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The hidden side of Jan Tinbergen's approach to economic policy (1934-1944)

Michaël Assous, Université Lumière Lyon 2 - Triangle
Vincent Carret, Université Lumière Lyon 2 - Triangle[1]

Abstract:

This article provides a comprehensive view of Tinbergen's macrodynamic models during the 1930s and early 1940s, to show how the economist's concerns evolved from problems of instability to the idea of reaching higher positions of equilibria.

Starting from the ideas developed around the first meetings of the Econometric Society, we show that Tinbergen built his own macrodynamic model with the aim to consider several problems of economic policy, in particular the effects of public expenditures and changes in money wages. One of the possibilities that Tinbergen underlined throughout his models was the threat of complete or partial collapse stemming from the presence of multiple equilibria.

From the mid-1930s, Tinbergen gained confidence in the power of economic policies to stabilize the economy, and his attention shifted to the final position of equilibrium, and the policies that could improve it. His most well-known models developed for the League of Nations addressed that issue and showed how the final equilibrium may be shifted by "permanent" policies. We argue that Tinbergen also considered the case that the economy could be lifted by temporary policies in the presence of multiple equilibria.

Based on papers published by Tinbergen in Dutch, French and German that for the most part have never been translated, this article offers a new perspective to the development of early macrodynamic modeling. From this literature the originality, breadth and pioneering ideas of Tinbergen clearly come out and explain many of his sometimes paradoxical policy positions.

Keywords: economic policy, Tinbergen, equilibria, macrodynamics, stability
JEL codes: B22, B23, B31, E60, C62

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[1] Contact: michael.assous@univ-lyon2.fr, vincent.carret@univ-lyon2.fr. The authors gratefully acknowledge the help of librarians in France and Germany who dusted off Tinbergen's articles and scanned them for us. We are also grateful to Erwin Dekker and Harald Hagemann who provided us with useful information about Tinbergen's early works.

Introduction

The importance of economic policy in Tinbergen's works has long been established, though, strikingly, some important aspects of his early approach went unnoticed.[2] The fact that Tinbergen introduced it in articles published in German, Dutch and French on the eve of a global conflict which profoundly transformed academic research has certainly contributed to it. The specific nature of this early approach to economic policy may however have played an even more important role.[3]

Tinbergen examined for the first time the importance of public interventions on the basis of a macrodynamic model in 1934. In that model which showed some similarities with the notion of the corridor concept introduced by Fisher in the 1930s[4], Tinbergen could demonstrate the possibility that the economy may oscillate in the neighbourhood of the equilibrium but cumulatively deviate from it beyond a critical value of some parameters. In this case - in which the capability of the economy for self adjustment definitely vanished and the risk that the economy may collapse dramatically increased - government interventions turned out to be a bulwark against instability with the implication that the government, by changing the structure of the economy, may strengthen its resilience. In addition it became apparent to Tinbergen that some policies - for example a change in money wages that he explored in detail in a 1935 article - may prove to be destabilizing and represent a serious danger.[5]

These two lines of thought ran through Tinbergen's most cited papers, as evidenced by his 1935 *Econometrica* survey or his 1939 study for the League of Nations as well as by other papers. But clearly it was not sufficient to bring about reactions from his contemporaries on the instability problems that he underlined.[6]

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[2] See Dekker (2021) for a recent account of Tinbergen's theory of economic policy making. Tinbergen ultimately aimed at stabilizing the economy by changing the structure of the economy much more than by correcting its movements through various countercyclical interventions. See also Arrow (1958) for an account of Tinbergen's 1950 policy analysis.

[3] It cannot be excluded that due to his strong interest in models with specific stability properties and multiple equilibria, Tinbergen may have met a strong opposition from Frisch who, at that time, advocated his own impulse-propagation approach and held total control over *Econometrica* where most of Tinbergen's english papers were written. See Dekker (2020) on the nature of the lasting collaboration between Tinbergen and Frisch and Carret (2020) on the dynamic properties of Frisch's model.

[4] See Dimand (1994) on the corridor. Beforehand, Tinbergen had written in 1932 a Pre-advice in which he discussed the role of policies but on the basis of a diagrammatic analysis (see Assous and Carret, 2020, section 1). It is only from 1934 and on the basis of his own macrodynamic model that Tinbergen developed his approach of economic policy. Among the pioneers of the econometric approach, Tinbergen was certainly the one who put the most emphasis on policy issues. The specificity of Tinbergen's approach is discussed in our forthcoming book on stability issues in the early history of macroeconomics (Assous and Carret, 2021).

[5] The role of a change in the money wage was debated in Germany (see Hagemann, 2019). Unlike his German contemporaries interested in the long run implications of such changes on technological unemployment and the problems of shortage of capital, Tinbergen was certainly the first to put the emphasis on the dynamic implication of a change on money wage.

[6] Although Tinbergen refers to these articles in his papers published in English like his 1935 survey or his 1938 paper on the control of the business cycle, most of them were very rarely quoted (see T. Koopmans 1952). In his

That dimension of Tinbergen's work was also hidden when he started to consider the possibility that economic policy may not only help control the movement of the economy but also its final position.[7] Along that line, Tinbergen associated public interventions to shocks likely to shift the equilibrium by acting on additional constants and initial conditions. By however postulating that the economy was constantly operating in the neighborhood of the new equilibrium, cases of instability he had examined in earlier analysis were ruled out from the start. Doing so in his works which have had the most important impact certainly contributed to overshadow his approach to multiple equilibria and large shocks.

Far from leading him to abandon his earlier insights, this reflection on the possibility to shift the equilibrium led Tinbergen to extend his nonlinear approach by considering the possibility that the economy may reach higher equilibria. Thus, Tinbergen could revisit the role of public policy as a way to lift the economy from a position of low equilibrium to a position of high equilibrium. In addition, he could identify the conditions under which a once and for all intervention ("temporary" shock) may permanently move the economy. It is worth noting that Tinbergen produced that result in 1937 and so three years before Kaldor's article on the trade cycle and multiple equilibria (Kaldor, 1940). Clearly, Tinbergen came to this idea independently. During that period, he constantly adjusted his models, disaggregating his 1934 model and integrating it to larger models which have made his world-wide reputation and whose numerous variants clarify the multiple facets of his view of economic policy.

The paper is organized in three parts. The first section presents Tinbergen's very early view on policy, based on his 1934 model and a paper published in French in 1935, in which he first raised the question of wage flexibility in relation to the stabilization of the economy. The second section shows how Tinbergen progressively came to pay attention to the possibility of shifting the equilibrium as he modified his model and transformed the way economic policy acted on it. The third section, finally, shows how Tinbergen built a new model of multiple equilibria which opened new possibilities for economic policies.

I. Staying close to equilibrium

With reference to the debates on public spending, reflation, competitive devaluation and the policy of the Roosevelt administration, Tinbergen underlined in the mid 1930s the "extraordinary importance"[8] of macroeconomic policies in two articles. The first was published in German and was devoted to the regulation of aggregate activity and employment through the manipulation of various "monetary factors". The second was published in French and focused on the impact of an

1936 paper on stability, Frisch did not rule out the case of instability and multiple equilibria but did not mention Tinbergen's approach.

[7] See the interview of Tinbergen by Morgan and Magnus (1987) which makes clear that in his early period, he put much more emphasis on the mechanism than on shocks and the final position of equilibrium.

[8] "In the contemporary economic situation the problems of economic policy - and especially those which relate to the "conjuncture" - are of extraordinary importance. To be sure, it suffices to recall the discussions on the politics of reflation, on that of devaluation, on that followed by Roosevelt" (Tinbergen 1935c: 17).

overall change in money wages. Both were based on a model with specific stability properties that Tinbergen used to show the existence of a critical threshold for which shocks, when they are "small", may cause automatic movements back to equilibrium but infinite downwards movements when they are "large".

That aspect of his analysis had two implications. The first was that policies were not only designed to cope with small shocks responsible for fluctuations in the vicinity of equilibrium but also - and maybe most importantly in the context of the 1930s - to tackle shocks strong enough to cause a complete collapse. As such, this made manifest that policies were ultimately intended to build strong economic foundations and enhance the resilience of the economy. The second was that some policies may be a source of adverse destabilizing effects to the point of causing a complete collapse as Tinbergen shows it in the case of a change in money wages.

The role of "Monetary factors"

Tinbergen thought that any economic policy, as most economic processes, was "essentially dynamic in nature"[9] meaning that its aim was less to help the economy reach a new hypothetical position than to affect its current trajectory.[10] Naturally, this required identifying the main dynamic relationships responsible for the economic movement and thus to put some distance with static models in which equilibrium is assumed to be instantaneously achieved. Only then the effect of various policies could be determined.

The macrodynamic model that Tinbergen built placed prices at its center.[11] From its evolution could hence be deduced the evolution of all variables. In an economy assumed to be made of two sectors, investment and consumer goods, this boiled down to determine the dynamics of two prices and eventually of only one price as soon as it is assumed that the activity of the investment goods sector moves in parallel with the activity of the consumer goods sector.

Tinbergen put a first emphasis on a case[12] including a production lag which entailed two consequences. The first was that the production coming on the market at any time resulted from decisions taken on the basis of prices set at time $t - \theta$ with θ being the production lag.

[9] "Cyclical policy issues are essentially of a dynamic nature" (Tinbergen 1935c: 17).

[10] "In problems of cyclical policy - and again in many other cases - the question is to find the course of certain economic quantities in a certain interval of time. This is a dynamic problem." (Tinbergen 1935c: 17).

[11] "The price, in our diagram, represents all the factors determining the economic situation" (Tinbergen 1935c: 21). Tinbergen also emphasized at several occasions the importance of other mechanisms such as those explored by Frisch and Kalecki the year before. At the time when they both presented it, Tinbergen had not yet integrated the mathematical apparatus he had already presented since 1931 to a macrodynamic model.

[12] Tinbergen studied various cases referred to in his 1935 *Econometrica* survey as the short and long lag schemes. Besides, he referred to cases integrating a speculative effect and the price of raw materials. These different variants formed the basis of the model estimated for the League of Nations.

