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Conflicting objectives in Groundwater Management

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Abstract

This paper studies the conflict between economic and environmental sustainability objectives faced by a water agency when she allocates water quotas to farmers. This conflict consists in a water allocation problem between the amount of water claimed by farmers to irrigate their crops and the water flows needed to the conservation and the preservation of the ecosystems. This conflict in objectives is analysed in a dynamic hydro-economic model in discrete-time using the viability approach. The viability kernel that defines the states of the resource yielding intertemporal feasible paths able to satisfy the set of constraints over time is analytically identified. The associated set of viable quota policies and the trade-off between food production and ecosystem conservation objectives are characterized. The theoretical results of the paper are illustrated with numerical simulations based on the **Western La Mancha aquifer in Spain**.

Keywords: Groundwater, Agriculture, Irrigation, Environmental flows, Individual permits, Sustainability, Dynamic model, Viability kernel, Water Budget.

JEL codes: Q15, Q25, C61.

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1. Introduction

In aquifers managed with quotas, water agencies are facing conflicting objectives between ensuring minimum environmental flows for the preservation of groundwater-dependent ecosystems and satisfying the water claims of the farmers to irrigate their crops (*Gomez-Limon et al.* 2002). This conflict can result from a lack of coordination between two distinct agencies trying to implement their own policies as in *Martin and Stahn* (2013) or arise when an agency wants to achieved simultaneously quantity and quality objectives as in *Roseta-Palma* (2003). Handling the potential conflict between economic and environmental objectives is a major concern to achieve sustainability (*Alley et al.* 1999, *Zhou* 2009). This paper analyses the existing trade-off between these objectives as a problem of constraints fulfillment.

Hydroeconomic models are generally developed as constraint optimization problems with environmental constraints represented by minimum flow requirements (*Booker et al.* 2012). At each period, the dynamics of an aquifer depend on the balance between the natural recharge, the natural discharge and the amount of extracted water. The natural discharge consists in water flows which sustain groundwater dependent-ecosystems, fish and other aquatic species, the health of riparian and wetland ecosystems and other environmental needs (*Bertrand et al.* 2012). Starting from an initial situation in which there is no water extraction, the natural discharge is equal to the natural recharge. Once water is extracted, the natural discharge decreases affecting groundwater-dependent ecosystems (*Lohman* 1972). As modelled by *Gisser and Mercado* (1972) and *Gisser and Sanchez* (1980a), this natural discharge is assumed to be an affine function of the water table. It allows to defined a critical boundary value of the water table for which the natural discharge is nil. A first requirement of a sustainable management is then to keep the water table above this critical value. This specification of stream-aquifer flows is an example of interaction between groundwater and surface water but differs from *Knapp and Olson* (1995) who consider the dynamics of two state variables, one for each water sources, or *Pulido-Velázquez et al.* (2006) who deal with embedded multi reservoirs. The protection of environmental flows has also been introduced as an objective management for regulating agency in terms of ecosystem damages (*Esteban and Albiac* 2011; 2012, *Esteban and Dinar* 2016). However the lack of consideration of natural drainage in the aquifer water balance in the above cited papers has promoted the persistent "water budget myth" that net withdrawals can equal groundwater recharge

on the long run without causing harmful damages (*Bredehoeft 2002, Devlin and Sophocleous 2004*). *Pereau and Pryet (2018)* shows how the introduction of natural drainage in the water budget impacts the amount of water extraction and the dynamics of the water table.

The allocation of water quotas to farmers is also a problem of constraint. The use of market-based instruments such as tradable permits has been proposed as a promising way to replenish an aquifer (*Provencher 1993*), or to efficiently manage groundwater aquifers for irrigated agriculture (*Latinopoulos and Sartzetakis 2015, Pereau et al. 2017*) or to protect streams (*Kuwayama and Brozovic 2013*). Tradable permits ensure that water will be used by farmers with maximum efficiency. However like all "cap and trade" systems, the way the "cap" which consists in the available amount of water for users is set, remains a difficult issue. As pointed out by *Gomez-Limon et al. (2002)*, irrigation contributes to maintain rural development and employment in arid regions. It can justify the objective to allocate minimum amount of quotas to farmers and to ensure that the least efficient farmers will remain active. A second requirement of a sustainable management is then to implement relevant strategies in the allocation of water quotas for every farmer by a regulating agency.

To deal with these two requirements which take the form of constraints, the use of the viability approach has shown to be well-adapted (*Aubin 1990*). The interest of this method is to focus on identifying inter-temporal feasible paths within a set of desirable objectives or constraints. This framework has been applied to renewable resource management (*Oubraham and Zaccour 2018*), like fisheries (*Doyen and Pereau 2012, Pereau et al. 2012, Cisse et al. 2015*) or groundwater (*Pereau et al. 2017*). With respect to the literature (*Tomini 2014, Esteban and Dinar 2016, Pereau et al. 2017*), a first contribution of the paper is to introduce explicitly the natural discharge in the water budget of the aquifer and as an objective for the water agency. A second contribution is to address the trade-offs that arise between ecosystem conservation and economic development.

The paper is structured as follows: Section 2 is devoted to the description of the dynamic hydro-economic model and the objectives of the water agency. Section 3 characterizes the feasible resource states and water policies under several constraints. An application based on the Western La Mancha aquifer (Spain) illustrates the main results in Section 4. The last section concludes.

2. The hydro-economic model

2.1. The aquifer dynamics

The dynamics of an aquifer are described by changes in the water table, measured by $H(t) \in [0; H^{\max}]$ at time t where H^{\max} stands for the maximum height of the aquifer. The water table increases with the constant natural recharge $R > 0$ and is reduced by water extraction $Q(t)$ dedicated to irrigation and flows for the natural discharge $W_f(t)$. The total volume of extracted water $Q(t)$ corresponds to the quota set by a water agency. A proportion μ of the water used for irrigation is assumed to come back to the aquifer where $0 < \mu < 1$ stands for the non-absorption coefficient. Total extraction is thus $(1 - \mu)Q(t)$. The natural discharge consists in water flows for groundwater dependent-ecosystems, fish and other aquatic species, the health of riparian and wetland ecosystems and other environmental needs.

Based on *Gisser and Sanchez* (1980a;b), the dynamics of the resource are

$$\begin{aligned} AS(H(t+1) - H(t)) &= R - (1 - \mu)Q(t) - W_f(H(t)), \\ H(0) &= H_0, \end{aligned} \quad (1)$$

where A stands for the area of the aquifer and S the storage coefficient. Following *Gisser and Mercado* (1972), *Gisser and Sanchez* (1980a), *Morel-Seytoux* (2009), *Cousquer et al.* (2017), the natural discharge depends on the water table as follows:

$$W_f(H(t)) = \begin{cases} 0 & \text{for } H = H_{\min}, \\ -\alpha + \gamma H(t) & \text{for } H > H_{\min}. \end{cases} \quad (2)$$

The assumption of a linear conductance model is mainly accepted in the literature for the description of stream-aquifer flow as shown by *Crosbie et al.* (2005) or *Fenicia et al.* (2006).

