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Sovereign default and imperfect tax enforcement

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Yanos Zylberberg**

JEL Codes: E02, E32, E62, F41, H20

Keywords: Sovereign default, Imperfect tax enforcement, Fiscal policy.



Sovereign default and imperfect tax enforcement*

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February 15, 2021

Abstract

The effect of fiscal policy on default risk is mitigated by the response of tax compliance. To explore the consequences of this stylized fact, we build a model of sovereign debt with limited commitment and imperfect tax enforcement. Fiscal policy persistently affects the size of the informal economy, which impacts future fiscal revenues and default risk. The interaction of imperfect tax enforcement and limited commitment strongly constrains the dynamics of optimal fiscal policy and leads to costly fluctuations in consumption.

JEL: E02, E32, E62, F41, H20.

Keywords: sovereign default, imperfect tax enforcement, fiscal policy.

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1 Introduction

When tax enforcement is weak, the relationship between fiscal policy and default risk is mitigated by the size of the informal economy. A fiscal consolidation leads to an expansion of the informal sector, thereby limiting fiscal surpluses, but also hampering future tax collection and failing to reduce default risk. For instance, the European sovereign debt crisis of 2009–2014 saw large fiscal consolidations being implemented in economies with relatively low tax enforcement. These policies led to significant welfare costs but limited effects on default risk.

In this paper, we study the dynamics of fiscal policy and default risk when tax enforcement is imperfect. The contribution of the paper is threefold. We first document stylized facts about tax compliance and its dynamics in economies with imperfect tax enforcement, most notably the relationship with fiscal policy and default risk. We then provide a model of sovereign debt with limited commitment in order to understand how the dynamics of tax compliance—disciplined by the empirical moments—affects optimal fiscal policy and default risk. Finally, we quantify the ignored, yet important, welfare cost associated with imperfect tax enforcement: a *responsive* tax compliance significantly constrains optimal fiscal policies which, ultimately, has an impact on consumption-smoothing.

We uncover novel empirical facts about the dynamics of tax compliance and its impact on default risk. First, we show that tax compliance is volatile and there is large heterogeneity in volatility across countries. Tax compliance is volatile because it strongly responds to fiscal policy and business cycle fluctuations.¹ The heterogeneous volatilities across economies reflect large heterogeneity in such responses. In some economies with imperfect tax enforcement, a larger share of taxpayers hide their activity in downturns and in periods of austerity. In contrast with the standard behavioral response, the magnitude of fluctuations in tax compliance implies sharply decreasing returns to taxes, and some economies display an extreme form of fiscal fatigue (Ghosh et al., 2013). Second, the response of tax compliance to fiscal policy alters the relationship between fiscal policy and default risk. We find that fiscal consolidations are associated with a marked decrease in default risk, but only in countries where tax compliance is inelastic. Instead, when tax compliance strongly

¹We use a measure of VAT (Value-Added-Taxes) compliance built in Pappadà and Zylberberg (2017) as a proxy for general tax compliance: there may be differential responses to different tax instruments that we ignore. The rationale for analyzing VAT is manifold: it constitutes a large share of tax revenues; it is an important adjustment tool at business cycle frequency; reconstructing counterfactual tax revenues under perfect tax compliance requires very few assumptions. The measure uses two different sources, i.e., taxes as received by the government and the reported consumption of goods at a highly disaggregated level (thus measuring frequent, minor changes in VAT), and capture any discrepancies between these two sources.

responds to taxes, this adjustment directly affects default risk and significantly limits the returns to fiscal consolidations.

We explore the implications of fluctuations in tax compliance on the dynamics of optimal fiscal policy in a model of sovereign debt where a benevolent government uses fiscal policy as a consumption-smoothing instrument. The key ingredients of the model are (i) imperfect tax enforcement and (ii) limited commitment to repay sovereign debt. The government stabilizes consumption on behalf of the household and does so through a distortionary tax. Entrepreneurs can adopt two technologies in order to produce the final good: an unobserved technology—which constitutes the informal sector of the economy—and a verifiable technology—the formal sector—in which there are production complementarities.² These two technologies may be interpreted as two different production lines, where one involves intermediaries and thus needs to be transparent. We assume that this choice is staggered, in a similar manner as [Calvo \(1983\)](#), and each entrepreneur may be able to reset technology with a certain probability in each period. The government has limited commitment such that debt prices reflect future incentives to default, and thus the degree to which the economy is expected to be distorted.³

The novelty of the model is to introduce dynamic distortions which interact with sovereign default. In standard models of sovereign debt with limited commitment (e.g., [Eaton and Gersovitz, 1981](#); [Arellano, 2008](#)), default risk depends on an endogenous state variable, the debt level, and exogenous state variables, e.g., productivity. In such a benchmark, a tax increase reduces future debt levels and unambiguously lowers default risk. By contrast, our model adds another endogenous state variable—the current technological choice—which affects the default risk through the future cost of raising tax revenues.⁴ In our setting, a tax increase diverts entrepreneurs

²One key modeling choice relates to imperfect tax enforcement and its relationship with the technological choices of entrepreneurs. Our modeling borrows from the literature on shadow economies with dual technology ([Rauch, 1991](#); [Enste and Schneider, 2000](#); [Straub, 2005](#)). We thus assume that tax monitoring is perfect for the formal technology and absent for the informal technology. This assumption can be relaxed, as long as the effort by the government in uncovering undeclared activity is exogenous, reflecting that *changes* in tax enforcement result from structural reforms implemented at a low frequency. Another important feature of the model is that there exist complementarities across producers of the formal sector. This could be generated by spillovers across production units through innovation, but also be related to increasing returns to tax compliance and built-in incentives to report VAT. A recent contribution, [Pomeranz \(2015\)](#), presents evidence in favor of enforcement spillovers along the production chain.

³There are two punishments following a default which give (limited) commitment to the government: market exclusion and a direct output cost possibly reflecting a failure of the domestic banking sector ([Mendoza and Yue, 2012](#)).

⁴Reciprocally, default risk distorts the decision of entrepreneurs: (i) a future default would induce low taxes in the future; (ii) persistent default risk without default would induce high taxes in the future. These two effects also govern the incentives to operate in the informal sector.

away from the formal sector and lowers returns in the formal sector through production complementarities. This response decreases the contemporary and future cost of raising tax revenues—because of staggered adjustments in technological choice—and tilts the future trade-off between repayment and default. This indirect effect mitigates the gains in debt service through the standard “fiscal surplus” effect: a tax increase may not unambiguously lower default risk.

We calibrate and simulate our model with aggregate productivity shocks and public expenditure shocks. The fundamentals underlying the choice of entrepreneurs are calibrated using three moments uncovered in the empirical section: the level of tax compliance, its elasticity of tax rates, and its persistence. These moments pin down parameters characterizing the domestic economy, i.e., the distribution of returns in the informal sector, the level of complementarities in the formal sector, and the period probability to reset technology.⁵ The remaining parameters of the model are calibrated on a benchmark economy (as in standard models of sovereign defaults, see, e.g., [Aguiar and Gopinath, 2006](#); [Arellano, 2008](#)).

In our quantitative analysis, we evaluate how the *dynamic properties* of tax compliance affect optimal fiscal policy and welfare by comparing two economies differing along the tax compliance response to fiscal policy and business cycle fluctuations around the (same) steady-state level. The *baseline* economy differs from the *low-response* economy in two important dimensions. First, the *baseline* economy is ten times more likely to experience a default (with a yearly probability of 0.2%, and a yearly probability to be excluded from financial markets of 1.8%). Default is more likely, even though the *baseline* economy accumulates far less debt on average (10% of output versus 21%). Second, fiscal policy in the *baseline* economy is less able to smooth fluctuations in consumption: household consumption is much more volatile around the same average levels. We use the model to quantify the costs of such fluctuations and find that they are equivalent to a 2.2% decrease in certainty equivalent consumption. These findings illustrate that fluctuations in tax compliance constrain the set of feasible fiscal policies and significantly lower welfare.⁶

⁵Tax compliance in the model is jointly characterized by the distribution of returns in the informal sector and the level of complementarities in the formal sector. Consequently, it relates to the production technology rather than a tax collection technology. In practice, economies may also differ along the latter dimension, a possibility that we ignore.

⁶The impact of dynamic distortions through tax evasion is not qualitatively different from that of more standard fiscal multipliers ([Doda, 2007](#); [Cuadra et al., 2010](#); [Pouzo and Presno, 2015](#)). Its quantitative implications are however very different. The standard behavioral response to tax policy is one order of magnitude lower than that implied by tax evasion in economies with imperfect tax enforcement. Moreover, standard distortions are quite comparable across economic environments. The elasticity of tax receipts t to tax rates τ , which determines the slope of the Laffer curve, is a function of the elasticities of output y and tax compliance γ , i.e., $\varepsilon_t = 1 + \varepsilon_y + \varepsilon_\gamma$. We find that differences in the latter is what drives variation in the dynamics of fiscal policies

This paper relates to the literature on sovereign default and limited commitment (Eaton and Gersovitz, 1981; Arellano, 2008). As in Arellano (2008), we assume market exclusion upon default, and default mostly occurs in bad times. In contrast with the two previous contributions, however, we explicitly model fiscal policy as a distortionary instrument which affects the future cost of raising tax revenues. One novelty of our approach, compared to numerous contributions (see for instance Aguiar et al., 2005; Cuadra et al., 2010; Bi, 2012; D’Erasmus and Mendoza, 2013; Arellano and Bai, 2016; Pouzo and Presno, 2015), is that distortions induced by fiscal policy do not only affect the contemporaneous choice of fiscal policies, but also future repayments and the debt pricing schedule.⁷ Some economies are at risk of falling into a tax evasion overhang and periods of severe austerity.⁸

We provide empirical evidence on the dynamics in tax compliance and its consequences. In particular, we document a novel stylized fact: the tax compliance response distorts the relationship between fiscal policy and default risk. We rely on a measure of tax compliance constructed in Pappadà and Zylberberg (2017), and document that tax compliance is, in some countries, very volatile, strongly counter-cyclical and very responsive to tax rates.⁹ Our paper relates to the literature having investigated the role of tax evasion and its dynamics during recent debt crises (see Pappa et al., 2015; Dellas et al., 2017; Pappadà and Zylberberg, 2017). In Pappa et al. (2015), tax hikes increase the incentives to conceal part of the activity and produce in the less productive informal sector, thus increasing output and welfare losses. This mechanism affects the size of the fiscal multiplier and explains the failure of the recent consolidation plans in Greece, Italy, Portugal and Spain. The contribution of the present analysis is to study the interaction of such a mechanism with sovereign default.

Our framework rationalizes the use of pro-cyclical fiscal policies in economies with imperfect tax enforcement. In the model, the government generally adopts a counter-cyclical fiscal policy which maintains reasonable debt levels in most future across institutional environments.

⁷Mendoza and Yue (2012) and Bocola (2016) study how default distorts the domestic economy through the collapse of the banking sector, thereby providing additional commitment for governments to repay. A recent literature investigates the interaction between sovereign default or fiscal policy and the private sectors through the movement of workers (Bandeira et al., 2019; Alessandria et al., 2019), instead of capital.

⁸Dovis et al. (2015) develops a model in which a similar dynamic component affects the future cost of fiscal policies. Their framework relies on inequality across domestic citizens as the endogenous state variable affecting default risk, thereby describing an *inequality overhang*.

⁹These findings, also partly highlighted in Pappadà and Zylberberg (2017); Dellas et al. (2017), may help rationalize differences in estimates of fiscal multipliers across environments and fiscal policy tools (Alesina and Ardagna, 2009; Romer and Romer, 2010; Favero et al., 2011; Auerbach and Gorodnichenko, 2012; Iizetzki et al., 2013; Alesina et al., 2015).

states of the World. However, in catastrophic states, economies implement costly pro-cyclical fiscal policies, a paradox that has been highlighted and discussed in the literature (Kaminsky et al., 2004; Ilzetski and Vegh, 2008; Frankel et al., 2013; Vegh and Vuletin, 2015). Many theoretical mechanisms have been discussed to explain this observation.¹⁰ In our model, limited commitment and the proximity to a “debt ceiling” is what drives the government to implement pro-cyclical fiscal policies (Aguiar et al., 2005; Doda, 2007; Cuadra et al., 2010).

The remainder of the paper is organized as follows. In Section 2, we present motivating stylized facts. We then introduce a model of sovereign debt augmented with imperfect tax enforcement in Section 3. Section 4 derives the qualitative predictions, while Section 5 presents the quantitative analysis. Section 6 briefly concludes.

2 Data

In this section, we provide descriptive statistics on the dynamics of taxes and tax compliance. We discuss the following stylized facts. First, the volatility of tax compliance is heterogeneous across countries; these differences are explained by different elasticities to the economic cycle and to tax rates. Second, this response has novel implications on the dynamics of fiscal policy and default risk.

