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# Pareto-improving structural reforms

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## ABSTRACT

Economists recommend to partly redistribute gains to losers from a structural reform, which in many cases may be required for making the reform politically viable. However, taxation is distortionary. Then, it is unclear that compensatory transfers can support a Pareto-improving reform. This paper provides sufficient conditions for this to occur, despite tax distortions. I consider an economy where workers have sector-specific skills and some sectors are regulated by a price floor. Transfers have to be financed by proportional taxation on firm's revenues or, equivalently, labor income. Labor supply is elastic to net post-tax real wages, and hence reduced by taxation. In a setting where preferences are isoelastic, deregulation is implementable in a Pareto-improving way through compensatory lump-sum transfers, despite that these are financed by distortionary taxes. In a more general setting, there always exist Pareto-improving reforms but they may involve tightening regulation for some goods. I provide sufficient conditions for deregulation, i.e. a general reduction in price floors, to be Pareto-improving. They imply that demand cross-price elasticities should not be too large and that the reform should not be too unbalanced. Finally, I consider counter-examples where some people earn rents associated with informational or institutional frictions, or where non homothetic preferences may imply that the schemes considered here are not viable.

Keywords: Structural reform, deregulation, price controls, Pareto optimality, rent seeking, taxation, compensatory transfers

JEL:E64, H21, P11.

# 1 Introduction

Structural reforms in labor and product markets are commonly advocated by economists. Regulations that distort prices, unless they are tailored to correcting an externality, lead to an inefficient allocation of resources. Common economic sense suggests they should be removed. However, the efficiency gains that are generally associated with such reforms are aggregate.<sup>1</sup> For example, removing a price floor on agricultural products allows additional units whose marginal cost is lower than the consumers' marginal willingness to pay to be produced; nevertheless, agricultural producers lose from such a reform, while more than 100% of the aggregate gains accrue to consumers. Economists generally recommend to redistribute some of the gains to losers from the reform, which in many cases may be required for making the reform politically viable.<sup>2</sup> If such redistribution can be financed through lump-sum taxes, aggregate gains can be reallocated so as to implement the reform in a Pareto-improving fashion. However, taxation is generally distortionary. If distortions are taken into account, it is unclear that a set of compensatory transfers can be designed so as to make the reform Pareto-improving. In such a situation, claims that the reform promotes economic efficiency are irrelevant; de facto, the reform benefits some groups at the expense of others, and it is legitimate for the losers to consider such claims as an ideology in favor of the winning social groups.

In this paper, I study a general equilibrium model of a regulated economy and show that under canonical conditions, a Pareto-improving reform with side transfers exists despite that it is financed by distortionary taxation. The

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<sup>1</sup>It is not the place here to discuss the huge literature attempting to evaluate the aggregate effects of structural reforms. For a recent survey, see for example Campos et al. (2017).

<sup>2</sup>For example, Delpla and Wyplosz (2007) propose to implement rent buyback schemes for a number of French regulations, including in particular barriers to entry in industries such as taxis and retail trade. A formal result regarding gains from trade can be found in Grandmont and MacFadden (1972). See also Castanheira et al. (2006) for case studies of structural reforms in Europe with an emphasis on the role of coalition building when implementing structural reforms.

key idea is that the distortionary tax that can be imposed upon the winners from the reform can be picked so as to mimic the effect of regulation, in that their utility *and* behavior are the same.

I first study a simple, symmetric model where each good is produced with a specific type of labor and a subset of goods are regulated by a price floor, implying that some rationing of labor supply must be forced upon the suppliers of those goods. I consider two types of deregulations: an extensive reform which entirely deregulates a subset of the regulated goods, and an intensive one which reduces regulated prices uniformly across the regulated goods. I show that each reform can be implemented in a Pareto-improving way through a combination of a distortionary proportional tax on labor or output and a nonnegative lump-sum transfer paid to each worker in the deregulated sectors.

I then generalize those results by doing away with the symmetry and functional forms assumptions. I show that a Pareto improvement exists provided one can tax each sector's specific labor at its own proportional rate and at the same time pay to it a fixed specific nonnegative lump-sum transfer. More precisely, I show that there exist reforms (called N-neutral) whose accompanying fiscal scheme leaves the behavior and net income of nonregulated agents unchanged. Consequently, if such a reform raises aggregate social welfare, it delivers positive total gains to the regulated agents. I then show that (i) these gains can be allocated among them in a Pareto-improving way which is also feasible, i.e. involves no negative transfers and that (ii) one can always pick an N-neutral reform such that aggregate social welfare goes up. Altogether, these results imply the existence of a Pareto-improving structural reform. However, such reform need not coincide with *deregulation*, i.e. it may in principle involve tightening regulation on some goods. I provide sufficient conditions on preferences and on the design of reform for deregulation (i.e. a reduction in all regulated prices) to be implementable in a Pareto-improving way. These conditions essentially mean that the price cross-elasticities of demand should not be too large and that the size of deregulation should not

be too uneven across goods.

Finally, I discuss some counter-examples, which come in three categories. First, it may be impossible for the transfer scheme to appropriate all the rents from the winners from the reform. I show that this may happen if taxes cannot be differentiated across groups while the indexation rule for setting regulated prices is such that workers who remain regulated gain less from deregulation, in consumption terms, than nonregulated citizens. In my example, this is because the CPI goes down due to deregulation, which triggers a reduction in all regulated prices, that are indexed on this CPI. For the sectors that remain regulated not to lose, then, their tax rate must be lower than the one that leaves the nonregulated indifferent. In the absence of tax differentiation across groups, therefore, rents accrue to nonregulated consumers, which reduces the total tax receipts available to compensate the deregulated groups for their losses from the reform. I show that this makes Pareto-improving reform far less likely. In another example, people differ in which goods they consume. Some consumers gain more than others because deregulated goods account for a bigger share of their consumption basket. Again, one cannot differentiate the tax rate across people and a Pareto-improvement is unlikely. I also show that the number of winners goes up with the scope of reform in that context, because broader reforms allow to partially compensate producers of a deregulated good by deregulating other goods that they consume. The second reason why the benchmark result may not hold is if there are resource costs of implementing the reform and/or compensating the losers. Again, I show that in such a world, there are economies of scope to reform: Even though the resource cost of reform is proportional to its size and therefore has no fixed cost component, there typically exists a minimum scope of reform (i.e. a minimum number of deregulated goods) above which it is viable. Finally, if preferences are not separable in labor and/or not homothetic, the linear tax scheme will typically fail at appropriating the gains from those who benefit from the reform and at the same time maintaining their labor supply unchanged. That is, there are

no N-neutral reforms. I construct an example where the tax rate which leaves the non regulated groups indifferent between the status quo and the reform reduces their labor supply at the same time. As a result the corresponding transfers may be too low to compensate losers.

The paper is organized as follows. The next section discusses the relationship between this paper and the theoretical public finance literature, and connects the present analysis with empirical policy debates on structural reforms. Section 3 sets up a tractable model based on symmetrical, isoelastic preferences; regulated goods are also assumed to face identical regulations. I then (Section 4) study the distributive effects of structural reforms and establish they can be implemented so as to Pareto-dominate the original allocation. In Section 5 I generalize the analysis by relaxing a number of symmetry assumptions and show how N-neutral reforms can be used to construct Pareto-improving schemes. Sufficient conditions for deregulation to be implementable in such a way are provided. Section 6 discusses some counterexamples. Section 7 concludes.

## 2 Relationship to the literature

### 2.1 Theoretical work on regulation and redistribution

The idea of implementing a Pareto-improvement through a tax which, although distortionary, leaves the vector of consumer prices unchanged has been proposed in the context of the analysis of trade restrictions by Dixit and Norman (1980a,b). In the context of the public finance literature, it underlies the proofs by Laroque (2005) and Kaplow (2006) of the traditional result that commodity taxation is dominated by income taxation (See Atkinson and Stiglitz (1976), Mirrlees (1976)).<sup>3</sup>

The present paper can be viewed as an application of these ideas to the context of reforming price regulations with entry barriers.<sup>4</sup> Another important related paper is Grüner (2002), who, focusing on labor market reform, shows that, even if lump-sum transfers are possible, compensatory tax schemes may fail to implement a Pareto-improvement if worker type is unobservable because of informational rents.<sup>5</sup> While my focus here is essentially on the distortionary effects of taxes on labor supply, rents play a role in the counter-examples of Section 6, where I discuss the effect of reform scope on

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<sup>3</sup>See also Bierbrauer et al. (2021) for an analysis of tax reforms in terms of political viability.

<sup>4</sup>Contrary to Laroque (2005), in the present paper commodity taxation is equivalent to income taxation, due to the assumption of sector specific labor. Also, the initial status quo is a situation where the regulation generates private rents, as opposed to public spending. This paper is concerned with the replacement of a regulation by a tax and the use of the proceeds to compensate the losers from deregulation, as opposed to the replacement of a tax by a more efficient one.

<sup>5</sup>In contrast, Saint-Paul (1994) finds that a redistributive system based on distortionary transfers and taxes generally dominates minimum wages, although this is not in general but for a wide range of parameter values. The differences between that paper and Grüner's are many. In particular, unlike Grüner, in that paper I do not have an unobservable disutility of labor, so there are no informational rents being reaped by a subset of the unskilled workers who potentially lose from a removal of the minimum wage.

Reform may also be blocked due to dynamic contractual failures that are associated with lack of commitment, as in for example Alesina and Drazen (1991), Fernandez and Rodrik (1991), Saint-Paul (1993), Dewatripont and Roland (1992), Saint-Paul et al. (2016). While any distortion, including the unavailability of lump-sum taxes, arguably comes from contractual failures, the issues discussed in the present paper are obviously very different from these paper's approaches.



reform support. The property that broadening the scope for reform enhances its political viability confirms the results found in substantially different contexts by Blanchard and Giavazzi (2003), Caselli and Gennaioli (2008) or Ilzetski (2018).<sup>6</sup>

An important related literature (which often focuses on the minimum wage) studies the extent to which, in a second best world, quantity constraints imposed by the government on some agents may have positive effects on welfare (See Guesnerie and Roberts (1984, 1987), Marceau and Broadway (1994), Allen(1987), Broadway and Cuff (2001), Lee and Saez (2012)).<sup>7</sup> The focus of the present paper is, somehow, opposite. The underlying economy has no structural market distortion. The equilibrium is inefficient because of a pre-existing regulation, modelled as quantity constraints. I ignore the reasons for such regulation to exist in the first place.<sup>8</sup> Therefore, this paper's research question is not the potential usefulness of quantity constraints, but how one can remove them while compensating losers. A limitation of this paper is that it ignores the reasons why those regulations exist in the first place. However in the public debate over reforming institutions such as the European Common Agricultural Policy or taxi licenses, such policies are generally interpreted as an inefficient way to transfer rents to special interest groups.<sup>9</sup> The approach taken here is consistent with this view. In

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<sup>6</sup>See also Coe and Snower (1997), who focus on complementarities across various dimensions of labor market reforms in their effect on employment, as well as Grüner (2013). For a less positive view about complementarities, see Amable et al. (2007).

<sup>7</sup>Contrarian prescriptions are obtained by Cahuc and Laroque (2014). Danziger and Danziger (2014) consider a rather special scheme such that the minimum wage paid by the firm is contingent upon the size of its (unskilled) workforce. This allows to decouple the level of the minimum wage from the marginal cost of labor. Dworzak et al. (2020) reach conclusions similar to those of Broadway and Cuff and Lee and Saez, in the very different context of a partial equilibrium exchange economy.

<sup>8</sup>In a number of contributions of the above mentioned literature, minimum wages may be useful because (i) the government has redistributive concerns, and (ii) its ability to use taxes and transfers is limited by institutions and/or microeconomic frictions such as imperfect observability of ability. In the present paper there is no redistributive concern of the government.

<sup>9</sup>Again, see Delpla and Wyplosz (2007).

In general, the tools available to the government for addressing distortions or correcting inequalities must be restricted for quantity constraints to be useful. Hence, for exam-

principle, the regulation under study here can be understood as having been historically motivated by welfare considerations, as in the above mentioned literature, or by political considerations (as in Saint-Paul (1996), Acemoglu and Robinson (2001), Djankov et al. (2002)), but my model ignores these considerations. Clearly, studying how the results would change when taking into account the rationale for the initial status quo is an important avenue for further research.

Another key difference between this paper and most of the literature on optimal taxation, is that it is concerned with compensation of losers under the Pareto criterion. In contrast, in the existing literature one is looking for a tax system which maximizes some aggregate social welfare function, which generally encompasses redistributive concerns.<sup>10</sup> A key aspect of the approach taken here is that I assume that the social groups that benefit from the initial regulation have some veto power on the reform (perhaps because they are apt at lobbying and mobilizing) and therefore must be compensated for the reform to go ahead. The Pareto criterion arises naturally when one tries to know whether any gains from the reform are left to those who were harmed by the regulation once the groups favored by the regulation have been compensated.

## 2.2 The policy debate

The quantitative relevance of the product market rigidities analyzed in this paper is discussed by an abundant literature. Typically, it has been established that such regulations, not surprisingly, have an important effect on consumer prices. For example, Combes and Lafourcade (2005) estimate that,

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ple, Guesnerie and Roberts (1984) look at the welfare effects of quantity constraints in a distorted economy, while ignoring the possibility of using alternative instruments. Allen (1987) shows that minimum wages may be a useful redistributive tool if taxes are constrained to be linear, but not if a non linear tax schedule is available.

<sup>10</sup>This is the case, for example, in Rothschild and Scheuer (2012) and Gomes et al. (2018), who consider how to allocate taxes across sectors when skills are sector-specific to some extent, a feature which resembles the assumptions made here. Beyond the many differences between these models and the current one, these contributions again address a different research question.

in France, the real cost of merchandise road transport fell by 39% between 1978 and 1988, of which they ascribe 22 points to deregulation.<sup>11</sup>

Structural rigidities are pervasive in many OECD countries and, altogether, are generally considered as having a large negative effect on employment. Cahuc and Kramarz (2004), among others, give many examples. They estimate that retail trade regulation alone destroys 3.4 million jobs in the French economy. This is 14 % of *total* salaried employment in 2004.<sup>12</sup>

Many such regulations are in the form of price floors, quantity constraints, barriers to entry, *numerus clausus*, and the like. The pervasiveness of such regulations is again documented in Cahuc and Kramarz (*op. cit.*), who extensively discuss the complex web of regulations that prevent entry in the French service industry, through licences, training requirements, and restrictions to price competition. The retail trade industry, for example, is subjected to many regulations that restrict entry for shops above a critical surface. An administrative commission delivers an authorization, depending on the pre-existing situation for supply and demand in the relevant administrative district. Explicit *numerus clausus* for new entrants exist for medical doctors, chiropractors and veterinaries. Explicit price controls exist for notarial fees, books, agricultural products, and so forth. These regulations have strong effects. For example, France accounts for 9 % of total veterinaries in Europe, while its population share is 13 % and its cattle share much higher. The number of taxis in Paris is smaller than in 1925, despite population and income growth. Overall, at the date of the Cahuc-Kramarz report, there were 11 broad industries that were subject to these regulations. The authors enumerate 70 activities and 28 non salaried occupations that are subject to rigidities preventing entry. To this should be added major price control schemes such as the minimum wage and the common agricultural policy.<sup>13</sup>

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<sup>11</sup>Card (1986) and Peoples (1998) find that in a number of industries, deregulation leads to a fall in the industry wage premium, consistently with the rent sharing hypothesis.

<sup>12</sup>See also Bertrand and Kramarz (2002), Boeri et al. (1999), Bassanini et al. (2000), Messina (2006), Nicoletti et al. (2000), Nicoletti and Scarpetta (2003), McKinsey Global Institute (1994,1997).

<sup>13</sup>While barriers to entry are more frequent than the price floors that are studied in the

While the aggregate effects of removing regulation in a small sector, such as taxis, may be negligible, the literature on policy complementarities (Blanchard and Giavazzi, 2003) provides a rationale for bundling product market reforms together. In addition, some sectors such as housing or retail trade are not small. These arguments suggest that there is some merit in studying the implementation of structural reforms in a general equilibrium setting.

### 3 The model

There is a continuum of consumers-producers indexed by  $i \in [0, 1]$ . Consumers with index  $i$  are endowed with a specific labor input which only allows them to produce good  $i$ .

All consumers have the same utility function, given by

$$U(\{c_{ij}\}, l_i) = \left[ \int_0^1 c_{ij}^\alpha dj \right]^{1/\alpha} - \frac{l_i^\gamma}{\gamma}, \quad (1)$$

where  $c_{ij}$  denotes consumption of good  $j$  by consumer  $i$  and  $l_i$  his labor supply. I assume  $-\infty < \alpha < 1$  and  $\gamma > 1$ .

Each good  $j$  is produced with a linear technology, using its corresponding specific labor as the only input:

$$y_j = l_j.$$

I assume that a fraction  $r$  of the goods are regulated. Goods such that  $i > r$  are unregulated. By symmetry, they all have the same equilibrium price, normalized to  $p_N = 1$ . Goods such that  $i < r$  are regulated: their price cannot fall below some  $p_R > 1$ , where  $p_R$  is fixed by law.<sup>14</sup> Accordingly, barriers to activity ensure that supply is rationed: people such that  $i < r$

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present paper, in terms of the model's notations a barrier to entry which caps employment in a sector at  $\bar{l}$  is equivalent to a price floor at  $p_R$ . The present model can therefore address both types of regulations, although its stylized features do not do justice to the regulatory complexity and diversity of these provisions in the real world.

<sup>14</sup>Equivalently, the law could specify the maximum labor supply  $\bar{l}$ . In practice, either type of specification exists, although, contrary to our stylized assumptions, regulations involve far many more clauses than just a price floor or a quantity cap.

cannot supply more than a maximum amount of labor  $\bar{l}$  to the market. Again, by symmetry,  $\bar{l}$  is the same for all goods such that  $i < r$ .

Clearly, the wage for labor  $i$  coincides with the price of the corresponding good.

