(a) \circ \pi$, $u_i(c) = u_i(a)$ for all $i \in F(a) = F(c)$, and $u_{f(c)(j)}(c) = u_{f(b)(j)}(b)$ for all $j \in \mathbb{N}$. Then $a \sim^* c$ by Pareto indifference and $c \sim^* b$ by Extended indifference, so that $a \sim^* b$ by transitivity of \succsim^* . Hence, \succsim^* satisfies Strong anonymity for future people. \square

Lemma 6. *Assume Richness and Unobservability of future identities, and suppose that a preorder \succsim^* on A satisfies Pareto indifference. Then the induced binary relation \succsim on \mathbf{X} defined in (3) is reflexive if and only if \succsim satisfies Strong anonymity.*

Proof. Assume Richness and Unobservability of future identities and suppose that a binary relation \succsim^* on A satisfies Pareto indifference. Let \succsim be the induced binary relation \succsim on \mathbf{X} defined in (3).

We begin by showing that for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$,

$$\mathbf{x} \succsim \mathbf{y} \text{ if and only if, for all } \pi \in \Pi, \mathbf{x}_\pi \succsim \mathbf{y}. \quad (4)$$

Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$. Since Π contains the identity mapping, it is straightforward that if $\mathbf{x}_\pi \succsim \mathbf{y}$ for all $\pi \in \Pi$, then $\mathbf{x} \succsim \mathbf{y}$ follows. We now suppose that $\mathbf{x} \succsim \mathbf{y}$. Let $\pi \in \Pi$ and consider any $a, b \in A$

such that $\mathbf{x}(a) = \mathbf{x}_\pi$ and $\mathbf{x}(b) = \mathbf{y}$. We show that $a \succsim^* b$, which implies $\mathbf{x}_\pi \succsim \mathbf{y}$ by (3). Since A is rich, there exists $c \in A$ such that $f(c) = f(a) \circ \pi^{-1}$ and $u_i(c) = u_i(a)$ for all $i \in F(a) = F(c)$. Note that $\mathbf{x} = \mathbf{x}(c)$. Since $\mathbf{x} \succsim \mathbf{y}$, we obtain $c \succsim^* b$ by (3). Moreover, $a \sim^* c$ by Pareto indifference. Since \succsim^* is transitive, $a \succsim^* b$ follows.

We now complete the proof. Let $\mathbf{x} \in \mathbf{X}$. Since \succsim satisfies (4), it follows that $\mathbf{x} \sim \mathbf{x}$ if and only if $\mathbf{x}_\pi \sim \mathbf{x}$ for all $\pi \in \Pi$. This means that \succsim is reflexive if and only if \succsim satisfies Strong anonymity. \square

Proof of Proposition 5. Assume Richness and Unobservability of future identities and suppose that a preorder \succsim^* on A satisfies Pareto indifference. Let \succsim be the induced binary relation on \mathbf{X} defined in (3).

From Lemma 5, statements (i) and (ii) are equivalent. Since \succsim is transitive, it follows from Lemma 6 that statements (iii), (iv) and (v) are equivalent. Furthermore, statements (vi) and (vii) are equivalent since it follows from (3) that, for any $a, b \in A$, $\mathbf{x}(a) \succsim \mathbf{x}(b)$ implies $a \succsim^* b$.

We show that (i) implies (vi). Assume that \succsim^* satisfies Extended indifference. Let $a, b \in A$ and suppose that $a \succsim^* b$. Consider any $c, d \in A$ such that $\mathbf{x}(c) = \mathbf{x}(a)$ and $\mathbf{x}(d) = \mathbf{x}(b)$. By Extended indifference, we obtain $a \sim^* c$ and $b \sim^* d$. Since \succsim^* is transitive, we obtain $c \succsim^* d$. By (3), $\mathbf{x}(a) \succsim \mathbf{x}(b)$ follows.