2.1 The dynamics of tax compliance

We describe the construction of our tax compliance measure over time and across countries in Appendix B. The measure is based on the comparison between actual Value-Added Taxes (VAT) revenues and revenues that are predicted by household consumption baskets.¹¹

The volatility of tax compliance Figure 1 (panel a) plots the within-country standard deviation of tax compliance between 1995 and 2013 and compares this measure of volatility to a more direct measure of imperfect tax enforcement, i.e., the size

¹⁰See Kaminsky et al. (2004) for instance on international capital flows, or von Hagen and Harden (1995); Aaron Tornell (1999); Alesina et al. (2008) for explanations based on the redistribution effect of increasing taxes; the competition among taxpayers to receive the proceeds from the positive shock; or the desire to limit rents that politicians could capture.

¹¹VAT is of interest not only because it allows for the construction of a credible measure of tax compliance across countries and over time, but also because it is the preferred tax instrument to adjust to economic conditions. We provide evidence in Appendix Figure A1 that VAT is more frequently adjusted than other taxes. The corporate tax rate and the income tax rate are adjusted once every 8-12 years; adjustments are then non-negligible. By contrast, the effective VAT rate is more often modified, reflecting frequent changes in the classification of goods across the different brackets. Furthermore, the systematic analysis of fiscal consolidations classified by Alesina et al. (2016) shows that almost half of the tax surplus is generated through reforms of indirect taxation.

of the informal sector (as computed in [Schneider and Enste, 2013](#), for 2005–2012). We find a wide disparity in the volatility of tax compliance across countries; interestingly, this disparity is only partly reflected by differences in the size of the informal sector. While there is a positive correlation between the two measures, countries with similar incidence of the informal sector markedly differ along fluctuations in tax compliance—maybe because of the structure of their production. In what follows, we focus on the *volatility in tax compliance* in order to define two groups of countries: those with above-median volatility in tax compliance (*high-response* countries) and those with below-median volatility (*low-response* countries), respectively in red and blue in [Figure 1](#).¹² One important conjecture is that these countries differ in the dynamics of tax compliance because their *response* to economic conditions differs. Panel (b) of [Figure 1](#) shows that variation in the volatility of tax compliance only partly reflects variation in the volatility of tax rates: for a given volatility in tax policy, there remains a large gap between the two groups of countries in terms of tax compliance dynamics. This gap, as we will see later, is explained by heterogeneous responses of tax compliance to economic conditions.

We report summary statistics for tax revenues, tax rates and tax compliance in [Table 1](#). For each variable, we display the sample average, the average in each group of countries, and we generate indicators of within-country fluctuations: the coefficient of variation to capture overall volatility, and the correlation with the cycle to estimate its cyclical component. VAT revenues represent about a third of total tax revenues in both groups of countries and appear to be orthogonal to economic conditions. However, while VAT *revenues* are mostly acyclical or mildly pro-cyclical, VAT *rates* are counter-cyclical. To reconcile the acyclicity of VAT revenues with the sharp counter-cyclicity of VAT rates, we need to analyze fluctuations in tax compliance.¹³ The within-country variations in tax compliance are of the same order of magnitude as variations in revenues. However, and in contrast to revenues, compliance is markedly pro-cyclical thereby counteracting the counter-cyclicity of tax rates. The acyclicity of VAT revenues reflects these two opposing forces: (i) a

¹²The high-response group is composed of the following countries: Bulgaria, Canada, Colombia, Cyprus, Czech Republic, Greece, Hungary, Ireland, Italy, Japan, Luxembourg, Macedonia, Poland, Portugal, Romania, Russia, Slovakia, Slovenia, South Africa, and Spain. The low-response group is composed of the following countries: Australia, Austria, Belgium, Chile, Denmark, Estonia, Finland, France, Germany, Israel, Malta, the Netherlands, New Zealand, Norway, Serbia, Sweden, Switzerland, and the United Kingdom.

¹³In our sample of countries, tax compliance is 0.86 on average, which is arguably quite high—maybe due to the over-representation of rich economies or the built-in incentives to report VAT compared to other taxes. In the sample of high-response economies, tax compliance is slightly lower, around 0.84, reflecting that fluctuations in tax compliance tend to be larger in economies with sizable tax evasion (as shown in [Figure 1](#)).

counter-cyclical VAT rate, and (ii) a pro-cyclical tax compliance.

The (heterogeneous) response of tax compliance to economic conditions

To quantify how tax compliance fluctuates with economic conditions, we estimate the following baseline specification:

$$\Delta \ln \gamma_{tc} = \varepsilon_{\tau} \Delta \ln \tau_{tc} + \beta \mathbf{X}_{tc} + \delta_t + \mu_c + e_{tc}, \quad (1)$$

where t indexes the year and c stands for the country. The dependent variable, $\Delta \ln \gamma_{tc}$, is the annual percentage change in VAT compliance, and $\Delta \ln \tau_{tc}$ is the annual percentage change in tax rates. The vector \mathbf{X} includes time-varying controls, such as changes in output, changes in the sectoral decomposition of economic activity and changes in trade (the ratio of exports and imports over GDP); μ_c captures country-specific trends in tax compliance and δ_t is a year fixed-effect. To capture heterogeneity in the elasticity across countries, we will add an interaction between $\Delta \ln \tau_{tc}$ and a dummy for being a high-response country.

Table 2 uncovers an important, yet overlooked, characteristic of tax compliance: it fluctuates markedly with economic conditions. We find that the average elasticity of tax compliance to the tax rate is about -0.35 (Panel A, see also Pappadà and Zylberberg, 2017), and, as shown in columns 2 and 3, the correlation is robust to the addition of controls (sectoral composition in column 2 and trade in column 3). This average elasticity masks non-negligible heterogeneity between the two groups of countries (see Panel B of Table 2). In our preferred specifications (columns 2 and 3, panel B), the elasticity is much lower for the low-volatility group, -0.13 , than for the high-volatility group, -0.42 .

We now describe a series of robustness checks and results that complement this finding. First, we estimate the elasticity of tax compliance to output, ε_y , by replicating the previous specification with the annual percentage change in GDP per capita as the dependent variable. An increase of one percent in output is associated with an increase of 0.4 percent in tax compliance (see Appendix Table A1). The response is larger for high-response countries: tax compliance is almost acyclical in the low-volatility group while economic conditions explain about 10% of the overall volatility in tax compliance in the high-volatility group. Second, we provide a sensitivity analysis in Appendix Table A2 with the effective VAT rate instead of the standard rate; this specification accounts for changes in the allocation of goods across tax brackets, and changes in reduced and super-reduced rates. Third, we show in Appendix Table A3 that the distinction between the two groups of economies does not overlap with a more general divide “developed”/“developing” economies as adopted in

Vegh and Vuletin (2015). Fourth, we explore *within*-country heterogeneity in the elasticity of tax compliance to tax rates. Two economies with similar fundamentals may respond differently to a tax hike, depending on economic conditions. We define dummies for periods of low output based on HP-filtered GDP growth per capita and interact them with $\Delta \ln \tau_{tc}$ in the previous specification. The results, presented in Appendix Table A4, show that the elasticity of tax compliance to tax rate is not significantly different during recessions. Finally, we quantify the persistence in the dynamics of tax compliance over time in Appendix Table A5. We regress tax compliance on its lag, and we instrument the lagged tax compliance by lagged changes in tax rates. Our findings indicate that tax compliance is persistent and follows an AR(1) process with coefficient between 0.20 and 0.40.

In summary, (i) tax compliance strongly responds to economic conditions, (ii) our dichotomy high/low-response countries based on the volatility of tax compliance reflects higher/lower *responses* to economic conditions and does not align with more standard classifications (e.g., developed/developing countries or relying on the size of the informal sector), (iii) changes in tax compliance are persistent. Distortions implied by an episode of fiscal consolidation could thus have further repercussions on *future* tax collection and default risk. We analyze this effect next.

2.2 Fiscal policy, the dynamics of tax compliance, and default risk

In this section, we analyze how fluctuations in tax compliance impact the relationship between fiscal policy and default risk.

Empirical analysis In a first step, we document the direct relationship between tax compliance and default risk, as measured by the yield on ten-year government bonds. Table 3 reports the regression of changes in bond yields on changes in tax compliance, instrumented in column 2 by changes in VAT rates and controlling for business cycle fluctuations. One percentage point increase in tax compliance decreases default risk by 0.13 percentage points.

The previous specification provides some insight about the impact of tax compliance on default risk, but its interpretation is not straightforward. In the IV specification, the variation in tax compliance is explained by variation in indirect taxation which, itself, may directly affect default risk. Letting $q(b, \gamma)$ denote the debt price as a function of contracted debt b and tax compliance γ , we have that:

$$\frac{dq}{d\tau} = \frac{\partial q}{\partial b} \frac{db}{d\tau} + \frac{\partial q}{\partial \gamma} \frac{d\gamma}{d\tau},$$

and any increase in tax rate τ affects default risk through two opposing effects: a *fiscal surplus* effect (through a decrease in debt) and a *tax compliance* effect (through a decrease in tax compliance). Our previous findings hint at a large *tax compliance* effect.

In a second step, we specifically focus on episodes of fiscal consolidations, and we estimate $dq/d\tau$ in environments where the extent of the tax compliance effect is expected to differ. We collect the episodes of fiscal consolidation classified by [Alesina et al. \(2016\)](#) across 15 countries and 20 years,¹⁴ and regress the change in 10-year bond spread on the total size of these announced and forward-looking fiscal consolidations. We provide the detailed estimates in Table 4. Fiscal consolidations are associated with a marked decrease in default risk in the group of low-response countries. A fiscal surplus of a percentage point of GDP decreases the spread by about 0.2-0.5 percentage points, depending on the definition of fiscal consolidation (including expenditures, and accounting for future fiscal surpluses). The effect of fiscal consolidations could not be more different within the group of high-response countries, pointing to large differences in the relative size of the *tax compliance* effect across environments. In the preferred specification (column 4), the *fiscal surplus* and *tax compliance* effects cancel each other in high-response countries.

Interpretation We have shown that (i) tax compliance is volatile, co-moves with fiscal policy, and (ii) there is a large heterogeneity across countries. These fluctuations in tax compliance strongly shape the relationship between fiscal policy and default risk, as investors appear to internalize future distortions in tax collection.

In what follows, we add such a tax compliance channel in a model of sovereign debt where the constraint induced by the response of the economy to distortionary tax instruments interacts with the constraint induced by the limited commitment to repay. The objective of the model is to rationalize the dynamics of fiscal policies and their impact on default risk. The empirical estimates will be used to discipline the calibration of the production side of the economy. These previous empirical observations do not however discipline our exact modeling of tax compliance.

Our modeling of tax compliance will have the following features. First, we will assume that entrepreneurs face a trade-off between declared and undeclared activity, and this trade-off will be tied to the choice of a production technology. There will be production complementarities in the formal sector, capturing innovation externalities or spillovers along the production chain. These complementarities could

¹⁴These countries are: Australia, Austria, Belgium, Canada, Germany, Denmark, Spain, Finland, France, the United Kingdom, Ireland, Italy, Japan, Portugal, Sweden.

also be interpreted as enforcement spillovers: each intermediary along the production chain has some incentives to declare a transaction and a higher share of formal entrepreneurs would provide incentives to operate in the formal sector (Pomeranz, 2015). The existence of production complementarities induces an inefficient decentralized allocation, with an inefficiently low tax compliance. Changes in the structure of taxes modify the extent to which the economic activity is distorted toward the informal sector. Second, we will posit that the choice of a production technology is staggered, and the dynamics of tax compliance will exhibit some stickiness: (i) some entrepreneurs will not be able to respond to contemporaneous fiscal policy, (ii) the entrepreneurs that do have the opportunity to respond account for the fact they may not be able to do so in subsequent periods.

3 A model of a small open economy

This section develops a model of a small open economy with a representative household and a government. The model embeds production with different technologies associated with different tax enforcement in an otherwise standard framework à la Eaton and Gersovitz (1981) where a benevolent government with limited commitment issues debt on behalf of the household.

The predictions of the model derive from the interaction of two frictions. There is imperfect tax enforcement: transfers from/to the household are made through a distortionary instrument that affects the cost of raising revenues in the future. There is limited commitment from the government to reimburse its debt, and default risk limits its capacity to transfer consumption from the future to the present. These two frictions discipline the joint dynamics of tax compliance and default risk. Tax compliance affects default risk through the future cost of raising tax revenues. The opposite mechanism also plays a role, albeit less important: Default risk affects future returns to the formal sector.

The theoretical predictions of the model do not rely on the exact modeling of distortions associated with imperfect tax enforcement as long as these distortions are persistent and interact with limited commitment. We rely on a simple model where entrepreneurs make a staggered choice between an informal technology and a formal technology, and there are production complementarities in the formal sector.¹⁵

¹⁵We could allow for a less binary tax monitoring, i.e., there would be imperfect tax monitoring associated to both technologies and entrepreneurs would then pay a fine conditional on being detected. We could also allow for some degree of production complementarities in both technologies. The important feature is that the least monitored technology is the one with relatively low production complementarities such as to generate a gap between the social returns to the two technologies, a quantity directly related to the fiscal multiplier.