For a consumer with income  $R$  and labor supply  $l$ , the indirect utility function is

$$V(R, l, p) = \frac{R}{p} - \frac{l^\gamma}{\gamma},$$

where  $p$  is the aggregate price index

$$p = (rp_R^{-\frac{\alpha}{1-\alpha}} + 1 - r)^{-\frac{1-\alpha}{\alpha}}. \quad (2)$$

The corresponding demand for a good of type  $k \in \{R, N\}$  is

$$c(R, p, p_k) = Rp^{\frac{\alpha}{1-\alpha}} p_k^{-\frac{1}{1-\alpha}}. \quad (3)$$

From there it is straightforward to compute the equilibrium. Non regulated households set their labor supply so as to maximize  $V(l, l, p)$ , yielding

$$l = l_N = p^{-\frac{1}{\gamma-1}}.$$

Consequently, from (3), they consume

$$c_{Nk} = p^{\frac{\alpha}{1-\alpha} - \frac{1}{\gamma-1}} p_k^{-\frac{1}{1-\alpha}} \quad (4)$$

of each good of type  $k$ . Their resulting utility is

$$u_N = \left(1 - \frac{1}{\gamma}\right) p^{-\frac{\gamma}{\gamma-1}}.$$

As for regulated households, their income is  $p_R \bar{l}$ , so that from (3) again they consume

$$c_{RN} = p^{\frac{\alpha}{1-\alpha}} p_R \bar{l} \quad (5)$$

of non regulated goods and

$$c_{RR} = p^{\frac{\alpha}{1-\alpha}} p_R^{-\frac{\alpha}{1-\alpha}} \bar{l}$$

of regulated ones.

To compute the equilibrium  $\bar{l}$ , we just write down that supply equals demand for any nonregulated good. By Walras's law, regulated markets will then also be in "equilibrium", in that demand will equal the regulated level of supply  $\bar{l}$ . If the government fails to set  $\bar{l}$  equal to this level, the rationing scheme will be inconsistent. Thus  $\bar{l}$  is such that

$$l_N = r c_{RN} + (1 - r) c_{NN}.$$

Indeed the LHS is the supply of a nonregulated good, while the RHS sums up the demand for that good across consumer types. Substituting (5), (4) and making use of (2), we get that

$$\bar{l} = p^{-\frac{1}{\gamma-1}} p_R^{-\frac{1}{1-\alpha}}, \quad (6)$$

which allows us to compute the regulated type's equilibrium utility:<sup>15</sup>

$$u_R = V(p_R \bar{l}, \bar{l}, p) = p^{-\frac{\gamma}{\gamma-1}} \left[ p_R^{-\frac{\alpha}{1-\alpha}} - \frac{1}{\gamma} p_R^{-\frac{\gamma}{1-\alpha}} \right]. \quad (7)$$

The rent  $u_R/u_N$  earned by the regulated agents in relative utility terms is denoted by  $J$ . We have that

$$J = \frac{\gamma p_R^{-\frac{\alpha}{1-\alpha}} - p_R^{-\frac{\gamma}{1-\alpha}}}{\gamma - 1}. \quad (8)$$

## 4 Reform

Now that the equilibrium of the regulated economy has been characterized, we can study the effect of reforms. I consider two alternative designs. First, an *extensive* reform by which a range of sectors are entirely deregulated. Second, an *intensive* reform by which the price floor is uniformly reduced across sectors. Clearly, both designs are stylized representations of the real

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<sup>15</sup>It can be shown that  $\frac{du_R}{dp_R} > 0$  at  $p_R = 1$ , implying there exist levels of regulation that make regulated people better-off.

world reform process, which is far more complex.<sup>16</sup> The reforms considered in this section preserve the symmetrical feature of the regulation. More general reforms, allowing an arbitrary distribution of price changes across sectors, are studied in the next section.

There are real world examples of both extensive reforms and intensive reforms. Extensive reforms include the deregulation of freight rail transport in Europe in 2007 (See for example European Commission, 2011) and the deregulation of taxis in Sweden in 1990 (See Kang, 1998 for a thorough discussion of taxi deregulation globally). Intensive reforms include the easing of conditions for establishing notarial offices in France in 2015 and the 1992 reform of the European Common Agricultural Policy which reduced the price floors.<sup>17</sup> However, the latter are more examples of partial deregulation of a subset of sectors, while the intensive reforms considered here apply across the whole spectrum of regulated sectors.

## 4.1 Extensive reform

Now consider a reform whereby the fraction of regulated sectors is reduced from  $r$  to  $r' < r$ . I will denote by  $\Delta r = r - r'$  the size of the reform. In order to compensate losers, the government transfers a lump-sum  $T$  to each household  $i$  such that  $r' < i \leq r$ . This transfer is financed by a general proportional tax on labor income  $\tau$ .<sup>18</sup> We want to know the conditions under which such a scheme allows to implement the reform in a Pareto-improving way.

Before studying the effects of reform, we need to compute the new equi-

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<sup>16</sup>See Section 2.2.

<sup>17</sup>See for example *L'Express* (2015), and <https://www.ers.usda.gov/topics/international-markets-us-trade/countries-regions/european-union/common-agricultural-policy/>

<sup>18</sup>This tax can be thought of, equivalently, as a proportional tax on goods. Also, for simplicity, taxes are paid by everybody, including the deregulated who also get the transfer. This is somewhat realistic in that a specific group may benefit from specific transfers meant to compensate them for a one-off event, while not being exempt from general taxation. But one may think that better outcomes might be obtained by not taxing the groups to whom one is paying a transfer at the same time. This turns out not to be the case, as Section 5.2.1 below makes clear.

librium, taking into account the link between  $\tau$  and  $T$  implied by the government's budget constraint. This is done in the following lemma.<sup>19</sup>

*LEMMA 1 – A. The new values of the price level and of employment (=output) in any sector  $i$  are given by*

$$p' = (r' p_R^{-\frac{\alpha}{1-\alpha}} + 1 - r')^{-\frac{1-\alpha}{\alpha}} = p'_E < p; \quad (9)$$

$$l_i = p'_E^{-\frac{1}{\gamma-1}} (1 - \tau)^{\frac{1}{\gamma-1}} = l'_{NE}, \quad i > r'; \quad (10)$$

$$l_i = p_R^{-\frac{1}{1-\alpha}} p'_E^{-\frac{1}{\gamma-1}} (1 - \tau)^{\frac{1}{\gamma-1}} = \bar{l}'_E, \quad i \leq r'. \quad (11)$$

*B. Nonregulated groups ( $i > r$ ) and nonderegulated groups ( $i \leq r'$ ), weakly gain from the reform if and only if*

$$1 - \tau \geq \frac{p'_E}{p}. \quad (12)$$

*C. Deregulated groups ( $r' < i \leq r$ ) weakly gain from the reform if and only if*

$$T \geq p_E^{-\frac{1}{\gamma-1}} \frac{\gamma - 1}{\gamma} \left[ \left( \frac{p'_E}{p} \right)^{\frac{\gamma}{\gamma-1}} J - (1 - \tau)^{\frac{\gamma}{\gamma-1}} \right], \quad (13)$$

where  $T$  is the equilibrium transfer level given by

$$T = \frac{\tau(1 - \tau)^{\frac{1}{\gamma-1}} p_E^{-\frac{1}{\gamma-1} - \frac{\alpha}{1-\alpha}}}{\Delta r}. \quad (14)$$

*D. Consequently, for any  $r' < r$ , the reform can be implemented in a Pareto-improving way if and only if there exists some  $\tau \in [0, 1]$  such that (12) and (13) hold, for  $T$  given by (14).*

This Lemma tells us the following:

Part A of Lemma 1 characterizes the new equilibrium.

Equation (12) implies that the nonregulated and nonderegulated groups are better-off provided the tax rate does not exceed a critical threshold

$$\tilde{\tau}_E = 1 - p'_E/p,$$

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<sup>19</sup>All omitted proofs are in the Appendix.



which is equal to their increase in pre-tax real income due to deregulated goods being cheaper.

Equation (13) states that for the deregulated groups to benefit from the reform, their transfer must be larger than their loss from deregulation, given by the RHS of (13). This loss is larger, (i) The greater the relative rent generated by the regulation,  $J$ , (ii) the higher the tax rate  $\tau$  (since by assumption all groups are subject to that tax), (iii), the higher the new price level  $p'_E$ , that is, the lower the scope of deregulation. This effect arises because conditional on being deregulated, a given group is partially compensated, as consumers, by deregulation in other sectors which reduces their CPI. Consequently, the amount of transfers that have to be paid to them is lower. This effect drives the economies of scope in reform that are discussed in Sections 6.1 and 6.3.

Equation (14) determines the equilibrium level of transfer from the government budget constraint. This level is higher (i) the higher the tax rate as long as one is on the left portion of the Laffer curve, and (ii) the lower the scope of reform  $\Delta r$ , since it determines the number of people who have to be compensated. Finally, the term  $p'_E{}^{-\frac{1}{\gamma-1}-\frac{\alpha}{1-\alpha}}$  is generally decreasing with  $p'_E$ , suggesting that for a given tax rate tax receipts are greater, the lower  $r'$ , i.e. the greater the scope for reform. This is because the tax base goes up with the scope of reform, as a lower  $p'_E$  raises labor supply in nonregulated sectors (Equation (10)) as well as employment in nonderegulated sector through an aggregate demand spillover (Equation (11)). However there is a countervailing effect: nonderegulated sectors sell their output at a higher price  $p_R$ , which as such raises the tax base. If there are strong complementarities between goods, i.e. if  $\alpha$  is sufficiently negative, and if labor supply is sufficiently inelastic, i.e.  $\gamma$  large enough, then this latter effect dominates: when the scope for reform goes up, activity does not go up sufficiently to compensate for the loss of tax receipts due to lower prices in the deregulated sectors. That is, in such a case,  $-\frac{1}{\gamma-1} - \frac{\alpha}{1-\alpha} > 0$ .

The next proposition shows that by picking a tax close enough to  $\tilde{\tau}_E$  the reform can be implemented in a Pareto-improving way:

*PROPOSITION 1 – All extensive structural reforms are viable in the sense that for any  $r' \leq r$  there exists  $\bar{\tau}_E \in (0, \tilde{\tau}_E]$  such that the post-reform allocation Pareto-dominates the pre-reform one if and only if  $\bar{\tau}_E \leq \tau \leq \tilde{\tau}_E$ .*

What is the interpretation of Proposition 1? For the nonregulated agents, both the gains from reform (ignoring transfers) and their labor supply entirely depend on their net real wage, equal here to  $(1 - \tau)/p'_E$ . Therefore, the level of  $\tau$  which entirely cancels their gains is the one that leaves their net real wage unchanged relative to the no reform case, and it also leaves their net labor supply unchanged. Setting  $\tau$  at that level transfers all their utility gains from reform to the deregulated agents.

This argument also holds for the nonderegulated agents, but that is for general equilibrium reasons: Preferences are homothetic and the consumption ratio between any regulated good and any nonderegulated good is  $p_R^{-\frac{1}{1-\alpha}}$ . This quantity is unaffected by an extensive reform. At a tax rate which leaves labor supply and therefore production unchanged in nonregulated goods compared to the pre-reform case, the demand-determined level of output in any remaining regulated good must therefore also be the same as in the pre-reform case. Nonderegulated households then have the same pre-tax income as before the reform. They benefit from a lower aggregate price level but these benefits are taxed proportionally, exactly the same way as for nonregulated households. Consequently, they are left indifferent about reform at exactly the same tax rate as the nonregulated.

Therefore, at  $\tau = \tilde{\tau}_E$  all the social gains from the reform are transferred to the deregulated. But these aggregate social gains are strictly positive both because the inefficient price dispersion between goods is reduced and because the labor supply of the deregulated agents, which was inefficiently low, goes up due to demand being reallocated in their favor.<sup>20 21</sup>

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<sup>20</sup>Comparing (10) for  $\tau = \tilde{\tau}_E$  with (6), we get  $l'_{NE} = p^{-\frac{1}{\gamma-1}} > \bar{l} = p^{-\frac{1}{\gamma-1}} p_R^{-\frac{1}{1-\alpha}}$ .

<sup>21</sup>It may be that some of the reforms in range  $[\bar{\tau}_E, \tilde{\tau}_E]$  are themselves Pareto-dominated by lower values of  $\tau$  because the tax rate lies on the downward sloping side of the "Laffer curve" which relates the tax rate to the net utility of the deregulated group. This possibility is discussed in the Appendix.

In short: for distortions to prevent the reform from being implemented, compensatory taxes should reduce labor supply relative to the no reform case. But labor supply is unchanged at the tax level that leaves nonderegulated agents exactly indifferent, while it goes up for the deregulated.

## 4.2 Intensive reform

Now consider an intensive reform: Instead of deregulating a fraction of the sectors, the price of all regulated sectors is reduced from  $p_R$  to  $p'_R < p_R$ . Again a proportional tax  $\tau$  is levied upon all incomes and a compensatory lump-sum transfer  $T$  is paid to the deregulated groups. The new equilibrium is summarized by Lemma 2:

*LEMMA 2 – A. The new values of the price level and of employment (=output) in any sector  $i$  are given by*

$$\begin{aligned} p' &= (rp_R^{\frac{1-\alpha}{1-\alpha}} + 1 - r)^{-\frac{1-\alpha}{\alpha}} = p'_I; \\ l'_i &= p_I^{\frac{1}{\gamma-1}} (1-\tau)^{\frac{1}{\gamma-1}} = l'_{NI}, \quad i > r \\ l'_i &= p_R^{\frac{1}{1-\alpha}} p_I^{\frac{1}{\gamma-1}} (1-\tau)^{\frac{1}{\gamma-1}} = \bar{l}'_I, \quad i \leq r. \end{aligned}$$

*B. Nonregulated groups ( $i > r$ ) weakly gain from the reform if and only if*

$$\tau \leq \tilde{\tau}_I = 1 - \frac{p'_I}{p}. \quad (15)$$

*C. Regulated groups ( $i \leq r$ ) weakly gain from the reform if and only if*

$$\tau(1-\tau)^{\frac{1}{\gamma-1}} p_I^{\frac{1-\alpha}{1-\alpha}} \geq r \frac{\gamma-1}{\gamma} \left( J - \left( \frac{p(1-\tau)}{p'_I} \right)^{\frac{\gamma}{\gamma-1}} J' \right) \left( \frac{p'_I}{p} \right)^{\frac{\gamma}{\gamma-1}}, \quad (16)$$

where

$$J' = \frac{\gamma p_R^{\frac{1-\alpha}{1-\alpha}} - p_R^{\frac{1-\gamma}{1-\alpha}}}{\gamma-1}. \quad (17)$$

*D. Consequently, for any  $p'_R < p_R$ , the reform can be implemented in a Pareto-improving way if and only if there exists some  $\tau \in [0, 1]$  such that (15) and (16) hold, with at least one strict inequality.*

Lemma 2 has a similar interpretation as Lemma 1. The maximum tax rate in (15) has the same expression as in (12), because it only depends on the change in the CPI induced by the reform, regardless of whether the reform is extensive or intensive. Condition (16) is similar to (13)-(14) but has to be modified to reflect the fact that regulated groups are left with a rent  $J'$  after the reform, since each sector is only partially deregulated.

We can again show that an intensive reform is always viable. Proposition 2 extends Proposition 1 to the case of an intensive reform.

*PROPOSITION 2 – All intensive structural reforms are viable in the sense that for any  $p'_R \leq p_R$  there exists  $\bar{\tau}_I \in (0, \tilde{\tau}_I]$  such that the post-reform allocation Pareto-dominates the pre-reform one if and only if  $\bar{\tau}_I \leq \tau \leq \tilde{\tau}_I$ .*

## 5 Some more general results

The preceding results rely on a model which is tractable at the cost of a number of special assumptions. The structure of preferences and technology is symmetrical, as is that of regulation and of the reforms that have been considered. We want to know whether the central insights – in particular regarding the implementability of a structural reform – still apply to a more general setting where these assumptions regarding symmetry are relaxed.

In this section I consider a more general framework: Utility is still linear in aggregate consumption (hence homothetic) but not necessarily symmetrical nor isoelastic. There is no restriction on the shape of regulation nor on the structure of reform (although I only consider small reforms). I assume that tax rates can be differentiated across groups, implying that the only distortions come from labor supply, as opposed to informational rents. Sectors may differ by size and productivity.

There are three main results: First, aggregate welfare goes up if and only if a weighted average of labor supply across regulated sectors goes up (Proposition 3). Second, despite the tax distortions, any vector of labor supply increases in the regulated sector can be implemented along with a

Pareto-improving schedule of (distortionary) taxes and (lump-sum) transfers (Proposition 5). This result can be understood as a generalization of Propositions 1 and 2. However, contrary to those propositions that explicitly analyzed a reduction in either the number of regulated sectors or the level of regulated prices, Proposition 5 is silent about which specific reforms are viable. It is actually possible that the Pareto-improving reforms that satisfy Proposition 5 involve *tightening* regulation for some goods. Proposition 6 spells out some sufficient conditions for an actual deregulation – i.e. (weakly) reducing all regulated prices – to be implementable in a Pareto-improving fashion.

## 5.1 A generalized model

In this section I introduce the basic assumptions of the generalized model as well as key notations.

There are  $q = n + r$  goods, indexed by  $i = 1, \dots, q$ . A vector will be denoted by lower case, bold letters,  $\mathbf{x}$  and its components by  $x_i$ . In particular,  $\mathbf{1}$  denotes a vector whose elements are all equal to 1. A matrix will be denoted by an upper-case, bold,  $\mathbf{M}$  and its components by  $m_{ij}$ . These conventions are extended to functions, for example for a collection of scalar functions  $(f_1, f_2, \dots)$ ,  $\mathbf{f}(\mathbf{x})$  is a vector whose  $i$ 'th element is  $f_i(x_1, \dots, x_q)$ . Similarly, for any scalar function  $f$  and vector  $\mathbf{x}$ ,  $f(\mathbf{x}) = (f(x_1), f(x_2), \dots)^T$ . By definition, the gradient  $\nabla \mathbf{f}$  is a matrix whose generic element is  $\partial f_i / \partial x_j$ . If  $f$  is a scalar,  $\nabla f$  is its gradient, defined as a line vector to remain consistent with the preceding definition. For any two vectors  $\mathbf{x}, \mathbf{y}$ , their scalar product is denoted by  $\langle \mathbf{x}, \mathbf{y} \rangle = \sum_i x_i y_i$ . For any  $q$ -vector  $\mathbf{x}$ , I will denote by  $\mathbf{x}_N = (x_1, \dots, x_n)$  the  $n$ -vector made of its first  $n$  components and by  $\mathbf{x}_R = (x_{n+1}, \dots, x_q)$  the  $r$ -vector made of the remaining values. For any two commensurate vectors,  $\mathbf{x}$  and  $\mathbf{y}$ ,  $\mathbf{z} = \mathbf{x} \bullet \mathbf{y}$  denotes their element by element product, i.e.  $z_i = x_i y_i$ . Element by element division is denoted by  $\div$ , and in particular for any three vectors  $\mathbf{x}, \mathbf{y}, \mathbf{z}$ ,  $z_i \neq 0$ ,  $\langle \mathbf{x}, \mathbf{y} \rangle = \langle \mathbf{x} \bullet \mathbf{z}, \mathbf{y} \div \mathbf{z} \rangle$ .

