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Brexit: Brinkmanship and Compromise*

Helios Herrera† Antonin Macé ‡ Matías Núñez §

May 11, 2021

Abstract

We study how do-or-die threats ending negotiations affect gridlock and welfare in the ratification of deals/treaties between opposing parties. Failure to agree in any period, as usual, implies a status-quo disagreement payoff and a continuation of the negotiation: a renegotiated amended agreement to be ratified next period. However, under brinkmanship, agreement failure in any period may precipitate instead a "hard" outcome, worse than the status-quo and than any feasible agreement. Such brinkmanship threats improve the scope for agreement, but also entail costs as we show. With symmetric parties only more extreme brinkmanship is beneficial: when an agreement is unlikely to begin with mild brinkmanship only reduces welfare by increasing the equilibrium chance of a hard outcome. If a party is advantaged it typically benefits even from mild threats, as the expected agreement shifts in its favor, while only extreme brinkmanship threats can benefit the disadvantaged party.

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"Brinkmanship...the threat that leaves something to chance" (Thomas Schelling)

1 Introduction

Since the Brexit referendum in June 2016 the everlasting negotiations for a withdrawal agreement with the EU (finally ratified in the UK in Jan. 2020) marked a low point in British democracy. For three consecutive times the UK Parliament rejected a negotiated agreement in 2019, which every time had to be renegotiated in Brussels. Every time, PM Theresa May, despite threatening not to do so, requested last minute deadline extensions (which were granted by the EU) to avoid a no-deal Brexit that would significantly hurt the UK (and EU) economies. Some experts came to believe that Brexit could be delayed forever,[1] and viewed Parliament not as the solution to Brexit but as the problem itself. Indeed, the possibility of a future re-vote on a new deal fostered the unwillingness to compromise of UK parties and factions. To try to solve what became known as the "kicking the can down the road problem" *threats* of no-extension to the ratification deadlines (precipitating no-deal Hard Brexit outcome) were made on several occasions by several EU countries.[2] On the UK side, in the summer 2019, PM Boris Johnson also vowed for the UK to leave the EU by Oct 31 2019 "do or die" pledging not to extend the deadline.[3] Only an early election with a Tory landslide broke the impasse and allowed the withdrawal agreement to be ratified in Jan. 2020 by the UK. But the saga just moved to a second, not less dramatic, stage, namely agreeing on a UK-EU trade deal, once again under the extremely tight "do or die" deadline of Dec. 31 2020, imposed unilaterally by the UK.[4,5]

Similar brinkmanship - "my way or the highway" - episodes characterized political

[1]See, for instance, Forbes April 12 2019: "Brexit Forever: Eventually People Will Grow Tired Of This And The U.K. Will Remain."

[2]For instance, from Ireland (see later) and from France (e.g. see The Guardian Oct 28 2019: "Macron against Brexit extension as Merkel keeps option open").

[3]Boris Johnson tried also to suspend the UK parliament (provoking a constitutional crisis), thus presenting to the UK MPs a Hard Brexit on Oct. 31 as the only alternative to the current agreement. See The Economist Aug. 29 2019: "Taking Back Control".

[4]The tightness of this agreement time-window is unprecedented for any yet-to-be negotiated trade deal. Once again, a failure to agree would have been an economic calamity hurting probably more the UK than its counterpart the EU.

[5]Unlike the domestic episode of October 2019, this time the UK's walk-away threat is targeted to the counterparts in Brussels. See for instance The Economist May 28th 2020 "Brexit: Deadlock looms at Brexit talks next week: the chances Britain will leave the EU without a trade deal are rising".

negotiations in other countries, notably in the US in the last decade, where they precipitated several government shutdowns and debt ceiling crises. For instance, the longest U.S. government shutdown in history occurred when the US Congress and President Donald Trump could not agree on an appropriations bill to fund the operations of the federal government for the 2019 fiscal year nor on a temporary continuing resolution that would extend the deadline for passing a bill.[6] Similar brinkmanship tactics were at the core of the US debt ceiling crises under presidents Obama (2011 and 2013) and Clinton 1995, both facing a republican congress.[7] For instance, in 2013: the Republican Party in Congress refused to raise the debt ceiling unless President Obama would have defunded the Affordable Care Act (Obamacare), his signature legislative achievement. As negotiations went on, the US Treasury stated that it would have to *delay payments* if funds could not be raised through these measures: the US defaulting on its debt became more likely as days passed without an agreement and would have resulted in permanent damage to the economy.[8] A similar Debt Ceiling episode happened in 2011.[9]

Political negotiations or treaty ratifications naturally occur under deadlines extendable under certain conditions and/or approval by the negotiating sides. Threatening not to extend this deadline or imposing 'do-or-die' conditions for an agreement to be reached, "or else", may result in a hard breakdown of negotiations or a government shutdown, namely an outcome worse than any agreement and, crucially, worse than the status-quo, i.e. the negotiations' limbo in which no agreement is (yet) reached. Putting this "time bomb"that can go off anytime over a negotiation may have common benefits of possibly breaking the impasse, but also private benefits for some parties. What is key about these brinkmanship threats though is that they

[6]This shutdown (from Dec. 22, 2018, for 35 days) stemmed from an impasse over Trump's demand for federal funds for a U.S.–Mexico border wall.

[7]These are designed to provide extra pressure on the counterparts. For instance, on January 2013, Paul Ryan, Chairman of the House Budget Committee argued that giving Treasury enough borrowing power to postpone default until mid-March would allow Republicans to gain an advantage over Obama and Democrats in debt ceiling negotiations.

[8]Treasury Secretary Timothy Geithner warned that "failure to raise the limit would precipitate a default by the United States. Default would effectively impose a significant and long-lasting tax on all Americans and all American businesses and could lead to the loss of millions of American jobs. Even a very short-term or limited default would have catastrophic economic consequences that would last for decades."

[9]As in the subsequent 2013 episode, U.S. government debt was downgraded for the first time in its history, the stock market fell, measures of volatility jumped, and credit risk spreads widened noticeably.

work by creating the *risk* of an accident. Namely, they generate probabilistic outcomes that hang over the negotiation like a Damocles's sword that can fall at any moment. Such risk was very salient in the case of the US debt ceiling crises for instance and had substantial effects on financial markets, thus brinkmanship episodes represent a true risk whose extent depends on many random factors beyond the credibility of the threatening side.[10] As Schelling (1960) observes referring to cold war impasses: "the key to these threats is that, though one may or may not carry them out if the threatened party fails to comply, the final decision is not altogether under the threatener's control....these risks could involve chance, accident, third-party influence, imperfection in the machinery of decision, or just processes that we do not entirely understand."

Such political brinkmanship cases which, as the word says, put the negotiation at the brink of a precipice, beg both normative and positive questions which we address here: for instance, whether imposing such a burden upon a negotiation can be welfare improving, and in particular which side would benefit from such a walk-away threat, if any, and lastly how credible should this threat be to be beneficial to one side. Namely, prima facie, there seems to be two possible benefits of walk-away threats precipitating calamitous potential costs to all negotiating sides. One is a common benefit: making both sides more willing to compromise thus reducing the cost of extended negotiations and delayed outcomes. The other is private: do-or-die threats can also be imposed strategically by parties inside (or even outside) the negotiation seeking an advantageous bargaining position, namely an advantage possibly when the threat of a hard outcome hurts more one side than the other. On the flip side, there are costs of imposing such threats if the time bomb happens to explode before the negotiation ends and hard outcome de facto materializes thereby hurting, possibly to a different extent, both parties.

To shed light on the above trade-offs, we present a model in which two parties, in our core setup, must repeatedly decide to ratify or not, a proposed agreement presented to them. These proposed agreements are randomly drawn every period out of a set of agreements that grant to both parties (weakly) better outcomes than

[10] Also in the Brexit case, the pound had one of its worse week of 2019 after on Dec. 16 Boris Johnson legislated a deadline for the UK's Brexit transition period, pledging to outlaw any extension to the UK's post-Brexit transition period beyond the end of 2020.

the status-quo. This randomness reflects the unavoidable underlying uncertainty on how proposals are generated from the point of view of the body that decides its final approval.[11] The benchmark core setup we analyze is a standard pie-sharing framework with costly delay, namely the pie is shrinking with time. In the presence of brinkmanship however, in addition if a proposal is rejected then, with some probability $h > 0$ (zero in the benchmark case), this causes the bargaining to end with a *hard outcome* which is *ruinous to both* parties: the two sides may differ in their utility from this hard outcome, but crucially this utility is always negative, i.e., a *worse outcome than the status-quo*. Conversely, $(1 - h)$ represents the chance that the negotiating process continues through one additional period in which new proposal is drawn to be voted on, and so forth.

In reality, the extent of brinkmanship h depends in part on the credibility of these "do or die" announcements, thus our premise is that these brinkmanship threats materialize only with some probability: while it is politically costly to renege on a "do or die" announcement, there are also clear incentives not to carry through with the threat if, ex-post, a deal is not reached and an extension is needed. A necessary *first step* to understand the effects of brinkmanship is to disregard the origin of such threats and take h as exogenous: we abstract from modeling the choice of h (and asking why and how threats are credible) as it necessarily implies imposing significant more structure to the model. Our goal is to understand the *effects* of brinkmanship, namely who may benefit from such threats and how this affects several outcomes besides welfare, such as the per period chance of a deal/delay, the expected location of a deal and the overall equilibrium chance of a hard outcome.

We find that the unique stationary equilibrium is characterized by an *agreement set*, representing the scope for agreement, namely the deals acceptable by both parties. In general, we show that a larger brinkmanship threat h (weakly) enlarges the equilibrium scope for agreement making an agreement more likely in every period:

[11] Our focus is the final approval/ratification of an agreement previously negotiated by a committee/delegation. This negotiation may entail bargaining between several factions, inside and/or outside the economy, as well as unforeseen economic and unanticipated institutional constraints becoming binding. In the case of Brexit, no political actor knew exactly what future proposed agreement will be in store next if the current withdrawal agreement is turned down, or what EU-UK trade deal will end up being ratified, if any. The ratification of a negotiated agreement may fail in general. For instance, the Trans-Pacific Parnership (TPP), signed by all twelve negotiating countries in 2016, never came into effect because most countries did not ratify it at home.

a higher h always succeeds in improving the chances of agreement forcing parties to compromise more.[12] While this could have been anticipated, the effects on welfare are subtle and differ case by case, as we explain. Yet, the overarching principle is that a larger brinkmanship h is always (weakly) effective in enlarging the scope for agreement, but at the same time more dangerous: the hard outcome may become more likely in equilibrium, which in turn reduces welfare.

To illustrate the relation between brinkmanship and incentives to compromise, we analyze first a one-agent decision model in which one agent must decide in each period whether to accept the deal she is presented with or wait for a fresh deal in the next period. Here the brinkmanship threat speeds up the agreement scope but is always welfare reducing. Second, we analyze the symmetric two agent case in which both parties would be equally affected by a hard outcome. We show that regardless of the parameters and the size of brinkmanship threats, welfare depends only on one sufficient statistic: the equilibrium agreement probability or scope for agreement, albeit in a convex, generically non-monotone way. This implies that, if we start from a default situation in which agreements are unlikely to begin with, then mild brinkmanship increases the agreement scope but only make things worse for either parties: only threats that are highly credible can improve welfare. Thus, in this case if either side (or third party) has no ability/credibility to make h high enough, it should avoid threats altogether.