3.1 Preferences and technology

The economy is populated by a continuum of infinitely lived households of measure one. Letting c_t denote her consumption at time t , the representative household in this economy maximizes expected utility as given by:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s),$$

where $\beta < 1$ denotes the discount factor and $u(\cdot)$ represents the period utility function, which satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Since all households are identical, we refer to them throughout as the representative household. In what follows, we may drop time indices for the sake of exposure.

There are two types of agents who populate households. Each household is composed of a unit mass of entrepreneurs who hold one unit of an investment good in each period, and a unit mass of final good producers. There is perfect redistribution within each household such that all agents consume the same in each period.

The consumption good can be produced with two technologies. First, it can be assembled by final good producers using investment goods. We assume that there are many varieties of such investment goods y_i and there are complementarities between the investment goods y_i when they are used as factors of production for the consumption good.¹⁶ Final good producers assemble investment goods y_i using the following CES technology,

$$y = z \left(\int_0^1 y_i^\phi di \right)^{\frac{1}{\phi}},$$

where $\phi < 1$ captures production complementarities and z is a technology shock which follows a Markov process. We assume that the final good producers are fully transparent. This technology represents the formal sector of our economy.

Second, the final good can be directly produced by entrepreneurs. Entrepreneurs can transform their investment unit into the final good using a private technology with unobservable period-specific return R . We assume that R is known to the entrepreneur only and is independently drawn from a continuous probability distribution $H(\cdot)$ in each period. This technology is not observable to the government and represents the informal sector of our economy.¹⁷ We assume that markets for

¹⁶The return to the formal technology increases with the mass of producers adopting this technology, possibly reflecting spillovers across transparent firms through innovation, complementarities along the production chain or (tax) enforcement spillovers along the production chain.

¹⁷The hypothesis that one technology is fully informal while the other one is fully transparent can be relaxed. There is a need for the technology in which there exist complementarities in production to be relatively more transparent.

investment goods and the final good are perfectly competitive.

Moreover, there are rigidities and technological choice is staggered over time, in the manner of [Calvo \(1983\)](#). In each period, there is an idiosyncratic draw determining whether an entrepreneur is allowed to change her technology. With probability $1 - \theta$, the entrepreneur can choose and either (i) adopt the formal technology, produce an investment good and sell her unit to the final good producer, or (ii) adopt the private technology. With $\theta = 0$, this choice would be completely flexible across periods and the contemporary choice does not commit the entrepreneur in the future.

There is a benevolent government whose objective is twofold. First, in each period, the government needs to produce and finance a public good whose cost g_t is exogenous and subject to shocks, following a Markov process. Second, the government maximizes the welfare of the representative household by issuing debt and purchasing assets on its behalf: the household's borrowing and saving are done exclusively through the government. In order to finance the public investment and transfer from and to the household, we assume that the benevolent government can only levy indirect taxes τ_t on final output.¹⁸

3.2 Financial markets

We assume that the economy is small relative to the international financial market, and that the government can issue and trade one-period bonds on these markets. The international financial market is willing and able to purchase any asset that yields an expected return at least as high as ρ .

The government stabilizes the consumption of the representative household by issuing debt and purchasing assets on its behalf. The debt is financed either through taxation on the household itself or through new debt. The government has imperfect commitment and may default on its obligations. Let b_{t+1} and q_{t+1} respectively denote the amount and price of debt issued by the government at time t , and let $D_t \in \{0, 1\}$ denote the decision to default on previous obligations. If tr_t denotes total indirect taxation levied by the government at time t , the resource constraint for the government is:

$$(1 - D_t) b_t - q_{t+1} b_{t+1} = tr_t - g_t \tag{2}$$

We suppose that there are two sources of punishment which gives some (limited) commitment to the government. As in [Arellano \(2008\)](#), there is an exclusion from

¹⁸We use an indirect tax to match the empirical estimates, but any distortionary instrument with different impact on returns across technologies would generate the same qualitative results.

the international market following a default and reintegration is stochastic and occurs with probability ν in each period. We further assume that, during market exclusion, there is an exogenous default cost Δ that is incurred by the household directly. This cost captures the fact that the domestic intermediation sector may be affected by a default through a capital flight from financial investors (Mendoza and Yue, 2012). Since transfers from the government are made through a distortionary instrument, it is not innocuous to assume that the default cost is paid by the household: the punishment does not distort the choice of entrepreneurs through a direct effect. A government with distortionary tax instruments has incentives to default because a default is then a relatively efficient way to redistribute to the representative household.

In order to solve for the equilibrium of the economy, we need to understand how the government makes savings and investment decisions. We turn to these next.

3.3 Timing of actions and equilibrium

We now specify the timing of actions within each period. At the beginning of each period, the aggregate shocks z_t and g_t are revealed and perfectly observed by all agents. If the government is excluded from international financial markets, a reintegration draw takes place. The government then decides to repay or default on its past obligations, b_t , and commits to an indirect taxation rate τ_t . Production takes place and taxes are paid by agents. International financial markets open, and sovereign bonds are traded. Finally, households consume.

In order to characterize the equilibrium of this economy, we need to explore the dynamic optimization problem of entrepreneurs, the dynamic optimization problem of the government and how investors price sovereign debt. We turn to these separate programs next.

Entrepreneurs and final good producers We first describe the (static) program of final good producers. In each period, these producers take prices p_i of each intermediate good variety as given and maximize:

$$\max_{y_i} \left\{ z(1 - \tau) \left(\int_0^1 y_i^\phi di \right)^{\frac{1}{\phi}} - \int_0^1 p_i y_i di \right\}.$$

The resulting demand for variety i is characterized by the following equation:

$$z(1 - \tau)y_i^{\phi-1} \left(\int_0^1 y_i^\phi di \right)^{\frac{1}{\phi}-1} = p_i.$$

In equilibrium, all the different varieties will be produced at the same price, which will be denoted thereafter by r_t —the return to the formal sector.

We now focus on the program of entrepreneurs. In each period, there is a draw determining the return to the private technology, a random variable R_t , and an idiosyncratic draw determining whether an entrepreneur can set her technology. With probability $1 - \theta$, the entrepreneur can freely choose to adopt any of the two technologies. With probability θ , the entrepreneur keeps the same technology as in period $t - 1$.

The reader interested in the exact derivation of technological choices can refer to Appendix C.1; the description below simplifies the argument. An entrepreneur with the opportunity to re-optimize in period t and with unobserved individual return R_t considers the path of future (and possibly stochastic) returns to the different technologies and chooses the technology with the highest expected revenues, which is equivalent to maximizing revenues in future states of nature where re-optimization is not possible. The expected revenues in the formal sector *in those states of nature* are:

$$E_t \left[\sum_{k=0}^{\infty} \theta^k \delta_{t,t+k} r_{t+k} \right],$$

where r_{t+k} is the price for one unit of the differentiated good in the formal sector in period $t + k$, θ^k is the probability of not having re-optimized after k periods and $\delta_{t,t+k} = \beta^k \frac{u'(c_{t+k})}{u'(c_t)}$ is the discount factor between period t and period $t + k$. Since returns to the private technology are independent across periods, the equivalent expected revenues in the informal sector are,

$$E_t \left[\sum_{k=1}^{\infty} \theta^k \delta_{t,t+k} R_{t+k} \right] = R_t + E_t \left[\sum_{k=1}^{\infty} \theta^k \delta_{t,t+k} \right] \bar{R}.$$

where $\bar{R} = \int R dH(R)$ is the expected return to the private technology. The previous expression heavily relies on the fact that draws of future returns are assumed to be idiosyncratic, independent shocks.

Let \tilde{R}_t denote the level of unobserved return in the informal sector for which an individual is indifferent between the two technologies. At first order, the discount factor between period t and period $t + k$ is β^k , and we have:

$$\tilde{R}_t + \frac{\theta\beta}{1 - \theta\beta} \bar{R} = \sum_{k=0}^{\infty} (\theta\beta)^k E_t r_{t+k},$$

The indifference threshold, \tilde{R}_t , thus verifies the following recursive equation (see

Appendix C.1),

$$\tilde{R}_t = r_t + \theta\beta \left(E[\tilde{R}_{t+1}] - \bar{R} \right). \quad (3)$$

Among entrepreneurs with the opportunity to modify their technology, the share of them adopting the formal technology, γ_t^* , should be equal to the ones with sufficiently low returns to the informal sector, i.e., $\gamma_t^* = H \left[\tilde{R}_t \right]$. Transparency, γ_t , defined as the aggregate share of entrepreneurs operating in the formal sector, verifies the following dynamics:

$$\gamma_t = (1 - \theta)\gamma_t^* + \theta\gamma_{t-1}. \quad (4)$$

We combine the previous equations to derive the (sluggish) dynamics of aggregate transparency, which governs distortions on the production side of the economy, as a function of returns to the formal sector:¹⁹

$$H^{-1} \left(\frac{\gamma_t - \theta\gamma_{t-1}}{1 - \theta} \right) = r_t + \theta\beta \left[E_t H^{-1} \left(\frac{\gamma_{t+1} - \theta\gamma_t}{1 - \theta} \right) - \bar{R} \right]. \quad (5)$$

where the return to the formal sector, r_t , is equal to $z_t(1 - \tau_t)\gamma_t^{\frac{1}{\phi}-1}$ (see the demand for the differentiated good with γ_t units produced in total).

Equation (5) describes the sluggish dynamics of transparency due to technological rigidities: (i) some entrepreneurs have not been given the opportunity to respond to economic conditions (see Equation 4), (ii) entrepreneurs with the opportunity to re-optimize internalize that they may not be able to do so in subsequent periods. The latter effect is best understood in Equation (3): the contemporary indifference threshold depends on the returns to the formal sector but requires a premium to compensate for future losses induced by not adjusting technology and the gap between the future indifference threshold and the expected returns to the informal technology. Two assumptions guarantee that past transparency, γ_{t-1} , is sufficient to keep track of past choices of entrepreneurs: (i) the independence of draws determining whether an entrepreneur can re-optimize, (ii) the independence of returns to the private technology across periods.

Equation (5) also shows how expectations about future transparency influence the current choice of entrepreneurs. A high transparency in future periods increase future returns to the formal sector and induce more re-optimizing entrepreneurs to opt for the formal sector today. Through this channel, default risk directly affects the current choices of entrepreneurs, independently from the current level of taxes.

¹⁹We restrict our analysis to distributions H with a support such that Equation (5) implicitly defines a unique solution γ_t as a function of $(\gamma_{t-1}, \gamma_{t+1}, r_t)$.

Households Households make no saving or borrowing decisions in our economy. Once they receive the output net of taxes w , their consumption is given by

$$c = w$$

In order to smooth consumption, the government needs to smooth output net of taxes through the level of these indirect taxes.

Bond prices Investors are ready to buy any bonds in period t as long as these bonds guarantee at least ρ in expectations in period $t + 1$. The bond price verifies

$$q_{t+1} = \rho^{-1} E_t [1 - D_{t+1}] \quad (6)$$

Bond prices range between 0, when default is expected with certainty next period, and ρ^{-1} .

Government The government is assumed to be benevolent and to maximize the welfare of consumers in each period t by choosing the levels of taxes τ_t , public savings b_{t+1} and default decisions D_t , subject to the budget constraint (2), the endogenous response of entrepreneurs, as characterized by Equation (5), and the bond price equation (6).

We are now ready to define a recursive equilibrium of our economy and shed light on the main trade-off underlying government decisions.

4 Recursive equilibrium and dynamic properties

In this section, we define the equilibrium characterizing the economy in a recursive form and provide some intuitions behind the main mechanisms at play in the model.

4.1 Recursive equilibrium

Our environment is one in which entrepreneurs and the government do not interact cooperatively over time: entrepreneurs form beliefs about future government decisions in order to set their current technology. As we suppose limited commitment towards foreign investors, we also assume that the government does not have any commitment device towards domestic entrepreneurs about its future choices. The set of equilibria that will be considered are thus Markov perfect equilibria in which agents perfectly observe a common state vector.

Let us define this state of the economy. Given the assumptions that (i) productivity and expenditure shocks follow a Markov process, (ii) technological choices are staggered in the manner of Calvo (1983), (iii) returns to the private sector are independently drawn across periods, (iv) market reintegration is idiosyncratic, the following quantities fully characterize the economy at the beginning of period t : the state of the economy with respect to international markets, $h_t \in \{A, X\}$, where A is access and X is exclusion; the inherited asset position b_t ; the inherited state of technological choices γ_{t-1} ; and current productivity z_t and public expenditures g_t . For convenience, let $\mathbf{s}_t = (b_t, \gamma_{t-1}, z_t, g_t)$ denote the state of the economy, except from access to international bond markets.