There is a continuum of agents of total mass 1, who have the same utility

function but differ by skills. For each  $i$ , a mass  $\theta_i$  of agents have skills specific to sector  $i$  and can only work in that sector. This is as in Section 3, except that the size of the workforce may now differ across sectors. Clearly,  $\sum_{i=1}^q \theta_i = 1$ . Utility is

$$U(\mathbf{c}, \lambda) = u(\mathbf{c}) - v(\lambda),$$

where  $\mathbf{c}$  is the vector of consumption goods and  $\lambda$  is labor supply. I assume  $v', v'' > 0$  and  $u$  is concave and homogeneous of degree one. This assumption guarantees that lump-sum transfers or taxes, when available, would not change labor supply. Therefore, any reform which raises total aggregate utility (in a utilitarian sense) can be implemented in Pareto-improving way if lump-sum taxes and transfers are available. Clearly, this formulation is more general than the symmetrical, isoelastic preferences defined by (1), although it remains homothetic in consumption, separable, and symmetrical across agents.

Let  $p_i$  be the price of good  $i$  and  $R$  the consumer's expenditure. The expenditure function is denoted by

$$c_i = R\psi_i(\mathbf{p}),$$

In particular, the consumer's budget constraint implies that

$$\langle \mathbf{p}, \boldsymbol{\psi}(\mathbf{p}) \rangle = 1, \forall \mathbf{p}. \quad (18)$$

I normalize the aggregate hedonic price level to 1, implying that  $u = R$ , i.e. utility can be rewritten as  $R - v(l)$ . Consequently total labor supply in sector  $i$  is

$$l_i = \theta_i \lambda(w_i),$$

where  $w_i$  is the real wage in sector  $i$  and  $\lambda = v'^{-1}$ .

As in Section 3, sectors are either regulated or nonregulated. I denote by  $\mathcal{N} = \{1, \dots, n\}$  the set of nonregulated sectors (or, rather, their indices) and  $\mathcal{R} = \{n+1, \dots, q\}$  the set of regulated sectors. In deregulated sectors, the price

adjusts so as to ensure competitive equilibrium. In regulated sectors, the price is fixed by law above the competitive level, so that output is determined by demand and supply is rationed.

Production  $y_i$  in sector  $i$  only uses the specific labor input  $l_i$  and productivity  $a_i$  may now differ across sectors:

$$y_i = a_i l_i.$$

In equilibrium, then,  $w_i = a_i p_i$  since the aggregate price index is equal to 1. The pre-reform allocation of resources is then determined by the following set of equations

$$\mathbf{a} \bullet \mathbf{l} = Y \psi(\mathbf{p}) \quad (19)$$

$$\mathbf{l}_N = \boldsymbol{\theta}_N \bullet \lambda(\mathbf{p}_N \bullet \mathbf{a}_N) \quad (20)$$

$$u(\boldsymbol{\psi}(\mathbf{p})) = 1. \quad (21)$$

The unknowns are (i)  $Y$ , aggregate GDP, (ii)  $\mathbf{p}_N$ , the vector of nonregulated prices, (iii)  $\mathbf{l}$ , the vector of employment (=output) in each sector. These are  $q + n + 1$  unknowns. The corresponding equations are (i) that output equals demand in all sectors (19), (ii) that output equals supply in all nonregulated sectors (20), and (iii) that the aggregate price level is normalized to 1 (21). We have that

*LEMMA 3 (Existence of equilibrium with rationing) – There exists a 3-uple  $(\mathbf{l}, \mathbf{p}_N, Y)$  which is solution to (19)-(21).*

By construction, from (19) and (18) it is always true that total income equals total expenditure, i.e.  $Y = \langle \mathbf{a} \bullet \mathbf{p}, \mathbf{l} \rangle$ . I will denote by

$$s_i = p_i \psi_i(\mathbf{p})$$

the income share of sector  $i$ . In this equilibrium, the distribution of income by sectors is summarized by

$$\mathbf{R} = \mathbf{a} \bullet \mathbf{p} \bullet \mathbf{l} = Y \mathbf{s}$$

while the vector of incomes of individuals by occupation is

$$\hat{\mathbf{R}} = \mathbf{a} \bullet \mathbf{p} \bullet \mathbf{l} \div \boldsymbol{\theta} = Y \mathbf{s} \div \boldsymbol{\theta}$$

For prices to be above their equilibrium value in regulated sectors, we need that  $v'(\lambda_i) < w_i$  for  $i \in \mathcal{R}$ . Let

$$\omega_i = 1 - \frac{v'(\lambda_i)}{w_i} = 1 - \frac{v'(l_i/\theta_i)}{a_i p_i} \in [0, 1).$$

Clearly,  $\omega_i$  is a measure of the regulatory distortion in sector  $i$ . The smaller  $\omega$ , the closer the sector is to competitive equilibrium. We have that  $\omega_i = 0$  for  $i \in N$ .

## 5.2 Reforms

I now consider a perturbation of the preceding equilibrium, called a reform. It is useful to consider various reform definitions:

1. A *pure structural reform* (PSR) is a change in the set of regulated prices from  $(p_{n+1}, \dots, p_q)$  to  $(p_{n+1} + dp_{n+1}, \dots, p_q + dp_q)$ , i.e.  $\mathbf{p}_R$  is replaced by  $\mathbf{p}_R + d\mathbf{p}_R$ .

2. A *structural reform with fiscal adjustment* (SRFA) consists of three items: (i) a change in the set of regulated prices from  $\mathbf{p}_R$  to  $\mathbf{p}'_R = \mathbf{p}_R + d\mathbf{p}_R$ , (ii) introducing (possibly negative) proportional taxes on labor (or production), income in each sector at a small rate  $d\tau_i$ , (iii) paying a (possibly negative) small lump-sum transfer to each individual in sector  $i$ ,  $dT_i$ . These instruments must satisfy the government budget constraint

$$\langle \mathbf{s}, d\boldsymbol{\tau} \rangle Y = \langle \boldsymbol{\theta}, d\mathbf{T} \rangle \quad (22)$$

Such a reform  $\rho$  is described by the triplet  $\rho = (d\mathbf{p}_R, d\boldsymbol{\tau}, d\mathbf{T})$ . An SRFA is the most general case we consider.

3. A *structural reform with side transfers* (SRST) is an SRFA such that  $d\boldsymbol{\tau} = \mathbf{0}$ . Consequently, the budget constraint is just

$$\langle \boldsymbol{\theta}, d\mathbf{T} \rangle = 0. \quad (23)$$



4. A *feasible structural reform* (FSR) is an SRFA such that  $\mathbf{dT} \geq \mathbf{0}$ . That is, we allow for proportional taxes and subsidies, as well as lump-sum transfers, but rule out lump-sum taxes.

In the model of Section 4, an extensive reform would be a reduction of  $r$  down to  $r' < r$ , such that  $dp_i = 1 - p_R < 0$  for  $i \in [r', r]$  and  $dp_i = 0$  otherwise. An intensive reform would be such that  $dp_i = p'_R - p_R$  for all  $i \in \mathcal{R}$ . In this section we allow for an arbitrary change in the set of regulated prices (which also includes regulating a subset of goods which were not regulated as long as we include them in the original set of regulated goods with a non binding regulation), but restrict the analysis to differential changes only. The reforms studied in Section 4 coincide with FSRs such that  $d\tau_i = \tau$  for all  $i$  and  $dT_i = T$  for  $i \in \mathcal{R}$  and  $dT_i = 0$  for  $i \notin \mathcal{R}$ . Again I allow to differentiate taxes and transfers by groups, contrary to the preceding section, but limit myself to differential compensation schemes.

The post-reform equilibrium allocation of labor, price vector, and GDP level solves the following conditions

$$\mathbf{a} \bullet \mathbf{l}' = Y\psi(\mathbf{p}') \quad (24)$$

$$\mathbf{l}'_N = \boldsymbol{\theta}_N \bullet \lambda(\mathbf{p}'_N \bullet \mathbf{a}_N \bullet (\mathbf{1}_N - \mathbf{d}\boldsymbol{\tau}_N)) \quad (25)$$

$$u(\psi(\mathbf{p}')) = 1, \quad (26)$$

where the new values are determined by primes.<sup>22</sup>The new distribution of income is

$$\mathbf{R}' = \mathbf{a} \bullet \mathbf{p}' \bullet \mathbf{l}' \bullet (\mathbf{1} - \mathbf{d}\boldsymbol{\tau}) + \boldsymbol{\theta} \bullet \mathbf{dT} = Y\mathbf{s}' \bullet (\mathbf{1} - \mathbf{d}\boldsymbol{\tau}) + \boldsymbol{\theta} \bullet \mathbf{dT}, \quad (27)$$

$$\hat{\mathbf{R}}' = Y\mathbf{s}' \bullet (\mathbf{1} - \mathbf{d}\boldsymbol{\tau}) \div \boldsymbol{\theta} + \mathbf{dT}. \quad (28)$$

### 5.2.1 The irrelevance of tax distortions in regulated sectors

A key feature of the analysis and of the proofs is that proportional taxes are only distortionary for nonregulated sectors. In regulated sectors, supply

<sup>22</sup>It is straightforward to prove existence of a solution to (24-26) along the same lines as Lemma 3; see the remarks after the proof of Lemma 3 in the Appendix.

is constrained by demand. Therefore activity is not reduced by a marginal proportional tax. As a result, the feasibility constraint  $dT_i \geq 0$  is irrelevant for regulated sectors. Any lump-sum tax can be replaced by a proportional tax, with no difference for the allocation of resources. We can give formal content to this observation:

*DEFINITION 1* – Let  $\rho_0 = (\mathbf{dp}_{R0}, \mathbf{d}\tau_0, \mathbf{dT}_0)$  and  $\rho_1 = (\mathbf{dp}_{R1}, \mathbf{d}\tau_1, \mathbf{dT}_1)$  be two SRFAs. They are **equivalent** [ $\rho_0 \sim \rho_1$ ] if and only if

$$\begin{aligned} \mathbf{l}'_0 &= \mathbf{l}'_1, \\ \mathbf{R}'_0 \psi(\mathbf{p}'_0)^T &= \mathbf{R}'_1 \psi(\mathbf{p}'_1)^T. \end{aligned}$$

That is, the post-reform labor supply and consumption vector of each group is unchanged, and therefore so is the distribution of welfare. It will be also useful to define production-equivalence:

*DEFINITION 2* – Let  $\rho_0 = (\mathbf{dp}_{R0}, \mathbf{d}\tau_0, \mathbf{dT}_0)$  and  $\rho_1 = (\mathbf{dp}_{R1}, \mathbf{d}\tau_1, \mathbf{dT}_1)$  be two SRFAs. They are **production-equivalent** [ $\rho_0 \approx \rho_1$ ] if and only if

$$\begin{aligned} \mathbf{l}'_0 &= \mathbf{l}'_1, \\ \mathbf{p}'_0 &= \mathbf{p}'_1. \end{aligned}$$

Clearly, if two reforms are production-equivalent and deliver the same distribution of income ( $\mathbf{R}'_0 = \mathbf{R}'_1$ ), then they are equivalent.

Lemma 4 shows that in the regulated sectors, nondistortionary transfers and distortionary taxes are equivalent. That is, I can substitute one for the other and get an equivalent allocation. Consequently, any reform with fiscal adjustment can be made feasible in those sectors, by replacing any lump-sum tax imposed on a regulated group (a negative  $T_i$ ) by a distortionary proportional tax. At the margin, the fact that regulation remains binding implies that those groups are not on their labor supply curve but are rationed instead, since their employment level is determined by the "numerus clausus"

consistent with the regulation and independent of the specific tax rate  $d\tau_i$  imposed upon those groups. Therefore, everything takes place as if these group's labor supply were inelastic, i.e. as if the proportional tax on their income were not distortionary.

*LEMMA 4* – Let  $\rho_0 = (\mathbf{dp}_{R0}, \mathbf{d}\tau_0, \mathbf{dT}_0)$  and  $\rho_1 = (\mathbf{dp}_{R1}, \mathbf{d}\tau_1, \mathbf{dT}_1)$  be two SRFAs. Assume that  $\mathbf{dp}_{R0} = \mathbf{dp}_{R1}$ ,  $\mathbf{d}\tau_{N0} = \mathbf{d}\tau_{N1}$ ,  $\mathbf{dT}_{N0} = \mathbf{dT}_{N1}$ , and  $\theta_R \bullet \mathbf{dT}_{R0} - \mathbf{Y}_{sR0} \bullet \mathbf{d}\tau_{R0} = \theta_R \bullet \mathbf{dT}_{R1} - \mathbf{Y}_{sR1} \bullet \mathbf{d}\tau_{R1}$ . Then  $\rho_0 \sim \rho_1$ .

Proof – Clearly, the system (24-26) is the same under both reforms. Therefore, the equilibrium set of prices and allocation of labor is identical, i.e.  $\mathbf{dp}_0 = \mathbf{dp}_1$ ,  $\mathbf{dl}_0 = \mathbf{dl}_1$ . Furthermore, by construction,  $\mathbf{R}'_0 = \mathbf{R}'_1$  from (27). Hence both conditions in Definition 1 hold. QED.

Therefore, everything takes place as if lump-sum taxes were available for regulated sectors. For  $i \in \mathcal{R}$ , I denote by  $d\tilde{T}_i = dT_i - Y_{s_i}d\tau_i/\theta_i$  the net income transfer to an individual of group  $i$ . Then

*LEMMA 5* – For any SRFA  $\rho$  such that  $dT_N \geq 0$ , there exists an FSR  $\tilde{\rho}$  such that  $\tilde{\rho} \sim \rho$ .

Proof – For any  $i \in R$  such that  $dT_i < 0$ , replace  $dT_i$  by 0 and  $d\tau_i$  by  $-\theta_i d\tilde{T}_i/(Y_{s_i})$ . Then apply Lemma 4.

Lemma 4 and 5 are important here, since due to the generality of the price changes we consider, some regulated groups may experience direct gains from the reform, either because regulation is tightened for them or because their direct gains from other sectors being deregulated offsets their losses from deregulation in their own sector. In such a situation one might want to extract resources from them so as to transfer part of their gains to the losers from the reform, as it is the case for the nonregulated groups. Lemma 4 and 5 tell us that we can do it as if it were possible to impose lump-sum taxes upon those groups.<sup>23</sup>

<sup>23</sup>In the analysis of Section 4, it is unclear whether results such as Lemmas 4 and 5 would apply. The reforms considered there are not marginal and sectors are fully deregulated in an extensive reform. However due to the symmetrical properties of the reforms, deregulated

### 5.2.2 The case for Pareto-improving structural reforms

In this subsection I provide some basic results on the conditions for structural reforms to be welfare-improving. My starting point is utilitarian social welfare, which is defined as

$$W = \sum_i \theta_i U(\mathbf{c}_i, \lambda_i) = \sum_i (R_i - \theta_i v(\lambda_i)) = Y - \langle \boldsymbol{\theta}, v(\boldsymbol{\lambda}) \rangle. \quad (29)$$

*PROPOSITION 3 – Consider an SRFA*

(i) *The effect on utilitarian social welfare is*

$$\frac{dW}{Y} = \langle \mathbf{s} \bullet \boldsymbol{\omega}, \mathbf{dl} \div \mathbf{l} \rangle.$$

*Consequently, aggregate social welfare goes up if and only if*

$$\langle \mathbf{s} \bullet \boldsymbol{\omega}, \mathbf{dl} \div \mathbf{l} \rangle > 0. \quad (30)$$

(ii) *The change in output is*

$$\frac{dY}{Y} = \langle \mathbf{s}, \mathbf{dl} \div \mathbf{l} \rangle \quad (31)$$

(iii) *The change in income distribution ( $R = sY$ ) is*

$$\mathbf{dR} \div \mathbf{R} = \mathbf{dp} \div \mathbf{p} + \mathbf{dl} \div \mathbf{l} - \mathbf{d\tau} + \boldsymbol{\theta} \bullet \mathbf{dT} \div \mathbf{R} \quad (32)$$

(iv) *For any PSR such that (30) holds, there exists a Pareto-improving production-equivalent SRST.*

Proposition 3 tells us that regardless of the accompanying fiscal scheme, aggregate social welfare goes up if and only if a weighted average of employment growth in regulated sectors is positive, with weights proportional to the product between their share in GDP  $s_i$  and their distortion index  $\omega_i$ . Claim (iv) is just a verification of the standard result that the utilitarian and

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sectors must get a lump-sum transfer, not a tax, so such results are not needed. Indeed we have shown that a Pareto improvement is possible despite the deregulated sectors having to pay the same income tax as all other groups.

Pareto criteria coincide if utility is linear in consumption and unrestricted side-transfers are available.

I now show that Pareto-improving feasible structural reforms are always possible, despite that lump-sum transfers can only be financed by distortionary taxes. The argument rests upon the same device as in the preceding section, i.e. imposing a proportional tax on the deregulated that leaves them indifferent about the reform, or equivalently such that their net after tax real wage is unchanged.

*DEFINITION 3 – An SRFA is N-neutral if and only if*

$$\mathbf{dT}_N = \mathbf{0}$$

and

$$\mathbf{d}\tau_N = \mathbf{dp}_N \div \mathbf{p}_N. \quad (33)$$

An N-neutral reform taxes away any welfare gains made by a nonregulated group, by imposing a proportional tax equal to the increase in their real wage (i.e. the price of their product). In particular, this implies that their labor supply, and consequently their employment level, is unchanged, i.e.  $\mathbf{dl}_N = \mathbf{0}$ .

In the analysis of Section 4, the N-neutral reforms are those that tax income at the maximum rate which leaves the nonregulated and nonderegulated indifferent between the reform and the status quo, and which simultaneously leaves their employment level unchanged, that is  $\tau = 1 - p'/p$ , a condition which is equivalent to (33) in the context of the uneven, differential policies that are considered here.

*PROPOSITION 4 – Assume an SRFA  $\rho_0$  is N-neutral and satisfies (30). Then there exists an N-neutral FSR  $\rho_1$  such that (i)  $\rho_0 \approx \rho_1$ , and (ii)  $\rho_1$  is Pareto-improving.*

In other words, if an N-neutral reform satisfies the utilitarian criterion, I can construct a *feasible* reform which has the same effect on the productive

allocation of resources and is Pareto-improving. Note that whether or not a reform is N-neutral depends on whether the endogenous equilibrium prices satisfy (33). It is not a priori obvious that an N-neutral reform exists. But if it does, Proposition 4 tells us that it can be implemented in a Pareto-improving way. The reason is that if (30) holds, the aggregate utility gain of the regulated agents is strictly positive (since that of the nonregulated is zero by construction), and Lemma 5 tells us that utility can be freely transferred between them in a feasible way.<sup>24</sup>

Consequently, to establish the existence of Pareto-improving, feasible structural reforms, it is enough to show that an N-neutral reform which satisfies (30) can be constructed.