Coefficients $\alpha, \gamma > 0$ respectively stand for the intercept and the slope of the natural discharge function (2) and are equal to

$$\alpha = \frac{RH_{\min}}{H^{\max} - H_{\min}}; \gamma = \frac{R}{H^{\max} - H_{\min}},$$

where $H_{\min} = \alpha/\gamma$ refers to the minimum level of the water table for which the natural discharge is nil.

It is worth mentioning that the natural discharge $W_f(t)$ appears in *Gisser and Sanchez* (1980b) but is not modelled in the sequel of that paper. However this is done in the model of *Gisser and Sanchez* (1980a) which is an integration of the two aquifers model of *Gisser and Mercado* (1972) into a single one. It can be checked that this is not the net natural recharge or the exploitable recharge, equal to annual recharge minus annual discharge, which is considered, but the natural recharge minus evapotranspiration. It implies that environmental flows and the water extraction problem cannot be monitored separately, without being subject to the water myth budget criticism.

Substitute Eq. (2) into Eq. (1) yields

$$H(t+1) = \left(1 - \frac{\gamma}{AS}\right) H(t) + \frac{R + \alpha}{AS} - \frac{1 - \mu}{AS} Q(t), \quad (3)$$

with $0 < \gamma/AS < 1$.

At the steady state, the water balance equation yields a negative relationship between Q and H

$$Q = \frac{R + \alpha}{1 - \mu} - \frac{\gamma}{1 - \mu} H. \quad (4)$$

The water balance at the steady-state takes the form of a decreasing line in the state-control (H, Q) space as shown in Fig. (1). Above the line given by Eq. (4), the water table diminishes with extraction and below the water table increases. However Eq. (4) does not correspond to the whole set of the steady-state water extraction due to the natural discharge term. The condition $H \geq H_{\min}$ ensuring a positive natural discharge $W_f(t) \geq 0$ also implies a constraint of the amount of extracted water such that $Q \leq R/(1 - \mu)$. The set of steady-state water extraction and water table satisfying the environmental constraint consists in $H^{\max} \geq H \geq H_{\min}$ and $0 \leq Q \leq R/(1 - \mu)$ because all the value of Q greater than $R/(1 - \mu)$ (located between $(R + \alpha)/(1 - \mu)$ and $R/(1 - \mu)$) implies $H < H_{\min}$.

2.2. The farmers

A set of n farmers are assumed to use water extracted from the aquifer, denoted by w_i , as the only input to irrigate their crops. The total crop income is given by the product of the crop yield y_i and the constant price of

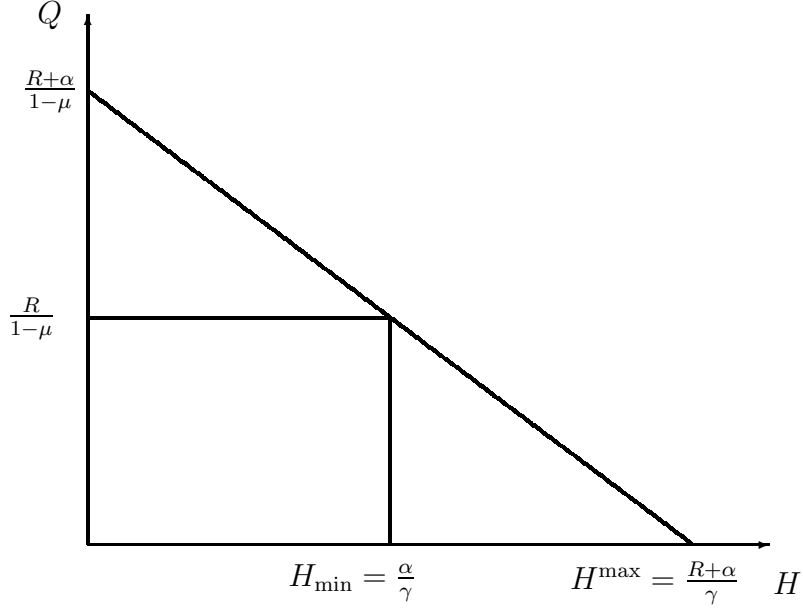


Figure 1: Water Budget at the steady state. The amounts of extraction which maintained a water table above its minimal level H_{\min} are located below $Q = R/(1 - \mu)$.

the agricultural product p_y . The crop-water production function $y_i(w_i)$ is assumed to be quadratic (*Burness and Brill 2001, Latinopoulos and Sartzetakis 2015*).

$$y_i(w_i(t)) = aw_i(t) - \frac{b_i}{2}w_i^2(t), \quad (5)$$

where $a > 0$ is a technical parameter of production common to all farmers. Heterogeneity between farmers relies on the parameter $b_i > 0$ associated to the quadratic term of the production function (5). Individual production reaches a maximum for $\bar{w}_i = a/b_i$ yielding $\bar{y}_i = a^2/2b_i = a\bar{w}_i/2$. It implies that individual water extraction $w_i \in [0, \bar{w}_i]$. The maximum amount of water consumption is $\bar{W} = \sum_{i=1}^n \bar{w}_i$ and the maximum amount of production is then $\bar{Y} = \sum_{i=1}^n \bar{y}_i$. Farmers can be ranked according their efficiency $\bar{y}_1 > \bar{y}_2 > \dots > \bar{y}_n$ for $b_1 < b_2 < \dots < b_n$. Farmer n is the less efficient while farmer 1 is the most productive. Farmer 1 is able to produce more agricultural output than other farmers with the same amount of water. Lastly, farmers are supposed to be price takers on the product market.

The pumping cost function is assumed to be decreasing and convex with respect to the water table. The externality occurs through the extraction

cost, which increases when the water table becomes lower. Moreover, the unit extraction cost is assumed to be the same for farmers at each point of the aquifer. The unitary cost $c(H(t))$ is given by

$$c(H(t)) = c_0 - c_1 H(t), \quad (6)$$

where $c_0 = c_1 H^{max}$ and c_1 respectively stand for the intercept and the slope of the pumping cost function (6).