Next, we show that (vi) implies (iii). Let $\mathbf{x} \in \mathbf{X}$. Since A is rich, there exists $a \in A$ such that $\mathbf{x} = \mathbf{x}(a)$. Since \succsim^* is reflexive, we obtain $a \sim^* a$. Since \succsim satisfies the property in statement (vi), $a \succsim^* a$ implies $\mathbf{x} \succsim \mathbf{x}$.

Finally, to complete the proof, we show that (iii) implies (i). Suppose that \succsim is reflexive. Let $a, b \in A$ such that $\mathbf{x}(a) = \mathbf{x}(b)$. Since \succsim is reflexive, we obtain $\mathbf{x}(a) \sim \mathbf{x}(b)$. By (3), $a \sim^* b$ follows. \square

To illustrate that there are binary relations on \mathbf{X} that satisfy neither reflexivity of \succsim nor Strong anonymity of \succsim when \succsim is defined by (3), consider the following example.

Example 1. Consider the binary relation \succsim_1^* on A defined as follows: For any $a, b \in A$,

$$a \succsim_1^* b \text{ if and only if (i) } F(a) = F(b) \text{ and } u_i(a) \geq u_i(b) \text{ for all } i \in F(a) \\ \text{or, (ii) for all } \pi \in \Pi, u_{f(a)(\pi(j))}(a) \geq u_{f(b)(j)}(b) \text{ for all } j \in \mathbb{N}.$$

One can show that \succsim_1^* is a preorder which satisfies Pareto indifference and the following axiom.

Egalitarian indifference. For any $a, b \in A$, if there exists $u^* \in \mathbb{R}$ such that $u_{f(a)(j)}(a) = u_{f(b)(j)}(b) = u^*$ for all $j \in \mathbb{N}$, then $a \sim^* b$.

Proposition 6. Assume Richness and Unobservability of future identities, and suppose that the preorder on A is given by \succsim_1^* . Let \succsim_1 denote the induced binary relation on \mathbf{X} defined in (3). Then \succsim_1 is given by

$$\mathbf{x} \succsim_1 \mathbf{y} \text{ if and only if, for all } \pi \in \Pi, x_{\pi(j)} \geq y_j \text{ for all } j \in \mathbb{N}, \quad (5)$$

it is transitive and satisfies (4), but it is neither reflexive nor complete, and it does not satisfy Strong anonymity. Furthermore, any preorder \succsim^* on A inducing \succsim_1 satisfies Egalitarian indifference and does not satisfy Extended indifference.

For the proof of Proposition 6, the following lemma is useful.

Lemma 7. *The induced relation \succsim_1 is characterized by (5). Furthermore, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$,*

- (i) $\mathbf{x} \succsim_1 \mathbf{y}$ if and only if $\inf_{j \in \mathbb{N}} x_j \geq \sup_{j \in \mathbb{N}} y_j$.
- (ii) $\mathbf{x} \sim_1 \mathbf{y}$ if and only if $\sup_{j \in \mathbb{N}} x_j = \inf_{j \in \mathbb{N}} x_j = \sup_{j \in \mathbb{N}} y_j = \inf_{j \in \mathbb{N}} y_j$.
- (iii) $\mathbf{x} \succ_1 \mathbf{y}$ if and only if $\inf_{j \in \mathbb{N}} x_j \geq \sup_{j \in \mathbb{N}} y_j$ and $\sup_{j \in \mathbb{N}} x_j > \inf_{j \in \mathbb{N}} y_j$.

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$. We begin by showing that \succsim_1 is characterized by (5). It follows from the definition of \succsim_1^* that $\mathbf{x} \succsim_1 \mathbf{y}$ if, for all $\pi \in \Pi$, $x_{\pi(j)} \geq y_j$ for all $j \in \mathbb{N}$. By way of contraposition, suppose that there exists $\pi \in \Pi$ such that $x_{\pi(j)} < y_j$ for some $j \in \mathbb{N}$. Since A is rich, there exist $a, b \in A$ with $F(a) \neq F(b)$ such that $\mathbf{x}(a) = \mathbf{x}$ and $\mathbf{x}(b) = \mathbf{y}$. Hence, there exists $\pi \in \Pi$ such that $u_{f(a)(\pi(j))}(a) < u_{f(b)(j)}(b)$ for some $j \in \mathbb{N}$, implying that $a \succsim_1^* b$ does not hold. By (3), $\mathbf{x} \succsim_1 \mathbf{y}$ does not hold.