*PROPOSITION 5 – For any  $r$ -vector  $\mathbf{dz}$  there exists an N-neutral SRFA such that  $\mathbf{dl}_R = \mathbf{dz}$ .*

*Corollary – There exists a Pareto-improving structural reform.*

Proposition 5 tells us that we can solve the inverse problem, that is, pick a reform which delivers any arbitrary vector of employment growth for the regulated sector, and make sure it is N-neutral. Since the choice of  $\mathbf{dl}_R$  is arbitrary, we can always pick that vector so that there are aggregate welfare gains, i.e. so that (30) holds. This will be the case, in particular, if  $\mathbf{dl}_R \geq \mathbf{0}$  with at least one strict inequality. In this case, since the reform is N-neutral,  $\mathbf{dl}_N = \mathbf{0}$ , so that we also have  $dY > 0$  by (31): Output goes up as well as aggregate welfare. The corollary follows from Proposition 4, which tells us that we can then always design the tax treatment of the deregulated so as to make each of them better-off in a feasible way.

While Propositions 4 and 5 tell us that many structural reforms exist that are both feasible and Pareto-improving, they are mute about the structure

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<sup>24</sup>Note that while reform  $\rho_1$  implements the same changes in regulated prices as  $\rho_0$  and the same equilibrium vector of prices and production, the distribution of income is obviously different between the two reforms, since reform  $\rho_0$  need not be feasible. Hence these reforms are production equivalent to each other but not equivalent.

of those reforms. They need not be deregulations in the sense that all price controls are loosened: Regulation could in principle be tightened for some goods, i.e.  $dp_i > 0$ .

The conditions for *deregulation* to be implementable in a Pareto-improving way are far more stringent. The following Proposition provides a sufficient condition.

*PROPOSITION 6 – Let*

$$s_R = \langle \mathbf{s}_R, \mathbf{1} \rangle,$$

$$\sigma = \min_i \sigma_i = -\frac{p_i}{\psi_i} [\nabla \psi]_{ii} > 0,$$

and

$$\eta = \max_{i,j,i \neq j} \left| \frac{p_j}{\psi_i} [\nabla \psi]_{ij} \right| \geq 0.$$

*Consider a nonzero  $r$ -vector of price reductions  $\mathbf{dx} \geq \mathbf{0}$ . Let  $x_m = \max \mathbf{dx} > \mathbf{0}$ ,  $\bar{x}_Y = \langle \mathbf{s}_R, \mathbf{dx} \rangle / s_R > 0$ , and  $\sigma_R = \langle \mathbf{s}_R, \boldsymbol{\sigma}_R \bullet \mathbf{dx} \rangle / \langle \mathbf{s}_R, \mathbf{dx} \rangle > 0$ .*

*Assume*

$$\eta < \frac{1}{q-1} \min\left(\frac{1-s_R}{2} \sigma, \frac{\bar{x}_Y}{x_m} s_R \sigma_R\right) \quad (34)$$

*Then (i) there exists an  $N$ -neutral SRFA such that  $\mathbf{dp}_R \div \mathbf{p}_R = -\mathbf{dx}$  and*

*(ii) These reforms are such that  $\mathbf{dl}_R > \mathbf{0}$ , implying that they satisfy (30) and (31).*

This proposition tells us that (i) if the price cross-elasticities of demand (whose maximum absolute value across pairs of goods is  $\eta$ ) are *small enough* relative to the own price elasticities (whose minimum absolute value across all goods is  $\sigma$ , and whose income share weighted average across regulated goods is  $\sigma_R$ ); and (ii) if the structural reform is *not too unbalanced*, in the sense that the maximum reduction in a regulated price  $x_m$  is not too large relative to its income share weighted average counterpart  $\bar{x}_Y$ , then an  $N$ -neutral SRFA exists which implements the planned structural reform, and it

is such that employment grows in all regulated sectors. In turn, this implies that it satisfies (30), so that by Proposition 4 we know that the planned structural reform can be implemented in a Pareto-improving fashion.

Note that in the case of isoelastic utility, studied in the preceding section,  $\eta = 0$ , so that Proposition 6 applies.<sup>25</sup>

Intuitively, if cross-elasticities are too large, upon deregulation the increase in the relative price of some nonregulated good (resp. fall in the price of some regulated good) may reduce total demand for a complement (resp. substitute) regulated good by enough so that employment in that sector would fall. If distortions are particularly large in that sector, total social welfare could well be reduced.

In principle, the conditions in Proposition 6 are testable for a given projected deregulation package. A demand system could be estimated, from which estimates of  $\eta$  and  $\sigma$  could be obtained. Whether condition (34) holds could then be tested.

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<sup>25</sup>Also, for the reform to be implemented in a Pareto-improving way, we do not need that  $\mathbf{dl}_R > 0$ . Lemma A4 in the Appendix provides a weaker condition than (34), with analogous properties, for the N-neutral reform to satisfy (30).



## 6 Some counter-examples

I now discuss some counter-examples where a Pareto-improvement is not possible. I first consider the case where the reform has resource costs, which highlights the role of economies of scope. I then discuss the case where the tax scheme cannot extract all the rents from winners, either for institutional reasons (indexation of regulated prices and uniform taxation) or for informational reasons (heterogeneous preferences). Finally, I show that my benchmark results no longer hold if preferences are not homothetic in consumption and separable.

### 6.1 Cost of reform

It is obvious that if the reform involves resource costs, these may be too large for transfers to compensate losers. What is more interesting, though, is the fact that our framework predicts that when reform is costly, it is more likely to be viable, the broader its scope. This, even though there is no fixed cost of reform. The following Proposition shows that if the cost of reform is linear in the number of sectors that are deregulated (or equivalently in the number of people who have to be compensated), then there exists a minimum reform scope for the reform to be viable.

*Proposition 7 – Assume  $p \leq \gamma - r(\gamma - 1)$ . Assume the resource cost of an extensive reform is  $c\Delta rp'_E$ . Then there exists  $c_-$  and  $c_+$ ,  $0 \leq c_- < c_+$ , such that*

- (i) if  $c \leq c_-$  all reforms are viable*
- (ii) if  $c_- < c \leq c_+$  there exists  $r_v > 0$  such that the reform is viable if and only if  $r' \leq r_v$*
- (iii) if  $c > c_+$  no reform is viable*
- (iv) in case (ii), the minimum reform scope goes up as the unit cost of reforms go up:  $\partial r_v / \partial c < 0$ .*

The interesting part of that proposition is claim (ii): for the reform to be

implementable in a Pareto-improving way, it must be large enough—despite that the cost of reform is not fixed but simply proportional to the number of sectors that are deregulated. The mechanism comes from the economies of scope in reform. Losers are compensated not only by the transfer but also by the fall in prices in the deregulated goods other than the one they produce. As a result their utility gains, expressed in terms of the aggregate consumption good, grow faster than the scale of the reform.<sup>26</sup>

## 6.2 Indexation on the aggregate price level

In the second-best world we consider, the welfare effects of a given partial reform depend on the specifics of regulation. The way regulation is specified has a general equilibrium effect on the distribution of gains from the reform. In particular, the indexation rule for the prices of the regulated sectors matters. In Section 4, when considering an extensive reform, I have assumed that the price of regulated sectors is automatically indexed on the price of unregulated sectors. This generates a built-in mechanism through which nonderegulated sectors benefit from the reform through a higher purchasing power, since the price of unregulated sectors (and therefore their own price) goes up relative to the aggregate price level when there is a reform.<sup>27</sup> I have shown that for any given tax rate, the utility gains of the nonderegulated groups are then proportional to those of the regulated groups, implying that the same tax rate leaves both groups indifferent between the reform and the status quo. In other words, it is possible to finance a compensatory transfer by a uniform proportional tax rate on all incomes, in a way that does not leave any rent to the groups that are not directly affected by the reform. Whenever the regulated price is indexed on the aggregate price level, though,

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<sup>26</sup>Proposition A3 in the Appendix extends the results of Proposition 7 to the case where  $p > \gamma - r(\gamma - 1)$ . Characterization is less straightforward but the results are qualitatively similar.

<sup>27</sup>The indexation rule is irrelevant in the case of an intensive reform, since there is only one relative price in the economy we consider. However, under extensive reform, relative prices may change along two dimensions, since there are three groups that are differently affected by the reform: nonregulated, deregulated, nonderegulated.

this conclusion no longer holds. As the price of the nonderegulated groups is indexed on the aggregate price level, their real wage is unchanged by the reform. Their potential gains only come from the fact that they work more, due to the general equilibrium effect of the reform on labor demand. Since their gains are smaller than under the indexation rule considered in Section 4, it turns out that the maximum tax rate that prevents them from being net losers is lower than its counterpart for the nonregulated groups. Unless tax rates can be differentiated among groups, the latter must therefore earn rents compared to the nonderegulated ones, which reduces the amount that can be transferred to the losers of the reform.<sup>28</sup> As a result, a Pareto-improving reform becomes impossible under some configurations.

I now describe how the equilibrium is changed by this new indexation rule and discuss its consequences for the viability of reform.

It is now natural to normalize the aggregate price level, instead of the nonregulated price, to 1. As a result,  $p_R$  can again be treated as fixed. The nonregulated price  $p_N$ , for any given  $r$ , solves

$$rp_R^{-\frac{\alpha}{1-\alpha}} + (1-r)p_N^{-\frac{\alpha}{1-\alpha}} = 1. \quad (35)$$

Note that since  $p_R > 1$ ,  $p_N < 1$ . Furthermore,  $\partial p_N / \partial r < 0$ . The greater the scope of regulation, the lower the price (i.e. the real wage), of the non-regulated groups.

As in Section 4, I consider an extensive reform with the same features as above.  $r$  falls to  $r' = r - \Delta r$ . A proportional tax  $\tau$  is levied on all incomes. The proceeds are rebated to the groups whose index falls between  $r'$  and  $r$  in a lump-sum, uniform fashion.

We can then prove the following

*LEMMA 6 – A. The new values of the price of nonregulated goods and of*

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<sup>28</sup>Here there is no a priori reason why tax rates could not be differentiated across groups, as different groups produce different goods. Still, policy may be constrained by the complexity of the compensatory tax scheme. Also, although this is outside the scope of this model, uniform tax rates may be considered as especially legitimate in the presence of disagreements about the distributional effects of the reform.

employment (=output) in any sector  $i$  are given by

$$p'_i = p'_N = \left( \frac{1 - r' p_R^{-\frac{\alpha}{1-\alpha}}}{1 - r'} \right)^{-\frac{1-\alpha}{\alpha}}, \quad i > r'; \quad (36)$$

$$l_i = l'_N = (p'_N(1 - \tau))^{\frac{1}{\gamma-1}}, \quad i > r' \quad (37)$$

$$l_i = \bar{l}' = p_N^{\frac{1}{\gamma-1} + \frac{1}{1-\alpha}} p_R^{-\frac{1}{1-\alpha}} (1 - \tau)^{\frac{1}{\gamma-1}}, \quad i \leq r'. \quad (38)$$

*B. Nonregulated groups ( $i > r$ ) weakly gain from the reform if and only if*

$$1 - \tau \geq \frac{p_N}{p'_N}, \quad (39)$$

*C. Nonderegulated groups ( $i \leq r'$ ) weakly gain if and only if*

$$1 - \tau \geq \frac{p_N}{p'_N} \left( \frac{J}{J'} \right)^{\frac{\gamma-1}{\gamma}}, \quad (40)$$

where

$$J = \frac{u_R}{u_N} = \frac{\gamma(p_R/p_N)^{-\frac{\alpha}{1-\alpha}} - (p_R/p_N)^{-\frac{\gamma}{1-\alpha}}}{\gamma - 1}, \quad (41)$$

$$J' = \frac{u'_R}{u'_N} = \frac{\gamma(p_R/p'_N)^{-\frac{\alpha}{1-\alpha}} - (p_R/p'_N)^{-\frac{\gamma}{1-\alpha}}}{\gamma - 1}. \quad (42)$$

*D. Deregulated groups ( $r' < i \leq r$ ) weakly gain from the reform if and only if*

$$\tau(1 - \tau)^{\frac{1}{\gamma-1}} p_N^{\frac{1}{\gamma-1} + \frac{1}{1-\alpha}} \geq \Delta r \frac{\gamma - 1}{\gamma} p_N^{\frac{\gamma}{\gamma-1}} \left[ J - \left( \frac{p'_N(1 - \tau)}{p_N} \right)^{\frac{\gamma}{\gamma-1}} \right]. \quad (43)$$

The effect of the reform on the relative rent of the regulated (which was unchanged under extensive reform in the analysis of section 4 except of course for the deregulated) can be obtained by examining (41) and (42). Typically, the regulated's rent  $J$  will be locally increasing in  $p_R/p_N$ . Otherwise,  $p_R/p_N$  is too high, in that regulated groups would gain from being less regulated. Therefore, since  $p'_N > p_N$ ,  $J' < J$ . The reform reduces the utility rent of the regulated groups, due to the mechanical indexation of  $p_R$  on the CPI.

That is,  $p_N$  goes up while  $p_R$  is unchanged – nonderegulated groups tend to gain less from the reform than nonregulated ones. Consequently, the former accept the reform for lower tax rates than the latter. That is, (40) is more stringent than (39) as long as  $J' < J$ . Relative to the analysis of Section 4.1, this feature reduces the maximum possible value of  $\tau$ , which in turn makes it more likely that  $T$  falls short of the level that is necessary to compensate losers. Indeed, an analytical result can be proved:

*PROPOSITION 8 – There exists  $\bar{p}_R > 1$  such that for any  $p_R \in (1, \bar{p}_R)$ , the only viable reform is  $r' = 0$ .*

Full reform remains viable here: As no nonderegulated sector remains, indexation becomes irrelevant and no rent has to be paid. Therefore the results of Section 4 apply in this case. For any partial extensive reform, however, regardless of its scope, a Pareto-improvement is impossible if regulation is initially not too tight. Numerical simulations in the working paper version of this article (Saint-Paul, 2018) suggest that if rents cannot be eliminated by discretionary taxation, it is not difficult to end up in a situation where consensus over a reform is impossible (unless this reform is complete).<sup>29</sup>

### 6.3 Heterogeneous preferences

If preferences are heterogeneous and compensatory taxes cannot be indexed on them, then Pareto-improving reform may be impossible. Agents who benefit less from the reform because they consume less of the deregulated goods can only be better-off if they pay lower taxes. This in turn delivers rents to those who benefit more from the reform, which makes it impossible to

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<sup>29</sup>In a real world situation, provided a demand system is estimated, one could use a computable general equilibrium model to estimate the values of the rents  $J$  and  $J'$  as well as the maximum tax rate which satisfies (40). In principle one could compute the associated transfer and check whether (43) holds. Of course, the estimation and specification issues would be far more complex than in our simple symmetrical setting. Furthermore if (43) were violated, the reform might still be viable if supplemented with a reconsideration of the regulatory level of  $p_R$  in relationship to the CPI.

design it in an N-neutral way. The maximum tax rate that can be imposed on an individual is now  $\tau_i = 1 - p'_i/p_i$ , where  $p_i$  and  $p'_i$  are the individual-specific consumer price indices before and after the reform. Clearly, if some groups in the population do not consume the deregulated goods at all, for example, then (assuming general equilibrium effects are zero) Pareto-improvement is possible only if those can be identified and exempted from compensatory taxation. One can show that if this effect is strong enough, it generates economies of scope in reforms, in that more people gain from the reform, the broader its scope. This is because broader reforms reduce the proportion of the population who do not consume any of the deregulated groups, and who lose regardless of the level of the compensatory tax rate.<sup>30</sup>

## 6.4 Non homotheticity

If preferences are not homothetic in consumption goods and/or not separable between consumption and labor, the tax rate which leaves the winners from the reform indifferent does not generally leave their behavior unchanged. It is no longer possible to implement an N-neutral reform with proportional tax rates. If such a tax rate reduces the nonregulated's labor supply relative to the no reform case, this effect reduces the transfer that may be paid to the deregulated and has a negative effect on the aggregate demand for the regulated goods and therefore on regulated employment  $\bar{l}$ . For example, assume there are only two goods, good 1 which is nonregulated and is the numéraire, and good 2 which is regulated and whose price is  $p_R$ . The reform reduces the price from  $p_R$  to  $p'_R$  and the transfer to group 2 is financed by a proportional tax  $\tau$  on the income of the producers of good 1. Utility is

$$U(c_1, c_2, l) = \frac{1}{\alpha} \left( c_2^\alpha + \left( c_1 - \frac{l^\gamma}{\gamma} \right)^\alpha \right) \quad (44)$$

In this formulation, the disutility cost of labor is in terms of the nonregulated numéraire instead of utils. As a result the price index relevant for labor

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<sup>30</sup>See the working paper version of this paper, Saint-Paul (2018), for a formal analysis.

supply is that of good 1. Hence it differs from the consumption price index and the labor supply of group 1 is

$$l = (1 - \tau)^{\frac{1}{\gamma-1}}$$

which is independent of  $p'_R$ . Therefore, the tax rate which leaves this group indifferent between reform and the status quo always reduces its labor supply relative to the status quo, since taxes are nil in the latter case. One can show numerically that for  $\alpha$  low enough, i.e. for enough complementarity between the two goods, and  $\gamma$  low enough, i.e. an elastic enough labor supply curve. the labor supply distortions at the tax rate which is best for the deregulated groups are such that the resulting transfers fail to compensate them for the direct losses from reform (See the Appendix for details). Essentially, when complementarities are strong, the consumer welfare gains from the reduction in the relative price of deregulated goods are not strong enough to outweigh the employment reduction in the sectors that are taxed.

## 7 Conclusion

I believe this paper has established some useful benchmark results regarding the viability of structural reforms under compensatory transfers and distortionary taxation. While these results can be overturned, especially under informational rents, they at least suggest that structural reforms cannot be undermined by their budgetary cost, provided these costs are correctly evaluated in welfare terms. Hence, why some reforms appear difficult to implement remains a productive avenue for future research. In that respect, one may mention two important avenues. First, dynamic aspects could be analyzed by introducing imperfect labor mobility between sectors as well as the ability for the government to smooth the tax burden of compensatory transfers over time by using public debt. Second, the analysis could take into account microeconomic foundations for both the rigidities studied here and institutions such as the structure of taxation and indexation rules. For example, price floors may initially be established as an imperfect device to provide insurance

to producers with sector-specific human capital in the presence of productivity shocks. Reform could come about on the policy agenda as an outcome of financial development (which would provide better means of insurance), or of secular structural decline in an industry that would weaken its lobbying power.



## 8 Appendix

### 8.1 Proof of Lemma 1

The first formula in claim A is straightforward from the definition of the aggregate price level.