Accordingly, the supply of consumer goods is equal to the sum of a constant A and a part proportional to the price at time $t - \theta$.[13]

The second consequence was that spendings on consumer goods at time t depends on incomes resulting from output produced between $t - \theta$ and t and thus on the level of prices during that interval. The sensitivity of spendings to the evolution of prices between t and $t - \theta$ was represented by the coefficient ε, in the following equation:

$$K + k(t) = K + \varepsilon \int_{t-\theta}^{t} p(\tau)d\tau$$

This equation expresses the fact that spendings depend on the income generated during the whole period of production, with K the equilibrium level of (nominal) spending and $k(t)$ a function of time giving the deviation from this level, the value of spending at each moment being called by Tinbergen the purchasing power applied per unit of time.

In the absence of unsold stocks, equilibrium on the goods market implies that real spendings $\frac{K+k(t)}{P+p(t)}$ are equal to the whole volume of goods produced. One obtains hence the final equation from which the whole movement of the economy can be derived once initial conditions are specified:

$$[P + p(t)][A + ap(t - \theta)] = K + \varepsilon \int_{t-\theta}^{t} p(\tau)d\tau \qquad (1)$$

where a is a coefficient determining the sensitivity of supply to changes in prices.

Depending on the combination of parameters a and ϵ and constants A, $P$and K, Tinbergen identified different types of movements corresponding either to cycles around the equilibrium or collapsing trajectories. Through the manipulation of the coefficient ϵ, public authorities are assumed to be able to regulate the level of spendings. Tinbergen did not provide much detail on the way this was supposed to be done, mentioning only in passing the possibility that this could be achieved through public works financed with new money.[14] The challenge was rather to demonstrate the ability of the public authorities to control the trajectory of the economy whatever its nature. It was in particular clear that by manipulating ϵ, the authorities may help controlling the movement of the economy hence revealing the two sides of any policy: to counter the effects

[13]Tinbergen pointed out that one may perfectly assume in the line of Hayek that the lag may change along the cycle. "One could assume with Hayek (Production and Prices, Vienna 1931) that θ changes during the cycle. In my view, such an assumption should be seen as a second approach to a problem, where we are not yet even in agreement on the first approach. But the thought is very interesting." (Tinbergen, 1934: 299).

[14] Tinbergen did not think that the rate of interest played a great role and took a very critical view on Hayek to which he constantly referred in his 1934 article. In the terms of his model, Tinbergen did not think that monetary authorities could have any impact on the coefficient ε through the manipulation of the rate of interest.

of large shocks and hence to ensure that the stability threshold would not be attained,[15] and to make sure that as long as the economy is operating below that threshold value, it quickly stabilizes.

In his 1935 *Econometrica* survey, Tinbergen went back to the issue with a similar model but only to consider "injections of purchasing power" transferred to the economy by a shock on initial conditions and no more by a change in structural parameters. At this occasion, he introduced the distinction between a temporary and a permanent shock, the former referring to a once and for all change in initial conditions, the latter to a permanent one, a problem that subsequently became central to his approach. In both cases, though, Tinbergen based his analysis on a linear approximation, thus restricting his reflection to the study of small shocks and to stable cyclical movements. In this way, a whole stretch of his thought was left aside as it is shown in section 2.[16]

He however adopted a different view in his discussion of the effect of a change in money wages, the other very important aspect of the "politiques conjoncturelles" in which he introduced the possibility that a shock on initial conditions may lead to destabilizing movements to the point of causing a collapse of the economy.

Destabilizing effects of money wage changes

Tinbergen discussed the effects of deflationary and inflationary wage policies on the basis of a simplified version of his 1934 model. The main simplification concerned the treatment of the expression for purchasing power: as we have seen, income spent at time t is continuously generated during the period of production, from $t - \theta$ to the time it is brought on the market. Instead of taking the integral which introduces some difficulties in finding an analytical solution for the model, Tinbergen (following Kalecki [1935b], see Tinbergen [1935c: 22]) took an average value over the whole period, which amounted to the simplification $\varepsilon \int_{t-\theta}^{t} p(\tau)d\tau = \theta\epsilon p(t - \frac{\theta}{2})$.[17] If one now takes θ as two units of time (Tinbergen 1935c: 22), it turns out that income spent at t depends on the price set at time $t - 1$ while production supplied at t depends on price set at $t - 2$. Substituting ε by the constant k used in 1935 and 1936, one gets Tinbergen's simplified final equation:

$$[P + p(t)][A + ap(t - 2)] = K + 2kp(t - 1) \qquad (2)$$

[15] To use a term later introduced by Leijonhuvfud (1973), this amounts to increasing the width of the corridor of stability.

[16] Tinbergen points out the possibility of multiple equilibria but the idea that a macroeconomic policy could be seen as a way to ward off strong instability moved to the background.

[17] This move signaled his willingness to abandon mixed difference-differential equations that were the predominant feature of the early models of Frisch, Kalecki and himself, and to adopt the tools of difference equations.

whose solutions are quite similar to those defined in his 1934 article. Again, Tinbergen can underline that depending on the value of the constants k and a, the system may either generate cycles or downward or upwards movements: "it depends on the numerical value of some constants if a cyclic movement or a unilateral movement occurs" (Tinbergen 1935c: 24). Any change in money wages, which amounts to a change in initial conditions through its impact on prices, may thus - and this for any constant k and a - transform a cyclical movement into a downward cumulative movement, a case that Tinbergen illustrated in his discussion.[18] [19] In sum, as long as the economy remains close to the equilibrium and does not reach the stability threshold, a wage change has only the effect of changing the period. If, however, that change is strong, then, movements may cease to be cyclical: "It is above all by increasing k that such transformations are obtained, or by increasing the initial disturbance" (Tinbergen 1935c: 24).

Tinbergen illustrated the influence of disturbances on his model with an example of a change in money wages, that impacted both spendings and costs of production. On the supply side, it was assumed that wages were equal to prices in the long term, so that an increase in wages would translate into an increase of prices. The new equation for supply after an increase of α percent in wages is assumed to be $A + a(p(t-2) - \alpha P) = A - a\alpha P + ap(t-2)$. On the demand side, a change in wages causes a change in income in the proportion $1 + \alpha$ so that we get the final equation:

$$[A - a\alpha P + ap(t-2)][P + p(t)] = K(1+\alpha) + 2k(1+\alpha)[p(t-1) - \alpha P]$$

[18] The period becomes shorter when k^2 is much lower than a. It increases when k^2 becomes closer to a. It can even become very long when k^2=a. Cycles are damped when a is lower than 1, regular when a is higher than 1 and explosives when it is greater than 1. The amplitude of the cycle depends on the initial conditions, that is on the price at time 0 and 1. The period and the nature of the moment is independent of the size of the perturbation.

[19] In the case of "non-small movements", "cyclical motions no longer have a period independent of amplitude - that is, of the initial disturbance - but this period increases with this amplitude and exceeds each limit as this initial disturbance approaches a moderate finite number. The shape of the oscillations is also transformed: the peaks become higher, the minima more flattened" (Tinbergen 1935c: 23).

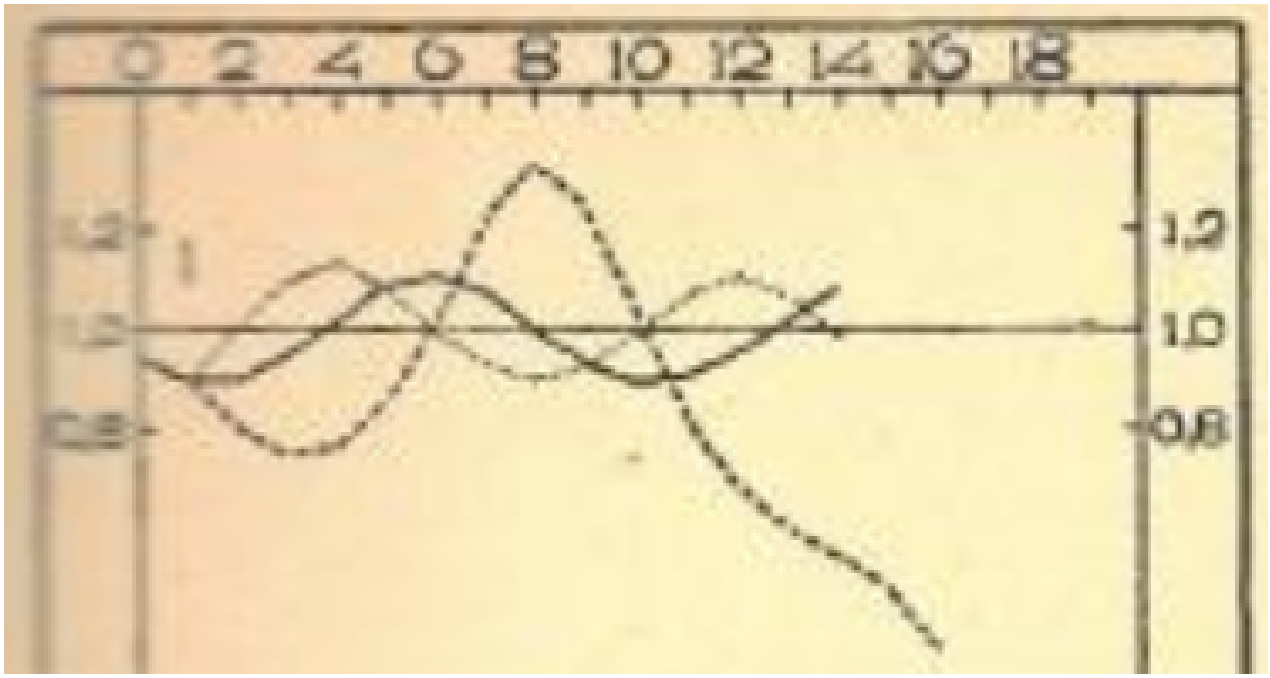

Figure 1: Movement of supply after a shock on wages in Tinbergen (1935)

Because processes of production begin in period $t - 2$, there is a transition period between the shock on wages and the moment where all production in progress is paid at the new wages. Tinbergen considers two cases, the first one where the production processes cannot be interrupted and continue until their end at their old wages, the second where they are interrupted and restarted with the new wages. Tinbergen proceeds to compute several numerical examples to show the influence of wage changes on a model showing endogenous fluctuations.[20] What interests him is the effect on production of wage changes, according to the moment of the cycle when it is decided, and he is able to show that "there is a great difference according to the phase of the cycle in which wage changes are decided" (1935c: 26). The most "desirable influence", that is, the change yielding the highest damping of fluctuations, comes when prices are going through the position of equilibrium,[21] in spite of the fact that "at this point, the movement's velocity is maximal" (Tinbergen, 1935c: 27), while changes at the peaks or troughs of cycles generally have the effect of increasing (at least at first) the amplitude of oscillations.