As in *Pereau et al.* (2017), it is further assumed that farmers receive for free individual water quotas $q_i^-(t)$ at the beginning of each period t . Each farmer can buy or sell quota units on the market at a unit price $m(t)$ to have the right to extract water. The amount of quota held by farmer i after trade is $q_i(t)$. The individual profit of farmer i can be written as follows:

$$\pi_i(H(t), w_i(t)) = p_y y_i(w_i(t)) - c(H(t))w_i(t) - m(t)(w_i(t) - q_i^-(t)). \quad (7)$$

Using Eqs. (5) and (6), each farmer maximizes his payoff. It gives the individual quota water demand

$$q_i^*(t) = \frac{a}{b_i} - \frac{1}{b_i p_y} (c_0 - c_1 H(t) + m(t)). \quad (8)$$

It is a decreasing function of the quota price and an increasing function of the water table.

2.3. The water agency

A water agency has to manage the quota market by setting the supply of quota $Q^s(t) = Q(t)$. Using Eq. (8), the aggregate water demand for quotas $Q^*(t) = \sum_{i=1}^n q_i^*(t)$ is given by

$$Q^*(t) = a\beta - \frac{\beta}{p_y} (c_0 - c_1 H(t) + m(t)), \quad (9)$$

with $\beta = \sum_{i=1}^n \frac{1}{b_i}$. The market clearing condition $Q^*(t) = Q(t)$ yields the equilibrium water quota price $m^*(t)$

$$m^*(t) = a p_y - c_0 + c_1 H(t) - \frac{p_y}{\beta} Q(t). \quad (10)$$

It shows that the equilibrium water quota price decreases with the quota supply and increases with the water table. As we show in subsection 2.4,

the water quota market is solved by assuming that the less efficient farmer is active.

The water agency needs to ensure a positive demand for quotas, implying a positive price for the water permit in Eq. (10):

$$m^*(t) \geq 0. \quad (11)$$

This positivity condition on the quota price yields a state-control constraint

$$Q(t) \leq Q_M(H(t)). \quad (12)$$

with

$$Q_M(H(t)) = \beta \left(a - \frac{c_0}{p_y} \right) + \frac{\beta c_1}{p_y} H(t). \quad (13)$$

The tradable water permit constraint entails a superior limit for the total water extraction $Q(t)$. This superior bound $Q_M(H(t))$ is an increasing function of the state variable $H(t)$ in the state-control (H, Q) space. This bound depends on the economic parameters of farmers. Below the bound $Q_M(H(t))$, the low pumping costs increase the demand for quotas and the quota price. In the opposite case, the pumping cost is so high that there is no demand for quotas.

2.4. The objectives

We assume that the water agency faces conflicting objectives between maintaining minimum environmental flows to sustain dependent ecosystems and satisfying the individual water claims of farmers for irrigation. The environmental objective of the water agency takes the form

$$W_f(t) \geq \widetilde{W}_f \quad (14)$$

The value \widetilde{W}_f may be thought as a norm being set up by a decision maker or by the European Water Framework Directive for instance. Such a value remains exogenous in our framework but its choice will impact the associated trade-off for water irrigation. Substitute $W_f(H(t))$ by its expression (2) implies a value on the level of the water table such that

$$H(t) \geq H_W = \frac{\widetilde{W}_f + \alpha}{\gamma}. \quad (15)$$

Eq (15) means that to satisfy the environmental objective, the level of the water table has to be greater than H_W . It turns out that H_W is always greater or equal than the minimal value H_{\min} since for $\widehat{W}_f = 0$, $H_W = H_{\min}$.

Concerning the amount of individual water claimed by the farmers to irrigate their crops, we assume that farmers aim at ensuring an agricultural production target denoted by \tilde{y} such that

$$y_i(t) \geq \tilde{y}, \quad (16)$$

As mentioned in the introduction, *Gomez-Limon et al.* (2002) argued that the social impact of irrigation can justify to maintain employment in arid areas. This can take the form of constraint like eq. (16). From (5), it implies

$$q_i(t) \geq \frac{a}{b_i} - \frac{\sqrt{a^2 - 2b_i\tilde{y}}}{b_i}. \quad (17)$$

Substitute (10) in (8) gives

$$q_i^*(t) = \frac{1}{b_i\beta}Q(t). \quad (18)$$

Combining (17) and (18) implies a corresponding amount of water quotas needed for irrigation since water is the only input used by farmers. To ensure that all the farmers will be active, condition (16) implies

$$Q(t) \geq \sup_i \beta \left(a - \sqrt{a^2 - 2b_i\tilde{y}} \right),$$

which can be rewritten as

$$Q(t) \geq \beta \left(a - \sqrt{a^2 - 2\tilde{y} \sup_i b_i} \right).$$

It states that the amount of water quotas has to be such that the least efficient farmer is active since $\sup_i b_i = b_n$. Condition (16) entails the following bound on the water supply $Q(t)$

$$Q(t) \geq Q_A, \quad (19)$$

where Q_A is constant and independent of the state variable

$$Q_A = a\beta - \beta\sqrt{2b_n(\bar{y}_n - \tilde{y})}, \quad (20)$$

It shows that Q_A increases with the agricultural production target \tilde{y} . The payoff of farmer i can be written as

$$\pi_i^*(t) = \frac{1}{2}b_i p_y q_i^*(t)^2 + m^*(t)q_i^-(t). \quad (21)$$

2.5. The economic and environmental trade-off

Two cases have to be considered concerning the maximum feasible target value for the agricultural production the water agency can choose.

Assume that the water agency has no environmental constraint to be achieved, the maximum target value for the agricultural production, denoted by \tilde{y}^{\max} , is obtained when the tradable water permit constraint given by Eq. (12) intercepts the steady-state water balance given by Eq. (4) giving

$$Q = \frac{\frac{c_1\beta}{p_y}(R + \alpha) + \gamma\beta\left(a - \frac{c_0}{p_y}\right)}{\frac{c_1\beta}{p_y}(1 - \mu) + \gamma}. \quad (22)$$

Substituting the above expression in Eq. (20) yields

$$\tilde{y}^{\max} = \bar{y}_n - \frac{1}{2b_n} \left(a - \frac{\frac{c_1}{p_y}(R + \alpha) + \gamma\left(a - \frac{c_0}{p_y}\right)}{\frac{c_1\beta}{p_y}(1 - \mu) + \gamma} \right)^2. \quad (23)$$

In such a situation, this maximum value can imply an extraction of water which makes the water table below H_{\min} .

However when the environmental constraint is binding, the water agency has to keep the water table above H_{\min} . In that case, she faces a conflict between the productive and the environmental objectives. To determine the existing trade-off between the two targets, substitute the value of H_W given by Eq. (15) in the steady-state water balance Eq. (4), it gives

$$Q = \frac{R + \alpha}{1 - \mu} - \frac{\gamma}{1 - \mu} \left(\frac{\widetilde{W}_f + \alpha}{\gamma} \right),$$

which can be rewritten as

$$Q = \frac{R - \widetilde{W}_f}{1 - \mu}. \quad (24)$$

Substitute Eq. (24) in Eq. (20) yields

$$\tilde{y}_f = \bar{y}_n - \frac{1}{2b_n} \left(a - \frac{R - \widetilde{W}_f}{\beta(1 - \mu)} \right)^2. \quad (25)$$

Eq. (25) implies a decreasing and concave relationship between the two targets. An increase in the agricultural production can only be obtained at

the expense of a decrease in the environmental flows. Figure (2) illustrates such a trade-off between the two targets or constraints. The value of \tilde{y}_f^{\max} is obtained when $\tilde{W}_f = 0$ in eq (25).