Thirdly, we show that mild brinkmanship (low credibility threats) can generate private benefits in the asymmetric model. If one party is advantaged in the sense that it perceives a lower cost of the hard outcome then mild brinkmanship shifts the whole agreement set to its advantage. We show that as h increases the agreement set moves gradually through three qualitatively different regimes: two-sided compromise, one-sided compromise and full agreement. Only in the two-sided compromise region more brinkmanship benefits the advantaged party as the threat shifts the expected agreement more to its side of the agreement set thus hurting the disadvantaged side. High brinkmanship may benefit the latter party despite its disadvantage if it is credible enough to get close to provoke an immediate agreement.

In the following, after the literature review, we introduce the model, analyzing the

[12] Evidently, for a high enough h all agreements are accepted which implements the first best in terms of total welfare: no delay and no hard outcome in equilibrium. This amounts to an ex-ante commitment of both parties to accept immediately the first deal put on the table.

one agent case and the symmetric case before the general case and some extensions. All proofs are relegated to the appendix.

2 Related Literature

This paper touches on several strands of literature, which we outline below.

Collective search. Our modeling strategy borrows from the collective search and experimentation models, in which a group chooses every period between accepting the current negotiation outcome or wait for a new outcome next period[13]. For instance, Compte and Jehiel [2010] show that more stringent majority requirements select more efficient proposals but take more time to do so and Albrecht et al. [2010] find that committees are more permissive than a single decision maker facing an otherwise identical search problem.[14] Compte and Jehiel [2017] push further the same approach for large committees characterizing the optimal majority rule. Similarly, several authors assume different types of stochasticity. Strulovici [2010] and Messner and Polborn [2012] focus on committee decisions in which preferences are unknown and only learned over time, thus the option to delay happens in equilibrium albeit with different degrees of efficiency depending on the majority rule. Ortner [2017] analyzes how shocks to popularity of politicians affect bargaining outcomes. Moldovanu and Rosar [2019] study voting in a Brexit-like model with one irreversible option and compare the effect of different voting rules. They show that voting by supermajority over two consecutive periods dominates voting by simple majority. Basak and Deb [2020] focus on a bargaining environment in which public opinion shocks provide leverage by making compromises costly in the presence of deadlocks.

Stochastic bargaining. In our model offers/deals are exogenous, but there is a vast literature of legislative bargaining models with endogenous offers in which elements of stochasticity generate inefficient delays in agreements or gridlock in the presence of an endogenous status-quo. Several papers analyze stochastically evolving

[13]This literature is somehow related to a classic literature on bargaining where players are allowed to search for outside options, see Wolinsky [1987] and Chikte and Deshmukh [1987] for classic treatments on the question. See Muthoo [1995] that analyzes the role of players being able to leave temporarily the negotiation and Manzini and Mariotti [2004] where bilateral bargaining occurs between players that can agree on a joint outside option.

[14]In a related model with common values, Moldovanu and Shi [2013] study costly search for a committee and study how acceptance thresholds and welfare depend on the degree of conflict within the committee.

preferences, see Dziuda and Loeper [2016] or Bowen et al. [2017]. Other works explore the case of delay with a stochastic total surplus, such as Eraslan and Merlo [2002], Merlo and Wilson [1998], Merlo and Wilson [1995].

Timing games. Several authors have looked at the effect of hard deadlines in negotiations, which nicely complements our stationary setup. Namely, while we study dynamic negotiation between two parties in the presence of a stationary stochastically extendable deadline, in most of the literature, the deadline is tight in the sense that no extension is possible.[15] This generates incentives to reach agreements in the "eleventh hour", that is at or very close to the deadline (see Simsek and Yildiz [2016] for the role of optimism in these models). Such (non-stationary) timing games have been studied by Fuchs and Skrzypacz [2013] and others[16].

Ratification. Lastly, conceptually, our work speaks to the interaction between an executive branch which negotiated an agreement (possibly with an outside party/ country) and the legislative branch that needs to ratify it. For instance, Humphreys [2007] studies strategic ratification touching upon the seminal ideas of Putnam [1988] and Schelling [1960]. However, we look at this interaction once a deal has been negotiated, not before, thus for instance at how an executive branch, who has the power to seek extensions to deadline and renegotiation, can put pressure on the legislative branch who has the power to ratify the current deal.

3 One-agent model

This section presents a simple one-agent model which illustrates the relation between brinkmanship and incentives to compromise and will help understand our core two-agent model. An agent has single-peaked preferences over possible deals on the real line, represented by the function u:

$$\forall x \in \mathbb{R}, \qquad u(x) = 1 - |b - x|,$$

[15] See also Ellingsen and Miettinen [2008] who study a model of bilateral bargaining where negotiators can write binding contracts and show that conflict is frequently the unique equilibrium outcome when commitments technologies are highly credible.

[16] See Cramton and Tracy [1992] for empirical evidence or Güth et al. [2001] for experimental evidence on this observation.

where $b \in \mathbb{R}$ corresponds to the agent's bliss point. The least acceptable deal on the left side of b is located at $b-1$, it is such that $u(b-1) = 0$.[17] The agent receives proposals from a set of deals X. This set X is bounded above by a proposal at a distance $g \in [0,1]$ from her bliss point b; X is bounded below by the least acceptable deal $b-1$.

Normalizing the origin of the line at the center of the set X, we may write $X = [-a, a]$, with $a = 1 - b = \frac{1-g}{2}$, and the utility function on X can be written as $u(x) = a + x$.

The final outcome for the agent may be a deal in X or the *hard outcome* d^*. The option d^* does not lie in the set X and yields a utility $u(d^*) = d < 0$. The main parameters of the model are represented on Figure 1.

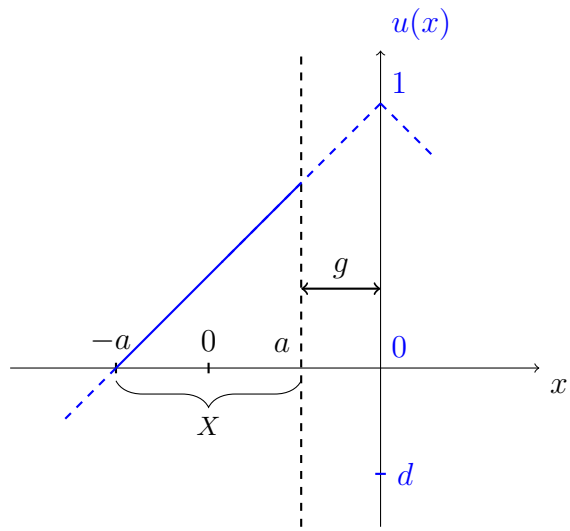

Figure 1: One-agent model

The decision procedure takes place sequentially. The agent does not control the agenda, which is stochastic. At each period $t \in \mathbb{N}$, a proposed deal $x_t \in X$ is drawn from the uniform distribution on X, independently from previous draws. Then, the agent chooses to accept or reject the proposal. If the agent accepts the proposal at period t, the final outcome is x_t. Otherwise, the *(brinkmanship) threat* $h \in [0,1]$ represents the probability that the hard outcome (with utility $d < 0$) is implemented. Formally, a Bernoulli variable H_t of parameter $h \in [0,1]$ is drawn. If $H_t = 1$, the procedure stops and the outcome is the hard outcome d^*, obtained at period t. If

[17] In the model (see below), 0 corresponds to the status-quo instantaneous payoff.

$H_t = 0$, an extension is granted and the agent moves to the next period $t+1$. The agent's discount factor is $\beta \in (0,1)$.

The strategy of the agent consists in accepting or rejecting deals as they arrive. We restrict our attention to stationary strategies. For a given (stationary) strategy, we denote by $A \subseteq X$ its *agreement set*, i.e. the set of deals that are accepted if proposed, and by w the agent's *reservation value*, i.e. her expected utility when she rejects a deal. By stationarity, w satisfies the following recursive equation:

$$w = \overbrace{hd}^{\text{hard outcome } d} + \overbrace{\beta(1-h)\mathbb{P}(x \in A)\mathbb{E}[u(x) \mid x \in A]}^{x \text{ lies in the agreement set}} + \overbrace{\beta(1-h)\mathbb{P}(x \notin A)w}^{x \text{ does not lie in the agreement set}} .$$

3.1 Optimal strategy

For a stationary strategy to be optimal, the agent accepts a deal $x \in X$ if and only if its utility exceeds her reservation value, i.e. $u(x) \geq w$. Thus, a strategy is optimal if and only if its agreement set satisfies $A = A_w = \{x \in X \mid u(x) \geq w\}$. The condition for the stationary strategy with reservation value w to be optimal can thus be summarized by:

$$w = \frac{hd + \beta(1-h)\mathbb{P}(x \in A_w)\mathbb{E}[u(x) \mid x \in A_w]}{1 - \beta(1-h)\mathbb{P}(x \notin A_w)}. \tag{1}$$

Building on equation (1), our first result characterizes the optimal strategy of the agent. To ease notations, we introduce two parameters that we use throughout: $\Phi = \frac{h}{1-\beta(1-h)}$ and $\Delta = \frac{\beta(1-h)}{1-\beta(1-h)}$. We let c denote the center of the agreement set A.

Proposition 1 *There exists a unique optimal stationary strategy. The agreement set A is a non-empty closed interval with $c \geq 0$. There is a threshold $h_1 \in (0,1)$ such that:*

- *for $h \in [0, h_1)$, the agreement set is $A = [a-l, a]$ with a length $l = 2a\lambda$, such that $\lambda = \frac{1}{\Delta}\left(\sqrt{1 + 2\Delta(1 - \frac{\Phi d}{2a})} - 1\right)$. The instantaneous probability of accepting a proposal λ is increasing with the threat h.*

- *for $h \in [h_1, 1]$, the agreement set is $A = [-a, a]$. Any proposal is immediately accepted.*

The main lesson of Proposition 1 is that when the threat h increases, the agent compromises more, as her instantaneous probability of agreement λ increases. Intuitively, the agent rejects proposals too far from her bliss point when the threat is low, but when the threat reaches or exceeds the threshold h_1, then the agent accepts any proposal to avoid the risk of the hard outcome.

As a result, the center of the agreement set c, which is also the expected location of an accepted proposal, is always positive but decreases with the threat h. Observe also that the center c decreases (λ increases) when the hard outcome becomes more severe (d decreases).

To illustrate Proposition 1, we draw the agreement set associated to the optimal strategy on a first example.

Example 1

We focus on the example where $\beta = 0.95$, $g = 1/5$ and $d = -1/2$. For these parameters, we draw the bounds and the center of the agreement set for all values of h between 0 and 1 on Figure 2.