The following figure reproduces the graphs presented by Tinbergen in 1935 from the trajectories described by his equations, either resulting from an increase or a decrease in money wages at different moments of the cycle. The heavy lines represent the movement of the economy in the absence of a change in money wages. The dotted lines (red) represent the trajectories of the

[20] Tinbergen gives conditions for obtaining endogenous fluctuations in this nonlinear model in his first paper on the model (1934) and in the exploration of the multiple equilibria published in (1936). They are presented in more detail in Assous and Carret (2020: 10 ff.).

[21] An increase will have the best effect during the ascendant phase of the cycle, while the inverse is true for a decrease, somewhat counterintuitively.

economy given a rise in money wages, while the dashed lines (blue) represent the movement resulting from a fall in money wages.[22]

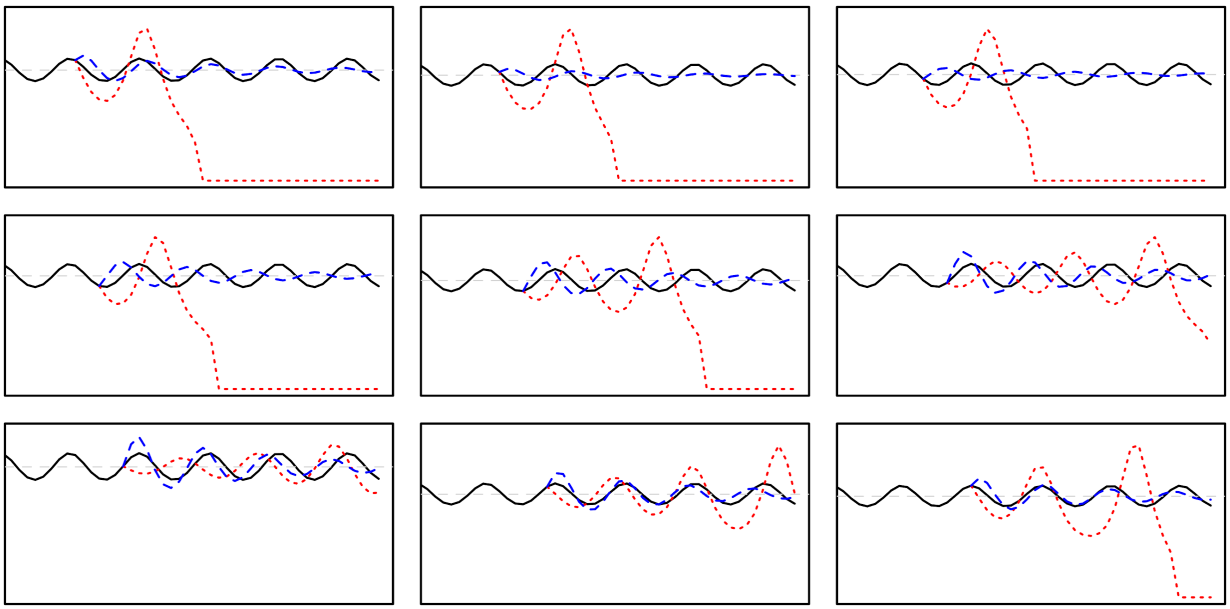

Figure 2: Reproduction of the different wage shocks in Tinbergen (1935)

Tinbergen also observes that wage changes, whether upward or downward, will have several dynamic effects in "the period, the degree of damping, the initial amplitude and the phase" (Tinbergen 1935c: 27). It is also evident that they change the stability properties of the equilibrium positions, and transform the original endogenous fluctuations in damped or explosive oscillations. While static analysis would have compared two positions of equilibrium parameterized by different levels of wages, this dynamic approach thus allows him to show that a change in wages, while modifying the equilibrium level of prices, can make it impossible to attain this new position because it is unstable.[23]

In particular, wage increases of about ten percent have the effect of diminishing production at first, before a reversal takes place and a boom of larger amplitude takes place. In fact, in all cases examined by Tinbergen, the wage increase has a destabilizing effect on production, by producing fluctuations of larger and larger amplitude; because there is a second, unstable equilibria below

[22] An application reproducing those trajectories for Tinbergen's two cases can be found here: https://economic-instability.com/wp-content/applications/tinbergen_1935.

[23] The change in equilibrium is not visible in the figures above, the same as in Tinbergen's article, because he limited himself to simple cases where A=K=P=a=1, that entailed the peculiar fact that production remained at the same level after a change in wages / prices, while prices and (nominal) purchasing power adapted to the new level. Other values of parameters and constants can very well give rise to a change in equilibrium for all three variables (prices, production and demand) after a change in wages.

the high equilibria around which the economy was fluctuating, wage increases (almost[24]) always have the effect of collapsing the economy in this model!

This conclusion, rather strange for a socialist such as Tinbergen, actually explains several of his policy positions, in particular against the policies taken by Roosevelt in the early years of his first term. This article on wages actually began with a reference to Roosevelt policy, and it was not an isolated reference in Tinbergen's work of the time. In an article published around the same time in a German magazine (Tinbergen, 1935b), Tinbergen thus argued that "disruptive experiments" such as the depreciation of the dollar and changes in wages were abandoned in the next wave of measure (after 1935) because of their unsettling character. The question that unsettled Tinbergen was the unintentional, unforeseen effects of a policy "to what extent are guarantees created that a possible revival will not 'shoot through its equilibrium' in a short time and bring another crisis?" (1935b: 96).

The problems raised by changes in money wages in Tinbergen's papers are also picked up by Dekker (2021: 169 ff.) who gives other references to non-academic forays by Tinbergen on the question and underlines the tension existing between rigid wages ensuring their stability, and flexible wages ensuring their adaptation to new economic conditions. Tinbergen took this position not only against Roosevelt's policies, but also against the Dutch Plan of Labor of the early 1930s. While we agree with Dekker that Tinbergen "saw the dangers of too much stimulus at one point in time clearly" (Dekker, 2021: 169), we believe that the explanation of such circumspection directly came from the models he developed at the same time, rather than from the "traditional worries of classical economists [that] Tinbergen had absorbed in the 1920s." (Dekker, 2021: 169). The model built since 1934 and used in 1935 to study the effect of changes in money wages showed precisely this, without having to resort to classical economics; on the contrary, the originality of Tinbergen was to introduce dynamical considerations, that raised in turn the problem of stability of an equilibrium, and ultimately the possibility to attain it. Tinbergen always kept those stability issues somewhere in his subsequent articles, although it became clear that the focus shifted to successful policies ensuring that the economy stayed close to equilibrium. In turn, this put a new question at the center of inquiries: how to raise this level of equilibrium?

II. Shifting equilibrium

From 1935, Tinbergen pushed forward with his analysis of economic policies on the basis of a model presented at the fifth meeting of the *Econometric Society* in september 1935 in Namur (Staehle, 1937), and published the same year in the November issue of the *Weltwirtschaftliches Archiv* (Tinbergen, 1935d). This model was not different in spirit from the earlier model, although it became much more detailed and formed the basis for subsequent statistical

[24] Tinbergen underlined that for a smaller increase in wages, the movement of production was at first upward rather than downward. This led him to hypothesize that there existed a finite value in wage increase, somewhere between five and ten percent, that would perfectly stabilize the economy and eliminate the cycle.

estimations conducted in the second part of the 1930s. From then on, Tinbergen confined himself to the study of small deviations from equilibrium with the aim to determine the parameters likely to damp the effect of external shocks in the most efficient way. The problem raised by multiple equilibria, at least for some time, moved to the background.

At the same time, Tinbergen was able to derive the coefficients of his equations from historical data, the basis for his study of the American business cycle conducted at the League of Nations and for several macroeconometric works on the Dutch economy. But as Tinbergen explored this model, he also started to consider another role for economic policy: the idea that it could be used to attain a new, higher position of equilibrium, via permanent rather than one-time interventions that would only have temporary effects. This was discussed in several articles, and in more exploratory papers Tinbergen also started to find new ways to explain the mechanism behind the improvement of the final position of equilibrium: this opened the door back to multiple equilibria, in several articles presented in the third section of this paper.

Economic policy as a linearisation of the model

Tinbergen did not really build a new model, but rather deepened and put together the mechanisms explored separately during the preceding years in papers and in the meetings of the Econometric Society. In particular, Tinbergen kept the idea - central in 1934 model - that "Sales come about by dividing the available purchasing power by the price" (1937b: 46) on the basis of which the relation between prices, production and the value of consumer goods sold, as well as the movement of employment was reconsidered. In addition, speculation, that was already broached upon as early as 1931 at the Lausanne meeting, takes a center stage to the problem of fluctuations via its impact on the evolution of profits.

The earliest version of the extended model was presented and published in 1935, and gave the basis for several variants explored afterward. Many of these variants counted more than a dozen equations, up to almost four dozens for the League of Nations study. Because this aspect of the macroeconometric program developed by Tinbergen has already been explored elsewhere,[25] we will not present it here, and we concentrate on the different manner in which economic policy impacts this model (although the next section will present in more detail smaller variants that were used by Tinbergen to understand the impact of multiple equilibria on this extended model).