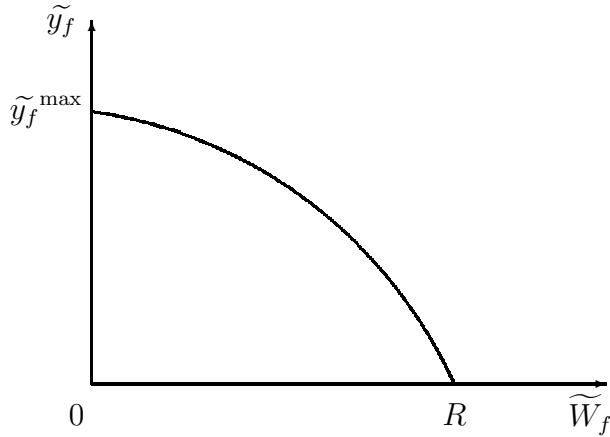


Figure 2: Decreasing trade-off between the two objectives \tilde{y}_f and \tilde{W}_f .

Figure (2) illustrates the antagonism between economic and ecological performances in water policies and extends the trade-off put forward by *Polasky et al.* (2005) or *Barraquand and Martinet* (2011) for agricultural policies. Starting from \tilde{y}_f^{\max} , an increase in \tilde{W}_f implies a small cost in terms of production loss. However for larger value of \tilde{W}_f , the production cost become more important. When the environmental flow objective is equal to the natural recharge $\tilde{W}_f = R$, it means that the water table is at its highest value $H(t) = H^{\max}$ and then no water extraction for irrigation is allowed.

3. Viability analysis

3.1. The resource constraints

The first constraint for the water agency concerning the environmental flow requirement on the form $H(t) \geq H_W$ has already be introduced in the above section. A second constraint results from the constraint $Q(t) \geq Q_A$

due to the productive objective of the farmers and the water permit price constraint $Q(t) \leq Q_M(H(t))$. Combining Eqs. (12) and (19) gives

$$Q_A \leq Q(t) \leq Q_M(H(t)). \quad (26)$$

This yields a second critical threshold on the water table

$$H(t) \geq H_Y, \quad (27)$$

where H_Y is such that

$$H_Y = H^{\max} - \frac{p_y}{c_1} \sqrt{2b_n (\bar{y}_n - \tilde{y})}.$$

It shows that H_Y increases with the target on agricultural production \tilde{y} . The resource constraint becomes

$$H(t) \geq H_C, \quad (28)$$

where the critical value of the water table H_C is defined as

$$H_C = \begin{cases} H_Y & \text{when } H_Y < H_{\min}, \\ H_W & \text{when } H_W \geq H_{\min}. \end{cases} \quad (29)$$

When $H_Y < H_{\min}$ the constraint derived from the environmental flows objective is not binding. The decrease in the water table below the minimal value H_{\min} is such that no environmental flows can sustain the ecosystem, implying its collapse. However when $H_W \geq H_{\min}$, the environmental constraint holds and there exists a trade-off between satisfying the environmental flows for the ecosystem and the water available for agricultural irrigation as shown by Fig (2).

3.2. The viability kernel

The dynamics of the aquifer given by Eq. (1) are taken into account together with the resource constraint (28).

In an infinite horizon context, the viability kernel can be formally defined as the set of initial situations H_0 such that there exists water extraction $Q(t)$ and resources $H(t)$, which satisfy the previous constraints, for all time $t = 0, 1, \dots, \infty$. It can be written as

$$Viab = \{H_0 | \exists Q(.) \text{ satisfying (28)} \forall t = 0, 1, \dots, \infty\}. \quad (30)$$

According to the values of Q_A and the associated H_C , several cases can be distinguished. Based on the water balance at the steady state (4), the steady state amount of extracted water Q_C related to the water table stock level H_C is defined as follows:

$$Q_C = \frac{R + \alpha}{1 - \mu} - \frac{\gamma}{1 - \mu} H_C. \quad (31)$$

Since H_C can take two values H_Y and H_W , the associated steady-state water extraction H_C can also take two values Q_Y and Q_W . This gives the following proposition for the viability kernel:¹

Proposition 1. *Assuming that $\tilde{y}_i \leq \tilde{y}^{\max}$ when $H_Y < H_{\min}$ or $\tilde{y}_i \leq \tilde{y}_f^{\max}$ when $H_W \geq H_{\min}$, it can be shown that*

- *If $Q_A > Q_C$ the viability kernel is empty $Viab = \emptyset$.*
- *If $Q_A \leq Q_C$ the viability kernel is $Viab = [H_C, H^{\max}]$.*

In the no viability case, the viability kernel is empty. It means that for every initial water table state H_0 , every trajectory Q_A greater than $Q_C = Q_Y$ violates the constraints at least for a period of time. However, if and only if $H_0 \geq H_C$, there exists a water quota strategy complying with the constraints over time and the definition of the viability kernel (30). Figure (3) illustrates two viable cases when there is no conflict between the two objectives of the water agency, $H_Y < H_{\min}$ (case a) and when the two objectives are conflicting, $H_W \geq H_{\min}$ (case b). In case a), the critical value of the water table H_Y is obtained at the intersection of the $Q_M(H(t))$ line and the horizontal Q_A line. For this value H_Y corresponds the steady state extraction Q_Y which is higher than Q_A and the viability kernel is $Viab = [H_C, H^{\max}]$. The viability domain corresponds to the area which lies above Q_A and below $Q_M(H(t))$. In this area, the viability kernel allows increasing and decreasing water dynamics depending on whether the system is above or below the steady-state water balance. In case b), the environmental target yields a critical H_W for which the value of the steady state extraction is $Q_A = Q_W$. The viability kernel is then $Viab = [H_W, H^{\max}]$.

¹The proof is given in the Appendix.