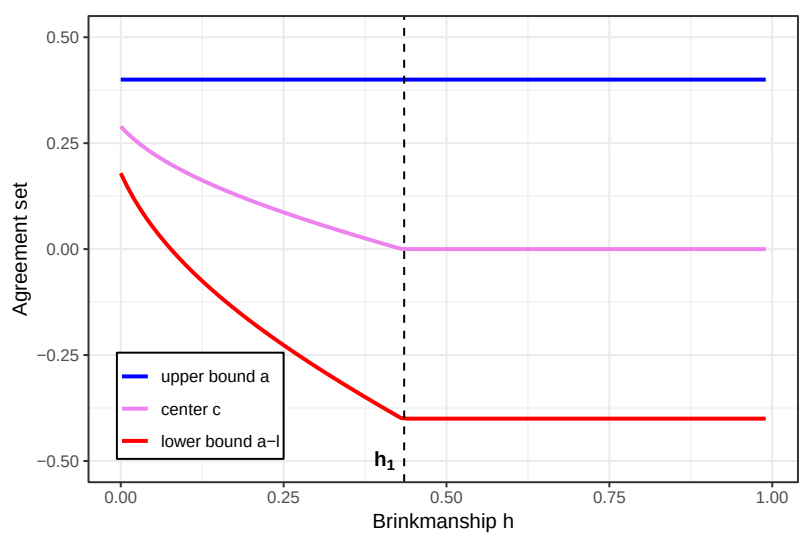

Figure 2: Agreement set in Example 1

We observe two regimes on this picture. For $h \leq h_1 \approx 0,43\%$, the agent rejects some deals. As h increases, the length of the agreement set increases up to $h = h_1$, where the agent accepts all deals. The center of the agreement set decreases, up to 0 when $h = h_1$.

3.2 Welfare

We now characterize the agent's welfare when she plays her optimal strategy. We denote by W her expected utility, which satisfies the following recursive equation:

$$W = \mathbb{P}(x \in A)\mathbb{E}[u(x) \mid x \in A] + \mathbb{P}(x \notin A)\left(hd + \beta(1-h)W\right).$$

In this formula, the welfare is computed at the beginning of a period: either the randomly selected deal x belongs to the agreement set, in which case it yields $\mathbb{E}[u(x) \mid x \in A]$ in expectation, or it fails to do so and hence, either the hard outcome is selected or a new period starts, with expected utility W. Therefore, the welfare of the agent is given by:

$$W = \frac{(1-\lambda)hd + \lambda\mathbb{E}[u(x) \mid x \in A]}{1-\beta(1-h)(1-\lambda)}. \tag{2}$$

The following result asserts that this welfare only depends on the instantaneous probability of accepting a proposal.

Proposition 2 *The agent's welfare when she plays her optimal strategy solely depends on the instantaneous probability of accepting a proposal λ, and follows a convex function, given by:*

$$W = 2a\left(1 - \lambda + \frac{\lambda^2}{2}\right).$$

As a function of the endogenous variable λ, the agent's welfare is decreasing and convex.[18] As λ increases with the threat h, this means that, while a higher threat might be desirable as it implements an agreement sooner, the agent is always hurt by threats. This is not surprising in this one-agent benchmark, since increasing the threat only decreases the payoffs associated to actions in an otherwise similar decision problem. Finally, observe that the agent's welfare improves when the upper bound of the agreement set becomes closer to her bliss point (when g decreases, or equivalently, a increases).

To illustrate Proposition 2, we plot on Figure 3 the agent's welfare and the overall probability of the hard outcome d^* as a function of the threat h.

The agent's welfare is strictly decreasing with the threat up to $h = h_1$, after which it remains constant, equal to $a = 0.4$. The decrease in welfare is particularly steep for

[18] Since $\lambda \in [0,1]$, we have $\dfrac{\partial W}{\partial \lambda} = 2a(\lambda - 1) \leq 0$ and $\dfrac{\partial^2 W}{\partial \lambda^2} = 2a \geq 0$.

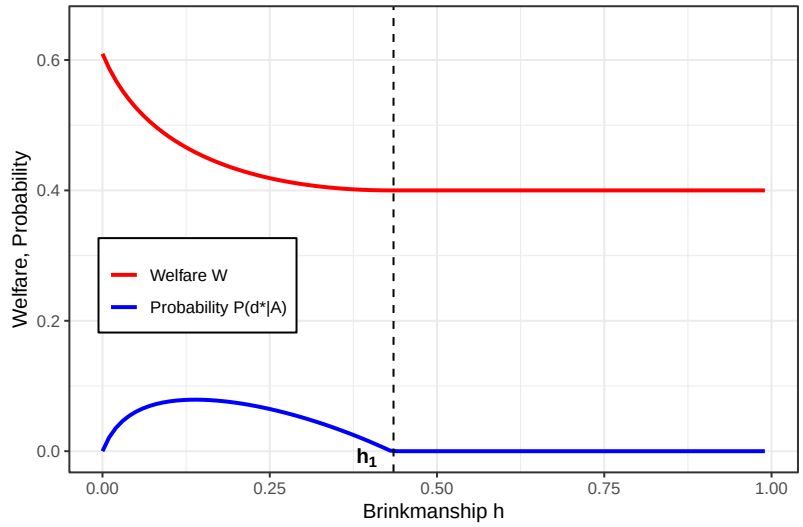

Figure 3: Welfare in Example 1

low values of the threat, reflecting two forces: the center of the agreement set moves away from the agent's bliss point (Figure 2) and the probability of a hard outcome occurring (instead of an accepted proposal) increases (Figure 3).

4 Two-agent model

We now introduce our main model with two agents, denoted by $\theta = 0$ and $\theta = 1$. Each agent $\theta \in \{0, 1\}$ has single-peaked preferences over possible deals on the real line, represented by the function $u_\theta(x) = 1 - |b_\theta - x|$. We assume that $b_0 < b_1$ and $b_1 - b_0 = 1 + g$, with $g \in (0, 1)$ representing the extent of the disagreement between the two sides. The proposal set X consists of all deals that are acceptable for both agents.

As in the one-agent model, we normalize the origin of the line at the center of the set X. We may thus write $X = [-a, a]$ with $a = 1 - b_1 = b_0 + 1 = \frac{1-g}{2}$. The utility functions on X can then be written as $u_0(x) = a - x$ and $u_1(x) = a + x$.

The final outcome may be a deal in X or the *hard outcome* d^*. The option d^* does not lie in the set X and yields a utility $d_\theta < 0$ for each agent $\theta \in \{0, 1\}$. We denote by $D = d_0 + d_1$ the hard outcome's *total value*, and by $B = d_1 - d_0$ the hard outcome's *bias*. Without loss of generality, we assume that $B \geq 0$, and if $B > 0$, we refer to 1 as the *advantaged* agent and to 0 as the *disadvantaged* one. The main

parameters of the model are represented on Figure 4.

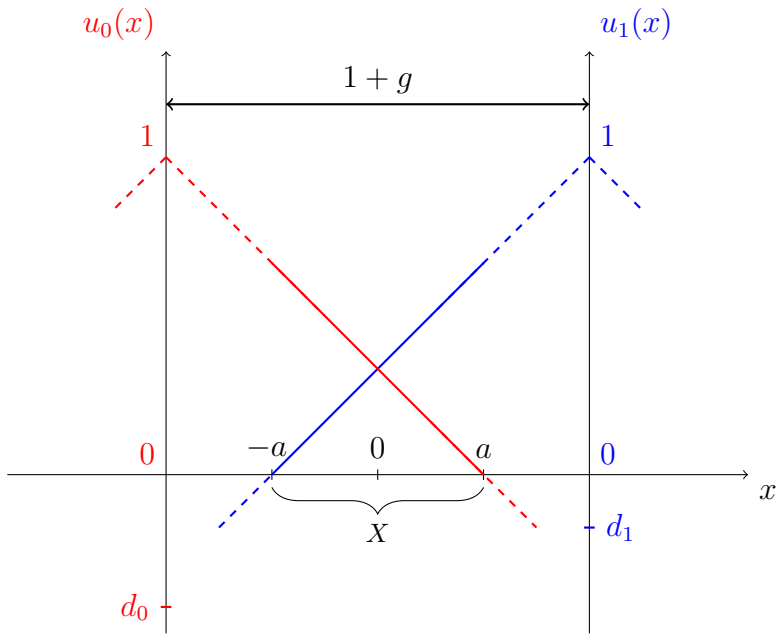

Figure 4: Two-agent model

As in the single-agent model, the bargaining procedure is sequential and the agenda is stochastic. For each $t \in \mathbb{N}$, a deal $x_t \in X$ is drawn from the uniform distribution on X, independently from previous draws. Then, agents simultaneously choose to accept or reject the proposal. If both accept it at period t, the final outcome is x_t. Otherwise, the resulting outcome is governed by a Bernoulli variable H_t which takes value 1 with probability h and value 0 with probability $1-h$. If $H_t = 1$, the procedure stops and the hard outcome d^* is obtained at period t. If, on the contrary, $H_t = 0$, an extension is granted and both players move to the next period $t+1$.

For a profile of stationary strategies, we denote by w_θ agent θ's *reservation value* and by $A \subseteq X$ the agreement set, i.e. the deals that are accepted by both agents if proposed. The condition for a stationary profile with reservation values (w_0, w_1) to be an equilibrium is that its agreement set satisfies $A = A_w = \{x \in X \mid u_\theta(x) \geq w_\theta,\ \forall \theta \in \{0,1\}\}$. Hence, this profile is an equilibrium if (1) is satisfied for each $\theta \in \{0,1\}$, where w_θ, d_θ and u_θ respectively play the role of w, d and u. Applying a fixed-point argument, we first obtain the existence of a stationary equilibrium.

Theorem 1 *A stationary equilibrium exists. Moreover, at each such equilibrium, the agreement set A is a non-empty closed interval.*

4.1 Symmetric model

We start by analyzing the symmetric version of the model, i.e. we set $d_0 = d_1$, or equivalently $B = 0$.

4.1.1 Equilibrium

Our first result characterizes the unique stationary equilibrium of the game.

Proposition 3 *In the symmetric model, for any $h \in [0, 1]$, there exists a unique stationary equilibrium. There is a threshold $0 < h_1 < 1$, such that:*

- *for $h \in [0, h_1)$, the agreement set A is centered in $c = 0$ and has a length $l = 2a\lambda$ such that $\lambda = \frac{1}{2\Delta}\left(\sqrt{1 + 4\Delta(1 - \frac{\Phi D}{2a})} - 1\right)$. The instantaneous probability of accepting a proposal λ is increasing in the threat h.*

- *for $h \in [h_1, 1]$, the agreement set is $A = [-a, a]$.*

The main take-away of Proposition 3 is that the willingness of both agents to compromise, as measured by the instantaneous agreement probability λ, always increases with the threat h. As in the one-agent model, each agent rejects deals too far from her bliss point when the threat h is low enough, but accepts all deals when the threat reaches the threshold h_1. The agreement probability λ also depends on the severity of the hard outcome: when it becomes more severe (D decreases), agents become more likely to compromise (λ increases). Finally, note that λ decreases with a, or equivalently increases with $g = 1 - 2a$: when the agent's bliss points are further apart (g increases), agents compromise more easily (λ increases).

Contrary to what happens in the one-agent model, the center of the agreement set c does not depend on the threat. Indeed, by symmetry of the model, c always coincides with 0, the center of the proposal set X.

We illustrate the insights of Proposition 3 by drawing the equilibrium agreement set on an example.