The first question Tinbergen tried to answer with the model presented in 1935 was: how to ensure that economic policy will damp out the effect of external shocks as quickly as possible, so that large deviations from equilibrium do not happen? This justified the use of linear equations for his model, and thus economic policy can be viewed here in some way as a “linearisator” simplifying the economic relations, a line he introduced with the action of economic policy on coefficients in 1934 as we have seen in section 1.

[25] See for instance Morgan (1990) chapter 4, Dekker (2021), especially chapters 6 and 10.

In the article presented at the Namur meeting, Tinbergen proposed two approaches of economic policy against the cycle: either using "counter-shocks" against every external impact, or modifying the structure of the economy so that it reacts with more damping against shocks.[26] This second approach clearly had his favors: "The question, around which everything revolves, is: how should one determine or change the elementary coefficients so that the economic process reacts as favorably as possible to the external shocks." (Tinbergen, 1935d: 380). Based on his model that contained seventeen equations, Tinbergen presented several conditions on the coefficients that would ensure a quick return to equilibrium, and protect the economy against "an unfavorable impact" that could "lead to dangerous movements of the system" (Tinbergen, 1935d: 381), a clear reference to the movements described on the basis of the nonlinear model.

Tinbergen was also urged to clarify the role of economic policy in his model after the summer of 1936, during which he crossed over Europe and attended at least two international conferences on business cycles (in Geneva and Vienna[27]) before the annual meeting of the Econometric Society in late September, the famous Oxford meeting where Hicks, Harrod and Meade presented their interpretations of the *General Theory*. After those meetings, Tinbergen wrote a very pedagogical article to explain to his fellow literary economists the basics of dynamical systems.[28] Tinbergen goes on to describe in detail the mathematical form of his system, with several figurative representations aimed at explaining as simply as possible the origin of movement in a mathematical model, as well as the statistical relationships on which linear or nonlinear relationships are based.

Some of the incomprehensions of his colleagues can be surmised from his text: for instance, he thought it was necessary to explain that a "closed" system of equations "can of course relate very well to an 'open economy'" (Tinbergen, 1937a: 2). Other important points were the simplifications used (macroeconomic, temporal and mathematical). Tinbergen also took care of explaining the advantage of using deviations from equilibrium rather than absolute values, to reinforce the linearity of the system.

Tinbergen also defined several approaches to equilibrium, including two that included a possible trend of the economy. When he presented the solution of a general system of difference equations, Tinbergen pointed out that it could include these trends directly alongside the cycles arising under certain conditions, as long as the model included at least three lags. This was

[26] During the discussion after his presentation, the nature of shocks was discussed and Tinbergen proposed changes in wages or a devaluation as examples of a shock originating from economic policy. See the summary of the meeting in *Econometrica* (Staehle, 1937).

[27] The meeting in Geneva was organized by the League of Nations to discuss the forthcoming book of Haberler (Boianovsky and Trautwein, 2006), and the statistical study of Tinbergen that was just beginning his work at the League. See also Hagemann (2020: 369) on this meeting and another one a few days later in Vienna, "the fifth international conference of the research institutes on business cycles" (*ibid*.: 378), organized by Oskar Morgenstern. Tinbergen also explicitly referred to the Oxford meeting (1937a: 2).

[28] "The immediate cause of this essay is a discussion I had in the summer of 1936 with a number of leading economic theorists. It turned out that there are serious misunderstandings in business cycle theory about the mathematical method to be discussed here" (Tinbergen, 1937a: 1).

represented in a figure, where Tinbergen could again illustrate what he considered to be one of the main goals of economic policy: damp out fluctuations around the trend line.

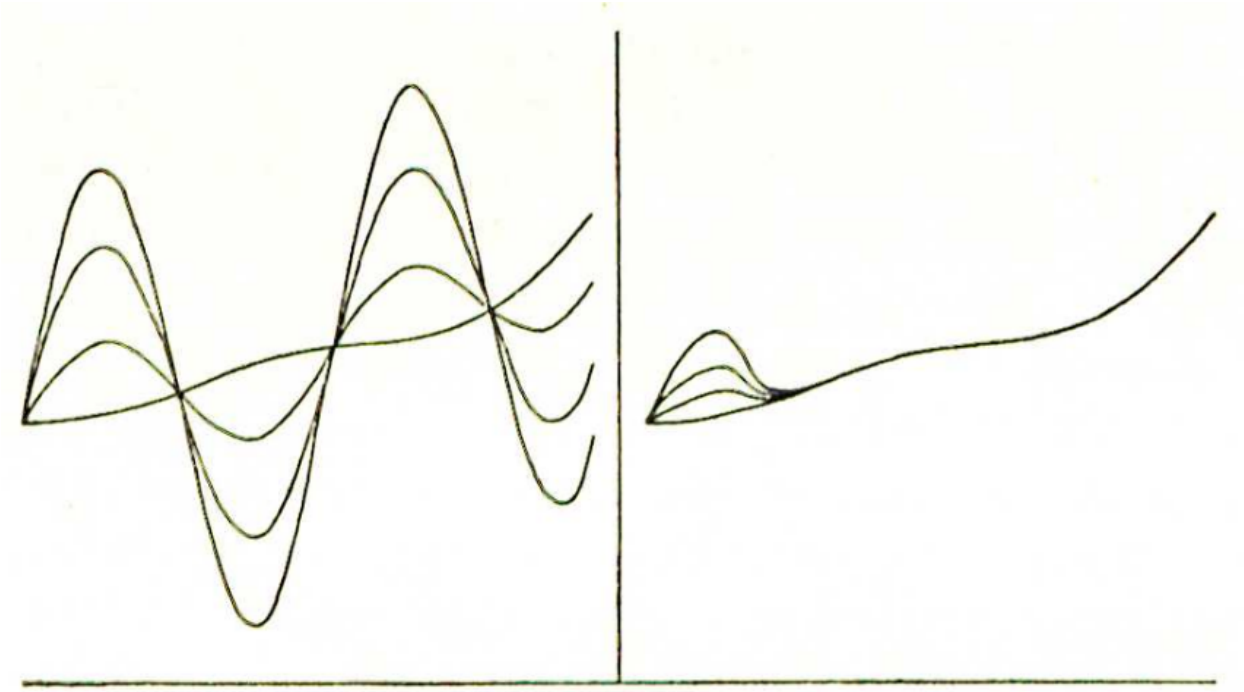

Figure 3: Oscillations around a trend in Tinbergen (1937a)

Tinbergen underlined the link between stability and economic policy, in a paragraph concerned with "Questions of stability and economic policy", where he noted that stability could not be taken for granted, but should be actively pursued. He argued that there were different degrees of stability, and that more unstable movements were a feature of recent business cycles, that should be eliminated by an "ideal" economic policy (Tinbergen, 1937a: 91).

More explicitly than in other earlier articles, Tinbergen described how an economic policy could act on coefficients and which coefficients were the most efficient to damp out fluctuations.[29] Among them, one finds in first place the limitation of speculation opportunities, something very important in Tinbergen's model and that surely stemmed from the excess of the "roaring twenties", a decade of financial excess and widespread speculation.[30]

[29] "a sufficient condition for a successful economic policy can also easily be established for these elementary coefficients." (1937a: 92)

[30] "The importance of a decrease of z [the coefficient of sensitivity of speculation to profits] is evidently that the amount of realized and financed speculative profits, which correspond to a given increase in total profits, is decreased. This could be done by limiting the opportunity for speculation altogether. A reduction in [the coefficient of sensitivity of investment to profits] could be achieved through an appropriate public works policy; namely, by one in which a relatively large amount of such work is carried out in the depression and relatively little in the boom. What the exact directives for the public should be is not discussed further here. A decrease in 1 and 2 [the coefficient of sensitivity of consumption to profits] would mean greater consistency in consumer spending among non-workers." (1937a: 92)

In an article published in 1938 in *Econometrica*, and suggestively titled "On the theory of business cycle control" (Tinbergen, 1938b), this approach was also developed on the basis of a similar model. While he referred the reader to his papers on multiple equilibria for the implications of large shocks away from equilibria, he preferred to keep a linearised version of the model as a simplification in mathematical form. Again, Tinbergen emphasised the role played by speculation in creating oscillations in the economic model, and he remarked that acting on the sensitivity of its parameters gave the best results to damp out the cycle. Finally, Tinbergen explored the role of wages and the rate of interest in this model, showing that they were mostly following the cyclical fluctuations determined by profits and did not have a clear impact on the cycle, somewhat of a break from his conclusion of 1935.

At the same time, he explored the idea (already present in 1935), that additional terms brought on by economic policies may have an important impact through a change of the position of equilibrium, in addition to their influence on cyclical movements.[31]

Economic policy and permanent changes in the equilibrium position

Some of the consequences of permanent changes resulting from economic policy have already been presented. For instance, in 1935, Tinbergen wrote about the "undreamt of problems" that were entailed by "more or less permanent changes in data" (Tinbergen, 1935d: 382). These problems were reconsidered in the monograph he was preparing during this period, that was published in French in 1938 (Tinbergen, 1938a),[32] and in a pre-advice written for the Dutch Economic Association and published in 1936 (translated in Tinbergen [1959a], the version from which we quote thereafter).

The pre-advice was written as an answer to the question of economic recovery in the Netherlands, particularly in view of the problems arising from international exchanges, and the lessons that could be learned from other countries. The discussion was based on the 1935 extended model and considered again deviations from the trend, although the reflexion on the position of equilibrium is more developed in this paper: "Viewed more closely from the theoretical angle, what really matters is not so much the ordinary (mathematically established) trend but a consecutive series of figures which I would like to call an 'equilibrium development'." (1959a: 45 et 49). This was in line with Tinbergen's shifting interest toward the evolution of equilibrium positions, and how economic policies could impact it.