A recursive Markov perfect equilibrium is given by a sequence of debt $\{b_{t+1}\}$, transparency $\{\gamma_t\}$, default decisions $\{D_t\}$ and bond prices $\{q_{t+1}\}$ satisfying the following conditions in all periods and histories:

Definition 1. *In each period t , the government debt and default decisions maximize the representative household's welfare, given the state (h_t, \mathbf{s}_t) of the economy, and subject to the period budget constraint (2) and the bond price equation (6). The entrepreneurs maximize their profits, and Equation (5) defines the dynamics of transparency, tax receipts and output.*

The government value function in period t is,

$$v(\mathbf{s}_t) = \mathbb{1}_{h_t=A} \cdot v_A(\mathbf{s}_t) + \mathbb{1}_{h_t=X} \cdot v_X(\mathbf{s}_t),$$

where the conditional value functions v_A and v_X follow the following recursive formulations:²⁰

$$v_A(\mathbf{s}_t) = \max_{D_t, b_{t+1}, \gamma_t, \tau_t} \{u(c_t) + \beta(1 - D_t)E_t v_A(\mathbf{s}_{t+1}) + \beta D_t E_t v_X(\mathbf{s}_{t+1})\},$$

$$v_X(\mathbf{s}_t) = (1 - \nu) [u(c_t(1 - \Delta)) + \beta E_t v_X(\mathbf{s}_{t+1})] + \nu \max_{b_{t+1}, \gamma_t, \tau_t} \{u(c_t) + \beta E_t v_A(\mathbf{s}_{t+1})\},$$

subject to Equations (2), (5) and (6).²¹ Note that, as the government is not allowed to return the proceeds of its external borrowing to the representative household as a lump-sum transfer, there is only one instrument to transfer from and to the household: in effect, the choice of future debt pins down the choice of the tax rate (through the budget constraint) and reciprocally. The government does face a debt

²⁰Without market access, the government makes no decisions because the period tax rate, and thus the level of transparency, are pinned down by the budget constraint $tr_t = g_t$.

²¹The interested reader can refer to the Appendix C.2 for a description of the default sets with two endogenous state variables and two variables characterizing the state of nature.

price schedule which depends on our two endogenous state variables, $q(b_{t+1}, \gamma_t, z_t, g_t)$, but the two arguments are always tied through the budget constraint.

4.2 Static and dynamic distortions

In this section, we describe the static distortions induced by fiscal policy on the allocation of entrepreneurs across technologies, and their impact on default risk and fiscal policy.

Static distortions We first analyze the static distortions implied by a fiscal consolidation on transparency and total output. In the model, the relationship between transparency and fiscal policy is disciplined by Equation (5).²² Letting ε denote the elasticity of transparency to tax *rate*, the elasticity of transparency to tax *receipts*, verifies

$$\frac{\partial \gamma_t}{\partial tr_t} \frac{tr_t}{\gamma_t} = \frac{1}{1/\phi + 1/\varepsilon}.$$

The degree of complementarity in the formal sector, ϕ , determines the extent to which fiscal policy (through a change in the fiscal surplus) distorts the allocation of entrepreneurs. Specifically, when spillovers across entrepreneurs are large and ϕ is sizable relative to $-\varepsilon$, a change in fiscal surplus has a large effect on the allocation of entrepreneurs. As the tax rate increases, entrepreneurs switch to the less productive informal sector, which, with complementarities in production, sharply drives down the returns to the formal sector and creates a multiplier effect captured in $1/\phi$. Such a change in the allocation of entrepreneurs across technologies affects consumption $c_t = w_t$. In Appendix C.3, we show that the effect of a change in fiscal surplus on consumption (i.e., the fiscal multiplier) is:

$$\frac{\partial c_t}{\partial tr_t} = -1 + \left[\frac{r_t}{\phi} - \tilde{R}_t \right] \frac{\partial \gamma_t}{\partial tr_t}$$

With lump-sum transfers, a change in fiscal surplus would generate a one-to-one loss in consumption and total output would be left unchanged. Instead, when taxes are distortionary ($\partial \gamma_t / \partial tr_t < 0$), consumption drops even further, and the size of leakages depends upon (i) the shift in the allocation of entrepreneurs between the formal and informal sector and (ii) the difference between the social returns in the formal and informal sectors $r_t/\phi - \tilde{R}_t$. The degree of complementarity in the formal sector impacts both quantities. When complementarity is high in the formal sector, there are large differences between the social returns in the formal

²²The reader interested in the derivation of the elasticity of tax compliance to the tax rate, $\varepsilon < 0$, may refer to Appendix C.3.

and informal sectors at equilibrium and transparency sharply responds to changes in fiscal surplus. In our model with non-Ricardian households, the previous equation characterizes the size of the fiscal multiplier, which is always greater than 1.

Static distortions induced by fiscal policy have dynamic consequences, which materialize in the response of default risk to fiscal policy and ultimately in the choice of optimal fiscal policy. Section 5 will shed light on these consequences using a quantitative model, we briefly discuss the mechanisms at play next.

Distortions, default risk and fiscal policy In this section, we describe some dynamic properties of the recursive equilibrium and show how our mechanism modifies the conclusions of a model without distortionary redistribution between the government and households.

To understand the trade-off faced by the government, one can use two polar cases (developed in Appendix C.4): full commitment in which default risk does not constrain fiscal policy; and market exclusion in which the government cannot smooth consumption on behalf of agents. When the government is fully unconstrained in international debt markets, fiscal policy is countercyclical and mitigates fluctuations in output and distortions over time. When the government is fully constrained in international debt markets, fiscal policy is procyclical and amplifies fluctuations in output and distortions. We now explain the interaction of distortions with limited commitment in the general case.

A fiscal consolidation affects default risk as follows. While an increase in tax rates leads to higher fiscal surplus, it also lowers the contemporary return r_t to the formal sector through two channels. There is a direct effect deriving from higher taxes. There is an indirect channel deriving from entrepreneurs switching to the informal sector thereby lowering the returns to the formal sector. Through Equation (5), this affects the indifference threshold between the formal and the informal sector, and thus the number of entrepreneurs operating in the informal sector. With staggered technological choices, this response is persistent and increases the future costs of raising tax revenues. Investors anticipate both the decrease in debt levels and the more indirect effect on future tax collection. The latter reduces the gains in fiscal consolidations, and is absent from any benchmark model à la [Arellano \(2008\)](#) with non-distortionary redistribution.

While the previous mechanism is always at work, at least qualitatively, its quantitative relevance crucially depends on fundamentals and the state of the economy. The production technology determines the elasticity of transparency to the return r_t in the formal sector. A large elasticity would make fiscal consolidations more distor-

tionary. The probability to set technology influences how contemporary shocks affect future transparency. With fully flexible or fully rigid choices, a fiscal consolidation would have little influence on future transparency and would not be distortionary in the long-run. Finally, the elasticity of default risk to the future costs of raising tax revenues crucially depends on the debt level. It is only above a certain debt level that the future default sets are not empty.

The *tax compliance* effect impacts the return to fiscal consolidations, and the dynamics of optimal fiscal policies. Following a negative shock to productivity, the government has incentives to lower tax rates through three distinct channels. First, the government is willing to smooth available income to the household. Second, low productivity induces a low return to the formal sector and thus larger inefficiencies. Third, low tax rates would increase the returns to tax rates in the future. Lowering tax rates would be optimal, but only if debt price is not very sensitive to future debt. When, instead, debt prices markedly respond to future debt, an expansionary policy in bad times would increase debt service thereby mitigating the effect of lower distortions. In states of the economy where both the *fiscal surplus* and *tax compliance* channels play a role, current and future tax revenues are not very sensitive to fiscal policy which has three implications: (1) default risk is not sensitive to fiscal policy, (2) optimal fiscal policy may be (weakly) pro-cyclical; (3) the dynamics of the system is stable.

5 Quantitative analysis

In this section, we calibrate the model to match the key facts documented in the empirical section, and we evaluate its capacity to describe the joint dynamics of fiscal policy and default risk.

5.1 Calibration and solution method

Calibration In order to calibrate the model, we proceed as follows. We set all parameters but the ones characterizing the production side of the economy for a benchmark economy (as is standard in models of sovereign default, see, e.g., [Aguiar and Gopinath, 2006](#)). We then separately estimate the production parameters by matching key empirical moments: (a) the transparency level, (b) the elasticity of transparency to tax rate, (c) the auto-correlation of transparency and (d) the fiscal multiplier, all evaluated at the steady state. Table 6 lists all parameter values.

We set a discount factor β corresponding to an annual discount rate of 10%. We assume, in contrast with the theoretical section, that the government is more

impatient than external investors: the risk-free interest rate, $\rho-1$ is set equal to 2.5%. We use a Constant-Relative-Risk-Aversion utility function with a parameter $\sigma = 2$. The output cost Δ is set at 4% of the steady-state output, and the reintegration probability ν is set such that the average exclusion length is 10 years. As in [Aguilar and Gopinath \(2006\)](#), we consider auto-correlated productivity and spending shocks, and we set the standard deviations of innovation shocks to match the volatility of output and public expenditures in the data. We normalize steady-state productivity z to be equal to 1, and public expenditures g to be 28% of output—an average between the mean public expenditure and the mean tax revenue in the sample of countries used in [Section 2](#).

The model still requires parameter values for fundamentals governing the production side of the domestic economy. There is no direct empirical counterpart, and we use indirect moments to set these parameters. More specifically, we use a uniform distribution with bounds a_1 and a_2 to model the distribution of returns H . We then set a_1 , a_2 , the degree of complementarity within the formal sector ϕ and the Calvo parameter, θ , such as to match (a) the steady-state transparency of 0.85 (see [Table 1](#)), (b) the elasticity of transparency at the steady-state of -0.35 (see [Table 2](#)), (c) the fiscal multiplier at the steady-state of 1.5, as estimated by [Pappa et al. \(2015\)](#) in high-evasion countries, (d) the persistence of transparency at the steady-state of 0.30 (see [Appendix Table A5](#)). The steady-state and volatility of transparency mostly pin down the shape of returns in the informal sector; the fiscal multiplier disciplines complementarities in the formal sector; the persistence of transparency pins down the period probability to reset technology.²³

Solution method We solve numerically the model as a fixed-point problem. The government does not have commitment on future policy and takes it as given when deciding upon contemporary choices. The problem is solved by iteration and we find the fixed point by looping over the future behavior of the government and debt pricing. More specifically, we initially set two policy functions $\mathbf{s} \mapsto \gamma(\mathbf{s})$ and $\mathbf{s} \mapsto q(\mathbf{s})$, which characterize future transparency, as a function of the vector \mathbf{s} of state variables, and future expected default, as a function of contemporary transparency and debt. Given the policy functions $\mathbf{s} \mapsto \gamma(\mathbf{s})$ and $\mathbf{s} \mapsto q(\mathbf{s})$, we solve the dynamic problem of the government through value function iteration. We then update the policy functions $\mathbf{s} \mapsto \gamma(\mathbf{s})$ and $\mathbf{s} \mapsto q(\mathbf{s})$, using the choice of transparency and default obtained in the previous step, and we iterate until we converge to the fixed point.

In practice, we need to keep track of four state variables: the inherited debt level,

²³We provide the formulas for these theoretical moments in [Appendix C.5](#).

the inherited transparency and the two shocks. We create rough grids for each of the state variables: The AR(1) processes for productivity and public expenditures are discretized using the [Tauchen \(1986\)](#) method. At each iteration of the previous algorithm, we use cubic spline interpolation to approximate the value function and the (given) policy functions, following [Hatchondo et al. \(2010\)](#), such that the government effectively solves a continuous problem.

5.2 Properties of the calibrated economy

Policy functions We first analyze the properties of the calibrated model by analyzing the policy functions $(D_t, q_{t+1}, \gamma_t, b_{t+1})$. For the sake of exposition, we display each policy response as a function of the two endogenous state variables (b_t, γ_{t-1}) —averaged over the possible exogenous state variables (see [Figure 2](#)).

Default risk is nil for low levels of inherited debt b_t , and steadily increases for b_t between 10% and 15% of output. This steady increase of default with inherited debt implies that: (i) there exists a large region in the set of state variables where default is likely, and (ii) the government may be tempted to accumulate debt within this region, as the marginal cost of doing so is not immediately prohibitive. The relationship between debt b_{t+1} and inherited debt b_t is initially close to being linear. For higher levels of inherited debt, however, debt reaches a maximum just before the default region and then sharply decreases. This pattern derives from two effects: (a) the threat of default restrains government choices; (b) the government sometimes does default and future debt is then reset to 0.

The dynamics of transparency—a key property of the calibrated economy—exhibits stickiness, as expected from the staggered technological adjustments of entrepreneurs. Transparency decreases with inherited debt until it reaches a minimum just before the default region and then increases slowly (because of actual defaults). The dynamics of transparency is also important to understand the dynamics of default risk. At high levels of debt, inherited transparency affects default: a low inherited transparency significantly lowers the returns to taxes, and transfers from the household are costly. As a consequence, low inherited transparency generates lower fiscal surplus, higher debt levels, and a higher default risk. The combination of the previous effects constrains fiscal policy: the threat of default induces higher taxes, which in turn lower transparency and raise the risk of default in the future (the *tax compliance* effect). This optimal policy response will give rise to costly fluctuations in consumption, as we see next.