Example 2

We focus on the example where $\beta = 0.95$, $g = 1/5$, $D = -1/2$ and $B = 0$. For these parameters, we draw the bounds and the center of the agreement set for all values of h between 0 and 1 on Figure 5.

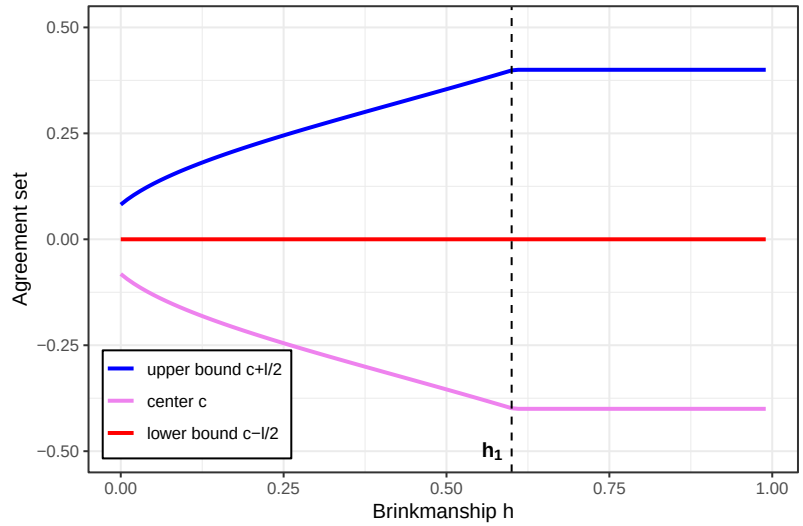

Figure 5: Agreement set in Example 2

We observe two regimes on Figure 5. For $h \leq h_1 \approx 0.6$, both agents reject some deals at equilibrium. As h increases, the length of the agreement set increases up to $h = h_1$, where both agents accept all deals. As the model is symmetric, no agent is advantaged, and the center is equal to 0.

4.1.2 Welfare

We denote by W_θ the expected utility of each agent $\theta \in \{0, 1\}$. As for the one-agent model, the welfare of agent θ at a stationary equilibrium satisfies (2), where W_θ, d_θ and u_θ respectively play the role of W, d and u.

Theorem 2 *In the symmetric model, agents' equilibrium welfare solely depends on the instantaneous probability of accepting a proposal λ, and follows a convex function, given by:*

$$W_0 = W_1 = a\left(1 - \lambda + \lambda^2\right).$$

As a function of the endogenous variable λ, welfare is convex, symmetric around $1/2$, and reaches its maximum either at $\lambda = 0$ or $\lambda = 1$. Since λ is increasing in h, the main lesson of Theorem 2 is that welfare is not monotonic as a function of the threat h, contrary to what we observed in the single-agent model. While welfare is maximal when the threat h is high enough to enforce full agreement, this does not mean that decreasing the threat always reduces welfare. On the contrary, when the

agreement probability λ is below 1/2, then decreasing the threat increases welfare. The insight is particularly relevant when the party choosing the threat is constrained, for instance if threats above some threshold $\overline{h}$ are deemed non-credible. In such case, the optimal value of the threat is either to fix a maximal threat $\overline{h}$ or no threat at all ($h = 0$).

Note the discrepancy between Theorem 2 and the observation made in the one-agent model that the welfare was always decreasing with h (Proposition 2). To shed more light on the non-monotonicity result in Theorem 2, we introduce a decomposition of the agents' welfare in the next section.

4.1.3 Welfare decomposition: instant utility and delay factor

In this section, we decompose the agent's welfare as the product of two factors: *instant utility* and *delay factor*. The instant utility is the undiscounted expected utility of the eventual outcome. As the agreement set is centered in 0, the average utility of an accepted deal is a for each agent. Thus, the instant utility of an agent $\theta \in \{0, 1\}$ can be written as the convex combination of a and d_θ, this last term being weighted by the *overall probability of hard outcome* $\mathbb{P}(d^* \mid A)$. The delay factor is the ratio of welfare to instant utility, it accounts for the delay incurred in reaching an agreement through the bargaining process. Equation (2) implies the following welfare decomposition:[19]

$$W_\theta \quad = \quad \underbrace{(a + (d_\theta - a)\mathbb{P}(d^* \mid A))}_{\text{instant utility}} \quad \times \quad \underbrace{\frac{1-(1-h)(1-\lambda)}{1-\beta(1-h)(1-\lambda)}}_{\text{delay factor}}.$$

The shape of welfare as a function of the threat h thus depends on the relative importance of those two terms. The delay factor is increasing in the threat h, as agents reach an agreement sooner with a higher threat.[20] By contrast, the instant utility displays a U-curve, because the overall probability of hard outcome is hump-shaped, as it is null both when there is no threat and when the threat is high enough to force full agreement. The evolution of the agent's welfare reflects the relative weight of these two terms.

[19] To see this, note that $\mathbb{P}(d^* \mid A) = (1-\lambda)h \sum_{k=0}^{+\infty}(1-\lambda)^k(1-h)^k = \dfrac{(1-\lambda)h}{1-(1-\lambda)(1-h)}$.

[20] Indeed, $(1-h)(1-\lambda)$ decreases with h, while $\frac{1-x}{1-\beta x}$ decreases with x, as $\beta < 1$.

When the discount factor β is so small that the agreement probability is higher than one half in the absence of threat (i.e. $\lambda(\beta, h = 0) \geq \frac{1}{2}$), then welfare is increasing in h by Theorem 2. This arises because, when β is low, the evolution of the delay factor dominates that of the instant utility. For larger β however (such that $\lambda(\beta, h = 0) < \frac{1}{2}$), the welfare displays a U-curve as a function of h. This is because, when β is large, the evolution of the instant utility dominates that of the delay factor. This second case corresponds to what arises in Example 2 (where $\lambda \approx 20.5\%$ when $h = 0$), as illustrated in Figure 6.

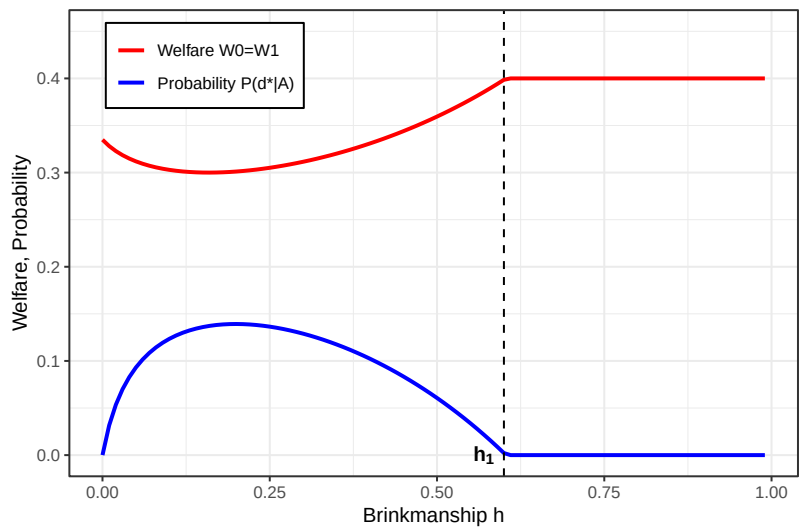

Figure 6: Welfare and probability of a hard outcome in Example 2.

4.2 Asymmetric model

We now consider the asymmetric version of the model, where the hard outcome affects both agents in a different manner. More precisely, we let $B > 0$ in the rest of the section so that $d_1 > d_0$, making player 1 less affected in the event of a hard outcome. The payoff in case of hard outcome is the only asymmetry/advantage assumed: our goal is to see what kind of advantage this may bring in equilibrium, if any, to side 1.

4.2.1 Equilibrium

To ease their description, we divide stationary equilibria into types, corresponding to the number of agents that reject some proposals at equilibrium. At a *two-sided equilibrium*, both agents reject some proposals: for any $\theta \in \{0,1\}$, $w_\theta > 0$. This type of equilibrium has already been encountered in the symmetric model. At a *one-sided equilibrium*, only agent 1 rejects some proposals: $w_0 \leq 0$ and $w_1 > 0$. Such equilibrium coincides with the optimal agreement set obtained in the single-agent model. Finally, the model also admits *full-agreement equilibria*, for which both agents accept all deals: for any $\theta \in \{0,1\}$, $w_\theta \leq 0$. In the following result, we employ the notation h_θ to denote the threshold at which agent θ decides to accept all proposals.

Proposition 4 *In the asymmetric model, for any $h \in [0,1]$, there exists a unique stationary equilibrium. There are thresholds $0 < h_0 < h_1 < 1$, such that:*

- *for $h \in [0, h_0)$, the equilibrium is two-sided. The agreement set is centered in $c = \frac{\Phi B}{2}$ and has length $l = 2a\lambda$ with $\lambda = \dfrac{1}{2\Delta}\left(\sqrt{1 + 4\Delta\left(1 - \frac{\Phi D}{2a}\right)} - 1\right)$. Both λ and c are increasing in h.*

- *for $h \in [h_0, h_1)$, the equilibrium is one-sided. The agreement set is centered in $c = a - \dfrac{l}{2}$ and has length $l = 2a\lambda$ with $\lambda = \dfrac{1}{\Delta}\left(\sqrt{1 + 2\Delta\left(1 - \frac{\Phi(B+D)}{4a}\right)} - 1\right)$. Moreover, λ is increasing in h and c is decreasing in h.*

- *for $h \in [h_1, 1]$, the equilibrium displays full agreement: the agreement set is centered in $c = 0$ and has length $l = 2a$.*

As in the symmetric model, the agreement probability λ always increases with the threat h. Moreover, the location of the expected deal, c, varies non-monotonically with h, as it is equal to 0 for both $h = 0$ and $h = 1$. Starting from $h = 0$, the expected deal first increases with h, up to the point h_0 at which agent 0 accepts any deal, and then decreases to reach 0 when $h = h_1$, the point where both agents fully compromise.

At a two-sided equilibrium, the expected location of the deal, c, increases with the relative advantage of agent 1 at the hard outcome, B. By contrast, the length of the agreement set does not depend on B, and it behaves as in the symmetric model.

At a one-sided equilibrium, agent 0 accepts any proposed deal. As a result, both the center and the length of the agreement set depend on agent 1's value for the hard outcome $d_1 = (D + B)/2$. As in the single-agent model, c increases and λ decreases when agent 1's value for the hard outcome increases ($D + B$ increases).

To illustrate Proposition 4, we draw the equilibrium agreement set as a function of h on a third (asymmetric) example.

Example 3

We focus on the example where $\beta = 0.95$, $g = 1/5$, $D = -1/2$ and $B = 1/4 > 0$. For these parameters, we draw the bounds and the center of the agreement set for all values of h between 0 and 1 on Figure 7.