Households such that  $i > r$  now pay a proportional tax  $\tau$ . Their income is  $l(1 - \tau)$  and consequently their labor supply is<sup>31</sup>

$$l = l'_N = p'^{-\frac{1}{\gamma-1}} (1 - \tau)^{\frac{1}{\gamma-1}}. \quad (45)$$

Furthermore, their utility is

$$u'_N = \left(1 - \frac{1}{\gamma}\right) \left(\frac{1 - \tau}{p'}\right)^{\frac{\gamma}{\gamma-1}}. \quad (46)$$

They benefit from the reform scheme iff  $u'_N \geq u_N$ , or equivalently

$$1 - \tau \geq \frac{p'}{p}, \quad (47)$$

which proves claim B for the nonregulated groups.

Households such that  $r' < i \leq r$  are now deregulated and face the same price vectors as those with  $i > r$ . By linearity of the indirect utility function with respect to consumption, their labor supply is the same,  $l'_N = (p'(1 - \tau))^{-\frac{1}{\gamma-1}}$ . This, together with (45), proves claim A's second formula. Also, their disposable income is  $R'_D = T + p'^{-\frac{1}{\gamma-1}} (1 - \tau)^{\frac{\gamma}{\gamma-1}}$  and their utility is

$$u'_D = \frac{T}{p'} + \left(1 - \frac{1}{\gamma}\right) p'^{-\frac{\gamma}{\gamma-1}} (1 - \tau)^{\frac{\gamma}{\gamma-1}}. \quad (48)$$

These people benefit from the reform provided  $u'_D \geq u_R$ , i.e.

$$T \geq p'^{-\frac{1}{\gamma-1}} \frac{\gamma - 1}{\gamma} \left[ \left(\frac{p'}{p}\right)^{\frac{\gamma}{\gamma-1}} J - (1 - \tau)^{\frac{\gamma}{\gamma-1}} \right]. \quad (49)$$

This proves formula (13) in the text.

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<sup>31</sup>In this Appendix, the subscripts  $I, E$  that distinguish between the equilibrium levels of variables pertaining to intensive reforms and those pertaining to extensive reforms are dropped, without ambiguity.

Finally, households such that  $i \leq r'$  have an income equal to  $R'_R = \bar{l}' p_R (1 - \tau)$  and their labor supply is constrained to be equal to  $\bar{l}'$ . Consequently, their utility is

$$u'_R = \frac{p_R \bar{l}' (1 - \tau)}{p'} - \frac{\bar{l}'^\gamma}{\gamma}. \quad (50)$$

Again, the equilibrium rationed labor supply in the regulated sectors can be obtained from the equilibrium condition in any of the non regulated good:

$$l'_N = (1-r)c(p'^{-\frac{1}{\gamma-1}}(1-\tau)^{\frac{\gamma}{\gamma-1}}, p', 1) + \Delta r c_N(p'^{-\frac{1}{\gamma-1}}(1-\tau)^{\frac{\gamma}{\gamma-1}} + T, p', 1) + r' c_N(\bar{l}' p_R (1 - \tau), p', 1),$$

where again  $c(\cdot, \cdot, \cdot)$  is defined by (3). This expression is equivalent to

$$\begin{aligned} p'^{-\frac{1}{\gamma-1}}(1-\tau)^{\frac{1}{\gamma-1}} &= (1-r)p'^{\frac{\alpha}{1-\alpha}-\frac{1}{\gamma-1}}(1-\tau)^{\frac{\gamma}{\gamma-1}} \\ &+ \Delta r [p'^{\frac{\alpha}{1-\alpha}-\frac{1}{\gamma-1}}(1-\tau)^{\frac{\gamma}{\gamma-1}} + T p'^{\frac{\alpha}{1-\alpha}}] + r' \bar{l}' p_R (1 - \tau) p'^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Using (2), this can be rearranged as

$$r' \bar{l}' p_R (1 - \tau) = p'^{-\frac{1}{\gamma-1}}(1-\tau)^{\frac{1}{\gamma-1}} \left[ r' p_R^{-\frac{\alpha}{1-\alpha}} + \tau(1-r') \right] - T \Delta r. \quad (51)$$

Total tax revenues are equal to  $\tau[(1-r')l'_N + r' p_R \bar{l}']$ , implying that the government budget constraint can be written as

$$T \Delta r = \tau(1-r') p'^{-\frac{1}{\gamma-1}}(1-\tau)^{\frac{1}{\gamma-1}} + \tau r' p_R \bar{l}'. \quad (52)$$

We can eliminate  $T$  between (51) and (52) to get an expression for  $\bar{l}'$ :

$$\bar{l}' = p_R^{-\frac{1}{1-\alpha}} p'^{-\frac{1}{\gamma-1}} (1 - \tau)^{\frac{1}{\gamma-1}} \quad (53)$$

This expression coincides with the third formula of claim A. Furthermore, the utility of the non deregulated groups can be rewritten as, using (8), (50), and (53)

$$u'_R = \frac{\gamma - 1}{\gamma} p'^{-\frac{1}{\gamma-1}} (1 - \tau)^{\frac{1}{\gamma-1}} J. \quad (54)$$

This group gains from the reform provided  $u'_R \geq u_R$ , or equivalently, from (54), (7), and (8):

$$1 - \tau \geq \frac{p'}{p},$$

which is clearly the same condition as (47), proving claim B for the non-deregulated groups.

The equilibrium value of  $\bar{l}'$  can be substituted into (52) to reexpress the government's budget constraint:

$$T = \frac{\tau(1 - \tau)^{\frac{1}{\gamma-1}} p'^{-\frac{1}{\gamma-1} - \frac{\alpha}{1-\alpha}}}{\Delta r}. \quad (55)$$

This coincides with formula (14) in the text, which together with (49) proves claim C. Finally, claim D is straightforward from the preceding results.

## 8.2 Proof of Proposition 1

A sufficient condition for the existence of viable reforms is that (49) hold for the maximum tax rate compatible with the nonderegulated groups participation constraint, i.e.  $\tilde{\tau} = 1 - p'/p$ . Using (49) and (55) and rearranging, this is equivalent to

$$pp'^{-\frac{1}{1-\alpha}} - p'^{-\frac{\alpha}{1-\alpha}} \geq \Delta r \frac{\gamma - 1}{\gamma} (J - 1), \quad (56)$$

or equivalently

$$h(r') = pp'^{-\frac{1}{1-\alpha}} - (r' p_R^{-\frac{\alpha}{1-\alpha}} + 1 - r') - \Delta r \frac{\gamma - 1}{\gamma} (J - 1) \geq 0$$

Note that  $h(r) = 0$ . Furthermore, from (61)

$$\frac{d}{dr'} h(r') = \frac{\gamma - 1}{\gamma} (J - 1) + (1 - p_R^{-\frac{\alpha}{1-\alpha}}) \left( 1 - \frac{1}{\alpha} \frac{p}{p'} \right). \quad (57)$$

As  $r'$  goes, up, so does  $p'$ . Clearly, then, since  $(1 - p_R^{-\frac{\alpha}{1-\alpha}})/\alpha > 0$ ,

$$\frac{d^2}{dr'^2} h(r') > 0.$$

Therefore, the  $h$  function is convex. From (57) and (8) we get that  $dh/dr' \leq 0$  at  $r' = r$  iff

$$\frac{1}{\alpha} p_R^{-\frac{\alpha}{1-\alpha}} - \frac{1}{\gamma} p_R^{-\frac{\gamma}{1-\alpha}} \leq \frac{1}{\alpha} - \frac{1}{\gamma} \quad (58)$$

This holds with equality for  $p_R = 1$ . Furthermore, the derivative of the LHS with respect to  $p_R$  has the same sign as  $p_R^{\frac{\alpha-\gamma}{1-\alpha}} - 1 < 0$ . Therefore, (58) always holds. Therefore,  $h'(r) \leq 0$ , implying from convexity that  $h'(r') < 0$  for all  $r' < r$ . It follows that since  $h(r) = 0$ ,  $h(r') > 0$  for all  $r' < r$ . This proves that the reform is viable for  $\tau = \tilde{\tau}_E$  regardless of the value of  $r'$ .

Next, observe that (i)  $u'_N$  and  $u'_R$  are decreasing functions of  $\tau$  and (ii)  $u'_D$  is hump-shaped in  $\tau$ . Therefore, the reform is Pareto improving if and only if  $\tau$  lies in an interval  $[\tau_c, \tilde{\tau}_E]$ .

QED

### 8.2.1 Characterizing dominated fiscal schemes

It may be that some of the Pareto improving schemes are themselves dominated by lower tax rates because they lie on the wrong side of the "Laffer curve" which relates the net utility of the deregulated group to the tax rate. In such a situation there exists a subinterval of the Pareto-improving interval of tax rates,  $[\tau_c, \hat{\tau}_E]$ , which defines the compensatory schemes that are not dominated by another compensatory scheme. This possibility is analyzed in the following Proposition:

*PROPOSITION A1 – (i) There exists a unique tax rate  $\hat{\tau}_E \in (0, 1)$  which maximizes the utility of the deregulated agents,  $u'_D$ , which is hump-shaped in  $\tau$ .*

*(ii) This tax rate is equal to*

$$\hat{\tau}_E = \frac{(\gamma - 1)(p_E^{\frac{\alpha-\gamma}{1-\alpha}} - \Delta r)}{\gamma p_E^{\frac{\alpha-\gamma}{1-\alpha}} - (\gamma - 1)\Delta r}. \quad (59)$$

*(iii)*

*Assume*

$$p \leq \gamma - r(\gamma - 1), \quad (60)$$

*then*

$$\hat{\tau}_E \geq \tilde{\tau}_E \text{ for all } r' \leq r.$$

(iv) Assume (60) is violated. Then there exists a unique  $r_c \in (0, r)$  such that

$$r' \geq r_c \Leftrightarrow \hat{\tau}_E \geq \tilde{\tau}_E.$$

Proof – Substituting (55) into (48) we get that

$$u'_D \propto \gamma\tau(1-\tau)^{\frac{1}{\gamma-1}} p'^{-\frac{\alpha}{1-\alpha}} + \Delta r(\gamma-1)(1-\tau)^{\frac{\gamma}{\gamma-1}},$$

where the proportionality factor is  $> 0$  and independent of  $\tau$ . Clearly then

$$\frac{d}{d\tau} u'_D \propto (p'^{-\frac{\alpha}{1-\alpha}} - \Delta r) - \frac{1}{\gamma-1} \frac{\tau}{1-\tau} p'^{-\frac{\alpha}{1-\alpha}}.$$

Since  $p'^{-\frac{\alpha}{1-\alpha}} - \Delta r = 1 - r + r' p_R^{-\frac{\alpha}{1-\alpha}} > 0$ , claims (i) and (ii) follow immediately from this expression.

From (59) the condition  $\hat{\tau} \geq 1 - p'/p$  is equivalent to

$$-pp'^{-\frac{1}{1-\alpha}} + \gamma p'^{-\frac{\alpha}{1-\alpha}} - \Delta r(\gamma-1) \geq 0,$$

which from the definition of the price level can be rearranged as

$$h(r') = -pp'^{-\frac{1}{1-\alpha}} + \gamma(r' p_R^{-\frac{\alpha}{1-\alpha}} + 1 - r') - \Delta r(\gamma-1) \geq 0.$$

Let  $X = r' p_R^{-\frac{\alpha}{1-\alpha}} + 1 - r' = p'^{-\frac{\alpha}{1-\alpha}}$ . We note that:

$$\frac{dp'}{dr'} = \frac{1-\alpha}{\alpha} X^{-\frac{1}{\alpha}} (1 - p_R^{-\frac{\alpha}{1-\alpha}}) > 0 \quad (61)$$

and

$$\frac{d^2 p'}{dr'^2} = \frac{1-\alpha}{\alpha^2} X^{-\frac{1}{\alpha}-1} (1 - p_R^{-\frac{\alpha}{1-\alpha}})^2 > 0.$$

Next, observe that  $h(r) = (\gamma-1)p^{-\frac{\alpha}{1-\alpha}} > 0$  and that

$$\begin{aligned} \frac{d^2 h}{dr'^2} &= \frac{\alpha-2}{(1-\alpha)^2} pp'^{\frac{2\alpha-3}{1-\alpha}} \left( \frac{dp'}{dr'} \right)^2 + \frac{1}{1-\alpha} pp'^{\frac{\alpha-2}{1-\alpha}} \frac{d^2 p'}{dr'^2} \\ &\propto \frac{\alpha-2}{1-\alpha} \left( \frac{dp'}{dr'} \right)^2 \frac{1}{p'} + \frac{d^2 p'}{dr'^2} \\ &= (1 - p_R^{-\frac{\alpha}{1-\alpha}})^2 X^{-\frac{1}{\alpha}-1} \frac{(1-\alpha)(\alpha-1)}{\alpha^2} < 0. \end{aligned}$$

Therefore  $h$  is concave. Consequently, since  $h(r) > 0$ , either  $h(0) \geq 0$  and  $h(r') \geq 0, \forall r' \in [0, r]$ , or  $h(0) < 0$ , in which case  $h(r) > 0$  for  $r < r_c$  and  $\geq 0$  for  $r \geq r_c$ , where  $r_c$  is the unique solution in  $(0, r)$  to  $h(r_c) = 0$ . To complete the proof, we just need to check that the condition  $h(0) \geq 0$  coincides with (60), which is straightforward. This proves claims (ii) and (iv).

QED

It is straightforward to observe that whenever  $\hat{\tau}_E < \tilde{\tau}_E$ , the interval of Pareto improving reforms that are themselves Pareto optimal is of the form  $[\tau_c, \hat{\tau}_E]$

### 8.3 Proof of Lemma 2

It is straightforward to see that (45) and (46) still hold. Therefore, the participation constraint of the nonregulated groups, (47), is the same, and this proves claim B, as well as the second formula of claim A, while the first one is obvious. Also, the nonregulated groups' demand for nonregulated goods is  $c(p'^{-\frac{1}{\gamma-1}}(1-\tau)^{\frac{\gamma}{\gamma-1}}, p', 1)$ . As for regulated groups, their income is  $T + p'_R \bar{l}'(1-\tau)$ , and their demand for nonregulated goods is  $c_N(\bar{l}' p'_R(1-\tau) + T, p', 1)$ . These observations allow us to rewrite the equilibrium condition for nonregulated goods:

$$l'_N = (1-r)c(p'^{-\frac{1}{\gamma-1}}(1-\tau)^{\frac{\gamma}{\gamma-1}}, p', 1) + rc(\bar{l}' p'_R(1-\tau) + T, p', 1).$$

The government's budget constraint is

$$rT = \tau \left[ (1-r)p'^{-\frac{1}{\gamma-1}}(1-\tau)^{\frac{\gamma}{\gamma-1}} + r\bar{l}' p'_R(1-\tau) \right]. \quad (62)$$

Eliminating  $T$  between these two equations allows us again to compute the new value of employment in regulated sectors:

$$\bar{l}' = p_R'^{-\frac{1}{1-\alpha}} p'^{-\frac{1}{\gamma-1}} (1-\tau)^{\frac{1}{\gamma-1}}, \quad (63)$$

which proves the third expression in claim B.

Substituting into (62) yields

$$T = \frac{\tau}{r} (1-\tau)^{\frac{1}{\gamma-1}} p'^{-\frac{1}{\gamma-1} - \frac{\alpha}{1-\alpha}}. \quad (64)$$

This in turn allows us to compute the regulated group's utility level:

$$u'_R = \frac{\tau}{r}(1-\tau)^{\frac{1}{\gamma-1}} p'^{-\frac{\gamma}{\gamma-1}-\frac{\alpha}{1-\alpha}} + p'^{-\frac{\gamma}{\gamma-1}}(1-\tau)^{\frac{\gamma}{\gamma-1}} \frac{\gamma-1}{\gamma} J', \quad (65)$$

where

$$J' = \frac{\gamma p_R'^{-\frac{\alpha}{1-\alpha}} - p_R'^{-\frac{\gamma}{1-\alpha}}}{\gamma-1}. \quad (66)$$

Comparing (65) with (7), using the expressions for  $J$  and  $J'$ , delivers the expression in claim C. Finally, claim D derives from preceding statements.

## 8.4 Proof of Proposition 2

We proceed as in the proof of Proposition 1.

First, we show that (16) holds with strict inequality if one picks  $\tau = 1 - p'_I/p$ . Substituting into (16) and simplifying, we have to prove the following inequality:

$$p p_I'^{-\frac{1}{1-\alpha}} - p_I'^{-\frac{\alpha}{1-\alpha}} > r \frac{\gamma-1}{\gamma} (J - J'). \quad (67)$$

We note that both sides coincide at  $p'_R = p_R$ . We next show that

$$\varphi(p'_R, p_R) \equiv \frac{\partial}{\partial p'_R} \left[ p p_I'^{-\frac{1}{1-\alpha}} - p_I'^{-\frac{\alpha}{1-\alpha}} - r \frac{\gamma-1}{\gamma} (J - J') \right] < 0 \text{ for } 1 \leq p'_R \leq p_R. \quad (68)$$

To see this, note that

$$\frac{\partial p}{\partial p_R} = r p_R^{-\frac{1}{1-\alpha}} p^{\frac{1}{1-\alpha}}$$

and that

$$\frac{\partial J}{\partial p_R} = \frac{\gamma}{(\gamma-1)(1-\alpha)} \left[ p_R^{-\frac{\gamma}{1-\alpha}-1} - p_R^{-\frac{1}{1-\alpha}} \right].$$

It then follows from straightforward calculations that

$$\varphi \propto \alpha - 1 - p/p'_I + p_R'^{\frac{\alpha-\gamma}{1-\alpha}}.$$

Since  $p'_I \leq p$  and  $p'_R \geq 1$ , this expression is clearly smaller than  $\alpha - 1 - 1 + 1 = \alpha - 1 < 0$ . Since  $\varphi < 0$  and (67) holds with equality in the limit case where  $p'_R = p_R$ , clearly (67) holds for any  $p'_R \in [1, p_R)$ .

The rest of the proof is the same as for Proposition 2.

QED.