Simulated moments Section 2 uncovers two key empirical relationships: (i) how economic conditions affect tax compliance, (ii) how fluctuations in tax compliance impact the relationship between fiscal policy and default risk. We now evaluate the ability of the model to capture these quantitative relationships. To this purpose, we simulate 800 economies characterized by the same primitives as our baseline calibrated economy and hit by productivity and expenditure shocks over 100 periods (years). As primitives of production are set to replicate how tax compliance (i.e., transparency in the model) varies with fiscal policy, we derive an untargeted moment, the average elasticity of transparency to *output*, and find that a one percentage increase in output is associated with a 0.50 percentage increase in transparency (versus 0.40 in the data).²⁴

The second key empirical relationship is the sensitivity of default risk to the fiscal surplus; this moment is disciplined by the separate effects of debt levels and transparency on default risk in the simulated economy—the *fiscal surplus* and *tax compliance* effects mentioned in Section 2. In order to quantify the separate contribution of the partial dependence of default risk on (i) debt level and (ii) transparency, we regress (log) debt prices on (log) debt level and (log) transparency in the simulated data and we jointly instrument the endogenous regressors with the expenditure shock and the productivity shock.²⁵ We find that a one percent decrease in debt, as a fraction of output, increases debt prices by 1.15 percent. A higher tax rate does not only induce a decrease in debt, but also a decrease in transparency. The *tax compliance* effect counteracts the direct effect of taxes on *fiscal surplus*: the increase in debt prices implied by an increase in tax rate is 60% lower because of the dynamics of tax compliance. The calibrated model thus replicates the fact that the government does not succeed in markedly lowering default risk by increasing the tax burden—when it does so.

5.3 The dynamics of tax compliance and its welfare cost

We next explore the costs induced by the dynamics of tax compliance and its impact on optimal fiscal policies. To do so, we compare two (simulated) environments: the previous economy in which the behavioral response of the economy is large—a *baseline* economy, calibrated using the average elasticity of tax compliance to

²⁴Our simulated economies exhibit an average fiscal multiplier of 1.45, close to the (targeted) value as evaluated locally around the steady-state economy.

²⁵The correlation between fiscal surplus and default risk differs from causal estimates. Indeed, the government mostly implements fiscal consolidation when the other covariates (e.g., the shocks in the simulations) make them less distortionary. In other words, the timing and magnitude of fiscal policy are not exogenous.

the tax rate ($\varepsilon_\gamma = -0.35$); an economy with similar steady-state environment but smaller response of entrepreneurs—a *low-response* economy, calibrated using the lower bound for the elasticity of tax compliance to the tax rate ($\varepsilon_\gamma = -0.13$). The two economies only differ through the dispersion of informal returns, governing the response of tax compliance to economic conditions: the shockless steady-state economies would coincide. We simulate 800 economies over 100 periods (years) in both cases, using the same primitives—apart from the ones disciplining the volatility of tax compliance—and the same shock structure.

We first illustrate the relationship between debt, transparency and default across the two environments in Figure 3. The top panels (a) and (b) display the distribution of debt and transparency in the *baseline* economy; the red lines represent average expected default as a function of inherited debt and transparency. We contrast these distributions and default schedules with those of the *low-response* economy—panels (c) and (d). The dynamics of debt and fiscal policy in the two environments differ along two key dimensions. First, the government faces a steep debt price schedule for high levels of debt and low levels of transparency in the *baseline* economy; the *low-response* economy does not appear to exhibit such strong dependence of default risk on past transparency. Second, the *low-response* economy exhibits a much less dispersed distribution of transparency. Debt can be, and is indeed, stabilized by fiscal policy in this environment; the stabilization allows to sustain debt levels very close to a “debt ceiling” without inducing large variations in tax rates. This does not hold in the *baseline* economy where the government more frequently chooses extreme fiscal policies leading to high but also very low levels of transparency.

We better quantify the differences between the two environments in Table 6. We first report the means and standard deviations for debt, transparency, consumption (Panel A) and the yearly probabilities of default and market exclusion (Panel B). The *low-response* economy is able to accumulate more debt on average (21.5% of output, versus 10%); this difference is non-negligible as it only arises from the sensitivity of technological choices to economic conditions. The *low-response* economy does accumulate more debt, but not at the expense of default risk. Indeed, the *baseline* economy is ten times more likely to experience a default and the probability to be excluded from debt markets is 1.8% versus 0.2% in the *low-response* economy.

The low incidence of default in the *low-response* economy illustrates that fiscal policy stabilizes default risk. This stabilization is however done in parallel with consumption smoothing. Indeed, while average transparency and consumption are similar across environments, their volatilities markedly differ. In the *low-response* economy, taxes are stable. In the *baseline* economy, taxes are volatile, which in-

duces fluctuations in consumption. To quantify the cost of these fluctuations, we report the average welfare in Panel C and calculate the associated average certainty equivalent consumption. In the *low-response* economy, fluctuations in consumption induce a wedge between the average consumption and the average certainty equivalent consumption of 0.8%. The same wedge is 3% in the *baseline* economy. The difference-in-wedges provides an estimate for the cost of the consumption volatility implied by the dynamics of tax compliance interacted with limited commitment: this effect accounts for 2.2% in certainty equivalent consumption—one order of magnitude higher than the consumption costs of (rather infrequent) defaults.

In conclusion, the tax compliance channel—as disciplined by the sensitivity of technological choices to taxes—does not only produce different default rates across environments but also implies distinct fiscal policies under default risk. The dynamics of tax compliance forces the government to implement volatile fiscal policies which are still unable to prevent default. These findings are consistent with fiscal consolidations having limited impacts on default risk, like those observed in peripheral European economies plagued by tax evasion (see for instance Greece and Portugal between 2008 and 2014).

6 Final remarks

In this paper, we study one implication of imperfect tax enforcement—the dynamics in tax compliance—on default risk and optimal fiscal policy. We show that there are important differences across countries in the volatility of tax compliance, partly reflecting its response to economic conditions. These fluctuations significantly alter the relationship between fiscal policy and default risk. To explore the implications of this empirical regularity, we introduce imperfect tax enforcement in a simple model of sovereign debt with limited commitment. We show that the interaction of limited commitment and imperfect tax enforcement strongly influences the dynamics of fiscal policy during default crises.

With imperfect tax enforcement, fiscal policy impacts the technological choice of entrepreneurs, which affects the future trade-off between repayment and default. This effect lowers the returns to fiscal consolidations and constrains the dynamics of optimal fiscal policies with non-negligible implications on the capacity of governments to smooth fluctuations in consumption. We show that optimal fiscal policy markedly differs across environments that are similar in all dimensions but the behavioral response of the informal sector to economic conditions. The response of tax compliance strongly affects the government capacity to (i) accumulate debt, (ii) avoid default, and (iii) smooth economic shocks.

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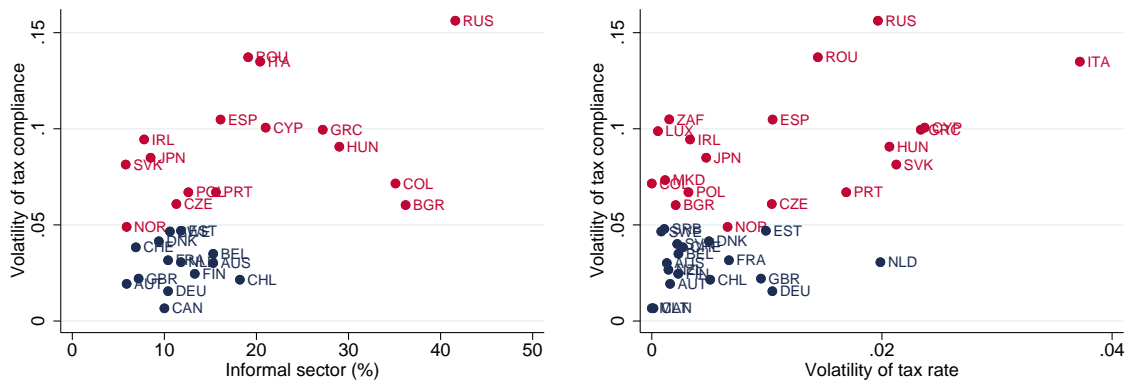
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Figures and tables

Figure 1. Volatility in VAT compliance, volatility in effective VAT rate, and size of the informal sector (red: above-median volatility in tax compliance, blue: below-median volatility).

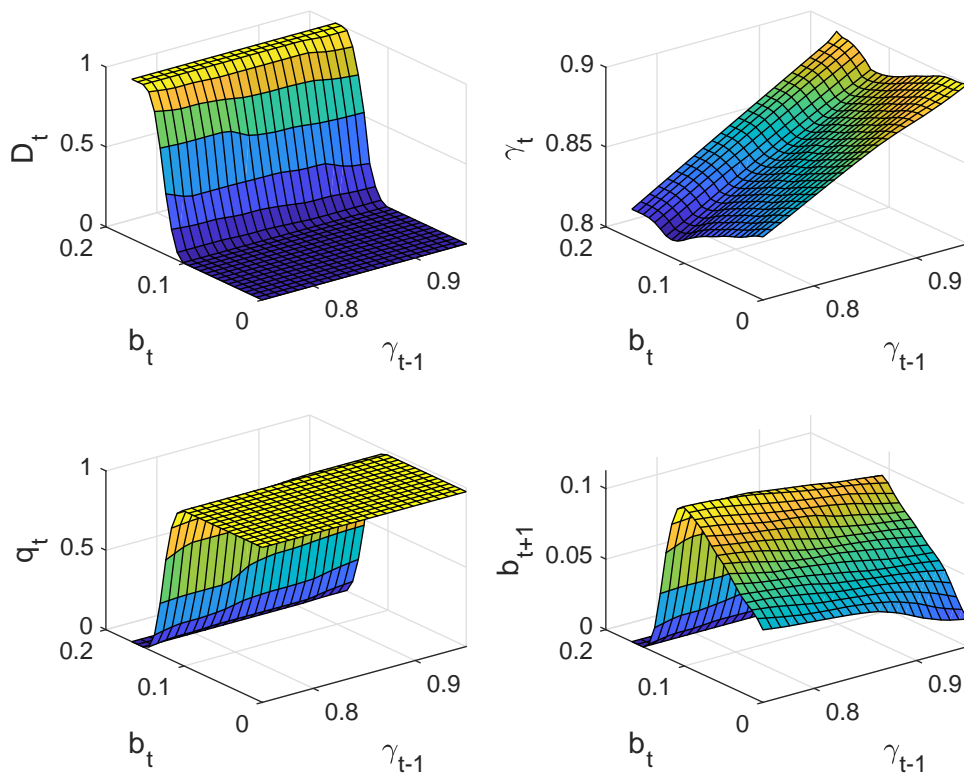


(a) Volatility in VAT compliance and size of the informal sector.

(b) Volatility in VAT compliance and volatility in effective VAT rate.

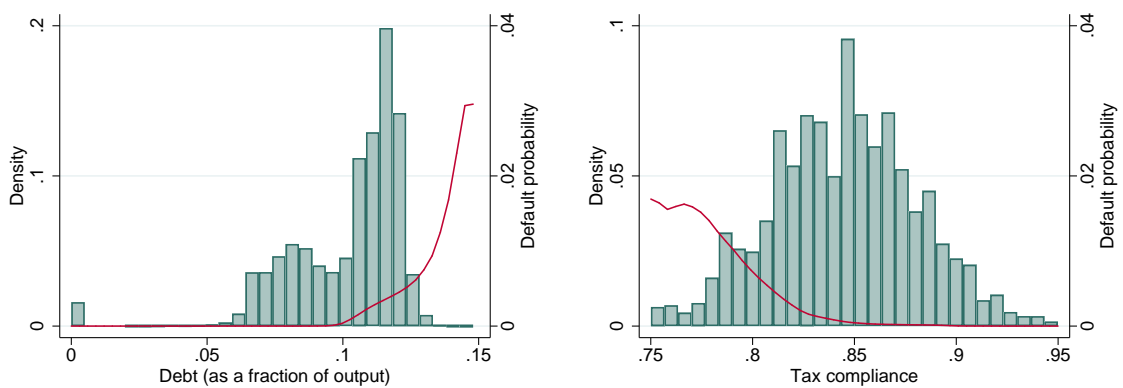
Notes: Panel (a) (resp. b) displays the within-country standard deviation of VAT compliance along the y-axis (1995–2013), and the size of the informal sector (2005–2012) as computed in [Schneider and Enste \(2013\)](#) (resp. within-country standard deviation of the effective VAT rate) along the x-axis.

Figure 2. Policy functions and debt price schedule as a function of inherited transparency and debt.