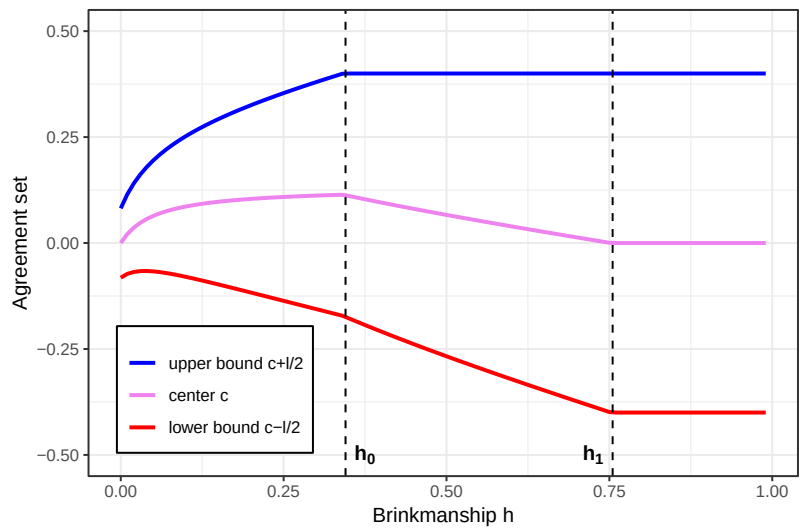

Figure 7: Agreement set in Example 2

We observe three regimes on this picture. For $h \leq h_0 \approx 0.34$, we have a two-sided equilibrium where both agents reject some deals. As the threat h increases, the length of the agreement set increases. Moreover, the center of the agreement set increases as well. This means that the advantage of agent 1 (in terms of the expected deal) becomes stronger when the hard outcome becomes more likely.

For $h \geq h_0$ and $h \leq h_1 \approx 0.75$, we enter into a second regime where only agent 1 rejects some deals, while agents 0 accepts all. In this regime, it remains true that λ increases with h, i.e. that both agents become more willing to compromise. However,

as agent 0 already accepts all deals, all this compromise effort is borne by agent 1, and the expected deal becomes closer to 0 as h increases.

Finally, when h_1 is reached, both agents accept all deals, and full agreement remains for any further increase in the threat h.

4.3 Welfare

In this section, we characterize the welfare of each agent in the asymmetric model for each type of equilibrium. First, note that the first proposed deal is accepted at a full-agreement equilibrium, so that $W_0 = W_1 = a$.

We then consider the welfare associated to two-sided equilibria.

Proposition 5 *At any two-sided equilibrium, agents' welfare is given by:*

$$W_0 = a(1 - \lambda + \lambda^2) - \frac{B\Phi}{2} \qquad \textit{and} \qquad W_1 = a(1 - \lambda + \lambda^2) + \frac{B\Phi}{2}.$$

The main observation of Proposition 5 is that agents' welfare at a two-sided equilibrium can be split in two parts. First, a common-value part is the average welfare, given by $\overline{W} = a(1-\lambda+\lambda^2)$, as for the symmetric model. To obtain an agent's welfare, one needs to add a second, zero-sum and private-value part, of magnitude $\frac{B\Phi}{2} = c$, so that we have $W_0 = \overline{W} - c$ and $W_1 = \overline{W} + c$. When the agreement set is centered in 0, both players share the same welfare since the zero-sum part vanishes. If the center is closer to agent 1's bliss point, agent 1's welfare increases by the same amount as agent 0's welfare decreases.

The following result focuses on one-sided equilibrium welfare.

Proposition 6 *At any one-sided equilibrium, agents' welfare is such that:*

$$W_1 = 2a(1 - \lambda + \frac{\lambda^2}{2}) > a \qquad \textit{and} \qquad W_0 < 2a - W_1 < a.$$

We observe that agent 1's welfare decreases with the agreement probability λ, and thus with h, at a one-sided equilibrium. Nevertheless, agent 1 always remains advantaged, in the sense that she achieves a welfare higher than a, the welfare level reached under full agreement. By contrast, the welfare of agent 0 remains below a. Finally, note that the advantage conferred to agent 1 is costly at a social level, as the average welfare is below a.

4.4 Welfare-maximizing threats

In this section, we describe how each agent would choose the level of threat h, if she were to choose it unilaterally.

Theorem 3 *In the asymmetric model,*

- *the equilibrium welfare of agent* 1 *is maximal for a threat* $h \in (0, h_0]$*. For this threat level,* $W_1 > a$ *and the hard outcome occurs with positive probability in equilibrium.*
- *the equilibrium welfare of agent* 0 *is maximal for any threat* $h \in [h_1, 1]$*. For this threat level,* $W_0 = a$ *and the hard outcome never occurs in equilibrium.*

We observe in Theorem 3 a discrepancy between the two agents. Agent 0 would always choose a threat h high enough to enforce full agreement, so that the hard outcome never occurs. This is not the case for agent 1, who would always choose an intermediate threat $h \in (0, h_0]$. This means that agent 1 prefers to use the threat at her advantage, at the risk of seeing the hard outcome occurring. Indeed, for such threat h, the hard outcome arises on-path with positive probability. Note one surprising feature of the result: the level of threat preferred by agent 0, who suffers the most from the hard outcome, is always higher than the level preferred by agent 1.

To illustrate the optimal choice of threats, we depict on Figure 8 the welfare of each agent and the average welfare as a function of the threat h in Example 3.

In this example, agent 0's welfare displays a U-curve, as in the symmetric model, and is maximal for $h \geq h_1 \approx 0.75$, when agents reach full agreement. However, the welfare of agent 1 is maximized for a threat $h = h_0 \approx 0.34$. This means that agent 1 would choose a threat just high enough to make her opponent accept every deal.[21]

[21] We note that this is not a general result: for the same parameter values but $\beta = 0.999$, agent 1's welfare W_1 is maximal for $h < h_0$.

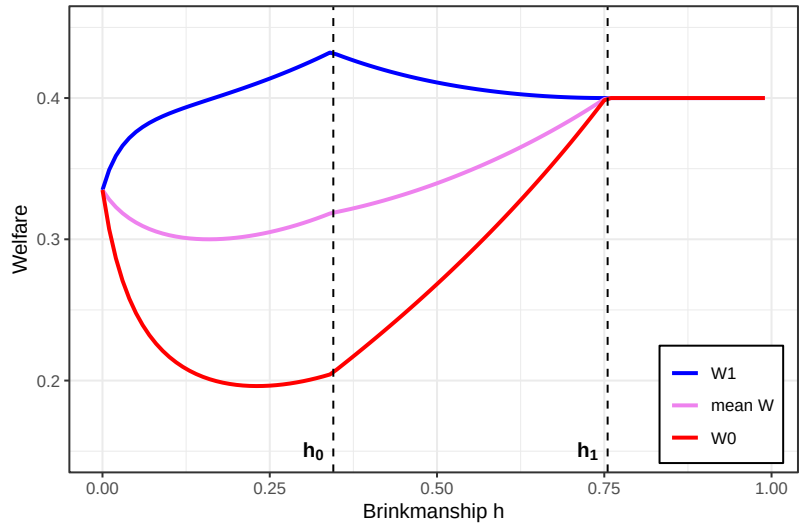

Figure 8: Agents' welfare in Example 3.

5 Extensions and Discussion

5.1 Discount factor and brinkmanship threat

As mentioned in the introduction, there are two forces that push players to compromise in our model. First, as usual, there is a discount factor $\beta \in (0, 1)$, the term by which the surplus is to be divided between the two agents shrinks. Second, the brinkmanship threat $h \in [0, 1]$ which represents the risk of the hard outcome d^* at any period and can be interpreted as a "stationary time-bomb".

The model would be quite different without a standard discount factor, i.e. if we assume $\beta = 1$, for the following reasons. Note that in the symmetric model the expected deal c is the same with either $h = 0$[22] or $h = h_1$. Since there is no loss from delay/discounting with $\beta = 1$, the welfare of the two agents must also be the same for $h = 0$ and $h = h_1$. In this case, the advantage of the maximal threat disappears, and both players would be equally satisfied with no threat at all.

By contrast, in the asymmetric model, since $\Phi = 1$ when $\beta = 1$, the location of the expected deal when $h < h_0$ becomes $c = \frac{B}{2}$. As c does not depend on the threat h in this region, the advantage of a higher/positive threat disappears for agent 1, while the risk she bears from such a threat remains. Agent 1 is thus better off when there is no threat at all, unlike in our model. One way to understand the discrepancy between

[22] To be precise, we consider here the limit when $h \to 0$.

the two assumptions is to look at the location of the expected deal when h tends to 0, and to interpret $1-\beta$ as a per-period probability to reach an outcome for which both players get 0 (*limbo outcome*). With $\beta = 1$, the hard outcome is always more likely to arise than the limbo outcome. Therefore, the asymmetric threat dominates which, in turn, implies that $c > 0$. On the other hand, with $\beta < 1$, the risk of the limbo outcome dominates the threat of the hard outcome for h small enough, so that $c = 0$ at the limit.

5.2 Costly threat

We consider the same model as above, in which we add the possibility of agreement failure at the negotiation deal-making stage. At each period, there is a fixed probability $k \in (0,1)$ that *no deal is drawn*, in which case agents directly face the lottery where the hard outcome realizes with probability h, while the game continues to the next period with probability $(1-h)$. With remaining probability $(1-k)$, a deal is drawn and the period is the same as in the baseline model.

Proposition 7 *For any $h, k \in [0,1]$, there exists a unique stationary equilibrium. Moreover:*

- *the thresholds h_θ decrease with k: the possibility of failure pushes both players to compromise*
- *there exists $\overline{k}$ such that for $k \leq \overline{k}$:*
  - *the welfare-maximizing threat is h_1 for both agents in the symmetric model and for the disadvantaged agent in the asymmetric model*
  - *the welfare-maximizing threat lies in $(0, h_0]$ for the advantaged agent in the asymmetric model*

We interpret Proposition 7 as a robustness check to our results. If the possibility of agreement failure k is small enough, then all of our results on agents' welfare remain true. The only nuance comes from the full-agreement region. In this region, agents' welfare becomes decreasing in h rather than constant: even if both agents decide to compromise fully, they still bear a risk k of no-agreement, which itself yields a risk h of hard outcome. As a result, all agents are better off for $h = h_1$ in this region. Hence,

the robust feature of our results is that, for both agents in the symmetric model or for the disadvantaged agent in the asymmetric one, the welfare-maximizing threat is the one that makes all agents fully compromise, but not necessarily the highest possible threat.

6 Conclusions

The broad question of how threats during a negotiation may alter actual and potential outcomes is central to many political and non political negotiations. We analyzed how these threats can affect the negotiation gridlock, the welfare of the negotiating parties and finally when they can be used strategically by either party. Our model is extendable to more general distributions of potential agreements and to asymmetric information in which the payoffs to hard outcomes are private information of the two sides. Finally, the results of our model may be used as the last stage of a broader model which endogenizes the credibility walk-away do-or-die threat announcements.