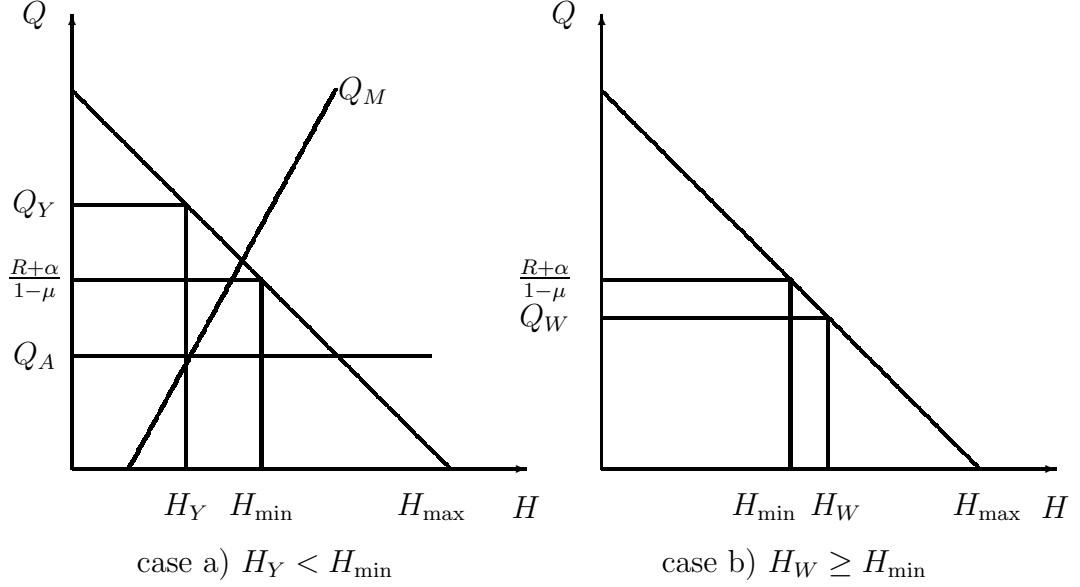


Figure 3: The two viable cases when the environmental constraint is not binding (case a) and is binding (case b). In case a) H_Y is the intersection point between Q_M and Q_A . In case b) H_W results from the environmental target and implies a maximum sustainable extraction Q_W .

3.3. The viable quota corridor

The water quotas $Q(t)$ set by the water agency have to comply with the additional dynamic viability condition $H(t+1) \geq H_C$. It is assumed that at each period, farmers buy the amount of quotas $q_i^*(t)$ they need to maximise their annual rent at the equilibrium quota price $m^*(t)$. Since farmers received an equal fraction of the quota supply $q_i^-(t)$, some farmers will sell their quotas when $q_i^*(t) < q_i^-(t)$ while others will buy them when $q_i^*(t) > q_i^-(t)$. Using Eq. (3), condition $H(t+1) \geq H_C$ yields

$$Q(t) \leq \frac{R + \alpha}{1 - \mu} - \frac{AS}{1 - \mu} H_C + \frac{AS}{1 - \mu} \left(1 - \frac{\gamma}{AS}\right) H(t).$$

Using the steady-state water budget given by Eq. (31) the dynamic condition entails a superior bound for the water extraction

$$Q(t) \leq Q_D(H(t)), \tag{32}$$

with

$$Q_D(H(t)) = Q_C + \frac{AS}{1 - \mu} \left(1 - \frac{\gamma}{AS}\right) (H(t) - H_C). \tag{33}$$

This superior limit is an affine and increasing function of the state variable $H(t)$. By definition, $H(t) = H_C$ entails $Q(t) = Q_C$.

The following proposition specifies the set of viable water quota policies Q_{Viab} associated with the viable water tables.

Proposition 2. *Considering that $Q_A \leq Q_C$, the viable quotas associated with $\text{Viab} = [H_C, H^{\max}]$ are*

- when $H_Y < H_{\min}$

$$Q_{\text{Viab}} = [Q_A, \min(Q_M(H(t)), Q_D(H(t)))] .$$

- when $H_W \geq H_{\min}$

$$Q_{\text{Viab}} = [Q_A, Q_D(H(t))] .$$

When $H_Y < H_{\min}$, the comparison between the slopes of $Q_D(H(t))$ and $Q_M(H(t))$ shows² that the dynamic viability constraint $Q_D(H(t))$ is binding and reduces the viability domain under the following condition:

$$\frac{\frac{\beta c_1}{p_y}(1 - \mu)}{AS - \gamma} > 1. \quad (34)$$

Condition (34) ensures that the water agency is dynamically constrained in the setting of the available water quota. When this condition is not binding, it implies that the viability domain and the viable quota policies domain are the same. This case refers to the Gisser-Sanchez Effect stating the inefficiency of a water policy that occurs when the term in condition (34) is closed to zero.

Figure (4) shows the set of viable quotas when condition (34) is binding. Consider a configuration in which Q_A is equal to $Q_A = Q_Y$. While the viability domain was lying in the area above Q_A and below Q_M , the viable quota domain is now reduced due to the dynamic viability condition $H(t + 1) \geq H_Y$. The set of viable quotas lies now in the area between Q_Y and Q_D . To illustrate the objective conflict faced by the water agency, assume that the environmental objective of the water agency implies a critical water table level equal to $H_W > H_{\min}$ and corresponding to higher environmental

²See Appendix.

flows for the ecosystem. Such an objective means a lower production target such that $Q_A = Q_W$ and a new constraint represented by Q_D derived from the condition $H(t+1) \geq H_W$. It turns out that the viable quota domain is reduced and lies now in the area between Q_W and Q_D .

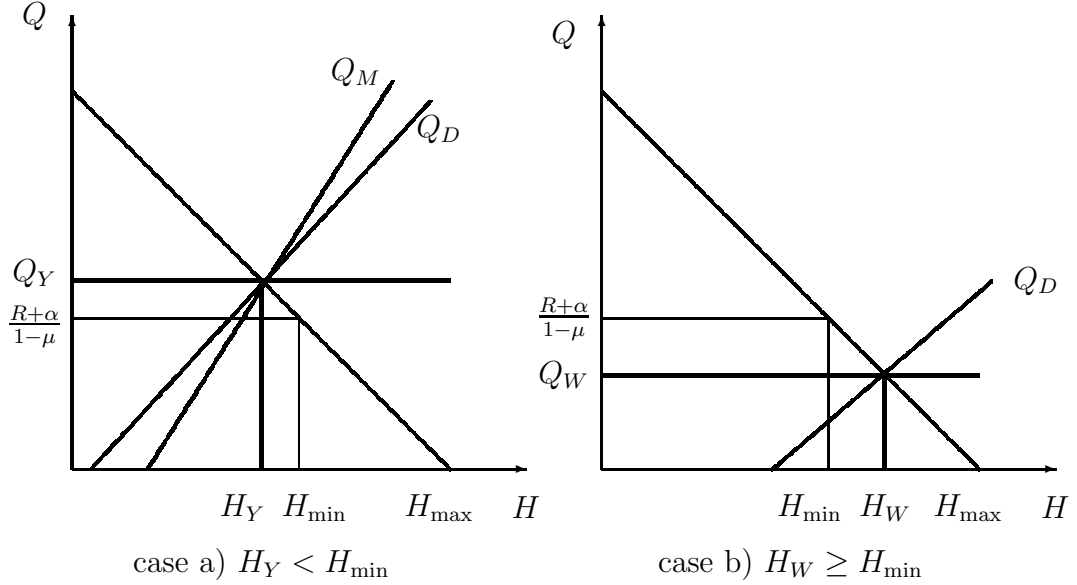


Figure 4: The viable quota policies are located in the area Q_D and Q_Y in case a) and in the area Q_D and Q_W in case b).