### 8.4.1 Characterizing dominated fiscal schemes in the intensive reform case

A result similar to Proposition A1 can be established for intensive reforms:

*Proposition A2* –

(i) *There exists a unique tax rate  $\hat{\tau}_I \in (0, 1)$  which maximizes the utility of the regulated agents,  $u'_R$ , which is hump-shaped in  $\tau$ .*

(ii) *This tax rate is equal to*

$$\hat{\tau}_I = \frac{(\gamma - 1)(p_I'^{\frac{1}{1-\alpha}} - rJ')}{\gamma p_I'^{\frac{1}{1-\alpha}} - (\gamma - 1)rJ'}. \quad (69)$$

(iii) *Assume*

$$p \leq \gamma - r\alpha^{\frac{\alpha}{\gamma-\alpha}}(\gamma - \alpha), \quad (70)$$

*Then*

$$\hat{\tau}_I \geq \tilde{\tau}_I \text{ for all } p'_R \leq p_R.$$

*Proof* – Claims (i) and (ii) are straightforward from (65). Using (69) and (15) and rearranging we get that

$$\hat{\tau}_I \geq \tilde{\tau}_I \Leftrightarrow p \leq \gamma p'_I - rJ'(\gamma - 1)$$

Note that  $p'_I \geq 1$  and that  $J'$  reaches its maximum for  $p'_R = \alpha^{-\frac{1-\alpha}{\gamma-\alpha}}$ , and this maximum is equal to  $(\gamma\alpha^{\frac{\alpha}{\gamma-\alpha}} - \alpha^{\frac{\gamma}{\gamma-\alpha}})/(\gamma - 1)$ . Consequently, the expression  $\gamma p'_I - rJ'(\gamma - 1)$  is always larger than  $\gamma - r(\gamma\alpha^{\frac{\alpha}{\gamma-\alpha}} - \alpha^{\frac{\gamma}{\gamma-\alpha}})$  which is the RHS of (70). This proves (iii).

## 8.5 Proof of Lemma 3

To prove existence, we show that this equilibrium with rationing is the same as the Walrasian equilibrium of another economy. To construct this alternative economy, replace the utility of an agent in sector  $i \in \mathcal{R}$  by  $u(c) - p_i l$ .



This is clearly possible, since  $p_i$  is exogenous for regulated goods. Denote by superscript  $A$  the equilibrium values for this alternative economy. Use the same price normalization. Then equilibrium in goods markets is given by  $\mathbf{a} \bullet \mathbf{l}^A = Y^A \psi(\mathbf{p}^A)$ ; equilibrium in labor markets is determined by  $\mathbf{a}_N \bullet \mathbf{l}_N^A = \theta_N \bullet \lambda(\mathbf{p}_N^A)$  for non regulated goods and  $\mathbf{p}_R^A = \mathbf{p}_R$  for regulated goods; finally the same price normalization  $u(\psi(\mathbf{p}^A)) = 1$  holds. By construction, then, the Walrasian equilibrium of this alternative economy mimics the rationing equilibrium of our regulated economy. Existence follows from standard results on Walrasian equilibria (Debreu, 1959). QED.

Remark – Existence of equilibrium with proportional taxation can be proved by a similar method. In addition to replacing preferences in regulated groups by their alternative counterparts, one also has to replace preferences in nonregulated groups by  $u(c) - \frac{v(l)}{1-\tau_i}$ . It is then straightforward to show that the conditions for the Walrasian equilibrium in the alternative economy match (24)-(26), along with  $\mathbf{p}_R^A = \mathbf{p}_R$ .

## 8.6 Proof of Proposition 3

First, note that claim (iii) follows straightforwardly from the definition of the income of group  $i$ .

Next, let us prove claims (i) and (ii). Let  $\mathbf{S}$  be the matrix whose generic element is

$$s_{ij} = \frac{1}{u} \frac{\partial^2 u}{\partial c_i \partial c_j} - \frac{1}{u^2} \frac{\partial u}{\partial c_i} \frac{\partial u}{\partial c_j}.$$

Then  $\mathbf{S}$  is the Hessian of  $\ln u$ , which is strictly concave. Consequently  $\mathbf{S}$  is negative definite and therefore invertible. Furthermore, the FOC for the consumer problem is  $(\nabla u)^T = \mu \mathbf{p}$  (where  $\mu$  here is a scalar denoting the consumer's problem's Lagrange multiplier). By Euler's theorem  $\langle \mathbf{c}, (\nabla u)^T \rangle = u$ . The consumer's income is  $y = \langle \mathbf{p}, \mathbf{c} \rangle > 0$ . Therefore,  $\mu = u/y$ . Hence  $\frac{1}{u} (\nabla u)^T = \frac{\mathbf{p}}{y}$ . Since  $\mathbf{S} = \nabla \frac{1}{u} (\nabla u)^T$ , we have that the derivative of  $\mathbf{c}$  with respect to  $\mathbf{p}$  is  $y \nabla \psi = \mathbf{S}^{-1}/y$ . Therefore

$$\nabla\psi = \frac{1}{y^2}\mathbf{S}^{-1},$$

implying that  $\nabla\psi$  is invertible. By definition, our price normalization is such that  $u = y$ . Hence  $\mu = 1$  and  $(\nabla u)^T = \mathbf{p}$ . Also  $\langle \mathbf{p}, \mathbf{c} \rangle = y = u$ , implying  $du = \langle \mathbf{p}, \mathbf{dc} \rangle + \langle \mathbf{dp}, \mathbf{c} \rangle$ . But, by definition of the gradient,  $\langle \mathbf{dc}, (\nabla u)^T \rangle = du$ . Therefore

$$\langle \mathbf{dp}, \mathbf{c} \rangle = 0. \quad (71)$$

Equivalently

$$\langle \mathbf{s}, \mathbf{dp} \div \mathbf{p} \rangle = 0. \quad (72)$$

From (29), we have that

$$dW = dY - \langle v'(\boldsymbol{\lambda}), \mathbf{dl} \rangle \quad (73)$$

We have that

$$dY = \langle \mathbf{1}, \mathbf{dR} \rangle$$

and from (32)

$$\mathbf{dR} = \mathbf{a} \bullet \mathbf{p} \bullet \mathbf{dl} + \mathbf{a} \bullet \mathbf{l} \bullet \mathbf{dp} - Y \mathbf{s} \bullet \mathbf{d\tau} + \boldsymbol{\theta} \bullet \mathbf{dT},$$

implying, by virtue of the government's budget constraint (22), that

$$\begin{aligned} dY &= \langle \mathbf{1}, \mathbf{dR} \rangle \\ &= \langle \mathbf{1}, \mathbf{a} \bullet \mathbf{p} \bullet \mathbf{dl} + \mathbf{a} \bullet \mathbf{l} \bullet \mathbf{dp} \rangle \end{aligned} \quad (74)$$

$$\begin{aligned} &= \langle \mathbf{p}, \mathbf{a} \bullet \mathbf{dl} \rangle + \langle \mathbf{a} \bullet \mathbf{l}, \mathbf{dp} \rangle \\ &= \langle \mathbf{p}, \mathbf{a} \bullet \mathbf{dl} \rangle \end{aligned} \quad (75)$$

$$\begin{aligned} &= \langle \mathbf{a} \bullet \mathbf{p} \bullet \mathbf{l}, \mathbf{dl} \div \mathbf{l} \rangle \\ &= Y \langle \mathbf{s}, \mathbf{dl} \div \mathbf{l} \rangle \end{aligned} \quad (76)$$

Where, since  $\mathbf{a} \bullet \mathbf{l} = \mathbf{c}$  at the aggregate level, (71) has been used. This proves (ii). Similarly, from (73) and (75),

$$\begin{aligned} dW &= \langle \mathbf{a} \bullet \mathbf{p} - v'(\boldsymbol{\lambda}), \mathbf{dl} \rangle \\ &= \langle \mathbf{a} \bullet \mathbf{p} \bullet \boldsymbol{\omega}, \mathbf{dl} \rangle \\ &= Y \langle \mathbf{s} \bullet \boldsymbol{\omega}, \mathbf{dl} \div \mathbf{l} \rangle. \end{aligned}$$

This proves point (i).

Now consider a PSR such that (30) holds. Observe that an SRST with the same price changes for regulated goods results in the same equilibrium for prices and labor supply, and therefore the same  $dY$ . For any  $i$ , the corresponding change in welfare is

$$dU_i = dy_i - v'(\lambda_i)d\lambda_i.$$

For all groups,

$$dy_i = dT_i + a_i p_i d\lambda_i + \lambda_i a_i dp_i$$

For non regulated groups,

$$v'(\lambda_i) = a_i p_i.$$

Therefore, for non regulated groups:

$$\begin{aligned} dU_i &= dT_i + \lambda_i a_i dp_i \\ &= dT_i + \frac{s_i Y}{\theta_i} \frac{dp_i}{p_i}. \end{aligned}$$

For regulated groups,

$$\begin{aligned} dU_i &= dT_i + \lambda_i a_i dp_i + (a_i p_i - v'(\lambda_i))d\lambda_i \\ &= dT_i + \frac{s_i Y}{\theta_i} \frac{dp_i}{p_i} + a_i p_i \omega_i d\lambda_i \\ &= dT_i + \frac{s_i Y}{\theta_i} \frac{dp_i}{p_i} + \frac{a_i p_i \omega_i l_i}{\theta_i} \frac{dl_i}{l_i} \\ &= \frac{s_i Y}{\theta_i} \left( \frac{dp_i}{p_i} + \omega_i \frac{dl_i}{l_i} \right) + dT_i. \end{aligned} \tag{77}$$

Since  $\omega = 0$  for non regulated groups, this formula actually holds for both groups.

Let  $x = \langle \mathbf{s} \bullet \boldsymbol{\omega}, \mathbf{dl} \div \mathbf{l} \rangle Y > 0$ . Consider the transfer scheme

$$\mathbf{dT} = -Y (\mathbf{dp} \div \mathbf{p} + \boldsymbol{\omega} \bullet \mathbf{dl} \div \mathbf{l}) \bullet \mathbf{s} \div \boldsymbol{\theta} + x \mathbf{1}.$$

Clearly,  $dU_i = x > 0$ . Furthermore

$$\begin{aligned} \langle \boldsymbol{\theta}, \mathbf{dT} \rangle &= -Y \langle \mathbf{s}, \mathbf{dp} \div \mathbf{p} \rangle - Y \langle \mathbf{s} \bullet \boldsymbol{\omega}, \mathbf{dl} \div \mathbf{l} \rangle + x \\ &= 0, \end{aligned}$$

by (72). The transfer scheme satisfies the budget constraint (23). Thus the proposed SRST implements the same productive allocation as the PSR in a Pareto-improving way. This proves point (iv). QED.

## 8.7 Proof of Proposition 4

Assume an N-neutral reform  $(\mathbf{dp}_R, \mathbf{d}\tau, \mathbf{dT})$  such that (30) holds exists. Let us construct a Pareto-improving FSR  $(\mathbf{d}\tilde{\mathbf{p}}_R, \mathbf{d}\tilde{\tau}, \mathbf{d}\tilde{\mathbf{T}})$ .

First, we assume that  $\mathbf{d}\tilde{\tau}_N = \mathbf{d}\tau_N$  and  $\mathbf{d}\tilde{\mathbf{p}}_R = \mathbf{dp}_R$ , implying that the equilibrium values of  $\mathbf{dp}$  and  $\mathbf{dl}$  are also unchanged.

Second, we assume  $\mathbf{d}\tilde{\mathbf{T}}_N = 0$ . Hence the alternative reform is also N-neutral, and the feasibility constraint  $d\tilde{T}_i \geq 0$  holds for  $i \in N$ . From Lemmas 4 and 5, we know that we can restrict ourselves to the case  $d\tilde{\tau}_R = 0$  and that  $d\tilde{T}_i$  can be chosen independently of the feasibility constraint since it is always possible to then pick instead, for example  $d\tilde{\tau}_i = -d\tilde{T}_i/(s_i Y)$  and  $d\tilde{T}_i = 0$ . Since the reform is then an SRST from the point of view of the regulated group, the derivations that lead to (77) apply. Therefore, for  $i \in \mathcal{R}$ , the change in welfare is, from (77)

$$dU_i = \frac{s_i Y}{\theta_i} \left( \frac{dp_i}{p_i} + \omega_i \frac{dl_i}{l_i} \right) + d\tilde{T}_i.$$

Let  $x = \langle \mathbf{s} \bullet \boldsymbol{\omega}, \mathbf{dl} \div \mathbf{l} \rangle Y / \langle \boldsymbol{\theta}_R, \mathbf{1}_R \rangle > 0$ . Let us pick

$$\mathbf{d}\tilde{\mathbf{T}}_R = -Y (\mathbf{dp}_R \div \mathbf{p}_R + \boldsymbol{\omega}_R \bullet \mathbf{dl}_R \div \mathbf{l}_R) \bullet s_R \div \boldsymbol{\theta}_R + x \mathbf{1}_R.$$

Clearly,  $dU_i = x > 0$  for all  $i \in \mathcal{R}$ . Furthermore,

$$\begin{aligned} \langle \mathbf{s}, \mathbf{d}\tau \rangle Y - \langle \boldsymbol{\theta}, \mathbf{dT} \rangle &= \langle \mathbf{s}_N, \mathbf{d}\tau_N \rangle Y - \langle \boldsymbol{\theta}_R, \mathbf{d}\tilde{\mathbf{T}}_R \rangle \\ &= \langle \mathbf{s}_N, \mathbf{dp}_N \div \mathbf{p}_N \rangle Y + \langle \mathbf{s}_R, \mathbf{dp}_R \div \mathbf{p}_R \rangle Y \\ &\quad + \langle \mathbf{s}_R \bullet \boldsymbol{\omega}_R, \mathbf{dl}_R \div \mathbf{l}_R \rangle Y - \langle \boldsymbol{\theta}_R, \mathbf{1}_R \rangle x \\ &= \langle \mathbf{s}, \mathbf{dp} \div \mathbf{p} \rangle Y + \langle \mathbf{s} \bullet \boldsymbol{\omega}, \mathbf{dl} \div \mathbf{l} \rangle Y - \langle \boldsymbol{\theta}_R, \mathbf{1}_R \rangle x \\ &= 0, \end{aligned}$$

by (72), (33), and the fact that  $\omega_i = 0$  for  $i \in \mathcal{N}$ . Hence, the proposed transfer scheme satisfies the government's budget constraint.

Our constructed FSR clearly matches all requirements.

QED.

## 8.8 Proof of Proposition 5

Let us first write the equilibrium conditions in differential form. From (24), we have that

$$\mathbf{dl} \div \mathbf{l} = \mathbf{1} \frac{dY}{Y} + \mathbf{M}(\mathbf{dp} \div \mathbf{p}), \quad (78)$$

with  $\mathbf{M} = (m_{ij})$  such that

$$m_{ij} = \frac{p_j}{\psi_i} [\nabla \psi]_{ij}.$$

Clearly,  $\mathbf{M}$  is invertible since  $\nabla \psi$  is.

Let

$$\eta_i = w_i \frac{\lambda'(w_i)}{\lambda(w_i)}$$

be the elasticity of labor supply for group  $i$ . The labor supply conditions for the nonregulated can be written as

$$\mathbf{dl}_N \div \mathbf{l}_N = \boldsymbol{\eta} \bullet (\mathbf{dp}_N \div \mathbf{p}_N - \mathbf{d}\boldsymbol{\tau}_N).$$

Finally, the price normalization condition boils down to (72).

For an N-neutral reform,  $\mathbf{d}\boldsymbol{\tau}_N$  becomes endogenous, and the set of equations that characterize the equilibrium is

$$\left[ \mathbf{1} \frac{dY}{Y} + \mathbf{M}(\mathbf{dp} \div \mathbf{p}) \right]_N = \mathbf{0}_N, \quad (79)$$

$$\langle \mathbf{s}, \mathbf{dp} \div \mathbf{p} \rangle = 0, \quad (80)$$

that is,  $n+1$  equations in  $n+1$  unknowns:  $\mathbf{dp}_N$  and  $dY$ , as  $\mathbf{dp}_R$  is exogenous. The allocation of labor can then be computed residually from (78) and it will obviously satisfy  $\mathbf{dl}_N = 0$ . The tax rates imposed upon the nonregulated are

computed residually as  $\mathbf{d}\tau_N = \mathbf{d}\mathbf{p}_N \div \mathbf{p}_N$ . Generically, then, one can always construct an N-neutral fiscal scheme for any change in  $\mathbf{p}_R$ ,  $\mathbf{d}\mathbf{p}_R$ . This does not tell us, however, whether it matches (30).

To show that such reforms exist, consider the reverse problem. Pick any vector of employment growth in regulated sectors,  $\mathbf{d}\mathbf{l}_R \div \mathbf{l}_R$ . Obviously, it is always possible to choose it so that (30) holds. This will in particular be the case if  $\mathbf{d}\mathbf{l}_R \geq 0$ , with at least one strict inequality.

Next, choose  $dY/Y = \langle \mathbf{s}_R, \mathbf{d}\mathbf{l}_R \div \mathbf{l}_R \rangle$ . We are going to check that this choice is indeed consistent with equilibrium. For this, let  $v_i = -dY/Y$  if  $i \in \mathcal{N}$  and  $v_i = dl_i/l_i - dY/Y$  for  $i \in \mathcal{R}$ . Clearly, from (78), one must have

$$\mathbf{d}\mathbf{p} \div \mathbf{p} = \mathbf{M}^{-1}\mathbf{v}.$$

Since  $\mathbf{M}$  is invertible, we can always choose this vector of price changes. Its restriction to  $\mathcal{R}$ ,  $[\mathbf{d}\mathbf{p} \div \mathbf{p}]_R$ , delivers the set of changes in regulated prices necessary to deliver the desired level of employment growth under an N-neutral reform.

By construction, the candidate equilibrium matches (79). To complete the proof, we just have to check that it satisfies (80).

Differentiating (18) we find that for any price vector

$$\nabla\psi(\mathbf{p})\mathbf{p} + \psi(\mathbf{p}) \equiv \mathbf{0}. \quad (81)$$

We have that

$$\begin{aligned} \langle \mathbf{s}, \mathbf{M}(\mathbf{d}\mathbf{p} \div \mathbf{p}) \rangle &= \langle \mathbf{p}, \nabla\psi(\mathbf{p})\mathbf{d}\mathbf{p} \rangle \quad (\text{by definition of } \mathbf{M}) \\ &= \langle \nabla\psi(\mathbf{p})\mathbf{p}, \mathbf{d}\mathbf{p} \rangle \quad (\text{since } \nabla\psi(\mathbf{p}) \text{ is symmetrical}) \\ &= -\langle \psi(\mathbf{p}), \mathbf{d}\mathbf{p} \rangle \quad (\text{by (81)}) \\ &= -\langle \mathbf{s}, \mathbf{d}\mathbf{p} \div \mathbf{p} \rangle. \end{aligned} \quad (82)$$

Finally by construction  $\langle \mathbf{s}, \mathbf{M}(\mathbf{d}\mathbf{p} \div \mathbf{p}) \rangle = \langle \mathbf{s}, \mathbf{v} \rangle = -dY/Y + \langle \mathbf{s}_R, \mathbf{d}\mathbf{l}_R \div \mathbf{l}_R \rangle = 0$ . Therefore, from (82), (80) holds.

## 8.9 Proof of Proposition 6

We prove a series of Lemmas, which provide bounds for different properties of a reform to hold, and together imply Proposition 6.