A Proofs

A.1 Proof of Proposition 1

Proof. Let ψ be the function defined on $\mathbb{R}$ by:

$$\psi(w) = hd + \beta(1-h)\mathbb{P}(x \in A_w)\mathbb{E}[u(x) \mid x \in A_w] + \beta(1-h)\mathbb{P}(x \notin A_w)w.$$

We have that for any $w \in (-\infty, 1]$,

$$hd + \beta(1-h)w \leq \psi(w) \leq hd + \beta(1-h).$$

Let $w^{\max} = hd + \beta(1-h) \leq 1$ and $w^{\min} = \frac{hd}{1-\beta(1-h)}$. As $d \leq 0$, we obtain that $w^{\max} \geq w^{\min}$. Now, for $w \in [w^{\min}, w^{\max}]$, we have $\psi(w) \leq w^{\max}$ and:

$$\psi(w) \geq hd + \beta(1-h)w^{\min} = hd + \beta(1-h)\frac{hd}{1-\beta(1-h)} = \frac{hd}{1-\beta(1-h)} = w^{\min}.$$

Hence, the function $\psi : [w^{\min}, w^{\max}] \to [w^{\min}, w^{\max}]$ is continuous, from a non-empty convex compact set onto itself, so it admits a fixed point w^* by Brouwer's theorem. By application of (1), the stationary strategy with reservation value w^* is optimal.

At any optimal strategy with reservation value w, the agreement set can be written as $A = \{x \in X \mid u(x) \geq w\} = \{x \in X \mid a + x \geq w\} = [-a + \max(w, 0), a]$.

If the agreement set A was empty, we would have by application of equation (1), $w = hd/(1 - \beta(1 - h)) \leq 0$, a contradiction with $A = [-a + \max(w, 0), a]$. Thus, A is a non-empty, closed interval centered in $c = \max(w, 0)/2 \geq 0$.

Now, we may apply (1), by noting $\lambda := \mathbb{P}(x \in A) = \dfrac{l}{2a}$ and observing that $\mathbb{E}[u(x) \mid x \in A] = u(c) = a + c$. We obtain:

$$w = \frac{hd + \beta(1-h)\lambda(a+c)}{1 - \beta(1-h) + \beta(1-h)\lambda} = \frac{\Phi d + \Delta\lambda(a+c)}{1 + \Delta\lambda}.$$

We consider in turn two possible sorts of agreement sets: full agreement and partial agreement.

Partial agreement. We first consider the case $w > 0$, where $A = [-a + w, a]$, so that some proposals that are too far from the agent's bliss point are rejected. We have $w = 2a - l = 2a(1 - \lambda)$ and $a + c = a + \frac{w}{2} = 2a(1 - \frac{\lambda}{2})$. We obtain:

$$2a(1-\lambda)(1+\Delta\lambda) = \Phi d + 2a\Delta\lambda(1 - \frac{\lambda}{2}) \quad \Leftrightarrow \quad 1 - \frac{\Phi d}{2a} = \lambda + \Delta\frac{\lambda^2}{2}.$$

Hence, we get:

$$\lambda = \frac{1}{\Delta}\left(\sqrt{1 + 2\Delta\left(1 - \frac{\Phi d}{2a}\right)} - 1\right).$$

Full agreement. The remaining case to consider is $w \leq 0$. In that case, $A = [-a, a]$, so that any proposal is immediately accepted. We can write $w = \dfrac{\Phi d + \Delta a}{1 + \Delta}$.

To conclude, we show that there is a threshold value $h_1 \in (0, 1)$, below which the unique optimal strategy is of the partial agreement type, and above which the unique optimal strategy is of the full agreement type. Observe that full agreement is optimal if and only if $\Phi d + \Delta a \leq 0$. By contrast, partial agreement is optimal if and only if there exists $\lambda \in (0, 1)$ such that $1 - \frac{\Phi d}{2a} = \lambda + \Delta\frac{\lambda^2}{2}$, which is equivalent to $1 - \frac{\Phi d}{2a} < 1 + \frac{\Delta}{2}$, or $\Phi d + \Delta a > 0$.

Let $F(h) = \Phi(h)d + \Delta(h)a$. We have $\frac{\partial \Phi}{\partial h} = \frac{1-\beta}{(1-\beta(1-h))^2} > 0$ and $\frac{\partial \Delta}{\partial h} = \frac{-\beta}{(1-\beta(1-h))^2} < 0$. As $d < 0$ and $a > 0$, we obtain that F is decreasing in h, with $F(0) = \frac{\beta a}{1-\beta} > 0$ and $F(1) = d < 0$. Let $h_1 \in (0,1)$ be the unique value such that $F(h_1) = 0$. We have obtained that for $h < h_1$, $F(h) > 0$, so that partial agreement is the sole optimal stationary strategy. Similarly, for $h \geq h_1$, $F(h) \leq 0$, so that full agreement is the sole optimal stationary strategy. ■

A.2 Proof of Proposition 2

Proof. If $h \geq h_1$, we have full agreement. Then, the first proposed deal is accepted and it is located at $c = 0$ on average, so that $W = a$. As $\lambda = 1$, the formula in Proposition 2 is thus valid.

If $h < h_1$, we have partial agreement, let A be the agreement set. Noting $\lambda = \mathbb{P}(x \in A)$, we know that:

$$\begin{aligned} W &= \frac{(1-\lambda)\, hd + \lambda \mathbb{E}[u(x) \mid x \in A]}{1 - \beta(1-h)\,(1-\lambda)}, \\ w &= \frac{hd + \beta(1-h)\lambda \mathbb{E}[u(x) \mid x \in A]}{1 - \beta(1-h)\,(1-\lambda)}. \end{aligned}$$

Solving for $\lambda \mathbb{E}[u_\theta(x) \mid x \in A]$ in both expressions, we have:

$$\begin{aligned} \frac{\lambda \mathbb{E}[u(x) \mid x \in A]}{1 - \beta(1-h)\,(1-\lambda)} &= W - \frac{(1-\lambda)\, h}{1 - \beta(1-h)\,(1-\lambda)} d \\ &= \frac{1}{\beta(1-h)} \left(w - \frac{h}{1 - \beta(1-h)\,(1-\lambda)} d \right). \end{aligned}$$

Thus, simplifying, we have the affine relation:

$$W = \frac{1}{\beta(1-h)} \left(w - hd \right).$$

As we have partial agreement, we know that $w = 2a - l = 2a(1-\lambda)$, and we may write

$$W = \frac{2a\,(1-\lambda) - hd}{\beta(1-h)}.$$

As we have partial agreement, we also know that λ is the solution of:

$$\begin{aligned}
1-\frac{\Phi d}{2a}=\lambda+\Delta\frac{\lambda^2}{2} \quad &\Leftrightarrow \quad 1-\frac{h}{1-\beta(1-h)}\frac{d}{2a}=\lambda+\frac{\beta(1-h)}{1-\beta(1-h)}\frac{\lambda^2}{2}\\
&\Leftrightarrow \quad 1-\beta(1-h)-\frac{hd}{2a}=(1-\beta(1-h))\,\lambda+\beta(1-h)\frac{\lambda^2}{2}\\
&\Leftrightarrow \quad (1-\lambda)-\frac{hd}{2a}=\beta(1-h)\left(1-\lambda+\frac{\lambda^2}{2}\right).
\end{aligned}$$

We thus obtain as desired:

$$W=\frac{2a}{\beta(1-h)}\left((1-\lambda)-\frac{hd}{2a}\right)=2a\left(1-\lambda+\frac{\lambda^2}{2}\right).$$

■

A.3 Proof of Theorem 1

Proof. The argument follows the one in the proof of Proposition 1. For $\theta\in\{0,1\}$, let ψ_θ be the function defined on $\mathbb{R}^2$ by: for any $w=(w_0,w_1)$,

$$\psi_\theta(w)=hd_\theta+\beta(1-h)\mathbb{P}(x\in A_w)\mathbb{E}[u_\theta(x)\mid x\in A_w]+\beta(1-h)\mathbb{P}(x\notin A_w)w_\theta.$$

As in the proof of Proposition 1, we note $w_\theta^{\max}=hd_\theta+\beta(1-h)$ and $w_\theta^{\min}=\frac{hd_\theta}{1-\beta(1-h)}$. We obtain similarly that for any $w_{1-\theta}$, we have $w_\theta^{\min}\leq\psi_\theta(w)\leq w_\theta^{\max}$. The function $\psi(w)=(\psi_0(w),\psi_1(w))$ is thus continuous, from the non-empty convex compact set $[w_0^{\min},w_0^{\max}]\times[w_1^{\min},w_1^{\max}]$ onto itself. By Brouwer's theorem, ψ admits a fixed point w^*. Hence, the game admits $w^*=(w_0^*,w_1^*)$ as a stationary equilibrium.

At any stationary equilibrium w, the agreement set can be written as:

$$\begin{aligned}
A=\{x\in X\mid\forall\theta,\mathbb{E}[u_\theta(x)\mid x\in A]\geq w_\theta\}&=\{x\in X\mid a-x\geq w_0 \text{ and } a+x\geq w_1\}\\
&=\{x\in X\mid -a+w_0\leq x\leq a-w_1\}.
\end{aligned}$$

Thus, A is an interval. If A were empty, we would have by application of (1): for any θ, $w_\theta=\frac{hd_\theta}{1-\beta(1-h)}<0$, which would imply $A=[-a,a]$, a contradiction. Hence, A is a non-empty interval. ■

A.4 Proof of Proposition 3 and Proposition 4

Proof. Since Proposition 3 deals with the symmetric model ($B = 0$), and Proposition 4 deals with the asymmetric one ($B > 0$), we give directly the proof for the general case ($B \geq 0$).

A. Agreement sets in equilibrium

We use the typology of stationary equilibria, and we focus in turn on two-sided equilibria ($w_0 > 0$ and $w_1 > 0$), full-agreement equilibria ($w_0 \leq 0$ and $w_1 \leq 0$) and one-sided equilibria ($w_0 \leq 0$ and $w_1 > 0$; or $w_0 > 0$ and $w_1 \leq 0$).

Two-sided equilibrium. Let w be a two-sided equilibrium. As in the proof of Proposition 1, we may apply (1), by noting $\lambda := \mathbb{P}(x \in A) = \dfrac{l}{2a}$ and observing that, since $u_1(x) = x + a$, we have $\mathbb{E}[u_1(x) \mid x \in A] = u_1(c) = a + c$. We obtain the reservation value for agent 1:

$$w_1 = \frac{\Phi d_1 + \Delta\lambda(a + c)}{1 + \Delta\lambda}.$$

Similarly, as $u_0(x) = a - x$, we have $\mathbb{E}[u_0(x) \mid x \in A] = u_0(c) = a - c$. By application of (1), we obtain:

$$w_0 = \frac{\Phi d_0 + \Delta\lambda(a - c)}{1 + \Delta\lambda}.$$

As the equilibrium is two-sided, we have by assumption $w_0 > 0$ and $w_1 > 0$. The agreement set is thus $A = [c - l/2, c + l/2] = [-a + w_1, a - w_0]$. We obtain

$$\begin{cases} w_1 + w_0 & = 2a - l = 2a(1 - \lambda) \\ w_1 - w_0 & = 2c. \end{cases}$$

Solving for c first, we get:

$$2c = \frac{\Phi(d_1 - d_0) + \Delta\lambda 2c}{1 + \Delta\lambda} \quad \Leftrightarrow \quad c = \frac{\Phi(d_1 - d_0)}{2} = \frac{\Phi B}{2}.$$

Solving for λ, we obtain :

$$2a(1 - \lambda) = \frac{\Phi(d_0 + d_1) + 2a\Delta\lambda}{1 + \Delta\lambda} = \frac{\Phi D + 2a\Delta\lambda}{1 + \Delta\lambda} \quad \Leftrightarrow \quad 1 - \frac{\Phi D}{2a} = \lambda + \Delta\lambda^2.$$

Hence, we get:

$$\lambda = \frac{1}{2\Delta}\left(\sqrt{1+4\Delta\left(1-\frac{\Phi D}{2a}\right)}\right).$$

The formula for l is obtained by writing $l = 2a\lambda$.