Part (i). If $\inf_{j \in \mathbb{N}} x_j \geq \sup_{j \in \mathbb{N}} y_j$, then, for all $\pi \in \Pi$, $x_{\pi(j)} \geq y_j$ for all $j \in \mathbb{N}$, implying that $\mathbf{x} \succsim_1 \mathbf{y}$. By way of contraposition, suppose that $\inf_{j \in \mathbb{N}} x_j < \sup_{j \in \mathbb{N}} y_j$. Then there exist $\pi \in \Pi$ such that $x_{\pi(j)} < y_j$ for some $j \in \mathbb{N}$, implying that $\mathbf{x} \succsim_1 \mathbf{y}$ does not hold.

Part (ii). If $\sup_{j \in \mathbb{N}} x_j = \inf_{j \in \mathbb{N}} x_j = \sup_{j \in \mathbb{N}} y_j = \inf_{j \in \mathbb{N}} y_j$, then, by part (i), $\mathbf{x} \succsim_1 \mathbf{y}$ and $\mathbf{y} \succsim_1 \mathbf{x}$ follow, that is, $\mathbf{x} \sim_1 \mathbf{y}$. Next, suppose $\mathbf{x} \sim_1 \mathbf{y}$. Then, by part (i), we obtain $\sup_{j \in \mathbb{N}} x_j \geq \inf_{j \in \mathbb{N}} x_j \geq \sup_{j \in \mathbb{N}} y_j \geq \inf_{j \in \mathbb{N}} y_j \geq \sup_{j \in \mathbb{N}} x_j$, implying that all inequalities are in fact equalities.

Part (iii). If $\inf_{j \in \mathbb{N}} x_j \geq \sup_{j \in \mathbb{N}} y_j$ and $\inf_{j \in \mathbb{N}} y_j < \sup_{j \in \mathbb{N}} x_j$, then, by part (i), $\mathbf{x} \succsim_1 \mathbf{y}$ follows, while $\mathbf{y} \succsim_1 \mathbf{x}$ does not hold. Thus, $\mathbf{x} \succ_1 \mathbf{y}$. If $\mathbf{x} \succ_1 \mathbf{y}$, then, by part (i), $\inf_{j \in \mathbb{N}} x_j \geq \sup_{j \in \mathbb{N}} y_j$ and $\inf_{j \in \mathbb{N}} y_j < \sup_{j \in \mathbb{N}} x_j$. \square

Proof of Proposition 6. As was shown in Lemma 7, \succsim_1 is characterized by (5). To show that \succsim_1 is transitive, let $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbf{X}$ and suppose that $\mathbf{x} \succsim_1 \mathbf{y}$ and $\mathbf{y} \succsim_1 \mathbf{z}$. By part (i) of Lemma 7, $\inf_{j \in \mathbb{N}} x_j \geq \sup_{j \in \mathbb{N}} y_j$ and $\inf_{j \in \mathbb{N}} y_j \geq \sup_{j \in \mathbb{N}} z_j$ follow. Since $\sup_{j \in \mathbb{N}} y_j \geq \inf_{j \in \mathbb{N}} y_j$, we obtain $\inf_{j \in \mathbb{N}} x_j \geq \sup_{j \in \mathbb{N}} z_j$. By part (i) of Lemma 7, $\mathbf{x} \succsim_1 \mathbf{z}$ follows. Thus, \succsim_1 is transitive. Furthermore, it follows from part (i) of Lemma 7 that \succsim_1 satisfies (4) since, for all $\pi \in \Pi$, $\inf_{j \in \mathbb{N}} x_j = \inf_{j \in \mathbb{N}} x_{\pi(j)}$.