- Lemma A1 shows that under a condition weaker than (34), there exists a set of proportional tax rates on nonregulated groups that, if applied, make a given structural reform N-neutral. This proves the first part of Proposition 6.
- Lemma A2 shows that under a condition weaker than (34), such a reform is such that output goes up. This result is not needed, per se, for proving Proposition 6 but we build on the bounds for  $dY/Y$  established in the proof to prove Lemma A3.
- Lemma A3 shows that under (34), the N-neutral reform is such that employment goes up in all regulated sectors. This proves the second part of Proposition 6.
- Finally, Lemma A4 provides weaker conditions than (34) for the N-neutral reform to increase aggregate social welfare (while it may reduce employment in a subset of the regulated sectors).

First recall that  $\sigma_i = -m_{ii} > 0$ , that  $\sigma = \min_i \sigma_i >$ , and that  $\eta = \max_{i,j,i \neq j} |m_{ij}| \geq 0$ . Then

*LEMMA A1 – If*

$$\eta/\sigma < \frac{1}{2(n-1)} \quad (83)$$

*then for any  $r$ -vector  $\mathbf{dx}$  there exists an  $N$ -reform such that  $\mathbf{dp}_R \div \mathbf{p}_R = -\mathbf{dx}$ .*

*Proof – Let*

$$\mathbf{M} = \begin{pmatrix} \mathbf{M}^{NN} & \mathbf{M}^{NR} \\ \mathbf{M}^{RN} & \mathbf{M}^{RR} \end{pmatrix}$$

*Let*

$$\mathbf{z} = \begin{pmatrix} \mathbf{dp}_N \div \mathbf{p}_N \\ dY/Y \end{pmatrix}$$

and let

$$\mathbf{Q} = \begin{pmatrix} \mathbf{M}^{NN} & \mathbf{1} \\ \mathbf{s}_N^T & 0 \end{pmatrix}$$

The system (79-80) can be rewritten as

$$\mathbf{Q}\mathbf{z} = \begin{pmatrix} \mathbf{M}^{NR}\mathbf{dx} \\ \langle \mathbf{s}_R, \mathbf{dx} \rangle \end{pmatrix}.$$

To show existence of an N-reform, we show that  $\mathbf{Q}$  is invertible. First, since  $\mathbf{M}$  differs from a negative definite matrix by a change of variables,  $\det \mathbf{M}^{NN} \neq 0$ .

Second, let  $\mathbf{v} = \mathbf{M}^{NN-1}\mathbf{1}_N$ . Let  $\xi = \max_{i \in N} |v_i|$ . Since  $\sum_{j=1}^n m_{ij}v_j = 1$ , clearly

$$1 \geq (\sigma - (n-1)\eta)\xi. \quad (84)$$

To see this, first note that from (83),  $\sigma - (n-1)\eta > 0$ . Let  $k$  such that  $\xi = |v_k|$ . Assume  $v_k < 0$ . Then  $1 = \xi|m_{kk}| + \sum_{j \neq k} m_{kj}v_j \geq (\sigma - (n-1)\eta)\xi$ . Now if  $v_k > 0$ , one has  $1 = \xi m_{kk} + \sum_{j \neq k} m_{kj}v_j \leq -\xi\sigma + (n-1)\eta\xi$  which is negative. This case is clearly ruled out.

Next, from (84) we then have that  $\xi \leq \frac{1}{\sigma - (n-1)\eta}$ . For all  $i$ ,  $1 \leq m_{ii}v_i + (n-1)\eta\xi \leq m_{ii}v_i + \frac{(n-1)\eta}{\sigma - (n-1)\eta}$ . By (83) again, we have that  $\frac{(n-1)\eta}{\sigma - (n-1)\eta} < 1$ . Therefore, since  $m_{ii} < 0$ ,  $v_i < 0$ . Since that is true for all  $i$ , we trivially have that  $\langle \mathbf{s}_N, \mathbf{v} \rangle < 0$ , implying in particular that this quantity is different from zero. One can then check that the inverse of  $\mathbf{Q}$  is

$$\mathbf{Q}^{-1} = \begin{pmatrix} \mathbf{M}^{NN-1} - \mathbf{v}\mathbf{s}_N^T\mathbf{M}^{NN-1}/\langle \mathbf{s}_N, \mathbf{v} \rangle & \mathbf{v}/\langle \mathbf{s}_N, \mathbf{v} \rangle \\ \mathbf{s}_N^T\mathbf{M}^{NN-1}/\langle \mathbf{s}_N, \mathbf{v} \rangle & -1/\langle \mathbf{s}_N, \mathbf{v} \rangle \end{pmatrix}.$$

QED

Since (34) implies (83), point (i) of Proposition 6 follows.

Next, recall that  $x_m = \max_i dx_i$ , and let

$$\mu = \langle \mathbf{s}_R, \boldsymbol{\sigma}_R \bullet \mathbf{dx} \rangle / s_R > 0.$$

*LEMMA A2 – If*

$$\eta < \frac{1}{q-1} \min\left(\frac{\mu}{x_m}, \sigma \frac{1-s_R}{1+s_R}\right), \quad (85)$$



then any  $N$ -neutral reform such that  $\mathbf{dp}_R \div \mathbf{p}_R = -\mathbf{dx}$  satisfies  $dY > 0$ .

Proof – Let

$$K = \max_i \left| \frac{dp_i}{p_i} \right| > 0.$$

From (78) and (31), and the fact that  $\mathbf{dl}_N = \mathbf{0}$  by construction,

$$dY/Y = \frac{1}{1 - s_R} \langle \mathbf{s}_R, (\mathbf{M}(\mathbf{dp} \div \mathbf{p}))_R \rangle \quad (86)$$

$$\geq \frac{s_R}{1 - s_R} (\mu - K\eta(q - 1)). \quad (87)$$

For  $i \in \mathcal{N}$ , we have that (from (78) again),

$$\begin{aligned} -m_{ii} \frac{dp_i}{p_i} &= \frac{dY}{Y} + \sum_{j \neq i} m_{ij} \frac{dp_j}{p_j} \\ &\geq \frac{s_R}{1 - s_R} (\mu - K\eta(q - 1)) - K\eta(q - 1) \\ &= \frac{1}{1 - s_R} (s_R \mu - K\eta(q - 1)). \end{aligned} \quad (88)$$

Assume  $K$  is reached for  $i \in \mathcal{N}$  and  $dp_i/p_i < 0$ . Then for this  $i$ ,  $K = -dp_i/p_i$  and the preceding inequality reads  $(-m_{ii})K \leq \frac{1}{1 - s_R} (-s_R \mu + K\eta(q - 1))$ , implying  $s_R \mu \leq K(\eta(q - 1) - \sigma(1 - s_R))$ . From (85), the RHS is negative. Since  $\mu > 0$ , we then have a contradiction, so this case can be ruled out.

Assume next that  $K$  is reached for  $i \in \mathcal{R}$ , implying  $dp_i/p_i < 0$  for this  $i$  (since  $\mathbf{dx} \geq \mathbf{0}$  by assumption). Then by definition  $K = x_m$  and from (87),

$$dY/Y \geq \frac{s_R}{1 - s_R} (\mu - x_m \eta(q - 1)), \quad (89)$$

which is  $> 0$  from (85).

Now assume that  $K$  is reached for  $i \in \mathcal{N}$ , so that for this particular  $i$   $dp_i/p_i > 0$ . From (86), we have that

$$dY/Y \leq \frac{s_R}{1 - s_R} (\mu + K\eta(q - 1)) \quad (90)$$

Together with (88), this implies that, for any  $i \in \mathcal{N}$ ,

$$\sigma_i \frac{dp_i}{p_i} \leq \frac{s_R \mu + K\eta(q - 1)}{1 - s_R}.$$

Indeed, either  $dp_i/p_i \geq 0$ , in which case the LHS of (88) coincides with  $\sigma_i \frac{dp_i}{p_i}$ , or  $dp_i/p_i < 0$ , in which case the preceding inequality trivially holds.

Therefore, since  $\sigma = \min \sigma_i$ , for  $i \in \mathcal{N}$ ,  $\frac{dp_i}{p_i} \leq \frac{s_R \mu + K \eta (q-1)}{\sigma_i (1-s_R)} \leq \frac{s_R \mu + K \eta (q-1)}{\sigma (1-s_R)}$ . Since  $dp_i/p_i = K$  for some  $i \in \mathcal{N}$ , it follows that

$$K \leq \frac{s_R \mu}{\sigma (1-s_R) - \eta (q-1)}.$$

Substituting into (87) and rearranging, we get

$$\frac{dY}{Y} \geq \frac{s_R \mu}{1-s_R} \frac{\sigma (1-s_R) - (1+s_R) \eta (q-1)}{\sigma (1-s_R) - \eta (q-1)}. \quad (91)$$

This is again  $> 0$  from (85).

QED.

Note that since  $\mu = \sigma_R \bar{x}_Y$ , condition (85) can be rewritten as

$$\eta < \frac{1}{q-1} \min\left(\frac{\bar{x}_Y}{x_m} \sigma_R, \sigma \frac{1-s_R}{1+s_R}\right). \quad (92)$$

*LEMMA A3 – If*

$$\eta < \frac{1}{q-1} \min\left(\frac{s_R \mu}{x_m}, \sigma \frac{1-s_R}{2}\right), \quad (93)$$

*then any  $N$ -neutral reform such that  $\mathbf{dp}_R \div \mathbf{p}_R = -\mathbf{dx}$  is such that  $\mathbf{dl}_R > 0$ .*

*Proof – Start again with the case where  $K$  is reached for  $i \in \mathcal{R}$ . Using (78) and (89) we have that for  $i \in \mathcal{R}$*

$$\begin{aligned} \frac{dl_i}{l_i} &\geq -\sigma_i \frac{dp_i}{p_i} + \frac{dY}{Y} - \eta (q-1) K \\ &\geq \sigma_i dx_i + \frac{s_R \mu - x_m \eta (q-1)}{1-s_R}. \end{aligned} \quad (94)$$

Since  $\sigma_i > 0$  and  $dx_i \geq 0$ , it follows from (93) that the RHS is  $> 0$ .

Assume now that  $K$  is reached for  $i \in \mathcal{N}$ . Using (78) together with (91) we get that for  $i \in \mathcal{R}$ ,

$$\begin{aligned}
\frac{dl_i}{l_i} &\geq -\sigma_i \frac{dp_i}{p_i} - \eta(q-1)K + \frac{dY}{Y} \\
&\geq \sigma_i dx_i + \frac{s_R \mu}{\sigma(1-s_R) - \eta(q-1)} \left( \sigma - \frac{2\eta(q-1)}{1-s_R} \right). \quad (95)
\end{aligned}$$

Again this is  $> 0$  from (93).

QED.

Note that since  $\mu = \sigma_R \bar{x}_Y$ , condition (93) is equivalent to (34), which proves point (ii) in Proposition 6.

Finally the following Lemma derives a weaker condition than (93) under which  $dW > 0$ , in which case we know that the N-reform can be implemented in Pareto-improving way.

Let

$$v = \langle \mathbf{s}_R \bullet \boldsymbol{\omega}_R, \boldsymbol{\sigma}_R \bullet \mathbf{d}\mathbf{x} \rangle / \langle \mathbf{s}_R, \boldsymbol{\omega} \rangle \geq 0.$$

LEMMA A4 – If

$$\eta < \frac{1}{q-1} \min\left(\frac{s_R \mu + (1-s_R)v}{x_m}, \sigma(1-s_R) \frac{s_R \mu + (1-s_R)v}{2s_R \mu + (1-s_R)v}\right), \quad (96)$$

then any N-neutral reform such that  $\mathbf{d}\mathbf{p}_R \div \mathbf{p}_R = -\mathbf{d}\mathbf{x}$  is such that (30) holds, i.e.  $dW > 0$ .

Proof – Assume  $K$  is reached for  $i \in \mathcal{R}$ . Then (94) holds. Furthermore, we know that  $dW > 0$  iff  $\langle \mathbf{s} \bullet \boldsymbol{\omega}, \mathbf{d}\mathbf{l} \div \mathbf{l} \rangle > 0$ . Multiplying both sides of (94) by  $s_i \omega_i$  and summing over  $i \in \mathcal{R}$ , we get that

$$\langle \mathbf{s} \bullet \boldsymbol{\omega}, \mathbf{d}\mathbf{l} \div \mathbf{l} \rangle \geq \langle \mathbf{s}_R, \boldsymbol{\omega} \rangle v + \langle \mathbf{s}_R, \boldsymbol{\omega} \rangle \frac{s_R \mu - x_m \eta (q-1)}{1-s_R}.$$

From (96) the RHS is clearly  $> 0$ .

Assume now  $K$  is reached for  $i \in \mathcal{N}$ . Then (95) holds and performing the same steps on (95) as on (94) yields

$$\langle \mathbf{s} \bullet \boldsymbol{\omega}, \mathbf{d}\mathbf{l} \div \mathbf{l} \rangle \geq \langle \mathbf{s}_R, \boldsymbol{\omega} \rangle v + \langle \mathbf{s}_R, \boldsymbol{\omega} \rangle \frac{s_R \mu}{\sigma(1-s_R) - \eta(q-1)} \left( \sigma - \frac{2\eta(q-1)}{1-s_R} \right).$$

Since  $v + \frac{s_R \mu}{\sigma(1-s_R) - \eta(q-1)} \left( \sigma - \frac{2\eta(q-1)}{1-s_R} \right) \propto v(\sigma(1-s_R) - \eta(q-1)) + s_R \mu \sigma - \frac{2\eta(q-1)}{1-s_R} s_R \mu \propto \sigma(1-s_R) [v(1-s_R) + \mu s_R] - \eta(q-1) [v(1-s_R) + 2\mu s_R]$ , the RHS is clearly  $> 0$  by (96).

## 8.10 Proof of Proposition 7

The only novelty with respect to the analysis of Propositions 1 and A1 is that the government budget constraint is now given by

$$T = \frac{\tau(1-\tau)^{\frac{1}{\gamma-1}} p'^{-\frac{1}{\gamma-1} - \frac{\alpha}{1-\alpha}}}{\Delta r} - cp'. \quad (97)$$

The reform is viable if and only if the transfer scheme which is most favorable to the deregulated while leaving the nonregulated and nonderegulated weakly better-off raises the utility of the deregulated. From Proposition A1, under the assumption that  $p \leq \gamma - r(\gamma - 1)$ , this is the case for  $\tau = 1 - p'/p$ .

Then from (48) we can compute  $u'_D$  as

$$u'_D = -c + \frac{1}{p' \Delta r} \left( 1 - \frac{p'}{p} \right) p'^{-\frac{\alpha}{1-\alpha}} + \frac{\gamma-1}{\gamma} p^{\frac{\gamma}{\gamma-1}}.$$

The reform is viable iff  $u'_D \geq u_R$ . Using the same steps as in the proof of Proposition 1, this is equivalent to

$$h(r') = pp'^{-\frac{1}{1-\alpha}} - (r' p_R^{-\frac{\alpha}{1-\alpha}} + 1 - r') - \Delta r \left[ \frac{\gamma-1}{\gamma} (J-1) + cp^{\frac{\gamma}{\gamma-1}} \right] \geq 0.$$

As in Proposition 1, we have that  $h(r) = 0$  and that  $h'' > 0$ . It follows that the maximum of  $h$  over  $r' \in [0, r]$  is  $\max(h(0), 0)$ .

Clearly, no reform is viable if and only if  $h(0) < 0$ , that is

$$p - 1 - r \left[ \frac{\gamma-1}{\gamma} (J-1) + cp^{\frac{\gamma}{\gamma-1}} \right] < 0,$$

or equivalently

$$c > c_+ = \frac{p - 1 - r^{\frac{\gamma-1}{\gamma}} (J-1)}{rp^{\frac{\gamma}{\gamma-1}}}. \quad (98)$$

This proves claim (iii).

Now assume that (98) is violated. Clearly, either  $h'(r) \leq 0$  and all reforms are viable, or  $h'(r) > 0$  and  $h$  is U-shaped, so that the range of reforms is an interval  $[0, r_v]$ ,  $r_v > 0$ .

The condition  $h'(r) \geq 0$  reads

$$\frac{\gamma - 1}{\gamma}(J - 1) + (1 - p_R^{-\frac{\alpha}{1-\alpha}})(1 - \frac{1}{\alpha}) + cp^{\frac{\gamma}{\gamma-1}} \leq 0$$

This is equivalent to

$$c \leq c_- = - \left[ \frac{\gamma - 1}{\gamma}(J - 1) + (1 - p_R^{-\frac{\alpha}{1-\alpha}})(1 - \frac{1}{\alpha}) \right] p^{-\frac{\gamma}{\gamma-1}} \geq 0,$$

where the last inequality comes from the fact that the expression  $\frac{\gamma-1}{\gamma}(J-1) + (1 - p_R^{-\frac{\alpha}{1-\alpha}})(1 - \frac{1}{\alpha})$  has been proved to be negative in the proof of Proposition 1.

This proves claims (i) and (ii).

Finally in case (ii) the critical value of  $r'$   $r_v$  is such that  $h(r_v) = 0$  and  $h'(r_v) < 0$ . Since  $\partial h / \partial c < 0$ , claim (iv) follows.

QED

### 8.10.1 The case when (60) is violated

*Proposition A3 – Assume (60) is violated. Then there exists  $\tilde{c} \in (c_-, c_+)$  such that, if  $S$  denotes the set of viable reforms*

- (i) If  $c < c_-$  then  $S = [0, r]$
- (ii) If  $c_- \leq c \leq \tilde{c}$  then  $S = [0, r_v]$
- (iii) If  $\tilde{c} < c \leq c_+$  then  $[0, r_v] \subset S \subset [0, r_c]$
- (iv) If  $c > c_+$  then  $S \subset [0, r_c]$ .

*Proof –* From Prop. A1 we know that in this case,  $\hat{\tau} \leq 1 - p'/p$  if and only if  $r' \leq r_c$ . Clearly,  $r_c$  is independent of  $c$ . Recall that the N-neutral reform is the best for group  $D$  iff  $\hat{\tau}_E \geq 1 - p'/p$ , which is always true if (60) holds, and true iff  $r' \geq r_c$  if (60) is violated.

In case (i), all N-neutral reforms are viable, therefore all reforms are viable.

From Proposition 7 and its proof,  $r_v$  is a decreasing function of  $c$  which is continuous and such that  $r_v(c_-) = r$  and  $r_v(c_+) = 0$ . Let  $\tilde{c} \in (c_-, c_+)$  such that  $r_v(\tilde{c}) = r_c$ .

In case (ii),  $r_v \geq r_c$ . All N neutral reforms such that  $r \leq r_v$  are viable. Furthermore, all N neutral reforms such that  $r > r_v$  are also such that  $r > r_c$ , implying they are best for group D. However they do not raise their utility relative to the status quo, implying they are not viable. Therefore  $S$  must coincide with  $[0, r_v]$ .