Full-agreement equilibrium. If w is a full-agreement equilibrium, then $A = [-a, a]$. We have $c = 0$, $l = 2a$ and $\lambda = 1$.

One-sided equilibrium. Let w be a one-sided equilibrium. If $w_0 \leq 0$ and $w_1 > 0$, then we have $A = [-a, a - w_1]$. The rationality condition for agent 1 is exactly the same as in the one-agent model. As in the proof of Proposition 1, we obtain:

$$\lambda = \frac{1}{\Delta}\left(\sqrt{1+2\Delta\left(1-\frac{\Phi d_1}{2a}\right)}-1\right) = \frac{1}{\Delta}\left(\sqrt{1+2\Delta\left(1-\frac{\Phi(B+D)}{4a}\right)}-1\right).$$

The length and center of the agreement set are given by $l = 2a\lambda$ and $c = a - \frac{l}{2}$.

Similarly, if we have $w_0 > 0$ and $w_1 \leq 0$, then we have $A = [-a + w_0, a]$, and we get:

$$\lambda = \frac{1}{\Delta}\left(\sqrt{1+2\Delta\left(1-\frac{\Phi d_0}{2a}\right)}-1\right) = \frac{1}{\Delta}\left(\sqrt{1+2\Delta\left(1-\frac{\Phi(D-B)}{4a}\right)}-1\right).$$

In that case, the length and center of the agreement set are given by $l = 2a\lambda$ and $c = -a + \frac{l}{2}$.

B. Conditions for existence

We first provide conditions for the existence of each type of equilibrium. In particular, we observe that one-sided equilibria with $w_0 > 0$ and $w_1 \leq 0$ cannot exist.

Two-sided equilibria: existence. Two-sided equilibria are characterized by the system of equations: $c = \frac{\Phi B}{2}$ and $1 - \frac{\Phi D}{2a} = \lambda + \Delta\lambda^2$. As $c = \frac{\Phi B}{2} \geq 0$, a necessary and sufficient condition for such an equilibrium to exist is that the previous system admits a solution with $c + l/2 \leq a$, or equivalently $c \leq a(1-\lambda)$. This condition can be written as $\lambda \leq 1 - \frac{\Phi B}{2a}$. Hence, a two-sided equilibrium exists if and only if:

$$\exists \lambda < 1 - \frac{\Phi B}{2a}, \qquad 1 - \frac{\Phi D}{2a} = \lambda + \Delta\lambda^2.$$

This condition is equivalent to $1 - \frac{\Phi D}{2a} < 1 - \frac{\Phi B}{2a} + \Delta(1 - \frac{\Phi B}{2a})^2$, as $\lambda \mapsto \lambda + \Delta\lambda^2$ is

increasing and continuous on $[0, 1 - \frac{\Phi B}{2a}]$. The condition can then be written:

$$\frac{\Phi(B-D)}{2a} < \Delta \left(1 - \frac{\Phi B}{2a}\right)^2.$$

Let $F_0(h) = \frac{\Phi(h)(B-D)}{2a} - \Delta(h)\left(1 - \frac{\Phi(h)B}{2a}\right)^2$. We have obtained that a two-sided equilibrium exists if and only if $F_0(h) < 0$.

Full-agreement equilibria: existence. In a full agreement equilibrium, we have $c = 0$, $l = 2a$ and $\lambda = 1$. As $B \geq 0$, we have $w_1 \geq w_0$, and a necessary and sufficient condition for existence is then $w_1 \leq 0$. This can be written $w_1 = \frac{\Phi d_1 + \Delta a}{1+\Delta} \leq 0$. This condition is equivalent to $\Phi d_1 + \Delta a \leq 0$ or $\frac{\Phi(B+D)}{2a} + \Delta \leq 0$. Let $F_1(h) = \frac{\Phi(h)(B+D)}{2a} + \Delta(h)$.[23] We have shown that a full-agreement equilibrium exists if and only if $F_1(h) \leq 0$.

One-sided equilibria: existence. Let us first consider the case $w_0 \leq 0$ and $w_1 > 0$. As shown in the proof of Proposition 1, one-sided equilibria are characterized by the equation $1 - \frac{\Phi d_1}{2a} = \lambda + \Delta\frac{\lambda^2}{2}$, or equivalently $1 - \frac{\Phi(B+D)}{4a} = \lambda + \Delta\frac{\lambda^2}{2}$. Such an equilibrium exists if and only if $w_0 \leq 0$ and $\lambda \leq 1$. The lower bound of the agreement set is $-a + w_1 = a - l$, it follows that $w_1 = 2a - l = 2a(1-\lambda)$. We may thus write:

$$\begin{aligned} w_0 = (w_0 + w_1) - w_1 &= \frac{\Phi(d_0 + d_1) + \Delta\lambda[(a-c) + (a+c)]}{1 + \Delta\lambda} - 2a(1-\lambda) \\ &= \frac{\Phi D + 2a\Delta\lambda - 2a(1-\lambda)(1+\Delta\lambda)}{1+\Delta\lambda}. \end{aligned}$$

Hence, we can write :

$$\begin{aligned} w_0 \leq 0 \;\Leftrightarrow\; \frac{\Phi D}{2a} + \Delta\lambda \leq (1-\lambda)(1+\Delta\lambda) &\Leftrightarrow\; \lambda + \Delta\lambda^2 \leq 1 - \frac{\Phi D}{2a} \\ &\Leftrightarrow\; 2\left(\lambda + \Delta\frac{\lambda^2}{2}\right) - \lambda \leq 1 - \frac{\Phi D}{2a} \\ &\Leftrightarrow\; 2\left(1 - \frac{\Phi(B+D)}{4a}\right) - \lambda \leq 1 - \frac{\Phi D}{2a} \\ &\Leftrightarrow\; 1 - \frac{\Phi B}{2a} \leq \lambda. \end{aligned}$$

[23] The function F_1 plays the same role here as the function F in the proof of Proposition 1.

To conclude, a one-sided equilibrium exists if and only if:

$$\exists \lambda \in [1 - \frac{\Phi B}{2a}, 1), \qquad 1 - \frac{\Phi(B+D)}{4a} = \lambda + \Delta \frac{\lambda^2}{2}.$$

This condition is equivalent to

$$\left(1 - \frac{\Phi B}{2a}\right) + \frac{\Delta}{2}\left(1 - \frac{\Phi B}{2a}\right)^2 \leq 1 - \frac{\Phi(B+D)}{4a} < 1 + \frac{\Delta}{2},$$

since $\lambda \mapsto \lambda + \Delta\frac{\lambda^2}{2}$ is increasing and continuous on $[1 - \frac{\Phi B}{2a}, 1]$. Thus, such a one-sided equilibrium exists if and only if two conditions are jointly satisfied. The first condition is $\frac{\Phi(B+D)}{2a} + \Delta > 0$, equivalent to $F_1(h) > 0$. The second condition can be written as $\frac{\Phi(B-D)}{2a} \geq \Delta\left(1 - \frac{\Phi B}{2a}\right)^2$, equivalent to $F_0(h) \geq 0$.

Consider now the case $w_0 > 0$ and $w_1 \leq 0$. We obtain as above (replacing d_1 by d_0) that $1 - \frac{\Phi(D-B)}{4a} = \lambda + \Delta\frac{\lambda^2}{2}$. Following the same steps as above, we also obtain that

$$w_1 \leq 0 \quad \Leftrightarrow \quad 2\left(1 - \frac{\Phi(D-B)}{4a}\right) - \lambda \leq 1 - \frac{\Phi D}{2a} \quad \Leftrightarrow \quad 1 + \frac{\Phi B}{2a} \leq \lambda.$$

This last condition is incompatible with $\lambda < 1$, which holds at a one-sided equilibrium. Hence, such a one-sided equilibrium cannot exist.

C. Equilibrium uniqueness.

As Φ is increasing in h and Δ is decreasing in h, we have that $F_0(h) = \frac{\Phi(h)(B-D)}{2a} - \Delta(h)\left(1 - \frac{\Phi(h)B}{2a}\right)^2$ is increasing in h, with $F_0(0) = -\Delta(0) = -\frac{\beta}{1-\beta} < 0$ and $F_0(1) = \Phi(1)\frac{B-D}{2a} = \frac{B-D}{2a} > 0$. Hence, there exists a unique value $h_0 \in (0,1)$ for which $F_0(h_0) = 0$.

Similarly, the function $F_1(h) = \frac{\Phi(h)(B+D)}{2a} + \Delta(h)$ is decreasing in h, with $F_1(0) = \Delta(0) = \frac{\beta}{1-\beta} > 0$ and $F_1(1) = \Phi(1)\frac{B+D}{2a} = \frac{B+D}{2a} < 0$. Hence, there exists a unique value $h_1 \in (0,1)$ for which $F_1(h_1) = 0$.

Now, observe that:

$$\frac{\Phi}{\Delta}(h_1) = \frac{2a}{-B-D} \geq \frac{2a}{B-D} \geq \frac{2a}{B-D}\left(1 - \frac{\Phi(h_0)B}{2a}\right)^2 = \frac{\Phi}{\Delta}(h_0).$$

As $\frac{\Phi}{\Delta}(h)$ is decreasing on $(0,1)$, we have $h_0 \leq h_1$, with an equality if and only if $B = 0$. To conclude, for any $h \in [0,1]$, there exists a unique equilibrium:

- if $h < h_0$, there is a two-sided equilibrium, and only this equilibrium, since $F_0(h) < 0$ and $F_1(h) > 0$.
- if $h_0 \leq h < h_1$, there is a one-sided equilibrium, and only this equilibrium, since $F_0(h) \geq 0$ and $F_1(h) > 0$.
- if $h \geq h_1$, there is a full-agreement equilibrium, and only this equilibrium, since $F_0(h) < 0$ and $F_1(h) \leq 0$

Two-sided equilibria: comparative statics. As Φ is increasing in h, it is immediate that $c = \frac{\Phi B}{2}$ is a non-decreasing function of h, strictly increasing whenever $B > 0$.

The agreement probability λ is obtained as the solution of the equation $1 - \frac{\Phi D}{2a} = \lambda + \Delta\lambda^2$. As shown on Figure 9, λ increases as h increases.