We next show that \succsim_1 is neither reflexive nor complete. From part (ii) of Lemma 7, it follows that $\mathbf{x} \sim_1 \mathbf{x}$ if and only if \mathbf{x} is egalitarian ($x_i = x_j$ for all $i, j \in \mathbb{N}$). Hence, $\mathbf{x} \sim_1 \mathbf{x}$ does not hold if there exist $i, j \in \mathbb{N}$ such that $x_i \neq x_j$. Thus, \succsim_1 is not reflexive. Furthermore, \succsim_1 is not complete since it follows from part (i) of Lemma 7 that \mathbf{x} and \mathbf{y} are not comparable by \succsim_1 if $\inf_{j \in \mathbb{N}} x_j < \sup_{j \in \mathbb{N}} y_j$ and $\sup_{j \in \mathbb{N}} x_j > \inf_{j \in \mathbb{N}} y_j$.

It is straightforward that \succsim_1 does not satisfy Strong anonymity since \succsim_1 is not reflexive, so that $\mathbf{x}_\pi \sim_1 \mathbf{x}$ need not follow even when π is the identity permutation. Finally, from part (ii) of Lemma 7 and (3), it follows that any preorder \succsim^* inducing \succsim_1 satisfies Egalitarian indifference but does not satisfy Extended indifference. \square

Remark 2. Condition (4), which is implied by Pareto indifference under Richness and Unobservability of future identities, might be considered a principle of impartiality in the sense that the potential future identities are treated equally. Indeed, the binary relation \succsim_1 of Example 1 illustrates this. However, it is not by itself a fairness principle as adding well-being to one identity need not be equally good as adding the same well-being to another identity, even if one starts with an egalitarian stream. The binary relation \succsim_1 simply requires that one well-being stream is preferred to another if it is known to be a Pareto improvement for future people conditional on their identities being the same, even though one cannot observe how well-beings are allocated to identities within this set of identities. In terms of indifference of \succsim^* , \succsim_1 respects Pareto indifference and Egalitarian indifference under Richness and Unobservability of future identities.

Also, one does not normally think that reflexivity is a fairness principle. However, in our setting reflexivity of \succsim means that two streams are equally good if it is possible that they give the same well-being to the same future identities, even though the identities to which well-beings accrue cannot actually be observed. In terms of indifference of \succsim^* , a reflexive binary relation \succsim respects also Extended indifference under Richness and Unobservability of future identities. This induces equivalence classes that are much wider than those induced by Pareto indifference alone under these assumptions. This *is* a fairness principle since it asserts that it does not matter which identities are the bearers of well-being. Indeed, the combination of condition (4) and reflexivity of \succsim implies Strong anonymity (both of \succsim and, in terms of \succsim^* , for future people), which is a principle of impartiality and a fairness principle.

Appendix B: Weakening continuity.

In the appendix we show that our main results, i.e., Propositions 3 and 4, can be strengthened by replacing Continuity by weaker continuity axioms. To this end, we prove stronger variants of Lemma 3 and Proposition 2 with these weaker continuity axioms.

Let $d_1: \mathbf{X} \times \mathbf{X} \rightarrow \mathbb{R}_+$ be the distance function given by, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$,

$$d_1(\mathbf{x}, \mathbf{y}) = \min \left\{ 1, \sum_{i=1}^{\infty} |x_i - y_i| \right\}.$$

Using the distance function d_1 , we define weak continuity as follows; see Svensson (1980).

Weak continuity. For any $\mathbf{x} \in \mathbf{X}$, the sets $\{\mathbf{y} \in \mathbf{X} : \mathbf{y} \succsim \mathbf{x}\}$ and $\{\mathbf{y} \in \mathbf{X} : \mathbf{x} \succsim \mathbf{y}\}$ are closed in (\mathbf{X}, d_1) .

Clearly, Weak continuity is implied by Continuity.

The following lemma shows that Lemma 3 can be strengthened by using Weak continuity instead of Continuity.