In case (iii),  $r_v < r_c$ . All N-neutral reforms are viable for  $r \leq r_v$ , implying  $[0, r_v] \subset S$ . All N-neutral reforms are best for group D if  $r > r_c$ , yet since  $r > r_v$  too, they do not raise its utility and are therefore nonviable, therefore,  $S \subset [0, r_c]$ . The same holds in case (iv) where no N-reform is viable.

QED

## 8.11 Proof of Lemma 6

The demand curves now are given by  $c(R, 1, p_k)$ . The real wage in nonregulated sectors is just  $p_N$ , so that

$$l_N = p_N^{\frac{1}{\gamma-1}}.$$

From this it is straightforward to derive the equilibrium utility of the nonregulated:

$$u_N = p_N^{\frac{\gamma}{\gamma-1}} \frac{\gamma - 1}{\gamma}. \quad (99)$$

As for the regulated groups, their income is  $p_R \bar{l}$ . Their constrained employment level  $\bar{l}$  can again be computed from the equilibrium condition in the nonregulated sector,  $l_N = (1 - r)c(p_N^{\frac{\gamma}{\gamma-1}}, 1, p_N) + rc(p_R \bar{l}, 1, p_N)$ , yielding

$$\bar{l} = p_N^{\frac{1}{\gamma-1} + \frac{1}{1-\alpha}} p_R^{-\frac{1}{1-\alpha}}.$$

Therefore, the regulated groups' utility is equal to

$$u_R = J u_N, \quad (100)$$

with

$$J = \frac{\gamma(p_R/p_N)^{-\frac{\alpha}{1-\alpha}} - (p_R/p_N)^{-\frac{\gamma}{1-\alpha}}}{\gamma - 1}.$$

Now consider an extensive reform. By inverting (35), we get (36). Next, the nonregulated post-reform labor supply is now clearly equal to

$$l'_N = (p'_N(1 - \tau))^{\frac{1}{\gamma-1}}, \quad (101)$$

which proves (37). Consequently their net income is  $R'_N = (p'_N(1 - \tau))^{\frac{\gamma}{\gamma-1}}$  and their utility is

$$u'_N = (p'_N(1 - \tau))^{\frac{\gamma}{\gamma-1}} \frac{\gamma - 1}{\gamma}. \quad (102)$$

Clearly, for these agents not to lose, it must be that

$$1 - \tau \geq \frac{p_N}{p'_N}, \quad (103)$$

which coincides with (39), thus proving claim B.

Let  $T$  be the transfer to the deregulated groups and  $\bar{l}'$  the new employment level of the nonderegulated groups. Equilibrium in the nonregulated sectors reads

$$l'_N = (1-r)c((p'_N(1-\tau))^{\frac{\gamma}{\gamma-1}}, 1, p'_N) + \Delta r.c((p'_N(1-\tau))^{\frac{\gamma}{\gamma-1}} + T, 1, p'_N) + r'.c(p_R\bar{l}'(1-\tau), 1, p'_N). \quad (104)$$

The government's budget constraint is

$$T\Delta r = \tau(r'p_R\bar{l}' + (1 - r')p'_N{}^{\frac{\gamma}{\gamma-1}}(1 - \tau)^{\frac{1}{\gamma-1}}). \quad (105)$$

Substituting (105), (101) and (3) into (104) allows to compute the equilibrium value of  $\bar{l}'$ , which yields

$$\bar{l}' = p'_N{}^{\frac{1}{\gamma-1} + \frac{1}{1-\alpha}} p_R{}^{-\frac{1}{1-\alpha}} (1 - \tau)^{\frac{1}{\gamma-1}},$$

which yields (38), thus completing the proof of claim A. We can substitute (38) into (105) and get the equilibrium value of  $T$  :

$$T = \frac{\tau(1 - \tau)^{\frac{1}{\gamma-1}} p'_N{}^{\frac{1}{\gamma-1} + \frac{1}{1-\alpha}}}{\Delta r}. \quad (106)$$

The income of the nonderegulated group is  $p_R \bar{l}'(1 - \tau)$ , allowing us to compute its utility:

$$u'_R = u'_N J', \quad (107)$$

where

$$J' = \frac{\gamma(p_R/p'_N)^{-\frac{\alpha}{1-\alpha}} - (p_R/p'_N)^{-\frac{\gamma}{1-\alpha}}}{\gamma - 1}.$$

It follows from the above analysis and (100) and (107) that for the non-deregulated to support the reform, the proportional tax rate must satisfy

$$1 - \tau \geq \frac{p_N}{p'_N} \left( \frac{J}{J'} \right)^{\frac{\gamma-1}{\gamma}},$$

which proves claim C.

Finally, the deregulated group's utility is simply equal to  $T + u'_N$ , implying from (106), (102) and (99) that for the reform to make the deregulated weakly better-off, the following must hold:

$$\tau(1 - \tau)^{\frac{1}{\gamma-1}} p_N'^{\frac{1}{\gamma-1} + \frac{1}{1-\alpha}} \geq \Delta r \frac{\gamma - 1}{\gamma} p_N'^{\frac{\gamma}{\gamma-1}} \left[ J - \left( \frac{p'_N(1 - \tau)}{p_N} \right)^{\frac{\gamma}{\gamma-1}} \right]. \quad (108)$$

This establishes claim D and completes the proof.

## 8.12 Proof of Proposition 8

Note that  $\lim_{p_R \rightarrow 1} p_N = \lim_{p_R \rightarrow 1} p'_N = 1$ , implying  $\lim_{p_R \rightarrow 1} J = \lim_{p_R \rightarrow 1} J' = 1$ . Denote

$$\tilde{\tau} = 1 - \frac{p_N}{p'_N} \left( \frac{J}{J'} \right)^{\frac{\gamma-1}{\gamma}}.$$

Clearly,  $\lim_{p_R \rightarrow 1} \tilde{\tau} = 0$ . Observe from (106) and (102) that the deregulated group's utility,  $u'_D = u'_N + T$  is hump-shaped in  $\tau$  and reaches its maximum at

$$\hat{\tau} = \frac{(\gamma - 1)(p_N'^{\frac{\alpha}{1-\alpha}} - \Delta r)}{\gamma p_N'^{\frac{\alpha}{1-\alpha}} - (\gamma - 1)\Delta r}.$$



Clearly,  $\lim_{p_R \rightarrow 1} \hat{\tau} = \frac{(\gamma-1)(1-\Delta r)}{\gamma-(\gamma-1)\Delta r} > 0$ . Consequently, for  $p_R$  close enough to 1,  $\hat{\tau} > \tilde{\tau}$ , implying that  $u'_D$  is monotonically increasing with  $\tau$  over  $[0, \tau_{\max}]$ .

To prove Proposition 8, we show that for  $p_R$  small enough, (108) is violated at  $\tau = \tilde{\tau}$ , meaning the deregulated groups are made worse-off by the reform. Since  $p_R$  can be chosen such that  $\tau_{\max} > \tilde{\tau}$ , it follows that the deregulated are also worse-off for any  $\tau < \tilde{\tau}$ , while by definition the nonderegulated are worse-off for  $\tau > \tilde{\tau}$ . Clearly, then, no value of  $\tau$  can implement the reform in a Pareto-improving way.

Next, substituting the value of  $\tilde{\tau}$  into condition (108), and rearranging, we get the following

$$p'_N{}^{\frac{1}{1-\alpha}} p_N^{-1} J'^{\frac{\gamma-1}{\gamma}} - J^{\frac{\gamma-1}{\gamma}} p'_N{}^{\frac{\alpha}{1-\alpha}} \geq \frac{\gamma-1}{\gamma} \Delta r \cdot J^{\frac{\gamma-1}{\gamma}} (J' - 1). \quad (109)$$

This equation holds with equality at  $p_R = 1$ . We compute the derivative with respect to  $p_R$  of the difference between the LHS and the RHS of (109), denoted by  $E$ , at  $p_R = 1$  and show it is negative. For this, note that

$$\frac{dp_N}{dp_R} = -\frac{r}{1-r} \left( \frac{p_R}{p_N} \right)^{-\frac{1}{1-\alpha}} = -\frac{r}{1-r} \text{ at } p_R = 1,$$

implying similarly that  $dp'_N/dp_R = -r'/(1-r')$  at  $p_R = 1$ . This allows to compute<sup>32</sup>

$$\begin{aligned} \frac{dJ}{dp_R} &= \frac{\gamma}{(\gamma-1)(1-\alpha)(1-r)} \left( p_R^{-\frac{\gamma}{1-\alpha}-1} p_N^{\frac{\gamma+\alpha}{1-\alpha}} - \alpha p_R^{-\frac{1}{1-\alpha}} p_N^{\frac{2\alpha}{1-\alpha}} \right) \\ &= \frac{\gamma}{(\gamma-1)(1-r)} \text{ at } p_R = 1. \end{aligned}$$

Similarly  $\frac{dJ'}{dp_R} = \frac{\gamma}{(\gamma-1)(1-r')}$  at  $p_R = 1$ . Next, we have that

$$\begin{aligned} E &= \frac{d}{dp_R} \left[ p'_N{}^{\frac{1}{1-\alpha}} p_N^{-1} J'^{\frac{\gamma-1}{\gamma}} - J^{\frac{\gamma-1}{\gamma}} p'_N{}^{\frac{\alpha}{1-\alpha}} - \frac{\gamma-1}{\gamma} \Delta r \cdot J^{\frac{\gamma-1}{\gamma}} (J' - 1) \right] \\ &= \frac{\gamma-1}{\gamma} J'^{-\frac{1}{\gamma}} p'_N{}^{\frac{1}{1-\alpha}} p_N^{-1} \frac{dJ'}{dp_R} + \frac{1}{1-\alpha} p_N^{-1} J'^{\frac{\gamma-1}{\gamma}} p'_N{}^{\frac{\alpha}{1-\alpha}} \frac{dp'_N}{dp_R} - p_N^{-2} J'^{\frac{\gamma-1}{\gamma}} p'_N{}^{\frac{1}{1-\alpha}} \frac{dp_N}{dp_R} \\ &\quad - \frac{\gamma-1}{\gamma} p'_N{}^{\frac{\alpha}{1-\alpha}} J'^{-\frac{1}{\gamma}} \frac{dJ}{dp_R} - \frac{\alpha}{1-\alpha} J^{\frac{\gamma-1}{\gamma}} p'_N{}^{\frac{2\alpha-1}{1-\alpha}} \frac{dp'_N}{dp_R} \\ &\quad - \left( \frac{\gamma-1}{\gamma} \right)^2 \Delta r \cdot J^{-\frac{1}{\gamma}} (J' - 1) \frac{dJ}{dp_R} - \frac{\gamma-1}{\gamma} \Delta r \cdot J^{\frac{\gamma-1}{\gamma}} \frac{dJ'}{dp_R}. \end{aligned}$$

<sup>32</sup>A useful intermediate step consists in showing that  $\frac{d}{dp_R} \left( \frac{p_R}{p_N} \right) = \frac{1}{1-r} p_N^{\frac{2\alpha-1}{1-\alpha}}$ .

Using the preceding expressions, we compute  $E$  at  $p_R = 1$  and get

$$\begin{aligned} E &= \frac{1}{1-r'} - \frac{1}{1-\alpha} \frac{r'}{1-r'} + \frac{r}{1-r} \\ &\quad - \frac{1}{1-r} + \frac{\alpha}{1-\alpha} \frac{r'}{1-r'} - 0 - \frac{\Delta r}{1-r'} \\ &= -\frac{\Delta r}{1-r'} < 0. \end{aligned}$$

Therefore, by continuity, for any given  $r' > 0$ , (109) is violated for  $p_R$  greater than and close enough to 1, which completes the proof. Note that the constraint  $\tau < \tilde{\tau}$  does not apply for the case where  $r' = 0$  since the nonderegulated group then becomes empty. Proposition 2 then holds, since the analysis differs from that of Section 4 only by the choice of the numéraire. QED.

### 8.13 Non homothetic preferences: derivations

Group 1, the nonregulated group, maximizes (44) subject to its budget constraint

$$p'_R c_2 + c_1 \leq (1-\tau)l.$$

The solution is

$$\begin{aligned} l &= (1-\tau)^{\frac{1}{\gamma-1}}, & (110) \\ c_1 &= c_{1N} = \frac{(1-\tau)^{\frac{\gamma}{\gamma-1}}}{\gamma} \frac{\gamma + p_R^{\frac{\alpha}{\alpha-1}}}{1 + p_R^{\frac{\alpha}{\alpha-1}}}, \\ c_2 &= c_{2N} = \frac{\gamma-1}{\gamma} \frac{(1-\tau)^{\frac{\gamma}{\gamma-1}}}{1 + p_R^{\frac{\alpha}{\alpha-1}}} p_R^{\frac{1}{\alpha-1}}. \end{aligned}$$

From there we can compute the utility of this group:

$$u'_N = \frac{1}{\alpha} \left( \frac{\gamma-1}{\gamma} \right)^\alpha \left( 1 + p_R^{\frac{\alpha}{\alpha-1}} \right)^{1-\alpha} (1-\tau)^{\frac{\alpha\gamma}{\gamma-1}}.$$

Since the status quo is equivalent to the special case  $\tau = 0$ ,  $p'_R = p_R$ , the tax rate which leaves this group indifferent,  $\tilde{\tau}$ , is

$$\tilde{\tau} = 1 - \left[ \frac{1 + p_R^{\frac{\alpha}{\alpha-1}}}{1 + p_R^{\frac{\alpha}{\alpha-1}}} \right]^{\frac{1-\alpha}{\alpha} \frac{\gamma-1}{\gamma}}. \quad (111)$$

From (110), the transfer paid to the deregulated group 2 is

$$T = \tau(1 - \tau)^{\frac{1}{\gamma-1}}. \quad (112)$$

Let  $\bar{l}'$  be the constrained employment level in group 2. The deregulated group, Group 2, maximizes (44) subject to

$$p'_R c_2 + c_1 \leq p'_R \bar{l}'$$

The solution is

$$\begin{aligned} c_1 &= c_{1R} = \frac{p'_R \bar{l}' + T + p_R'^{\frac{\alpha}{\alpha-1}} \bar{l}'^\gamma / \gamma}{1 + p_R'^{\frac{\alpha}{\alpha-1}}}, \\ c_2 &= c_{2R} = \frac{p'_R \bar{l}' + T - \bar{l}'^\gamma / \gamma}{1 + p_R'^{\frac{\alpha}{\alpha-1}}} p_R'^{\frac{1}{\alpha-1}}. \end{aligned}$$

These derivations allow to compute  $\bar{l}'$ , which must be solution to

$$\bar{l}' = c_{2N} + c_{2R},$$

or equivalently

$$\bar{l}' = p_R'^{\frac{1}{\alpha-1}} \left( T - \bar{l}'^\gamma / \gamma + \frac{\gamma-1}{\gamma} (1-\tau)^{\frac{\gamma}{\gamma-1}} \right). \quad (113)$$

The resulting utility for group 2 is

$$u'_D = \frac{1}{\alpha} \left( 1 + p_R'^{\frac{\alpha}{\alpha-1}} \right)^{1-\alpha} (p'_R \bar{l}' + T - \bar{l}'^\gamma / \gamma)^\alpha \quad (114)$$

Again, the status quo can be computed as the special case,  $\tau = 0$ ,  $p'_R = p_R$ .

In particular, the status quo value of  $\bar{l}$  is given by the solution to

$$\bar{l} = p_R^{\frac{1}{\alpha-1}} \left( \frac{\gamma-1}{\gamma} - \bar{l}^\gamma / \gamma \right), \quad (115)$$

while the utility of the regulated group under the status quo is

$$u_R = \frac{1}{\alpha} \left( 1 + p_R^{\frac{\alpha}{\alpha-1}} \right)^{1-\alpha} (p_R \bar{l} - \bar{l}^\gamma / \gamma)^\alpha.$$

Finally, to perform the numerical analysis, we need to make sure that  $p'_R$  and  $p_R$  are above the Walrasian equilibrium price for good 2, which now differs from 1 since there is no symmetry between the two goods. The deregulated group's maximization program implies that in the status quo, its Walrasian labor supply is

$$l^s(p_R) = p_R^{\frac{1}{\gamma-1}}. \quad (116)$$

The Walrasian equilibrium price for good 2 is the value of  $p_R$  such that  $\bar{l} = l^s(p_R)$ . Using (115) and (116) it follows that the Walrasian equilibrium price  $p_W$  is the unique solution to

$$\gamma p_W^{\frac{\gamma-\alpha}{(\gamma-1)(1-\alpha)}} + p_W^{\frac{\gamma}{\gamma-1}} = \gamma - 1.$$

The second thing to be checked is that the tax rate  $\tau = \tilde{\tau}$  cannot be improved upon for group 2, i.e. that it is on the upward sloping side of that group's utility Laffer curve. For this I numerically check that  $\frac{du'_D}{d\tau} \geq 0$  at  $\tau = \tilde{\tau}$ , where the required expression is obtained from differentiating (114):

$$\begin{aligned} \frac{du'_D}{d\tau} &\propto \frac{d}{d\tau} (p'_R \bar{l}' + T - \bar{l}'^\gamma / \gamma) \\ &= (p'_R - \bar{l}'^{\gamma-1}) \frac{d\bar{l}'}{d\tau} + \frac{dT}{d\tau}. \end{aligned}$$

The two derivatives on the RHS are computed from (113) and (112), yielding

$$\frac{d\bar{l}'}{d\tau} = -\frac{1}{\gamma-1} \tau(1-\tau)^{\frac{2-\gamma}{\gamma-1}} \frac{p'_R{}^{\frac{1}{\alpha-1}}}{1 + p'_R{}^{\frac{1}{\alpha-1}} \bar{l}'^{\gamma-1}}$$

and

$$\frac{dT}{d\tau} = (1-\tau)^{\frac{1}{\gamma-1}} - \frac{1}{\gamma-1} \tau(1-\tau)^{\frac{2-\gamma}{\gamma-1}}$$

The following Table reports the relative utility gain,  $\frac{u'_D - u_R}{|u_R|}$ , of the deregulated for a number of values of  $\alpha$  and  $\gamma$ . It was assumed that  $p_R = 1.3p_W$  and  $p'_R = 1.2p_W$ . The tax rate is the maximum one that does not make the nonregulated worse-off  $\tilde{\tau}$ , and using the preceding derivations it was checked numerically that  $\frac{du'_D}{d\tau} \geq 0$  at  $\tau = \tilde{\tau}$ .

$\alpha \backslash \gamma$	1.1	1.2	2	4
0.5	2.8	2.4	2.2	4.0
-0.1	0.2	0.2	0.2	0.2
5	0.3	0.7	1.1	1.3
10	-1.6	-0.6	0.9	1.2

Table A1 – Relative utility gain (%) as a function of  $\alpha$  and  $\gamma$ .

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