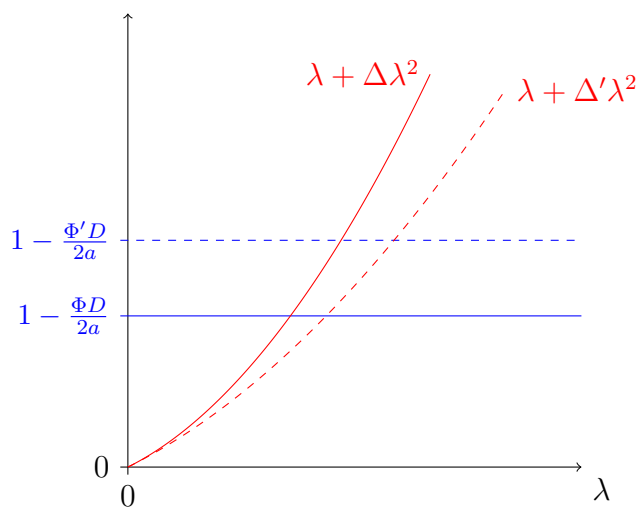

Figure 9: Characterization of λ (h increases)

One-sided equilibria: comparative statics. The agreement probability λ is obtained as the solution of the equation: $1 - \frac{\Phi d_1}{2a} = \lambda + \Delta\frac{\lambda^2}{2}$. As $d_1 < 0$, we may apply the same argument as for the two-sided equilibria, depicted on Figure 9. We obtain that λ increases with h, and as a result, $c = a - \frac{l}{2} = a(1 - \lambda)$ decreases with h. ■

A.5 Proof of Theorem 2

Proof. As $B = 0$, we have either full-agreement or a two-sided equilibrium. For a full-agreement equilibrium, we know that the first proposed deal will be accepted with probability one, so that $W_0 = W_1 = a$. As we also have $\lambda = 1$, the formula $W_0 = W_1 = a(1 - \lambda + \lambda^2)$ is valid.

Let w be a two-sided equilibrium and let A be its agreement set. As in the proof of Proposition 2, we obtain:

$$W_\theta = \frac{1}{\beta(1-h)} \left(w_\theta - hd_\theta\right).$$

As w is two-sided, we have $w_0 = w_1 = a - \frac{l}{2} = a(1 - \lambda)$, and we may write

$$W_0 = W_1 = \frac{a(1-\lambda) - \frac{hD}{2}}{\beta(1-h)}.$$

As w is two-sided, we know that λ is the solution of:

$$\begin{aligned}
1 - \frac{\Phi D}{2a} = \lambda + \Delta\lambda^2 \quad &\Leftrightarrow \quad 1 - \frac{h}{1-\beta(1-h)}\frac{D}{2a} = \lambda + \frac{\beta(1-h)}{1-\beta(1-h)}\lambda^2 \\
&\Leftrightarrow \quad 1 - \beta(1-h) - \frac{hD}{2a} = \left(1 - \beta(1-h)\right)\lambda + \beta(1-h)\lambda^2 \\
&\Leftrightarrow \quad 1 - \lambda - \frac{hD}{2a} = \beta(1-h)\left(1 - \lambda + \lambda^2\right).
\end{aligned}$$

We obtain as desired:

$$W_0 = W_1 = \frac{a(1-\lambda) - \frac{hD}{2}}{\beta(1-h)} = a\left(\frac{1 - \lambda - \frac{hD}{2a}}{\beta(1-h)}\right) = a(1 - \lambda + \lambda^2).$$

■

A.6 Proof of Proposition 5

Proof. Let w be a two-sided equilibrium and let A be its agreement set. As w is two-sided, we have $w_0 + w_1 = 2a(1 - \lambda)$. Then, using the same formulas as in the

proof of Theorem 2, we may write:

$$W_0 + W_1 = \frac{2a(1-\lambda) - hD}{\beta(1-h)} = 2a(1-\lambda+\lambda^2).$$

As w is two-sided, we have $c = \dfrac{(-a+w_1)+(a-w_0)}{2}$, so that $w_0 + w_1 = 2c = \frac{\Phi B}{2}$. Using the formulas from the proof of Theorem 2, we may write:

$$\begin{aligned} W_1 - W_0 &= \frac{1}{\beta(1-h)}\left(w_1 - w_0 - h(d_1 - d_0)\right) \\ &= \frac{1}{\beta(1-h)}\left(\Phi B - hB\right) = \frac{hB}{\beta(1-h)}\left(\frac{1}{1-\beta(1-h)} - 1\right) = \Phi B. \end{aligned}$$

To conclude, we write $W_0 = \dfrac{W_0+W_1}{2} - \dfrac{W_1-W_0}{2}$ and $W_1 = \dfrac{W_0+W_1}{2} + \dfrac{W_1-W_0}{2}$, and we obtain the desired formulas. ■

A.7 Proof of Proposition 6

Proof. Let w be a one-sided equilibrium and let A be its agreement set. As w is one-sided, the welfare of agent 1 is the same as in the single-agent model. Applying Proposition 2, we obtain $W_1 = 2a(1-\lambda+\frac{\lambda^2}{2})$. As $\lambda < 1$, we have that $W_1 > a$.

Then, we may write, by application of (2), and using the decomposition of total welfare in instant payoff and delay factor:

$$\begin{aligned} W_0 + W_1 = \frac{(1-\lambda)hD + 2a\lambda}{1-\beta(1-h)(1-\lambda)} &= 2a\frac{\lambda + (1-\lambda)\frac{hD}{2a}}{1-\beta(1-h)(1-\lambda)} \\ &= 2a\frac{\lambda + (1-\lambda)\frac{hD}{2a}}{\lambda + (1-\lambda)h} \times \frac{\lambda + (1-\lambda)h}{1-\beta(1-h)(1-\lambda)} \\ &= 2a\frac{\lambda + (1-\lambda)\frac{hD}{2a}}{\lambda + (1-\lambda)h} \times \frac{1-(1-h)(1-\lambda)}{1-\beta(1-h)(1-\lambda)} \\ &< 2a. \end{aligned}$$

Thus $W_0 < 2a - W_1 < a$. ■

A.8 Proof of Theorem 3

Proof. For agent 1, the welfare W_1 is constant, equal to a on $[h_1, 1]$. Moreover, W_1 is decreasing on $[h_0, h_1]$, as we know that λ is increasing with h in this regime (by Proposition 4) and that $W_1 = 2a(1 - \lambda + \frac{\lambda^2}{2})$ is a decreasing function of λ (by Proposition 6). For $h = 0$, we have $W_0 = W_1$ by symmetry, and we know from the proof of Proposition 6 that $W_0 + W_1 < 2a$ whenever $\lambda < 1$, we thus have $W_1 < a$. We thus obtained that W_1 is maximal for some $h \in (0, h_0]$.

For agent 0, we have $W_0 \leq W_1$ (since for any equilibrium, $c \geq 0$) and we know that whenever $\lambda < 1$, we have $W_0 + W_1 < 2a$. Hence, for $\lambda < 1$, $W_0 < 1/2$. Thus, W_0 is maximal for $\lambda = 1$, i.e. $h \in [h_1, 1]$. ■

A.9 Proof of Proposition 7

Proof. In the extended model with a risk of agreement failure k, the equilibrium reservation values associated to an agreement set A are given by:

$$w_\theta = hd_\theta + \beta(1-h)(1-k)\mathbb{P}(x \in A)\mathbb{E}[u_\theta(x) \mid x \in A] + \beta(1-h)[(1-k)\mathbb{P}(x \notin A) + k]w_\theta.$$

Noting that $(1-k)\mathbb{P}(x \notin A) + k = 1 - (1-k)\mathbb{P}(x \in A)$, we obtain an extension of (1) for the equilibrium conditions:

$$\forall \theta \in \{0, 1\}, \qquad w_\theta = \frac{hd_\theta + \beta(1-h)(1-k)\mathbb{P}(x \in A_w)\mathbb{E}[u_\theta(x) \mid x \in A_w]}{1 - \beta(1-h) + \beta(1-h)(1-k)\mathbb{P}(x \in A_w)}. \tag{3}$$

Noting $\Phi = \frac{h}{1-\beta(1-h)}$ and $\Delta_k = (1-k)\Delta = \frac{\beta(1-h)(1-k)}{1-\beta(1-h)}$, equation (3) can be written:

$$w_\theta = \frac{\Phi d_\theta + \Delta_k \lambda \mathbb{E}[u_\theta(x) \mid x \in A_w]}{1 + \Delta_k \lambda}.$$

Hence, we obtain the same results as for the equilibrium agreement set (Theorem 1, Proposition 3, Proposition 4), but the parameter Δ must be replaced by Δ_k.

We have that Δ_k decreases with k. Using the definition of h_0 and h_1 in the proof of Proposition 3 and Proposition 4, one can see that h_0 and h_1 must also decrease with k.

As for reservation values, agents' equilibrium welfare generalizes as follows:

$$\forall \theta \in \{0,1\}, \qquad W_\theta = \frac{(1-(1-k)\lambda)hd_\theta + (1-k)\lambda \mathbb{E}[u_\theta(x) \mid x \in A]}{1-\beta(1-h)+\beta(1-h)(1-k)\lambda}. \tag{4}$$

We obtain as in the baseline model the relation $W_\theta = \dfrac{1}{\beta(1-h)}(w_\theta - hd_\theta)$.

In the symmetric model ($B = 0$), the formula for agent's welfare obtained in Theorem 2 generalizes as $W_0 = W_1 = a\,(1-\lambda+(1-k)\lambda^2)$. This function remains convex as a function of λ. Moreover, as $\lambda(h=0) > 0$ and $\lambda(h=h_1) = 1$, we have, for k small enough, $W_\theta(h=h_1) > W_\theta(h=0)$ for each $\theta \in \{0,1\}$. Therefore, for k small enough, $\arg\max_{h \le h_1} W_\theta(h) = \{h_1\}$.

In the asymmetric model ($B > 0$), for $h \le h_1$, the instant utility for agent 0 is always lower or equal to that obtained at $h = h_1$ and the delay factor is always strictly lower than that obtained at $h = h_1$. Therefore, for k small enough, $\arg\max_{h \le h_1} W_0(h) = \{h_1\}$.

In the asymmetric model ($B > 0$), the formula for agent 1's welfare at a two-sided equilibrium obtained in Proposition 5 generalizes as $W_1 = a\,(1-\lambda+(1-k)\lambda^2) + \frac{\Phi B}{2}$. The formula for agent 1's welfare at a one-sided equilibrium obtained in Proposition 6 generalizes as $W_1 = 2a\left(1-\lambda+(1-k)\frac{\lambda^2}{2}\right)$. We know that W_1 is decreasing in h over $[h_0, h_1]$ and that, when k is small enough:

$$W_1(h=0) = a\left(1-\lambda(h=0)+(1-k)\lambda(h=0)^2\right) < a(1-k) = W_1(h=h_1).$$

Therefore, for k small enough, $\arg\max_{h \le h_1} W_1(h) \subseteq (0, h_0]$.

For $h \ge h_1$, we have full agreement ($\lambda = 1$, $c = 0$), and we obtain by application of (4):

$$\forall \theta \in \{0,1\}, \qquad W_\theta = \frac{khd_\theta + (1-k)a}{1-\beta k(1-h)}.$$

For k small enough, the numerator is positive and decreasing in h, the denominator is positive and increasing in h. It follows, that for$k$ small enough and for each $\theta \in \{0,1\}$, W_θ decreases with h on $[h_1, 1]$. We thus obtain:

- if $B = 0$, then $\arg\max_{h \in [0,1]} W_\theta(h) = \{h_1\}$ for each $\theta \in \{0,1\}$. If $B > 0$, then $\arg\max_{h \in [0,1]} W_0(h) = \{h_1\}$.
- if $B > 0$, then $\arg\max_{h \in [0,1]} W_1(h) \subseteq (0, h_0]$.

■

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