Lemma 8. *Consider $\mathbf{x} \in \mathbf{X}$ and z be a cluster point of \mathbf{x} . If \succsim is a reflexive and transitive binary relation satisfying Strong anonymity and Weak continuity, then $\mathbf{x} \sim (z, \mathbf{x}) \sim ((z)_{\text{con}}, \mathbf{x})$.*

Proof. Let $\mathbf{x} \in \mathbf{X}$ and let z be a cluster point for \mathbf{x} . Assume that \succsim is a reflexive and transitive binary relation satisfying Strong anonymity and Weak continuity.

We show that $\mathbf{x} \sim (z, \mathbf{x})$ and $\mathbf{x} \sim ((z)_{\text{con}}, \mathbf{x})$, which imply $(z, \mathbf{x}) \sim ((z)_{\text{con}}, \mathbf{x})$ by the transitivity of \succsim .

Since z is a cluster point of \mathbf{x} , there exists a subsequence of \mathbf{x} that converges to z . Furthermore, it is well-known that any sequence of real numbers has a monotone subsequence and that every subsequence of a convergent sequence has the same limit. Therefore, there exists a monotone subsequence of \mathbf{x} that converges to z , that is, there exists an increasing function $f: \mathbb{N} \rightarrow \mathbb{N}$ such that $(x_{f(k)})_{k \in \mathbb{N}} \in \mathbf{X}$ is monotone and converges to z . Without loss of generality, we assume that $(x_{f(k)})_{k \in \mathbb{N}}$ is non-decreasing.

Let $\tilde{\mathbf{x}} = (x_{f(k)})_{k \in \mathbb{N}}$ and, for any $n \in \mathbb{N}$, define $\tilde{\mathbf{x}}(n)$ by

$$\tilde{\mathbf{x}}(n) = (\tilde{x}_1, \dots, \tilde{x}_{n-1}, z, \tilde{x}_n, \tilde{x}_{n+1}, \dots).$$

Then, we obtain that

$$\begin{aligned} \lim_{n \rightarrow +\infty} \sum_{i=1}^{\infty} |\tilde{x}_i - \tilde{x}(n)_i| &= \lim_{n \rightarrow +\infty} \left(z - \tilde{x}_n + \lim_{N \rightarrow +\infty} \sum_{i=n+1}^N |\tilde{x}_i - \tilde{x}(n)_i| \right) \\ &= \lim_{n \rightarrow +\infty} \left(z - \tilde{x}_n + \lim_{N \rightarrow +\infty} (\tilde{x}_N - \tilde{x}_n) \right) \\ &= \lim_{n \rightarrow +\infty} 2(z - \tilde{x}_n) \\ &= 0. \end{aligned}$$

Thus, for any $\varepsilon \in (0, 1)$, there exists an increasing sequence $\{n_t\}_{t \in \mathbb{N}}$ in \mathbb{N} satisfying

$$n_t + t < n_{t+1} \quad \text{for each } t \in \mathbb{N}$$

and we can define the sequence $\{\tilde{\mathbf{x}}^n\}_{n \in \mathbb{N} \cup \{0\}}$ in \mathbf{X} that satisfies

$$\sum_{i=1}^{\infty} |\tilde{x}_i^n - \tilde{x}_i^{n-1}| < \frac{\varepsilon}{2^n} \quad \text{for each } n \in \mathbb{N}$$

as follows:

$$\begin{aligned}\tilde{\mathbf{x}}^0 &= (\tilde{x}_1, \dots, \tilde{x}_{n_1-1}, \tilde{x}_{n_1}, \tilde{x}_{n_1+1}, \dots, \tilde{x}_{n_2-1}, \tilde{x}_{n_2}, \tilde{x}_{n_2+1}, \dots), \\ \tilde{\mathbf{x}}^1 &= (\tilde{x}_1, \dots, \tilde{x}_{n_1-1}, z, \tilde{x}_{n_1}, \dots, \tilde{x}_{n_2-2}, \tilde{x}_{n_2-1}, \tilde{x}_{n_2}, \dots), \\ \tilde{\mathbf{x}}^2 &= (\tilde{x}_1, \dots, \tilde{x}_{n_1-1}, z, \tilde{x}_{n_1}, \dots, \tilde{x}_{n_2-2}, z, \tilde{x}_{n_2-1}, \dots),\end{aligned}$$