In fact, the whole analysis of economic policy in this paper was organized around this idea: which economic policy will lead to the best position of final equilibrium in the short term? Breaking with previous analyses mainly aimed toward damping the cycle, Tinbergen integrated

[31] "unique circumstances and sudden changes can be incorporated into the picture by being added as additional terms in the equations. An autonomous change in wages, as for instance The New Deal brought about can be treated in this way; every crop change, a big strike, etc." (1937a: 96).
[32] The "avant-propos" of this monograph is dated from January 1936.

economic policy via additional variables in his equations, that led the economy to higher or lower positions.[33] The earlier warnings on the impact of small changes in coefficients on the movement of the economy were not completely shed off however, and were reminded as a caution to take against extrapolations of the model in the too distant future,[34] and also against transposing successful measures from one country to another, where structural differences could cause large differences in reactions to similar measures.[35]

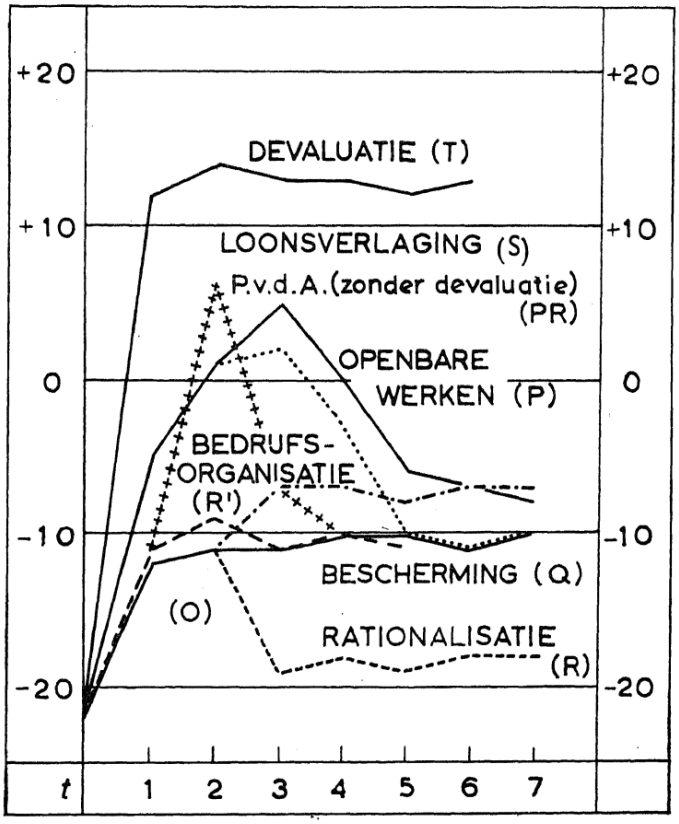

Figure 4: Effects of different policies on the level of employment in Tinbergen (1936 [1959a])

The figure reproduced here summarised his results on the consequences of several types of economic policy on employment (where deviations from employment are represented on the vertical axis); it is clear that devaluation presented the best improvement of all the policies

[33] "I have assumed that this affected only the constant term and not the coefficients of the equations concerned. Taken strictly this will be the case only if a number of the governmental measures to cope with the present slump were again removed." (Tinbergen, 1959a: 69)

[34] "A slight change in some coefficients is sometimes enough to turn a periodical into an aperiodical movement and *vice-versa*, of which the first parts almost coincide but whose further development increasingly diverges." (Tinbergen, 1959a: 75)

[35] "At the present stage of the investigation it cannot yet be said whether small structural differences might not prove to result in large differences in reaction to certain measures. Striking examples could be given of the great influence which may be exercised by a moderate structural change." (Tinbergen, 1959a: 84).

examined.[36] The effects of changes in wages were less conducive to an improvement in employment as can be seen in the diagram, where after an initial increase employment goes down to a level not very different from the level obtained without any intervention.

The potential to reach higher equilibria with economic policies was also explored in the famous 1935 survey published in *Econometrica* (Tinbergen, 1935a), where Tinbergen used the same model as in 1934, but where he considered "money injections" financing public works expenses to change the final level of employment.[37] At this occasion, he introduced the distinction between a temporary and a permanent shock, the former referring to a once and for all "injection," the latter to a permanent one.

A distinction on the differences of permanent, structural and temporary policies was developed in the monograph published in French by Tinbergen in 1938. Changes in coefficients represented changes in the economic structure and Tinbergen argued that changes in additive constants or in initial conditions were symptomatic of "circonstances modifiées" (1938a: 34). An important distinction was made between a permanent change or a temporary one in those additive constants: in the latter case, this could be represented by a change in initial conditions giving rise to a movement determined by the systems' equations. In the case of a permanent change however, "l'interprétation la plus simple de ce fait est que la valeur d'équilibre de certaines variables a changé" (1938a: 35).

Tinbergen took the example of a fall in prices, something he had already done with wages as we saw earlier (section I). The accent was put on "additive constants" and their impact on the equilibrium position and the movement of the economy. Among the cases considered were an economic invention, an exceptional crop or, more importantly from the point of view of economic policy, the devaluation of a foreign currency, which acted in particular on investments via an adverse effect on expectations (1938a: 38), a new mechanism compared to the questions explored in the 1936 pre-advice.

It was clear for Tinbergen that in the case of a permanent change, two consequences would ensue: an initial deviation from equilibrium, giving rise to an oscillation under certain general conditions, and the formation of a new equilibrium. New oscillations could stem from an economic policy as the consequence of the inevitable delays in taking action (Tinbergen, 1938a: 109). But, not all shocks and not all changes in variables would lead to the same kind of oscillations or modifications of the equilibrium position.

[36] While it sometimes argued that Tinbergen had little impact on Dutch economic policy in the 1930s (Hansen, 1969), we can note that the Netherlands finally exited the "Gold bloc" in September of 1936, just a few months after Tinbergen's pre-advice showing the opportunity of devaluing the guilder.

[37] "An amount K'(t) is added to the amount spent at the consumers' goods market (e.g., when public works are paid for with created money)" (Tinbergen, 1935a: 304).

In this context, changes in money-wages did not have the same influence as in the purchasing power model, where they could give rise to destabilizing oscillations.[38] He pointed out that, because they acted both on supply and demand, this influence would be mitigated, but that they had a much more important effect on equilibrium.[39] He underlined that this was in particular the case in relation to the economic policy of the New Deal in the United States; while the transformation induced by this set of policy was very complex, Tinbergen thought that some factors tended to increase the level of production, such as devaluation and increased government expenditures, while others, like the increase in wage rates, led to a fall in the level of production (Tinbergen, 1938a: 45).

It seems that with the new model, there was less room for multiple equilibria and possible instability, because Tinbergen was interested in the linear form of his system and consequently on small deviations around the equilibrium. However, multiple equilibria made a comeback in this model through the problem of obtaining a higher position of equilibrium, something that became the center of his attention and led him to consider the possibility that economic policy could act on the final position of equilibrium through permanent and also temporary changes.

It was particularly in the 1938 *Econometrica* paper cited above that Tinbergen suggested that nonlinear models could be used to explain changes in the equilibrium positions: "In the case of a nonlinear final equation there exist other equilibrium positions or developments than the one chosen as reference developments and much depends on the situation of these other equilibria, which may be stable or unstable." (1938b: 33). He did not fully explore these ideas in this paper, but did so in other articles presented in the next section.

III. Reaching new heights

After Tinbergen established the baseline of his new model, he was led to explore new implications of multiple equilibria and nonlinearities for economic policies. This was done in several articles from the late 1930s and early 1940s, where he determined with his usual pedagogical style[40] when an economy may be stuck in a low equilibrium with the possibility that shocks may help reach a higher equilibrium.[41] An important lesson from these articles became that the equilibrium obtained was at least as important as the trajectory; this had been a bit lost in the importance given to the (trendless) study of the business cycle as fluctuating around a stationary position.

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[38] “une politique des prix ou des salaires, par exemple une stabilisation directe des prix ou une stabilisation des salaires, ou bien une augmentation de la flexibilité des salaires, aura peu d'influence sur le mouvement cyclique” (1938a: 91)

[39] “les salaires prennent de l'importance si l'on considère leurs rapports avec le niveau d'équilibre de la production dans un pays en concurrence avec le marché mondial" (1938a: 92)

[40] Dekker (2020) underlined that of the Frisch - Tinbergen duo, Tinbergen was a much clearer expositor than Frisch.

[41] Tinbergen mentions for the first time that possibility in 1932 but only with the help of a diagram in a somewhat cryptic way (see Assous and Carret, 2020, first section).

In fact, the problem of obtaining a new, higher equilibrium was becoming one of Tinbergen's most important pursuits. Multiple equilibria were one possible approach, used to show how a temporary economic policy could lead the economy to a higher equilibrium. Another one that he developed was the study of trends, which we only mention here to make clear that Tinbergen's different pursuits were very much connected. The subject of trends is first examined in 1936, in a French article on the "statistical determination of the position of cyclical equilibrium", and then expanded upon in several articles, eventually leading to the famous 1942 German article (translated in Tinbergen [1959b]) "On the theory of trend movements". While an interesting approach in itself, this is not our concern here, because our main focus is on policies and the problem of stability, instability and multiple equilibria in Tinbergen's work, that was developed at the same time in a parallel line of thought.

High and low equilibria

Tinbergen elaborated his ideas on economic policy and multiple equilibria in a 1937 German article (Tinbergen, 1937b). This article was explicitly written as a rebuke of the too simplified theory of Kahn's multiplier. Although he underlined the important steps toward dynamic analysis that is found in the concept of the multiplier,[42] Tinbergen did not think it was enough to provide a relevant explanation of the dynamics of the economy. Instead, he suggested using a more detailed model such as the one he had already built in 1935, but that had yet to be estimated on actual data.

While there are ten variables in the model, it can be reduced to four main equations, as Tinbergen does himself in Tinbergen (1937a). It is then always possible to find the aggregate level of employment from these equations and the employment in both sectors, with a relation given by Tinbergen between the level of employment in the two sectors and profits. All the variables are measured as deviations from their equilibrium value, and the unit of time is assumed to be four months.