3.4. Some viable quota strategies

The set of viable quotas when $H_Y < H_{\min}$ under condition (34) and when $H_W \geq H_{\min}$ can be rewritten as

$$Q_{\text{Viab}} = \rho Q_A + (1 - \rho) Q_D(H) \quad (35)$$

with $0 \leq \rho \leq 1$. Eq. (35) shows that several viable quota policies may exist depending the value of the parameter ρ . The minimal viable quota Q_A is obtained when $\rho = 1$. This quota corresponds to an ecological and conservation viewpoint in the sense that it favors the resource. On the opposite, the maximal viable quota consists in implementing $Q_D(H)$ when $\rho = 0$. Such a policy

is more economic-oriented since it allows a high water extraction. However this quota can only be implemented one time. Consider the dynamics given in Eq. (3)

$$H(t+1) = \left(1 - \frac{\gamma}{AS}\right) H(t) + \frac{R + \alpha}{AS} - \frac{1 - \mu}{AS} Q_{\text{Viab}}(t). \quad (36)$$

Assuming $Q_{\text{Viab}}(t) = Q_D$ and using the expression of Q_D given by (33) yields

$$H(t+1) = H_C. \quad (37)$$

After implementing Q_D in period t , the water table is set at H_C and the amount of water extraction is constrained to Q_C for the next period $t+1$.

4. Numerical illustration

The model is numerically tested using data on the Western la Mancha aquifer, Spain. Parameters values are provided by *Martinez-Santos et al.* (2008), *Esteban and Albiac* (2011; 2012), *Esteban and Dinar* (2013; 2016)

Concerning the hydrology parameters, we consider the following values

Parameters	Description	Units	Value
AS	aquifer area \times storativity	Mm^3	126.5
R	natural recharge	Mm^3/y	360
α	intercept of the nat. discharge fct	Mm^3/y	3323.0769
γ	slope of the nat. discharge fct	Mm^3/m	5.5384615
μ	return flow coefficient		0.20
$H_0 = H^{\max}$	initial stock level	m	665
H_{\min}	minimum water table level	m	600

From the value of H_{\min} , we can obtain the values of α and γ . For the economic parameters, we assume

Parameters	Description	Units	Value
g	intercept of the water demand fct	$\text{€}/Mm^3$	4400.73
k	slope of the water demand fct	$\text{€}/Mm^3$	0.097
c_0	intercept of the pumping cost fct	$\text{€}/Mm^3/y$	266000
c_1	slope of the pumping cost fct	$\text{€}/Mm^3/m$	400

From the values of g and k of the water demand function, it is possible to obtain particular values of the coefficients a , b n and p_y with $a = (g*b)/n =$

907.36, $b = n/(kp_y) = 20.61$ with $n = 100$ and $p_y = 50e/\text{ton}$. *Heterogeneity between farmers is* as a uniform random variable over the interval $[b * (1 - \delta), b * (1 + \delta)]$ with a dispersion rate $\delta = 10\%$. The numerical application remains illustrative since it doesn't rely on a specific crop. The initial state refers to a full aquifer $H_0 = H^{\max}$ implying a maximum discharge $W_f = R$ and no water for irrigation. The policy parameter ρ in Eq. (35) is set to $\rho = 0.5$

We first consider the case when the environmental constraint is not binding. It corresponds to case a) in Figures (3) and (4). The maximum production target is set to $\tilde{y}^{\max} = 5321.8$ tons and entails a quota $Q_A = Q_Y = 666.2Mm^3$ and a water table $H_Y = 568.7m$ which is lower than the critical value $H_{\min} = 600m$. Figure (5) displays different trajectories of the water table, the quota, the quota price, the natural discharge, the mean individual rent and the pumping cost. When the environmental constraint becomes binding according case b) in Figures (3) and (4), the water table is constraint to remain above $H_{\min} = 600m$. For a target $\tilde{W}_f = 150Mm^3$, Figure (6) displays the trajectories for the different variables of interest. For that particular value, the production objective is set at its maximum value $\tilde{y}^{\max} = 2201$ tons and requires a steady-state water extraction quota $Q_W = 262.5Mm^3$. This quota is lower than the extraction value one will expect when environmental flows are not explicitly introduced in the water budget equation (equal to $(R/(1 - \mu)) = 450Mm^3$). To satisfy this objective, the critical value of the water table is $H_W = 627m$.

The following table shows the existing trade-off between the two conflicting objectives as suggested by Figure (2).

\tilde{W}_f	\tilde{y}_f^{\max}	H_W	Q_W
0	3691	$H_{\min} = 600$	$\frac{R}{1-\mu} = 450$
50	3202	609	387
100	2706	618	325
150	2201	627	262
200	1689	636	200
250	1170	645	137
300	643	654	75
$R = 360$	0	$H^{\max} = 665$	0

For an average value of $\tilde{W}_f = 100Mm^3$, a one-percent increase of \tilde{W}_f implies a production loss of 0.37% for \tilde{y}_f . This cost increases to 0.89% for the average

value $\widetilde{W}_f = 250Mm^3$ and 1.66% for $\widetilde{W}_f = 300Mm^3$.

5. Conclusion

This paper has analysed the conflict faced by a water agency between economic and environmental objectives. We assumed that a water agency aims at sustaining minimum environmental flows for the conservation of the ecosystems and satisfying a production objective for farmers. In a dynamic viability framework including natural drainage in the water budget equation, these two objectives have been interpreted in terms of constraints to be satisfied over time. Results show that the trade-off water agencies have to deal with when setting and allocating quotas to farmers can be measured in terms of agricultural production losses for a rise in water flows dedicated to the preservation of ecosystems.

Future extensions could be considered. A first one consists in introducing other water demands than irrigated agricultural like urban water services (Hansen 2012) and addressing the trade-off of these demands with environmental objectives (Babel *et al.* 2005) in a global water market. A second extension will be to relax the assumption of deterministic framework and consider different kinds of uncertainties (de Frutos Cachorro *et al.* 2014, Krishnamurthy 2017).