and so forth. Formally, $\tilde{\mathbf{x}}^0 = \tilde{\mathbf{x}}$ and for each $n \in \mathbb{N}$, $\tilde{\mathbf{x}}^n$ is defined as follows. For each $i \in \{n_t : t \in \{1, \dots, n\}\}$,

$$\tilde{x}_i^n = z$$

and the subsequence of $\tilde{\mathbf{x}}^n$ composed of all the other components coincides with $\tilde{\mathbf{x}}$. Analogously, we define $\tilde{\mathbf{x}}^\infty$ by, for each $i \in \{n_t : t \in \mathbb{N}\}$,

$$\tilde{x}_i^\infty = z$$

and the subsequence of $\tilde{\mathbf{x}}^\infty$ composed of all the other components coincides with $\tilde{\mathbf{x}}$. By the definitions of $\{\tilde{\mathbf{x}}^n\}_{n \in \mathbb{N} \cup \{0\}}$ and $\tilde{\mathbf{x}}^\infty$, we obtain that

$$\sum_{i=1}^{\infty} |\tilde{x}_i^\infty - \tilde{x}_i^0| \leq \lim_{N \rightarrow +\infty} \sum_{n=1}^N \sum_{i=1}^{\infty} |\tilde{x}_i^n - \tilde{x}_i^{n-1}| < \varepsilon. \quad (6)$$

We now define the sequence $\{\mathbf{x}^n\}_{n \in \mathbb{N} \cup \{0\}}$ using the sequence $\{\tilde{\mathbf{x}}^n\}_{n \in \mathbb{N} \cup \{0\}}$ as follows. For each $n \in \mathbb{N} \cup \{0\}$,

$$x_i^n = x_i \text{ for all } i \in \mathbb{N} \setminus \{f(k) : k \in \mathbb{N}\}$$

and

$$(x_{f(k)}^n)_{k \in \mathbb{N}} = \tilde{\mathbf{x}}^n.$$

Analogously, we define \mathbf{x}^∞ by

$$x_i^\infty = x_i \text{ for all } i \in \mathbb{N} \setminus \{f(k) : k \in \mathbb{N}\}$$

and

$$(x_{f(k)}^\infty)_{k \in \mathbb{N}} = \tilde{\mathbf{x}}^\infty.$$

Note that $\mathbf{x}^0 = \mathbf{x}$. From (6) and the definitions of $\{\mathbf{x}^n\}_{n \in \mathbb{N} \cup \{0\}}$ and \mathbf{x}^∞ , it follows that

$$\sum_{i=1}^{\infty} |x_i^1 - x_i^0| = \sum_{i=1}^{\infty} |\tilde{x}_i^1 - \tilde{x}_i^0| \leq \sum_{i=1}^{\infty} |\tilde{x}_i^\infty - \tilde{x}_i^0| = \sum_{i=1}^{\infty} |x_i^\infty - x_i^0| < \varepsilon. \quad (7)$$

Note that there exist permutations $\pi, \rho \in \Pi$ such that $\mathbf{x}^1 = (z, \mathbf{x})_\pi$ and $\mathbf{x}^\infty = ((z)_{\text{con}}, \mathbf{x})_\rho$. Thus,

it follows from (7) that for any $\varepsilon \in (0, 1)$, there exist $\pi, \rho \in \Pi$ such that

$$d_1((z, \mathbf{x})_\pi, \mathbf{x}) \leq d_1(((z)_{\text{con}}, \mathbf{x})_\rho, \mathbf{x}) < \varepsilon. \quad (8)$$

Let $m \in \mathbb{N}$. By (8), there exist $\pi^m, \rho^m \in \Pi$ such that

$$d_1((z, \mathbf{x})_{\pi^m}, \mathbf{x}) \leq d_1(((z)_{\text{con}}, \mathbf{x})_{\rho^m}, \mathbf{x}) < \frac{1}{m}.$$