The most important equation is the equation for profits, Z_t, or as Tinbergen calls it more generally, the "total income of all non-workers" (1937b: 42). This equation depends on the revenues from production in the two sectors, and from the revenues given by "speculation", which depends positively on the rate of increase of profits: the quicker profits accumulate, the more speculation adds to current profits, a mechanism at the root of economic fluctuations as we will see.[43] The equation for profits thus has the form:

[42] "The theoretically and practically important new feature in Kahn's calculation is the fact that a dynamic, no matter how simple it is, is created, that is, a piece of economic theory that does not deal with stationary states and their comparison with one another." (1937b: 40)

[43] Speculation came back frequently in Tinbergen's models. As soon as the 1931 Lausanne meeting (for the summary of the meeting see Staehle [1933], Tinbergen's communication was published in the first issue of *Econometrica* in Tinbergen [1933]), he introduced those effects (originally via a rate of change but after 1935 via a first order difference). The process of elimination followed by Tinbergen meant that it became one of the most important drivers of the cycle as he progressively abandoned the explanations relying on wages or the interest rate.

$$Z_t = \underbrace{U'_t - L_t}_{Cap.\,cons.} + \underbrace{b_t + 2.2b_{t-1}}_{Investment} + \underbrace{1.2(Z_{t-1} - Z_{t-2})}_{Speculation}$$

Where U'_t is "the value of all consumer goods", L_t the overall real wage bill, and b_t is the amount of employment in the investment sector, which is determined by the profits in the previous period:

$$b_t = 0.12\, Z_{t-1}$$

While very simple, this equation is important because in this model, economic policy acted directly through "job creation" (1937b: 39) via an exogenous increase in b_t. In the profit equation, we see that b_{t-1} is even more important than b_t: this is the effect of the production lag of investment goods. Tinbergen adds an equilibrium condition relating the value of consumer goods and wages and profits expenditures:

$$U'_t = L_t + F'_t \Rightarrow U'_t - L_t = F'_t$$

This (static) equation is an equilibrium condition that means that all wages are spent in consumption, an hypothesis introduced by Kalecki in a macrodynamic model in 1933 (Kalecki, 1935a,b), and that was appropriated by Tinbergen in the new model. F'_t is a consumption function for "non-workers," that explains how non-invested profits are spent. The form on the right makes apparent that we can replace this in the profit equation, and we see immediately that profits depend now positively on the consumption of capitalists, as well as on investment and speculation, again, something that was clearly presented at the Leyden meeting in Kalecki's 1933 model.[44] This also shows that only one more equation for F'_t is required to close the system.

Tinbergen presents several numerical examples based on different specifications of this consumption function, which always depends on the profits of the past two periods.[45] In the first two cases examined, the effect of job creation is ambiguous because after an initial (cumulative) increase above the amount of jobs created, the economic variables showed a reversal and a crisis settled in the model.[46] This oscillation resulted from the dependence of profits on its values in the last two periods via consumption, investment and speculation. But in his third case, he introduced another idea that showed under which conditions job creation could have an unmistakably positive effect on the level of employment. To that end, he complexified the

[44] There is a close proximity between Kalecki's profit equation which involves that profits are equal to capitalists' consumption and investment spending; the idea is the same here, plus the gains coming from speculation. Tinbergen does not go as far as saying that capitalists only earn what they spend, even if it appears to be the case..

[45] Tinbergen usually justifies this by the fact that (i) profits are often distributed only a few times a year (ii) in connection to (i), there is a lag in the spending of those profits.

[46] "The result of our calculation could perhaps be viewed as unfavorable for job creation. But that would be premature because it neglects some very important factors for the assessment." (1937b: 52).

consumption function of capitalists to give it a nonlinear form in profits, while it still depends on the last two lags:

$$F'_t = 0.45\, Z_{t-1} - 0.06\, Z_{t-2} - 1.53\, Z^2_{t-2} - 2.56\, Z^3_{t-2}$$

This form was used rather as a methodological starting point to explore consequences of economic policies than as a true representation of reality.[47] When inserted into the equation for profits along with the other equations above, this will give rise to three equilibria. It is worth quoting Tinbergen at length on the consequences of the presence of multiple equilibria to shed a new light on economic policies:

> Job creation as an instrument for overcoming a crisis has its greatest value when it is absolutely necessary as an initial spark, that is, when without it the ascent is a long way off. This is especially the case when the economy has several (higher and lower) stable states of equilibrium and when it is in equilibrium at a relatively low level of employment at the end of the crisis. This then means that there is no longer any recovery through the free forces, but only the possibility of some 'external impulse' distancing oneself from the low level of equilibrium so far that further automatic movements bring the economy to a higher equilibrium position. Job creation is the surest method for this external impetus. (1937b: 54)

Tinbergen clearly shows here that when there are several equilibria, an external impulse can in fact lead to a higher level of employment and profits, without necessarily maintaining the initial expenditure at the source of the shock. It is very interesting that Tinbergen chose here to build a model with three equilibria, instead of two as in the 1934 model, because it means that (under certain conditions) the two extreme equilibria will be stable, while the middle equilibria will be unstable. In the case where the economy is stuck at the bottom equilibria, which seems to be that envisaged by Tinbergen in the above quotation, an economic policy geared toward increasing employment will be able to improve the final position of the economy much more than the initial increase.

In his numerical example, Tinbergen took a more questionable case: he started his economy from the middle, unstable equilibrium, where an infinitely small amount of increase in profits or in jobs will yield an upward movement that will stabilize around the high equilibrium (and conversely, an infinitely small downward deviation from the unstable equilibrium will lead the economy to the bottom equilibrium). The result of both deviations and the consequent fluctuations given by Tinbergen's model are illustrated in the next figure, where Tinbergen's numerical example is extended for more periods. At time $t = 150$, there is a small positive shock, inferior to the distance between the low and middle equilibria, and we see that cumulative

[47] This posture is explicitly stated in the Survey (1935a: 274), and can be inferred as well in this article.

effects make this shock sufficient to go over the unstable equilibrium and attain the high equilibrium: a rather optimistic view of economic policy.

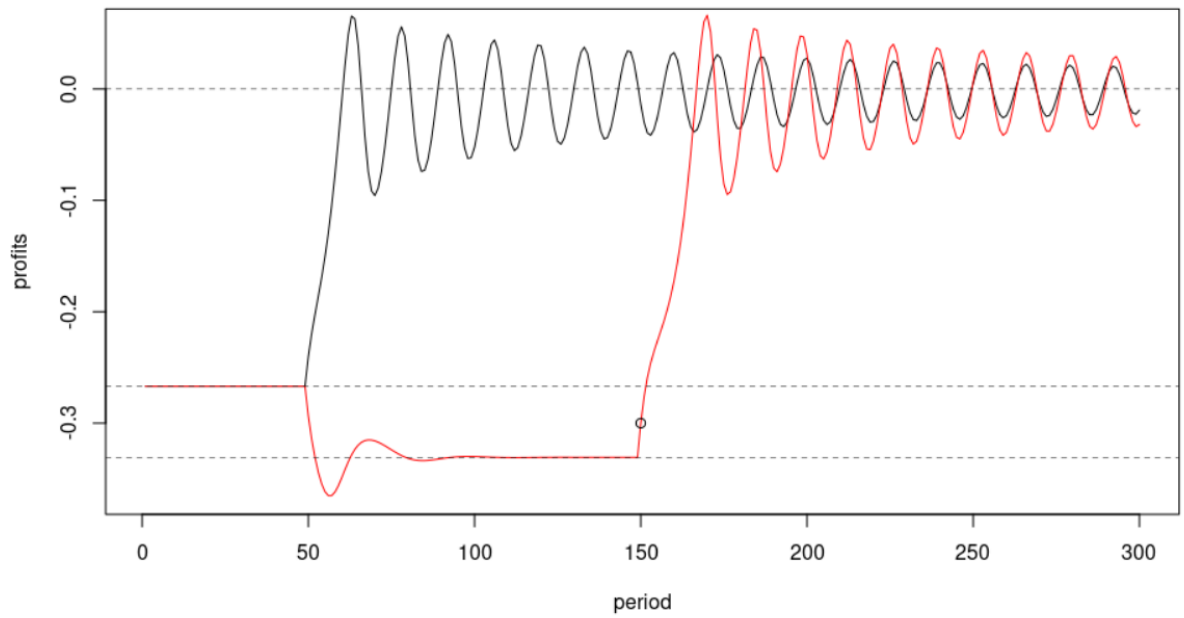

Figure 5: Effects of shocks in a model with three equilibria (Tinbergen, 1937b)

From this figure, it is apparent that upward and downward deviations do not have the same effect on cyclical fluctuations: profits take a longer time to stabilize around the high equilibrium than around the low equilibrium (at least for the numbers given by Tinbergen). But it is also apparent that however low the fluctuations around the high equilibrium can get, they remain largely above the original level and obviously above the level of the low equilibrium.

Tinbergen insisted on the importance of economic policy to obtain a higher equilibrium at the end of his article: "We hope to have shown by this example that the study of the possibility of several equilibrium positions, and above all the concrete research of economic figures, is related in this direction to very important basic features of the problem of economic policy." (1937b: 56). This is an important step in the evolution of Tinbergen's approach to macroeconomic problems, because it was a break from previous studies where the cycle was the essential part of the analysis: it is obvious here that the final position is at least as much important, and that this final position can be influenced by economic policy.

Interestingly, Tinbergen also suggests in the last footnote that "We believe that a stabilization of the economy at the highest level, as it is defended by authoritative authors, is impossible under certain conditions, which are related to the plurality question of the equilibrium, without constant new government expenditure" (1937b: 56). Although he does not expand on this idea, from the developments above it is very possible that he thought of the possibility that there may exist several equilibria, among which the highest one could be unstable and impossible to maintain (in

view of the constant shocks hitting the economy) without a policy striving to push the economy toward this unstable equilibrium. This case was in fact presented several years later, in a 1944 paper presented below.