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8. Appendix

8.1. Proof of the viability kernel

Proof: Consider the dynamics

$$H(t+1) = \left(1 - \frac{\gamma}{AS}\right) H(t) + \frac{R + \alpha}{AS} - \frac{1 - \mu}{AS} Q(t),$$

From the water balance at the steady state (4), we have

$$Q_C = \frac{R + \alpha}{1 - \mu} - \frac{\gamma}{1 - \mu} H_C.$$

Substitute in the dynamics give

$$H(t+1) = \left(1 - \frac{\gamma}{AS}\right) H(t) + \frac{\gamma}{AS} H_C - \frac{1-\mu}{AS} (Q(t) - Q_C),$$

We first show that $Q_A \leq Q_C$ implies $Viab = [H_C, H^{\max}]$.

Assume that $H_0 \geq H_C$, choose $Q(t) = Q_A$ then it yields a recursive equation

$$\begin{aligned} H(1) &= \left(1 - \frac{\gamma}{AS}\right) H(0) + \frac{\gamma}{AS} H_C - \frac{1-\mu}{AS} (Q_A - Q_C), \\ H(2) &= \left(1 - \frac{\gamma}{AS}\right) H(1) + \frac{\gamma}{AS} H_C - \frac{1-\mu}{AS} (Q_A - Q_C), \\ &\vdots \\ H(t) &= \left(1 - \frac{\gamma}{AS}\right) H(t-1) + \frac{\gamma}{AS} H_C - \frac{1-\mu}{AS} (Q_A - Q_C), \end{aligned}$$

which gives the solution

$$\begin{aligned} H(t) &= \left(1 - \frac{\gamma}{AS}\right)^t H(0), \\ &+ \frac{\gamma}{AS} \left(1 + \left(1 - \frac{\gamma}{AS}\right) + \dots + \left(1 - \frac{\gamma}{AS}\right)^{t-1}\right) H_C, \\ &- \frac{1-\mu}{AS} \frac{\gamma}{AS} \left(1 + \left(1 - \frac{\gamma}{AS}\right) + \dots + \left(1 - \frac{\gamma}{AS}\right)^{t-1}\right) (Q_A - Q_C). \end{aligned}$$

When $t \rightarrow \infty$, the term $\left(1 - \frac{\gamma}{AS}\right)^t$ is nil and $\sum_{t=0}^{\infty} \left(1 - \frac{\gamma}{AS}\right)^t = \frac{\gamma}{AS}$ It implies

$$H(t) = H_C - \frac{1-\mu}{AS} (Q_A - Q_C).$$

Since $Q_A < Q_C$ we have shown that

$$H(t) = H_C + \frac{1-\mu}{AS} (Q_C - Q_A) > H_C.$$

Hence $[H_C, H^{\max}]$ is viable and $Viab = [H_C, H^{\max}]$.

Now if $Q_A > Q_C$, we show by forward induction that $\forall H_0 \geq H_C$

$$H(t) = H_C - \frac{1-\mu}{AS} (Q_A - Q_C) < H_C.$$

It implies that $Viab = \emptyset$.

8.2. Dynamic constraint $Q_D(H)$

The constraint on the state variable $H(t+1) \geq H_C$ implies

$$H_C \leq \left(1 - \frac{\gamma}{AS}\right) H(t) + \frac{\gamma}{AS} H_C - \frac{1-\mu}{AS} (Q(t) - Q_C).$$

By denoting $Q_D(H(t)) = Q_C + \frac{AS}{(1-\mu)} \left(1 - \frac{\gamma}{AS}\right) (H(t) - H_C)$, it gives $Q(t) \leq Q_D(H(t))$. We look at the conditions depending on the sign of $Q_M(H(t)) - Q_D(H(t))$ under which this dynamic constraint is binding and reduces the viability kernel. By definition, $Q_D(H_C) = Q_C$ and since $Q(t)$ is bounded by Q_C , it implies that for $H(t) = H_C$

$$Q_M(H_C) < Q_D(H_C).$$

It yields

$$g - Q_C < k(c_0 - c_1 H_C).$$

The expression of $Q_M(H(t)) - Q_D(H(t))$ is given by

$$Q_M(H(t)) - Q_D(H(t)) = g - Q_C - k c_0 + k c_1 H(t) - \frac{AS}{1-\mu} \left(1 - \frac{\gamma}{AS}\right) (H(t) - H_C).$$

Using the two previous expressions gives

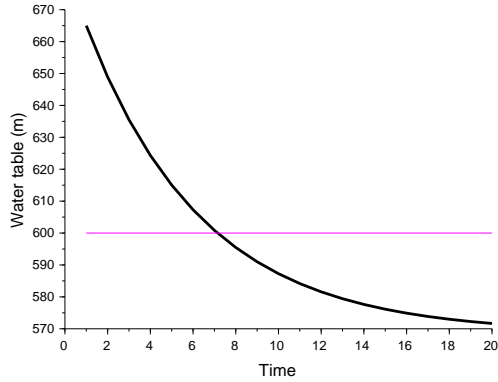
$$Q_M - Q_D < k(c_0 - c_1 H_C) - k c_0 + k c_1 H(t) - \frac{AS}{1-\mu} \left(1 - \frac{\gamma}{AS}\right) (H(t) - H_C),$$

$$Q_M - Q_D < k c_1 (H(t) - H_C) - \frac{AS}{1-\mu} \left(1 - \frac{\gamma}{AS}\right) (H(t) - H_C).$$

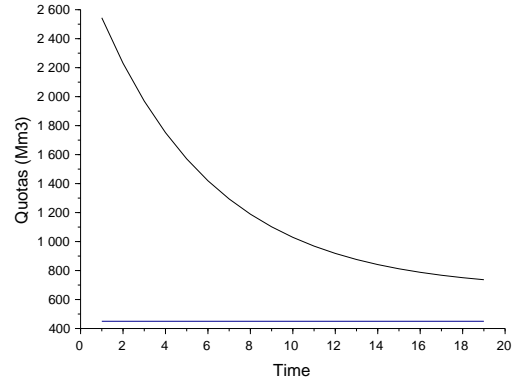
When $H(t) > H_C$ the condition ensuring $Q_M(H(t)) - Q_D(H(t)) > 0$ is

$$\frac{k c_1 (1-\mu)}{AS - \gamma} > 1,$$

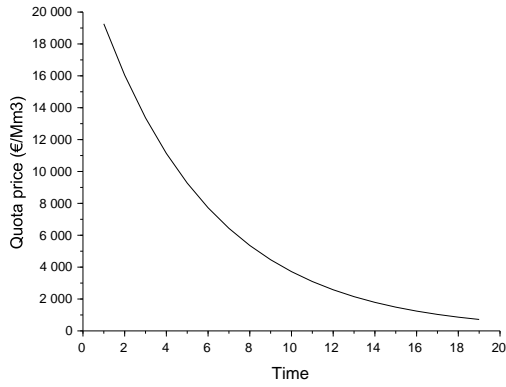
and corresponds to condition (34) in the text.