Consider the sequences $((z, \mathbf{x})_{\pi^m})_{m \in \mathbb{N}}$ and $(((z)_{\text{con}}, \mathbf{x})_{\rho^m})_{m \in \mathbb{N}}$ in \mathbf{X} . By Strong anonymity, we obtain that $(z, \mathbf{x})_{\pi^m} \sim (z, \mathbf{x})$ and $((z)_{\text{con}}, \mathbf{x})_{\rho^m} \sim ((z)_{\text{con}}, \mathbf{x})$ for each $m \in \mathbb{N}$. Since \succsim satisfies Weak continuity and

$$\lim_{m \rightarrow +\infty} d_1((z, \mathbf{x})_{\pi^m}, \mathbf{x}) = \lim_{m \rightarrow +\infty} d_1(((z)_{\text{con}}, \mathbf{x})_{\rho^m}, \mathbf{x}) = 0,$$

we obtain $(z, \mathbf{x}) \sim \mathbf{x}$ and $((z)_{\text{con}}, \mathbf{x}) \sim \mathbf{x}$. □

Note that a variant of Lemma 4 that uses Weak continuity instead of Continuity can be proved by using Lemma 8 instead of Lemma 3. Thus, in what follows, we will use Lemma 4 to establish a variant of Proposition 2.

To state the variant of Proposition 2, we introduce another continuity axiom: *Restricted continuity* requires an evaluation be continuous with respect to streams that have a constant subsequence. Furthermore, it only asserts that indifference relations must be robust with respect to small changes of constant well-being levels in a very weak form by requiring social indifference must hold for all constant well-being levels in a neighborhood of a given constant well-being level.

Restricted continuity For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\ell \in \mathbb{R}$, and $\underline{\ell} < \bar{\ell}$ such that $\underline{\ell} \leq \ell \leq \bar{\ell}$, if $((\ell')_{\text{con}}, \mathbf{x}) \sim ((\ell')_{\text{con}}, \mathbf{y})$ for all $\ell' \in (\underline{\ell}, \bar{\ell})$, then $((\ell)_{\text{con}}, \mathbf{x}) \sim ((\ell)_{\text{con}}, \mathbf{y})$.

The following lemma shows that Continuity implies Restricted continuity for any reflexive and transitive binary relation satisfying Monotonicity.

Lemma 9. *Let \succsim be a reflexive and transitive binary relation satisfying Monotonicity. If \succsim satisfies Continuity, then \succsim satisfies also Restricted continuity.*

Proof. Let \succsim be a reflexive and transitive binary relation satisfying Monotonicity and Continuity. Consider any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\ell \in \mathbb{R}$, and $\underline{\ell} < \bar{\ell}$ such that $\underline{\ell} \leq \ell \leq \bar{\ell}$. Suppose that $((\ell')_{\text{con}}, \mathbf{x}) \sim ((\ell')_{\text{con}}, \mathbf{y})$ for all $\ell' \in (\underline{\ell}, \bar{\ell})$. Without loss of generality, we assume that $\bar{\ell} > \ell$. Then there exists a sequence $\{\ell^k\}_{k \in \mathbb{N}}$ in $(\underline{\ell}, \bar{\ell})$ that converges to ℓ . Since $[\ell, \bar{\ell}] \subseteq (\underline{\ell}, \bar{\ell})$, it follows that $((\ell^k)_{\text{con}}, \mathbf{x}) \sim ((\ell^k)_{\text{con}}, \mathbf{y})$ for all $k \in \mathbb{N}$. Furthermore, by Monotonicity, $((\ell^k)_{\text{con}}, \mathbf{y}) \succsim ((\ell)_{\text{con}}, \mathbf{y})$ for all $k \in \mathbb{N}$. Since \succsim is transitive, we obtain $((\ell^k)_{\text{con}}, \mathbf{x}) \succsim ((\ell)_{\text{con}}, \mathbf{y})$ for all $k \in \mathbb{N}$. Since the sequence $\{((\ell^k)_{\text{con}}, \mathbf{x})\}_{k \in \mathbb{N}}$ converges to $((\ell)_{\text{con}}, \mathbf{x})$ in the sup metric, we obtain by Continuity