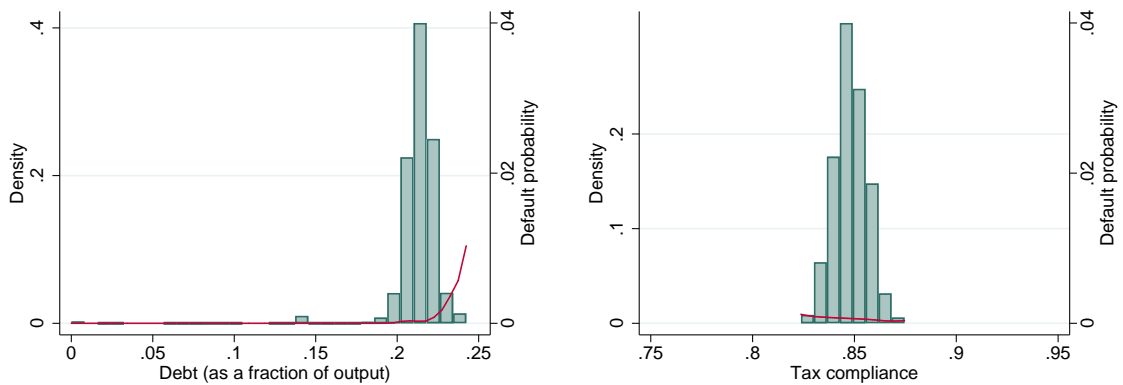
Notes: This Figure displays the policy choice as a multivariate function of b_t and γ_{t-1} , and averaged over the other (exogenous) state variables, z_t and g_t .

Figure 3. Distribution of debt and transparency in the *baseline* economy and in the *low-response* economy.



(a) Debt (% of output, baseline).

(b) Transparency (baseline).



(c) Debt (% of output, low-response).

(d) Transparency (low-response).

Notes: The *baseline* (top panels) and *low-response* results (bottom panels) are derived from simulating 800 economies characterized, and hit by productivity and expenditure shocks over 100 periods (years). The red lines are local regressions using expected default as the dependent variable.

Table 1. Descriptive statistics.

	Mean			Cyclicalit
	All	High-response	Low-response	
Tax revenue (% GDP)	21.52 [0.124]	21.09 [0.113]	21.91 [0.132]	0.092
VAT revenue (% tax revenue)	32.20 [0.078]	31.88 [0.097]	32.45 [0.059]	0.024
VAT rate (%)	14.44 [0.081]	14.12 [0.107]	14.75 [0.049]	-0.100
VAT compliance	0.862 [0.082]	0.847 [0.106]	0.878 [0.052]	0.212

Notes: Coefficient of variations, i.e., the ratio of within-country standard deviation to the mean, are reported between square brackets. The *Cyclicalit* index is computed by taking the annual change in the dependent variable, and estimate its correlation with the annual change in HP filtered GDP per capita. See Section 2 for the definition of *high-response* and *low-response* countries.

Table 2. Elasticity of tax compliance to VAT standard rates.

Tax compliance	(1)	(2)	(3)
Panel A: Average effect			
Tax rate	-.348 (.066)	-.352 (.067)	-.351 (.067)
Panel B: Heterogeneous effect			
Tax rate	-.132 (.139)	-.132 (.140)	-.129 (.140)
Tax rate \times High-response	-.277 (.157)	-.283 (.158)	-.285 (.158)
Controlling for sectoral composition	No	Yes	Yes
Controlling for trade	No	No	Yes
Observations	527	527	527

Notes: Robust standard errors are reported between parentheses. All specifications include year- and country-fixed effects. The dependent variable is the yearly percentage change in VAT compliance. *Tax rate* is the yearly percentage change in VAT standard rate. See Section 2 for the definition of *high-response* countries.

Table 3. Default risk and tax compliance.

Δ Spread (10 yrs)	(1)	(2)
Δ Tax compliance	-.037 (.016)	-.126 (.053)
Observations	334	334
Sample	All	All
Specification	OLS	IV
F-stat (first stage)	-	31.79
Extended controls	Yes	Yes

Notes: Robust standard errors are reported between parentheses. All specifications are in differences and include country-fixed effects. The instrument in the IV specification is the yearly change in the effective VAT rate.

Table 4. Default risk and fiscal consolidations.

Δ Spread (10 yrs)	(1)	(2)	(3)	(4)
Fiscal consolidation	-.00215 (.00075)	-.00403 (.00172)	-.00439 (.0016)	-.00521 (.00292)
Fiscal consolidation \times High-response	.00371 (.00089)	.00457 (.00209)	.00762 (.00194)	.00563 (.00344)
Observations	209	209	209	209
Fiscal consolidations	Taxes and expenditures		Taxes	
Unexpected/anticipated	$u + a$	u	$u + a$	u
Extended controls	Yes	Yes	Yes	Yes

Notes: Robust standard errors are reported between parentheses. All specifications are taken in differences and include country-fixed effects. Units of observation are episodes of fiscal consolidations, as collected by [Alesina et al. \(2016\)](#). The dependent variable is the change in 10-year bond spread (in percent) and the explaining variable is the size of the fiscal consolidation (taxes and expenditures in columns 1 and 2, taxes only in column 3 and 4). Columns 1 and 3 use the total expected fiscal consolidation implemented in period t , including future discounted fiscal surpluses; columns 2 and 4 use contemporary fiscal surpluses, excluding future periods. See Section 2 for the definition of the *High-volatility* dummy.

Table 5. Calibrated parameters.

Parameter		Value
Preferences		
Discount factor	β	0.90
Interest rate	$\rho - 1$	0.025
Risk-aversion (CRRA)	σ	2
Technology		
Distribution (informal)	a_1	-0.72
Distribution (formal)	a_2	1.18
Complementarities	ϕ	0.90
Probability to set technology	θ	0.32
Default		
Probability of reintegration	ν	0.10
Output cost	Δ	0.04
Shocks		
TFP z , autocorrelation	ρ_z	0.85
TFP z , standard deviation	σ_z	0.015
Expenditures g , autocorrelation	ρ_g	0.85
Expenditures g , standard deviation	σ_g	0.035

Notes: See Section 5 and Appendix C.5 for a detailed description of the calibration target, in particular for the *Technology* parameters. The calibration targets $M1$, $M2$, $M3$, $M4$ are the elasticity of tax compliance to the tax rate ($\epsilon_\gamma = -0.35$), the steady-state level of tax compliance ($\bar{\gamma} = 0.85$), the fiscal multiplier at the steady state (1.50), and the persistence of tax compliance (0.30).

Table 6. Simulated moments in the *baseline* economy and in the *low-response* economy.

	Baseline $\epsilon_\gamma = -0.35$	Low-response $\epsilon_\gamma = -0.15$
Panel A: Debt level, transparency and consumption		
Debt level (% output)	0.102 [0.022]	0.215 [0.015]
Transparency	0.845 [0.036]	0.848 [0.008]
Consumption	0.668 [0.075]	0.672 [0.043]
Panel B: Episodes of default		
Default	.0018	.0002
Market exclusion	.0176	.0025
Panel C: Welfare		
Welfare	-15.53 [1.245]	-15.05 [0.528]
Equivalent consumption	0.647 [0.041]	0.665 [0.022]

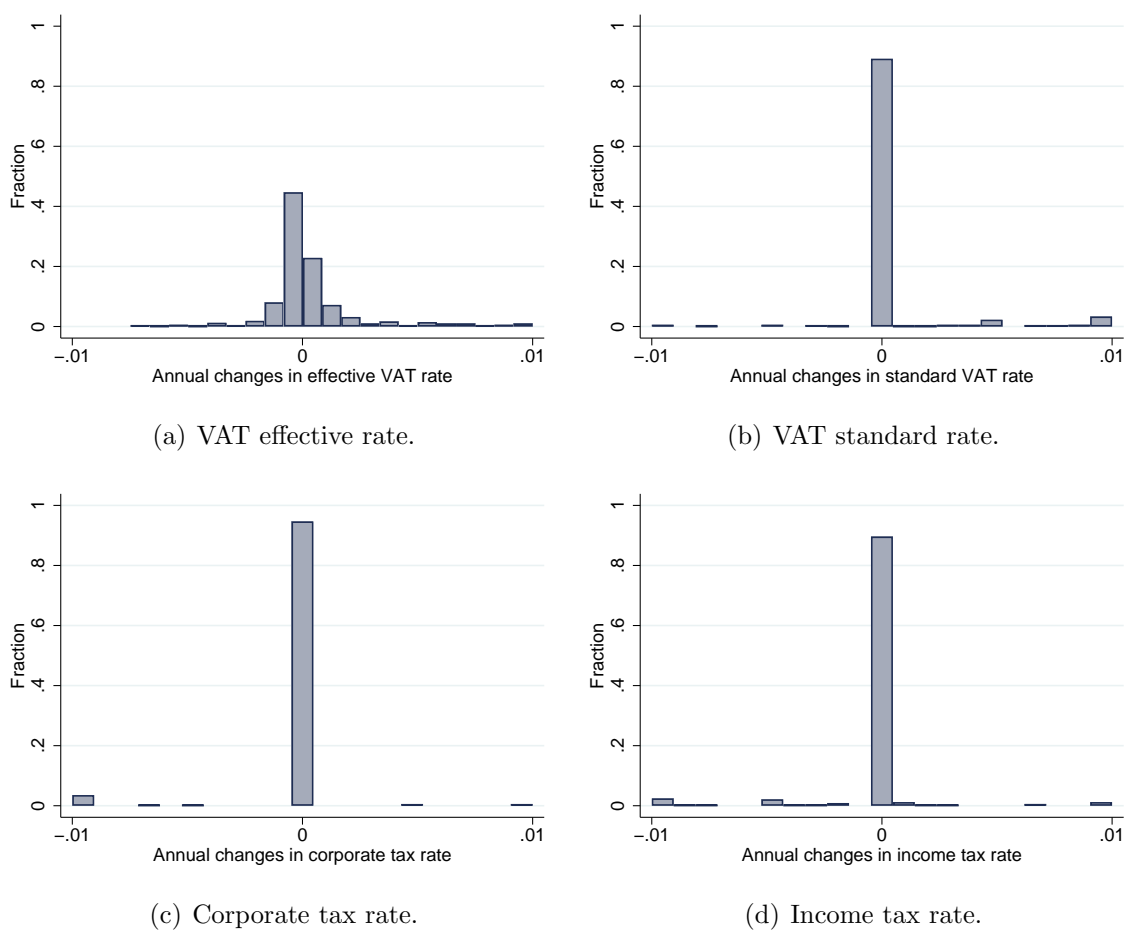
Notes: The *baseline* and *low-response* results are derived from simulating 800 economies characterized, and hit by productivity and expenditure shocks over 100 periods (years).

ONLINE APPENDIX—not for publication

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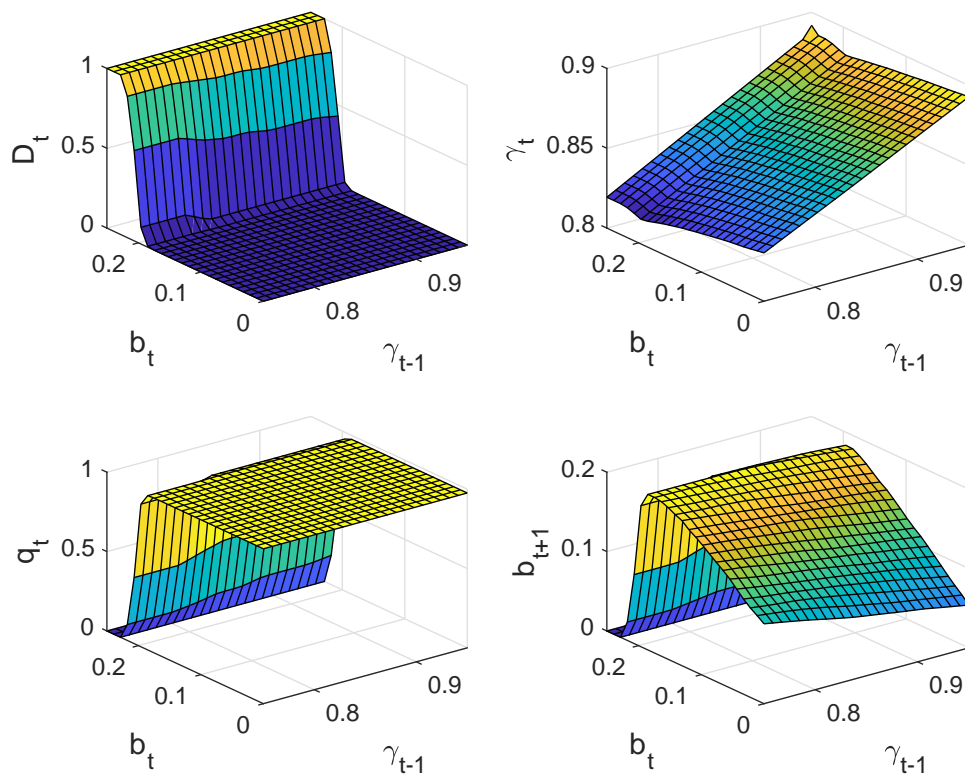
A Additional tables and figures

Figure A1. Distribution of annual changes for the different tax instruments (VAT, corporate tax, income tax).