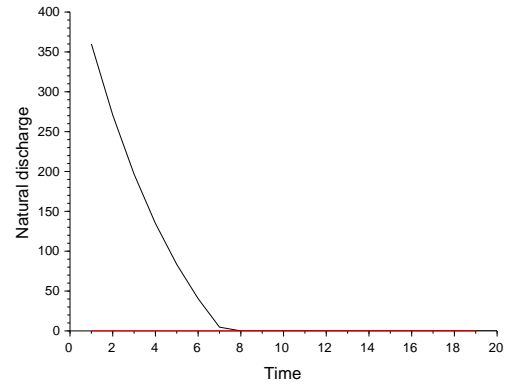
(a) Water table $H(t)$



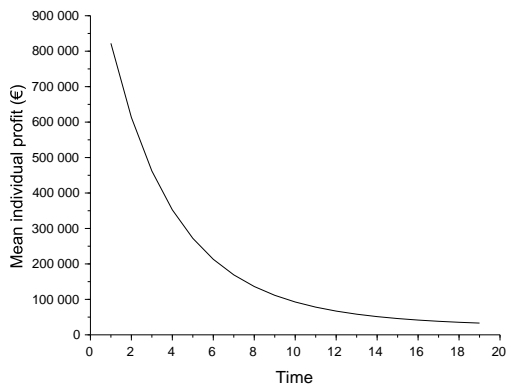
(b) Quota $Q(t)$



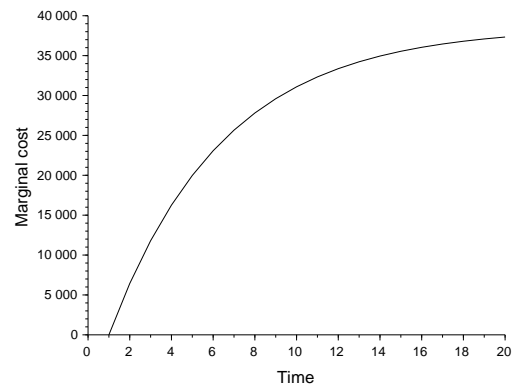
(c) Quota price $m(t)$



(d) Natural discharge $W_f(t)$

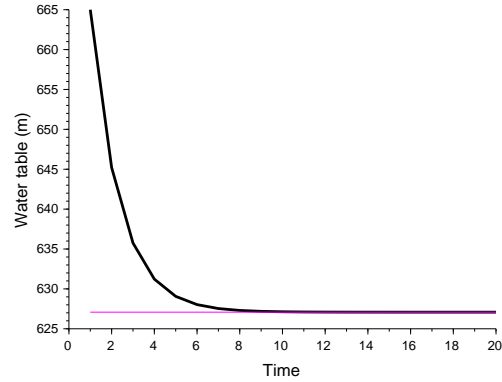


(e) Mean individual rent $m(t)$

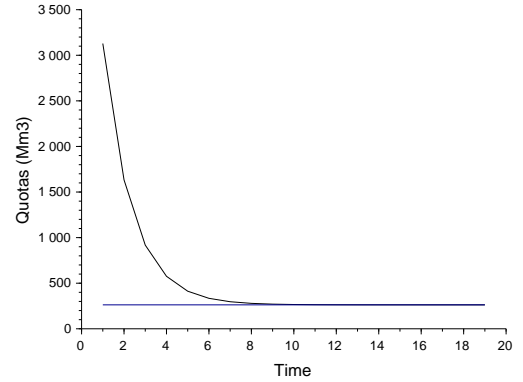


(f) Marginal Pumping cost $C(t)$

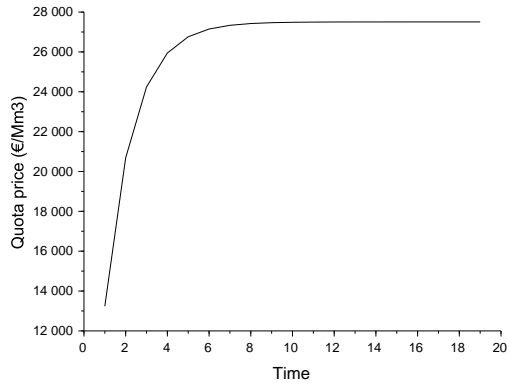
Figure 5: The environmental target is not binding with $\tilde{y}^{\max} = 5322t$, $H_Y = 569m$ and $Q_A = 666Mm^3$.



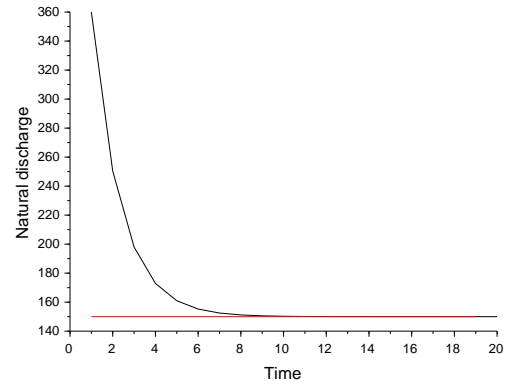
(a) Water table $H(t)$



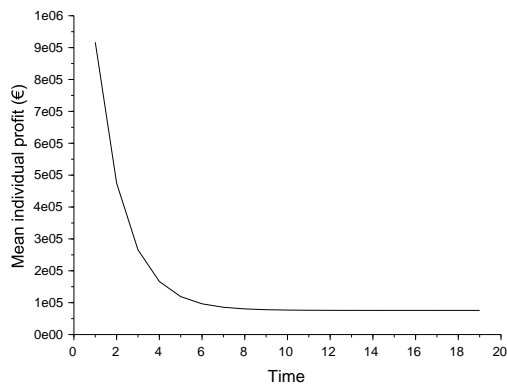
(b) Quota $Q(t)$



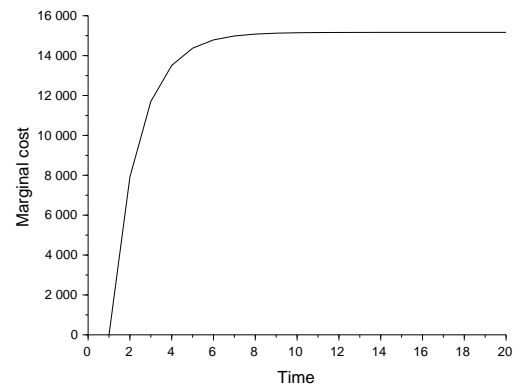
(c) Quota price $m(t)$



(d) Natural discharge $W_f(t)$



(e) Mean individual rent $m(t)$



(f) Marginal Pumping cost $C(t)$

Figure 6: The environmental target is binding and environmental flows are set to $\widetilde{W}_f = 150Mm^3$