that $((\ell)_{\text{con}}, \mathbf{x}) \succsim ((\ell)_{\text{con}}, \mathbf{y})$. By the same argument, we obtain $((\ell)_{\text{con}}, \mathbf{y}) \succsim ((\ell)_{\text{con}}, \mathbf{x})$. Thus, $((\ell)_{\text{con}}, \mathbf{x}) \sim ((\ell)_{\text{con}}, \mathbf{y})$, thereby establishing that \succsim satisfies Restricted continuity. \square

The following proposition is a stronger variant of Proposition 2 using Weak continuity and Restricted Continuity instead of Continuity.

Proposition 7. *Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$. The two following statements are equivalent:*

- (1) *For all reflexive and transitive binary relations \succsim satisfying Strong anonymity, Monotonicity, Weak continuity, Restricted continuity and Very weak Pigou-Dalton transfer, we have that $\mathbf{x} \sim \mathbf{y}$.*
- (2) *$y_i \geq \liminf_{j \in \mathbb{N}} x_j$.*

Proof. Since Continuity implies Weak continuity and Restricted continuity, the proof that (1) implies (2) is analogous to that of Proposition 2.

We prove that (2) implies (1). Assume that \succsim is a reflexive and transitive binary relation on \mathbf{X} that satisfies Strong anonymity, Monotonicity, Weak continuity, Restricted continuity and Very weak Pigou-Dalton transfer. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that there exists $i \in \mathbb{N}$ such that $x_i > y_i \geq \liminf_{j \in \mathbb{N}} x_j$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$. There are two subcases:

Subcase (i): $\limsup_{j \in \mathbb{N}} y_j > \liminf_{i \in \mathbb{N}} x_i$. The result follows from Lemma 4.

Subcase (ii): $\limsup_{j \in \mathbb{N}} y_j = \liminf_{i \in \mathbb{N}} x_i = \ell$. Since ℓ is a cluster point of \mathbf{x} and \mathbf{y} , it follows from Lemma 8 that $((\ell)_{\text{con}}, \mathbf{x}) \sim \mathbf{x}$ and $((\ell)_{\text{con}}, \mathbf{y}) \sim \mathbf{y}$. Let $\underline{\ell}, \bar{\ell} \in \mathbb{R}$ be such that $\underline{\ell} = \ell < \bar{\ell}$. By Lemma 4, we obtain $((\ell')_{\text{con}}, \mathbf{x}) \sim ((\ell')_{\text{con}}, \mathbf{y})$ for all $\ell' \in (\underline{\ell}, \bar{\ell})$. By Restricted continuity, $((\ell)_{\text{con}}, \mathbf{x}) \sim ((\ell)_{\text{con}}, \mathbf{y})$ follows. By the transitivity of \succsim , we obtain $\mathbf{x} \sim \mathbf{y}$. \square

Using Lemma 8 and Proposition 7, we obtain the following stronger variants of Propositions 3 and 4 with the weaker continuity axioms. Since their proofs are analogous to those of Propositions 3 and 4, we state them without proof.

Proposition 8. *Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} \geq \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$. The two following statements are equivalent:*

- (1) *For all reflexive and transitive binary relations \succsim satisfying Strong anonymity, Monotonicity, Weak continuity, Restricted continuity, Very weak Pigou-Dalton transfer and Critical-level consistency, we have that $\mathbf{x} \sim \mathbf{y}$.*
- (2) *$\ell = \liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ and $y_i \geq \ell$ for all $i \in \mathbb{N}$ such that $x_i > y_i$.*

Proposition 9. *Assume that \succsim is a reflexive and transitive binary relation satisfying Strong anonymity, Monotonicity, Weak continuity, Restricted continuity, Very weak Pigou-Dalton transfer and Critical-level consistency. Then \succsim satisfies Non-positive value of additional lives.*

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