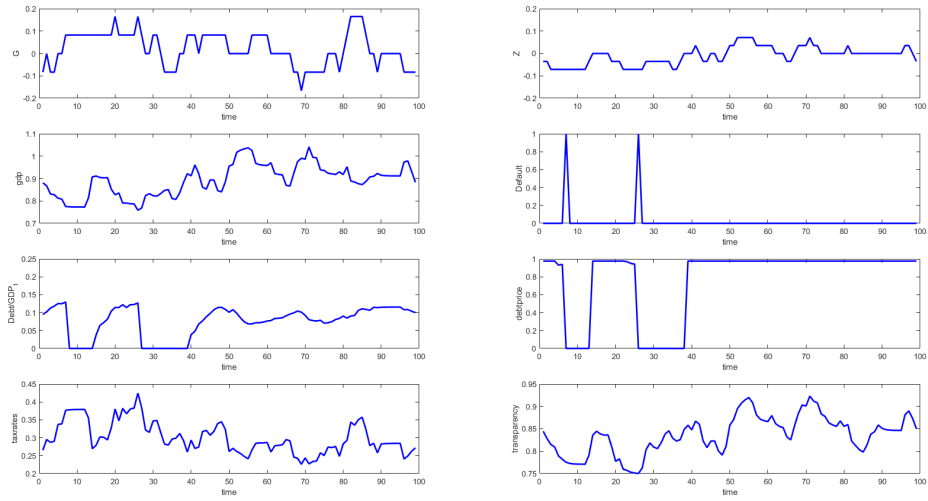
Notes: This Figure displays the distribution of annual changes between 1995 and 2013 in the rates of VAT (panels a and b), corporate tax (panel c) and income tax (panel d).

Figure A2. Policy functions and debt pricing schedule as a function of inherited transparency and debt (*low-response economy*).

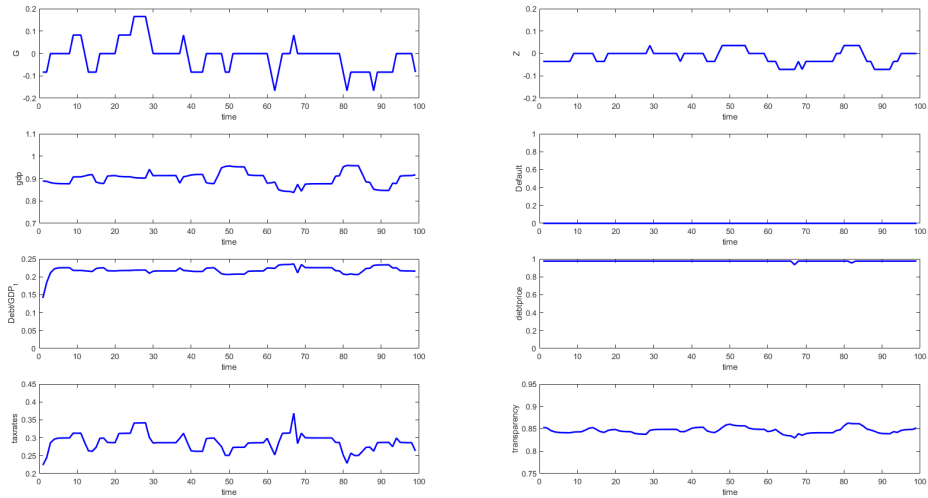


Notes: This Figure displays the policy choice as a multivariate function of b_t and γ_{t-1} , and averaged over the other (exogenous) state variables, z_t and g_t .

Figure A3. Examples of simulated economies—*baseline* economy and *low-response* economy.



(a) Baseline economy.



(b) Low-response economy.

Notes: This Figure displays two examples of a simulated economy in the *baseline* calibration (panel a) and in the *low-response* calibration (panel b).

Table A1. Elasticity of tax compliance to cyclical fluctuations.

Tax compliance	(1)	(2)	(3)
Panel A: Average effect			
GDP growth	.305 (.138)	.316 (.142)	.325 (.142)
Panel B: Heterogeneous effect			
GDP growth	.043 (.173)	.043 (.178)	.059 (.178)
GDP growth \times High-response	.474 (.190)	.486 (.192)	.472 (.193)
Controlling for sectoral composition	No	Yes	Yes
Controlling for trade	No	No	Yes
Observations	527	527	527

Notes: Robust standard errors are reported between parentheses. All specifications include year- and country-fixed effects. The dependent variable is the yearly percentage change in VAT compliance. GDP growth is the yearly percentage change in GDP. See Section 2 for the definition of *high-response* countries.

Table A2. Robustness analysis—elasticities of VAT rates to VAT *effective* rates.

Tax compliance	(1)	(2)	(3)
Panel A: Average effect			
Tax rate	-.501 (.053)	-.505 (.053)	-.503 (.053)
Panel B: Heterogeneous effect			
Tax rate	-.240 (.120)	-.240 (.121)	-.237 (.121)
Tax rate \times High-response	-.320 (.133)	-.323 (.133)	-.325 (.133)
Controlling for sectoral composition	No	Yes	Yes
Controlling for trade	No	No	Yes
Observations	527	527	527

Notes: Robust standard errors are reported between parentheses. All specifications include year- and country-fixed effects. The dependent variable is the yearly percentage change in VAT compliance. *Tax rate* is the yearly percentage change in VAT effective rate, accounting for changes in exempt, reduced, and super-reduced rates as well as compositional changes across categories of goods. See Section 2 for the definition of *high-response* countries.

Table A3. Robustness analysis—heterogeneity along the volatility of VAT compliance versus “developed” economies.

Tax compliance			
	(1)	(2)	(3)
Panel A: Cycle			
GDP growth	.042 (.201)	.069 (.204)	.086 (.205)
GDP growth × High-response	.601 (.198)	.600 (.201)	.583 (.202)
GDP growth × Industrial	-.217 (.208)	-.237 (.214)	-.233 (.214)
Panel B: Tax rate			
Tax rate	-.293 (.197)	-.311 (.199)	-.300 (.199)
Tax rate × High-response	-.169 (.168)	-.166 (.168)	-.170 (.168)
Tax rate × Industrial	.158 (.146)	.176 (.148)	.169 (.148)
Controlling for sectoral composition	No	Yes	Yes
Controlling for trade	No	No	Yes
Observations	488	488	488

Notes: Robust standard errors are reported between parentheses. All specifications include year- and country-fixed effects. The dependent variable is the yearly percentage change in VAT compliance. *Tax rate* is the yearly percentage change in VAT standard rate. See Section 2 for the definition of *high-response* countries.

Table A4. Elasticity of tax compliance to VAT standard rates—heterogeneity along the cycle.

Tax compliance			
	(1)	(2)	(3)
Tax rate	-.411 (.087)	-.336 (.072)	-.399 (.085)
Tax rate × Recession 1	.138 (.130)		
Tax rate × Recession 2		-.090 (.197)	
Tax rate × Recession 3			.126 (.132)
Observations	527	527	527
Extended controls	Yes	Yes	Yes

Notes: Robust standard errors are reported between parentheses. All specifications include year- and country-fixed effects. The dependent variable is the yearly percentage change in VAT compliance. *Tax rate* is the yearly percentage change in VAT standard rate. *Recession 1* is equal to 1 when the HP-filtered GDP is negative; *Recession 2* is equal to 1 when the HP-filtered GDP is below the 25th percentile; *Recession 3* dummy is equal to 1 when the HP-filtered GDP is below the 50th percentile.

Table A5. Persistence of VAT compliance.

	OLS	IV
Tax compliance	(1)	(2)
Tax compliance in $t - 1$.416 (.052)	.228 (.150)
Observations	451	451
F-stat (first stage)	-	42.40

Notes: Robust standard errors are reported between parentheses. All specifications include year- and country-fixed effects, controls for GDP growth and controls for yearly changes in industrial composition. The instrument in the IV specification is the yearly (log) change in effective tax rates.

B Construction of the measure of tax compliance

B.1 A measure of tax compliance

The construction of a tax compliance measure relies on [Pappadà and Zylberberg \(2017\)](#). To measure tax compliance, we rely on a simple flat tax, the Value-Added Tax, which is the preferred instrument to adjust fiscal policy to economic fluctuations. Our measure of tax compliance compares tax receipts to expected receipts as predicted by tax rates and actual expenditures. Letting $tr_{i,t,c}$, $\tau_{i,t,c}$ and $c_{i,t,c}$ denote VAT revenues, VAT rate and consumption of good i in year t and country c , the measure of VAT compliance is defined as:

$$\gamma_{t,c} = \frac{\sum_i tr_{i,t,c}}{\sum_i (\tau_{i,t,c} c_{i,t,c})}.$$

The gap between tax revenues and expected tax revenues, as captured by the distance between $\gamma_{t,c}$ and 1, reflects imperfect tax enforcement from tax authorities.²⁶ The measure accounts for possible changes in consumption patterns $c_{i,t,c}$ as a response to differential tax rates across goods: Fluctuations in tax compliance can only arise from changes in tax compliance *within* good categories.

We use distinct data sources for total tax revenues, and for reported consumption of 48 disaggregated good categories between 1979 and 2013 in about 40 countries.²⁷ We observe total VAT receipts, $tr_{t,c} = \sum_i tr_{i,t,c}$, in national accounts. We use annual household expenditure surveys to create actual consumption, $c_{i,t,c}$, in each sub-category of good.²⁸ The information in household surveys comes from the purchaser side thereby alleviating under-reporting of undeclared transactions. We also extract from the European Commission documentation and national sources the different tax rates and we reference good categories that are subject to these rates for each country/year. Categories like medical services, international public transport, basic food products or cultural services are subject to reduced rates or exemptions. There are frequent adjustments in the composition of exempted categories. Volatility in the effective VAT rate, $\sum_i (\tau_{i,t,c} c_{i,t,c}) / \sum_i c_{i,t,c}$, derives from large, infrequent changes in standard rates, and from smaller, frequent adjustments

²⁶Such measure cannot however shed light on the nature of tax leakages, whether they come from informal exemptions, corruption of tax authorities or non-cooperative tax evasion from agents.

²⁷We rely on OECD and Eurostat and their harmonized 48 COICOP (Classification of Individual Consumption by Purpose) sub-categories of goods.

²⁸These household surveys are standardized across countries, and they follow similar methodology (i.e., sampling and questionnaire). The aggregate consumption constructed from these surveys strongly correlate with total output, but there exists (standard) measurement error (see [Aguiar and Bils, 2015](#); [Kolsrud et al., 2019](#), for a correction method and a comparison with registry data). This error will translate into measurement error in the denominator of our expression for $\gamma_{t,c}$.

in the composition of exempted or reduced-rates categories.

B.2 Corrections and adjustments

We implement the following corrections in order to clean our measure of tax compliance. First, we are interested in short-term fluctuations, and we cannot allow for “high-frequency” measurement error. Tax reforms are often implemented during the year, while national accounts are closed at the end of each period, i.e., year or quarter. When the tax structure is changed during the course of the year, we construct the annual effective tax rate by weighting each tax rate by the consumption observed during its spell. When consumption could not be observed at a higher frequency than annually, we construct the annual effective tax rate by weighting each tax rate with the time during which it was enforced. Second, some tax reforms may affect differently sub-categories of goods within a 2-digit classification. Assume, for instance, that we do not observe consumption in art galleries, but we observe consumption for a larger category (“cultural goods”). For many countries entering in the European Union, art galleries would pass from category 1 to category 3. We would reconstruct an expected tax revenue for cultural goods by considering the average share of sub-categories over the period. Along the same lines, VAT can be collected for all registered firms or there may exist a minimum threshold. In the case of a reform affecting this threshold, we recreate the new expected revenue by subtracting the average share of value added created by firms below the threshold. Third, some reforms modify the tax environment without modifying tax rates for any category of goods, e.g., adopting online forms. We collect this information and control for such reforms in the empirical specification.

C Complements to the theory

In this section, we describe the detailed derivation of dynamic technological choices, we provide a characterization of the response to fiscal policy, we describe default sets, we provide a brief analysis of the model dynamics through two extreme cases and we derive theoretical moments which will discipline the calibration of the model.

C.1 Technological choices

Consider an entrepreneur with the opportunity to choose technology and with current return to the private technology, R .