The idea of multiple equilibria, and how they influenced a policy destined to improve the level of production and employment, resurfaced several times in Tinbergen's work during the following years.[48] Tinbergen was nevertheless aware of the difficulty of building a theory on nonlinear equations. For instance, in his 1938 article "On the Theory of Business Cycle Control", he referred the reader to his 1936 *Wagemann Festschrift* article as well as to the 1937 article presented above on the behavior of the model under nonlinear assumptions. But he also suggested several simplifications to study nonlinear cases, in particular he gave a small example where he used a piecewise linear function to study the behavior of a model with maximum and minimum values. Interestingly he added that this made it possible to mathematically translate the distinction made by Haberler "between the analysis of the cumulative movement and the turning-point analysis" (1938b: 33). There is a strong link between this idea and what Tinbergen developed in the 1937 paper, where he started from the (cumulative) analysis of the multiplier, to develop a model of cyclical fluctuations and eventually a model where the economy was able to fluctuate around a higher equilibrium.

In 1938, this was presented in the framework of the phenomena of "saturation" which had already been discussed in the 1935 survey, along with other possible simplifications. In the survey saturation was defined by the following: "Its essential feature is that certain variables are not allowed, by natural, technical, or institutional circumstances, to surpass a given limit." (1935a: 296). In other words, to take the example given in 1938, employment "cannot continue to be the same linear function of [profits], after the state of full employment … has been reached." (1938b: 32). To simplify the study of the behavior of the model, and avoid the difficulties related with nonlinear equations, Tinbergen approximated the asymptotic function by two functions, one valid before full employment is reached and the other after. In fact several other equations in the system would have to be changed, to reflect for instance that if production could not be increased anymore, a still growing demand for investment goods would start to increase prices (Tinbergen, 1938b: 32).

This approach of nonlinearities and its consequences on fluctuations culminated in a 1944 article, an apex of multiple equilibria in Tinbergen's thought, that takes on new meanings when viewed from the perspective of all the leads he had already explored. Indeed, his explicit reference in this article to Kaldor's 1940 paper may have contributed to obscure the much more direct reference to his own earlier works on multiple equilibria, often written in German, French and Dutch. Nevertheless, the approach developed is the same as in the small example of the 1938

[48] Although it seems to have been superseded by the search for trends and their mechanisms to explain higher position of equilibrium.

Econometrica article, and more importantly, Tinbergen examines the cases of one, two, or three equilibria, that had all been explored in his previous articles presented above.

Multiple equilibria, cycles and policies

In this article,[49] Tinbergen explored the implications of multiple equilibria in a Keynesian cross diagram and showed how, depending on the slope of the expenditure curve, different types of equilibrium can be defined. When the expenditure curve intersects the 45° line from above, the equilibria are stable while if it intersects from below, they are unstable. With reference to Goudriaan with whom Tinbergen had debated several times in the 1930s (see Rodenbourg, 2010), Tinbergen considered the possibility that the expenditure curve may be represented by a discontinuous curve assumed to be flat for extreme values of output.[50] The following figure, taken from the article, summarizes most of the cases that can arise when the demand curve has two kinks at maximum and minimum levels. Two types of equilibrium are then identified. The "Goudriaan point" defined for extremes values of output and the "Keynes point" when the equilibrium is unique and the expenditure curve intersects the 45° line only once.[51]

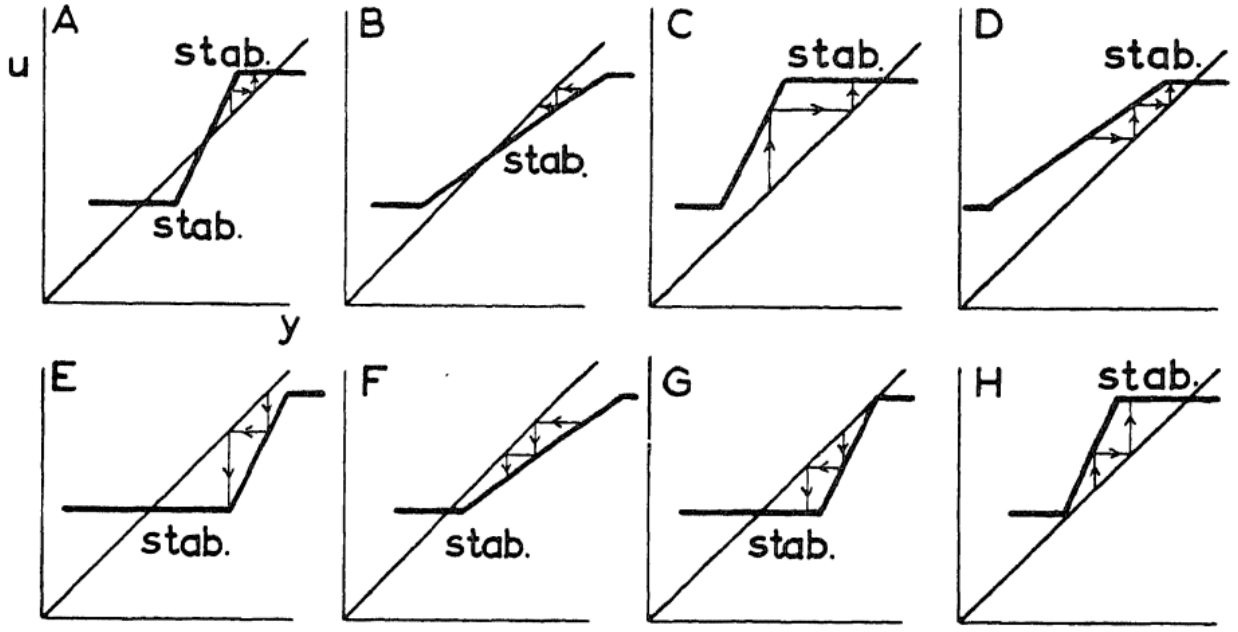

Figure 6: Possible configurations for multiple equilibria in Tinbergen (1944 [1959c])

[49] There exists at least three versions of this article: the first one was published in 1943 in *De Economist* (in Dutch), the second in 1944 in Norwegian in the *Nordisk Tidsskrift for Teknisk Økonomi*, which has been translated in a third English version in Tinbergen (1959c), from which we quote in the rest of this section.

[50] Even if Tinbergen recognized that in practice, the demand curve is rather S-shaped and continuous, he argued that the qualitative solutions obtained do not change while his approach simplifies the mathematical treatment of the model.

[51] According to Goudriaan, summarised by Tinbergen, the positions of equilibria arise from the maximum and minimum values that real expenditures can take: "Even if real income, for some reason or other, should exceed this maximum or fall under this minimum, real expenditure would still remain within these limits." (1959c: 24).

We see that there are three main possibilities in such a system: a unique position of equilibrium, such as in cases B, C, D, E, and F, a system with two equilibria, as in cases G and H, and a case with three equilibria, A. It is worth here pointing out that Tinbergen excluded the other case with three equilibrium, where only the middle one would be stable.[52] Clearly, Tinbergen thought that the movements of the economy were confined, which shows that he moved from a vision of a world likely to collapse when pushed out of the corridor to the vision of a world oscillating inside a corridor where only partial collapse may occur. The two cases G and H are situations where one of the "Goudriaan-point" (an extremum value for the demand function) is confounded with the "Keynes-point" that arises in the linear case. The case G with a downward unstable high equilibrium is a direct representation of the idea mentioned in the last footnote of the 1937 article (mentioned above), that policies of permanent expenditures may be necessary to maintain the high equilibrium. On the other hand, the case H is of course evocative of the 1934-36 model studied above, with a stable upper equilibrium, and a half-stable lower equilibrium, in the sense that it is downward stable but upward unstable; it is in reference to those "especially interesting" points of "mixed equilibria" (Tinbergen, 1959c: 24-25) that Tinbergen referred to Kaldor's 1940 theory,[53] although it should be noted that the mechanism underlying the two theories are different.

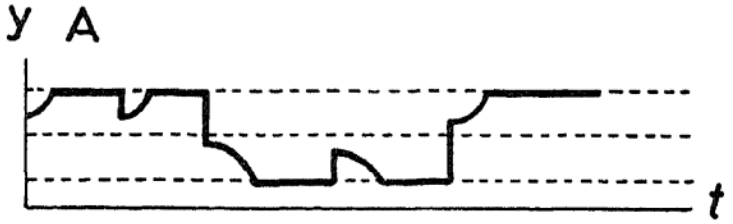

Figure 7: Transitions between high and low equilibria due to shocks (Tinbergen, 1944 [1959c])

Unlike Kaldor, Tinbergen does not explore how the curves can be shifted up and down by changes in the stock of capital, but rather how external shocks (that can arise from economic policy), may push the economy from one equilibrium to the other: "A movement from one Goudriaan-point to the other will take place only if there occurs a disturbance strong enough to carry the system across the Keynes-point." (1959c: 25). This is illustrated in a schematic

[52] Had he explored it, he would have found something very similar to an extension examined by Solow in his famous 1956 model, where the variability in population growth transforms the curve of break-even investment into a curvilinear form with two unstable equilibria and a stable middle equilibrium. In a paragraph that can only reminisce of what Tinbergen wrote a dozen of years earlier, Solow argued that: "The interesting thing about this case is that it shows how, in the total absence of indivisibilities or of increasing returns, a situation may still arise in which small-scale capital accumulation only leads back to stagnation but a major burst of investment can lift the system into a self-generating expansion of income and capital per head" (Solow, 1956: 91). Or in other words, pump-priming on steroids.

[53] "For deviations to one side they are stable positions of equilibrium, for deviations to the other side unstable, and in such a way that the positions are unstable for disturbances in the direction of the other equilibrium, stable for disturbances in the opposite direction. This sort of equilibria, whose importance for business-cycle explanation has already been stressed by Kaldor, we shall name *mixed* equilibria." (1959c: 24-25).

representation of the movements of the economy between equilibria. Case A, reproduced in figure 7, shows in particular that a sufficient shock can allow the economy to go past the unstable "Keynesian-point" of equilibrium and attain a high (or low) equilibrium.