Let $\rho_{t'}$ denote her expected discounted revenues if she could re-optimize in any future period t' and let $R_{t'}$ denote the random draw for the private technology in period t' . Her expected discounted revenues if she were to choose the private technology in period t would be:

$$E_t \left[R + \sum_{k=1}^{\infty} \theta^k \delta_{t,t+k} R_{t+k} + \sum_{k=1}^{\infty} \theta^{k-1} (1-\theta) \delta_{t,t+k} \rho_{t+k} \right]$$

Her expected discounted revenues if she were to choose the formal technology in period t would be:

$$E_t \left[r_t + \sum_{k=1}^{\infty} \theta^k \delta_{t,t+k} r_{t+k} + \sum_{k=1}^{\infty} \theta^{k-1} (1-\theta) \delta_{t,t+k} \rho_{t+k} \right]$$

Letting R_t denote the indifference threshold, i.e., the level of private returns for which the entrepreneur is currently indifferent between the two technologies, and $\bar{R} = \int R dH(R)$ the average expected return to the private technology, we have:

$$R_t + \sum_{k=1}^{\infty} \theta^k \delta_{t,t+k} \bar{R} = E_t \left[r_t + \sum_{k=1}^{\infty} \theta^k \delta_{t,t+k} r_{t+k} \right]$$

At first order, we have:

$$\begin{cases} R_t + \frac{\theta\beta}{1-\theta\beta} \bar{R} & = r_t + \sum_{k=1}^{\infty} (\theta\beta)^k E_t r_{t+k} \\ R_{t+1} + \frac{\theta\beta}{1-\theta\beta} \bar{R} & = r_{t+1} + \sum_{k=1}^{\infty} (\theta\beta)^k E_{t+1} r_{t+k+1} \\ \dots & \end{cases}$$

Differencing the first and second equations (and using the law of iterated expectations), we get:

$$R_t - \theta\beta E_t R_{t+1} + \theta\beta \bar{R} = r_t$$

The indifference threshold thus verifies the following recursive equation,

$$R_t = r_t + \theta\beta (E[R_{t+1}] - \bar{R}).$$

Among entrepreneurs with the opportunity to modify their technology, the share of them adopting the formal technology, γ_t^* , should be equal to the ones with sufficiently low returns to the informal sector, i.e., $\gamma_t^* = H[R_t]$. Transparency, γ_t , defined as the aggregate share of entrepreneurs operating in the formal sector, verifies the following dynamics:

$$\gamma_t = (1 - \theta)\gamma_t^* + \theta\gamma_{t-1}.$$

We combine the previous equations to derive the (sluggish) dynamics of aggregate transparency, which governs distortions on the production side of the economy, as a function of returns to the formal sector:

$$H^{-1}\left(\frac{\gamma_t - \theta\gamma_{t-1}}{1 - \theta}\right) = r_t + \theta\beta \left[E_t H^{-1}\left(\frac{\gamma_{t+1} - \theta\gamma_t}{1 - \theta}\right) - \bar{R} \right].$$

The previous equation describes the sluggish dynamics of technological choices due to the staggered nature of such choices. Two effects underlie these dynamics: (i) a backward-looking rigidity, because some entrepreneurs have not been given the opportunity to respond to economic conditions in the current period, (ii) a forward-looking rigidity, because entrepreneurs with the opportunity to re-optimize internalize that they may not be able to do so in subsequent periods.

Given the previous allocation of entrepreneurs across technologies, tax receipts and consumption can be written as:

$$\begin{cases} tr_t = \tau_t z_t \gamma_t^{1/\phi} \\ c_t = w_t = (1 - \tau_t) z_t \gamma_t^{1/\phi} + \theta(1 - \gamma_{t-1})\bar{R} + (1 - \theta) \int_{R_t}^{\infty} R dH(R) \end{cases} \quad (7)$$

Production is the sum of production in the formal sector and production with the private technology. The latter is a combination of (i) an unbiased selection of entrepreneurs—the number of entrepreneurs operating with the private technology in the previous period and with expected return \bar{R} in period t , and (ii) a biased selection of entrepreneurs—the number of entrepreneurs having just re-optimized and with high returns to the private technology.

The previous equations imply that: (i) past transparency, γ_{t-1} , is sufficient to keep track of past choices of entrepreneurs, (ii) expectations about future transparency influence the current choice of entrepreneurs because they will affect future returns to the formal sector.

The general formulation of current choices as a function of choices in $t - 1$ and in $t + 1$ (see Equation 5) relies on the independence of draws determining whether an entrepreneur can re-optimize as in the original model of staggered price-setting (Calvo, 1983). Importantly, the independence of returns to the private technology across periods ensures that production in the informal sector is independent of previous draws and its dependence on past choices is fully captured by the number of entrepreneurs operating with the private technology in the previous period.

C.2 Default sets

As in Eaton and Gersovitz (1981) or Arellano (2008), the decision to default can be fully described by a default set $D(b, \gamma)$, which is a set of states of nature (z, g) under which the government prefers to default, as a function of the endogenous state variables (b, γ) . The recursive equilibrium of this economy is then defined as a set of price functions for bonds, policy functions for the government including $D(b, \gamma)$ such that (i) the government policy functions solve the government problem taking as given price functions for bonds and the dynamics of transparency as defined by Equation (5), and (ii) bond prices reflect the default probabilities implied by the policy functions $D(b, \gamma)$.

These default sets defined above satisfy the following monotonicity property: default sets are monotonous in inherited debt. If $b_1 < b_2$, then $D(b_1, \gamma) \subseteq D(b_2, \gamma)$. The proof of this property is immediate by contradiction. Assume that there exists a state of nature (z, g) such that $(z, g) \in D(b_1, \gamma)$ but $(z, g) \notin D(b_2, \gamma)$. The maximum utility reached after a default is independent of current debt b_1 or b_2 . By contrast, the maximum utility that can be reached with reimbursement depends on current debt. Let b' and γ' denote the chosen debt level, transparency and indifference threshold conditional on reimbursing for the state (b_2, γ) . We have that $u(w(b', b_2, \gamma, z, g)) + \beta E_t v_A(b', \gamma', z', g')$ is greater than the value of default because $(z, g) \notin D(b_2, \gamma)$ by assumption. However, the utility from reimbursement associated with inherited debt b_1 and the same targets (b', γ') would be $u(w(b', b_1, \gamma, z, g)) + \beta E_t v_A(b', \gamma', z', g')$ and would be higher than the utility from default because

$$w(b', b_1, \gamma, z, g) \geq w(b', b_2, \gamma, z, g)$$

As a consequence, reimbursement is preferred to default, and $(z, g) \notin D(b_1, \gamma)$, which contradicts the initial hypothesis.

C.3 Response to fiscal policy

Few elasticities Consider the relationship relating tax receipts to tax rates and transparency, $tr_t = \tau_t z_t \gamma_t^{1/\phi}$. We now derive few equations which are used in the paper, notably to analyze the effect of fiscal policy.

Let ε_x denote the elasticity of x with respect to the tax rate τ and let η_x denote the elasticity of x with respect to tax receipts t . As z_t is exogenous, we have:

$$\varepsilon_x = 1 + \varepsilon_\gamma / \phi$$

and

$$1 = \eta_\tau + \eta_\gamma / \phi$$

We can combine these two equations to retrieve the elasticity of transparency to tax receipts (see Section 4 in the paper):

$$\eta_\gamma = \frac{1}{1/\varepsilon_\gamma + 1/\phi}$$

We now provide a formula to characterize a crucial moment of the analysis, the elasticity of transparency to the tax rate. We differentiate Equation (5) with respect to the tax rate τ_t :

$$\frac{\partial \gamma_t}{\partial \tau_t} = \varepsilon_\gamma \frac{\gamma_t}{\tau_t} = \frac{-z_t \gamma_t^{1/\phi-1}}{\frac{1}{1-\theta} \left(\frac{1}{h(R_t)} + \theta^2 \beta E_t \frac{1}{h(R_{t+1})} \right) - (1-\tau_t) z_t (1/\phi - 1) \gamma_t^{1/\phi-2}}$$

Effect of fiscal policy and fiscal multiplier The previous formula helps us understand the impact of fiscal policy on fiscal receipts and consumption:

$$\begin{cases} \frac{\partial tr_t}{\partial \tau_t} = z_t \gamma_t^{1/\phi} + \tau_t z_t \frac{1}{\phi} \gamma_t^{1/\phi-1} \frac{\partial \gamma_t}{\partial \tau_t} \\ \frac{\partial c_t}{\partial \tau_t} = \frac{\partial w_t}{\partial \tau_t} = -\tau_t z_t \gamma_t^{1/\phi} + (1-\tau_t) z_t \frac{1}{\phi} \gamma_t^{1/\phi-1} \frac{\partial \gamma_t}{\partial \tau_t} - (1-\theta) R_t h(R_t) \frac{\partial R_t}{\partial \tau_t} \end{cases}$$

As $R_t = H^{-1} \left(\frac{\gamma_t - \theta \gamma_{t-1}}{1-\theta} \right)$, we have $\frac{\partial R_t}{\partial \tau_t} = \frac{1}{1-\theta} \frac{1}{h(R_t)} \frac{\partial \gamma_t}{\partial \tau_t}$, and the previous expressions become:

$$\begin{cases} \frac{\partial tr_t}{\partial \tau_t} = z_t \gamma_t^{1/\phi} + \tau_t z_t \frac{1}{\phi} \gamma_t^{1/\phi-1} \frac{\partial \gamma_t}{\partial \tau_t} \\ \frac{\partial c_t}{\partial \tau_t} = \frac{\partial w_t}{\partial \tau_t} = -\tau_t z_t \gamma_t^{1/\phi} + (1-\tau_t) z_t \frac{1}{\phi} \gamma_t^{1/\phi-1} \frac{\partial \gamma_t}{\partial \tau_t} - R_t \frac{\partial \gamma_t}{\partial \tau_t} \end{cases}$$

The previous expressions allow us to quantify the output losses arising from tax distortions on technological choices:

$$\frac{\partial (tr_t + c_t)}{\partial \tau_t} = (r_t / \phi - R_t) \frac{\partial \gamma_t}{\partial \tau_t} < 0$$

Output losses depend upon (i) the shift in the allocation of entrepreneurs between the formal and informal sector and (ii) the difference between the social returns in the formal and informal sectors $r_t/\phi - R_t$. At the steady state, *private* returns in the two sectors should be equal, i.e., $r - R = 0$, but there would remain a gap between the social returns to the different technologies.

A similar approach allows us to derive a formula for the fiscal multiplier in the economy:

$$\frac{\partial c_t}{\partial tr_t} = -1 + (r_t/\phi - R_t) \frac{\partial \gamma_t}{\partial tr_t} < -1$$

C.4 Dynamics in two polar cases

Full commitment When the government has perfect commitment, debt price is independent of fiscal policy and the solution to the government program verifies a slightly modified Euler equation, i.e., $\lambda_t = E_t \lambda_{t+1}$, where:

$$\lambda_t = -\frac{\partial w_t / \partial \gamma_t}{\partial tr_t / \partial \gamma_t} u'(w_t) - \beta \frac{1}{\partial tr_t / \partial \gamma_t} E_t \frac{\partial v_A(\mathbf{s}_{t+1})}{\partial \gamma_t}.$$

The first term is the marginal utility of consumption weighted by a factor accounting for tax leakages. The second term captures the expected future gains of a marginal increase in aggregate transparency. The leakages implied by taxes depend on the elasticity ε of transparency to tax rates as follows:

$$\frac{\partial w_t / \partial \gamma_t}{\partial tr_t / \partial \gamma_t} = \frac{1/\varepsilon - \frac{1-\tau_t}{\tau_t} \frac{1-\phi}{\phi}}{1/\varepsilon + 1/\phi}.$$

In addition to the desire to smooth consumption across time and states of nature—implying counter-cyclical fiscal policy—the government takes into account (i) tax leakages and (ii) future distortions. Given that the capacity to raise tax revenues is pro-cyclical, the elasticity is more negative in bad times and leakages would be higher. The optimal tax rates implemented by the government should be even more counter-cyclical than in a world without distortions, in order to equalize the weighted marginal utility of consumption over time.

No commitment When the government cannot save nor borrow, then it needs to satisfy in each period a balanced budget and

$$tr_t = \tau_t z_t \gamma_t^{\frac{1}{\phi}} = g_t,$$

which implies, combined with Equation (5), that fiscal policy is pro-cyclical. In stark contrast with the perfect commitment case, the higher the distortions related to tax collection, and the higher the differences between the (high) tax rates in bad times and the (low) tax rates in good times.

C.5 Targeted moments

We now derive few theoretical moments, evaluated at the steady state, which will be used to discipline the quantitative analysis and the calibration of the model. The first moment is the elasticity of transparency to the tax rate, evaluated at the steady state:

$$\varepsilon_\gamma = \frac{-z\tau\gamma^{1/\phi-2}}{\frac{(1+\theta^2\beta)}{(1-\theta)h(R)} - (1-\tau)z(1/\phi-1)\gamma^{1/\phi-2}} \quad (\text{M1})$$

The second moment is the level of the steady-state transparency,

$$\theta\beta\bar{R} + (1-\theta\beta)H^{-1}(\gamma) = (1-\tau)z\gamma^{1/\phi-1} \quad (\text{M2})$$

The third moment is the steady-state fiscal multiplier, evaluated at the steady-state:

$$\frac{\partial c_t}{\partial tr_t}\Big|_{ss} = -1 + (r/\phi - R)\frac{\gamma}{t}\eta_\gamma \quad (\text{M3})$$

The fourth moment is the persistence of transparency, evaluated at the steady state:

$$\frac{\partial \gamma_t}{\partial \gamma_{t-1}}\Big|_{ss} = \frac{\frac{\theta}{(1-\theta)h(R)}}{\frac{(1+\theta^2\beta)}{(1-\theta)h(R)} - (1-\tau)z(1/\phi-1)\gamma^{1/\phi-2}} \quad (\text{M4})$$