Tinbergen addresses mathematically these problems on the basis of the following equation implying two lags:[54]

$$y_t = a\, y_{t-1} - b\, y_{t-2}$$

The stability space, as Tinbergen illustrates it, is delimited by the parameters a and b and the particular impact of the upper and lower limits on the movement of the system for the different regions delimited.[55] Six cases in particular are presented in diagrammatic form from numerical computations: three of them where the roots are complex and give cyclical fluctuations around the zero level. The differences between the undamped case and the explosive case is not very important because it is stopped at the limit (supposed to be at five and minus five in Tinbergen's numerical example). There are no surprises in those trajectories, but the movements arising from real roots are more interesting. When the roots are both damped, the movement slowly goes back to the Keynes equilibrium, but also shows a cumulative increase after the initial shock away from this equilibrium. Then there is the case when both roots are real and above one: in this case neither Goudriaan-points and Keynes-points are stable and the system shows periodic, self-sustained oscillations between the two high and low points of equilibrium. The next figure represents this case by extending Tinbergen's computation with the same initial conditions and values for a and b (see case U, Tinbergen, 1959c: 30-31, the circles represent the two initial conditions).

[54] The system studied is a simplification of the equation given in the opening section of the paper, where income is taken as a deviation from the Keynes-point of equilibrium, supposed to be at the zero level. Tinbergen used a standard accelerator relationship that originates from the fact that "expenditure is directly dependent on price increases (gains from speculation), while prices move parallel to income." (1959c: 20), and he obtains the final equation:

$$y'(t)=a'\, y(t-1)+b[y'(t-1)-y'(t-2)]$$

We see that the accelerator relationship comes again from the speculation mechanism, but this should not hide the fact that the model is formally identical to Samuelson's 1939 accelerator-multiplier model, although Tinbergen strangely makes no reference to him. In 1942, Tinbergen gave the following definition of an accelerator relationship, in line with his use of it in this model: "By an acceleration principle we mean quite generally the influence of the rate of increase of any variable on that variable itself." (1942: 146)

[55] An application reproducing the trajectories and the stability space of the model can be found here: https://economic-instability.com/wp-content/applications/tinbergen_1944/.

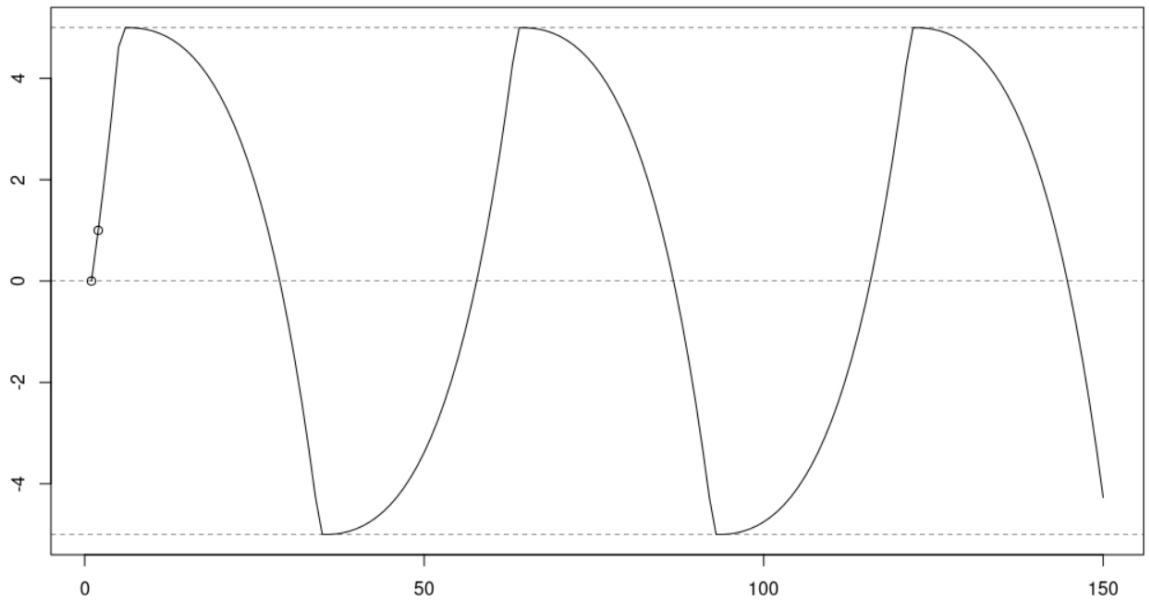

Figure 8: Endogenous cycles in Tinbergen (1944 [1959c])

Finally, and in direct continuity with the 1936 nonlinear model, Tinbergen examined a case where one root is superior to one and the other inferior to one. He shows that in this case both Goudriaan-points are stable equilibrium points, toward which the economy converges after being disturbed from the Keynes-point equilibrium. However those equilibria are only locally stable and a shock of sufficient magnitude can push the economy beyond the unstable Keynes-point equilibrium toward the other Goudriaan-point equilibrium. Tinbergen analyzes the conditions under which a locally stable equilibrium would become unstable, by expressing shocks as a function of the distance between equilibria:

> If thus the system gets a small deflection from the Goudriaan-equilibrium, a movement starts towards the equilibrium position and the original equilibrium is a stable one; for greater departures it is unstable, however. The critical value of this departure ... is thus proportional to the distance from the Goudriaan-point to the Keynes-point. (1959c: 29).

He named those equilibria "*meta-stable* equilibrium, by analogy with the physical equilibrium of the oversaturated solution: this situation survives small departures but in case of greater departures a sudden crystallization takes place." (1959c: 30). This case is illustrated by figure 9 taken from his text. The important difference with the cases presented in figure 7, is that a shock of a relatively small magnitude compared to the distance between equilibria can be sufficient to pass over the unstable equilibria and be attracted to the lower equilibrium; this represents the action of the multiplier, and we see in the figure that a shock of magnitude one is in fact sufficient to fall back to the bottom equilibrium. The argument is very similar to the one derived from the 1936 model presented in the *Wagemann Festschrift*, where we can remark that Tinbergen implicitly supposed a minimum value for prices at zero, that introduced a third

equilibrium similar to the one in this example (although the other equilibria were mathematically different in 1936: they arose from a quadratic form rather than from limit values and "saturation"). In doing so however, Tinbergen rules out the possibility of a complete collapse by concentrating only on situations of a partial fall.

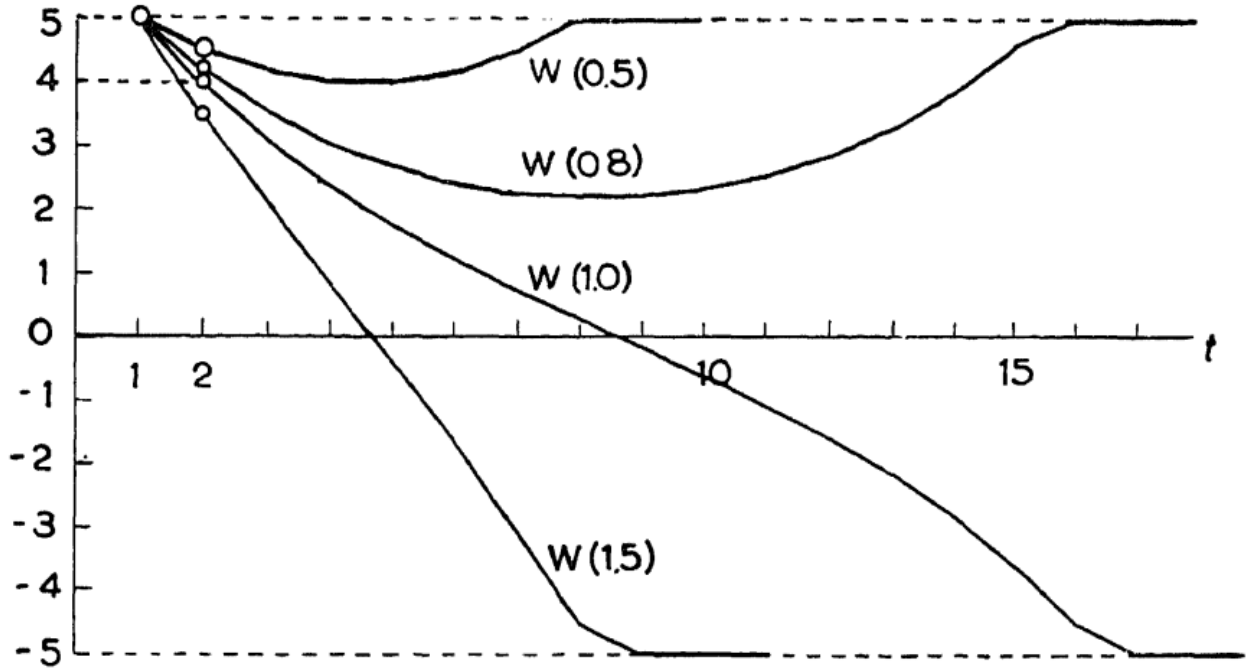

Figure 9: Transitions between equilibria after cumulative shocks in Tinbergen (1944 [1959c])

This exploration allowed him to propose a typology of the possibilities of cyclical reversal, as either perfectly endogenous, extended endogenous (as a consequence of the presence of Goudriaan-points) or exogenous. He ended by a note to his fellow literary economists: "The most difficult ones to 'explain' verbally are the perfectly endogenous turning-points. This is perhaps the reason why they have claimed so little attention in literary theory, in spite of the fact that they are likely to be of the utmost importance for the understanding of economic dynamics." (1959c: 36).

Conclusion

The possibility of multiple equilibria was an important thread followed by Tinbergen throughout the 1930s and early 1940s to understand the behaviour of dynamical economic systems. Presenting it provides new insights on his contribution and the way he understood the potentiality - and dangers - of economic policy.

Subsequently, Tinbergen emphasized in the 1950s the intertwinement between dynamic models and economic policy and kept putting forward the idea that the mathematical structure was not bound to any particular economic mechanisms, while underlining the fact that small changes in

coefficients could profoundly impact the trajectory of the system.[56] But it is clear that the most radical aspects of his initial approach never became central to macrodynamic analysis, which certainly explains their relative neglect.

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[56] "Des modèles identiques en ce qui concerne leur structure qualitative peuvent permettre des mouvements tout à fait différents au fur et à mesure que certains coefficients surpassent certaines valeurs critiques." (1949: 648).

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